

# Hypothesis Testing

## Example

You are the production manager at a beverage manufacturer and you receive a bottling unit that has been recently re-adjusted so that it puts 200 milliliter of beverage in disposable plastic bottles.

You need to test that indeed the bottling unit puts in 200 milliliter of beverage.

For that you fill out 10 bottles using the unit at different times so as to obtain a random sample and very carefully measure the amount of beverage inside each bottle.

# Hypothesis Testing

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# Hypothesis Testing

## Example

You are the production manager at a beverage manufacturer and you receive a bottling unit that has been adjusted so that it puts 200 milliliter of beverage in disposable bottles.

Population mean  
claimed by the  
bottling unit

You need to test that indeed the bottling unit puts in 200 milliliter of beverage.

For that you fill out 10 bottles using the unit at different times so as to obtain a random sample and very carefully measure the amount of beverage inside each bottle.



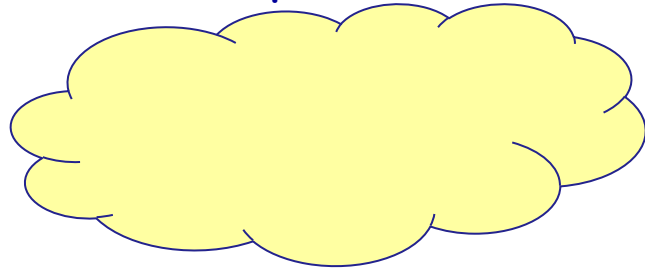
# Hypothesis Testing

## *The Central Limit Theorem*

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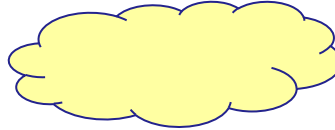
**Population**



**Mean =  $\mu$**   
**Std =  $\sigma$**



**Sample (of size  $n$ )**

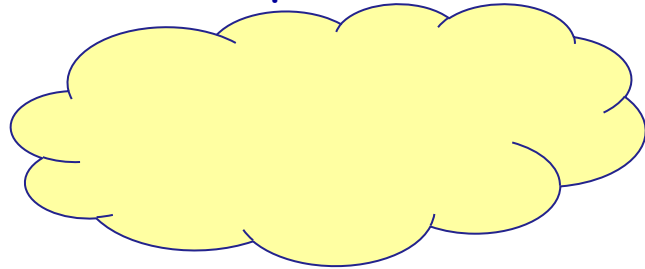


$\bar{x} \sim \text{Normal} \left( \mu, \frac{\sigma}{\sqrt{n}} \right)$

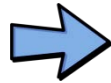
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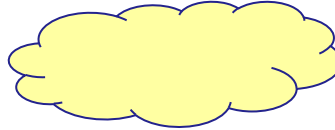
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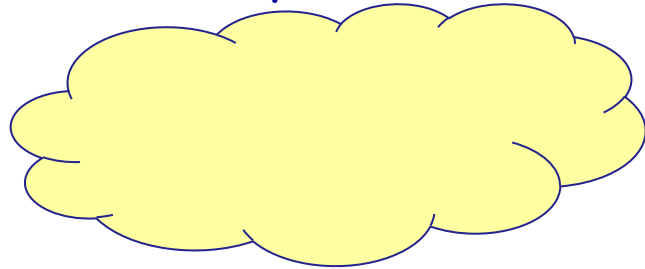


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↑

# Hypothesis Testing

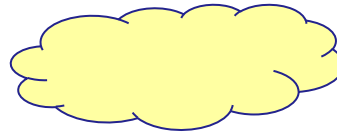
## *The Central Limit Theorem*

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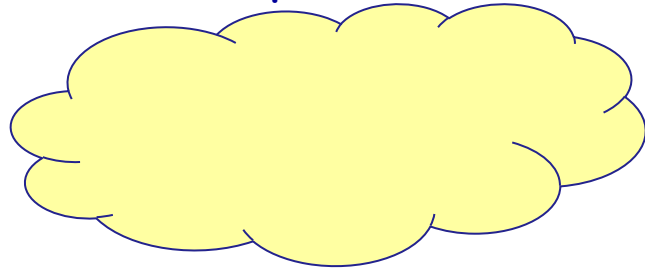
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# Hypothesis Testing

