CS401PC : DISCRETE MATHEMATICS

9/12/2019

BTech II year II sem

Unit – I

Propositional Logic: A statement or proposition is a declarative sentence that is either true or false but not both.

Example: p: 4 is an integer. — true
q: $\sqrt{5}$ is an integer. — false

Compound proposition: When one or more proposition are connected through various connectivities is called compound proposition.

Eg: $\underbrace{\text{Roses are red}}_{\text{proposition 1}}$ and $\underbrace{\text{Violets are blue}}_{\text{proposition 2}}$

→ A proposition is said to be primitive if it cannot be broken down into similar propositions

Eg: Roses are red → Unbreakable.

Basic logic Operations

① Conjunction → AND – $\wedge$

② disjunction → OR – $\vee$

③ NOT – $\sim$

④ NAND $\rightarrow$ NOT + AND

⑤ NOR $\rightarrow$ NOT + OR

⑥ EXOR

⑦ EXNOR

$\rightarrow$ ① Conjunction: Any two proposition can be combined by the word "and" to form a compound proposition called Conjunction.

Truth Table.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Eg: p: 2 is an even no.
q: 7 divides 14

② Disjunction: Any two propositions combined by word '$\wedge$' is called disjunction.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Eg: p: 2 is an integer
q: 3 is greater than 5.

Negation:

The negation of a proposition p is It is not the case that p, denoted by $\sim p$.

Eg: p: Paris is the capital of France

q: Apollo is a Hindu god

$\sim p$: It is not the case that paris is the capital of France.

Applications of propositional logic

1. Translating English sentence:

Decompose each part of the statement into individual propositional variables and write as a compound proposition.

Eg: you drink coffee only if you put the ground coffee into the maker and don't forget to press brew.

d: you can drink coffee.

g: you put ground coffee into the maker.

f: you forgot to press brew

$d \longrightarrow (g \wedge \sim f)$

(2) Boolean Searches:

Search engines using boolean searches on logical connectivities.

(1) AND requires record match both terms

(2) OR returns records that match one or both of the term.

(3) NOT excludes a term.

(3) logic Puzzles.

A large class of elementary logical puzzles can be solved using the laws of Boolean algebra and logic truth table.

(4) logic Circuits:

Propositional logic can be applied to the design of computer hardware. A logic circuit receives input signals $P_1, P_2 \ldots P_n$, each bit produces output signals $S_1, S_2, \ldots S_n$, each a bit.

Laws of algebra of propositions

1. Idempotent law $\Rightarrow$ $p \vee p \equiv p$
2. Associative law $\Rightarrow$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Commutative law $\Rightarrow$ $(p \vee q) \equiv (q \vee p)$
4. Distributive law $\Rightarrow$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

5. Identity law $\Rightarrow$ $p \vee T \equiv T$, $p \wedge T \equiv p$
6. Complement law $\Rightarrow$ $p \vee \sim p \equiv t$, $\sim T = f$

Implication:

Let p & q be two statements, then 'if p, then q' is a statement called an implication / condition

$p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

① The statement $q \to p$ is called the Conv[erse]
of the implication $p \to q$.

② The statement $\sim p \to \sim q$, is called is the
inverse of the implication $p \to q$

③ The statement $\sim q \to \sim p$ is called the
Contrapositive of the implication $p \to q$.

Example: p: Today is Sunday.
q: I will go for a walk.

① Converse implication
$q \to p$: If I will go for a walk, then
today is Sunday.

② Inverse implication
$\sim p \to \sim q$: If today is not sunday,
then I will not go for a walk

③ Contrapositive:
$\sim q \to \sim p$: If I will not go for a walk,
then today is not Sunday.

Biimplication

let p and q be the two statement, then "p if
and only if q" $p \leftrightarrow q$. is called the Biimplicati[on]
or bicoditional statements p & q

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(1) $(\sim p \wedge q) \rightarrow (\sim(q \rightarrow p))$

p	$\sim p$	q	$\sim p \wedge q$	$q \rightarrow p$	$\sim(q \rightarrow p)$	A
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	F	F	T	F	T
F	T	T	T	F	T	T

(2) $\sim p \wedge p$

p	$\sim p$	$\sim p \wedge p$
T	F	F
F	T	F

Problems (1) $p \wedge (p \rightarrow q)$

(2) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

Problems: Proposition and logical Operations.

(1) p: It is cold q: It is raining

write the verbal sentences for the given statements

(a) $\sim p$ (b) $p \wedge q$ (c) $p \vee q$ d) $q \vee \sim p$

$\sim p$: It is not cold

$p \wedge q$: It is cold and It is raining.

$p \vee q$: It is cold or raining.

$q \vee \sim p$: It is raining and It is not cold.

(2) Truth Values and Truth tables.

(a) $4+2=5$ F
and $\wedge$
$6+3=9$ T
$= F$

(b) $3+2=5$ and $4+7=11$
$T \wedge T = T$

(c) $3+2=5$ or $6+1=7$
$T \vee T = T$

(3) Verify the proposition $p \vee \sim(p \wedge q)$ is tautology.

Show that $\sim(p \wedge q)$ & $\sim p \vee q$ are logically equivalent. (5)

(6) $p \wedge (\sim q \vee r) \rightarrow$ tautology / contradiction / Contigency

(7) Write the negation of each of the following statements

(a) 13 is an even number

(b) $5+8 < 18$

(c) This flower is beautiful.

(8) Construct the truth table for each following statements formulas

(a) $(\sim p \vee q) \wedge p$ (b) $(\sim p \wedge q) \rightarrow p$

(c) $(p \rightarrow q \wedge r) \vee (\sim p)$

(d) $(p \vee q) \Leftrightarrow (q \rightarrow r)$

e) $(p \rightarrow r) \Leftrightarrow (q \rightarrow r)$

Problem: Consider the following argument. If sheela solved seven problems correctly, then sheela obtained the grade A. Sheela solved seven problems correctly, Therefor, sheela obtained grade A.

p: sheela solved seven problems

q: sheela obtained the grade A

So Arguments takes the form

$$p \to q$$
$$p$$
$$q$$

$$((p \to q) \wedge p) \to q.$$

p	q	$p \to q$	$(p \to q) \wedge p$	$((p \to q) \wedge p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Because $((p \to q) \wedge p) \to q$ is a tautology, it follows that the given ~~st~~ argument is valid.

