§1.5 First Order Linear Equations

Goals

- To understand that any first-order linear differential equation

$$y' + p(t)y = g(t)$$

  can be solved by multiplying each side of the equation by an integrating factor

$$\mu(t) = e^{\int p(t)\,dt}.$$

- To understand the existence and uniqueness of solutions to first-order *linear* differential equations.

To Prepare for Class on §1.5

1. Read §1.5.1 and §1.5.2 through Example 1.5.2.

2. Consider the differential equation $y' - 5y = \sin 2t$ with initial condition $y(0) = 1$.

   (a) Multiply both sides of the equation $y' - 5y = \sin 2t$ by e^{-5t} and show that

$$e^{-5t}(y' - 5y) = \frac{d}{dt}\left(e^{-5t}y\right).$$

   (b) Integrate both sides of

$$e^{-5t}(y' - 5y) = \frac{d}{dt}\left(e^{-5t}y\right) = e^{-5t}\sin 2t$$

   with respect to t and solve for y. *Think integration by parts. You can always use Sage to check that you integrated correctly.*

   (c) Use the results from part (b) to solve the initial value problem

$$y' - 5y = \sin 2t$$
$$y(0) = 1.$$