THE DOT PRODUCT AND CONVOLUTION

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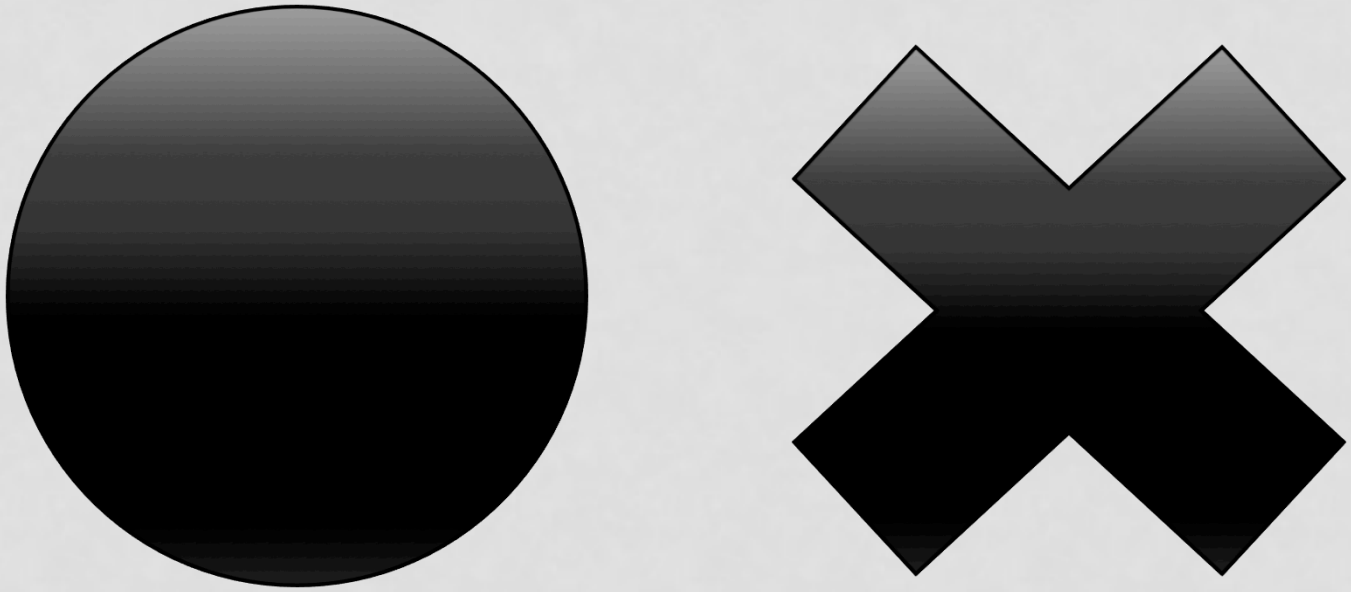
PSY 696B – NEURAL TIME SERIES ANALYSIS

SPRING 2014



THE EQUATION

$$dotproduct_{ab} = \sum_{i=1}^{n} a_i b_i$$

THE INTERPRETATIONS

- **Signal-processing:** "sum of elements in one vector weighted by elements of another vector"
- **Statistics:** "covariance or similarity between two vectors"
- **Geometry:** "mapping between vectors (product of the magnitudes of the two vectors scaled by the cosine of the angle between them"
- **In any case…** two vectors of equal length

GEOMETRY VISUALIZATION

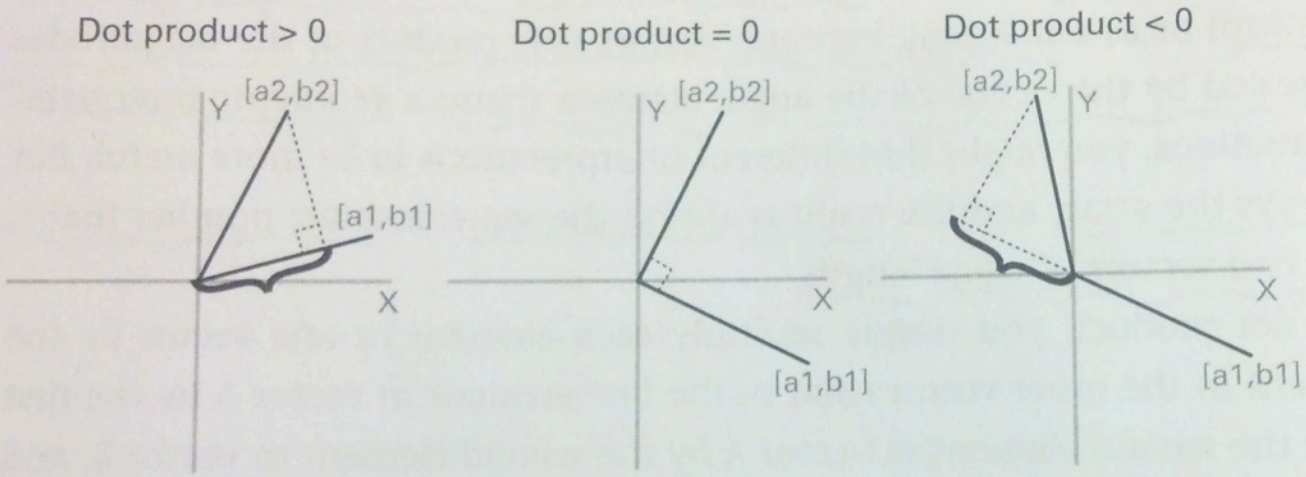
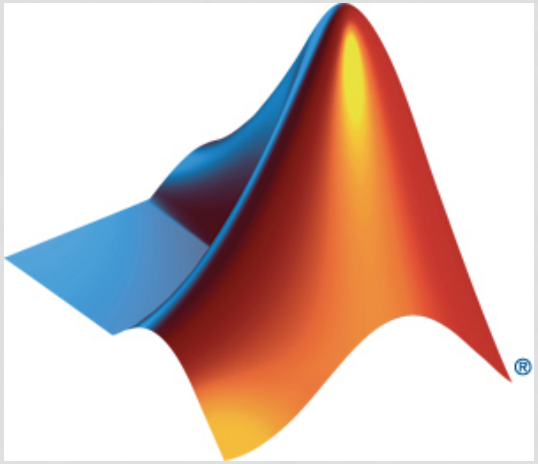