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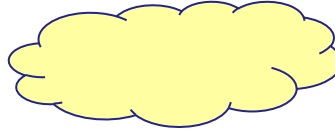
Population

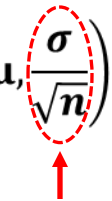


Mean =  $\mu$   
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Sample (of size  $n$ )



$$\bar{x} \sim \text{Normal} \left( \mu, \frac{\sigma}{\sqrt{n}} \right)$$




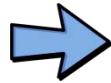
# Hypothesis Testing

## *The Central Limit Theorem*

### Population

all possible number  
of bottles that the  
unit can fill

Mean =  $\mu$   
Std =  $\sigma$



### Sample (of size 10)

random sample  
of 10 bottles

$\bar{x} \sim \text{Normal} \left( \mu, \frac{\sigma}{\sqrt{10}} \right)$

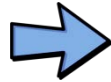
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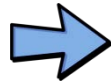
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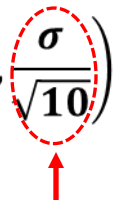
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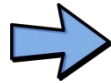
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$\bar{x} \sim \text{Normal} \left( \mu, \frac{\sigma}{\sqrt{10}} \right)$

If the claim is correct:  $\bar{x} \sim \text{Normal} \left( \mathbf{200}, \frac{\sigma}{\sqrt{10}} \right)$

