Mathematical Foundations of Time Series Classification

Wigner Summer Camp
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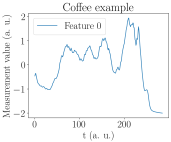

12	3
2	13

What is a Time Series?

A **time series** is a sequence of observations indexed by time.

Examples include:

- Daily temperature readings
- Stock prices over time
- Sensor outputs in experiments

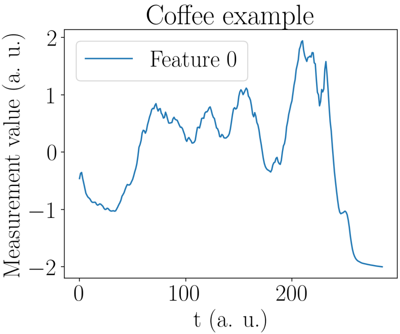

Figure: Example of a univariate time series – Coffee[1].

What is Time Series Classification?

Time Series Classification (TSC) is the task of assigning a label to an entire time series.

Goal: Learn a function $f : \text{Series} \rightarrow \text{Class}$.

Applications:

- Activity recognition (e.g., walking vs. running).
- Fault detection in engines.
- Medical diagnosis based on EEG/ECG.

(Common) Methods for TSC

1. **K-Nearest Neighbors (KNN)**:
   - Classifies based on similarity.
2. **Neural Networks**:
   - CNNs learn local patterns
   - RNNs capture temporal dependencies.

+1. **ALT (Adaptive Law-Based Transformation)**[2, 3]:
   - Transforms data into a linearly separable space.
   - Transparent and interpretable.

The Confusion Matrix

	Predicted: +	**Predicted: -**
Actual: +	True Positive (TP)	False Negative (FN)
Actual: -	False Positive (FP)	True Negative (TN)

Table: Confusion Matrix

TP: correctly predicted positive case.
TN: correctly predicted negative case.
FP: incorrectly predicted positive case.
FN: missed positive case.

12	3
2	13

Table: Example confusion matrix.

Evaluating Classifier Performance 2

Metric Definitions:

- **Accuracy**: Proportion of all correct predictions.
- **Precision**: $\mathrm{TP}/(\mathrm{TP}+\mathrm{FP})$: Fraction of predicted positives that are true positives.
- **Recall (Sensitivity)**: $\mathrm{TP}/(\mathrm{TP}+\mathrm{FN})$: Fraction of actual positives that are correctly identified.
- **F1-score**: Harmonic mean of precision and recall.

Important Notes:

- High accuracy can be misleading in imbalanced datasets.
- If the model predicts only the majority class, accuracy might still be high, but recall or precision will be low.
- F1-score helps balance precision and recall in such cases.

Example Calculation

Confusion Matrix:

	Predicted +	Predicted -
Actual +	40 (TP)	5 (FN)
Actual -	10 (FP)	50 (TN)

Metrics:

- Accuracy: $\frac{40+50}{105} = 0.857$.
- Precision: $\frac{40}{40+10} = 0.8$.
- Recall: $\frac{40}{40+5} \approx 0.889$.
- F1-score: $2 \cdot \frac{0.8 \cdot 0.889}{0.8+0.889} \approx 0.842$.

Exercise: Confusion Matrix Practice

Confusion Matrix:

	Predicted +	Predicted -
Actual +	30 (TP)	15 (FN)
Actual -	5 (FP)	60 (TN)

Calculate:

- Accuracy
- Precision
- Recall
- F1-score

Solution

Metrics:

- Accuracy: $\frac{30+60}{110} = 0.818$.
- Precision: $\frac{30}{30+5} = 0.857$.
- Recall: $\frac{30}{30+15} = 0.667$.
- F1-score: $2 \cdot \frac{0.857 \cdot 0.667}{0.857+0.667} \approx 0.75$.

Case 1: Imbalanced Dataset

Confusion Matrix:

	Predicted +	Predicted -
Actual + (Minority)	5 (TP)	5 (FN)
Actual - (Majority)	10 (FP)	180 (TN)

Metrics:

- Accuracy: $\frac{5+180}{200} = 0.925$.
- Precision: $\frac{5}{5+10} \approx 0.333$.
- Recall: $\frac{5}{5+5} = 0.5$.
- F1-score: $2 \cdot \frac{0.333 \cdot 0.5}{0.333+0.5} \approx 0.4$.

High accuracy hides poor minority class performance.

Case 2: Predicting All as One Class

Confusion Matrix:

	Predicted +	Predicted -
Actual +	0 (TP)	50 (FN)
Actual -	0 (FP)	150 (TN)

Metrics:

- Accuracy: $\frac{150}{200} = 0.75$.
- Precision: Undefined ($\frac{0}{0}$), often set to 0.
- Recall: 0.
- F1-score: 0.

Classifier completely ignores positive class.

Case 3: Both Imbalanced and All One Class

Confusion Matrix:

	Predicted -	Predicted +
Actual + (10)	0 (TP)	10 (FN)
Actual - (990)	0 (FP)	990 (TN)

Metrics:

- Accuracy: $\frac{990}{1000} = 0.99$.
- Precision: Undefined $(\frac{0}{0})$, often set to 0.
- Recall: 0.
- F1-score: 0.

Extremely misleading: near-perfect accuracy, but model fails completely on minority class.

References

- R. Briandet and et al., "Coffee dataset," https://www.timeseriesclassification.com/description.php?Dataset=Coffee, 1996, discrimination of Arabica and Robusta in Instant Coffee by Fourier Transform Infrared Spectroscopy and Chemometrics. J. Agric. Food Chem. 44(1), Briandet et al.
- M. T. Kurbucz, B. Hajós, B. P. Halmos, V. Á. Molnár, and A. Jakovác, "Adaptive law-based transformation (alt): A lightweight feature representation for time series classification," *arXiv preprint arXiv:2501.09217*, 2025.
- B. P. Halmos, B. Hajós, V. Á. Molnár, M. T. Kurbucz, and A. Jakovác, "Alt: A python package for lightweight feature representation in time series classification," *arXiv preprint arXiv:2504.12841*, 2025.