

dm25s1on

Topic 10 : Classification2

Motivating Example

Part 02 : NaiveBayes

Preparation

Data Handling

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Exploring Data 1

Exploring Data 2

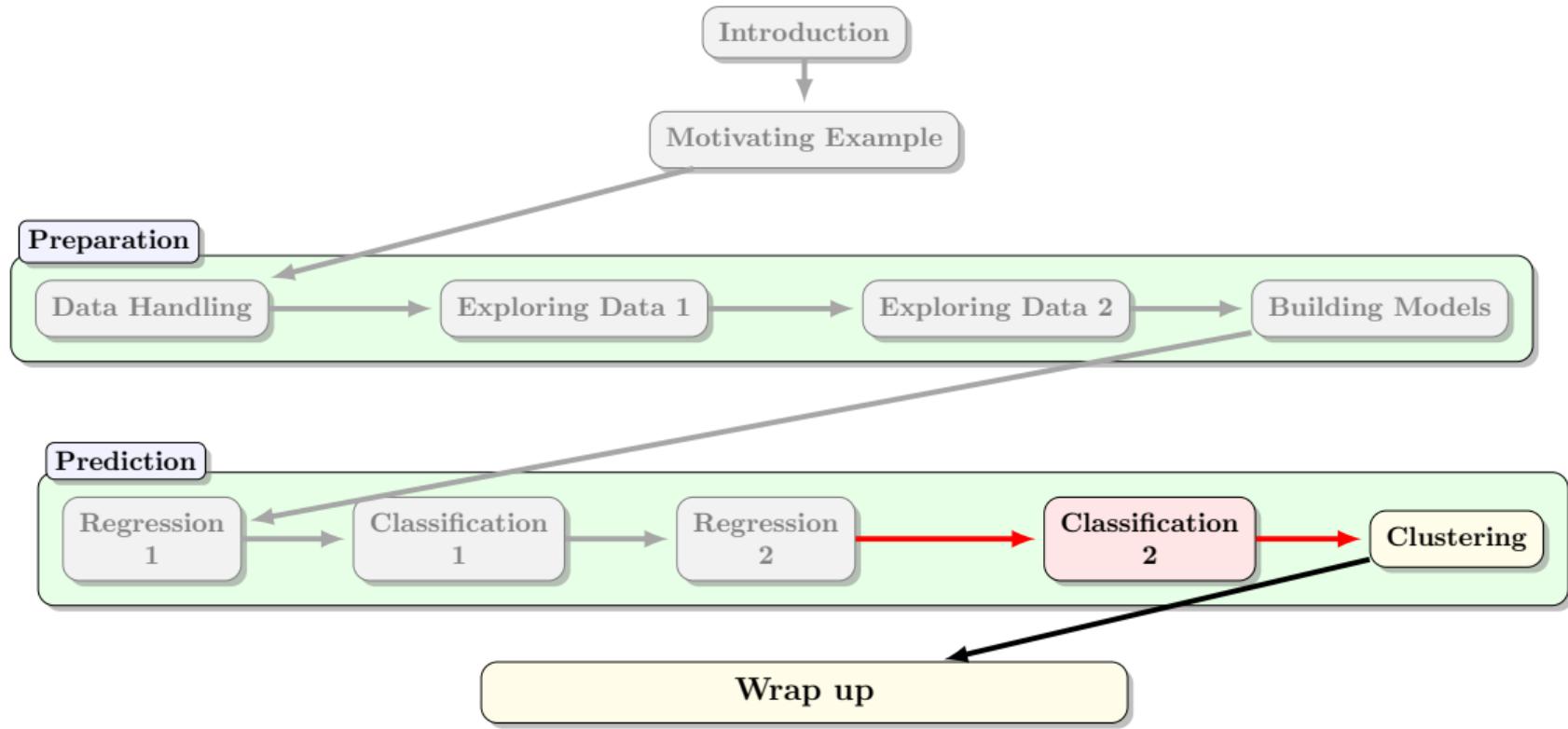
Outline

- Naive Bayes
- Ordinal Classification/Regression

Autumn Semester, 2025

Wrap up

# Data Mining (Week 10)



# Outline

1. Naive Bayes classification	3
2. Ordinal targets	16
3. Resources	18

# Rev. Bayes and his theorem



Rev. Thomas Bayes, 1702–1761

## Usage

Given  $P(E|H)$  (Probability of Evidence (attributes) given the Hypothesis (the known classes) in the *training* set), Bayes theorem shows how to invert this relationship to get  $P(H|E)$  (Probability of the Hypothesis (class) given the evidence (attributes) with an (unseen) *test case*).