# Hypothesis Testing

## *The Central Limit Theorem*

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Mean =  $\mu$   
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random sample  
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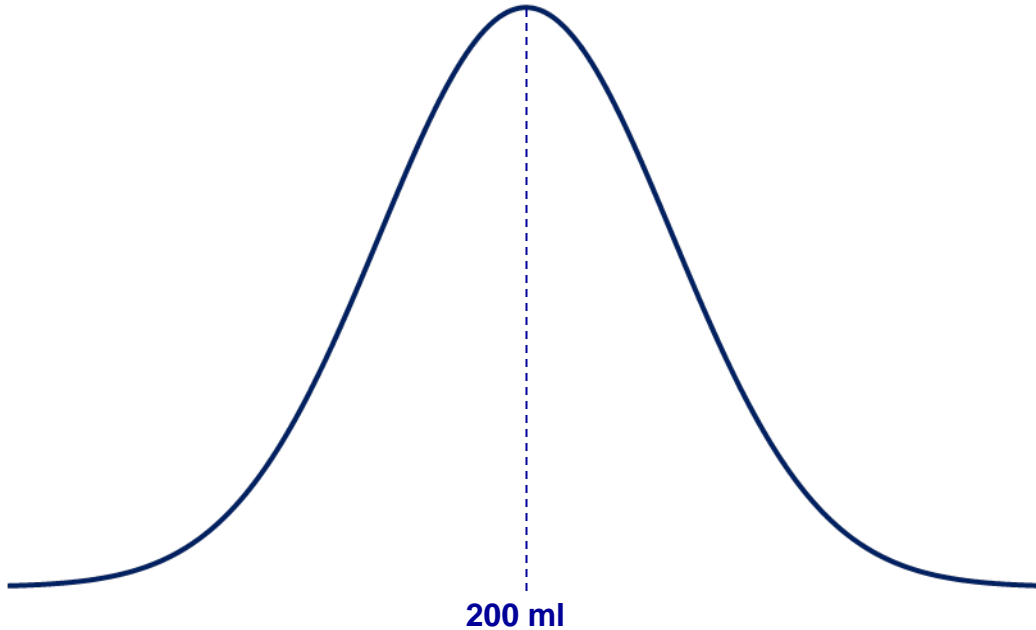
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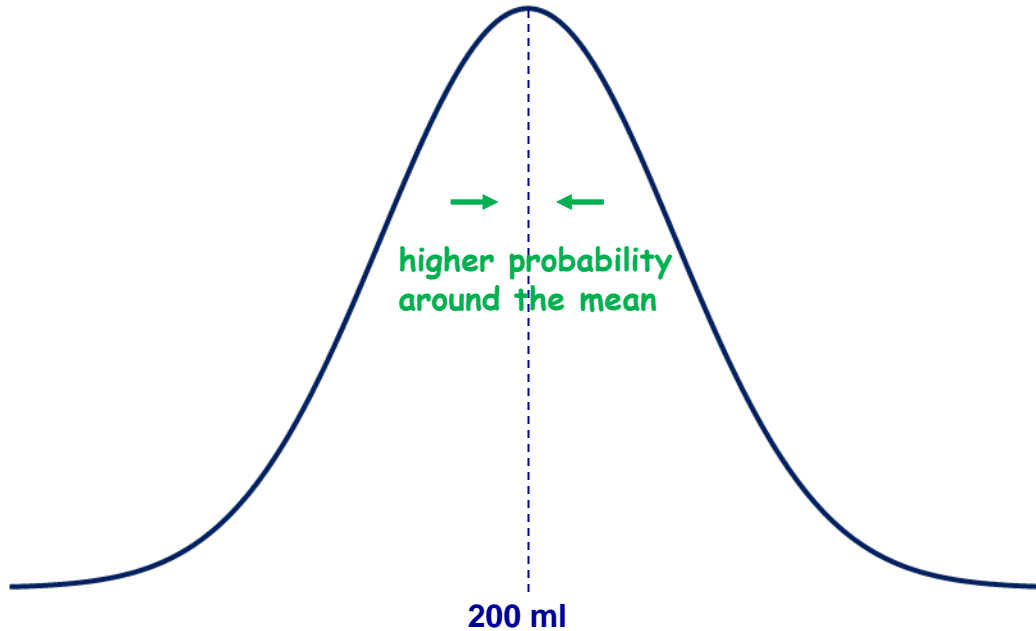
# Hypothesis Testing