A finite sequence $A_1, A_2, A_3, \ldots A_{n-1}, A_n$ of statement is called an argument. The final statement A_n is the conclusion, and the statements $A_1, A_2, A_3 \ldots A_{n-1}$ are called the premises of the argument. An argument $A_1, A_2, A_3 \ldots A_{n-1}, A_n$ is called logically valid if the statement formula

$$(A_1 \wedge A_2 \wedge A_3 \wedge \ldots \wedge A_{n-1}) \rightarrow A_n$$ is a tautology.

Some Valid Argument forms

(1) Consider the following argument form

$$p \rightarrow q$$
$$p$$
$$\therefore q$$

$((p \rightarrow q) \wedge p) \rightarrow q$ is tautology. So this is a valid argument form. This argument form is called modus ponens. The latin meaning of modus ponens is method of affirming.

(2)
$$p \rightarrow q$$
$$\sim q$$
$$\therefore \sim p$$

$((p \rightarrow q) \wedge (\sim q)) \rightarrow \sim p$ is tautology.

This argument form is called modus tollens. Latin meaning of modus tollen is method of denying.

(3) (a) $p \vee q$

$\sim p$

$\therefore q$

b. $p \vee q$

$\sim q$

p

These two argument forms are also valid forms. They are called disjunctive syllogisms.

(4) $p \to q$ $q \to r$ $\therefore p \to r$.

This is called hypothetical syllogism.

(5) $p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Problem:

If Peter solved seven problem correctly, then Peter obtained the grade A. Peter obtained the grade A. Therefore, Peter solved seven problems correctly. To check the validity of this argument, we first write the arguments by statements letters and logical connectivities.

Sol: p: Peter solved seven problems correctly.

q: Peter obtained the grade A.

$p \to q$

q

$\therefore p$

$((p \to q) \wedge q) \to p$.

p	q	$p \to q$	$(p \to q) \wedge q$	$((p \to q) \wedge q) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Quantifiers and First Order logic

Let x be a Variable and D be a set; P(x) is Sentence, Then P(x) is Called a Predicate or Propositional function with respect to the set D if for each value of x in D, P(x) is a statement i.e., P(x) is true or false. Moreover, D is Called the domain of the discourse. and x is Called the free Variable.

Example: Consider the sentence P(x)

P(x) : x is an even integer.

where the domain of the discourse is the set of integers. Then

P(4) : 4 is an even integer, is T.

&

P(3) : 3 is an even integer is F.

$P(x, y)$: x equals $y+1$

$P(2,1)$: $x=2$ $y=1$

$x=2 = 1+1 = y+1$

it follows that $P(2,1)$ is T.

Let $Q(x,y)$ denote the Sentence.

$Q(x,y)$: x^2 is greater than or equal to y.

→ Let the domain be the set of integers. Here the predicate $Q(x,y)$ involves two variables.

Consider $Q(2,3)$ $x=2$ $y=3$

$x^2 = 4>3$. It is true.

$Q(2,5)$ $x=2$ $y=5$

4 is neither greater than 5 nor equal to 5.

→ Let $x_1, x_2 \ldots x_n$ be n variables. An n-place Predicate is a sentence $P(x_1, x_2 \ldots x_n)$ Containing $x_1, x_2, x_3 \ldots x_n$ Such that on assignment of values to the variables $x_1, x_2 \ldots x_n$ from appropriate domains, a statement results

→ Let $P(x)$ be a predicate and let D be the domain of the discourse. The universal Quanti-fication of $P(x)$ is the statement.

for all x, P(x)

or

for every x, P(x)

→ The symbol used to denote the adjective for all is $\forall$, it is called "Universal Quantifier".

$\forall x\, P(x)$

Eg: $P(x) : x^2 \geq x$ → True.

$\forall x\, P(x)$.

→ Let P(x) be a predicate and let D be the domain of the discourse, The existential quantification of P(x) is the statement.

There exists x, P(x)

→ The Symbol used to denote "there exists" is $\exists$ and it is called the existential quantifier.

$\exists x\, P(x)$

Predicate:

"x is greater than 3"

→ It has two parts. The first part, the variable x, is the subject of the statement. The second part "is greater than 3", is the predicate.

→ P(x) = x is greater than 3

where P denotes the predicate, "is greater than 3" and x is the variable.

P(x) = P(x_1, x_2 … x_n) Here P is also referred to as n-place predicate or n-ary predicate.

Eg: P(x) x>10

P(11) is equivalent to the statement 11> 10 which is true. P(5) is false.

Quantifiers

In Predicate logic, Predicates are used alongside quantifiers to express the extent to which a predicate is true over a range of elements. Using quantifiers to create such propositions is called quantification.

(1) Universal Quantification:

Mathematical statements sometimes assert that a property is true for all the values of a variable in a particular domain, called the domain of discourse.

→ The Universal quantification of P(x) is the statement P(x) for all values of x in the domain.

$$\forall x\, P(x)$$

Eg: Let P(x) be the statement "$x+2 > x$". what is the truth value of the statement $\forall x P(x)$?

Sol: $x+2$ is greater than x for any real number

$$P(x) \equiv T$$

$$\forall x(P(x)) \equiv T.$$

(2) Existential Quantification:

Some mathematical statements assert that there is an element with a certain property. Such statements are expressed by existential quantification.

P(x) is true for at least One Value of x in the domain.

The existential quantification of $P(x)$ is the statement "There exists an element x in the domain such that $P(x)$". $\exists P(x) \rightarrow$ There is atleast one such x such that $P(x)$.

Eg: let $P(x)$ be the statement "$x>5$". what is the truth value of the statement $\exists x P(x)$?

Sol: $P(x)$ is true for all real numbers greater than 5 and false for all real numbers less than 5. $\exists x P(x) \equiv T$.

$\forall P(x) \leftrightarrow Q(x)$

$P(x)$: x is 18 years or older

$Q(x)$: x is eligible to vote.

→ Restriction of Universal quantification is the same as the Universal quantification of a Conditional statement.

