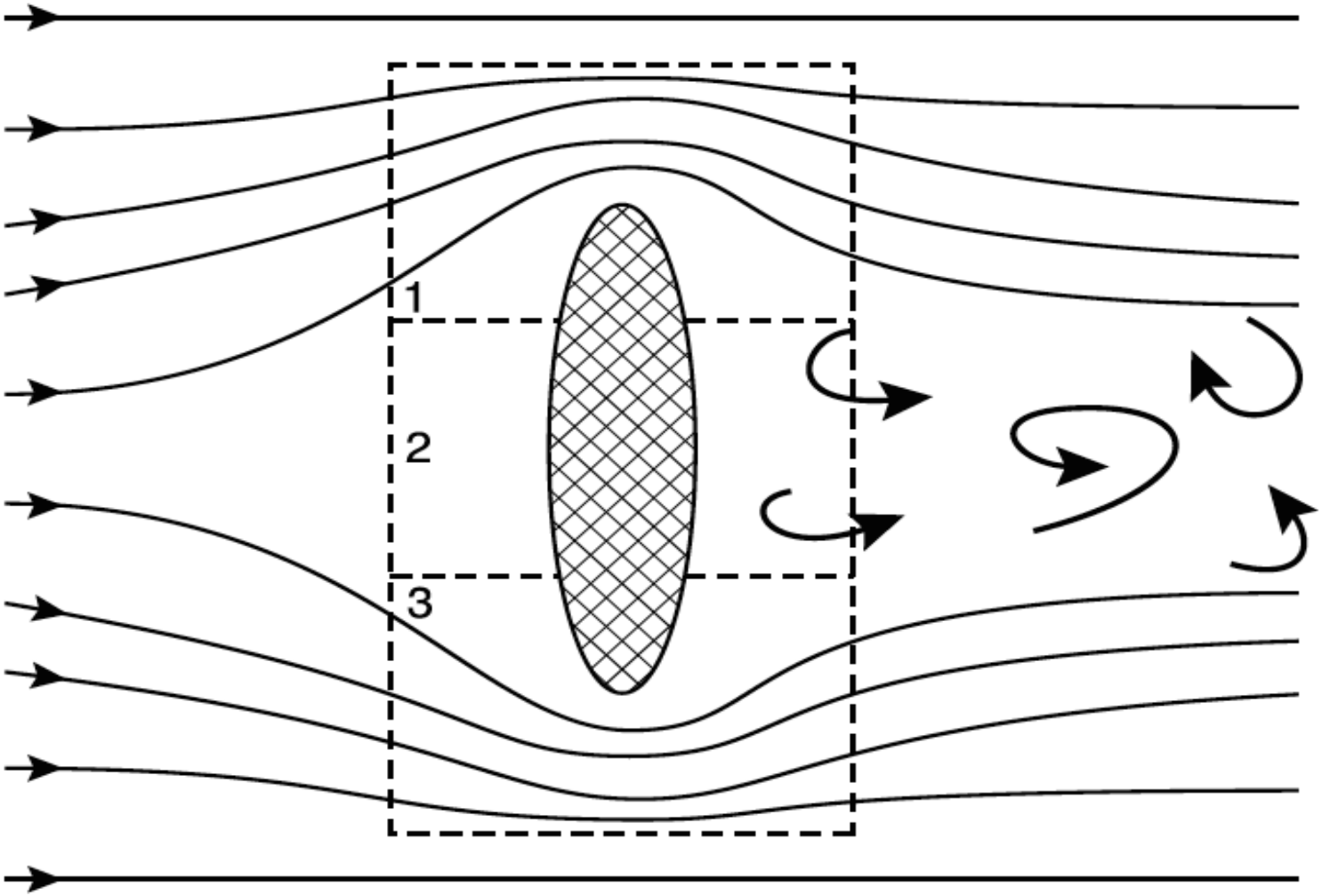
An Efficient Perturbed Parameter Scheme in the Lorenz system for Quantifying Model Uncertainty

