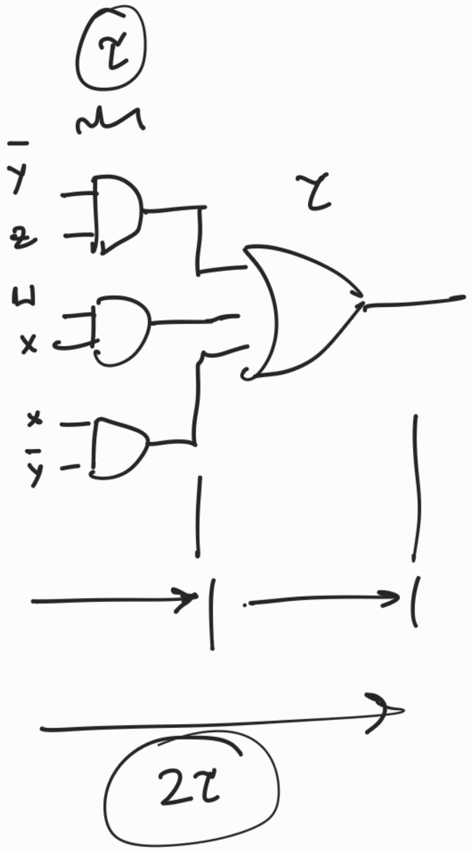
Propagation delay

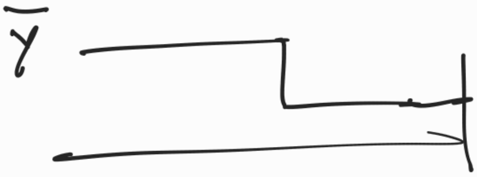
$$F = \bar{y}z + wx + x\bar{y}$$

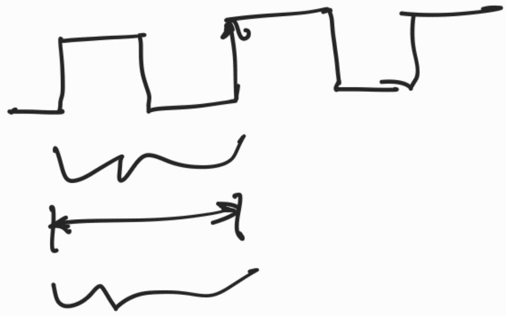


2 level logic circuit

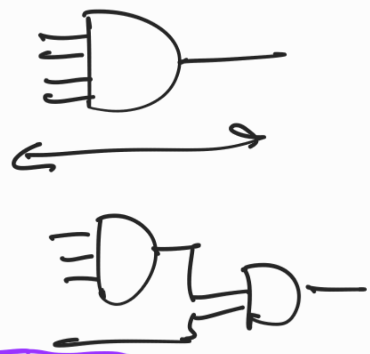
picoseconds.

nanoseconds → 10^{-9} seconds.
picoseconds → 10^{-12} seconds

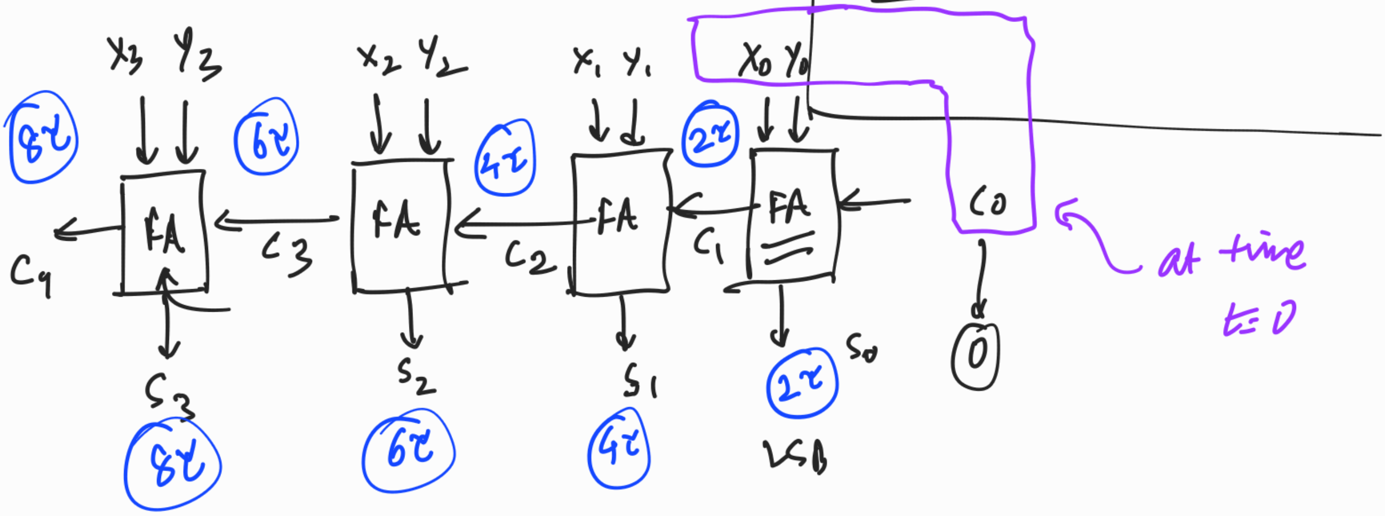




→ level
→ fan-in.