→ Restriction of an existential quantification is the same as the existential quantification of a conjunction.

① All men are mortal

$\forall x \{ M(x) \rightarrow H(x) \}$

② Every Apple is Red

$\forall x \ (A(x) \rightarrow R(x))$

③ Any integer is either true or false.

$\forall x \ (I(x) \rightarrow \{ P(x) \vee N(x) \})$

Negation:

$\forall(x) \Rightarrow \sim \forall(x) \Rightarrow \exists(x)$

④ Not every graph is Conneited

$Graph(x) \Rightarrow \forall x \{ G(x) \rightarrow C(x) \}$

$\underbrace{\sim \forall(x)}_{\text{Not all}} (G(x) \rightarrow C(x))$

Example: ① All integers are rational numbers.

$\forall x \ (I(x) \rightarrow R(x))$

② Some rational numbers are integers.

$\exists(x) \ (r(x) \rightarrow I(x))$

→ for all pairs of real numbers; x, y does not the identity $x-y = y-x$ hold? express this using quantifiers.

$\forall x \forall (y) \ (x-y \equiv y-x)$

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Problem: " There is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression.

Solution: Let $P(x,y)$ be the expression

$$x+y=y.$$

let $Q(x,y)$ be the expression

$$xy=x.$$

Then the expression is

$$\exists x \forall y (P(x,y) \land Q(x,y))$$

$\Rightarrow$ Everyone is a lion and everyone is fierce.

$$\forall (x) \; lion(x) \Longrightarrow fierce(x)$$

$$\forall (x) \; lion(x) \land fierce(x)$$

$\Rightarrow$ When you have at least one lion that drink coffee

$$\exists (x) \; lion(x) \land Drinks(x, Coffee)$$

$\Rightarrow$ you have people even though no lion drinks coffee.

$$\exists (x) \; lion(x) \Longrightarrow Drinks(x, coffee)$$

Nested Quantifiers:

$$xy((x+y)=0)$$

→ Two quantifiers are nested if one is within, the scope of the other.

$$P(x,y)$$

$$\forall x \boxed{\exists x((x+y)=0)}$$

$$Q(x)$$

$$\forall x\, Q(x)$$

Rules of Inference.

The arguments are chained together using Rules of inference to deduce new statement and ultimately prove that the theorem is valid

① **Argument:** A sequence of statements; premises that end with a conclusion.

② **Validity:** A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.

Fallacy: An incorrect reasoning or mistake which leads to invalid arguments. (15)

→ An argument is a sequence of statements called premises which ends with a conclusion.

Premises: $P_1, P_2, P_3, \ldots P_n$

Conclusion: q

if $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \rightarrow q$. is tautology then argument is termed valid.

Rules of Inference:

Simple arguments can be used as building blocks to construct more complicated valid arguments. Certain simple arguments that have been established as valid are very important interms of their usage. These arguments are called rules of inference

Rule Of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus Ponens
$\sim q$ $p \to q$ $\therefore \sim p$	$(\sim q \wedge (p \to q)) \to \sim p$	Modus Tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical Syllogism
$\sim p$ $p \vee q$ $\therefore q$	$(\sim p \wedge (p \vee q)) \to q$	Disjunctive Syllogism
p $\therefore (p \vee q)$	$p \to (p \vee q)$	Addition
$(p \wedge q) \to r$ $\therefore p \to (q \to r)$	$((p \wedge q) \to r) \to (p \to (q \to r))$	Exportation
$p \vee q$ $\sim p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\sim p \vee r)) \to q \vee r$	Resolution

Rule of Inference	Name
$\forall x\, P(x)$ / $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitary c / $\therefore \forall x\, P(x)$	Universal generalization
$\exists x\, P(x)$ / $\therefore P(c)$ for some c	Existential instantiation
$P(c)$ for some c / $\therefore \exists x\, P(x)$	Existential generalization.

Problem: "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a goa trip", and "If we take a goa trip, then we will be home by sunset",

lead the conclusion:

"we will be home by sunset".

p — "It is sunny this afternoon

q — It is colder than yesterday

r — we will go swimming

s — we will take a goa trip

t — we will be home by sunset

The hypotheses are

$\sim p \wedge q$, $r \rightarrow p$, $\sim r \rightarrow s$, $s \rightarrow t$.

Conclusion is t.

Step	Reason
① $\sim p \wedge q$	Hypothesis
② $\sim p$	Simplification
③ $r \rightarrow p$	Hypothesis
④ $\sim r$	Modus Tollens using ② & ③
⑤ $\sim r \rightarrow s$	Hypothesis
⑥ s	Modus Ponens using ④ & ⑤
⑦ $s \rightarrow t$	Hypothesis
⑧ t	Modus Ponens using ⑥ & ⑦

(a)

Examples of propositions with truth values

1. five is a prime no : T
2. Dog is a animal : T
3. $8 \div 4 = 2$: T
4. The earth is round : T
5. $1+2=5$: f
6. chennai is an England : f

Example of Non-Proposition

1. How old are you ?
2. What is height of Himalaya ?
3. $x+3=7$?
4. Please Open the door. (request)
5. The Peacock is very beautiful (!)

$P \to Q$

* If P, then Q
* P implies Q
* P only if Q
* Q is necessary for P
* P is sufficient for Q

$P \leftrightarrow Q$

* P if and only if Q
* P iff Q
* P is necessary and sufficient for Q
* If P, then Q and conversely

Examples: ① If either Ram takes C++ or kumar takes Pascal then latha will take lotus.

Propositions: p: Ram takes C++

q: kumar takes Pascal

r: latha will take lotus

Logical Connectivities: $(p \vee q) \rightarrow r$.

② Anbl Can access the internet from Campus, Only if she is a Computer science mayor or she is not a ~~fresh~~ girl.

Propositions: p: Any Can access the internet from Campus.

q: she is a Computer Science mayor

r: she is not a ~~fresh~~ girl.

Logical Connectives: $p \rightarrow (q \vee \sim r)$

③ If the moon is out and it is not snowing, then Ram goes out for a walk.

Propositions: p: The moon is out

q: It is snowing

r: Ram goes out for a walk

$(p \wedge \sim q) \rightarrow r$.