Gino Chen, Ben Kirtman and Mohamed Iskandarani

University of Miami

Motivation

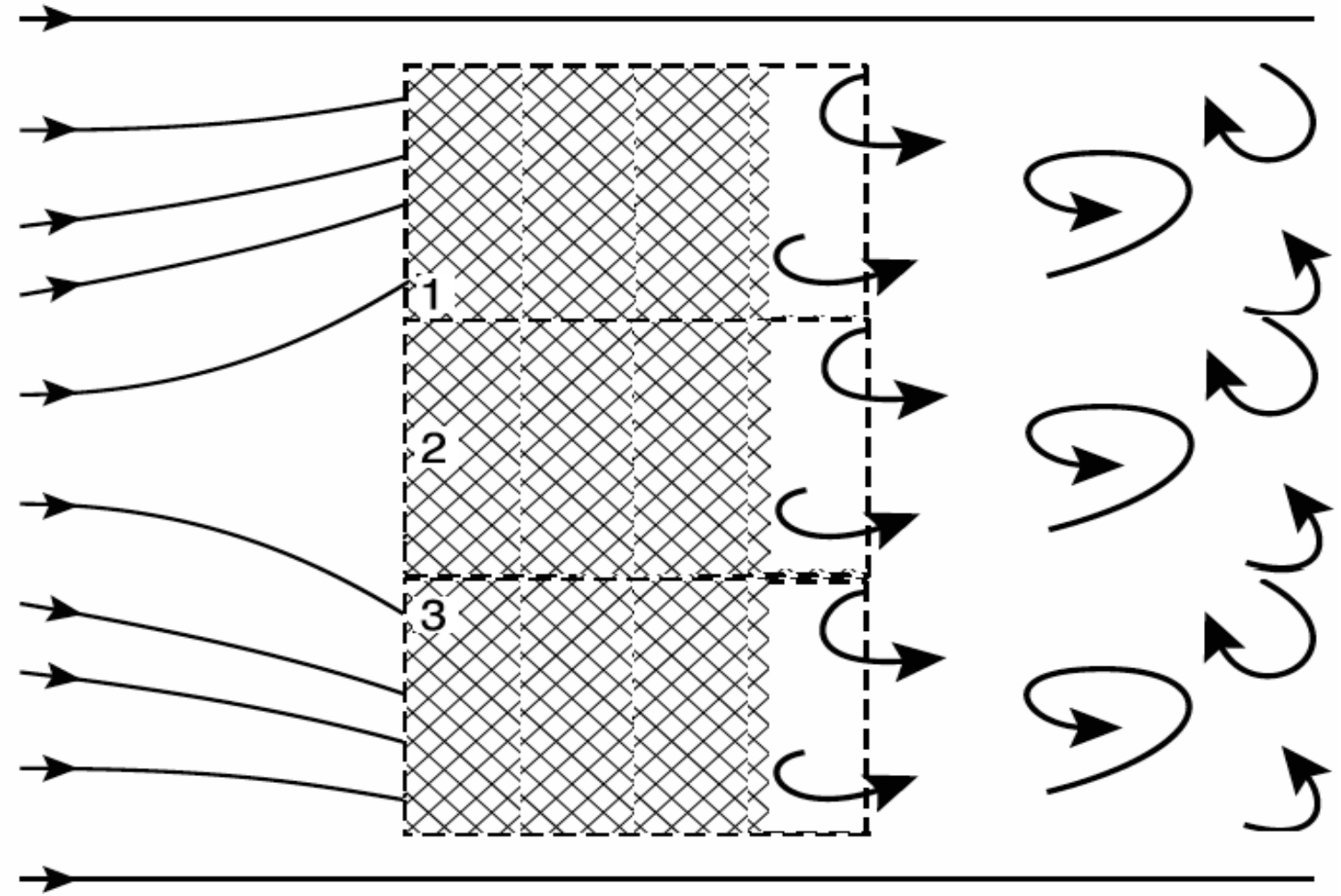
- **Models contain serious errors!**



Palmer 2006

Motivation

- **Bulk-parameterized wave-drag tendency causing error!**



Approach

- Add stochastisity to the bulk-parameterized term.
  - Additive **Stochastic Parameterization**

- Without stochastisity in time? Still reliable?
  - Proposed scheme: "Informative" **Perturbed Parameter** scheme

  $\Longrightarrow$ Cost reduction with spectral method!

Experimental Setup

- Model equations: Lorenz'63 & Lorenz'96
- Truth model & Parameterized forecast model
- 300 perfect initial conditions

Experimental Setup

- Truth model
  - Atmosphere

$$\frac{d\vec{X}}{dt} = F_x(\vec{X}, \vec{Y}) + \mathrm{U}_{\mathrm{truth}}(\vec{Z})$$
$$\frac{dZ_i}{dt} = F_z(\vec{Z}, X); \quad i = 1, \ldots, 4$$

  - Ocean

$$\frac{d\vec{Y}}{dt} = F_y(\vec{X}, \vec{Y})$$

Experimental Setup

- Parameterized forecast model
  - Atmosphere

$$\frac{d\vec{X}}{dt} = F_x(\vec{X}, \vec{Y}) + \mathrm{U}_{\mathrm{param}}(X)$$

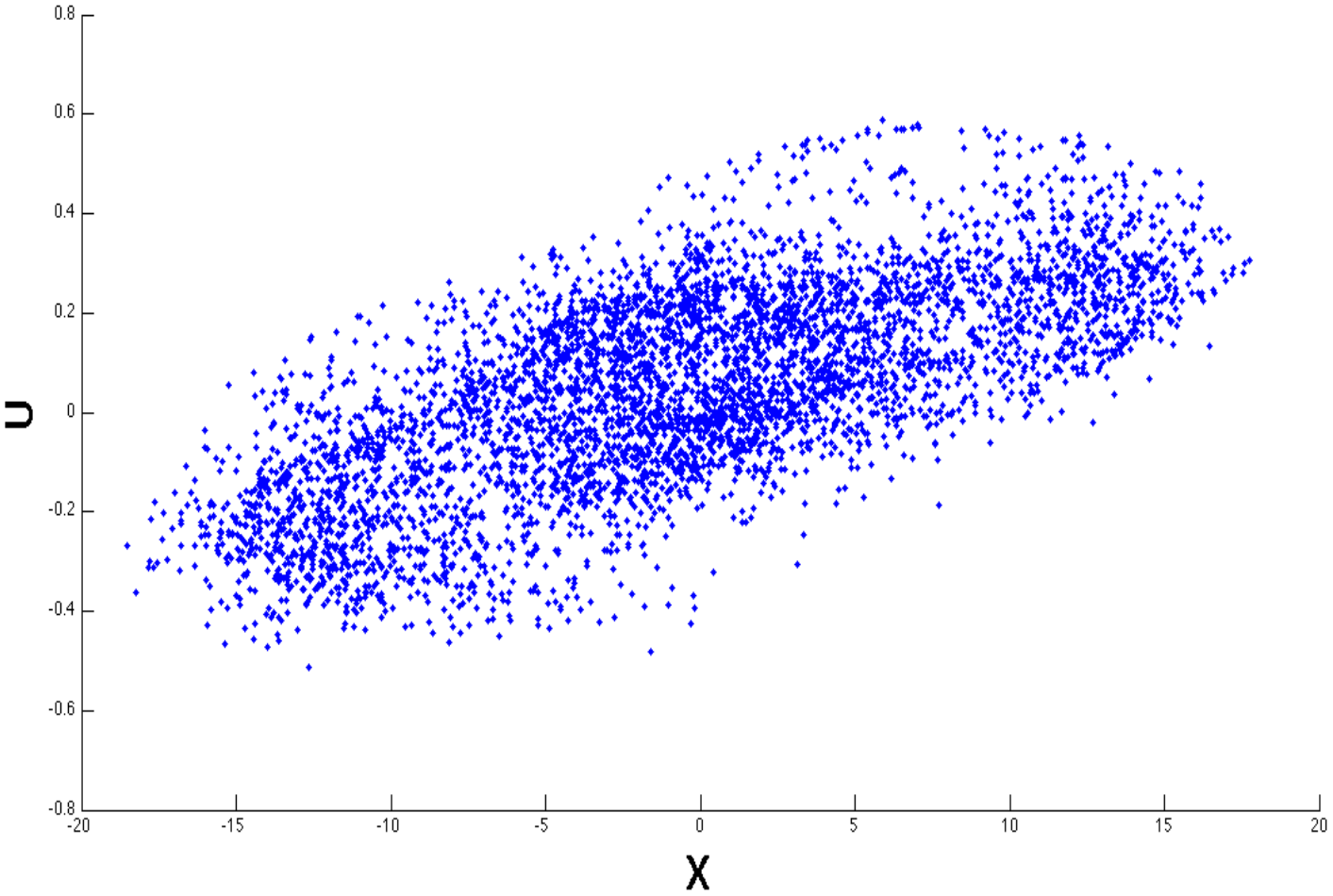
$$\cancel{\frac{dZ_i}{dt} = F_z(\vec{Z}, X); \quad i = 1, \ldots, 4}$$

