1st Order Op Amp Circuits

Subsystems

- Multipliers
  - Inverting and non-inverting amplifiers
    - Typically fixed number, which means fixed resistor values in amplifiers
- Adders and Subtractors
  - Summing and difference amplifiers

- Differentiators
- Integrators

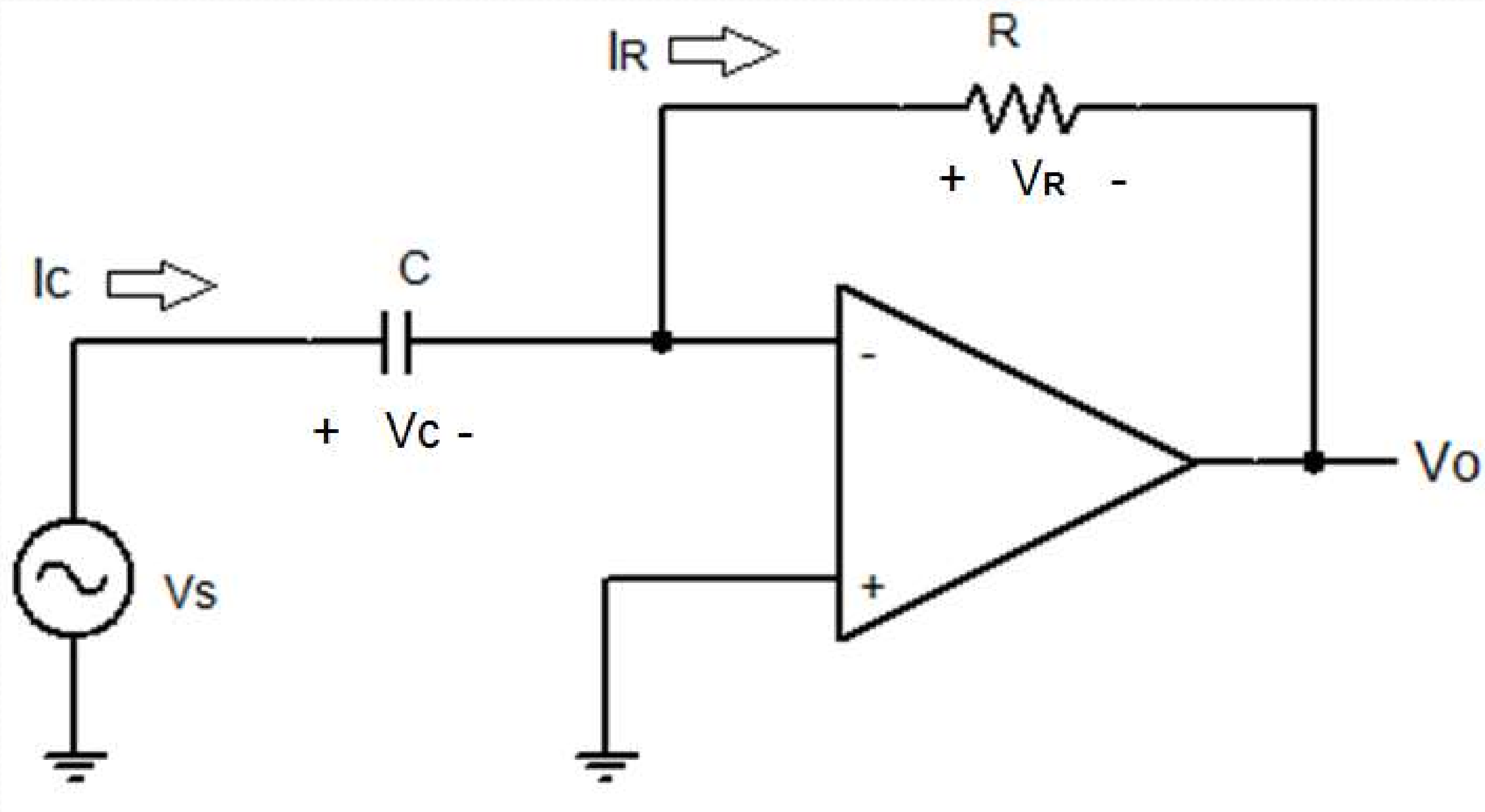
1st order op amp circuits

Capacitors