(4) If the moon is out, then if it is not snowing Ram goes out for a walk.

Propositions: p: The moon is out

q: It is snowing

r: Ram goes out for a walk.

$$p \rightarrow (\sim q \rightarrow r)$$

(5) If is not the Case that ram goes out for a walk, iff it is not snowing or the moon is out.

$$\sim (r \leftrightarrow (\sim q \vee p))$$

(6) Meenu is poor but Happy

If p: Meenu is rich, q: Meenu is happy

$$\sim p \wedge q.$$

(7) Meenu is rich or Unhappy

$$p \vee \sim q.$$

(8) Meenu is neither rich nor happy.

$$\sim p \wedge \sim q.$$

(9) Meenu to be poor is to be Unhappy

$$\sim p \leftrightarrow \sim q.$$

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Propositional Equivalence

① Idempotent laws

$p \wedge p \Leftrightarrow p$

$p \vee p \Leftrightarrow p$

② Associative law:

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

③ Commutative law:

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

④ Demorgans law:

$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

⑤ Distributive laws:

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

⑥ Complement laws

$p \wedge \sim p \Leftrightarrow F$, $p \vee \sim p \Leftrightarrow T$

⑦ Dominance laws:

$p \wedge F \Leftrightarrow F$, $p \vee T \Leftrightarrow T$

⑧ identity laws:

$p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

Problems ① If x is not a real number, then it is not a rational number and not an irrational number.

Ans p: x is a real number.

q: x is a rational number

r: x is an irrational number.

$\Rightarrow \sim p \to (\sim q \wedge \sim r)$

$\Rightarrow \sim(\sim p \to (\sim q \wedge \sim r))$

$\equiv \sim p \wedge (\sim(\sim q \wedge \sim r))$

$\equiv \sim p \wedge (\sim\sim q \vee \sim\sim r)$

$\equiv \sim p \wedge (q \vee r)$

Negation of the given Conditional is x is not a real number and it is a rational number or an irrational number.

② $(p \vee q) \wedge [\sim(\sim p) \wedge q]$

$\equiv (p \vee q) \wedge \{\sim\sim p \vee \sim q\}$ Demorgan's law

$\equiv (p \vee q) \wedge (p \vee (\sim q))$ by law of Double negation

$\equiv p \vee (q \wedge \sim q)$ by Distributive law

$\equiv p \vee F$ inverse law

$\equiv p$ identity law

(2) $(p \vee q) \wedge \sim((\sim p) \vee q)$

$\equiv (p \vee q) \wedge (p \wedge \sim q)$

$\equiv \{(p \vee q) \wedge p\} \wedge (\sim q)$ Using Associative law

$= (p \wedge (p \vee q)) \wedge (\sim q)$ Using Commutative law

$= p \wedge (\sim q)$ Using absorption law.

$\Rightarrow$ Associative law.

$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$.

Commutative law

$(p \vee q) \Leftrightarrow (q \vee p)$

Absorption law

$(p \vee (p \wedge q)) \Leftrightarrow p$

$(p \wedge (p \vee q)) \Leftrightarrow p$.

(3) $\sim[\sim((p \vee q) \wedge r) \vee \sim q]$

$\equiv \sim[\sim\{((p \vee q) \wedge r) \wedge q\}]$ — Demorgan's law

$\equiv ((p \vee q) \wedge r) \wedge q$ — law of double Negation

$\equiv (p \vee q) \wedge (q \wedge r)$ — Using associative law & Commutative law

$\equiv ((p \vee q) \wedge q) \wedge r$ — Associative law

$= q \wedge r$ Absorption law.

$p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

$\equiv p \vee [p \wedge (p \vee q)]$

$\equiv p \vee p$ by an Absorption law

$\equiv p$ by Idempotent law.

5) $(p \vee q \vee (\sim p \wedge \sim q \wedge r)) \Leftrightarrow (p \vee q \vee r)$

$\sim p \wedge \sim q \wedge r \Leftrightarrow (\sim p \wedge \sim q) \wedge r$ — Associative law

$\sim (p \vee q) \wedge r$ — Demorgan's law

$\therefore \; p \vee q \vee (\sim p \wedge \sim q \wedge r)$

$\equiv \underbrace{(p \vee q)}_{p} \vee (\underbrace{\sim(p \vee q)}_{q} \wedge r)$

$\equiv \underbrace{[(p \vee q) \vee \sim(p \vee q)]} \wedge [(p \vee q) \vee r]$ — Distributive law

$\equiv T \wedge (p \vee q \vee r) \Longrightarrow$ Inverse law & Associative law

$\equiv p \vee q \vee r$ — Commutative & Identity law

6) $[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$

$\equiv (\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)$

$\equiv \sim (\sim p \vee \sim q) \vee (p \wedge q \wedge r)$ $[u \rightarrow v \Leftrightarrow \sim u \vee v]$

$\equiv \underbrace{(p \wedge q)}_{a} \vee [\underbrace{(p \wedge q)}_{a} \wedge \underbrace{r}_{b}]$ — Demorgan law & Associative law

$\equiv p \wedge q \longrightarrow$ Absorption law

(7) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$

$\equiv (p \vee q) \wedge (p \vee \sim q) \Leftrightarrow p \vee (q \wedge \sim q) \Rightarrow$ Distributive law

$p \vee F$ ↳ Contradiction

$p \rightarrow$ Identity law

$= p$

$\therefore [(p \vee q) \wedge (p \vee \sim q)] \Leftrightarrow p$

$\equiv p \vee q$.

(8) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

$\equiv p \rightarrow (q \rightarrow r)$

$\equiv \sim p \vee (\sim q \vee r)$

$\equiv (\sim p \vee \sim q) \vee r$

$\equiv \sim (p \wedge q) \vee r$

$\equiv (p \wedge q) \rightarrow r$.