  - Ocean

$$\frac{d\vec{Y}}{dt} = F_y(\vec{X}, \vec{Y})$$

A cloud of U_{truth}

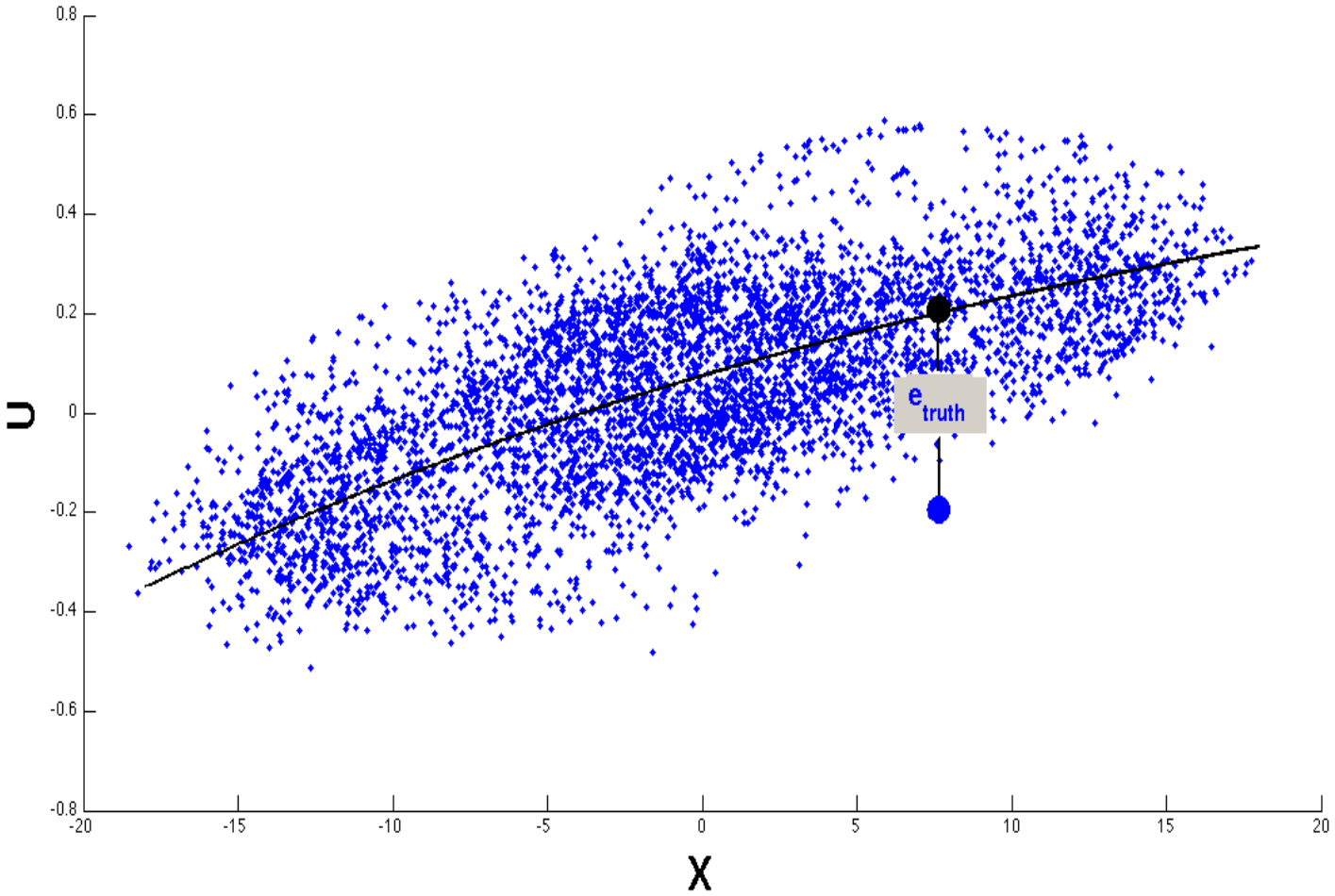
$U_{truth}(X)$



U_{param}: Deterministic & Stochastic

$$U_{det} = b_0 + b_1 X + b_2 X^2 + b_3 X^3,$$

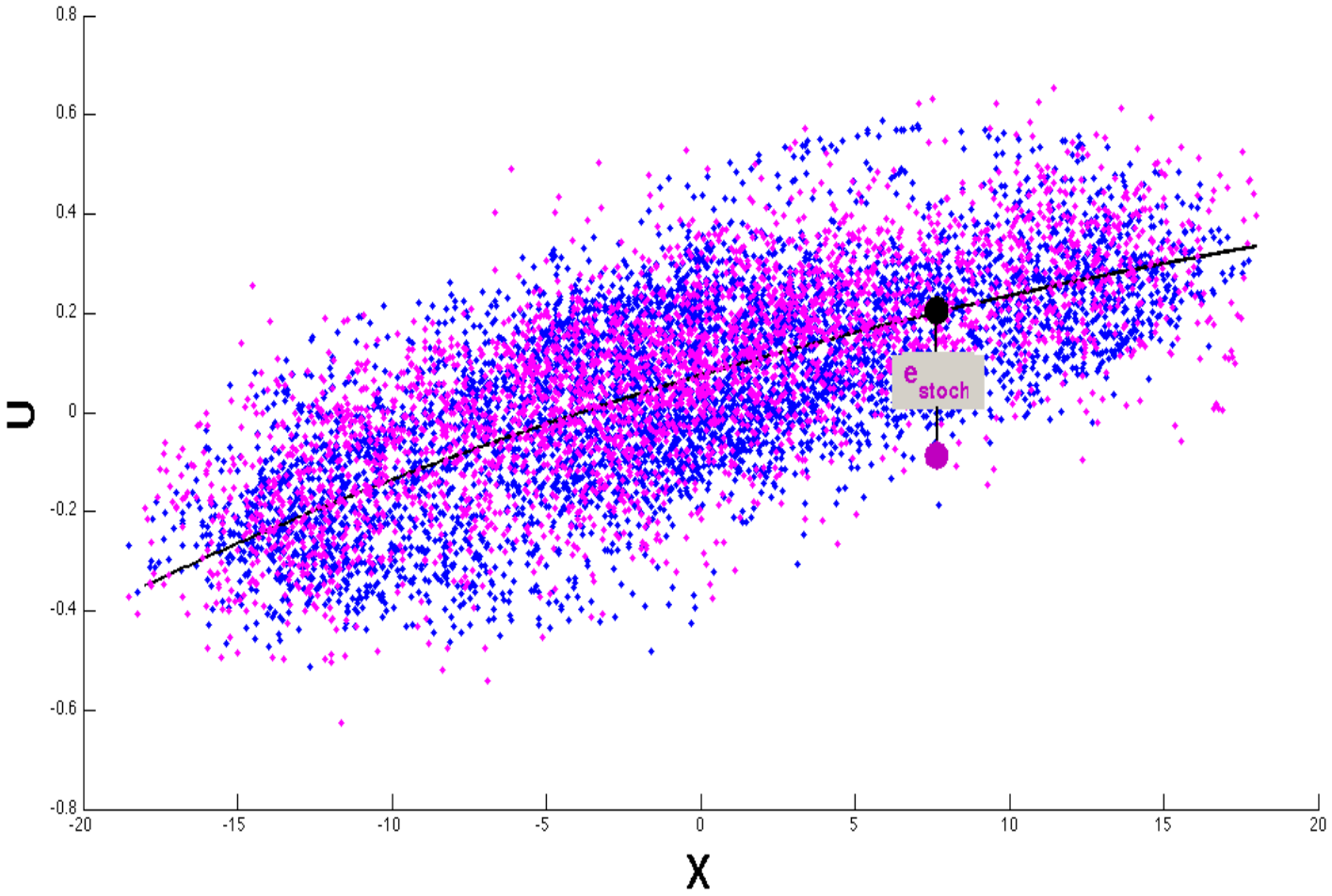
