1. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(1) $\det(A) = (6+0+0) - (0+0+0) = 6$

선형변환의 크기 $= 6$

(2) $(A - \lambda I)x = 0$

$$\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$\det(A - \lambda I) = (1-\lambda)(2-\lambda)(3-\lambda)$

$\therefore \ \lambda = 1, \ 2, 3$

$\lambda = 1$ $\begin{bmatrix} 0 & 0 & \cancel{1} \\ 0 & 1 & 3 \\ 0 & 0 & \cancel{2} \end{bmatrix} \rightarrow \begin{bmatrix} 0 & & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{aligned} x_1 &= x_1 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$ $\therefore \ v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda = 2$ $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & \cancel{3} \\ 0 & 0 & \cancel{1} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{aligned} x_1 &= 0 \\ x_2 &= x_2 \\ x_3 &= 0 \end{aligned}$ $\therefore \ v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda_3 = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{aligned} x_1 - \tfrac{1}{2}x_3 &= 0 \\ x_2 - 3x_3 &= 0 \\ x_3 &= x_3 \end{aligned}$ $\therefore \ v_3 = \begin{pmatrix} \frac{1}{2} \\ 3 \\ 1 \end{pmatrix}$

2)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} = A$$

$$\begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{bmatrix} = A - \lambda I$$

$$\det(A) = \lambda^2(1-\lambda)$$

$\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \begin{aligned} 3x_1 &= -x_3 \\ x_2 &= x_2 \\ x_3 &= x_3 \end{aligned} \quad \frac{x_3}{3} \Rightarrow x_2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} -\frac{1}{3} \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} \cancel{-1} & 0 & 0 \\ 0 & -1 & 0 \\ \cancel{3} & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= x_3 \end{aligned} \quad x_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & -\frac{1}{3} & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad P^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -3 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

∴ 대각화

$$\begin{pmatrix} 0 & 1 & 0 \\ -3 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -\frac{1}{3} & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. $P(\text{스팸}|\text{당첨}) = ?$ → $\dfrac{P(\text{스팸} \cap \text{당첨})}{P(\text{당첨})} = \dfrac{P(\text{스팸})P(\text{당첨}|\text{스팸})}{P(\text{당첨})}$

$$= \frac{0.2 \times 0.5}{P(\text{당첨})}$$

$$\frac{P(\text{당첨}|\text{스팸}) \cdot P(\text{스팸})}{P(\text{스팸})P(\text{당첨}|\text{스팸}) + P(\sim\text{스팸})P(\text{당첨}|\sim\text{스팸})} = \frac{0.5 \times 0.2}{(0.2 \times 0.5) + (0.8 \times 0.01)} = 0.9259\ldots$$

4-2) hit: 9 / out: 6

$$\text{entropy} = -P(A)\log(P(A)) - P(B)\log(P(B))$$

$$= -\frac{9}{15}\cdot\log\left(\frac{9}{15}\right) - \frac{6}{15}\cdot\log\left(\frac{6}{15}\right) = 0.292285\ldots$$

4-3) hit: 4 / out: 11

$$\text{entropy} = -\frac{4}{15}\cdot\log\left(\frac{4}{15}\right) - \frac{11}{15}\cdot\log\left(\frac{11}{15}\right) = 0.25853\ldots$$

$$\text{Cross}: -\frac{9}{15}\cdot\log\left(\frac{4}{15}\right) - \frac{6}{15}\cdot\log\left(\frac{11}{15}\right) = 0.3982\ldots$$

4.

loss function of Logistic regression: $\min_w \sum_{i=1}^{m} -y^{(i)}\log(\hat{y}^{(i)}) - (1-y^{(i)})\log(1-\hat{y}^{(i)})$

$$= \min_w \sum_{i=1}^{m}\left(-y^{(i)}\log\frac{1}{1+e^{-w^T x^{(i)}}} - (1-y^{(i)})\log\frac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}}\right) \rightarrow J(w)$$

$J(w)$가 convexity 만족하려면 둘 다 convex in w 만족

$\log\dfrac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}} = w^T x - \log\dfrac{1}{1+e^{-w^T x^{(i)}}}$, Hessian이 positive semi define

∴ convex 하다.