0.1 Thermodynamic equation

From [1], the thermodynamic equation

$$\frac{D(c_pT)}{Dt} - \alpha\frac{Dp}{Dt} = Q \tag{1}$$

with $w = \frac{Dz}{Dt}$ and the hydrostatic condition

$$\frac{\partial p}{\partial z} = -\frac{g}{\alpha} \tag{2}$$

becomes

$$\begin{aligned} \frac{D(c_pT)}{Dt} - \alpha\Big(\frac{\partial p}{\partial t} + \vec{V}_h \cdot \vec{\nabla}_h p + w\frac{\partial p}{\partial z}\Big) = Q \\ \frac{D(c_pT + gz)}{Dt} = \frac{Ds}{Dt} = Q + \alpha\Big(\frac{\partial p}{\partial t} + \vec{V}_h \cdot \vec{\nabla}_h p\Big). \end{aligned} \tag{3}$$

The local tendency could be written as

$$\frac{\partial s}{\partial t} = \Big(\frac{\partial s}{\partial t}\Big)_{\text{advec}} + \Big(\frac{\partial s}{\partial t}\Big)_{\text{work}} + \Big(\frac{\partial s}{\partial t}\Big)_{\text{heat}} = \dot{s_1} + \dot{s_2} + \dot{s_3}. \tag{4}$$

Suppose the state is updated sequentially through each process, from advection ($\dot{s_1}$), work ($\dot{s_2}$), to apparent heating $\dot{s_3}$. Then $\frac{\partial s}{\partial t} = \frac{s_I - s_0}{\delta t} = \dot{s_1} + \dot{s_2} + \dot{s_3} \Rightarrow s_I = s_0 + \delta t\dot{s_1} + \delta t\dot{s_2} + \delta t\dot{s_3} = s_1 + \delta t\dot{s_2} + \delta t\dot{s_3} = s_2 + \delta t\dot{s_3}$, where the accumulated state is updated according to $s_i = s_{i-1} + \delta t\dot{s}_i = s_0 + \delta t \sum_{j=1}^{i} \dot{s}_j$. Notice that the accumulated heating $\dot{s} = \sum_{i=1}^{3} \dot{s}_i = \frac{s_I - s_0}{\delta t}$, but the individual heating $\dot{s}_i \neq \frac{s_i - s_0}{\delta t}$, instead $\dot{s}_i = \frac{s_i - s_0}{\delta t} - \sum_{j=1}^{i-1} \dot{s}_j$, indicating that s_i is the accumulated state. Notice that $\dot{s_3} = Q$ is a sum of different heating processes, $Q = \sum \dot{q}_i$, where the accumulated states could also be sequentially updated, $q_i = q_{i-1} + \delta t\dot{q}_i$, and $Q = \frac{q_I - q_0}{\delta t}$.

0.1.1 Summarize

- Obtain the *accumulated state*, s_i, through each process.
- Use the accumulated state as the input for the next process tendency, $\dot{s}_i(s_{i-1})$ (instead of the initial state s_0, e.g., radiative heating require cloud states).
- Obtain the final tendency from the final accumulated state, $\frac{\partial s}{\partial t} = \frac{s_I - s_0}{\delta t}$.

References

[1] B. A. Boville and C. S. Bretherton, "Heating and kinetic energy dissipation in the ncar community atmosphere model," *Journal of climate*, vol. 16, no. 23, pp. 3877–3887, 2003.