



*DS-GA 3001.009: Responsible Data Science*

## Privacy and Data Protection

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<http://stoyanovich.org/>  
<https://dataresponsibly.github.io/>

# Truth or dare?

**Did you go out drinking over the weekend?**

let's call this property **P** (Truth=Yes) and estimate **p**, the fraction of the class for whom **P** holds

1. flip a coin **C1**

1.if **C1** is tails, then **respond truthfully**

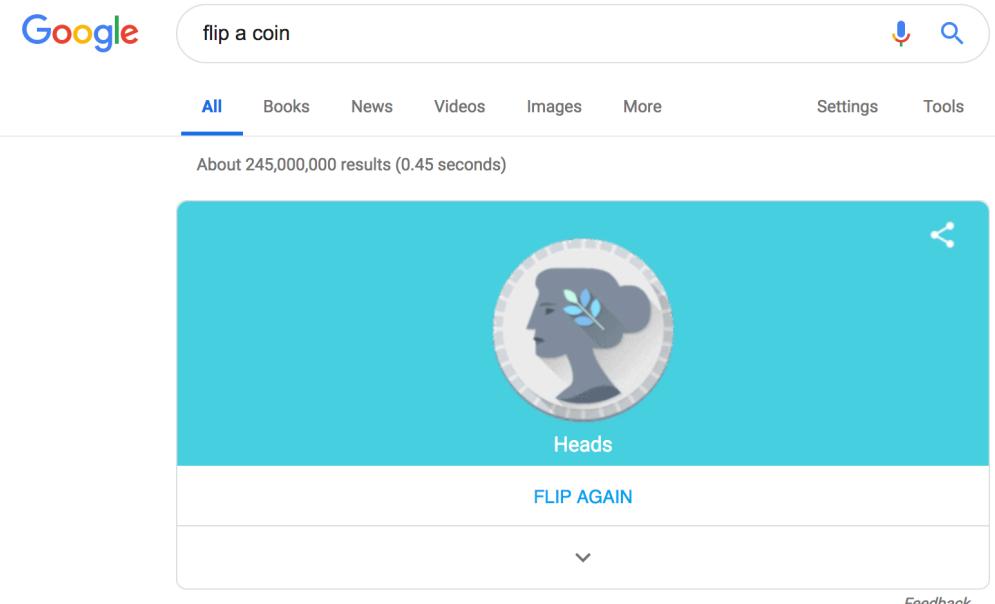
2.if **C1** is heads, then flip another coin **C2**

1.if **C2** is heads then **Yes**

2.else **C2** is tails then respond **No**

the expected number of **Yes** answers is:

$$A = \frac{3}{4}p + \frac{1}{4}(1-p) = \frac{1}{4} + \frac{p}{2}$$



thus, we estimate **p** as:

$$\tilde{p} = 2A - \frac{1}{2}$$

# Randomized response

Did you go out drinking over the weekend?

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}

randomization - adding noise - is what gives plausible deniability a process privacy method

the expected number of **Yes** answers is:

$$A = \frac{3}{4}p + \frac{1}{4}(1-p) = \frac{1}{4} + \frac{p}{2}$$



privacy comes from plausible deniability

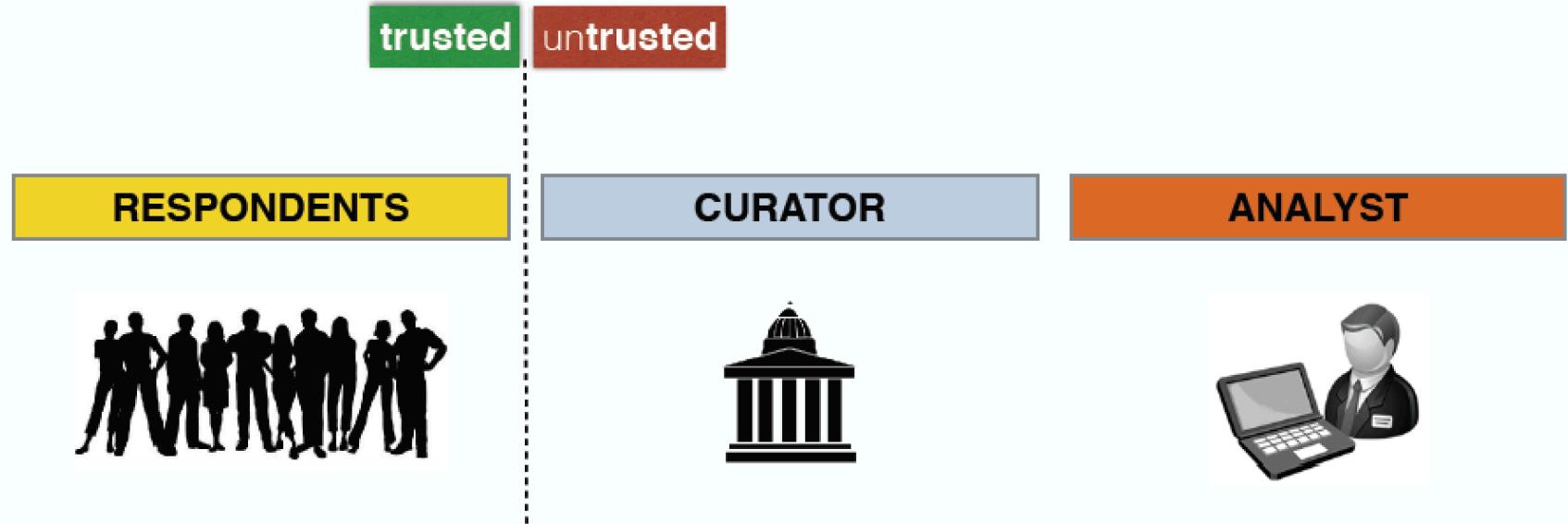
# Privacy: two sides of the coin

protecting an individual  
\_\_\_\_\_  
plausible deniability



learning about the population  
\_\_\_\_\_  
noisy estimates

# Privacy-preserving data analysis



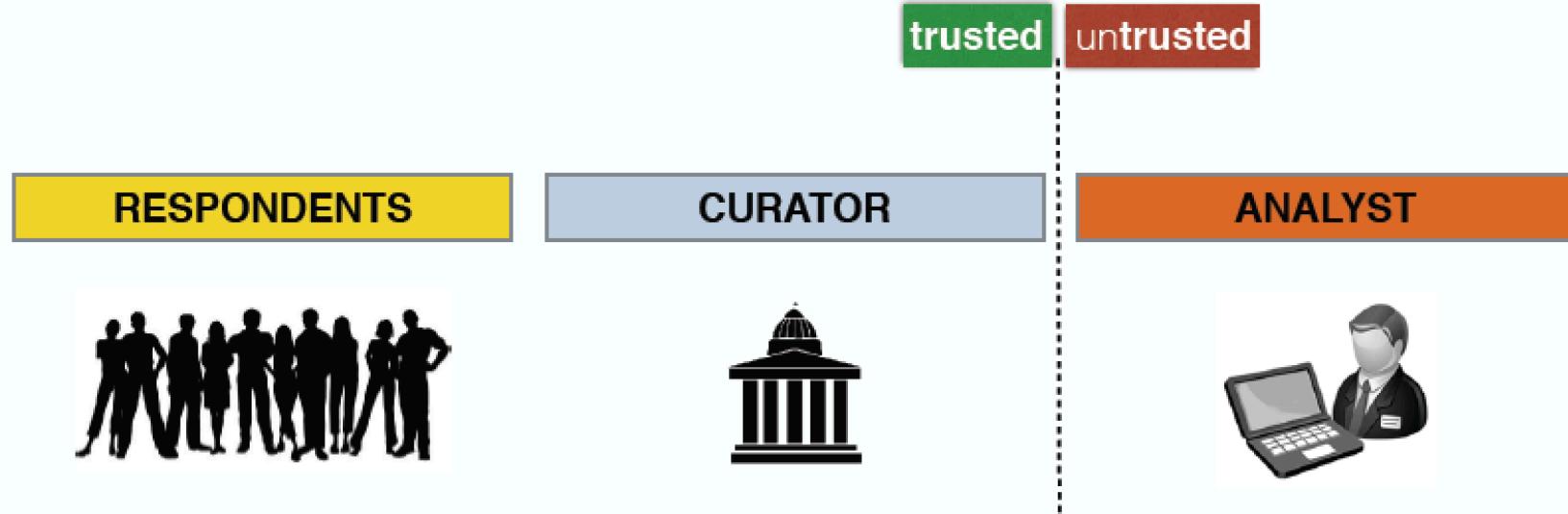
**respondents** contribute their personal data

the **curator** is **untrusted**, collects data, releases it to analysts

the **analyst** is **untrusted**, extracts value from data

slide by Jerome Miklau

# Privacy-preserving data analysis



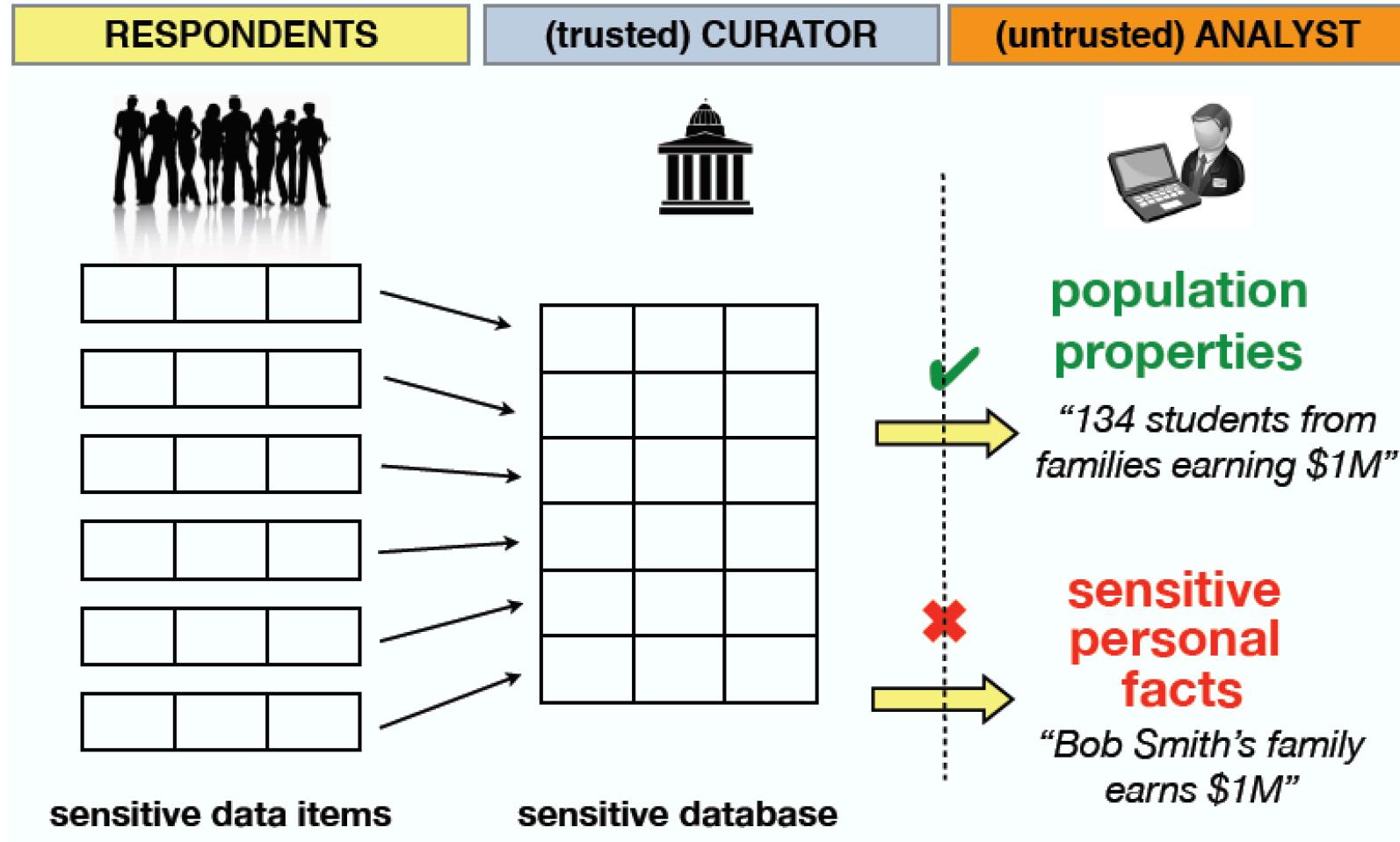
**respondents** in the population seek protection of their personal data

the **curator** is **trusted** to collect data and is responsible for safely releasing it

the **analyst** is **untrusted** and wants to gain the most accurate insights into the population

