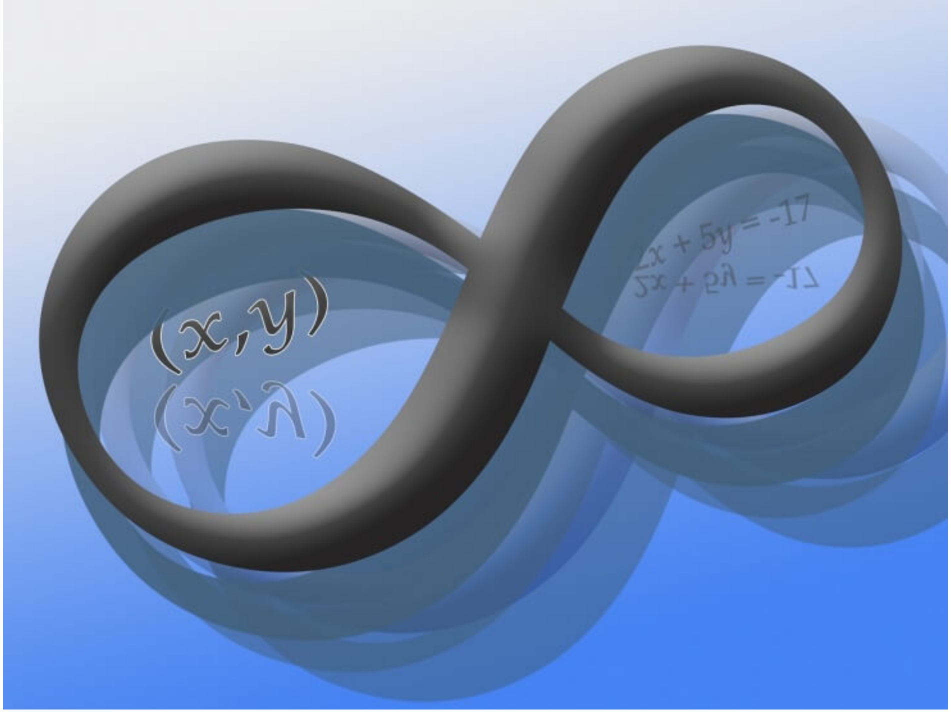
Step-by-step guide to execute Linear Regression in R



One of the most popular and frequently used techniques in statistics is linear regression where you predict a real-valued output based on an input value. Technically, linear regression is a statistical technique to analyze/predict the linear relationship between a dependent variable and one or more independent variables.

Let's say you want to predict the price of a house, the price is the dependent variable and factors like size of the house, locality, and season of purchase might act as independent variables. This is because the price depends on other variables.

R comes with many default data sets and it can be seen using MASS library.

Install.packages("MASS")

Library(MASS)

Data()

This will give you a list of available data sets using which you can get can a clear idea of linear regression problems.

Analysing a default data set in R

In this post, I will use a default data set called "airquality" data. The data set has various air quality parameters in New York city.

These are the parameters in the data set:

- Daily temperature from May to August
- Solar radiation data
- Ozone data
- Wind data

Our goal is to predict the temperature for a particular month in New York using solar radiation, ozone and wind data. I am going to use Linear Regression (LR) to make the prediction.

To start using LR or any other algorithm, first and foremost step is to generate a Hypothesis.

The hypothesis is: "Temperature of house depends on ozone, wind and solar radiations". Now, the null hypothesis of linear regression says there is no relation between dependent and independent variables; and all coefficients are zero. i.e. if equation is Temp=a1.Solar.R +a2.Ozone + a3.Wind + error.

On the other hand, alternate hypothesis says there is at least one non-zero coefficient and hence relationship exists between dependent and independent variables.

In mathematical notations it can be written as:

H_0: a1=a2=a3=0

H_a: a1≠a2≠a3≠0

Let's test the hypothesis using a linear regression model and draw a conclusion.

To test the hypothesis, we would check the level of significance of variables in support of

our hypothesis. If the significance is higher than accepted level (generally 95%), we would reject the null hypothesis and hence there is a relation between depend

If the significance is less than the accepted level, we will reject the null hypothesis and hence there is no relationship between dependent and independent variables.

