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3.3 CONSTRUCTING BOOTSTRAP CONFIDENCE INTERVALS

The distributions of sample statistics considered so far in this chapter require us to either already know the value of the population parameter or to have the resources to take thousands of different samples. In most situations, neither of these is an option.

In this section we introduce a method for estimating the variability of a statistic that uses only the data in the original sample. This clever technique, called *bootstrapping*,[31]The term *bootstrap* was coined by Brad Efron and Robert Tibsharani to reflect the phrase “pulling oneself up by one's own bootstraps.” allows us to approximate a sampling distribution and estimate a standard error using just the information in that one sample.

DATA 3.3 **Commuting in Atlanta**

© Pavel Losevsky/iStockphoto

What is the average commute time in Atlanta?

What is the average commuting time for people who live and work in the Atlanta metropolitan area? It's not very feasible to contact *all* Atlanta residents and ask about their commutes, but the US Census Bureau regularly collects data from carefully selected samples of residents in many areas. One such data source is the American Housing Survey (AHS), which contains information about housing and living conditions for samples from the country as a whole and certain metropolitan areas. The data in **CommuteAtlanta** includes cases where the respondent worked somewhere other than home in the Atlanta metropolitan area.[32]Sample chosen using DataFerret at http://www.thedataweb.org/index.html. Among the questions asked were the time (in minutes) and distance (in miles) that respondents typically traveled on their commute to work each day.

The commute times for this sample of 500 Atlantans are shown in the dotplot of Figure 3.14. The sample mean is $\bar{x} = 29.11$ minutes and the standard deviation in the sample is s=20.7 minutes . The distribution of commute times is somewhat right skewed with clusters at regular intervals that reflect many respondents rounding their estimates to the nearest 5 or 10 minutes. Based on this sample we have a point estimate of 29.11 minutes for μ, the mean commute time for all workers in metropolitan Atlanta. How accurate is that estimate likely to be?

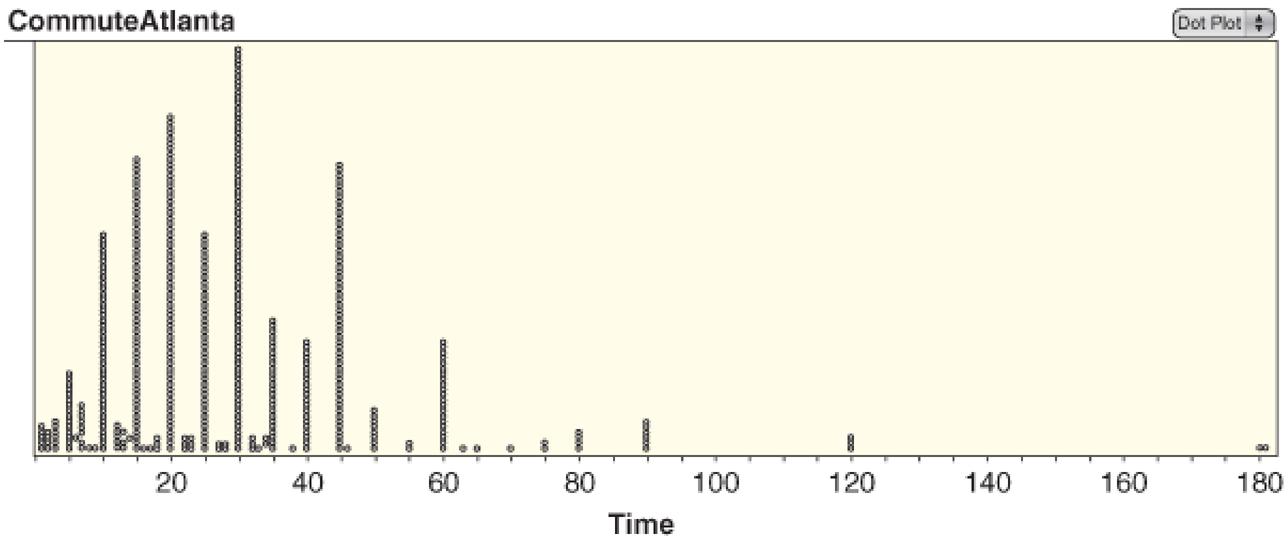


Figure 3.14 ***Sample of 500 Atlanta commute times***

To get a range of plausible values for the mean commute time of all Atlantans it would help to see a sampling distribution of means for many samples of size 500. However, we don't have data for the population of all Atlanta commuters, and if we did we could easily find the population mean exactly! We only have the commuting times for the single sample of 500 workers. How can we use the information in that sample to assess how much the means for other samples of 500 Atlantans might vary?

Bootstrap Samples

Ideally, we'd like to sample repeatedly from the population to create a sampling distribution. How can we make the sample data look like data from the entire population? The key idea is to assume for the moment that the population of all commuters in Atlanta is basically just many, many copies of the commuters in our original sample. See Figure 3.15, which illustrates this concept for a very small sample of six stick figures, in which we assume the population is just many copies of the sample. If we make lots of copies of the sample and then sample repeatedly from this hypothetical "population," we are coming as close as we can to mimicking the process of sampling repeatedly from the population.

