An Efficient Perturbed Parameter Scheme in the Lorenz system for Quantifying Model Uncertainty

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1. Introduction

The goal of this study is to develop a reliable and efficient perturbed parameter scheme that is comparable to stochastic parameterizations. The Lorenz model is a simple testbed for numerical simulations. We have implemented a two time-scale Lorenz 63 (L63) model to mimic the ocean-atmosphere coupled system, and a spatially-resolved convective-scale Lorenz 96 (L96) model coupled to the L63 atmospheric component. The full set of equations is defined as the "truth". The spatially-resolved system will be parameterized by three different schemes: (i) deterministic, (ii) additive stochastic parameterization, and (iii) perturbed parameter. Perfect initial conditions are applied to investigate model error. By applying "informative" probability distributions to the perturbed parameter scheme, we have reduced systematic biases where previous implementations have failed to do so. Despite the fact that stochastic parameterizations are shown to resolve systematic errors, they become computationally expensive in a GCM with more grid points to generate correlated stochasticity. The proposed scheme uses a stochastic spectral method, Polynomial Chaos Expansion (PCE), to approximate the exact ensemble states with minimum model simulations. Once built, PCE is implemented as a low cost sampling surrogate to generate large ensemble forecasts without actually integrating the model. The PCE-accelerated informative perturbed parameter scheme generates competitive and reliable ensemble forecasts.

2. Experimental Setup

2.1. The Coupled Lorenz System

We use a two time-scale, coupled L63 model (Lorenz 1963; Siqueira and Kirtman 2012) where the fast component is analogous to the atmosphere and the slow component is analogous to the ocean. The atmospheric component is further coupled to a small-scale spatially resolved system (high frequency small amplitude). This spatially resolved system is composed of four identical dynamical equations of the convective-scale L96 model (Lorenz 2006) each representing a spatial grid point with nonlinear interaction with the neighbors. The first and last grid point share the boundary. The small-scale system could be seen as a convective process in the atmospheric component. Therefore, a deterministic parameterization of the spatially resolved system is analogous to the bulk parameterization of the subgrid-scale processes in the weather and climate models.

The atmospheric component:

$$\begin{aligned}\frac{dX_1}{dt} &= \sigma(X_2 - X_1) - a(Y_1 + k) - \mathbf{U}^* \\ \frac{dX_2}{dt} &= rX_1 - X_2 - X_1X_3 + a(Y_2 + k) \\ \frac{dX_3}{dt} &= X_1X_2 - bX_3 + aY_3\end{aligned} \tag{1}$$

The oceanic component:

$$\begin{aligned}\frac{dY_1}{dt} &= \tau\Big(\sigma(Y_2 - Y_1)\Big) - a(X_1 + k) \\ \frac{dY_2}{dt} &= \tau(rY_1 - Y_2 - Y_1Y_3) + a(X_2 + k) \\ \frac{dY_3}{dt} &= \tau(Y_1Y_2 - bY_3) + aX_3\end{aligned} \tag{2}$$

where $\mathbf{U}^* = \frac{a_z\tau_z}{s_z}\sum_{i=1}^{4} Z_i$ (the term to be parameterized) is associated to the spatially resolved system:

$$\frac{dZ_i}{dt} = \tau_z\Big(- s_z Z_{i+1}(Z_{i+2} - Z_{i-1}) - Z_i + \frac{a_z}{s_z}X_1\Big); \ i = 1, \ldots, 4 \tag{3}$$

where $Z_0 = Z_4$, and $Z_5 = Z_1$. The spatially resolved system represents the subgrid-scale processes after $\mathbf{U}^*$ is parameterized.

2.2. Truth Model

The true states are from the outputs of the entire set of equations (1), (2) and (3). The equations are integrated by an adaptive fourth-order Runge-Kutta (RK4) time-stepping scheme. A true time series of 1600 Model Time Units (MTU) is generated (transient phase is removed) for the forecast models to compare with. This study conducts a total of 300 forecast events (each of 25 MTU) with initial conditions selected from the 1600 MTU truth time series at intervals of 5 MTU where the atmospheric states generally lose correlation.

