기수: 14기　　이름: 신지원

1-1

1) $H(X) = -\sum_{i=1}^{6} p_i \log_2 p_i$

$H(X) = -6 \cdot \frac{1}{6} \log_2 \frac{1}{6} = -\log_2 \frac{1}{6} \approx \log_2 6 \approx 2.585$

→ 모든 사건의 확률이 같을 때이기 때문에 최대

2) $H(X) = -1 \cdot \log_2 1 = 0$

3) 짝수 $p = x$

홀수 $p = \frac{x}{2}$

$3x + 3 \cdot \frac{x}{2} = \frac{9}{2}x = 1$

$\therefore x = \frac{2}{9}$

짝수 $\frac{2}{9}$, 홀수 $\frac{1}{9}$

$H(X) = -3 \cdot \frac{2}{9} \log_2 \frac{2}{9} - 3 \cdot \frac{1}{9} \log_2 \frac{1}{9} \approx 2.530$

4) $H(X) = -0.5 \log_2 0.5 - 5 \cdot 0.1 \cdot \log_2 0.1 \approx 0.5 + 5 \cdot 0.332 \approx 0.5 + 1.66$

$= 2.16$

1-2

$0 \le H(X) \le \log_2 |X|$ 증명

i) $H(X) = -\sum_{x \in X} P(x) \log_2 P(x)$

$-p(x) \log p(x) \ge 0$,

$\therefore -\sum_{x \in X} P(x) \cdot \log_2 P(x) \ge 0$

$\therefore 0 \le H(X)$

ii) by Jensen's Inequality,

$f(p) = -p\log_2 p$ → Concave 함수

$H(X) = -\sum p(x) \log_2 p(x) \le -\sum \frac{1}{|X|} \log_2 \frac{1}{|X|} = \log_2 |X|$

by i), ii), $0 \le H(X) \le \log_2 |X|$

1-3

1) $P(X=0) = 0.45 + 0.15 = 0.60$

$P(X=1) = 0.40$

$\therefore H(X) = -0.6\log_2 0.6 - 0.4\log_2 0.4 \approx \underline{0.971}$

$P(Y=0) = 0.55$

$P(Y=1) = 0.45$

$H(Y) = -0.55 \cdot \log_2 0.55 - 0.45\log_2 0.45 \approx \underline{0.993}$

2) $H(X,Y) = -\Sigma p(x,y)\log_2 p(x,y) \approx 1.846$

3) $H(X|Y) = H(X,Y) - H(Y)$

$\approx 1.846 - 0.993 = 0.853$

4) $I(X;Y) = H(X) - H(X|Y) = 0.971 - 0.853 = 0.118$

5) Y를 알고 있을때 X에 대한 불확실성을 나타내는 척도

$I(X;Y)=0$일때: 독립

$I(X;Y)=H(X)$일때: Y로 X를 예측가능

2-1

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{bmatrix}$$

$$\mathrm{Det} = (2-\lambda) \cdot \begin{vmatrix} 2-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (2-\lambda)\left((2-\lambda)(1-\lambda) - 0\right)$$

$$= (2-\lambda)^2(1-\lambda)$$

$$\therefore \lambda_1 = 2, \ \lambda_2 = 1$$

i) $\lambda = 2$

$$(A - 2I) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\rightarrow x_1 + x_3 = 0$$

$$\vec{v_1} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

ii) $\lambda = 1$

$$(A - I) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \quad \rightarrow \quad x_2 + x_3 = 0$$

$$\vec{v_3} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\therefore P = [\vec{v_1}, \vec{v_2}, \vec{v_3}] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$ //

$$D = P^{-1} \cdot AP = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ //

2-3 $\quad U, \Sigma, V^T$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B^TB = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BB^T = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

B^TB: $\lambda_1 = 2, \ \lambda_2 = 1, \ \lambda_3 = 0$

$\therefore \ \sigma_1 = \sqrt{2}, \ \sigma_2 = 1, \ \sigma_3 = 0$

$$\Sigma = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\lambda = 2 \ : \ \vec{v_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$

$\lambda = 1 \ : \ \vec{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 0 \ : \ \vec{v_3} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$

$$\therefore \ V = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}, \quad V^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\vec{u_1} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{u_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \ U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3-1

$'' > 0$ 증명

$f_1(\hat{y}) = -y\log\hat{y}$

$f_2(\hat{y}) = -(1-y)\log(1-\hat{y})$

> 둘다 convex 이면 convex (목적함수)

i) $f_1(\hat{y}) = -y\underline{\log\hat{y}}$ concave

$-\log\hat{y}$ 는 convex

$\therefore y \geq 0$ 이므로 convex

ii) $f_2(\hat{y}) = -(1-y)\underline{\log(1-\hat{y})}$ concave

$-\log(1-\hat{y}) \Rightarrow$ convex

$(1-y) \geq 0 \rightarrow$ convex

$\therefore$ convex

3-2 $f(x) = x^4 - 2x^3 - 3x^2 + x$

$x_0 = 1,\ \gamma = 0.2$

$f'(x) = 4x^3 - 6x^2 - 6x + 1$

$x_1 = x_0 - \gamma f'(x_0)$

$f'(1) = 4 - 6 - 6 + 1 = -7$

$x_1 = 1 - 0.2 \cdot (-7) = 1 + 1.4 = 2.4$

$x_2 =$

$f'(2.4) = 7.336$

$x_2 = 2.4 - 0.2 \cdot 7.336 = 2.4 - 1.467$

$= \underline{0.933}$ //

3-3

$\min f(x,y) = x^2+y^2$

$x+y \geq 2$

1) $L(x,y,\lambda) = x^2+y^2-\lambda(x+y-2)$

2) $\frac{\partial L}{\partial x} = 2x-\lambda = 0,$

$\therefore 2x=\lambda, \ x=\frac{\lambda}{2}$

$\frac{\partial L}{\partial y} = 2y-\lambda=0$

$\therefore y=\frac{\lambda}{2}$

$\frac{\partial L}{\partial \lambda} = -x-y+2=0, \ \therefore x+y=2$

3) $\rightarrow x=y=1,$

$\lambda=2$

4) $f(x,y) = x^2+y^2 = 1+1 = \underline{2}$