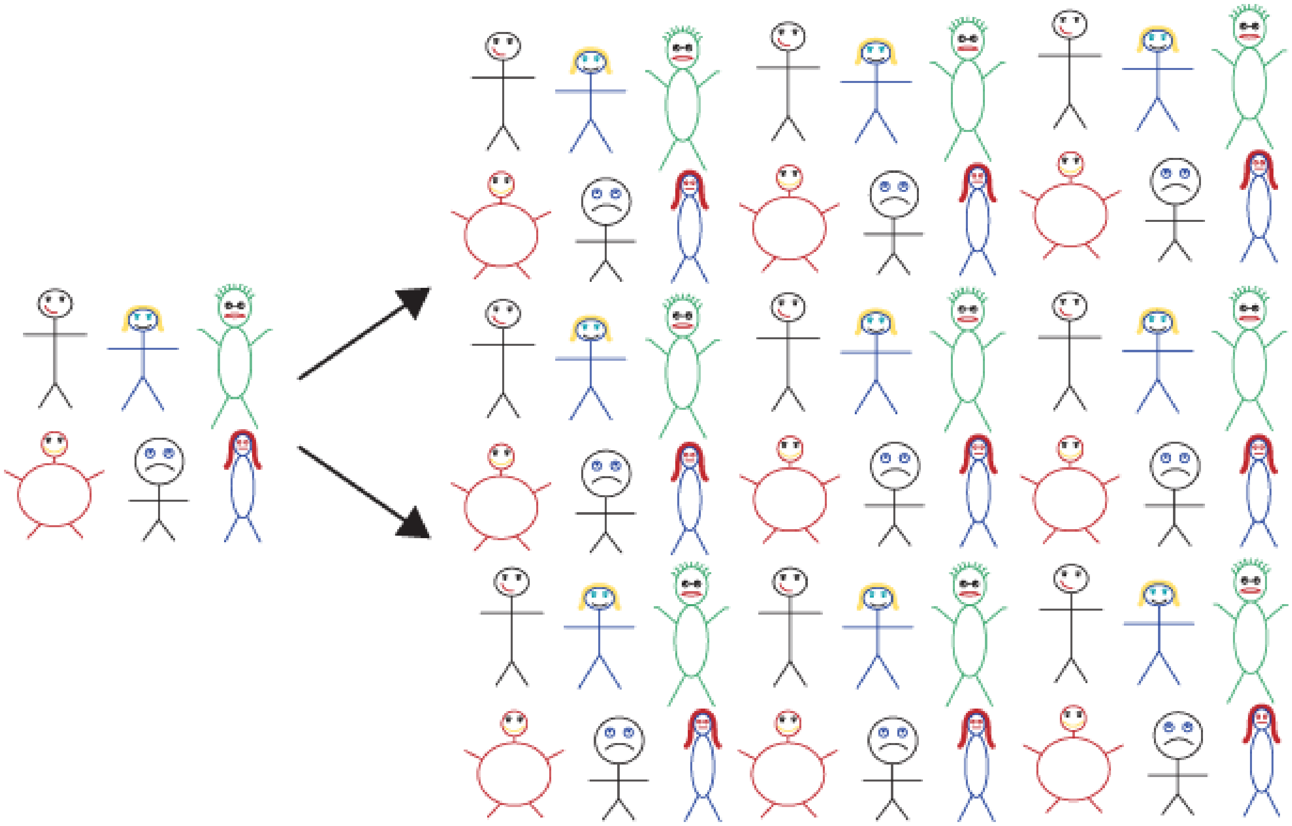


Figure 3.15 ***Using a sample to represent a population***

In practice, instead of actually making many copies of the sample and sampling from that, we use a sampling technique that is equivalent: we sample *with replacement* from the original sample. Sampling with replacement means that once a commuter has been selected for the sample, he or she is still available to be selected again. This is because we're assuming that each commuter in the original sample actually represents many fellow Atlantans with a similar commute time. Each sample selected in this way, with replacement from the original sample, is called a *bootstrap sample*.

Recall from Section 3.1 that the variability of a sample statistic depends on the size of the sample. Because we are trying to uncover the variability of the sample statistic, it is important that each bootstrap sample is the same size as the original sample. For the Atlanta commuters, each bootstrap sample will be of size $n=500$.

For each bootstrap sample, we compute the statistic of interest, giving us a *bootstrap statistic*. For the Atlanta commuters, we compute a bootstrap statistic as the sample mean commute time for a bootstrap sample. If we take many bootstrap samples and compute a bootstrap statistic from each, the distribution of these bootstrap statistics will help us understand the distribution of the sample statistic. Table 3.7 shows the sample means for 10 different bootstrap samples of size 500 taken with replacement from the original commute times in **CommuteAtlanta**.

Table 3.7 ***Mean commute times for 10 bootstrap samples of n = 500 Atlantans***

28.06	29.21	28.43	28.97	29.95	28.67	30.57	29.22	27.78	29.58

Bootstrap Distribution

Based on just the 10 bootstrap statistics in Table 3.7, we can begin to get some feel for how accurately we can estimate the mean commute time based on a sample of size 500. Note that, for the hypothetical population we simulate when sampling with replacement from the original sample, we *know* that the "population" mean is the sample mean, 29.11 minutes. Thus the bootstrap sample means give us a good idea of how close means for samples of size 500 should be to a "true" mean. For the 10 samples in Table 3.7 the biggest discrepancy is the seventh sample mean (30.57), which is still within 1.46 minutes of 29.11.

Of course, with computer technology, we aren't limited to just 10 bootstrap samples. We can get a much better picture of the variability in the means for samples of size 500 by generating many such samples and collecting the sample means. Figure 3.16 shows a dotplot of the sample means for 1000 samples of size 500, taken with replacement, from the original sample of Atlanta commute times. This gives a good representation of the *bootstrap distribution* for mean Atlanta commute times. We see that the distribution is relatively symmetric, bell-shaped, and centered near the original sample mean of 29.11.

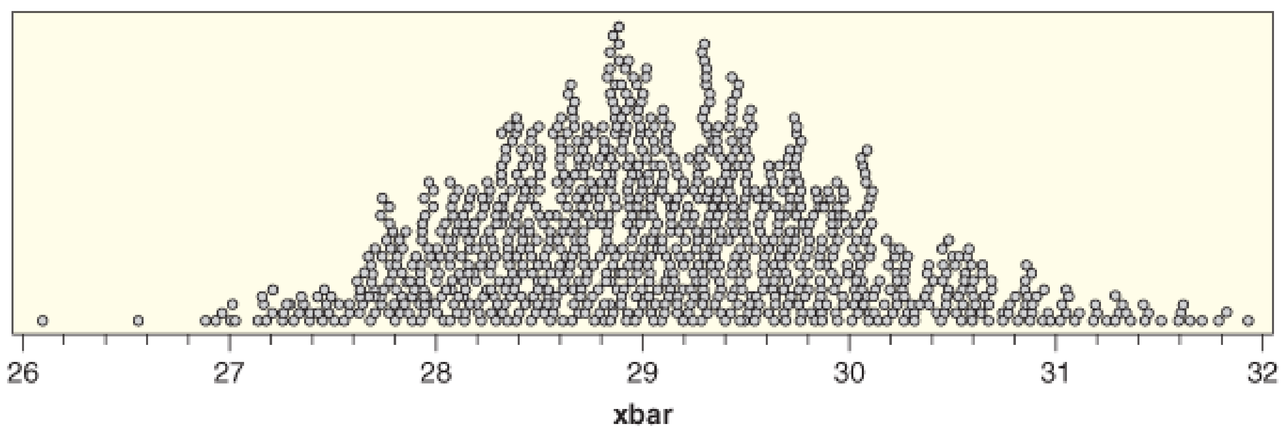


Figure 3.16 ***Commuting time means for 1000 bootstrap samples of size $n=500$***

Generating a Bootstrap Distribution

To generate a bootstrap distribution, we:

- Generate **bootstrap samples** by sampling with replacement from the original sample, using the same sample size.
- Compute the statistic of interest, called a **bootstrap statistic**, for each of the bootstrap samples.
- Collect the statistics for many bootstrap samples to create a **bootstrap distribution**.

