Q No: 01

ii) $f(x) = \cos^3\left(\frac{x}{x+1}\right)$

Suppose: $u = \frac{x}{x+1} \implies \frac{du}{dx} = \frac{d}{dx}\left[\frac{x}{x+1}\right]$

$$\frac{du}{dx} = \frac{(x+1)\frac{d}{dx}x - x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$y = \cos^3 u \implies \frac{dy}{du} = \frac{d}{du}\cos^3 u$$

$$\frac{dy}{du} = 3\cos^2 u \frac{d}{dx}\cos u$$

$$\frac{dy}{du} = -3\cos^2 u \sin u$$

By Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = -3\cos^2 u \sin u \times \frac{1}{(x+1)^2}$$

Put value of u: $\frac{dy}{dx} = \frac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}$

Q No: 01

i) $f(x) = (5x+8)^7 (1-\sqrt{x})^6$

by Product Rule

$$\frac{dy}{dx} = (5x+8)^7 \frac{d}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6 \frac{d}{dx}(5x+8)^7$$

$$= (5x+8)^7 \times 6(1-\sqrt{x})^5 \frac{d}{dx}(1-\sqrt{x}) + (1-\sqrt{x})^6 \times 7(5x+8)^6 \frac{d}{dx}(5x+8)$$

$$= 6(5x+8)^7 (1-\sqrt{x})^5 \left(\frac{-1}{2\sqrt{x}}\right) + 35(1-\sqrt{x})^6 (5x+8)^6$$

$$= \frac{-3}{\sqrt{x}}(5x+8)^7 (1-\sqrt{x})^5 + 35(1-\sqrt{x})^6 (5x+8)^6$$

Q No: 02

i) $\int \frac{e^x}{1+e^{2x}} dx$

Suppose: $u = e^x \Rightarrow \frac{du}{dx} = e^x$

$du = e^x dx$ $\quad\quad \because e^{2x} = (e^x)^2 = u^2$

$$\int \frac{1}{1+u^2} du = \tan^{-1} u + C$$

Put value of u.

$$= \tan^{-1} e^x + C$$

Q No: 02

ii) $\int \frac{\sin\frac{1}{x}}{3x^2}\,dx = \frac{1}{3}\int \frac{\sin 1/x}{x^2}\,dx$

Suppose $u = \frac{1}{x} \Rightarrow \frac{du}{dx} = \frac{x\frac{d}{dx}(1) - 1\frac{d}{dx}x}{x^2}$

$\frac{du}{dx} = \frac{x(0) - 1}{x^2}$

$du = \frac{-1}{x^2}\,dx$

$\boxed{-du = \frac{1}{x^2}\,dx}$

$\frac{1}{3}\int \frac{\sin 1/x}{x^2}\,dx = \frac{-1}{3}\int \sin u\,du$

$= \frac{-1}{3}(-\cos u) + C$

$= \frac{1}{3}\cos u + C$

Put value of u

$\int \frac{\sin 1/x}{3x^2}\,dx = \frac{1}{3}\cos\left(\frac{1}{x}\right) + C$

Q No: 03

a) Find Terminal point

$v = 3i - 2j$

initial point $(2, -3)$

Let: (x, y) be the terminal point

Then,

$x - 2 = 3$

$\boxed{x = 5}$

and

$y - (-3) = -2$

$y + 3 = -2$

$\boxed{y = -5}$

So, The Terminal Point is $(5, -5)$

Q No: 03

b) Find Scalar C_1 and C_2

$W = C_1V_1 + C_2V_2$

$V_1 = 2i - j$

$V_2 = 4i + 2j$

$W = C_1(2i - j) + C_2(4i + 2j)$

$= (2C_1 + 4C_2)i + (-C_1 + 2C_2)j$

Express vector $2i + 4j$

So, $2C_1 + 4C_2 = 2 \longrightarrow (A)$

$-C_1 + 2C_2 = 4 \longrightarrow (B)$

$\boxed{C_2 = \frac{4 + C_1}{2}} \longrightarrow (C)$

Put in eq (A) value of C_2

$2C_1 + 4\left(\frac{4 + C_1}{2}\right) = 2$

$2C_1 + 8 + 2C_1 = 2$

$4C_1 = 2 - 8$

$C_1 = \frac{-6}{4}$

$\boxed{C_1 = -\frac{3}{2}}$

Put value of C_1 in eq (C)

$$C_2 = \frac{4 + (-3/2)}{2}$$

$$C_2 = \frac{8 - 3}{4}$$

$$\boxed{C_2 = \frac{5}{4}}$$