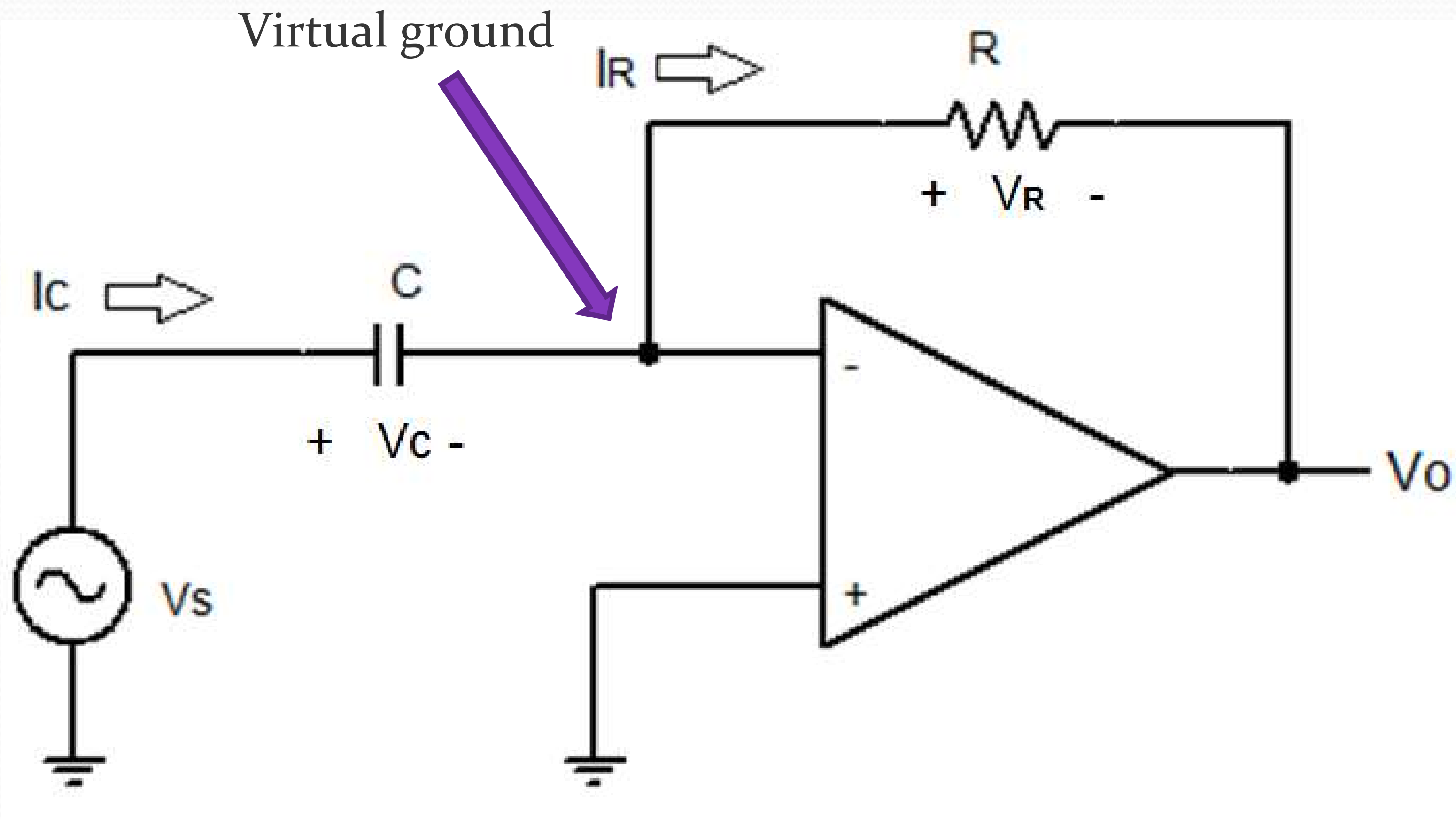
$$i_C(t) = C\frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C}\int_{t_o}^{t_1} i_C(t)\,dt + v_C(t_o)$$

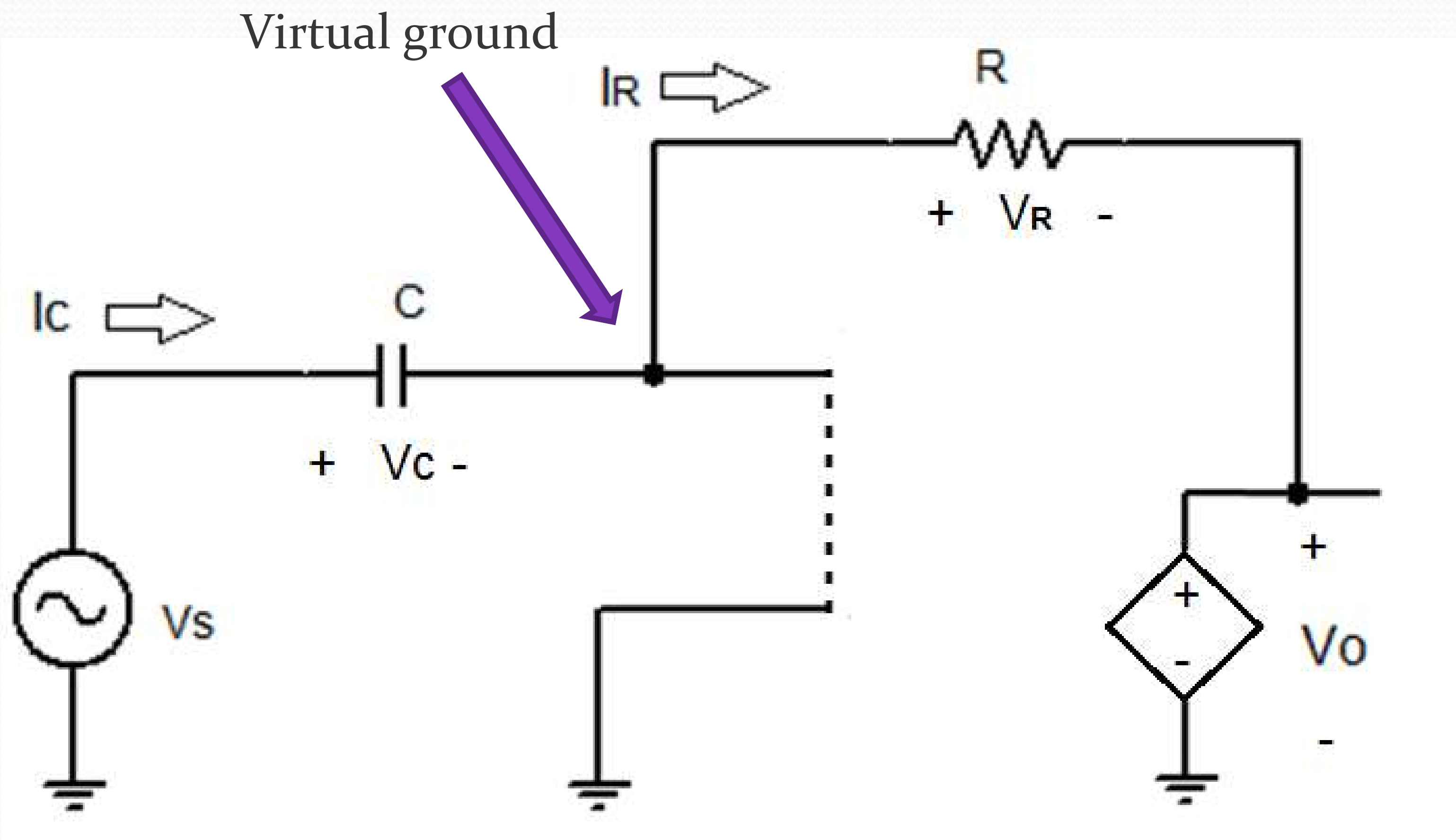
Differentiator



Ideal Op Amp Model



Op Amp Model



Analysis

- Since current is not allowed to enter the input terminals of an ideal op amp.