Before that, let's understand the data by exploring it in R.

data(airquality)# to call the data
attach(airquality)
head(airquality,10)# to see first 10 rows

##	Ozone	Solar.R	Wind	Temp	Month	Day
## 1	41	190	7.4	67	5	1
## 2	36	118	8.0	72	5	2
## 3	12	149	12.6	74	5	3
## 4	18	313	11.5	62	5	4
## 5	NA	NA	14.3	56	5	5
## 6	28	NA	14.9	66	5	6
## 7	23	299	8.6	65	5	7
## 8	19	99	13.8	59	5	8
## 9	8	19	20.1	61	5	9
## 10	NA	194	8.6	69	5	10

Attach () function makes the data available to the R search path.

Summary function gives you the range, quartiles, median and mean for numerical variables and table with frequencies for categorical variables.

summary(airquality)

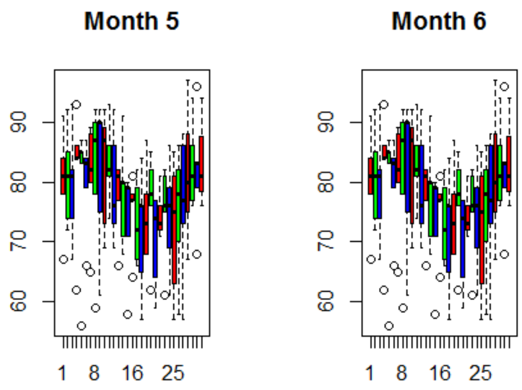
```
##      Ozone           Solar.R          Wind             Temp
## Min.   :  1.00    Min.   :  7.0    Min.   : 1.700    Min.   :56.00
## 1st Qu.: 18.00    1st Qu.:115.8    1st Qu.: 7.400    1st Qu.:72.00
## Median : 31.50    Median :205.0    Median : 9.700    Median :79.00
## Mean   : 42.13    Mean   :185.9    Mean   : 9.958    Mean   :77.88
## 3rd Qu.: 63.25    3rd Qu.:258.8    3rd Qu.:11.500    3rd Qu.:85.00
## Max.   :168.00    Max.   :334.0    Max.   :20.700    Max.   :97.00
## NA's   :37        NA's   :7

##      Month           Day
## Min.   :5.000    Min.   : 1.0
## 1st Qu.:6.000    1st Qu.: 8.0
## Median :7.000    Median :16.0
## Mean   :6.993    Mean   :15.8
## 3rd Qu.:8.000    3rd Qu.:23.0
## Max.   :9.000    Max.   :31.0
```

Data visualization

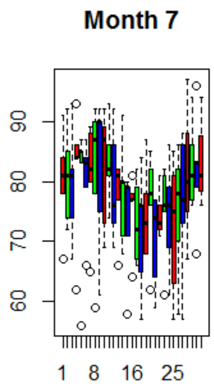
I use a boxplot to visualize the daily temperature for month 5, 6, 7, 8 and 9.

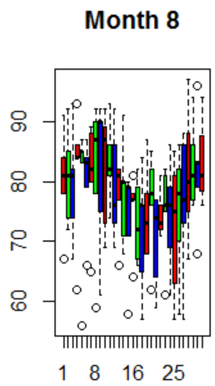
month5=subset(airquality,Month=5)
month6=subset(airquality,Month=6)
month7=subset(airquality,Month=7)
month8=subset(airquality,Month=8)
month9=subset(airquality,Month=9)

par(mfrow = c(1,2)) # 3 rows and 2 columns
boxplot((month5$Temp~airquality$Day),main="Month 5",col=rainbow(3))
boxplot((month6$Temp~airquality$Day),main="Month 6",col=rainbow(3))

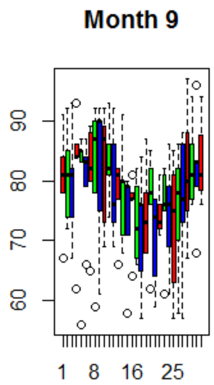


boxplot((month7$Temp~airquality$Day),main="Month 7",col=rainbow(3))
boxplot((month8$Temp~airquality$Day),main="Month 8",col=rainbow(3))



