Bayesian statistics with R

8. Heterogeneity and multilevel models (aka mixed models)

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Multilevel (aka mixed-effect) models

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- Measurement that come in clusters or groups.
- Come up with examples of clusters or groups.

Clusters might be:

- Classrooms within schools
- Students within classrooms
- Chapters within books
- Individuals within populations
- Populations within species
- Trajectories within individuals
- Fishes within tanks
- Frogs within ponds
- PhD applicants in doctoral schools
- Nations in continents
- Sex or age are not clusters per se (if we were to sample again, we would take the same levels, e.g. male/female and young/old)

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- Interested in variance components (environmental vs. genetic variance).
- Control for bias due to pseudoreplication (time, space, individual).

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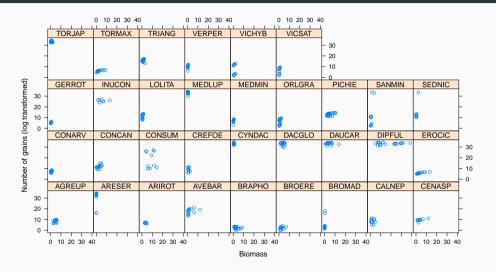
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- No information passed among clusters.
- Multilevel models remember and pool information. They have memory.
- Properties of clusters come from a population.
- If previous clusters improve your guess about a new cluster, you want to use pooling.

Plant experiment in the field at CEFE



Courtesy of Pr Eleni Kazakou

Number of grains per species (cluster) as a function of biomass



GLM with complete pooling

$$\begin{aligned} \mathsf{Y}_i &\sim \mathsf{Distribution}(\mathsf{mean}_i) & & & [\mathsf{likelihood}] \\ \mathsf{link}(\mathsf{mean})_i &= \alpha + \beta \; x_i & & & [\mathsf{linear} \; \mathsf{model}] \\ &\alpha &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & [\mathsf{prior} \; \mathsf{for} \; \mathsf{intercept}] \\ &\beta &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & [\mathsf{prior} \; \mathsf{for} \; \mathsf{slope}] \end{aligned}$$

Model with complete pooling. All clusters the same.

GLM with no pooling

Model with no pooling. All clusters unrelated (fixed effect).

GLMM or **GLM** with partial pooling

$$\begin{aligned} \mathsf{Y}_i &\sim \mathsf{Distribution}(\mathsf{mean}_i) & & & & & & \\ \mathsf{link}(\mathsf{mean})_i &= \alpha_{\mathsf{CLUSTER}[i]} + \beta \; x_i & & & & & & \\ \alpha_j &\sim \mathsf{Normal}(\bar{\alpha},\sigma) & & & & & & & \\ \bar{\alpha} &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & & & & & \\ \sigma &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & & & & \\ \beta &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{determined} & \\ \rho &\sim \mathsf{to} \; \mathsf{be} \; \mathsf{dete$$

Model with partial pooling. Clusters are somehow related (random effect).

Back to the plant example

Model with complete pooling (all species are the same)

[likelihood]	$nseeds_i \sim Normal(\mu_i, \sigma^2)$
[linear model]	$\mu_i = \alpha + \beta \text{ biomass}_i$
[prior for intercept]	$lpha \sim Normal(0, 1000)$
[prior for slope]	$eta \sim Normal(0, 1000)$
[prior for standard deviation]	$\sigma \sim Uniform(0,100)$

Read in and manipulate data

nhanecies <- length(levels(Sn))

```
# read in data
VMG <- read csv2(here::here("slides","dat","VMG.csv")) %>%
  mutate(Sp = as_factor(Sp), Vm = as.numeric(Vm))
# nb of seeds
v <- log(VMG$NGrTotest)</pre>
# biomass
x <- VMG$Vm
x \leftarrow (x - mean(x))/sd(x)
# species name
Sp <- VMG$Sp
# species label
species <- as.numeric(Sp)</pre>
# species name
```

Specify the model in Jags

```
model <-
paste("
model{
for(i in 1:n){
    y[i] ~ dnorm(mu[i], tau.y)
    mu[i] <-a+b*x[i]
tau.y <- 1 / (sigma.y * sigma.y)</pre>
sigma.y ~ dunif(0,100)
a \sim dnorm(0, 0.001)
b \sim dnorm(0, 0.001)
")
```

writelines (model here: here ("slides" "code" "complete modling hug"))

Prepare ingredients for running Jags

```
\# d.a.t.a.
allom.data \leftarrow list(y = y, n = n, x = x)
# initial values
init1 <- list(a=rnorm(1), b=rnorm(1), sigma.y=runif(1))</pre>
init2 <- list(a=rnorm(1), b=rnorm(1), sigma.y=runif(1))</pre>
inits <- list(init1.init2)</pre>
# parameters to be estimated
allom.parameters <- c("a", "b", "sigma.y")</pre>
```