$$i_C(t) = i_R(t)$$

$$v_C(t) = v_S(t)$$

$$i_C(t) = C\frac{dv_C}{dt} = C\frac{dv_S}{dt}$$

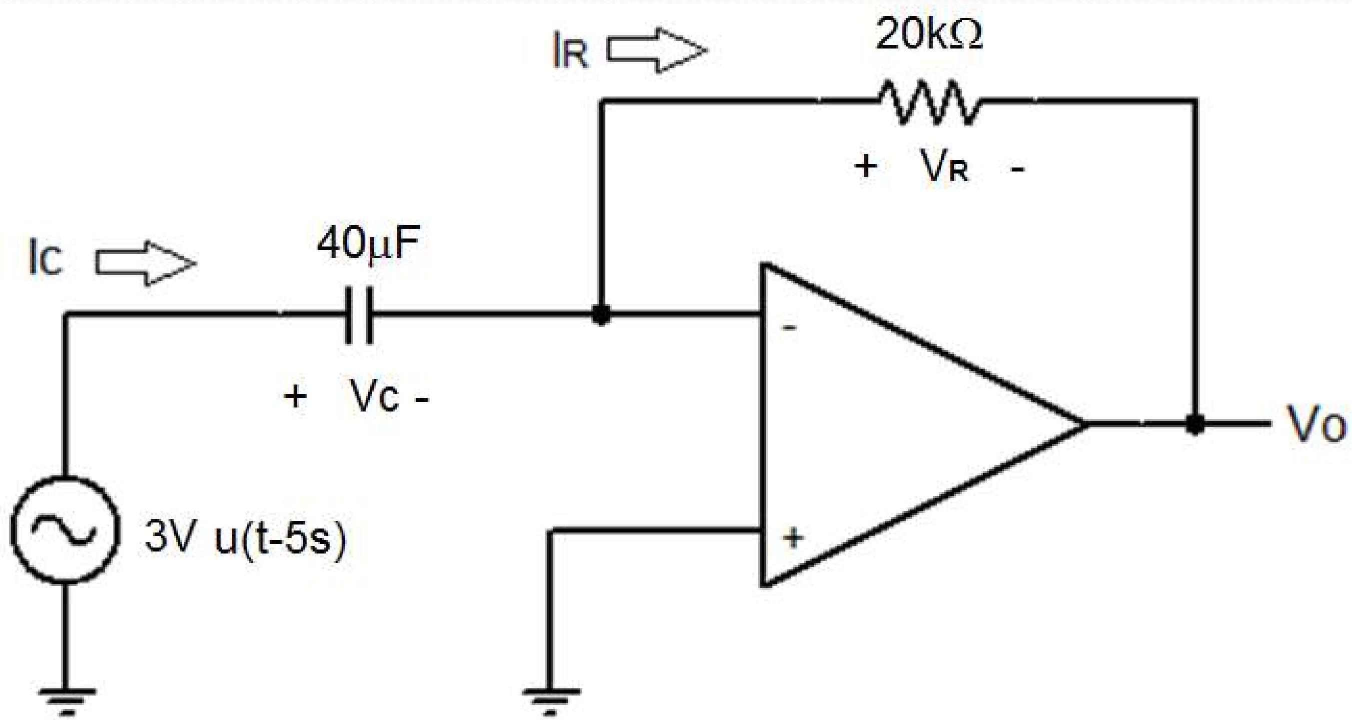
$$i_R(t) = -\frac{v_o}{R}$$

$$-\frac{v_o}{R} = C\frac{dv_S}{dt}$$

$$v_o(t) = -RC\frac{dv_S(t)}{dt}$$

Example #1

- Suppose $v_S(t) = 3V\ u(t\text{-}5s)$
  - The voltage source changes from 0V to 3V at t = 5s.
    - Initial condition of V_C = 0V when t <5s.
    - Final condition of V_C = 3V when t > 5RC.