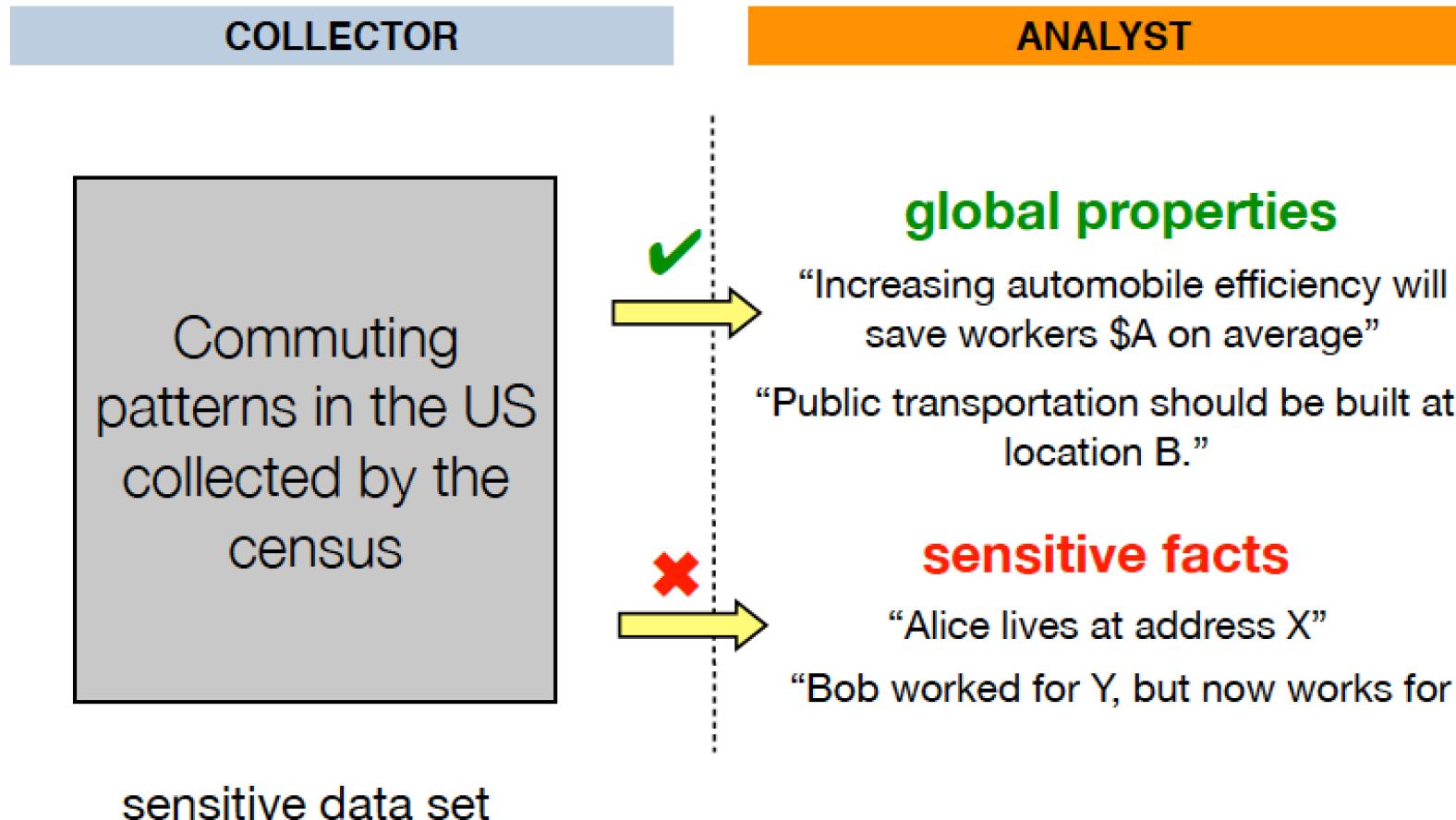
slide by Jerome Miklau

# Privacy-preserving data analysis



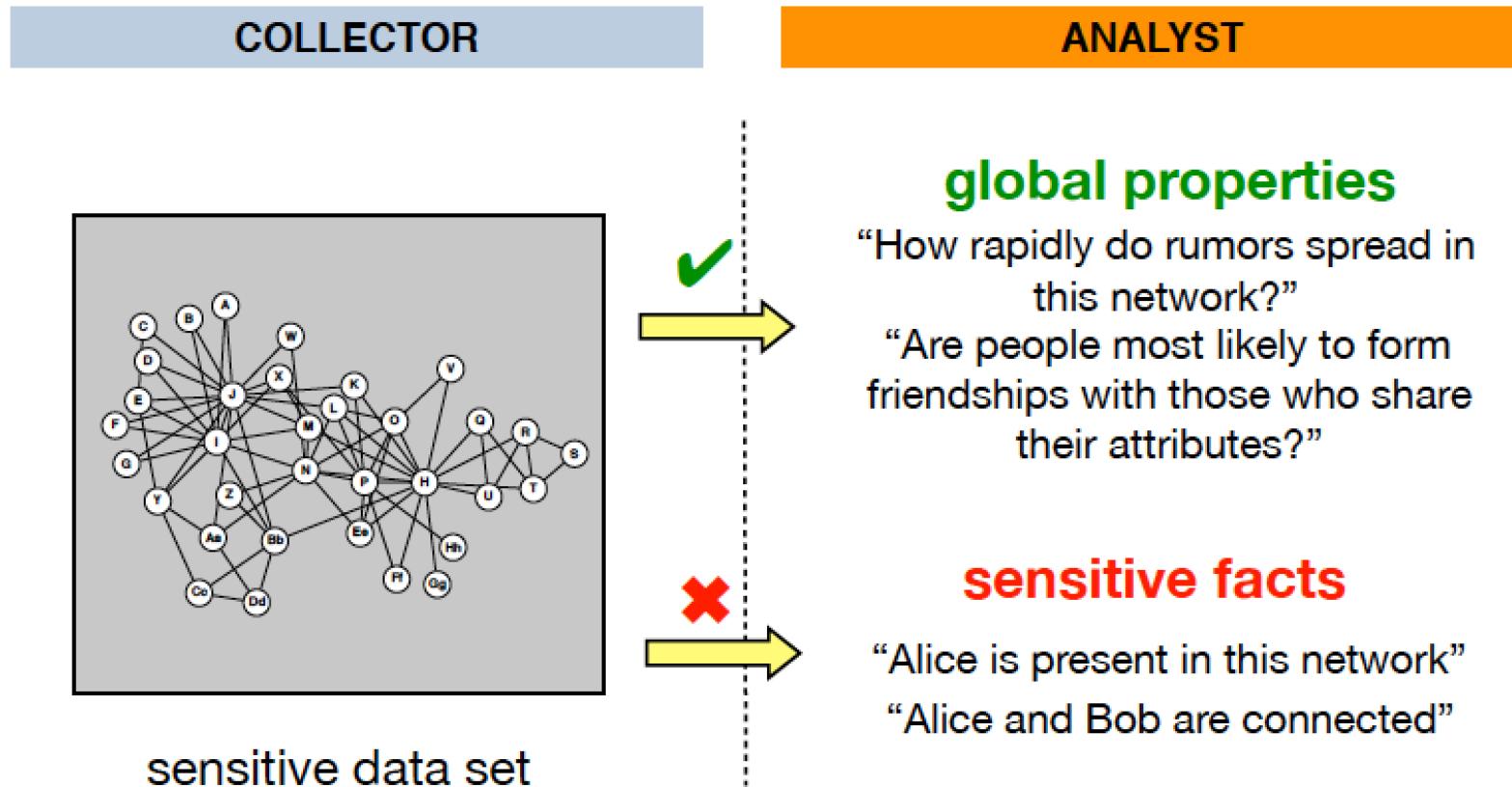
slide by Jerome Miklau

# Example: Census data



slide by Jerome Miklau

# Example: social networks



slide by Jerome Miklau

# Defining private data analysis

- Take 1: If **nothing is learned** about any individual in the dataset, then no individual can be harmed by analysis.
  - **Dalenius' Desideratum:** an *ad omnia* (Latin: “for all”) privacy goal for statistical databases, as opposed to *ad hoc* (Latin: “for this”). Anything that can be learned about a respondent from the statistical database should be learnable without access to the database.
  - Put another way, the adversary’s prior and posterior views about an individual should not be different.
  - This objective is **unachievable** because of auxiliary information.
  - **Example:** Alice knows that John smokes. She read a medical research study that found a causal relationship between smoking and lung cancer. Alice concludes, based on study results and her prior knowledge about John that he has a heightened risk of developing lung cancer.
  - Further, the risk is to everyone in a particular group (smokers, in this example), **irrespective of whether they participated in the study**. We’ll return to this when discussing the **Barrow, Alaska alcohol study**.

# Defining private data analysis

- Take 1: If **nothing is learned** about any individual in the dataset, then no individual can be harmed by analysis.
  - **Dalenius' Desideratum:** an “*ad omnia*” (opposed to *ad hoc*) privacy goal for statistical databases: Anything that can be learned about a respondent from the statistical database should be learnable without access to the database.
  - Put another way, the adversary’s prior and posterior views about an individual should not be different.
- Take 2: The information released about the sensitive data set is virtually indistinguishable **whether or not a respondent’s data is in the dataset**. This is an informal statement of **differential privacy**: that no information **specific to an individual** is revealed.

# Defining private data analysis

## review articles

DOI:10.1145/1866739.1866758

**What does it mean to preserve privacy?**

BY CYNTHIA DWORCK

## A Firm Foundation for Private Data Analysis

Communications of the ACM [CACM](#)

[Homepage archive](#)

Volume 54 Issue 1, January 2011

Pages 86-95

“A natural approach to defining privacy is to require that accessing the database teaches the analyst nothing about any individual. But this is problematic: **the whole point of a statistical database is to teach general truths**, for example, that smoking causes cancer. Learning this fact teaches the data analyst something about the likelihood with which certain individuals, not necessarily in the database, will develop cancer. We therefore **need a definition that separates the utility of the database** (learning that smoking causes cancer) **from the increased risk of harm due to joining the database. This is the intuition behind differential privacy.**”

# Differential privacy: the formalism

We will define privacy with respect to a database  $\mathbf{D}$  that is made up of rows (equivalently, tuples) representing individuals. Tuples come from some universe of datatypes (the set of all possible tuples).

The  $\ell_1$  norm of a database  $\mathbf{D}$ , denoted  $\|\mathbf{D}\|_1$ , is the number of tuples in  $\mathbf{D}$ .

The  $\ell_1$  distance between databases  $\mathbf{D}_1$  and  $\mathbf{D}_2$  represents the number of tuples on which they differ.  $\|\mathbf{D}_1 - \mathbf{D}_2\|_1$

We refer to a pair of databases that differ in at most 1 tuple as  
**neighboring databases**  $\|\mathbf{D}_1 - \mathbf{D}_2\|_1 \leq 1$

Of these  $\mathbf{D}_1$  and  $\mathbf{D}_2$ , one, say  $\mathbf{D}_2$ , is a subset of the other, and, when a proper subset, the larger database  $\mathbf{D}_2$  contains 1 extra tuple.

# Differential privacy: the formalism

The information released about the sensitive data set is virtually indistinguishable **whether or not a respondent's data is in the dataset**. This is an informal statement of **differential privacy**. That is, no information **specific to an individual** is revealed.

A randomized algorithm  $M$  provides  **$\epsilon$ -differential privacy** if, for all neighboring databases  $D_1$  and  $D_2$ , and for any set of outputs  $S$ :

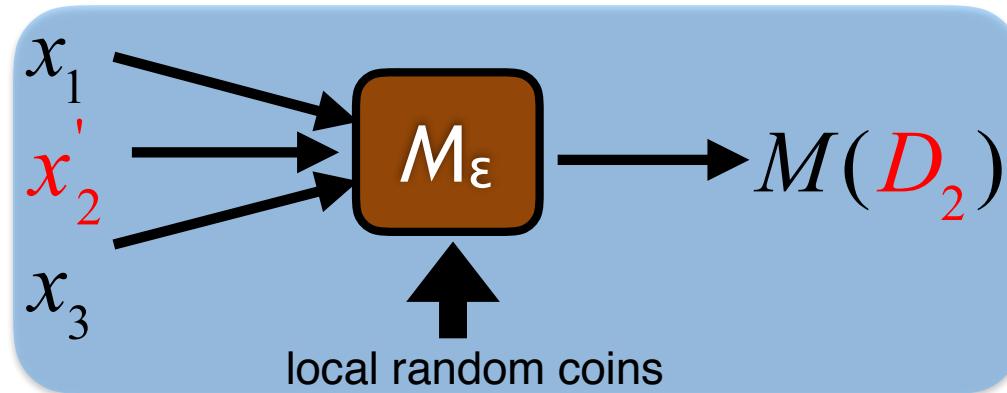
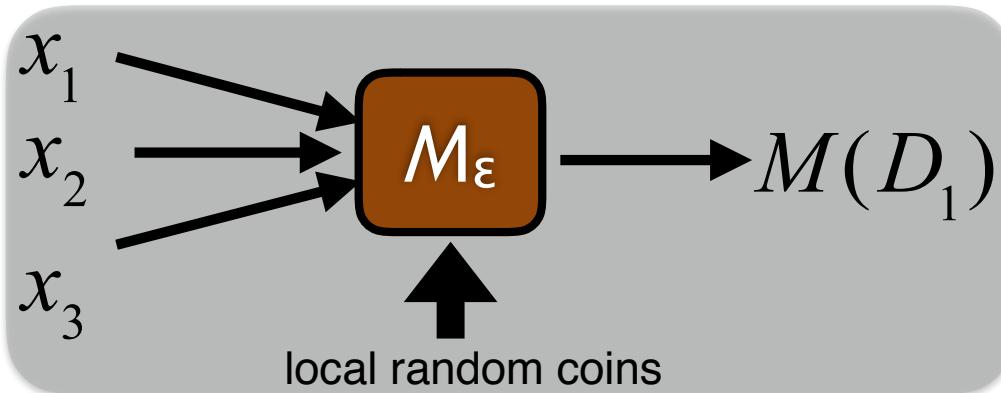
$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S]$$

**$\epsilon$  (epsilon) is a privacy parameter**

 **lower  $\epsilon$  = stronger privacy** 

The notion of **neighboring databases** is integral to plausible deniability:  $D_1$  can represent a database with a particular respondent's data,  $D_2$  can represent a neighboring database but without that respondent's data

# Differential privacy: neighboring databases



A randomized algorithm  $\mathbf{M}$  provides  **$\epsilon$ -differential privacy** if, for all neighboring databases  $\mathbf{D}_1$  and  $\mathbf{D}_2$ , and for any set of outputs  $\mathbf{S}$ :

$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S]$$

Think of database of respondents  $\mathbf{D} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$  as **fixed** (not random),  
 $\mathbf{M}(\mathbf{D})$  is a random variable distributed over possible outputs

**Neighboring databases** induce **close distributions** on outputs

based on a slide by Adam Smith

# Back to randomized response

**Did you go out drinking over the weekend?**

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Denote:

- Truth=Yes by **P**
- Response=Yes by **A**
- **C1**=tails by **T**
- **C1**=heads and **C2**=tails by **HT**
- **C1**=heads and **C2**=heads by **HH**

