Camera Calibration

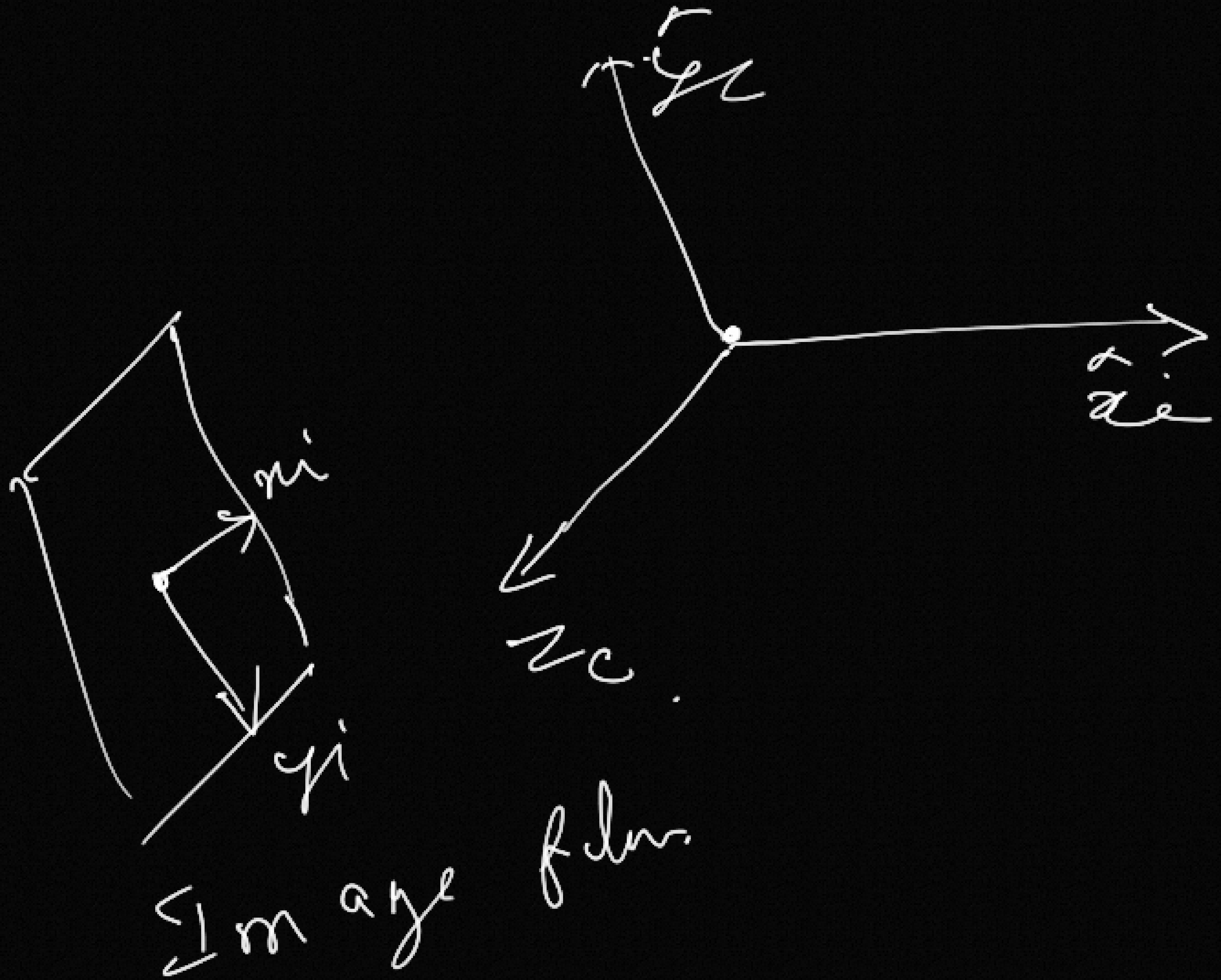
1. Linear Camera Model

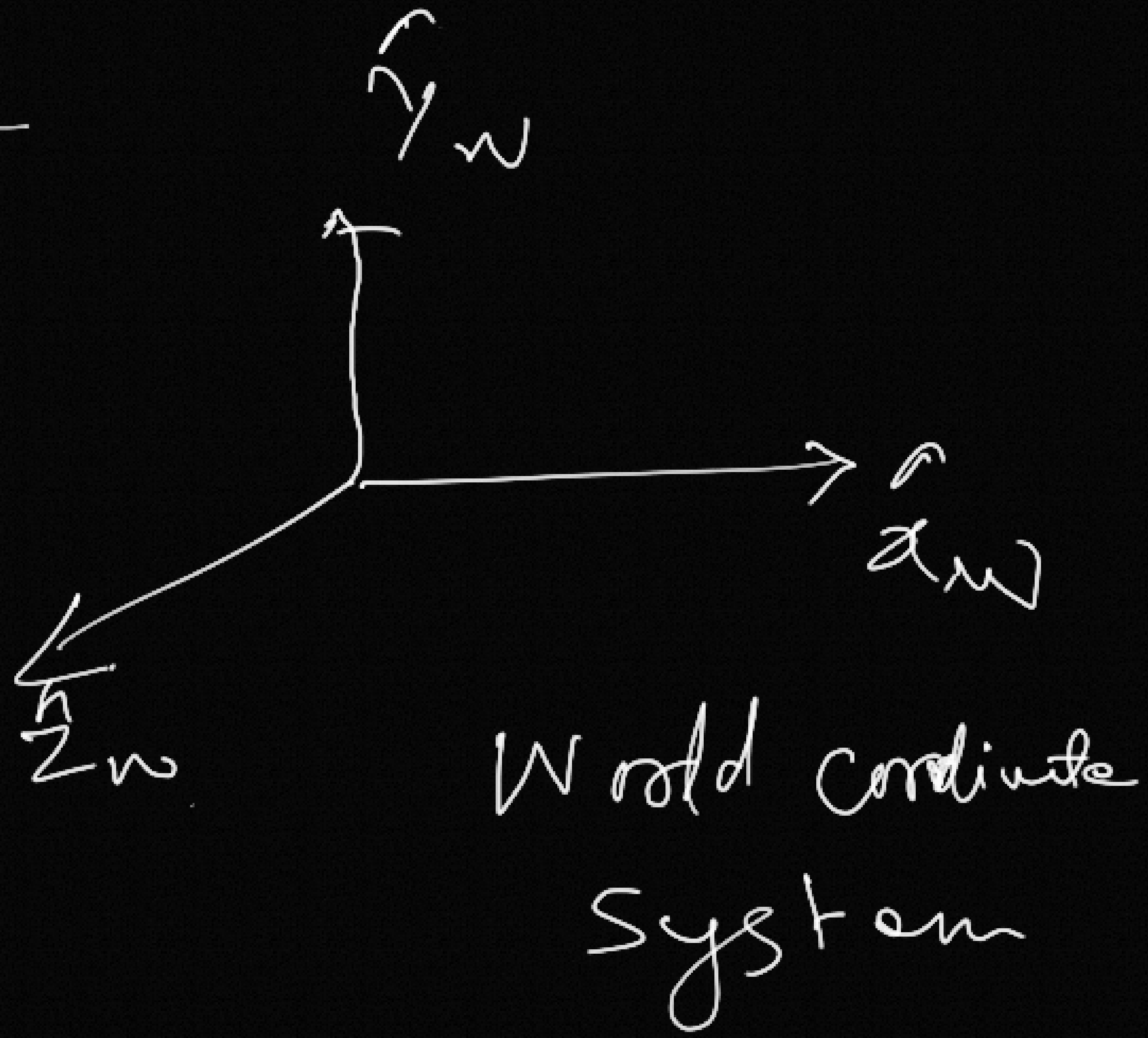
2. C.C.

3. Intrinsic & extrinsic parameters of camera

4. Stereo

Linear Camera Model





$$X_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \longrightarrow X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \longrightarrow X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

