

# Medical Image Analysis



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*Department of Computer Science & Engineering*

# Syllabus

- **Classical Approaches:** Introduction to image processing and medical imaging modalities, denoising and enhancement, Tissue and Cell Segmentation: clustering, active contours and level sets based approaches, Medical Image alignment: rigid and deformable registration
- **Machine Learning and Deep Learning Approaches:** Fundus Image analysis, Retinal Vessel Segmentation, MRI image analysis and segmentation, 3D brain reconstruction from MRI slices and analysis, Microscopic image analysis and interpretation, Ultrasonography image analysis, X-Ray and CT image segmentation, diagnosis and prognosis of various diseases, Correlation between different medical imaging modalities and conversions, augmenting clinical measurements with medical imaging modalities for diseases diagnosis and prognosis

# Course Logistics

- Instructor: Angshuman Paul
- Contact: [apaul@iitj.ac.in](mailto:apaul@iitj.ac.in)
- TA: Jayant Mahawar ([mahawar.2@iitj.ac.in](mailto:mahawar.2@iitj.ac.in))
- Class hours:
  - Tuesday: 3-3:50 PM
  - Wednesday: 2-2:50 PM
  - Friday: 2-2:50 PM

# Course Logistics

- Course project
- Reading group
- Quiz
- Viva
- Minor and Major
- All are compulsory

# Books and Study Materials

- Book:
  - Fundamentals of Medical Imaging by Paul Suetens
  - Medical Imaging Signals and Systems by Jerry L. Prince and Jonathan M. Links
- Other useful books:
  - Introduction to Medical Image Analysis, by Rasmus R. Paulsen and Thomas B. Moeslund
- Other books & Online materials: Will be informed from time to time

# Prerequisites

- Digital Image Processing
- Computer Vision
- ML
- DL

# Medical Images



# Medical Images



James Clerk Maxwell

# Medical Images



James Clerk Maxwell

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

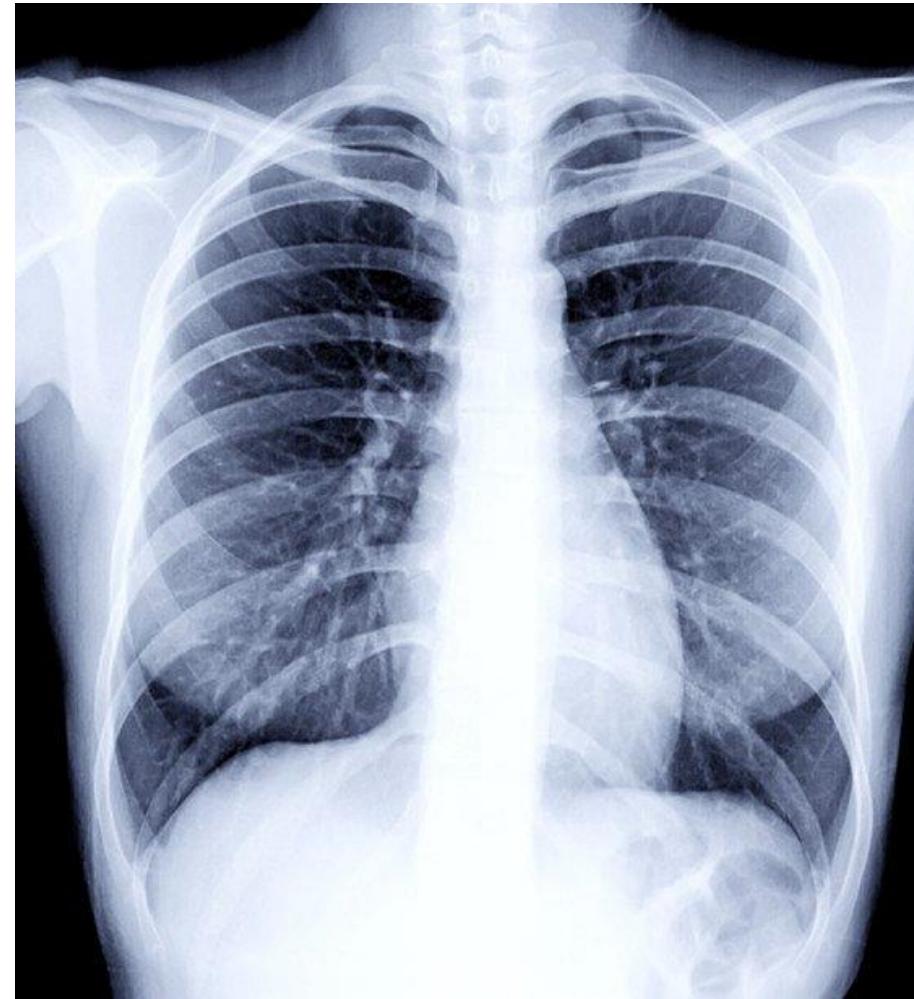
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

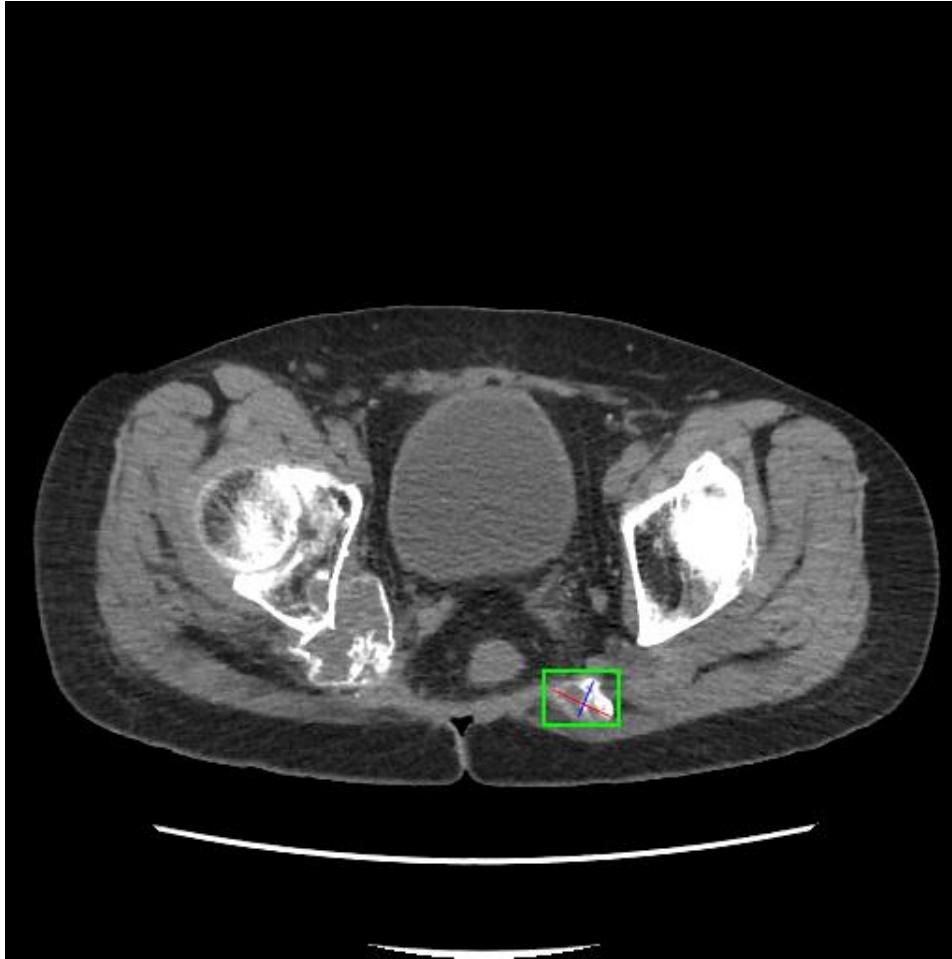
# Medical Images

- Most medical images are results of EM waves
  - X-ray and CT
  - MRI
  - Histopathology & other optical microscopy images
  - PET
  - OCT
- Some medical images use sound waves
  - Ultrasound

# Medical Images: X-ray



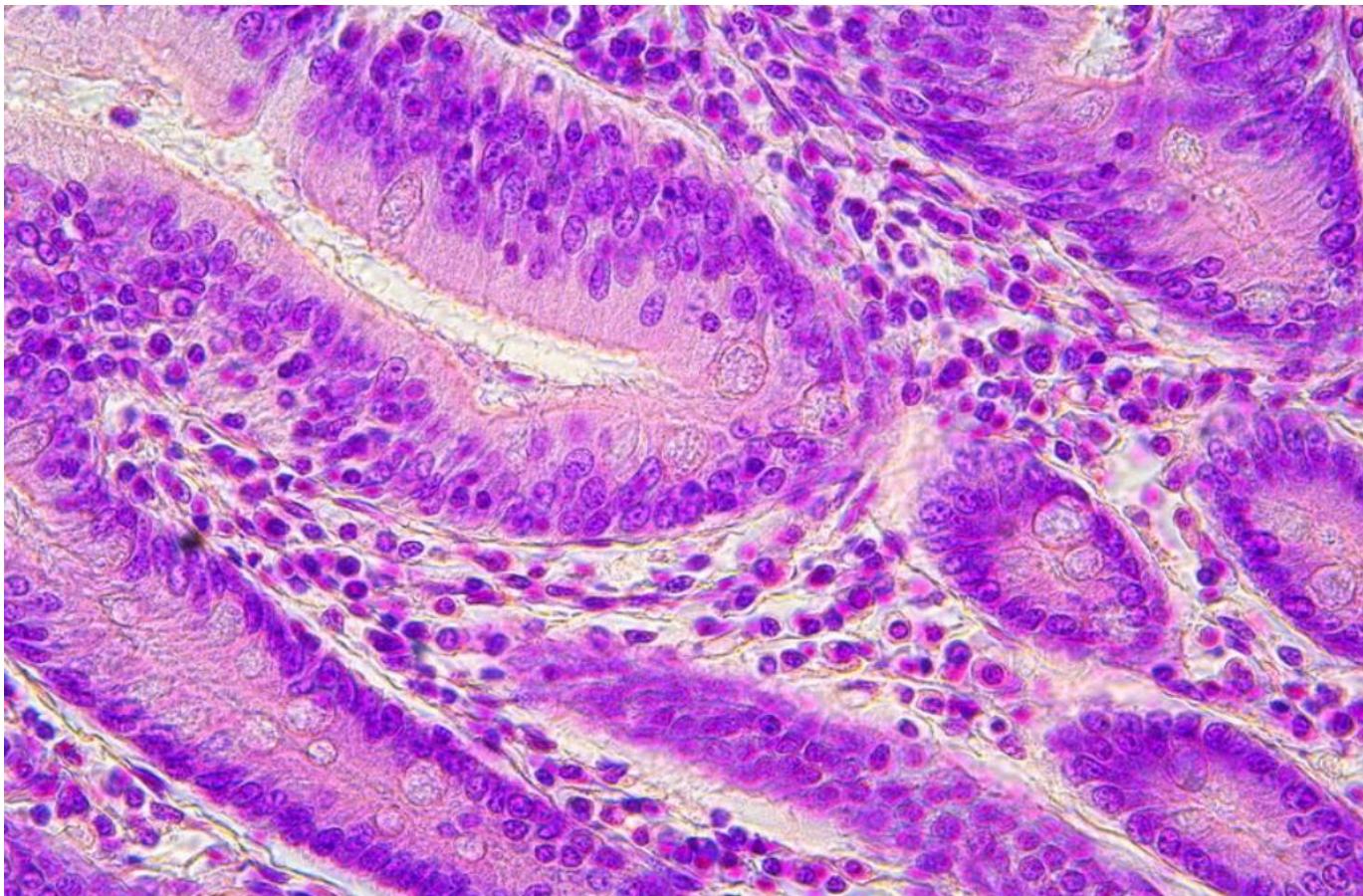
# Medical Images: CT



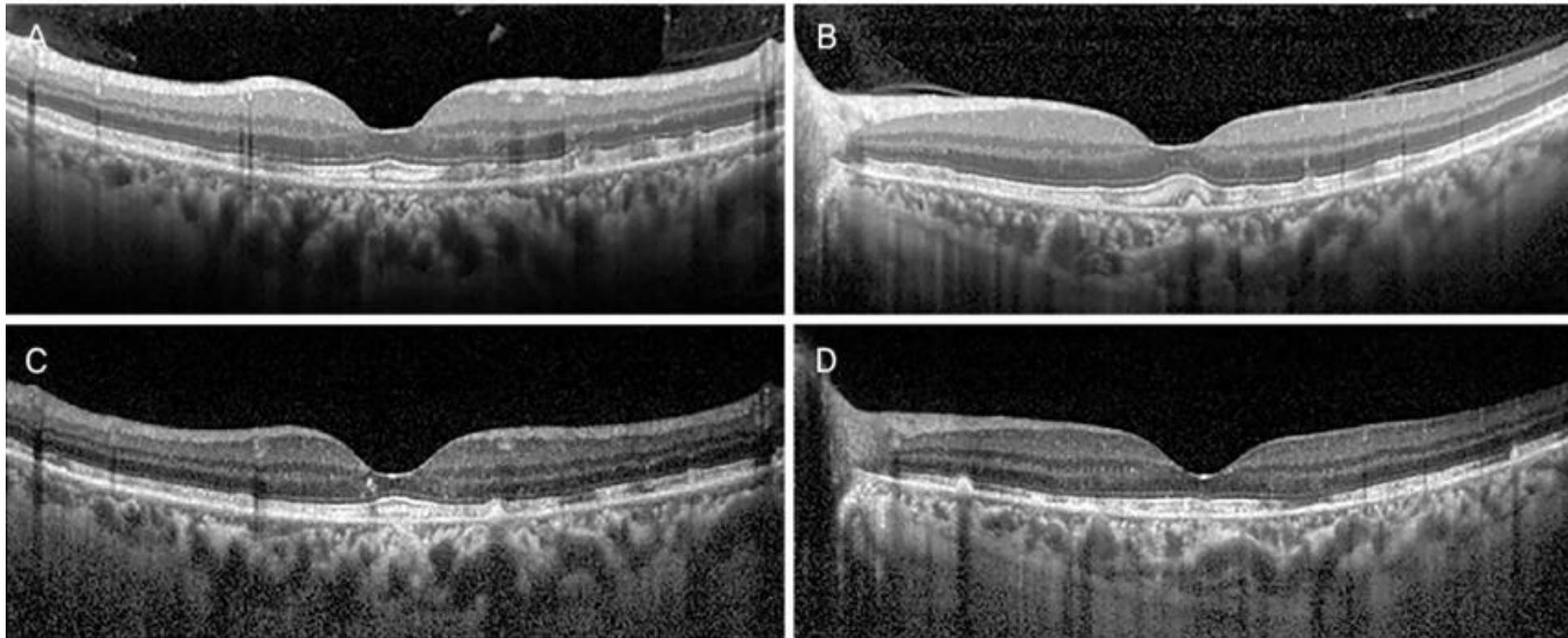
# Medical Images: MRI



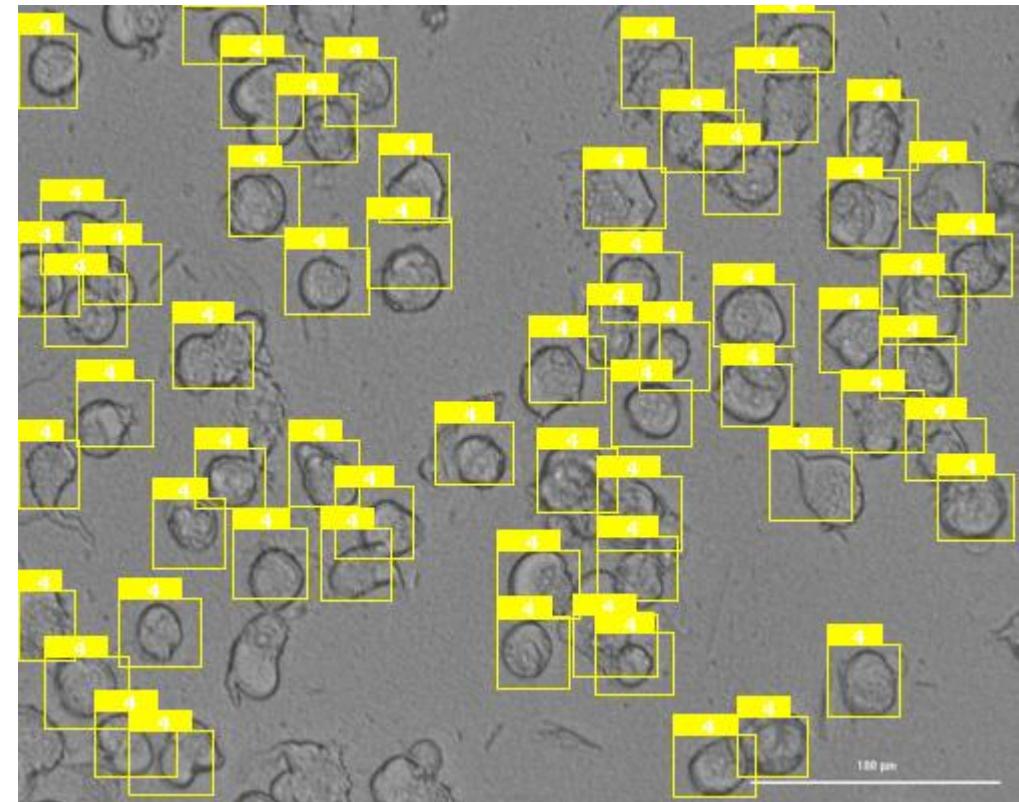
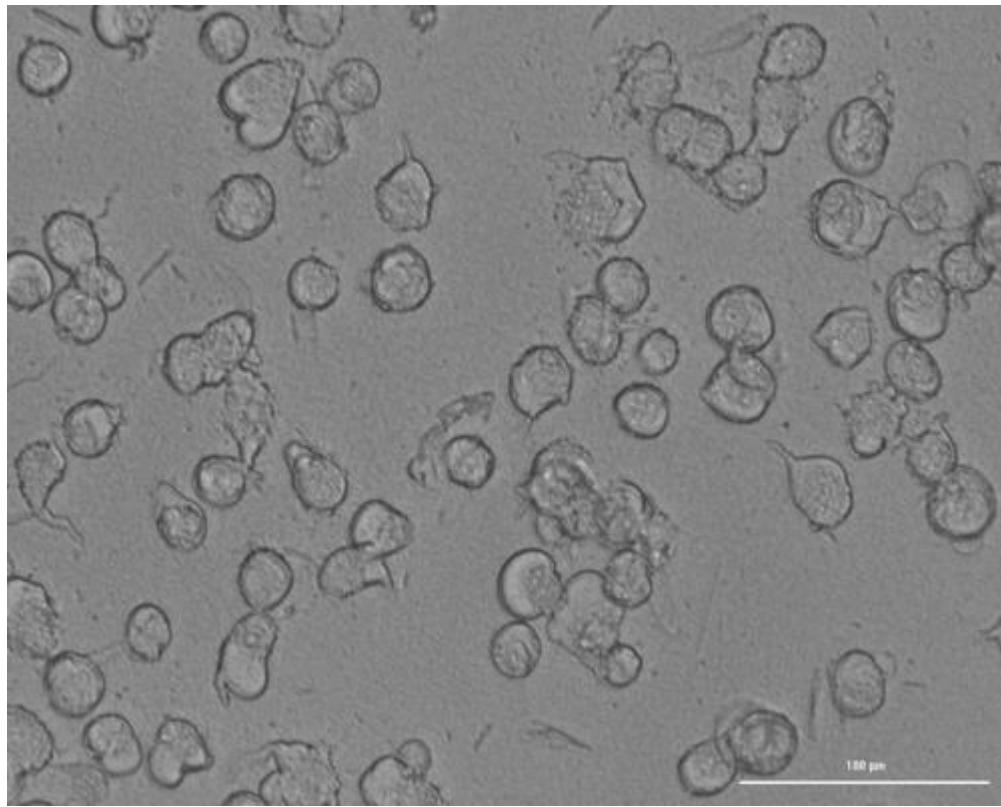
# Medical Images: Histopathology



