Linear Algebra Revision Test with Solutions

Wigner Data and Compute Intensive Sciences Group



8 July 2025

Part 1: Dot Product

1.
$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 1 \\ -1 \\ 2 \end{bmatrix} = 1 \cdot 4 + (-2) \cdot 1 + 3 \cdot (-1) + 0 \cdot 2 = \boxed{-1}$$

$$2. \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 2 \\ 0 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \\ 5 \\ -1 \end{bmatrix}$$

Part 2: Vector-Matrix Multiplication

3.
$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ -2 & 0 & 5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + (-1) \cdot (-2) + 3 \cdot 3 \\ 0 \cdot 1 + 4 \cdot (-2) + 1 \cdot 3 \\ -2 \cdot 1 + 0 \cdot (-2) + 5 \cdot 3 \\ 1 \cdot 1 + 1 \cdot (-2) + 1 \cdot 3 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \\ 13 \\ 2 \end{bmatrix}$$

4.
$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 3 & -2 & 1 & 0 \\ 0 & 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \cdot 2 + 0 \cdot (-1) + 2 \cdot 0 + (-1) \cdot 3 \\ 3 \cdot 2 + (-2) \cdot (-1) + 1 \cdot 0 + 0 \cdot 3 \\ 0 \cdot 2 + 1 \cdot (-1) + 4 \cdot 0 + 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 8 \\ 5 \end{bmatrix}$$

Part 3: Matrix-Matrix Multiplication

$$5. \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + (-1) \cdot (-2) + 0 \cdot 3 & 2 \cdot 2 + (-1) \cdot 0 + 0 \cdot (-1) \\ 3 \cdot 1 + 1 \cdot (-2) + 2 \cdot 3 & 3 \cdot 2 + 1 \cdot 0 + 2 \cdot (-1) \\ -1 \cdot 1 + 4 \cdot (-2) + 1 \cdot 3 & -1 \cdot 2 + 4 \cdot 0 + 1 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 7 & 4 \\ -6 & -3 \end{bmatrix}$$

$$6. \begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 3 & -1 & 0 \\ 0 & 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 3 \\ -2 & 4 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 6 & 1 \\ 5 & -6 & 8 \\ -7 & 16 & 5 \end{bmatrix}$$

Part 4: Matrix Determinant

7.
$$\det \begin{pmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 0 & 1 & -1 \end{pmatrix} = 3(2 \cdot (-1) - 2 \cdot 1) - 1((-1) \cdot (-1) - 2 \cdot 0) + 0 = \boxed{-11}$$

8.
$$\det \begin{pmatrix} 2 & -1 & 0 & 1 \\ 1 & 3 & 2 & 0 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 3 & 1 \end{pmatrix} = \boxed{-42}$$

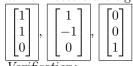
(Expansion along first row recommended)

Part 5: Eigenproblems

- 9. For $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$:
 - Eigenvalues: $\boxed{5}$, $\boxed{-3}$, $\boxed{2}$
 - \bullet Corresponding eigenvectors:

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$	$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
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10. Which vectors are eigenvectors?



 $\overline{Verification:}$