

# Bayesian statistics with R

## 4. A detour with priors

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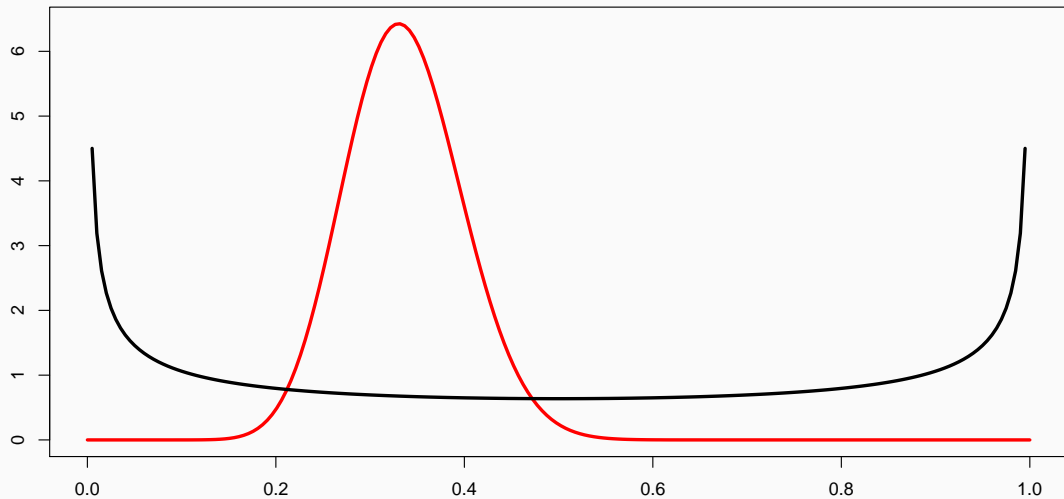
## A detour to explore priors

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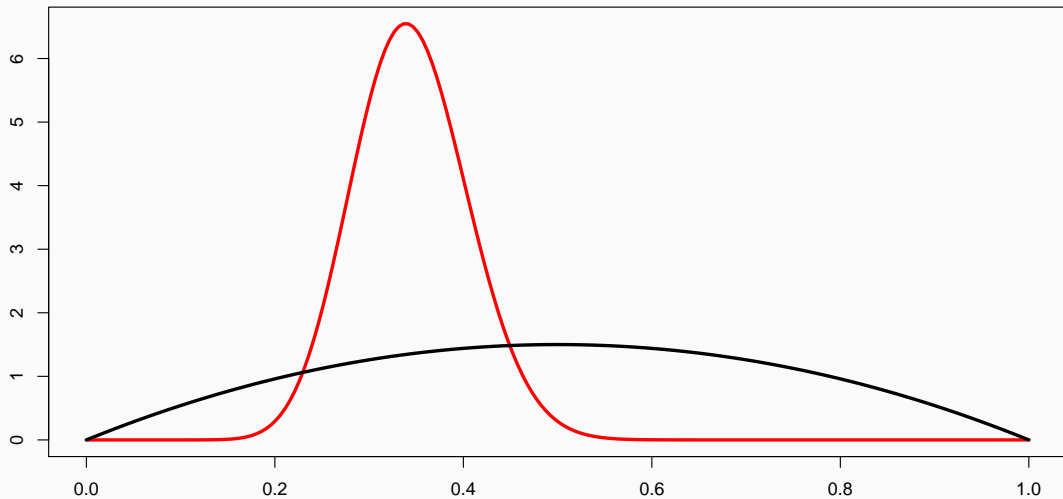
## Influence of the prior

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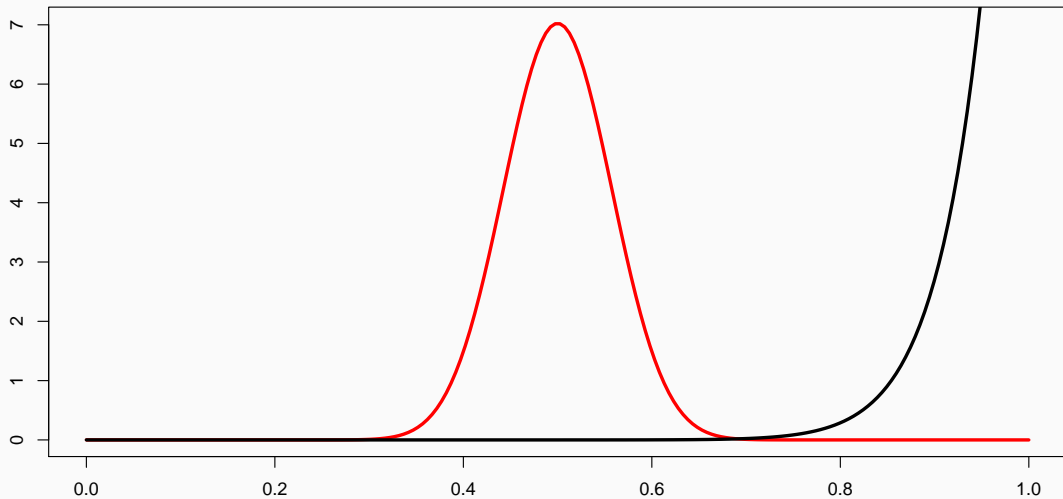
## Prior $Beta(0.5, 0.5)$ and posterior survival $Beta(19.5, 38.5)$



## Prior $Beta(2, 2)$ and posterior survival $Beta(21, 40)$



## Prior $Beta(20, 1)$ and posterior survival $Beta(39, 49)$



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- With sufficiently large and informative datasets the prior typically has little effect on the results.
- Always perform a sensitivity analysis.

