Practice problem 1:Optimization in ML

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1. Justify whether the following least problems $\min_{x\in\mathbb{R}^n} \frac{1}{2}\|Ax-b\|^2$, $A_{m\times n}$,$b_{m\times 1}$ has unique solution or not. If yes find it.
   - $A = \begin{bmatrix} 2 & 3 \\ 4 & 4 \\ 8 & 6 \end{bmatrix}, b = (10, 15, 20)^T.$
   - $A = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 4 & 7 \\ 3 & 9 & 6 \\ 1 & 8 & 4.5 \end{bmatrix}, b = (4, 2, 3, 1)^T.$
   - $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, b = (4, 0, 2)^T.$
   - $A = \begin{bmatrix} 1 & -1 \\ -5 & 5 \\ 3 & -3 \end{bmatrix}, b = (0, 0, 0)^T.$
   - $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ -1 & 0 \end{bmatrix}, b = (3, 4, 5)^T.$
   - $A = \begin{bmatrix} 2 & 3 & 4 \\ 17 & 6 & 7 \\ 5 & 4 & 5 \\ 4 & 5 & 3 \end{bmatrix}, b = (3, 2, 1, 7)^T.$

   Hint: Least square problem will have unique solution iff column vectors of A are linearly independent.

2. Find the best fitting curve $y = mx + c$ for the set of points $S = \{(1, 3), (4, 2), (5, 3), (2, 7)\}$.