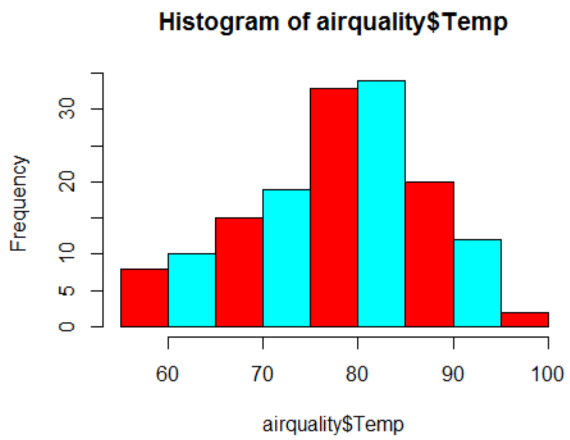


boxplot((month9$Temp~airquality$Day),main="Month 9",col=rainbow(3



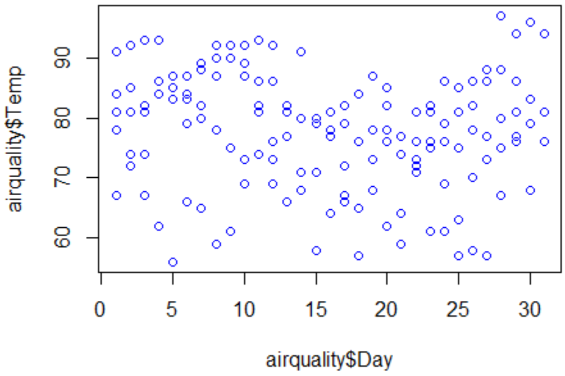
I use a histogram to see the distribution of temperature data.

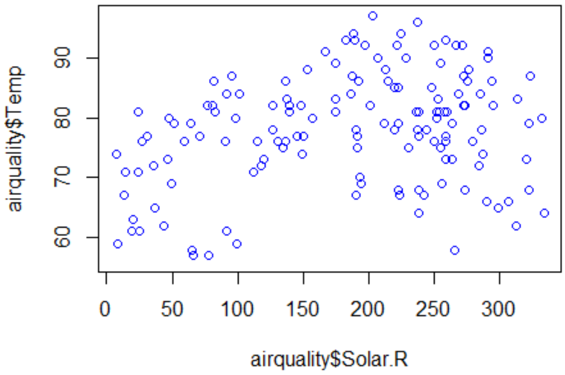
hist(airquality$Temp,col=rainbow(2))

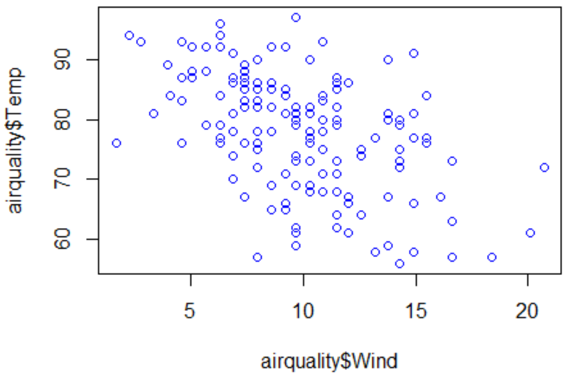


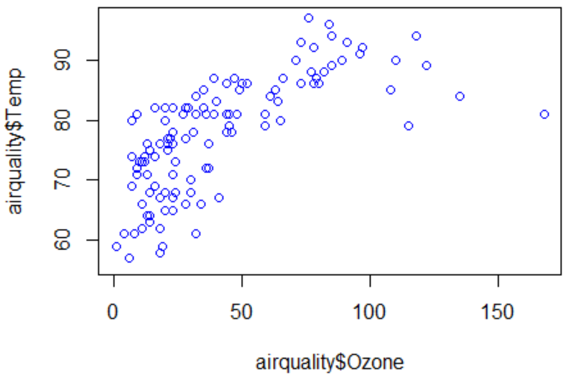
I use a scatter plot to see if there is a linear pattern between the 'temperature rise' and other variables.

