DEPARTMENT OF PHYSICS
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PH1020 Physics II | **Problem Set 1** | **17.1.2024**
(Tutorial on 30.1.2024)

The Electric field and its flux

1. Find the electric field above the center of a square sheet (side a), carrying a uniform charge σ. Once you have evaluated this charge density, verify that your result reproduces the limiting cases $a \to \infty$, and $z \gg a$ correctly.

2. (a) A sphere of radius R centred at the origin carries a *surface* charge density $\sigma(\mathbf{r}) = (\mathbf{K} \cdot \mathbf{r})$, where $\mathbf{K}$ is a given constant vector of appropriate dimensions. Find the electric field at the centre of the sphere.

   (b) Repeat the calculation for the case in which the sphere has a *volume* charge density $\rho(\mathbf{r}) = (\mathbf{K} \cdot \mathbf{r})$ $(0 \leq |\mathbf{r}| \leq R)$, rather than a surface charge density. (Again, $\mathbf{K}$ has the appropriate physical dimensions.)

3. Using Gauss' law in the integral form,

$$\oint_S \mathbf{E} \cdot \hat{n} dS = \frac{Q_{\text{enclosed}}}{\epsilon_0} \ ,$$

   where $\hat{n}$ is the outward normal to the Gaussian surface S, obtain the electric field $\mathbf{E}$ due to the following volume charge distributions:

   (a)

$$\rho(\varrho, \varphi, z) = \begin{cases} \beta\varrho/a & 0 < \varrho \leq a \\ 0 & a < \varrho < \infty \ . \end{cases}$$

   Here (ϱ, φ, z) denote cylindrical polar coordinates. Using cylindrical symmetry one can show that $\mathbf{E}(\mathbf{r})$ must necessarily be of the form $E(\varrho)\hat{e}_\varrho$. So the problem reduces to a choice of Gaussian surface, S.

   (b)

$$\rho(r, \theta, \varphi) = \begin{cases} \beta[1 - (r^2/a^2)] & 0 < r \leq a \\ 0 & a < r < \infty \ . \end{cases}$$

   Here (r, θ, φ) denote spherical polar coordinates. Using spherical symmetry one can show that $\mathbf{E}(\mathbf{r})$ must necessarily be of the form $E(r)\hat{e}_r$.

   Note: β is a constant of appropriate dimensions in each case.

4. Consider two point charges q that are located at positions $x = \pm\ell$.

   (a) At points close to the origin on the x axis, find E_x. At points close to the origin on the y axis, find E_y. Make suitable approximations with $x \ll \ell$ and $y \ll \ell$.

(b) Consider a small infinitesimal cylinder centered at the origin, with its axis along the x axis. The radius is r_0 and the length is $2x_0$. Using your results from part (a), verify that there is zero flux through the cylinder, as required by Gauss's law.

5. Two infinite lines of charge, each of uniform charge density λ, are located along the x and y axes respectively. Consider a cubical Gaussian surface with edge length L, centred at the origin of the coordinates O with its faces perpendicular to the coordinate axes. Find the electric flux through *each* of the six faces of the cube.

6. A ring of radius R has a uniform line charge density λ ($\lambda > 0$). The ring is located in the x-y plane with its centre at the origin.

   (a) What is the electric field at any point along the z-axis?

   (b) A point charge -Q is initially placed at the origin and is constrained to move along the z axis. If it is displaced a small distance ($\ll$ R) from the origin show that it undergoes simple harmonic motion and determine its period.