

# Bayesian statistics with R

## 6. Bayesian analyses in R with the Jags software

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## Bayes in practice

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## Software implementation (R friendly)

Oldies but goodies:

- WinBUGS, OpenBUGS: Where it all began.
- Jags: What we will use in this course.

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If you're not into coding:

- brms: Bayesian regression models with Stan.
- MCMCglmm: Generalised Linear Mixed Models.
- Check out the CRAN Task View: Bayesian Inference for more.

# Introduction to JAGS (Just Another Gibbs Sampler)

Martyn Plummer



## Real example

Impact of climatic conditions on white stork breeding success



mangl.at

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- We need to build a model - write down the likelihood.
- We need to specify priors for parameters.

## Read in the data

```
nbchicks <- c(151,105,73,107,113,87,77,108,118,122,112,120,122,89,69,71,
              53,41,53,31,35,14,18)

nbpairs <- c(173,164,103,113,122,112,98,121,132,136,133,137,145,117,90,80,
             67,54,58,39,42,23,23)

temp <- c(15.1,13.3,15.3,13.3,14.6,15.6,13.1,13.1,15.0,11.7,15.3,14.4,14.4,
          12.7,11.7,11.9,15.9,13.4,14.0,13.9,12.9,15.1,13.0)

rain <- c(67,52,88,61,32,36,72,43,92,32,86,28,57,55,66,26,28,96,48,90,86,
          78,87)

datax <- list(N = 23, nbchicks = nbchicks, nbpairs = nbpairs,
              temp = (temp - mean(temp))/sd(temp),
              rain = (rain - mean(rain))/sd(rain))
```

## Write down the model

$$\text{nbchicks}_i \sim \text{Binomial}(\text{nbpairs}_i, p_i) \quad [\text{likelihood}]$$

$$\text{logit}(p_i) = a + b_{\text{temp}} \text{temp}_i + b_{\text{rain}} \text{rain}_i \quad [\text{linear model}]$$

$$a \sim \text{Normal}(0, 1000) \quad [\text{prior for } a]$$

$$b_{\text{temp}} \sim \text{Normal}(0, 1000) \quad [\text{prior for } b_{\text{temp}}]$$

$$b_{\text{rain}} \sim \text{Normal}(0, 1000) \quad [\text{prior for } b_{\text{rain}}]$$

# Build the model

```
{  
# Likelihood  
  for( i in 1 : N){  
    nbchicks[i] ~ dbin(p[i],nbpairs[i])  
    logit(p[i]) <- a + b.temp * temp[i] + b.rain * rain[i]  
  }  
# ...
```

## Specify priors

```
# Priors  
a ~ dnorm(0,0.001)  
b.temp ~ dnorm(0,0.001)  
b.rain ~ dnorm(0,0.001)  
}
```

**Warning:** Jags uses precision for Normal distributions (1 / variance)

## You need to write everything in a file

```
model <-  
paste("  
model  
{  
  for( i in 1 : N)  
  {  
    nbchicks[i] ~ dbin(p[i],nbpairs[i])  
    logit(p[i]) <- a + b.temp * temp[i] + b.rain * rain[i]  
  }  
a ~ dnorm(0,0.001)  
b.temp ~ dnorm(0,0.001)  
b.rain ~ dnorm(0,0.001)  
}  
")  
writeLines(model,"code/logistic.txt")
```



## Alternatively, you may write a R function

```
logistic <- function() {  
  for( i in 1 : N)  
  {  
    nbchicks[i] ~ dbin(p[i], nbpairs[i])  
    logit(p[i]) <- a + b.temp * temp[i] + b.rain * rain[i]  
  }  
  
  # priors for regression parameters  
  a ~ dnorm(0, 0.001)  
  b.temp ~ dnorm(0, 0.001)  
  b.rain ~ dnorm(0, 0.001)  
}
```

## Let us specify a few additional things

```
# list of lists of initial values (one for each MCMC chain)
init1 <- list(a = -0.5, b.temp = -0.5, b.rain = -0.5)
init2 <- list(a = 0.5, b.temp = 0.5, b.rain = 0.5)
inits <- list(init1,init2)

# specify parameters that need to be estimated
parameters <- c("a","b.temp","b.rain")

# specify nb iterations for burn-in and final inference
nb.burnin <- 1000
nb.iterations <- 2000
```

