

# Multipliers.

$$\begin{array}{c} Z = X \times Y \\ \downarrow \quad \quad \quad \nearrow \\ \text{product} \quad \quad \text{multiplicant} \quad \quad \text{multiplier} \end{array}$$

Paper and pencil method.

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline 225 \end{array}$$

(PP)  
Partial product for 5.  
Partial product for 1.

$$\begin{array}{r}
 1111 \quad (15)_{10} \\
 \times 1111 \quad (15)_{10} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11\textcircled{1}\textcircled{1} \quad PP_0 \\
 1110 \quad PP_1 \\
 11100 \quad PP_2 \\
 111000 \quad PP_3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11100001 \quad \leftarrow \textcircled{225}_{10} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \longrightarrow \textcircled{255}
 \end{array}$$

A  
B

$\nwarrow$  unsigned  
 $A_3 \ A_2 \ A_1 \ A_0$   
 $B_3 \ B_2 \ B_1 \ B_0$

$P_7 \ P_6 \ P_5 \ P_4 \ P_3 \ P_2 \ P_1 \ P_0$

for  $n$ -bit multiplier/multiplicand  
 $2n$  bits are needed to hold the  
largest possible product.

0	0	1	1
$\times 0$	$\times 1$	$\times 0$	$\times 1$
$\hline 0$	$\hline 0$	$\hline 0$	$\hline 1$

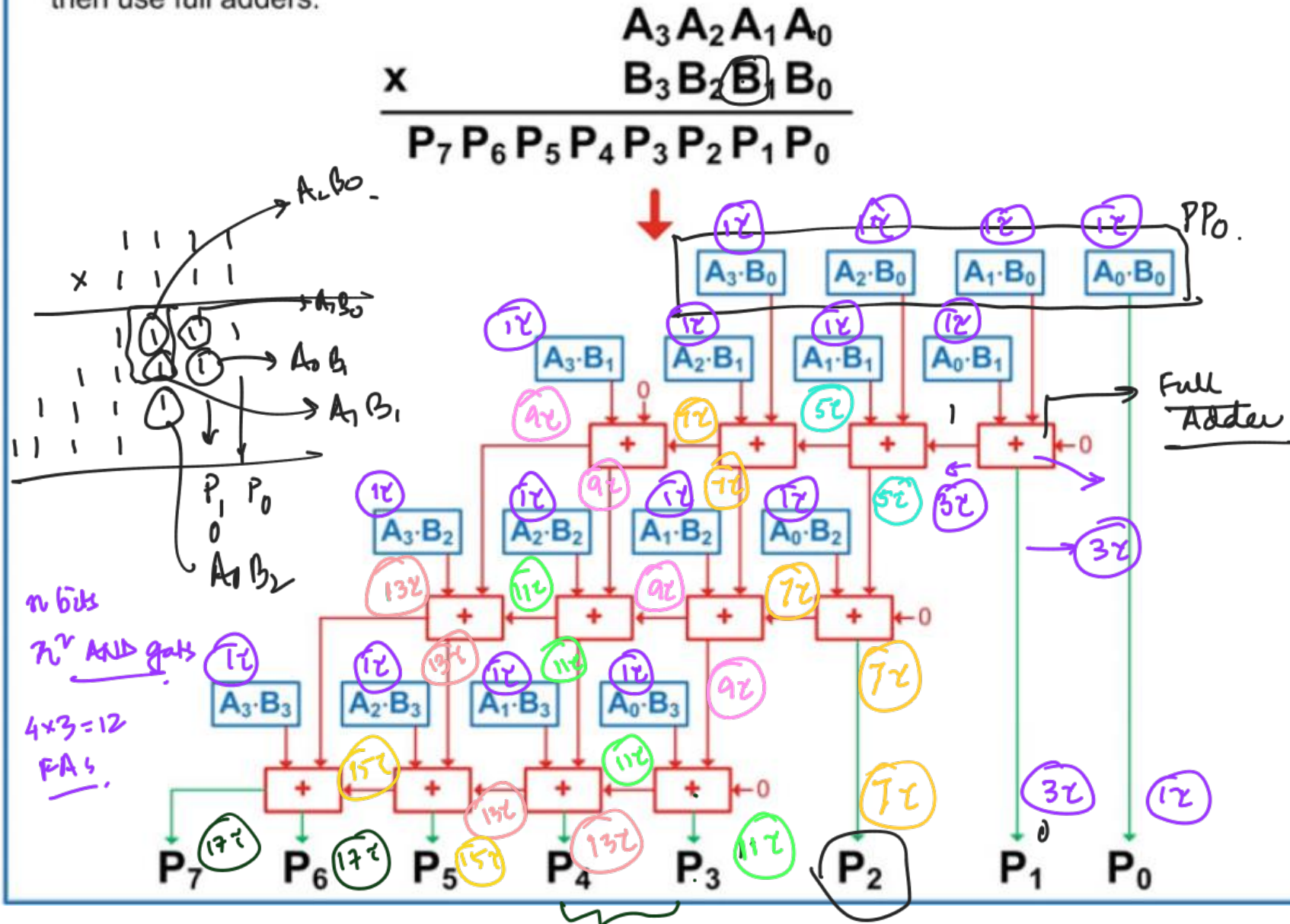
Combinational Array Multiplier.

