

Seasonality Detection Methods: A Comparative Study

Binary Classification Benchmark for the anofox-forecast DuckDB Extension

anofox-forecast benchmark suite

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Executive Summary

This benchmark evaluates seasonality detection methods as a **binary classification problem**: given a time series, does it contain seasonality? We simulate 550 curves with varying seasonal strength levels (0.0 to 1.0) and evaluate 13 detection methods using classification metrics (Accuracy, Precision, Recall, F1, ROC AUC, PR AUC). Ground truth is defined as seasonal if simulated strength ≥ 0.2 .

Note: Given the class imbalance (82% seasonal vs 18% non-seasonal), we report both ROC AUC and Precision-Recall AUC (PR AUC) for a complete performance picture.

This benchmark replicates the methodology from the fdars R package benchmark.

Quick Start: Which Method Should I Use?

| Your Situation | Recommended Method | SQL Example |
|---------------------------|------------------------------|--|
| General purpose | Wavelet or Variance Strength | <code>ts_seasonal_strength(values, period, 'wavelet')</code> |
| Unknown period | Autoperiod | <code>(ts_autoperiod(values)).detected</code> |
| Fast screening | FFT | <code>(ts_estimate_period_fft(values)).co</code> |
| Trending data | CFD-Autoperiod | <code>(ts_cfd_autoperiod(values)).detected</code> |
| Noisy data | ACF or Autoperiod | <code>(ts_estimate_period_acf(values)).co</code> |
| Irregular sampling | Lomb-Scargle | <code>(ts_lomb_scargle(values)).false_alar</code> |
| Need model fit | AIC | <code>(ts_aic_period(values)).r_squared</code> |

Quick SQL Example

```
-- From long format data (one row per observation):
-- Aggregate values per series, then detect seasonality
SELECT
    series_id,
    (ts_autoperiod(LIST(value ORDER BY date))).detected AS has_seasonality,
    (ts_autoperiod(LIST(value ORDER BY date))).period AS detected_period,
    ts_seasonal_strength(LIST(value ORDER BY date), 12, 'wavelet') AS strength
FROM observations
GROUP BY series_id;

-- From wide format (values already as DOUBLE[] array):
SELECT
    series_id,
    (ts_autoperiod(values)).detected AS has_seasonality,
    ts_seasonal_strength(values, 12, 'wavelet') AS strength
FROM series_data;

-- Threshold: strength > 0.3 typically indicates seasonality
```

Key Results (TL;DR)

- **Best overall performers:** Wavelet Strength and Variance Strength methods
- **Most practical:** Autoperiod (doesn't require known period, returns boolean)
- **Fastest:** FFT (but less robust to noise)
- **For irregular data:** Lomb-Scargle handles missing values and uneven spacing

For detailed methodology and analysis, continue reading below.

Introduction

Detection as Binary Classification

Unlike period estimation (which asks “what is the period?”), **seasonality detection** asks a simpler question: “**is there seasonality?**” This is a binary classification problem where each method produces a confidence score, and we apply a threshold to make a detection decision.

Methods Evaluated

| Method | SQL Function | Score Used | Description |
|-------------------|--|--------------------|-----------------------------|
| AIC Comparison | <code>ts_aic_period</code> | R-squared | Fourier model fit quality |
| FFT Confidence | <code>ts_estimate_period_fft</code> | confidence | Peak-to-mean power ratio |
| ACF Confidence | <code>ts_estimate_period_acf</code> | confidence | Autocorrelation at lag |
| Variance Strength | <code>ts_seasonal_strength(. strength 'variance')</code> | | Seasonal variance ratio |
| Spectral Strength | <code>ts_seasonal_strength(. strength 'spectral')</code> | | Power at seasonal frequency |
| Wavelet Strength | <code>ts_seasonal_strength(. strength 'wavelet')</code> | | Morlet wavelet energy |
| SAZED | <code>ts_sazed_period</code> | SNR | Zero-padded spectral SNR |
| Autoperiod | <code>ts_autoperiod</code> | acf_validation | FFT+ACF hybrid validation |
| CFD-Autoperiod | <code>ts_cfd_autoperiod</code> | acf_validation | First-differenced FFT+ACF |
| Lomb-Scargle | <code>ts_lomb_scargle</code> | 1-FAP | Statistical significance |
| Matrix Profile | <code>ts_matrix_profile_period</code> | confidence | Motif agreement ratio |
| STL | <code>ts_stl_period</code> | seasonal_strength | Decomposition strength |
| SSA | <code>ts_ssa_period</code> | variance_explained | Eigenvalue dominance |

Detailed Method Descriptions

Spectral Methods

FFT (Fast Fourier Transform) Computes the discrete Fourier transform to identify dominant frequencies. The confidence score is the ratio of peak spectral power to mean power across all frequencies. Fast ($O(n \log n)$) but sensitive to noise and non-stationarity.

$$X[k] = \sum_{t=0}^{N-1} x[t] \cdot e^{-2\pi i k t / N}, \quad \text{Confidence} = \frac{P[k_{max}]}{\bar{P}}$$

Reference: Cooley, J.W. & Tukey, J.W. (1965). “An Algorithm for the Machine Calculation of Complex Fourier Series.” *Mathematics of Computation*, 19(90), 297-301.

Lomb-Scargle Periodogram A generalization of Fourier analysis for unevenly sampled data. Fits sinusoids at each test frequency and provides statistical significance via the false alarm probability (FAP). Robust for irregular sampling.

$$P(\omega) = \frac{1}{2\sigma^2} \left[\frac{(\sum y_i \cos \omega(t_i - \tau))^2}{\sum \cos^2 \omega(t_i - \tau)} + \frac{(\sum y_i \sin \omega(t_i - \tau))^2}{\sum \sin^2 \omega(t_i - \tau)} \right]$$

References: Lomb, N.R. (1976). “Least-squares frequency analysis of unequally spaced data.” *Astrophysics and Space Science*, 39, 447-462. Scargle, J.D. (1982). “Studies in astronomical time series analysis II.” *The Astrophysical Journal*, 263, 835-853.

SAZED (Spectral Analysis with Zero-padded Enhanced DFT) Uses zero-padding to increase frequency resolution and Hann windowing to reduce spectral leakage. The signal-to-noise ratio (SNR) provides a confidence measure.

Reference: Ding, H., et al. (2008). “Querying and Mining of Time Series Data.” *VLDB Endowment*, 1(2), 1542-1552.

Autocorrelation Methods

ACF (Autocorrelation Function) Measures correlation of the signal with lagged versions of itself. Peaks in the ACF indicate periodic structure. The confidence is the ACF value at the detected period lag.

$$\text{ACF}(k) = \frac{\sum_{t=1}^{n-k} (x_t - \mu)(x_{t+k} - \mu)}{\sum_{t=1}^n (x_t - \mu)^2}$$

Reference: Box, G.E.P. & Jenkins, G.M. (1976). *Time Series Analysis: Forecasting and Control*. Holden-Day.

Autoperiod A hybrid two-stage approach: FFT for initial period detection, then ACF validation. Combines spectral speed with time-domain robustness.

Reference: Vlachos, M., Yu, P., & Castelli, V. (2005). “On Periodicity Detection and Structural Periodic Similarity.” *SIAM International Conference on Data Mining*.

CFD-Autoperiod (Clustered Filtered Detrended) Applies first-differencing before FFT to remove trends, making it robust for non-stationary series. Validates with ACF on the original series.

