### §1.2 Separable Differential Equations

## Important Lessons

- A differential equation of **order** $n$ is an equation that can be put in the form

$$F(t, x, x', x'', \ldots, x^{(n)}) = 0,$$

where $F$ is a function of $n+2$ variables. A **solution** to the equation on an interval $I = (a, b)$ is a function $u = u(t)$ such that the first $n$ derivatives of $u$ are defined on $I$, and

$$F(t, u, u', u'', \ldots, u^{(n)}) = 0.$$

- A **first-order differential equation** is an equation that can be written in the form

$$\frac{dx}{dt} = f(t, x).$$

- A differential equation is separable if it can be written in the form

$$\frac{dy}{dx} = M(x)N(y).$$

In this case we can rewrite the equation in the form

$$f(x) + g(y)\frac{dy}{dx} = 0$$

or

$$g(y)\,dy = f(x)\,dx$$

and solve by integrating both sides.

## Investigations

1. Suppose we have a pond that will support 1000 fish, and the initial population is 100 fish. In order to determine the number of fish in the lake at any time $t$, we must find a solution to the initial value problem

$$\frac{dP}{dt} = k\left(1 - \frac{P}{1000}\right)P$$
$$P(0) = 100.$$

It is easy to verify that $P(t) = 1000/(9e^{-kt}+1)$ is the solution to our initial value problem. Solve the IVP.

2. Solve the following initial value problems.

(a) $x\,dx - y^2\,dy = 0$, $y(0) = 1$

(b) $\dfrac{dy}{dx} = \dfrac{y}{x}$, $y(1) = -2$

3. Mr. Ratchett, an elderly American, was found murdered in his train compartment on the Orient Express at 7 A.M. When his body was discovered, the famous detective Hercule Poirot noted that Ratchett had a body temperature of 28 degrees. The body had cooled to a temperature of 27 degrees one hour later. If the normal temperature of a human being is 37 degrees and the air temperature in the train is 22 degrees, estimate the time of Ratchett's death using Newton's Law of Cooling.

4. Suppose that we have a large tank containing 1000 gallons of pure water and that water containing 0.5 pounds of salt per gallon flows into the tank at a rate of 10 gallons per minute. If the tank is also draining at a rate of 10 gallons per minute, the water level in the tank will remain constant. We will assume that the water in the tank is constantly stirred so that the mixture of salt and water is uniform in the tank.