Figure 10.1
Graphical illustration of the geometric interpretation of the dot product between two two-element vectors. Curly brackets illustrate the magnitude of the projection of one vector onto the other (this is the dot product).

MATLAB (YAYYYY!)

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con·vo·lu·tion noun \kän-və-'lü-shən\
: something that is very complicated and difficult to understand
: a twist or curve
Full Definition of CONVOLUTION
Cite!
g+1
1 : a form or shape that is folded in curved or tortuous windings
2 : one of the irregular ridges on the surface of the brain and especially of the cerebrum of higher mammals
3 : a complication or intricacy of form, design, or structure
See convolution defined for English-language learners »
See convolution defined for kids »

TIME-FREQUENCY (SQUIGGLY LINE) ANALYSIS DEFINITION

- **"Extension of the dot product, in which the dot product is computed repeatedly over time"**
- **Algorithm:** "compute the dot product between two vectors, shift one vector in time relative to the other vector, compute the dot product again, and so on."
- Terminology (a la MXC):
  - **Signal** = EEG data
  - **Kernel** = wavelet or sine wave

$$(a * b)_k = \sum_{i=1}^{n} a_i b_{k-i}$$

THE INTERPRETATIONS

- **Signal-processing:** "time series of one signal weighted by another signal that slides along the first signal"
- **Statistics:** "cross-variance (similarity between two vectors over time"
- **Geometry:** "time series of mappings between two vectors"
- **Other:** "frequency filter"

HOW TO CONVOLVE: BASIC

1. Create a kernel (e.g. 1Hz sine wave)
2. Flip kernel backwards (why?)
3. Compute dot product at beginning of signal
4. Move across the signal, computing dot products along the way

