

dm24s1

Topic 10 : Classification2

Part 01 : Classification2-Overview

Preparation

Data Handling

Exploring Data

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Exploring Data 2

Building Models

Prediction

Autumn Semester, 2024

Outline

- Decision Trees
- Naive Bayes

Wrap up

Data Mining (Week 10)

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Motivating Example

Preparation

Data Handling

Exploring Data 1

Exploring Data 2

Building Models

Prediction

Clustering

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Classification
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Regression

Classification
2

Wrap up

Classification2-Overview — Summary

1. Introduction	4
2. Classification Trees	6
3. Naive Bayes classification	27
4. Ordinal targets	38
5. Resources	40

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These are two of the Top 10 algorithms in data mining (**WuKumarRossQuinlanEtAl2008**), each with its own strengths and weaknesses.

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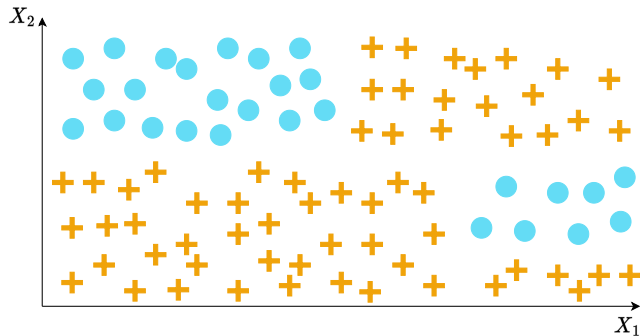
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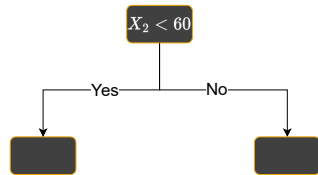
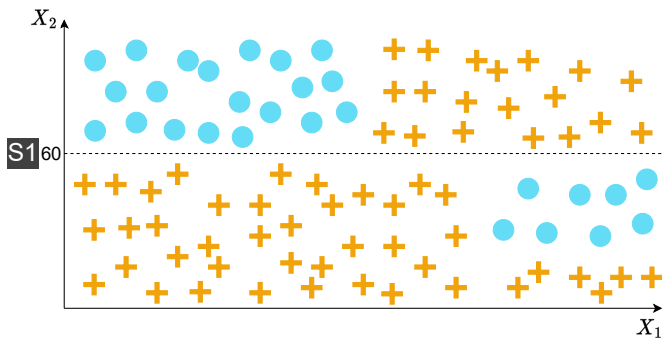
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- The rules which generate the binary splits are applied in a greedy fashion and are intended to reduce the *impurity* in each nodes' children as quickly as possible
- the algorithm proceeds top-down from the root (all data), recursively generating rules as it goes
- Prediction is simple: the rules are applied along the path from root to leaf. The predicted class value is either the most frequent value at the leaf, or the leaf's probability vector.

Classification tree: Example Data



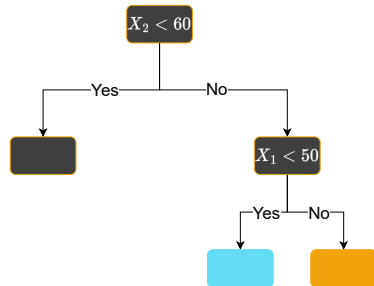
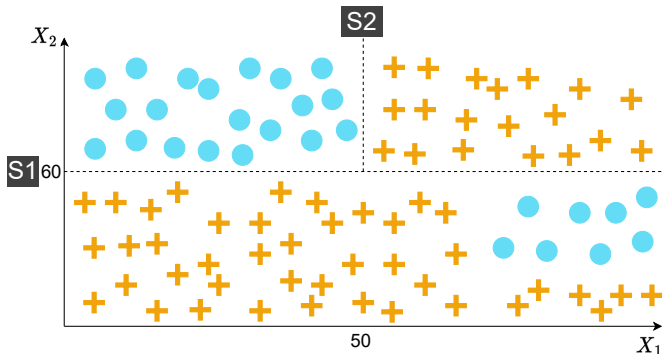
Task: learn from this training data, to classify new data as either orange cross or blue disk