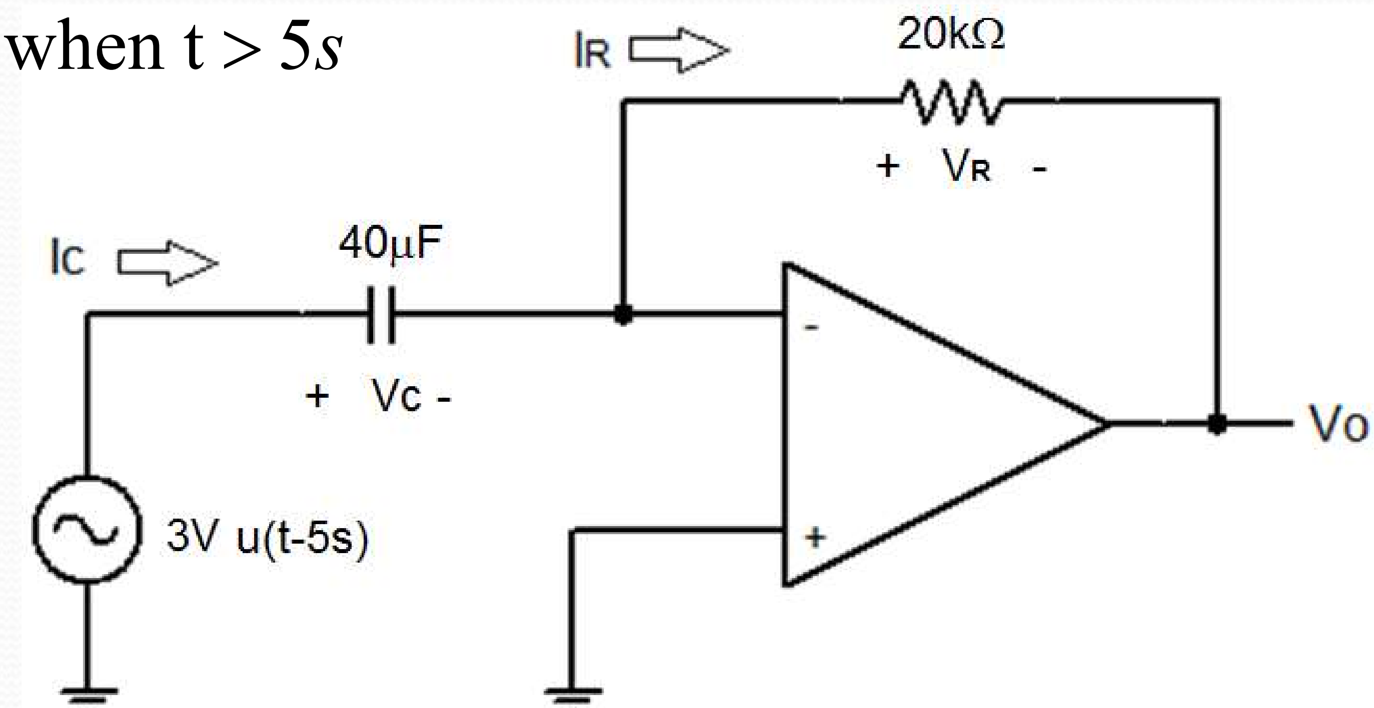
Example #1 (con't)

$$v_C(t) = 0V \quad \text{when } t < t_o$$

$$v_C(t) = V_{C_{initial}} + \left(V_{C_{final}} - V_{C_{initial}}\right)e^{-(t-t_o)/\tau} \quad \text{when } t > t_o$$

$$v_C(t) = 0V + \left(3V - 0V\right)e^{-(t-5s)/0.8s} \quad \text{when } t > t_o$$

$$v_C(t) = 3V\, e^{-(t-5s)/0.8s} \quad \text{when } t > 5s$$



Example #1 (con't)

$$v_o(t) = -RC\frac{dv_C(t)}{dt}$$

$$v_o(t) = 0V \quad \text{when t} < 5\text{s}$$

$$v_o(t) = 0V \quad \text{when t} > \text{t}_\text{o} + 5\tau, \text{ where } \tau = \text{RC}$$

$$v_o(t) = 0V \quad \text{when t} > 5\text{s} + 5(20\text{k}\Omega)(40\mu\text{F}) = 9s$$

$$v_o(t) = \frac{-1}{0.8s}(-20x10^3\,\Omega)(40x10^{-6}\,F)(3V)\,e^{-(t-5s)/0.8s}$$