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## Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (1)$$

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## Application to classification

By convention,  $A = H$  and  $B = E$ , where  $H$  is the **hypothesis** (observation has specific class) and  $E$  is the **evidence** from the training data in support of that hypothesis.

With this interpretation, the Bayes identity can be used to predict class probabilities (hypothesis) from features (evidence).

# Conditional probabilities and Bayes terminology

## Definition 1 (Conditional Probability)

If  $A$  and  $B$  are events, the Probability of  $A$ , given that  $B$  is true (has happened), written  $P(A|B)$  is defined as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (2)$$

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Given Bayes Theorem 1, we have

Term	Description
$P(A)$	Class prior; Prior probability
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- In data mining,  $B$  might represent the features (derived from the Data) for a given instance and  $A$  might represent the *predicted label* for these features.
- If  $A$  and  $B$  are independent events,  $P(A \cap B) \equiv P(A)P(B)$ , so  $P(A|B) = P(A)$  and  $P(B|A) = P(B)$ .

# Extended Naive Bayes

In practice, there could be multiple features/evidence so  $B = \{B_1, B_2, \dots, B_n\}$

## Definition 2 (Extended Bayes Theorem)

The **extended form**, when  $\{B_j\}$  partition  $B$ , so  $B = \cup_j B_j$  and  $B_p \cap B_q \equiv \emptyset$  unless  $p = q$ , is

$$P(A|\{B_i\}) = \frac{P(\{B_i\}|A)P(A)}{P(\{B_i\})} \quad (3)$$

which is the component-wise version of the standard Bayes Theorem.

## Side note: Prosecutor's Fallacy

Note that  $P(A|B) \neq P(B|A)$  in general. If the ratio  $\frac{P(A)}{P(B)}$  is not close to 1, lawyers can mislead jurors regarding guilt or innocence. *Probability of Guilt given the evidence is not the same as the probability of the evidence assuming the defendant is guilty.* “Since the defendant is probably guilty, and we have proved he had the opportunity, he must have committed the crime.”

# Naive Extended Bayes

## Definition 3 (Naive Bayes)

If the features  $B$  are assumed to be independent of each other, it can be shown that

$$P(B) = P(B_1 \cap B_2 \cap \dots \cap B_n) = \prod_i P(B_i) \quad (4)$$

$$P(B|A_j) = \prod_k P(B_i|A_j) \quad (5)$$

The **naïve** form of Bayes theorem becomes

$$P(A_j|B) = \frac{\prod_i P(B_i|A_j)P(A_j)}{\prod_i P(B_i)} \quad (6)$$

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- Bayes theorem provides the expression used to predict a new observation’s membership of each class (associated with a label).
- The observation is then assigned to the class for which its conditional probability is greatest.

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  - $P(B_i|A_j)$  (the evidence that the feature valued  $B_i$  predicts the class label  $A_j$ ).
- From this, we can use the Naive version of the extended Bayes Theorem 6 to predict  $P(A_j|B)$ , the posterior probability of class label  $A_j$  given all the evidence from the features  $B$ .

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- One aspect of Naïve Bayes (with  $P(A|B)$ ), like decision trees (with  $P(A \cap B)$ ), is the direct role played by probability
- When training Naïve Bayes, it is convenient to compute a table of *marginal counts*, as seen in the next slide, and to use these for prediction.

# Fruit classification example

## Example: Fruit classification

Type	Long	$\neg$ Long	Sweet	$\neg$ Sweet	Yellow	$\neg$ Yellow	Total
Banana	400	100	350	150	450	50	500
Orange	0	300	150	150	300	0	300
Other	100	100	150	50	50	150	200
<i>Total</i>	<i>500</i>	<i>500</i>	<i>650</i>	<i>350</i>	<i>800</i>	<i>200</i>	<i>1000</i>

Source: [stackoverflow](#)

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Fruit classification : Precalculations

$$P(\text{Fruit}) = \text{Total}_{\text{Fruit}} / \text{Total}_{\text{*}}$$

$$\rightarrow P(\text{Other}) = 200/1000 = 0.2$$

$$P(\text{Feature}) = \text{Total}_{\text{Feature}} / \text{Total}_{\text{*}}$$

$$\rightarrow P(\text{Sweet}) = 650/1000 = 0.65$$

$$P(\text{Feature} | \text{Fruit}) = \langle \text{Fruit}, \text{Feature} \rangle / \text{Total}_{\text{Fruit}}$$

$$\rightarrow P(\text{Sweet} | \text{Other}) = 150/200 = 0.75$$

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Given the 3 binary-valued attributes, there are  $2^3 = 8$  possible combinations - Naïve Bayes will classify each of these 8 combinations as one of the 3 fruit classes.