Run Jags

#>

#>

```
allom.1 <- jags(allom.data,
                inits,
                allom.parameters,
                n.iter = 2500,
                model.file = here::here("slides","code","completepooling.b
                n.chains = 2,
                n.burn = 1000
#> Compiling model graph
#>
     Resolving undeclared variables
#>
     Allocating nodes
#> Graph information:
      Observed stochastic nodes: 488
#>
```

Unobserved stochastic nodes: 3

Total amount circo 1056

Display results

#> deviance 3000

allom.1 #> Inference for Bugs model at "/Users/oliviergimenez/Dropbox/OG/GITHUB/ba 2 chains, each with 2500 iterations (first 1000 discarded) # n.sims = 3000 iterations saved #> mu.vect sd.vect 2.5% 25% 50% 75% 97.5% #> a 13.927 13.615 13.929 14.245 14.825

0.464 13.009 0.473 2.662 3.269 3.582 3.898 #> b 3.581 #> sigma.y 10.431 0.346 9.803 10.205 10.417 10.651 #> deviance 3672.061 2.533 3669.221 3670.245 3671.432 3673.199 3678.795 #> n.eff#> a 3000 #> b 3000 #> sigma.y 3000

16

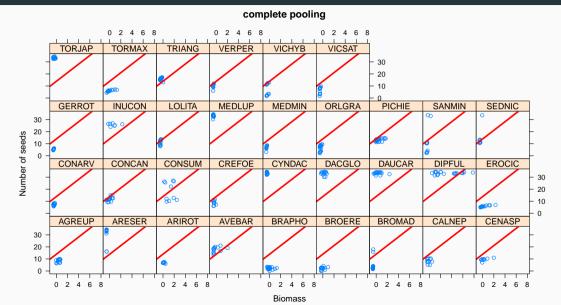
4.491

11.156

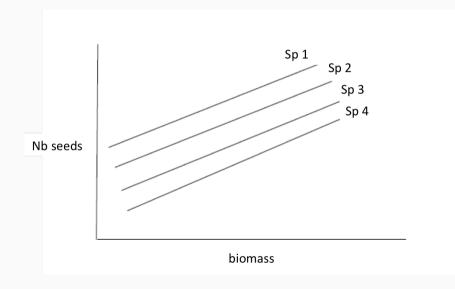
Compare with Frequentist approach

```
freq_lm <- lm(y \sim x, data = allom.data)
freq_lm
#>
#> Call:
\# lm(formula = y \sim x, data = allom.data)
#>
#> Coefficients:
#> (Intercept)
#> 13.927 3.578
```

Output



Model with partial pooling (species random effect)



Model with partial pooling (all species related in some way)

$nseeds_i \sim Normal(\mu_i, \sigma^2)$	[likelihood]
$\mu_i = lpha_{species[i]} + eta$ biomass $_i$	[linear model]
$lpha_j \sim Normal(ar{lpha}, \sigma_lpha)$	[prior for varying intercepts]
$ar{lpha} \sim Normal(0, 1000)$	[prior for population mean]
$\sigma_{lpha} \sim {\sf Uniform}(0,100)$	[prior for σ_{lpha}]
$eta \sim Normal(0, 1000)$	[prior for slope]
$\sigma \sim Uniform(0,100)$	[prior for σ]

Implementation in Jags

```
model <- paste("</pre>
model {
  for (i in 1:n){
    y[i] ~ dnorm(mu[i], tau.y)
    mu[i] \leftarrow a[species[i]] + b * x[i]
  tau.y <- 1/ (sigma.y * sigma.y)</pre>
  sigma.v \sim dunif(0, 100)
  for (j in 1:nbspecies){
    a[j] ~ dnorm(mu.a, tau.a)
  mu.a \sim dnorm(0, 0.001)
  tau.a <- 1/(sigma.a * sigma.a)
  giorn = a = dunif(0 100)
```

Prepare ingredients for running Jags

```
allom.data \leftarrow list(n = n.
                    nbspecies = nbspecies,
                    x = x
                    v = v
                    species = species)
init1 <- list(a = rnorm(nbspecies), b = rnorm(1), mu.a = rnorm(1),
               sigma.y = runif(1), sigma.a=runif(1))
init2 <- list(a = rnorm(nbspecies), b = rnorm(1), mu.a = rnorm(1),
               sigma.y = runif(1), sigma.a = runif(1))
inits <- list(init1,init2)</pre>
allom.parameters <- c("b", "mu.a", "sigma.y", "sigma.a")</pre>
```