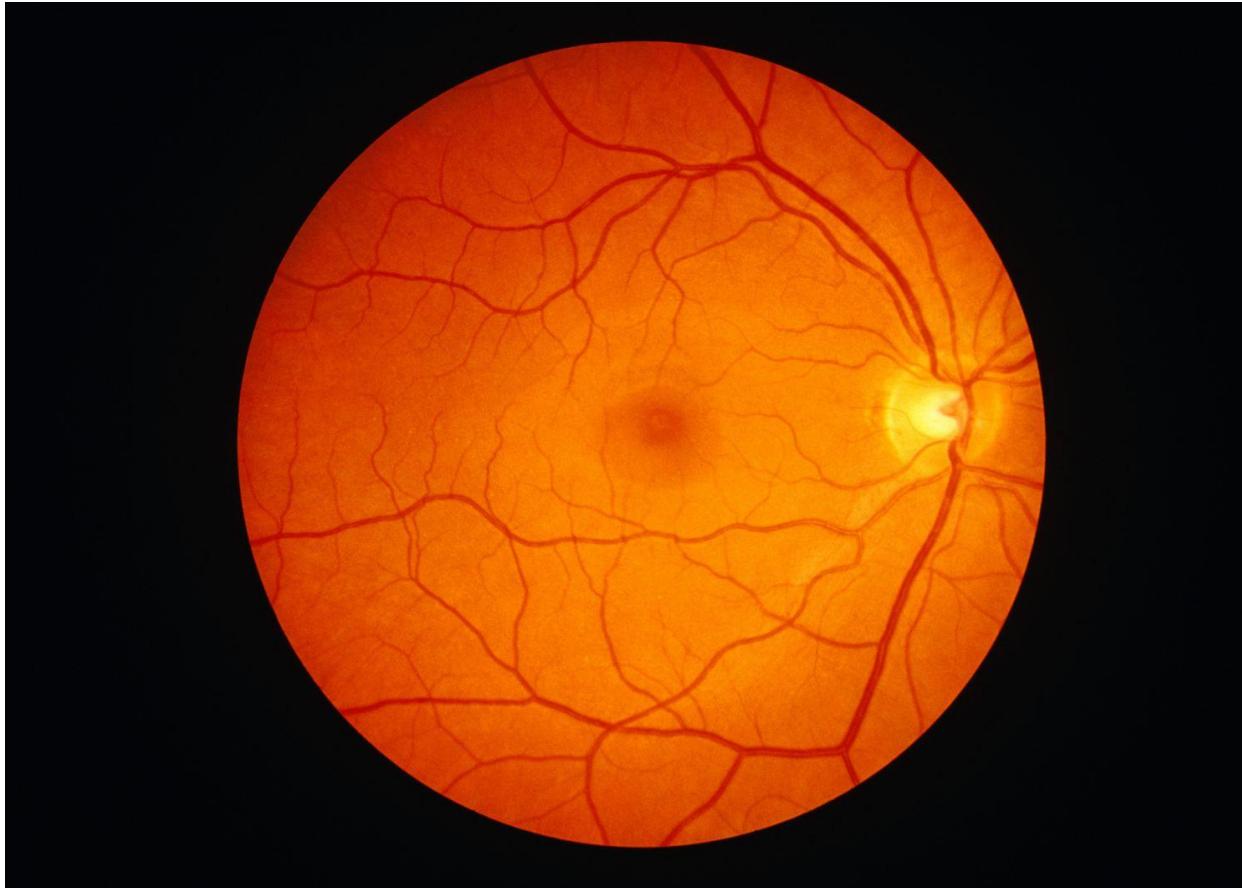
# Medical Images: SDOCT



# Medical Images: Cell Images



# Medical Images: Retinal Images



# What is Medical Image Analysis

- Use of ML, CV, IP techniques for the diagnosis of medical images

# Why Medical Image Analysis

- To make decision support systems
  - Better diagnosis
    - By making an automated diagnosis to assist the clinician
    - By improving the quality of the images
  - Faster
  - Facilitating telemedicine applications
  - Affordable health care

# Present Status

- Unprecedented progress in recent years
- Human level or super-human accuracies in several occasions
- FDA approval for decision support systems

# Present Status: Challenges

- Generalizability
- Explainability
- Cost of annotation
- Computational requirements
- Others

# Why Study MedIA?

- To know the classical and SOTA approaches
- Improve the existing approaches
  - Push the boundary of our ability
- Designing new approaches
  - To address the major challenges
- Use the understandings/ innovations from this area in solving problems of other fields

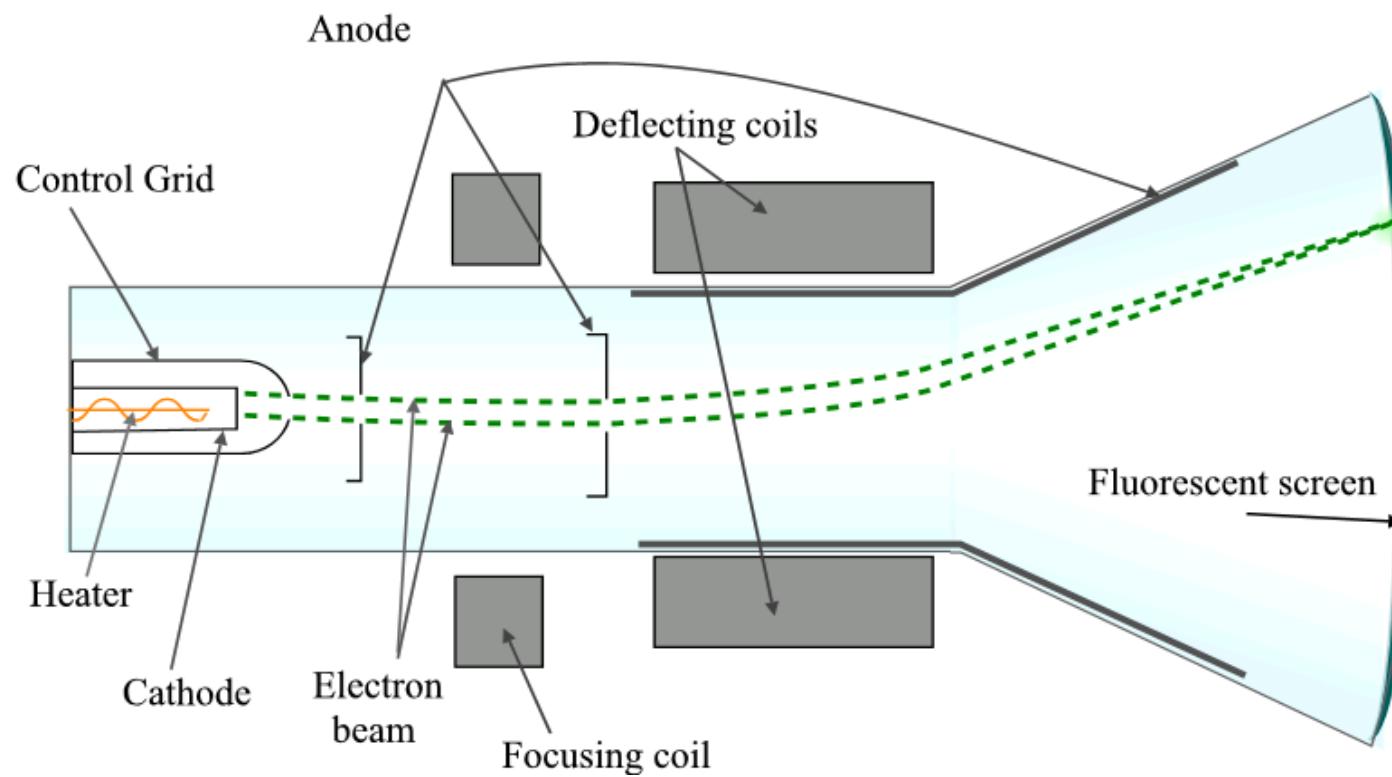
# X-ray

- Serendipity

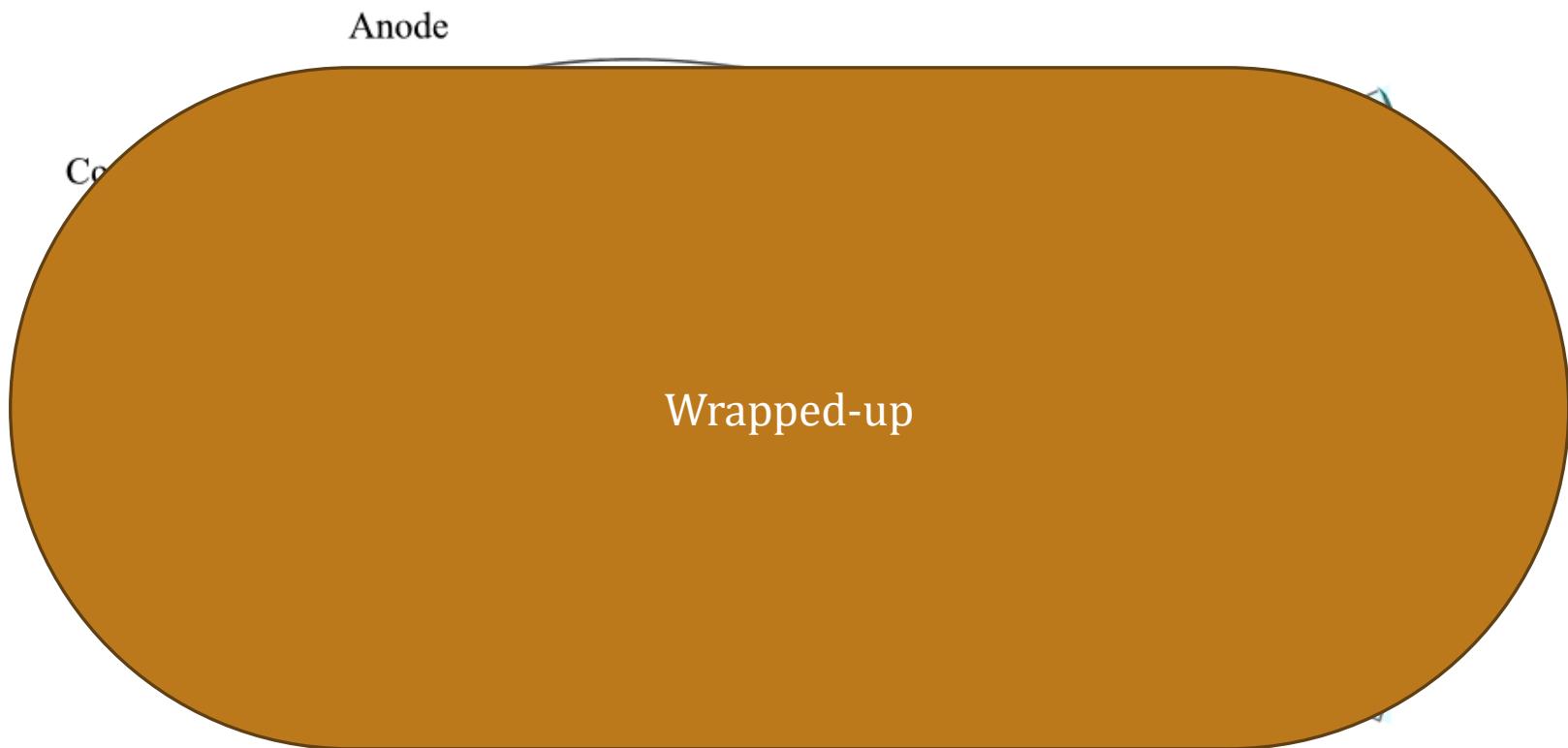
# X-ray

- Wilhelm Conrad Röntgen
- 1895
- First observed: 1869

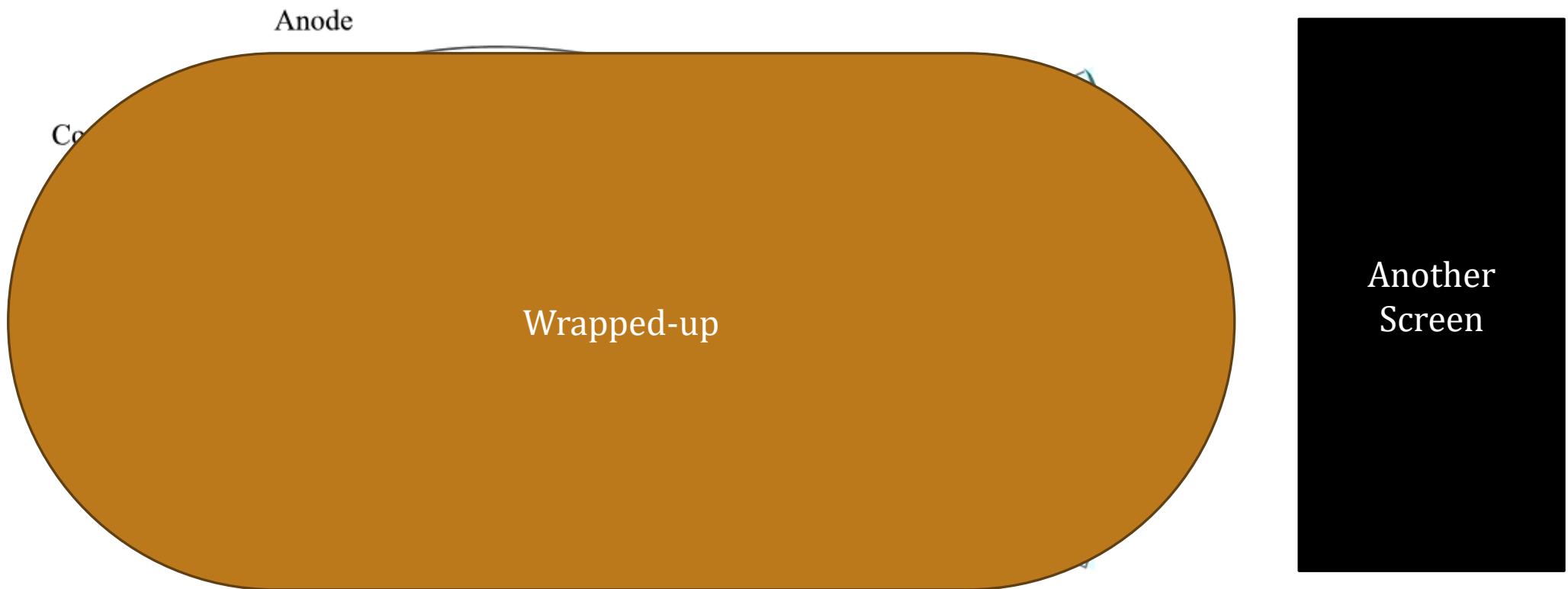
# What is X-ray



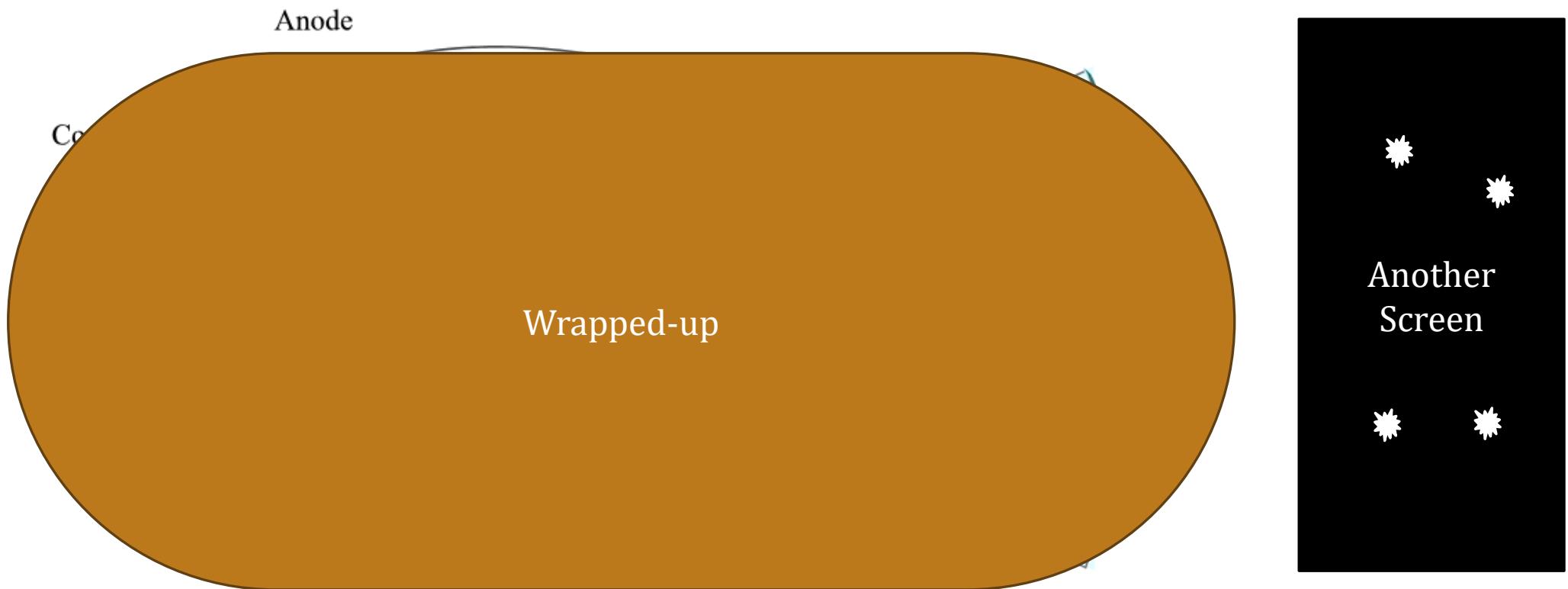
# What is X-ray



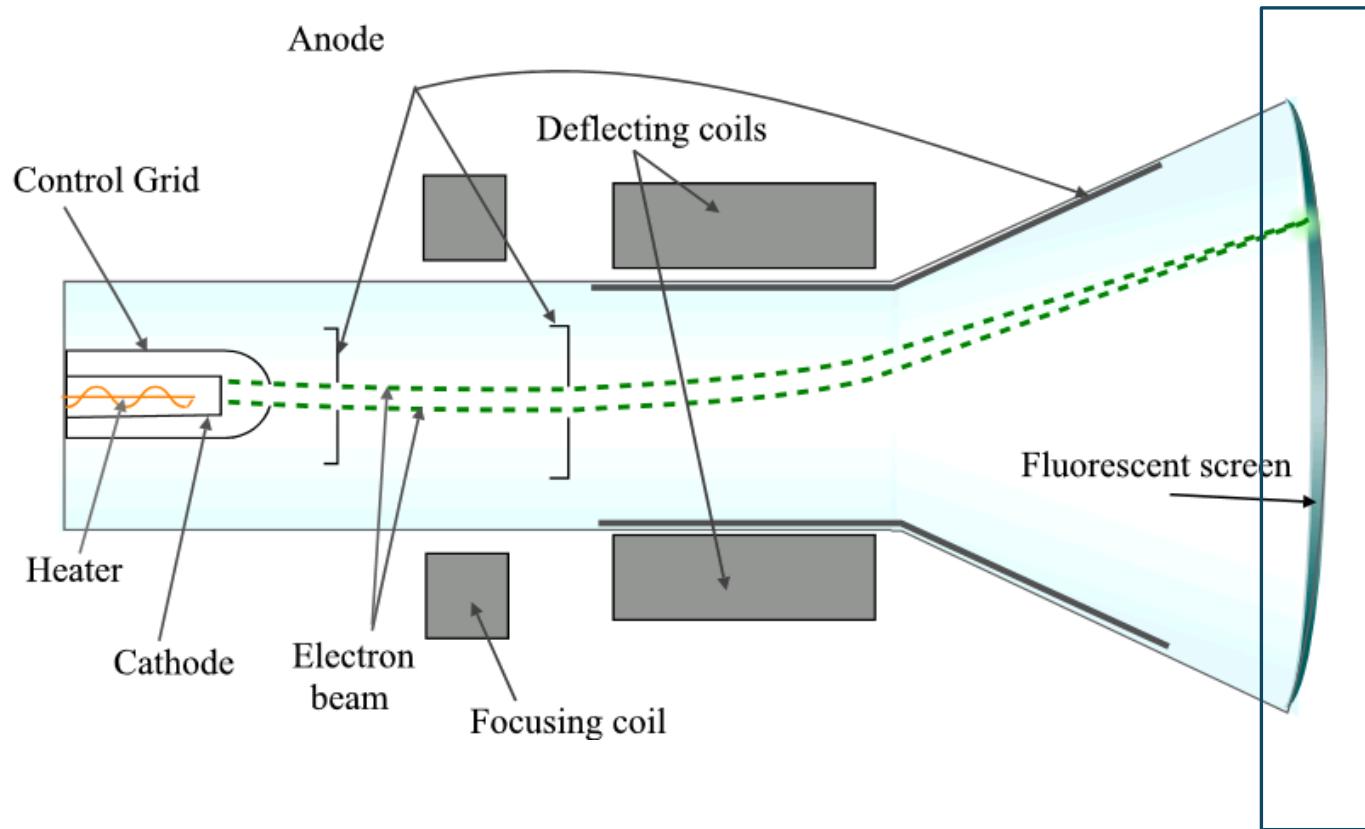
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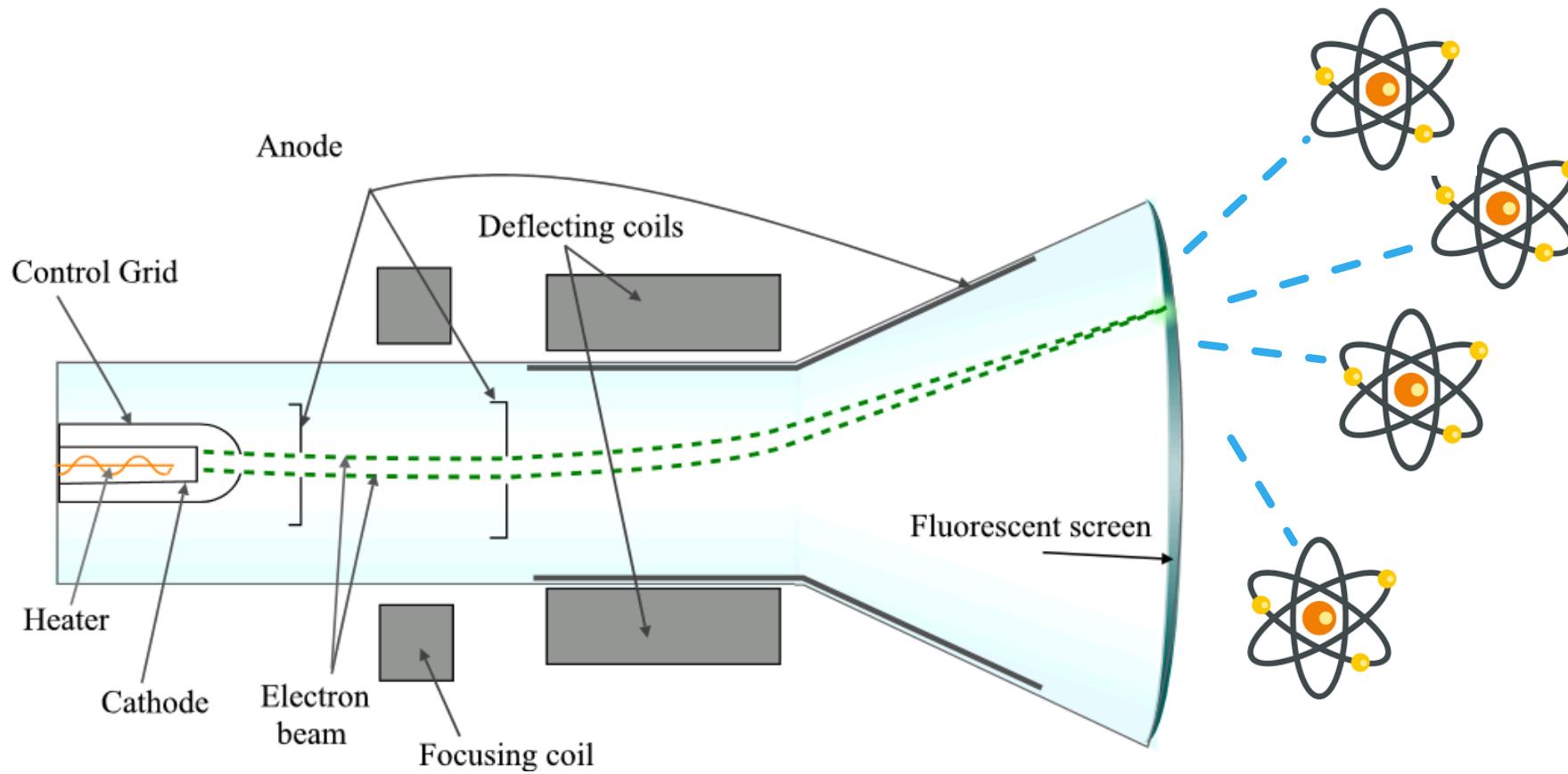
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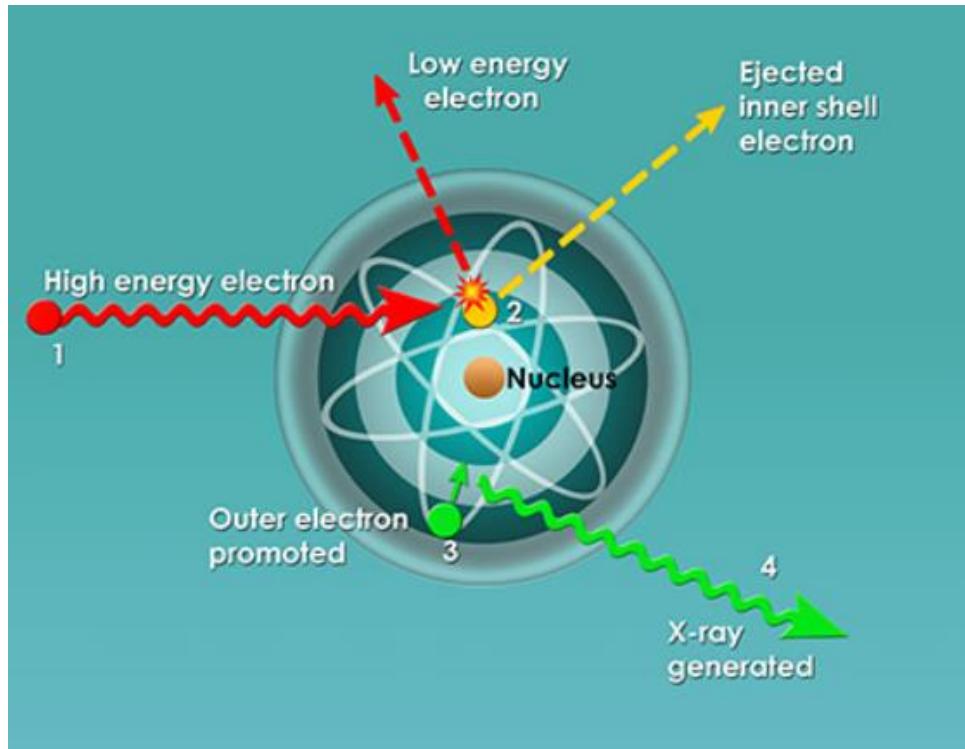
# What Happens Internally