A randomized algorithm **M** provides  **$\epsilon$ -differential privacy** if, for all neighboring databases **D<sub>1</sub>** and **D<sub>2</sub>**, and for any set of outputs **S**:

$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S]$$

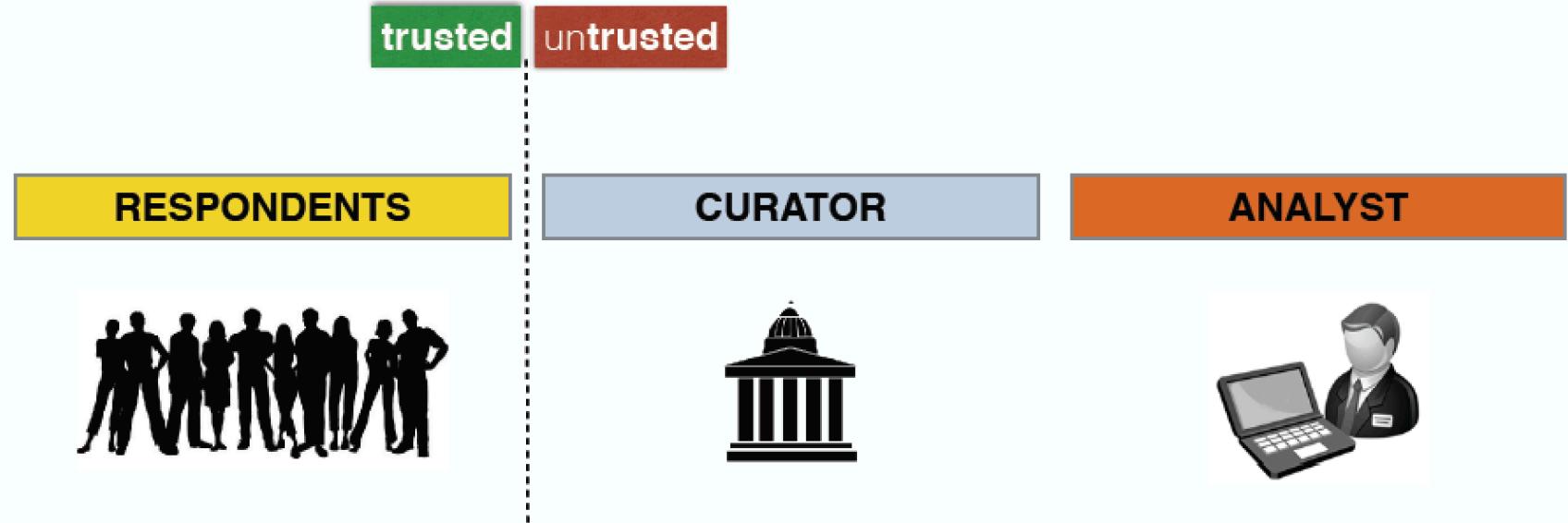
$$\Pr[A|P] = \Pr[T] + \Pr[HH] = \frac{3}{4}$$

$$\begin{aligned}\Pr[A | P] &= 3\Pr[A | \neg P] \\ \Rightarrow \epsilon &= \ln 3\end{aligned}$$

$$\Pr[A | \neg P] = \Pr[\cancel{H}T] = \frac{1}{4}$$
$$\Pr[H\cancel{H}]$$

our version of randomized response is  
( $\ln 3$ )-differentially private

# Local differential privacy



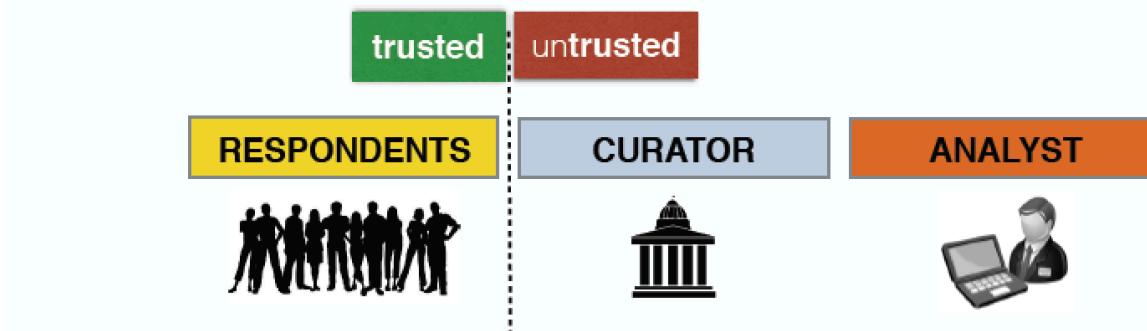
**respondents** contribute  
their personal data

the **curator** is **untrusted**,  
collects data, releases it to  
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slide by Jerome Miklau

# Differential privacy in the field



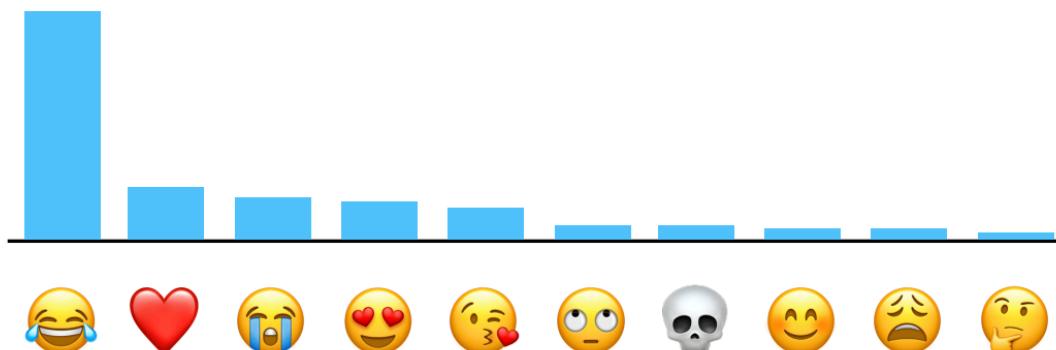
slide by Jerome Miklau

# Apple uses local differential privacy

**What's your favorite emoji?**

A privacy-preserving system

Apple has adopted and further developed a technique known in the academic world as *local differential privacy* to do something really exciting: gain insight into what many Apple users are doing, while helping to preserve the privacy of individual users. It is a technique that enables Apple to learn about the user community without learning about individuals in the community. Differential privacy transforms the information shared with Apple before it ever leaves the user's device such that Apple can never reproduce the true data.

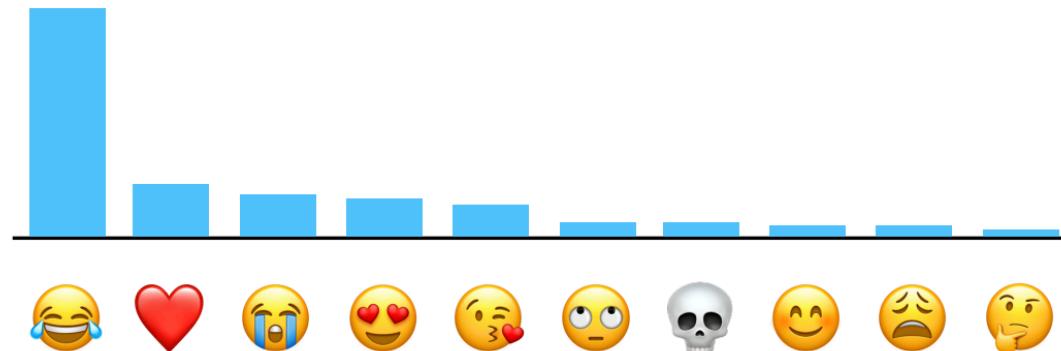


[https://www.apple.com/privacy/docs/Differential\\_Privacy\\_Overview.pdf](https://www.apple.com/privacy/docs/Differential_Privacy_Overview.pdf)

# Apple uses local differential privacy

Apple uses local differential privacy to help protect the privacy of user activity in a given time period, while still gaining insight that improves the intelligence and usability of such features as:

- QuickType suggestions
- Emoji suggestions
- Lookup Hints
- Safari Energy Draining Domains
- Safari Autoplay Intent Detection (macOS High Sierra)
- Safari Crashing Domains (iOS 11)
- Health Type Usage (iOS 10.2)

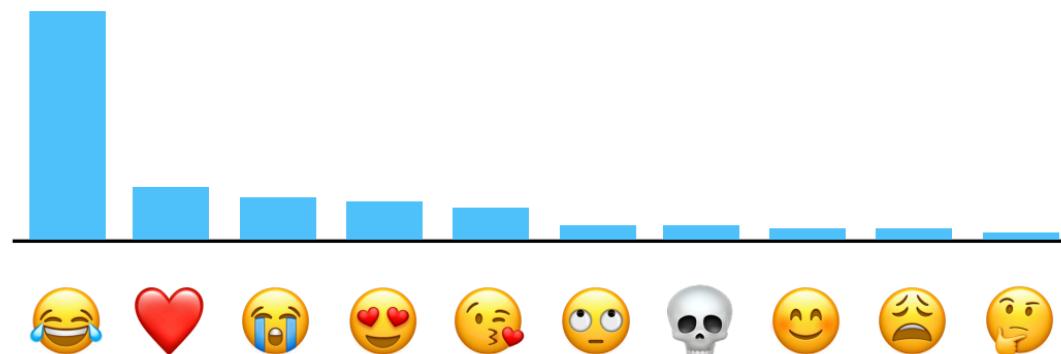


[https://www.apple.com/privacy/docs/Differential\\_Privacy\\_Overview.pdf](https://www.apple.com/privacy/docs/Differential_Privacy_Overview.pdf)

# Apple uses local differential privacy

## Privacy budget

The Apple differential privacy implementation incorporates the concept of a per-donation *privacy budget* (quantified by the parameter epsilon), and sets a strict limit on the number of contributions from a user in order to preserve their privacy. The reason is that the slightly-biased noise used in differential privacy tends to average out over a large numbers of contributions, making it theoretically possible to determine information about a user's activity over a large number of observations from a single user (though it's important to note that Apple doesn't associate any identifiers with information collected using differential privacy).



[https://www.apple.com/privacy/docs/Differential\\_Privacy\\_Overview.pdf](https://www.apple.com/privacy/docs/Differential_Privacy_Overview.pdf)

# Apple uses local differential privacy

## Count Mean Sketch

In our use of the Count Mean Sketch technique for differential privacy, the original information being processed for sharing with Apple is encoded using a series of mathematical functions known as *hash functions*, making it easy to represent data of varying sizes in a matrix of fixed size.

The data is encoded using variations of a SHA-256 hash followed by a privatization step and then written into the sketch matrix with its values initialized to zero.

The noise injection step works as follows: After encoding the input as a vector using a hash function, each coordinate of the vector is then flipped (written as an incorrect value) with a probability of  $1/(1 + e^{\varepsilon/2})$ , where  $\varepsilon$  is the privacy parameter. This assures that analysis of the collected data cannot distinguish actual values from flipped values, helping to assure the privacy of the shared information.

[https://www.apple.com/privacy/docs/Differential\\_Privacy\\_Overview.pdf](https://www.apple.com/privacy/docs/Differential_Privacy_Overview.pdf)

# Do we really need randomization?

- Data release approaches that fail to protect privacy (these are prominent classes of methods, there are others):
  - **aggregation** (e.g., **k-anonymity** - each record in the release is indistinguishable from at least  $k-1$  other records)
  - **sampling** (“just a few”) - release a small subset of the database
  - **query auditing** - stop answering queries when they become unsafe
  - **de-identification** - mask or drop personal identifiers

# Aggregation without randomization

- Alice and Bob are professors at State University. Both underwent “data safety” training.
- In March, Alice publishes an article: “.... the current freshman class at State University is made up of **3,005** students, **202** of whom are from families earning over \$1M per year.”
- In April, Bob publishes an article: “... **201** families in State University’s freshman class of **3,004** have household incomes exceeding \$1M per year.”
- Neither statement discloses the income of the family of any one student. But, taken together, they state that **John, a student who dropped out at the end of March**, comes from a family that earns \$1M. Anyone who has this **auxiliary information** will be able to learn about the income of John’s family.

this is known as a problem of **composition**, and can be seen as a kind of a **differencing attack**

- **A basic differencing attack:** (1) **X**: count the number of HIV-positive people in **D**; (2) **Y**: count the number of HIV-positive people in **D** not named *Freddie*; (3) **X - Y** tells you whether *Freddie* is HIV-positive

# Reconstruction: death by a 1000 cuts

- Another serious issue for aggregation without randomization, or with an insufficient amount of randomization: **reconstruction attacks**
- **The Fundamental Law of Information Recovery** (starting with the seminal results by Irit Dinur & Kobbi Nissim, PODS 2003): overly accurate estimates of too many statistics can completely destroy privacy
- Under what conditions can an adversary reconstruct a candidate database  $\mathbf{D}'$  that agrees with the real database  $\mathbf{D}$  in **99%** of the entries?
- Suppose that  $\mathbf{D}$  has  $n$  tuples, and that noise is bounded by some quantity  $E$ . Then there exists an adversary that can reconstruct  $\mathbf{D}$  to within  $4E$  positions, issuing all possible  $2^n$  queries

$$4E = \frac{4n}{401} < \frac{n}{100}$$

- Put another way: if the magnitude of the noise is less than  $n/401$ , then 99% of  $\mathbf{D}$  can be reconstructed by the adversary. Really, any number higher than 401 will work
- There are also reconstruction results under a limited number of queries

# Reconstruction: death by a 1000 cuts

## Privacy-Preserving Data Analysis for the Federal Statistical Agencies

January 2017

*John Abowd, Lorenzo Alvisi, Cynthia Dwork, Sampath Kannan, Ashwin Machanavajjhala, and Jerome Reiter*

we'll discuss the use  
of differential privacy  
by the 2020 US  
Census later today



The Fundamental Law of Information Recovery has troubling implications for the publication of large numbers of statistics by a statistical agency: it says that the confidential data may be vulnerable to database reconstruction attacks based entirely on the data published by the agency itself. **Left unattended, such risks threaten to undermine, or even eliminate, the societal benefits inherent in the rich data collected by the nation's statistical agencies.** The most pressing immediate problem for any statistical agency is how to modernize its disclosure limitation methods in light of the Fundamental Law.