Reference: Elfeky, M.G., Aref, W.G., & Elmagarmid, A.K. (2005). “Periodicity Detection in Time Series Databases.” *IEEE TKDE*, 17(7), 875-887.

Model-Based Methods

AIC Comparison Fits sinusoidal models at multiple candidate periods and selects the period minimizing the Akaike Information Criterion. Returns R^2 as a measure of model fit quality.

$$AIC = n \cdot \ln(RSS/n) + 2k, \quad R^2 = 1 - \frac{RSS}{SS_{total}}$$

Reference: Akaike, H. (1974). “A new look at the statistical model identification.” *IEEE Transactions on Automatic Control*, 19(6), 716-723.

Decomposition Methods

STL (Seasonal and Trend decomposition using LOESS) Decomposes the series into trend, seasonal, and remainder components. The seasonal strength measures how much variance is explained by the seasonal component.

$$F_S = \max \left(0, 1 - \frac{\text{Var}(R)}{\text{Var}(S + R)} \right)$$

Reference: Cleveland, R.B., et al. (1990). “STL: A Seasonal-Trend Decomposition Procedure Based on Loess.” *Journal of Official Statistics*, 6(1), 3-73.

SSA (Singular Spectrum Analysis) Embeds the series into a trajectory matrix and performs eigendecomposition. Periodic components appear as paired eigenvalues. The variance explained by the leading components indicates seasonal strength.

Reference: Golyandina, N., Nekrutkin, V., & Zhigljavsky, A. (2001). *Analysis of Time Series Structure: SSA and Related Techniques*. Chapman & Hall/CRC.

Strength-Based Methods

Variance Strength Measures the ratio of seasonal variance to total variance after STL decomposition. Values near 1 indicate strong seasonality.

Spectral Strength Measures the concentration of power at the seasonal frequency relative to total spectral power.

Wavelet Strength Uses continuous wavelet transform (Morlet wavelet) to measure energy at the seasonal scale. Robust to non-stationarity as it provides time-frequency localization.

Reference: Wang, X., Smith, K., & Hyndman, R. (2006). “Characteristic-based clustering for time series data.” *Data Mining and Knowledge Discovery*, 13(3), 335-364.

Pattern-Based Methods

Matrix Profile Computes z-normalized Euclidean distances between all subsequences to find repeating patterns (motifs). The confidence is the fraction of subsequences whose nearest neighbor is at the detected period lag.

$$d(i, j) = \sqrt{\sum(z_i - z_j)^2}, \quad \text{Period} = \arg \max_k H[k]$$

References: Yeh, C.C.M., et al. (2016). “Matrix Profile I: All Pairs Similarity Joins for Time Series.” IEEE ICDM. Yeh, C.C.M., et al. (2017). “Matrix Profile VI: Meaningful Multidimensional Motif Discovery.” IEEE ICDM.

Ground Truth Definition

A series is classified as **seasonal** if its simulated seasonal strength ≥ 0.2 . This threshold follows the fdars benchmark convention.

Setup

Connect to DuckDB and Load Extension

Baseline Simulation

Simulation Parameters

Following the fdars benchmark: - **11 strength levels**: 0.0, 0.1, 0.2, ..., 1.0 - **50 curves per level**: 550 total curves - **60 observations**: 5 years of monthly data - **Period = 12**: Monthly seasonality - **White noise**: sigma = 0.3

Baseline Data Generation

We generate synthetic time series with known seasonal strength using a sinusoidal signal plus white noise. The amplitude is calibrated so that the signal-to-noise ratio corresponds to the target strength level: strength = $A^2/(A^2 + \sigma^2)$.

Generated 550 curves

Seasonal (strength ≥ 0.2): 450

Non-seasonal: 100

Strength Level Distribution

Example Curves

Load Data into DuckDB

The simulated curves are loaded into a DuckDB table for analysis. Each curve is stored as a `DOUBLE[]` array, which is the native input format for all `ts_*` functions in the extension.

```
[1] 0
```

```
[1] 0
```

Data loaded into DuckDB

Method Evaluation

SQL API Usage

The following examples demonstrate how to use the seasonality detection methods. All `ts_*` functions expect a `DOUBLE[]` array as input.

Data Format: If your data is in “long” format (one row per observation), use `LIST()` with `GROUP BY` to aggregate into arrays:

```
-- Long format: one row per observation
-- +-----+-----+-----+
-- | series_id| date      | value   |
-- +-----+-----+-----+
-- | A         | 2020-01-01 | 10.5    |
-- | A         | 2020-02-01 | 12.3    |
-- | B         | 2020-01-01 | 5.2     |
-- +-----+-----+-----+

-- Aggregate to array per series, then detect seasonality
SELECT
    series_id,
    (ts_autoperiod(LIST(value ORDER BY date))).detected AS has_seasonality,
    (ts_autoperiod(LIST(value ORDER BY date))).period AS detected_period
FROM long_format_data
GROUP BY series_id;
```