3D ⇒ 2D

3D – 3D

$$x_i = f\,\frac{x_c}{z_c}, \quad y_i = f\,\frac{y_c}{z_c}$$

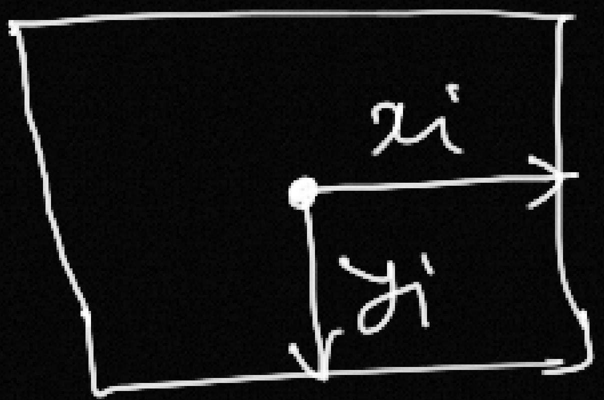
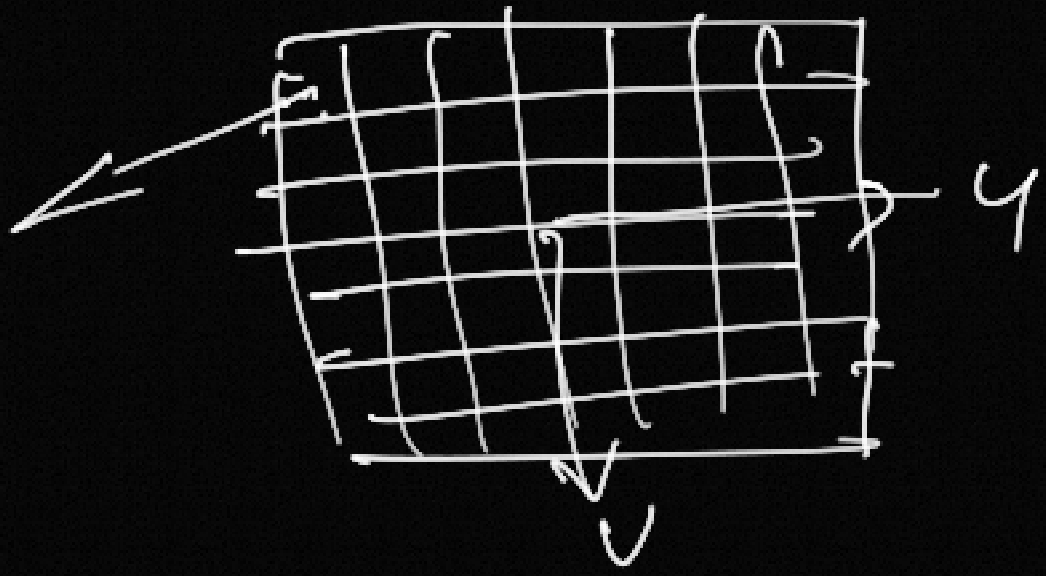


Image plane



$\# \text{pixels}/\text{mm} = m_x$ (in X-Axis)

m_y (in Y-Axis)

$u = m_x \cdot x_i$ $\qquad$ $v = m_y \cdot y_i$

$$u = m_x f \cdot \frac{x_c}{Z_c} \qquad v = m_y \cdot f \cdot \frac{y_c}{Z_c}$$

$$u = \underline{m_x f} \frac{x_c}{Z_c} + O_x \qquad v = m_y f \frac{y_c}{Z_c} + O_y$$

$$\boxed{u = f_x \frac{x_c}{Z_c} + O_x} \qquad \boxed{v = f_y \frac{y_c}{Z_c} + O_y}$$