Classification tree: Example Data - First Split



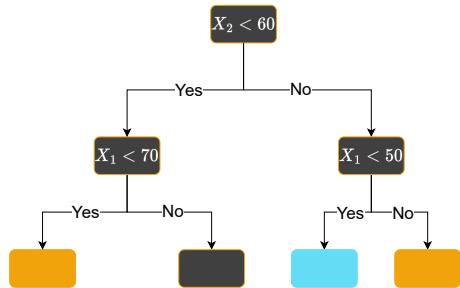
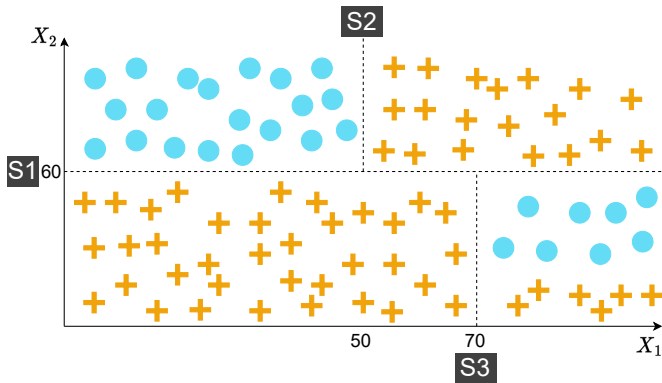
First split is on X_2 ; purity is improved (less mixing in each subset)

Classification tree: Example Data - Second Split



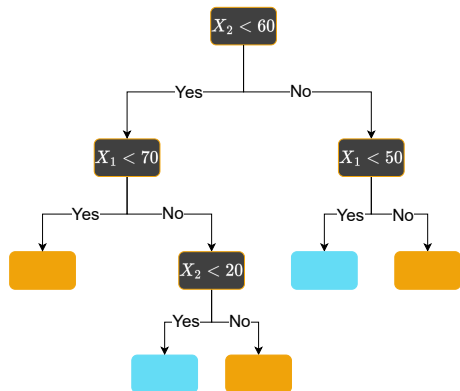
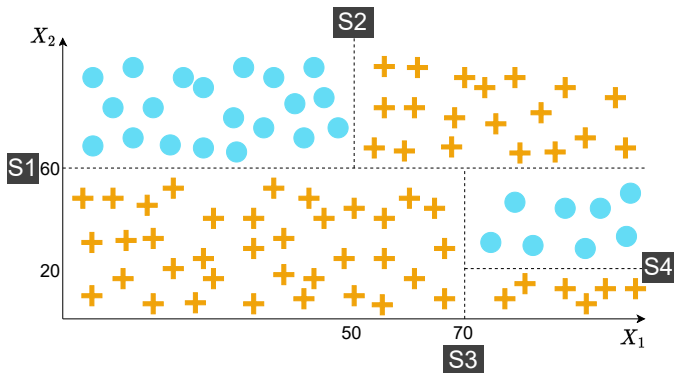
Second split is on X_1 so one region is pure (all blue disks) - can continue.

Classification tree: Example Data - Third Split



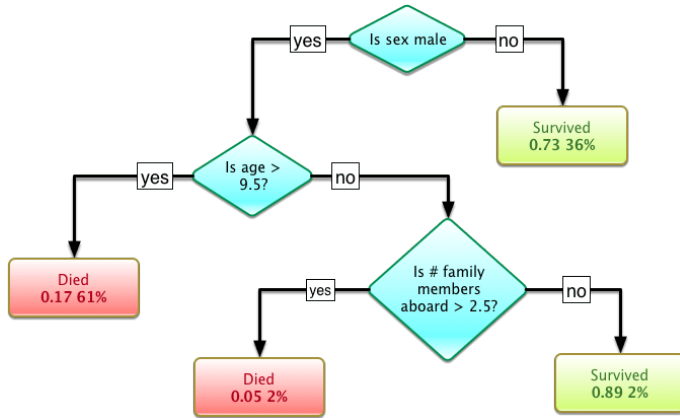
Third split on X_1 adds two extra pure regions.

Classification tree: Example Data - Fourth Split



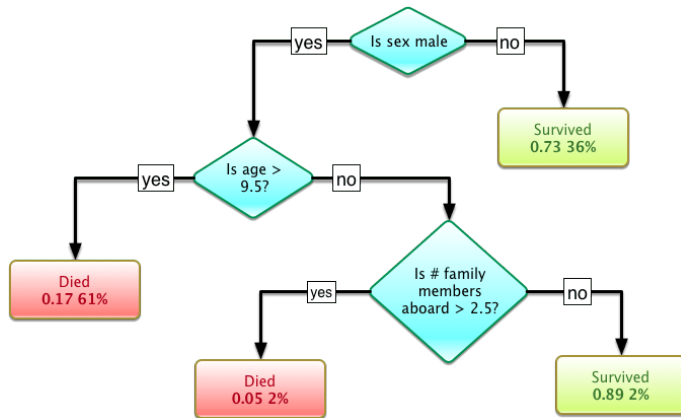
After fourth split on X_2 , all regions are pure, so we stop.