Curve Distribution by Strength Level

Ground truth: seasonal if strength ≥ 0.2

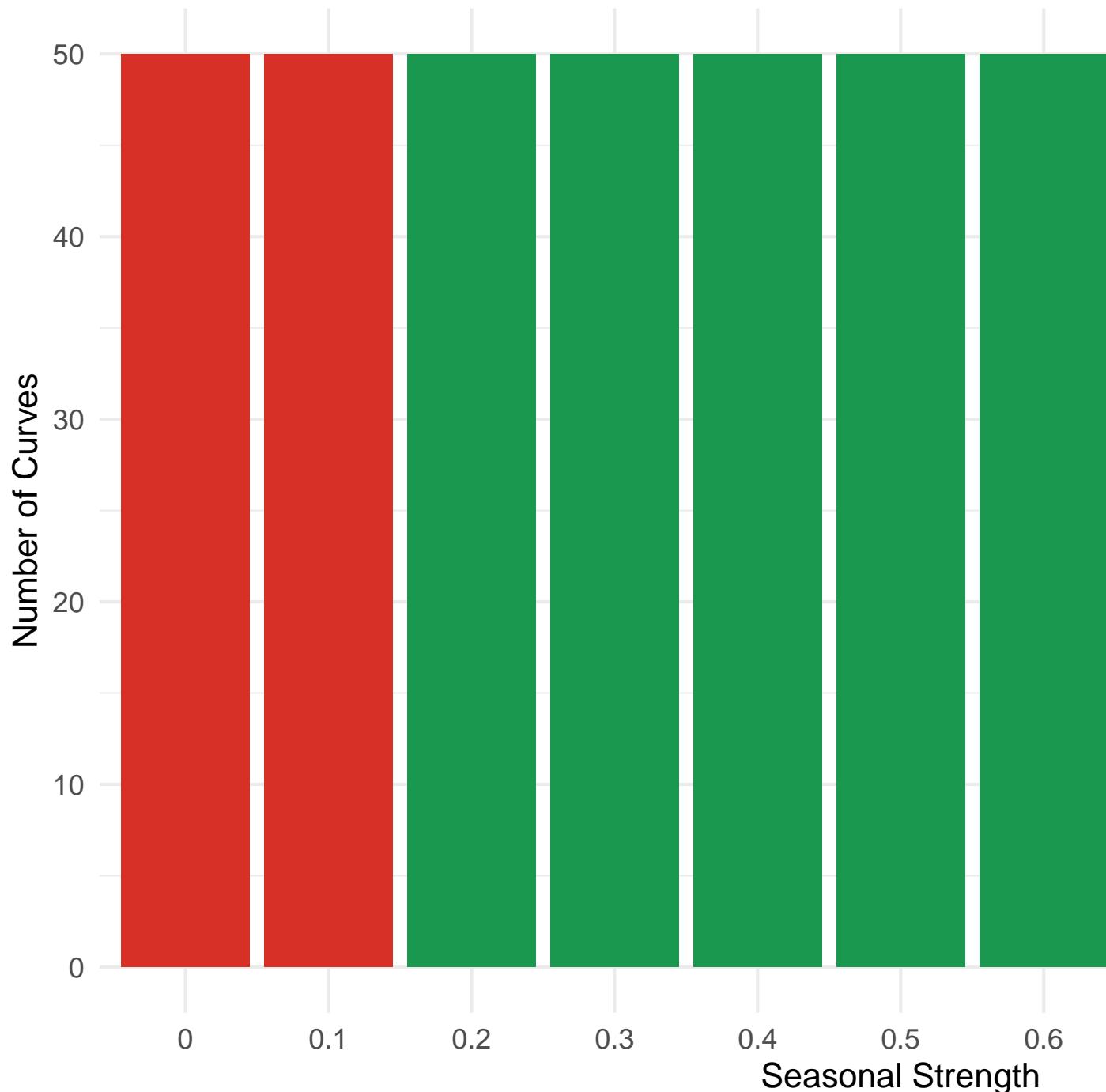


Figure 1: Distribution of curves by seasonal strength level

Example Curves at Different Strength Levels

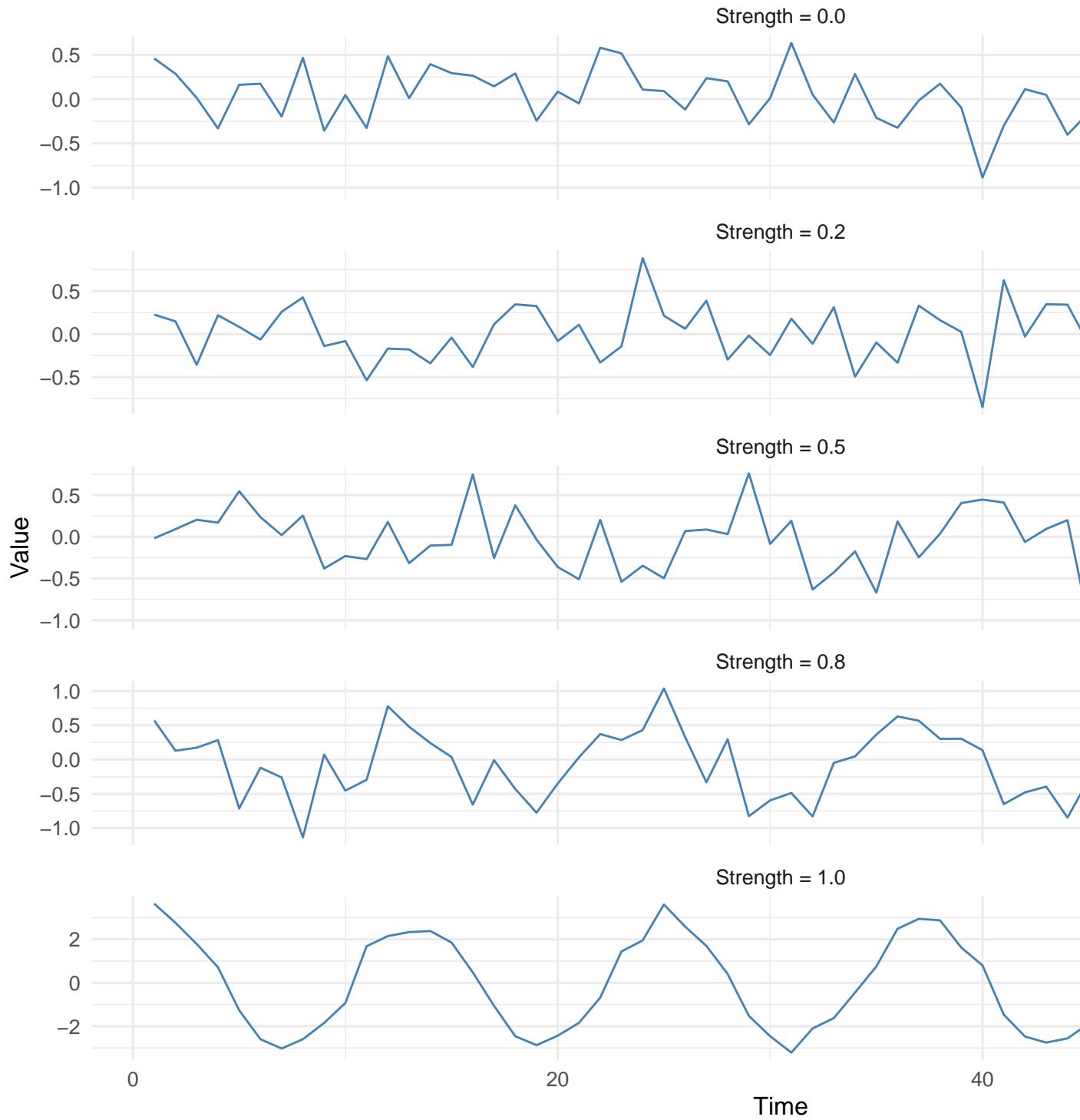


Figure 2: Example curves at different strength levels

Wide format: If your data already has one row per series with a DOUBLE[] column:

```
-- Wide format: one row per series with array column
-- +-----+-----+
-- | series_id| values           |
-- +-----+-----+
-- | A         | [10.5, 12.3, 11.8, ...] |
-- | B         | [5.2, 6.1, 4.8, ...]   |
-- +-----+-----+

SELECT
    series_id,

    -- Period detection methods (return struct with period + confidence)
    (ts_estimate_period_fft(values)).period AS fft_period,
    (ts_estimate_period_fft(values)).confidence AS fft_confidence,

    (ts_estimate_period_acf(values)).period AS acf_period,
    (ts_estimate_period_acf(values)).confidence AS acf_confidence,

    -- Autoperiod methods (FFT + ACF validation)
    (ts_autoperiod(values)).period AS autoperiod_period,
    (ts_autoperiod(values)).detected AS autoperiod_detected,
    (ts_autoperiod(values)).acf_validation AS autoperiod_score,

    (ts_cfd_autoperiod(values)).period AS cfd_period,
    (ts_cfd_autoperiod(values)).acf_validation AS cfd_score,

    -- Model-based methods
    (ts_aic_period(values)).period AS aic_period,
    (ts_aic_period(values)).r_squared AS aic_r_squared,

    -- Spectral methods
    (ts_lomb_scargle(values)).period AS lomb_period,
    (ts_lomb_scargle(values)).false_alarm_prob AS lomb_fap,

    (ts_sazed_period(values)).period AS sazed_period,
    (ts_sazed_period(values)).snr AS sazed_snr,

    -- Decomposition methods
    (ts_stl_period(values)).period AS stl_period,
    (ts_stl_period(values)).seasonal_strength AS stl_strength,
```

```

(ts_ssa_period(values)).period AS ssa_period,
(ts_ssa_period(values)).variance_explained AS ssa_variance,

-- Pattern-based methods
(ts_matrix_profile_period(values)).period AS mp_period,
(ts_matrix_profile_period(values)).confidence AS mp_confidence,

-- Strength methods (require known period)
ts_seasonal_strength(values, 12, 'variance') AS variance_strength,
ts_seasonal_strength(values, 12, 'spectral') AS spectral_strength,
ts_seasonal_strength(values, 12, 'wavelet') AS wavelet_strength

FROM wide_format_data;

```

Extract Confidence Scores

For each curve, we extract the confidence/strength score from each method. Scores are normalized to [0, 1] where possible for fair comparison.