# Run Jags

```
# load R2jags
library(R2jags)
# run Jags
storks <- jags(data = datax,
               inits = inits,
               parameters.to.save = parameters,
               model.file = "code/logistic.txt",
               # model.file = logistic, # if a function was written
               n.chains = 2,
               n.iter = nb.iterations,
               n.burnin = nb.burnin)

storks
```

## Inspect parameter estimates

```
#> Compiling model graph
#>   Resolving undeclared variables
#>   Allocating nodes
#> Graph information:
#>   Observed stochastic nodes: 23
#>   Unobserved stochastic nodes: 3
#>   Total graph size: 181
#>
#> Initializing model
#> Inference for Bugs model at "code/logistic.txt", fit using jags,
#> 2 chains, each with 2000 iterations (first 1000 discarded)
#> n.sims = 2000 iterations saved
```

#>	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
#> a	1.552	0.088	1.432	1.515	1.557	1.599	1.669	1.182	330
#> b.rain	-0.160	0.070	-0.297	-0.206	-0.158	-0.115	-0.037	1.058	46
#> b.temp	0.025	0.062	-0.105	-0.011	0.028	0.064	0.138	1.049	37
#> deviance	206.973	31.728	201.791	202.984	204.308	206.290	213.802	1.073	2000

## **Your turn: Practical 5**

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## Assess convergence

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## Reminder – MCMC Algorithm

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- For the MCMC algorithm, the posterior distribution is only needed to be known up to proportionality.
- Once the stationary distribution is reached we can regard the realisations of the chain as a (dependent) sample from the posterior distribution (and obtain Monte Carlo estimates).
- We consider some important implementation issues.

## MCMC – Proposal Distribution

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- This typically involves
  - specifying a given distribution family (e.g. normal, uniform), and then,
  - setting the parameters of the given distribution.
- Although the exact distribution specified is essentially arbitrary – it will have a significant effect on the performance of the MCMC algorithm.

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- In order to balance the size of the proposed moves with the chance of accepting them the proposal variance is often tuned to obtain a mean acceptance probability of 20 – 40%.



## Why is the proposal distribution so important?

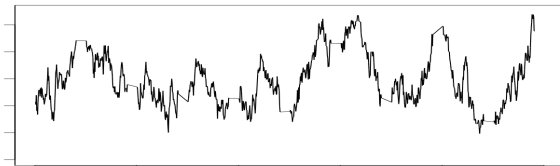
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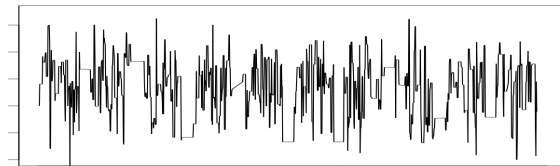
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- Automatic in Jags – ouf!
- The movement around the parameter space is often referred to as **mixing**.

## Good/Bad Traces

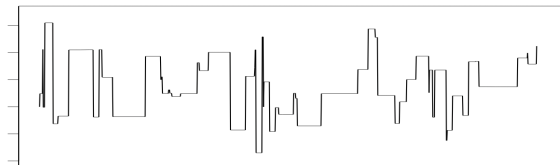
*Small  
moves -  
bad*



*good*



*Large  
moves -  
bad*



0

500

1000

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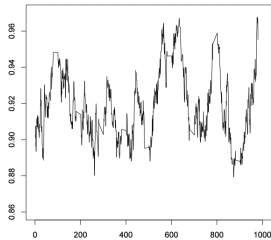
## Autocorrelation functions

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- Autocorrelation function (ACF) plots are a convenient way of displaying the strength of autocorrelation in the given sample values.