Classification tree example: Titanic survival



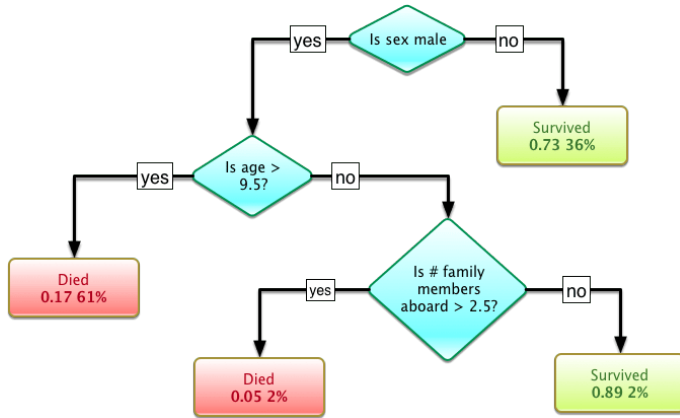
- First split is on Sex, as that attribute was the most important predictor of survival.

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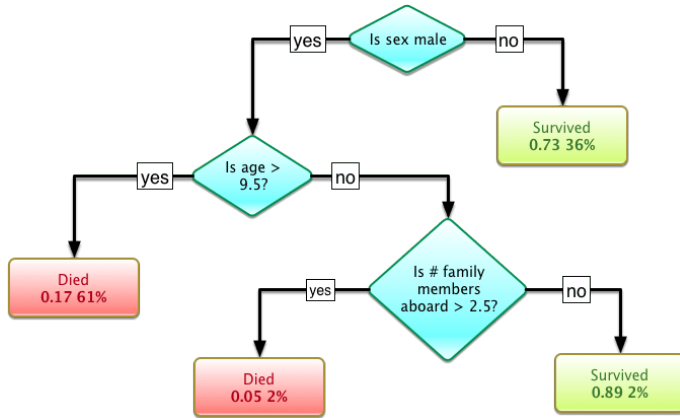
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- Leaf colour indicates $p(\text{survival}) \approx 1$ (green) or $p(\text{survival}) \approx 0$ (red)

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 - For each impure set
 - For each candidate split

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➤ Entropy is average amount of information conveyed by an event, considering all possible outcomes.

How information is measured

Information is measured in bits, and is computed from the probability $P(x)$ using $h(x) = -\log_2(P(x))$.

Information Entropy: Applied to classification

Classification and entropy

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$$H(X) = - \sum_{i=1}^n P(x_i) \log_2(P(x_i))$$

where $X = \{x_i\}$. If all probabilities are equal (X is uniformly distributed), $H(X) = 1$. If they differ, $H(X) < 1$. Remember the weather forecasting example!

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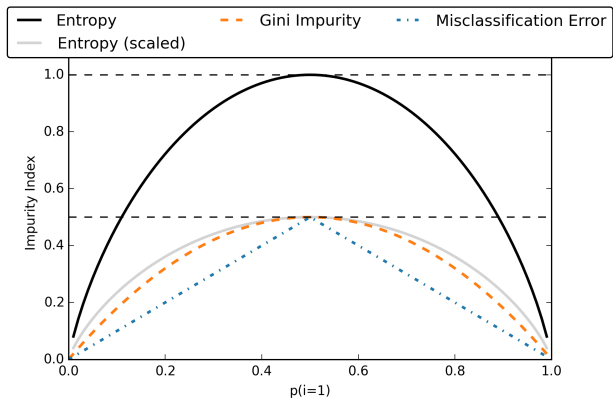
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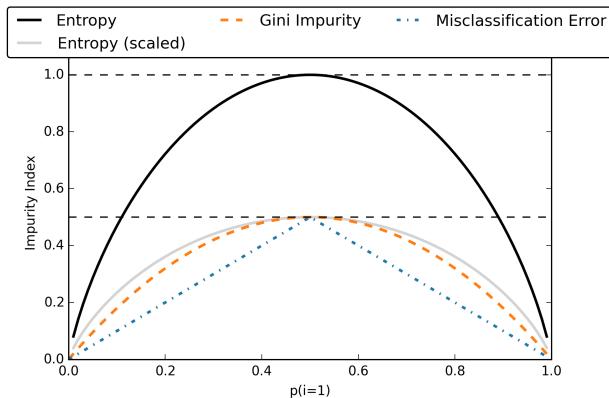
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A decision tree recursively partitions a set so as to increase the purity (equivalently: reduce the mixing) of the set of observations X at each node as we move from the root to the leaves.