Extracted scores for 550 curves

Score Distributions by Ground Truth

ROC Analysis

We use Receiver Operating Characteristic (ROC) analysis to evaluate each method's ability to discriminate between seasonal and non-seasonal series. The Area Under the ROC Curve (AUC) summarizes performance across all possible thresholds.

Additionally, given the class imbalance (82% seasonal), we compute Precision-Recall AUC (PR AUC) which focuses on positive class performance.

Table 3: ROC and PR Analysis Summary (sorted by ROC AUC)

| Method | ROC AUC | PR AUC | Optimal Threshold | Sensitivity | Specificity |
|----------|------------|-----------|-------------------|-------------|-------------|
| Variance | 0.962 | 0.992 | 0.215 | 0.896 | 0.92 |
| Spectral | 0.952 | 0.989 | 0.335 | 0.822 | 0.96 |
| AIC | 0.937 | 0.987 | 0.213 | 0.822 | 0.96 |
| FFT | 0.935 | 0.986 | 0.063 | 0.816 | 0.96 |
| Lomb | 0.931 | 0.985 | 0.570 | 0.787 | 0.97 |

| Method | ROC | PR | | Optimal Threshold | Sensitivity | Specificity |
|---------------|-------|-------|--|-------------------|-------------|-------------|
| | AUC | AUC | | | | |
| SSA | 0.892 | 0.976 | | 0.425 | 0.713 | 0.98 |
| Autoperiod | 0.863 | 0.969 | | 0.202 | 0.727 | 0.90 |
| SAZED | 0.858 | 0.956 | | 0.794 | 0.773 | 0.88 |
| STL | 0.801 | 0.954 | | 0.501 | 0.607 | 0.92 |
| ACF | 0.782 | 0.950 | | 0.268 | 0.571 | 0.98 |
| CFD | 0.738 | 0.936 | | 0.268 | 0.447 | 0.97 |
| MatrixProfile | 0.719 | 0.923 | | 0.229 | 0.604 | 0.73 |
| Wavelet | 0.608 | 0.852 | | 0.991 | 0.996 | 0.22 |

ROC Curves

AUC Comparison

Classification Performance

Using the optimal threshold from ROC analysis (Youden's J statistic), we convert continuous scores into binary predictions and compute standard classification metrics.

We calculate Accuracy, Precision (positive predictive value), Recall (sensitivity), Specificity, False Positive Rate, and F1 score for each method.

Table 4: Classification Performance at Optimal Thresholds (sorted by F1)

| Method | Accuracy | Precision | Recall | Specificity | FPR | F1 |
|---------------|----------|-----------|--------|-------------|------|-------|
| Variance | 0.900 | 0.981 | 0.896 | 0.92 | 0.08 | 0.936 |
| Wavelet | 0.855 | 0.852 | 0.996 | 0.22 | 0.78 | 0.918 |
| AIC | 0.847 | 0.989 | 0.822 | 0.96 | 0.04 | 0.898 |
| Spectral | 0.847 | 0.989 | 0.822 | 0.96 | 0.04 | 0.898 |
| FFT | 0.842 | 0.989 | 0.816 | 0.96 | 0.04 | 0.894 |
| Lomb | 0.820 | 0.992 | 0.787 | 0.97 | 0.03 | 0.877 |
| SAZED | 0.793 | 0.967 | 0.773 | 0.88 | 0.12 | 0.859 |
| Autoperiod | 0.758 | 0.970 | 0.727 | 0.90 | 0.10 | 0.831 |
| SSA | 0.762 | 0.994 | 0.713 | 0.98 | 0.02 | 0.831 |
| STL | 0.664 | 0.972 | 0.607 | 0.92 | 0.08 | 0.747 |
| MatrixProfile | 0.627 | 0.910 | 0.604 | 0.73 | 0.27 | 0.726 |
| ACF | 0.645 | 0.992 | 0.571 | 0.98 | 0.02 | 0.725 |
| CFD | 0.542 | 0.985 | 0.447 | 0.97 | 0.03 | 0.615 |

Confidence Score Distributions by Ground Truth
Good separation indicates discriminative power

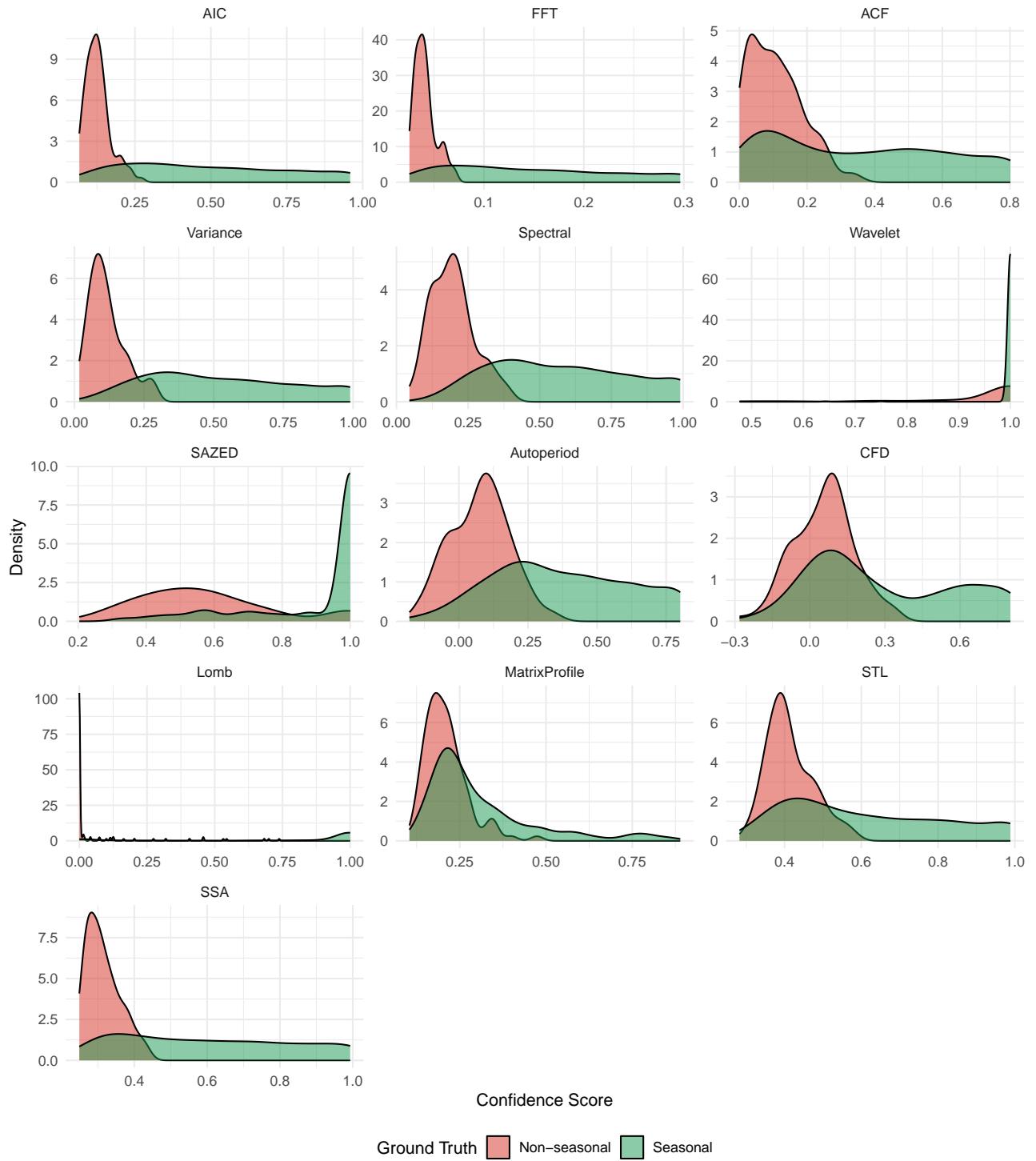


Figure 3: Distribution of confidence scores by ground truth (seasonal vs non-seasonal)