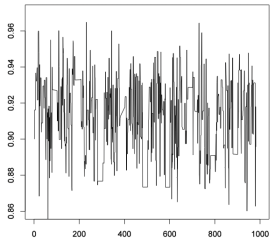
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- Autocorrelation function (ACF) plots are a convenient way of displaying the strength of autocorrelation in the given sample values.
- ACF plots provide the autocorrelation between successively sampled values separated by  $k$  iterations, referred to as lag, (i.e.  $\text{cor}(\theta_t, \theta_{t+k})$ ) for increasing values of  $k$ .

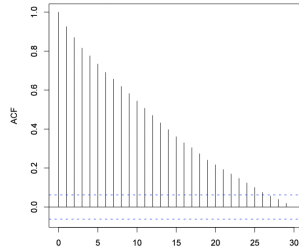
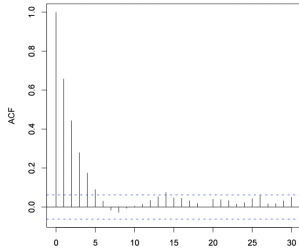
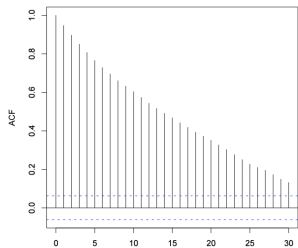
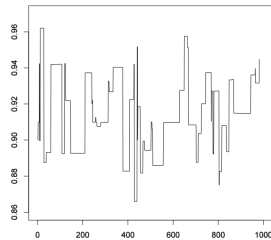
*Small moves*



*OK*



*Big moves*

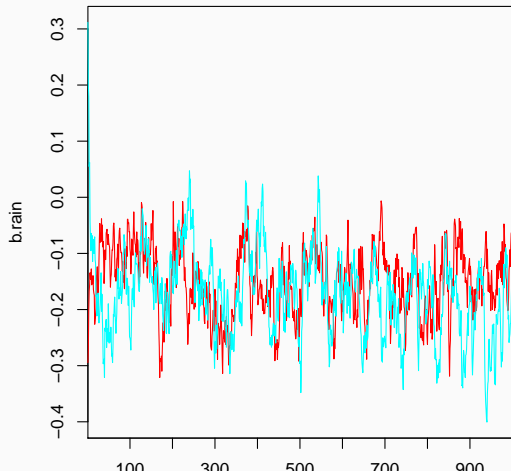




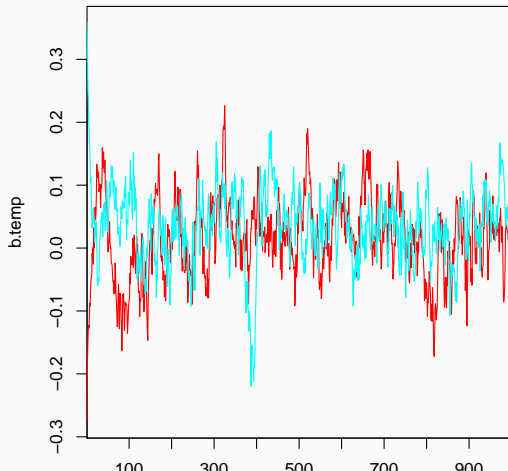
# Traceplots for the storks

```
traceplot(storks,mfrow = c(1, 2), varname = c('b.rain','b.temp'), ask = FALSE)
```

**b.rain**

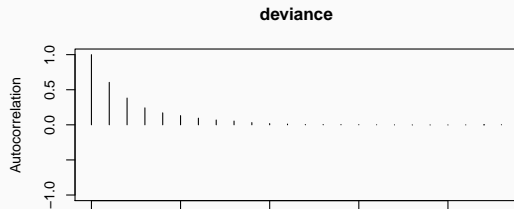
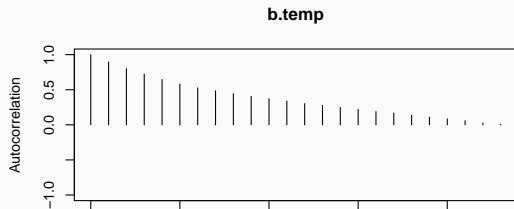
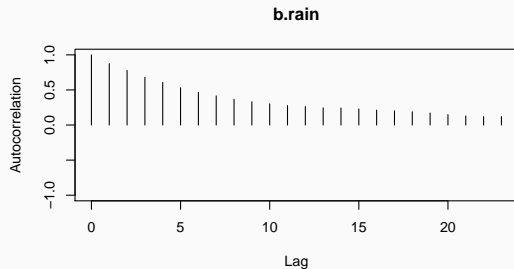
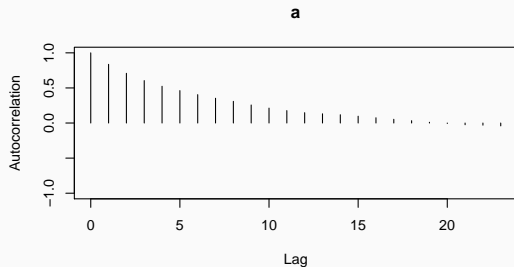


