

**Example 0.1.** Shifted Function: A function viewed in a fixed coordinate  $x$  is centered at zero  $f(x - 0) = f(x) = 1 + x^2$ , and shifted to any center at  $x = z$  according to  $f(x - z) = 1 + (x - z)^2$ . Therefore the shifted  $f(x - z = 0) = f(x = z) = 1 + (z - z)^2 = 1$  evaluated at the center  $x = z$  gives the same function value as the unshifted  $f(x = 0) = 1 + x^2 = 1$  centered at  $x = 0$ . Notice the two functions are indistinguishable when given a dummy variable  $y = x - z$  and evaluate in separate coordinates, so  $f(y) = 1 + y^2$  is essentially the same as  $f(x)$ . Hence, only if viewed in the same  $x$  coordinate will we mention the "shifted function".

**Example 0.2.** Reflected Function (Asymmetric): A function centered at zero  $f(x) = x - 1$  being reflected gives  $f(-x) = -x - 1$ , where simply plugging in  $x = 1$  will give the corresponding function values for the unreflected  $f(1) = 0$  which is not equal to the reflected  $f(-1) = -2$ . But, the reflected  $x = -1$  at  $f(-x)$  gives the same function value as  $x = 1$  at  $f(x)$ .

**Example 0.3.** Reflected Function (Symmetric): The function centered at zero  $f(x) = x^2$  being reflected gives  $f(-x) = x^2$  implies  $f(x) = f(-x)$ .

**Example 0.4.** Shifted Reflected Function (Asymmetric):  $f(x) = x - 1 \rightarrow f(2 - x) = f(-(x - 2)) = 1 - x$  is centered at  $x = 2$  and reflected (Figure 1).

Graph for  $1-x$ ,  $x-1$

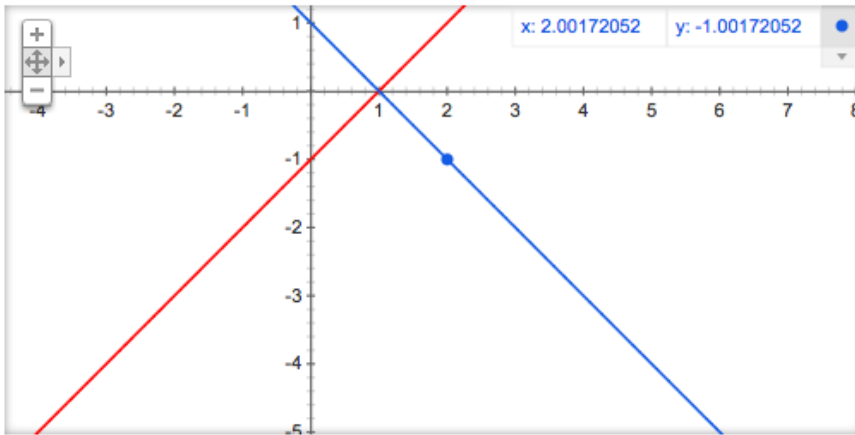


Figure 1