



UNIVERSITAT DE
BARCELONA



MSc in Fundamental Principles of Data Science

Ethical Data Science

Bias and Discrimination II: Causality

Jordi Vitrià

2020-2021

Fairness from a causality perspective

Observational Data

Source: <https://fairmlbook.org/>

UC Berkeley admissions data from 1973.

Department	Men		Women	
	Applied	Admitted (%)	Applied	Admitted (%)
A	825	62	108	82
B	520	60	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

Is there a fair admission process?

Fairness from a causality perspective

Observational Data

Source: <https://fairmlbook.org/>

UC Berkeley admissions data from 1973.

	Men		Women	
	Applied	Admitted (%)	Applied	Admitted (%)
A	825	62	108	82
B	520	60	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

First Observation:
four of the six largest
departments show a
higher acceptance ratio
among women, while
two show a higher
acceptance rate for
men.

Fairness from a causality perspective

Observational Data

Source: <https://fairmlbook.org/>

UC Berkeley admissions data from 1973.

	Men		Women	
Department	Applied	Admitted (%)	Applied	Admitted (%)
A	825	62	108	82
B	520	60	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7
		44%	1157/2651	30%
				556/1835

First Observation:
four of the six largest
departments show a
higher acceptance ratio
among women, while
two show a higher
acceptance rate for
men.

Second Observation:
The acceptance rate
across all six departments
(aggregate admission
decisions) for men is about
44%, while it is only
roughly 30% for women,
again, a significant
difference.

Such reversals are sometimes called *Simpson's paradox*

Fairness from a causality perspective

Simpson's paradox causes discomfort to some, because intuition suggests that a trend which holds for all subpopulations should also hold at the population level.

What is evident from the data is that gender influences department choice. Women and men appear to have different preferences for different fields of study. Moreover, different departments have different admission criteria. Some have lower acceptance rates, some higher.

Therefore, one explanation for the data we see is that **women chose to apply to more competitive departments, hence getting rejected at a higher rate than men.**

Fairness from a causality perspective

Indeed, this is the conclusion an original study (Bickel, Hammel, O'Connell, and others, "Sex Bias in Graduate Admissions.", 1975) drew:

The bias in the aggregated data stems not from any pattern of discrimination on the part of admissions committees, which seems quite fair on the whole, but apparently from prior screening at earlier levels of the educational system. Women are shunted by their socialization and education toward fields of graduate study that are generally more crowded, less productive of completed degrees, and less well funded, and that frequently offer poorer professional employment prospects.

In other words, the article concluded that the source of gender bias in admissions was a pipeline problem: Without any wrongdoing by the departments, women were “*shunted by their socialization*” that happened at an earlier stage in their lives.

Fairness from a causality perspective

It is difficult to debate this conclusion on the basis of the available data alone. The question of discrimination, however, is far from resolved.

We can ask why women applied to more competitive departments in the first place.

There are several possible reasons.

- Perhaps less competitive departments, such as engineering schools, were unwelcoming of women at the time. This may have been a general pattern at the time or specific to the university.
- Perhaps some departments had a track record of poor treatment of women that was known to the applicants.
- Perhaps the department advertised the program in a manner that discouraged women from applying.

Fairness from a causality perspective

It is difficult to debate this conclusion on the basis of the available data alone. The issue, is far from resolved.

We can ask whether the departments in engineering have possible reasons.

- Perhaps less of schools, were design a new study and collect more data in a manner that might lead to a more conclusive outcome. The other is to argue over which scenario is more likely based on our beliefs and plausible assumptions about the world. This may lead to the poor treatment
- Perhaps some of women that
- Perhaps the department in a manner that discouraged women from applying.

There is no way of knowing what was the case from the data we have. We see that at best the original analysis leads to a number of follow-up questions.

At this point, we have two choices. One is to design a new study and collect more data in a manner that might lead to a more conclusive outcome. The other is to argue over which scenario is more likely based on our beliefs and plausible assumptions about the world.

Causal inference is helpful in either case.

Causality

“We can distinguish two kinds of intentionality. Primary intentionality is the repetition of causes that worked in the past. This is the intentionality of adaptation by natural selection and of conditioned reflexes. Past effects are anticipated to occur again. Secondary intentionality is choice of action after simulation of possible choices and their effects. Simulated effects are anticipated to occur when the action is performed. Secondary intentionality requires imagination, an ability to “hold in mind” and evaluate virtual outcomes. Primary intentionality is “primary” in the sense that anticipation evolved before imagination.”

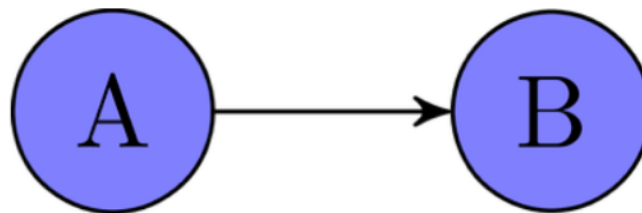
Fragment from: Haig, David; Dennett, Daniel C.: “Selfish Genes, Social Selves, and the Meanings of Life”.

Causality: Intuition

If I randomly picked a person from a population, and found out that she owns a Tesla, I'd find that she was more likely than the average person to have a college degree.

This means that owning a Tesla is **correlated** with having a college degree, i.e. *knowing if she has a college degree changes the likelihood that she also owns a Tesla.*

Does this mean that owning a Tesla **causes** people to have a college degree?



Getting answers from data

OBSERVATIONAL (passive observation of the world) DATASET

	Sex	Race	Height	Income	Marital Status	Years of Educ.	Liberal-ness
R1001	M	1	70	50	1	12	1.73
R1002	M	2	72	100	2	20	4.53
R1003	F	1	55	250	1	16	2.99
R1004	M	2	65	20	2	16	1.13
R1005	F	1	60	10	3	12	3.81
R1006	M	1	68	30	1	9	4.76
R1007	F	5	66	25	2	21	2.01
R1008	F	4	61	43	1	18	1.27
R1009	M	1	69	67	1	12	3.25

Let's consider 3 different features in this dataset, (X, Y, Z) .

Which can of questions can we answer from this dataset?

Association vs. Causality

Association (or prediction) is using data to map some features of the world (the inputs) to other features of the world (the outputs). For example, $\mathbb{E}(Y|X, Z)$.

All we need to do prediction is a dataset sampled from $p(X, Y, Z)$ and some inference tools (statistical inference & machine learning).

Mapping observed inputs to observed outputs is a natural candidate for automated data analysis because this task only requires 1) a large dataset with inputs and outputs, 2) an algorithm that establishes a mapping between inputs and outputs, and 3) a metric to assess the performance of the mapping, often based on a gold standard.

Association vs. Causality

Causal Inference is using data to predict certain features of the world if the world had been different. We cannot get these data by passive observation of the world! The world was different!

Answers to causal questions cannot be derived exclusively from $p(X, Y, Z)$. Answering a causal question (yes, sometimes is possible!) typically requires a combination of data, analytics, and expert **causal knowledge**.

For example, estimate the **income** that would have been observed if **all individuals** had **race=1** vs. if they had **race=2** or **race= 3**.

Association vs. Causality

Let's say we have i.i.d. data sampled from some joint $p(X, Y, Z)$.

Say we are ultimately interested in how variable Y behaves given X .
At a high level, one can ask this question in two ways:

- **observational**, based on $p(Y | X)$:
What is the distribution of Y given that I **observe** variable X takes value x ?