Run Jags

#>

```
allom.2 <- jags(allom.data,
                inits,
                allom.parameters,
                n.iter = 2500,
                model.file = here::here("slides", "code", "varint.bug"),
                n.chains = 2,
                n.burn = 1000
#> Compiling model graph
#>
      Resolving undeclared variables
#>
      Allocating nodes
#> Graph information:
      Observed stochastic nodes: 488
#>
#>
      Unobserved stochastic nodes: 37
```

Total amount ciaco 2/8/

Display results

allom.2

#>

#> b

#> mu.a

```
#> Inference for Bugs model at "/Users/oliviergimenez/Dropbox/OG/GITHUB/ba
```

2 chains, each with 2500 iterations (first 1000 discarded)

#>
$$mu.vect sa.vect 2.5\%$$
 25% 50%

#> siama a 1200

n.eff

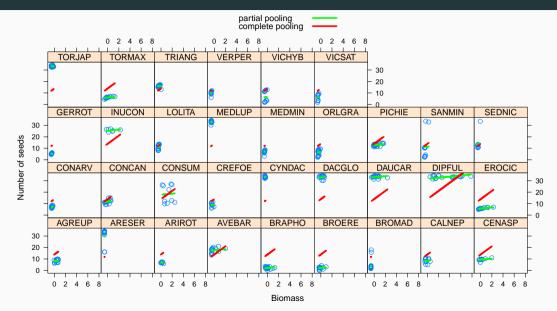
97.5%

0.938

Compare with Frequentist approach

```
library(lme4)
freq lmm <- lmer(y ~ x + (1 | species), allom.data, REML = FALSE)
freq 1mm
#> Linear mixed model fit by maximum likelihood ['lmerMod']
\# Formula: y \sim x + (1 \mid species)
#> Data: allom.data
#>
        AIC BIC logLik deviance df.resid
#> 2652.606 2669.368 -1322.303 2644.606 484
#> Random effects:
#> Groups Name Std.Dev.
#> species (Intercept) 10.472
#> Residual 3.058
#> Number of obs: 488, groups: species, 33
#> Finad Efforts.
```

Compare complete pooling vs partial pooling



Model with no pooling (all species unrelated)

```
\begin{split} \operatorname{nseeds}_i &\sim \operatorname{Normal}(\mu_i, \sigma^2) & \text{[likelihood]} \\ \mu_i &= \alpha_{\operatorname{species}[i]} + \beta \operatorname{biomass}_i & \text{[linear model]} \\ \alpha_j &\sim \operatorname{Normal}(0, 1000) & \text{[prior for intercepts]} \\ \beta &\sim \operatorname{Normal}(0, 1000) & \text{[prior for slope]} \\ \sigma &\sim \operatorname{Uniform}(0, 100) & \text{[prior for} \sigma \end{split}
```

Implementation in Jags

```
model <- paste("</pre>
model {
  for (i in 1:n){
    v[i] ~ dnorm (mu[i], tau.y)
    mu[i] \leftarrow a[species[i]] + b * x[i]
  tau.y <- 1 / (sigma.y * sigma.y)</pre>
  sigma.v \sim dunif(0, 100)
  for (j in 1:nbspecies){
    a[j] ~ dnorm(0, 0.001)
  b \sim dnorm(0,0.1)
}")
uritolings(model here::here("glides" "code" "nepoeling hug"))
```

Prepare ingredients

```
allom.data <- list(n = n, nbspecies = nbspecies, x = x, y = y, species = s;
init1 <- list(a = rnorm(nbspecies), b = rnorm(1), sigma.y = runif(1))
init2 <- list(a = rnorm(nbspecies), b = rnorm(1), sigma.y = runif(1))
inits<-list(init1, init2)
allom.parameters <- c("a", "b", "sigma.y")</pre>
```

Run JAGS

#>

```
allom.3 <- jags(data = allom.data,
                inits = inits.
                parameters.to.save = allom.parameters,
                n.iter = 2500.
                model.file = here::here("slides", "code", "nopooling.bug"),
                n.chains = 2.
                n.burn = 1000
#> Compiling model graph
#>
     Resolving undeclared variables
#>
     Allocating nodes
#> Graph information:
#>
      Observed stochastic nodes: 488
#>
      Unobserved stochastic nodes: 35
```

