

# Improving Kidney Exchange Programs

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**DSPT – Porto Meetup**

- 1 Introduction: Kidney Exchange Programmes (KEPs)
- 2 KEP: cycle formulation
- 3 KEP: possible objectives
- 4 Conclusion

# Kidney Failure Treatments

- Kidney failure
  - One kidney → OK
  - Both kidneys → Dialysis or Transplantation
- Dialysis vs Transplantation
  - Transplantation yields longer survivability
  - Transplantation yields a better quality of life
  - Dialysis is more expensive than transplantation; values for Portugal:
    - Hemodialysis → 30K euro per year per person
    - Transplantation: 30K euro once + 10K euro year

# Sources of kidneys for transplantation

- Deceased donors
  - very large waiting lists (5 years or more waiting)
- Living donors:
  - relatives, spouse, friends, altruistic donors
  - many ethical and legal issues (varies with country)
    - e.g. no commercial transaction of kidneys is generally accepted

# Sources of incompatibility

- Blood type compatibilities

Donor	Recipient			
	O	A	B	AB
O	✓	✓	✓	✓
A	✗	✓	✗	✓
B	✗	✗	✓	✓
AB	✗	✗	✗	✓

- Tissue type incompatibility
  - HLA (Human Leukocyte Antigens)
  - ...

# Introduction: background

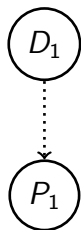


# Introduction: background

$D_1$

$P_1$

# Introduction: background

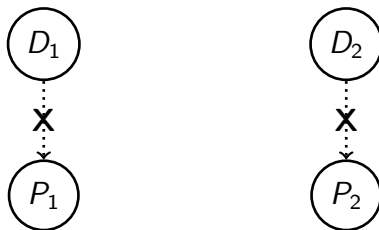




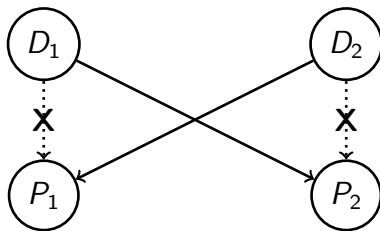
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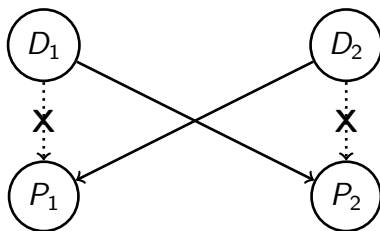
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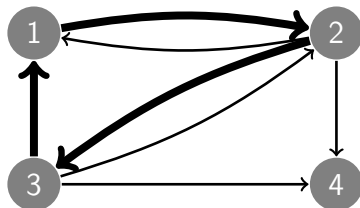
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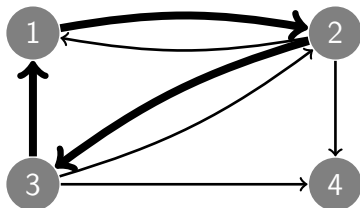
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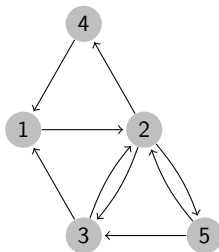


- Exchange between incompatible pairs in a directed graph
  - thin arrows → preliminary assessment compatibility
  - thick arrows → matching

# Kidney exchange programs

- KEPs were first proposed by (Rapaport, 1986)
- First transplants within a KEP were done in South Korea, 1991
- Many countries have now KEPs (USA, Switzerland, Turkey, Romania, Netherlands, UK, Canada, Australia, New Zealand, Spain)
- A KEP started in Portugal in 2011; presently, transplants are routinely performed

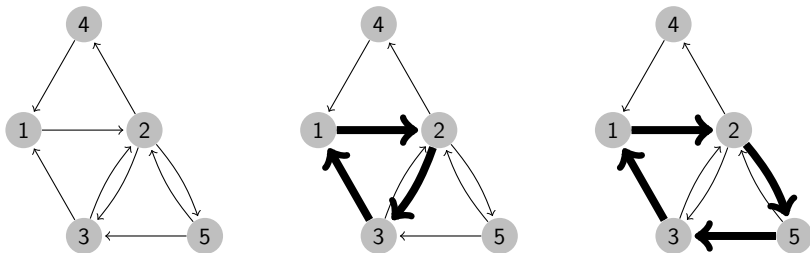
# Kidney exchanges: example



- instance with five pairs
- what is the **maximum number** of transplants?
- what if the allowed number of **simultaneous** transplants is limited?