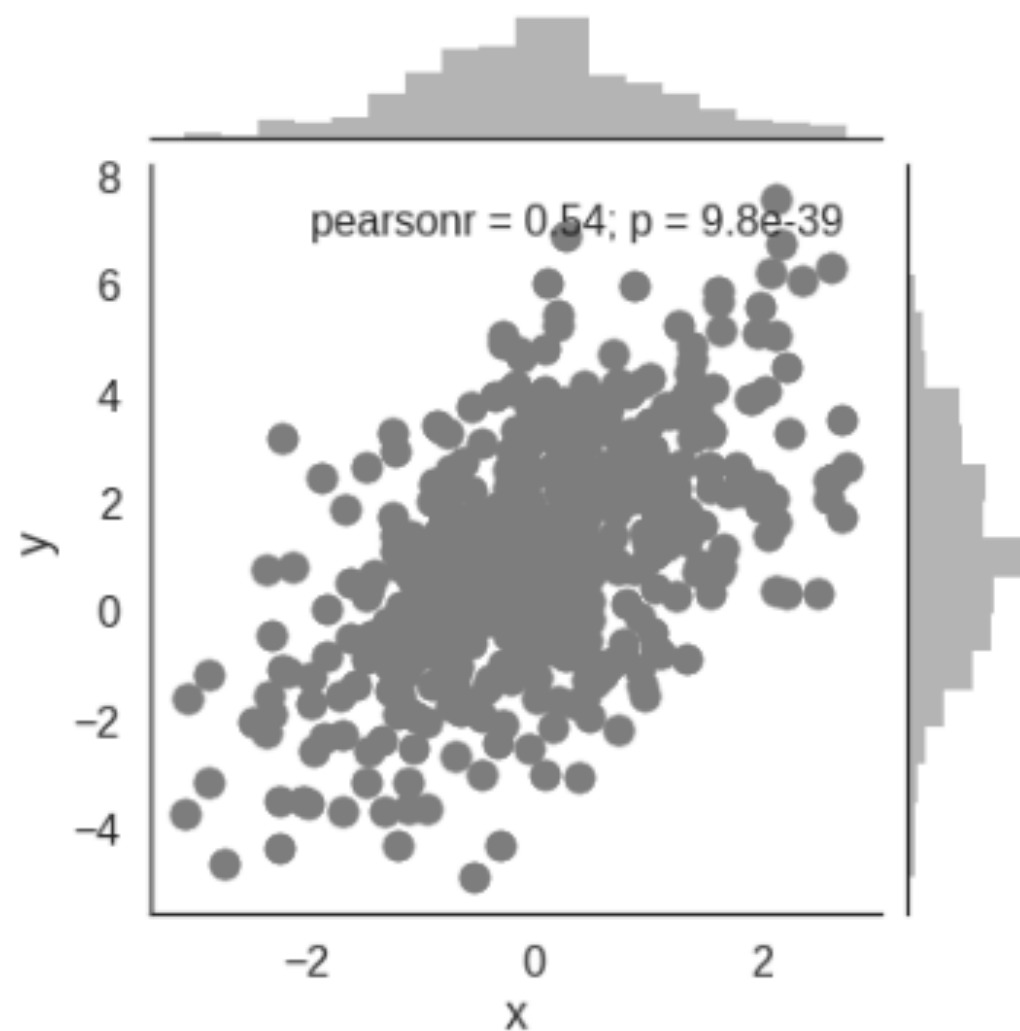
For example, estimate the **income** that would have been observed if **all individuals** had **race=1** vs. if they had **race=2** or **race=3**.

- **interventional**, based on $p(Y | do(X)) \neq p(Y | X)$:
What is the distribution of Y if I were to **set** the value of X to x .

This describes the distribution of Y I would observe if I **intervened** in the data generating process by artificially forcing the variable X to take value x , but otherwise **simulating the rest of the variables according to the original process that generated the data**.

Association vs. Causality

In order to understand what is $p(Y | do(X))$, let's suppose I have observed $p(X, Y)$.



<https://www.inference.vc/untitled/>

This is all we need to compute $p(Y | X)$. We can give an answer to any associational question.

For example:

- What is the expected value of Y if we observe $X = 3$? (Regression)
- What is the expected MAX/MIN/MEDIAN value of Y if we observe $X = 3$? (Quantile regression)

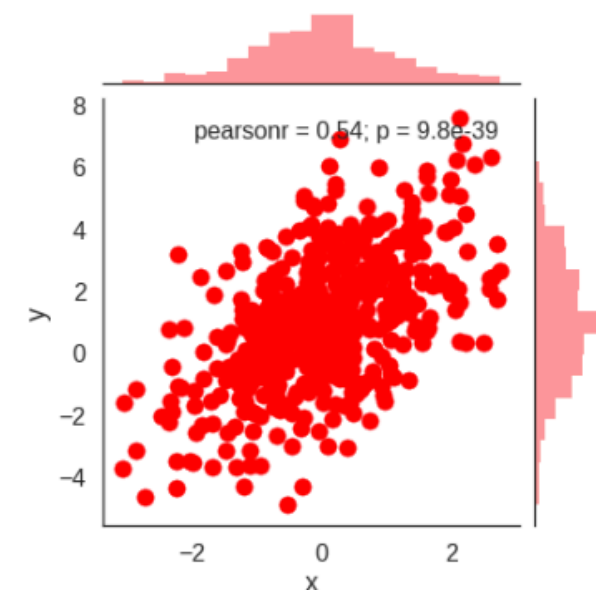
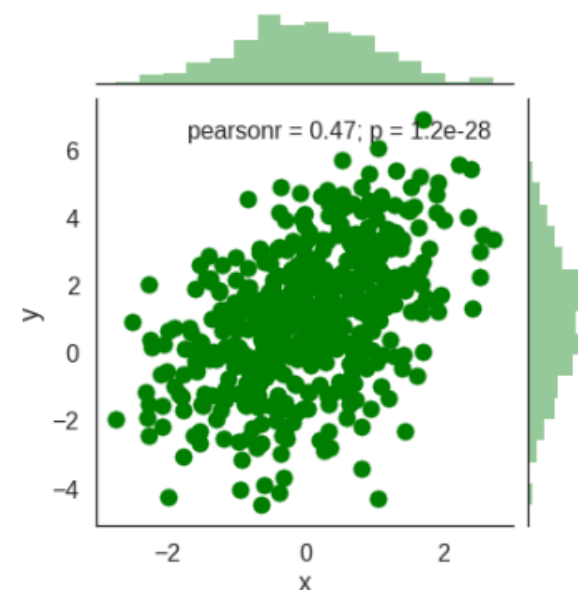
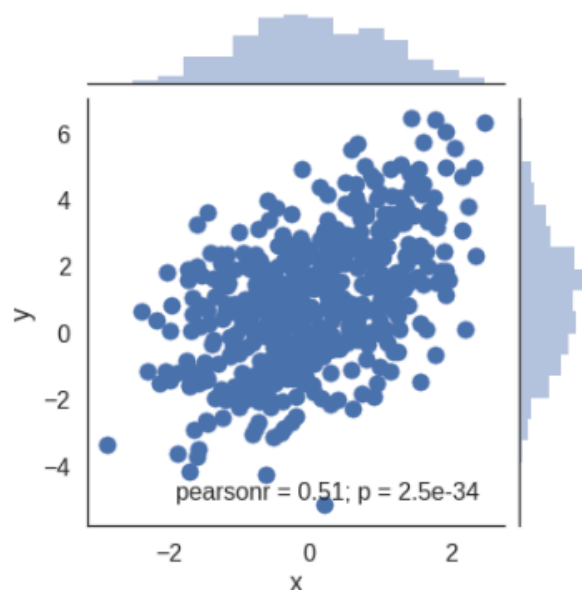
Association vs. Causality

Given $p(X, Y)$, there are several generative models that are compatible with $p(X, Y)$:

```
x = randn()
y = x + 1 + sqrt(3)*randn()
```

```
y = 1 + 2*randn()
x = (y-1)/4 + sqrt(3)*randn()/2
```

```
z = randn()
y = z + 1 + sqrt(3)*randn()
x = z
```



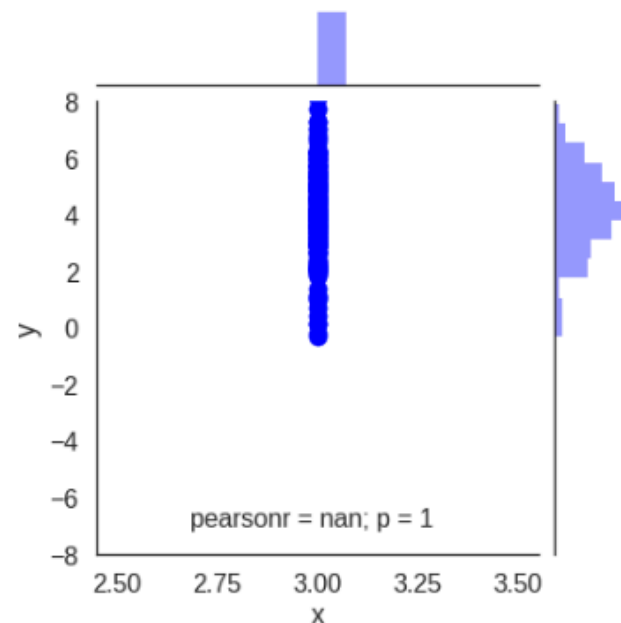
<https://www.inference.vc/untitled/>

Based on the joint distribution the three scripts are indistinguishable.

