Homework Assignment #1

Due: 2:20pm, March 25, 2023

Late submission:

within 24 hours after its due will incur 20% penalty,

after 24 hours will not be graded.

Note: One minute late is the same as 24 hours late.

1.2-16. A box of candy hearts contains 52 hearts, of which 19 are white, 10 are tan, 7 are pink, 3 are purple, 5 are yellow, 2 are orange, and 6 are green. If you select nine pieces of candy randomly from the box, without replacement, give the probability that

(a) Three of the hearts are white.

(b) Three are white, two are tan, one is pink, one is yellow, and two are green.

1.3-3. Let A_1 and A_2 be the events that a person is left-eye dominant or right-eye dominant, respectively. When a person folds his or her hands, let B_1 and B_2 be the events that the left thumb and right thumb, respectively, are on top. A survey in one statistics class yielded the following table:

	B_1	B_2	Totals
A_1	5	7	12
A_2	14	9	23
Totals	19	16	35

If a student is selected randomly, find the following probabilities: **(a)** $P(A_1 \cap B_1)$, **(b)** $P(A_1 \cup B_1)$, **(c)** $P(A_1 \mid B_1)$, **(d)** $P(B_2 \mid A_2)$. **(e)** If the students had their hands folded and you hoped to select a right-eye-dominant student, would you select a "right thumb on top" or a "left thumb on top" student? Why?

1.3-9. An urn contains four balls numbered 1 through 4. The balls are selected one at a time without replacement. A match occurs if the ball numbered m is the mth ball selected. Let the event A_i denote a match on the ith draw, $i = 1, 2, 3, 4$.

(a) Show that $P(A_i) = \dfrac{3!}{4!}$ for each i.

(b) Show that $P(A_i \cap A_j) = \dfrac{2!}{4!}$, $i \neq j$.

(c) Show that $P(A_i \cap A_j \cap A_k) = \dfrac{1!}{4!}$, $i \neq j, i \neq k, j \neq k$.

(d) Show that the probability of at least one match is

$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!}.$$

(e) Extend this exercise so that there are n balls in the urn. Show that the probability of at least one match is

$$\begin{aligned} &P(A_1 \cup A_2 \cup \cdots \cup A_n) \\ &\quad = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots + \frac{(-1)^{n+1}}{n!} \\ &\quad = 1 - \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right). \end{aligned}$$

(f) What is the limit of this probability as n increases without bound?

1.4-12. Flip an unbiased coin five independent times. Compute the probability of

(a) $HHTHT$.

(b) $THHHT$.

(c) $HTHTH$.

(d) Three heads occurring in the five trials.

1.5-4. Assume that an insurance company knows the following probabilities relating to automobile accidents (where the second column refers to the probability that the policyholder has at least one accident during the annual policy period):

Age of Driver	Probability of Accident	Fraction of Company's Insured Drivers
16–25	0.05	0.10
26–50	0.02	0.55
51–65	0.03	0.20
66–90	0.04	0.15

A randomly selected driver from the company's insured drivers has an accident. What is the conditional probability that the driver is in the 16–25 age group?

2.1-7. Let a random experiment be the casting of a pair of fair six-sided dice and let X equal the minimum of the two outcomes.

(a) With reasonable assumptions, find the pmf of X.

(b) Draw a probability histogram of the pmf of X.

(c) Let Y equal the range of the two outcomes (i.e., the absolute value of the difference of the largest and the smallest outcomes). Determine the pmf $g(y)$ of Y for $y = 0, 1, 2, 3, 4, 5$.

(d) Draw a probability histogram for $g(y)$.