# What Happens Internally

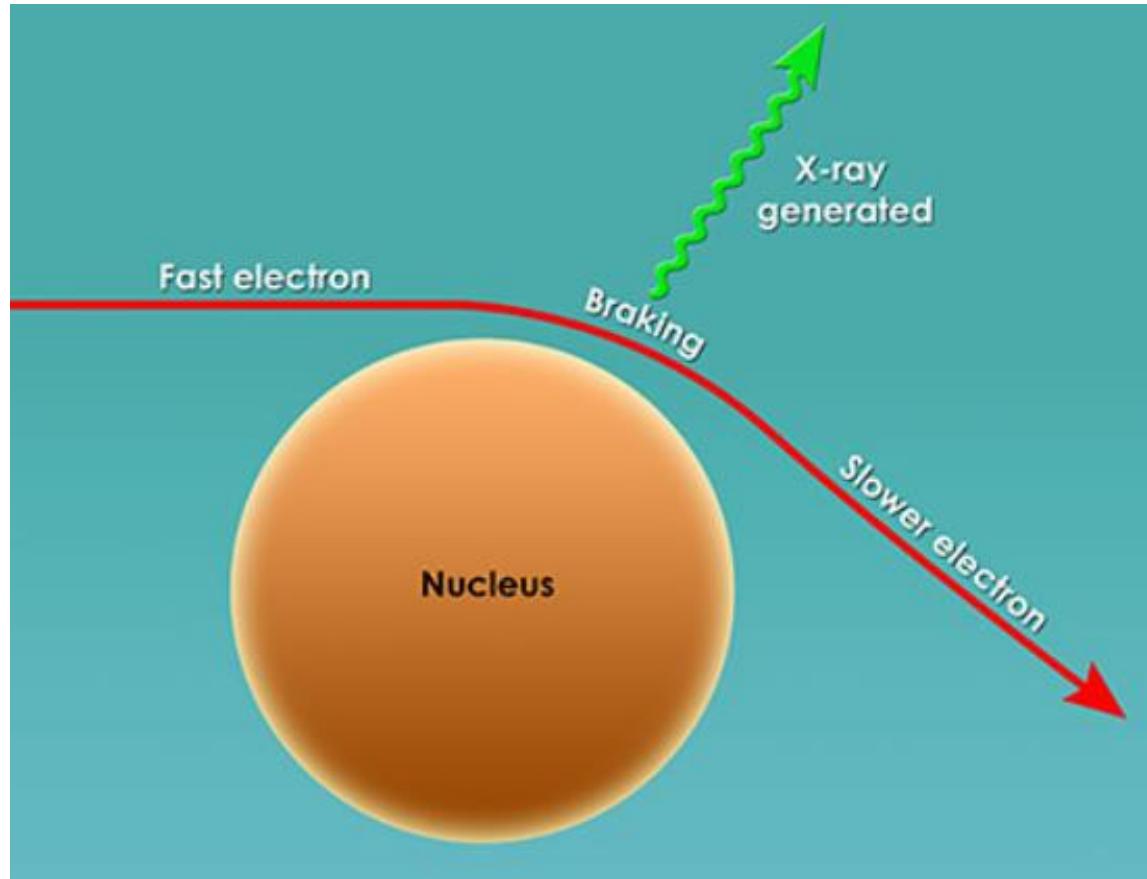


# Characteristics Generation

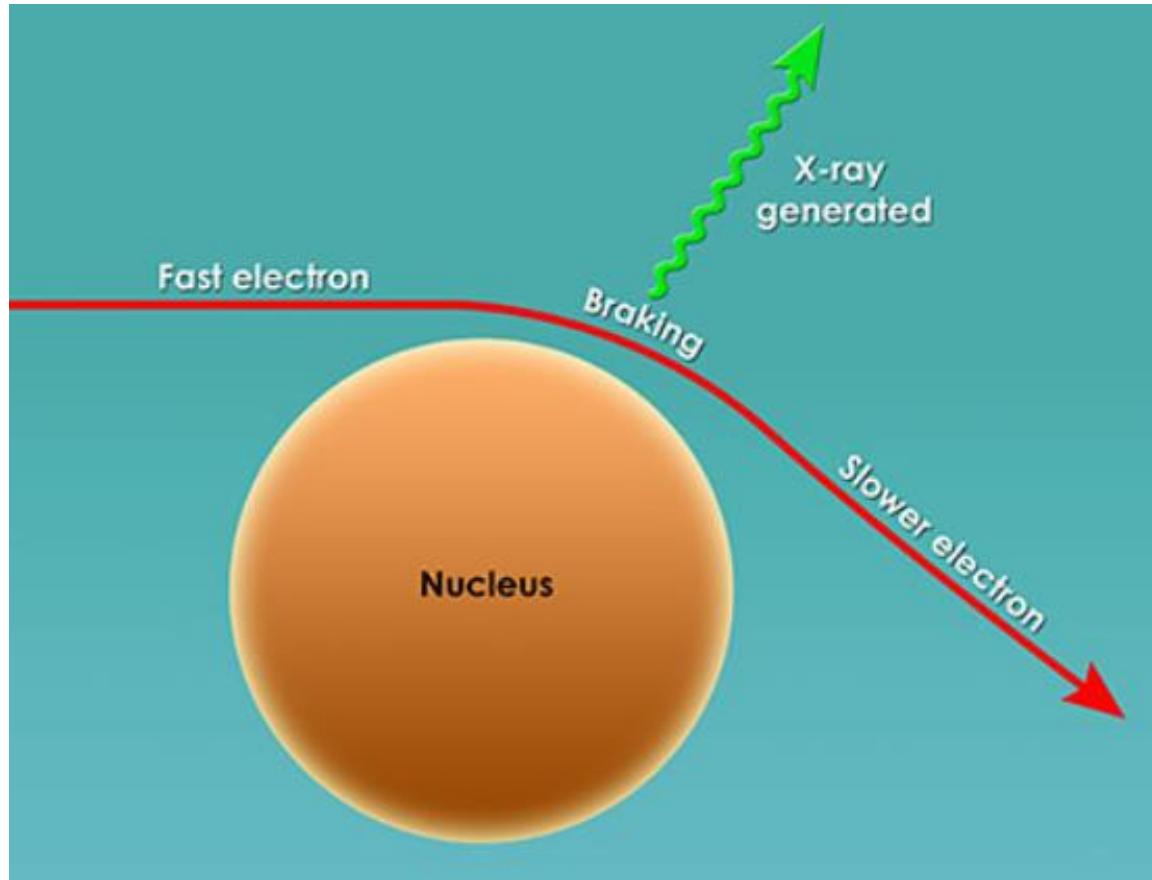


When a high energy electron (1) collides with an inner shell electron (2) both are ejected from the tungsten atom leaving a 'hole' in the inner layer. This is filled by an outer shell electron (3) with a loss of energy emitted as an X-ray photon (4)

# Bremsstrahlung/Braking X-ray Generation

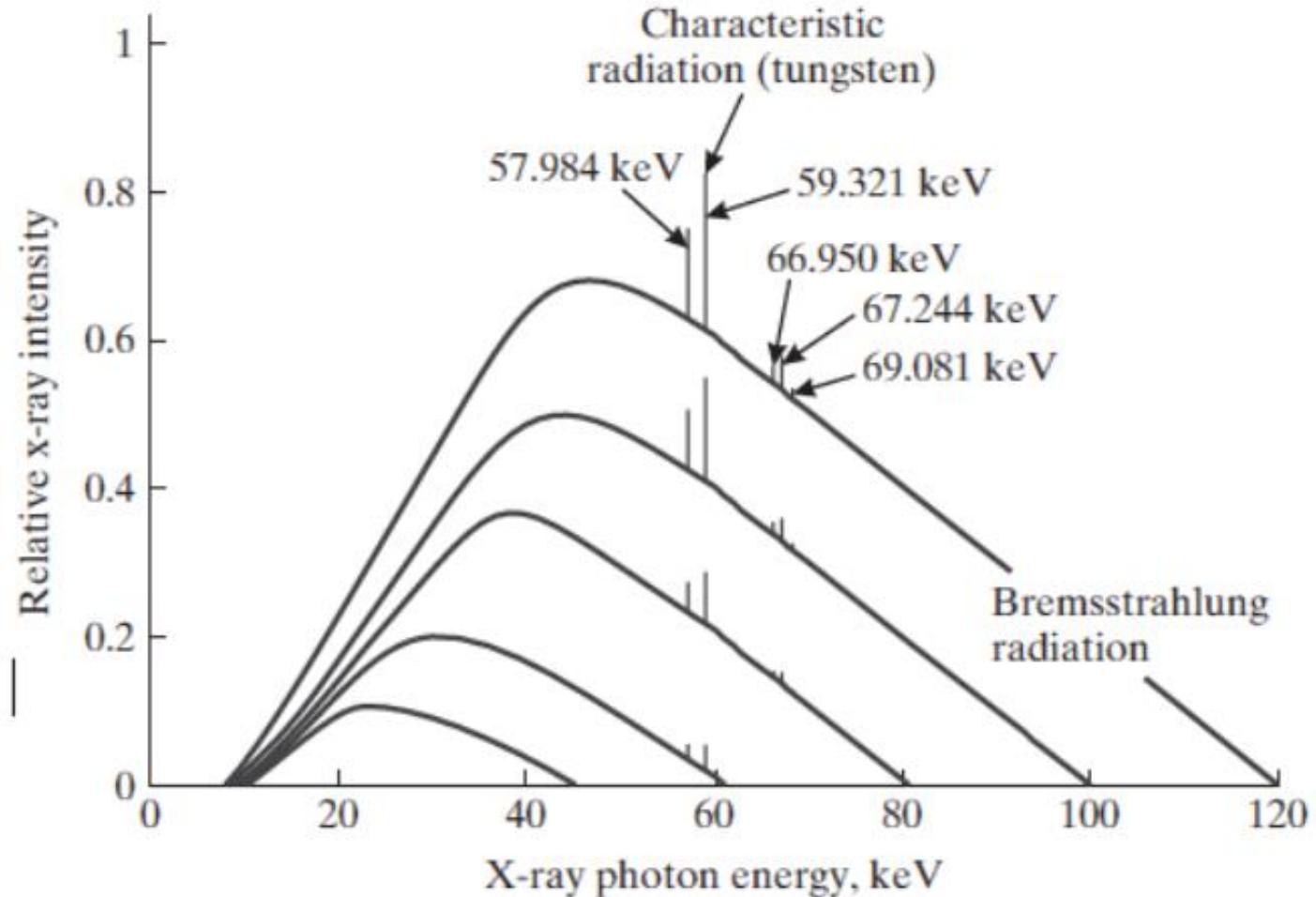


# Bremsstrahlung/Braking X-ray Generation

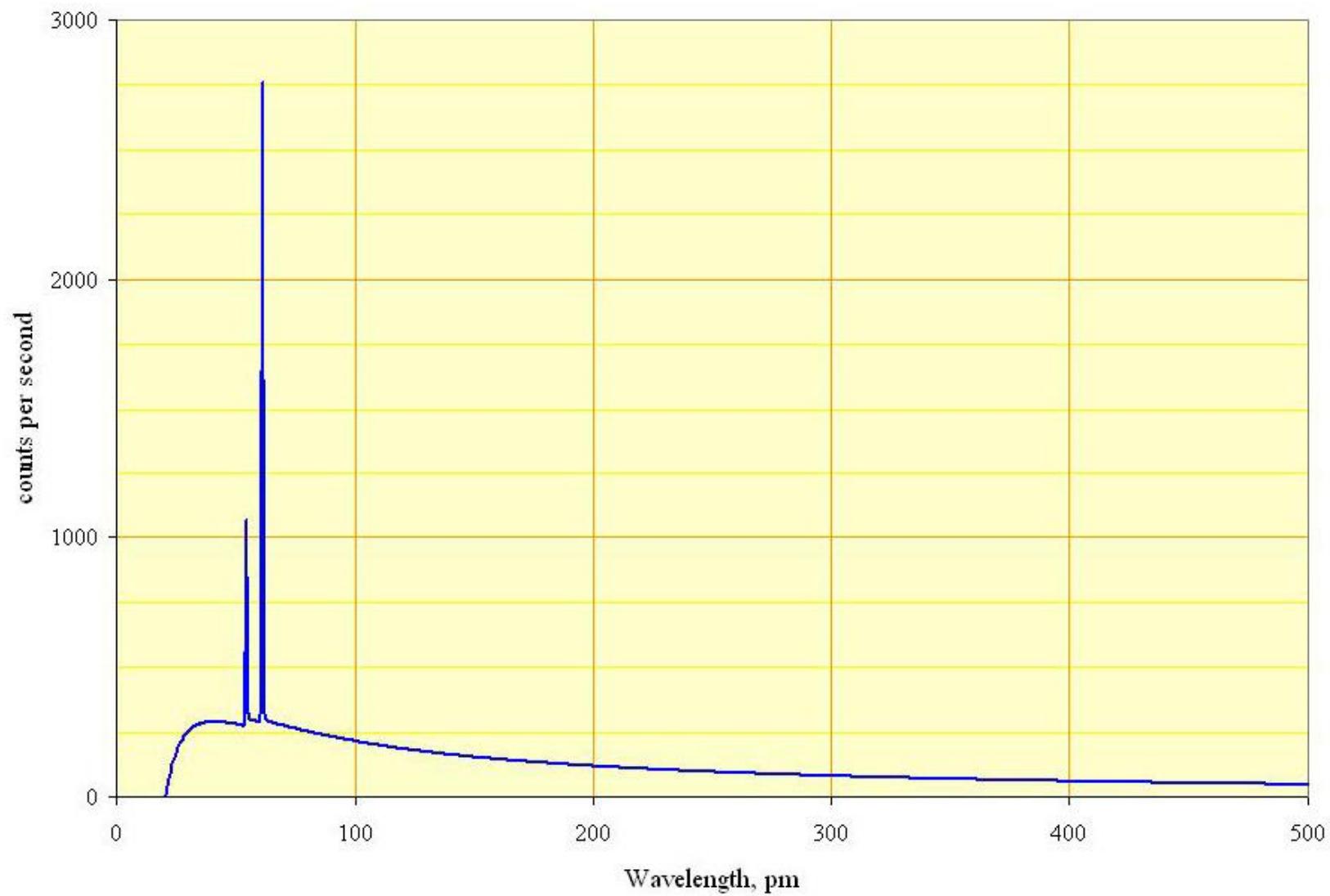


- When an electron passes near the nucleus it is slowed and its path is deflected. Energy lost is emitted as a bremsstrahlung X-ray photon.
- Bremsstrahlung = Braking radiation
- Approximately 80% of the population of X-rays within the X-ray beam consists of X-rays generated in this way.
- **Primary source of x-ray from x-ray tube**