$$U_{truth} = U_{det} + e_{truth}$$



U_{param}: Deterministic & Stochastic

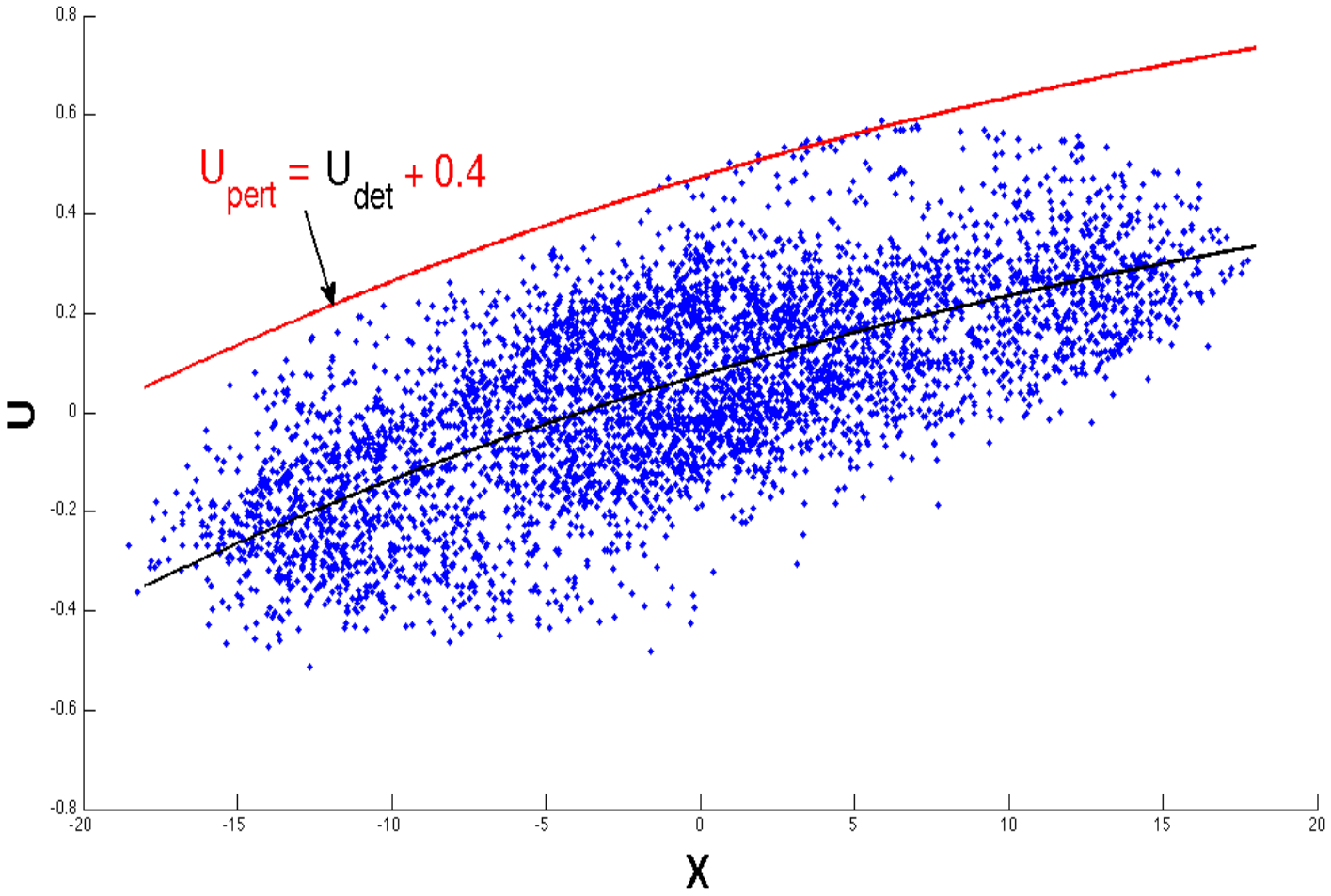
$$U_{det} = b_0 + b_1 X + b_2 X^2 + b_3 X^3,$$

$$U_{truth} = U_{det} + e_{truth} \Longrightarrow U_{stoch} = U_{det} + e_{stoch}.$$