Association vs. Causality

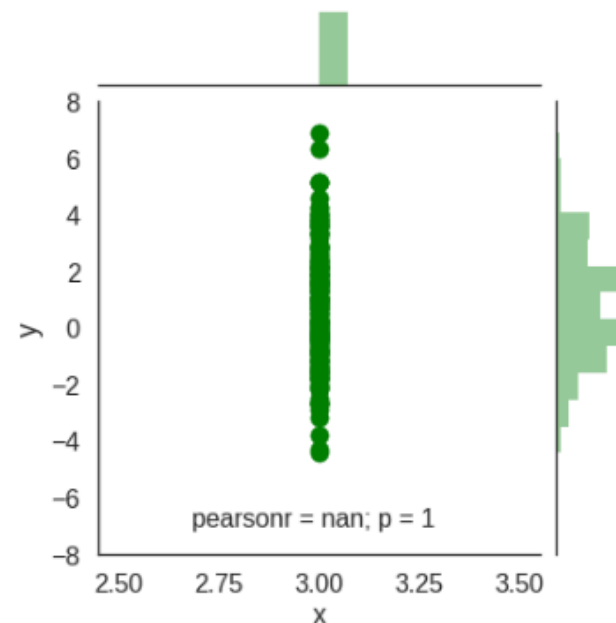
Let's now consider an **intervention** $p(Y | do(X = 3))$

```
x = randn()
x = 3
y = x + 1 + sqrt(3)*randn()
x = 3
```



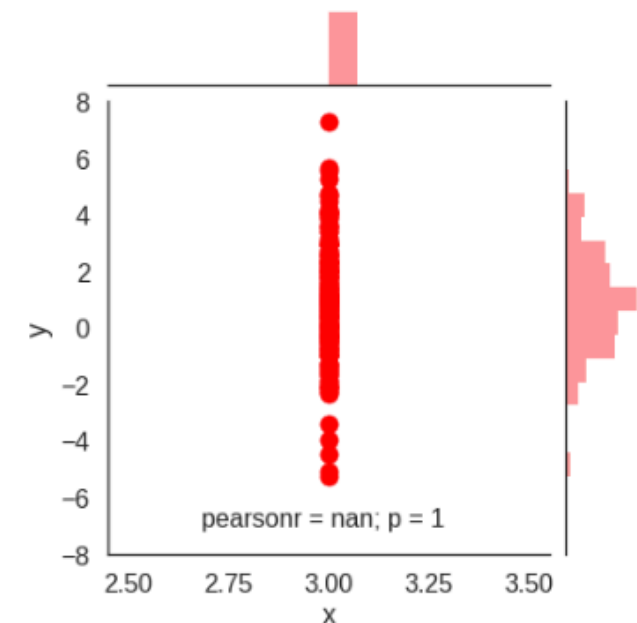
$$p(Y | do(X)) \neq p(Y | X)$$

```
y = 1 + 2*randn()
x = 3
x = (y-1)/4 + sqrt(3)*randn()/2
x = 3
```



$$p(Y | do(X)) = p(Y | X)$$

```
z = randn()
x = 3
x = z
x = 3
y = z + 1 + sqrt(3)*randn()
x = 3
```



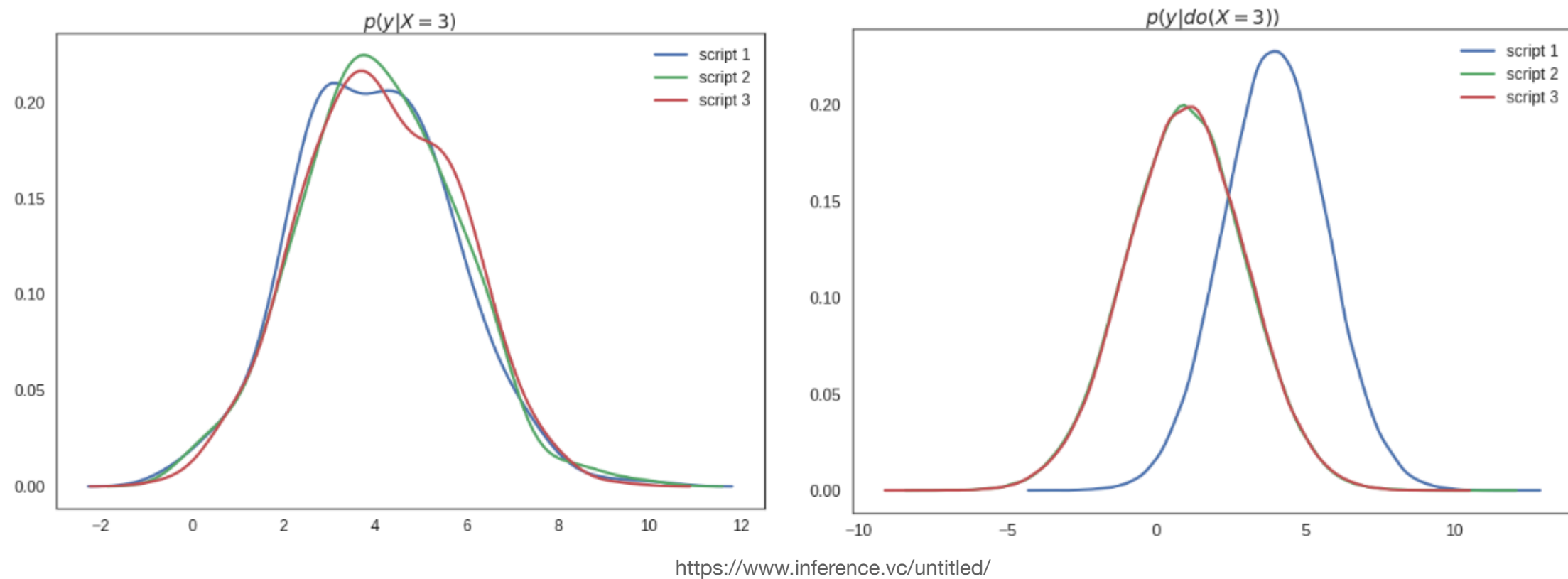
$$p(Y | do(X)) = p(Y | X)$$

Now the first model is distinct!

<https://www.inference.vc/untitled/>

Association vs. Causality

Here is a better look at the marginal distribution of Y under the intervention. The scripts behave differently under intervention.



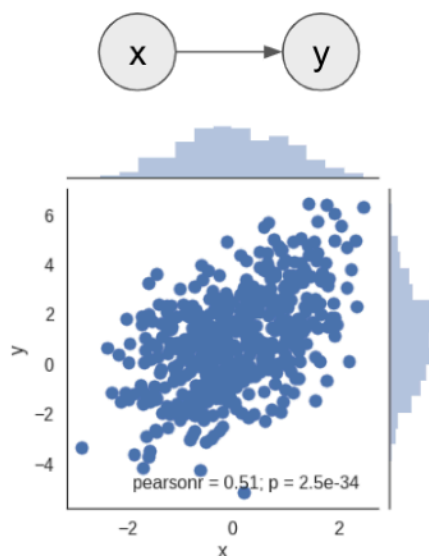
The joint distribution of data $p(X, Y, Z)$ alone is insufficient to predict behavior under interventions.

Association vs. Causality

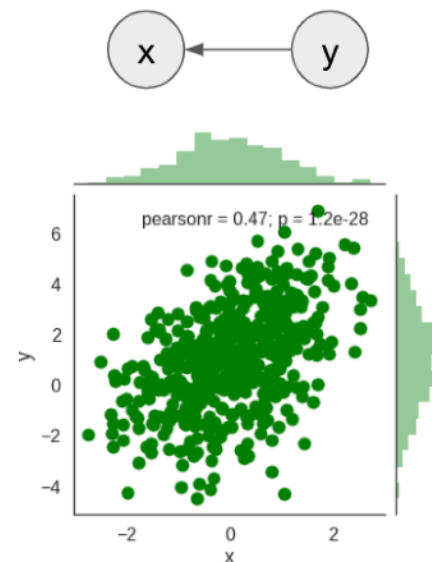
An intervention can be understood as a **modification of the generative model of the data, producing a different probability distribution** $p(\text{do}(X), Y, Z)$.

The resulting distribution depends on the original model:

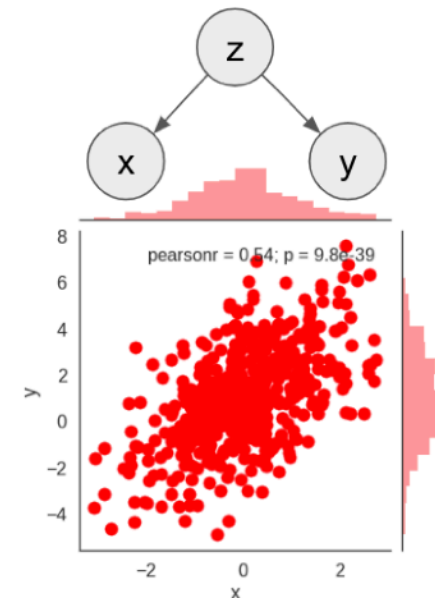
```
x = randn()
y = x + 1 + sqrt(3)*randn()
```



```
y = 1 + 2*randn()
x = (y-1)/4 + sqrt(3)*randn()/2
```



```
z = randn()
y = z + 1 + sqrt(3)*randn()
x = z
```



Directed Acyclic Graphs (DAG).