$\boxed{\begin{array}{l} u \rightarrow v \Leftrightarrow \sim u \vee v \\ \sim (u \rightarrow v) \Leftrightarrow u \wedge \sim v \end{array}}$

$\boxed{\begin{array}{l} u \rightarrow v \Leftrightarrow \sim u \vee v \\ \alpha u \rightarrow v \Leftrightarrow \alpha u \wedge v \end{array}}$

⟹ Duality

Suppose u is a compound proposition that contains no connectives other than ∧ & ∨. Suppose we replace each occurrence of ∧ & ∨ in u by ∨ and ∧ respectively. Also if u has T_0 & F_0 as components, suppose we replace each occurrence of T_0 & F_0 by F_0 & T_0 respectively. Then the resulting compound proposition is called the dual of u and is denoted by u^d.

Eg: $U : p \wedge (q \vee \sim r) \vee (s \wedge T_0)$

$U^d : p \vee (q \wedge \sim r) \wedge (s \vee F_0)$

The following two results are importance.

① $(U^d)^d \Leftrightarrow U$

② $U \Leftrightarrow V$ then $U^d \Leftrightarrow V^d$.

Examples

① $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

$\sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

② $(p \wedge q) \vee [(\sim p \vee q) \wedge (\sim r \vee s)] \vee (r \wedge s)$

$(p \vee q) \wedge [(\sim p \wedge q) \vee (\sim r \wedge s)] \wedge (r \vee s)$

Rule of Inference

① If sachin hits a Century, then he gets free Car.

Sachin hits a Century

$\therefore$ Sachin gets a free Car

Sol p: Sachin hits a Century q: sachin gets free car

$p \to q$

p

$\therefore q$

Modus Ponen Rule — this is valid argument

(2) If Sachin hits a Century, then he gets a free car
Sachin does not get a Car

∴ Sachin has not hit a Century

Sol: P: Sachin hits a Century q: Sachin gets a free Car

$p \to q$

$\sim q$

$\therefore \sim q$

Modus Tollens Rule, the argument is Valid.

(3) If I drive to work, then I will arrive tired
I am not tired

I do not drive to work.

P: I drive to work q: I arrive tired

$p \to q$

$\sim q$

$\sim p$

∴ Modus Tollen rule, then it is valid argument.

(4) If I study, then I do not fail in the examination
If I do not fail in the examination, my father gifts a two-wheeler to me.

Therefore If I study then my father gifts a two-wheeler to me.

Sol: P: I study q: I donot fail in the examination
r: My father gifts a two-wheeler to me

ree

$p \rightarrow q$

$q \rightarrow r$

$p \rightarrow r$

Rule of Syllogism, this is a valid argument.

(5) If I study, I will not fail in the examination.

If I do not watch TV in the evenings, I will study.

I faid in the examination

$\therefore$ I must have watched TV in the evenings

let p: I study q: I fail in the examination

r: I watch TV p q ~p

$p \rightarrow \sim q$

$\sim r \rightarrow p$

q

$\therefore r$.

$p \rightarrow \sim q$ $(p \rightarrow \sim q \Leftrightarrow (\sim\sim q \rightarrow \sim p))$

$\Downarrow$

$q \rightarrow \sim p$

$\sim r \rightarrow p \; [(\sim p \rightarrow r) \Leftrightarrow (\sim\sim r \rightarrow \sim\sim p)]$

$\Downarrow$

$\sim p \rightarrow r$

$\Rightarrow$ $\left.\begin{array}{l} q \rightarrow \sim p \\ \sim p \rightarrow r \end{array}\right\} \rightarrow q \rightarrow r$ (Rule of Syllogism)

q

$q \rightarrow r$

q

$\therefore r$ $\rightarrow$ Modus Ponens

(6) $p \to r$
$q \to r$

$\therefore (p \vee q) \to r$

$\equiv (p \to r) \wedge (q \to r)$

$\equiv (\sim p \vee r) \wedge (\sim q \vee r)$

$\Rightarrow (r \vee \sim p) \wedge (r \vee \sim q)$ by Commutative law.

$\Rightarrow r \vee (\sim p \wedge \sim q)$ distributive law.

$= \sim (p \vee q) \vee r \to$ Commutative law & Demorgan's law.

$\Rightarrow (p \vee q) \to r$

argument is Valid.

(7) Show that $R \vee S$ follows logically from the Premises $C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$ and $(A \wedge \sim B) \to R \vee S$.

$\Rightarrow (C \vee D) \wedge \{(C \vee D) \to \sim H\} \wedge (\sim H \to (A \wedge \sim B)) \wedge (A \wedge \sim B) \to R \vee S$

$\equiv C \vee D \wedge \left\{ \begin{array}{l} (C \vee D) \to \sim H \\ \sim H \to A \wedge \sim B \end{array} \right\}$ $\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$ || Syllogism

$A \wedge \sim B \to R \vee S$ $\begin{array}{l} r \to s \\ \hline p \to s \end{array}$

$\equiv \dfrac{C \vee D}{p} \wedge \left(\dfrac{(C \vee D) \to (R \vee S)}{p \to q} \right)$

$\begin{array}{l} p \\ p \to q \\ \hline q \end{array}$ $\therefore$ modus Ponen

$\equiv \underline{\underline{R \vee S}}$

(19)

Show that $R \wedge (P \vee Q)$ is a Valid Conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, & $\neg M$

Soln $(P \vee Q) \wedge (Q \rightarrow R) \wedge (P \rightarrow M) \wedge (\neg M)$

$\equiv (\neg P \rightarrow Q) \wedge (Q \rightarrow R) \wedge (\neg P)$

using $P \vee Q \Leftrightarrow \neg P \rightarrow Q$ Modus Tollen

$\equiv (\neg P \rightarrow R) \wedge (\neg P)$ Rule of Syllogism

$\equiv R$ Modus Ponen Rule

$= R \wedge (P \vee Q)$ because $P \vee Q$ is true.