# Sampling (“just a few”)

- Suppose that we take a random small sample  $\mathbf{D}'$  of  $\mathbf{D}$  and release it without any modification
- If  $\mathbf{D}'$  is much smaller than  $\mathbf{D}$ , then every respondent is unlikely to appear in  $\mathbf{D}'$
- This technique provides protection for “the typical” (or for “most”) members of the dataset
- It may be argued that atypical individuals are the ones needing stronger protection
- In any case, this method is problematic because a respondent who does appear has **no plausible deniability!**
- Suppose next that appearing in the sample  $\mathbf{D}'$  has terrible consequences. Then, every time subsampling occurs - some individual suffers horribly!

# Query auditing

- Monitor queries: each query is granted or denied depending on what other queries were answered in the past
- If this method were to work, it could be used to detect that a differencing attack is about to take place
- But:
  - **Query auditing is computationally infeasible**

[Kleinberg, Papadimitriou, Raghavan, *PODS 2000*]

- Refusal to respond to a query may itself be disclosive
- We refuse to execute a query, then what? No information access at all?

# Query auditing is infeasible

[Kleinberg, Papadimitriou, Raghavan, *PODS 2000*]

- We have a set of (secret) Boolean variables  $\mathbf{X}$  and the result of some *statistical queries* over this set
- A *statistical query*  $\mathbf{Q}$  specifies a subset  $\mathbf{S}$  of the variables in  $\mathbf{X}$ , and returns the sum of the values of all variables in  $\mathbf{S}$
- **The auditing problem:** Decide whether the value of any Boolean variable is determined by the results of the queries
- **Main result:** The Boolean auditing problem is coNP-complete
  - coNP-complete is the hardest class of problems in coNP: all coNP problems can be formulated as a special case of any coNP-complete problem
  - if P does not equal NP, then there does not exist a polynomial time algorithm that solves this problem

# De-identification

- Also known as **anonymization**
- Mask or drop identifying attribute or attributes, such as social security number (SSN), name, mailing address
- Turns out that this also doesn't work because **auxiliary information** is available
- Fundamentally, this is due to **the curse of dimensionality**: high-dimensional data is sparse, the more you know about individuals, the less likely it is that two individuals will look alike

**de-identified data can be re-identified with a linkage attack**

# A linkage attack: Governor Weld

In 1997, Massachusetts Group Insurance Commission released "anonymized" data on state employees that showed every single hospital visit!

Latanya Sweeney, a grad student, sought to show the ineffectiveness of this "anonymization."

She knew that Governor Weld resided in Cambridge, Massachusetts, a city of 54,000 residents and seven ZIP codes.

Only six people in Cambridge shared his birth date, only three of them men, and of them, only he lived in his ZIP code.

For twenty dollars, she purchased the complete voter rolls from the city of Cambridge, a database containing, among other things, the name, address, ZIP code, birth date, and sex of every voter.

*Follow up: ZIP code, birthdate, and sex sufficient to identify 87% of Americans!*

<https://arstechnica.com/tech-policy/2009/09/your-secrets-live-online-in-databases-of-ruin/>

slide by Bill Howe

# The Netflix prize linkage attack

[Narayanan and Shmatikov, *IEEE S&P 2008*]

- In 2006, Netflix released a dataset containing ~100M **movie ratings** by ~500K users (about 1/8 of the Netflix user base at the time)
- **FAQ:** “Is there any customer information in the dataset that should be kept private?”

*“No, all customer identifying information has been removed; all that remains are ratings and dates. This follows our privacy policy, which you can review here. Even if, for example, you knew all your own ratings and their dates you probably couldn’t identify them reliably in the data because only a small sample was included (less than one-tenth of our complete dataset) and that data was subject to perturbation. Of course, since you know all your own ratings that really isn’t a privacy problem is it?”*

**The real question:** How much does the adversary need to know about a Netflix subscriber to identify her record in the dataset, and thus learn her complete movie viewing history?

# The Netflix prize linkage attack

[Narayanan and Shmatikov, *IEEE S&P 2008*]

- Very little auxiliary information is needed to de-anonymize an average subscriber record from the Netflix Prize dataset
- **Perturbation, you say?** With 8 movie ratings (of which 2 may be completely wrong) and dates that may have a 14-day error, 99% of records be uniquely identified in the dataset
- For 68%, two ratings and dates (with a 3-day error) are sufficient
- **Even without any dates, a substantial privacy breach occurs, especially when the auxiliary information consists of movies that are not blockbusters:** Two movies are no longer sufficient, but 84% of subscribers can be uniquely identified if the adversary knows 6 out of 8 moves outside the top 500

**We cannot assume a priori that any data is harmless!**

# The Netflix prize linkage attack

WIRED

An in-the-closet lesbian mother is suing Netflix for privacy invasion, alleging the movie rental company made it possible for her to be outed when it disclosed insufficiently anonymous information about nearly half-a-million customers as part of its \$1 million contest to improve its recommendation system.

The suit known as [Doe v. Netflix \(.pdf\)](#) was filed in federal court in California on Thursday, alleging that Netflix violated fair-trade laws and a federal privacy law protecting video rental records, when it launched its popular contest in September 2006.

The suit seeks more than \$2,500 in damages for each of more than 2 million Netflix customers.

RYAN SINGEL SECURITY 12.17.09 04:29 PM

## NETFLIX SPILLED YOUR BROKEBACK MOUNTAIN SECRET, LAWSUIT CLAIMS



# The Netflix prize linkage attack

WIRED

RYAN SINGEL SECURITY 03.12.10 02:48 PM

## NETFLIX CANCELS RECOMMENDATION CONTEST AFTER PRIVACY LAWSUIT



Netflix is canceling its second \$1 million Netflix Prize to settle a legal challenge that it breached customer privacy as part of the first contest's race for a better movie-recommendation engine.

# A closer look at differential privacy

A randomized algorithm  $M$  provides  **$\epsilon$ -differential privacy** if, for all neighboring databases  $D_1$  and  $D_2$ , and for any set of outputs  $S$ :

$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S]$$

**$\epsilon$  (epsilon) is a privacy parameter**

**lower  $\epsilon$  means stronger privacy**



- The state-of-the-art in privacy technology, first proposed in 2006
- Has precise mathematical properties, captures cumulative privacy loss over multiple uses with the concept of a **privacy budget**
- Privacy guarantee encourages participation by respondents
- Robust against strong adversaries, with auxiliary information, including also **future auxiliary information!**
- Precise error bounds that can be made public

# A closer look at differential privacy

A randomized algorithm  $M$  provides  **$\epsilon$ -differential privacy** if, for all neighboring databases  $D_1$  and  $D_2$ , and for any set of outputs  $S$ :

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**$\epsilon$  (epsilon) is a privacy parameter**

 **lower  $\epsilon$  means stronger privacy** 

**$\epsilon$  (epsilon)** cannot be too small: think 1/10, not 1/2<sup>50</sup>

Differential privacy is a condition on the **algorithm M** (process privacy),  
Saying “the output is safe” doesn’t take into account how it was computed,  
is insufficient

# Query sensitivity

The  $\ell_1$  sensitivity of a query  $q$ , denoted  $\Delta q$ , is the maximum difference in the result of that query on a pair of neighboring databases

$$\Delta q = \max_{D,D'} |q(D) - q(D')|$$

- Example 1: counting queries
  - “How many elements in  $D$  satisfy property  $P$ ? ” **What’s  $\Delta q$ ?**
  - “What fraction of the elements in  $D$  satisfy property  $P$ ? ”
- Example 2: max / min
  - “What is the maximum employee salary in  $D$ ? ” **What’s  $\Delta q$ ?**

**Intuition: for a given  $\epsilon$ , the higher the sensitivity, the more noise we need to add to meet the privacy guarantee**

# Query sensitivity

The sensitivity of a query  $q$ , denoted  $\Delta q$ , is the maximum difference in the result of that query on a pair of **neighboring databases**

$$\Delta q = \max_{D,D'} |q(D) - q(D')|$$

query $q$	query sensitivity $\Delta q$
-----------	------------------------------

select count(\*) from D

1

select count(\*) from D  
where sex = Male and age > 30

?

# Query sensitivity

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query $q$	query sensitivity $\Delta q$
select count(*) from D	1
select count(*) from D where sex = Male and age > 30	1
select MAX(salary) from D	?

# Query sensitivity

The sensitivity of a query  $q$ , denoted  $\Delta q$ , is the maximum difference in the result of that query on a pair of neighboring databases

$$\Delta q = \max_{D,D'} |q(D) - q(D')|$$

query $q$	query sensitivity $\Delta q$
select count(*) from D	1
select count(*) from D where sex = Male and age > 30	1
select MAX(salary) from D	$MAX(salary)-MIN(salary)$
select gender, count(*) from D group by gender	?

# Query sensitivity

The sensitivity of a query  $q$ , denoted  $\Delta q$ , is the maximum difference in the result of that query on a pair of neighboring databases

$$\Delta q = \max_{D,D'} |q(D) - q(D')|$$

**query  $q$**

select gender, count(\*)  
from D group by gender

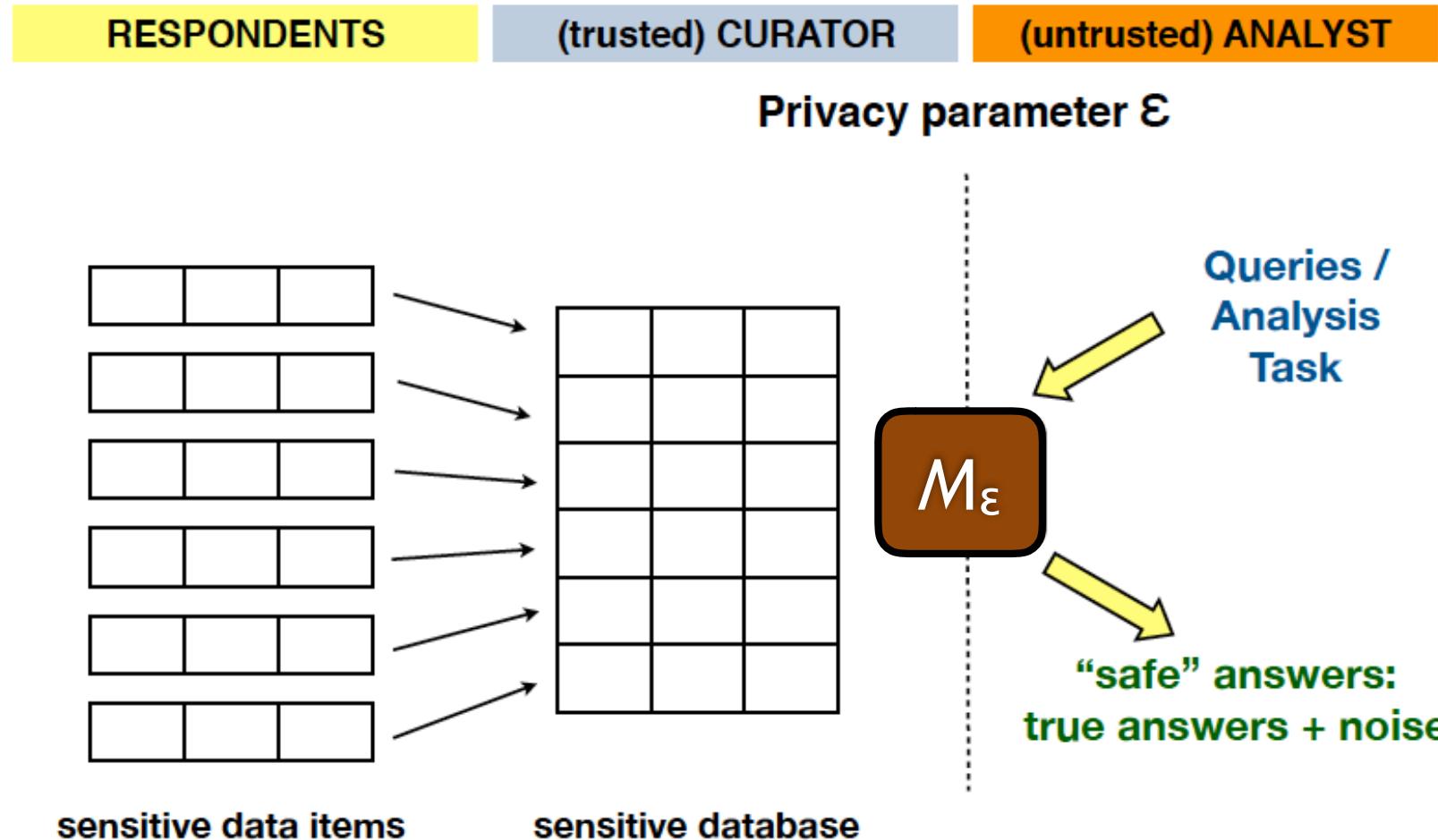
**query sensitivity  $\Delta q$**

1 (disjoint groups, presence or absence  
of one tuple impacts only one of the  
counts)

an arbitrary list of  $m$  counting  
queries

?