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WE ARE ALL BAYESIANS,...

Based on my priors  
I will be just fine

A black and white cartoon illustration. A man is shown falling backwards, his body arched in mid-air. He is wearing a long-sleeved shirt, trousers, and shoes. His arms are outstretched, and his legs are bent. Above his head is a thought bubble containing the text "Based on my priors I will be just fine". The background is simple, with some vertical lines suggesting a ground surface.

## How to incorporate prior information?

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- We assume a vague prior:

$$\phi_{prior} \sim \text{Beta}(1, 1) = \text{Uniform}(0, 1)$$

# Notation

- $y_{i,t} = 1$  if individual  $i$  detected at occasion  $t$  and 0 otherwise
- $z_{i,t} = 1$  if individual  $i$  alive between occasions  $t$  and  $t + 1$  and 0 otherwise

$$y_{i,t} \mid z_{i,t} \sim \text{Bernoulli}(p \mid z_{i,t}) \quad [\text{likelihood (observation eq.)}]$$

$$z_{i,t+1} \mid z_{i,t} \sim \text{Bernoulli}(\phi \mid z_{i,t}) \quad [\text{likelihood (state eq.)}]$$

$$\phi \sim \text{Beta}(1, 1) \quad [\text{prior for } \phi]$$

$$p \sim \text{Beta}(1, 1) \quad [\text{prior for } p]$$

## European dippers in Eastern France (1981-1987)



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- Mean posterior  $\phi_{posterior} = 0.56$  with credible interval  $[0.52, 0.60]$ .
- No increase of precision in posterior inference.

## How to incorporate prior information?

- Now if you had only the three first years of data, what would have happened?

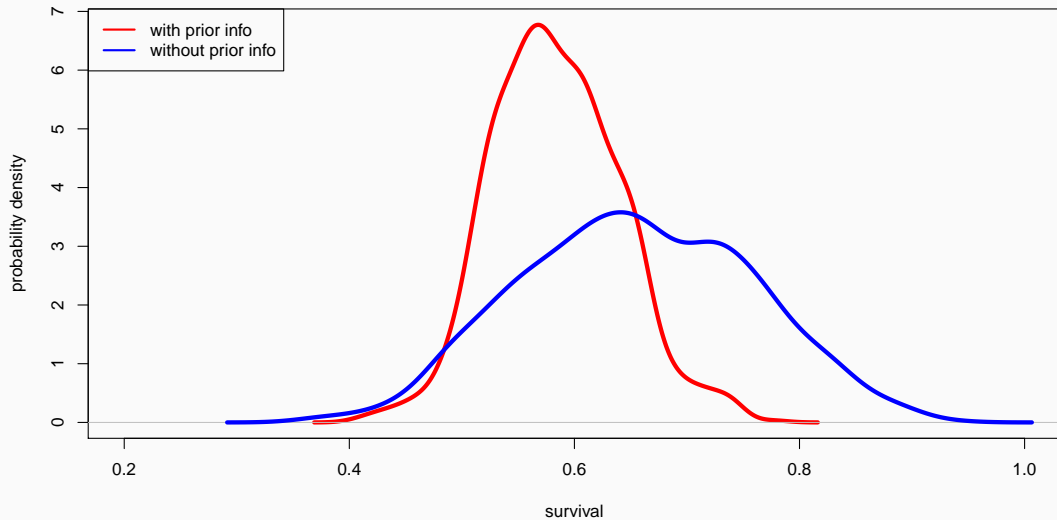
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- Now if you had only the three first years of data, what would have happened?
- Width of credible interval is 0.47 (vague prior) vs. 0.30 (informative prior).
- Huge increase of precision in posterior inference (40% gain)!

## Compare **vague** vs. **informative** prior



## Prior elicitation via moment matching

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## Remember the Beta distribution

- Recall that the Beta distribution is a continuous distribution with values between 0 and 1. Useful for modelling survival or detection probabilities.