# Naïve Bayes using scikit-learn

## Setup

```
from sklearn.naive_bayes import GaussianNB  
from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
```

## Fit and Predict

```
gnb = GaussianNB()  
gnb.fit(Xtrain, ytrain)  
y_gnbTest = gnb.predict(Xtest)  
print(accuracy_score(ytest, y_gnbTest))  
print(confusion_matrix(ytest, y_gnbTest))  
print(classification_report(ytest, y_gnbTest, digits=3, target_names=target_names))
```

➤ Note that `GaussianNB` rarely has arguments and numeric features do not need to be scaled before use

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- Categorical features are used to group counts of obaservations when computing probabilities
- But what about numerical features?
- Assume each numeric feature has a Gaussian distribution, characterised by its mean ( $\mu$ ) and standard deviation ( $\sigma$ ) parameters
- GaussianNB fits this prior distribution to each numeric feature, when computing the marginal counts for the categorical features
- For a given test instance, its z-score for each numeric feature is computed from the fitted  $\mu$  and  $\sigma$  for that feature (scaling is implicit).
- Hence, its likelihood for that probability distribution can be obtained, and substituted in the Naive Bayes (NB) expression over all the features, for each class value.
- The predicted class value is just the class value with the largest NB prediction over the features.

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  - Prone to underfitting (high bias) because so much aggregation happens that observation-specific information is lost

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- Implementations exist in `sklearn`: `from sklearn.naive_bayes import GaussianNB, etc.`

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2. Ordinal targets	16
3. Resources	18

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In the meantime, either Regression or Classification is used, with caveats...

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  - Support Vector Machines (SVM) were state of the art (1985-2000, say) and are still extremely effective for very high dimensional problems like document classification: a small number of support vectors define the decision boundary, so the classification decision collapses to 1D

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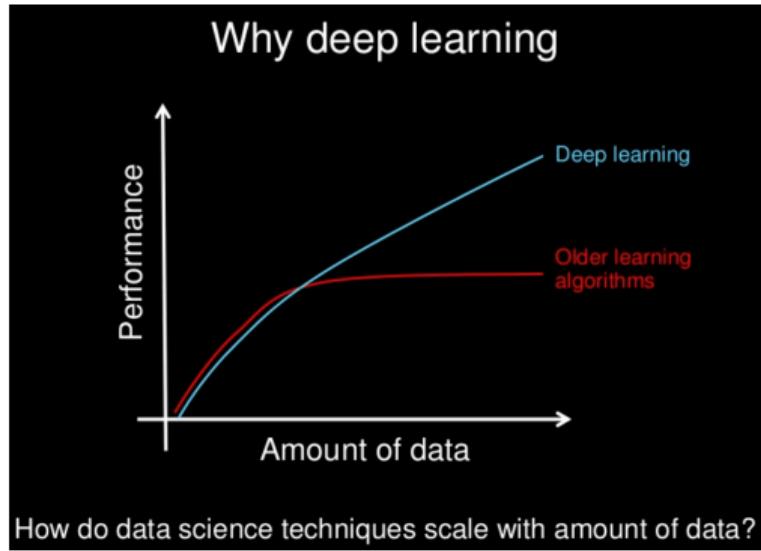
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Classification is sometimes confused with clustering - will cover *clustering* next week.

