Problems for Practice

September 25, 2024

Problem 1

Let G be a connected graph with atleast 3 vertices. Form G' from G by adding an edge with end vertices u and v whenever $d_G(x, y) = 2$. Prove that G' is 2-connected.

Problem 2

Prove that a connected simple graph with blocks $B_1, B_2, \cdots, B_k$ has $\sum_{i=1}^{k} n(B_i) - k + 1$ vertices.

Problem 3

Prove that every 3-regular simple graph with connectivity 1 has atleast 10 vertices.

Problem 4

Prove that for all graph G, number of cut vertices is smaller than the number of blocks of G.

Problem 5

What is the vertex connectivity of $K_{m,n}$, the complete bipartite graph with m and n vertices on the two parts. Explain your answer.

Problem 6

Let G be a simple graph of diameter two. Show that the edge connectivity of G is equal to its minimum degree.

Problem 7

Show that if G is simple and the minimum degree $\delta(G) \geq n-2$, (n being the number of vertices in G) then the vertex connectivity $\kappa(G) = \delta(G)$.

Problem 8

Show that if G is simple, with $n \geq k+1$, and $\delta(G) \geq \frac{(n+k-2)}{2}$, then G is k-connected.

Problem 9

Prove or Disprove : There exists a simple connected graph G such that both G and G^c are Eulerian.

Problem 10

Prove or Disprove : Every Eulerian graph in which the degree of every vertex is atleast 3 is Hamiltonian.

Problem 11

Prove or Disprove : Every Eulerian bipartite graph have an even number of edges.

Problem 12

Let G be a simple graph such that $|V(G)| \geq 3$ and $|E(G)| = \binom{n-1}{2} + 2$. Show that G is Hamiltonian.

Problem 13

G is a simple connected graph with 13 vertices and 76 edges. Show that G is Hamiltonian.

Is G Eulerian? Why? Prove that

True or False??

1. If a graph contains an Euler tour, then it does not have a cut-edge.
2. In any graph G, the existence of a perfect matching is equivalent to the condition that $|N(S)| \geq |S|$ for every $S \subseteq V(G)$.
3. Every 3-regular graph has a perfect matching.
4. If all cycles in a graph G are of even length, then G contains a matching whose size is equal to the minimum cardinality of a vertex cover of its edges.
5. If a graph is 100-edge-connected, then it must be 3-connected.

Problem 14

Prove or Disprove : Let M be a matching in G, and let C be a cycle of length $2k$ that contains exactly k edges of M. Let G' be the graph formed by contracting C to a single vertex. Then M is maximum in G if and only if $M - E(C)$ is maximum in G'.

Problem 15

Prove or Disprove : Every tree has atmost one perfect matching.

Problem 16

Let G be a simple graph with $2n$ vertices and $\delta(G) \geq n+1$. Prove that G has a perfect matching.

Problem 17

Give an example of a k-regular simple graph with no perfect matching for all $k \geq 2$.

Problem 18

Let G be a connected graph with atleast 4 vertices. Suppose every edge of G is contained in some perfect matching, Prove that G is 2-connected.

Problem 19

Find the number of perfect matchings in K_{2n}.

Problem 20

Prove that a tree has a perfect matching iff $o(T - v) = 1$ for all $v \in V(T)$.

Problem 21

Let G be a bipartite graph with n vertices in each partition. Prove that G has a matching of size atleast $min\{n,\ 2\delta(G)\}$.