CSIE 5452, Fall 2021: Quiz 1 Solution

Due November 8 (Monday) 2:20pm

There are totally 50 points. You are expected to use X minutes for a question with X points. When you submit your solutions on Gradescope, please select the corresponding page(s) of each question.

1 CAN Timing Analysis (12pts)

Given a set of periodic messages $\mu_0, \mu_1, \mu_2, \mu_3$ with their priorities, transmission times, and periods as follows:

Message	Priority (P_i)	Transmission Time (C_i) (msec)	Period (T_i) (msec)
μ_0	0	20	100
μ_1	1	60	200
μ_2	2	40	170
μ_3	3	20	400

The worst-case response time R_i of μ_i can be computed as

$$R_i = Q_i + C_i,$$

and

$$Q_i = B_i + \sum_{\forall j, P_j < P_i} \left\lceil \frac{Q_i + \tau}{T_j} \right\rceil C_j,$$

where $\tau = 0.1$, Q_i is the worst-case waiting time of μ_i, B_i is the maximum blocking time of μ_i, which is equal to the maximum transmission time of all lower or same (μ_i itself) priority messages. Perform the timing analysis for μ_2 by completing the following table (you may or may not use all rows).

Answer: Q_2 is computed as follows:

Iteration	LHS (Q_2)	B_2	j	$Q_2 + \tau$	T_j	$\left\lceil \frac{Q_2+\tau}{T_j} \right\rceil$	C_j	RHS	Stop?
1	40	40	0	40.1	100	1	20	120	No
			1		200	1	60		
2	120	40	0	120.1	100	2	20	140	Yes
			1		200	1	60		(Constraint Violation)

The RHS is 140, and $140 + C_2 = 140 + 40 = 180 > T_2 = 170$. Therefore, the timing analysis returns a constraint violation.

2 TDMA Timing Analysis (12pts)

Follow the assumptions (each time slot has the same length, each time slot serves exactly one frame, and a frame is transmitted only if the whole time slot is available) in the lecture and Homework 1. Answer the following questions for the "asynchronous" message with the following frame arrival pattern and the schedule pattern, based on the last digit of your student ID number, D. EXCEPT the last question, no explanation is required.

- If D is 0 or 5, the frame arrival pattern is $(2, 5, 0, 2)$ and the schedule pattern is $(4, 10, 0, 3, 7, 8)$.
- If D is 1 or 6, the frame arrival pattern is $(2, 5, 0, 3)$ and the schedule pattern is $(4, 10, 1, 2, 4, 7)$.
- If D is 2 or 7, the frame arrival pattern is $(2, 5, 1, 3)$ and the schedule pattern is $(4, 10, 0, 4, 5, 7)$.
- If D is 3 or 8, the frame arrival pattern is $(2, 5, 1, 4)$ and the schedule pattern is $(4, 10, 0, 2, 5, 9)$.
- If D is 4 or 9, the frame arrival pattern is $(2, 5, 2, 4)$ and the schedule pattern is $(4, 10, 2, 3, 5, 8)$.

1. (1pt) Write down the last digit of your student ID number.
2. (1pt) Duplicate the arrival pattern (hint: $(4, 10, W, X, Y, Z)$).

   Answer: $(4, 10, 0, 2, 5, 7)$, $(4, 10, 0, 3, 5, 8)$, $(4, 10, 1, 3, 6, 8)$, $(4, 10, 1, 4, 6, 9)$, $(4, 10, 2, 4, 7, 9)$.
3. (1pt) Duplicate the arriving times of frames in the frame arrival pattern but fix $m = 4$ and $p = 10$.

   Answer: $(4, 10, 0, 2, 5, 7, 10, 12, 15, 17)$, $(4, 10, 0, 3, 5, 8, 10, 13, 15, 18)$, $(4, 10, 1, 3, 6, 8, 11, 13, 16, 18)$, $(4, 10, 1, 4, 6, 9, 11, 14, 16, 19)$, $(4, 10, 2, 4, 7, 9, 12, 14, 17, 19)$.
4. (1pt) Duplicate the starting times of time slots in the schedule pattern but fix $n = 4$ and $q = 10$.

   Answer: $(4, 10, 0, 3, 7, 8, 10, 13, 17, 18)$, $(4, 10, 1, 2, 4, 7, 11, 12, 14, 17)$, $(4, 10, 0, 4, 5, 7, 10, 14, 15, 17)$, $(4, 10, 0, 2, 5, 9, 10, 12, 15, 19)$, $(4, 10, 2, 3, 5, 8, 12, 13, 15, 18)$.
5. (4pts) Complete the following table.