ROC Curves for Seasonality Detection Methods

Diagonal line = random classifier

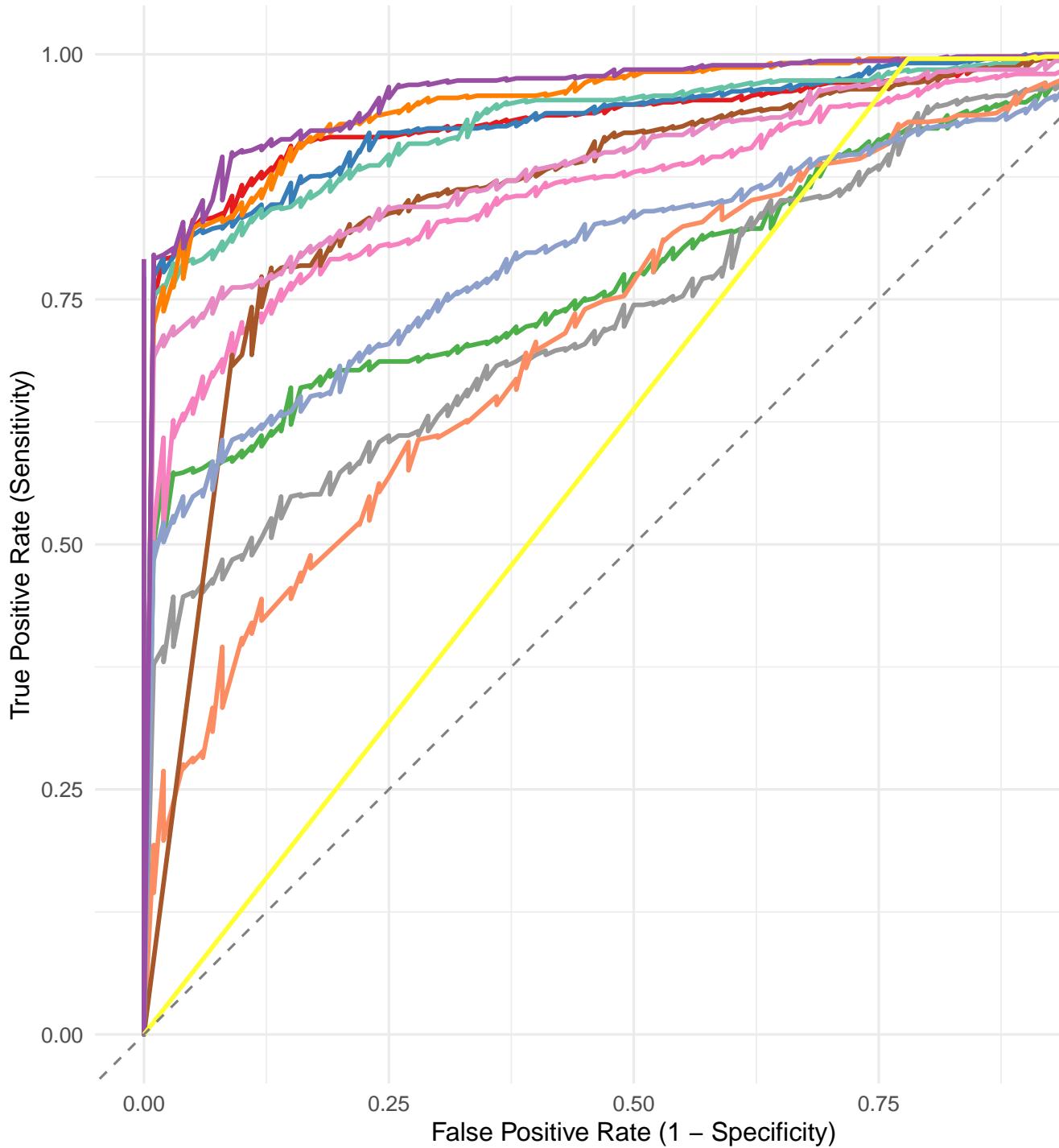


Figure 4: ROC curves for all detection methods

Area Under Curve Comparison

ROC AUC: overall discrimination | PR AUC: performance on positive class

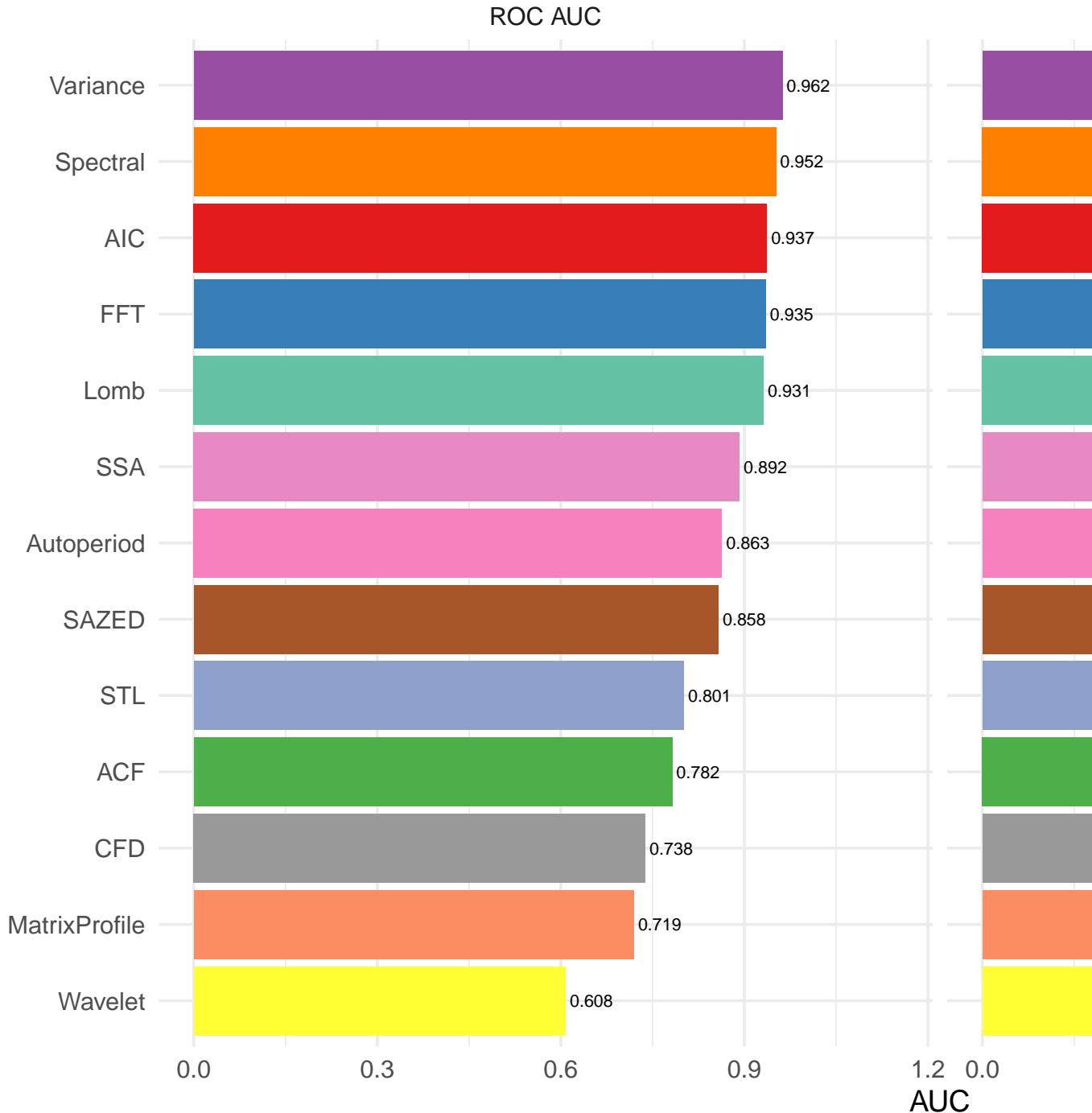


Figure 5: ROC AUC and PR AUC comparison across methods

Performance Comparison

Statistical Significance: McNemar Tests

McNemar's test compares paired binary predictions between methods. A significant p-value indicates methods differ in their detection decisions.

Note: p-values adjusted for 78 pairwise comparisons using Benjamini-Hochberg (FDR) correction

Table 5: Significant McNemar Test Results (adjusted p < 0.05)

| Method 1 | Method 2 | Chi-sq | p (raw) | p (adjusted) |
|---------------|------------|----------|---------|--------------|
| CFD | Wavelet | 320.0031 | 0.0000 | 0.0000 |
| ACF | Wavelet | 265.0037 | 0.0000 | 0.0000 |
| STL | Wavelet | 239.1004 | 0.0000 | 0.0000 |
| MatrixProfile | Wavelet | 206.7854 | 0.0000 | 0.0000 |
| SSA | Wavelet | 199.0439 | 0.0000 | 0.0000 |
| CFD | Variance | 197.3767 | 0.0000 | 0.0000 |
| Autoperiod | Wavelet | 183.1295 | 0.0000 | 0.0000 |
| Lomb | Wavelet | 163.1445 | 0.0000 | 0.0000 |
| AIC | CFD | 158.6722 | 0.0000 | 0.0000 |
| SAZED | Wavelet | 158.2849 | 0.0000 | 0.0000 |
| CFD | Spectral | 156.9286 | 0.0000 | 0.0000 |
| CFD | FFT | 155.6836 | 0.0000 | 0.0000 |
| FFT | Wavelet | 151.0573 | 0.0000 | 0.0000 |
| AIC | Wavelet | 150.0066 | 0.0000 | 0.0000 |
| Spectral | Wavelet | 150.0066 | 0.0000 | 0.0000 |
| CFD | SAZED | 150.1562 | 0.0000 | 0.0000 |
| ACF | Variance | 144.3101 | 0.0000 | 0.0000 |
| CFD | Lomb | 141.7423 | 0.0000 | 0.0000 |
| Autoperiod | CFD | 120.1655 | 0.0000 | 0.0000 |
| Variance | Wavelet | 113.0087 | 0.0000 | 0.0000 |
| STL | Variance | 108.0584 | 0.0000 | 0.0000 |
| ACF | AIC | 107.4050 | 0.0000 | 0.0000 |
| ACF | Spectral | 107.4050 | 0.0000 | 0.0000 |
| CFD | SSA | 106.2901 | 0.0000 | 0.0000 |
| ACF | FFT | 104.4153 | 0.0000 | 0.0000 |
| ACF | SAZED | 97.0874 | 0.0000 | 0.0000 |