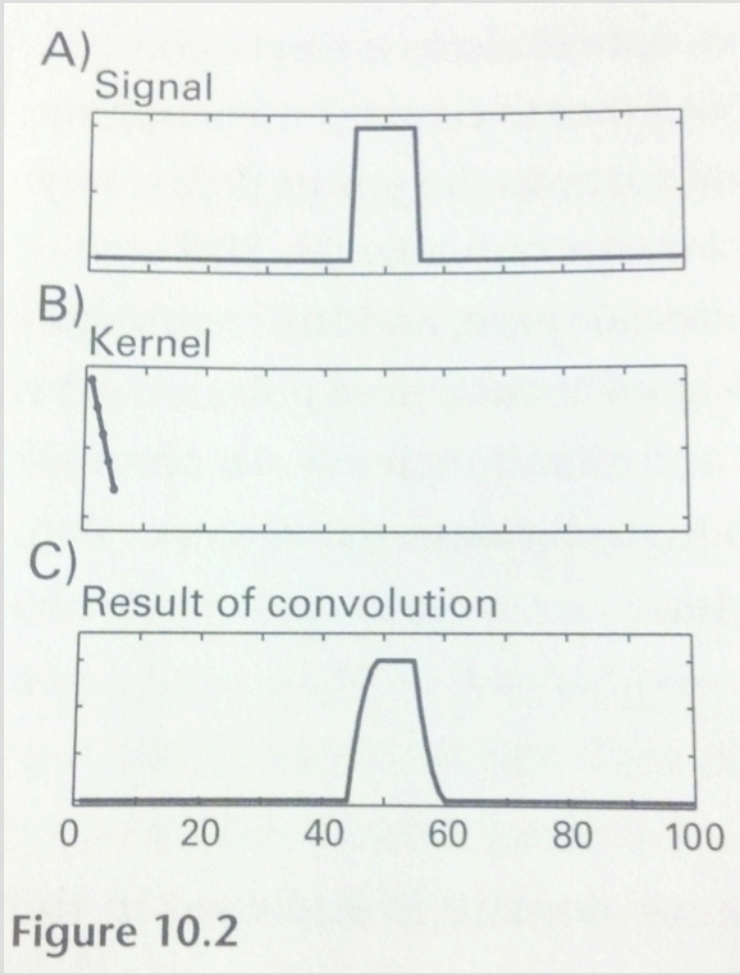
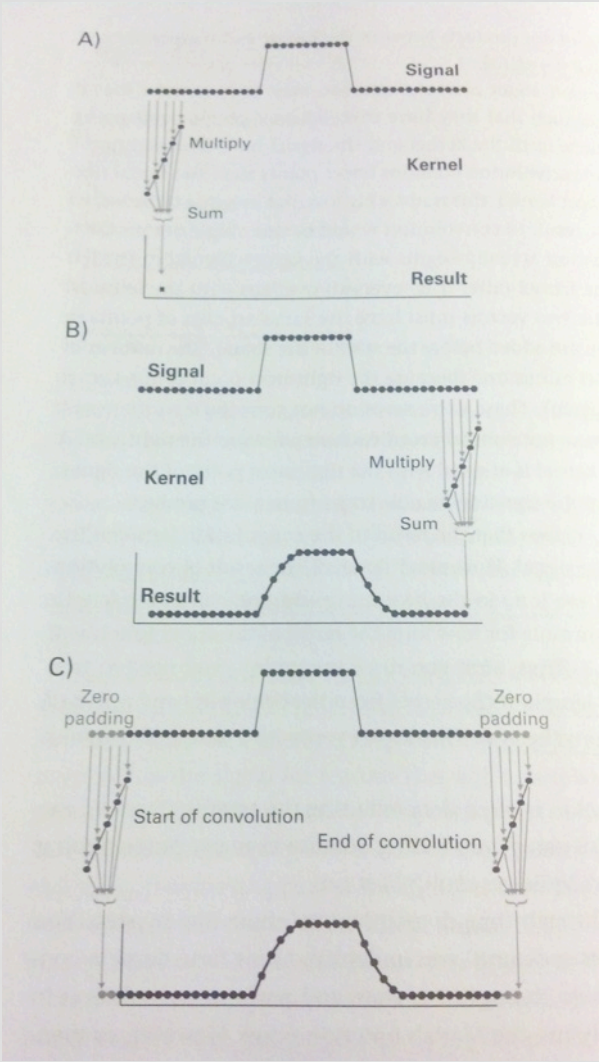


Figure 10.2

HOW TO CONVOLVE: ADVANCED

1. Create a kernel (e.g. 1Hz sine wave)
2. Flip kernel backwards
3. **Zero-pad signal at beginning and end**
4. Compute dot product at beginning of signal **(yielding a point at center of kernel)**
5. Move across the signal, computing dot products along the way
6. **Remove zero-padding**

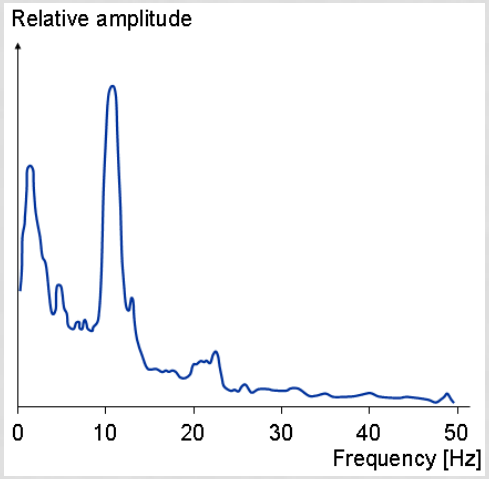


CONVOLUTION VS. CROSS-COVARIANCE

- **Convolution:** kernel is reversed
- **Cross-correlation** (cross-covariance scaled by the variances)**:** kernel kept in original orientation

APPLICATION TO EEG DATA ANALYSIS

- Use wavelets consisting of a sine wave for each frequency bin across the frequency spectrum
- Convolution for each frequency bin provides info of bin-specific and time-specific activity
- In MXC words, "…it reveals when and to what extent the EEG data contain features that look like the wavelet."



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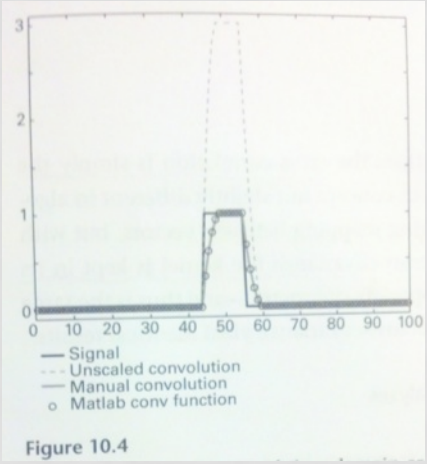

MATLAB (YAYYYY!)