Tutorial 2 Solutions

Wednesday, 12 February 2025

1. $$A = \{x \in \{0,1\}^* \mid x = 0^+1\}$$

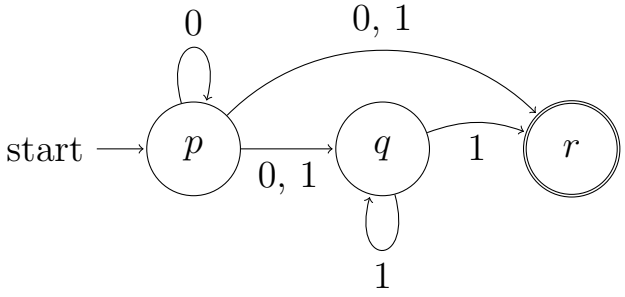
2. The epsilon-closure of a state is the set of states reachable from it using only epsilon transitions.

   Epsilon-closure$(p) = \{p, q, r\}$, Epsilon-closure$(q) = \{q, r\}$, Epsilon-closure$(r) = \{r\}$

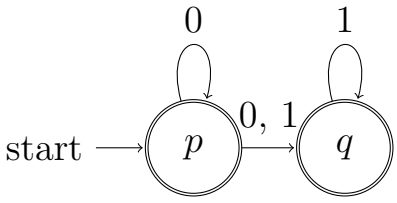
   Transition Table for the NFA Without Epsilon Transitions We replace the epsilon transitions with direct transitions based on the epsilon-closure. The updated transitions are:

State	**0**	**1**
p	$\{p, q, r\}$	$\{q, r\}$
q	$\emptyset$	$\{q, r\}$
r	$\emptyset$	$\emptyset$

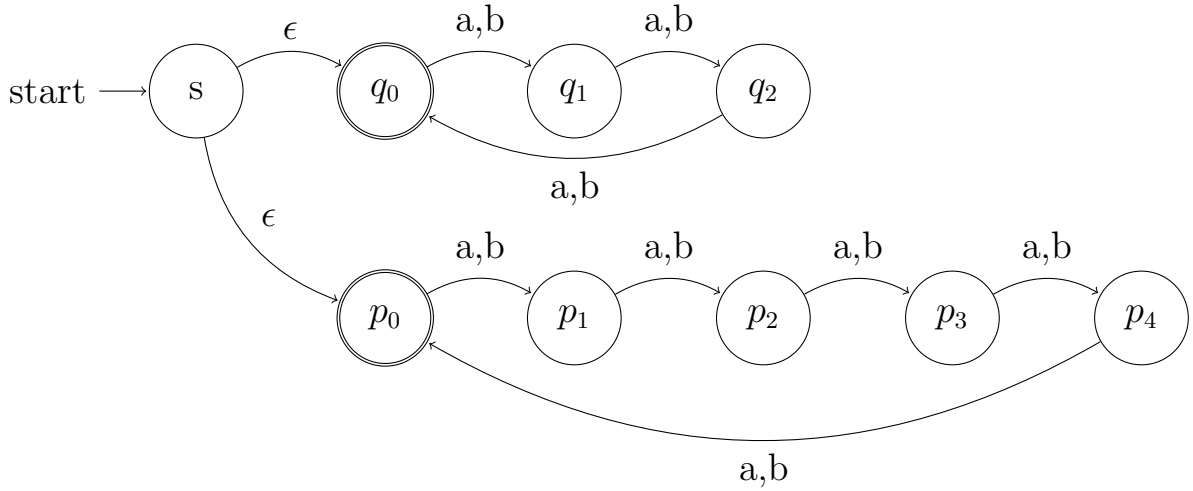
   The equivalent NFA without epsilon transitions is constructed as follows: (This doesn't accept the empty string)