This process is illustrated in Figure 3.17.

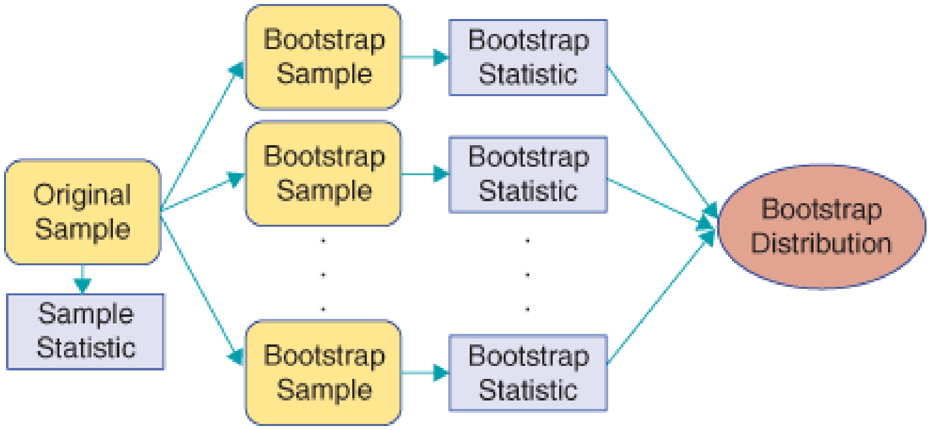
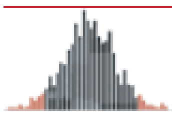


Figure 3.17 ***Generating a bootstrap distribution***

Assuming the original sample is chosen randomly from the population, the bootstrap distribution generally gives a good approximation to a sampling distribution that we might see if we were able to collect lots of samples from the entire population, but is centered around the sample statistic rather than the population parameter. This allows us to get a good idea of how variable our sample statistic is, and how close we can expect it to be to the population parameter. In Figure 3.16 we see that none of the 1000 sample means are more than three minutes away from the center of the bootstrap distribution. Thus, we are quite confident that a sample of 500 Atlanta commuters will give an estimate that is within three minutes of the mean commute time for the entire population.



Example 3.19

Mixed Nuts with Peanuts

Containers of mixed nuts often contain peanuts as well as cashews, pecans, almonds, and other nuts. For one brand, we want to estimate the proportion of mixed nuts that are peanuts. We get a jar of the nuts and assume that the nuts in that container represent a random sample of all the mixed nuts sold by that company. We open the jar and count 100 nuts of which 52 are peanuts. The estimated proportion of peanuts is $\hat{p} = 52 / 100 = 0.52$.

(a) How could we physically use the jar of nuts to construct one bootstrap sample? What would we record to find the bootstrap statistic?

(b) If we create a bootstrap distribution by collecting many bootstrap statistics, describe the center and anticipated shape of the distribution.

(c) Use *StatKey* or other technology to create a bootstrap distribution.

Solution

(a) To find a bootstrap sample we need to select 100 nuts from the original sample with replacement. To accomplish this we could shake the nuts in the jar, reach in and pick one at random, record whether or not it is a peanut, and *put it back in the jar*. (This is what sampling with replacement means.) Repeat this process 99 more times to simulate a new sample of 100 nuts. The bootstrap statistic is the proportion of peanuts among the 100 nuts selected.

(b) Since the bootstrap statistics come from the original sample with a sample proportion of 0.52, we expect the bootstrap distribution to be centered at 0.52. Since we are simulating a sampling distribution, we think it is likely that the distribution will be bell-shaped.

(c) While it would be time consuming to repeat the physical sampling process described in part a many times, it is relatively easy to use *StatKey* or other technology to simulate the process automatically. Figure 3.18 shows a dotplot of the bootstrap distribution of sample proportions for 1000 samples of size 100, simulated from the original sample with 52 peanuts out of 100. As expected, we see a symmetric, bell shape distribution, centered near the value of the statistic in the original sample (0.52).

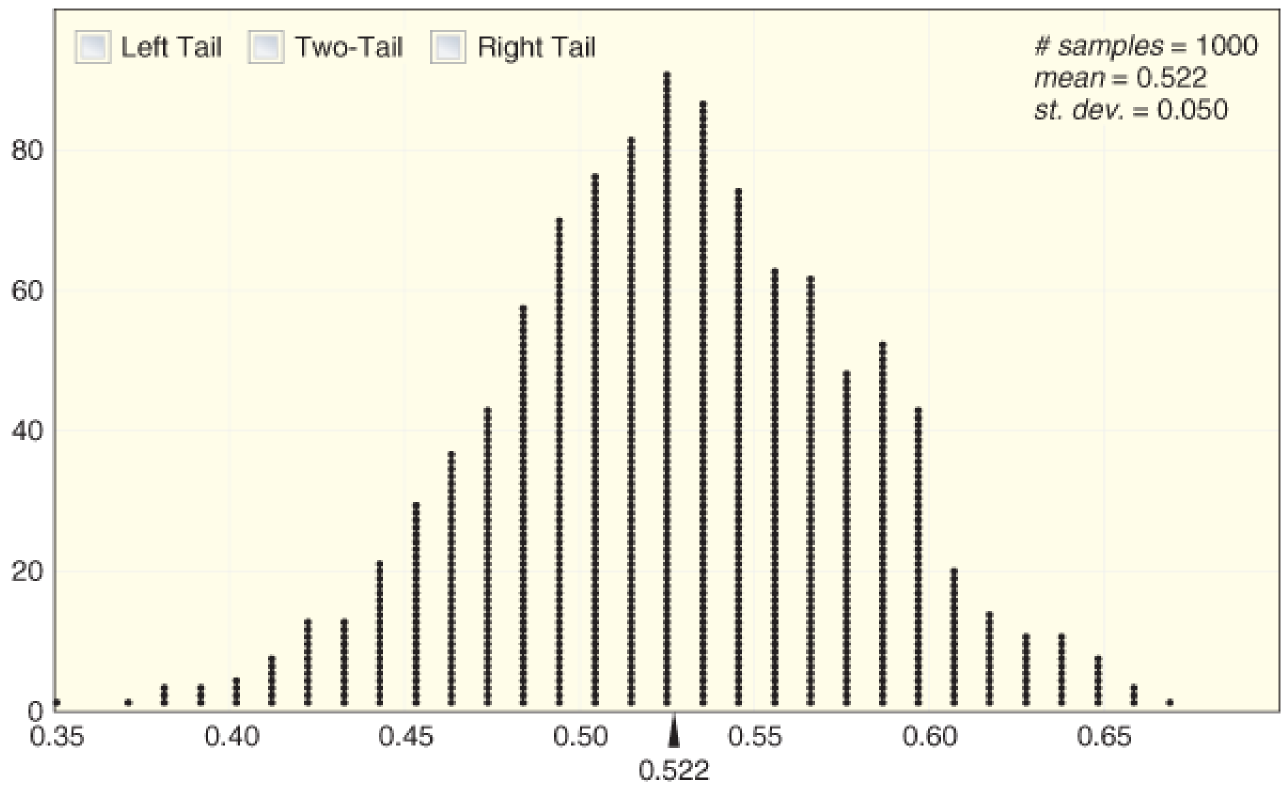


Figure 3.18 ***Bootstrap proportions for 1000 samples simulated from a sample with*** $\hat{p} = 0.52$ ***and n=100***

