

Q1

$$(1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix} \quad 1 \times 2 \times 3 = 6 //$$

복합된 6인 4면체 생성

$$(2) A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda = 1, 2, 3$$

① $\lambda = 1$

$$(A - \lambda I)X = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_3 = 0 \\ x_2 + 3x_3 = 0 \\ 2x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases} \Rightarrow x_2 = x_3 = 0$$

$$X = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_1$$

② $\lambda = 2$

$$(A - \lambda I)X = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x_1 + x_3 = 0 \\ 3x_3 = 0 \\ x_3 = 0 \end{cases} \Rightarrow x_1 = x_3 = 0$$

$$X = \begin{bmatrix} 0 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_2$$

③ $\lambda = 3$

$$(A - \lambda I)X = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2x_1 + x_3 = 0 \\ -x_2 + 3x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_3 = 2x_1 \\ x_2 = 3x_3 \end{cases}$$

$$X = \begin{bmatrix} x_1 \\ 6x_1 \\ 2x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} x_1$$

eigen value: 1, 2,

Q2.

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$B - \lambda I = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{bmatrix}$$

$$\det(B - \lambda I) = (1-\lambda)\lambda^2 = 0$$

$$\lambda = 0, 1$$

① $\lambda = 0$

$$(B - \lambda I)X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x_1 + x_3 = 0 \Rightarrow x_3 = -3x_1$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ -3x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

② $\lambda = 1$

$$(B - \lambda I)X = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x_1 = 0 \\ -x_2 = 0 \\ 3x_1 = 0 \end{cases} \Rightarrow x_1 = x_2 = 0$$

$$X = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -3 & 0 & 1 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array} \right]$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q3

$$P(\text{Spam}) = 0.2$$

$$P(\text{당첨} | \text{Spam}) = 0.5$$

$$P(\text{당첨} | \text{not Spam}) = 0.01$$

$$P(\text{Spam} | \text{당첨}) = ?$$

$$P(\text{Spam} | \text{당첨}) = \frac{P(\text{당첨} | \text{Spam}) P(\text{Spam})}{P(\text{당첨})}$$

$$P(\text{당첨}) = P(\text{당첨} | \text{Spam}) P(\text{Spam}) + P(\text{당첨} | \text{not Spam}) P(\text{not Spam})$$

$$P(\text{당첨}) = 0.5 \times 0.2 + 0.01 \times 0.8$$

$$0.5 = 0.10 + 0.008 = 0.108$$

$$P(\text{Spam} | \text{당첨}) = \frac{0.5 \times 0.2}{0.108}$$

$$= \frac{0.1}{0.108} = 0.926$$

92.6%

Q4.

$$(2) P(H, t) = \frac{8}{15}, P(O, t) = \frac{7}{15}$$

$$H(S) = -\left(\frac{8}{15} \log_2 \frac{8}{15} + \frac{7}{15} \log_2 \frac{7}{15}\right)$$

$$\approx 0.997$$

$$(3) P(H, t) = \frac{5}{15}, P(O, t) = \frac{10}{15}$$

$$H(S) = -\left(\frac{5}{15} \log_2 \frac{5}{15} + \frac{10}{15} \log_2 \frac{10}{15}\right)$$

$$\approx 0.918$$

$$KL(p||q) = 0.122$$

$$KL(q||p) = 0.117$$

$$JSD(p||q) = 0.120$$

Q5. Convex Optimization

$$\min \sum -y_i \log y_i - (1-y_i) \log (1-y_i)$$

$$= \min \sum -y_i \log \frac{1}{1+e^{-w^T x_i}} - (1-y_i) \log \frac{e^{-w^T x_i}}{1+e^{-w^T x_i}}$$

$$= \min \sum y_i \left(\log(1+e^{-w^T x_i}) - \frac{e^{-w^T x_i}}{1+e^{-w^T x_i}} \right) - \log \frac{e^{-w^T x_i}}{1+e^{-w^T x_i}}$$

$$= \min \sum y_i \log e^{-w^T x_i} + w^T x_i - \log \frac{1}{1+e^{-w^T x_i}}$$

$$= \min \sum -y_i w^T x_i + w^T x_i - \log \frac{1}{1+e^{-w^T x_i}}$$

Hessian of positive semi definite

\Rightarrow convex

$\Rightarrow \nabla J(w) = 0$ 인 것이 optimum