Distribution of sample mean *if the claim is correct*



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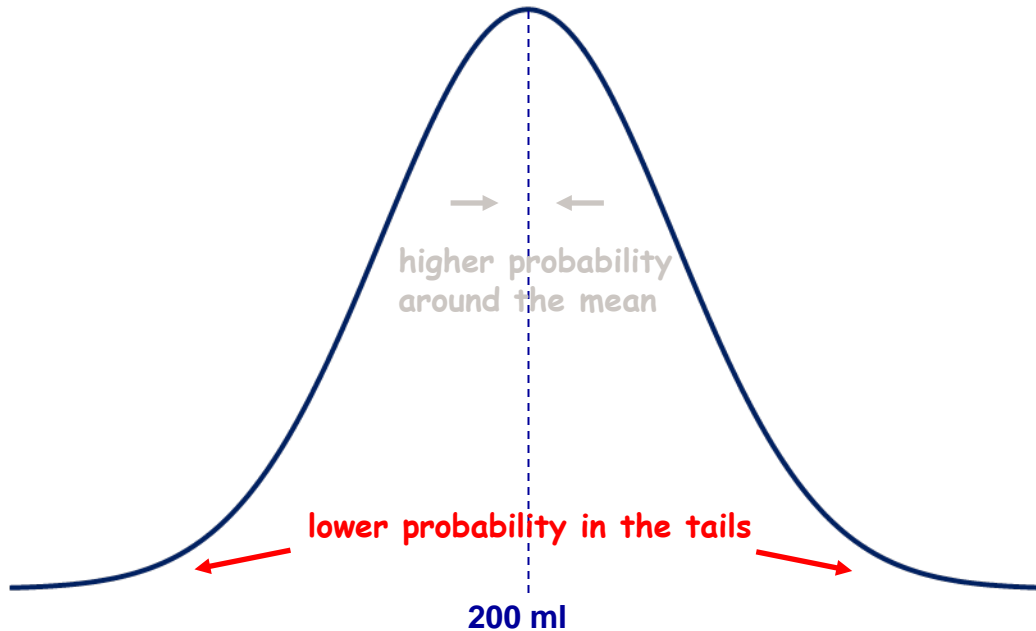
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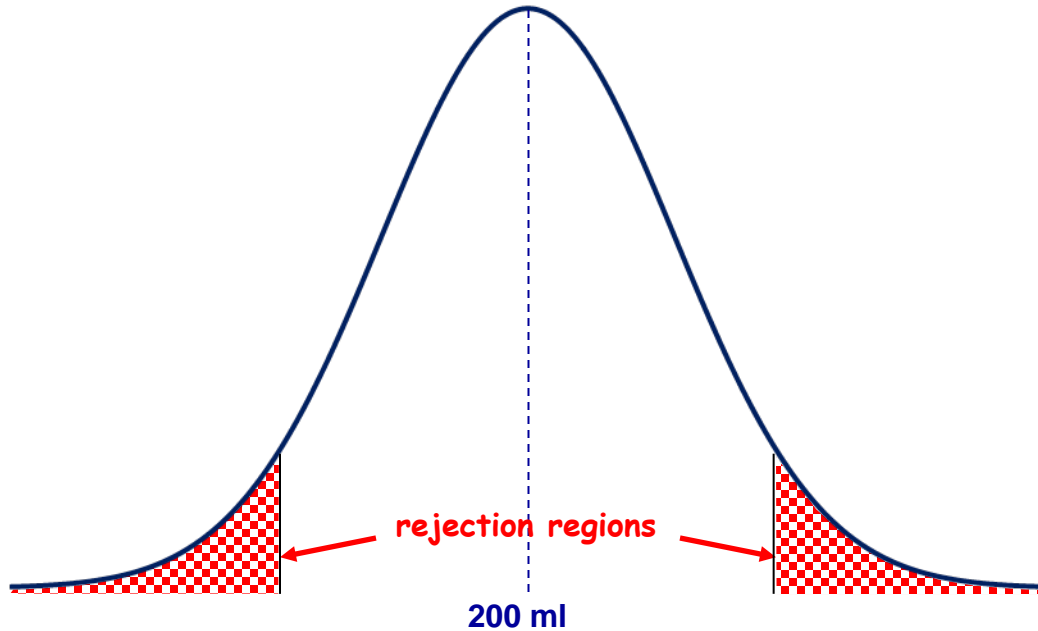