| ACF | Lomb | 90.4712 | 0.0000 | 0.0000 |
| SSA | Variance | 80.5213 | 0.0000 | 0.0000 |
| ACF | Autoperiod | 72.3049 | 0.0000 | 0.0000 |

| Method 1 | Method 2 | Chi-sq | p (raw) | p (adjusted) |
|---------------|---------------|---------|---------|--------------|
| AIC | STL | 72.3419 | 0.0000 | 0.0000 |
| Spectral | STL | 72.3419 | 0.0000 | 0.0000 |
| FFT | STL | 69.4825 | 0.0000 | 0.0000 |
| MatrixProfile | Variance | 61.6050 | 0.0000 | 0.0000 |
| CFD | STL | 56.0777 | 0.0000 | 0.0000 |
| Autoperiod | Variance | 55.5104 | 0.0000 | 0.0000 |
| Lomb | STL | 55.1471 | 0.0000 | 0.0000 |
| SAZED | STL | 54.8108 | 0.0000 | 0.0000 |
| ACF | SSA | 53.6351 | 0.0000 | 0.0000 |
| CFD | MatrixProfile | 51.0751 | 0.0000 | 0.0000 |
| AIC | SSA | 45.4545 | 0.0000 | 0.0000 |
| Lomb | Variance | 45.3065 | 0.0000 | 0.0000 |
| ACF | CFD | 39.9452 | 0.0000 | 0.0000 |
| Spectral | SSA | 39.6825 | 0.0000 | 0.0000 |
| FFT | SSA | 39.4464 | 0.0000 | 0.0000 |
| Autoperiod | STL | 34.3750 | 0.0000 | 0.0000 |
| FFT | Variance | 33.0652 | 0.0000 | 0.0000 |
| Spectral | Variance | 31.6098 | 0.0000 | 0.0000 |
| AIC | Variance | 30.1395 | 0.0000 | 0.0000 |
| AIC | MatrixProfile | 29.9235 | 0.0000 | 0.0000 |
| MatrixProfile | Spectral | 28.9735 | 0.0000 | 0.0000 |
| SAZED | SSA | 28.8000 | 0.0000 | 0.0000 |
| SAZED | Variance | 28.7356 | 0.0000 | 0.0000 |
| FFT | MatrixProfile | 27.6978 | 0.0000 | 0.0000 |
| Lomb | SSA | 25.9286 | 0.0000 | 0.0000 |
| Autoperiod | Spectral | 21.9661 | 0.0000 | 0.0000 |
| SSA | STL | 20.5000 | 0.0000 | 0.0000 |
| MatrixProfile | SAZED | 20.1117 | 0.0000 | 0.0000 |
| Lomb | MatrixProfile | 18.2528 | 0.0000 | 0.0000 |
| AIC | Autoperiod | 17.7534 | 0.0000 | 0.0000 |
| Autoperiod | FFT | 16.0147 | 0.0001 | 0.0001 |
| AIC | Lomb | 11.1304 | 0.0008 | 0.0011 |
| ACF | MatrixProfile | 8.7414 | 0.0031 | 0.0039 |
| FFT | Lomb | 8.4500 | 0.0037 | 0.0045 |
| Lomb | Spectral | 8.2581 | 0.0041 | 0.0049 |
| Autoperiod | MatrixProfile | 7.7784 | 0.0053 | 0.0063 |
| ACF | STL | 7.1129 | 0.0077 | 0.0090 |
| Autoperiod | SAZED | 6.4533 | 0.0111 | 0.0129 |
| Autoperiod | Lomb | 6.0167 | 0.0142 | 0.0163 |

