Causal Data Science for Business Decision Making

Graphical Causal Models II

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Causal Effects

Definition: Causal Effect (Pearl, 2009, p. 70)

Given two disjoint sets of variables, X and Y, the causal effect of X on Y, denoted either as $P(y|\hat{x})$ or $P(y|do(x))$ is a function from X to the space of probability distributions on Y. For each realization x of X, $P(y|\hat{x})$ gives the probability of $Y = y$ induced by deleting from the [structural causal] model [...] all equations corresponding to variables in X and substituting $X = x$ in the remaining equations.

- Sometimes, causal effects of interventions are defined as

$$E(Y = y|do(X = x'')) - E(Y = y|do(X = x'))$$

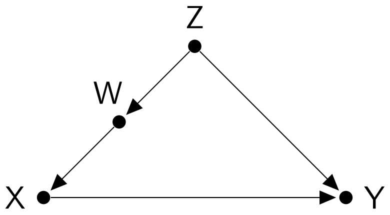
which can always be computed from the general function $P(y|do(x))$

The Identification Problem

- Carrying out the intervention ourselves, in a randomized control trial, is not always feasible (too expensive, impractical, or unethical)
- How can we then identify the effect of interventions purely from observational data?
  - We want to know $P(y|do(x))$ but all we have is data $P(x, y, z)$
  - And we know that $P(y|do(x)) \neq P(y|x)$ ("correlation is not causation")
  - No fancy machine learning algorithm will ever solve this problem
- We need to find a way to transform $P(y|do(x))$ into an expression that only contains, observed, "do-free" quantities
- What if we only have data that is measured with selection bias or that stems from a different population (topic of later lectures)?

Confounding Bias

- Problem of confounding: Paths between treatment X and outcome Y that are not emitted by X and which create an association between X and Y that is not causal



- In this example there is one confounding path: $X \leftarrow W \leftarrow Z \rightarrow Y$
  - Because confounding paths always point into X, they are also called *backdoor paths* (i.e., they "enter through the backdoor")
- The other path between X and Y ($X \rightarrow Y$) is emitted by X and therefore causal

Blocking Backdoor Paths

- So confounding paths create spurious correlations between treatment and outcome. But remember the d-separation criterion from the previous lecture

Definition: *d*-separation (Pearl et al., 2016, p. 46)

A path p is blocked by a set of nodes Z if and only if

1. p contains a chain of nodes $A \rightarrow B \rightarrow C$ or a fork $A \leftarrow B \rightarrow C$ such that the middle node B is in Z (i.e., B is conditioned on), or
2. p contains a collider $A \rightarrow B \leftarrow C$ such that the collision node B is not in Z, and no descendant of B is in Z

- We can block biasing paths by conditioning on intermediate variables on these paths that are not colliders or descendants of colliders

Backdoor Adjustment

Definition: The Backdoor Criterion (Pearl et al., 2016, p. 61)

Given an ordered pair of of variables (X, Y) in a directed acyclic graph G, a set of variables Z satisfies the backdoor criterion relative to (X, Y) if no node in Z is a descendant of X, and Z blocks every path between X and Y that contains an arrow into X.

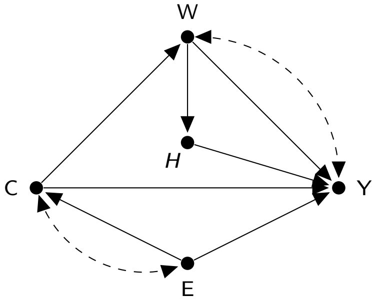
- If a set of variables Z satisfies the backdoor criterion fo X and Y, then the causal effect is given by

$$P(Y = y | do(X = x)) = \sum_z P(Y = y | X = x, Z = z) P(Z = z)$$

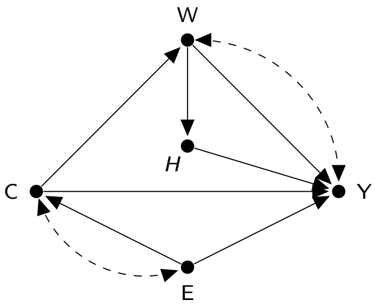
- I.e., condition on the values of Z and average over their joint distribution ($=$ adjusting for Z)

Example 1: College Wage Premium

- Take the stylized example of the college wage premium
  - C: college degree
  - Y: earnings
  - W: occupation
  - H: work-related health
  - E: other socio-economic factors