plot(airquality$Temp~airquality$Day+airquality$Solar.R+airquality$Wind+airquality$Ozone,col="blue")



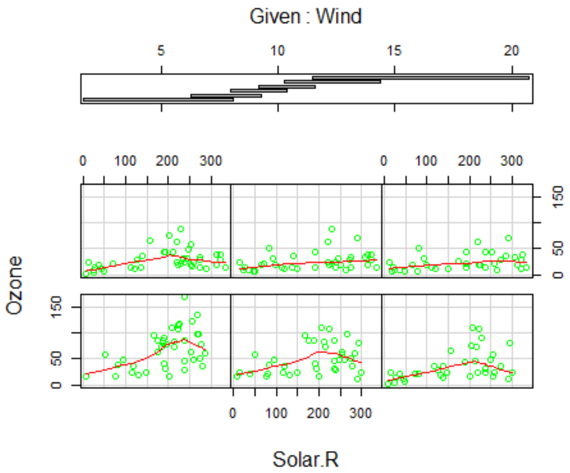






It seems that solar.R , Ozone, and wind have a linear pattern with temperature. Solar and Ozone have a positive relationship and wind has a negative one.
I use Co-plot to see the effect of wind and solar radiations combined on Temperature

coplot(Ozone~Solar.R|Wind,panel=panel.smooth,airquality,col ="green")



It's time to execute to Linear Regression on our data set

I use lm function to run a linear regression on our data set. The function lm fits a linear model to the data where Temperature (dependent variable) is on the left hand side separated by a ~ from the independent variables.

Data preparation:

The input data needs be processed before we use them in our algorithm. This means, deleting rows that has no values, checking correlation and outliers. While building the model, R inherently takes care of the null values, and drops the rows where the data is missing. This eventually results in data loss.

There are different methods to deal with data loss like imputing mean for numerical variables and mode for categorical variables. Another method is to replace null values with any value way larger than other values in the range.

For e.g. we can replace a null value with -1 when the variable is age. Since age cannot be negative, R considers the negative value as an outlier while building the model.

We can use the following command to find column wise count of null values in the data.

sapply(airquality,function(x){sum(is.na(x))})

Ozone Solar.R Wind Temp Month Day
37 7 0 0 0 0

You can see that there are missing values in Ozone and Solar.R. We can drop those rows but that would result in data loss since there are just 153 rows in our data, dropping 37 would be almost a 20% loss. Hence, we will replace the null values with mean (since both of the variables are numerical).

airquality$Ozone[is.na(airquality$Ozone)]=mean(airquality$Ozone,na.rm=T)

airquality$Solar.R[is.na(airquality$Solar.R)]=mean(airquality$Solar.R,na.rm=T)

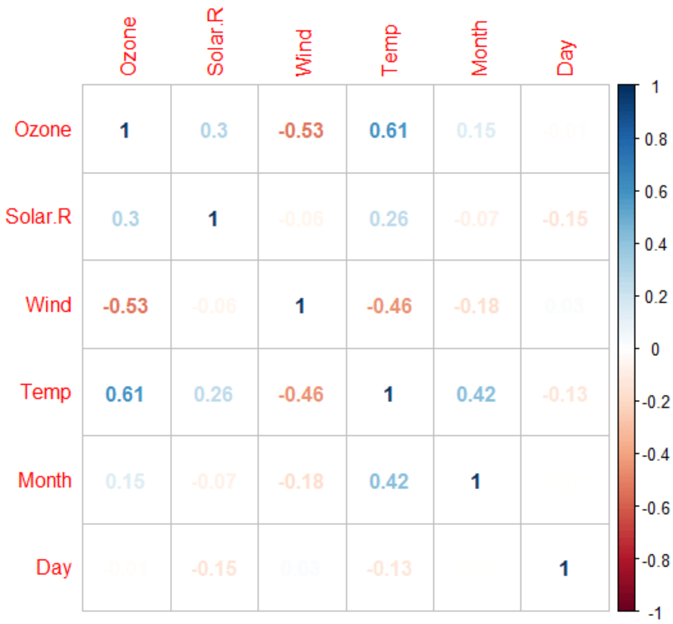
sapply(airquality,function(x){sum(is.na(x))})

Ozone Solar.R Wind Temp Month Day
0 0 0 0 0 0

Now, let's check the correlation between independent variables. We use corrplot library to visualize the correlation between variables.

library(corrplot)

o=corrplot(cor(airquality),method='number') # this method can be changed try using method='circle'



We can drop one of the two variables that has high correlation but if we have a good knowledge about data then we can form a new variable by taking the difference of two. For example, if ‘expenditure and income’ as variables have high correlation then we can create a new variable called ‘savings’ by taking the difference of ‘expenditure’ and ‘income’. We can do this only if we have domain knowledge.