   Answer:

k	$\max_{1\le j\le n}(s_{j+k} - s_j)$	=	$\min_{1\le i\le m}(a_{i+k-1} - a_i)$	=	(Column-3) − (Column-5)
1	$\max_{1\le j\le 4}(s_{j+1} - s_j)$	4	$\min_{1\le i\le 4}(a_i - a_i)$	0	4
2	$\max_{1\le j\le 4}(s_{j+2} - s_j)$	7	$\min_{1\le i\le 4}(a_{i+1} - a_i)$	2	5
3	$\max_{1\le j\le 4}(s_{j+3} - s_j)$	9	$\min_{1\le i\le 4}(a_{i+2} - a_i)$	5	4
4	$\max_{1\le j\le 4}(s_{j+4} - s_j)$	10	$\min_{1\le i\le 4}(a_{i+3} - a_i)$	7	3

6. (4pts) Explain the scenario that the worst-case happens (which frame misses which time slot, and which frame suffers the worst case).

   Answer: The first/second/first/second/first frame (whose arrival time is "0/3/1/4/2") misses the first/third/fourth/second/third time slot (whose starting time is "0/4/7/2/5"), and the second/third/second/third/second frame (whose arrival time is "2/5/3/6/4") suffers the worst case. When the first/second/first/second/first frame's arrival time is aligned with the first/third/fourth/second/third time slot's starting time (shifting "0/4/7/2/5" to "0/3/1/4/2"), the second/third/second/third/second frame arrives at "2/5/3/6/4" and is served at "7/10/8/11/9" (shifting "7/11/14/9/12" to "7/10/8/11/9"), so the waiting time is 5.

3 Short Answers (18pts)

1. (6pts) What are the main differences between anti-lock braking systems (ABS), traction control systems (TCS), and electronic stability control (ESC)?

   Answer: ABS, TCS, and ESC prevent the wheels from from loss of traction when braking, acceleration, and turning, respectively.

2. (6pts) During the process of simulate annealing, if we get a new solution violating N constraints, what should we do?

   Answer: Add a penalty (larger than the objectives of feasible solutions) to the cost for "each" constraint violation.

3. (6pts) Assume that we perform CAN timing analysis for a set of periodic messages $\mu_0, \mu_1, \mu_2, \mu_3$, where a smaller index means a higher priority, *i.e.*, μ_0 is the highest-priority message, and μ_3 is the lowest-priority message. Also assume that we perform the analysis from μ_3 to μ_0 one by one. After analyzing μ_3 and μ_2 (without analyzing the other messages), we find that, for $i = 3, 2$, the worst-case response time R_i of μ_i is smaller than its period T_i. How should we interpret the results, *i.e.*, are μ_3 and μ_2 schedulable? Explain your answer.

   Answer: We cannot decide it at this point. We can only decide it after the other messages are analyzed because the analysis of lower-priority messages depends on the schedulability of higher-priority messages (to compute how many times the lower-priority messages are blocked). If the other messages are schedulable; μ_3 and μ_2 are schedulable; otherwise, we cannot decide it and the system needs to be redesigned.

4 MILP Formulation (8pts)

Given integers $A_1, A_2, \ldots, A_N, A_{N+1}, \ldots, A_{2N}$ and $S_1, S_2, \ldots, S_N, S_{N+1}, \ldots, S_{2N}$. Use an MILP formulation to compute

$$\max_{1 \le k \le N} \left(\max_{1 \le j \le N} (S_{j+k} - S_j) - \min_{1 \le i \le N} (A_{i+k-1} - A_i) \right).$$

Warning: this question is challenging, so you should complete other questions first.

Answer: The decision variables are a positive integer k, integers s, a, and binaries $y_1, y_2, \ldots, y_N$, $x_1, x_2, \ldots, x_N$, $y'_1, y'_2, \ldots, y'_{2N}$, $x'_1, x'_2, \ldots, x'_{2N}$. The formulation is

$$\begin{aligned}
\max \quad & s - a, \\
& y_1 + y_2 + \cdots + y_N = 1, \\
& y'_1 + y'_2 + \cdots + y'_{2N} = 1, \\
& 1 \cdot y'_1 + 2 \cdot y'_2 + \cdots + 2N \cdot y'_{2N} - 1 \cdot y_1 - 2 \cdot y_2 - \cdots - N \cdot y_N = k, \\
& S_1 \cdot y'_1 + S_2 \cdot y'_2 + \cdots + S_{2N} \cdot y'_{2N} - S_1 \cdot y_1 - S_2 \cdot y_2 - \cdots - S_N \cdot y_N = s, \\
& x_1 + x_2 + \cdots + x_N = 1, \\
& x'_1 + x'_2 + \cdots + x'_{2N} = 1, \\
& 1 \cdot x'_1 + 2 \cdot x'_2 + \cdots + 2N \cdot x'_{2N} - 1 \cdot x_1 - 2 \cdot x_2 - \cdots - N \cdot x_N = k - 1, \\
& A_1 \cdot x'_1 + A_2 \cdot x'_2 + \cdots + A_{2N} \cdot x'_{2N} - A_1 \cdot x_1 - A_2 \cdot x_2 - \cdots - A_N \cdot x_N = a, \\
& 1 \le k \le N.
\end{aligned}$$

Note that y'_1 and x'_{2N} are actually redundant—they will always be 0.