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# Forecast Evaluation

## BSAD 8310: Business Forecasting — Lecture 6

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Spring 2026

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## The Evaluation Problem

Five model families, one retail series — how do we know which one to use?

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Every lab in Lectures 1–5 held out a single final window of  $H$  periods.

### Three failure modes:

1. **Lucky split:** one origin may favor one model by chance
2. **Test-set leakage:** tuning on the test set inflates performance
3. **Horizon conflation:**  $h = 1$  and  $h = 12$  accuracy are very different — averaging hides the pattern

A single split is a photograph. Walk-forward evaluation is a movie.

### What rigorous evaluation requires:

- Multiple forecast origins (not one  $T$ )
- Horizon-specific accuracy ( $h = 1, \dots, H$ )
- A loss function matched to business costs
- Statistical testing to separate signal from noise

*This lecture delivers all four. (See L01 footnote: “walk-forward evaluation: Lecture 6.”)*

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Before evaluating, a map of what we now have:

Family	Lecture	Strength	Weakness
Benchmarks	L1	Simple, fast	No dynamics
Regression / AR	L2	Interpretable	Assumes linearity
ETS / Exp. smooth.	L3	Trend & seasonality	Univariate only
ARIMA / SARIMA	L4	Box-Jenkins; flexible	Complex identification
VAR / ARIMAX / ECM	L5	Multivariate dynamics	Parameter-heavy

Evaluation is the final step in the modeling cycle: build → fit → **evaluate out-of-sample** → decide. No model is universally best across all series, horizons, and loss functions.

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## Error Metrics

RMSE, MAE, MAPE, and MASE each answer a different business question.

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Let  $e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$ .

$$\text{RMSE}_h = \sqrt{\frac{1}{n} \sum e_{T+h}^2} \quad \text{scale-dep.}$$

$$\text{MAE}_h = \frac{1}{n} \sum |e_{T+h}| \quad \text{scale-dep.}$$

$$\text{MAPE}_h = \frac{100}{n} \sum |e_{T+h}/y_{T+h}| \quad \text{scale-free}$$

$$\text{MASE}_h = \text{MAE}_h / \bar{q} \quad \text{scale-free}$$

$\bar{q}$  = in-sample MAE of seasonal naïve.

### Decision guide:

RMSE: large errors costly

MAE: equal cost per unit

MASE < 1: beats naïve

MAPE: only when  $y_t \gg 0$

Never report RMSE alone — show MAE alongside.

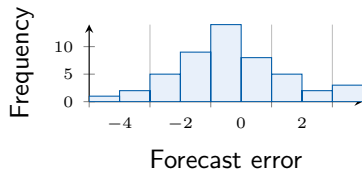
(Hyndman and Koehler 2006) *Socratic: two series,  $\text{RMSE}(A) < \text{RMSE}(B)$  but  $\text{MASE}(A) > \text{MASE}(B)$ .*

*Which model is “better”?*

RMSE penalizes large errors quadratically; MAE treats all errors equally.

### When RMSE $\gg$ MAE:

- Error distribution has heavy tails
- Occasional very large misses
- If those misses are costly  $\Rightarrow$  RMSE is the right metric



Stockout cost =  $5 \times$  overstock cost. The loss function is *asymmetric* and favors penalizing large underestimates.

$\Rightarrow$  RMSE aligns better than MAE.

$\Rightarrow$  Ideally: use asymmetric loss  $L(e) = c_1 e^+ + c_2 e^-$ , where  $e^+ = \max(e, 0)$ ,  $e^- = \max(-e, 0)$ .