# Adding noise



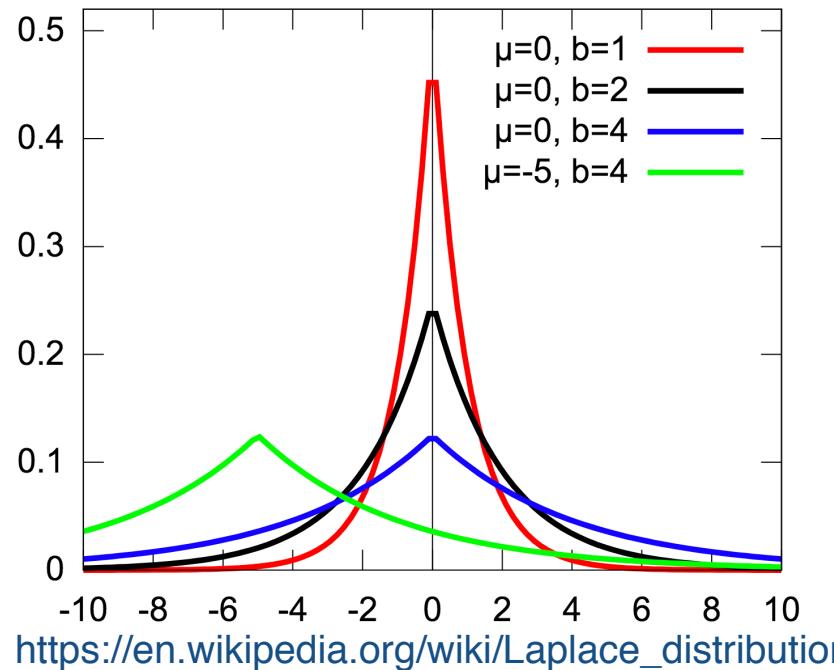
slide by Jerome Miklau

# Adding noise

Use the **Laplace mechanism** to answer  $\mathbf{q}$  in a way that's  $\epsilon$ -differentially private

$$M(\epsilon) : q(D) + \text{Lap}\left(\frac{\Delta q}{\epsilon}\right)$$

The Laplace distribution, centered at 0 with scale  $\mathbf{b}$ , denoted **Lap( $\mathbf{b}$ )**, is the distribution with probability density function:



fix sensitivity  $\Delta q$ , verify that more noise is added for lower  $\epsilon$

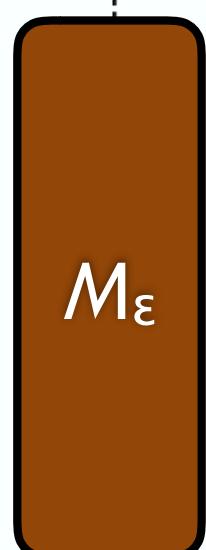
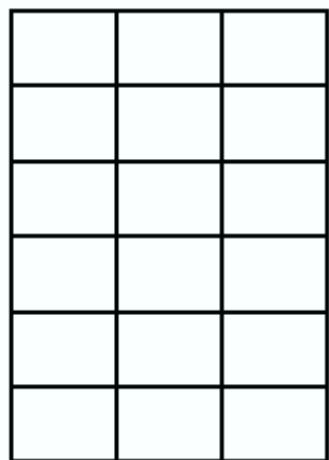
↓ lower  $\epsilon$  = stronger privacy ↑

# Adding noise

(trusted) CURATOR

(untrusted) ANALYST

$\epsilon = 1$



$\epsilon = 1$

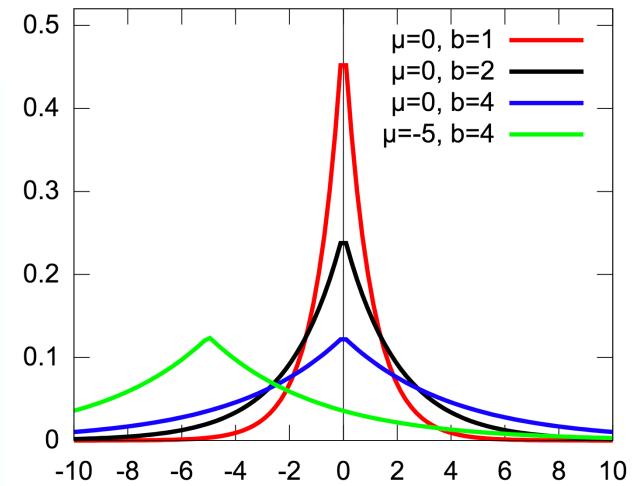
Count(sex=Male, age=18)

true answer + noise(-3,3)

sensitive database

Laplace noise centered at 0,  
in interval (-3,3) with 95% prob.

slide by Jerome Miklau



# Query sensitivity

The sensitivity of a query  $q$ , denoted  $\Delta q$ , is the maximum difference in the result of that query on a pair of neighboring databases

$$\Delta q = \max_{D,D'} |q(D) - q(D')|$$

query $q$	query sensitivity $\Delta q$
select gender, count(*) from D group by gender	1 (disjoint groups, presence or absence of one tuple impacts only one of the counts)
an arbitrary list of $m$ counting queries	$m$ (no assumptions about the queries, and so a single individual may change the answer of every query by 1)

# Composition

If algorithms M1 and M2 are  $\epsilon$ -differentially private, then outputting results of both algorithms is  $2\epsilon$ -differentially private

query $q$	query sensitivity $\Delta q$
select gender, count(*) from D group by gender	<b>parallel composition</b> 1 (disjoint groups, presence or absence of one tuple impacts only one of the counts)
an arbitrary list of $m$ counting queries	<b>sequential composition</b> $m$ (no assumptions about the queries, and so a single individual may change the answer of every query by 1)

# Sequential composition

- Consider 4 queries executed in sequence
  - Q1: select count(\*) from D under  $\epsilon_1 = 0.5$
  - Q2: select count(\*) from D where sex = Male under  $\epsilon_2 = 0.2$
  - Q3: select count(\*) from D where sex = Female under  $\epsilon_3 = 0.25$
  - Q4: select count(\*) from D where age > 20 under  $\epsilon_4 = 0.25$
- $\epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 = 1.2$  That is: all queries together are  $\epsilon$ -differentially private for  $\epsilon = 1.25$ . **Can we make a stronger guarantee?**
- This works because **Laplace noise is additive**

More generally: set a **cumulative privacy budget**, and split it between all queries, pre-processing, other data manipulation steps of the pipeline

# Parallel composition

- If the inputs are disjoint, then the result is  $\epsilon$ -differentially private for  $\epsilon = \max(\epsilon_1, \dots, \epsilon_k)$ 
  - Q1: select count(\*) from D under  **$\epsilon_1 = 0.5$**
  - Q2: select count(\*) from D where sex = Male under  **$\epsilon_2 = 0.2$**
  - Q3: select count(\*) from D where sex = Female under  **$\epsilon_3 = 0.25$**
  - Q4: select count(\*) from D where age > 20 under  **$\epsilon_4 = 0.25$**
- **$\epsilon = \epsilon_1 + \max(\epsilon_2, \epsilon_3) + \epsilon_4 = 1$**  That is: all queries together are  $\epsilon$ -differentially private for  $\epsilon = 1$ .

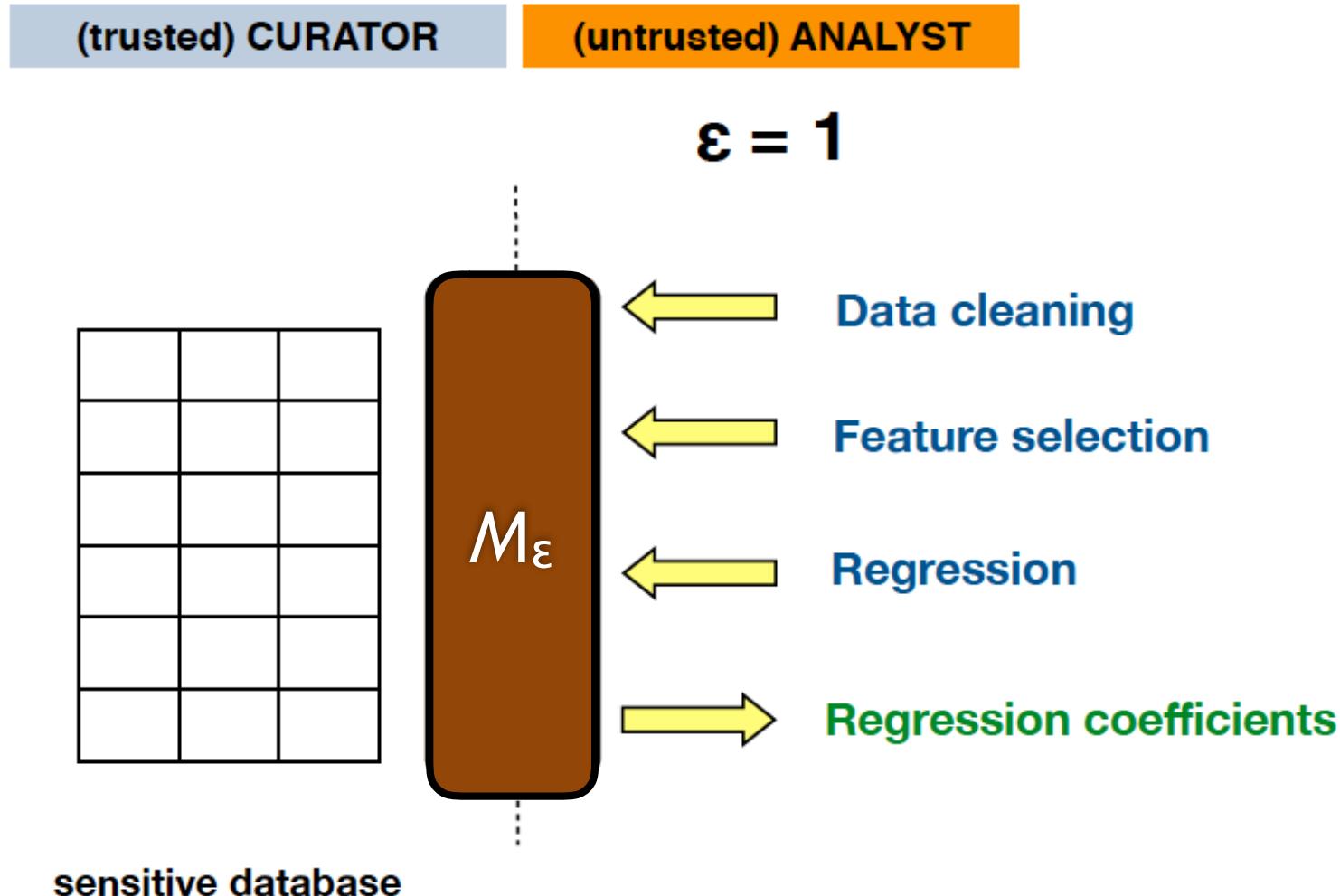
# Composition and consistency

- Consider again 4 queries executed in sequence
  - Q1: select count(\*) from D under  $\varepsilon_1 = 0.5$  returns **2005**
  - Q2: select count(\*) from D where sex = Male under  $\varepsilon_2 = 0.2$  returns **1001**
  - Q3: select count(\*) from D where sex = Female under  $\varepsilon_3 = 0.25$  returns **995**
  - Q4: select count(\*) from D where age > 20 under  $\varepsilon_4 = 0.25$  returns **1789**

Assuming that there are 2 genders in D, Male and Female, there is **no database consistent with these statistics!**

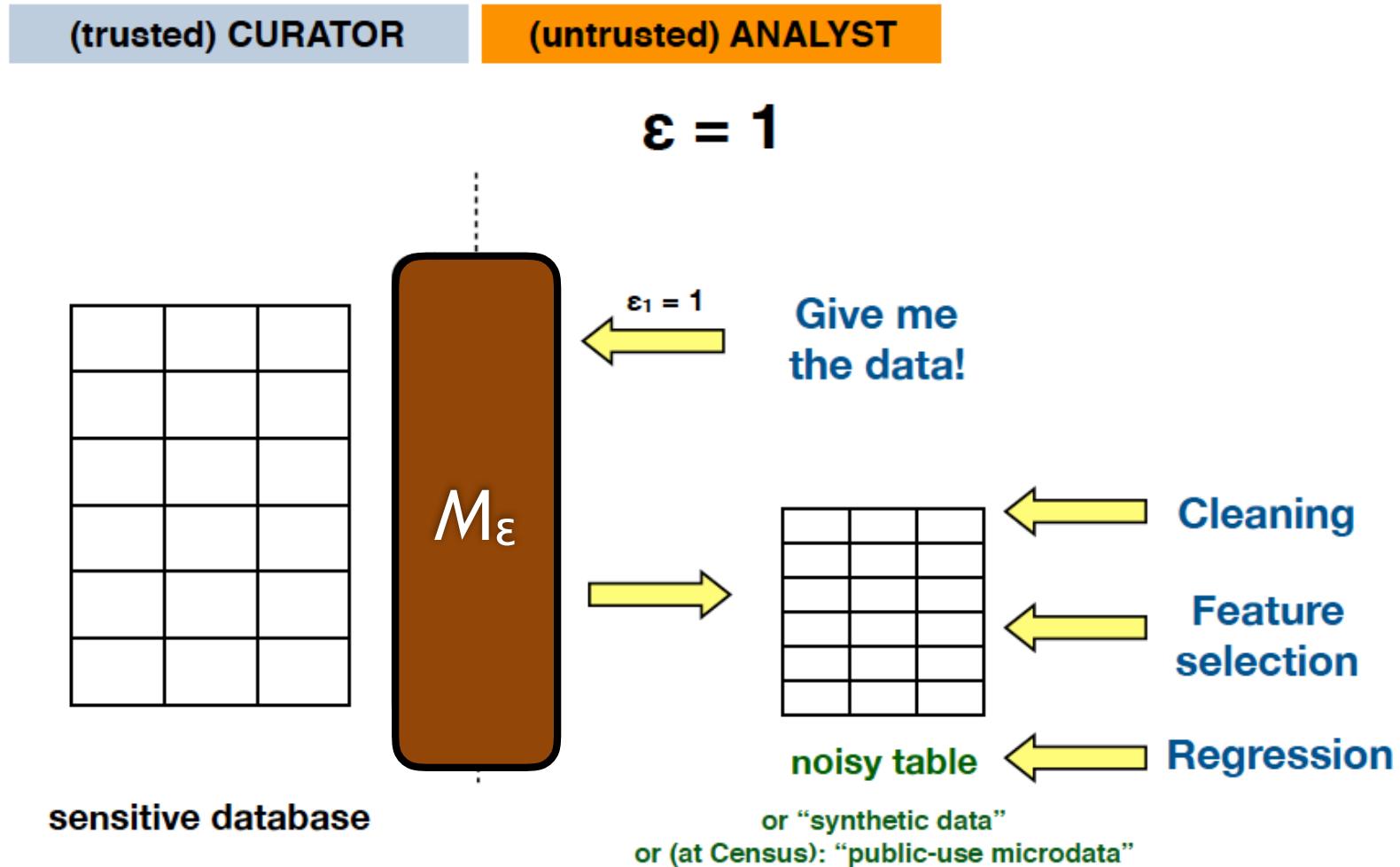
Also don't want any negative counts + may want to impose datatype checks, e.g., no working adults with age = 5 etc.