**b.temp**



# Autocorrelation for the storks

```
autocorr.plot(as.mcmc(storks), ask = FALSE)
```



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- Once there, explore efficiently: The post-convergence sample size required for suitable numerical summaries.

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- In practice, we must discard observations from the start of the chain and just use observations from the chain once it has converged.
- The initial observations that we discard are referred to as the **burn-in**.
- The simplest method to determine the length of the burn-in period is to look at trace plots.

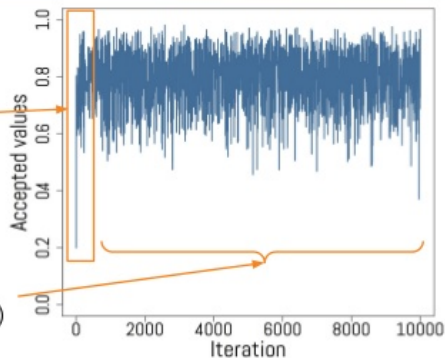


## Burn-in (if simulations cheap, be conservative)

Discard initial guesses that are still far from optimum: the

**BURN-IN**

These numbers should be a good  
**sample of the Posterior**  $P(\phi \mid \text{data})$



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- The effective sample size ( $n_{\text{eff}}$ ) measures chain length while taking into account the autocorrelation of the chain.
  - $n_{\text{eff}}$  is less than the number of MCMC iterations.
  - Check the  $n_{\text{eff}}$  of every parameter of interest.
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  - Check the  $n_{\text{eff}}$  of any interesting parameter combinations.
- We need  $n_{\text{eff}} \geq 100$  independent steps.

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## Potential scale reduction factor

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- Measures the ratio of the total variability combining multiple chains (between-chain plus within-chain) to the within-chain variability. Asks the question is there a chain effect? Very much alike the  $F$  test in an ANOVA.



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- Values near 1 indicates likely convergence, a value of  $\leq 1.1$  is considered acceptable.
- Necessary condition, not sufficient; In other words, these diagnostics cannot tell you that you have converged for sure, only that you have not.

## n.eff and $\hat{R}$ for the storks

storks

```
#> Inference for Bugs model at "code/logistic.txt", fit using jags,
#> 2 chains, each with 2000 iterations (first 1000 discarded)
#> n.sims = 2000 iterations saved
#>      mu.vect sd.vect   2.5%   25%   50%   75%  97.5% Rhat n.eff
#> a      1.552  0.088   1.432   1.515   1.557   1.599   1.669 1.182  330
#> b.rain  -0.160  0.070  -0.297  -0.206  -0.158  -0.115  -0.037 1.058   46
#> b.temp   0.025  0.062  -0.105  -0.011   0.028   0.064   0.138 1.049   37
#> deviance 206.973 31.728 201.791 202.984 204.308 206.290 213.802 1.073 2000
#>
#> For each parameter, n.eff is a crude measure of effective sample size,
#> and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
#>
#> DIC info (using the rule, pD = var(deviance)/2)
#> pD = 503.4 and DIC = 710.4
#> DIC is an estimate of expected predictive error (lower deviance is better).
```

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- Run multiple chains from arbitrary starting places (initial values).
- Assume convergence when all chains reach same regime.
- Discard initial burn-in phase.
- Check autocorrelation, effective sample size and  $\hat{R}$ .

