Simulation Project

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The Central Limit Theorem and the exponential distribution

In this project, I will investigate the exponential distribution and compare it to the Central Limit Theorem. I will compare the sample mean and variance of simulations with the theoretical mean and variance of the exponential distribution. For this purpose, I will do 1000 simulations of the averages of 40 exponentials. I will set the parameter λ to 0.2.

Comparison between the sample mean and the theoretical mean

Since I used $\lambda = 0.2$, the theoretical mean is $\mu = \frac{1}{\lambda} = \frac{1}{0.2} = 5$.

I created a vector of 1000 simulations for the means of 40 exponentials.

```
# Simulations of the means
set.seed(60)
mexp <- NULL
for (i in 1:1000) mexp <- c(mexp, mean(rexp(40,0.2)))
```

The mean of these 1000 simulations is :

```
mean(mexp)
```

```
## [1] 5.027802
```

We can see that it's really close to the theoretical mean of 5.

Comparison between the sample variance of the mean and the theoretical variance of the mean

We know that the variance of the sample mean is the population variance divided by n. The sample variance of the mean is $\frac{\sigma^2}{n} = \frac{(1/\lambda)^2}{n} = \frac{(1/0.2)^2}{n} = \frac{25}{40} = 0.625$.

```
var(mexp)
```

```
## [1] 0.6633712
```

We can see that the sample variance of the mean is close to the theoretical variance of the mean of 0.625.

Comparison between the sample variance and the theoratical variance

We know that the variance of the exponential distribution is σ^2. Since $\lambda = 0.2$ and $\sigma = \frac{1}{\lambda}$, the theoretical variance is $\sigma^2 = (\frac{1}{\lambda})^2 = (\frac{1}{0.2})^2 = 5^2 = 25$.

Let's do 1000 simulations of the variance of 40 exponentials.

```
# Simulations of the variances
set.seed(60)
vexp <- NULL
for (i in 1:1000) vexp <- c(vexp, var(rexp(40,0.2)))
```

Let's compute the mean of the variances.

```
mean(vexp)
```

```
## [1] 25.22624
```

We can see that this value is close to the theoretical variance of 25.

Distribution

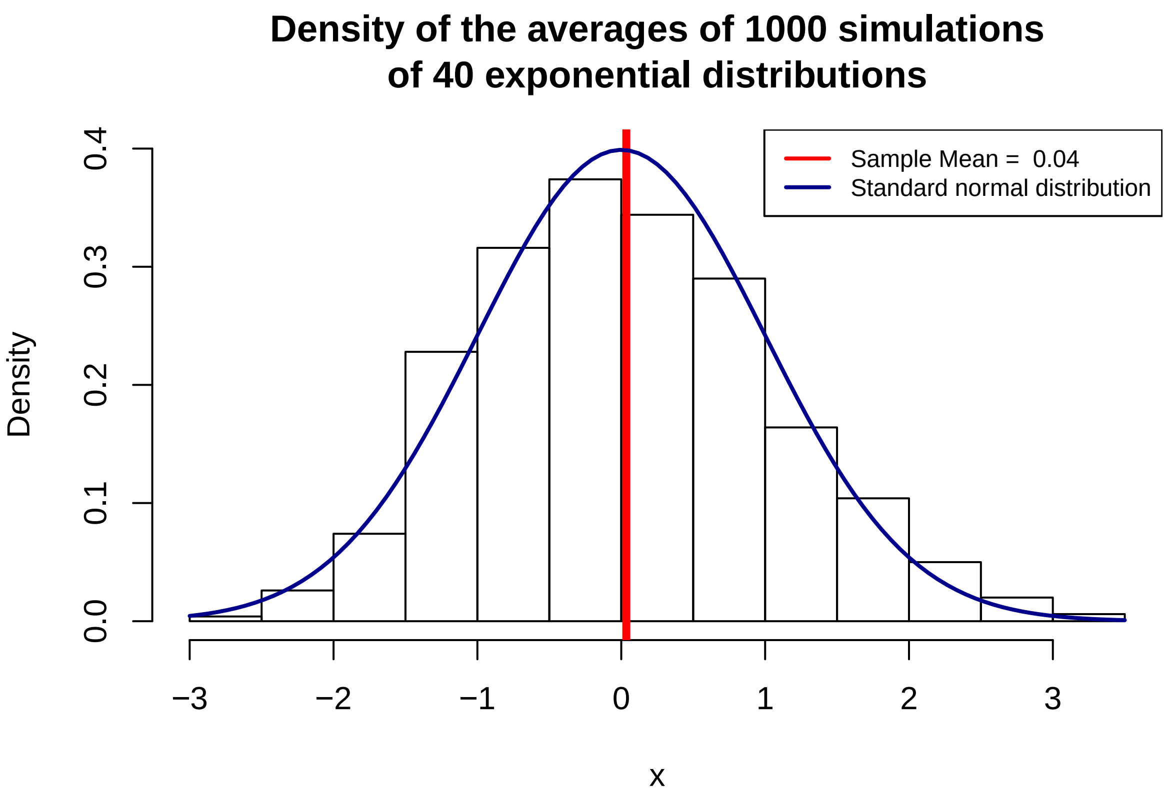
We are now going to check that the distribution of the means of the simulations follows a normal distribution.

We know that $\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}$ follows a standard normal distribution. To verify this, I computed $\frac{mexp-\mu}{\sigma/\sqrt{n}} = \frac{mexp-5}{5/\sqrt{40}}$. I plotted the density of this vector and compared it to a standard normal distribution.

```
sigma <- 5
mu <- 5
n <- 40
norm_mexp <- (mexp - mu) / (sigma / sqrt(n))

# Plot
hist(norm_mexp,
     freq = FALSE,
     breaks = 20,
     main = "Density of the averages of 1000 simulations\nof 40 exponential distributions",
     xlab = "x",
     ylim = c(0,0.4))
# Vertical line on the sample mean
abline(v = mean(norm_mexp),
       col = "red",
       lwd = 4)
# Normal distribution reference
curve(dnorm(x,0,1),
      add = TRUE,
      col = "darkblue",
      lwd=2)
legend("topright",
       legend = c(paste("Sample Mean = ", round(mean(norm_mexp), digits = 2)),
                  "Standard normal distribution"),
       col = c("red", "darkblue"),
```

```
        lty = c(1,1),
        lwd = 2,
        cex = 0.75)
```



We can see that the sample mean is centered on 0, the theoretical mean of the standard normal distribution, and follow roughly a standard normal distribution of $\mu = 0$ and $\sigma = 1$.