Machine Learning
Exercise 2: Manual implementation of linear regression in python

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Abstract

This week's tasks focus on deepening your knowledge about linear regression by manually implementing the underlying algorithm in python.

Task 1: Understand the algorithm

- Additional information about implementing linear regression in python can again be found here.

- Make sure you understand how the algorithm works before you start implementing any python code.

- Try to structure your code in a clear way, e.g. by using functions and object oriented programming.

Task 2: Implement a general least squares optimizer

- consider an array of x-values with a variable length with corresponding y-values and statistical errors y_{err}

- x = [1.47,1.50 ,1.52,1.55 ,1.57, 1.60,1.63,1.65,1.68,1.70,1.73,1.75,1.78,1.80,1.83]

- y = [52.21,53.12,54.48,55.84,57.20,58.57,59.93,61.29,63.11,64.47,66.28,68.10,69.92,72.19,74.46]

- y_{err} is one for each y

- define an empty matrix A of shape (len(x values), 2)

- fill the matrix with the values $A_{i,0} = 1$ and $A_{i,1} = x$

- define a diagonal matrix W from the y_{err} such that $W_{i,i} = \frac{1}{y_{err}^2}$

- reshape the array of y vectors into a column vector

- calculate the result of the function $p = (A^T W A)^{-1} A^T y$ and $cov = (A^T W A)^{-1}$ (Hint: use the Invert-Method for numpy matrices)

- display the coeffictients results and plot the data and fit function (Hint: for errorbars use plt.errorbar)

Task 3: Bayes Theorem

A newly developed machine can detect counterfeit bills. We define the event A: "The machine sounds an alarm", and event F: "The banknote is counterfeit". We would now like to find out how high the probability is that a banknote is actually a counterfeit, given the machine sounds the alarm. So what we are looking for is $P(F|A)$. The machine was tested with many real and fake bills. It was found that the machine alarms on a counterfeit bill with 96% confidence. with 96% certainty. However, the machine also gives an alarm for 1% of the real bills. alarm. In addition, it is known that 0.01% of all banknotes in circulation are counterfeits. are counterfeits. This also means that 99.99% of the banknotes are not counterfeits.