$$f_x = m_x f, \quad f_y = m_y f$$

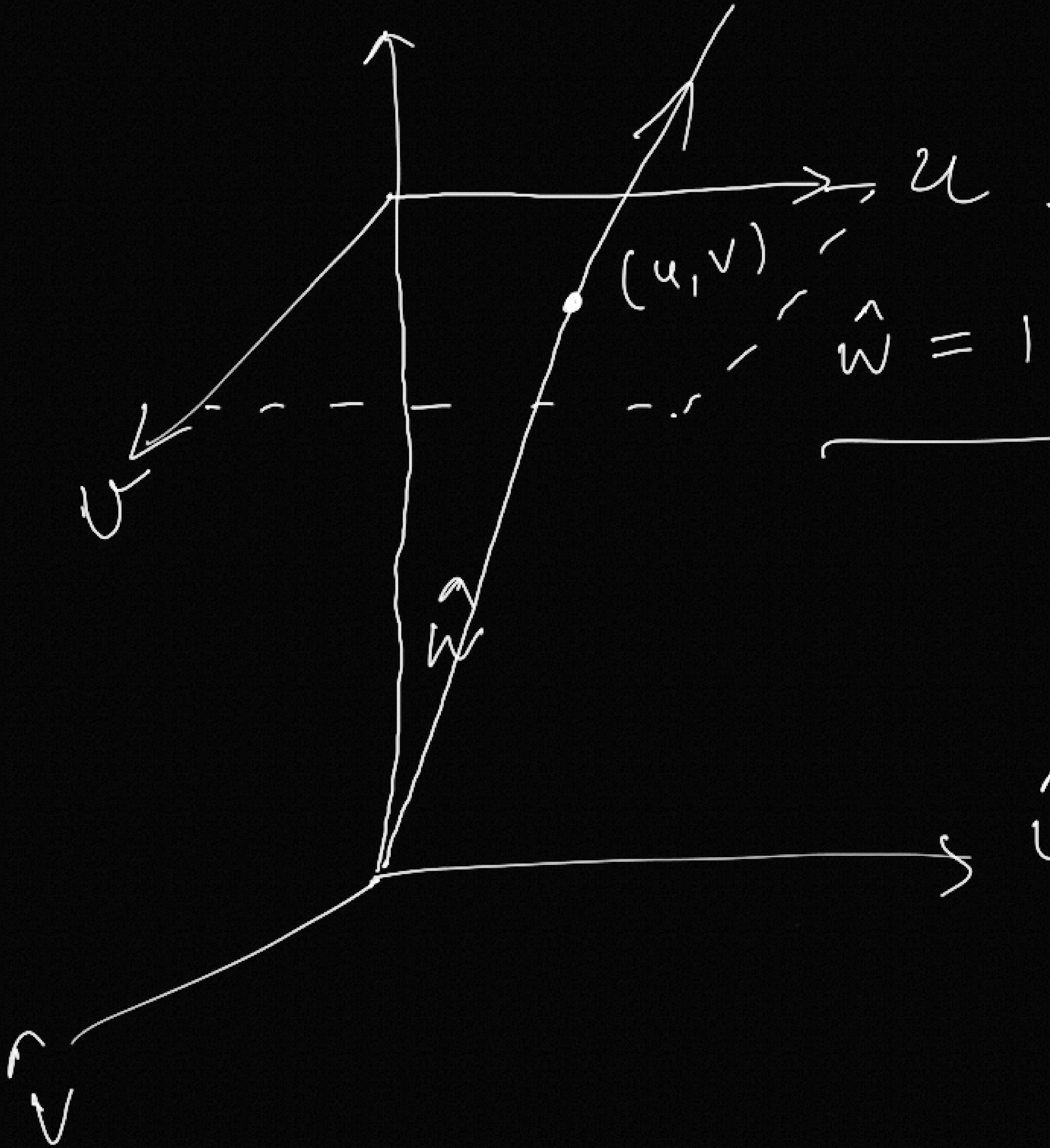
Homogeneous Coordinate System :-

The Homo. Representation of a 2D-point $U = (u, v)$ is a 3D-point $\hat{U} = (\hat{u}, \hat{v}, \hat{w})$

where $\hat{w} \neq 0$ is fictitious s.t.

$$u = \frac{\hat{u}}{\hat{w}}, \quad v = \frac{\hat{v}}{\hat{w}}$$

$$U = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \hat{w}\, u \\ \hat{w}\, v \\ \hat{w} \end{bmatrix} \equiv \begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{pmatrix} = \hat{U}$$



1

2

3D → 4D Homo. Representation

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ \hat{w} \end{bmatrix}$$

$$x = \frac{\hat{x}}{\hat{w}}, \quad y = \frac{\hat{y}}{\hat{w}}, \quad z = \frac{\hat{z}}{\hat{w}}$$

$$u = f_x \cdot \frac{x_c}{z_c} + O_x \quad , \quad v = f_y \cdot \frac{y_c}{z_c} + O_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix}$$

$$\equiv \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \begin{bmatrix} f_x \cdot x_c + O_x z_c \\ f_y \, y + O_y z_c \\ z_c \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} f_x & 0 & O_x & 0 \\ 0 & f_y & O_y & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

