

10's complement.

9's complement.

9's complement.

$$\begin{array}{r} 470 \\ - 231 \\ \hline \end{array} \rightarrow 470 \rightarrow 768 \rightarrow \boxed{1}238$$

extra step

$$\begin{array}{r} 238 \\ + 1 \\ \hline 239 \end{array}$$

10's complement.

$$\begin{array}{r} 470 \\ - 231 \\ \hline \end{array} \rightarrow 470 \rightarrow 769 \rightarrow \boxed{1}239$$

9 - each digit
↓
add 001 at the end.

→ Signed magnitude
→ 1's complement
→ 2's complement

Representation step.

$$\begin{array}{c} -6 \\ \downarrow \\ 0110 \\ \downarrow \\ 1001 \\ \downarrow \\ 1010 \end{array}$$

n=4

Addition

$$\begin{array}{r} 0010 \\ 0011 \\ \hline 0101 \end{array} \quad \begin{array}{r} +2 \\ +3 \\ \hline \end{array}$$

5.2
→ 5 + (-2)

7.3

-8 4 2 1

$$\begin{array}{r} 0000 \\ 1010 \\ \hline 1110 \\ 0001 \\ + 1 \\ \hline 0010 \end{array}$$

$$\begin{array}{r} +4 \\ -6 \\ \hline -2 \end{array} \quad \begin{array}{r} 1001 \\ \downarrow \\ 1010 \end{array}$$

+6 + 3

-7 + 1

Overflow

range

[-8 to +7]

Adding

$$\begin{array}{r} +5 \\ +6 \\ \hline +11 \end{array}$$

$$\begin{array}{r} 0101 \\ 0110 \\ \hline 1011 \end{array}$$

→ -5

$$\begin{array}{c} 1011 \\ \downarrow \\ 0100 \\ \downarrow \\ 0101 \end{array} \quad (5)$$

$$\begin{array}{r}
 -5 \\
 -6 \\
 \hline
 -11
 \end{array}
 \quad
 \begin{array}{r}
 \textcircled{1} \textcircled{0} \textcircled{1} \textcircled{1} \\
 \downarrow \\
 1011 \\
 \downarrow \\
 1010 \\
 \downarrow \\
 (1) 0101 \rightarrow +5
 \end{array}$$

$$\begin{array}{r}
 \text{Sign position } (-8) \\
 \uparrow \\
 1 \textcircled{0} 101
 \end{array}$$

if signs are different \rightarrow no overflow.

An overflow occurs if and only if the carry out of the sign position is NOT equal to the carry into the sign position.

Adding two positives gives a negative.
 Adding two negatives gives a positive.

Exercise

10's complement

Sign Extension method.

$$n=4 \rightarrow n=8$$

4 bit

0101

+5

8 bit

0000 0101

1011

-5

1111 1011

$x = 5$

$x \cdot 2$

$x \cdot 4$

$x \cdot 8$

0 0 0 0 0 1 0 1
0 0 0 0 1 0 1 0
0 0 0 1 0 1 0 0
0 0 1 0 1 0 0 0

+5

+10.

+20.

+40.



$x \cdot 2^i \rightarrow$ shift left by i bits

$x = -5.$

$x \cdot 2$

$x \cdot 4$

1 1 1 1 1 0 1 1

1 1 1 1 0 1 1 0

- 5.

- 10

- 20

- 40.

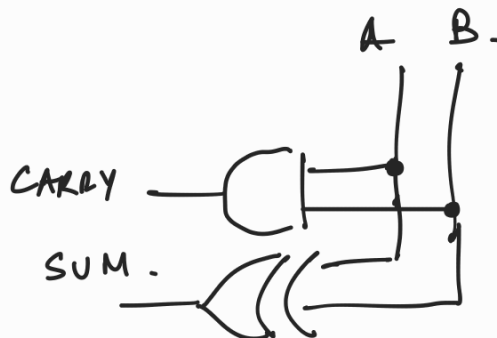
decimal

Half-adder

A	B	CARRY	SUM
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$SUM = A \oplus B.$$

$$CARRY = A \cdot B.$$



not very useful.

Full adder

Full adder

Full Adder

x_i	y_i	C_i	C_{i+1}	S_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$C_{i+1} = \sum m(3, 5, 6, 7)$$

$$S_i = \sum m(1, 2, 4, 7)$$

C_{i+1}

$x_i \backslash y_i$	00	01	11	10
0			3	
1		5	7	6

$$C_{i+1} = x_i y_i + x_i C_i + y_i C_i$$

S_i

$x_i \backslash y_i$	00	01	11	10
0		1		1
1	1		1	

$$S_i = x_i \oplus y_i \oplus C_i$$