(9) Show that the premises $p \rightarrow q$, $p \rightarrow r$, $q \rightarrow \neg r$, r are Consistent whereas $p \rightarrow q$, $p \rightarrow r$, $q \rightarrow \neg r$, p are inconsistent

$S = (p \rightarrow q) \wedge (p \rightarrow r) \wedge (q \rightarrow \neg r) \wedge r$

$t = (p \rightarrow q) \wedge (p \rightarrow r) \wedge (q \rightarrow \neg r) \wedge p$

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$q \rightarrow \neg r$	s	t
T	T	T	F	T	F		
T	T	F	F	F	T		
T	F	T	F	T	T		
T	F	F	F	F	T		
F	T	T	F	T	F		
F	T	F	T	T	T		
F	F	T	T	T	T		
F	F	F	T	T	T		

p	q	r	p→q	p→r	q→~r	s	t
F	F	F	T	T	T	F	F
F	F	T	T	T	T	T	F
F	T	F	T	T	T	F	F
F	T	T	T	T	F	F	F
T	F	F	F	F	T	F	F
T	F	T	F	T	T	F	F
T	T	F	T	F	T	F	F
T	T	T	T	T	F	F	F

We Observe that there is a situation where the Conjunction s of the premises $p \to q$, $p \to r$, $q \to \sim r$, & is true. Therefore the premises are Consistent.

On the Other hand, the Conjunction t of the premises $p \to q$, $p \to r$, $q \to \sim r$ and p is false. in all possible situations, these premises are inconsistent.

⇒ Problem ① Consider the following Open statements with the set of all real numbers as the Universe

$P(x): |x| \geq 3$, $q(x): x > 3$

Truth table

$$\forall x \,[\, \overset{T}{P(x)} \longrightarrow \overset{F}{q(x)} \,] \to F$$

$P(-4)$: $|4| > 3$

$4 > 3$ T

$q(-4)$: $-4 > 3$ — False.

Converse of the statement

$F \rightarrow T = F$

$\forall(x)\, [q(x) \rightarrow P(x)]$.

Inverse of the statement

$F \rightarrow T = F$

$\forall(x)\, [\sim P(x) \rightarrow \sim q(x)]$

Contrapositive of the statement

$\forall(x)\, [\sim q(x) \rightarrow \sim P(x)]$

$T \rightarrow F = F$.

Negation and simplify the following.

① $\exists x, [P(x) \vee q(x)]$

$\sim [\exists x\, (P(x) \vee q(x)]$

$\forall x\ \sim (P(x) \vee q(x))$

$\forall x\, [\sim P(x) \wedge \sim q(x)]$

② $\forall x\, [P(x) \wedge \sim q(x)]$

$\sim [\forall x\, [P(x) \wedge \sim q(x)]$

$\exists x\, [\sim (P(x) \wedge \sim q(x)]$

$\exists x\, [\sim Px \vee q(x)]$

(3) $\forall x\, [P(x) \rightarrow q(x)]$

$\sim \forall x\, [P(x) \rightarrow q(x)]$

$\exists x\, [\sim (P(x) \rightarrow q(x))]$

$\exists x\, (P(x) \wedge \sim q(x)]$

(4) $\exists x\, [(P(x) \vee q(x)) \rightarrow r(x)]$

$\sim \exists x\, [(P(x) \vee q(x)] \rightarrow r(x)]$

$\forall x\, [(\sim P(x) \vee q(x)) \wedge \sim r(x)]$

(5) Write down the following proposition in Symbolic form and find its negation

(1) All integers are rational numbers and Some rational numbers are not integers'.

Sol: • $P(x)$: x is a rational number

$q(x)$: x is an integer.

Z: Set of All integers

Q: set of all rational numbers

$\therefore \{\forall x \in Z,\ P(x)\} \wedge \{\exists x \in Q,\ \sim q(x)\}$

Negation :-

$\{\exists x \in Z,\ \sim P(x)\} \vee \{\forall x \in Q,\ q(x)\}$

"If all triangles are right-angled, then no triangle is equiangular.

T: set of all triangles

P(x): x is right-angled

q(x): x is equiangular

$\{\forall x \in T, P(x)\} \rightarrow \{\forall x \in T, \sim q(x)\}$

Negation

$\{\forall x \in T, P(x)\} \wedge \{\exists x \in T, q(x)\}$

All integers are right angled and some triangles are equiangular.

(7) for All integers n, if n is not divisible by 2 then n is odd

Sol: Z: set of all integers

R: set of real numbers

P(n): n is divisible by 2

q(n): n is odd

$\forall n \in Z, \sim P(n) \rightarrow q(n)$

Negation: $\exists n \in Z, \sim P(n) \wedge \sim q(n)$

for some integer n, n is not divisible by 2 and n is not odd.

⟹ Prove that the following argument is valid

All men are mortal

Sachin is a man

$\therefore$ Sachin is a mortal

Sol S : Set of All men $P(x)$: mortal

a : Sachin.

given argument reads

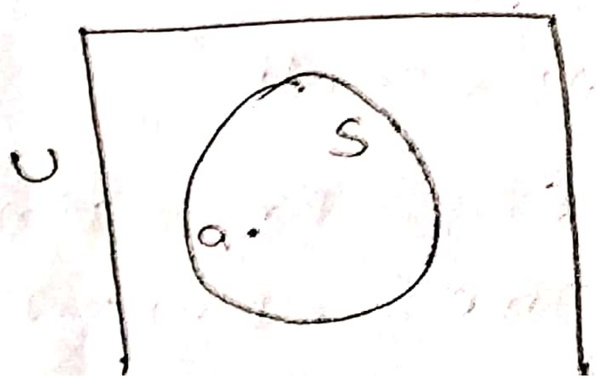
$$\forall x \in S,\ P(x)$$

$$a \in S$$

$$\therefore P(a)$$

Since the statement $\forall x \in S,\ P(x)$ is true, and $a \in S$, it follows by the rule of Universal Simplification that $P(a)$ is true. Thus, the given argument is valid.

Venn Diagram



Q. Find whether the following is a valid argument (22)
for which the Universe is the set of all students.

No Engineering student is bad in studies.