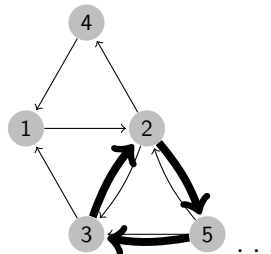
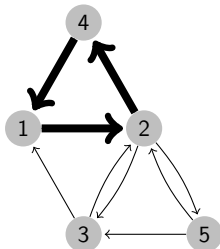
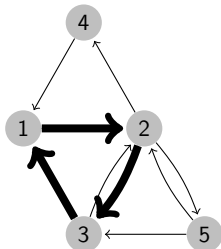
# Kidney exchanges: example



- **feasible exchange:** a set of vertex-disjoint cycles (e.g.,  $1 - 2 - 3 - 1$ )
- size of an exchange: sum of the lengths of its cycles
- maximum exchange in this example: 4 (cycle  $1 - 2 - 5 - 3 - 1$ )

# Kidney exchanges: maximum cycle size

- In many situations the **length of each cycle is limited**
- If maximum cycle size is  $K = 3$ , several solutions are possible.



# Kidney exchanges: why limiting size

- Two main reasons:
  - usually, all transplants in a cycle should be done **at same time**
    - someone could withdraw from the program
  - **last-minute** incompatibility test (crossmatch, just before transplantation)
    - if positive, no transplantation can be done for **any pair** in this cycle
    - (**rearrangements** may change the previous limitation)
- However, optimum number of transplants **increases** with maximum size allowed
- Most programs have  $k = 2$  or  $k = 3$

# Kidney Exchange Model

## Problem

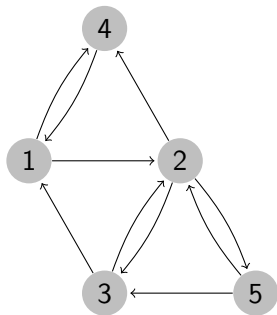
- Given:
  - a pool of  $n$  incompatible donor-patient pairs
  - the compatibility between all donors and all patients
- find the **maximum number** of kidney exchanges with cycles of size at most  $k$

## Complexity

- EASY, if  $k = 2$  or no limit is imposed on the size of the cycles
- HARD, if  $k = 3, 4, 5, \dots$

# Mathematical programming formulations

- There are several possibilities for modeling the problem in mathematical programming
- One of the most successful is the **cycle formulation**:
  - enumerate all cycles in the graph with length at most  $K$
  - for each cycle  $c$ , let variable  $x_c$  be 1 if  $c$  is chosen, 0 otherwise
  - every feasible solution corresponds to a set of vertex-disjoint cycles



Cycles for  $K = 3$ :

- ① [1,2,3]
- ② [1,2,4]
- ③ [1,4]
- ④ [2,3]
- ⑤ [2,5]
- ⑥ [2,5,3]
- ⑦ ~~[1,2,5,3]~~

# Cycle formulation

$$\text{maximize} \quad \sum_c w_c x_c \quad (1)$$

$$\begin{aligned} \text{subject to} \quad & \sum_{c:i \in c} x_c \leq 1 \quad \forall i \in V \\ & x_c \in \{0, 1\} \quad \forall c \end{aligned} \quad (2)$$

- case of 0 – 1 weights:  $w_c = |c|$ , (length of cycle  $c$ )
- objective: maximize the weight of the exchange
- constraints: every vertex is at most in one cycle
  - (i.e., donate/receive at most one kidney)
- difficulty: exponential number of variables
  - in our experience: not an issue in practice