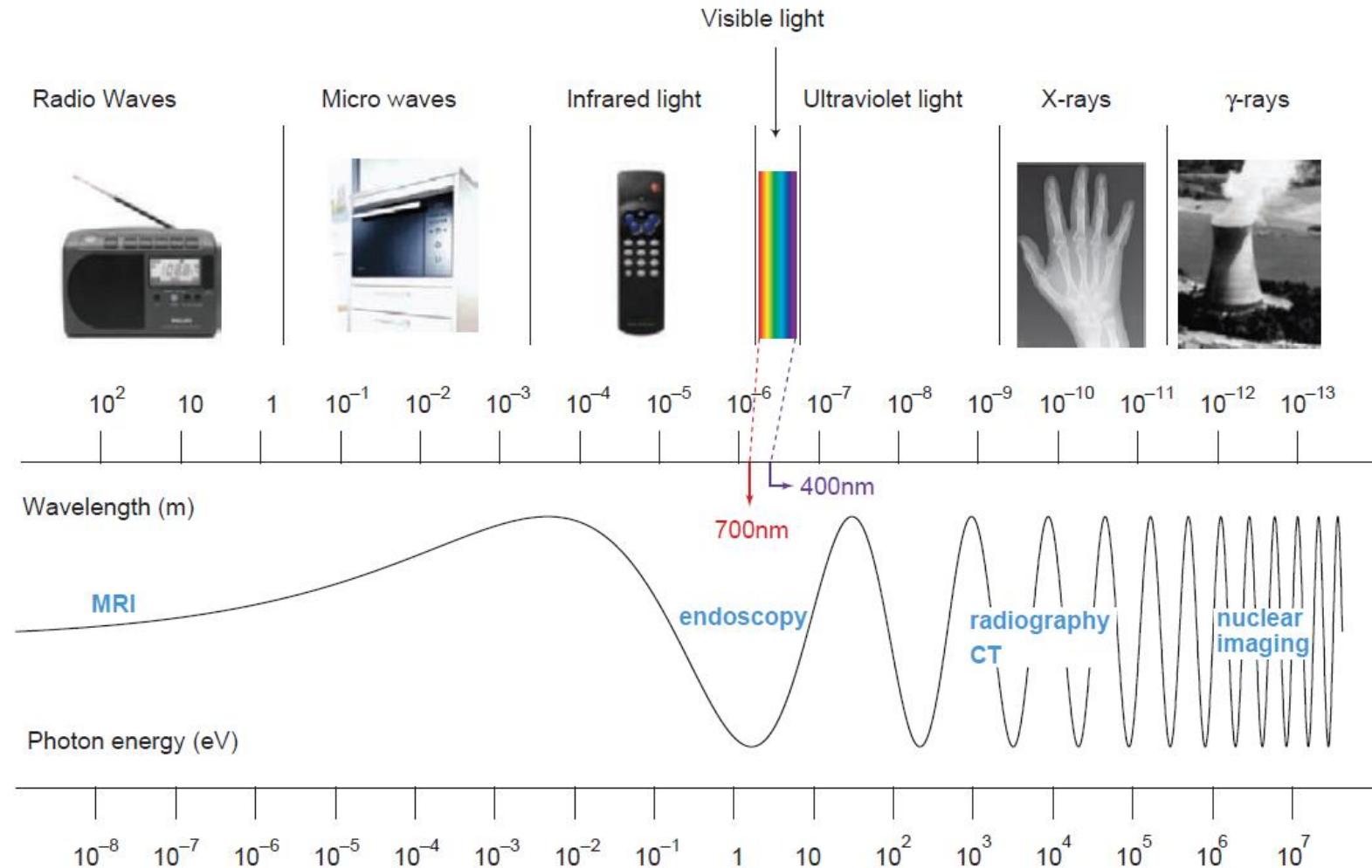
# X-ray Generation



# X-ray Generation



# EM Spectrum



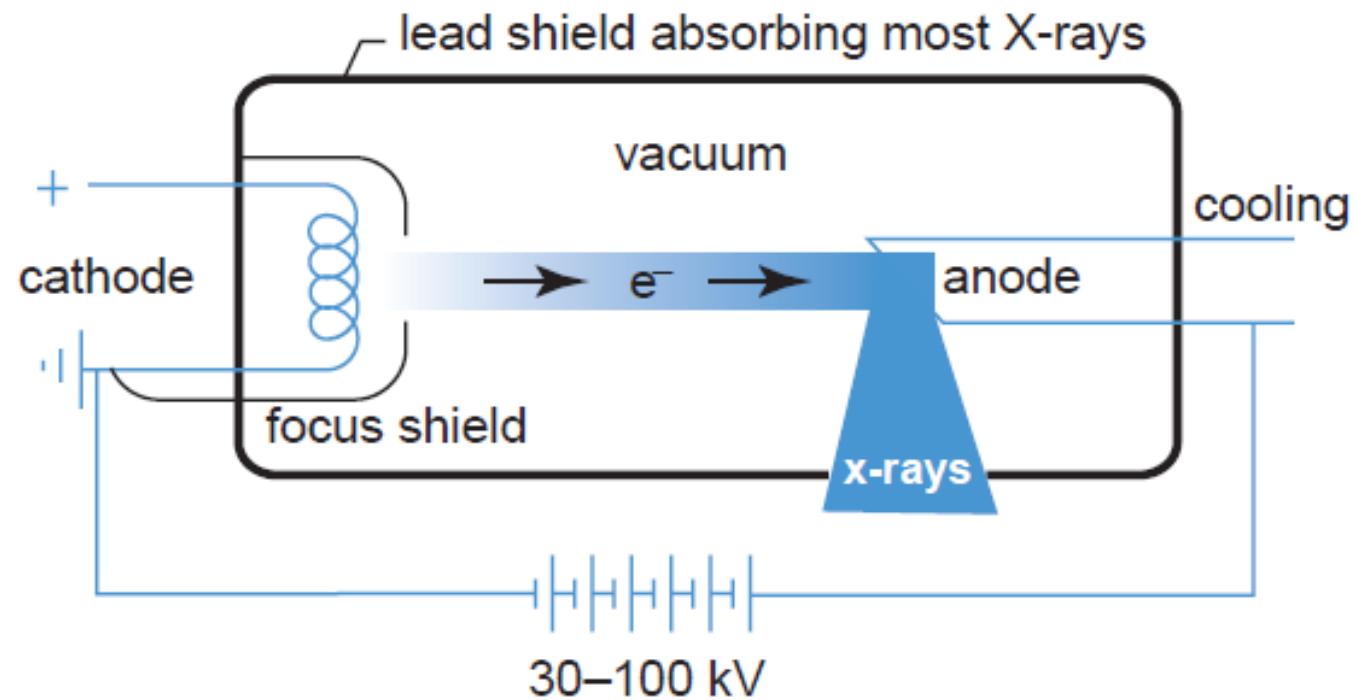
# EM Spectrum

TABLE 4.2

The EM Spectrum			
Frequency Range	Wavelengths	Photon Energies	Description
$1.0 \times 10^5$ – $3.0 \times 10^{10}$ Hz	3 km–0.01 m	413 peV–124 $\mu$ eV	Radio waves
$3.0 \times 10^{12}$ – $3.0 \times 10^{14}$ Hz	100–1 $\mu$ m	12.4 meV–1.24 eV	Infrared radiation
$4.3 \times 10^{14}$ – $7.5 \times 10^{14}$ Hz	700–400 nm	1.77–3.1 eV	Visible light
$7.5 \times 10^{14}$ – $3.0 \times 10^{16}$ Hz	400–10 nm	3.1–124 eV	Ultraviolet light
$3.0 \times 10^{16}$ – $3.0 \times 10^{18}$ Hz	10 nm–100 pm	124 eV–12.4 keV	Soft x-rays
$3.0 \times 10^{18}$ – $3.0 \times 10^{19}$ Hz	100–10 pm	12.4–124 keV	Diagnostic x-rays
$3.0 \times 10^{19}$ – $3.0 \times 10^{20}$ Hz	10–1 pm	124 keV–1.24 MeV	Gamma rays

Adapted from Johns and Cunningham, 1983.

# Generation of X-ray



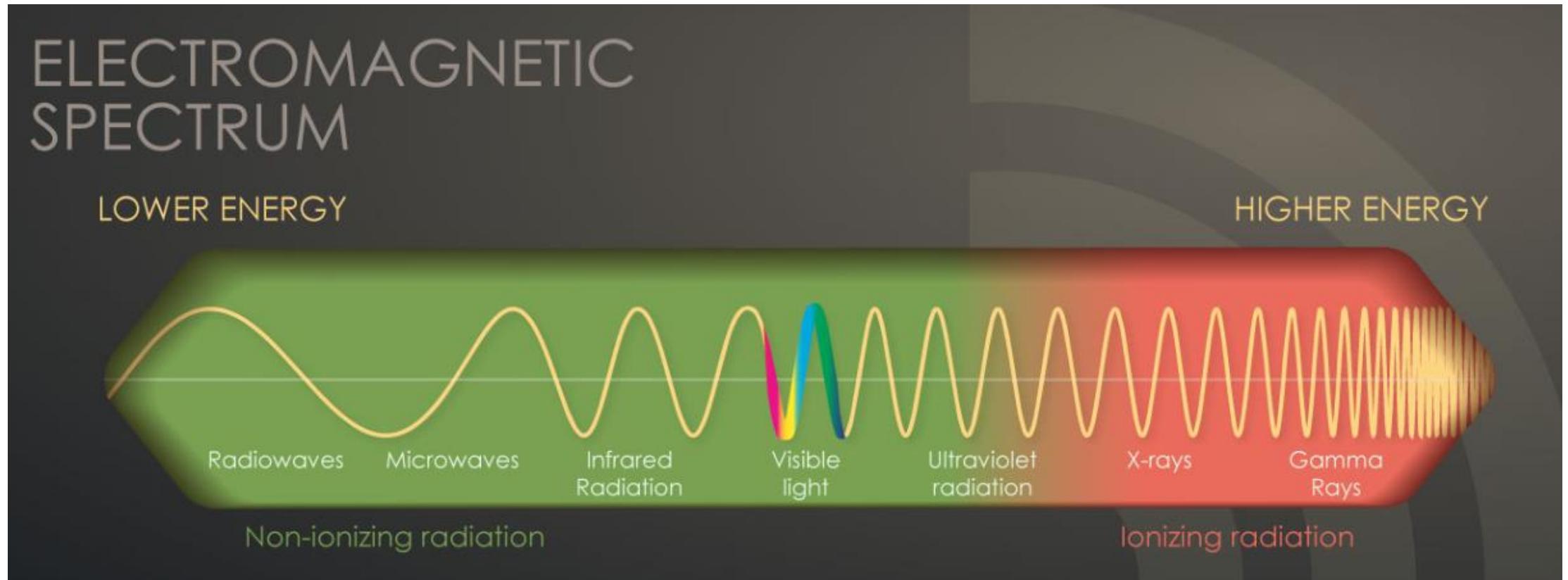
# Interaction of X-ray with Matter

- $E = h\nu$
- X-ray Photons
- May be absorbed by electrons of atoms
  - Electron will get sufficient energy to escape the atom
  - Creation of ions
- Ionizing radiation

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- Photons with energy less than 13.6 eV are nonionizing
  - Cannot eject an electron from its atom, but are only able to raise it to a higher energy shell (excitation)

# Ionizing Radiation



# Ionizing Radiation

- Useful for diagnostic imaging (such as x-ray)
- Ionizing activity can alter molecules within the cells of our body
  - May cause eventual harm (such as cancer)
- Intense exposures to ionizing radiation may produce skin or tissue damage.

# Processes of X-ray Interaction

- Photons can interact in different ways
- Rayleigh scattering or coherent scattering
  - Low energy photons:  $< 30 \text{ keV}$
  - Absorbed by an electron and immediately released again as of a new photon with the same energy but traveling in a different direction
  - Non ionizing
  - Usually not observed during standard x-ray imaging
  - May be observed during mammography

# Processes of Ionization

- Photoelectric absorption
  - Higher energy photon
  - Absorbed by an electron and the electron escapes the atom
  - Travels in the same direction as that of the incident photon
  - Ionizing

# Processes of Ionization

- Compton scattering
  - Higher energy photon
  - Incident photon transfers partial energy to an electron
  - The electron escapes the atom
  - A photon with remaining energy travels in a different compared to that of the incident photon
  - Ionizing

# Processes of Ionization

- Pair production
  - Electron-positron pair is created from an incident photon
  - Positron meets another electron and annihilates producing energy
  - The energy is released in the form of a photon pair travelling in opposite directions
  - Used in nuclear medicine

# Processes of Ionization

- At even higher energy, photon may cause nuclear reactions
- Not used in medical applications

# Medical Usage



First x-ray of human body

# Medical Usage

- X-ray is attenuated by different tissues
- The more the atomic number of an element, the more the attenuation
  - Bone: Ca ( $Z=20$ )
    - More attenuation, output photon with less energy
    - Bright regions in x-ray films
  - Soft tissue: C ( $Z=6$ ), H ( $Z=1$ ), O ( $Z=8$ )
    - Less attenuation, output photon with more energy
    - Dark regions in x-ray films

# Medical Usage

Intensity of outgoing beam       $I_{\text{out}}$  :       $I_{\text{in}}$  Intensity of incoming beam

# Medical Usage

$$I_{\text{out}} = I_{\text{in}} e^{-\mu d}$$

**Homogeneous medium, photons of single energy**

$\mu$ : Attenuation coefficient

Function of photon energy and medium

$$\mu(10 \text{ keV}, \text{H}_2\text{O}) = 5 \text{ cm}^{-1}$$

$$\mu(100 \text{ keV}, \text{H}_2\text{O}) = 0.17 \text{ cm}^{-1}$$

$$\mu(10 \text{ keV}, \text{Ca}) = 144 \text{ cm}^{-1}$$

$$\mu(100 \text{ keV}, \text{Ca}) = 0.40 \text{ cm}^{-1}.$$

# Medical Usage

$$I_{\text{out}} = I_{\text{in}} e^{-\mu d}$$

**Homogeneous medium, photons of single energy**

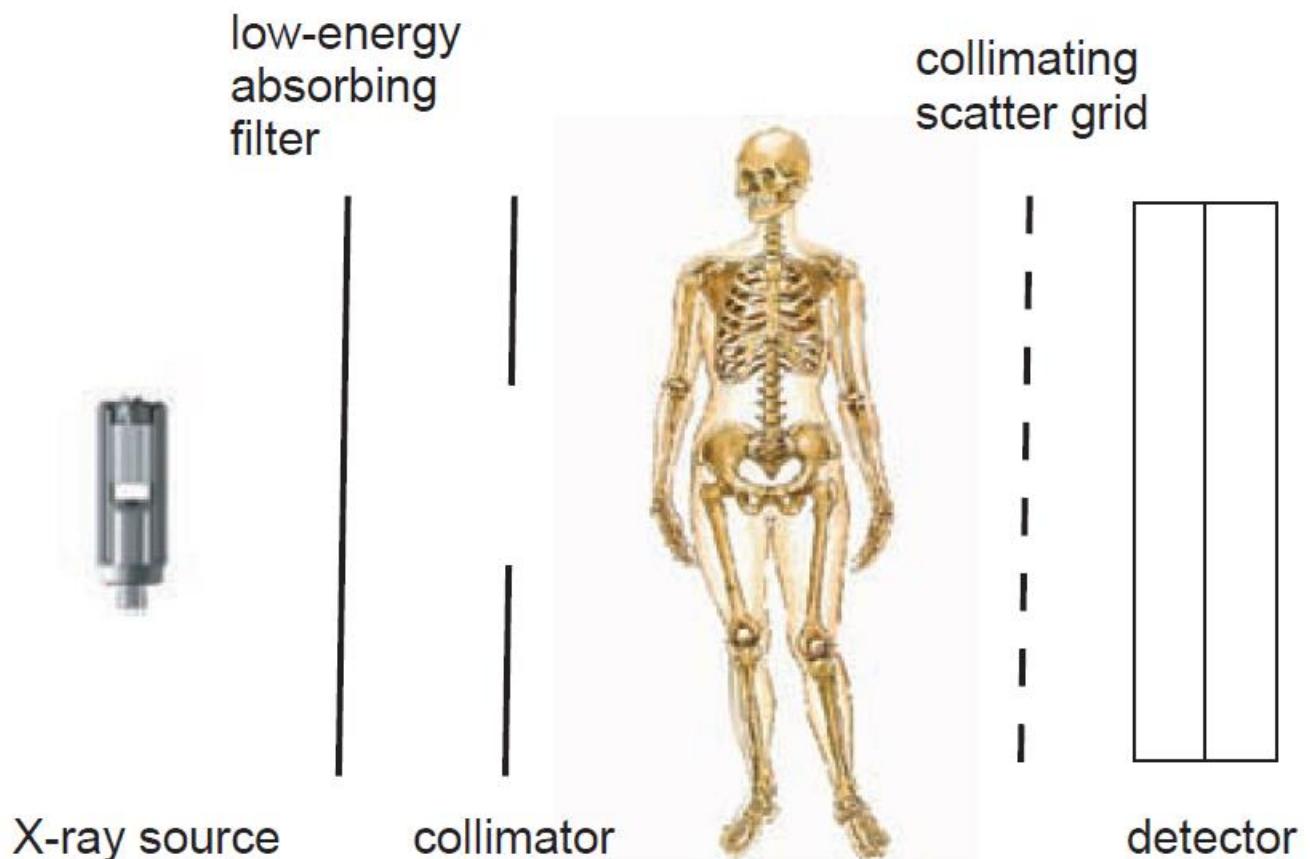
$$I_{\text{out}} = I_{\text{in}} e^{-\int_{x_{\text{in}}}^{x_{\text{out}}} \mu(x) dx}$$

**Heterogeneous medium, photons of single energy**

$$I_{\text{out}} = \int_0^{\infty} \sigma(E) e^{-\int_{x_{\text{in}}}^{x_{\text{out}}} \mu(E,x) dx} dE$$

**Heterogeneous medium, photon of multiple energy**

# X-ray Equipment



**Figure 2.9** Schematic representation of the radiographic imaging chain.



(a)



(b)



(c)



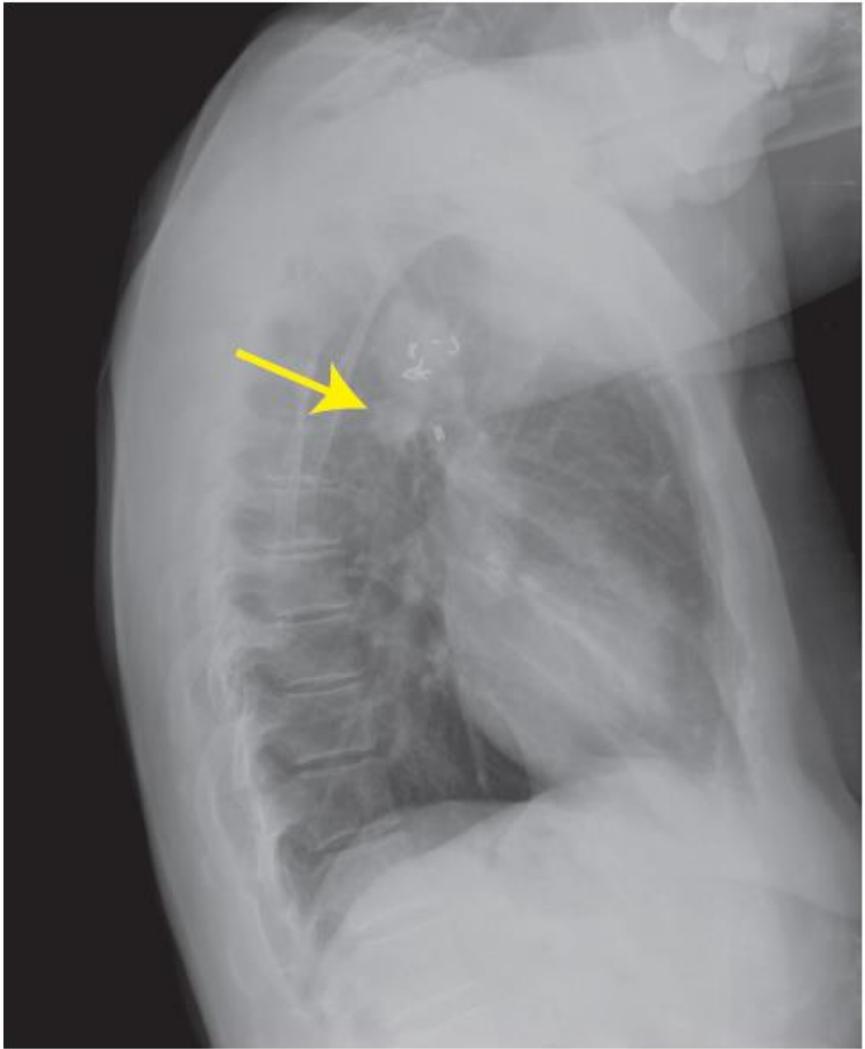
(d)



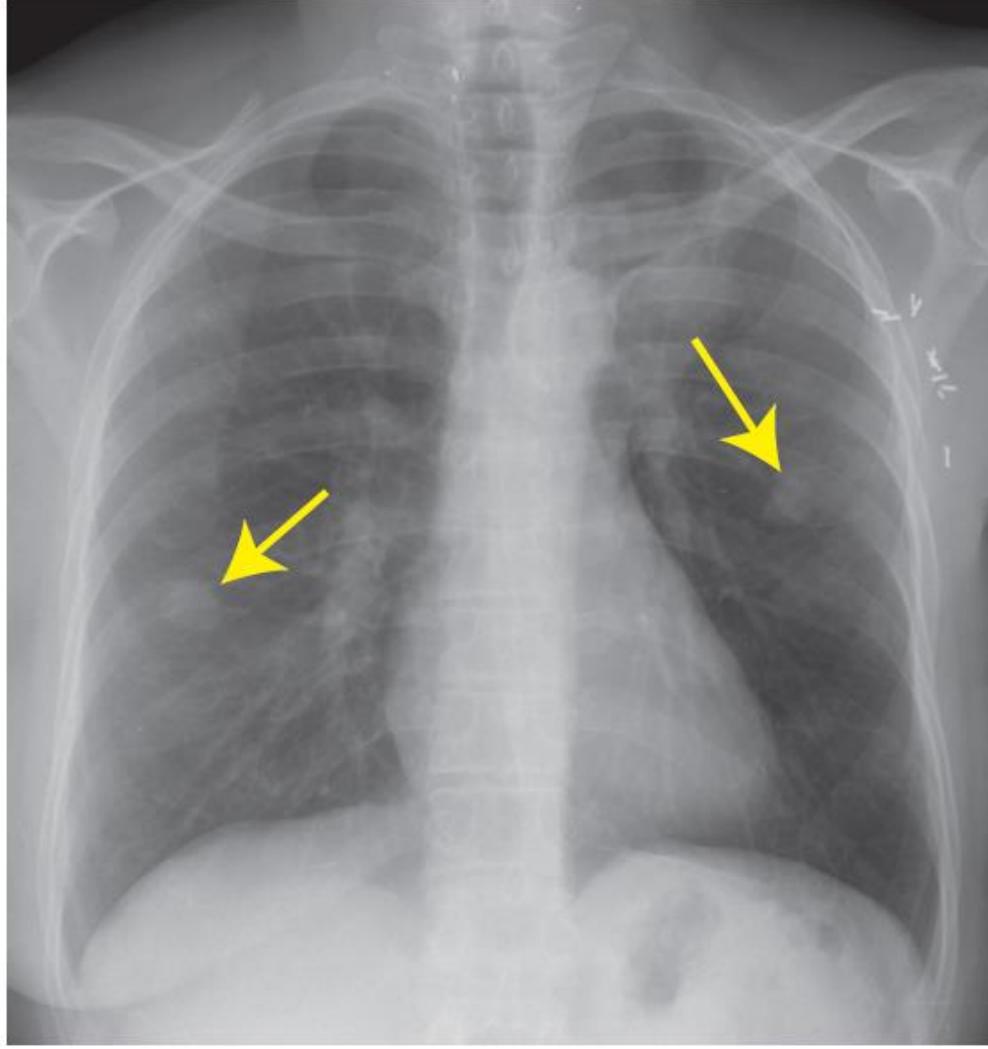
(e)



(f)



(a)



(b)

**Figure 2.15** Radiographic chest image showing multiple lung metastases. (Courtesy of Professor J. Verschakelen, Department of Radiology.)