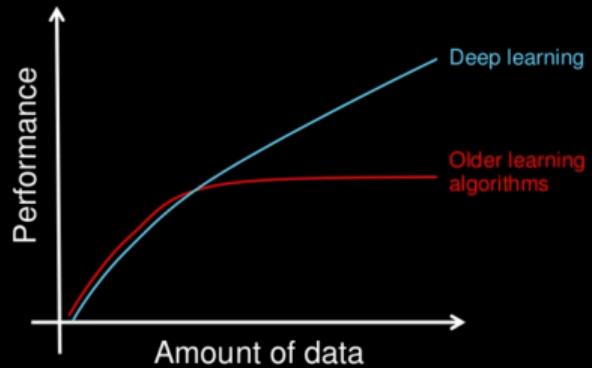
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Source: Andrew Ng, *Why Deep Learning*

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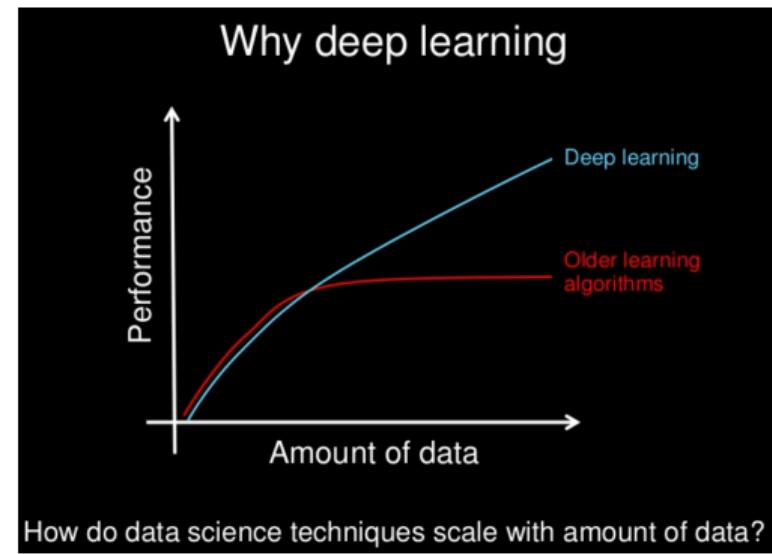


How do data science techniques scale with amount of data?

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## Learning from big data

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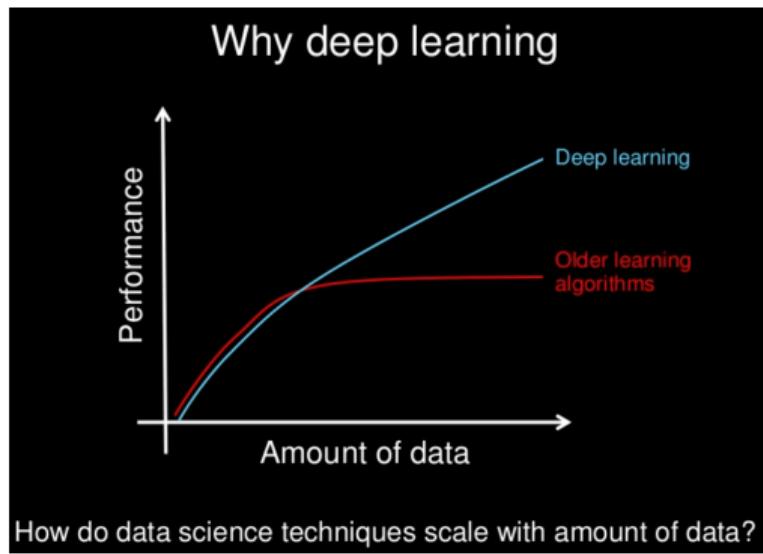


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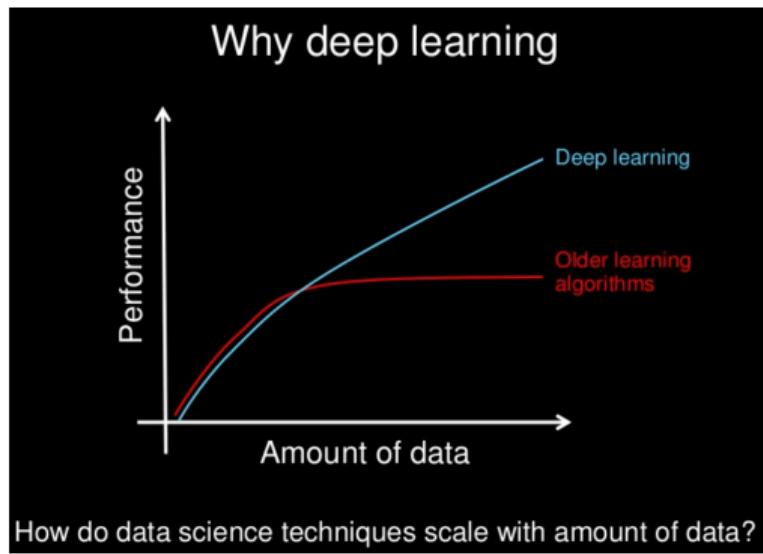