Classification tree metrics for rule building

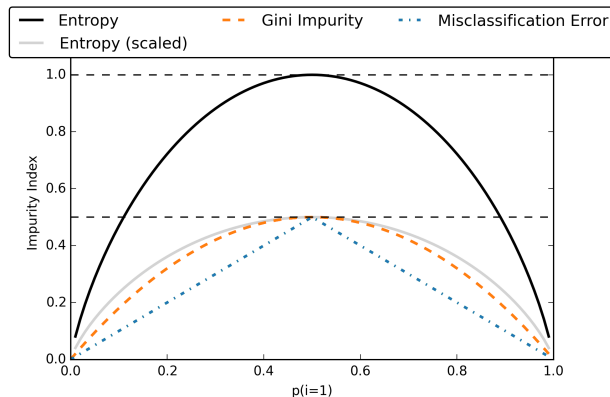


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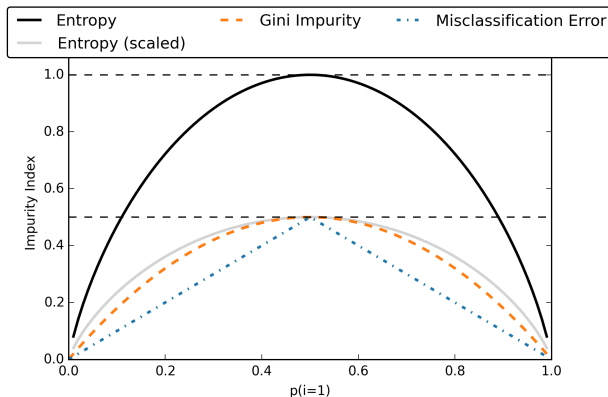
Let p_i be the probability of an item with label $1 < i < J$ being chosen.

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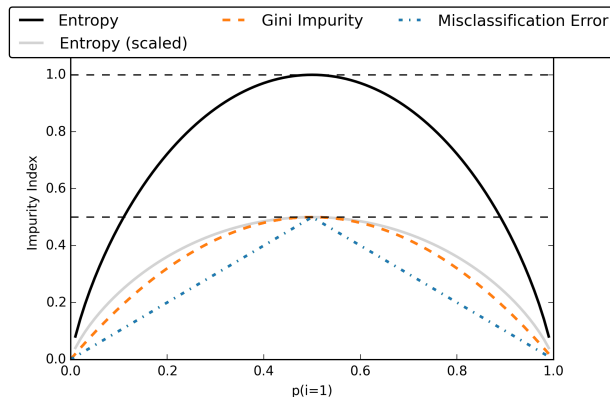
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- For *two attributes* T and X , $H(T, X) = \sum_{c \in X} P(c) E(c)$ where each c represents a level of the X attribute.

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Example: PlayTennis example data

outlook	temp	humidity	windy	play
sunny	hot	high	false	no
sunny	hot	high	true	no
overcast	hot	high	false	yes
rainy	mild	high	false	yes
rainy	cool	normal	false	yes
rainy	cool	normal	true	no
overcast	cool	normal	true	yes
sunny	mild	high	false	no
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rainy	mild	high	true	no

Source: Mitchell, Machine Learning, 1997.

PlayTennis example calculations

Example 4 ($H(\text{play})$)

$$\begin{aligned} H(\text{play}) &= - (p(\text{play} = \text{yes}) \log_2 p(\text{play} = \text{yes}) + p(\text{play} = \text{no}) \log_2 p(\text{play} = \text{no})) \\ &= H_{9,5} \\ &= - \left(\frac{9}{14} \log_2 \left(\frac{9}{14} \right) + \frac{5}{14} \log_2 \left(\frac{5}{14} \right) \right) \approx 0.94 \end{aligned}$$

