Homework Assignment #2

Textbook Exercises (from textbook 9th ed.)

2.1-16. (Michigan Mathematics ...

2.2-9. A roulette wheel …

2.3.16. Let X equal the number of …

2.4-17. Your stockbroker is free to …

2.5-10. In 2012, Red Rose …

Due: 2:20pm, April 13, 2023

2.1-16. (Michigan Mathematics Prize Competition, 1992, Part II) From the set $\{1, 2, 3, \ldots, n\}$, k distinct integers are selected at random and arranged in numerical order (from lowest to highest). Let $P(i, r, k, n)$ denote the probability that integer i is in position r. For example, observe that $P(1, 2, k, n) = 0$, as it is impossible for the number 1 to be in the second position after ordering.

(a) Compute $P(2, 1, 6, 10)$.

(b) Find a general formula for $P(i, r, k, n)$.

2.2-9. A roulette wheel used in a U.S. casino has 38 slots, of which 18 are red, 18 are black, and 2 are green. A roulette wheel used in a French casino has 37 slots, of which 18 are red, 18 are black, and 1 is green. A ball is rolled around the wheel and ends up in one of the slots with equal probability. Suppose that a player bets on red. If a \$1 bet is placed, the player wins \$1 if the ball ends up in a red slot. (The player's \$1 bet is returned.) If the ball ends up in a black or green slot, the player loses \$1. Find the expected value of this game to the player in

(a) The United States.

(b) France.

2.3-16. Let X equal the number of flips of a fair coin that are required to observe the same face on consecutive flips.

(a) Find the pmf of X. HINT: Draw a tree diagram.

(b) Find the moment-generating function of X.

(c) Use the mgf to find the values of **(i)** the mean and **(ii)** the variance of X.

(d) Find the values of **(i)** $P(X \leq 3)$, **(ii)** $P(X \geq 5)$, and **(iii)** $P(X = 3)$.

2.4-17. Your stockbroker is free to take your calls about 60% of the time; otherwise, he is talking to another client or is out of the office. You call him at five random times during a given month. (Assume independence.)

(a) What is the probability that he will take every one of the five calls?

(b) What is the probability that he will accept exactly three of your five calls?

(c) What is the probability that he will accept at least one of the calls?

2.5-10. In 2012, Red Rose tea randomly began placing 1 of 12 English porcelain miniature figurines in a l00-bag box of the tea, selecting from 12 nautical figurines.

(a) On the average, how many boxes of tea must be purchased by a customer to obtain a complete collection consisting of the 12 nautical figurines?

(b) If the customer uses one tea bag per day, how long can a customer expect to take, on the average, to obtain a complete collection?

Homework Submission Policy

- **Late submission:**
  - within 24 hours after its due will incur 20% penalty,
  - after 24 hours and within seven days of its due will incur 50% penalty, and
  - after seven days of its due will not be graded.

Note: One minute late is the same as 23 hours late.