Independent Sets and Cliques

October 8, 2024

Definition

1. A subset S of V is called an independent set of G if no two vertices of S are adjacent in G.
2. An independent set is maximum if G has no independent set S' with $|S'| > |S|$.
3. Recall that a subset K of V such that every edge of G has at least one end in K is called a covering of G.

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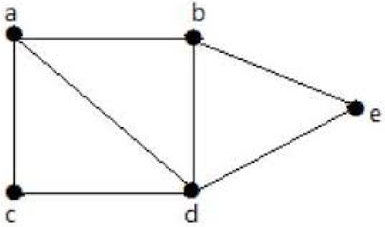
Example



1. Independent set $S_1 = \{a, e\}$, and Covering $S_1^c = \{c, b, d\}$
2. Independent set$S_2 = \{b, c\}$, and Covering $S_1^c = \{a, d, e\}$.

Theorem

A set $S \subseteq V$ is an independent set of G if and only if $V \setminus S = S^c$ is a covering of G.

Proof.

1. By definition, S is an independent set of G
2. if and only if no edge of G has both ends in S
3. if and only if each edge has at least one end in $V \setminus S = S^c$.
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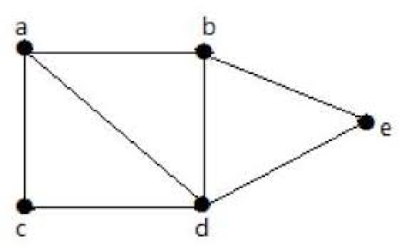
Definition

1. The number of vertices in a maximum independent set of G is called the independence number of G and is denoted by a $\alpha(G)$.

2. The number of vertices in a minimum covering of G is the covering number of G and is denoted by $\beta(G)$.



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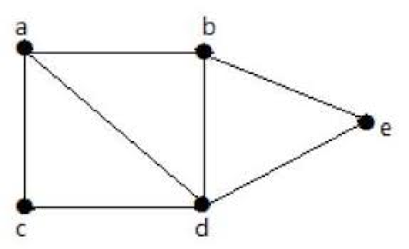

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Corollary

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$\alpha + \beta = |V(G)| = n.$

Proof.

1. Let S be a maximum independent set of G.
2. Let K be a minimum covering of G.
3. By Previous theorem, $V \setminus K$ is an independent set and $V \setminus S$ is a covering of G.
4. $\beta \leq |V \setminus S| = n - \alpha =, \Rightarrow n \geq \alpha + \beta.$
5. $\alpha \geq |V \setminus K| = n - \beta, \Rightarrow n \leq \alpha + \beta.$
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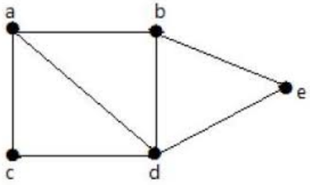
Edge Analogues

Definition

1. The edge analogue of an independent set is a set of links no two of which are adjacent, that is, a matching.
2. The edge analogue of a covering is called an edge covering. An edge covering of G is a subset L of E such that each vertex of G is an end of some edge in L.
3. An edge-cover F is called a minimal edge-cover if there is no edge-cover which is properly contained in F.



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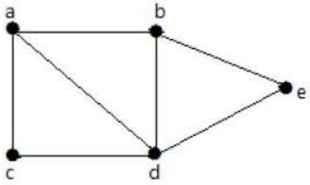

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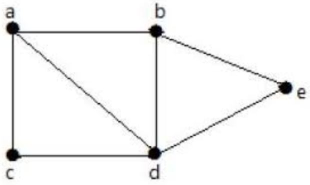

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Observations

1. Note that edge coverings do not always exist; a graph G has an edge covering if and only if $\delta(G) > O$.
2. $\alpha'(G)$ = number of edges in a maximum matching of G
3. $\beta'(G)$ = number of edges in a minimum edge covering of G
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2. The complement of a matching need not be an edge cover.
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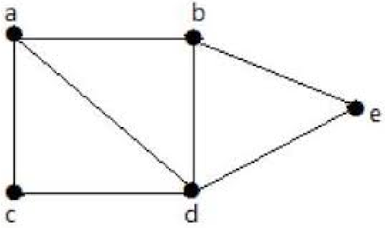

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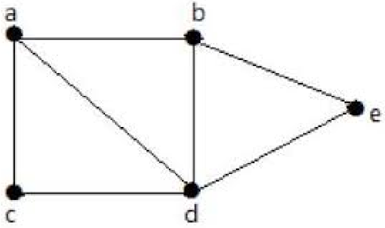

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If $\delta(G) > 0$*, then* $\alpha'(G) + \beta'(G) = n = |V(G)|$.

Proof.

Proof consists of two steps.

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Claim : $\alpha'(G) + \beta'(G) \leq n = |V(G)|$

1. Let M be a maximum matching.
2. Let U be the set of all M-unsaturated vertices.
3. Then, $|U| = n - 2|M|$.
4. Let $U = \{v_1, v_2, \cdots, v_p\}$, where $p = n - 2|M|$.
5. Let e_i be an edge incident to v_i, $i = 1, \cdots, p$.
6. such an edge exists, since $\delta(G) > 0$.
7. Then $M \cup \{e_1, e_2, \cdots e_p\}$ is an edge-cover.
8. So,
   $\beta'(G) \leq |M| + p \leq |M| + (n - 2|M|) = n - |M| = n - \alpha'(G)$
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Claim : $\alpha'(G) + \beta'(G) \geq n = |V(G)|$

1. Let F be a minimum edge cover.
2. Let H be the spanning subgraph of G with edge set F.
3. Let M_H be a maximum matching in H.
4. Let U be the set of M_H unsaturated vertices in H.
5. As before, $|U| = n - 2|M_H|$.
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2. Let H be the spanning subgraph of G with edge set F.
3. Let M_H be a maximum matching in H.
4. Let U be the set of M_H unsaturated vertices in H.
5. As before, $|U| = n - 2|M_H|$.
6. Let $U = \{v_1, v_2, \cdots, v_p\}$ where $p = n - 2|M_H|$.

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Proof Contd.

1. Since, F is an edge-cover, it contains an edge e_i incident with v_i, $1 \leq i \leq p$.
2. Since, M_H is a maximum matching, U is an independent set in H.
3. Therefore $e_i \neq e_j$, if $v_i \neq v_j$.
4. Hence, $e_1, e_2, \cdots e_p$ are all distinct edges.
5. It follows that $|F| \geq |M_H| + p = |M_H| + n - 2|M_H| = n - |M_H|$.
6. Hence, $\beta'(G) = |F| \geq n - 2|M_H| \geq n - \alpha'(G)$, since $\alpha'(G) \geq |M_H|$.
7. So, $\alpha'(G) + \beta'(G) \geq n$ $\square$

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In a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.

Proof.

1. $\alpha(G) + \beta(G) = n$
2. $\alpha'(G) + \beta'(G) = n$
3. G is bipartite.
4. In a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering. (Proved earlier)
5. which implies, $\alpha' = \beta$.
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