# Objective

- Ethically, a sensitive subject
- Most common: **maximize total number of transplant**
  - easy to explain/justify
  - difficult to dispute
- Our claim: **this is not adequate**
  - KEPs are typically run periodically
  - most of the pairs will eventually be matched, after waiting a few periods
  - hence, we should look for alternatives
    - somehow access the **quality** of the transplants

# Objective: our proposal

- For each pair  $i$ 
  - for each possible matching  $j$  for that pair
    - $i$ 's patient may receive kidney of  $j$ 's donor
    - determine **expected survival time** for patient  $s_{ji}$
- Then, maximize  $\sum s_{ji}$ , for all arcs in the solution
- This can be done by assigning the weight  $w_c$  for each cycle:

$$w_c = \sum_{ji \in c} s_{ji}$$

- **Problem:**
  - $n^2$  weights to determine  $\rightarrow$  **not practical, if done manually**
  - solution?



# Predicting survival time

- There is a considerable amount of **historical data**
- Idea: **use *historical data* to train a machine learning model**
- This has been done in the past, but to our knowledge not to parameterize a KEP optimization model
- See, eg., Living Kidney Donor Risk Index (LKDPI)  
<http://www.transplantmodels.com/lkdpi/>

## LKDPI Score:

9

This model calculates a risk score for a recipient of a potential live donor kidney.

### Live Donor Characteristics:

Donor age:

43



Donor sex:

male



Recipient sex:

female



Donor eGFR:

95



Donor SBP:

130



Donor BMI:

24



Donor is African-American:

No



Donor history of cigarette use:

No



Donor and recipient biologically related:

Yes



Donor and recipient are ABO incompatible:

No



Donor Weight:

70 kg/155 lb



Recipient Weight:

80 kg/178 lb



Donor and recipient HLA-B mismatches:

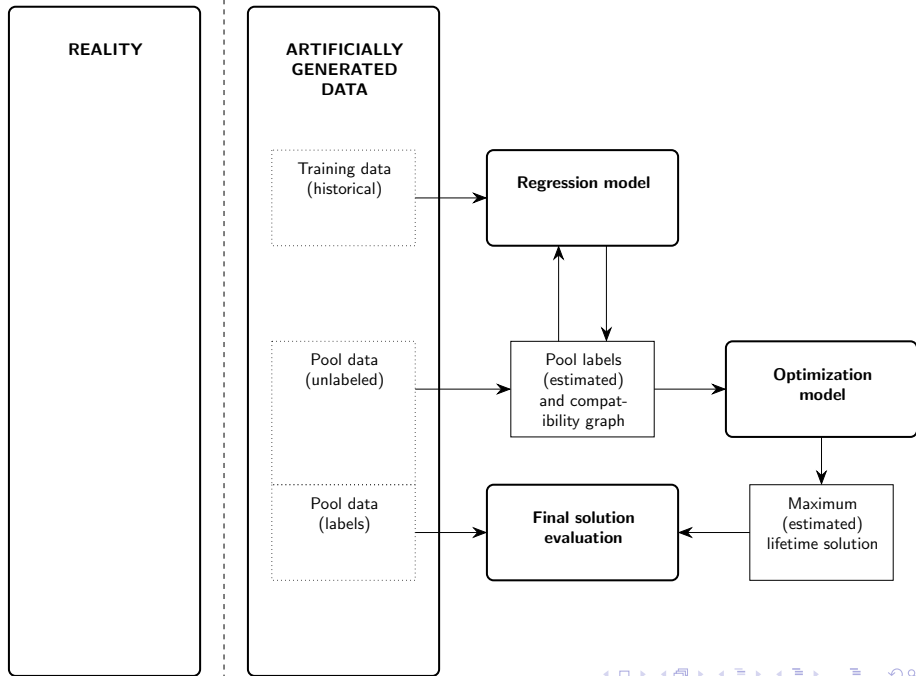
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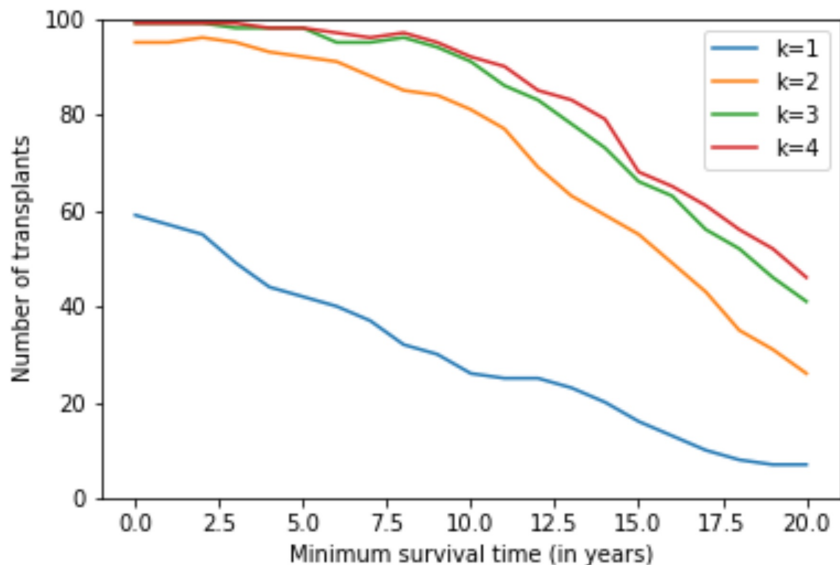
Donor and recipient HLA-DR