$$v_o(t) = 3V\,e^{-(t-5s)/0.8s}$$

Example #2

- Let R = 2 kΩ, C = 0.1μF, and $v_S(t) = 2V \sin(500t)$ at t = 0s

Since $v_C(t) = v_S(t)$

$$v_o(t) = -RC\frac{dv_S}{dt}$$

$$v_o(t) = -(2000\,\Omega)(10^{-7}F)\frac{d\left[2V\sin(500t)\right]}{dt}$$

$$v_o(t) = \left(-0.2ms\right)\left(2V\right)\left(500\right)\cos(500t)$$

$$v_o(t) = -0.2V\cos(500t) \quad \text{when t} > 0\text{s}$$

$$v_o(t) = 0V \quad \text{when t} < 0\text{s}$$

Cosine to Sine Conversion

$$v_o(t) = -0.2V\cos(500t)$$

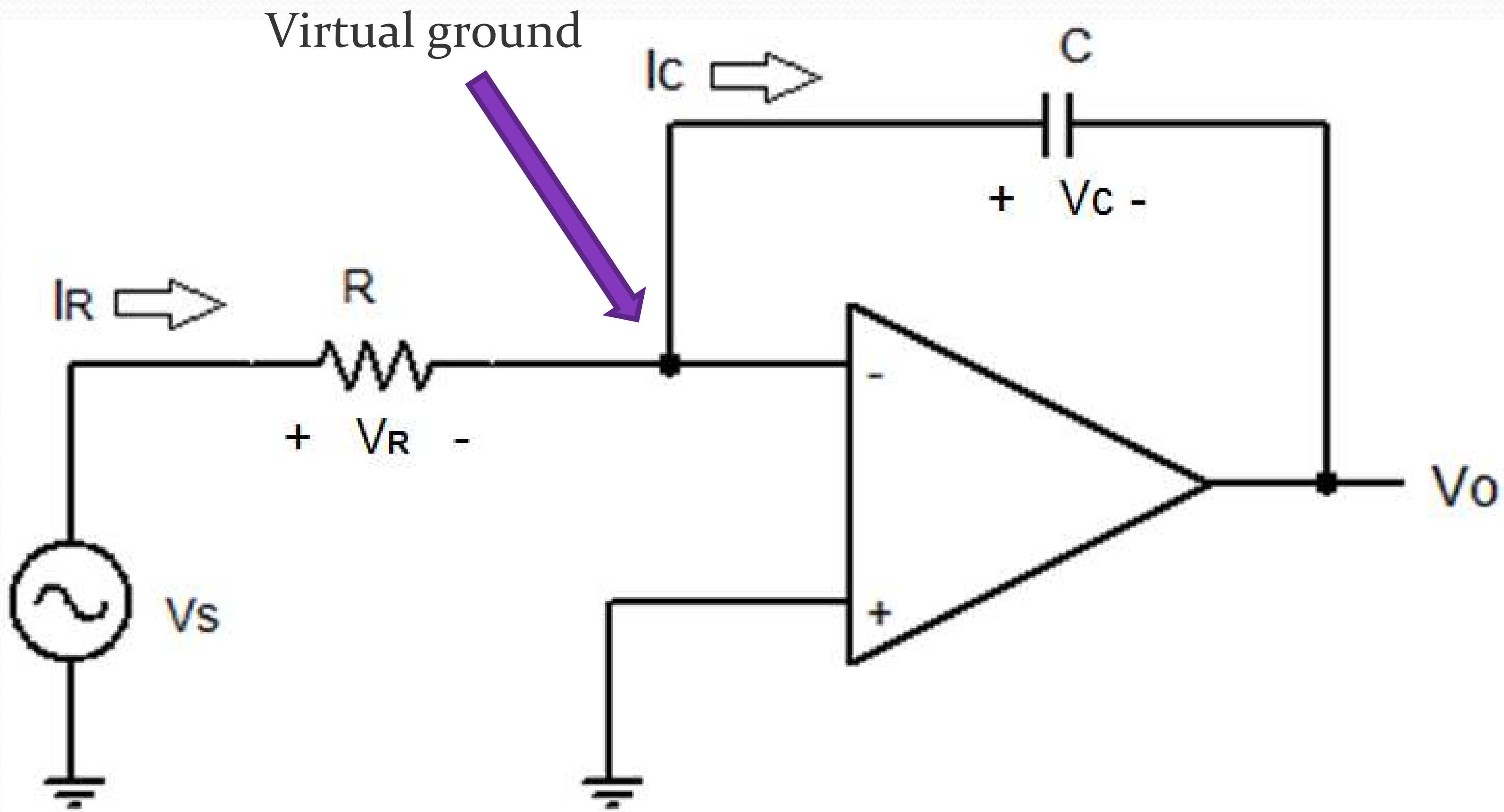
$$v_o(t) = -0.2V\sin(500t + 90^o)$$

$$v_o(t) = 0.2V\sin(500t + 90^o - 180^o)$$

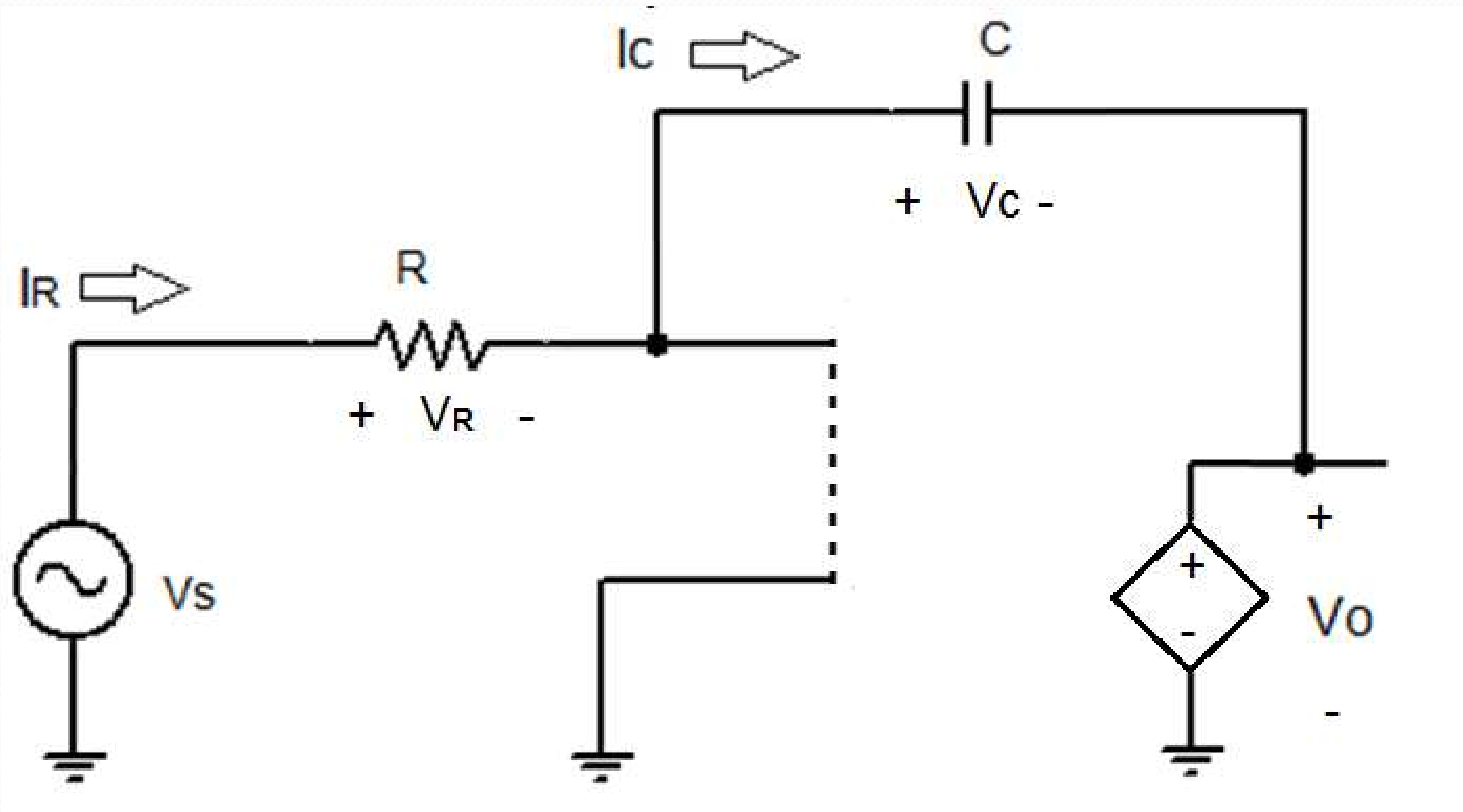
$$v_o(t) = 0.2V\sin(500t - 90^o)$$

As $v_S(t) = 2V\sin(500t)$, the output voltage lags the input voltage by 90 degrees.

Integrator



Op Amp Model



Integrator