Example 3.20

Laughter in Adults

How often do you laugh? Estimates vary greatly in how often, on average, adults laugh in a typical day. (Different sources indicate that the average is 10, or 15, or 40, depending on the source, although all studies conclude that adults laugh significantly less than children.) Suppose that one study randomly selects six adults and records how often these adults laugh in a day, with the results given in Table 3.8.

Table 3.8 ***Number of laughs in a day***

16	22	9	31	6	42

(a) Define the parameter we are estimating and find the best point estimate from this sample.

(b) Describe how to use cards to generate one bootstrap sample. What statistic would we record for this sample?

(c) Generate several bootstrap samples this way, and compute the mean for each.

(d) If we generated many such bootstrap statistics, where will the bootstrap distribution be centered?

Solution

(a) We are estimating μ, the average number of laughs in a typical day for all adults. The best point estimate is the mean from our sample, which we calculate to be $\bar{x} = 21.0$.
(b) Since there are six values in the sample, we use six cards and put the six values on the cards. We then mix them up, pick one, and write down the value. (Since there are six values, we could also roll a six-sided die to randomly select one of the numbers.) Then we put the card back (since we are sampling with replacement), mix the cards up, and draw out another. We do this six times to obtain a bootstrap sample of size 6. Since we are interested in the mean, the statistic we record is the mean of the six values.
(c) Several bootstrap samples are shown in Table 3.9. Answers will vary, but all bootstrap samples will have the same sample size, $n=6$, and will only include values already present in the original sample.
(d) If we calculated many bootstrap statistics to generate a bootstrap distribution, it would be centered at the value of the original sample statistic, which is $\bar{x} = 21.0$.

Table 3.9 ***Three bootstrap samples***

Bootstrap Sample 1:	16	31	9	16	6	42	Mean = 20.0
Bootstrap Sample 2:	31	16	16	6	31	22	Mean = 20.33
Bootstrap Sample 3:	42	31	42	9	42	22	Mean = 31.33

Practice Problems 3.3G

Estimating Standard Error Based on a Bootstrap Distribution

The variability of bootstrap statistics is similar to the variability of sample statistics if we were to sample repeatedly from the population, so we can use the standard deviation of the bootstrap distribution to estimate the standard error of the sample statistic.

Standard Error from a Bootstrap Distribution

The standard deviation of the bootstrap statistics in a bootstrap distribution gives a good approximation of the standard error of the statistic.

Example 3.21

Use the information in Figure 3.18 to find the standard error of the sample proportion when estimating the proportion of peanuts in mixed nuts with a sample of size 100.

Solution

The information in the upper corner of Figure 3.18 indicates that the standard deviation of those 1000 bootstrap proportions is 0.050, so we use that value as an estimate of the standard error for the proportion.

The 1000 bootstrap means for Atlanta commute times in Figure 3.16 have a standard deviation of 0.915 minutes, so we have $SE = 0.915$ for the sample mean commute time based on samples of size $n = 500$. The standard error depends on the size and variability of the original sample, but not on the number of bootstrap samples (provided we use enough bootstrap samples to obtain a reasonable estimate).

Because the estimated SE is based on simulated bootstrap samples, it will vary slightly from simulation to simulation. A different set of 1000 commute bootstrap means produced a standard error estimate of 0.932 (similar to the previous estimate of 0.915), and 1000 new simulated mixed nut samples gave an estimated standard error of 0.048 (also similar to the previous estimate of $SE = 0.050$.) In practice, these subtle differences are almost always negligible. However, a more accurate estimate can easily be achieved by simulating more bootstrap samples: The more bootstrap samples, the more accurate the estimated SE will be. In this text we often use 1000 bootstrap samples so that the individual bootstrap statistics are visible in plots, but 10,000 or more bootstrap samples are more often used in practice.[33]The number of bootstrap samples you find may depend on the speed of your technology. If we create 100,000 bootstrap samples for the Atlanta commute times, the SE is 0.927 in one simulation, 0.928 in another simulation, and 0.927 in a third simulation: We are now estimating within 1 one-thousandth of a minute.

95% Confidence Interval Based on a Bootstrap Standard Error

Recall from Section 3.2 that we can use the standard error to construct a 95% confidence interval by going two standard errors on either side of the original statistic. Now, we can use this idea more practically by using the bootstrap distribution to estimate the standard error.

A 95% Confidence Interval Using a Bootstrap Standard Error

When a bootstrap distribution for a sample statistic is symmetric and bell-shaped, we estimate a 95% confidence interval using

$$\text{Statistic} \pm 2 \cdot SE$$

where SE denotes the standard error of the statistic estimated from the bootstrap distribution.

Example 3.22

Use the standard errors found in previous examples to find and interpret 95% confidence intervals for

(a) the mean Atlanta commute time, and

(b) the proportion of peanuts in mixed nuts.

In addition, give the margin of error for both intervals.

Solution

(a) The sample mean from the original sample of 500 Atlanta commuters is $\bar{x} = 29.11 \text{ minutes}$ and the estimated standard error for this mean from the bootstrap distribution in Figure 3.16 is 0.915. Going two standard errors on either side of the sample statistic gives

$$\begin{array}{rcl} \bar{x} & \pm & 2 \cdot SE \\ 29.11 & \pm & 2(0.915) \\ 29.11 & \pm & 1.83 \end{array}$$

or an interval from $29.11 - 1.83 = 27.28$ minutes to $29.11 + 1.83 = 30.94$ minutes. The margin of error is 1.83 minutes, and we are 95% confident that the mean commute time for all Atlanta commuters is between 27.28 minutes and 30.94 minutes.

