CS1200 — Discrete Mathematics for Computer Science

Practice questions

Part A

1. Let $Q(x, y)$ denote "$x+y = 0$". What is the truth value of the proposition $\exists A \forall M Q(M, A)$?
2. Express the statement "The product of two negative real numbers is not negative" using quantifiers.
3. Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t. Then, express the statement "No one can fool everyone all the time" using quantifiers.
4. Given the three premises: $\neg A \rightarrow (C \wedge D)$, $A \rightarrow B$ and $\neg B$,
   Use the rules of inference to prove: C
5. Given the three premises: $P \wedge Q$, $P \rightarrow \neg(Q \wedge R)$ and $S \rightarrow R$,
   Use the rules of inference to prove: $\neg S$
6. Prove that if n is an integer, then n is odd if and only if $7n + 3$ is odd.
7. Prove that if $n = ab$ where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$ using proof by contraposition.
8. Find a counter-example to the statement that every positive integer can be written as the sum of squares of three integers.

Part B

9. Suppose you are given the premises - "All lions are fierce" and "Some lions do not drink coffee". Can you conclude the statement "Some fierce creatures do not drink coffee" using the rules of inference?
10. A multivariable polynomial function $f : \mathcal{R}^n \rightarrow \mathcal{R}$ is said to be symmetric if its value remains the same irrespective of the order of its arguments. Note that this is clearly a definition, not an implication.
    In other words, for $f : \mathcal{R}^n \rightarrow \mathcal{R}$, $Sym(f)$ is the value of the predicate "f remains the same irrespective of the order of its arguments".
    (a) For example it is easy to see that for 2 variable function $f : \mathcal{R}^2 \rightarrow \mathcal{R}$, $Sym(f) := \forall x_1 \forall x_2, f(x_1, x_2) = f(x_2, x_1)$ or equivalently $\forall (x_1, x_2) \in \mathcal{R}^2, f(x_1, x_2) = f(x_2, x_1)$. Give an example of an asymmetric (not symmetric) and symmetric function of 2 variables.

(b) Write a similar reformulation of $Sym(f)$ for a 3-variable function.

(c) You can observe that in both the reformulations we check the different permutations of the variables, in other words, a universal quantifier on the different permutations of the variables involved. Let P_1 be the set of different permutations of the variables $\{x_1, x_2, ..., x_n\}$ and for a permutation $p = (x_i, ..., x_j)$, the r^{th} variable is represented by $x_{p(r)}$. Rewrite $Sym(f)$ for an n variable function explicitly using quantifiers.

11. In the year 2037, the graduating batch of 2027 has a 10-year get-together in which $n(> 1)$ people turn up. They greet each other by shaking hands and catching up with their former classmates. Some are very enthusiastic about meeting as many people as possible, while others prefer talking to a few. Amit and Anuj, who still enjoy making sound observations, believe that there will be at least 2 people who have greeted (shook hands) with the same number of people. Is that true? Prove your answer and state the proof method you have used.