



## Two Popular Discrete Distributions

- The Binomial
- The Poisson

# The Binomial Distribution

## Bernoulli Process

A situation where the random variable has only two mutually exclusive outcomes.

Exam grade → pass / fail

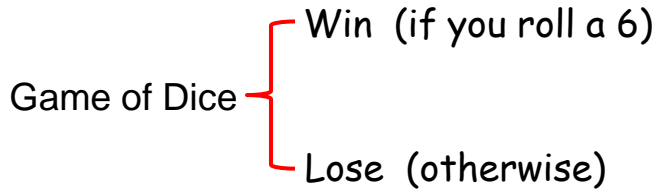
Coin toss → heads / tails

Lottery → win / do-not-win



# The Binomial Distribution

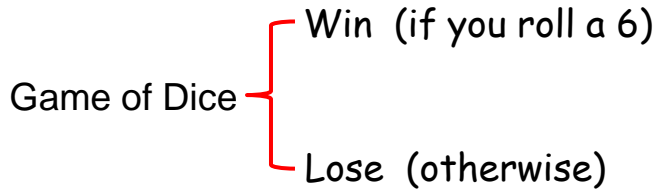
## Multiple Trials of the Bernoulli Process





# The Binomial Distribution

## Multiple Trials of the Bernoulli Process

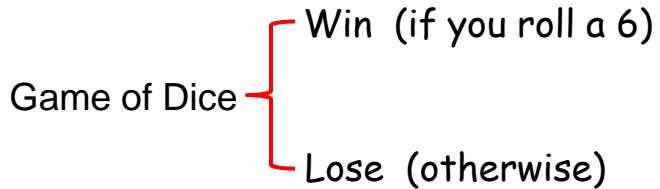


**Probability of winning =  $1/6 = 0.1667$**



# The Binomial Distribution

## Multiple Trials of the Bernoulli Process

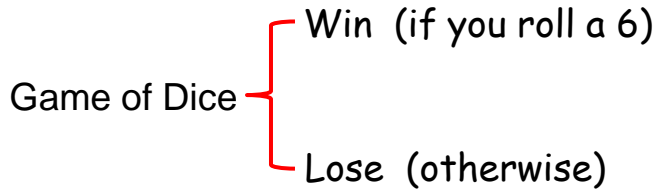


Probability of winning at least 4 times in 10 rolls of the dice?



# The Binomial Distribution

## Multiple Trials of the Bernoulli Process



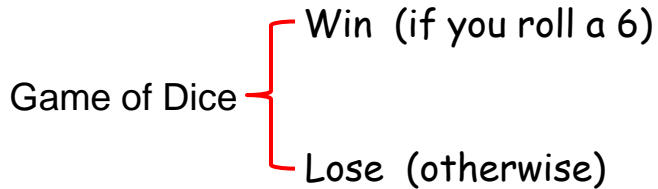
Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?



# The Binomial Distribution

## Multiple Trials of the Bernoulli Process



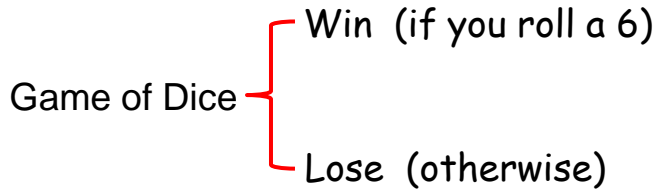
Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?

Probability of winning exactly 3 times in 10 rolls of the dice?

# The Binomial Distribution

## ~~Multiple~~ Ten Trials of the Bernoulli Process



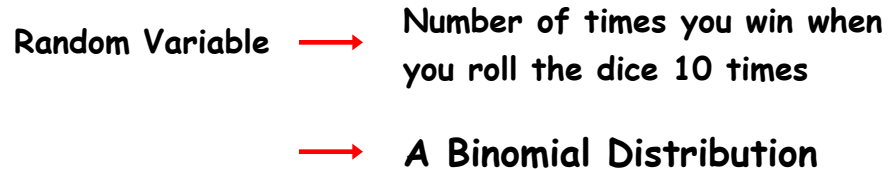
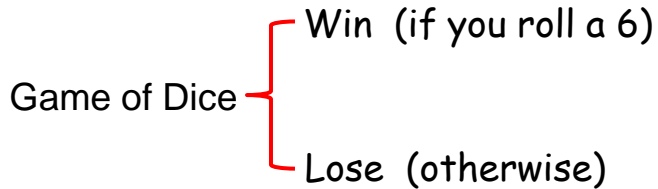
Random Variable → Number of times you win when  
you roll the dice 10 times





# The Binomial Distribution

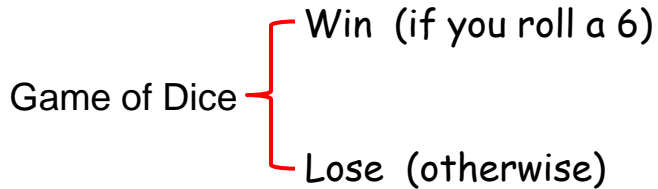
## ~~Multiple~~ Ten Trials of the Bernoulli Process





