Mathematical Foundations of Time Series Classification

Wigner Summer Camp
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Balázs, Paszkál, Vince, Levente, Antal
Éva, Hajni

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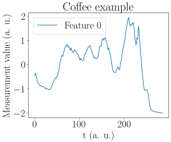

12	3
2	13

What is a Time Series?

A **time series** is a sequence of observations indexed by time.

Examples include:

- Daily temperature readings
- Stock prices over time
- Sensor outputs in experiments

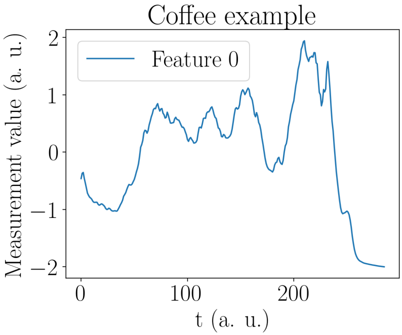

Figure: Example of a univariate time series – Coffee[1].

What is Time Series Classification?

Time Series Classification (TSC) is the task of assigning a label to an entire time series.

Goal: Learn a function $f : \text{Series} \rightarrow \text{Class}$.

Applications:

- Activity recognition (e.g., walking vs. running).
- Fault detection in engines.
- Medical diagnosis based on EEG/ECG.

(Common) Methods for TSC

1. **K-Nearest Neighbors (KNN)**:
   - Classifies based on similarity.
2. **Neural Networks**:
   - CNNs learn local patterns
   - RNNs capture temporal dependencies.

+1. **ALT (Adaptive Law-Based Transformation)**[2, 3]:
   - Transforms data into a linearly separable space.
   - Transparent and interpretable.

The Confusion Matrix

	Predicted: +	**Predicted: -**
Actual: +	True Positive (TP)	False Negative (FN)
Actual: -	False Positive (FP)	True Negative (TN)

Table: Confusion Matrix

TP: correctly predicted positive case. **TN**: correctly predicted negative case. **FP**: incorrectly predicted positive case. **FN**: missed positive case.

12	3
2	13

Table: Example confusion matrix.

Evaluating Classifier Performance 2

Metric Definitions:

- **Accuracy**: Proportion of all correct predictions.
- **Precision**: $\text{TP}/(\text{TP} + \text{FP})$: Fraction of predicted positives that are true positives.
- **Recall (Sensitivity)**: $\text{TP}/(\text{TP} + \text{FN})$: Fraction of actual positives that are correctly identified.
- **F1-score**: Harmonic mean of precision and recall.

Important Notes:

- High accuracy can be misleading in imbalanced datasets.
- If the model predicts only the majority class, accuracy might still be high, but recall or precision will be low.
- F1-score helps balance precision and recall in such cases.

Example Calculation

Confusion Matrix:

	Predicted +	Predicted -
Actual +	40 (TP)	5 (FN)
Actual -	10 (FP)	50 (TN)

Metrics:

- Accuracy: $\frac{40+50}{105} = 0.857$.
- Precision: $\frac{40}{40+10} = 0.8$.
- Recall: $\frac{40}{40+5} \approx 0.889$.
- F1-score: $2 \cdot \frac{0.8 \cdot 0.889}{0.8+0.889} \approx 0.842$.

Exercise: Confusion Matrix Practice

Confusion Matrix:

	Predicted +	Predicted -
Actual +	30 (TP)	15 (FN)
Actual -	5 (FP)	60 (TN)

Calculate:

- Accuracy
- Precision
- Recall
- F1-score

Solution

Metrics:

- Accuracy: $\frac{30+60}{110} = 0.818.$
- Precision: $\frac{30}{30+5} = 0.857.$
- Recall: $\frac{30}{30+15} = 0.667.$
- F1-score: $2 \cdot \frac{0.857 \cdot 0.667}{0.857+0.667} \approx 0.75.$

Case 1: Imbalanced Dataset

Confusion Matrix:

	Predicted +	Predicted -
Actual + (Minority)	5 (TP)	5 (FN)
Actual - (Majority)	10 (FP)	180 (TN)

Metrics:

- Accuracy: $\frac{5+180}{200} = 0.925$.
- Precision: $\frac{5}{5+10} \approx 0.333$.
- Recall: $\frac{5}{5+5} = 0.5$.
- F1-score: $2 \cdot \frac{0.333 \cdot 0.5}{0.333+0.5} \approx 0.4$.

High accuracy hides poor minority class performance.

Case 2: Predicting All as One Class

Confusion Matrix:

	Predicted +	Predicted -
Actual +	0 (TP)	50 (FN)
Actual -	0 (FP)	150 (TN)

Metrics:

- Accuracy: $\frac{150}{200} = 0.75$.
- Precision: Undefined ($\frac{0}{0}$), often set to 0.
- Recall: 0.
- F1-score: 0.

Classifier completely ignores positive class.

Case 3: Both Imbalanced and All One Class

Confusion Matrix:

	Predicted -	Predicted +
Actual + (10)	0 (TP)	10 (FN)
Actual - (990)	0 (FP)	990 (TN)

Metrics:

- Accuracy: $\frac{990}{1000} = 0.99$.
- Precision: Undefined $(\frac{0}{0})$, often set to 0.
- Recall: 0.
- F1-score: 0.

Extremely misleading: near-perfect accuracy, but model fails completely on minority class.

References

R. Briandet and et al., “Coffee dataset,” https://www.timeseriesclassification.com/description.php?Dataset=Coffee, 1996, discrimination of Arabica and Robusta in Instant Coffee by Fourier Transform Infrared Spectroscopy and Chemometrics. J. Agric. Food Chem. 44(1), Briandet et al.

M. T. Kurbucz, B. Hajós, B. P. Halmos, V. Á. Molnár, and A. Jakovác, “Adaptive law-based transformation (alt): A lightweight feature representation for time series classification,” *arXiv preprint arXiv:2501.09217*, 2025.

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