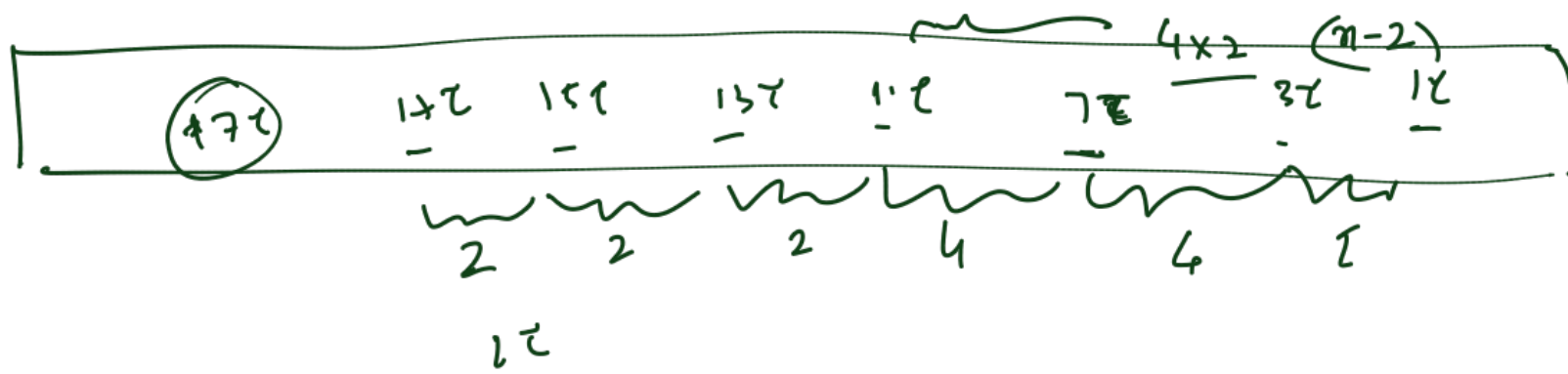
Ack for (unannotated version of)  
figure below:

Brock J. LaMeres - Introduction to Logic Circuits  
and Logic Design with Verilog - Springer (2023).

### Example: Design of a 4-Bit Unsigned Multiplier

If we break the sum of the partial product columns into incremental addition steps, we can then use full adders.





$$4(n-2)\tau \rightarrow \underline{n-2} \text{ times } \underbrace{4\tau}_{2 \text{ FAs}} \text{ delays}$$

$$\underline{2n\tau} \rightarrow$$

$$1\tau + 4(n-2)\tau + 2n\tau \rightarrow (6n-7)\tau.$$

$n$	$(6n-7)$
8	41 $\tau$
16	89 $\tau$
32	185 $\tau$
64	<span style="border: 1px solid black; padding: 2px;">377<math>\tau</math></span>



10 picoseconds