## The Binomial Distribution

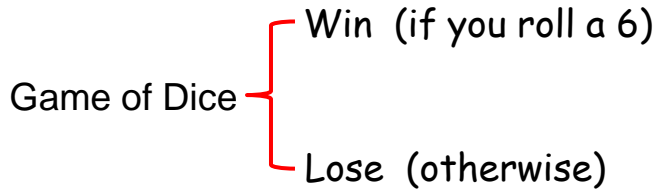
### ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0

# The Binomial Distribution

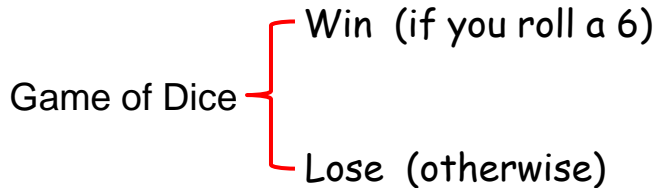
## ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1

# The Binomial Distribution

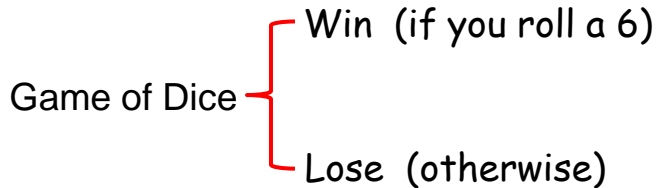
## ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2

# The Binomial Distribution

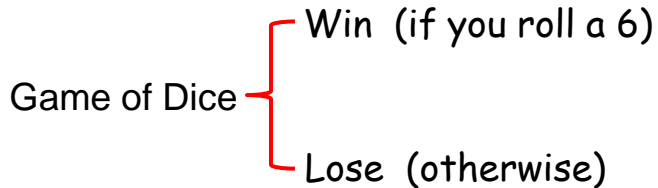
## ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2, 3

# The Binomial Distribution

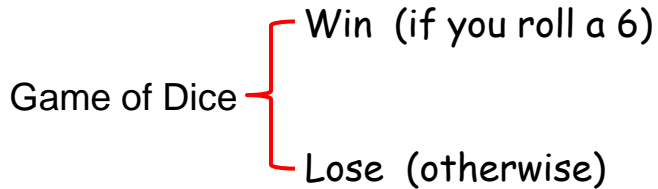
## ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2, 3, 4

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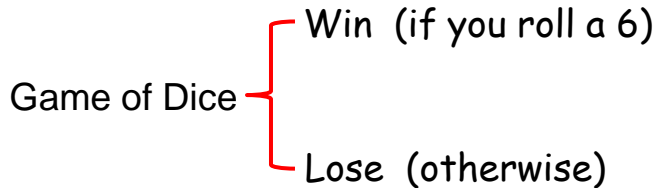
## ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2, 3, 4, 5

# The Binomial Distribution

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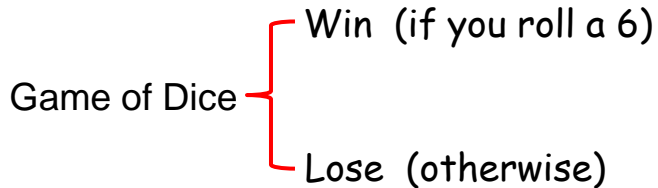


- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2, 3, 4, 5, 6



## The Binomial Distribution

### ~~Multiple~~ Ten Trials of the Bernoulli Process



- Random Variable → Number of times you win when you roll the dice 10 times
- A Binomial Distribution
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10



## The Binomial Distribution

Consider a situation where there are  $n$  independent trials, where the probability of **success** on each trial is  $p$  and the probability of **failure** is  $1-p$ .



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Then this random variable is said to have a Binomial distribution.

In our example...

$n = 10$ ,  $p = 1/6 = 0.1667$ , **success** = getting a 6 in one roll

$X$  = Number of times you win when the dice is rolled ten times



# The Binomial Distribution

## Probability Mass Function

$$P(X = x) = \frac{n!}{x! (n - x)!} p^x (1 - p)^{n-x}$$



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the `BINOM.DIST` function



## Binomial Distribution, the BINOM.DIST function

**=BINOM.DIST(x, n, p, FALSE/TRUE)**



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probability that you win three times in ten rolls of a dice?

$$P(X=3) = \text{BINOM.DIST}(3, 10, 0.1667, \text{FALSE}) = 0.1551$$

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probability that you win at most five times in ten rolls of a dice?

$$P(X \leq 5) = \text{BINOM.DIST}(5, 10, 0.1667, \text{TRUE}) = 0.9976$$

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$$P(X \leq 5) = \text{BINOM.DIST}(5, 10, 0.1667, \text{TRUE}) = 0.9976$$

probability that you win less than five times in ten rolls of a dice?

$$P(X < 5) = P(X \leq 4) = \text{BINOM.DIST}(4, 10, 0.1667, \text{TRUE}) = 0.9845$$

## Binomial Distribution, the BINOM.DIST function

=BINOM.DIST(x, n, p, FALSE / TRUE)

$P(\text{successes} = x)$

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probability that you win three times in ten rolls of a dice?

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probability that you win at least three times in ten rolls of a dice?

$$P(X \geq 3) = 1 - P(X \leq 2) = 1 - \text{BINOM.DIST}(2, 10, 0.1667, \text{TRUE}) = 0.2249$$