Assignment 1: Optimization for Data Sciences
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1. Justify whether the following sets are convex or not:
   - $S = \{x \in \mathbb{R}^2 : x_1 + x_2 \leq 10, x_1 + x_2 \geq 2, x_1, x_2 \geq 0\}$.
   - $S = \{x \in \mathbb{R}^2 : 2 \leq x_1 \leq 4, x_2 = 3\}$.
   - $S = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 4\}$.
   - $S = \{x \in \mathbb{R}^2 : \frac{x_1^2}{9} + \frac{x_2^2}{4} \leq 1\}$.
   - $S = \{x \in \mathbb{R}^2 : |x_1| + |x_2| = 9\}$.
   - $S = \{x\mathbb{R}^3 : x_1^2 + x_2^2 = x_3^2\}$.
2. Consider $S = \{(0,0), (4,1), (6,7), (2,5), (3,3)\}$. Plot S in 2-D graph and identify $Conv(S)$.
3. Suppose S be a nonempty convex set. Show that for some $\alpha > 0$, $S_1 = \{\alpha x : x \in S\}$ is a convex set.
4. Justify whether the following matrices are positive semi-definite/definite or not:
   - $H = \begin{bmatrix} 4 & 3 \\ 5 & 1 \end{bmatrix}$
   - $H = \begin{bmatrix} -2 & 1 \\ 1 & -5 \end{bmatrix}$
   - $H = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$
   - $H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$
5. Justify whether the following functions are convex or not:
   - $f(x) = x_1^2 + x_2^2$
   - $f(x) = 4x_1^2 + x_2^2 - 2x_1x_2$
   - $f(x) = -3x_1^2 + 4x_1x_2 - 3/2x_2^2$
   - $f(x) = x_1 \log(x_1) + x_2 \log(x_2)$ at $x = (2,4)$
   - $f(x) = (4,2,3)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \frac{1}{2}(x_1, x_2, x_3)\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
6. Check whether x^* is a stationary of f or not. If yes, check whether x^* is a local minima or not.
   - $f(x) = x_1^2 + 3(x_2 - 1)^2$, $x^* = (0,1)^T$
   - $f(x) = x_1^3 + x_2^2$, $x^* = (0,0)^T$
   - $f(x) = (x_1 - 1)^2 + (x_2 - x_3)^2$, $x^* = (2,1,3)^T$.
   - $f(x) = x_1^2 - 4x_1x_2 + 5x_2^2$, $x^* = (0,0)^T$
   - $f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$, $x^* = (1,1)^T$.
7. Define $f(x) = 2x_1^2 + 3x_2^2 + 4x_1x_2 + 2x_1 + 6x_2$. Perform two iterations of steepest descent method using initial approximation $x^0 = (2,2)^T$.
8. Define $f(x) = x_1^2 + x_2^2 - x_1x_2 - 2x_1 + x_2$. What is a stationary point of f? Can we apply Newton's method at $x^0 = (2,3)^T$. If yes, find next iterating point.
9. Define $f(x) = (x_2 - 2)^4 + (x_1 - 2x_2)^2$. Perform two iterations of quasi-Newton method using $x^0 = (0,0)^T$.
10. Define $f(x) = 3x_1^2 + 2x_2^2 - 2x_1x_2 + 4x_1 + 6x_2$. Perform 3 iterations of conjugate gradient method using $x^0 = (2,3)$.