# Proposed flow:

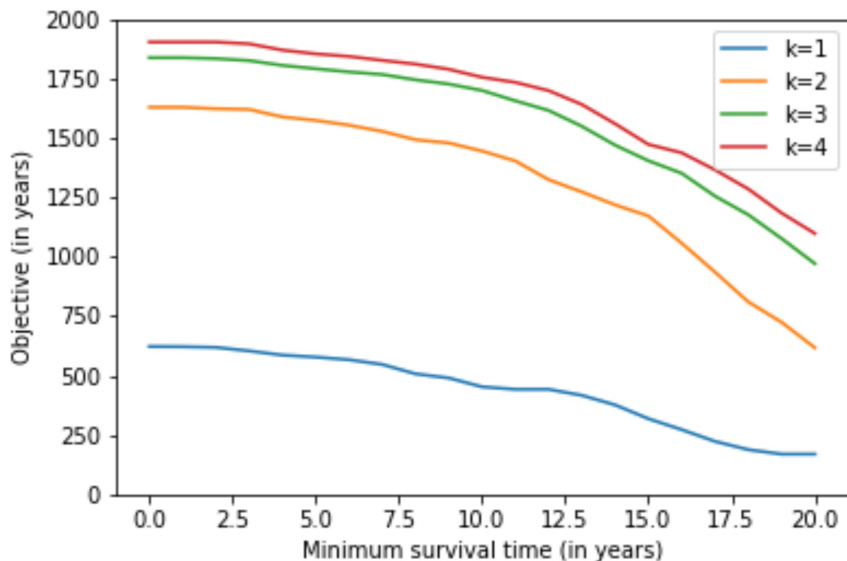
- 1 Use past data to train regression model
  - features: → patients' data
  - output: → observed **survival times**
- 2 Gather information concerning patients in current pool
- 3 Parameterize KEP optimization model using  
*pool information + predicted survival times*



# Results: number of transplants vs. minimum survival time



# Results: total survival time vs. minimum survival time



# Proposed flow: summary

- 1 Use past data to train regression model → **survival times**
- 2 Gather information concerning patients  
→ predict **survival time for each possible donor-patient** assignment
- 3 Find all cycles of desired length  
→ **filter cycles with unacceptably low-survival** patients
- 4 Prepare KEP optimization model using this information
- 5 Execute optimization model, retrieve and analyse solution
- 6 In the long run: follow transplanted patients, use survival times to improve regression model

# Conclusions

- Are current kidney exchange programs wrong?
  - is the objective correct?
- There seems to be room for improvement
  - machine learning + optimization
  - total survival time seems to be a better objective
- Ethics:
  - patient's *minimum acceptable survivable time* should be discussed

Thanks to my former KEP project colleagues and DDDM students!