No assumptions about the exact form of the functional relationships are needed. The only requirement is that causal relationships are **acyclic**.

<https://www.inference.vc/untitled/>

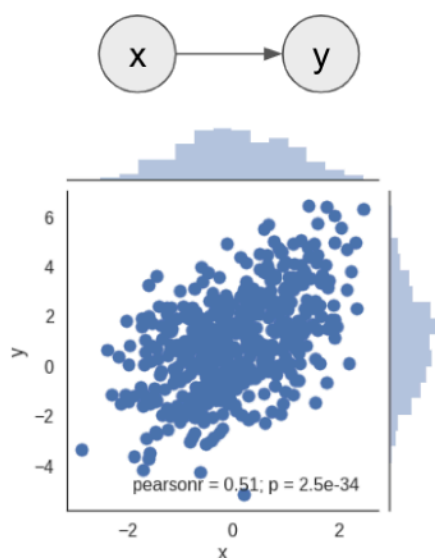
Association vs. Causality

An intervention can be understood as a **modification of the generative model of the data, producing a different probability distribution** $p(\text{do}(X), Y, Z)$.

The resulting distribution depends on the original model:

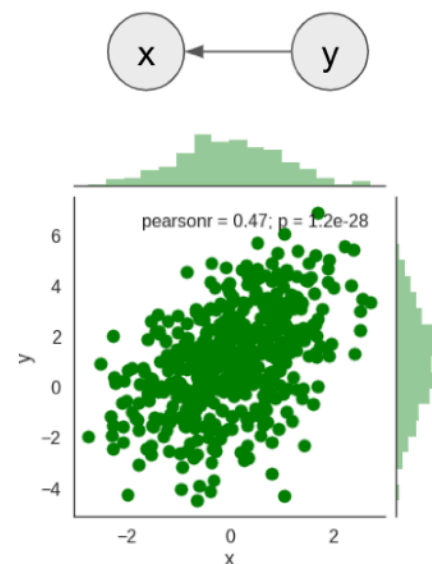
```
x = randn()
y = x + 1 + sqrt(3)*randn()
```

X is the cause of Y



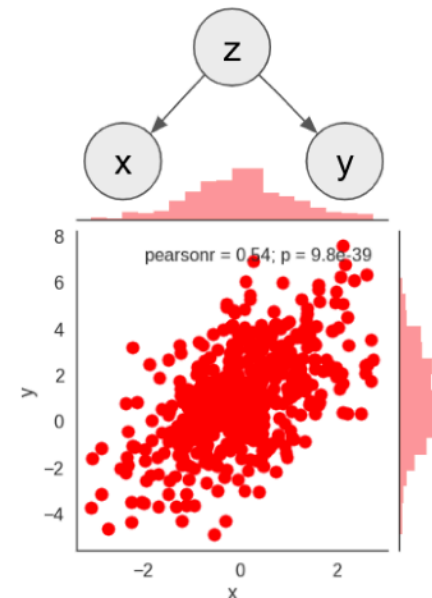
```
y = 1 + 2*randn()
x = (y-1)/4 + sqrt(3)*randn()/2
```

Y is the cause of X



```
z = randn()
y = z + 1 + sqrt(3)*randn()
x = z
```

X and Y are not causally related (but they are correlated!)



Directed Acyclic Graphs (DAG).

No assumptions about the exact form of the functional relationships are needed. The only requirement is that causal relationships are **acyclic**.

Association vs. Causality

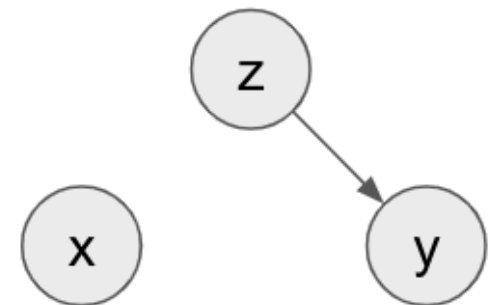
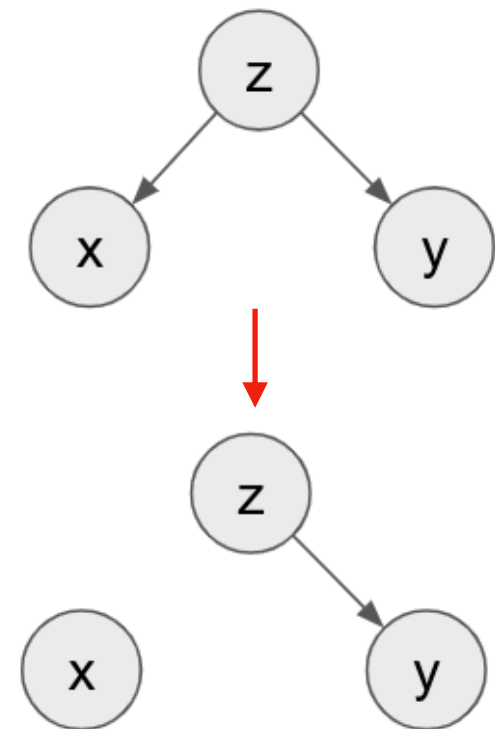
Graphically, to simulate the effect of an intervention, you **mutilate** the graph by removing all edges that point into the variable on which the intervention is applied, in this case X .



$$p(Y | do(X)) = p(Y | X = 3)$$



$$p(Y | do(X)) = p(Y)$$

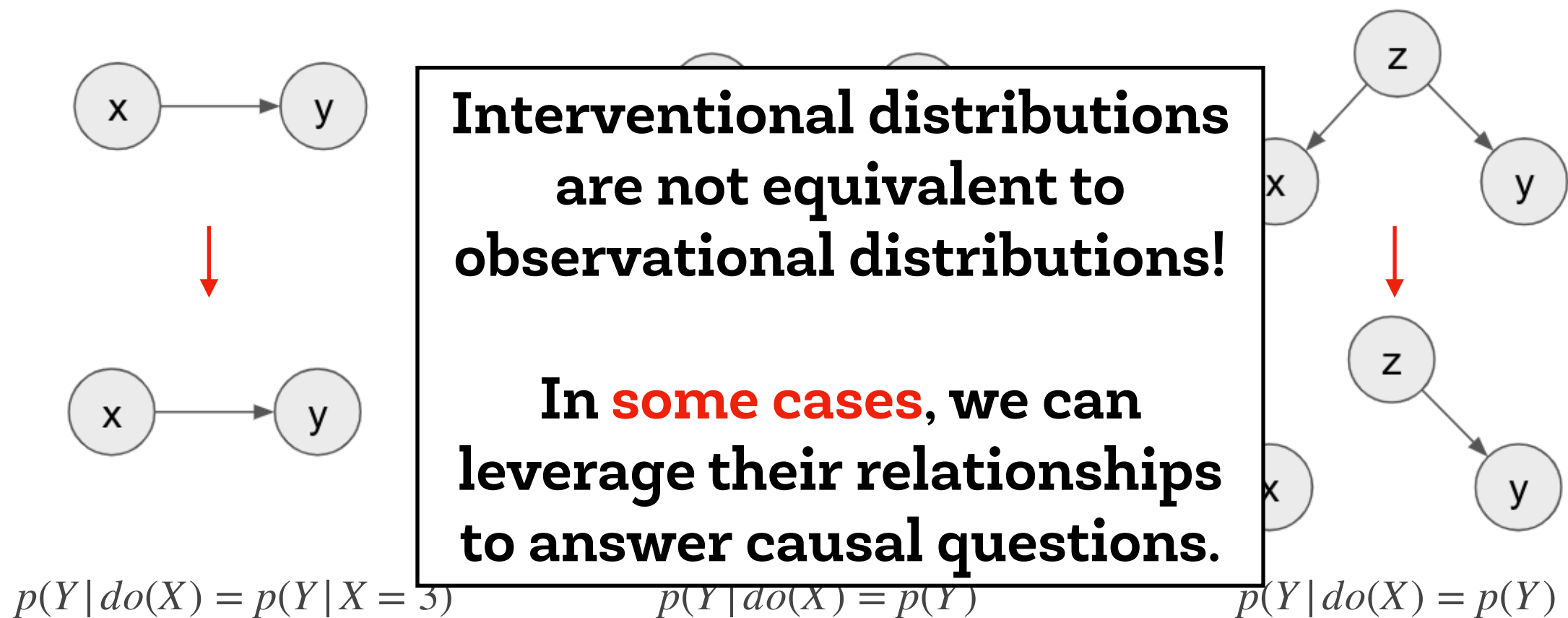


$$p(Y | do(X)) = p(Y)$$

Just by only looking at the causal diagram, we are now able to predict how the scripts are going to behave under the intervention $X = 3$.