Following Arnold et al. (2013), we designed two experiments by varying the time-scale ($\tau_z = 2$ and $\tau_z = 10$) for the spatially resolved system. The *error-doubling time*, approximately two atmospheric days by a GCM (Lorenz 2006), in the atmospheric component is 1.47 MTU for $\tau_z = 2$, and 1.10 MTU for $\tau_z = 10$. Therefore the atmospheric component maximum forecast lead time for both experiments is selected to be 5 MTU, which is in the range of 7 to 10 atmospheric days by a GCM. The results using L96 as the coupled model (Wilks 2005; Arnold et al. 2013) stated that the slow evolving case ($\tau_z = 2$ in our case) better represents a real atmospheric subgrid-scale process. Whereas the subgrid-scale system at $\tau_z = 10$ behaves like a white-noise process with less correlation between the time samples. The contrasting subgrid-scale dynamics gives us the opportunity to test the limits of our methodology.

2.3. Forecast Model

The forecast model uses the atmosphere (1) and ocean (2), and parameterizes $\mathbf{U}^*$ with U_p, thus truncating the small-scale system (3). Three schemes of U_p are compared for the forecast model: (i) deterministic, (ii) additive stochastic parameterization and (iii) the proposed perturbed parameter scheme.

3. Results

The forecast skill of the ensemble schemes is evaluated by three scores, Reliability (REL), Ignorance Skill Score (IGNSS) and Ranked Probability Skill Score (RPSS).

Figure 1 ($\tau_z = 2$) and Figure 2 ($\tau_z = 10$) show the time evolution of three scores RPSS (top), IGNSS (middle) and REL (bottom) generated by the schemes: (I) Uninformative perturbed parameter (e_{unif}), (II) Informative perturbed parameter (e_{clim}, e_{AR1}), (III) Additive stochastic parameterization (*Stoch*), (IIII) Deterministic parameterization (*Det*). The PCE for the perturbed parameter scheme in the two figures is approximated by 64^{th} degree polynomial. As expected, without a probabilistic feature, *Det* performed poorly over all scores. Therefore, we only compare the *Stoch*, and the perturbed parameter scheme using e_{clim}, e_{AR1} and e_{unif} in the following.

Results in Figure 1 and Figure 2 show consistent and nearly identical scores by e_{clim} and e_{AR1}. This supports the hypothesis of applying informative distributions, regardless of the highly bifurcated the states in $\tau_z = 10$, for the perturbed parameter scheme.

At $\tau_z = 10$, the uninformative e_{unif} performs poorly in Figure 2 (a) and (c). The failure is attributed to the wide ensemble spread (caused by strong bifurcations) at $\tau_z = 10$. Suppose the wide-spread case ($\tau_z = 10$) and the small-spread case ($\tau_z = 2$) both forecast low density at the category of occurrence by using e_{unif}. The small-spread case will remain reliable with the entire ensemble contained in (or in close proximity to) the category of occurrence. Whereas the wide-spread case using the uninformative input may have its highest density erroneously far from the category of occurrence, resulting in a penalty in RPSS. This explains why it is necessary to apply informative e_s, especially when forecasting the wide-spread cases.

4. Concluding Remarks and Future Work

We have implemented a coupled ocean-atmosphere system (L63) with a subgrid-scale system (L96) in the atmospheric component. The major result is to generate reliable forecasts by the proposed perturbed parameter scheme through the employment of informative input distributions. The informative perturbed parameter scheme especially outperformed an additive stochastic parameterization in a more realistic subgrid-scale system ($\tau_z = 2$). The low cost PCE surrogate guarantees unbiased statistics at large ensemble size, and effectively delivers the ensemble distributions without the need to integrate the actual forecast model. The numerical integration (i.e., quadrature) to build the PCE treats the model as a black box, which is an advantage for operational forecasts using complex GCMs.