(b) The original sample has a proportion of $\hat{p} = 0.52 \text{ peanuts}$, and the estimated standard error for this proportion from Example 3.21 is 0.050. Going two standard errors on either side of the estimate gives

$$\begin{array}{rcl} \hat{p} & \pm & 2 \cdot SE \\ 0.52 & \pm & 2(0.050) \\ 0.52 & \pm & 0.10 \end{array}$$

or an interval from $0.52 - 0.10 = 0.42$ to $0.52 + 0.10 = 0.62$. The margin of error is 0.10, and we are 95% confident that between 42% and 62% of all mixed nuts from this company are peanuts.

Practice Problems 3.3H

We now have a very powerful technique for constructing confidence intervals for a wide variety of parameters. As long as we can do the following:

- Find a sample statistic to serve as a point estimate for the parameter.
- Compute bootstrap statistics for many samples with replacement from the original sample.
- Estimate the standard error from the bootstrap distribution.
- Check that the bootstrap distribution is reasonably symmetric and bell-shaped.

Then we can use $statistic \pm 2 \cdot SE$ to estimate a 95% confidence interval for the parameter.

But what about other confidence levels, like 90% or 99%? We explore an alternate method for obtaining a confidence interval from a bootstrap distribution in the next section, which will address this question and provide even more general results.

SECTION LEARNING GOALS

You should now have the understanding and skills to:

- Describe how to select a bootstrap sample to compute a bootstrap statistic
- Recognize that a bootstrap distribution tends to be centered at the value of the original statistic
- Use technology to create a bootstrap distribution
- Estimate the standard error of a statistic from a bootstrap distribution
- Construct a 95% confidence interval for a parameter based on a sample statistic and the standard error from a bootstrap distribution

Exercises for Section 3.3

SKILL BUILDER 1

In Exercises 3.65 and 3.66, a sample is given. Indicate whether each option is a possible bootstrap sample from this original sample.

3.65 Original sample: 17, 10, 15, 21, 13, 18. Do the values given constitute a possible bootstrap sample from the original sample?

(a) 10, 12, 17, 18, 20, 21

ANSWER

WORKED SOLUTION

(b) 10, 15, 17

ANSWER

WORKED SOLUTION

(c) 10, 13, 15, 17, 18, 21

ANSWER

WORKED SOLUTION

(d) 18, 13, 21, 17, 15, 13, 10

ANSWER

WORKED SOLUTION

(e) 13, 10, 21, 10, 18, 17

ANSWER

WORKED SOLUTION

3.66 Original sample: 85, 72, 79, 97, 88. Do the values given constitute a possible bootstrap sample from the original sample?

(a) 79, 79, 97, 85, 88

(b) 72, 79, 85, 88, 97

(c) 85, 88, 97, 72

(d) 88, 97, 81, 78, 85

(e) 97, 85, 79, 85, 97

(f) 72, 72, 79, 72, 79

SKILL BUILDER 2

In Exercises 3.67 to 3.70, use the bootstrap distributions in Figure 3.19 to estimate the point estimate and standard error, and then use this information to give a 95% confidence interval. In addition, give notation for the parameter being estimated.