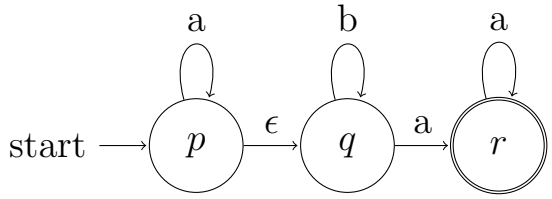
   An equivalent NFA that also accepts the empty string:



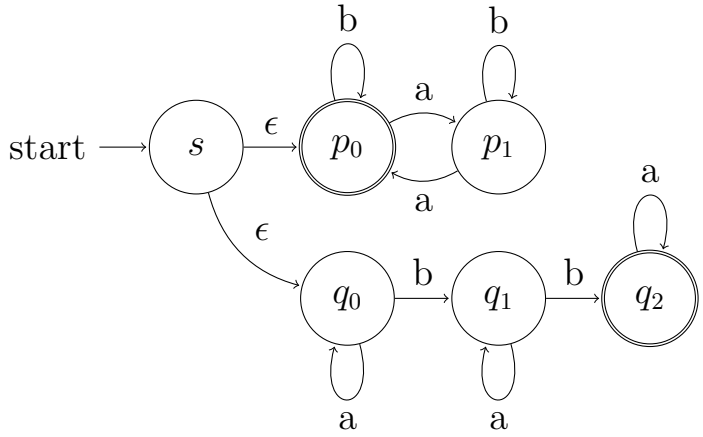
3. (a) $S = \{s \mid \#s \text{ is divisible by either 3 or 5}\}$



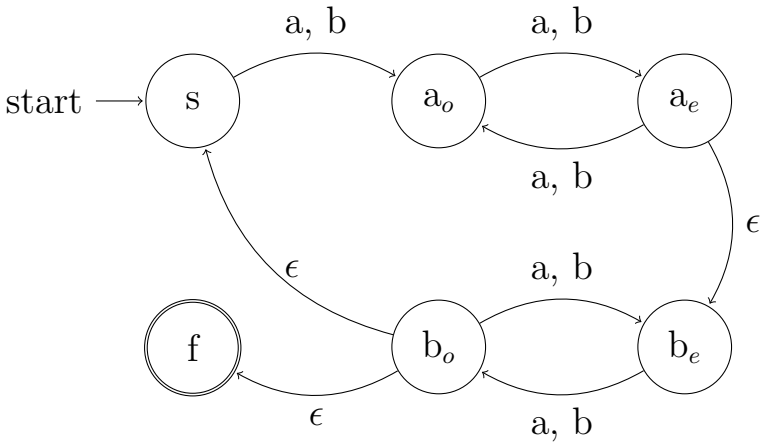
(b) Language $a^*b^*a^+$.



(c) The language comprising all strings that contain an even number of a's or contain exactly two b's.



4. L : Shuffle of languages A and B.



5.

$$A = \{x \in \{0,1\}^* \mid \text{the second symbol from the right is } 0\}.$$

	0	**1**
$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{p\}$	$\{p,q\}$	$\{p\}$
$\{q\}$	$\{r\}$	$\{r\}$
$\{r\}F$	$\emptyset$	$\emptyset$
$\{p,q\}$	$\{p,q,r\}$	$\{p,r\}$
$\{p,r\}F$	$\{p,q\}$	$\{p\}$
$\{q,r\}F$	$\{r\}$	$\{r\}$
$\{p,q,r\}F$	$\{p,q,r\}$	$\{p,r\}$

Starting from state $\{p\}$, we observe that the states $\{q,r\}$, $\{q\}$, $\{r\}$, and $\emptyset$ can never be reached. Therefore, these states can be ignored, and the transition table simplifies as follows:

	0	**1**
$\rightarrow \{p\}$	$\{p,q\}$	$\{p\}$
$\{p,q\}$	$\{p,q,r\}$	$\{p,r\}$
$\{p,r\}F$	$\{p,q\}$	$\{p\}$
$\{p,q,r\}F$	$\{p,q,r\}$	$\{p,r\}$