

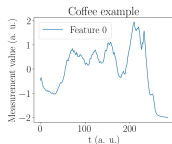
# Mathematical Foundations of Time Series Classification

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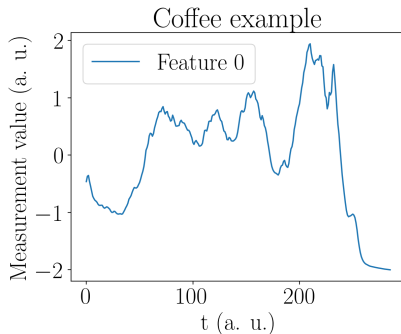
12	3
2	13

# What is a Time Series?

A **time series** is a sequence of observations indexed by time.

Examples include:

- ▶ Daily temperature readings
- ▶ Stock prices over time
- ▶ Sensor outputs in experiments



**Figure:** Example of a univariate time series – Coffee[1].

# What is Time Series Classification?

**Time Series Classification** (TSC) is the task of assigning a label to an entire time series.

**Goal:** Learn a function  $f : \text{Series} \rightarrow \text{Class}$ .

## **Applications:**

- ▶ Activity recognition (e.g., walking vs. running).
- ▶ Fault detection in engines.
- ▶ Medical diagnosis based on EEG/ECG.

# (Common) Methods for TSC

## 1. K-Nearest Neighbors (KNN):

- ▶ Classifies based on similarity.

## 2. Neural Networks:

- ▶ CNNs learn local patterns
- ▶ RNNs capture temporal dependencies.

## +1. ALT (Adaptive Law-Based Transformation)[2, 3]:

- ▶ Transforms data into a linearly separable space.
- ▶ Transparent and interpretable.

# The Confusion Matrix

	<b>Predicted: +</b>	<b>Predicted: -</b>
<b>Actual: +</b>	True Positive (TP)	False Negative (FN)
<b>Actual: -</b>	False Positive (FP)	True Negative (TN)

Table: Confusion Matrix

**TP:** correctly predicted positive case.

**TN:** correctly predicted negative case.

**FP:** incorrectly predicted positive case.

**FN:** missed positive case.

12	3
2	13

Table: Example confusion matrix.

# Evaluating Classifier Performance 2

## Metric Definitions:

- ▶ **Accuracy:** Proportion of all correct predictions.
- ▶ **Precision:**  $TP / (TP + FP)$ : Fraction of predicted positives that are true positives.
- ▶ **Recall (Sensitivity):**  $TP / (TP + FN)$ : Fraction of actual positives that are correctly identified.
- ▶ **F1-score:** Harmonic mean of precision and recall.

## Important Notes:

- ▶ High accuracy can be misleading in imbalanced datasets.
- ▶ If the model predicts only the majority class, accuracy might still be high, but recall or precision will be low.
- ▶ F1-score helps balance precision and recall in such cases.

# Example Calculation

## Confusion Matrix:

	Predicted +	Predicted -
Actual +	40 (TP)	5 (FN)
Actual -	10 (FP)	50 (TN)

## Metrics:

- ▶ Accuracy:  $\frac{40+50}{105} = 0.857$ .
- ▶ Precision:  $\frac{40}{40+10} = 0.8$ .
- ▶ Recall:  $\frac{40}{40+5} \approx 0.889$ .
- ▶ F1-score:  $2 \cdot \frac{0.8 \cdot 0.889}{0.8 + 0.889} \approx 0.842$ .

# Exercise: Confusion Matrix Practice

## Confusion Matrix:

	Predicted +	Predicted -
Actual +	30 (TP)	15 (FN)
Actual -	5 (FP)	60 (TN)

Calculate:

- ▶ Accuracy
- ▶ Precision
- ▶ Recall
- ▶ F1-score



# Solution

## Metrics:

- ▶ Accuracy:  $\frac{30+60}{110} = 0.818$ .
- ▶ Precision:  $\frac{30}{30+5} = 0.857$ .
- ▶ Recall:  $\frac{30}{30+15} = 0.667$ .
- ▶ F1-score:  $2 \cdot \frac{0.857 \cdot 0.667}{0.857 + 0.667} \approx 0.75$ .

## Case 1: Imbalanced Dataset

### Confusion Matrix:

	Predicted +	Predicted -
Actual + (Minority)	5 (TP)	5 (FN)
Actual - (Majority)	10 (FP)	180 (TN)

### Metrics:

- ▶ Accuracy:  $\frac{5+180}{200} = 0.925$ .
- ▶ Precision:  $\frac{5}{5+10} \approx 0.333$ .
- ▶ Recall:  $\frac{5}{5+5} = 0.5$ .
- ▶ F1-score:  $2 \cdot \frac{0.333 \cdot 0.5}{0.333+0.5} \approx 0.4$ .

*High accuracy hides poor minority class performance.*

## Case 2: Predicting All as One Class

### Confusion Matrix:

	Predicted +	Predicted -
Actual +	0 (TP)	50 (FN)
Actual -	0 (FP)	150 (TN)

### Metrics:

- ▶ Accuracy:  $\frac{150}{200} = 0.75$ .
- ▶ Precision: Undefined ( $\frac{0}{0}$ ), often set to 0.
- ▶ Recall: 0.
- ▶ F1-score: 0.

*Classifier completely ignores positive class.*

## Case 3: Both Imbalanced and All One Class

### Confusion Matrix:




	Predicted -	Predicted +
Actual + (10)	0 (TP)	10 (FN)
Actual - (990)	0 (FP)	990 (TN)

### Metrics:

- ▶ Accuracy:  $\frac{990}{1000} = 0.99$ .
- ▶ Precision: Undefined ( $\frac{0}{0}$ ), often set to 0.
- ▶ Recall: 0.
- ▶ F1-score: 0.

*Extremely misleading: near-perfect accuracy, but model fails completely on minority class.*

# References

-  R. Briandet and et al., “Coffee dataset,” <https://www.timeseriesclassification.com/description.php?Dataset=Coffee>, 1996, discrimination of Arabica and Robusta in Instant Coffee by Fourier Transform Infrared Spectroscopy and Chemometrics. J. Agric. Food Chem. 44(1), Briandet et al.
-  M. T. Kurbucz, B. Hajós, B. P. Halmos, V. Á. Molnár, and A. Jakovác, “Adaptive law-based transformation (alt): A lightweight feature representation for time series classification,” *arXiv preprint arXiv:2501.09217*, 2025.
-  B. P. Halmos, B. Hajós, V. Á. Molnár, M. T. Kurbucz, and A. Jakovác, “Alt: A python package for lightweight feature representation in time series classification,” *arXiv preprint arXiv:2504.12841*, 2025.