§1.1 Modeling with Differential Equations

Goals

- To understand that a **differential equation** is an equation relating a function to one or more of its derivatives.
- To understand when a function is a solution to a differential equation.
- To understand that some phenomenon, such as the relationship between a population of predators and a population of prey, are best modeled by systems of differential equations.
- To understand the three principle steps in modeling any phenomenon with differential equations:
  - Discovering the differential equation or equations that best describe a specified physical situation.
  - Finding—either exactly or approximately—the appropriate solution of the equation or equations.
  - Interpreting the solutions or behavior of solutions in terms of the phenomenon.

To Prepare for Class on §1.1

1. Read §1.1.1 Exponential Growth
2. Verify that $y(t) = -7e^{t^2} - \frac{1}{2}$ is a solution of $y' = 2ty + t$
3. Consider the DE
$$y'' - 3y' + 2y = 0$$
   (a) Is $y(t) = e^{-2t}$ a solution to the DE? Be sure to clearly state and explain your answer.
   (b) Find all values of a such that $y(t) = e^{at}$ is a solution to the DE.
4. The variables x and y are functions of time and correspond to either the population of predators or the population of prey. Consider the following predator-prey systems of differential equations
$$\frac{dx}{dt} = -\frac{x}{2} + 5xy$$
$$\frac{dy}{dt} = 3y - 2xy$$
   Which variable (either x or y) models the prey population and which variable (either x or y) models the predator population? Write a sentence or two to describe what specific part of the system of differential equations motivated your answer for your predator and prey variable choices.

Making Your Assignment Easy to Read and Easy to Grade

1. Make sure your handwriting is legible and large enough to be read easily.
2. Assignments with multiple pages must be stapled in the upper left-hand corner.
3. In the upper right-hand corner you should write (in this order):
   (a) Your name
   (b) section in the textbook that corresponds to the assignment.
   (c) The due date of the assignment.
4. Problems should be clearly labeled and numbered on the left side of the page. There should also be a visible separation between problems.
5. You should leave the entire left margin blank so that the grader can use this space for scoring and comments.
6. To ensure that each problem is graded, problems and solutions should be written in the order that they are assigned.
7. You should write up your solutions by yourself. You should always acknowledge any help received at the top of the assignment or in the right-hand margin. Copying from others is Academic Misconduct. Copied solutions will be awarded zero points and may result in a zero for the entire assignment.
8. Do not spend more than 60 minutes doing a prep assignment.