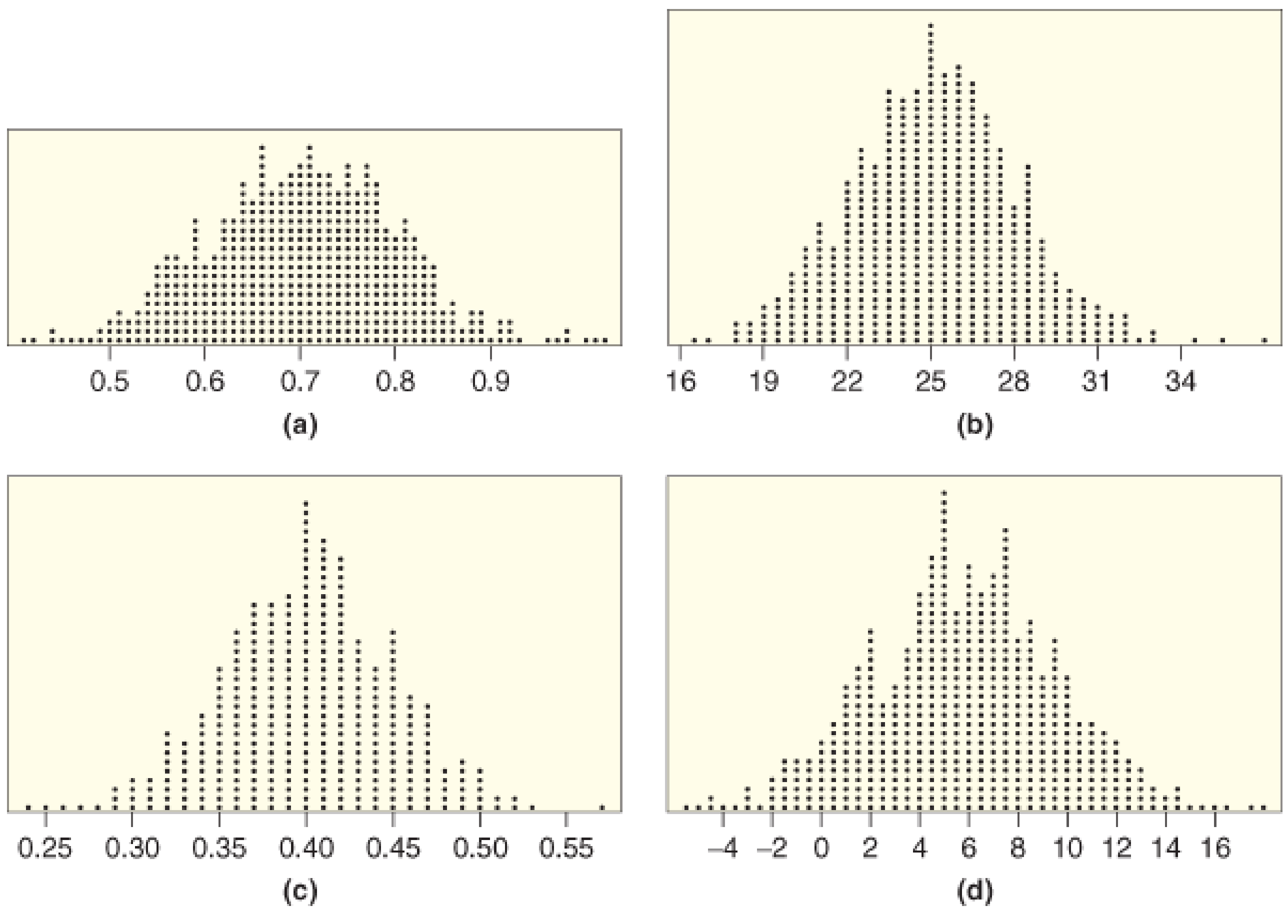


Figure 3.19 ***Four bootstrap distributions***

3.67 The bootstrap distribution in Figure 3.19(a), generated for a sample proportion

ANSWER

WORKED SOLUTION

3.68 The bootstrap distribution in Figure 3.19(b), generated for a sample mean

3.69 The bootstrap distribution in Figure 3.19(c), generated for a sample correlation

ANSWER

WORKED SOLUTION

3.70 The bootstrap distribution in Figure 3.19(d), generated for a difference in sample means

SKILL BUILDER 3

Exercises 3.71 to 3.74 give information about the proportion of a sample that agrees with a certain statement. Use *StatKey* or other technology to estimate the standard error from a bootstrap distribution generated from the sample. Then use the standard error to give a 95% confidence interval for the proportion of the population to agree with the statement. *StatKey* tip: Use“CI for Single Proportion” and then “Edit Data” to enter the sample information.

3.71 In a random sample of 100 people, 35 agree.

ANSWER

WORKED SOLUTION

3.72 In a random sample of 250 people, 180 agree.

3.73 In a random sample of 400 people, 112 agree and 288 disagree.

ANSWER

WORKED SOLUTION

3.74 In a random sample of 1000 people, 382 people agree, 578 disagree, and 40 are undecided.

3.75 **Hitchhiker Snails**

A type of small snail is very widespread in Japan, and colonies of the snails that are genetically similar have been found very far apart. Scientists wondered how the snails could travel such long distances. A recent study[34]Yong, E., “*The Scatological Hitchhiker Snail*,” *Discover*, October 2011, 13. provides the answer. Biologist Shinichiro Wada fed 174 live snails to birds and found that 26 of the snails were excreted live out the other end. The snails apparently are able to seal their shells shut to keep the digestive fluids from getting in.

(a) What is the best point estimate for the proportion of all snails of this type to live after being eaten by a bird?

ANSWER

WORKED SOLUTION

(b) Figure 3.20 shows a bootstrap distribution based on this sample. Estimate the standard error.

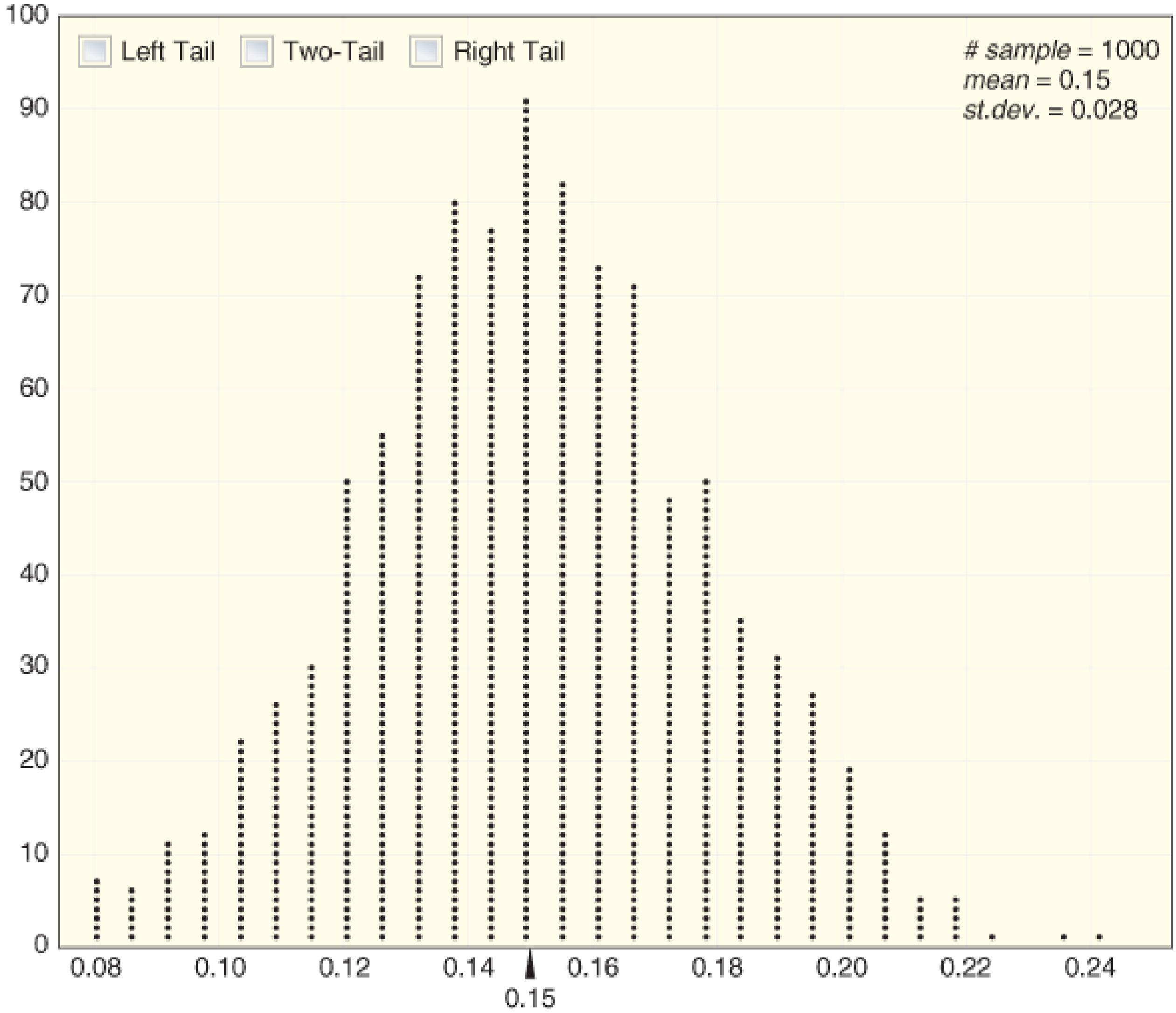


Figure 3.20 ***Bootstrap distribution of sample proportion of the snails that live***

ANSWER

WORKED SOLUTION

(c) Use the standard error from part b to find and interpret a 95% confidence interval for the proportion of all snails of this type to live after being eaten by a bird.

ANSWER

WORKED SOLUTION

(d) Using your answer to part c, is it plausible that 20% of all snails of this type live after being eaten by a bird?

