Date:

Assignment: 2.

Hamza Mehmood
SP-21-110
Linear Algebra
Sir Rizwan

(Question: 2)

$$14x + 3y = 24$$
$$3x + 4y - z = 30$$
$$-y + 4z = -24$$

By Jacobi Method
Initially:-

$$[x_0 = 0,\ y_0 = 0,\ z_0 = 0]$$

$$x_{k+1} = \frac{1}{4}(24 - 3y_k) \rightarrow A$$

$$y_{k+1} = \frac{1}{4}(30 - 3x_k + z_k) \rightarrow B$$

$$z_{k+1} = \frac{1}{4}(-24 + y_k) \rightarrow C$$

For $k = 0$

eq (A):

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$$x_1 = \frac{1}{4}(24 - 3y_0) = \frac{1}{4}(24 - 0)$$

$$\boxed{x_1 = 6}$$

eq (B) :-

$$y_1 = \frac{1}{4}(30 - 3x_0 + z_0) = \frac{1}{4}(30 - 0 + 0)$$

$$\boxed{y_1 = 7.5}$$

eq (C) :-

$$z_1 = \frac{1}{4}(-24 + y_0)$$

$$z_1 = \frac{-24}{4} \Rightarrow z_1 = -6$$

For K = 1 :-

eq (A) : $x_2 = \frac{1}{4}(24 - 3y_1)$

$$x_2 = \frac{1}{4}(24 - 3(7.5))$$

$$\boxed{x_2 = 0.375}$$

eq (B) :

$$y_2 = \frac{1}{4}(30 - 3x_1 + z_1)$$

$$\boxed{y_2 = 1.5}$$

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eq C: $z_2 = \frac{1}{4}(-24 + y_1)$

$z_2 = \frac{1}{4}(-24 + 7.5)$

$z_2 = 4.125$

For $k = 2$:-

eq (A): $x_3 = \frac{1}{4}(24 - 3y_2)$

$x_3 = \frac{1}{4}(24 - 3(1.5))$

$x_3 = 4.875$

eq (B): $y_3 = \frac{1}{4}(30 - 3x_2 + z_2)$

$y_3 = 6.1875$

eq (C): $\frac{1}{4}(-24 + y_2) \Rightarrow z_3 = 5.625$

For $k = 3$:-

eq (A):

$x_4 = \frac{1}{4}(24 - 3y_3)$

$x_4 = \frac{1}{4}(24 - 3(6.1875))$

$x_4 = 1.359375$

eq (B): $y_4 = \frac{1}{4}(30 - 3x_3 + z_3$

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$y_4 = .2.4375$

eq (C): $y_4 = \frac{1}{4}(-24 + y_3) \Rightarrow \frac{1}{4}(-24 + 6.18)$

$y_4 = -4.4453125$

For $k = 4$:

eq (A): $x_5 = \frac{1}{4}(24 - 3y_4)$

$= \frac{1}{4}(24 - 3)$

$x_5 = 4.171875$

eq (B): $y_5 = \frac{1}{4}(30 - 3x_4 + y_4)$

$= \frac{1}{4}(30 - 3(1.359375)$

$y_5 = 5.3671875$

eq (C): $y_5 = \frac{1}{4}(-24 + y_4)$

$y_5 = 5.390625$

For $k = 5$:

eq (A): $x_6 = \frac{1}{4}(24 - 3y_5)$

$\frac{1}{4}(24 - 3(5.3671875))$

$x_6 = 1.097460$

eq (B): $y_6 = \frac{1}{4}(30 - 3x_5 + y_5)$

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$$y_6 = \frac{1}{4}(30 - 3(4.171875))$$

$$y_6 = 3.0234375$$

eq (C): $$z_6 = \frac{1}{4}(-24 + y_5)$$

$$= \frac{1}{4}(-24 + 5.3671875)$$

$$z_6 = 4.6682203125$$

For $k = 6$:

$$x_7 = \frac{1}{4}(24 - 3y_6) \Rightarrow \frac{1}{4}(24 - 3(3.0234))$$

eq(A): $x_7 = 3.732421$

eq (B): $$y_7 = \frac{1}{4}(30 - 3x_6 + z_6)$$

$$y_7 = \frac{1}{4}(30 - 3(1.97460) - 4.6582))$$

$$y_7 = 4.854492188$$

eq(C): $$z_7 = \frac{1}{4}(-24 + y_6) = \frac{1}{4}(-24 + 3.023$$

$$z_7 = -5.244140$$

For $k = 7$:

eq (A) $$x_8 = \frac{1}{4}(30 - 3x_7 + 37)$$

$$x_8 = 2.3591$$

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eq (b): $y_8 = \frac{1}{4}(30 - 3x_7 + 37)$

$y_8 = 3.039650781$

eq (c): $z_8 = \frac{1}{4}(-24 + y_7)$

$z_8 = \frac{1}{4}(-24 + 4.8544)$

$z_8 = 4.786376953$

(Question: 1)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$|A - \lambda I_3| = 0$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

$$(a_{11}-\lambda)\begin{vmatrix} a_{22}-\lambda & a_{23} \\ a_{32} & a_{33}-\lambda \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33}-\lambda \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22}-\lambda \\ a_{31} & a_{32} \end{vmatrix} = 0$$

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$$(a_{11}-\lambda)\left((a_{22}a_{33}-a_{22}\lambda-a_{33}\lambda+\lambda^2)-a_{23}a_{32}\right) - a_{12}\left(a_{21}a_{33}-a_{21}\lambda-a_{23}a_{31}\right)+ a_{13}\left(a_{21}a_{32}-(a_{22}a_{31}-a_{31}\lambda)\right)=0$$

$$a_{11}a_{22}a_{33}-a_{11}a_{22}\lambda-a_{11}a_{33}\lambda-a_{11}a_{33}\lambda + a_{11}\lambda^2-a_{11}a_{23}a_{32}-a_{22}a_{33}\lambda + a_{22}\lambda^2+a_{33}\lambda^2-\lambda^3+a_{23}a_{32}\lambda-a_{12}a_{21}a_{33} + a_{12}a_{21}\lambda+a_{12}a_{23}a_{31}+a_{13}a_{21} a_{32}-a_{13}a_{22}a_{31}+a_{13}a_{31}\lambda=0$$

$$\lambda^3-\text{trace}(A)\lambda^2+(a_{11}a_{22}+a_{11}a_{33}+a_{22} a_{33}-a_{12}a_{21}-a_{23}a_{32}-a_{13}a_{31})$$

$$\lambda-|A|=0$$

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$y_1 = \frac{1}{4}(24 - y_1) = \frac{1}{4}(24 + 1.775510204)$

$z_1 = -5.556122449$

For $K = 1$

$x_2 = \frac{1}{4}(24 - 3y_1)$

$x_2 = 1.333819242$

$y_2 = \frac{1}{4}(30 - 3x_2 + z_1)$

$y_2 = 1.406172845$

$z_2 = \frac{1}{4}(24 + y_2)$

$z_2 = -5.634956789$

For $K = 2$

$x_3 = \frac{1}{4}(24 - 3y_2)$

$x_3 = 1.401391533$

$y_3 = \frac{1}{14}(30 - 3x_3 + z_2)$

$y_3 = \frac{1}{11}\ 1.44062044$

$z_3 = \frac{1}{4}(-24 + y_3)$

$z_3 = -5.639984489.$

For $K = 3$

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$$x_6 = \frac{1}{14}(24 - 3y_5)$$

$$x_6 = 1.405979356$$

$$y_6 = \frac{1}{14}(30 - 3x_6 + z_5)$$

$$y_6 = 1.438692454$$

$$z_6 = \frac{1}{4}(-24 + y_6)$$

$$z_6 = -5.640326887.$$