# Entire workflow must be DP



slide by Jerome Miklau

# Privacy-preserving synthetic data



slide by Jerome Miklau

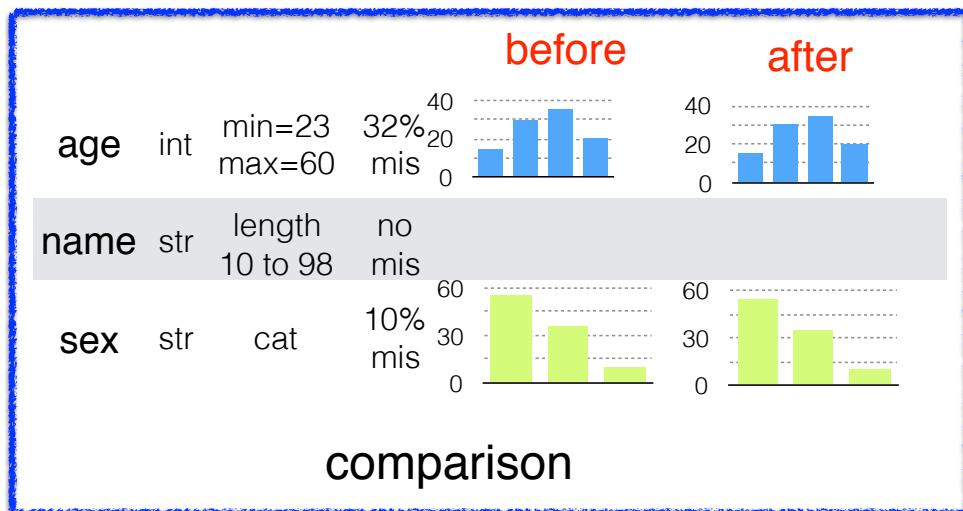
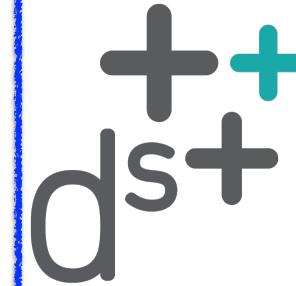
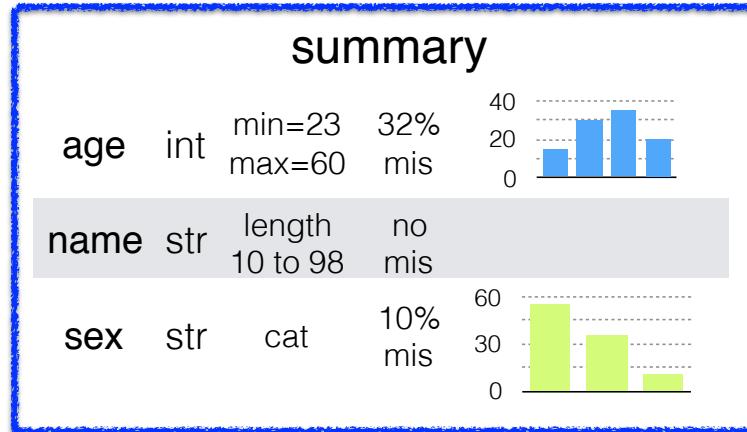
# An synthetic data tool: Data Synthesizer

[Ping, Stoyanovich, Howe 2017]

<http://demo.dataresponsibly.com/synthesizer/>

UID	sex	race	MarriageStatus	DateOfBirth	age	juv_fel	court	decile	score
2	1	0	2	1	4/18/47	69	0	1	1
3	2	0	2	1	1/22/82	34	0	3	3
4	3	0	2	1	1/24/82	24	0	4	4
5	4	0	2	1	1/21/93	23	0	8	8
6	5	0	1	2	1/22/73	43	0	1	1
7	6	0	1	3	8/24/71	44	0	1	10
8	7	0	1	3	1/31/73	45	0	1	6
9	8	0	1	2	2/25/73	43	0	4	4
10	9	0	3	1	6/10/94	21	0	3	4
11	10	0	3	1	6/10/94	27	0	1	4
12	11	1	3	2	8/22/78	37	0	1	1
13	12	0	2	1	1/2/74	41	0	4	4
14	13	1	3	1	4/24/85	47	0	3	3
15	14	0	2	1	3/25/85	31	0	1	1
16	15	0	4	1	2/25/79	37	0	1	1
17	16	0	2	1	6/27/84	25	0	10	10
18	17	0	3	1	12/24/84	33	0	5	5
19	18	0	3	1	8/8/85	31	0	3	3
20	19	0	2	3	6/28/51	64	0	6	6
21	20	0	2	1	11/28/82	21	0	9	9
22	21	0	3	1	8/6/88	27	0	2	2
23	22	1	3	1	3/22/95	21	0	4	4
24	23	0	4	1	1/1/01	24	0	4	4
25	24	0	3	1	1/10/73	43	0	1	1
26	25	0	1	1	8/24/83	32	0	3	3
27	26	0	2	1	2/8/89	27	0	3	3
28	27	1	3	1	3/21/89	36	0	3	3
29	28	1	3	1	9/27/89	36	0	3	3

Data  
Describer



Data  
Generator

Model  
Inspector

UID	A	B	C	D	E	F	G	H	I	J
2	1	0	1	1	4/18/47	69	0	1		
3	2	1	2	1	1/2/82	34	0	3		
4	3	0	2	1	1/24/82	24	0	4		
5	4	0	2	1	1/21/93	23	0	8		
6	5	0	1	2	1/22/73	43	0	1		
7	6	0	1	3	8/24/71	44	0	1		
8	7	0	3	1	1/31/73	45	0	6		
9	8	0	1	2	2/25/73	43	0	4		
10	9	0	2	3	6/10/94	21	0	3		
11	10	0	3	1	8/6/88	27	0	2		
12	11	1	3	2	8/22/78	37	0	1		
13	12	0	2	1	1/2/74	41	0	4		
14	13	1	3	1	4/24/85	47	0	3		
15	14	0	2	1	3/25/85	31	0	1		
16	15	0	4	1	2/25/79	37	0	1		
17	16	0	2	1	6/27/84	25	0	10		
18	17	0	3	1	12/24/84	33	0	5		
19	18	0	3	1	8/10/85	31	0	3		
20	19	0	2	1	1/3/89	44	0	9		
21	20	0	2	1	13/28/94	21	0	2		
22	21	0	3	1	8/6/88	27	0	2		
23	22	1	3	1	3/22/95	21	0	4		
24	23	0	4	1	1/1/01	24	0	4		
25	24	0	3	1	1/10/73	43	0	1		
26	25	0	2	1	8/24/83	32	0	3		
27	26	0	2	1	2/8/89	27	0	3		
28	27	1	3	1	9/3/79	36	0	3		

output

# Privacy-preserving synthetic data, generally

## Lots of advantages

- Consistency is not an issue
- Analysts can treat synthetic data as a regular dataset, run existing tools
- No need to worry about the privacy budget
- Can answer as many queries as they want, and any kind of a query they want, including record-level queries

## What's the catch?

**Recall the Fundamental Law of Information Recovery. It tells us that we cannot answer all these queries accurately and still preserve privacy!**

**Therefore, when releasing synthetic data, we need to document it with which queries it supports well**

# Data Synthesizer

[Ping, Stoyanovich, Howe 2017]

<http://demo.dataresponsibly.com/synthesizer/>

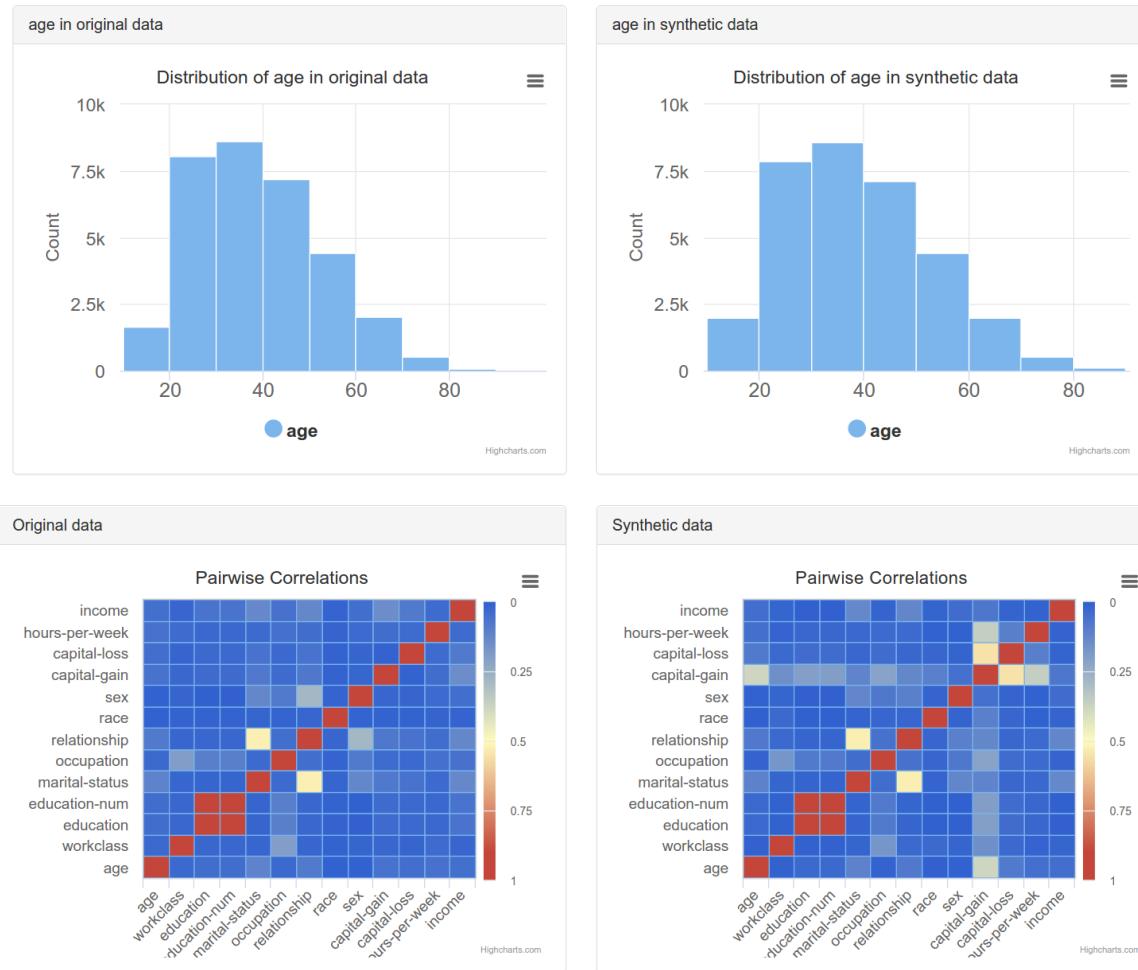
- Main goal: **usability first**
  - user is the data owner
  - the tool picks up data types from the input file: categorical / string / numerical (integer, float) / date-time
  - the tool computes the frequency of missing values per attribute
  - user can then inspect the result, over-ride what was learned about an attribute, e.g., whether it's categorical, or what its datatype is
- The tool generates an output dataset of a specified size, in one of three modes
  - **random** - type-consistent random output
  - **independent attribute** - learn a noisy histogram for each attribute
  - **correlated attribute** - learn a noisy Bayesian network (BN)



# Data Synthesizer

[Ping, Stoyanovich, Howe 2017]

<http://demo.dataresponsibly.com/synthesizer/>



# Data Synthesizer: independent attributes

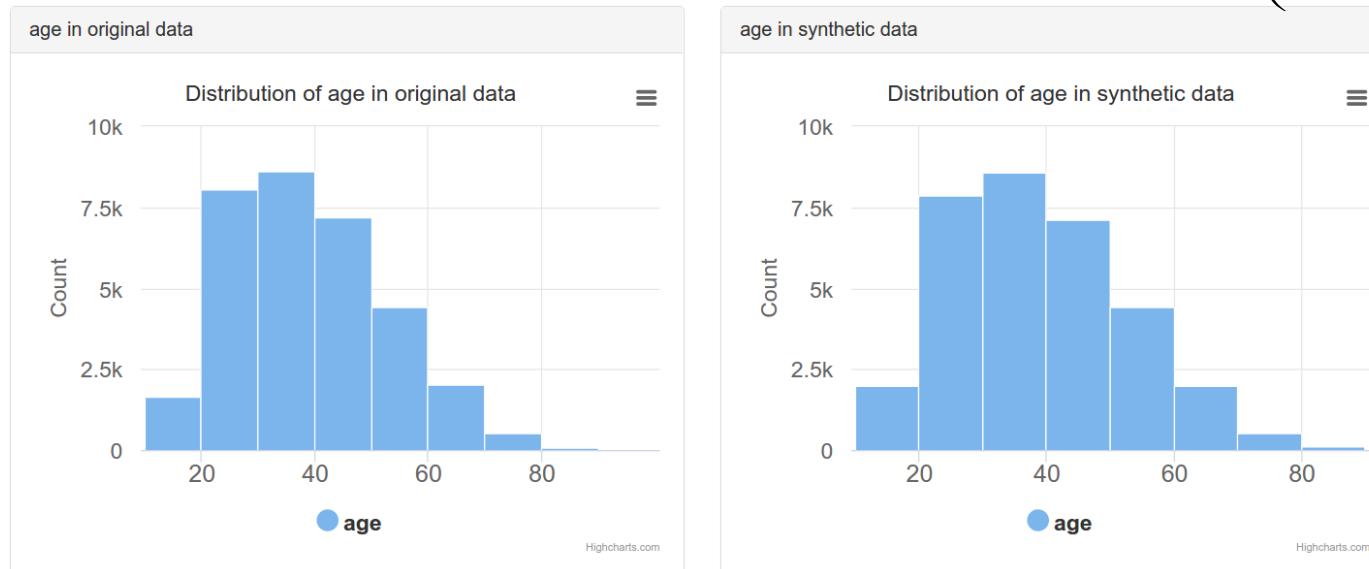
[Ping, Stoyanovich, Howe 2017]

<http://demo.dataresponsibly.com/synthesizer/>

Given the over-all privacy budget  $\epsilon$ , and an input dataset of size  $n$ .