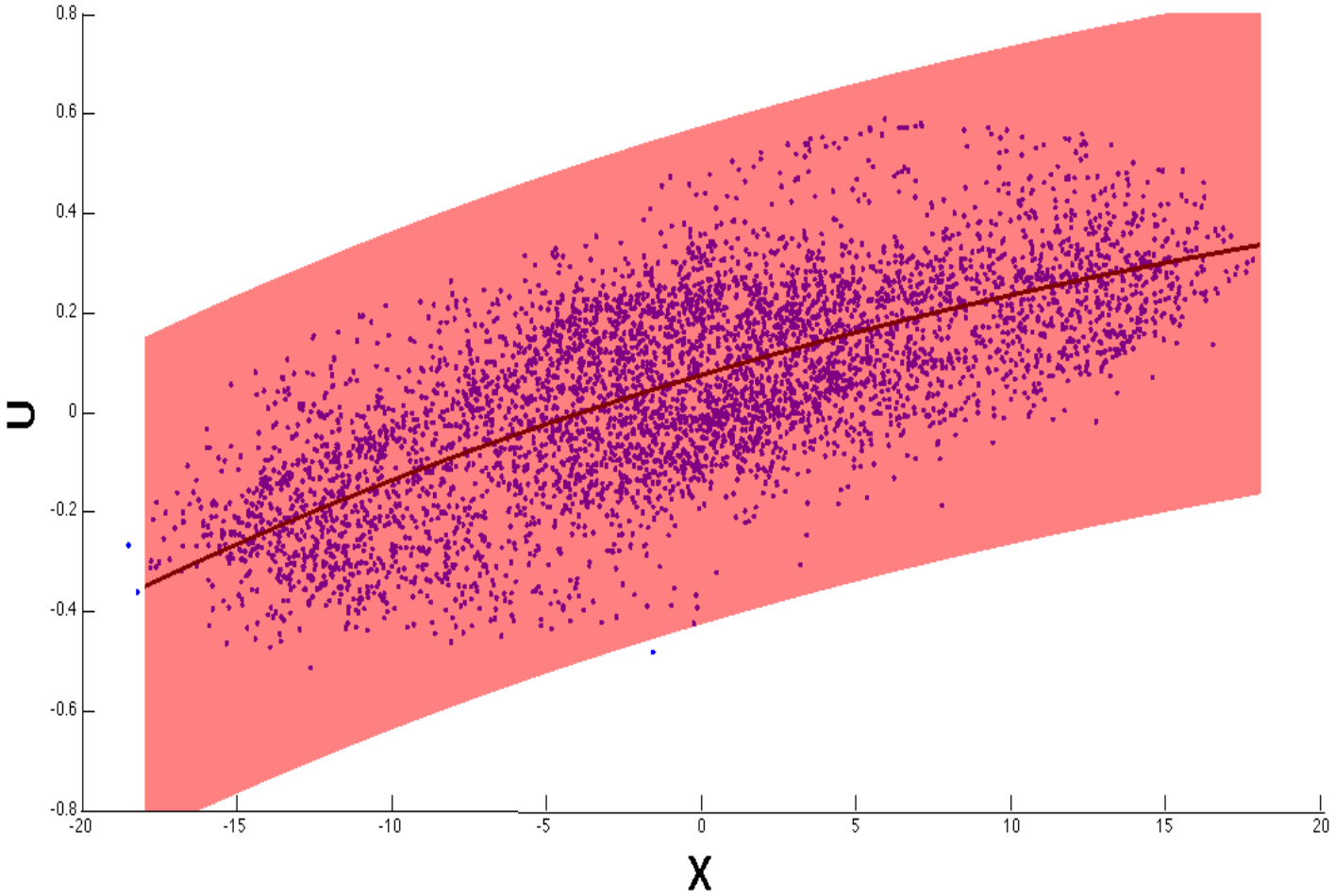
U_{param}: Perturbed Parameter

$$U_{pert} = U_{det} + e_{pert}.$$

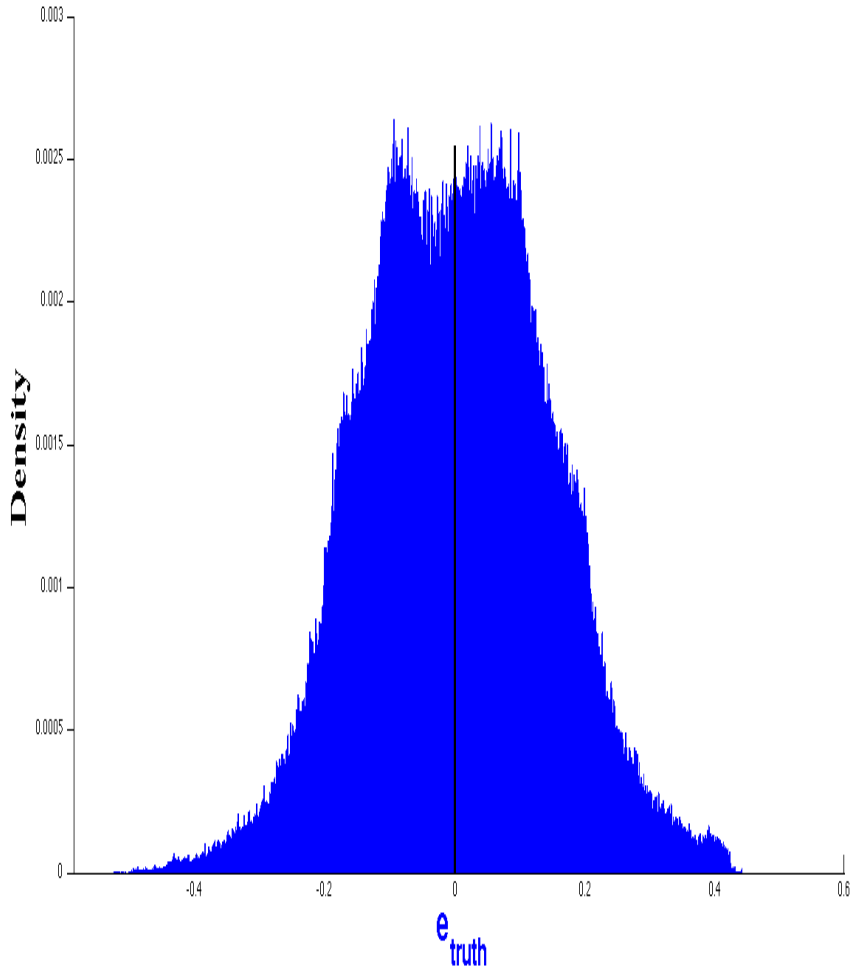


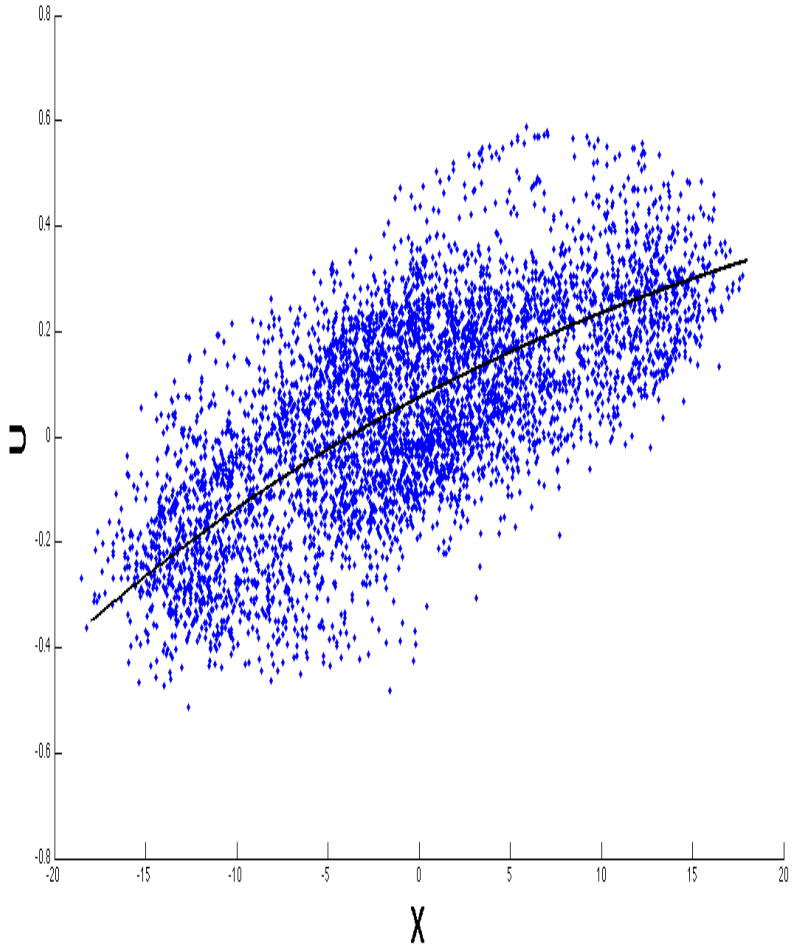
U_{param}: Perturbed Parameter