Anil is not bad in studies

$\therefore$ Anil is an engineering student

<u>Let:</u> $P(x)$: x is an engineering student

$q(x)$: x is bad in studies

a : Anil

the given argument reads

$$\forall x, [P(x) \rightarrow \sim q(x)]$$

$$\sim q(a)$$

$$\therefore P(a)$$

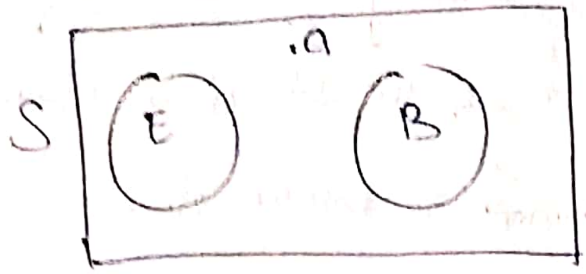
by the rule of Universal specification

$$\therefore \{\forall x \{P(x) \rightarrow \sim q(x)\}\} \wedge \sim q(x)$$

$$\Rightarrow \{P(a) \rightarrow \sim q(a)\} \wedge \sim q(a)$$

$$\not\Rightarrow P(a)$$

because $P(a)$ can be false, when both of $P(a) \rightarrow \sim q(a)$ & $\sim q(a)$ are true. the given argument is not valid.



⇒ Find whether the following argument is valid

No graduate student of Commerce or literature studies physics.

Anil is a graduate student who studies physics

$\therefore$ Anil is not a graduate student of literature.

Let us take the Universe to be the set of all graduate students, and let:

$P(x)$: x studies commerce

$q(x)$: x studies literature.

$r(x)$: x studies physics

a : Anil.

$$\forall x \left[\{P(x) \vee q(x)\} \rightarrow \sim r(x)\right]$$

$$r(a)$$

$$\therefore \sim q(a)$$

$$\Rightarrow \left\{\forall x \left[\{P(x) \vee q(x)\} \rightarrow \sim r(x)\right]\right\} \Rightarrow$$

$$\{P(a) \vee q(a)\} \rightarrow \sim r(a).$$

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$\{\forall x, [p(x) \vee q(x)] \to \sim r(x)\} \wedge r(a)$

$\Rightarrow \{[p(a) \vee q(a)] \to \sim r(a)\} \wedge r(a)$

$\Rightarrow r(a) \wedge \{r(a) \to \sim(p(a) \vee q(a))\}$ Commutative & Contrapositive

$\Rightarrow \sim\{p(a) \vee q(a)\}$ – Modus ponen law

$\Rightarrow \sim p(a) \wedge \sim q(a)$ Demorgan's law

$\Rightarrow \sim q(a)$ — Conjuctive simplification.

The given argument is valid.

<u>Statements with more than One Variable</u>

Consider the following declarative sentences

① $x-2y$ is a positive integer.

② $x+y-z=0$. These are open statements which contain more than one free Variable. These become prepositions if each variable is replaced by an element of a Certain Universe.

Example: $5, -3$

$5-2(-3) \Rightarrow -1$ T

<u>Illy</u> $\frac{1}{2}, \frac{1}{4}$ $\frac{1}{2} - \cancel{2}(\frac{1}{\cancel{4}_2}) = 0$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{4} \Rightarrow \frac{1}{2} + \frac{1}{4} - \frac{1}{4} = 0$ (which is false)

Ex: Let $P(x,y)$ and $q(x,y)$ denote the following Open statements.

$P(x,y): x^2 \geq y$ $q(x,y): (x+2) < y$

If the Universe for both of x,y is the set of all real numbers, determine the truth value of each of the following statements:

① $P(2,4)$ ② $q(1,\pi)$ ③ $P(-3,8) \wedge q(1,3)$.

$P(2,4) \equiv 2^2 \geq 4$ is true

$q(1,\pi) \equiv (1+2) < \pi$ is true

$P(-3,8) = [(-3)^2 \geq 8] \wedge [(1+2) < 3]$ false.

Quantified statements with more than One Variable

When an Open statements Contains more than One free Variables, quantification may be applied to each of the Variables. Thus, if $P(x,y)$ is an Open statement with Variables x,y we can have quantified statements of the following form:

① $\forall x, \forall y, P(x,y)$ ② $\exists x, \exists y, P(x,y)$

Write down the following statements in symbolic form using quantifiers

(1) Every real number has an additive inverse

(2) The set of real numbers has a multiplicative is identity.

(1) let: Every real number has an additive inverse.

x: real number (Any)

y: real number.

$x+y = y+x = 0$

$\forall x, \exists y [x+y = y+x = 0]$.

(2) $\exists x, \forall y [xy = yx = y]$.

Methods of Proofs

Given such a Conditional, the process of establishing that the Conditional is true, by using the rules laws of logic and other known facts Constitute a proof of the Conditional. The process of establishing that a proposition is a false Constitutes a disproof.

Direct Proof:

The Direct proof method of proving a Conditional $p \to q$ has the following lines of argument

① **Hypothesis**: first assume that p is true

② **Analysis**:- starting with the hypothesis and employing the rules/laws of logic and other known facts, infer that q is true.

③ **Conclusion**: $p \to q$ is true.

Example: The square of an odd integer is an odd integer.

⇒ Here, the Conditional to be proved is:

If n is an odd integer, then n^2 is an odd integer.

Assume that n is an odd integer (hypothesis) then $n = 2k+1$ for some integer k.

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$$

LHS = RHS

right hand side is not divisible by 2

∴ n^2 is not divisible by 2. This means that n^2 is an odd integer (conclusion)

Prove that, for all integers k and l, if k and l are both odd, then k+l is even and kl is odd. (25)

→ Take any two integers k and l, and Assume that both of these are odd (hypothesis)

then $k = 2m+1$ $l = 2n+1$

$k+l = 2m+1+2n+1 = 2(m+n+1)$

$kl = (2m+1)(2n+1) = 4mn+2m+2n+1$

$= 4mn+2(m+n)+1$

$(m,n) = (1,1)$

We Observe that k+l is divisible by 2 and kl is not divisible by 2. Therefore k+l is even integer and kl is an odd integer.

⇒ Prove that if m is an even integer, then m+7 is an odd integer.

the given statement is $p \to q$

<u>Proof</u>

p : m is even q : m+7 is odd.

Assume that $p \to q$ is false. that is assume that p is true and q is false. Since q is false. m+7 is even. Hence $m+7 = 2k$

$m = 2k-7$

$= (2k-8)+1$

$= 2(k-4)+1$ → m is odd

This means p is false then $p \to q$ is true.

⇒ Give ① a direct proof ② an indirect proof ③ proof by Contradiction for the following statement.

"If n is an odd integer, then n+11 is an even integer."

