Assignment : math for MLDL   Kim Yang ho

Q1-1 : $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

$$|A| = 1 \begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} - 0 \cdot \begin{vmatrix} 0 & 3 \\ 0 & 3 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 1 \cdot 6 - 0 + 0 = 6$$

Q1-2 : $A\mathbf{x} = \lambda \mathbf{x}$

$A\mathbf{x} - \lambda \mathbf{x} = (A - \lambda I)\mathbf{x} = 0 \iff \det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda) \cdot \{(2-\lambda)(3-\lambda) - 0\} - 0 + 1 \cdot 0$$
$$= (1-\lambda)(2-\lambda)(3-\lambda)$$

$\lambda = 1$ or 2 or 3

① $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_2 + 3x_3 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \mathbf{x}_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

② $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_1 + x_3 \\ 3x_3 \\ x_3 \end{bmatrix} \Rightarrow \mathbf{x}_{\lambda=2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

③ $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_1 + x_3 \\ -x_2 + 3x_3 \\ 0 \end{bmatrix} \Rightarrow \mathbf{x}_{\lambda=3} = \begin{bmatrix} \frac{1}{2} \\ 3 \\ 1 \end{bmatrix}$$

⇒ 이 3개의 고유벡터는 선형변환에 의해 방향이 바뀌지 않는 벡터이다.

2. $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$Bx = \lambda x \Rightarrow Bx - \lambda x = 0$

$\det\left(\begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 0-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{bmatrix} \right) = 0 \Rightarrow (0-\lambda)(0-\lambda)(1-\lambda)$

$\lambda = 0, 1$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

고유벡터, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3x_1 + x_3 = 0 \Rightarrow s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$

$x_2 = s$

$x_3 = 3t$

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} -x_1 \\ -x_2 \\ 3x_1 \end{matrix} \Rightarrow t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$x_3 = t$

$P = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$, $P^{-1} = \dfrac{1}{\underset{(=1)}{\det(P)}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$B = PDP^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

3.

Spam $= S$

당첨 $= C$. 라하면

$P(S) = 0.2$

$P(C \mid S) = 0.5$

$P(C \mid \sim S) = 0.01$

$$P(S \mid C) = \frac{P(S, C)}{P(C)} = \frac{P(C \mid S)\, P(S)}{P(C \mid S)\, P(S) + P(C \mid \sim S)\, P(\sim S)} = \frac{0.5 \times P(S)}{0.5 \times P(S) + 0.01\,(1 - P(S))}$$

$$= \frac{0.1}{0.1 + 0.008} = \frac{0.1}{0.108}$$

약 92.59%

4. ① $y =$ 히트 $\quad P(y) = \frac{9}{15} \quad P(1-y) = \frac{6}{15}$

$$\text{Entropy} = \sum_{x = y \text{ or } 1-y} P(x) \log \frac{1}{P(x)} = \frac{9}{15} \log \frac{15}{9} + \frac{6}{15} \log \frac{15}{6}$$

$$= \frac{3}{5}(\log 5 - \log 3) + \frac{2}{5}(\log 5 - \log 2)$$

$$= \log 5 - \frac{3}{5} \log 3 - \frac{2}{5} \log 2 \quad (\text{약 } 0.971)$$

② $y =$ 히트 $\quad Q(y) = \frac{4}{15}$

$$P(y) \log \frac{P(y)}{Q(y)} + P(1-y) \log \frac{P(1-y)}{Q(1-y)} = \frac{9}{15} \log \frac{9}{4} + \frac{6}{15} \log \frac{6}{11}$$

$$= \frac{18}{15} \log 3 - \frac{18}{15} + \frac{6}{15} + \frac{6}{15} \log 3 - \frac{6}{15} \log 11$$

$$= \frac{8}{5} \log 3 - \frac{2}{5} \log 11 - \frac{4}{5} \quad (\text{약 } 0.637)$$

5. $J(w) = \sum_{i=1}^{m} -y^{(i)} \log \frac{1}{1+e^{-w^T x^{(i)}}} - (1-y^{(i)}) \log \frac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}}$

$-\log \frac{e^{-w^T x^{(i)}}}{1+e^{-w^T x^{(i)}}} = w^T x - \frac{1}{\log 1+e^{-w^T x^{(i)}}}$ 이고,

↳ Hessian이 ≥ 0 이므로 $J(w)$는 $\underline{\text{convex}}$.

so,

$\Rightarrow \nabla J(w) = 0$ implies $w = \underbrace{w^*}_{\text{minimum}}$