Basic Graph Theory

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Problem 1

Given a graph G with

1. $|V(G)| = 24, \quad |E(G)| = 30$
2. Number of vertices of degree 5= 4.
3. Number of vertices of degree 1= 7.
4. Number of vertices of degree 2= 7.
5. All other vertices have degree 3 or 4.

How many vertices of degree 4 are there?

Answer : 1

Hint : Use Hand-shaking Lemma (degree sum Formula)

Problem 2

Can there exist a simple graph with the following?

1. $|V(G)| = 13, \quad |E(G)| = 31$
2. Number of vertices of degree 1= 3.
3. Number of vertices of degree 4= 7.

Answer : No

Hint : Use Hand-shaking Lemma and discuss all possible cases for the remaining three vertices.

Problem 3

Prove that if G is a simple graph woth n vertices and n edges with no vertices of degree 0 or 1, then the degree of every vertex is 2.

Answer : By contradiction. Assume that there exists a vertex of degree of greater than 2 and applying degree sum formula to reach a contradiction.

Problem 4

Prove that if G is a simple graph with n vertices and $n-1$ edges, G has atleast one vertex of degree less than 2.

Answer : Similar to previous problem. By contradiction. Assume that all vertices of the graph have degree ≥ 2 and apply degree sum formula to reach a contradiction.

Problem 4

Prove that in any simple graph (with atleast two vertices) there exists two vertices with same degree.

Answer : Use PHP.

1. We cannot have an isolated vertex and a vertex with degree $n-1$ if $|V| = n$.
2. Then, either $0 \leq deg(v) \leq n-2$ for all $v \in V(G)$.
3. or $1 \leq deg(v) \leq n-1$ for all $v \in V(G)$.
4. Then apply PHP to conclude.

Problem 5

Find all Non-isomorphic self complementary graphs on 5 vertices.

Hint :

1. Degree sequence and G and G^c are same if G is self complementary.
2. Find all degree sequences that satisfy the above condition. (There are three).
3. Sketch the graphs for these three sequences and decide which among them are self-complementary.