$$i_R = \frac{v_S(t) - v_1}{R} = \frac{v_S(t)}{R}$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C(t) = v_1 - v_o(t) = -v_o(t)$$

$$i_R - i_C = 0mA$$

$$\frac{v_S(t)}{R} - C\frac{d\left[-v_o(t)\right]}{dt} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_S(t)}{RC} = 0$$

$$v_o(t_2) = \frac{-1}{RC}\int_{t_1}^{t_2} v_S(t)dt + v_o(t_1)$$

Example #3

- Let R = 25 kΩ, C = 5nF, $v_S(t) = 3V \sin\left(6.24k\frac{rad}{s}t\right)$ at t=0s

$$V_o(t_2) = \frac{-1}{RC}\int_{t_1}^{t_2} V_{in}(t)dt + V_o(t_1)$$

$$V_o(t_2) = \frac{-1}{25\,k\Omega(5\,nF)}\int_{t_1}^{t_2} 3V\,\sin\left(6.24\,k\,\frac{rad}{s}t\right)dt$$

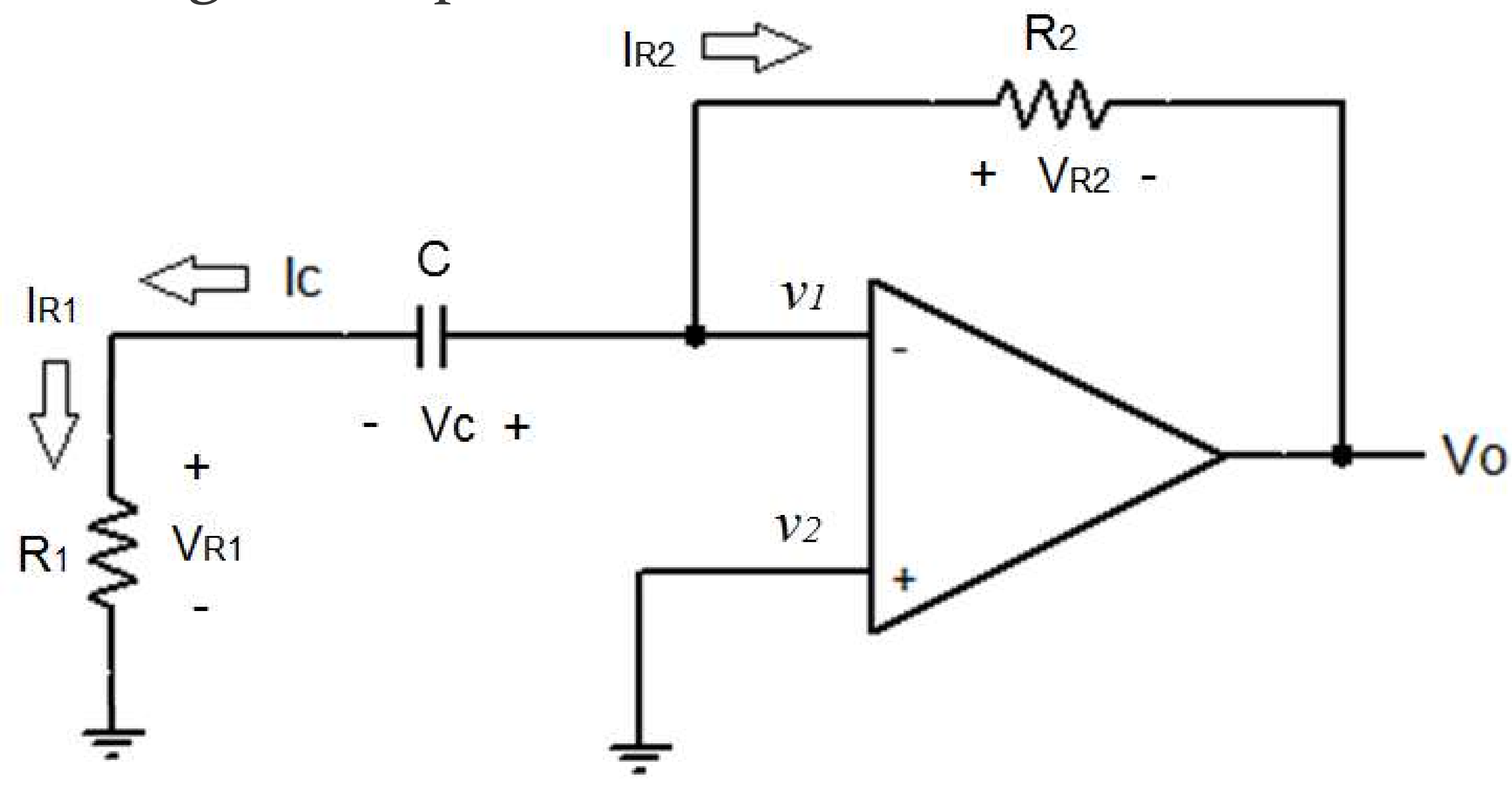
$$V_o(t_2) = 3.85V\,\cos\left(6.24\,k\,\frac{rad}{s}t\right)\Bigg|_{t_1}^{t_2} + V_o(t_1)$$

$$V_o(t_2) = 3.85V\,\sin\left(6.24\,k\,\frac{rad}{s}t_2 + 90^o\right) - 3.85V \text{ when } t_1 = 0s$$

since $v_o(t) = -v_C(t)$ and the voltage across a capacitor can't be discontinuous.

