# MLDL을 위한 수학 과제

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# Q1.

- $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(1) $\det(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix} = 1 \begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} = 6$ $\quad \therefore 6$

기하학적 의미 : 선형 변환을 통해서 basis vector가 바뀌었을때, 공간이 얼마나 늘어나고 줄어드는지.

basis vector의 직선으로 이루어진 공간의 넓이 (부피)

(2) Eigenvector : 정방행렬 A를 선형변환으로 봤을때, 선형변환 A에 의한 결과가 자기자신의 상수 배가 되는 0이 아닌 벡터

$\Leftrightarrow Av = \lambda v$

Eigenvalue : $\lambda$ ; 이 상수배 값을 의미

① Eigenvalue 구하기 : $\det(\lambda I - A) = 0$

$\det \begin{bmatrix} \lambda-1 & 0 & 1 \\ 0 & \lambda-2 & 3 \\ 0 & 0 & \lambda-3 \end{bmatrix} = (\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\therefore \lambda = 1, 2, 3$

$$\begin{bmatrix} \lambda-1 & 0 & 1 \\ 0 & \lambda-2 & 3 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

② Eigenvector 구하기 : $(\lambda I - A)\mathbf{x} = 0$

i) $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -x_2+3x_3 \\ -2x_3 \end{bmatrix} \qquad \therefore \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$3x_3 = x_2$

ii) $\lambda = 2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1+x_3 \\ x_3 \\ -x_3 \end{bmatrix} \qquad \therefore \Big[\ 0$$

iii) $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_1+x_3 \\ -x_2+3x_3 \\ 0 \end{bmatrix} = \emptyset \rightsquigarrow \begin{matrix} 2x_1 = x_3 \\ x_2 = 3x_3 \end{matrix}$$

$$\begin{bmatrix} \lambda-1 & 0 & 1 \\ 0 & \lambda-2 & 3 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

② Eigenvector 구하기 : $(\lambda I - A)x = 0$

i) $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{aligned} x_2 + x_3 &= 0 \\ x_3 &= 0 \\ x_3 &= 0 \end{aligned} \qquad \therefore \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

ii) $\lambda = 2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \therefore \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

iii) $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad \therefore \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

## Q2.

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

- Eigenvalue

$$\begin{vmatrix} \lambda-0 & 0 & 0 \\ 0 & \lambda-0 & 0 \\ 3 & 0 & \lambda-1 \end{vmatrix} = \lambda \begin{vmatrix} \lambda & 0 \\ 0 & \lambda-1 \end{vmatrix} = \lambda^2(\lambda-1) = 0 \qquad \therefore \lambda = 0, 1$$

$$\therefore D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Q3.

$P(\text{spam}) = 0.2$

$P(\text{당첨} \mid \text{spam}) = 0.5$

$P(\text{당첨} \mid \text{not spam}) = 0.01$

$*\ P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

spam 당첨

$$\Rightarrow P(\text{spam} \mid \text{당첨}) = \frac{(0.5)(0.2)}{0.51} = \frac{0.10}{0.51} = \frac{10}{51} \qquad \therefore \frac{10}{51}$$

# Q4.

(2) hit : 9 * 이항분포의 entropy : $H(p) = -p \cdot \log_2(p) - (1-p) \cdot \log_2(1-p)$

out : 6

$p = \frac{9}{15} = \frac{3}{5} = 0.6$

$1-p = \frac{6}{15} = \frac{2}{5} = 0.4$

$\Rightarrow H(p) = -0.6 \cdot \log_2(0.6) - (1-p)\log_2(0.4)$

$\fallingdotseq 0.971$

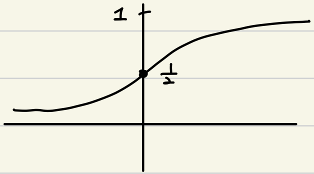
(3) 다른팀 hit = 4, out = 11 $\Big\}$ $q = \frac{4}{15}$, $1-q = \frac{11}{15}$

* KL Divergence : $KL(p\|q) = H(p,q) - H(p) = \Sigma p_i \log \frac{p_i}{q_i}$ ∵ 이산형

$\therefore KL(p\|q) = (0.6)\left(\log \frac{0.6}{4/15}\right) \times 15$

$= 9 \log \frac{9}{4}$

# Q5.




loss function : $-\log \frac{e^{w^Tx}}{1+e^{-w^Tx}}$

→ Hessian이 positive semi definite

⇒ convex 

∴  ← GD method 사용.

global minimum