Chapter 6 Expenses, Profit, and More

A. Introduction

Note: Bowers (*Actuarial Mathematics*) uses the term "expense-loaded premium" to denote premium that includes expenses in addition to the benefit premium. Cunningham (*Models for Quantifying Risks*) uses the terms "gross premium" and "expense augmented premium" to denote premium that includes expenses in addition to the benefit. Both texts use the symbol ***G*** to denote this premium.

The new textbook from Dickson (*Actuarial Mathematics for Life Contingent Risks*) doesn't address a chapter to Expenses, but does incorporate expenses into calculations (eventually). It's touched on in the Premium chapter (6.6) for the first time, then broadened in the Reserve chapter.

So far, we've looked at **Net** Present Values of all calculations. This assumed that there were:

- No expenses
- No dividends
- No profits

B. General Approach

Add information regarding expenses to what we've already done.

Equivalence Principle:

Present Value of Future Premiums = Present Value of Future Benefits + Present Value of Future Expenses

Loss Function:

$E\left[{}_0L\right]=0$ becomes $E\left[{}_0L^e\right]=0$ or $E\left[L_0^g\right]$ in *Dickson*

1) Side Note about Annuities

Expenses are often paid at the end of a year, so we will use the relationship $a_x = \ddot{a}_x - 1$

2) Expense Loaded Premiums

$$\underbrace{G}_{\substack{\text{Expense}\\\text{Loaded}\\\text{Premium}}} = \underbrace{P}_{\substack{\text{Net}\\\text{Premium}}} + \underbrace{e}_{\text{Expense}}$$

C. Types of Expenses

For our analysis, we'll assume only certain expenses exist:

1) **Percent of Gross Premium**
   a. Commissions to agents
      i. Typically, early year commissions are larger than later ("renewal") years
   b. State premium taxes
      i. Typically constant each year
2) **Fixed amount <u>per unit</u> of benefit**
   a. Underwriting expense (classifying risks…)
   b. Policy-issue expense (paper, computer time, …)
   c. Maintenance of policy expense
      i. Typically, higher in first year
3) **Fixed amount <u>per contract</u>**
   a. General Company expenses
4) **Fixed amount/percent of benefit payment <u>when payment made</u>**
   a. "Settlement" expense
   b. One time payment
   c. Paid to/by Insurance Company when benefit paid
   d. Examples: writing check, closing file

Example 6-1 – Expenses

An insurance company issues a $25,000 fully discrete whole life insurance to a 45-year old. The expense-loaded level annual premium is G. No withdrawals ("surrenders") are allowed.

Expenses occur at the beginning of the year (except for "settlement" expense):

	First Year	Renewal Years
Percent of Premium	66%	2%
Per Policy	$60	$20
Per $1,000 of insurance	$2	$1

Per Death Claim: $60

You are given A_{45}=0.350; d=0.065

Find G

PVPremium=PVB + PVExpenses

$$G\ddot{a}_{45} = 25000A_{45} + \left[0.66G + 0.03Ga_{45}\right] + \left[60 + 20a_{40}\right] + \left[\tfrac{2}{1000}\cdot 25000 + \tfrac{1}{1000}\cdot 25000a_{45}\right] + 60A_{45}$$

$$A_{45} = 0.350$$

$$\ddot{a}_{45} = \frac{1-A_{45}}{d} = \frac{1-0.35}{0.065} = 10.00$$

$$a_{45} = 9.00$$

$$G = \frac{25000A_{45} + 60 + 20a_{45} + 50 + 25a_{45} + 60A_{45}}{\ddot{a}_{45} - 0.66 - 0.02a_{45}} = 1012.65$$

$$25000P = 25000\frac{A_{45}}{\ddot{a}_{45}} = 875$$

$$Exp = (1012.65 - 875) = 137.65$$

Alternative

$$G = \frac{25000A_{45} + (40 + 20\ddot{a}_{45}) + (25 + 25\ddot{a}_{45}) + 60A_{45}}{\ddot{a}_{45} - (0.64 + 0.02\ddot{a}_{45})}$$

Problem 6-1

An insurance company issues fully-discrete whole life insurances of various face values to 40-year olds. Premiums are calculated using expense-augmented equivalence principle. The only decrement is death (i.e. no surrender).

Expenses occur at the beginning of the year (except for "settlement" expense):

	First Year	*Renewal Years*
Percent of Premium	80%	10%
Per Policy	\$50	\$15
Per \$1,000 *of insurance*	\$2.25	\$1.25

Per Death Claim: \$125

A_{40}=0.250; i=8%

*Find an expression for **G** if the policy size is an unknown **Y***

For \$100,000 *in Face value, find **G***

Problem 6-2

An insurance company issues a fully-discrete 20-year term insurance of ***Y*** *to x. The expense -augmented equivalence principle premium for a face amount of* ***Y*** *is $12 per $1,000 of insurance. The only decrement is death (i.e. no surrender).*

Expenses occur at the beginning of the year (except for "settlement" expense):

	First Year	Renewal Years
Percent of Premium	*90%*	*4%*
Per Policy	*$40*	*$10*
Per $1,000 of insurance	*$2.50*	*$1.25*