Example #4

Initial Charge on Capacitor



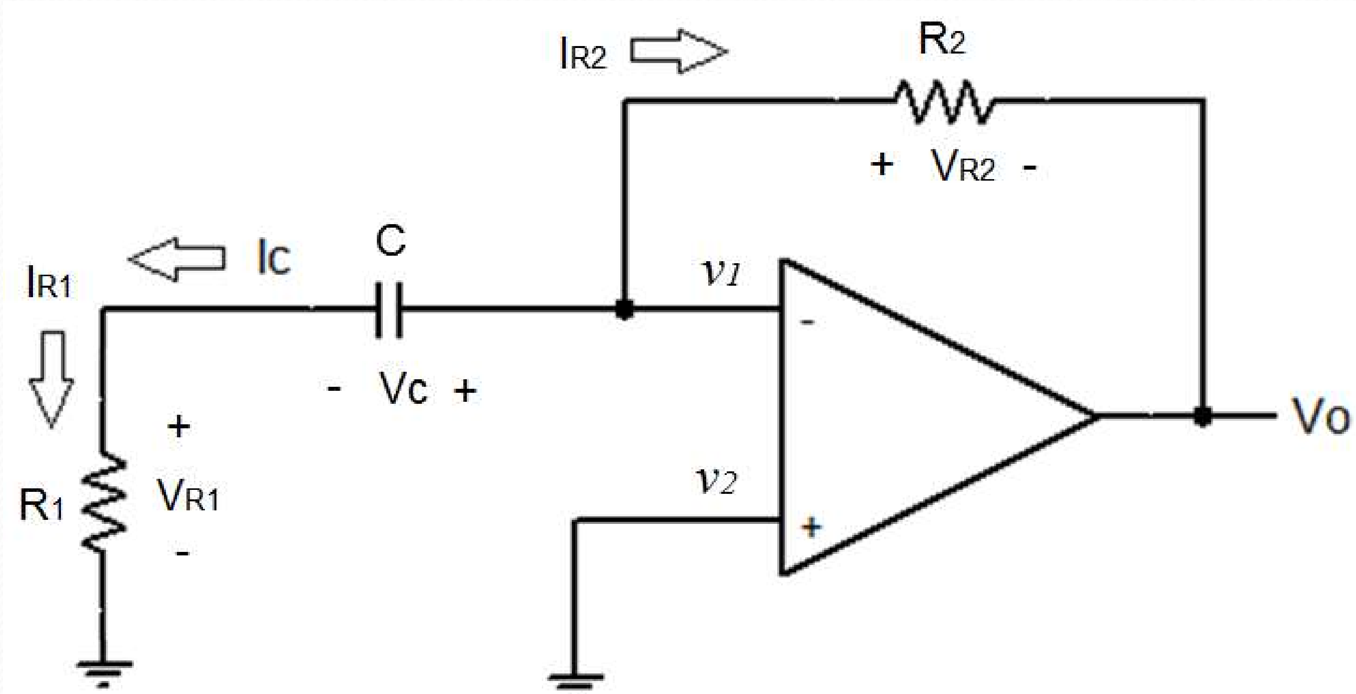
Example #4 (con't)

If there is an initial charge that produces a voltage on the capacitor at some time, t_o:

The voltage on the negative input of the op amp is:

$v_1 = V_C + V_{R1}$

$v_1 = v_2 = 0V$



Example #4 (con't)

The current flowing through R_1 is the same current flowing through C.

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$i_{R1}(t) = \frac{V_{R1}}{R_1} = \frac{[v_1 - v_C(t)]}{R_1} = \frac{[0V - v_C(t)]}{R_1} = -\frac{v_C(t)}{R_1}$$

$$\text{at t} = \text{t}_\text{o}, i_R(t_o) = -\frac{v_C(t_o)}{R_1}$$

$$\text{as t} \rightarrow \infty, v_C(t) \rightarrow 0V, i_C(t) \rightarrow 0mA$$