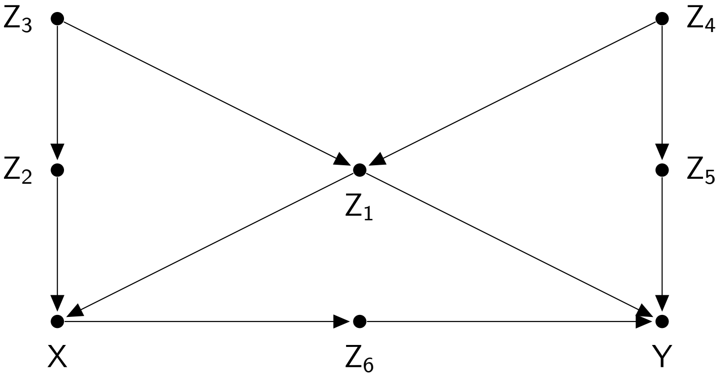


Example 1: College Wage Premium

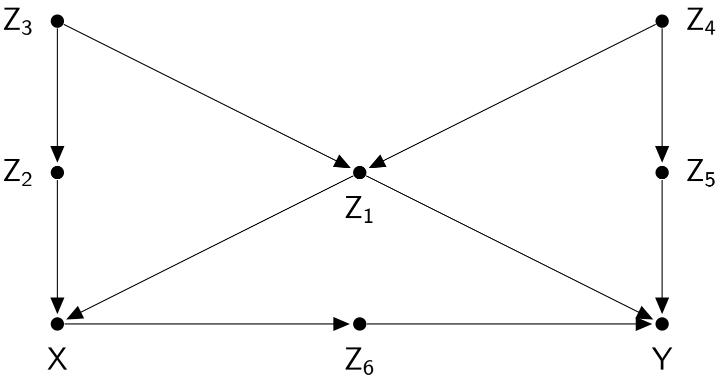


- Take the stylized example of the college wage premium
  - C: college degree
  - Y: earnings
  - W: occupation
  - H: work-related health
  - E: other socio-economic factors
- The graph contains two backdoor paths
  1. $C \leftarrow E \rightarrow Y$
  2. $C \dashleftarrow\dashrightarrow E \rightarrow Y$
- We can close both of these backdoor paths by adjusting for E

Example 2

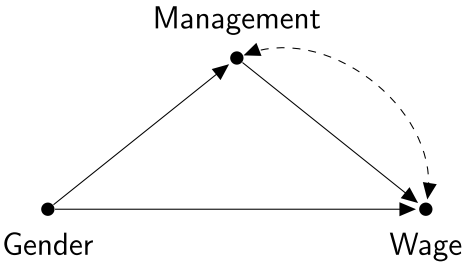


Example 2

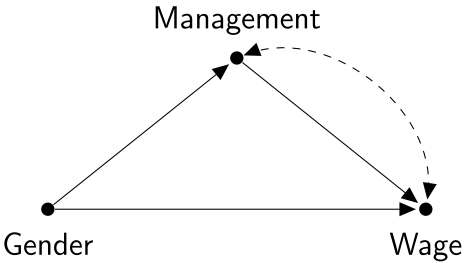


- Minimum sufficient adjustment sets: $\{Z_1, Z_2\}$, $\{Z_1, Z_3\}$, $\{Z_1, Z_4\}$, $\{Z_1, Z_5\}$

Example 3: Gender Wage Gap

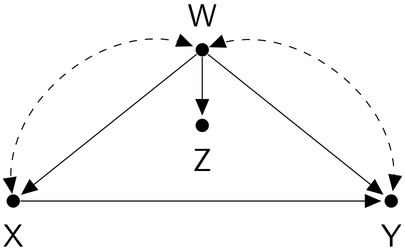


Example 3: Gender Wage Gap

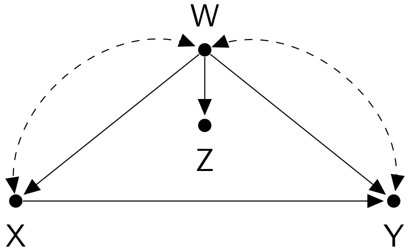


- Minimum sufficient adjustment sets: $\emptyset$ (empty set)
  - Causal effect is identified without adjusting for any covariate
- In fact, conditioning on *Management* would lead to collider bias in this case

Example 4

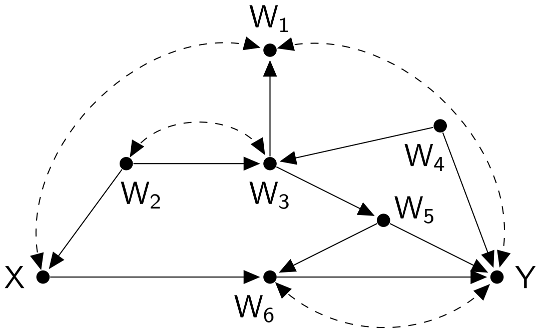


Example 4

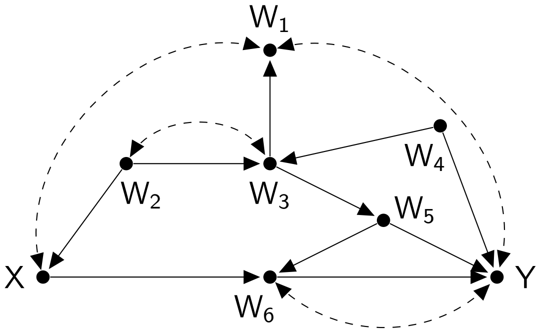


- There is no admissible adjustment set in this graph
  - W is a confounder on the path $X \leftarrow W \rightarrow Y$
  - But conditioning on W or Z leads to collider bias
- The causal effect of X on Y is not identifiable in this graph