Per Death Claim: $75

$A^{1}_{x:\overline{20}|} = 0.10; \ddot{a}_{x:\overline{20}|} = 12.00$

Find **Y**

1) Policy Fees and Average Policy Size

When looking at "fixed costs" and the average policy size, there are two broad ways to approach:

1) **Charge everyone a premium that has two pieces: (a) a "per $1,000" rate and (b) an additional policy fee.**

   This allows us to charge each policyholder the *same expense-loaded premium*, based on their policy size. This is true because the "exact" expense-loaded premium (***G***) as is a linear function of the face amount (***Y***). That is, ***G***=*a**Y***+*b*, where *a* and *b* are constants. The coefficient of the face amount (that is, "*a*") is the "per $1,000" premium, and the constant ("*b*") is the "policy fee." Since the exact expense loaded premium is a linear function of the death benefit, we can say:

   $G^{\text{Exact}}(F) = r_1 \cdot F + PolicyFee$

   If we have no insurance (i.e. *F*ace=0), then ***G***(0)=Policy Fee, which makes sense.

2) **Charge everyone the *same expense loaded premium per $1,000 of insurance*.**

   This is known as the "approximate premium rate method"

   $G^{\text{Approx}}(F) = r_2 \cdot F$

   Again, ***G***(0)=0, which makes sense.

   Also, at the average policy size, $G^{Approx}(AvgPolSize) = G^{Exact}(AvgPolSize)$

   We now know 2 points, so we can describe the line as

$$G^{Approx}(F) = \underbrace{\frac{G^{Exact}(AvgPolSize)}{AvgPolSize}}_{\text{This is } r_2} \cdot F$$

> Problem 6-3
>
> *Use Example 6-1 as a starting point. Average policy size is \$25,000. The insurance company uses the expense-loaded annual premium by the approximate premium rate method.*
>
> *Find the premium rate per \$1,000 of insurance.*

> Problem 6-4
>
> *Use Problem 6-1 as a starting point: Premiums are calculated using the expense-augmented equivalence principle and policy fee method.*
>
> *Find the policy fee*

D. Modification to Recursion formula

Now that we have expenses to deal with, we need to change the recursion formula from the Prior Topic to reflect this:

(New Notation is noted in the formula)

$$_{k+1}V = \frac{\left({}_kV + \overbrace{G_k}^{\text{Gross Premium}} - \overbrace{e_k}^{\text{Expense}} \right)(1+i_k) - \left(b_{k+1} + \overbrace{E_{k+1}}^{\text{Settlement Expense}} \right) q_{x+k}}{p_{x+k}}$$

$$\left({}_kV + G_k - e_k\right)(1+i) = \left(b_{k+1} - E_{k+1}\right) q_{x+k} + p_{x+k} \cdot {}_{k+1}V$$

E. Profit

Consider the period between years k and k+1

- A block of policies at the beginning of this period has a total of N_k (active) policies.
- ${}_kV$ and ${}_{k+1}V$ are the <u>gross</u> premium reserves at the beginning and end of the period, on a per policy basis.
- On an expected basis, the ending (total) gross premium reserve for this block of policies is ${}_{k+1}V^E = N_k \cdot {}_{k+1}V$
- Applying the recursion equation, we can express this total reserve as ${}_{k+1}V^E = \left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e_k\right)(1+i) - \left(b_{k+1} - {}_{k+1}V\right) \underbrace{N_k \cdot q_x}_{\text{Expected deaths}}$
- Call the actual gross premium for this block of business ${}_{k+1}\boldsymbol{V}^{\text{A}}$

Insurer's profit for the period is the difference $\text{Profit}_k = {}_{k+1}V^A - {}_{k+1}V^E$

1) Sources of Profit

Interest: Gain if better return; loss of worse

Mortality: Gain if few deaths; loss if more

Expense: Gain if lower expenses; loss if higher

2) Gain from Interest

Assume that for the period, the insurance company earned *i'* instead of *i*

Assume all other experience was as expected

Total Actual reserve is ${}_{k+1}V^A = \left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e_k\right)(1+i') - \left(b_{k+1} + E_{k+1} - {}_{k+1}V\right) N_k \cdot q_k$

So, the gain from interest is $N_k\left({}_kV + G_k - e_k\right)(i' - i)$

3) Gain from expenses

Assume that for the period, the insurance company has *e'* instead of *e* expenses

Assume all other experience was as expected

Total Actual reserve is ${}_{k+1}V^A = \left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e'_k\right)(1+i) - \left(b_{k+1} + E_{k+1} - {}_{k+1}V\right) N_k \cdot q_k$

So, the gain from expense is $N_k\left(e_k - e'_k\right)(1+i)$ (Note that it's a "gain" if expenses are lower than we think, so the ***e-e'*** is positive when this happens

4) Gain from mortality

Assume that for the period, there are ***D'$_k$*** deaths (instead of $D_k = N_k \cdot q_k$)