Total amount ciaco. 2/81

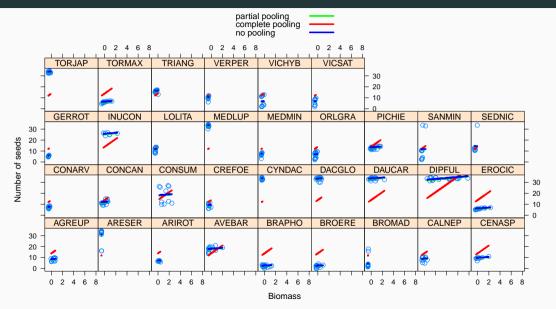
Display results

```
allom.3$BUGSoutput$summary[c(1:4, 32:33, 34), -c(4,6)]
                                                    97.5% Rhat n.e
#>
                         sd
                                 2.5%
                                            50%
              mean
#> a[1] 8.1659636 0.8157488 6.590577 8.1798267 9.8082250 1.000858
                                                                    30
#> a[2]
        30.7338212 0.8846737 29.017299 30.7379259 32.4311725 1.001322
                                                                    23
#> a[3]
        6.6512875 1.1509644 4.414152 6.6473330 8.9381659 1.004078
                                                                     4
#> a[4]
                                                                    30
        17.6301926 0.7809627 16.074789 17.6426587 19.1633082 1.000666
#> a[32]
         6.3626714 0.7948928 4.799545 6.3630696 7.9474871 1.003034
                                                                    6
#> a[33]
         6.5942650 0.8035074 5.034882 6.5921967
                                                                    30
                                                 8.1412549 1.000843
         0.4367871 0.2469271 -0.066436 0.4359984 0.9067317 1.000950
                                                                    30
#> b
```

Compare with Frequentist approach

```
lm(y ~ -1 + as.factor(species) + x, data = allom.data) %>%
  broom::tidy() %>%
  slice(c(1:4, 32:33, 34))
#> # A tibble: 7 x 5
#> term
                           estimate std.error statistic
                                                          p.value
#> <chr>
                              \langle db l \rangle
                                        \langle db l \rangle
                                                   \langle db l \rangle
                                                             \langle db l \rangle
#> 1 as.factor(species)1
                           8.17
                                        0.824
                                                   9.92 3.92e- 21
#> 2 as.factor(species)2
                             30.8
                                        0.895
                                                  34.4 1.67e-128
#> 3 as.factor(species)3
                           6.67
                                        1.16
                                                   5.76 1.56e- 8
#> 4 as.factor(species)4
                             17.6
                                        0.791
                                                  22.3 5.32e- 75
#> 5 as.factor(species)32
                              6.38
                                        0.797
                                                   8.01 9.95e- 15
#> 6 as.factor(species)33
                              6.63
                                        0.800
                                                   8.29 1.33e- 15
\#>7x
                              0.441
                                        0.243
                                                    1.81 7.06e- 2
```

Compare complete pooling vs partial pooling vs no pooling



Bonus: Model with varying intercept

and varying slope

Code: part 1

```
model <-
paste("
# varying-intercept, varying-slope allometry model
# with Vm as a species predictor
model {
  for (i in 1:n){
    y[i] ~ dnorm (mu[i], tau.y)
    mu[i] <- a[species[i]] + b[species[i]] * x[i]</pre>
  tau.y <- pow(sigma.y, -2)
  sigma.y ~ dunif (0, 100)
```

Code: part 2

```
for (j in 1:nbspecies){
  a[j] ~ dnorm (mu.a, tau.a)
  b[j] ~ dnorm (mu.b, tau.b)
mu.a ~ dnorm (0, .001)
tau.a <- pow(sigma.a, -2)
sigma.a ~ dunif (0, 100)
mu.b ~ dnorm (0, .001)
tau.b <- pow(sigma.b, -2)
sigma.b ~ dunif (0, 100)
```

Prepare ingredients

Run Jags

#>

#>

```
allom.4 <- jags(data = allom.data,
                inits = inits.
                parameters.to.save = allom.parameters,
                n.iter = 2500.
                model.file = here::here("slides", "code", "varintvarslope.bu
                n.chains = 2.
                n.burn = 1000
#> Compiling model graph
#>
     Resolving undeclared variables
#>
     Allocating nodes
#> Graph information:
#>
      Observed stochastic nodes: 488
```