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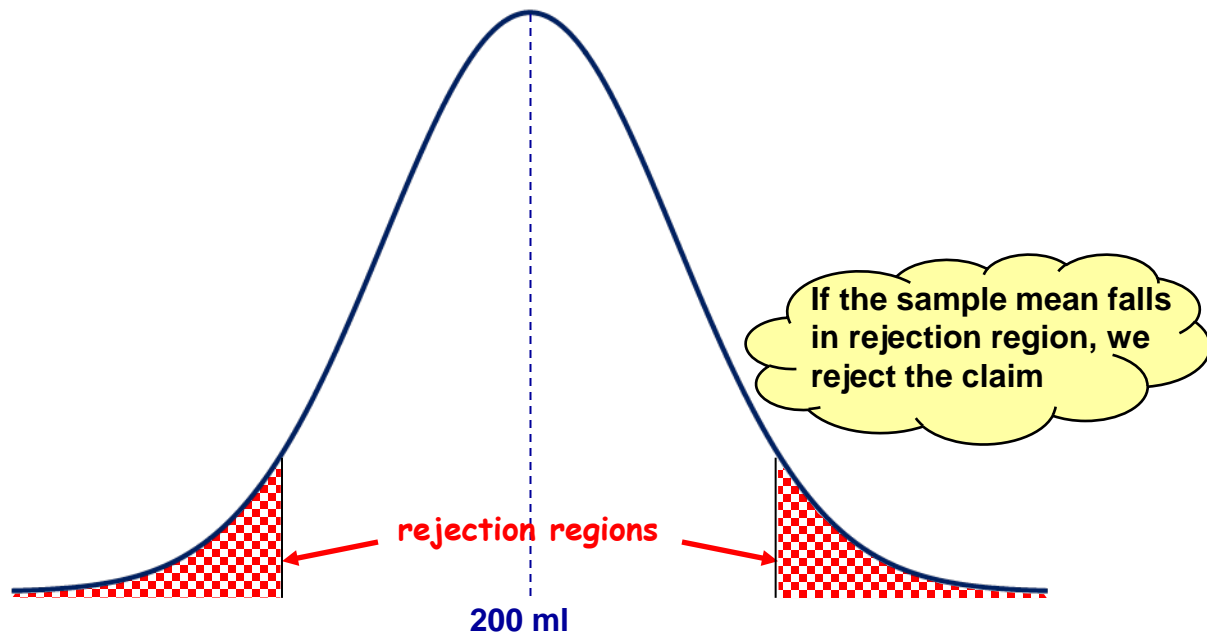
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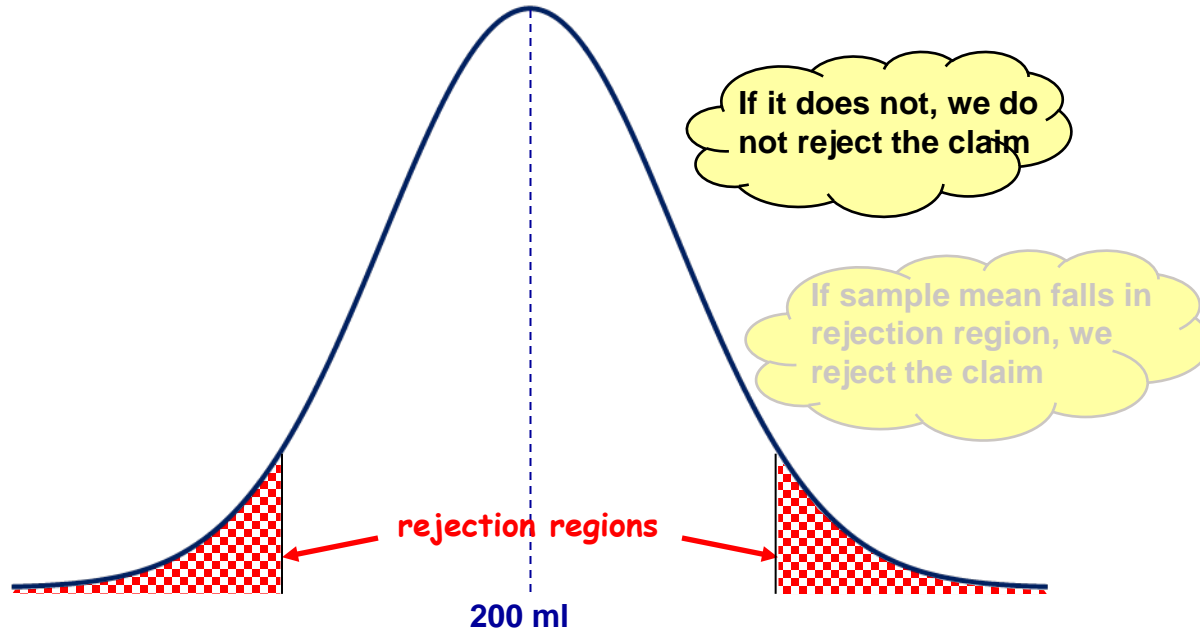
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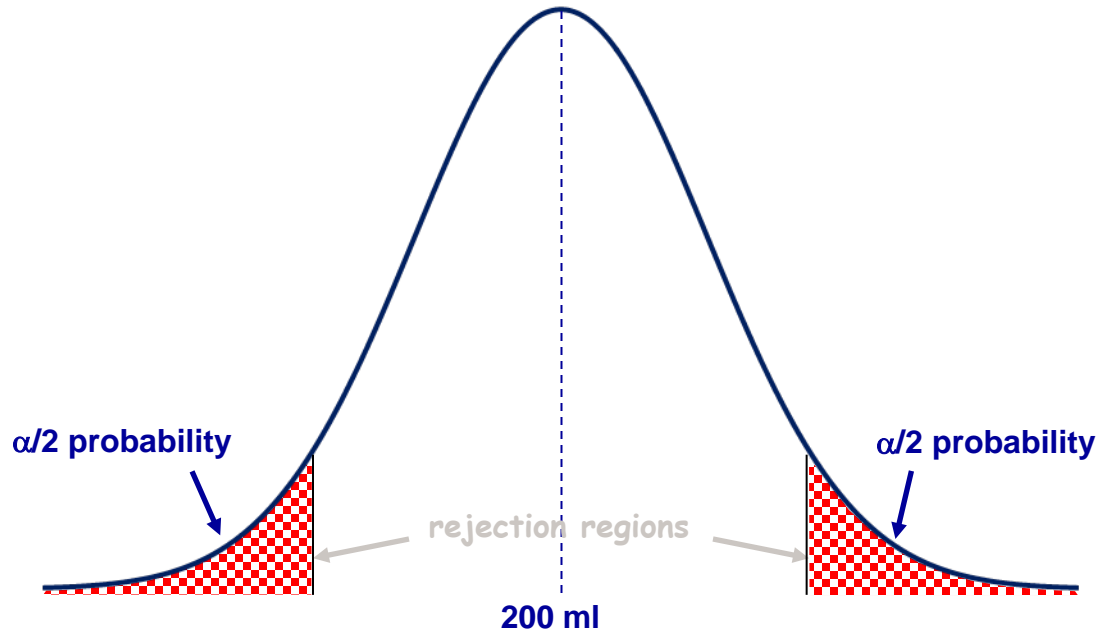
# Hypothesis Testing