Association vs. Causality

Graphically, to simulate the effect of an intervention, you **mutilate** the graph by removing all edges that point into the variable on which the intervention is applied, in this case X .

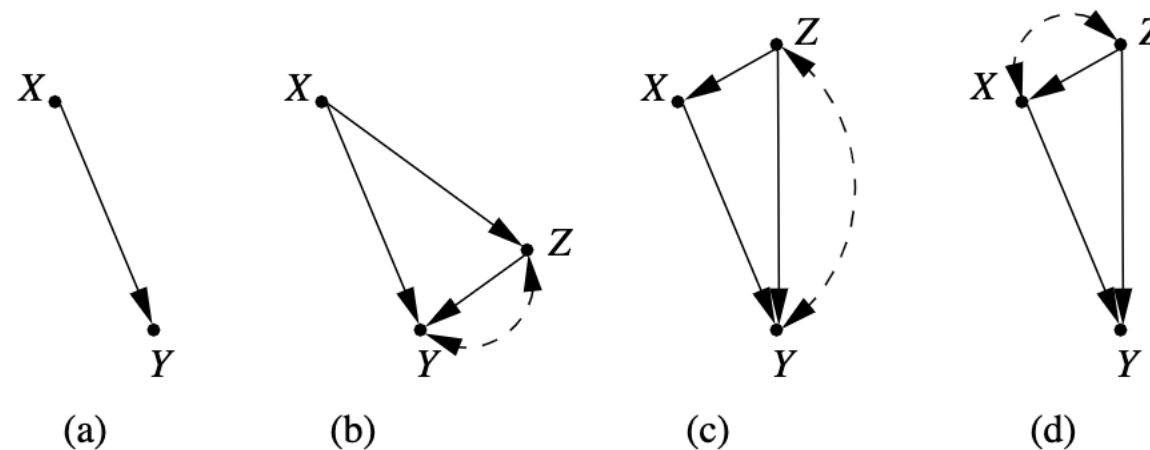


Just by only looking at the causal diagram, we are now able to predict how the scripts are going to behave under the intervention $X = 3$.

Association vs. Causality

An essential matter in causal inference is that of a query's **identifiability**.

Given a causal query for a certain DAG, we say it is **identifiable** if we can derive an statistical estimand (only using observational terms) for this query using the rules of do-calculus



Causal graphs where $P(y|do(x))$ is identifiable

Source: Complete Identification Methods for Causal Inference, PhD Thesis, University of California. I. Shpitser
https://ftp.cs.ucla.edu/pub/stat_ser/shpitser-thesis.pdf

Causal Inference and do-calculus

There are two ways to assess the existence of a **causal relationship** between two variables, X and Y :

1. The easiest way is an **intervention** in the real world: You randomly force X to have different values and you measure Y .

This is what we do in Randomized Clinical Trial (RCT) or in an A/B Test.

This is not always feasible (because of ethical or economical reasons)

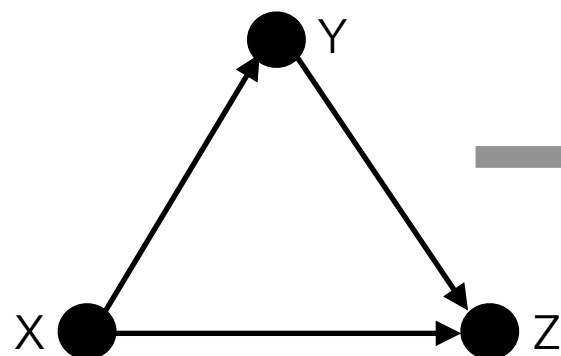
Causal Inference and do-calculus

There are two ways to assess the existence of a **causal relationship** between two variables, X and Y :

2. If the query is identifiable, **do-calculus**, allows us to massage $p(X, Y, Z)$ until we can express $p(Y | do(X))$ in terms of various marginals, conditionals and expectations under $p(X, Y, Z)$

The do-calculus is an axiomatic system for replacing probability formulas containing the do operator with ordinary conditional probabilities. It consists of three axiom schemas that provide **graphical criteria** for when certain substitutions may be made.

Example:



$$\Pr(z | do(x)) = \sum_y \Pr(y) \Pr(z | x, y)$$

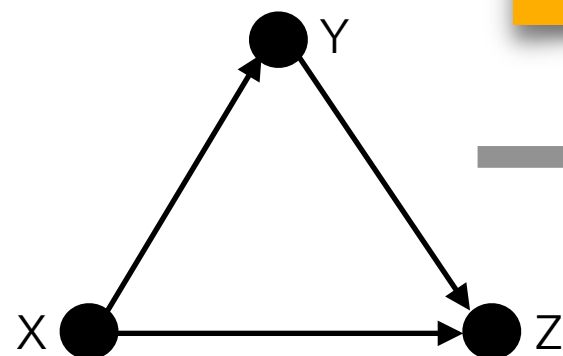
Causal Inference and do-calculus

There are two ways to assess the existence of a **causal relationship** between two variables, X and Y :

2. If the query is identifiable, **do-calculus**, allows us to massage $p(X, Y, Z)$ until we can express $p(Y | do(X))$ in terms of various marginals, conditionals and expectations under $p(X, Y, Z)$

The do-calculus is an axiomatic system for replacing probability formulas containing the do operator with ordinary conditional probabilities. It consists of three axiom schemas that provide **graphical criteria** for when certain substitutions may be made.

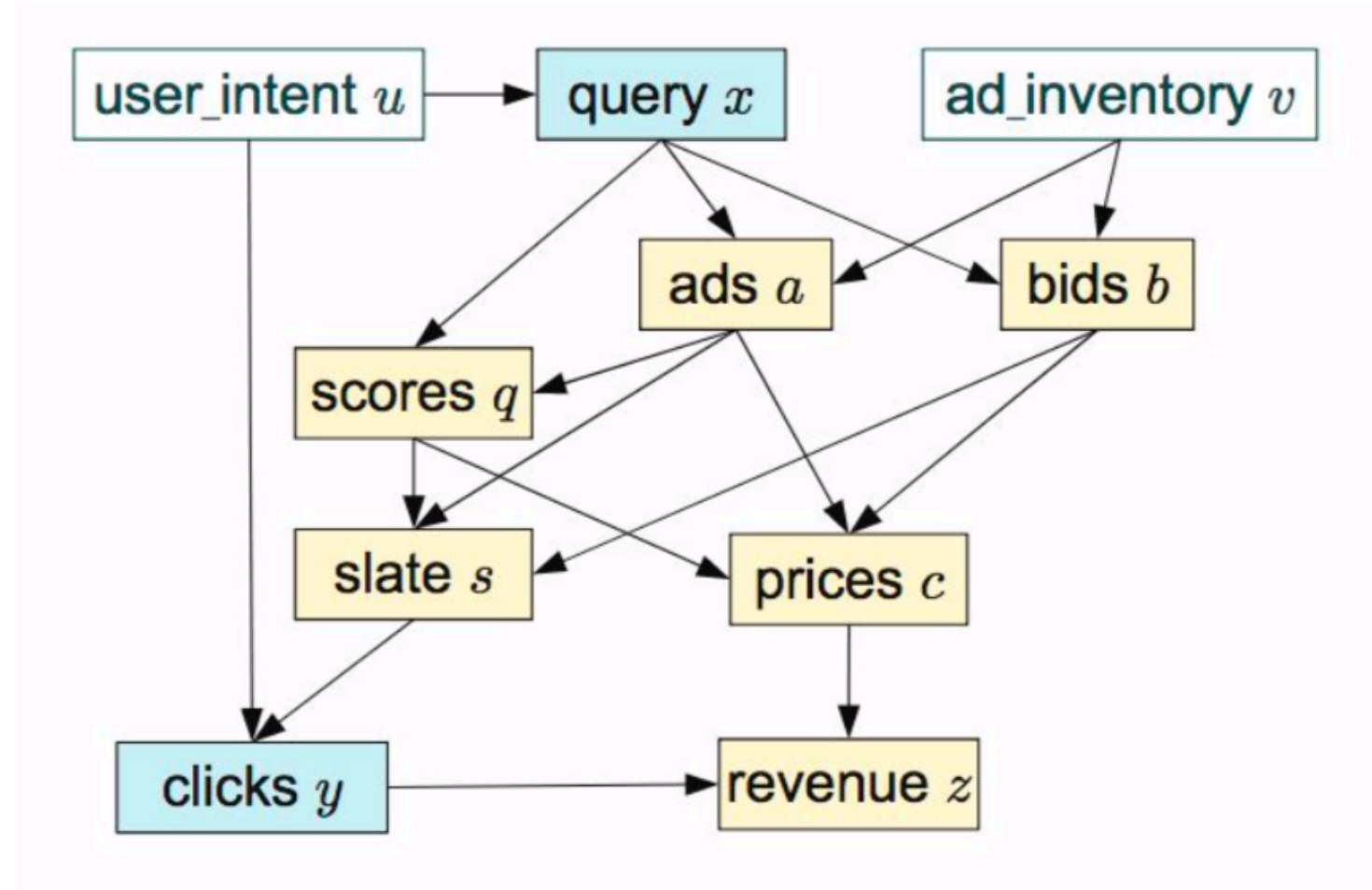
Example:



Only depends on the causal graph!

