Confidence Interval Construction

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A Stylized example…

A random sample of 20 observations from a population data had a mean equal to 70 The standard deviation of the population data is 10. Find an 85% confidence interval for the population mean.

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Probability outside the confidence interval is referred to as 'α'

… and we wish to construct a (1-α) confidence interval for the population mean

RICE
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Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$$

Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\underbrace{\bar{x} - \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}}}_{\text{Lower limit}} < \mu < \underbrace{\bar{x} + \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}}}_{\text{Upper limit}}$$

Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$$

margin of error = $\left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$$

margin of error $= \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} \; < \; \mu \; < \; \bar{x} + \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$$

margin of error = $\left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error = $|z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$

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$$\bar{x} - |z_{\alpha/2}| \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}| \frac{\sigma}{\sqrt{n}}$$

margin of error = $|z_{\alpha/2}| \frac{\sigma}{\sqrt{n}}$

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margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$

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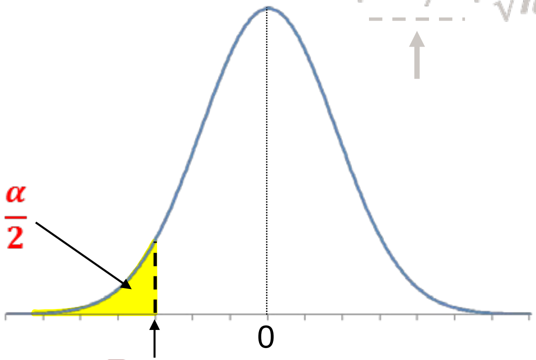
$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$



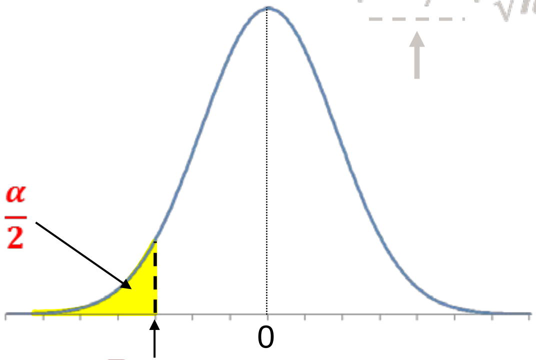
$z_{\alpha/2}$ **= NORM.INV(α/2, 0, 1)**

Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}}$$

margin of error $= \underline{\left|z_{\alpha/2}\right|}\frac{\sigma}{\sqrt{n}}$



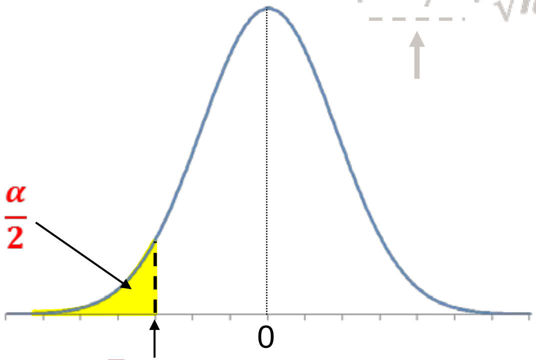
$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1) = NORM.INV(0.15/2, 0, 1)

Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} \quad < \quad \mu \quad < \quad \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error = $|z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$



$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1) = NORM.INV(0.15/2, 0, 1) = -1.4395

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} = 1.4395 \times \frac{10}{\sqrt{20}}$

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$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} \quad = 1.4395 \times \frac{10}{\sqrt{20}}$

Confidence Interval Construction

A (1 - α) confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}}$$

margin of error $= \left|z_{\alpha/2}\right|\frac{\sigma}{\sqrt{n}} = 1.4395 \times \frac{10}{\sqrt{20}} \leftarrow$

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} = 1.4395 \times \frac{10}{\sqrt{20}} = 3.2189$

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}}$$

margin of error $= \left|z_{\alpha/2}\right| \frac{\sigma}{\sqrt{n}} = 1.4395 \times \frac{10}{\sqrt{20}} = 3.2189$

$$\bar{x} - 3.2189 < \mu < \bar{x} + 3.2189$$

$$70 - 3.2189 < \mu < 70 + 3.2189$$

$$[\,66.78 < \mu < 73.22\,]$$

……a 85% confidence interval for the population mean

Confidence Interval Construction

A $(1 - \alpha)$ confidence interval for the population mean…

$$\bar{x} - |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$$

margin of error $= |z_{\alpha/2}|\frac{\sigma}{\sqrt{n}}$ $= 1.4395 \times \frac{10}{\sqrt{20}}$ $= 3.2189$

$$\bar{x} - 3.2189 < \mu < \bar{x} + 3.2189$$

$$70 - 3.2189 < \mu < 70 + 3.2189$$

$$[\, 66.78 < \mu < 73.22 \,]$$

……a 85% confidence interval
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Confidence Interval Construction

$$[\ 66.78 < \mu < 73.22\ ]$$

……a 85% confidence interval for the population mean

Confidence Interval Construction

$[\ 66.78 < \mu < 73.22\ ]$

……a 85% confidence interval for the population mean

……a **95%** confidence interval for the population mean

$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1)