*Never report only RMSE — always show MAE to reveal whether accuracy is driven by a few outlier periods.*



## MAPE failure cases:

- $y_{T+h} \approx 0 \Rightarrow \text{MAPE} \rightarrow \infty$  (e.g., new product launch, zero-demand months)
- **Asymmetric:** when  $y > 0$ , under-forecasts contribute at most 100% to MAPE (bounded:  $\hat{y} = 0 \Rightarrow |e/y| = 1$ ); over-forecasts are unbounded above ( $\hat{y} \rightarrow \infty \Rightarrow \text{MAPE} \rightarrow \infty$ )
- Percentage errors favor low-variance series

**MASE** (Hyndman and Koehler 2006):

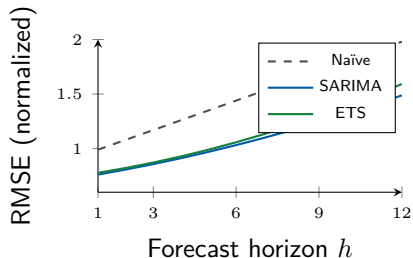
$\text{MASE} < 1 \Rightarrow$  beats seasonal naïve

$\text{MASE} > 1 \Rightarrow$  worse than naïve

Works for intermittent demand, zero values, and cross-series comparison.

**Practical recommendation:** report RMSE + MAE for absolute accuracy; MASE for benchmarking; MAPE only when  $y_t \gg 0$ . *Socratic: a new product launches in month 1 with  $y_1 = 0$ . Which metric can you not compute, and what do you use instead?*

**Key insight:** the winner at  $h = 1$  is not always the winner at  $h = 12$ .



***Always report accuracy by horizon, not just as a single average. Why profiles diverge:***

- At  $h = 1$ : model captures short-run dynamics
- At  $h = 12$ : model must extrapolate seasonal patterns
- Naïve accumulates error with horizon; ARIMA/ETS maintain seasonal anchor

*Lab 6 plots this profile on RSXFS with 36 walk-forward origins.*

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## Walk-Forward Validation

A single test set yields one error estimate. Walk-forward validation yields a distribution of errors across forecast origins.

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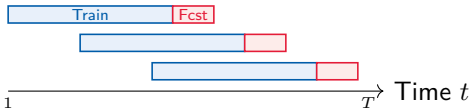
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At each **forecast origin**  $t$ : fit model on  $\{y_1, \dots, y_t\}$ , forecast  $\{y_{t+1}, \dots, y_{t+H}\}$ .

**Expanding:**



**Rolling:**



**Expanding:** train grows at each origin. **Rolling:** fixed-size window shifts forward. Both produce a time series of forecast errors per horizon  $h$ .

## Use expanding window when:

- Data-generating process is *stable* over time
- More training data  $\rightarrow$  better parameters
- Long series with no structural breaks

## Use rolling window when:

- Structural breaks (e.g., COVID-19 shock)
- Time-varying parameters / regime change
- Fixed window forces model to “forget” old patterns

**Minimum training size:** choose  $T_0$  large enough for reliable estimation. For  $\text{SARIMA}(p, d, q)(P, D, Q)_m$ : at least  $3m + p + P$  observations.

For monthly data:  $T_0 \geq 36$  months.

**Python:** `TimeSeriesSplit` in `sklearn.model_selection` implements expanding-window CV; rolling requires a manual loop.

1. Choose: first origin  $T_0$ , window type, horizon  $H$
2. **For** each origin  $t = T_0, T_0 + 1, \dots, T - H$ :
  - (a) Fit model on  $\{y_1, \dots, y_t\}$
  - (b) Generate forecasts  $\hat{y}_{t+h|t}$ ,  $h = 1, \dots, H$
  - (c) Store errors  $e_{t,h} = y_{t+h} - \hat{y}_{t+h|t}$
3. Average over origins per horizon:

$$\text{RMSE}_h = \sqrt{\frac{1}{n_h} \sum_t e_{t,h}^2}$$

4. Plot horizon profile; run DM test for significance

*Walk-forward errors are the closest thing to a **live forecast track record**: each  $e_{t,h}$  represents the error you would have made using this model in real time.  $e_{t,h}$  generalizes  $e_{T+h}$  from the Error Metrics section to multiple origins  $t$ ; the metric formulas are the same, averaged over  $t$ .*

*Number of usable origins at horizon  $h$ :  $n_h = T - T_0 - h + 1$ ; at the maximum horizon  $h = H$ :  $n_H = T - T_0 - H + 1$ ; with monthly data, 24–36 origins is typical.*