3×4

Intrinsic Matrix

(Internal parameters of camera

Calibration matrix (K)

$$\hat{U} = [K \mid \vec{0}]\, \hat{X}_c$$

$$\boxed{\hat{u} = M_{int}\,\hat{x}_c} \quad \text{—— } (A)$$

World $\rightarrow$ Camera

Two external parameters

$C_w \rightarrow$ Position

$R \rightarrow$ Rotation

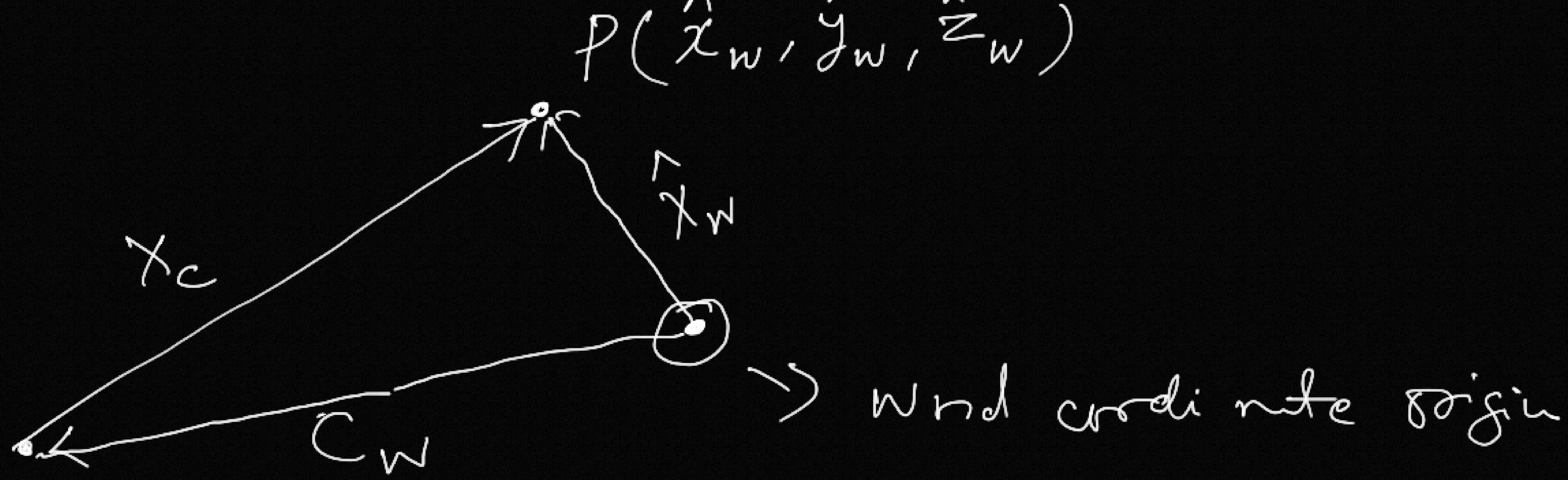
$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{array}{l} \rightarrow \text{direction of } \hat{x}_c \text{ is world co—} \\ \rightarrow \text{——— } \hat{y}_c \text{ in w—} \\ \rightarrow \hat{z}_c \text{ in co—} \end{array}$$

Orthonormal Vector :-

$$\boxed{u^T \cdot v = 0 \qquad u^T \cdot u = 1 \,,\; v^T \cdot v = 1}$$

Orthonormal Matrix

$$R^{-1} = R^T \,, \quad R^T R = I$$



$$
\begin{aligned}
X_c &= R\left(\hat{X}_w - C_w\right) \\
&= R\hat{X}_w - RC_w \\
&= \underline{R}\,\underline{\hat{X}}_w + \underline{t} \qquad \boxed{t = -RC_w}
\end{aligned}
$$

$$\hat{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\hat{X}_c = \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Extrinsic matrix}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_{3\times3} & t_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}$$