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Example 5 (H(play,outlook))

$$\begin{aligned}
 H(\text{play}, \text{outlook}) &= p(\text{outlook} = \text{sunny})H(\text{play} \& (\text{outlook} = \text{sunny})) + \dots \\
 &= p(\text{outlook} = \text{sunny})H_{3,2} + p(\text{outlook} = \text{overcast})H_{4,0} + \dots \\
 &\approx \frac{5}{14}0.97 + \frac{4}{14}0 + \frac{5}{14}0.97 \\
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PlayTennis example calculations

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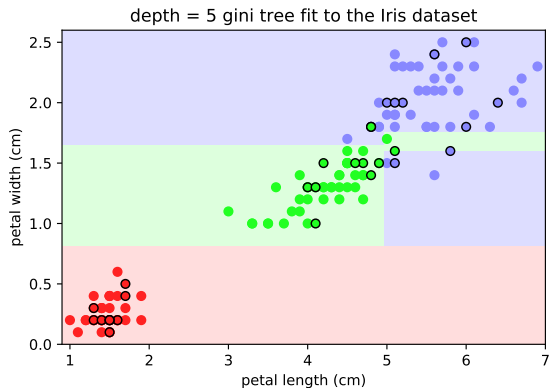
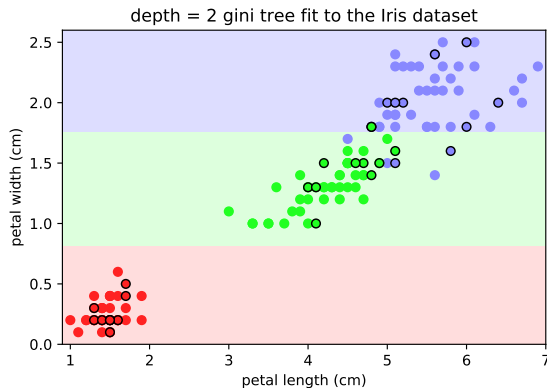
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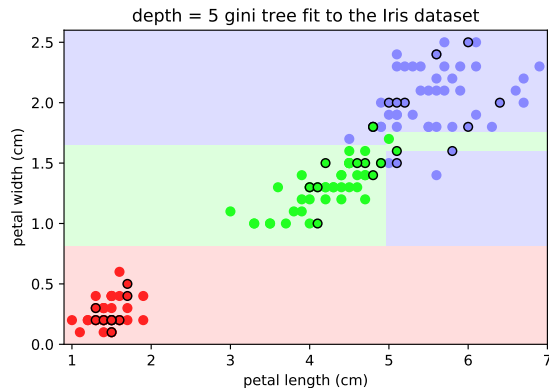
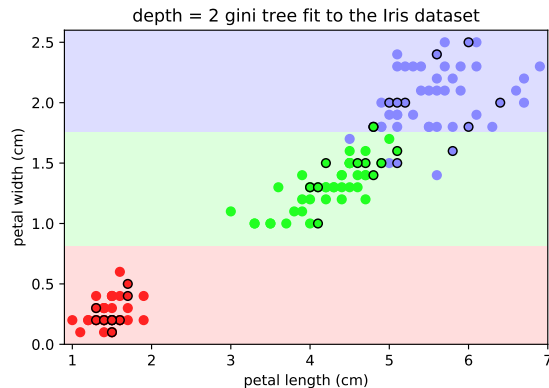
When growing decision trees, at a given node we search over the attributes for splitting, and choose the one that gives the maximum information gain, until we reach a leaf, which has an entropy of zero.

Classification tree examples: Iris Data



Note the rectangular regions (because each split is over one variable) and the greater complexity when the maximum depth of the tree increases.