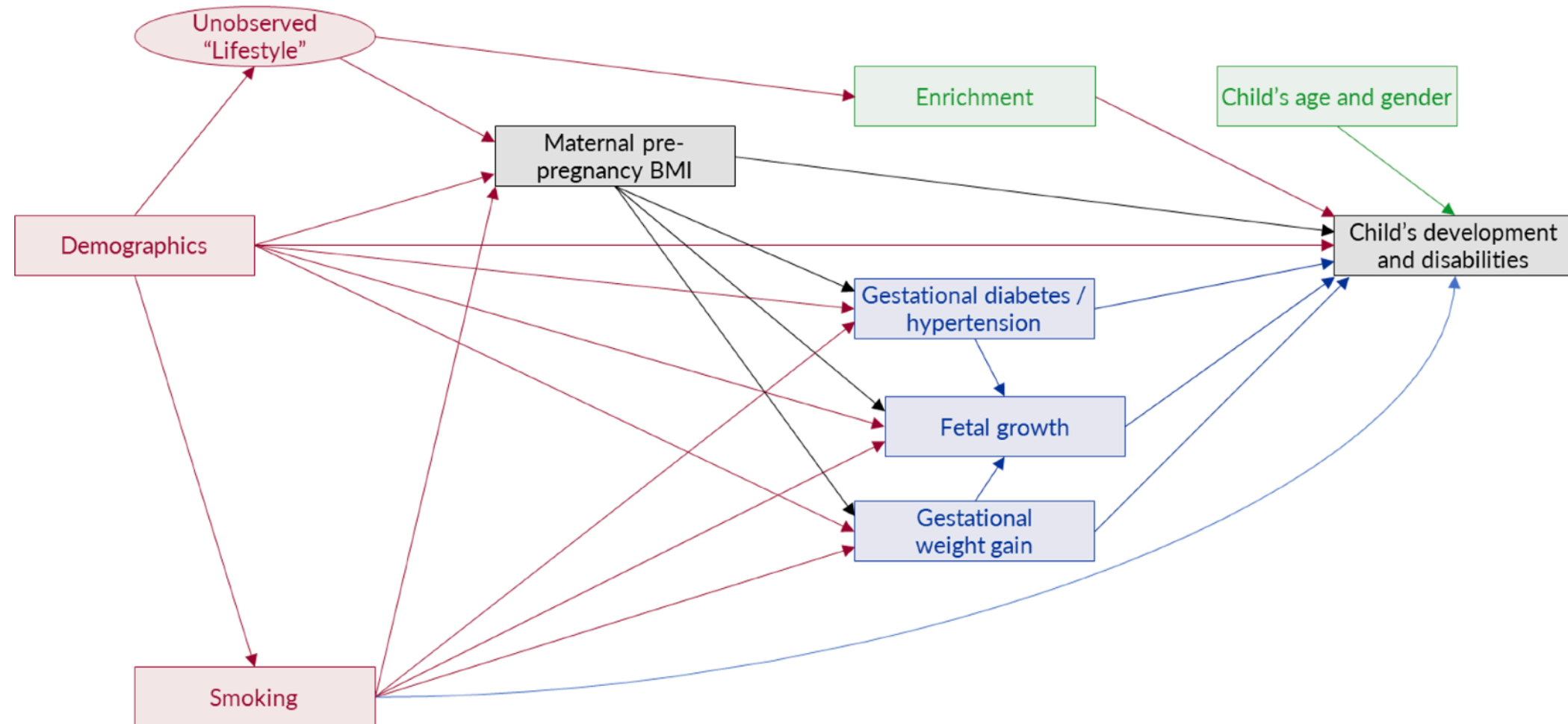
$$\Pr(z | do(x)) = \sum_y \Pr(y) \Pr(z | x, y)$$

Causal Inference and do-calculus



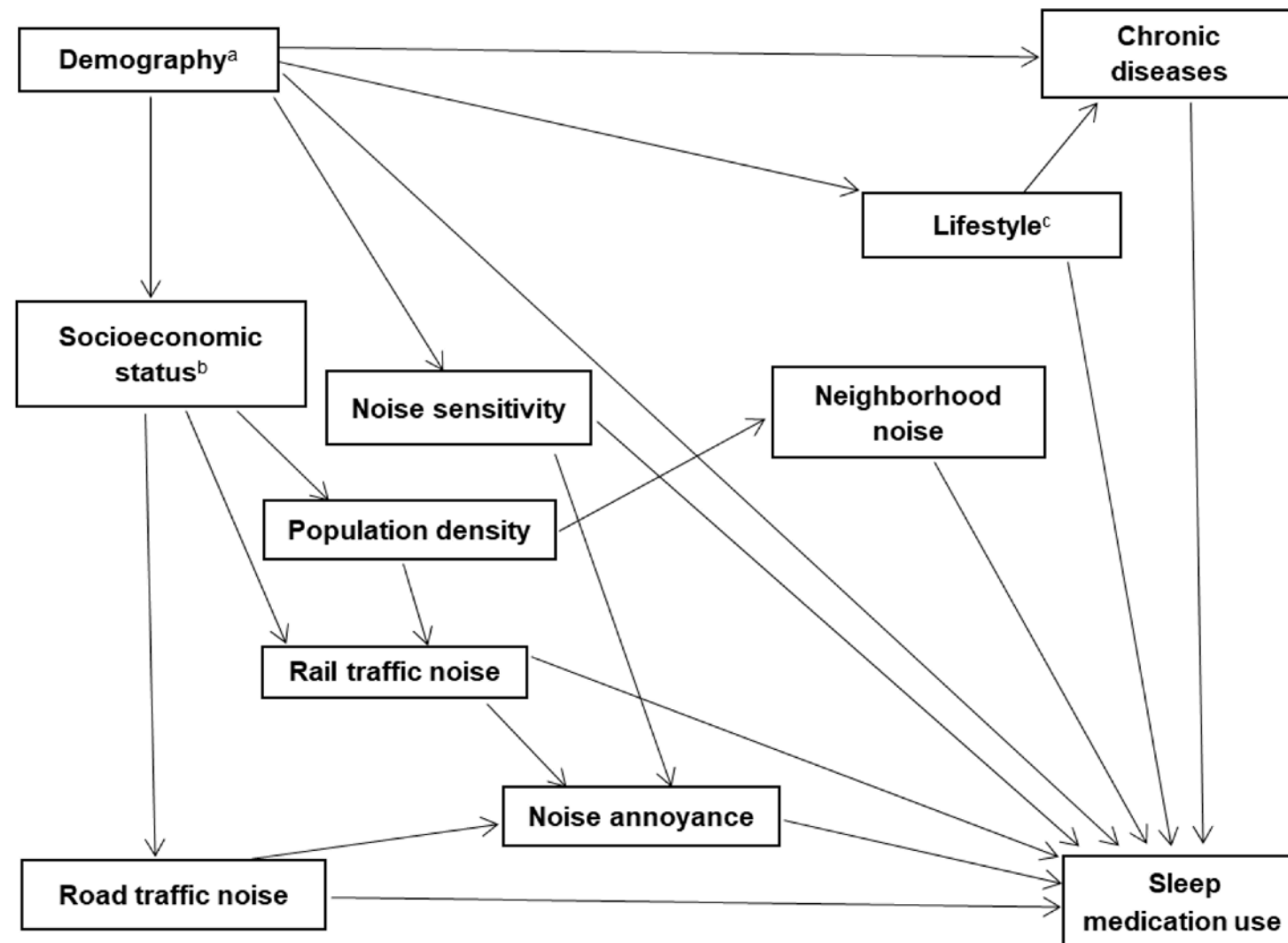
Bottou, Léon, et al. "Counterfactual reasoning and learning systems: the example of computational advertising." *The Journal of Machine Learning Research* 14.1 (2013): 3207-3260.

