Department of Mathematics,IIT Madras

MA 2040 – Probability, Statistics and Random Processes

Problem Sheet 1

1. A die is made in such a way that each even face is twice as likely as each odd face. All even faces are equally likely as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.
2. Let $S_1, S_2, \dots, S_n$ be a partition of a sample space Ω. Show that for any event A, $P(A) = \sum_{i=1}^{n} P(A \cap S_i)$. Hence, show that for any three events A, B and C, $P(A) = P(A \cap B) + P(A \cap C) + P(A \cap B' \cap C') - P(A \cap B \cap C)$.
3. Prove that for any two events A and B, $P(A \cap B) \geq P(A) + P(B) - 1$.
4. Two fair dice are rolled. Given that the roll results in a sum of 4 or less, find the conditional probability that both the dice show the same number. Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.
5. A batch of 100 items is inspected by testing 4 randomly selected items. If one of the four is defective, the batch is rejected. What is the probability that the batch is accepted, if it contains exactly five defectives.
6. Two players X and Y alternately roll a pair of fair dice. X wins if on a throw he gets a sum of 6 before Y gets a sum of 7; Y wins if he obtains a sum of 7 before X obtains a sum of 6. If X begins the game, prove that his probability of winning is 30/61.
7. In a deck of cards, let *A* be the event of drawing a spade and *B* be the event of drawing a king. Assuming that all cards are equally likely to be drawn. Obtain $P(A|B), P(B|A)$. Are *A* and *B* independent?
8. A fair die is rolled repeatedly till an odd prime appears. What is the probability that the number of rolls exceed 5?
9. It is observed that 5% of the individuals reserving tables at a restaurant will not appear. If the restaurant has 50 tables and takes 52 reservations, what is the probability that it will be able to accommodate everyone appearing?
10. The following table indicates the probabilities of weather and power cuts per day:

	Sunny	Rainy
Power cut	0.2	0.15
No power cut	0.6	p

(i) Find p.

(ii) What is the probability that there will not be rain for one week?

(iii) What is the probability that there will be at least one power cut in the next three days?

(iv) If A is the event of having a power cut and B is the event of the day being sunny, check if $P(A|B) = P(A)$.

(v) Also find $P(A'|B), P(A|B'), P(B|A), P(B|A')$.

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