Unobserved stochastic nodes: 71

Total amonh ciaco 2521

37

Display results

```
round(allom.4$BUGSoutput$summary[c(1:2, 32:33, 34:35, 65:66, 68:72), -c(4,
#>
                sd
                     2.5% 50% 97.5% Rhat n.eff
           mean
#> a[1]
         7.63 1.32 4.93 7.63 10.16 1.04
                                             57
#> a[2] 24.20 6.55 7.23 24.94 35.50 1.11 37
#> a[32]
       8.57 1.87 4.93 8.59 12.25 1.02
                                            120
#> a[33]
          12.97 3.84 4.67 13.12 20.37 1.10
                                           27
#> b [1]
         1.91 2.87 -3.58 1.85 7.56 1.03
                                             54
#> b[2]
         -10.91 11.36 -40.87 -9.57 8.86 1.12
                                             36
#> b[32]
       5.70 4.30 -2.77 5.75 13.87 1.02
                                            130
#> b[33]
        12.77 7.39 -2.96 12.93 26.96 1.11
                                            19
          16.70 2.02 12.80 16.67 20.71 1.02
                                             76
#> mu.a
#> mu.b 5.13 2.47 0.46 5.00 10.22 1.09
                                            28
#> sigma.a 10.91 1.48 8.46 10.76 14.26 1.01
                                            140
#> ciama h 12 21 2 68
                       7 96 11 00 19 19 1 11
```

Compare with Frequentist approach

#> Fixed Effects.

```
freq lmm2 <- lmer (y ~ x + (1 + x | species), allom.data, REML = FALSE)
freq lmm2
#> Linear mixed model fit by maximum likelihood ['lmerMod']
\# Formula: y \sim x + (1 + x \mid species)
#> Data: allom.data
       AIC BIC logLik deviance df.resid
#>
#> 2609.941 2635.083 -1298.971 2597.941 482
#> Random effects:
#> Groups Name Std.Dev. Corr
#> species (Intercept) 10.409
#> x 11.325 0.22
#> Residual 2.652
#> Number of obs: 488, groups: species, 33
```

Compare with Frequentist approach - with no correlation

#> Residual 2.661

#> Number of obs: 188 amounts: smaries 22

```
freq lmm wocorr <- lmer(y ~ x + (1 | species) +
                                 (0 + x \mid \text{species}), allom.data, REML = F.
freq lmm wocorr
#> Linear mixed model fit by maximum likelihood ['lmerMod']
\# Formula: y \sim x + (1 \mid species) + (0 + x \mid species)
#> Data: allom.data
#>
        AIC
            BIC logLik deviance df.resid
#> 2609.086 2630.037 -1299.543 2599.086 483
#> Random effects:
#> Groups Name Std.Dev.
#> species (Intercept) 10.203
#> species.1 x 10.632
```

• Varying effect estimates shrink towards mean $(\bar{\alpha})$.

- Varying effect estimates shrink towards mean $(\bar{\alpha})$.
- Avoids underfitting as in complete pooling model (null variance) or overfitting as in no pooling model (infinite variance).

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- Varying effects: adaptive regularization through cluster variance estimation.

- Varying effect estimates shrink towards mean $(\bar{\alpha})$.
- Avoids underfitting as in complete pooling model (null variance) or overfitting as in no pooling model (infinite variance).
- Varying effects: adaptive regularization through cluster variance estimation.
- Further from mean, more shrinkage.

- Varying effect estimates shrink towards mean $(\bar{\alpha})$.
- Avoids underfitting as in complete pooling model (null variance) or overfitting as in no pooling model (infinite variance).
- Varying effects: adaptive regularization through cluster variance estimation.
- Further from mean, more shrinkage.
- Fewer data in cluster, more shrinkage.

Multilevel models are awesome!

Shrinkage via pooling is desirable. The no-pooling model overstates variation
among clusters and makes the individual clusters look more different than they are
(overfitting). The complete-pooling model simply ignores the variation among
clusters (underfitting).

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 among clusters and makes the individual clusters look more different than they are
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- We may include predictors at the cluster level. Imagine we know something about functional traits, and wish to determine whether some species-to-species variation in the allometry relationship is explained by these traits.

Your turn: Practical 8

Conclusions

Take-home messages about Bayesian statistics

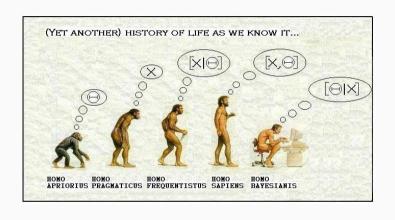
- Frees the modeler in you (M. Kéry)
 - Uses probability to quantify uncertainty for everything (propagation of uncertainty).
 - Allows use of prior information ('better' estimates).
 - Can fit complex (hierarchical) models with same MCMC algorithms.

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- With great tools come great responsabilities
 - Checking convergence is painful.
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- So what?
 - Make an informed and pragmatic choice.
 - Are you after complexity, speed, uncertainties, etc?
 - Talk to colleagues.



Why become a bayesian? Ask twitter!



Your turn: Practical 9