(a)



(b)



(c)

**Figure II.2**  
X-ray images of (a) the spine shown a surgical fixation device, (b) the pelvis showing two artificial hip joints, and (c) the knee showing bone fixation wires. Image (a) courtesy of Philips Healthcare. Images (b) and (c) courtesy of GE Healthcare.

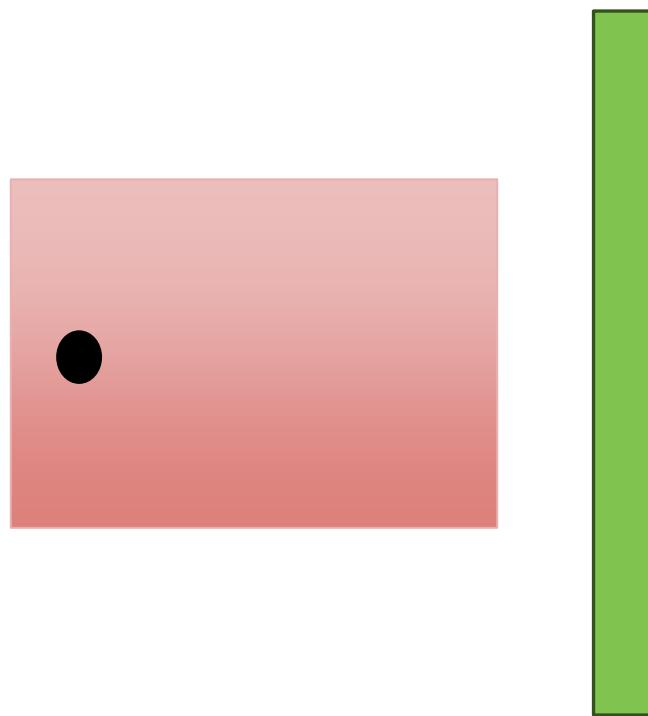
# X-ray Equipment



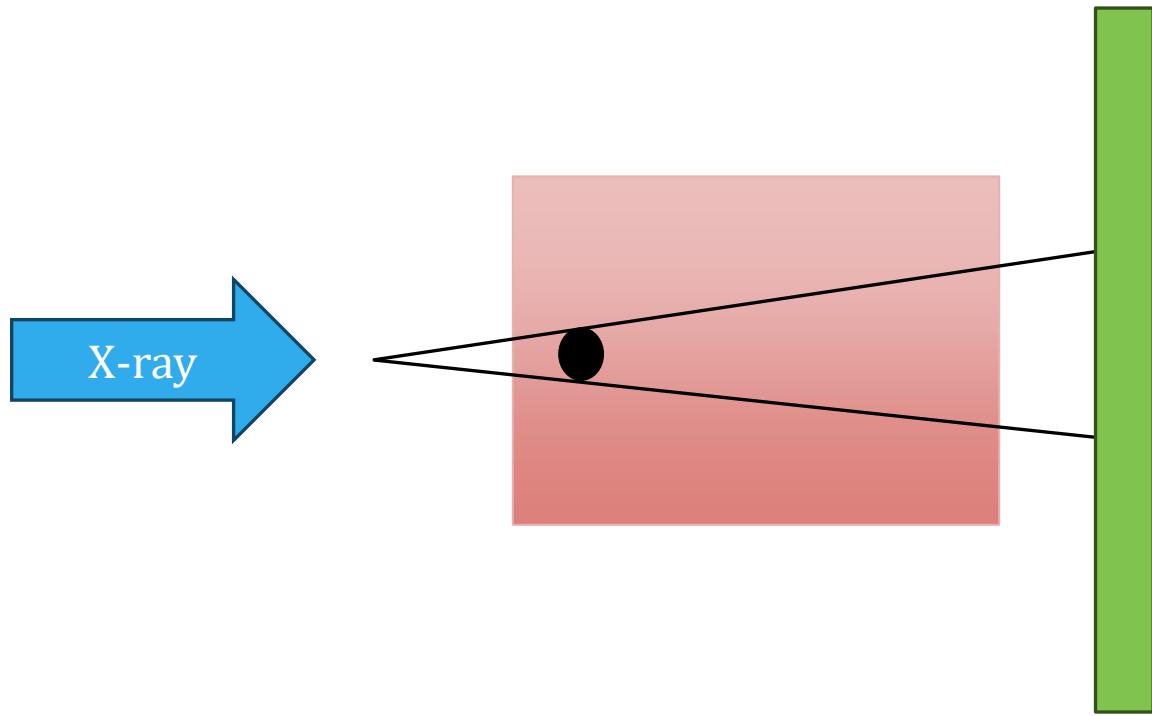
# Issues in X-ray

- Noise: Quantum noise
  - Due to the statistical nature of X-rays
- Artifacts: Due to scratches in the detector, dead pixels, unread scan lines, inhomogeneous X-ray beam intensity (heel effect), afterglow

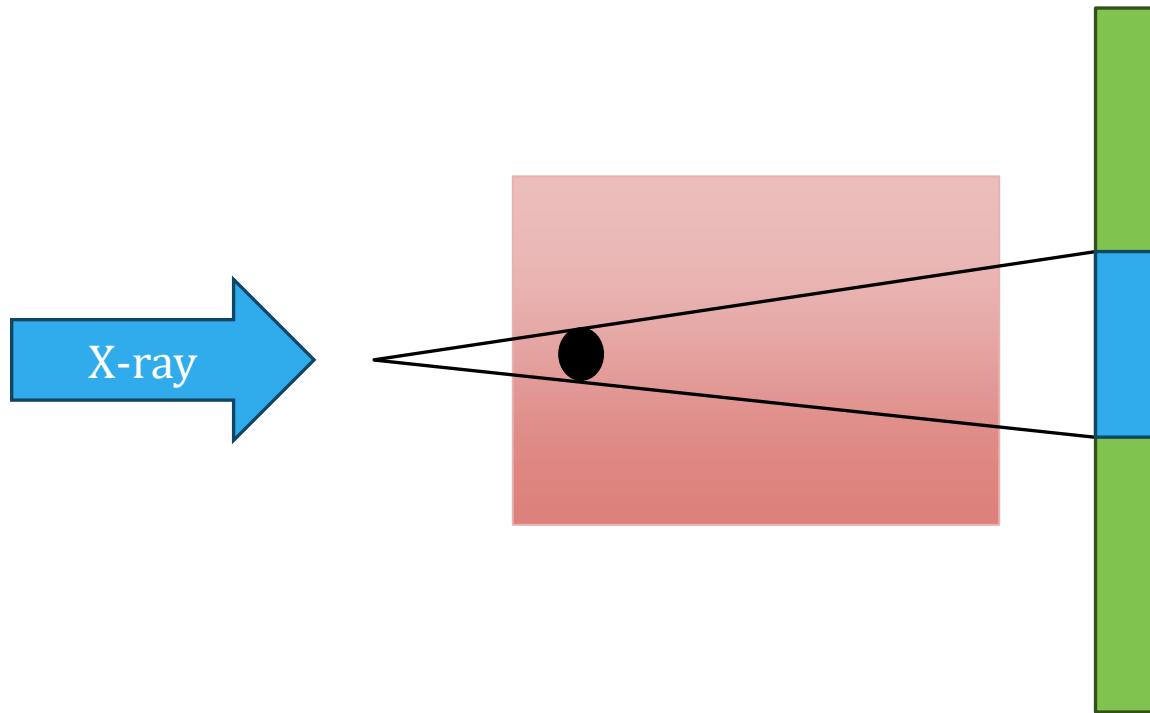
# Limitations in X-ray Imaging



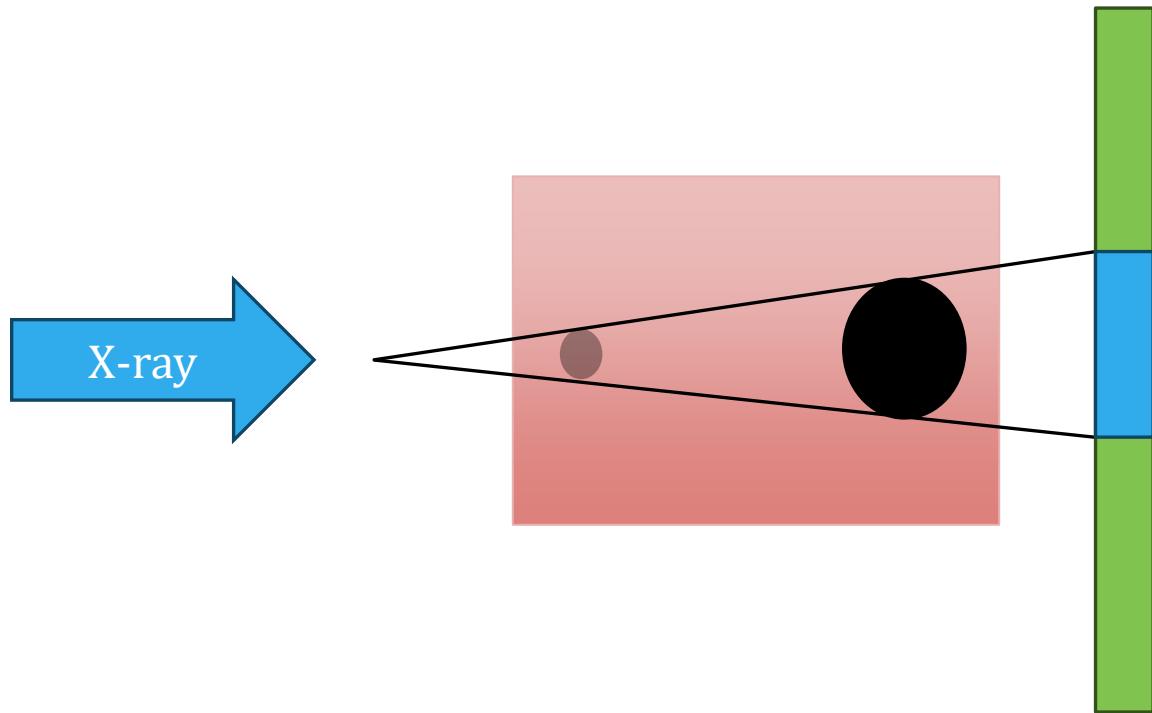
# Limitations in X-ray Imaging



# Limitations in X-ray Imaging



# Limitations in X-ray Imaging



# Attenuation

$$I_{\text{out}} = I_{\text{in}} e^{-\mu d}$$

**Homogeneous medium, photons of single energy**

$$I_{\text{out}} = I_{\text{in}} e^{-\int_{x_{\text{in}}}^{x_{\text{out}}} \mu(x) dx}$$

**Heterogeneous medium, photons of single energy**

$\mu$ : Attenuation coefficient

Function of photon energy and medium

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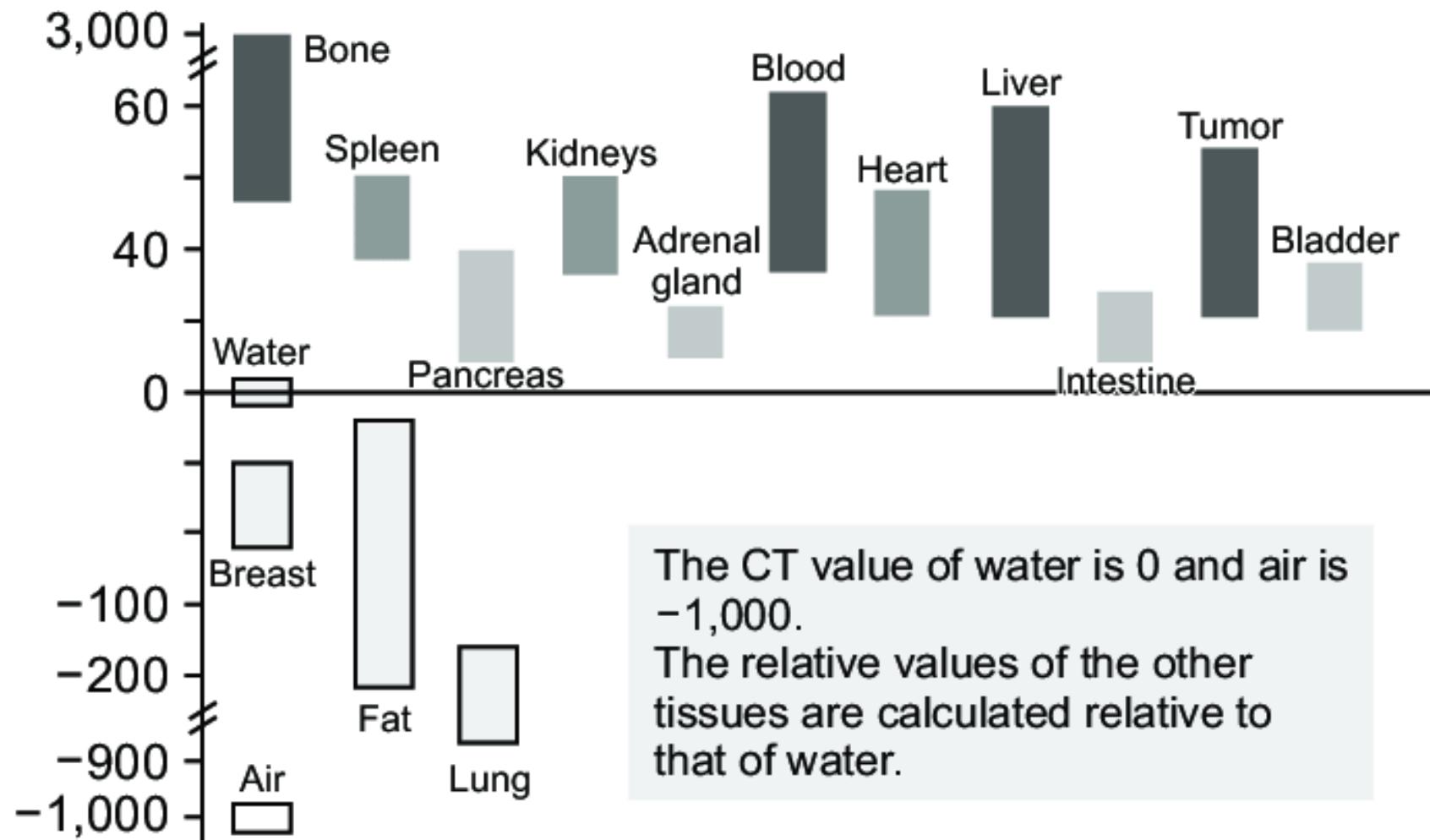
# CT Number

$\mu$ : Attenuation coefficient  
(Function of photon energy and medium)

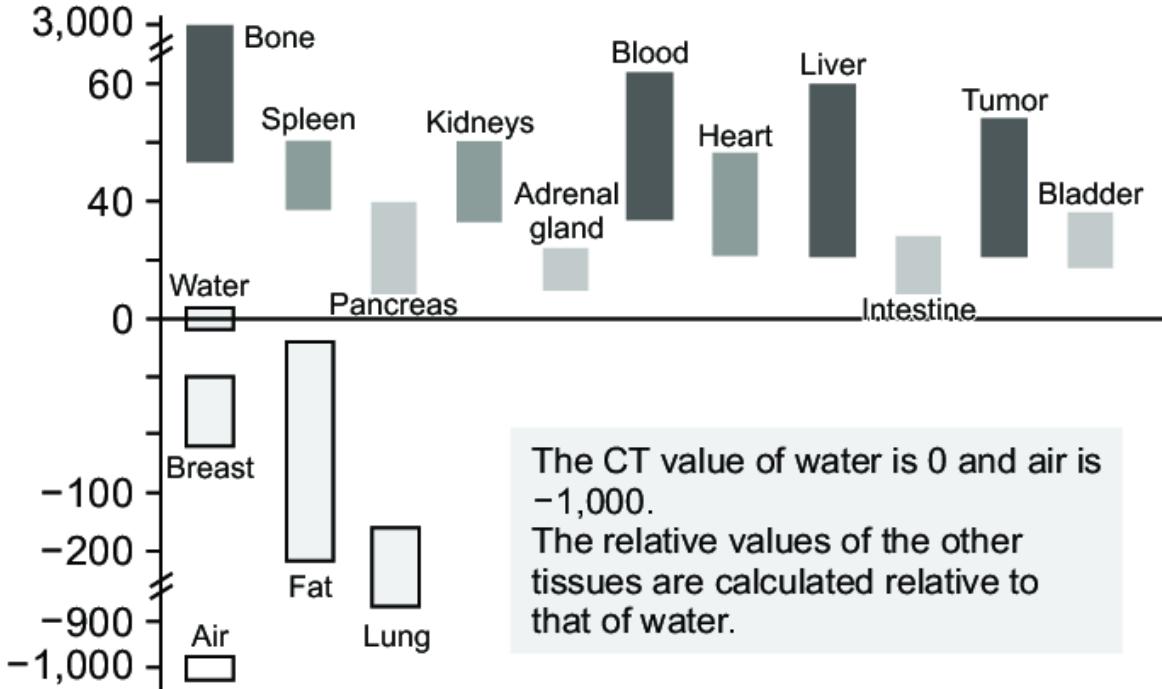
$$\text{CT Number (in HU)} = \frac{\mu - \mu_{H_2O}}{\mu_{H_2O}} \times 1000$$

**Hounsfield unit**

# CT Number

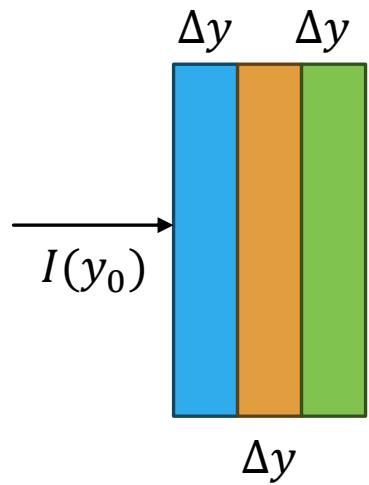


# CT Number

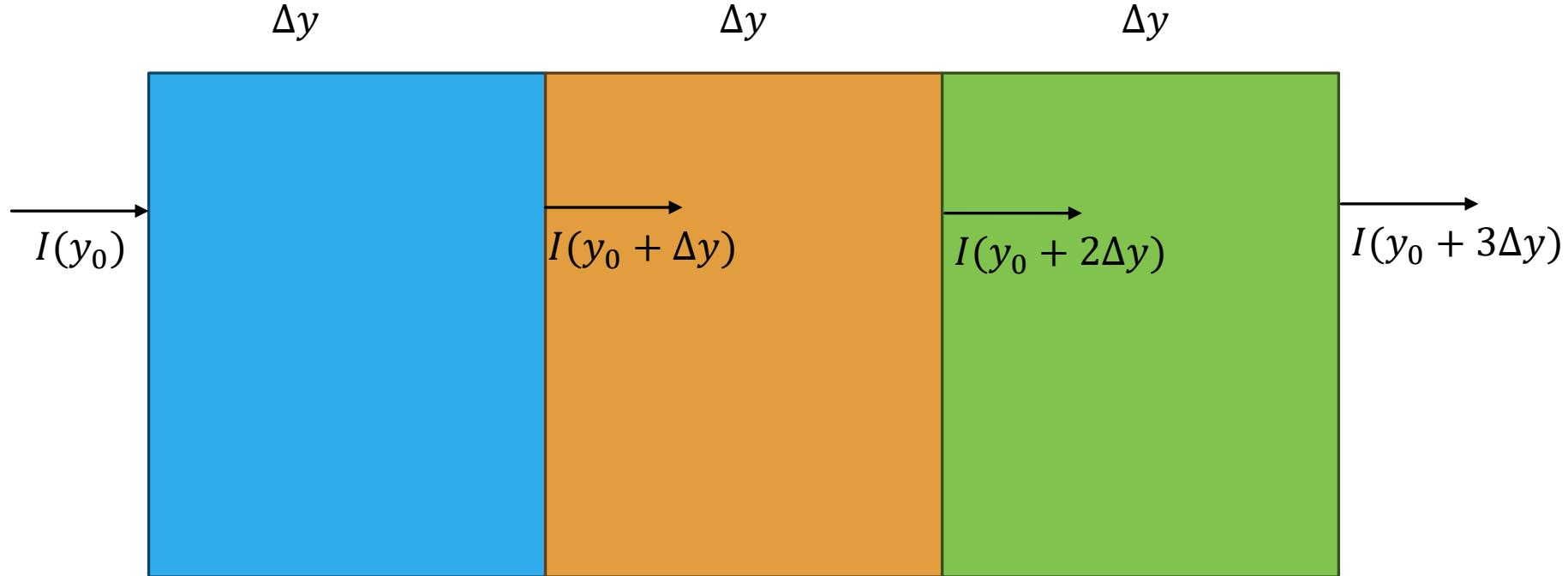


So, if we find  $\mu$  from an image, we can get information about tissues and abnormalities