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**Illustrative results** (36 expanding-window origins,  $H = 12$ ):

Model	RMSE <sub><math>h=1</math></sub>	RMSE <sub><math>h=3</math></sub>	RMSE <sub><math>h=12</math></sub>	MASE
Naïve (seasonal)	2,810	3,145	4,210	1.00
SARIMA(1,1,1)(0,1,1)	1,720	2,040	3,480	0.76
ETS (auto-AIC)	1,690	2,020	3,510	0.75
ARIMAX (+ sentiment)	1,640	1,980	3,530	0.73
VAR (BIC order)	1,660	2,100	3,620	0.77
Equal-weight combo	<b>1,600</b>	<b>1,920</b>	<b>3,390</b>	<b>0.71</b>

*Combination beats all individual models across every horizon. Is the ETS vs. SARIMA gap statistically real?*

→ Section 4.

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## The Diebold-Mariano Test

Visual inspection says Model A looks better. The DM test asks: is the gap statistically real?

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To compare two models, examine the **loss differential** at each origin.

For each origin  $t$ , let:

$$d_t = g(e_{1t}) - g(e_{2t})$$

where  $g(\cdot)$  is any loss function.

For MSE loss:  $d_t = e_{1t}^2 - e_{2t}^2$ .

Average loss differential:

$$\bar{d} = \frac{1}{n} \sum_{t=1}^n d_t$$

**Numeric example** (MSE loss,  $h = 1$ ):

Origin 1: SARIMA error = 3.2, ETS error = 2.8

$$d_1 = 3.2^2 - 2.8^2 = 10.24 - 7.84 = +2.40$$

If  $\bar{d} > 0$ : Model 1 has higher average loss  $\Rightarrow$  Model 2 is better.

But is  $\bar{d}$  statistically distinguishable from 0?

$\rightarrow$  **See next slide.**

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$$DM = \frac{\bar{d}}{\widehat{\text{se}}(\bar{d})} \xrightarrow{d} \mathcal{N}(0, 1)$$

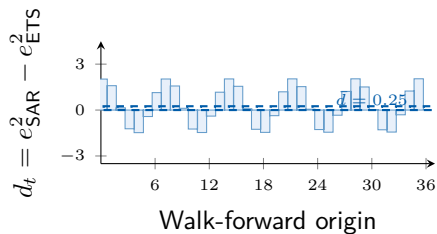
$\bar{d}$ : average loss differential (see prev. slide);  $\widehat{\text{se}}(\bar{d})$ : HAC standard error.

$H_0$ : equal predictive accuracy ( $\mathbb{E}[d_t] = 0$ ).

**Why HAC?** For an  $h$ -step forecast,  $d_t$  is serially correlated up to lag  $h - 1$  (overlapping forecast errors). Ignoring this understates the standard error.

Use Newey-West bandwidth  $= h - 1$  for the HAC variance; with  $h = 1$  (one-step forecasts),  $d_t$  is white noise under  $H_0$ .

## SARIMA vs. ETS on RSXFS ( $h = 1, 36$ origins, MSE loss):



### Decision rule:

- $DM > 1.645$  (one-sided): Model 2 significantly better
- $|DM| > 1.96$  (two-sided): models differ significantly
- Fail to reject  $H_0$ : insufficient evidence that models differ — do not interpret as evidence of equality

$\bar{d} > 0$ : SARIMA has slightly higher MSE. If  $DM > 1.645$ , use ETS.

## When the DM test applies:

- Non-nested models (e.g., SARIMA vs. ETS)
- Stationary loss differentials  $d_t$
- Sufficient number of origins ( $n \geq 20$ )

## When it does not apply:

- **Nested models** (e.g., AR(1) vs. AR(2)): under  $H_0$  the DM statistic is non-standard. Use the Clark-West correction (Clark and West 2007)
- Very small samples ( $n < 15$ ): use finite-sample correction (Harvey et al. 1997)

*Socratic: if  $\bar{d} > 0$  but  $DM = 0.8$ , what conclusion do you draw? What would increase power?*

DM tests **equal expected loss**, not model fit.