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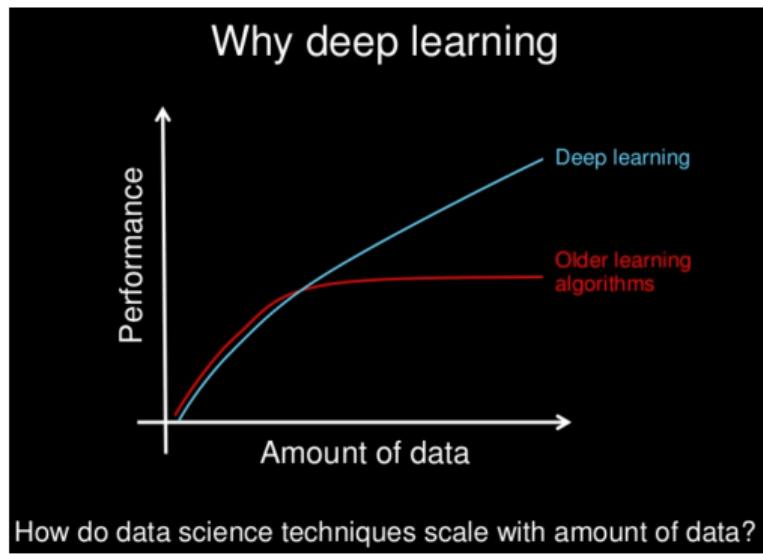


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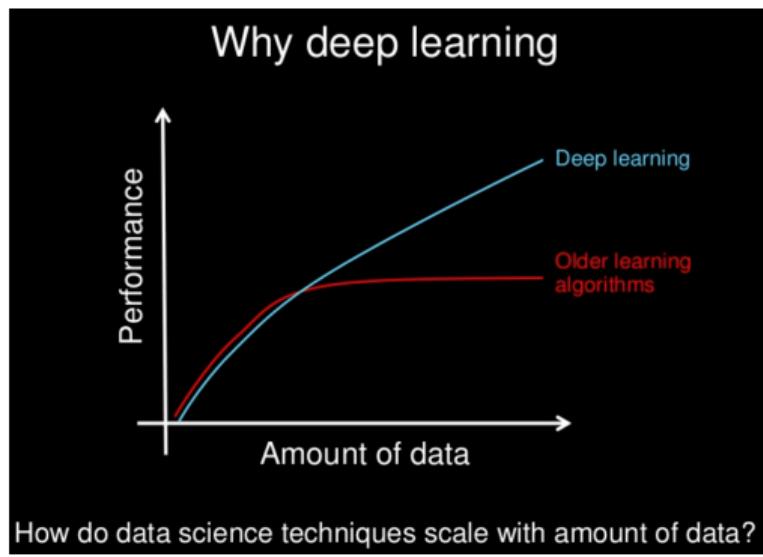


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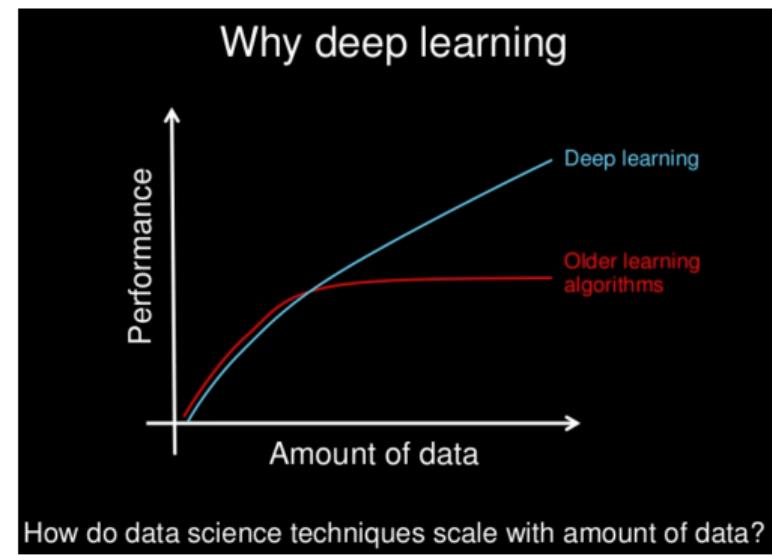


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Deep Learning will probably be covered in semester 2...

# General References