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- Reparameterize:
  - Standardize covariates.
  - Non-centering:  $\alpha \sim N(0, \sigma)$  becomes  $\alpha = z\sigma$  with  $z \sim N(0, 1)$ .

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- Something wrong with your model?
  - Start with a simpler model (remove complexities).
  - Use simulations.
- Change your sampler. Upgrade to Nimble or Stan.

**MCMC makes you queens and kings  
of the stats world**

---

## Get all values sampled from posteriors

```
res <- as.mcmc(storks) # convert outputs in a list
res <- rbind(res[[1]],res[[2]]) # put two MCMC lists on top of each other
head(res)

#>           a      b.rain      b.temp deviance
#> [1,] 0.05598852 -0.2944525 -0.2766720 1226.7430
#> [2,] 0.36816443 -0.2730678 -0.1820607  807.5734
#> [3,] 0.59604210 -0.1998024 -0.1478220  582.4418
#> [4,] 0.80826012 -0.1672283 -0.1263000  426.2800
#> [5,] 0.94492500 -0.1562201 -0.1231841  351.0118
#> [6,] 1.00817027 -0.1345293 -0.1270278  323.0022
tail(res)

#>           a      b.rain      b.temp deviance
#> [1995,] 1.463764 -0.2257593 0.1137950 206.4351
#> [1996,] 1.468269 -0.2241381 0.1760809 209.9852
#> [1997,] 1.446378 -0.1964248 0.1337033 208.1105
#> [1998,] 1.461178 -0.1576292 0.1465739 208.4039
#> [1999,] 1.479516 -0.1874332 0.1343823 206.2762
```

## Compute a posteriori $\Pr(\text{rain} < 0)$

```
# probability that the effect of rainfall is negative  
mean(res[, 'b.rain'] < 0)  
#> [1] 0.991
```

## Compute a posteriori $\Pr(\text{temp} < 0)$

```
# probability that the effect of temperature is negative  
mean(res[, 'b.temp'] < 0)  
#> [1] 0.306
```



## Get credible interval for the rain effect

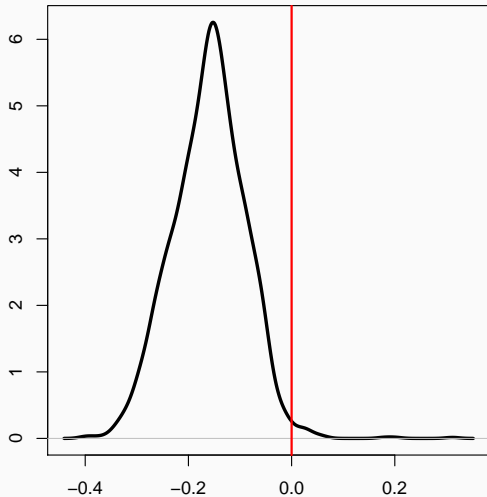
```
quantile(res[, 'b.rain'], c(0.025, 0.975))  
#>           2.5%           97.5%  
#> -0.29661171 -0.03665086
```

## Get credible interval for the temperature effect

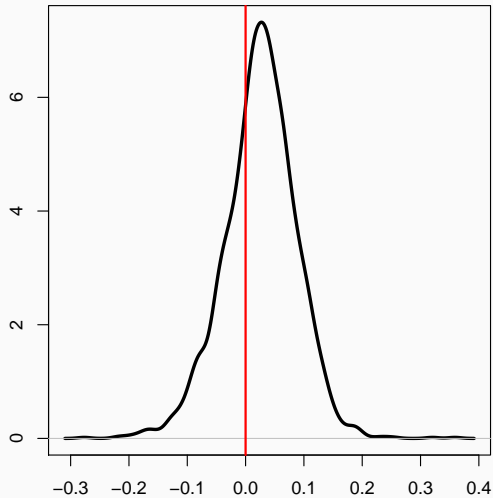
```
quantile(res[, 'b.temp'], c(0.025, 0.975))  
#>      2.5%      97.5%  
#> -0.1051933  0.1376384
```

# Graphical summaries

**Rainfall**



**Temperature**



## **Your turn: Practical 6**

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