Distribution of sample mean *if the claim is correct*



# Hypothesis Testing

Distribution of sample mean *if the claim is correct*





# Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}(\mu, \frac{\sigma}{\sqrt{n}})$$



# Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}(\mu, \frac{\sigma}{\sqrt{n}})$$

↑



# Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}(\mu, \frac{\sigma}{\sqrt{n}})$$

A red arrow points upwards to the parameter  $\mu$  in the normal distribution formula.





# Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}(\mu, \frac{\sigma}{\sqrt{n}})$$

↑



## Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$$



## Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$$


The fraction  $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$  is circled with a dashed red line, and a red arrow points from the text **z-statistic** to it.

# Hypothesis Testing

Distribution of sample mean

$$\bar{x} \sim \text{Normal}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$$

z-statistic



Sample mean falling  
in rejection region

**=**

The z-statistic falling  
in rejection region



# Hypothesis Testing

Distribution of sample mean

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$



$$\frac{\bar{x} - \mu}{s/\sqrt{n}}$$

**t-statistic**  
(n-1 degrees of freedom)

# Hypothesis Testing

Distribution of sample mean

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$\Rightarrow$

$$\frac{\bar{x} - \mu}{s/\sqrt{n}}$$

t-statistic

Sample mean falling  
in rejection region

$\equiv$

t-statistic  
The ~~z-statistic~~ falling  
in rejection region