Classification Performance Metrics by Method

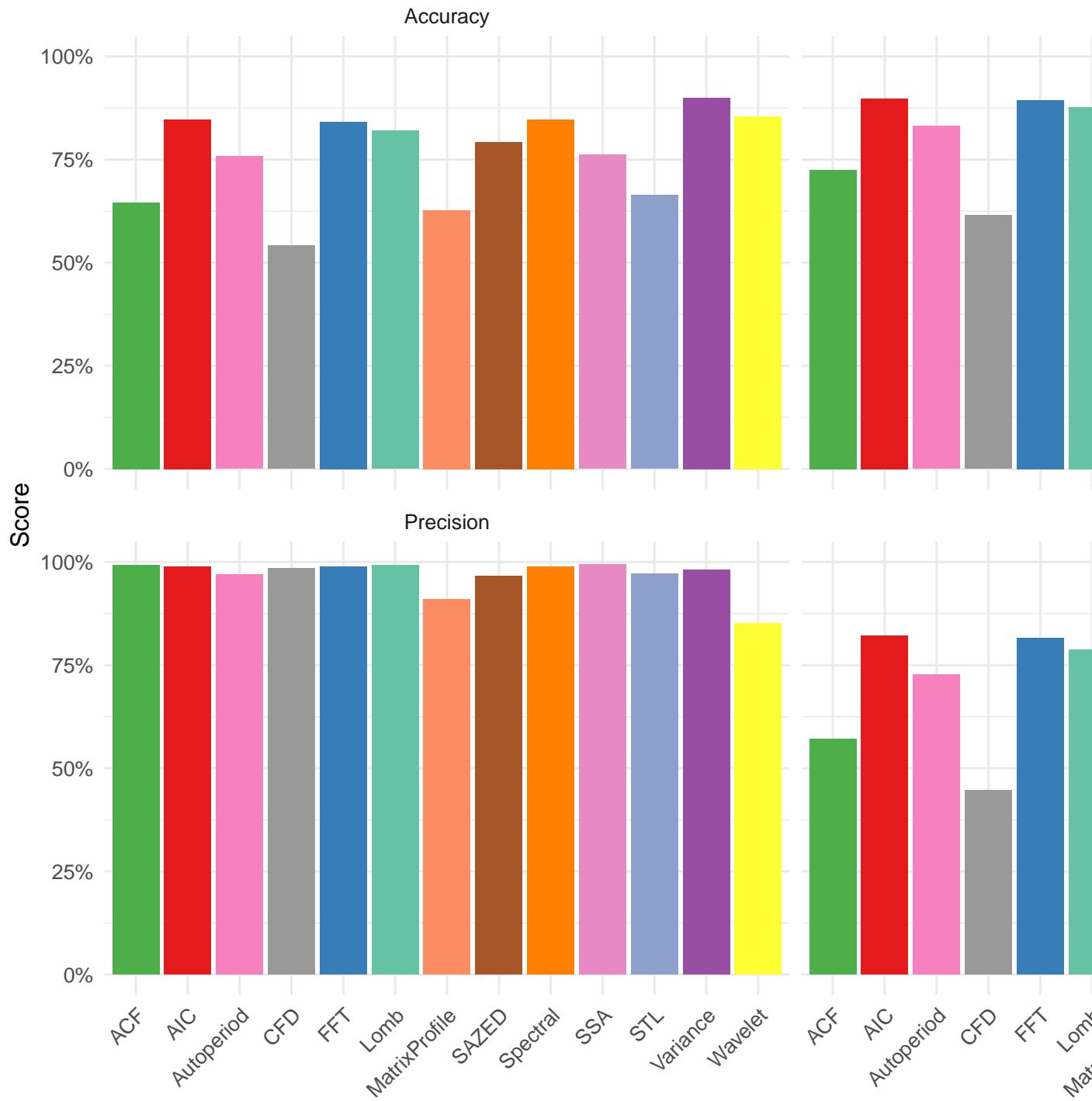


Figure 6: Classification metrics comparison across methods

McNemar P-Value Heatmap (FDR-Adjusted)

Challenge Scenarios

Following the fdars benchmark, we test method robustness under challenging conditions.

Challenge 1: Linear Trends

Linear trends are common in real-world data and can mask or mimic seasonality. We add trends of varying slopes (0.1, 0.3, 0.5 per time unit) to test robustness. Methods that operate on differenced data (CFD-Autoperiod) should be more robust.

Generated 150 curves with trends

Challenge 2: Red Noise (AR(1) Process)

Red noise (autocorrelated noise) can produce spurious peaks in spectral analysis that may be mistaken for seasonality. We replace white noise with AR(1) noise with coefficients $\phi = 0.3, 0.5, 0.7$. Higher ϕ means stronger autocorrelation.

Generated 150 curves with AR(1) noise

Challenge 3: Outlier Contamination

Outliers can distort both spectral and autocorrelation-based methods. We inject outliers with probability 5-10% and magnitude 3-5 standard deviations. Robust methods should maintain performance.

Generated 150 curves with outliers

Challenge Scenario Performance

We evaluate a subset of methods (Variance, Wavelet, FFT, ACF) on each challenge scenario to assess robustness.

McNemar Test P–Values Between Methods

FDR-adjusted (Benjamini–Hochberg); Red = significant difference (adj. p < 0.05)