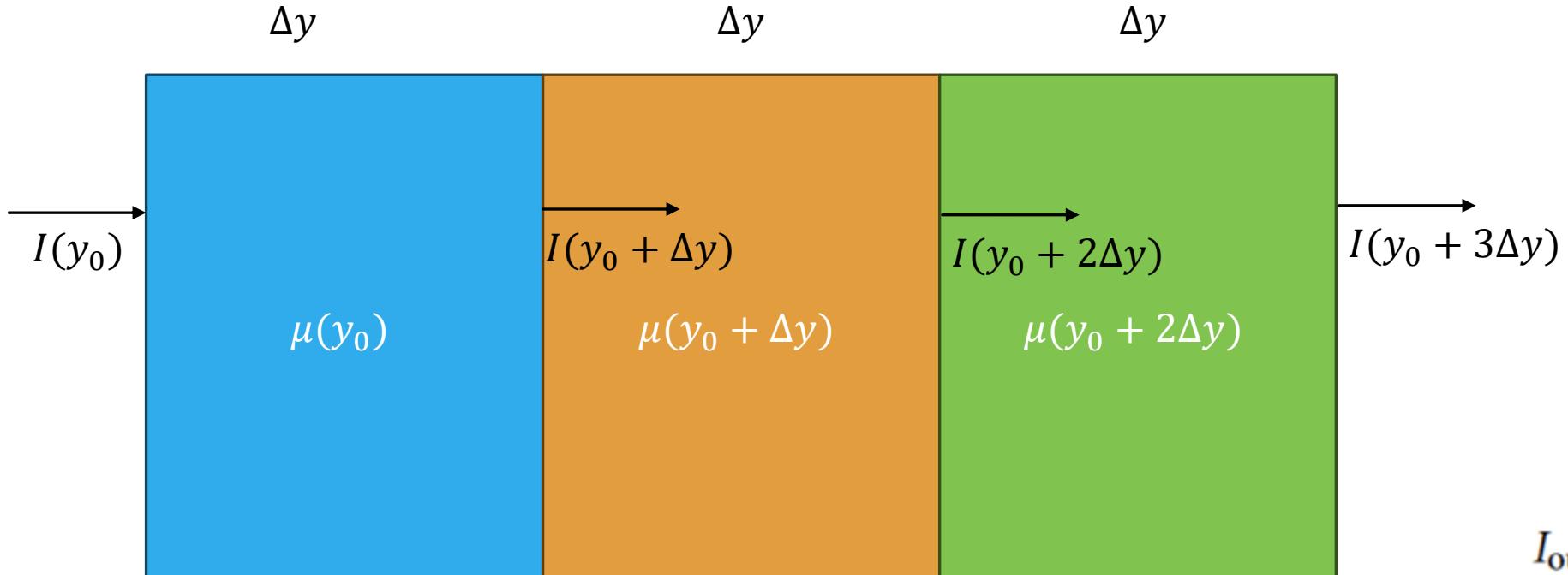
# CT Number



# CT Number

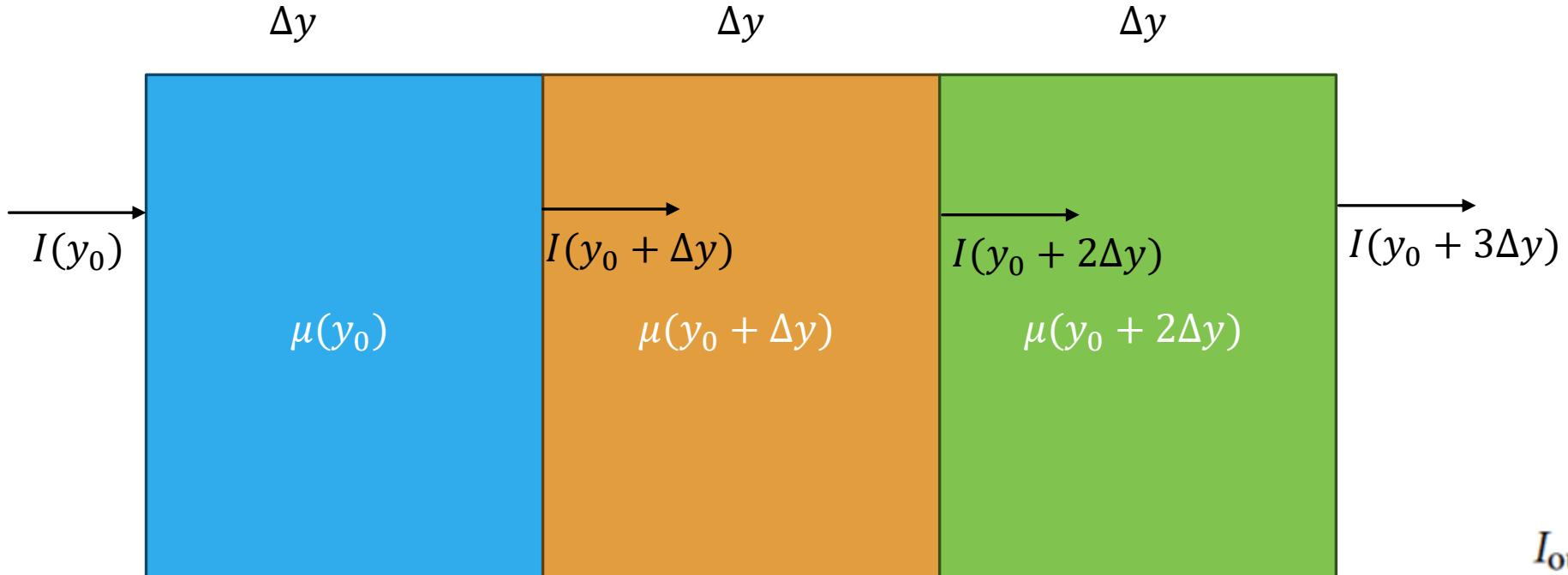


# CT Number



$$I(y_0 + \Delta y) = I(y_0) e^{-\mu(y_0) \Delta y}$$

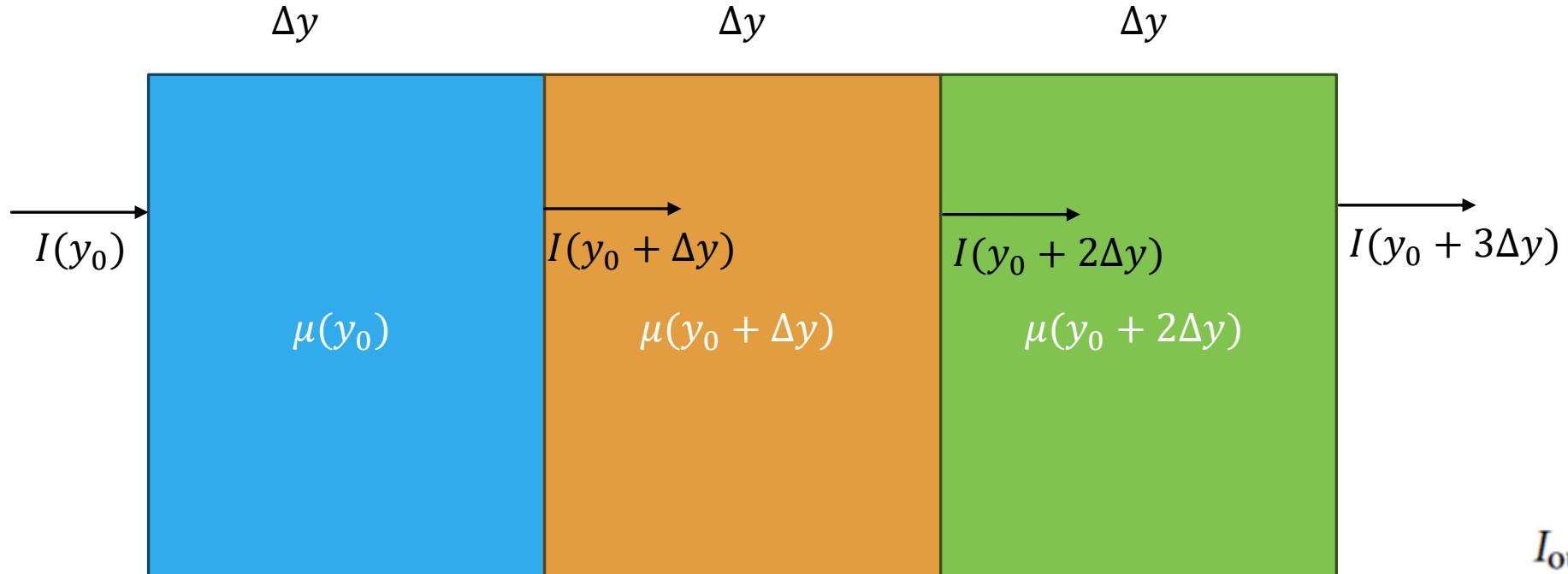
# CT Number



$$I(y_0 + \Delta y) = I(y_0) e^{-\mu(y_0)\Delta y}$$

$$I(y_0 + 2\Delta y) = I(y_0 + \Delta y) e^{-\mu(y_0 + \Delta y)\Delta y}$$

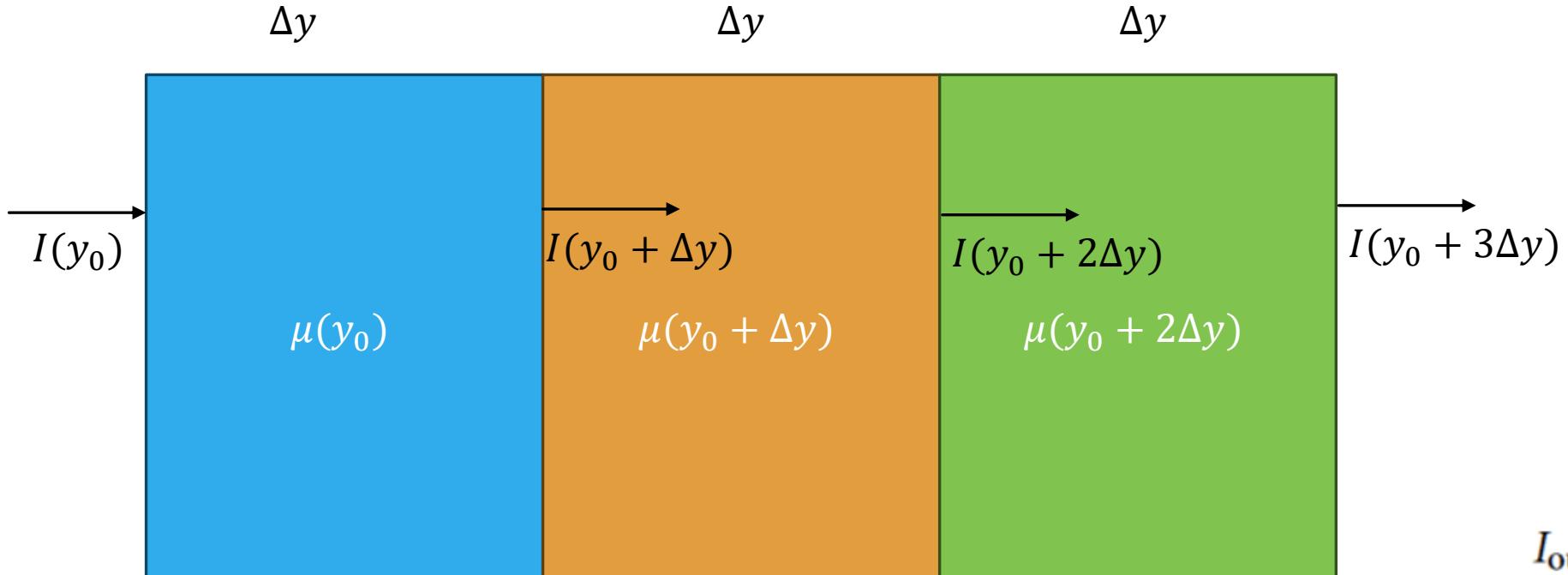
# CT Number



$$I(y_0 + \Delta y) = I(y_0) e^{-\mu(y_0)\Delta y}$$

$$I(y_0 + 2\Delta y) = I(y_0) e^{-\mu(y_0)\Delta y} e^{-\mu(y_0+\Delta y)\Delta y}$$

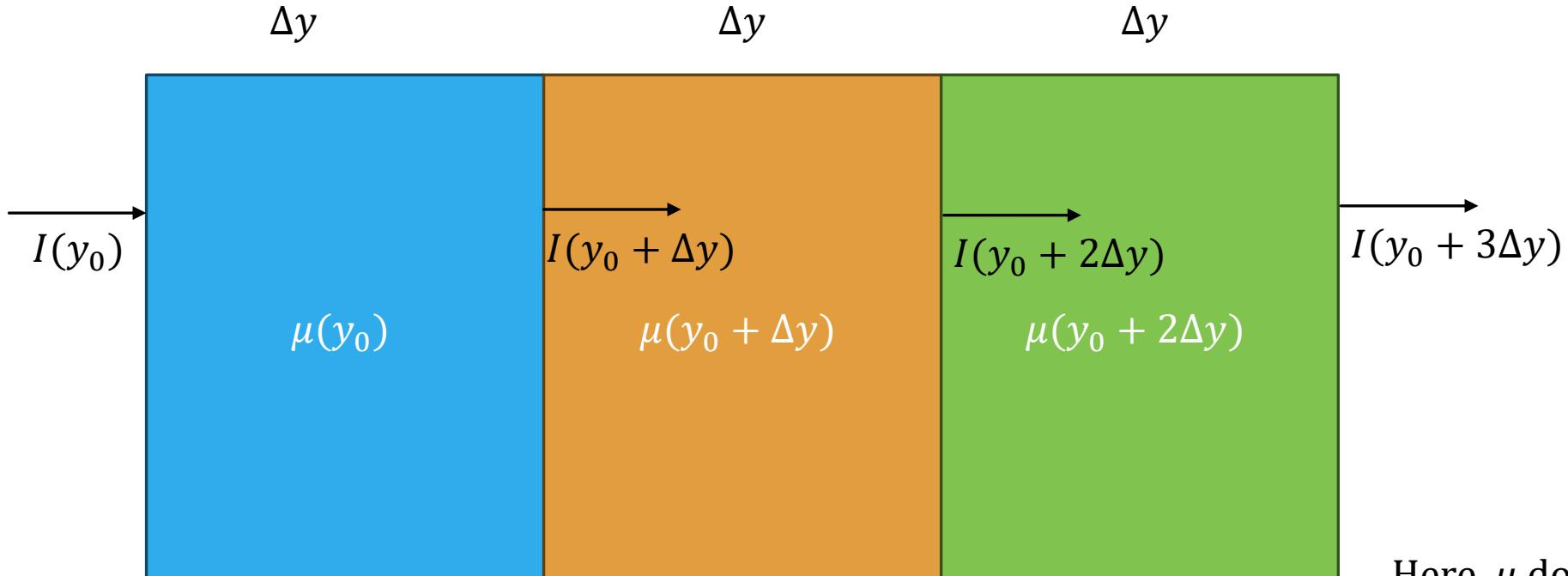
# CT Number



$$I(y_0 + 2\Delta y) = I(y_0) e^{-\mu(y_0)\Delta y} e^{-\mu(y_0 + \Delta y)\Delta y}$$

$$I_{\text{out}} = I(y_0) e^{-\int_{\text{source}}^{\text{detector}} \mu(y') dy'}$$

# CT Number

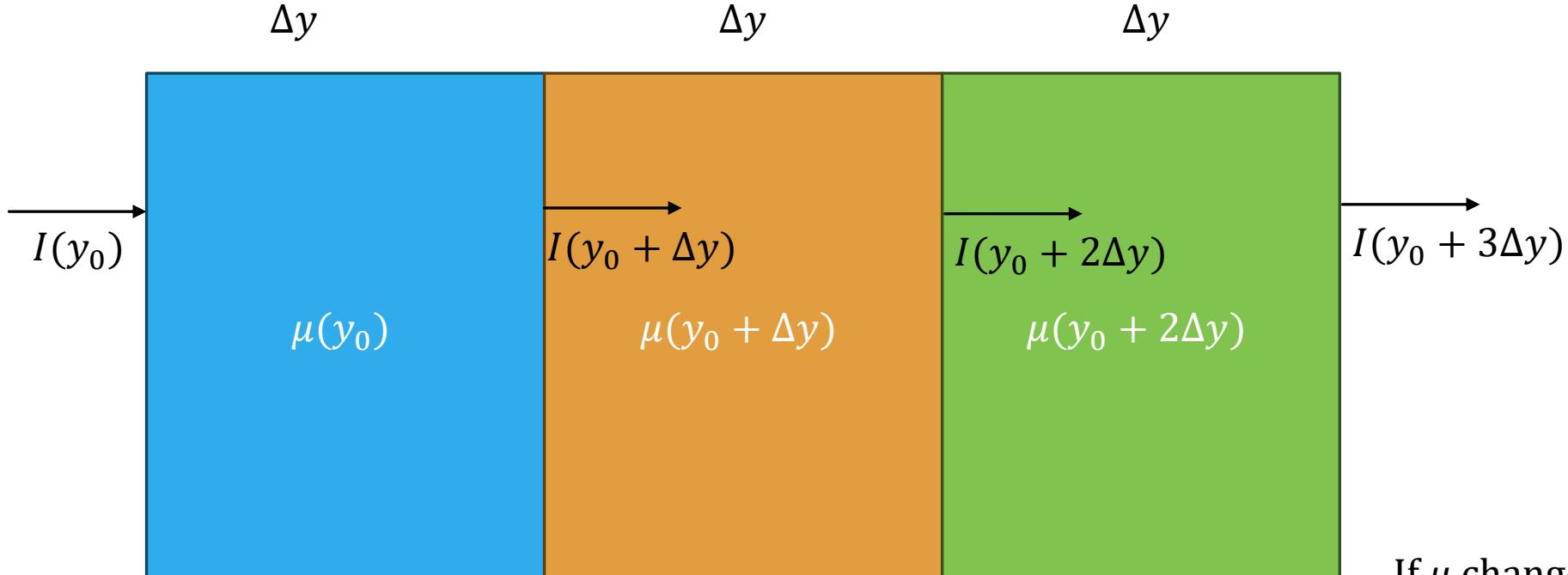


Here,  $\mu$  does not change  
in the vertical direction

$$I(y_0 + 2\Delta y) = I(y_0)e^{-\mu(y_0)\Delta y}e^{-\mu(y_0+\Delta y)\Delta y}$$

$$I_{out} = I(y_0)e^{-\int_{source}^{detector} \mu(y') dy'}$$

# CT Number

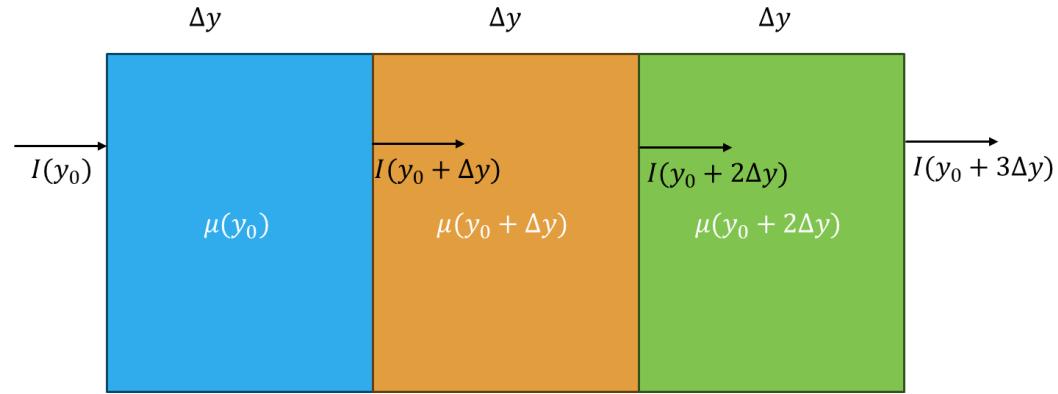


If  $\mu$  changes in the vertical direction as well

$$I(y_0 + 2\Delta y) = I(y_0)e^{-\mu(y_0)\Delta y}e^{-\mu(y_0+\Delta y)\Delta y}$$

