

0.1 State Function

A function that describes the *equilibrium* state of a system, irrespective of how the system arrived in the state. e.g, $f(p, V, T) = 0$. Whereas, mechanical work and heat depends on the *path* between two equilibrium states.

0.2 Ideal gas

For n moles of any gas

$$pV = nR^*T, \quad (1)$$

with the universal constant R^* [$JK^{-1} \text{ mol}^{-1}$]. Suppose $n = 1$ mole of dry air, then there is M_d (molecular weight for dry air) grams of dry air

$$p_d V_d = \frac{M_d[0.001kg]}{M_d} R^* [JK^{-1}(0.001kg)^{-1}] T = 0.001 M_d [kg] \left(\frac{1000 R^*}{M_d} [JK^{-1}kg^{-1}] \right) T \quad (2)$$

which becomes

$$p_d \alpha_d = R_d T. \quad (3)$$

This is the general relationship of any gas, hence the water vapor pressure

$$e \alpha_v = R_v T. \quad (4)$$

0.3 Thermodynamic equation

$$\frac{DI}{Dt} + p \frac{D\alpha}{Dt} = \dot{Q} \quad (5)$$

Q. How to relate the pressure (state) in the momentum equation by the thermodynamic equation?

A. $dI = c_v dT \Rightarrow I = c_v T$, and assuming ideal gas $p = \rho RT \Rightarrow p = \rho R I / c_v$, hence,

$$\frac{DI}{Dt} - \frac{p}{\rho^2} \frac{D\rho}{Dt} = \dot{Q}, \quad (6)$$

with the continuity equation it becomes

$$\frac{DI}{Dt} + \frac{p}{\rho} \nabla \cdot v = \dot{Q}. \quad (7)$$

Q. Assuming large scale in *hydrostatic balance*?

A. $\alpha dp = -g dz$, hence $dQ = c_v dT - \alpha dp = c_v dT + g dz = d(c_v T + g z) = ds$ (dry static energy)

$$\frac{Ds}{Dt} = \dot{Q}. \quad (8)$$

Q. What about general gas (non-ideal)?

A. Diagnostic equation for pressure is $p = -\frac{\partial I}{\partial \alpha}$.

0.4 Entropy

A measure of “difference” between adiabats (no heat exchange processes), and a measure of “work loss” in transferring heat (irreversible process).

In a closed system, when heat is added at a constant temperature (volume expands and pressure decreases), the amount of disorder increases which increases the potential temperature (the actual temperature is higher when forced back to original pressure). For a irreversible process, suppose a heat reservoir at T_2 transfers heat Q to a cooler reservoir at T_0 . By Carnot’s engine, we know that the maximum available energy the heat tranferred into work is

$$W_2 = (1 - \frac{T_0}{T_2})Q. \quad (9)$$

For a irreversible process (state variables (e.g., p , V , T) cannot go back to original values without additional work), where heat is first transferred to a middle reservoir at the state $T_1 < T_2$, and then transfers the same amount of heat Q to the cold reservoir at T_0 . Notice since $T_1 < T_2$, therefore the entropy is higher to transfer the same amount of Q . Therefore the maximum available work from the middle reservoir is

$$W_1 = (1 - \frac{T_0}{T_1})Q, \quad (10)$$

which is less than W_2 . Hence, the work loss from irreversible process is

$$W_2 - W_1 = Q(T_0/T_1 - T_0/T_2) = T_0(S_1 - S_2) = T_0dS, \quad (11)$$

in a form of heat loss, where the increase in dS becomes a measure of work loss. There is a net gain in entropy in the irreversible process. i.e., **the more entropy gain/more disorderness contributes to more work loss, which becomes a less efficient process.**