Causal Inference and do-calculus



ADAPTED FROM: Hinkle SN, Sharma AJ, Kim SY, Schieve LA. Maternal prepregnancy weight status and associations with children's development and disabilities at kindergarten. *Int J Obes (Lond)*. 2013;37(10):1344-51. DOI: 10.1038/ijo.2013.128 (Figure 1). Freely available at: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4407562>

Causal Inference and do-calculus



REPRODUCED UNDER CC-BY 4.0 LICENSE FROM: [Evandt J, Oftedal B, Krog NH, et al. Road traffic noise and registry based use of sleep medication. *Environ Health*. 2017;16\(1\):110. DOI: 10.1186/s12940-017-0330-5 \(Figure S1\). Freely available at: <https://www.doi.org/10.1186/s12940-017-0330-5>](#)

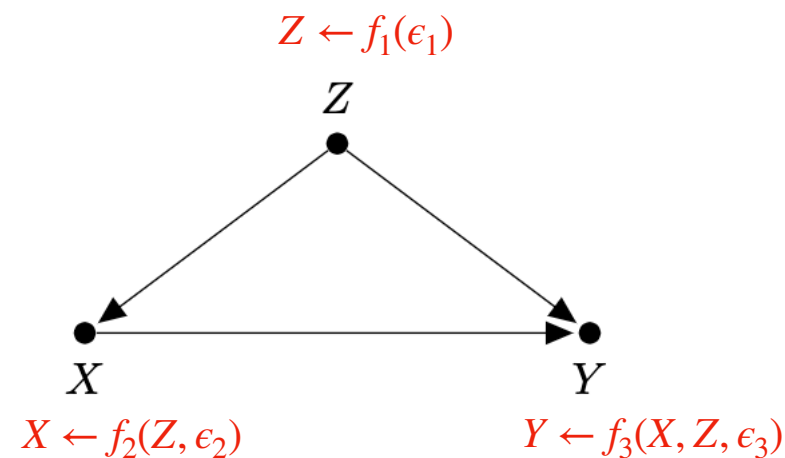
Causal Inference and SCM

The **causal diagram** can be seen as a representation of an underlying **structural causal model**:

Directed Acyclic Graphs (DAG):

No assumptions about the exact form of the functional relationships, as well as the distribution of background factors, are needed. The only requirement is that causal relationships are acyclic.

	Sex	Race	Height	Income	Marital Status	Years of Educ.	Liberal-ness
R1001	M	1	70	50	1	12	1.73
R1002	M	2	72	100	2	20	4.53
R1003	F	1	55	250	1	16	2.99
R1004	M	2	65	20	2	16	1.13
R1005	F	1	60	10	3	12	3.81
R1006	M	1	68	30	1	9	4.76
R1007	F	5	66	25	2	21	2.01
R1008	F	4	61	43	1	18	1.27
R1009	M	1	69	67	1	12	3.25



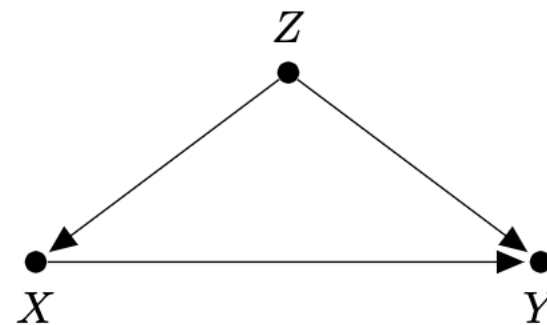
ϵ_i are **exogenous background factors** represented by an arbitrary noise distribution.

They exert an influence on the endogenous variables in the model. Because they are unobserved from the standpoint of the analyst, this renders the model stochastic with a probability distribution over the set of **endogenous variables**.

Causal Inference and SCM

We can estimate f_i from data by using predictive methods.

Actions can now be defined as interventions on variables in the model.



$$Z \leftarrow f_1(\epsilon_1)$$

$$X \leftarrow f_2(Z, \epsilon_2)$$

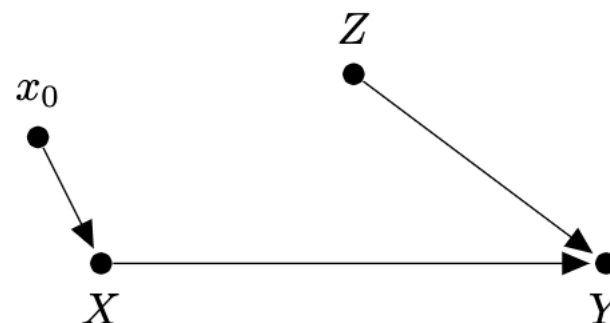
$$Y \leftarrow f_3(X, Z, \epsilon_3)$$

Structural Causal Model

For example, intervening on X amounts to deleting f_2 and setting X to a constant value x_0 .

$$\begin{aligned} Z &\leftarrow f_1(\epsilon_1) \\ X &\leftarrow x_0 \\ Y &\leftarrow f_3(X, Z, \epsilon_3) \end{aligned}$$

Modified
Structural Causal Model



This graph encodes the intervened distribution, from which we can **sample** and **compute causal queries** (if they are identifiable).

For example, compute **causal effects**:

$$p(Y | do(X = x_1)) - p(Y | do(X = x_0))$$

Counterfactuals

Counterfactuals in our life:

Source: <https://christophm.github.io/interpretable-ml-book/counterfactual.html>

Let's suppose that we want to rent an apartment and we train a model with real data to predict a price.

After entering all the details about size, location, whether pets are allowed and so on, the model tells us that we can charge 900€.

How could we get (by doing an intervention) 1000€? We can play with the feature values of the apartment to see how we can improve the value of the apartment!

We find out that the apartment could be rented out for over 1000 Euro, if it were 15 m² larger. Interesting, but non-actionable knowledge, because we cannot enlarge the apartment.

Finally, by tweaking only the feature values under our control (built-in kitchen yes/no, pets allowed yes/no, type of floor, etc.), we find out that if we allow pets and install windows with better insulation, we can charge 1000€.

Counterfactuals

Given a certain observational sample $e = (x_e, y_e, z_e)$ and an intervention $do(X = x_q)$, a **counterfactual** is the result of an hypothetical experiment in the past, what would have happened to the value of variable Y had we intervened on X by assigning value x_q .

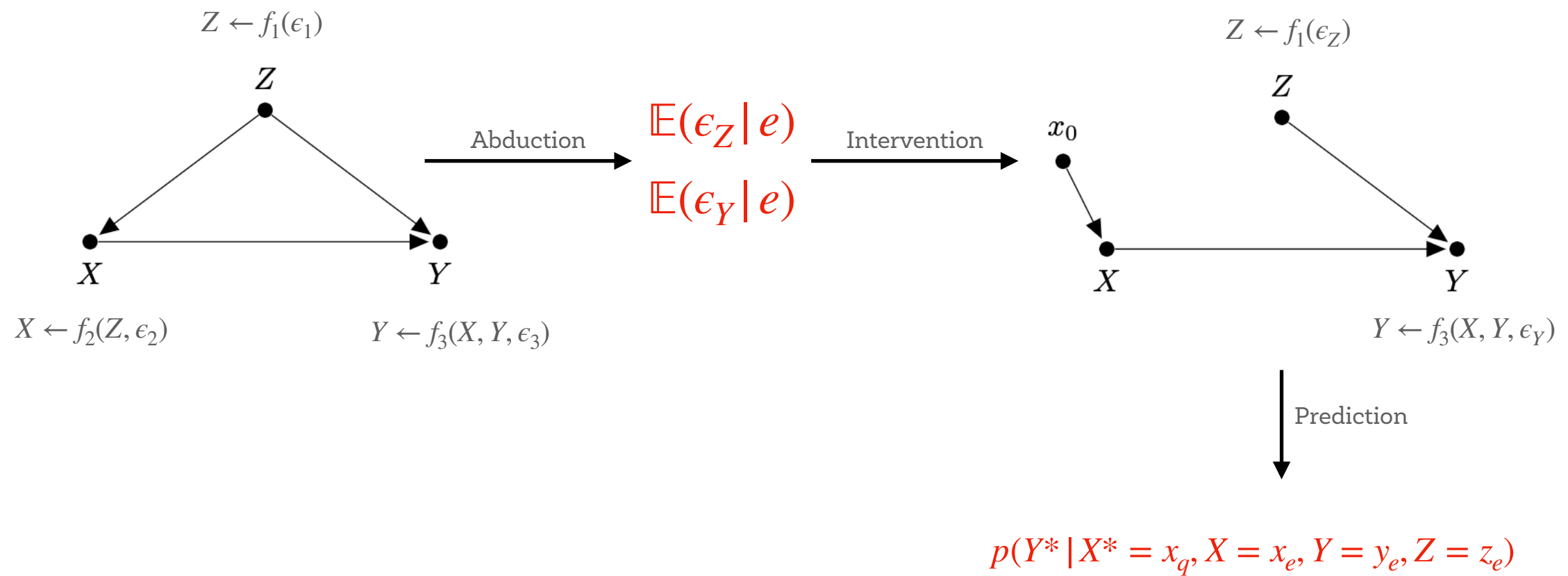
Identifiable counterfactuals can be computed as a three-step process:

- **Abduction**: compute the posterior distribution of ϵ conditioned on e .
- **Intervention**: apply the desired intervention $do(X = x)$
- **Prediction**: compute the required prediction in the intervened distribution.