Let’s see the effect of multicollinearity (without dropping a parameter) on our model.

```
Model_lm1=lm(Temp~.,data=airquality)
summary(Model_lm1)
```

```
##
## Call:
## lm(formula = Temp ~ ., data = airquality)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.110  -4.048   0.859   4.034  12.840
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 57.251830   4.502218  12.716  < 2e-16 ***
## Ozone        0.165275   0.023878   6.922 3.66e-10 ***
## Solar.R      0.010818   0.006985   1.549    0.124
## Wind        -0.174326   0.212292  -0.821    0.413
## Month        2.042460   0.409431   4.989 2.42e-06 ***
## Day         -0.089187   0.067714  -1.317    0.191
## —
## Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ‘ 1
##
## Residual standard error: 6.159 on 105 degrees of freedom
##   (42 observations deleted due to missingness)
## Multiple R-squared:  0.6013, Adjusted R-squared:  0.5824
## F-statistic: 31.68 on 5 and 105 DF,  p-value: < 2.2e-16
```

Before we interpret the results, I am going to the tune the model for a low AIC value.

The Akaike Information Criterion (AIC) is a measure of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Hence, AIC provides a means for model selection.

You can tune the model for a low AIC in two ways:

1) By eliminating some less significant variables and re-running the model

2) Using a ‘Step’ function in R. The step function runs all the possible parameters and checks the lowest value.

I am going to use the second method here.

```
Model_lm_best=step(Model_lm1)
```

```
## Start:  AIC=409.4
## Temp ~ Ozone + Solar.R + Wind + Month + Day
##
##           Df Sum of Sq    RSS    AIC
```

```
## – Wind     1     25.58 4008.2 408.11
## – Day      1     65.80 4048.5 409.22
## <none>                 3982.7 409.40
## – Solar.R  1     90.96 4073.6 409.91
## – Month    1    943.90 4926.6 431.01
## – Ozone    1   1817.27 5799.9 449.12
##
## Step:  AIC=408.11
## Temp ~ Ozone + Solar.R + Month + Day
##
##           Df Sum of Sq    RSS    AIC
## – Day      1      71.5 4079.8 408.07
## <none>                 4008.2 408.11
## – Solar.R  1      82.1 4090.3 408.36
## – Month    1     997.2 5005.5 430.77
## – Ozone    1    3242.4 7250.6 471.90
##
## Step:  AIC=408.07
## Temp ~ Ozone + Solar.R + Month
##
##           Df Sum of Sq    RSS    AIC
## <none>                 4079.8 408.07
## – Solar.R  1      92.1 4171.9 408.55
## – Month    1    1006.3 5086.1 430.55
## – Ozone    1    3225.5 7305.3 470.74
```

```
summary(Model_lm_best)
```

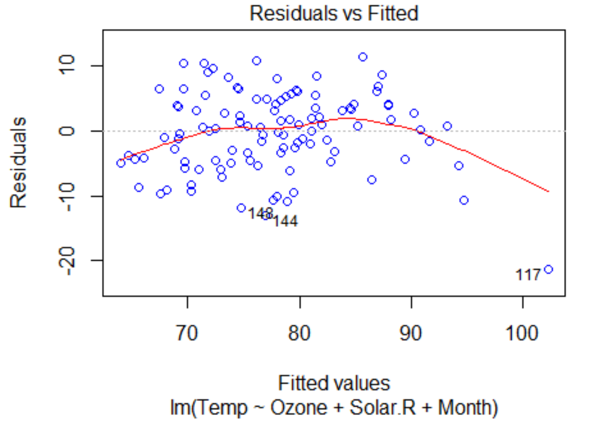
```
##
## Call:
## lm(formula = Temp ~ Ozone + Solar.R + Month, data = airquality)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -21.2300  -4.3645   0.6438   4.1479  11.3866
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 53.262897   3.268761  16.295  < 2e-16 ***
## Ozone        0.176503   0.019190   9.198 3.34e-15 ***
## Solar.R      0.010807   0.006953   1.554    0.123
## Month        2.092793   0.407364   5.137 1.26e-06 ***
## —
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.175 on 107 degrees of freedom
##   (42 observations deleted due to missingness)
## Multiple R-squared:  0.5916, Adjusted R-squared:  0.5802
## F-statistic: 51.67 on 3 and 107 DF,  p-value: < 2.2e-16
```