Example 5



Example 5



- Backdoor-admissible adjustment sets:

$$Z = \{\{W_2\}, \{W_2, W_3\}, \{W_2, W_4\}, \{W_3, W_4\}, \{W_2, W_3, W_4\}, \{W_2, W_5\},$$
$$\{W_2, W_3, W_5\}, \{W_4, W_5\}, \{W_2, W_4, W_5\}, \{W_3, W_4, W_5\}, \{W_2, W_3, W_4, W_5\}\}$$

Estimation

- We have already seen that once we have found a backdoor-admissible adjustment set Z, the causal effect is identified via the adjustment formula

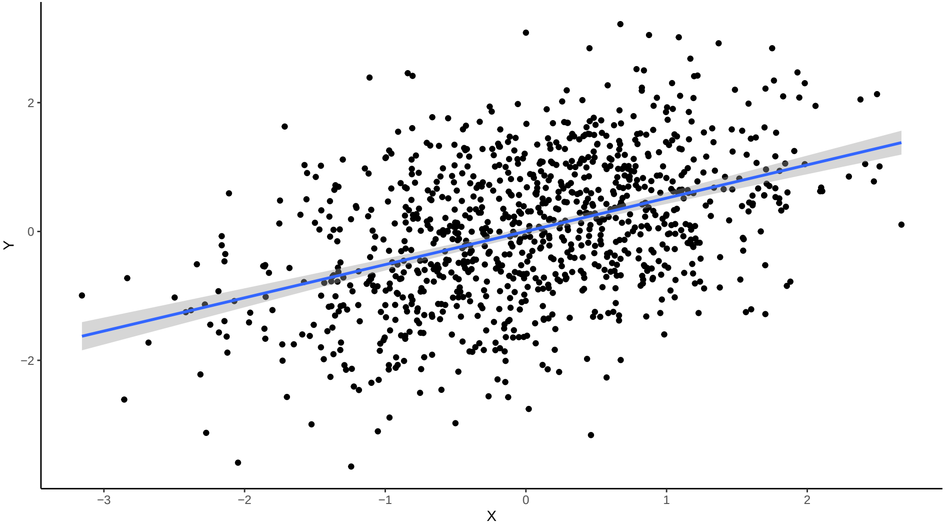
$$P(Y = y|do(X = x)) = \sum_z P(Y = y|X = x, Z = z)P(Z = z)$$

- This non-parametric regression formula can be hard to estimate directly
- In practice, we can relay on simpler estimation algorithms such as
  - Nearest-neighbor matching
  - Ordinary least squares (if we are willing to additionally assume $E[Y|X, Z]$ to be linear)

Nearest-neighbor matching

- The basic idea is very simple
  - Assume X takes two values: one (treated) and zero (untreated)
  - For every treated unit find an untreated neighbor in the data that has a similar value of Z
  - Estimate the correlation between X and Y in this matched sample
  - Due to the matching, the distribution of Z is balanced between treated and untreated units and thus cannot produce confounding anymore
- In practice, a few more complications arise
  - Do you always find a suitable neighbor for every treated unit (problem of common support)?
  - Should you match one or more neighbors (bias-variance tradeoff)
  - How to deal with continuous variables?

Refresher: Linear Regression



Refresher: Linear Regression (II)

- We assume that the conditional expectation of Y can be approximated by a linear function

$$y_i = a + bx_i$$

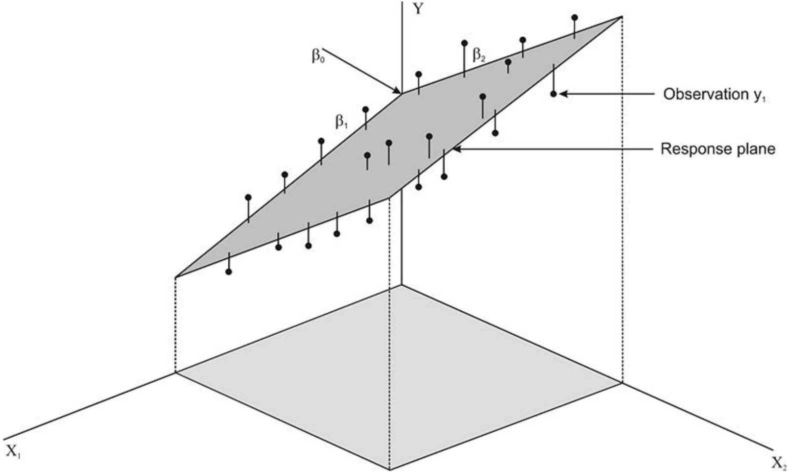
- To find the line of best fit we minimize the loss function

