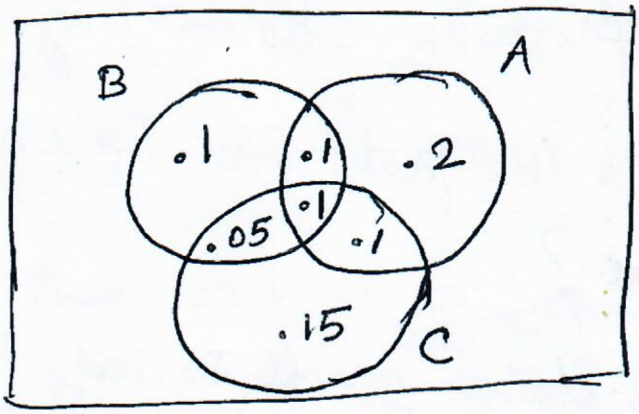
Problem Sheet 2.

1. Let A, B and C be three events with probabilities given below:



Find $P(A|B)$, $P(C|B)$, $P(B/A \cup C)$ and $P(B | A \cap C)$.

Ans. $4/7$, $3/7$, $5/14$ and $1/2$.

2. A family has five children. We pick one at random and find that the child is a girl. What is the prob. that all children are girls?

~~Let~~ Ans. $1/16$.

3. I have 3 bags that each contain 100 marbles. Bag 1 contains 75 red and 25 blue marbles.

Bag 2 has 60 red and 40 blue marbles

Bag 3 has 45 red and 55 blue marbles.

I choose one bag at random and pick a marble from the chosen bag, also at random

i) What is the prob. that the chosen marble is red?

ii) If the chosen marble is red, what is the prob. that bag 1 was chosen?

Ans. 0.6 and 5/12.

4. If A and B are two independent events, show that A and B^c are indep.

5. A box contains 2 coins, only one of them being fake with $P(\{H\}) = 1$. A coin is picked at random ~~at~~ and tossed twice.

Let $A = \{$ the 1st toss results in an H $\}$

$B = \{$ the 2nd toss " $\}$

$C = \{$ good coin is picked $\}$.

i) Show that A and B are ~~in~~ dependent.

ii) Show that A and B are conditionally independent given C.

6. Let a fair die be rolled once. Let $A = \{1,2\}$, $B = \{2,4,6\}$ & $C = \{1,4\}$

i) Show that A and B are independent.

ii) Show that A and B are NOT conditionally independent given C.

7. Let $C_1, C_2, \dots, C_n$ be a partition on a sample space Ω and let A and B be two events such that

(a) A & B are cond. indep. given C_i, $i=1,2,\dots n$

(b) B is indep. of all C_i's.

Show that A and B are indep.

Hint: Use total prob. for $A \cap B$.

8. A box contains 3 coins: 2 fair coins and one fake with $P(H) = 1$.

A coin is picked at random and tossed

i) What is prob. of getting head?

ii) If head is obtained, what is the prob. that it is a fake coin.

Ans. $2/3$ and $1/2$.

9. Let $x_1, x_2 \overset{\text{uniform}}{\in} \{1,2,3,4,5\}$. and $X = x_1 + x_2$. Find the pmf of the random variable X.

10. A coin with $P(H) = p$ ~~(~~ $0 < p < 1$) is tossed repeatedly until a head is obtained for the 1st time. Let Y be the no. of coin ~~tos~~ tossed needed. Find the pmf of Y.

Ans. $p_Y(y) = \begin{cases} (1-p)^{y-1} p & \text{for } y = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$