6. Big Omega Notation Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

sol:- $g(n) \geq c.n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constant c and n_0

$n^3 + 2n^2 + 4n \geq c.n^3$

divide on both sides with n^3

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $2/n$ and $4/n^2$ approaches 0

$1 + 2/n + 4/n^2$

e.g.:- $c = 1/2$

$1 + 2/n + 4/n^2 \geq 1/2$

$1 + 2/n + 4/n^2 \geq 1$ $(1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $(n \geq 1, n_0 = 1)$

Thus $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

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7. Big theta Notation determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

sol:- $C_1 n^2 \leq h(n) \leq C_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

Upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n$

$h(n) \leq C_2 n^2$

$4n^2 + 3n \leq C_2 n^2$

$4n^2 + 3n \leq 5n^2$

Let $C_2 = 5$

divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 \geq 1)$

Lower bound :-

$h(n) = 4n^2 + 3n$

$h(n) \geq c_1 n^2$

$4n^2 + 3n \geq c_1 n^2$

let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

divide both sides by n^2

$4 + 3/n \geq 4$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Omega(n^2)$

8. Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = -n^2$. show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer.

sol:- $f(n) > g(n)$

substituting $f(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n > c(-n^2)$

find c and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n \geq c \cdot n^2 \geq 0$

$n^3 + c(-2)n^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$ $(c = 2)$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

Therefore the statement $f(n) = \Omega(g(n))$ is true.

9. Determin whether $h(n) = n.\log n + n$ is $\Theta(n.\log n)$ prove a rigorous proof for your conclusion.

sol:- $C_1.n.\log n \leq h(n) \leq C_2 n.\log n$

Upper bound :-

$h(n) \leq c_2 n \log n$

$h(n) = n \log n + n$

$n \log n + n \leq c_2 n \log n$

divide on both sides by $n \log n$

$1 + \frac{n}{n \log n} \leq c_2$

$1 + \frac{1}{\log n} \leq c_2$

$1 + \frac{1}{\log n} \leq 2$ $(c_2 = 2)$

Then $h(n)$ is $O(n \log n)$ $(c_2 = 2, n_0 = 2)$

Lower bound :-

$h(n) \geq c_1 . n \log n$

$h(n) = n \log n + n$

$n \log n + n \geq c_1 n \log n$

divide both sides by $n \log n$

$1 + \frac{1}{\log n} \geq c_1$

$1 + \frac{1}{\log n} \geq c_1$

$1 + \frac{1}{\log n} \geq 1$ $(c_1 = 1)$

$\frac{1}{\log n} \geq n$ for all $n \geq 1$

$h(n)$ is $\Omega(n.\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n \log n + n$ is $\Theta(n \log n)$.

10 Solve the following recurrence relations and find the order of growth for solutions $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Sol: $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4,\ b = 2,\ f(n) = n^2$

applying master theorem

$T(n) = aT(n/b) + f(n)$ $\qquad \left(T(n) = O(n \cdot \log_b a)\right)$ $\epsilon > 0$

$f(n) = O(n \cdot \log_b a - 1)$

$f(n) = \Theta(n \cdot \log_b a)$, Then $T(n) = \Theta(n \log_b a \cdot \log n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$

$f(n) = \Theta(n^2) = \Theta(n \log_b a)$. (case 2)

$f(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n \log_b a \cdot \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$. with $T(1) = 1$ is $\Theta(n^2 \log n)$.