The forecast skill of using just a single perturbed parameter for the PCE-accelerated informative perturbed parameter scheme is surprisingly competitive with the additive stochastic parameterization. The challenge is to apply this to a realistic model with more parameters, where our forecast framework is developed to test multiple perturbed parameters effectively.

References

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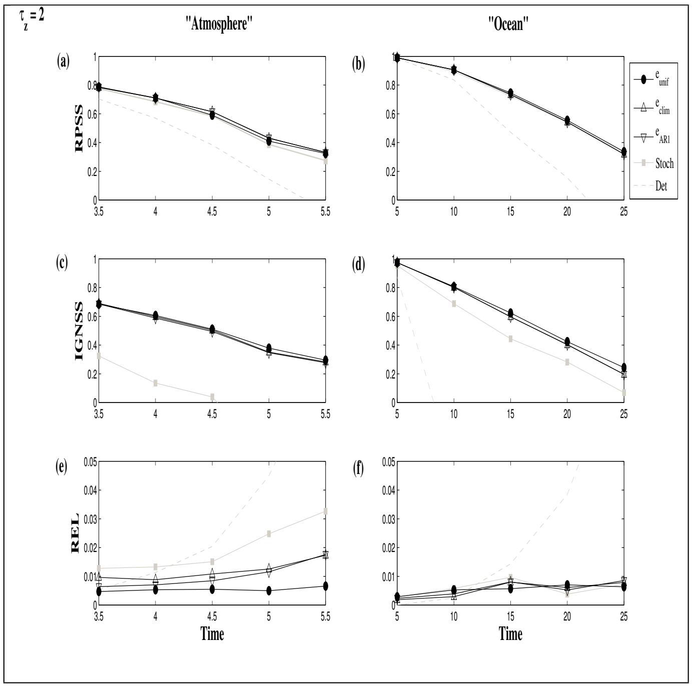

Figure 1. Forecast scores RPSS (top), IGNSS (middle) and REL (bottom) for the atmosphere (left) and ocean (right) components at $\tau_z = 2$ generated by e_{unif} (black circle), e_{clim} (black upper triangle), e_{AR1} (black lower triangle), *Stoch* (grey square), and *Det* (grey dashed-line). The PCE approximated by 64^{th} degree polynomial.

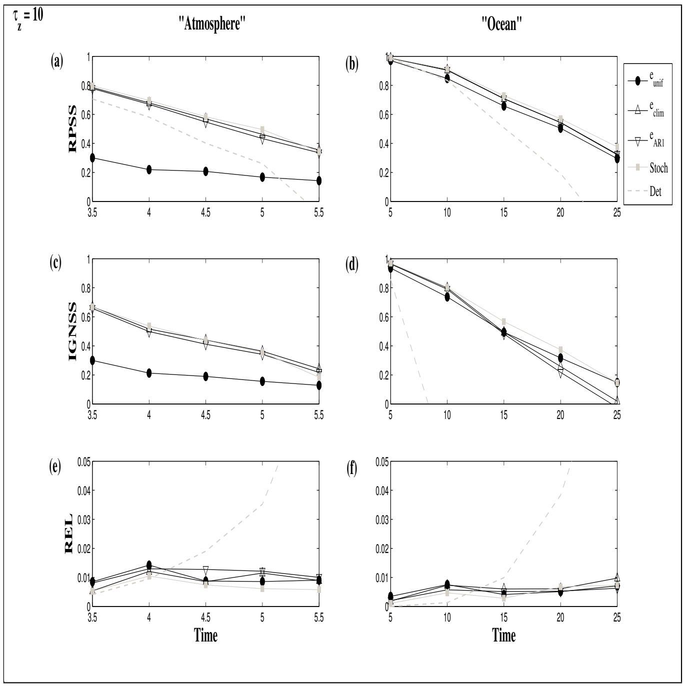

Figure 2. As Figure 1, forecast scores RPSS, IGNSS and REL at $\tau_z = 10$.