ANSWER

WORKED SOLUTION

3.76 Ants on a Sandwich

How many ants will climb on a piece of a peanut butter sandwich left on the ground near an ant hill? To study this, a student in Australia left a piece of a sandwich for several minutes, then covered it with a jar and counted the number of ants. He did this eight times, and the results are shown in Table 3.10. (In fact, he also conducted an experiment to see if there is a difference in number of ants based on the sandwich filling. The details of that experiment are given in Chapter 8, and the full dataset is in **SandwichAnts**.)[35]Mackisack, M., "*Favourite Experiments: An Addendum to What Is the Use of Experiments Conducted by Statistics Students?*," *Journal of Statistics Education*, 1994, http://www.amstat.org/publications/jse/v2n1/mackisack.supp.html.

Table 3.10 ***Number of ants on a sandwich***

Number of ants	43	59	22	25	36	47	19	21

(a) Find the mean and standard deviation of the sample.

(b) Describe how we could use eight slips of paper to create one bootstrap statistic. Be specific.

(c) What do we expect to be the shape and center of the bootstrap distribution?

(d) What is the population parameter of interest? What is the best point estimate for that parameter?

(e) A bootstrap distribution of 5000 bootstrap statistics gives a standard error of 4.85. Use the standard error to find and interpret a 95% confidence interval for the parameter defined in part d.

3.77 Skateboard Prices

A sample of prices of skateboards for sale online[36]Random sample taken from all skateboards available for sale on eBay on February 12, 2012. is shown in Table 3.11 and is available in the dataset **SkateboardPrices**.

Table 3.11 ***Prices of skateboards for sale online***

19.95	24.99	39.99	34.99	30.99	92.50	84.99	119.99	19.99	114.99
44.99	50	84.99	29.91	159.99	61.99	25	27.50	84.99	199

(a) What are the mean and standard deviation of the 20 skateboard prices?

ANSWER

WORKED SOLUTION

(b) Describe how to use the data to select one bootstrap sample. What statistic is recorded from the sample?

ANSWER

WORKED SOLUTION

(c) What shape and center do we expect the bootstrap distribution to have?

ANSWER

WORKED SOLUTION

(d) One bootstrap distribution gives a standard error of 10.9. Find and interpret a 95% confidence interval.

ANSWER

WORKED SOLUTION

3.78 Saab Sales

Saab, a Swedish car manufacturer, is interested in estimating average monthly sales in the US, using the following sales figures from a sample of five months:[37]http://www.saabsunited.com/saab-sales-data.

$$658,\ 456,\ 830,\ 696,\ 385$$

Use *StatKey* or other technology to construct a bootstrap distribution and then find a 95% confidence interval to estimate the average monthly sales in the United States. Write your results as you would present them to the CEO of Saab.

3.79 Rats with Compassion

The phrase "You dirty rat" does rats a disservice. In a recent study,[38]Bartal, I.B., Decety, J., and Mason, P., "*Empathy and Pro-Social Behavior in Rats*," *Science*, 2011; 224(6061): 1427-1430. rats showed compassion that surprised scientists. Twenty-three of the 30 rats in the study freed another trapped rat in their cage, even when chocolate served as a distraction and even when the rats would then have to share the chocolate with their freed companion. (Rats, it turns out, love chocolate.) Rats did not open the cage when it was empty or when there was a stuffed animal inside, only when a fellow rat was trapped. We wish to use the sample to estimate the proportion of rats to show empathy in this way. The data are available in the dataset **CompassionateRats**.

(a) Give the relevant parameter and its point estimate.

ANSWER

WORKED SOLUTION

(b) Describe how to use 30 slips of paper to create one bootstrap statistic. Be specific.

ANSWER

WORKED SOLUTION

(c) Use *StatKey* or other technology to create a bootstrap distribution. Describe the shape and center of the bootstrap distribution. What is the standard error?

ANSWER

WORKED SOLUTION

(d) Use the standard error to find and interpret a 95% confidence interval for the proportion of rats likely to show empathy.

ANSWER

WORKED SOLUTION

3.80 Are Female Rats More Compassionate Than Male Rats?

Exercise 3.79 describes a study in which rats showed compassion by freeing a trapped rat. In the study, all six of the six female rats showed compassion by freeing the trapped rat while 17 of the 24 male rats did so. Use the results of this study to give a point estimate for the difference in proportion of rats showing compassion, between female rats and male rats. Then use *StatKey* or other technology to estimate the standard error[39]In practice we should raise a caution here, since the proportion for female rats will be $\hat{p}=1$ for every bootstrap sample. and use it to compute an interval estimate for the difference in proportions. Use the interval to determine whether it is plausible that male and female rats are equally compassionate (i.e., that the difference in proportions is zero). The data are available in the dataset **CompassionateRats**.

3.81 Teens Are More Likely to Send Text Messages

Exercise 3.27 compares studies which measure the proportions of adult and teen cell phone users that send/receive text messages. The summary statistics are repeated below:

Group	Sample Size	Proportion
Teen	n_t=800	$\hat{p}_t=0.87$
Adult	n_a=2252	$\hat{p}_a=0.72$

Figure 3.21 shows a distribution for the differences in sample proportions $(\hat{p}_t - \hat{p}_a)$ for 5000 bootstrap samples (taking 800 values with replacement from the original teen sample and 2252 from the adults).

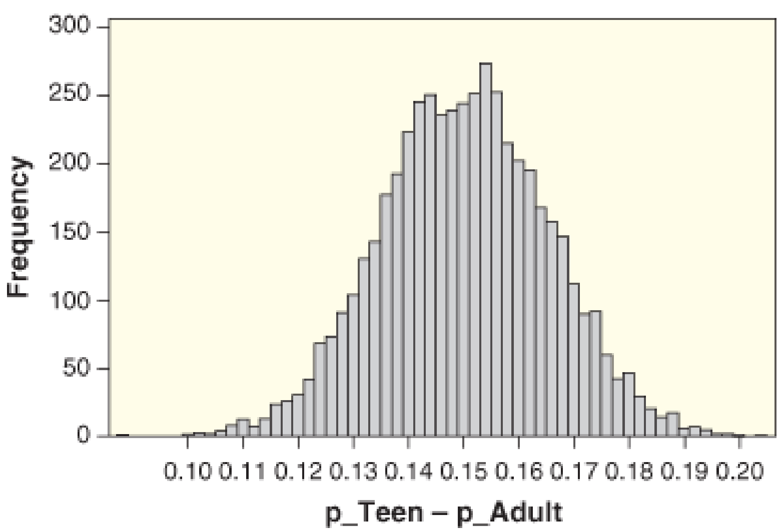


Figure 3.21 ***Bootstrap difference in sample proportions of teen and adult cell phone users who text***

(a) Based on the bootstrap distribution, which is the most reasonable estimate of the standard error for the difference in proportions: SE=0.015 , 0.030, 0.050, 0.10, or 0.15? Explain the reason for your choice.

