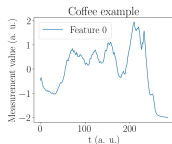# Mathematical Foundations of Time Series Classification

Wigner Summer Camp
Data and Compute Intensive Sciences Research Group

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2	13

What is a Time Series?

A **time series** is a sequence of observations indexed by time.

Examples include:
- ▶ Daily temperature readings
- ▶ Stock prices over time
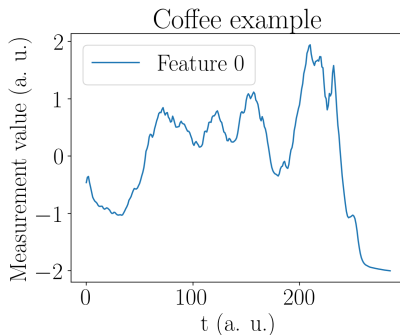- ▶ Sensor outputs in experiments

Figure: Example of a univariate time series – Coffee[1].

What is Time Series Classification?

Time Series Classification (TSC) is the task of assigning a label to an entire time series.

Goal: Learn a function $f : \text{Series} \rightarrow \text{Class}$.

Applications:

- ▶ Activity recognition (e.g., walking vs. running).
- ▶ Fault detection in engines.
- ▶ Medical diagnosis based on EEG/ECG.

(Common) Methods for TSC

1. K-Nearest Neighbors (KNN):
- ▶ Classifies based on similarity.

2. Neural Networks:
- ▶ CNNs learn local patterns
- ▶ RNNs capture temporal dependencies.

+1. ALT (Adaptive Law-Based Transformation)[2, 3]:
- ▶ Transforms data into a linearly separable space.
- ▶ Transparent and interpretable.

The Confusion Matrix

	Predicted: +	Predicted: -
Actual: +	True Positive (TP)	False Negative (FN)
Actual: -	False Positive (FP)	True Negative (TN)

Table: Confusion Matrix

TP: correctly predicted positive case.

TN: correctly predicted negative case.

FP: incorrectly predicted positive case.

FN: missed positive case.

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Table: Example confusion matrix.

Evaluating Classifier Performance 2

Metric Definitions:

- ▶ **Accuracy**: Proportion of all correct predictions.
- ▶ **Precision**: $TP/(TP + FP)$: Fraction of predicted positives that are true positives.
- ▶ **Recall (Sensitivity)**: $TP/(TP + FN)$: Fraction of actual positives that are correctly identified.
- ▶ **F1-score**: Harmonic mean of precision and recall.

Important Notes:

- ▶ High accuracy can be misleading in imbalanced datasets.
- ▶ If the model predicts only the majority class, accuracy might still be high, but recall or precision will be low.
- ▶ F1-score helps balance precision and recall in such cases.

Example Calculation

Confusion Matrix:

	Predicted +	Predicted -
Actual +	40 (TP)	5 (FN)
Actual -	10 (FP)	50 (TN)

Metrics:

- Accuracy: $\frac{40+50}{105} = 0.857$.
- Precision: $\frac{40}{40+10} = 0.8$.
- Recall: $\frac{40}{40+5} \approx 0.889$.
- F1-score: $2 \cdot \frac{0.8 \cdot 0.889}{0.8+0.889} \approx 0.842$.

Exercise: Confusion Matrix Practice

Confusion Matrix:

	Predicted +	Predicted -
Actual +	30 (TP)	15 (FN)
Actual -	5 (FP)	60 (TN)

Calculate:

- ▶ Accuracy
- ▶ Precision
- ▶ Recall
- ▶ F1-score

Solution

Metrics:

- Accuracy: $\frac{30+60}{110} = 0.818$.
- Precision: $\frac{30}{30+5} = 0.857$.
- Recall: $\frac{30}{30+15} = 0.667$.
- F1-score: $2 \cdot \frac{0.857 \cdot 0.667}{0.857+0.667} \approx 0.75$.

Case 1: Imbalanced Dataset

Confusion Matrix:

	Predicted +	Predicted -
Actual + (Minority)	5 (TP)	5 (FN)
Actual - (Majority)	10 (FP)	180 (TN)

Metrics:

- Accuracy: $\frac{5+180}{200} = 0.925$.
- Precision: $\frac{5}{5+10} \approx 0.333$.
- Recall: $\frac{5}{5+5} = 0.5$.
- F1-score: $2 \cdot \frac{0.333 \cdot 0.5}{0.333+0.5} \approx 0.4$.

High accuracy hides poor minority class performance.

Case 2: Predicting All as One Class

Confusion Matrix:

	Predicted +	Predicted -
Actual +	0 (TP)	50 (FN)
Actual -	0 (FP)	150 (TN)

Metrics:

- Accuracy: $\frac{150}{200} = 0.75$.
- Precision: Undefined ($\frac{0}{0}$), often set to 0.
- Recall: 0.
- F1-score: 0.

Classifier completely ignores positive class.

Case 3: Both Imbalanced and All One Class

Confusion Matrix:

	Predicted -	Predicted +
Actual + (10)	0 (TP)	10 (FN)
Actual - (990)	0 (FP)	990 (TN)

Metrics:

- Accuracy: $\frac{990}{1000} = 0.99$.
- Precision: Undefined ($\frac{0}{0}$), often set to 0.
- Recall: 0.
- F1-score: 0.

Extremely misleading: near-perfect accuracy, but model fails completely on minority class.

References

R. Briandet and et al., "Coffee dataset," https://www.timeseriesclassification.com/description.php?Dataset=Coffee, 1996, discrimination of Arabica and Robusta in Instant Coffee by Fourier Transform Infrared Spectroscopy and Chemometrics. J. Agric. Food Chem. 44(1), Briandet et al.

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