#1

(1) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

$$\det(A) = 1 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 1 \cdot (6 - 0) \qquad = 6$$

Det : 3차원 ⇒ 부피

(2) eigenvalue / eigenvector

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} 2-\lambda & 3 \\ 0 & 3-\lambda \end{vmatrix} + \begin{vmatrix} 0 & 2-\lambda \\ 0 & 0 \end{vmatrix}$$

$$= (1-\lambda)(2-\lambda)(3-\lambda)$$

$$\therefore \lambda = 1 \text{ or } 2 \text{ or } 3$$

① $\lambda = 1$.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_2 + x_3 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \therefore \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

② $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_1 + x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \therefore \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

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③ $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_1 + x_3 \\ -x_2 + x_3 \\ 0 \end{bmatrix} \qquad \therefore \begin{bmatrix} \frac{1}{2} \\ 3 \\ 1 \end{bmatrix}$$

#2

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

i) eigenvalue 구하기

$\det(B - \lambda I) = 0$

$$\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = -\lambda \begin{vmatrix} -\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = -\lambda(-\lambda)(1-\lambda)$$

$\therefore \lambda = 0$ or 1.

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

① $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \underline{3x_1 + x_3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \therefore \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$$

② $\lambda = 1$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \\ 3x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \therefore \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$$B = PDP^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

#3 $P(S|당) = \frac{P(당|S)\,P(S)}{P(당)} = \frac{P(당|S)\,P(S)}{P(당|S)\cdot P(S) + P(당|NS)\cdot(1-P(S))}$

$= \frac{0.5\times 0.2}{0.5\times 0.2 + 0.01\times 0.8} \approx 0.9259.$

#4 (2) $H(X) = \Sigma P(x)\log\frac{1}{P(x)}$

$P(HIT) = \frac{9}{15}$, $P(OUT) = \frac{6}{15}$

$\therefore H(X) = \frac{9}{15}\log\frac{15}{9} + \frac{6}{15}\log\frac{15}{6}$

$\frac{3}{5}(\log 5 - \log 3) + \frac{2}{5}(\log 5 - \log 2)$

$= \log 5 - \frac{3}{5}\log 3 - \frac{2}{5}\log 2$

(3) $KL = \Sigma P(x)\log_2\frac{P(x)}{Q(x)}$

$P(HIT) = \frac{4}{15}$ $P(OUT) = \frac{11}{15}$

$\therefore KL = \frac{9}{15}\log_2\frac{9}{4} + \frac{6}{15}\log_2\frac{6}{11}$

#5

i) $J(\omega) = \underbrace{-y^{(i)}\log\frac{1}{1+e^{-\omega^T x^{(i)}}}}_{①} \underbrace{-(1-y^{(i)})\log\frac{e^{-\omega^T x^{(i)}}}{1+e^{-\omega^T x^{(i)}}}}_{②}$

①, ②가 convex in ω 을 만족 $\Rightarrow$ $J(\omega)$: convex.

$-\log\frac{e^{-\omega^T x^{(i)}}}{1+e^{-\omega^T x^{(i)}}} = \omega^T x - \log\frac{1}{1+e^{-\omega^T x^{(i)}}}$ 이고,

Hessian이 positive semi definite $\Rightarrow$ convex