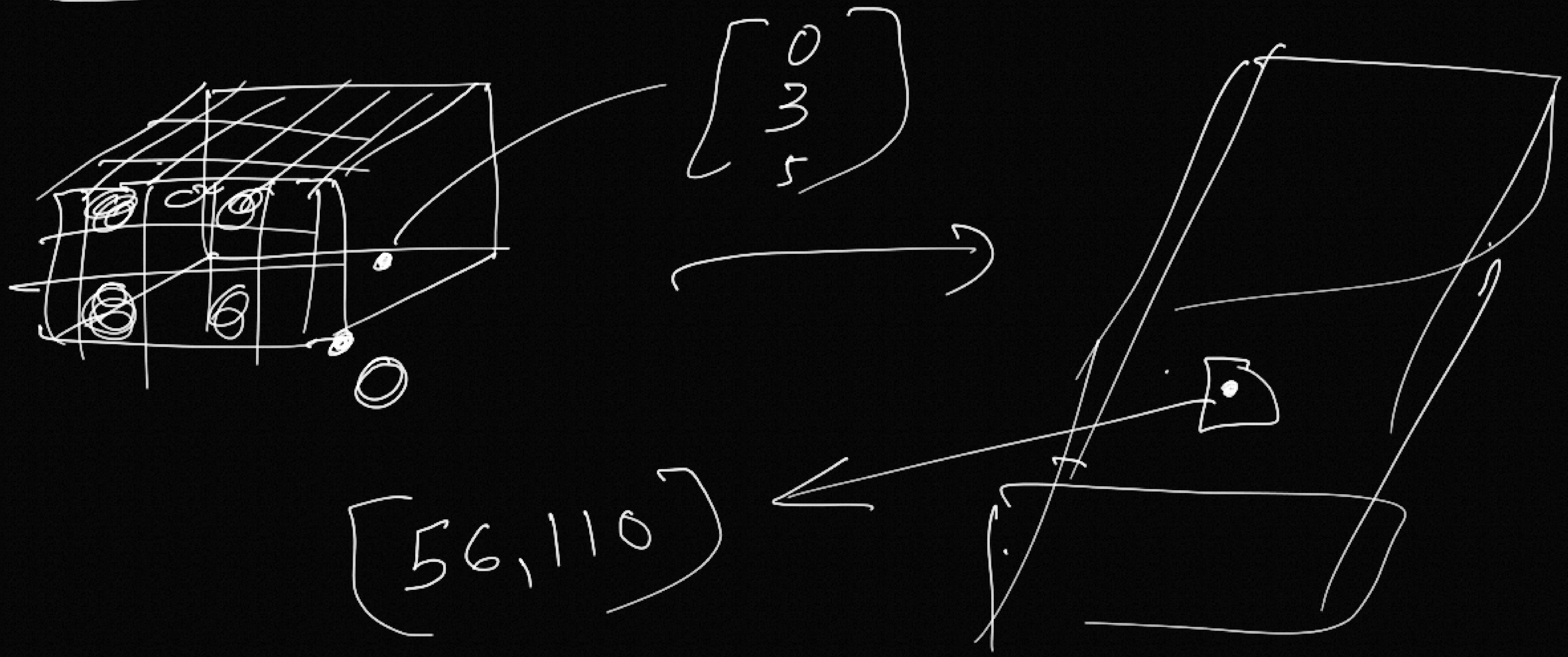
$$\hat{x}_c = M_{ext}\, \hat{x}_w \quad \text{—— } B$$

$$\hat{u} = M_{int}\, \hat{x}_c \quad \text{—— } A$$

$$\hat{U} = \underset{3\times4}{M_{int}} \cdot \underset{4\times4}{M_{ext}}\, \hat{X}_w$$

$$\hat{U} = P\, \hat{X}_w$$

Camera Calibration



$$\begin{bmatrix} u^{(i)} \\ v^{(i)} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_w^{(i)} \\ y_w^{(i)} \\ z_w^{(i)} \\ 1 \end{bmatrix}$$

$$\underset{\text{known}}{A}\,\underset{\text{unknown}}{p} = 0$$

Projection matrix is defined up to scale.

$$Ap = 0 \quad s.t. \quad \|p\|^2 = 1$$

$$\min_{p} \|Ap\|^2 \quad s.t. \quad \|p\|^2 = 1$$

(Constrained least square)

$$\equiv \quad p^T A^T A p \quad s.t. \quad p^T p = 1$$

$$L(p, \lambda) = p^T A^T A p - \lambda(\underline{p^T} p - 1)$$

$$\frac{\partial L}{\partial p} = 0, \quad 2A^T A p - 2\lambda p = 0$$

$$\boxed{A^T A p = \lambda p}$$

Eigen Vector corresponding to smallest eigen value will be the optimal

ϕ

σ

Recap

Step