$$\sum_i (y_i - y_i')^2 = \sum_i (y_i - a - bx_i)^2$$

- The name *rdinary least squares* (OLS) derives from the square loss function
- Solution of the minimization problem for the bivariate case
  - $\hat{b} = R_{YX} = \frac{\sigma_{XY}}{\sigma_X^2}$ and $\hat{a} = \bar{y} - \hat{b}\bar{x}$
- We can similarly find solution if there are more than one explanatory variable

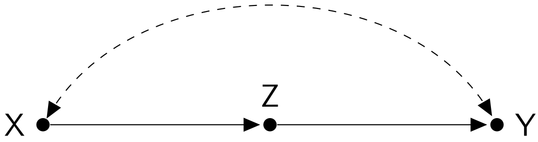
$$y_i = a + b_1 x_{1i} + b_2 x_{2i} \ldots + b_k x_{ki}$$

Multiple Regression



Source: `https://www.ck12.org/section/multiple-regression-of-regression-and-correlation/`

Front-door Criterion



- What happens if cannot measure all variables that we need for backdoor adjustment?
- We will see several solutions for dealing with unobservables throughout the course, but one particularly elegant one is the so-called front-door criterion (FC)
- For the FC to work, Z has to transmit the entire effect of X on Y
- Bellemare et al. (2020) use the FC to estimate whether sharing a ride on Uber and Lyft (X) leads to a lower propensity to tip (Y)
- The mediator Z in this case is whether a ride is actually shared after it has been requested by the app user

Front-door Criterion (II)

Definition: The Frontdoor Criterion (Pearl et al., 2016, p. 69)

A set of variables Z is said to satisfy the frontdoor criterion relative to an ordered pair of variables (X, Y) if

1. Z intercepts all directed paths from X to Y
2. There is no unblocked path from X to Z
3. All backdoor paths from Z to Y are blocked by X

Theorem: Frontdoor Adjustment (Pearl et al., 2016, p. 69)

If Z satisfies the frontdoor criterion relative to (X, Y) and if $P(x, z) > 0$, then the causal effect of X on Y is identifiable and is given by the formula

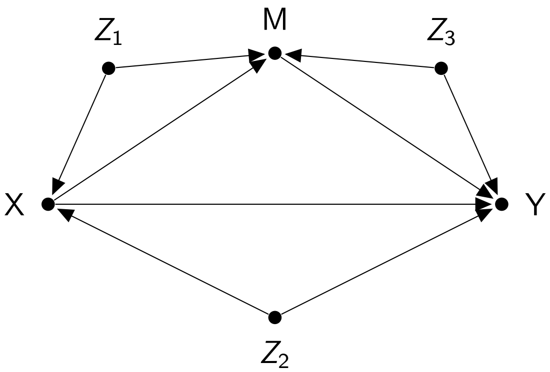
$$P(Y = y|do(X = x)) = \sum_{z} \sum_{x'} P(Y = y|Z = z, X = x')P(X = x')P(Z = z|X = x)$$

Where do DAGs come from?

Causal Modeling

- Causal diagrams are a model of how we think the world works
- We arrive at such a model by using our knowledge about the particular context under study
  - E.g., by consulting the relevant scientific literature for a topic
  - Or by interviewing domain experts
- The ladder of causation tells us that there is no way around these theoretical assumptions for causal inference: "*no causes in, no causes out*" (Cartwright, 1989)
- There some data-driven approaches, which are known under the rubric of *causal discovery*
  - They rely on the d-separation criterion we have already encountered and try to infer a compatible DAG from the conditional independence relationships found in the data
  - You can show, however, that you will never be able to perfectly determine a DAG from data alone. Some ex-ante causal assumption will always be needed

Causal Discovery and D-Separation



This graph implies the following conditional independence relationships in the data:

$$M \perp\!\!\!\perp Z_2 | X, Z_1$$
$$X \perp\!\!\!\perp Z_3$$
$$Y \perp\!\!\!\perp Z_1 | M, X, Z_2, Z_3$$
$$Z_1 \perp\!\!\!\perp Z_2$$
$$Z_1 \perp\!\!\!\perp Z_3$$
$$Z_2 \perp\!\!\!\perp Z_3$$

- The conditional independence relationships on the right can actually be used to learn the graph on the left from data

Thank you

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References I

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