Counterfactuals

e

R1001	M	1	70	50	1	12	1.73
R1002	M	2	72	100	2	20	4.53
R1003	F	1	55	250	1	16	2.99
R1004	M	2	65	20	2	16	1.13
R1005	F	1	60	10	3	12	3.81
R1006	M	1	68	30	1	9	4.76
R1007	F	5	66	25	2	21	2.01



Causal Graphs

Causal graphs are convenient when the exact assignments in a structural causal models are of secondary importance, but **what matters are the paths present and absent in the graph.**

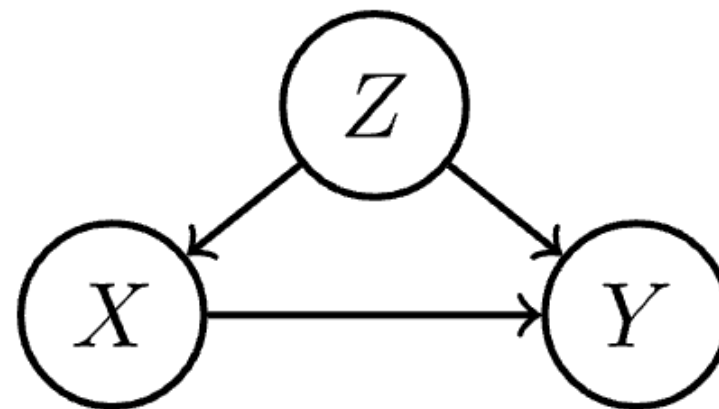
Graphs also let us import the established **language of graph theory to discuss causal notions.**

We can say, for example, that **an *indirect cause* of a node is any ancestor of the node in a given causal graph.** In particular, causal graphs allow us to distinguish cause and effect based on whether a node is an ancestor or descendant of another node.

Causal Graphs

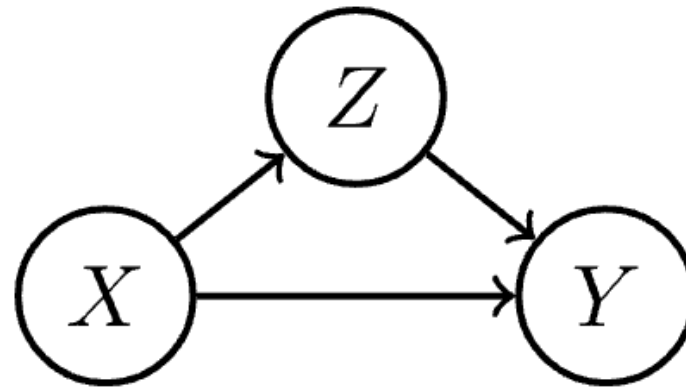
A **fork** is a node Z in a graph that has outgoing edges to two other variables X and Y .

Put differently, the node Z is a common cause of X and Y .



Z has a **confounding** effect: There is a positive correlation between X and Y (not due to $X \rightarrow Y$) because of Z .

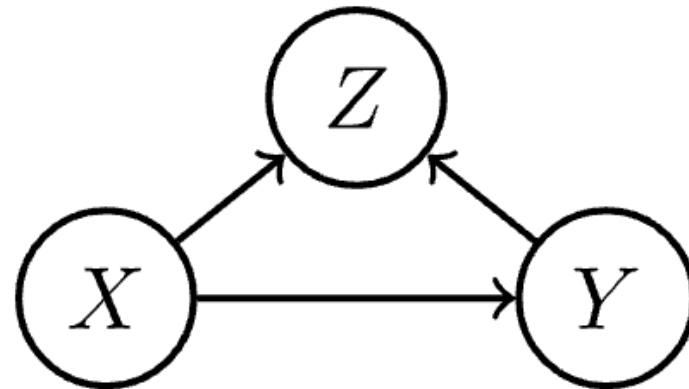
Causal Graphs



In this case, the path $X \rightarrow Z \rightarrow Y$ contributes to the total effect of X on Y. It's a causal path and thus one of the ways in which X causally influences Y. That's why Z is not a confounder. We call Z a **mediator** instead.

The notion of a mediator is particularly relevant to the topic of **discrimination analysis**.

Causal Graphs



This is a **collider**. Colliders aren't confounders. In fact, in the above graph, X and Y are unconfounded, meaning that we can replace do-statements by conditional probabilities.

However, something interesting happens when we condition on a collider. The conditioning step can create correlation between X and Y, a phenomenon called *explaining away*.

Causal Graphs

The causal effect of an action $X = x$ on a variable Y refers to the distribution of the variable Y in the model $do[X = x]$.

When we speak of the **causal effect of a variable X on another variable Y** we refer to all the ways in which setting X to any possible value x affects the distribution of Y .

Often we think of X as a binary treatment variable and are interested in a quantity such as:

$$\mathbb{E}_{do(X=1)}[Y] - \mathbb{E}_{do(X=0)}[Y]$$

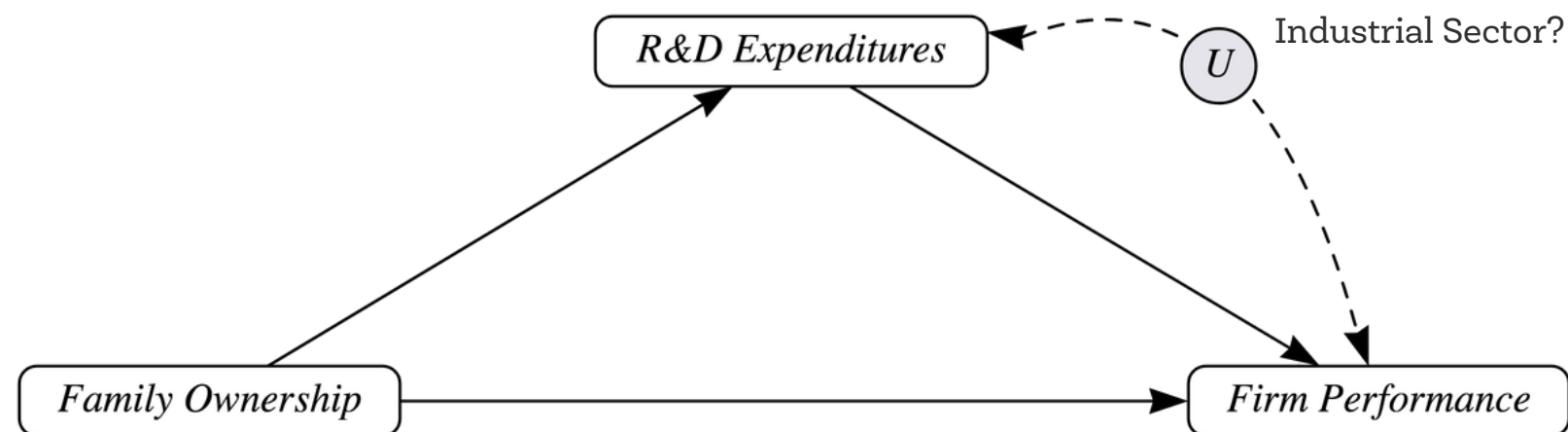
Causal effects are **population quantities**. They refer to effects averaged over the whole population. Often the effect of treatment varies greatly from one individual or group of individuals to another. Such treatment effects are called *heterogeneous*.

This quantity is called the **average treatment effect**.

Unobserved confounders

We have described causal inference with observational data under an assumption of no unobserved confounders.

This is not the case in most of the cases!



<https://p-hunermund.com/2018/08/27/why-you-shouldnt-control-for-post-treatment-variables-in-your-regression/>