① <u>Direct proof</u>: Assume that n is an odd integer. Then $n = 2k+1$ for some integer k. This gives $(n+11) =$

$$n+11 = (2k+1)+11$$
$$= 2k+12$$
$$= 2(k+6)$$ from which it is evident that n+11 is even.

② <u>Indirect proof</u>

Assume that n+11 is not even integer. Then $n+11 = 2k+1$ for some integer k. This gives $n = (2k+1)-11 \Rightarrow 2k-10$

$n = 2(k-5)$ which shows that n is even.

Thus if n+11 is not even then n is not odd. This proves the Contrapositive of the given statement.

③ <u>Contradiction</u>:

Assume that the given statement is false. That is, assume that n is odd and n+11 is odd. Since n+11 is odd, $n+11 = 2k+1$ for some integer k.

$$n = (2k+1)-11$$
$$= 2(k-5) \Rightarrow n \text{ is even.}$$

then n is odd ⇒ Contradiction.

) proof

$\underbrace{[(p \vee q) \wedge (\sim p \vee r)]}_{p} \rightarrow \underbrace{(q \vee r)}_{q}$ is a tautology. (26)

integr

$\sim \{(p \vee q) \wedge (\sim p \vee r)\} \vee (q \vee r)$ | $p \rightarrow q = \sim p \vee q$.

$[(\sim p \wedge \sim q) \vee (p \wedge \sim r)] \vee (q \vee r)$ | Demorgan's law.

$(\sim p \wedge \sim q) \vee [(p \wedge \sim r) \vee (q \vee r)]$ | Associative law.

$(\sim p \wedge \sim q) \vee \{(q \vee r) \vee (p \wedge \sim r)\}$ | commutative law

$(\sim p \wedge \sim q) \vee [(q \vee r \vee p) \wedge (q \vee r \vee \sim r)$ | Distributive law.

$(\sim p \wedge \sim q) \vee (q \vee r \vee p) \wedge (q \vee T))$ | $p \vee \sim p \equiv T$

$(\sim p \wedge \sim q) \vee (q \vee r \vee p) \wedge T)$ | $p \vee T \equiv T$

$(\sim p \wedge \sim q) \vee (q \vee r \vee p)$ | $p \wedge T = p$

$(\sim p \wedge \sim q) \vee (q \vee r \vee p)$

$\sim (p \vee q) \vee ((p \vee q) \vee r)$ | Demorgan's

$[\sim (p \vee q) \vee \{p \vee q\}] \vee r$ | Associative

$T \vee r$.

$\equiv T$

⇒ Prove or disprove that the sum of squares of non-zero integers is an even integer.

Proof "for any four non-zero integers a, b, c, d $a^2+b^2+c^2+d^2$ is an even integer"

a=1 b=1 c=1 d=2 then the proposition is false

- Thus the given proposition is disproved.

⇒ Normal forms

Given a compound proposition u, suppose we obtain a compound proposition v such that $u \equiv v$ and v is a disjunction of 2/ more compound propositions each of which is a conjunction involving the components of u or their negations. This v is called a disjunctive normal form of v.

⇒ Similarly, given a compound proposition u, if we obtain an equivalent compound proposition v which is a conjunction of 2 or more compound propositions each of which is a disjunction involving the components of u or their negations, then v is called a conjuctive normal form of u.

⇒ The disjuctive/conjunctive normal form of a compound proposition is not unique.

Example :

(1) $p \wedge (p \to q)$

$$p \wedge (p \to q) \equiv p \wedge (\sim p \vee q)$$

$$\equiv (p \wedge \sim p) \vee (p \wedge q)$$

(or)

that the compound proposition in the right hand side is the disjunction of $(p \wedge \sim p)$ and $(p \wedge q)$ each of which is a conjunction involving p or q or their negation. This is called disjunctive normal form

(2) $\sim (p \vee q) \leftrightarrow (p \wedge q)$

$r = \sim (p \vee q)$ $s = p \wedge q$

$$r \leftrightarrow s \Rightarrow (r \to s) \wedge (s \to r)$$

$$\equiv (\sim r \vee s) \wedge (\sim s \vee r)$$

$$= [(\sim r \vee s) \wedge \sim s] \vee [(\sim r \vee s) \wedge r]$$

$$= [(\sim r \wedge \sim s) \vee (s \wedge \sim s)] \vee [(\sim r \wedge r) \vee (s \wedge r)]$$

$$\equiv [(\sim r \wedge \sim s) \vee f_0] \vee [f_0 \vee (s \wedge r)]$$

$$= (\sim r \wedge \sim s) \vee (s \wedge r)$$

$$\equiv (r \wedge s) \vee (\sim r \wedge \sim s)$$

$$r \wedge s \equiv (\sim p \wedge \sim q) \wedge (p \wedge q)$$

$$= (\sim p \wedge q) \wedge (q \wedge \sim q)$$

$$\equiv f_0 \wedge f_0 \equiv F$$

$\sim r \wedge \sim s \equiv \sim r \wedge (\sim p \vee \sim q)$

$\equiv (\sim r \wedge \sim p) \vee (\sim r \wedge \sim q)$

$\equiv \{(p \vee q) \wedge \sim p\} \vee \{(p \vee q) \wedge \sim q\}$

$\equiv \{(p \wedge \sim p) \vee (q \wedge \sim p)\} \vee \{(p \wedge \sim q) \vee (q \wedge \sim q)\}$

$\equiv \{f_0 \vee (q \wedge \sim p)\} \vee \{(p \wedge \sim q) \vee f\}$

$\equiv \{(q \wedge \sim p) \vee (p \wedge \sim q)\}$

$\equiv f_0 \vee \{(q \wedge \sim p) \vee (p \wedge \sim q)\}$

$\equiv (q \wedge \sim p) \vee (p \wedge \sim q)$

disjunctive normar form.

$\Rightarrow$ (1) $p \wedge (p \rightarrow q)$

$\equiv p \wedge (\sim p \vee q)$

$\equiv (p \vee p) \wedge (\sim p \vee q)$

$\Rightarrow$ The Compound proposition in the right hand side is the Conjunction of propositions each of which is a disjunction involving the Components p and q or their negations. This is <u>Conjunctive normal form</u>