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- If  $X \sim \text{Beta}(\alpha, \beta)$ , then the first and second moments of  $X$  are:

$$\mu = E(X) = \frac{\alpha}{\alpha + \beta}$$

$$\sigma^2 = \text{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

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- Parameters  $\mu$  and  $\sigma^2$  are seen as the moments of a  $Beta(\alpha, \beta)$  distribution.
- Now we look for values of  $\alpha$  and  $\beta$  that match the observed moments of the Beta distribution ( $\mu$  and  $\sigma^2$ ).
- We need another set of equations:

$$\alpha = \left( \frac{1 - \mu}{\sigma^2} - \frac{1}{\mu} \right) \mu^2$$

$$\beta = \alpha \left( \frac{1}{\mu} - 1 \right)$$

- For our model, that means:

```
(alpha <- ( (1 - 0.57)/(0.073*0.073) - (1/0.57) )*0.57^2)
#> [1] 25.64636
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```

- Now use  $\phi_{prior} \sim \text{Beta}(\alpha = 25.6, \beta = 19.3)$  instead of  
 $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$



## Your turn: Practical 3

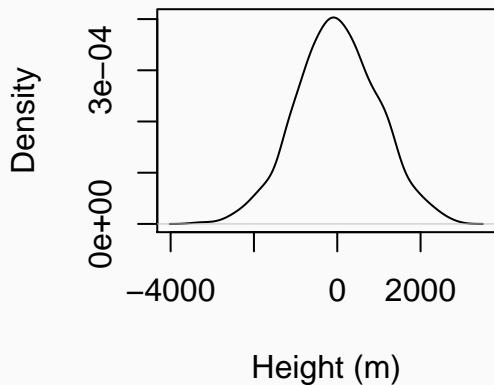
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## Prior predictive checks

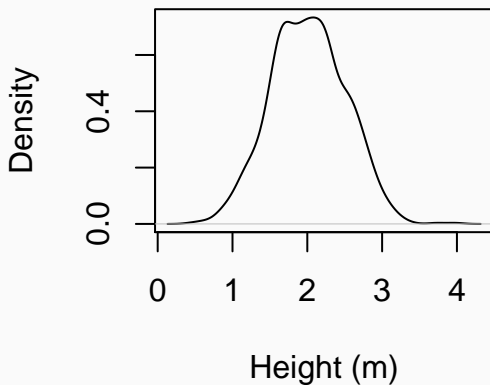
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# Linear regression

Unreasonable prior  $\beta \sim N(0, 1000^2)$

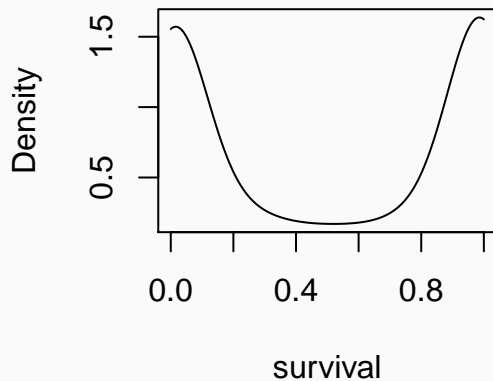


Reasonable prior  $\beta \sim N(2, 0.5^2)$

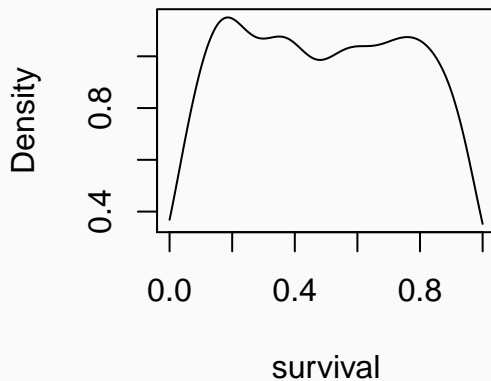


# Logistic regression

Unreasonable  $\text{logit}(\phi) = \beta \sim N(0, 10^2)$



Reasonable  $\text{logit}(\phi) = \beta \sim N(0, 1.5^2)$



## Your turn: Practical 4

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## Dynamic updating

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If you obtain more data, no need to redo all of the analysis. Your posterior from the first analysis simply becomes your prior for the next analysis (and so on).

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- Stage 0. Prior  $\Pr(\theta) \sim \text{Beta}(1, 1)$ .
- Stage 1. Observe  $y_1 = 22$  successes from  $n_1 = 29$  trials.
  - Likelihood is  $\Pr(y_1|\theta) \sim \text{Binomial}(n_1 = 29, \theta)$ .
  - Posterior is  $\Pr(\theta|y_1) \sim \text{Beta}(23, 8)$  with mean  $23/31 = 0.74$ .

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- Stage 2. Observe  $y_2 = 5$  succeeded from  $n_2 = 10$  new trials.
  - Likelihood is  $\Pr(y_2|\theta) \sim \text{Binomial}(n_2 = 10, \theta)$ .
  - Prior is  $\Pr(\theta) \sim \text{Beta}(23, 8)$  from stage 1.
  - Posterior is  $\Pr(\theta|y_1 \text{ and } y_2) \propto \Pr(\theta|y_1) \Pr(y_2|\theta) = \text{Beta}(28, 13)$  with mean  $28/41 = 0.68$ .