$$U_{pert} = U_{det} + e_{pert}.$$



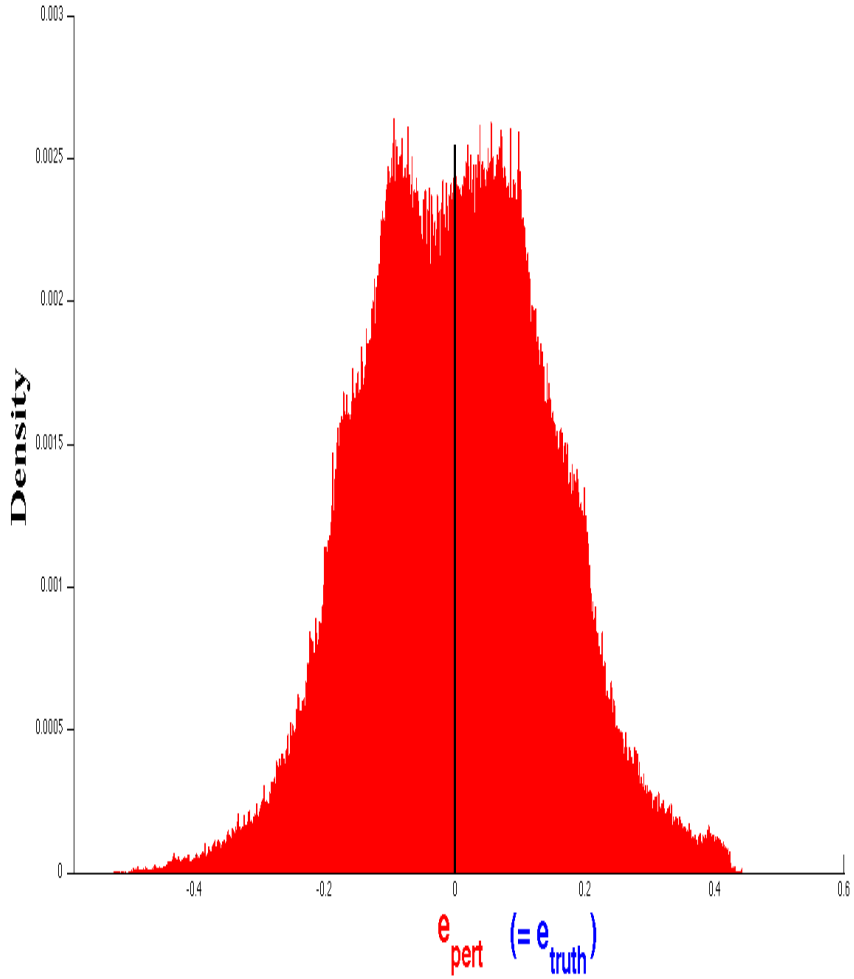
Informative Distribution

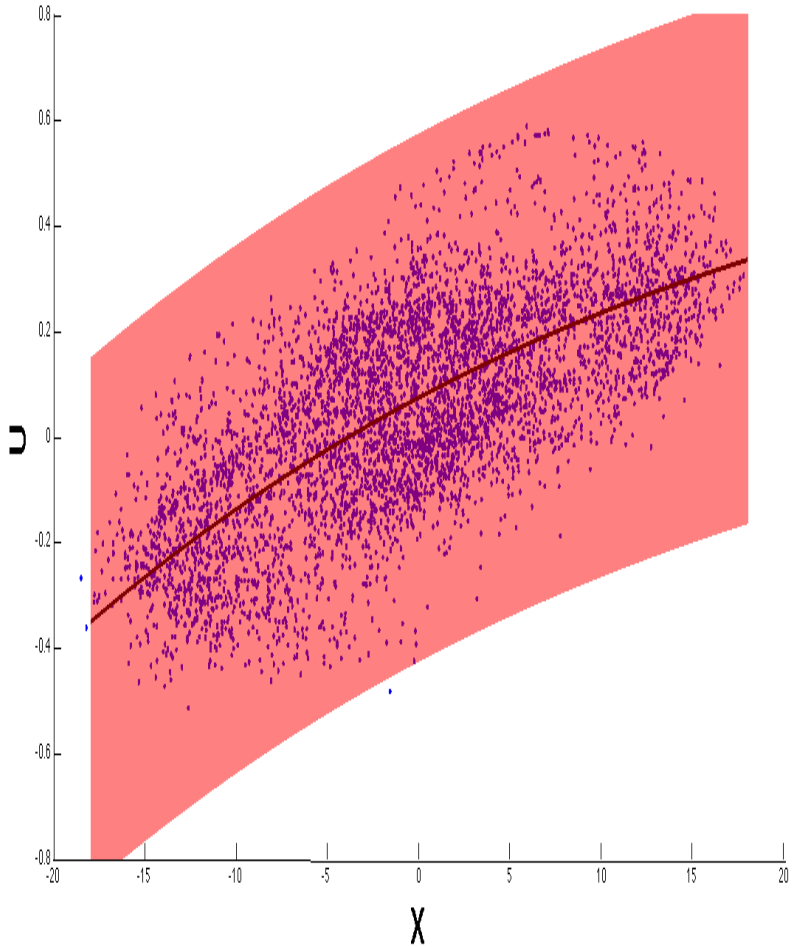




Informative Distribution

$$U_{pert} = U_{det} + e_{pert}.$$

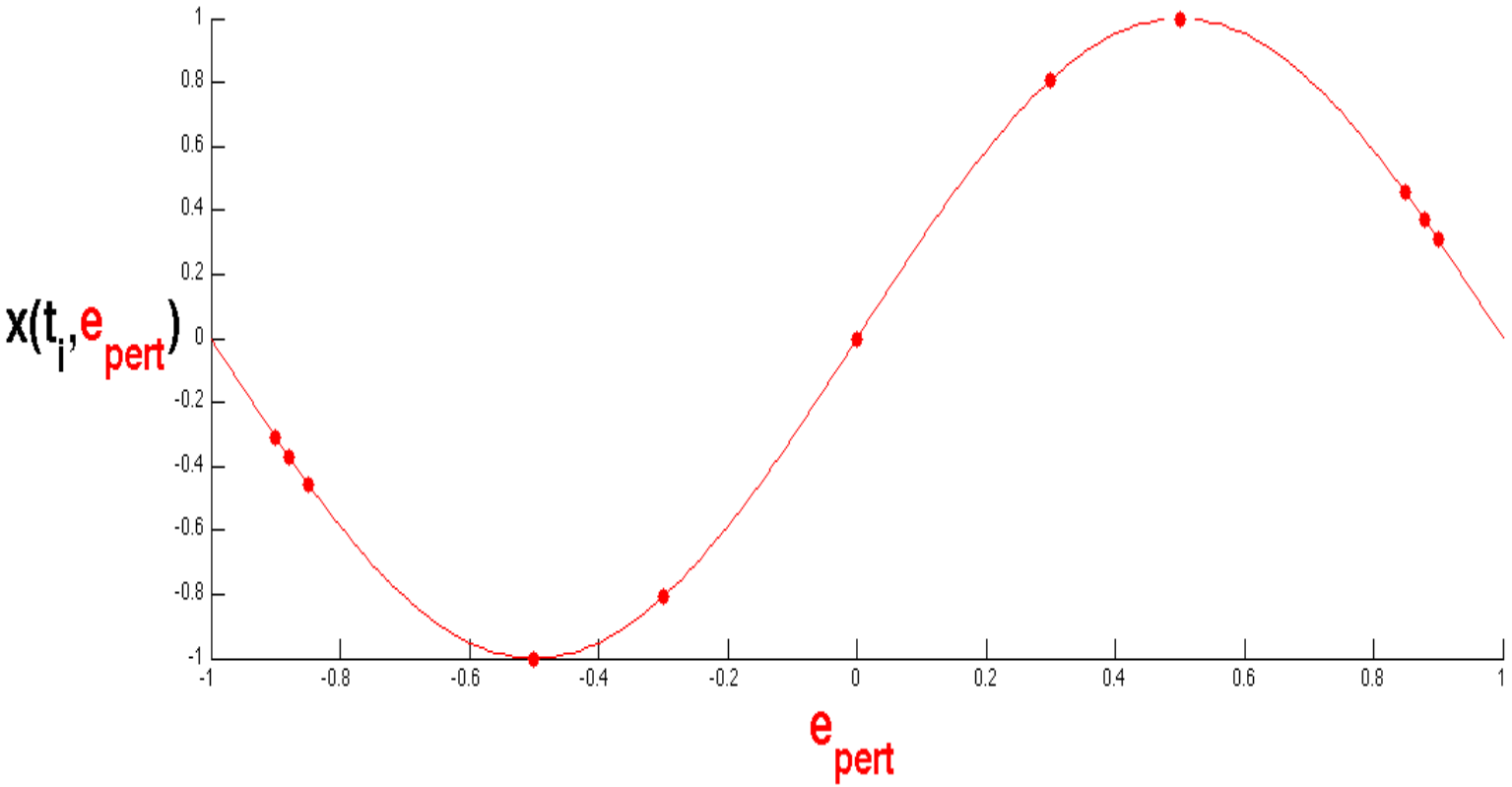




