

PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

Partial Marks : +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : -2 In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.37 If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle?

[A] $a, 4, 1$

[B] $a, 4, 2$

[C] $2a, 8, 1$

[D] $2a, 4, 1$

Q.38 If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k) , then which of the following is(are) possible value(s) of p, h and k ?

[A] $p = -2, h = 2, k = -4$

[B] $p = -1, h = 1, k = -3$

[C] $p = 2, h = 3, k = -4$

[D] $p = 5, h = 4, k = -3$

Space for rough work

Answers for the above questions

Ans for Q.37: (A), (B), and (C)	Ans for Q.38: (C)
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Q.39 Let $[x]$ be the greatest integer less than or equals to x . Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous?

- [A] $x = -1$ [B] $x = 0$ [C] $x = 1$ [D] $x = 2$

Q.40 Let $f: \mathbb{R} \rightarrow (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval $(0, 1)$?

- [A] $x^9 - f(x)$ [B] $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t dt$
[C] $e^x - \int_0^x f(t) \sin t dt$ [D] $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t dt$

Q.41 Which of the following is(are) NOT the square of a 3×3 matrix with real entries?

- [A] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ [B] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
[C] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ [D] $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Space for rough work

Answers for the above questions

Due to printing inconsistency in Q.39 in Code1 of Paper1 (and the corresponding question in all other codes), +4 marks will be awarded to all candidates for this question.

Ans for Q.40: (A) and (B)

Ans for Q.41: (B) and (D)

Q.42 Let a , b , x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

[A] $-1 + \sqrt{1 - y^2}$

[B] $-1 - \sqrt{1 - y^2}$

[C] $1 + \sqrt{1 + y^2}$

[D] $1 - \sqrt{1 + y^2}$

Q.43 Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

[A] $P(Y) = \frac{4}{15}$

[B] $P(X'|Y) = \frac{1}{2}$

[C] $P(X \cap Y) = \frac{1}{5}$

[D] $P(X \cup Y) = \frac{2}{5}$

Space for rough work

Answers for the above questions

Ans for Q.42: (A) and (B)

Ans for Q.43: (A) and (B)

SECTION 2 (Maximum Marks: 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened
Zero Marks : 0 In all other cases

Q.44 For how many values of p , the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

Q.45 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

for $x \in (0, \frac{\pi}{2}]$, then $\lim_{x \rightarrow 0} g(x) =$

Space for rough work

Answers for the above questions

Ans for Q.44: (2)	Ans for Q.45: (2)
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Q.46 For a real number α , if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Q.47 Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

Q.48 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

Space for rough work

Answers for the above questions

Ans for Q.46: (1)

Ans for Q.47: (5)

Ans for Q.48: (6)

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks: -1 In all other cases

Space for rough work

Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q.49 For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact $(-1, 1)$, then which of the following options is the only CORRECT combination for obtaining its equation?

- [A] (I) (i) (P) [B] (I) (ii) (Q) [C] (II) (ii) (Q) [D] (III) (i) (P)

Q.50 If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is $(8, 16)$, then which of the following options is the only CORRECT combination?

- [A] (I) (ii) (Q) [B] (II) (iv) (R) [C] (III) (i) (P) [D] (III) (ii) (Q)

Q.51 The tangent to a suitable conic (Column 1) at $(\sqrt{3}, \frac{1}{2})$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination?

- [A] (IV) (iii) (S) [B] (IV) (iv) (S) [C] (II) (iii) (R) [D] (II) (iv) (R)

Space for rough work

Answers for the above questions

Ans for Q.49: (B)	Ans for Q.50: (C)	Ans for Q.51: (D)
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Answer Q.52, Q.53 and Q.54 by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x \rightarrow \infty} f(x) = 0$	(P) f is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x \rightarrow \infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x \rightarrow \infty} f'(x) = -\infty$	(R) f' is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x \rightarrow \infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

Q.52 Which of the following options is the only CORRECT combination?

- [A] (I) (i) (P) [B] (II) (ii) (Q) [C] (III) (iii) (R) [D] (IV) (iv) (S)

Q.53 Which of the following options is the only CORRECT combination?

- [A] (I) (ii) (R) [B] (II) (iii) (S) [C] (III) (iv) (P) [D] (IV) (i) (S)

Q.54 Which of the following options is the only INCORRECT combination?

- [A] (I) (iii) (P) [B] (II) (iv) (Q) [C] (III) (i) (R) [D] (II) (iii) (P)

END OF THE QUESTION PAPER

Space for rough work

Answers for the above questions

Ans for Q.52: (B)	Ans for Q.53: (B)	Ans for Q.54: (C)
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SPACE FOR ROUGH WORK