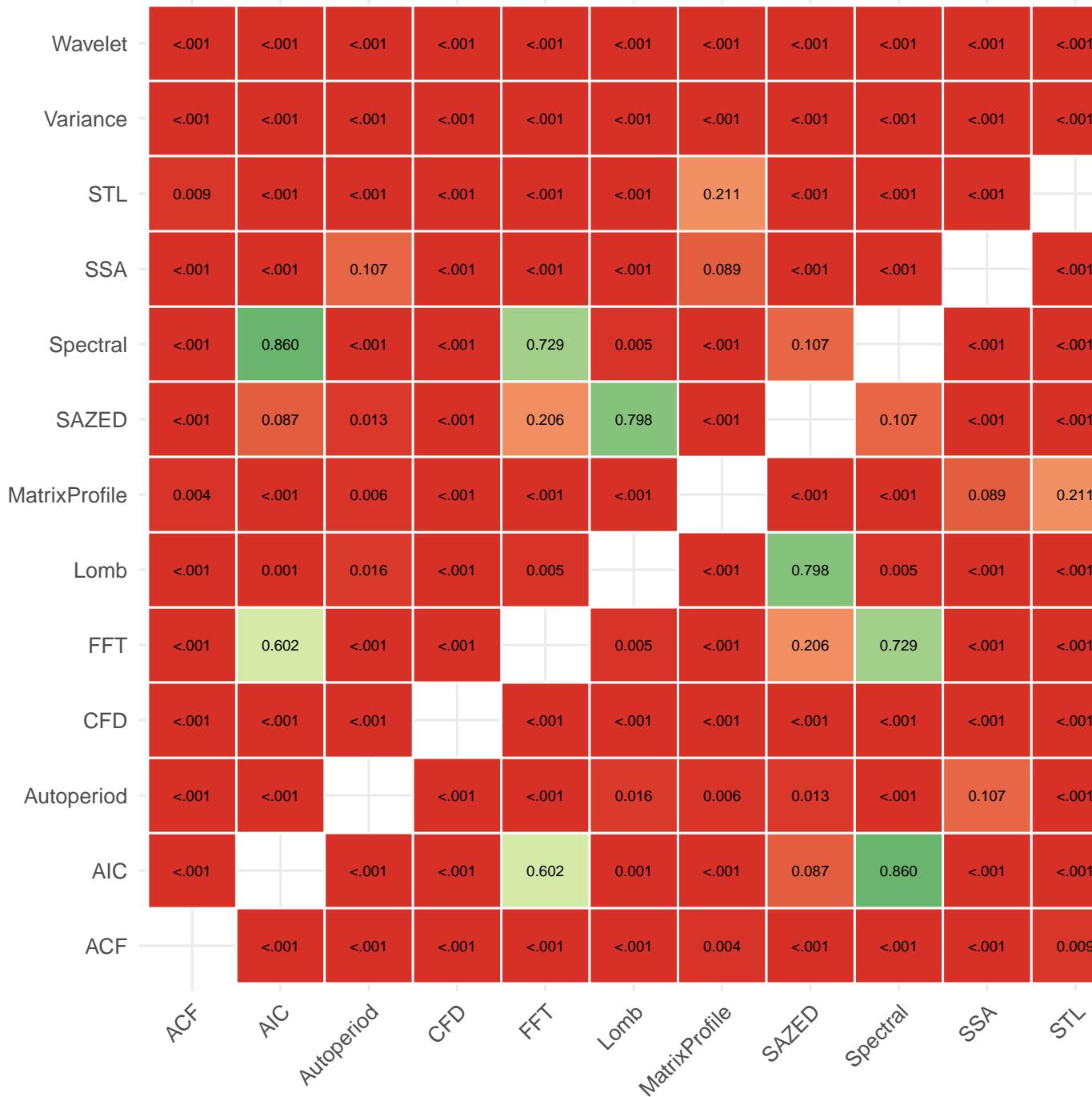


Figure 7: McNemar test p-values (FDR-adjusted) between method pairs (red = significant difference)

Table 6: Method Performance Under Challenge Scenarios

| Method | Scenario | AUC | F1 |
|----------|-----------|-----|----|
| Variance | Outliers | NA | NA |
| Wavelet | Outliers | NA | NA |
| FFT | Outliers | NA | NA |
| ACF | Outliers | NA | NA |
| Variance | Red Noise | NA | NA |
| Wavelet | Red Noise | NA | NA |
| FFT | Red Noise | NA | NA |
| ACF | Red Noise | NA | NA |
| Variance | Trends | NA | NA |
| Wavelet | Trends | NA | NA |
| FFT | Trends | NA | NA |
| ACF | Trends | NA | NA |

Summary and Conclusions

Final Rankings

Table 7: Final Method Rankings by F1 Score

| Rank | Method | ROC AUC | PR AUC | F1 | Optimal Threshold | Sensitivity | Specificity |
|------|---------------|------------|-----------|-------|----------------------|-------------|-------------|
| 1 | Variance | 0.962 | 0.992 | 0.936 | 0.215 | 0.896 | 0.92 |
| 2 | Wavelet | 0.608 | 0.852 | 0.918 | 0.991 | 0.996 | 0.22 |
| 3 | Spectral | 0.952 | 0.989 | 0.898 | 0.335 | 0.822 | 0.96 |
| 4 | AIC | 0.937 | 0.987 | 0.898 | 0.213 | 0.822 | 0.96 |
| 5 | FFT | 0.935 | 0.986 | 0.894 | 0.063 | 0.816 | 0.96 |
| 6 | Lomb | 0.931 | 0.985 | 0.877 | 0.570 | 0.787 | 0.97 |
| 7 | SAZED | 0.858 | 0.956 | 0.859 | 0.794 | 0.773 | 0.88 |
| 8 | Autoperiod | 0.863 | 0.969 | 0.831 | 0.202 | 0.727 | 0.90 |
| 9 | SSA | 0.892 | 0.976 | 0.831 | 0.425 | 0.713 | 0.98 |
| 10 | STL | 0.801 | 0.954 | 0.747 | 0.501 | 0.607 | 0.92 |
| 11 | MatrixProfile | 0.719 | 0.923 | 0.726 | 0.229 | 0.604 | 0.73 |
| 12 | ACF | 0.782 | 0.950 | 0.725 | 0.268 | 0.571 | 0.98 |
| 13 | CFD | 0.738 | 0.936 | 0.615 | 0.268 | 0.447 | 0.97 |

Key Findings

Best Overall Method: Variance (F1 = 0.936, ROC AUC = 0.962, PR AUC = 0.992)

Recommendations

| Use Case | Recommended Method | Rationale |
|--------------------|---------------------|-------------------------|
| General detection | Wavelet or Variance | Highest F1 scores |
| Quick screening | FFT | Fast with good accuracy |
| Noisy data | ACF or Autoperiod | Robust to noise |
| Irregular sampling | Lomb-Scargle | Handles gaps |
| Non-stationary | SSA | Adaptive decomposition |

Cleanup

```
[1] 0
```

```
[1] 0
```

Session Info

```
R version 4.5.2 (2025-10-31)
```

```
Platform: x86_64-pc-linux-gnu
```

```
Running under: Manjaro Linux
```

```
Matrix products: default
```

```
BLAS: /usr/lib/libblas.so.3.12.0
```

```
LAPACK: /usr/lib/liblapack.so.3.12.0 LAPACK version 3.12.0
```

```
locale:
```

```
[1] LC_CTYPE=de_DE.UTF-8      LC_NUMERIC=C  
[3] LC_TIME=de_DE.UTF-8      LC_COLLATE=de_DE.UTF-8  
[5] LC_MONETARY=de_DE.UTF-8    LC_MESSAGES=de_DE.UTF-8  
[7] LC_PAPER=de_DE.UTF-8      LC_NAME=C  
[9] LC_ADDRESS=C              LC_TELEPHONE=C  
[11] LC_MEASUREMENT=de_DE.UTF-8 LC_IDENTIFICATION=C
```

```
time zone: Europe/Berlin
```

```
tzcode source: system (glibc)
```

```
attached base packages:
[1] stats      graphics   grDevices utils      datasets  methods   base

other attached packages:
[1] pROC_1.19.0.1 scales_1.4.0  knitr_1.51    purrr_1.2.0  tidyverse_1.3.2
[6] dplyr_1.1.4   ggplot2_4.0.1  duckdb_1.4.3 DBI_1.2.3

loaded via a namespace (and not attached):
[1] gtable_0.3.6       jsonlite_2.0.0     compiler_4.5.2   tidyselect_1.2.1
[5] Rcpp_1.1.0         yaml_2.3.12      fastmap_1.2.0   R6_2.6.1
[9] labeling_0.4.3     generics_0.1.4    tibble_3.3.0     pillar_1.11.1
[13] RColorBrewer_1.1-3 rlang_1.1.6     xfun_0.54       S7_0.2.0
[17] otel_0.2.0        cli_3.6.5       withr_3.0.2     magrittr_2.0.4
[21] digest_0.6.39     grid_4.5.2       lifecycle_1.0.4 vctrs_0.6.5
[25] evaluate_1.0.5    glue_1.8.0       farver_2.1.2    codetools_0.2-20
[29] rmarkdown_2.30     tools_4.5.2      pkgconfig_2.0.3 htmltools_0.5.9
```