Allocate  $\epsilon/d$  of the budget to each attribute  $A_i$  in  $\{A_1, \dots, A_d\}$ . Then for each attribute:

- Compute the  $i^{th}$  histogram with  $t$  bins ( $t=20$  by default), with query  $q_i$
- The sensitivity  $\Delta q_i$  of this (or any other) histogram query is  $2/n$  **Why?**
- So, each bin's noisy probability is computed by adding  $Lap\left(\frac{2d}{\epsilon n}\right)$



# Data Synthesizer: correlated attributes

[Ping, Stoyanovich, Howe 2017]

<http://demo.dataresponsibly.com/synthesizer/>

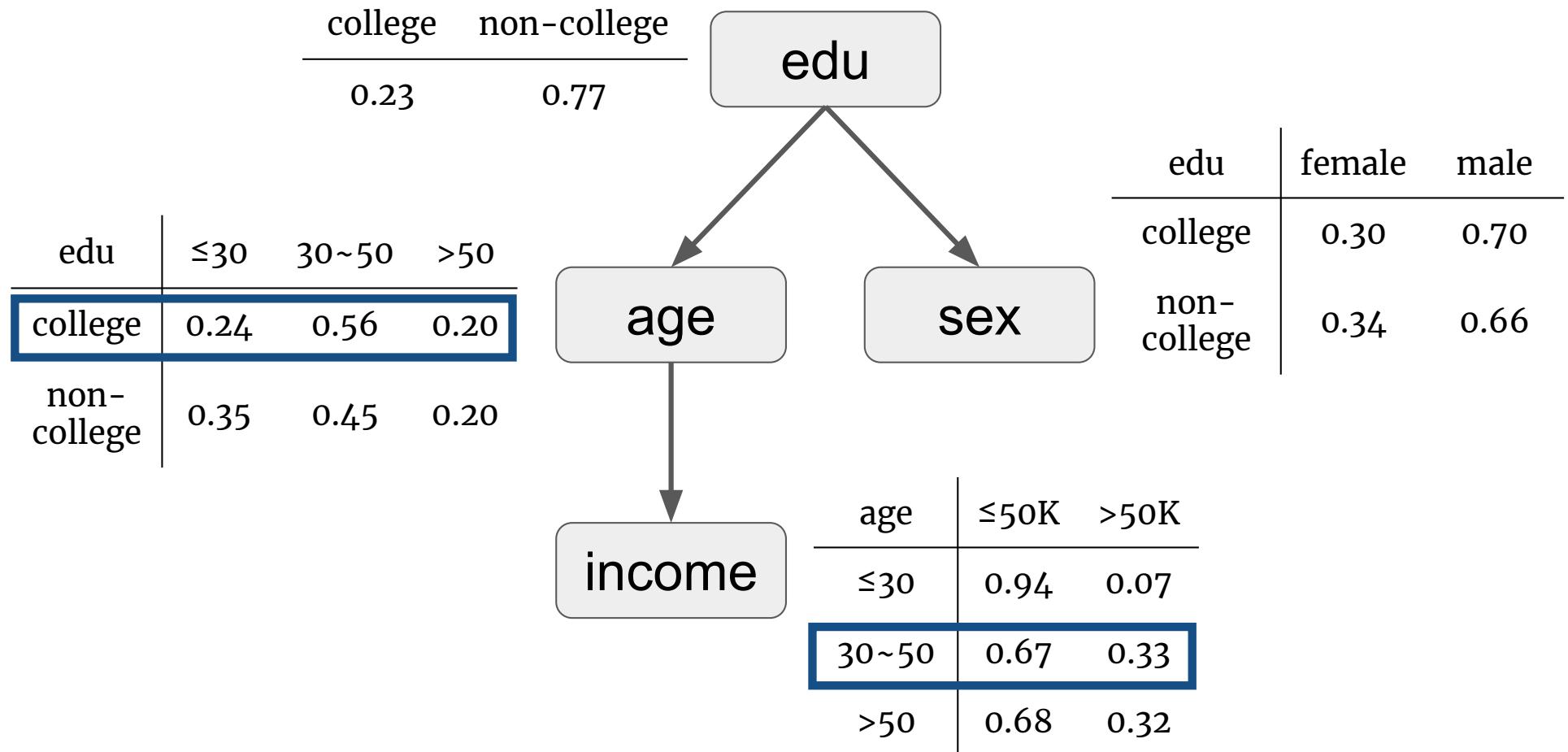


- Learn a differentially private Bayesian network (BN)
- Use the method called **PrivBayes** [Zhang, Cormode, Procopiuc, Srivastava, Xiao, 2016]
- Privacy budget is split equally between (a) network structure computation and (b) populating the conditional probability tables of each BN node
- User inputs privacy budget  $\epsilon$  and the maximum number of parents for a BN node  $k$  - you'll play with these settings as part of HW2
- The tool treats a missing attribute value as one of the values in the attribute's domain (not shown in the examples in the next two slides)

# Data Synthesizer: correlated attributes

K=1

note that this is not a causal, BN!

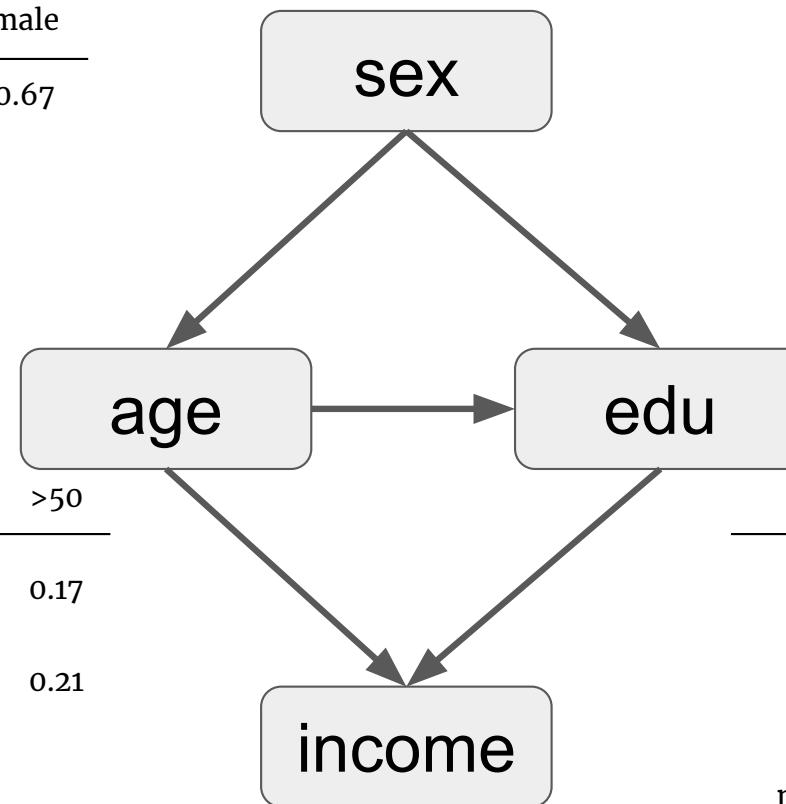


# Data Synthesizer: correlated attributes

K=2

note that this is not a causal, BN!

	female	male	
female	0.33	0.67	
sex	≤30	30~50	>50
female	0.40	0.43	0.17
male	0.29	0.59	0.21



age	sex	college	non-college
≤30	female	0.18	0.82
≤30	male	0.16	0.84
30~50	female	0.25	0.75
30~50	male	0.28	0.72
>50	female	0.17	0.83
>50	male	0.25	0.75

edu	age	≤50K	>50K
college	≤30	0.83	0.17
college	30~50	0.45	0.55
college	>50	0.41	0.59
non-college	≤30	0.96	0.04
non-college	30~50	0.76	0.24
non-college	>50	0.75	0.25

# Data Synthesizer



MAGAZINE EVENTS PAPERS TOPICS GOVTECH BIZ NAVIGATOR

## MetroLab “Innovation of the Month”

SECURITY

## University Researchers Use 'Fake' Data for Social Good

Virtually every interaction we have with a public agency creates a data point. Amass enough data points and they can tell a story. However, factors like privacy, data storage and usability present challenges for local governments and researchers interested in helping improve services. In this installment of MetroLab's Innovation of the Month series, we highlight how researchers at **Data Responsibly** are addressing those challenges by creating synthetic data sets for social good.

BY BEN LEVINE / NOVEMBER 7, 2017

**Howe:** Since its development, the tool has been receiving a lot of attention. For example: T-Mobile is interested in generating synthetic data to better engage with researchers and improve transparency for customers, the Colorado Department of Education has asked relevant agencies to use the tool to experiment with sharing sensitive data, and Elsevier is interested in using the tool to generate synthetic citation networks for research.

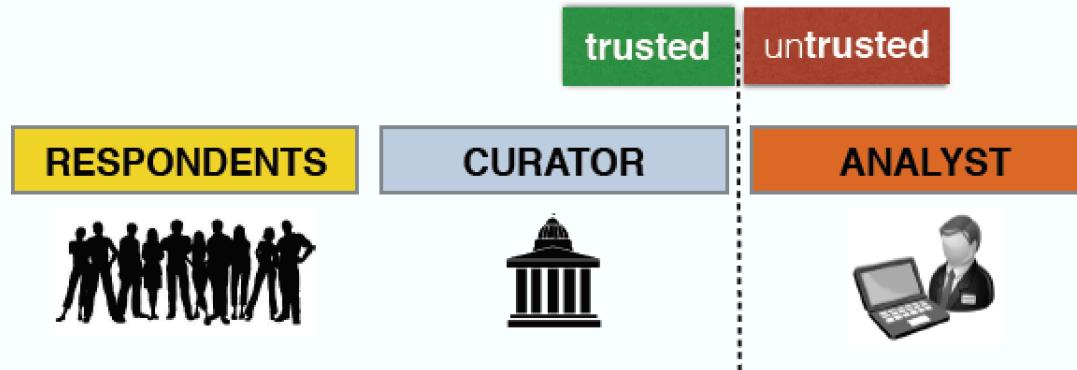
<http://www.govtech.com/security/University-Researchers-Use-Fake-Data-for-Social-Good.html>

Julia Stoyanovich

# Differential privacy in the field



United States<sup>TM</sup>  
**Census**  
Bureau



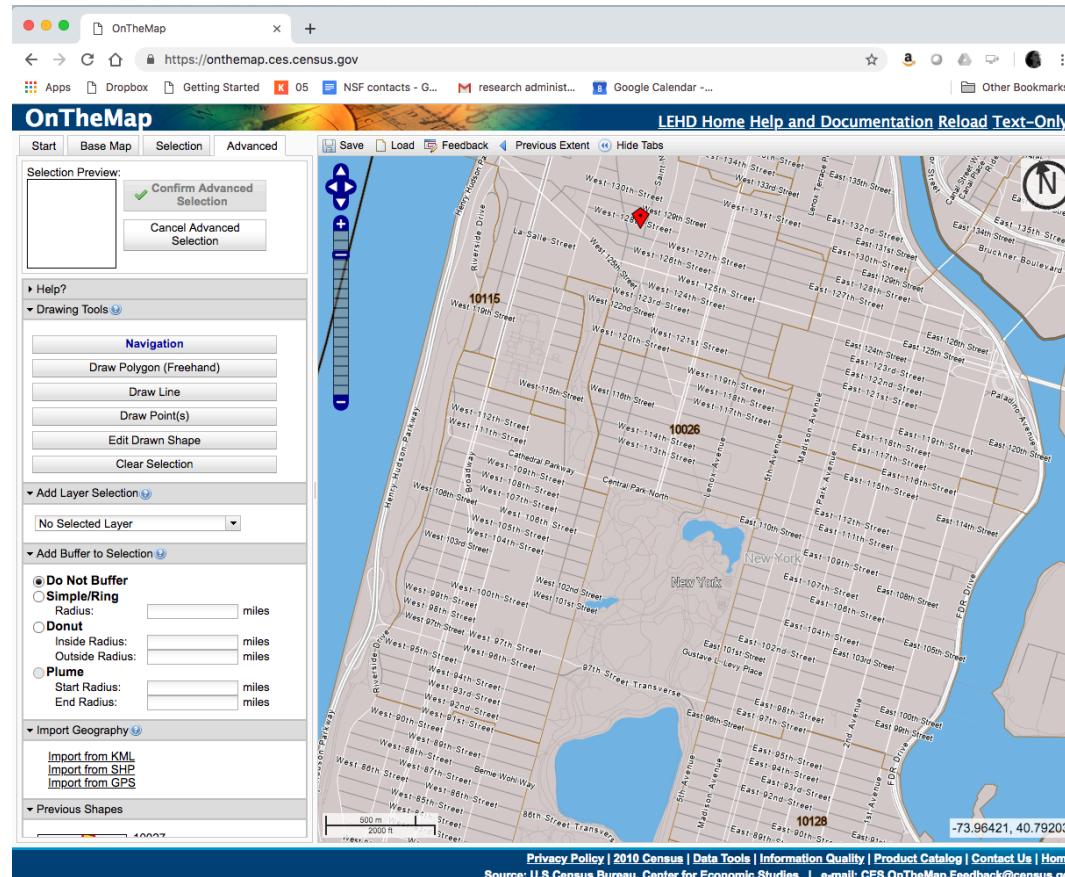
**current goals: Decennial Census 2020**

slide by Jerome Miklau

# Differential privacy in the field

First adoption by the US Census Bureau:

**OnTheMap** (2008), synthetic data about where people in the US live and work



# 2020 US Census to use differential privacy

TheUpshot

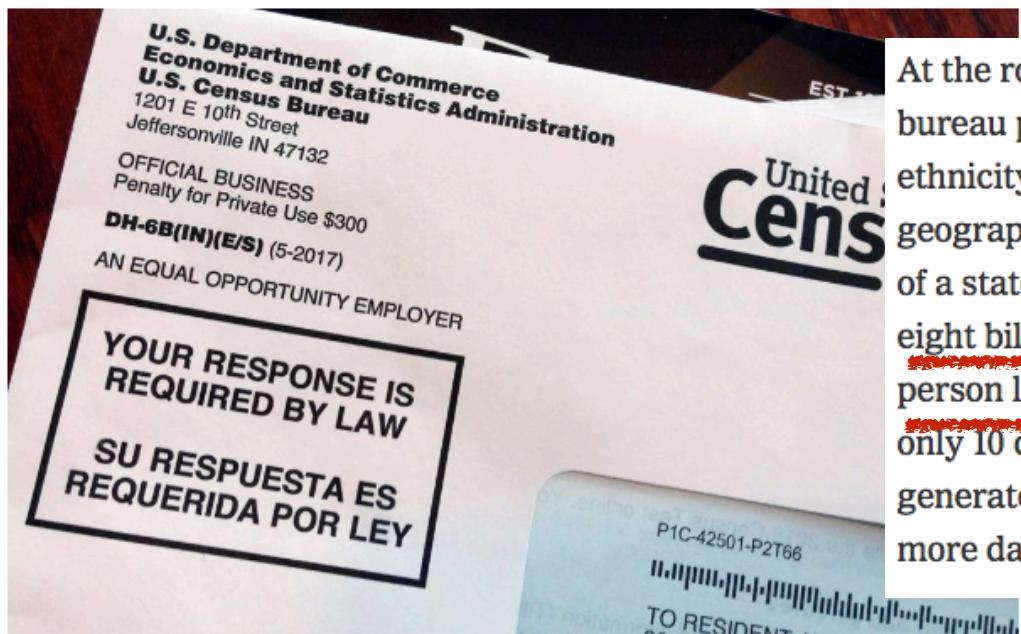
## To Reduce Privacy Risks, the Census Plans to Report Less Accurate Data

Guaranteeing people's confidentiality has become more of a challenge, but some scholars worry that the new system will impede research.

The New York Times

By Mark Hansen

Dec. 5, 2018



At the root of the problem are the tables of aggregate statistics that the bureau publishes. There are hundreds of tables — sex by age, say, or ethnicity by race — summarizing the population at several levels of geography, from areas the size of a city block all the way up to the level of a state or the nation. In 2010, the bureau released tables with nearly eight billion numbers in all. That was about 25 numbers for each person living in the United States, even though Americans were asked only 10 questions about themselves. In other words, the tables were generated in so many ways that the Census Bureau ended up releasing more data in aggregate than it had collected in the first place.

A 2018 census test letter mailed to a resident in Providence, R.I. The nation's test run of the 2020 Census is in Rhode Island. Michelle R. Smith/Associated Press

# Differential privacy in 2020 Census: pushback



## Implications of Differential Privacy for Census Bureau Data and Research

Task Force on Differential Privacy for Census Data †  
Institute for Social Research and Data Innovation (ISRDI)  
University of Minnesota

November 2018  
Version 2  
Working Paper No. 2018-6

- noisy data - **impact on critical decisions**
- difficult to explain differential privacy / privacy budget to the public - **how do we set epsilon?**
- disagreement about whether using differential privacy is legally required
- messaging is difficult to get right “the result doesn’t change whether or not you participate” - might discourage participation!

Revealing **characteristics** of individuals vs. their **identity**, is there a distinction?

But the Census collects “generic” **harmless data**, is this really a big deal?

**What sorts of trade-offs should we be aware of? Who should decide?**

# What does the law say?

**Title 13 of U.S. Code** authorizes data collection and publication of statistics by the Census Bureau.

**Section 9 of Title 13** requires privacy protections: “Neither the Secretary, nor any other officer or employee of the Department of Commerce or bureau or agency thereof, ... may ... make **any publication whereby the data furnished by any particular establishment or individual under this title can be identified**” (Title 13 U.S.C. § 9(a)(2), Public Law 87-813).

In 2002, Congress further clarified the concept of identifiable data: it is prohibited to publish “**any representation of information that permits the identity of the respondent to whom the information applies to be reasonably inferred by either direct or indirect means**” (Pub. L. 107–347, Title V, §502 (4), Dec. 17, 2002, 116 Stat. 2969).

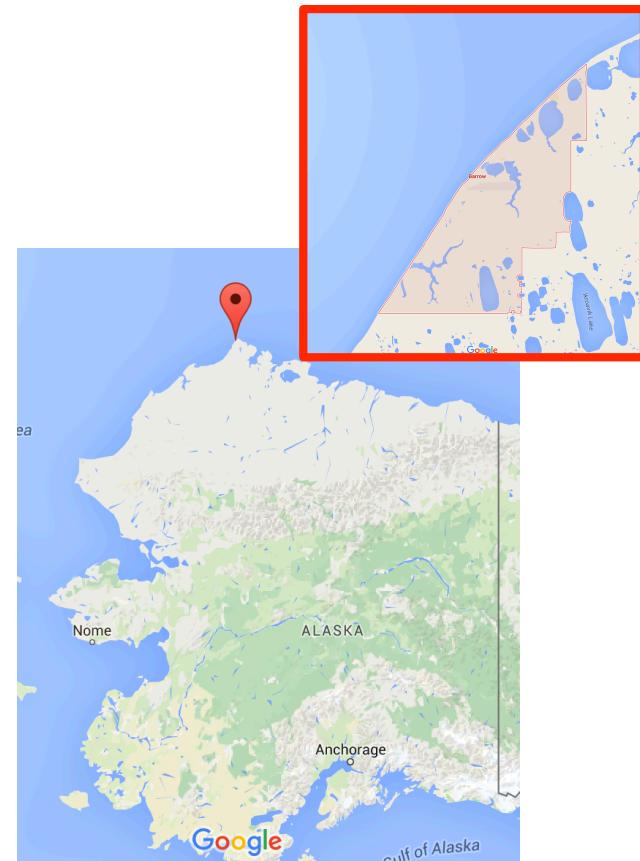
**Section 214 of Title 13** outlines penalties: fines up to \$5,000 or imprisonment up to 5 years or both per incident (data item), up to \$250,000 in total.

# Beyond re-identification: Barrow, Alaska, 1979

Native leaders and city officials, worried about drinking and associated violence in their community, **invited a group of sociology researchers** to assess the problem and work with them to devise solutions.

## Methodology

- 10% representative sample (N=88) of everyone over the age of 15 using a 1972 demographic survey
- Interviewed on attitudes and values about use of alcohol
- Obtained psychological histories & drinking behavior
- Given the Michigan Alcoholism Screening Test
- Asked to draw a picture of a person (used to determine cultural identity)



based on a slide by Bill Howe

# Study “results”

## Alcohol Plagues Eskimos; Alcoholism Plagues Eskimo Village

DAVA SOBEL ();  
January 22, 1980,  
, Section Science Times, Page C1, Column , words

 PERMISSIONS

[ DISPLAYING ABSTRACT ]

THE Inupiat Eskimos of Alaska's North Slope, whose culture has been overwhelmed by energy development activities, are "practically committing suicide" by mass alcoholism, University of Pennsylvania researchers said here yesterday. The alcoholism rate is 72 percent among the 2,000 Eskimo men and women in the village of Barrow, where violence is becoming the ...

At the conclusion of the study researchers formulated a report entitled **“The Inupiat, Economics and Alcohol on the Alaskan North Slope”**, released **simultaneously** at a press release and to the Barrow community.

The press release was picked up by the New York Times, who ran a front page story entitled **“Alcohol Plagues Eskimos”**

based on a slide by Bill Howe

# Harms and backlash

Study **results were revealed** in the context of a press conference that was held far from the Native village, and **without the presence, much less the knowledge or consent**, of any community member who might have been able to present any context concerning the socioeconomic conditions of the village.

**Study results suggested that nearly all adults in the community were alcoholics.** In addition to the shame felt by community members, the town's Standard and Poor bond rating suffered as a result, which in turn decreased the tribe's ability to secure funding for much needed projects.

## Article Preview

### Eskimos Irate Over Alcoholism Study

[ DISPLAYING ABSTRACT ]

BARROW, ALASKA HOT tempers and tension arising from a scientific report that found a high rate of alcoholism in this predominantly Eskimo community have abated somewhat after two days of meetings here at the northernmost point of Alaska.

 PERMISSIONS

based on a slide by Bill Howe

# Problems

## Methodological

Edward F. Foulks, M.D., "Misalliances In The Barrow Alcohol Study"

- "The authors once again met with the Barrow Technical Advisory Group, who stated their concern that only Natives were studied, and that outsiders in town had not been included." **any chance of selection bias?**
- "The **estimates of the frequency of intoxication based on association with the probability of being detained** were termed "ludicrous, both logically and statistically."

## Ethical

- Participants not in control of how their data is used
- Significant harm: social (stigmatization) and financial (bond rating)
- No laws were broken, and harms are not about individual privacy!
- **Who benefits? Who is harmed?**

**data protection .... responsibility .... trust**

based on a slide by Bill Howe

# We need an ethical framework

## THE BELMONT REPORT

Office of the Secretary

Ethical Principles and Guidelines for the Protection of Human Subjects of Research

The National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research

April 18, 1979

- **Boundaries between practice and research**
- Basic ethical principles
- Applications

# The Belmont Report: basic principles

## Respect for persons

- Individuals should be treated as autonomous agents

*"To respect autonomy is to give weight to autonomous persons' considered opinions and choices while refraining from obstructing their actions unless they are clearly detrimental to others. To show lack of respect for an autonomous agent is to repudiate that person's considered judgments, to deny an individual the freedom to act on those considered judgments, or to withhold information necessary to make a considered judgment, when there are no compelling reasons to do so. "*

- Persons with diminished autonomy are entitled to protection

*"In some situations, however, application of the principle is not obvious. The involvement of prisoners as subjects of research provides an instructive example. On the one hand, it would seem that the principle of respect for persons requires that prisoners not be deprived of the opportunity to volunteer for research. On the other hand, under prison conditions they may be subtly coerced or unduly influenced to engage in research activities for which they would not otherwise volunteer. Respect for persons would then dictate that prisoners be protected. Whether to allow prisoners to "volunteer" or to "protect" them presents a dilemma. Respecting persons, in most hard cases, is often a matter of balancing competing claims urged by the principle of respect itself. "*

# The Belmont Report: basic principles

## Beneficence

- Do not harm
- Maximize possible benefits and minimize possible harm

*"The Hippocratic maxim "do no harm" has long been a fundamental principle of medical ethics. **Claude Bernard extended it to the realm of research, saying that one should not injure one person regardless of the benefits that might come to others.** However, even avoiding harm requires learning what is harmful; and, in the process of obtaining this information, persons may be exposed to risk of harm. Further, the Hippocratic Oath requires physicians to benefit their patients "according to their best judgment." **Learning what will in fact benefit may require exposing persons to risk.** The problem posed by these imperatives is to decide when it is justifiable to seek certain benefits despite the risks involved, and when the benefits should be foregone because of the risks."*

# The Belmont Report: basic principles

## Justice

- Who ought to receive the benefits of research and bear its burdens?

*“Questions of justice have long been associated with social practices such as punishment, taxation and political representation. Until recently these questions have not generally been associated with scientific research. However, they are foreshadowed even in the earliest reflections on the ethics of research involving human subjects. For example, during the 19th and early 20th centuries the **burdens of serving as research subjects fell largely upon poor ward patients, while the benefits of improved medical care flowed primarily to private patients.** Subsequently, the **exploitation of unwilling prisoners as research subjects in Nazi concentration camps** was condemned as a particularly flagrant injustice. In this country, in the 1940's, the **Tuskegee syphilis study** used disadvantaged, rural black men to study the untreated course of a disease that is by no means confined to that population. **These subjects were deprived of demonstrably effective treatment in order not to interrupt the project, long after such treatment became generally available.**”*

# The Belmont Report: applications

## **Informed Consent: Information, Comprehension, Voluntariness**

*“Respect for persons requires that subjects, to the degree that they are capable, be given the opportunity to choose what shall or shall not happen to them. This opportunity is provided when adequate standards for informed consent are satisfied.*

*While the importance of informed consent is unquestioned, controversy prevails over the nature and possibility of an informed consent. Nonetheless, there is widespread agreement that **the consent process can be analyzed as containing three elements: information, comprehension and voluntariness.***

**A special problem of consent arises where informing subjects of some pertinent aspect of the research is likely to impair the validity of the research. ... In all cases of research involving incomplete disclosure, such research is justified only if it is clear that (1) incomplete disclosure is truly necessary to accomplish the goals of the research, (2) there are no undisclosed risks to subjects that are more than minimal, and (3) there is an adequate plan for debriefing subjects, when appropriate, and for dissemination of research results to them.**

# The Belmont Report: applications

**Assessment of Risks and Benefits**

**Selection of Subjects**

**GDPR**

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# General Data Protection Regulation GDPR

Welcome to gdpr-info.eu. Here you can find the official [PDF](#) of the Regulation (EU) 2016/679 (General Data Protection Regulation) in the current version of the OJ L 119, 04.05.2016; cor. OJ L 127, 23.5.2018 as a neatly arranged website. All Articles of the GDPR are linked with suitable recitals. The European Data Protection Regulation is applicable as of May 25th, 2018 in all member states to harmonize data privacy laws across Europe. If you find the page useful, feel free to support us by sharing the project.

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