Assume all other experience was as expected

Total Actual reserve is ${}_{k+1}V^A = \left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e_k\right)(1+i) - \left(b_{k+1} + E_{k+1} - {}_{k+1}V\right) D'_k$

So, the gain from expense is $\underbrace{\left(b_{k+1} + E_{k+1} - {}_{k+1}V\right)}_{\text{Remember...}}\left(N_k \cdot q_{x+k} - D'_k\right)$

5) Combine all sources of Gain/Loss

$$\begin{aligned}{}_{k+1}V^A = &\left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e'_k\right)(1+i') - \left(b_{k+1} + E'_{k+1} - {}_{k+1}V\right) D'_k \\ &- \left(N_k \cdot {}_kV + N_k \cdot G_k - N_k \cdot e_k\right)(1+i) - \left(b_{k+1} + E_{k+1} - {}_{k+1}V\right) N_k \cdot q_{x+k}\end{aligned}$$

Profit_k=Gain from interest + Gain from expense + Gain from mortality

Typically, it's (a) interest (b) expense (c) mortality when evaluating gain/loss

Problem 6-5

An insurer issued a large number of policies to women aged 60: 20-year endowment insurance with a sum insured of $100 000 payable at the end of the year of death or on survival to age 80, whichever occurs first.

Survival model: Standard Select Survival Model (from *Dickenson)*

Interest: 5% per year effective

Expenses: 10% of the first premium, 5% of subsequent premiums, and $200 on payment of the sum insured

An annual premium of $5,200 is payable for at most 10 years.

${}_5\boldsymbol{V}$=29,068; ${}_6\boldsymbol{V}$=35,324

Five years after they were issued, a total of 100 of these policies were still in force.

In the following year,

• expenses of 6% of each premium paid were incurred,

• interest was earned at 6.5% on all assets,

• one policyholder died, and

• expenses of $250 were incurred on the payment of the sum insured for the policyholder who died.

(a) Calculate the profit or loss on this group of policies for this year.

(b) Determine how much of this profit/loss is attributable to profit/loss from mortality, from interest and from expenses.

F. Asset Shares

“Asset Shares” are the accumulation of the amount of assets per policy that the insurance company will have available if all assumptions are met. Each year:

1. Premium is received at beginning of year
2. Expenses are paid at beginning of year
3. Interest is earned during year
4. Death benefits are paid at end of year for deaths during year
5. Payouts are made at end of year for surrender during year
6. Amount of remaining assets per policy increases, since number of policies is smaller, increases

We need to use a ***recursive approach*** to solve.

5.A.1 Example of Asset Share

Level premium ***G***.

Benefit is paid at death or withdrawal/surrender.

$b_k^{(d)}$ is death benefit in year k

$b_k^{(w)} = {}_k CV$ is cash surrender value ("CSV") in year k

Asset Share at time 0 is equal to 0 $\left({}_0 AS = 0\right)$

Benefits are paid at the end of each year (both death and surrender) $\underbrace{{}_k AS}_{\substack{\text{Asset}\\\text{Share}\\\text{at time}\\k}} = \underbrace{AVAP}_{\substack{\text{Accumulated}\\\text{Value of}\\\text{Accrued}\\\text{Premium}}} + \underbrace{AVAB/E}_{\substack{\text{Accumulated}\\\text{Value of}\\\text{Accrued}\\\text{Benefit/}\\\text{Expense}}}$

$$\left[{}_k AS + G\left(1 - \underbrace{c_k}_{\substack{\text{Percent}\\\text{of Premium}}}\right) - \underbrace{e_k}_{\substack{\text{Per}\\\text{Policy}}}\right](1+i) = q_{x+k}^{(d)} \cdot b_k^{(d)} + q_{x+k}^{(w)} \cdot {}_{k+1}CV + p_{x+k}^{(\tau)} \cdot {}_{k+1}AS$$

$$\left[{}_k AS + G\left(1-c_k\right) - e_k\right] = \underbrace{v \cdot q_{x+k}^{(d)} \cdot b_{x+k}^{(d)} + v \cdot q_{x+k}^{(w)} \cdot {}_{k+1}CV}_{\text{Benefit times } v \text{ times } q} + vp_{x+k}^{(\tau)} \cdot {}_{k+1}AS$$

$${}_{k+1}AS = \frac{\left[{}_k AS + G\left(1-c_k\right) - e_k\right]\left(1+i\right) - q_{x+k}^{(d)} \cdot b_{x+k} - q_{x+k}^{(w)} \cdot {}_{k+1}CV}{p_{x+k}^{(\tau)}}$$

Problem 6-6

A fully discrete whole life insurance policy with face value of $1,000 is issued to x

k	$q_{x+k}^{(d)}$	$q_{x+k}^{(w)}$	${}_k CV$	${}_k AS$
10	*0.025*	*0.075*	*105.00*	*85.00*
11	*0.028*	*0.070*	*120.00*	*111.70*
12	*0.028*	*0.065*	*136.00*	*Unknown*

The Gross premium payable at the beginning of each year is 45

Expenses are payable at the beginning of the year, and are equal to $6, plus 5% of the premium

What interest rate is used to compute the Asset Share?