ASSIGNMENT ⇒ 1

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Signature

ve the following recurrence relation.

a) $x(n) = x(n-1) + 5$ for $n>1$ with $x(1) = 0$

1) Write down the first two terms to identify the pattern.

$x(1) = 0$

$x(2) = x(1) + 5 = 5$

$x(3) = x(2) + 5 = 10$

$x(4) = x(3) + 5 = 15$

2) Identify the pattern (or) the general term.

→ The first term $x(1) = 0$

The common difference $d = 5$

The general formula for the n^{th} term of an A.P is

$x(n) = x(1) + (n-1)d$

substituting the given values

$x(n) = 0 + (n-1) \cdot 5 = 5(n-1)$

∴ The solution is $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n>1$ with $x(1) = 4$

step 1:- Write down the first two terms to identify the pattern

$x(1) = 4$

$x(2) = 3x(1) = 3 \times 4 = 12$

$x(3) = 3x(2) = 24$

$x(4) = 3x(3) = 36$

step 2:- Identify the general term

→ The first term $x(1) = 4$

→ The common ratio $r = 3$

The general formula for the n^{th} term of a G.P is

$x(n) = x(1) \cdot r^{n-1}$

substituting the given values

$x(n) = 4 \times 3^{n-1}$

The solution is $x(n) = 4 \times 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (Solve for $n = 2^k$)

for $n = 2^k$ can write recurrence interms of k.

1. Substitute $n = 2^k$ in the recurrence.

$$x(2^k) = x(2^{k-1}) + 2^k$$

2. Write down the first few terms to identify the pattern.

$$x(1) = 1$$

$$x(2) = x(2) = x(1) + 2 = 3$$

$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$

$$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$$

3. Identify the general term by finding the pattern we observe that $x(2^k) = x(2^{k-1}) + 2^k$

we sum the series

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$$

since $x(1) = 1$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$$

The generatic series with the term $a = 2$ and the last term 2^k except for the additional k term. The sum of geometric series with ratio $r = 2$ is given by

$$S = a \frac{r^n - 1}{r - 1}$$

here $a = 2$, $r = 2$ and $n = k$

$$S = \frac{2^{2k} - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1} - 2$$

adding the $+1$ term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

solution is $x(2^k) = 2^{k+1} - 1$

d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

For $n = 3^k$, we can write the recurrence interms of k

1) Substitute $n = 3^k$ in the recurrence $x(3^k) = x(3^{k-1}) + 1$

Write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1)+1 = 1+1 = 2$

$x(9) = x(3^2) = x(3)+1 = 2+1 = 3$

$x(27) = x(3^3) = x(9)+1 = 3+1 = 4$

3) Identify the general term

we observe that

$x(3^k) = x(3^{k-1}) + 1$

sum up the series

$x(3^k) = k+1$

The solution is $x(3^k) = k+1$

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2. Evaluate the following recurrence complexity

i) $T(n) = T(n/2)+1$, where $n = 2^k$ for all $k \geq 0$.

The recurrence relation can be solved using iteration method.

1) Substitute $n = 2^k$ in the recurrence.

2) Iterate the recurrence.

for $k=0$: $F(2^0) = T(1) = T(1)$.

$k=1$: $F(2^1) = T(1)+1$.

$k=2$: $T(2^2) = T(2)+1 = T(1)+2+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = (T(1)+2)+1 = T(1)+3$

3) Generalize the pattern

$T(2^k) = T(1)+k$

since $n = 2^k$, $k = \log n$

4) Assume $T(1)$ is a constant c

$T(n) = c + \log_2 n$

The solution is $T(n) = O(\log n)$.

ii) $T(n) = T(n/3) + T(2n/3) + cn$ where c is a constant and n is input size)

The recurrence can be solved using the master's theorem

For divide and conquer recurrence of the form

$T(n) = a + (n/b) + F(n)$

where $a = 2$, $b = 3$ and $F(n) = cn$

let's determine the value of $\log_b a$

$\log a = \log 3$

using properties of algorithm

$\log_3 2 = \frac{\log 2}{\log 3}$

now we compare $F(n) = cn$ within $\log_3 2$

$F(n) = O(n)$

$n = n$

since $\log_3 2$ we are in the third case of the master's theorem

$F(n) = O(n^c)$ with $c > \log_b a$

The solution is : $T(n) = O(f(n)) = O(cn) = O(n)$

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3. Consider the following recurrence algorithm ?

min [A(0.....n-2)]

if n=1 return A[0]

else temp = min (A[0.....n-2])

if temp <= A(n-1) return temp

else return A[n-1]

a) What does this algorithm compute

The given algorithm min [A[n,...,n-2]] computes the minimum value in the array 'A' from index '0' for (n-1) it does this by recursively finding the minimum value in the sub array A [0,...,n-2] and then comparing it with the last element A[n-1] to determine the overall minimum value.

b) Stepup a recurrence relation for the algorithm basic operation count and solveit.

The solution is $T(n) = n$

This means the algorithm perform 'n' basic operations for an input array of size 'n'.

Analyse the order of growth

i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$ Use the is $g(n)$ · notation

To analyse the order of growth and use the Ω notation, we need to compute and compare the given function $f(n)$ and $g(n)$ given functions $F(n) = 2n^2 + 5$

$g(n) = 7n$

Order the growth using $\Omega(g(n))$ notation

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, $F(n)$, grows atleast as far as $g(n)$

$F(n) = c \cdot g(n)$

Lets analyse $F(n) = 2n^2 + 5$ with respect to $g(n) = 7n$

1) Identify dominants terms:

→ The dominant term in $F(n)$ is $2n^2$ since it grows faster than the constant term as n increases.

→ The dominant term in $g(n)$ is $7n$

2) Establish the inequality.

→ We want to find constants c and n_0 such that:

$2n^2 + 5 \geq c \cdot 7n$ for all $n \geq n_0$

3) Simplify the inequality

→ Ignore the lower order than 5 for larger.

$2n^2 \geq 7c \cdot n$

→ Divide both sides by n

$2n \geq 7c$

→ solve for n:

$n \geq 7c/2$

4) Choose constants

let $c = 1$

$n \geq \frac{7 \cdot 1}{2} = 3.5$

∴ For $n \geq n_0$ the inequality holds.

$2n^2 + 5 \geq 7n$ for all $n \geq n$

we show that there exist constant $c = 1$ and $n_0 = n$ such that for all $n \geq n_0$

$$2n^2 + 5 \geq 7n$$

Thus. we can conclude that.

$$F(n) = 2n^2 + 5 = \Omega(7n).$$

in Ω notation, the dominant term $2n^2$ in $F(n)$ clearly grows faster than hence $F(n) = \Omega(n^2)$

However, for the specific comparision asked $F(n) = \Omega\ 7(n)$ is also constant.

Showing that $F(n)$ grows atleasts as Fast as $7n$.