Cost Reduction

Fourier-like expansion of the atmospheric and oceanic **state variables**

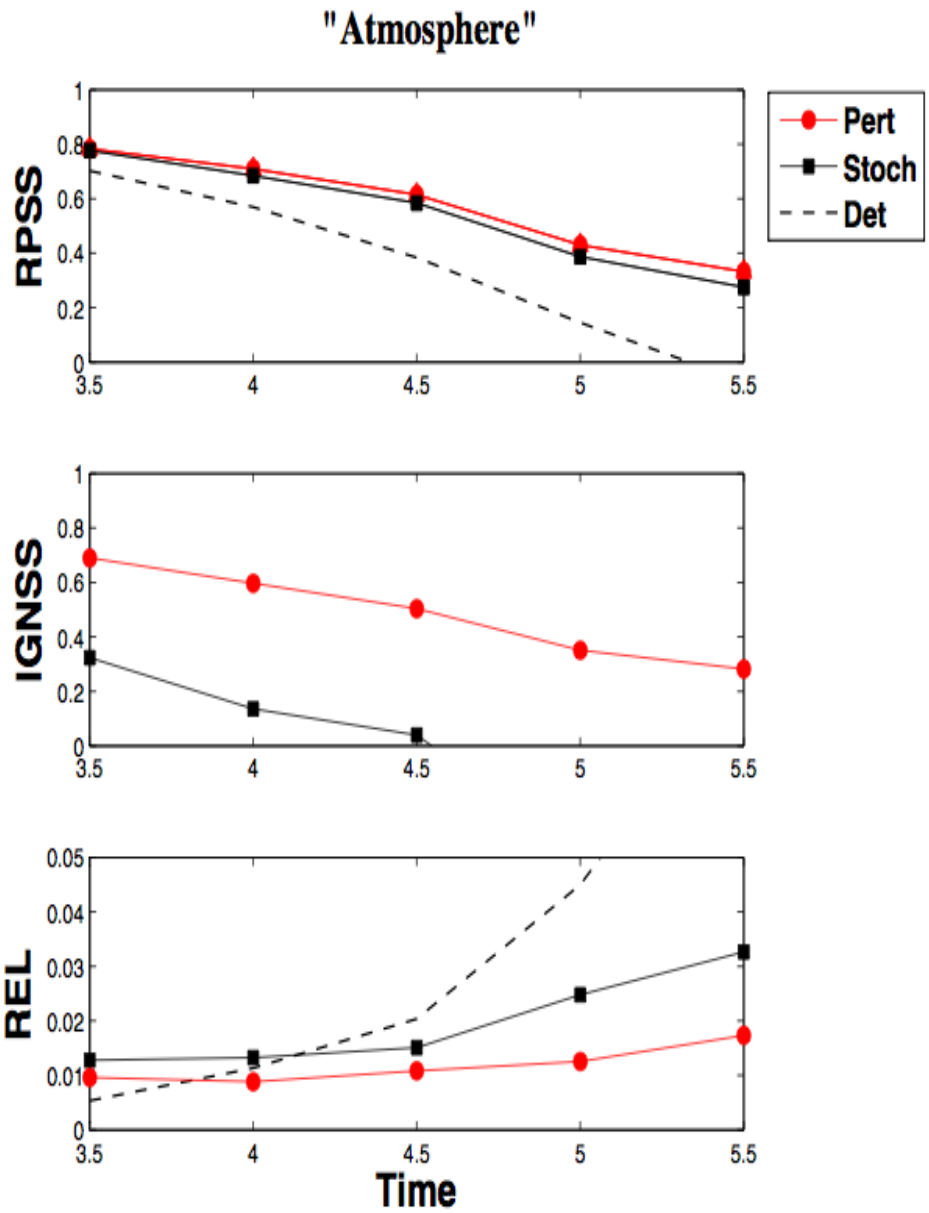
$$X \approx x(t, e_{\text{pert}}) = \sum_{i=1}^{N} x_i(t) P_i(e_{\text{pert}})$$