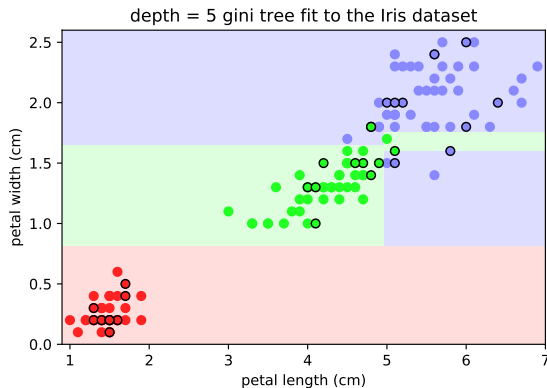
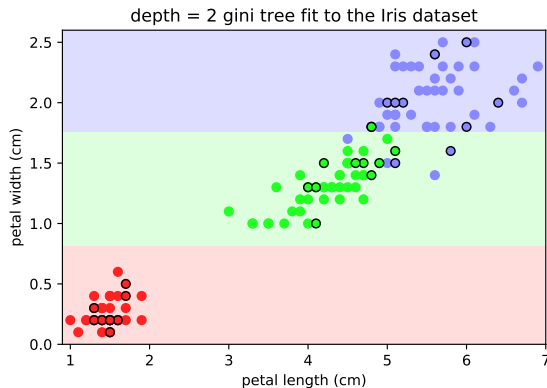
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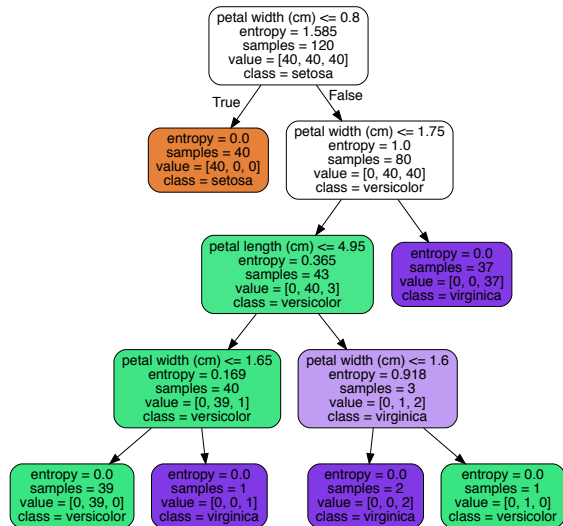


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Classification tree view: Iris Data

Note that the leaf nodes are pure (entropy=0) and are coloured according to predicted value (species label): brown for *I. setosa*, green for *I. versicolor* and purple for *I. virginica*.



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- Let Petal_Width = 1.5cm and Petal_Length = 5cm. The other (sepal) dimensions are ignored by the decision tree because they were not as useful for classification.

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Classification tree: Use for Prediction

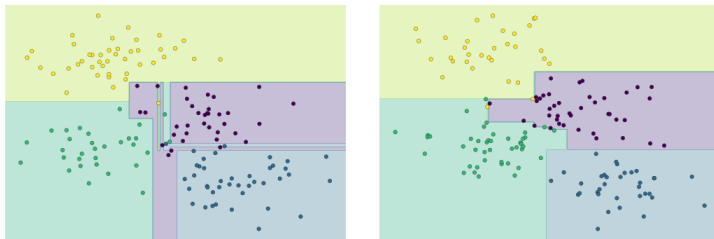
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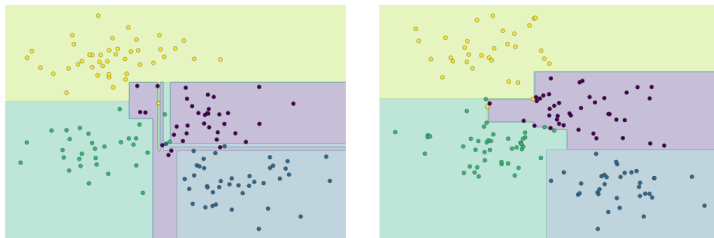
Python can extract paths from the root to each leaf as a set of if-then-else rules, to explain decisions.

Be careful of overfitting...



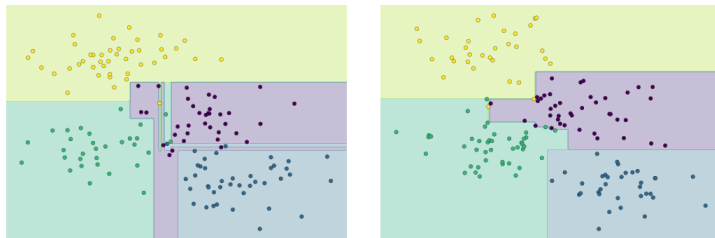
- Given two very similar (generated) data sets, all leaves in each fitted decision tree are *pure*.

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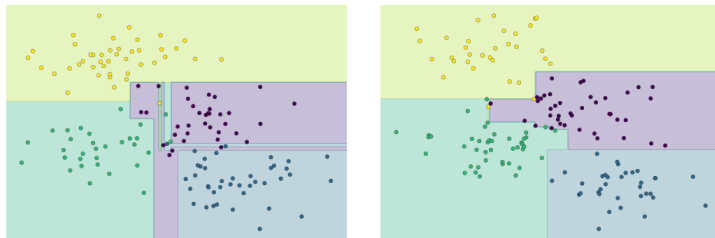
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- Given two very similar (generated) data sets, all leaves in each fitted decision tree are *pure*.
- The resulting trees look very different.
- This sensitivity to the noise in the data is characteristic of *overfitting* (high variance).
- Control by a) limiting depth or b) limiting number of leaves.

