1-1.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\det(A) = |A| = \sum_{j=1}^{3} (-1)^{1+j} a_{1j} \det A_{1j}$$

$$= 2 \cdot \det\left(\begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}\right) - 1 \cdot \det\left(\begin{bmatrix} 0 & 3 \\ 0 & 3 \end{bmatrix}\right) + \det\left(\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\right)$$

$$= 2 \cdot 6 - 1 \cdot 0 + 0 = 12$$

기하학적으로 부피의 의미를 가짐

1-2.

$A\mathbf{x} = \lambda\mathbf{x}$

$(A - \lambda I)\mathbf{x} = 0$

$\therefore \det(A - \lambda I) = 0$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$\therefore \lambda = 1, 2, 3$

$$(A-I)\mathbf{x}_{\lambda=1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \therefore \begin{matrix} x_3 = 0 \\ x_2 + 3x_3 = 0 \\ 2x_3 = 0 \end{matrix} \rightarrow \mathbf{x}_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$(A-I)\mathbf{x}_{\lambda=2} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \therefore \begin{matrix} -x_1 + x_3 = 0 \\ 3x_3 = 0 \\ x_3 = 0 \end{matrix} \rightarrow \mathbf{x}_{\lambda=2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A-I)\mathbf{x}_{\lambda=3} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \therefore \begin{matrix} -2x_1 + x_3 = 0 \\ -x_2 + 3x_3 = 0 \\ 0 = 0 \end{matrix} \rightarrow \mathbf{x}_{\lambda=3} = \begin{bmatrix} 1/2 \\ 3 \\ 1 \end{bmatrix}$$

- eigenvalue : eigenvector가 변환되는 크기
- eigenvector : 어떤 벡터에 선형변환을 취했을 때, 방향은 변하지 않고 크기만 변환되는 벡터

2.

$B\mathbf{x} = \lambda\mathbf{x}$

$(B - \lambda I)\mathbf{x} = 0$

$$|B - \lambda I| = \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = \lambda^2(1-\lambda) = 0$$

$\therefore \lambda = 0, 1$

$$\Rightarrow D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B\mathbf{x}_{\lambda=0} = 0 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \therefore 3x_1 + x_3 = 0 \rightarrow \mathbf{x}_{\lambda=0} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$$

$$(B-I)\mathbf{x}_{\lambda=1} = 0 \rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \therefore \begin{matrix} -x_1 = 0 \\ -x_2 = 0 \\ 3x_1 = 0 \end{matrix} \rightarrow \mathbf{x}_{\lambda=1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\det(P) = 1 \quad \therefore P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} = PDP^{-1}$$

#3.

$$P(\text{Spam}\mid\text{'당첨'}) = \frac{P(\text{spam \& '당첨'})}{P(\text{'당첨'})} = \frac{P(\text{'당첨'}\mid\text{Spam}) \times P(\text{Spam})}{P(\text{'당첨'}\mid\text{Spam}) \times P(\text{Spam}) + P(\text{'당첨'}\mid\text{Not Spam}) \times (1-P(\text{Spam}))} = \frac{0.5\times0.2}{0.5\times0.2+0.01\times0.8} \fallingdotseq 0.926$$

#4-2.

X : 안타, 아웃

$X \sim P$

$$H(X) = \sum_{x\in X} P(x)\log_2\frac{1}{P(x)} = \frac{3}{5}\cdot\log_2\frac{5}{3} + \frac{2}{5}\cdot\log_2\frac{5}{2}$$

$$= \frac{3}{5}(\log_2 5-\log_2 3) + \frac{2}{5}(\log_2 5-\log_2 2)$$

$$= \log_2 5 - \frac{3}{5}\log_2 3 - \frac{2}{5}$$

#4-3.

$$Q(x) = \begin{cases} 4/15 & \because x : \text{hit} \\ 11/15 & \because x : \text{out} \end{cases}$$

$$KL(P\|Q) = \sum_{x\in X} P(x)\cdot\log_2\frac{P(x)}{Q(x)}$$

$$= \frac{3}{5}\cdot\log_2\frac{9}{4} + \frac{2}{5}\cdot\log_2\frac{6}{11}$$

$$= \frac{3}{5}(\log_2 9 - \log_2 4) + \frac{2}{5}\cdot(\log_2 6 - \log_2 11)$$

$$= \frac{6}{5}\log_2 3 - \frac{6}{5} + \frac{2}{5} + \frac{2}{5}\log_2 3 - \frac{2}{5}\log_2 11$$

$$= \frac{8}{5}\log_2 3 - \frac{2}{5}\log_2 11 - \frac{4}{5}$$

#5.

loss function of logistic regression :

$$\min_{w}\sum_{i=1}^{m} -y^{(i)}\log(\hat{y}^{(i)}) - (1-y^{(i)})\log(1-\hat{y}^{(i)})$$

$$= \min_{w}\sum_{i=1}^{m} \underbrace{-y^{(i)}\log\frac{1}{1+e^{-w^Tx^{(i)}}} - (1-y^{(i)})\log\frac{e^{-w^Tx^{(i)}}}{1+e^{-w^Tx^{(i)}}}}_{: J(w)}$$

이 때 $-\log\frac{e^{-w^Tx^{(i)}}}{1+e^{-w^Tx^{(i)}}} = w^Tx - \log\frac{1}{1+e^{-w^Tx^{(i)}}}$ 이고

Hessian이 positive semi definite 하므로 convex.