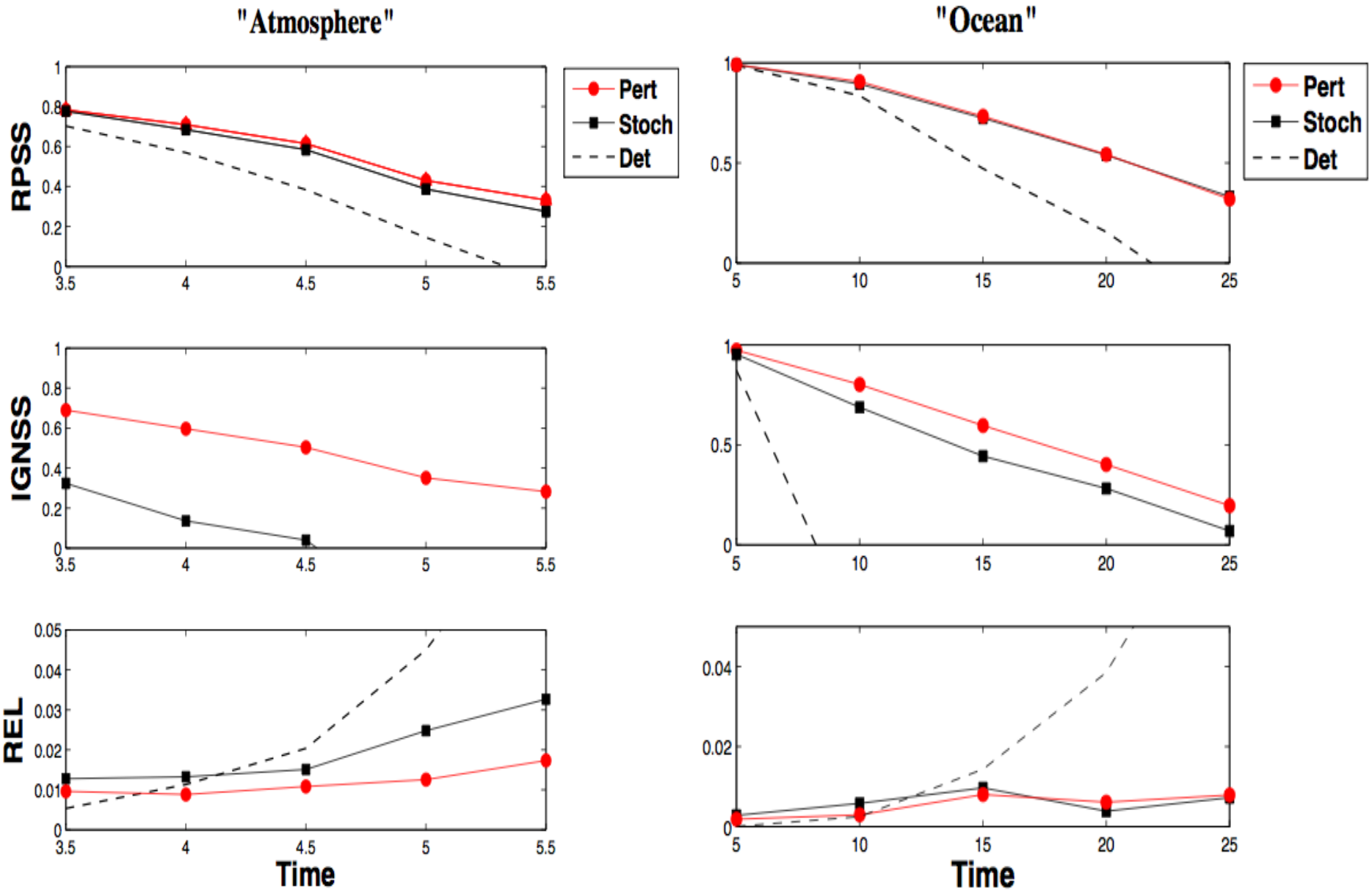
Summary of the Proposed Scheme

- Build the **surrogate model** for the forecast state variables.
- **Sample the "input"** e_{pert} with an **informative distribution**.
- **Obtain the "output"** ensemble forecast states.

Forecast Skill



Forecast Skill



Conclusion

- Perturbed Parameter Scheme is **reliable** with "*informative*" distribution!
- Polynomial Chaos Expansion further **reduces the cost**.
- **Easy to apply** to complex GCMs.

Reference

- G Chen, BP Kirtman, and M Iskandarani, *An Efficient Perturbed Parameter Scheme in the Lorenz system for Quantifying Model Uncertainty.* Q. J. Roy. Meteor. Soc. (submitted)

- Arnold HM, Moroz IM and Palmer TN. 2013, *Stochastic Parametrizations and Model Uncertainty in the Lorenz'96 System.*, Philos. Trans. A Math. Phys. Eng. Sci., 371

- TN Palmer, *Predictability of weather and climate.* Cambridge University Press, 2006