Classification trees in python

```
1 tree = DecisionTreeClassifier(criterion=criterion, max_depth=treeDepth, random_state=0)
2 tree.fit(Xtrain, ytrain)
3 y_treeTest = tree.predict(Xtest)
4 print(accuracy_score(ytest, y_treeTest))
5 print(confusion_matrix(ytest, y_treeTest))
6 print(classification_report(ytest, y_treeTest, digits=3))
```

After creating the classifier object, fit the training data and then use the fit to predict yTest from xTest. I have also shown how to get some diagnostic output. Similar diagnostics can be obtained for other classifiers.

Outline

1. Introduction	4
2. Classification Trees	6
3. Naive Bayes classification	27
4. Ordinal targets	38
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Rev. Bayes and his theorem



Rev. Thomas Bayes, 1702–1761

Usage

Given $P(E|H)$ (Probability of Evidence (attributes) given the Hypothesis (the known classes) in the *training* set), Bayes theorem shows how to invert this relationship to get $P(H|E)$ (Probability of the Hypothesis (class) given the evidence (attributes) with an (unseen) *test case*).

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Bayes' Theorem

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Application to classification

By convention, $A = H$ and $B = E$, where H is the **hypothesis** and E is the **evidence** in support of that hypothesis.

With this interpretation, the Bayes identity can be used to predict class probabilities (hypothesis) from features (evidence).

Conditional probabilities and Bayes terminology

Definition 6 (Conditional Probability)

If A and B are events, the Probability of A , given that B is true (has happened), written $P(A|B)$ is defined as follows:

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Given Bayes Theorem 1, we have

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$P(A)$	Class prior; Prior probability
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- In data mining, B might represent the features (derived from the **Data**) for a given instance and A might represent the *predicted label* for these features.
- If A and B are independent events, $P(A \cap B) \equiv P(A)P(B)$, so $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Extended and Bayes

➤ In practice, there would be multiple features/evidence so $B = \{B_1, B_2, \dots, B_n\}$

Definition 7 (Extended Bayes Theorem)

The **extended form**, when $\{B_j\}$ partition B , so $B = \cup_j B_j$ and $B_p \cap B_q \equiv \emptyset$ unless $p = q$, is

$$P(A|\{B_i\}) = \frac{P(\{B_i\}|A)P(A)}{P(\{B_i\})} \quad (3)$$

which is the component-wise version of the standard Bayes Theorem.

Side note: Prosecutor's Fallacy

Note that $P(A|B) \neq P(B|A)$ in general. If the ratio $\frac{P(A)}{P(B)}$ is not close to 1, lawyers can mislead jurors regarding guilt or innocence. *Probability of Guilt given the evidence is not the same as the probability of the evidence assuming the defendant is guilty.*

Naive Extended Bayes

Definition 8 (Naive Bayes)

If the features B are assumed to be independent of each other, it can be shown that

$$P(B) = P(B_1 \cap B_2 \cap \dots B_n) = \prod_i P(B_i) \quad (4)$$

$$P(B|A_j) = \prod_k P(B_i|A_j) \quad (5)$$

The **naïve** form of Bayes theorem becomes

$$P(A_j|B) = \frac{\prod_i P(B_i|A_j)P(A_j)}{\prod_i P(B_i)} \quad (6)$$

Naive Bayes classifier

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- It is then assigned to the class for which its conditional probability is greatest.

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 - $P(B_i|A_j)$ (the evidence that the feature valued B_i predicts the class label A_j).
- From this, we can use the naive version of the extended Bayes Theorem 6 to predict $P(A_j|B)$, the posterior probability of class label A_j given all the evidence from the features B .

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- One of the features of Naïve Bayes (with $P(A|B)$), like decision trees (with $P(A \cap B)$), is the direct role played by probability
- When training Naïve Bayes, it is convenient to compute a table of *marginal counts*, as seen in the next slide, and to use these for prediction.