Always use *out-of-sample* walk-forward errors — never in-sample residuals.

Always specify the loss function  $g(\cdot)$ : MSE, MAE, or asymmetric loss.

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## Forecast Combination

No single model wins every period. Combining forecasts diversifies model risk.

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**The portfolio analogy:** diversification reduces portfolio variance when assets are imperfectly correlated.

$$\text{Var}\left(\frac{1}{2}e_1 + \frac{1}{2}e_2\right) = \frac{1}{4}(\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2)$$

If  $\rho < 1$ : the combined error variance is smaller than the average of individual variances.

**Empirical finding:** Bates and Granger (1969) showed that weighted combinations *generally* outperform the best individual model in empirical comparisons.

*If two models have equal accuracy but errors are **not perfectly correlated**, the simple average outperforms either model alone. SARIMA-ETS error correlation  $\approx 0.72$  on RSXFS  $\Rightarrow$  significant gains from combining.*

Strategy	Formula	Key property	Practical note
Equal weights	$\hat{y}^C = \frac{1}{K} \sum_k \hat{y}^{(k)}$	No estimation needed	Surprisingly robust
RMSE weights	$w_k \propto \text{RMSE}_k^{-1}$	Rewards accuracy	Simple; no overfitting
OLS weights	$\hat{w}$ from regressing $y$ on $\hat{y}^{(k)}$	Optimal in theory	Prone to overfitting

**The combination puzzle (Timmermann 2006; Stock and Watson 2004):** estimated OLS weights *often underperform* equal weights out-of-sample. Three reasons:

1. Estimation error in  $\hat{w}$
2. Model instability / structural breaks
3. High collinearity among  $\hat{y}^{(k)}$

OLS weights can be negative or  $> 1$ . Constrain to  $w_k \geq 0$ ,  $\sum w_k = 1$  in practice. Alternatively: shrink toward equal weights via ridge.

## Extending our walk-forward results (RSXFS, 36 origins):

Method	RMSE ( $h = 1$ )	RMSE ( $h = 12$ )	MASE (avg)
Best individual (ARIMAX)	1,640	3,530	0.73
RMSE-weighted combo	1,620	3,410	0.72
<b>Equal-weight combo</b>	<b>1,600</b>	<b>3,390</b>	<b>0.71</b>
OLS combo (unconstrained)	1,680	3,580	0.76

**Decision rule:** when uncertain about which model to trust, **combine** (equal weights as default). When you have strong theoretical or empirical reasons to prefer one model, use it alone — but verify with the DM test.

*The M4 competition (Makridakis et al. 2020): top hybrid submissions combined diverse methods.*

*Equal-weight ensembles ranked in the top 25% of all 61 participating methods.*



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## Key Takeaways and Roadmap

Five evaluation principles — from metric choice to forecast combination — that apply across the full course.

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**Metric choice is a business decision:** align RMSE/MAE/MASE with the cost structure; MAPE fails near zero.

**Walk-forward validation** gives a distribution of errors across origins — one split is not sufficient.

**Diebold-Mariano test** determines whether accuracy differences are statistically real (use out-of-sample errors, HAC standard errors).

**Nested model comparisons** require the Clark-West correction; standard DM is invalid for nested models.

**Equal-weight combination** typically matches or beats the best individual model — the combination puzzle.







*We now have five model families + rigorous evaluation. The next step: machine learning for forecasting.*



Classical forecasting is now complete: benchmarks → regression → ETS → ARIMA → multivariate → **evaluation**.

**Lecture 7: Machine Learning Introduction** — bias-variance tradeoff, train/validation/test discipline, and how cross-validation extends walk-forward ideas to ML models.

Aspect	Classical forecasting	ML forecasting
Model structure	Specified (ARIMA, VAR)	Learned from data
Assumptions	Linearity, normality	Fewer formal (stationarity helpful)
Overfitting risk	Lower (few parameters)	High (mitigated by CV)
Evaluation	Walk-forward; DM test	Same + CV grid search

**Lab 6:** walk-forward on RSXFS, DM test (SARIMA vs. ETS), forecast combination.

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