Two Popular Discrete Distributions

- The Binomial
- The Poisson

The Binomial Distribution

Bernoulli Process

A situation where the random variable has only two mutually exclusive outcomes.

Exam grade → pass / fail

Coin toss → heads / tails

Lottery → win / do-not-win

The Binomial Distribution

Multiple Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

The Binomial Distribution

Multiple Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Probability of winning = 1/6 = 0.1667

The Binomial Distribution

Multiple Trials of the Bernoulli Process

$$\text{Game of Dice} \begin{cases} \text{Win (if you roll a 6)} \\ \\ \text{Lose (otherwise)} \end{cases}$$

Probability of winning at least 4 times in 10 rolls of the dice?

The Binomial Distribution

Multiple Trials of the Bernoulli Process

Game of Dice $\begin{cases} \text{Win (if you roll a 6)} \\ \text{Lose (otherwise)} \end{cases}$

Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?

The Binomial Distribution

Multiple Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?

Probability of winning exactly 3 times in 10 rolls of the dice?

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → Number of times you win when you roll the dice 10 times

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice $\begin{cases} \text{Win (if you roll a 6)} \\ \text{Lose (otherwise)} \end{cases}$

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

→ 0

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

→ **0, 1**

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → Number of times you win when you roll the dice 10 times

→ A Binomial Distribution

→ 0, 1, 2

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → Number of times you win when you roll the dice 10 times

→ A Binomial Distribution

→ 0, 1, 2, 3

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice $\begin{cases} \text{Win (if you roll a 6)} \\ \text{Lose (otherwise)} \end{cases}$

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

→ **0, 1, 2, 3, 4**

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

→ **0, 1, 2, 3, 4, 5**

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice → Win (if you roll a 6) / Lose (otherwise)

Random Variable → Number of times you win when you roll the dice 10 times

→ A Binomial Distribution

→ 0, 1, 2, 3, 4, 5, 6

The Binomial Distribution

~~Multiple~~ Ten Trials of the Bernoulli Process

Game of Dice
- Win (if you roll a 6)
- Lose (otherwise)

Random Variable → **Number of times you win when you roll the dice 10 times**

→ **A Binomial Distribution**

→ **0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10**

The Binomial Distribution

Consider a situation where there are ***n*** independent trials, where the probability of ***success*** on each trial is ***p*** and the probability of ***failure*** is ***1-p***.

The Binomial Distribution

Consider a situation where there are ***n*** independent trials, where the probability of ***success*** on each trial is ***p*** and the probability of ***failure*** is ***1-p***.

Define random variable ***X*** to denote number of successes in n trials.

The Binomial Distribution

Consider a situation where there are ***n*** independent trials, where the probability of ***success*** on each trial is ***p*** and the probability of ***failure*** is ***1-p***.

Define random variable ***X*** to denote number of successes in n trials.

Then this random variable is said to have a Binomial distribution.

The Binomial Distribution

Consider a situation where there are ***n*** independent trials, where the probability of ***success*** on each trial is ***p*** and the probability of ***failure*** is ***1-p***.

Define random variable ***X*** to denote number of successes in n trials.

Then this random variable is said to have a Binomial distribution.

In our example…

n = 10, ***p*** = 1/6 = 0.1667, ***success*** = getting a 6 in one roll

X = Number of times you win when the dice is rolled ten times

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!}\,p^{x}(1-p)^{n-x}$$

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!}p^{x}(1-p)^{n-x}$$

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!}p^x(1-p)^{n-x}$$

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

The Binomial Distribution

Probability Mass Function

$$P(X = x) = \frac{n!}{x!\,(n-x)!}\,p^x(1-p)^{n-x}$$

the BINOM.DIST function

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

RICE
JONES GRADUATE SCHOOL OF BUSINESS

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

FALSE: P(successes = x)

TRUE: P(successes ≤ x)

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

FALSE → P(successes = x)   TRUE → P(successes ≤ x)

probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

P(successes = x)   P(successes ≤ x)

probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

FALSE → P(successes = x)   TRUE → P(successes ≤ x)

probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

probability that you win at most five times in ten rolls of a dice?

P(X≤5) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9976

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE/TRUE)

FALSE → P(successes = x) TRUE → P(successes ≤ x)

probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

probability that you win at the most five times in ten rolls of a dice?

P(X≤5) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9976

probability that you win less than five times in ten rolls of a dice?

P(X<5) = P(X≤4) = BINOM.DIST(4, 10, 0.1667, TRUE) = 0.9845

Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE / TRUE)

P(successes = x)　　P(successes ≤ x)

probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

probability that you win at the most five times in ten rolls of a dice?

P(X≤5) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9976

probability that you win less than five times in ten rolls of a dice?

P(X<5) = P(X≤4) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9845

probability that you win at least three times in ten rolls of a dice?

P(X≥3) = 1- P(X≤2) = 1 - BINOM.DIST(2, 10, 0.1667, TRUE) = 0.2249