ML/DL을 위한 수학 9기 박서연

Q1 – (1) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow \det(A)$

$= 1 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 3 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 6$

linear transform이 일어났을 때 공간의 변화(이때는 부피)

(2) $Ax = \lambda x \Rightarrow (A - \lambda I)x = 0$

$\det(A - \lambda I) = 0$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{vmatrix}$

$= (1-\lambda)\begin{vmatrix} 2-\lambda & 3 \\ 0 & 3-\lambda \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 3-\lambda \end{vmatrix} + 1 \begin{vmatrix} 0 & 2-\lambda \\ 0 & 0 \end{vmatrix}$

$= (1-\lambda)(2-\lambda)(3-\lambda)$

e.values = 1, 2, 3

i) $\lambda = 1$ $A - \lambda I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ $\quad x_3 = 0$, $x_2 + 3x_3 = 0$, $2x_3 = 0$ $\quad X_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

ii) $\lambda = 2$

$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ $\quad -x_1 + x_3 = 0$, $3x_3 = 0$, $x_3 = 0$ $\quad X_{\lambda=2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

iii) $\lambda=3$

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0 \qquad \begin{matrix} -2x_1+x_3=0 \\ -x_2+3x_3=0 \end{matrix} \qquad X_{\lambda=3}=\begin{bmatrix} \frac{1}{2} \\ 3 \\ 1 \end{bmatrix}$$

eigenvector : linear transform에 의해 방향 변화X
eigenvalue : 벡터의 크기가 변화한 정도

Q2. $\det(B-\lambda I)=0$

$$|B-\lambda I| = \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = \lambda^2(1-\lambda)=0$$

$\lambda=1, 0$

i) $\lambda=1$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0 \qquad \begin{matrix} -x_1=0 \\ -x_2=0 \\ 3x_1=0 \end{matrix} \qquad X_{\lambda=1}=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

ii) $\lambda=0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0 \qquad 3x_1+x_3=0 \qquad X_{\lambda=0}=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$$

$$D=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P=\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \qquad P^{-1}=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\therefore B=PDP^{-1}=\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Q3. $P(\text{Spam}\mid\text{'당첨'})$

$P(\text{Spam})=0.2$
$P(\text{'당첨'}\mid\text{Spam})=0.5$
$P(\text{'당첨'}\mid\sim\text{Spam})=0.01$

$P(\text{Spam} \mid \text{'당첨'})$

$$= \frac{P(\text{'당첨'} \mid \text{spam})\,P(\text{spam})}{P(\text{'당첨'} \mid \text{spam})\,P(\text{spam}) + P(\text{'당첨'} \mid \sim\text{spam})\,P(\sim\text{spam})}$$

$$= \frac{0.5 \times 0.2}{0.5 \cdot 0.2 + 0.01 \times 0.8} \triangleq \frac{0.1}{0.1 + 0.008} \fallingdotseq 92.59\%$$

Q4. -(2) $P(\text{Hit}) = \frac{9}{15} = 0.6 \qquad P(\text{Out}) = \frac{6}{15} = 0.4$

$$H = -(0.4 \log_2(0.4) + 0.6 \log_2(0.6))$$
$$= -(0.4 \cdot (-1.322) + 0.6 \cdot (-0.7370))$$
$$= 0.971$$

(3) $P(\text{Hit}) = \frac{4}{15} \qquad P(\text{Out}) = \frac{11}{15}$

$$H = -\left(\frac{4}{15} \cdot \log_2\left(\frac{4}{15}\right) + \frac{11}{15} \cdot \log_2\left(\frac{11}{15}\right)\right) \fallingdotseq 0.8366$$

Q5. 첫번째 팀의 결과를 p, 두번째팀의 결과를 q라 하면,

$$KL(p \| q) = \sum_{x \in X} p(x) \log_2 \frac{p(x)}{q(x)}$$

$$= \frac{9}{15} \log_2 \frac{9}{4} + \frac{6}{15} \log_2 \frac{6}{11} \fallingdotseq 0.3522$$

Q5. loss func. of logistic regression

$$\min_w \sum_{i=1}^{m} \underbrace{-y^{(i)} \log \frac{1}{1+e^{-w^T x^{(i)}}} - (1-y^{(i)}) \log \frac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}}}_{J(w)}$$

$$-\log \frac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}} = w^T x - \log \frac{1}{1+e^{-w^T x^{(i)}}} \text{ 이고}$$

Hessian이 positive semi definite → $J(w)$ is convex

$\nabla J(w) = 0$ implies $w = w^*$ (minimum) 이므로 경사하강법 반복해 찾기