DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II **Problem Set 8** **April 2024**

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1. **The skin effect**: Recall the relations $\mathbf{D} = \epsilon\,\mathbf{E}$, $\mathbf{B} = \mu\,\mathbf{H}$, $\mathbf{J}_f = \sigma\,\mathbf{E}$ (Ohm's Law). For a metal under normal circumstances, $\mathbf{J}_f$ is much larger than $\partial\mathbf{D}/\partial t$.

   (a) Neglect ρ_f and show that for a metal, $\mathbf{E}$ satisfies the equation

   $$\nabla^2\mathbf{E} = \mu\,\sigma\,\frac{\partial\mathbf{E}}{\partial t}\,.$$

   (b) Consider a "plane wave" solution of the above equation of the form $\mathbf{E} = \mathbf{E}_0 \exp i(kz - \omega t)$, for $z > 0$. Find the allowed values of the wave number k as a function of the frequency ω.

   (c) Interpret the form of the solution. How does the amplitude of the electric field vary with k, and at what distance does it decay to $1/e$ of its value at $z = 0$?

2. An ideal parallel plate capacitor of capacitance C has circular plates located at $z = 0$ and $z = d$ respectively. The medium between the plates is a linear, homogeneous, isotropic dielectric of dielectric constant κ. The capacitor is connected to a resistance R in series, and a voltage V is applied to the circuit. The charge q on the capacitor plates increases with time according to $q = C\,V\,(1 - e^{-t/RC})$. Find the magnitude of the magnetic field H inside the dielectric.

3. An infinitely long straight non-magnetic conductor (wire) with a circular cross-section of radius a carries a steady current I. The current is distributed uniformly over the cross-section of the wire. The conductivity of the wire is σ. Find the rate at which energy flows into unit length of the conductor.

4. A beam of protons has a circular cross-section. Each proton has a velocity $\mathbf{v}$, and the beam constitutes a current I. Find the direction and magnitude of the Poynting vector $\mathbf{S}$ outside the beam, at a distance r from the axis of the beam.