$$I_{out} = I(y_0)e^{-\int_{source}^{detector} \mu(x,y')dy'}$$

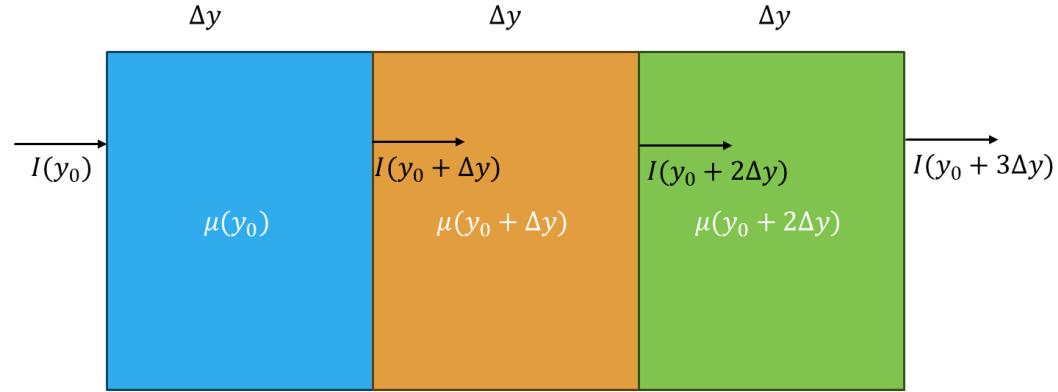
# Radon Transform



$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

$$g(x) = \int_{-\infty}^{\infty} \mu(x, y') dy'$$

# Radon Transform

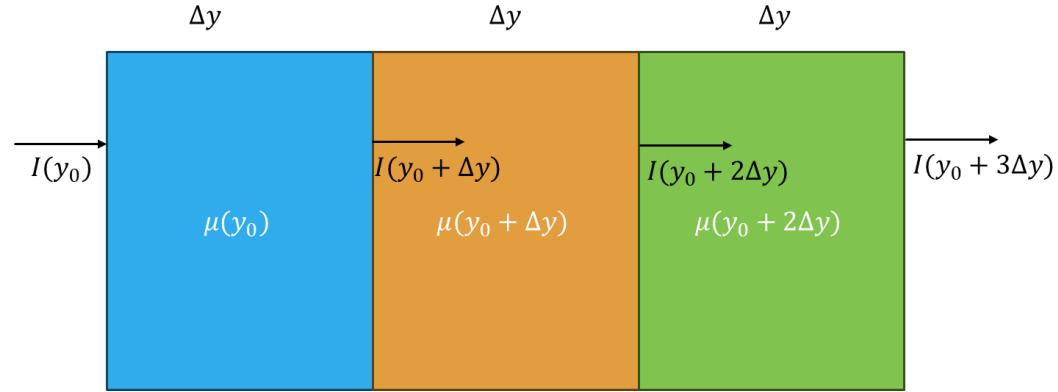


$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

More generally

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

# Radon Transform

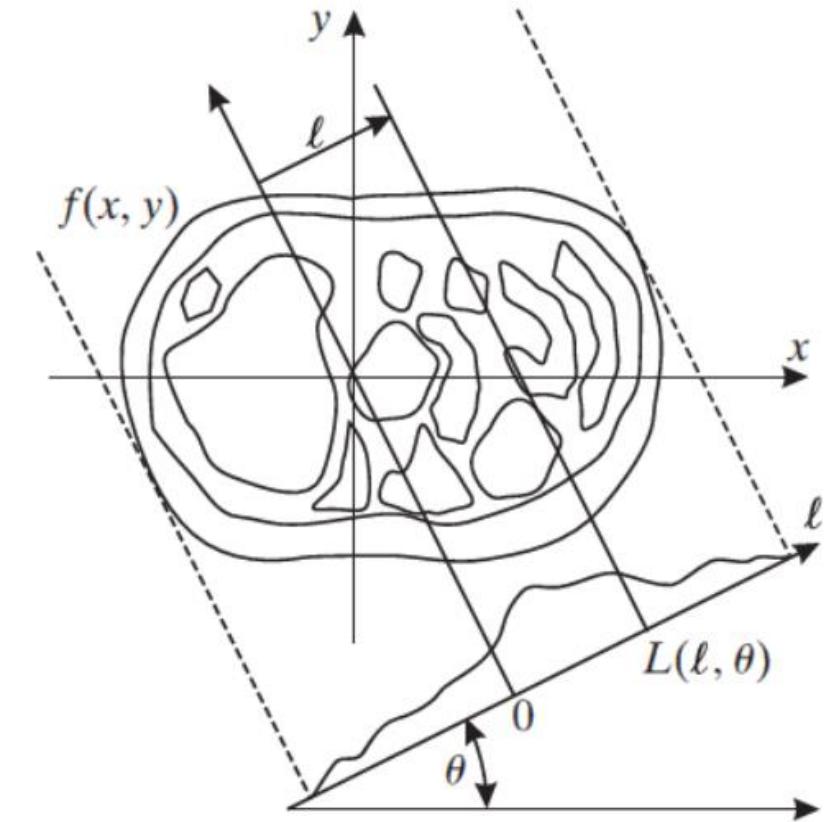


$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

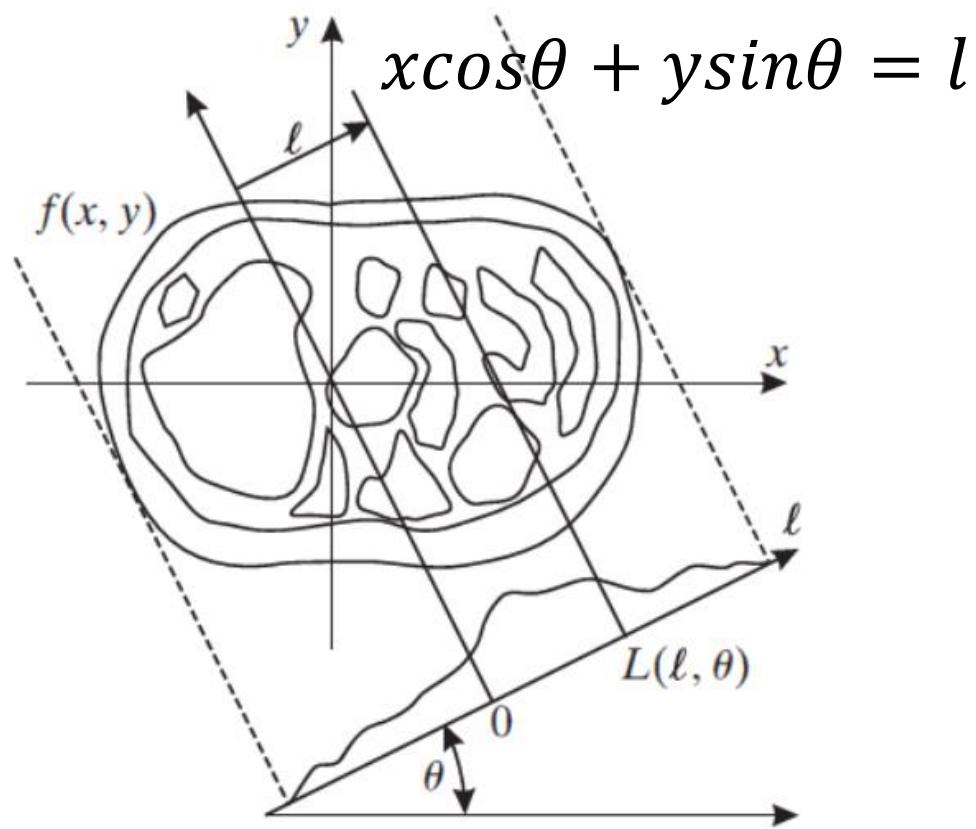
Line integral along the  
line  $x \cos \theta + y \sin \theta = l$

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

# Radon Transform



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# Radon Transform



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# Radon Transform

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

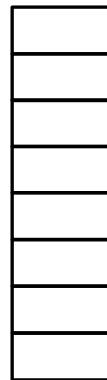
**For all values of  $\ell$  and  $\theta$**

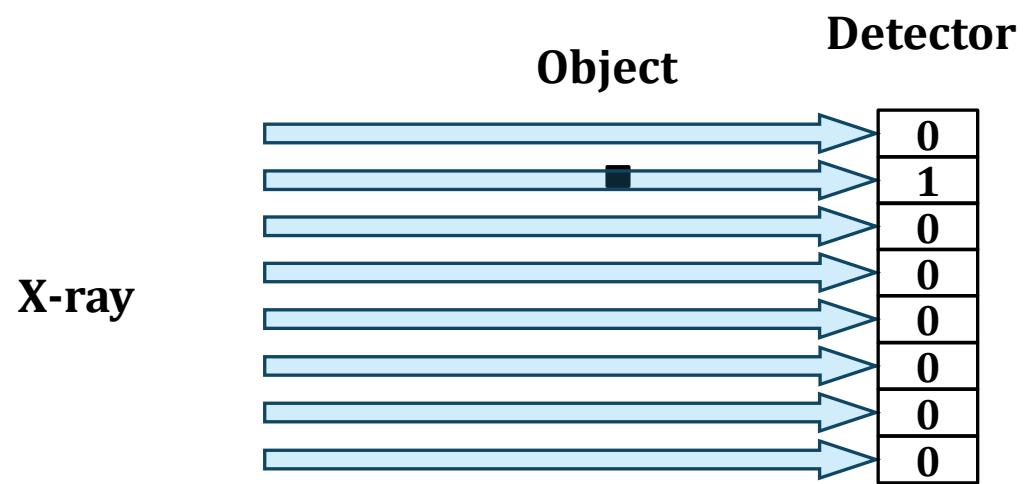
# Visual Impact of Radon Transform

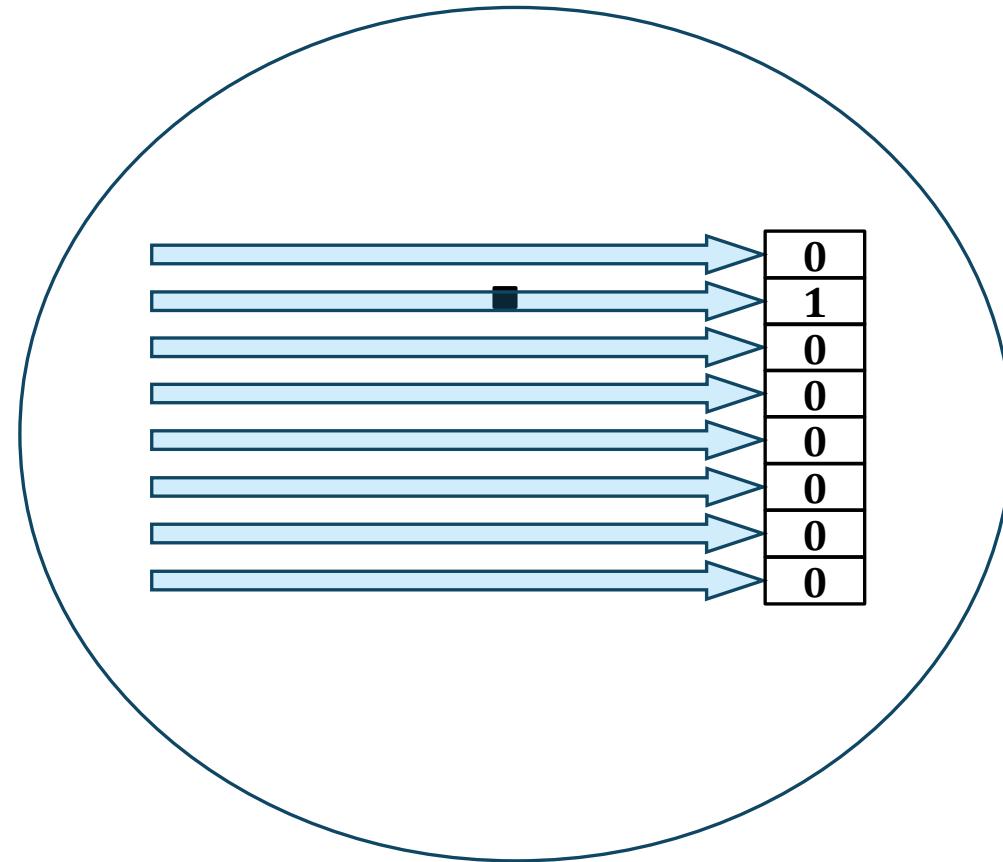
**Object**

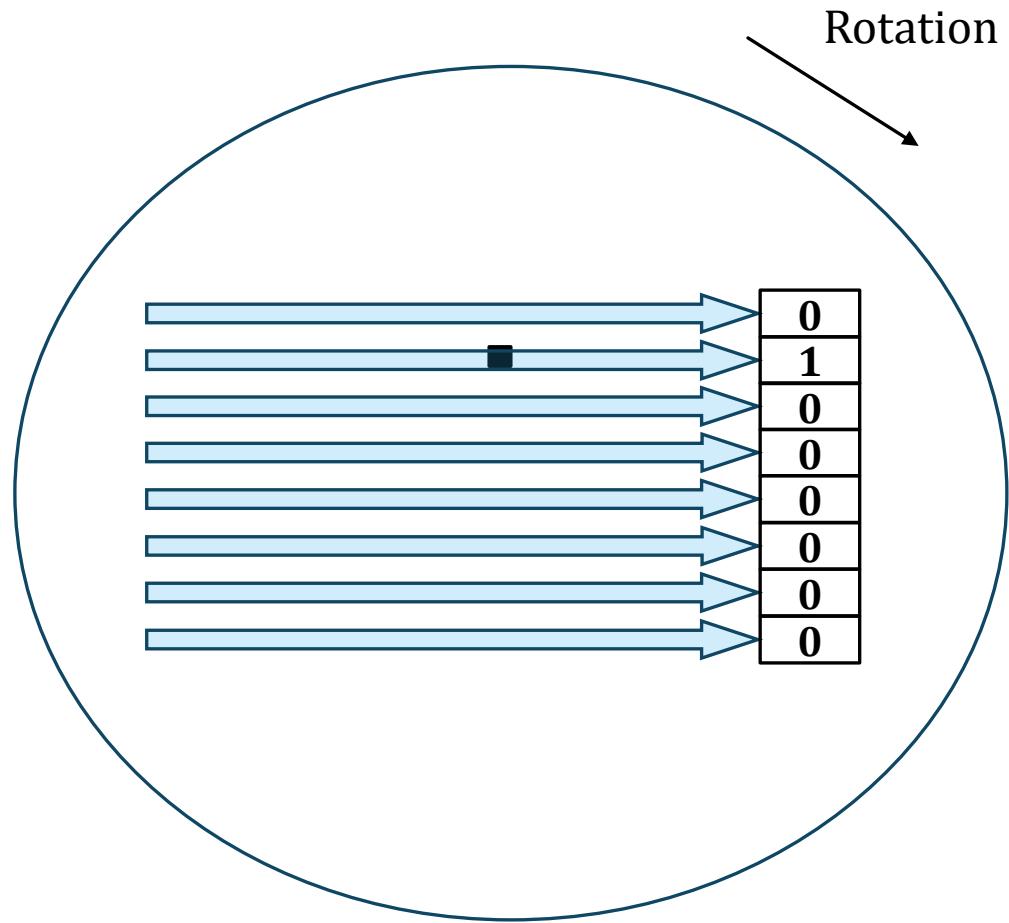


**Detector**

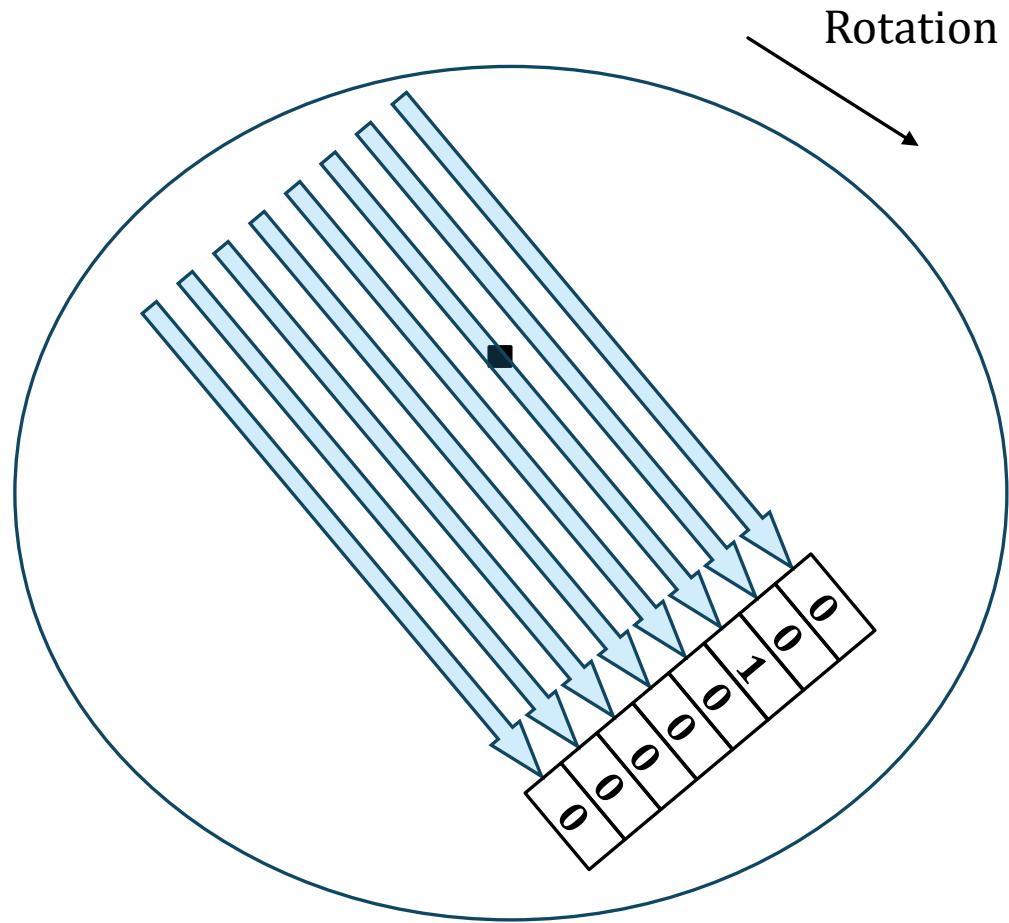




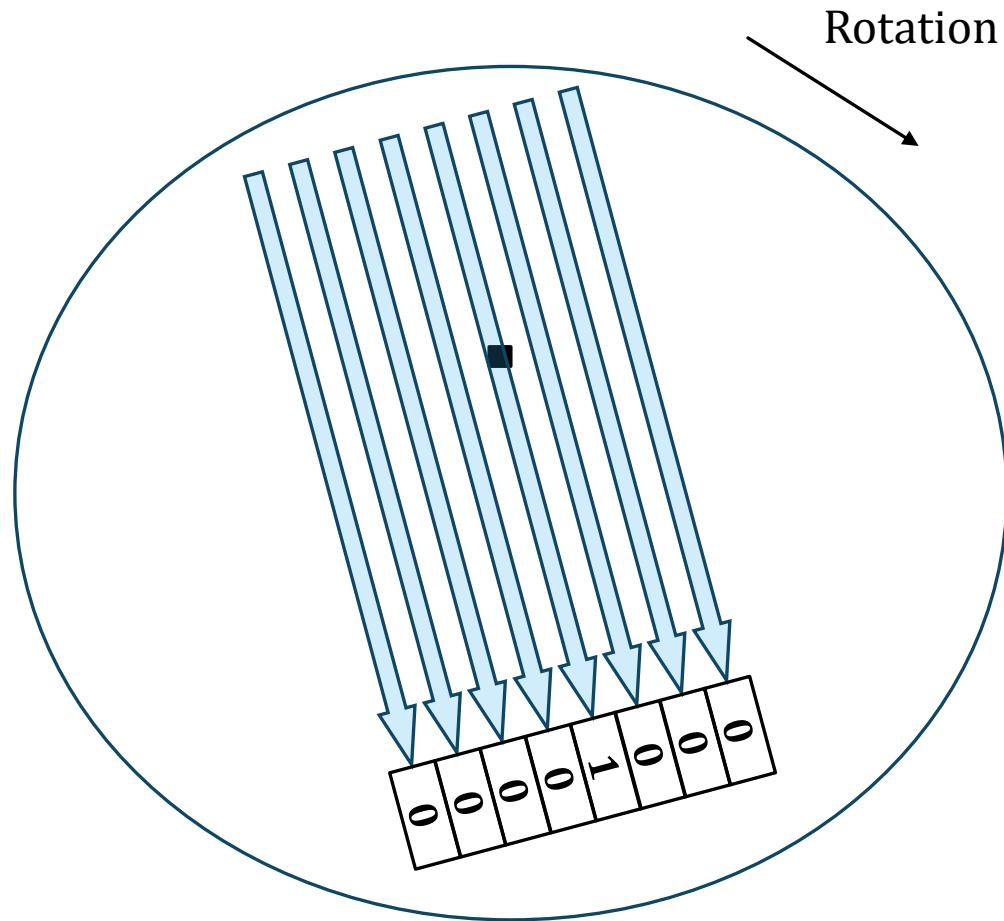




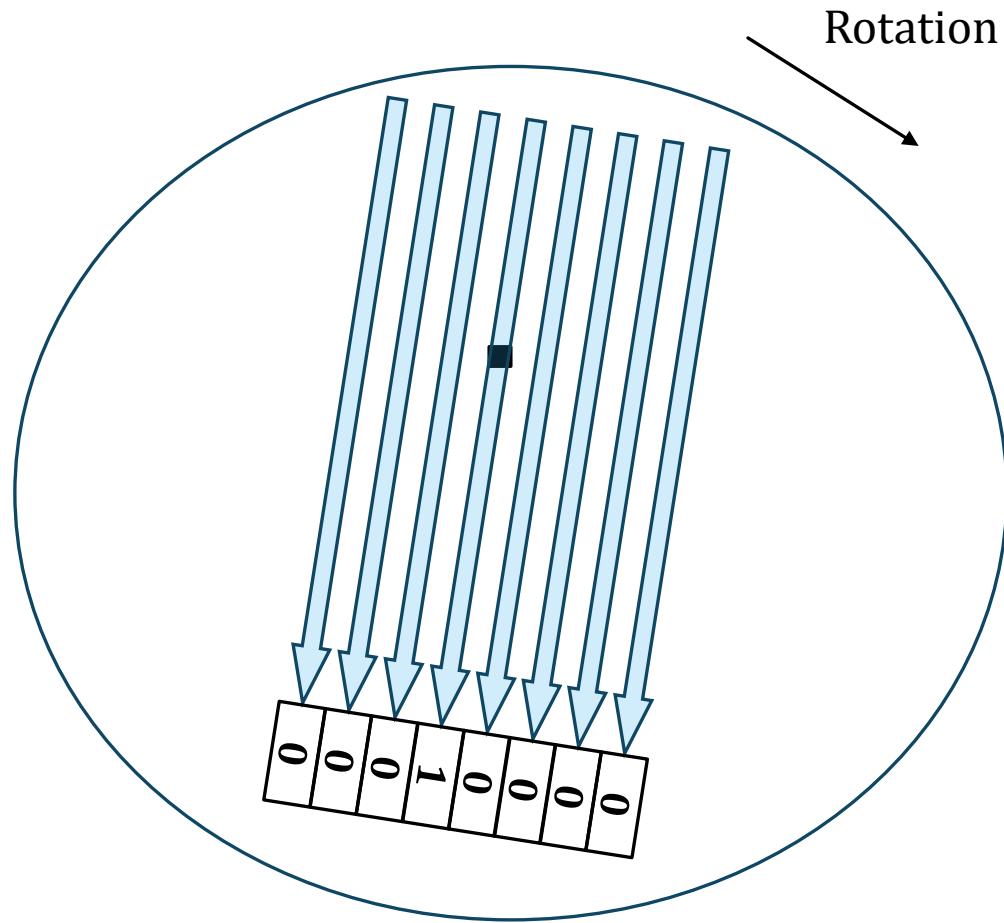
0	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---



