

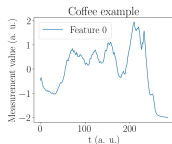
Mathematical Foundations of Time Series Classification

Wigner Summer Camp

Data and Compute Intensive Sciences Research Group

Balázs, Paszkál, Vince, Levente, Antal
Éva, Hajni

7–11 July 2025



| | |
|----|----|
| 12 | 3 |
| 2 | 13 |

What is a Time Series?

A **time series** is a sequence of observations indexed by time.

Examples include:

- ▶ Daily temperature readings
- ▶ Stock prices over time
- ▶ Sensor outputs in experiments

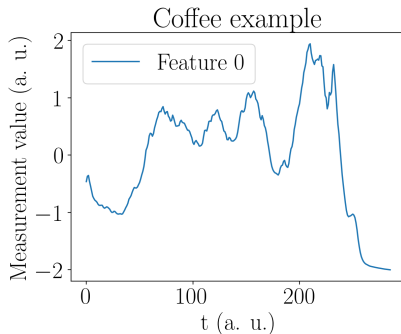


Figure: Example of a univariate time series – Coffee[1].

What is Time Series Classification?

Time Series Classification (TSC) is the task of assigning a label to an entire time series.

Goal: Learn a function $f : \text{Series} \rightarrow \text{Class}$.

Applications:

- ▶ Activity recognition (e.g., walking vs. running).
- ▶ Fault detection in engines.
- ▶ Medical diagnosis based on EEG/ECG.

(Common) Methods for TSC

1. K-Nearest Neighbors (KNN):

- ▶ Classifies based on similarity.

2. Neural Networks:

- ▶ CNNs learn local patterns
- ▶ RNNs capture temporal dependencies.

+1. ALT (Adaptive Law-Based Transformation)[2, 3]:

- ▶ Transforms data into a linearly separable space.
- ▶ Transparent and interpretable.

The Confusion Matrix

| | Predicted: + | Predicted: - |
|------------------|---------------------|---------------------|
| Actual: + | True Positive (TP) | False Negative (FN) |
| Actual: - | False Positive (FP) | True Negative (TN) |

Table: Confusion Matrix

TP: correctly predicted positive case.

TN: correctly predicted negative case.

FP: incorrectly predicted positive case.

FN: missed positive case.

| | |
|----|----|
| 12 | 3 |
| 2 | 13 |

Table: Example confusion matrix.

Evaluating Classifier Performance 2

Metric Definitions:

- ▶ **Accuracy:** Proportion of all correct predictions.
- ▶ **Precision:** $TP / (TP + FP)$: Fraction of predicted positives that are true positives.
- ▶ **Recall (Sensitivity):** $TP / (TP + FN)$: Fraction of actual positives that are correctly identified.
- ▶ **F1-score:** Harmonic mean of precision and recall.

Important Notes:

- ▶ High accuracy can be misleading in imbalanced datasets.
- ▶ If the model predicts only the majority class, accuracy might still be high, but recall or precision will be low.
- ▶ F1-score helps balance precision and recall in such cases.

Example Calculation

Confusion Matrix:

| | Predicted + | Predicted - |
|----------|-------------|-------------|
| Actual + | 40 (TP) | 5 (FN) |
| Actual - | 10 (FP) | 50 (TN) |

Metrics:

- ▶ Accuracy: $\frac{40+50}{105} = 0.857$.
- ▶ Precision: $\frac{40}{40+10} = 0.8$.
- ▶ Recall: $\frac{40}{40+5} \approx 0.889$.
- ▶ F1-score: $2 \cdot \frac{0.8 \cdot 0.889}{0.8 + 0.889} \approx 0.842$.

Exercise: Confusion Matrix Practice

Confusion Matrix:

| | Predicted + | Predicted - |
|----------|-------------|-------------|
| Actual + | 30 (TP) | 15 (FN) |
| Actual - | 5 (FP) | 60 (TN) |

Calculate:

- ▶ Accuracy
- ▶ Precision
- ▶ Recall
- ▶ F1-score

Solution

Metrics:

- ▶ Accuracy: $\frac{30+60}{110} = 0.818$.
- ▶ Precision: $\frac{30}{30+5} = 0.857$.
- ▶ Recall: $\frac{30}{30+15} = 0.667$.
- ▶ F1-score: $2 \cdot \frac{0.857 \cdot 0.667}{0.857 + 0.667} \approx 0.75$.

Case 1: Imbalanced Dataset

Confusion Matrix:

| | Predicted + | Predicted - |
|---------------------|-------------|-------------|
| Actual + (Minority) | 5 (TP) | 5 (FN) |
| Actual - (Majority) | 10 (FP) | 180 (TN) |

Metrics:

- ▶ Accuracy: $\frac{5+180}{200} = 0.925$.
- ▶ Precision: $\frac{5}{5+10} \approx 0.333$.
- ▶ Recall: $\frac{5}{5+5} = 0.5$.
- ▶ F1-score: $2 \cdot \frac{0.333 \cdot 0.5}{0.333+0.5} \approx 0.4$.

High accuracy hides poor minority class performance.

Case 2: Predicting All as One Class

Confusion Matrix:

| | Predicted + | Predicted - |
|----------|-------------|-------------|
| Actual + | 0 (TP) | 50 (FN) |
| Actual - | 0 (FP) | 150 (TN) |

Metrics:

- ▶ Accuracy: $\frac{150}{200} = 0.75$.
- ▶ Precision: Undefined ($\frac{0}{0}$), often set to 0.
- ▶ Recall: 0.
- ▶ F1-score: 0.

Classifier completely ignores positive class.

Case 3: Both Imbalanced and All One Class

Confusion Matrix:




| | Predicted - | Predicted + |
|----------------|-------------|-------------|
| Actual + (10) | 0 (TP) | 10 (FN) |
| Actual - (990) | 0 (FP) | 990 (TN) |

Metrics:

- ▶ Accuracy: $\frac{990}{1000} = 0.99$.
- ▶ Precision: Undefined ($\frac{0}{0}$), often set to 0.
- ▶ Recall: 0.
- ▶ F1-score: 0.

Extremely misleading: near-perfect accuracy, but model fails completely on minority class.

References

-  R. Briandet and et al., “Coffee dataset,” <https://www.timeseriesclassification.com/description.php?Dataset=Coffee>, 1996, discrimination of Arabica and Robusta in Instant Coffee by Fourier Transform Infrared Spectroscopy and Chemometrics. J. Agric. Food Chem. 44(1), Briandet et al.
-  M. T. Kurbucz, B. Hajós, B. P. Halmos, V. Á. Molnár, and A. Jakovác, “Adaptive law-based transformation (alt): A lightweight feature representation for time series classification,” *arXiv preprint arXiv:2501.09217*, 2025.
-  B. P. Halmos, B. Hajós, V. Á. Molnár, M. T. Kurbucz, and A. Jakovác, “Alt: A python package for lightweight feature representation in time series classification,” *arXiv preprint arXiv:2504.12841*, 2025.