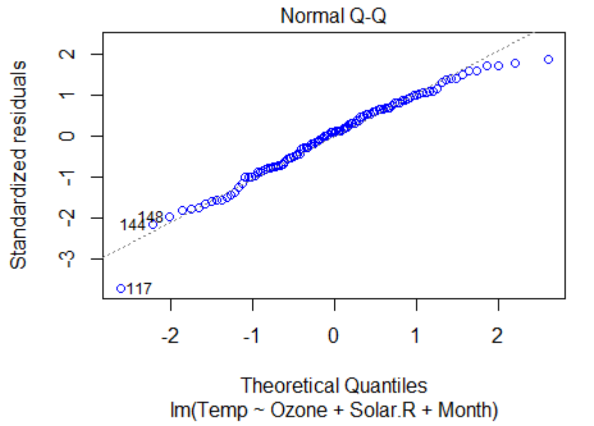
This summary gives coefficients of dependent variables and error term with the significance level (confidence level). The highlighted line in the result shows how to read the level of significance. A three asterisks means 99.99% significant (check the corresponding value. If it is less than 0.01 means variable is 99.99% significant).

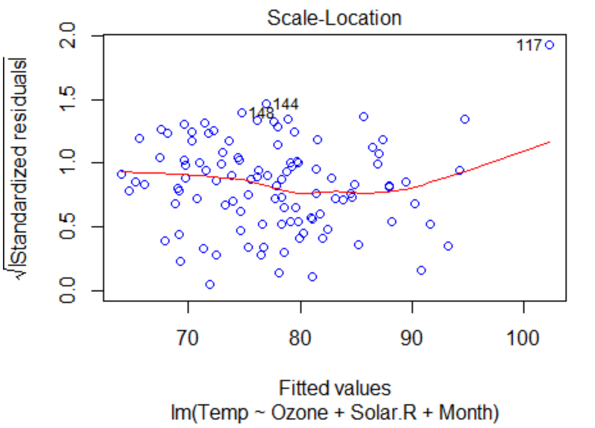
The R square and adjusted R square values defines how much variance of the dependent variable is explained by the model and the rest is explained by the error term. Hence, higher the R square or adjusted R square better the model.

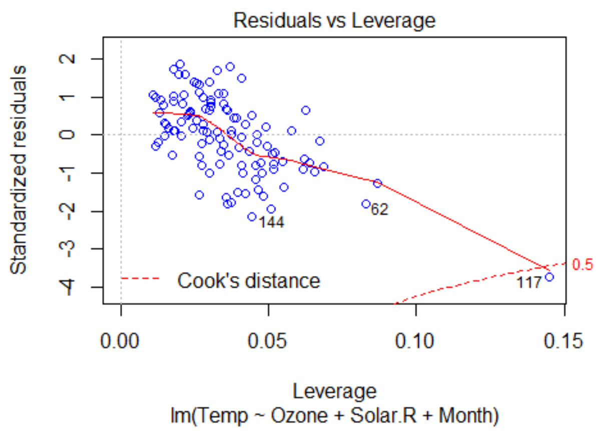
Adjusted R square is a better indicator of explained variance because it considers only important variables and extra variables are deliberately dropped by adjusted R square. In other words, adjusted R square penalizes the inclusion of many variables in the model for the sake of high percentage of variance explained.

```
plot(Model_lm_best,col="blue")
```









VIF and Multicollinearity

Variable Inflation factor is an important parameter regarding value of coefficient of determination (R2). If two independent variables are highly correlated then it inflates the model's variance (estimated error).