Confidence Interval Construction

$[\ 66.78 < \mu < 73.22\ ]$

……a 85% confidence interval for the population mean

……a **95%** confidence interval for the population mean

$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1)

= NORM.INV(0.05/2, 0, 1)

= -1.9600

Confidence Interval Construction

[66.78 $<$ μ $<$ 73.22] ……a 85% confidence interval for the population mean

[65.62 $<$ μ $<$ 74.38] ……a **95%** confidence interval for the population mean

$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1)

= NORM.INV(0.05/2, 0, 1)

= -1.9600

Confidence Interval Construction

$[\ 66.78 < \mu < 73.22\ ]$ ......a 85% confidence interval for the population mean

$[\ 65.62 < \mu < 74.38\ ]$ ......a 95% confidence interval for the population mean

$z_{\alpha/2}$ = NORM.INV(α/2, 0, 1)

= NORM.INV(0.05/2, 0, 1)

= -1.9600

$[\ 64.24 < \mu < 75.76\ ]$ ......a 99% confidence interval for the population mean

Confidence Interval Construction

$[\, 66.78 < \mu < 73.22 \,]$ ……a 85% confidence interval for the population mean

$[\, 65.62 < \mu < 74.38 \,]$ ……a **95%** confidence interval for the population mean

$[\, 64.24 < \mu < 75.76 \,]$ ……a **99%** confidence interval for the population mean

Confidence Interval Construction

[66.78 < μ < 73.22] ……a 85% confidence interval for the population mean

[65.62 < μ < 74.38] ……a **95%** confidence interval for the population mean

[64.24 < μ < 75.76] ……a **99%** confidence interval for the population mean

Tradeoff between 'precision' and 'uncertainty'

Confidence Interval Construction

$[\,66.78 < \mu < 73.22\,]$ ……a 85% confidence interval for the population mean

$[\,65.62 < \mu < 74.38\,]$ ……a **95%** confidence interval for the population mean

$[\,64.24 < \mu < 75.76\,]$ ……a **99%** confidence interval for the population mean

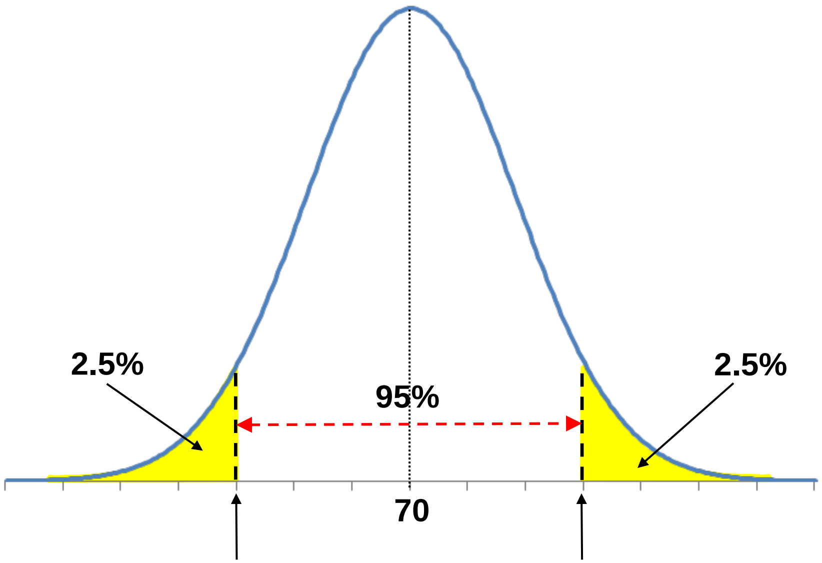
Tradeoff between 'precision' and 'uncertainty'

$[\,-\infty < \mu < +\infty\,]$ ……a **100%** confidence interval for the population mean

Confidence Interval Construction

$[\ 65.62 < \mu < 74.38\ ]$ ……a **95%** confidence interval for the population mean

Confidence Interval Construction



[65.62 < μ < 74.38]

……a **95%** confidence interval for the population mean