ANSWER ⊕

WORKED SOLUTION ⊕

(b) Using your choice for the SE estimate in part a, find and interpret a 95% confidence interval for the difference in proportion of teen and adult cell phone users who send/receive text messages.

ANSWER

WORKED SOLUTION

3.82 Tea, Coffee, and Your Immune System

Researchers suspect that drinking tea might enhance the production of interferon gamma, a molecule that helps the immune system fight bacteria, viruses, and tumors. A recent study[40]Adapted from Kamath et.al., "*Antigens in Tea-Beverage Prime Human* $V\gamma2V\delta2$ T *Cells in vitro and in vivo for Memory and Non-memory Antibacterial Cytokine Responses*," Proceedings of the National Academy of Sciences, May 13, 2003. involved 21 healthy people who did not normally drink tea or coffee. Eleven of the participants were randomly assigned to drink five or six cups of tea a day, while 10 were asked to drink the same amount of coffee. After two weeks, blood samples were exposed to an antigen and production of interferon gamma was measured.[41]To be specific, peripheral blood mononuclear cells were cultured with the antigen alkylamine ethylalamine in an enzyme-linked immunospot assay to the frequency of interferon-gamma-producing cells. The results are shown in Table 3.12 and are available in **ImmuneTea**. We are interested in estimating the effect size, the increase in average interferon gamma production for drinking tea when compared to coffee. Use *StatKey* or other technology to estimate the difference in mean production for tea drinkers minus coffee drinkers. Give the standard error for the difference and a 95% confidence interval. Interpret the result in context.

Table 3.12 ***Immune system response in tea and coffee drinkers***

Tea	5	11	13	18	20	47
	48	52	55	56	58	
Coffee	0	0	3	11	15	16
	21	21	38	52		

3.83 Better Traffic Flow

Exercise 2.144 introduces the dataset **TrafficFlow**, which gives delay time in seconds for 24 simulation runs in Dresden, Germany, comparing the current timed traffic light system on each run to a proposed flexible traffic light system in which lights communicate traffic flow information to neighboring lights. On average, public transportation was delayed 105 seconds under the timed system and 44 seconds under the flexible system. Since this is a matched pairs experiment, we are interested in the difference in times between the two methods for each of the 24 simulations. For the $n=24$ differences D, we saw in Exercise 2.144 that $\bar{x}_D = 61$ seconds with $s_D=15.19$ seconds . We wish to estimate the average time savings for public transportation on this stretch of road if the city of Dresden moves to the new system.

(a) What parameter are we estimating? Give correct notation.

ANSWER

WORKED SOLUTION

(b) Suppose that we write the 24 differences on 24 slips of paper. Describe how to physically use the paper slips to create a bootstrap sample.

ANSWER

WORKED SOLUTION

(c) What statistic do we record for this one bootstrap sample?

ANSWER

WORKED SOLUTION

(d) If we create a bootstrap distribution using many of these bootstrap statistics, what shape do we expect it to have and where do we expect it to be centered?

ANSWER

WORKED SOLUTION

(e) How can we use the values in the bootstrap distribution to find the standard error?

ANSWER

WORKED SOLUTION

(f) The standard error is 3.1 for one set of 10,000 bootstrap samples. Find and interpret a 95% confidence interval for the average time savings.

ANSWER

WORKED SOLUTION

3.84 Commuting Distances in Atlanta

In addition to the commute *time* (in minutes), the **CommuteAtlanta** dataset gives the *distance* for the commutes (in miles) for 500 workers sampled from the Atlanta metropolitan area.

(a) Find the mean and standard deviation of the commute distances in **CommuteAtlanta**.

(b) Use *StatKey* or other technology to create a bootstrap distribution of the sample means of the distances. Describe the shape and center of the distribution.

(c) Use the bootstrap distribution to estimate the standard error for mean commute distance when using samples of size 500.

(d) Use the standard error to find and interpret a 95% confidence interval for the mean commute distance of Atlanta workers.

3.85 Correlation between Distance and Time for Atlanta Commutes

The data in **CommuteAtlanta** contains information on both the *Distance* (in miles) and *Time* (in minutes) for a sample of 500 Atlanta commutes. We expect the correlation between these two variables to be positive, since longer distances tend to take more time.

(a) Find the correlation between *Distance* and *Time* for the original sample of 500 Atlanta commutes.

ANSWER

WORKED SOLUTION

(b) The file **BootAtlantaCorr** contains the correlations of *Distance* vs *Time* for 1000 bootstrap samples using the Atlanta commuting data, or use *StatKey* or other technology to create your own bootstrap distribution. Create a plot and describe the shape and center of the bootstrap distribution of these correlations.

ANSWER

WORKED SOLUTION

(c) Use the statistics in the bootstrap distribution to estimate the margin of error and create an interval estimate for the correlation between distance and time of Atlanta commutes.

ANSWER

WORKED SOLUTION

(d) Mark where the interval estimate lies on your plot in part b.

ANSWER

WORKED SOLUTION

3.86 NHL Penalty Minutes

Table 3.4 shows the number of points scored and penalty minutes for 24 ice hockey players on the Ottawa Senators NHL team for the 2009-2010 season. The data are also stored in **OttawaSenators**. Assume that we consider these players to be a sample of all NHL players.

(a) Create a dotplot of the distribution of penalty minutes (*PenMin*) for the original sample of 24 players. Comment on the shape, paying particular attention to skewness and possible outliers.

(b) Find the mean and standard deviation of the penalty minute values for the original sample.

(c) Use *StatKey* or other technology to construct a bootstrap distribution for the mean penalty minutes for samples of size $n=24$ NHL players. Comment on the shape of this distribution, especially compared to the shape of the original sample.

(d) Compute the standard deviation of the bootstrap means using the distribution in part c. Compare this value to the standard deviation of the penalty minutes in the original sample.

(e) Construct an interval estimate for the mean penalty minutes of NHL players.

(f) Give a reason why it might *not* be reasonable to use the players on one team as a sample of all players in a league.

3.87 Standard Deviation of NHL Penalty Minutes

Exercise 3.86 describes data on the number of penalty minutes for Ottawa Senators NHL players. The sample has a fairly large standard deviation, $s=49.1$ minutes. Use *StatKey* or other technology to create a bootstrap distribution, estimate the standard error, and give a 95% confidence interval for the *standard deviation* of penalty minutes for NHL players. Assume that the data in **OttawaSenators** can be viewed as a reasonable sample of all NHL players.

ANSWER

WORKED SOLUTION