To deal with this, we can check VIF of the model before and after dropping one of the two highly correlated variables.

Formula for VIF:

VIF$_{(k)}$= 1/1+R$_k$^2

Where R^2 is the value obtained by regressing the kth predictor on the remaining predictors.

So to calculate VIF, we make model for each independent variable and consider all other variables as predictors. Then we calculate VIF for each variable. Whenever VIF is high, it means that set of variables have high correlation with the selected variable.

We will use an R library called 'fmsb' to calculate VIF.

So we can check VIF for our final linear model.

Library(fmsb)

Model_lm1=lm(Temp~ Ozone+Solar.R+Month,data=airquality)

VIF(lm(Month ~ Ozone+Solar.R,data=airquality))

[1] 1.039042

VIF(lm(Ozone ~ Solar.R+Month, data=airquality))

[1] 1.137975

VIF(lm(Solar.R ~ Ozone +Month, data=airquality))

[1] 1.118629

As a general rule, VIF < 5 is acceptable (VIF = 1 means there is no multicollinearity), and VIF > 5 is not acceptable and we need to check our model.

In our example, VIF < 5 and hence there is no need of any additional verification needed.

Interpretation of results

Basic assumptions of linear regression:

- Linear relationship between variables
- Normal distribution of residuals
- No or little multi-collinearity: we have seen this using VIF
- Homoscedasticity: Variance across the regression line should be uniform

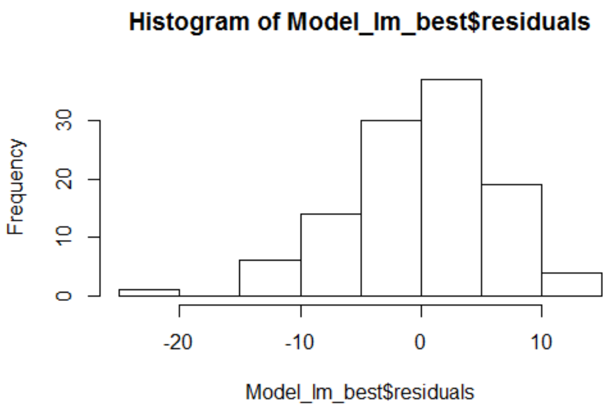
R displays the summary of the model and gives intercept values of all independent variables along with error terms (or residuals).

The Linear relationship between variables has been verified by the significance (p value) of variables.

In 'Residuals vs fitted values' plot it can be seen that residuals are linearly distributed and hence variance is uniform.

In 'Normal Q-Q' plot it can be seen that residuals are normally distributed. It can be seen by plotting histogram of residuals also

hist(Model_lm_best$residuals)



To measure the quality of the model there are many ways and residual sum of squares is the most common one.

There are many ways to measure the quality of a model, but residual sum of squares is the most common one.

Residual sum of squares attempts to make a 'line of best fit' in the scattered data points so that the line has least error with respect to the actual data points.

If Y is the actual data point and Y' is the predicted value by the equation, then the error is Y-Y'. But this has a bias towards 'sign' because when you sum up the error positive and negative values would cancel each other so the resultant error would be less than the actual value. To overcome this, a general method is to take square which serves two purposes:

1) Cancel out the effect of signs

2) Penalize the error in prediction

Prediction

To make a prediction, let's build a data frame for new values of Solar.R, Wind and Ozone.

Solar.R=185.93 Wind=9.96 Ozone=42.12

```
Solar.R=185.93
Wind=9.96
Ozone=42.12
Month=9
new_data=data.frame(Solar.R,Wind,Ozone,Month)
new_data
```

```
##   Solar.R Wind Ozone Month
## 1  185.93 9.96 42.12     9
```

```
pred_temp=predict(Model_lm_best,newdata=new_data)
```

```
## [1] "the predicted temperature is:  81.54"
```

Conclusion

The regression algorithm assumes that the data is normally distributed and there is a linear relation between dependent and independent variables. It is a good mathematical model to analyze relationship and significance of various variables.

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