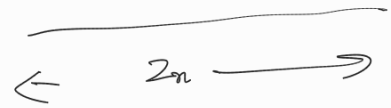


## Unsigned integers

Signed multiplicand  $n$  (✓) multiplicand  
 $\times$  ( ) multiplier  
 $n$



## Integer Unit

Operations: add, sub, mul, div.

Operands: Signed unsigned.

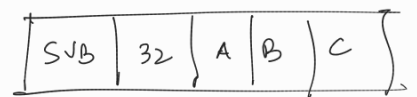
Sizes: 8, 16, 32, 64.

unsigned short int w, x, y, z;

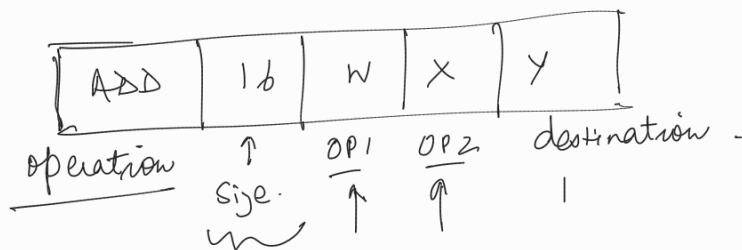
$$y = w * x;$$

int A, B, C, D

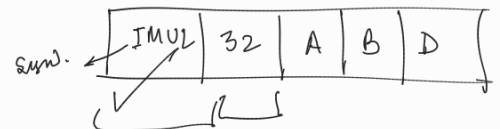
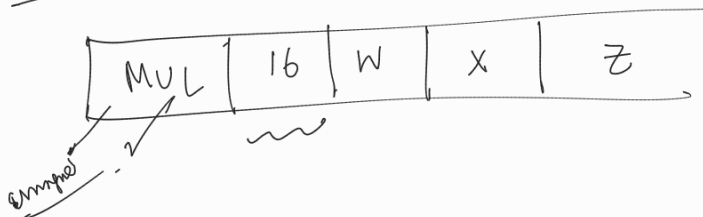
$$C = A - B$$



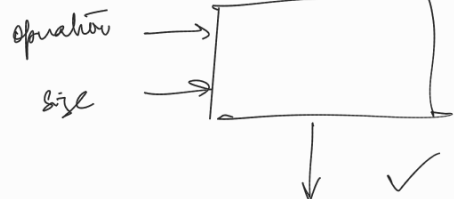
$$D = A * B.$$



$$Z = W * X$$



OP1      OP2



## Real valued numbers

### n-bit representation

Unsigned: 0 to  $2^n - 1$

Signed:  $-2^{n-1}$  to  $2^{n-1} - 1$

$n = 4 \rightarrow$  Range  $\rightarrow -8$  to  $+7$

64 bits.

unsigned  
signed.  $\left\{ \begin{array}{l} 0 \text{ to } 2^{64}-1 \\ -2^{63} \text{ to } 2^{63}-1 \end{array} \right\}$   
 $\downarrow$   
 $\sim 10^{-19} \text{ to } 10^{19}$

- ① motivation: extend the range
- ② motivation: obvious.

Range  
Resolution  $\rightarrow$  integer (1)

fixed point representation.

$\boxed{37.75}$   
integer fraction.

$i$  digits for integer  
 $f$  digits for fraction.

37.75  
 $i=2$   
 $f=2$   
Smallest  $00.00$   
~~10.00~~ Resolution:  $10^{-f}$   
largest  $99.99$  Range  
 $\begin{array}{r} 100.00 \rightarrow 10^i \\ - 00.01 \rightarrow 10^{-f} \\ \hline 99.99 \end{array}$   
 $10^i - 10^{-f}$

Binary

$i$  bits for integer  
 $f$  bits for the fraction.

$2^{-f} \rightarrow$  resolution/precision.  
 $2^i - 2^{-f} \rightarrow$  highest value

64 bit  
 $i=20$   
 $f=44$   
Resolution  $\rightarrow 2^{-44} \rightarrow 10^{-13}$   
Range  $\rightarrow [2^{20} - 2^{-44}] \rightarrow 10^6$

decimal to binary.

$\boxed{21.375}$

Repeated division.

$\begin{array}{r} 2 \overline{) 21} \\ \underline{20} \phantom{0} \\ 1 \phantom{0} \\ 2 \overline{) 10} \\ \underline{8} \phantom{0} \\ 2 \phantom{0} \\ 2 \overline{) 2} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \\ 2 \overline{) 0} \\ \underline{0} \phantom{0} \\ 0 \phantom{0} \end{array}$   
backwards.

$\boxed{10101}$   
integer.

$0.375 \times 2$   
 $\begin{array}{r} 0.75 \\ \underline{0.5} \\ 0.25 \\ \underline{0.25} \\ 0 \end{array}$

Repeated multiplication.

$\begin{array}{r} 0.75 \times 2 \\ 0.5 \times 2 \\ 0 \times 2 \end{array}$   
forward.  
 $\boxed{011}$

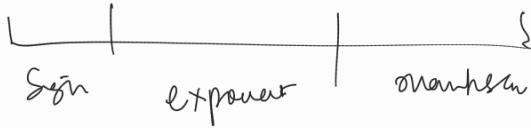
$\boxed{10101.011}$   
integer

Trade-off between Range & Resolution

Goal: realize the flexible tradeoff.

Scientific Notation.

$$X = 325.178$$



only one nonzero digit to the left of the decimal point

1.F.F.F

Mantissa

Exponent

$$\frac{325.178}{10^0} \times 10^0$$

radix

$$\frac{325.178}{10^{-1}} \times 10^{-1}$$

$$325.178 \times 10^{-1}$$

$$32517.8 \times 10^{-2}$$

Normalized representation