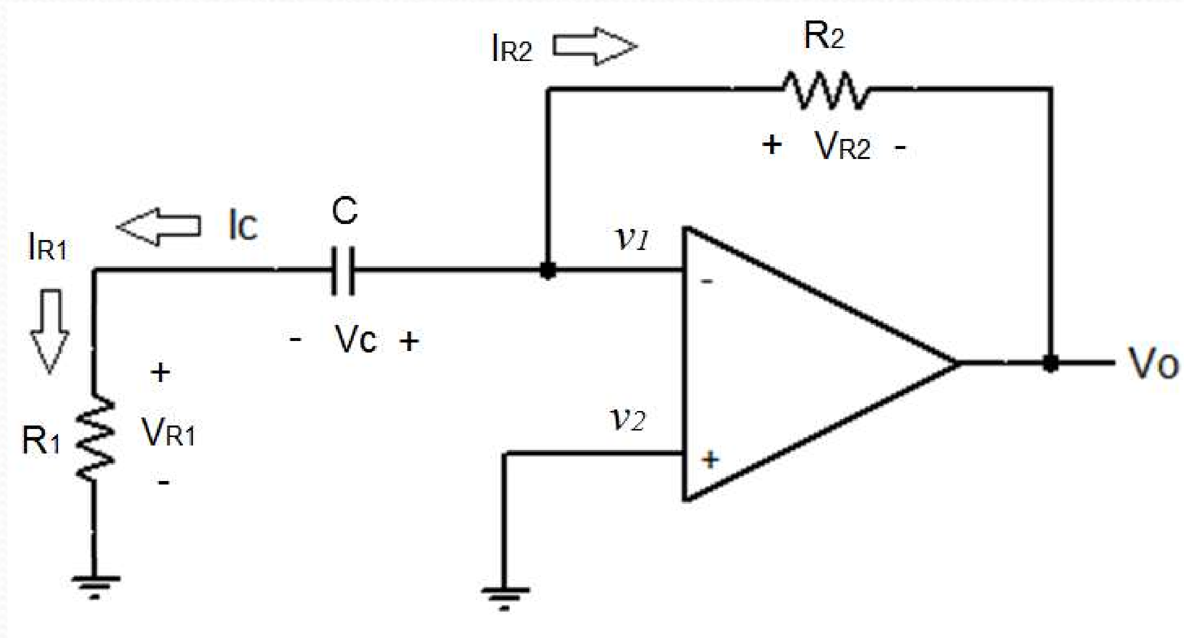
$$i_C(t) - i_{R1}(t) = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1} = 0$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1 C} = 0$$

$$v_C(t) = v_C(t_o)e^{-\frac{t-t_o}{R_1 C}}$$

$$i_C = C\frac{dv_C(t)}{dt} = -\frac{1}{R_1}v_C(t_o)e^{-\frac{t-t_o}{R_1 C}}$$

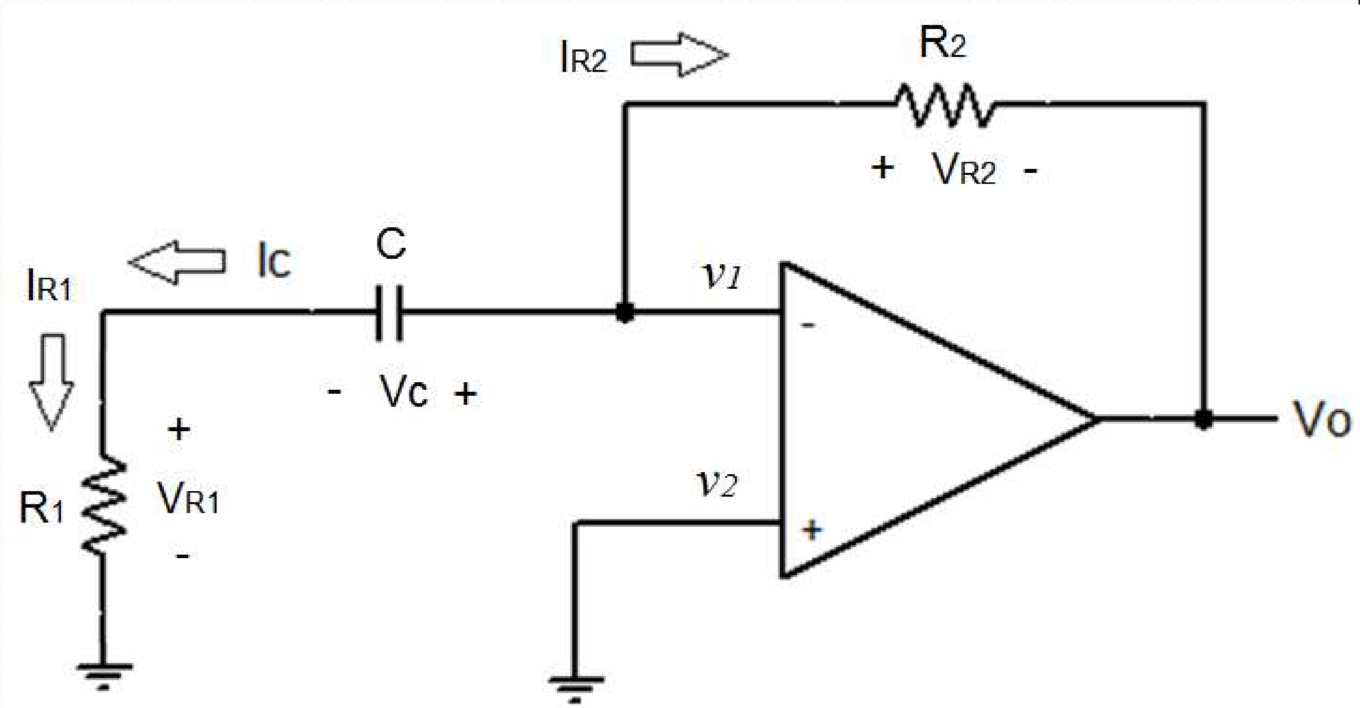


R_1C is the time constant, τ.

$$i_{R2} = -i_C$$

$$i_{R2} = \frac{0V - v_o(t)}{R_2} = -\frac{v_o(t)}{R_2}$$

$$v_o(t) = \frac{R_2}{R_1} v_C(t_o) e^{-\frac{t-t_o}{R_1 C}}$$



Summary

- Differentiator and integrator circuits are 1st order op amp circuits.
  - When the C is connected to the input of the op amp, the circuit is a differentiator.
    - If the input voltage is a sinusoid, the output voltage lags the input voltage by 90 degrees.
      - The output voltage may be discontinuous.
  - When the C is connected between the input and output of the op amp, the circuit is an integrator.
    - If the input voltage is a sinusoid, the output voltage leads the input voltage by 90 degrees.
      - The output voltage must be continuous.