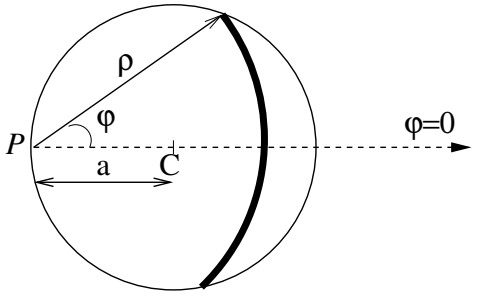
DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II **Problem Set 2** **Feb-2024**

Potential Problems

1. A circular sheet of radius a has uniform surface charge density σ. Calculate the potential at a point P on the circumference of the sheet, and also at C, the centre of the circle.
   Hint: Taking P as the origin and PC as the line corresponding to $\varphi = 0$, the equation of the circle in polar coordinates is $\varrho = 2a\cos\varphi$, where $-\pi/2 \leq \varphi \leq \pi/2$. Take elementary strips of area $2\varrho\varphi d\varrho$ as shown in the figure in order to evaluate the potential at P. Choose the limits of integration carefully.



2. Two parallel line charges A and B have the same linear charge density λ and the same length **L**. A extends from $(0, 0)$ to $(0, L)$, and B extends from $(a, 0)$ to (a, L) in the xy plane. Find the total force on B due to A.

3. The electric field in a certain region in space is given by

$$\mathbf{E}(x, y, z) = \frac{k}{\epsilon_0 a^2}\left(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z\right) \quad ,$$

   where k and a are constants with appropriate physical dimensions. Is the field conservative? If so, calculate the electrostatic potential as well as the charge density at an arbitrary point $P(x, y, z)$.

4. Prove the **Mean-Value Theorem:** If S_R is the surface of a (mathematical) sphere of radius R whose interior contains no charge, then the potential at the center is equal to the average potential over the surface S_R. Show this by establishing that the average is independent of the radius of the sphere.

5. A spherically symmetric charge distribution (density depends only on the radial coordinate) is placed in a given electric field produced by charges external to the spherical distribution. Prove that the net force experienced by the spherical distribution is as though all of the charge were concentrated at the center of the distribution. (*Hint:* Use the mean-value theorem.)

6. A triangular sheet as shown in the figure has a uniform charge density σ. Calculate the potential at point P.