

# Computing Depth

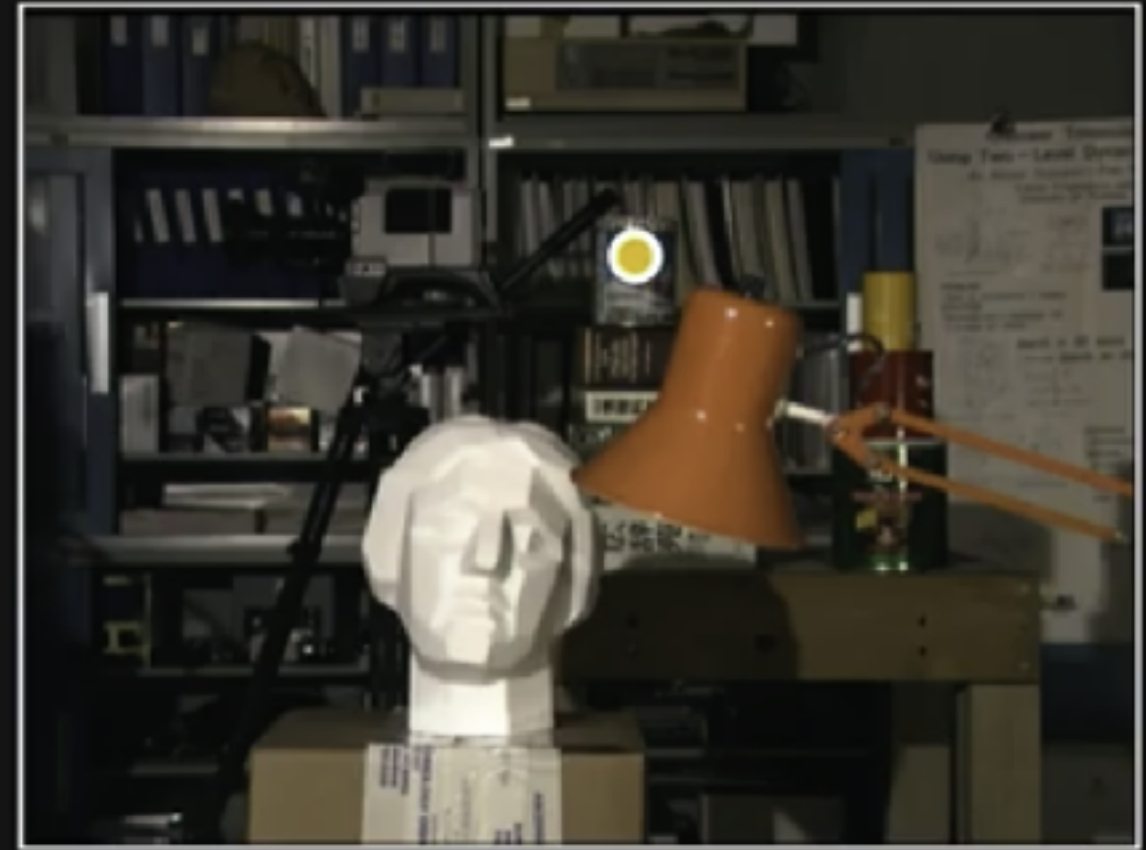


# Simple Stereo: Finding Correspondences

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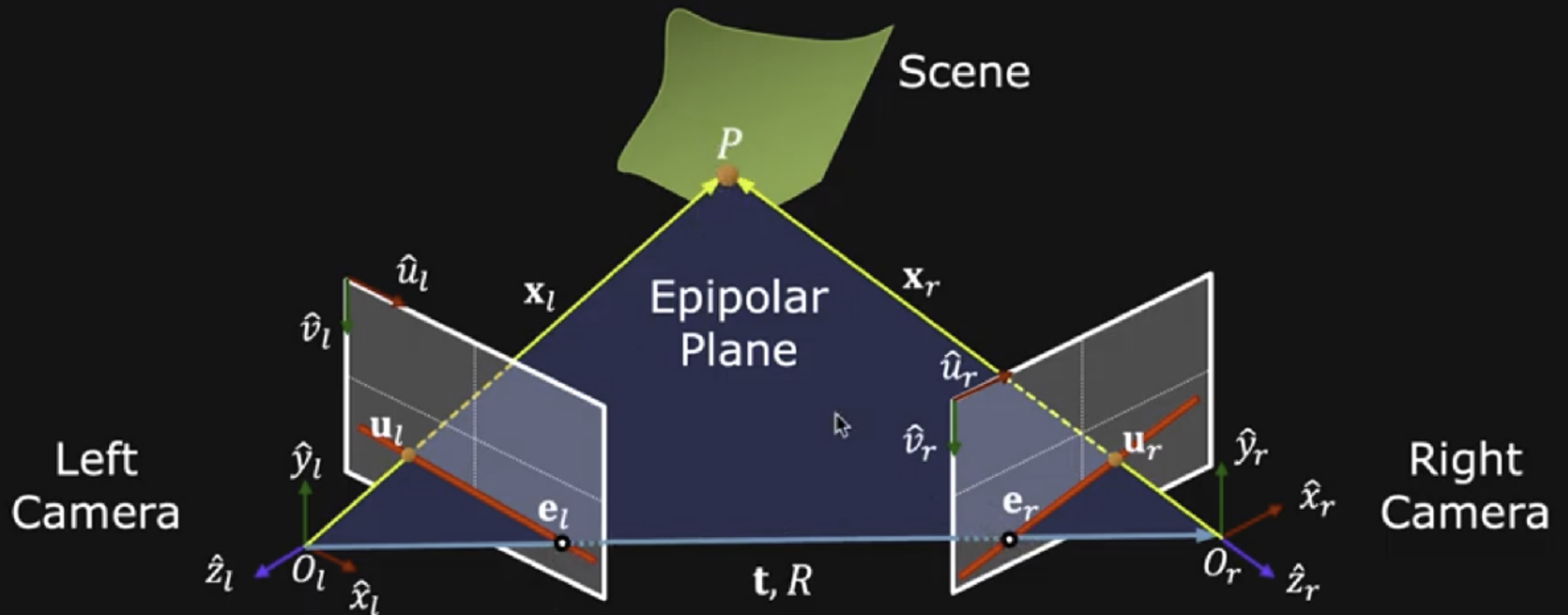


Left Camera Image



Right Camera Image

# Epipolar Geometry: Epipolar Line



**Epipolar Line:** Intersection of image plane and epipolar plane.



Finding Epipolar line

$$\underline{\begin{bmatrix} u_l & v_l & 1 \end{bmatrix}} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \underbrace{(f_{11} u_l + f_{21} v_l + f_{31})}_{a_l} u_r + \underbrace{(f_{12} u_l + f_{22} v_l + f_{32})}_{b_l} v_r + \underbrace{(f_{13} u_l + f_{23} v_l + f_{33})}_{c_l} = 0$$

$$\Rightarrow \boxed{a_l u_r + b_l v_r + c_l = 0} \quad \checkmark$$



# Finding Epipolar Lines: Example

Given the Fundamental matrix,

$$F = \begin{bmatrix} -.003 & -.028 & 13.19 \\ -.003 & -.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

and the **left** image point

$$\tilde{u}_l = \begin{bmatrix} 343 \\ 221 \\ 1 \end{bmatrix}$$

Left Image



Right Image



The equation for the epipolar line in the **right** image is

$$.03u_r + .99v_r - 265 = 0$$



Computing Depth :-

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^l & 0 & 0 & 0 \\ 0 & f_y^l & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^r & 0 & 0 & 0 \\ 0 & f_y^r & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

→ left camera image eq.

①

②

②

$$\begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = \begin{bmatrix} f_x^l & 0 & 0 & 0 \\ 0 & f_y^l & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & \textcircled{3} \\ - & - \\ - & - \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

4x4

3x4



$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = P_l \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \Rightarrow \boxed{\hat{U}_l = P_l \hat{X}_r}$$

$3 \times 4$

$$\boxed{\hat{U}_r = Mint \hat{X}_r}$$

$3 \times 4$

known

Cor

$$\begin{bmatrix} \phantom{x_r} \\ \phantom{y_r} \\ \phantom{z_r} \\ \phantom{1} \end{bmatrix}$$

$4 \times 3$

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} =$$

$3 \times 1$

$$\begin{bmatrix} \phantom{x_r} \\ \phantom{y_r} \\ \phantom{z_r} \\ \phantom{1} \end{bmatrix}$$

$4 \times 1$

$$\boxed{A X_r = b}$$



$$\underline{\underline{A^T A \hat{x}_r = A^T b}}$$

$$\boxed{\hat{x}_r = (A^T A)^{-1} A^T b}$$