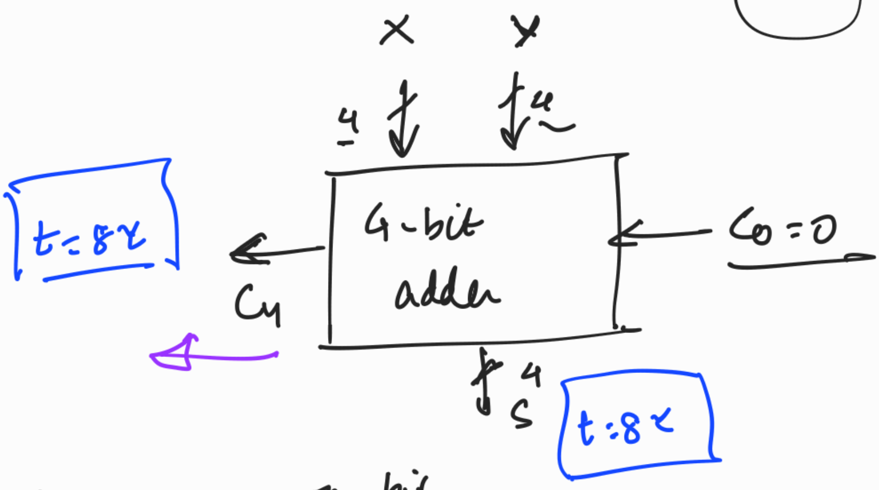


Ripple Carry Adder.



FA

$$\begin{cases} C_{i+1} = x_i y_i + x_i C_i + y_i C_i & (2\tau) \\ S_i = \bar{x}_i \bar{y}_i C_i + \bar{x}_i y_i \bar{C}_i + x_i \bar{y}_i \bar{C}_i + x_i y_i C_i & (2\tau) \end{cases}$$



For a RCA over n bits.

$$\boxed{\text{Delay is } 2n\tau} \longrightarrow$$

Carry look ahead adder.

Bottleneck: Serial dependence.

$$C_{i+1} = \sum m(3,5,6,7)$$

$$= \bar{x}_i y_i C_i + x_i \bar{y}_i C_i + x_i y_i \bar{C}_i + x_i y_i C_i$$

$$= (\bar{x}_i y_i + x_i \bar{y}_i) C_i + x_i y_i$$

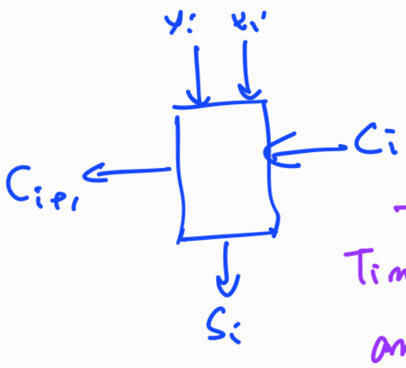
$$C_{i+1} = \underbrace{(x_i \oplus y_i)}_{P_i} C_i + \underbrace{x_i y_i}_{G_i}$$

$$S_i = \underbrace{(x_i \oplus y_i)}_{P_i} \oplus C_i$$

$P_i \rightarrow$ propagate term

$G_i \rightarrow$ generate term

$$\boxed{\begin{aligned} C_{i+1} &= P_i C_i + G_i \\ S_i &= P_i \oplus C_i \end{aligned}}$$



Timing analysis?

C_{i+1} is 1 if [(any one of x_i & y_i is 1) and C_i is 1]
or (both x_i & y_i are 1)
in which case we don't care about C_i.

$$C_1 = P_0 C_0 + G_0$$
$$C_2 = P_1 C_1 + G_1$$
$$C_3 = P_2 C_2 + G_2$$
$$C_4 = P_3 C_3 + G_3.$$

$$C_1 = P_0 C_0 + G_0$$
$$C_2 = P_1 (P_0 C_0 + G_0) + G_1$$
$$= P_1 P_0 C_0 + P_1 G_0 + G_1$$
$$C_3 = P_2 [P_1 P_0 C_0 + P_1 G_0 + G_1] + G_2$$
$$= P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2$$
$$C_4 = \boxed{P_3 P_2 P_1 P_0 C_0 + P_3 P_2 P_1 G_0 + P_3 P_2 G_1 + P_3 G_2 + G_3}$$

Key idea

C_1, C_2, C_3 do NOT appear in the above expressions. → eliminates serial dependence.

Homework: Timing analysis for 4-bit CLA-based adder.