Fruit classification example

Example: Fruit classification

Type	Long	\neg Long	Sweet	\neg Sweet	Yellow	\neg Yellow	Total
Banana	400	100	350	150	450	50	500
Orange	0	300	150	150	300	0	300
Other	100	100	150	50	50	150	200
Total	500	500	650	350	800	200	1000

Source: [stackoverflow](#)

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Fruit classification : Precalculations

$$P(\langle \text{Fruit} \rangle) = \text{Total}_{\langle \text{Fruit} \rangle} / \text{Total}_{*}$$

$$P(\langle \text{Feature} \rangle) = \text{Total}_{\langle \text{Feature} \rangle} / \text{Total}_{*}$$

$$P(\langle \text{Feature} \rangle \mid \langle \text{Fruit} \rangle) = \langle \text{Fruit}, \text{Feature} \rangle / \text{Total}_{\langle \text{Fruit} \rangle}$$

$$\rightarrow P(\text{Other}) = 200/1000 = 0.2$$

$$\rightarrow P(\text{Sweet}) = 650/1000 = 0.65$$

$$\rightarrow P(\text{Sweet} \mid \text{Other}) = 150/200 = 0.75$$

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Banana - B

$$P(B|L, S, Y) =$$

$$\frac{P(L|B)P(S|B)P(Y|B)P(B)}{P(L)P(S)P(Y)}$$

$$= \frac{0.8 \times 0.7 \times 0.9 \times 0.5}{0.5 \times 0.65 \times 0.8}$$

$$= 0.97$$

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Given the 3 binary-valued attributes, there are $2^3 = 8$ possible combinations - Naïve Bayes will classify each of these 8 combinations as one of the 3 fruit classes.

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- Implementations exist in `sklearn`: `from sklearn.naive_bayes import GaussianNB, etc.`

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4. Ordinal targets	38
5. Resources	40

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➤ In the meantime, either Regression or Classification is used, with caveats... ➤

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 - SVM was state of the art (1985-2000, say) and is still extremely effective for very high dimensional problems like document classification

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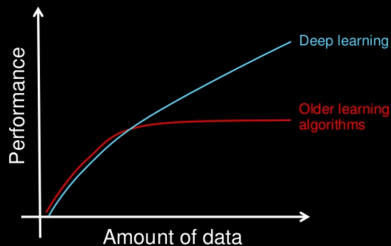
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Classification is sometimes confused with clustering - will cover this clustering next lecture.

But is that the last word on Classification?

Why deep learning

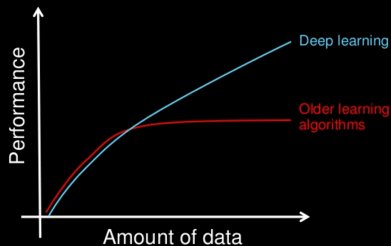


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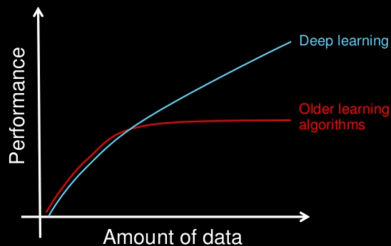
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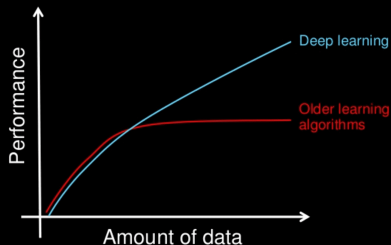
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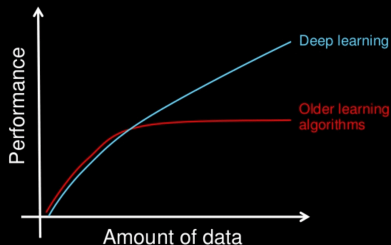
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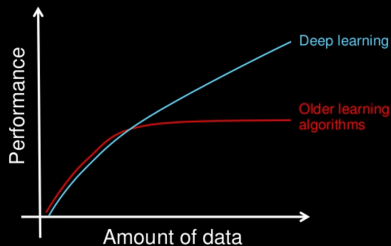
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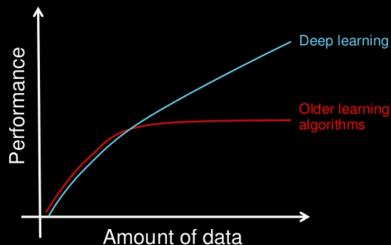
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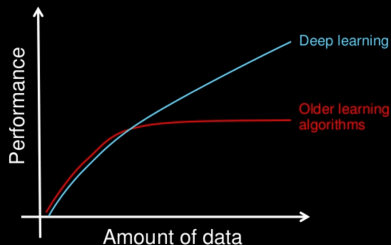
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Deep Learning will probably be covered in semester 2...

General References