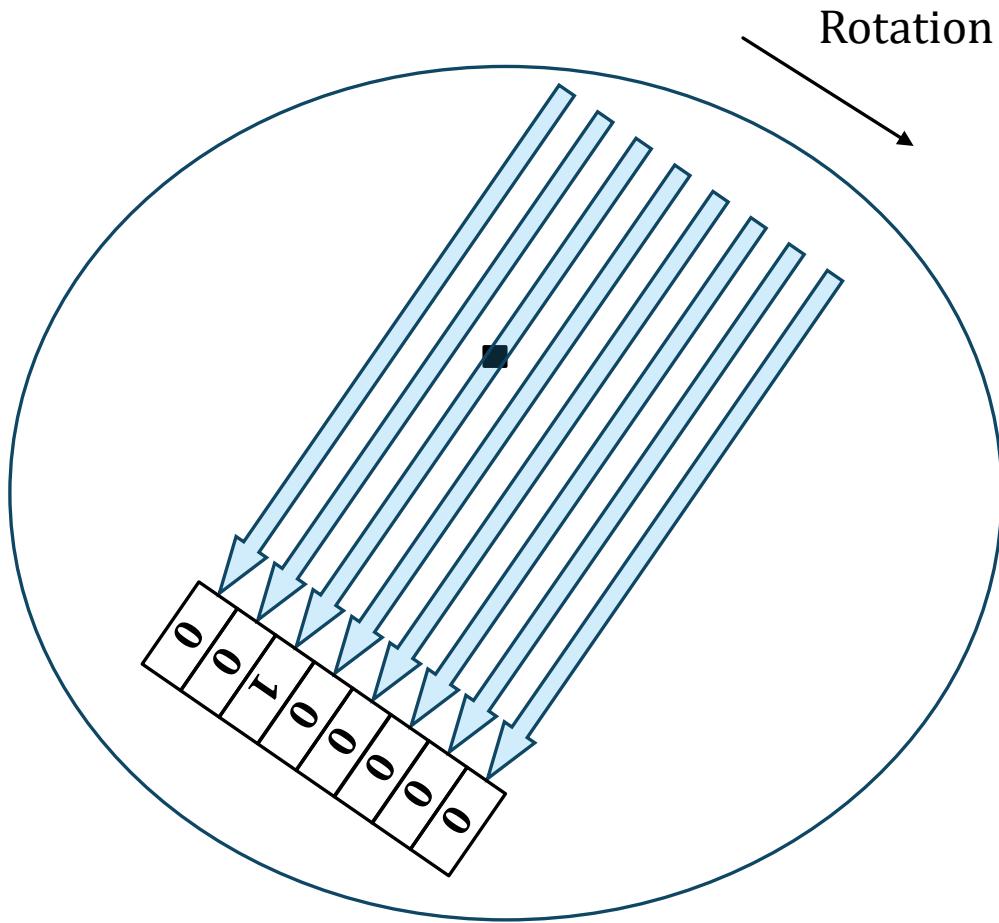
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0



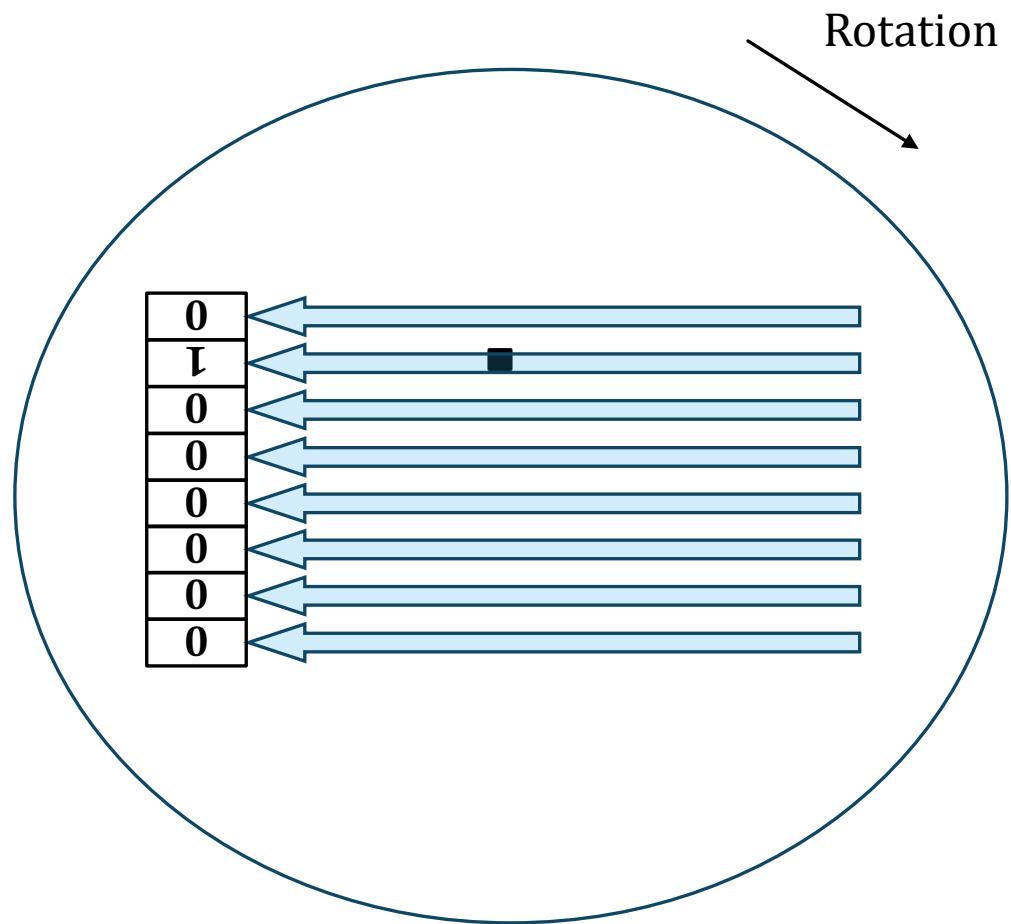
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0



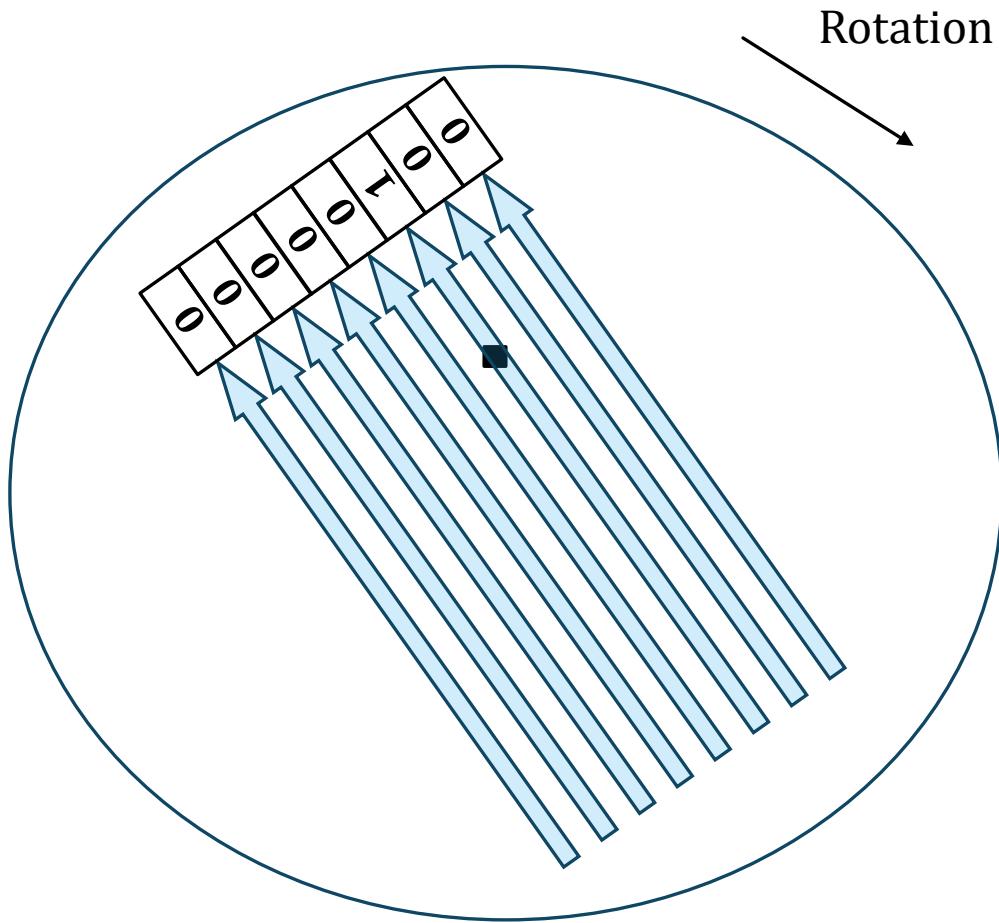
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0



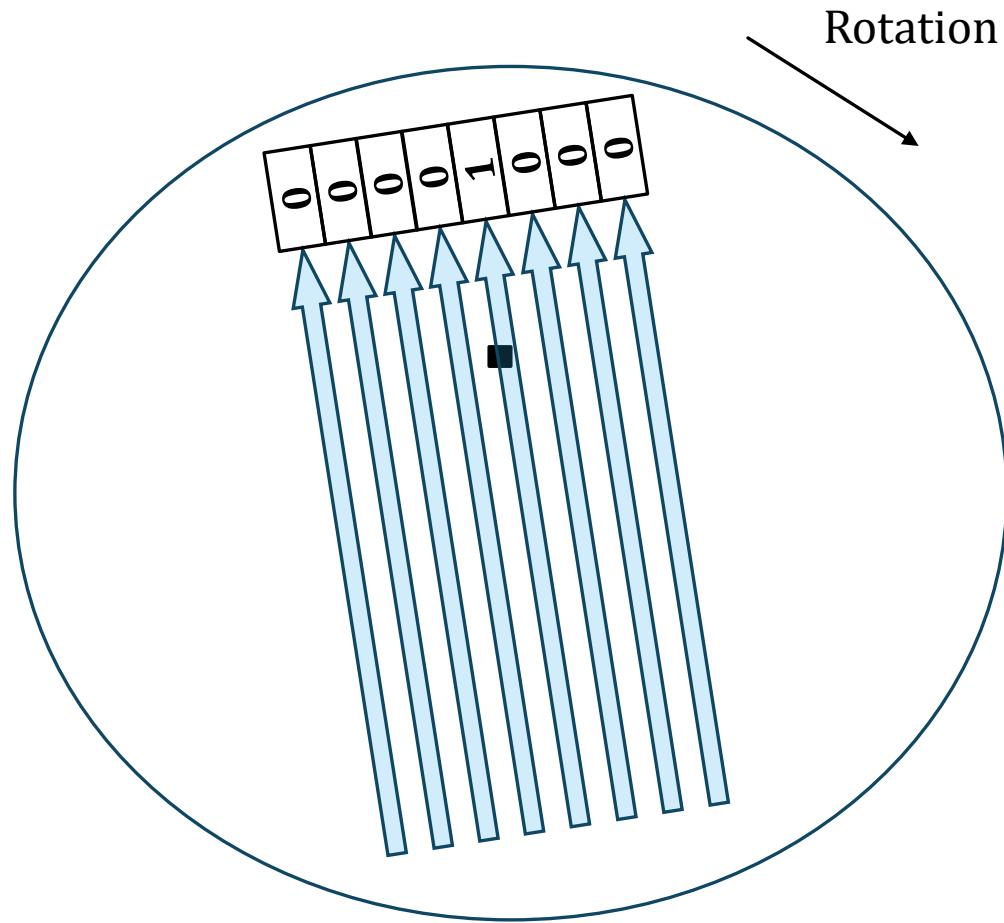
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0



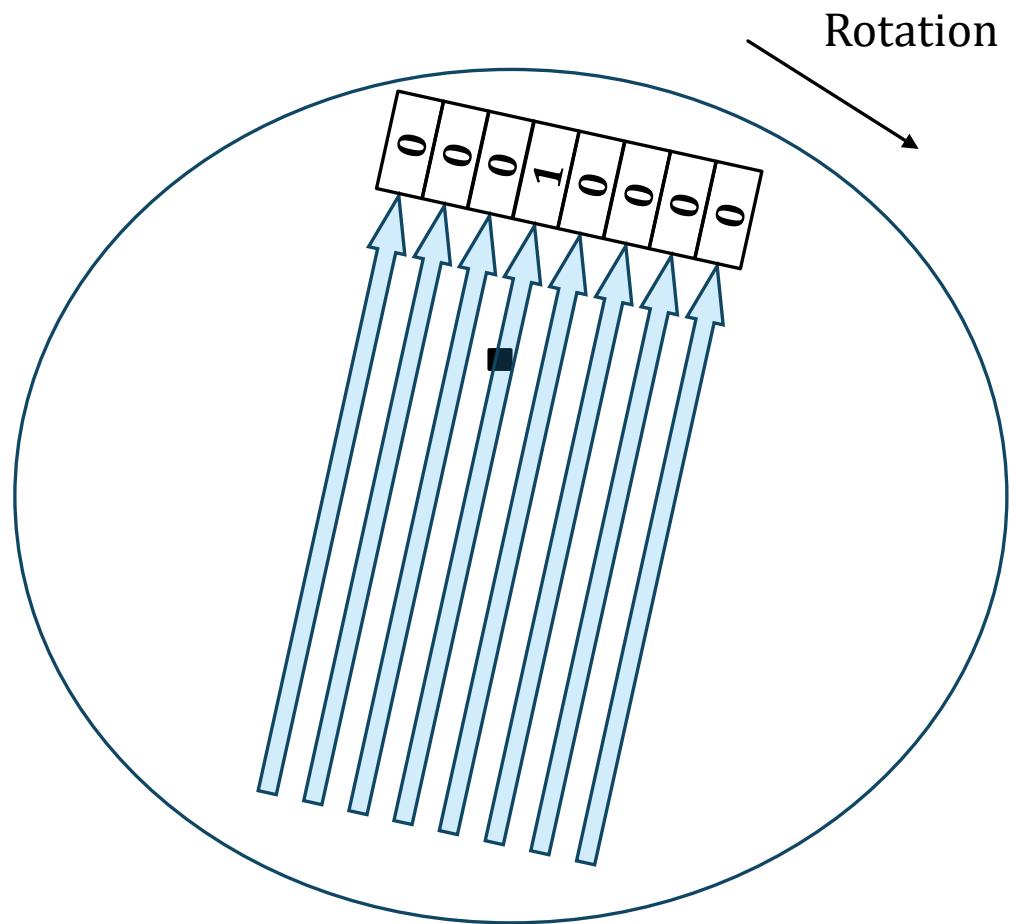
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0



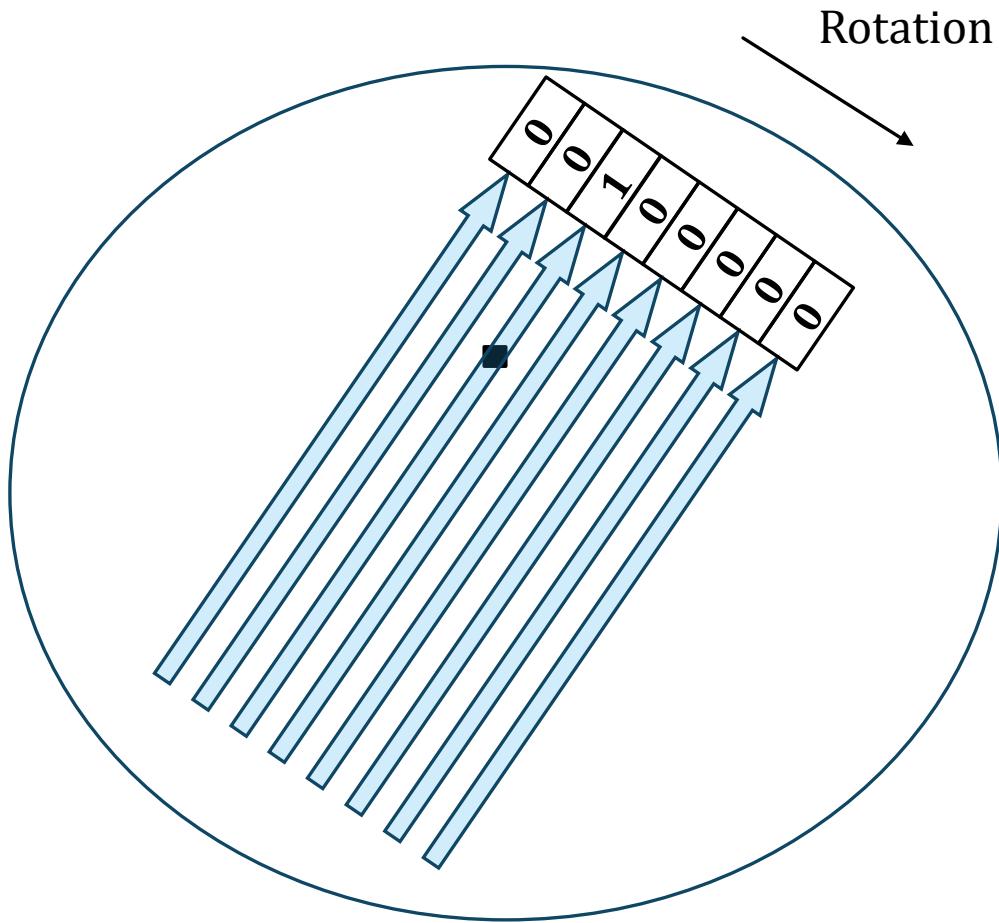
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0



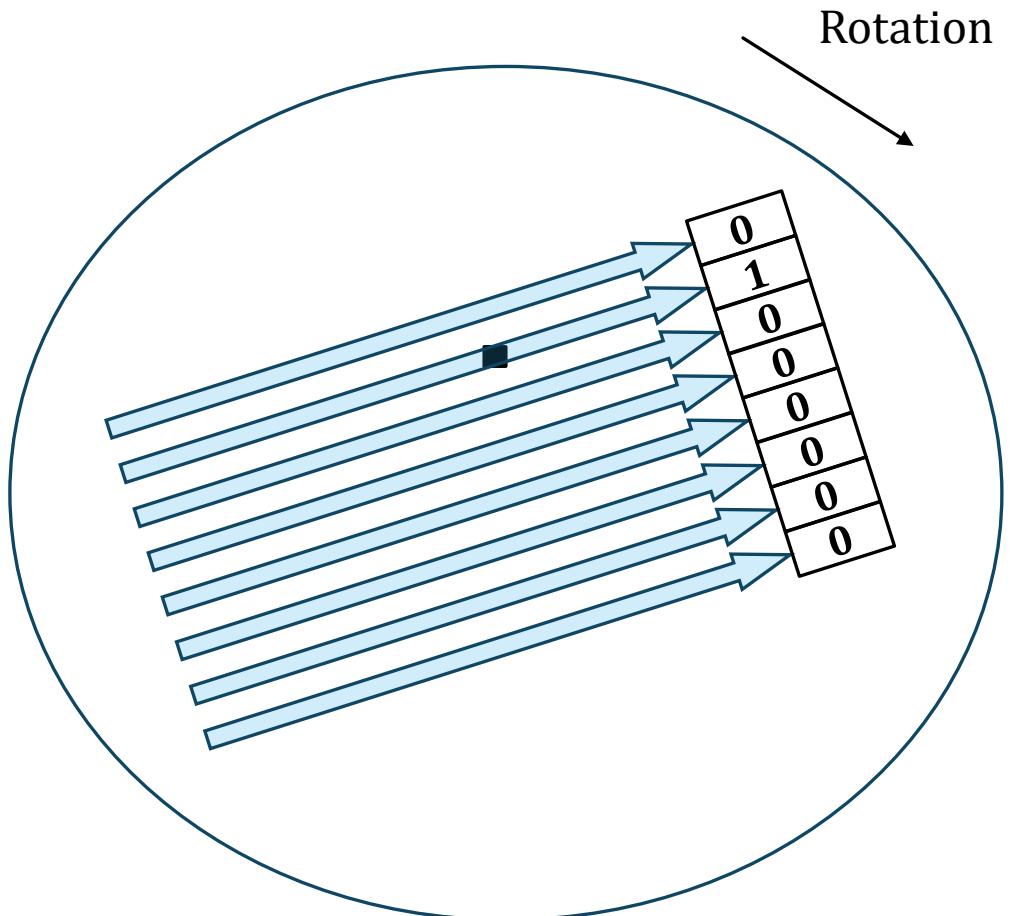
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0



0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0

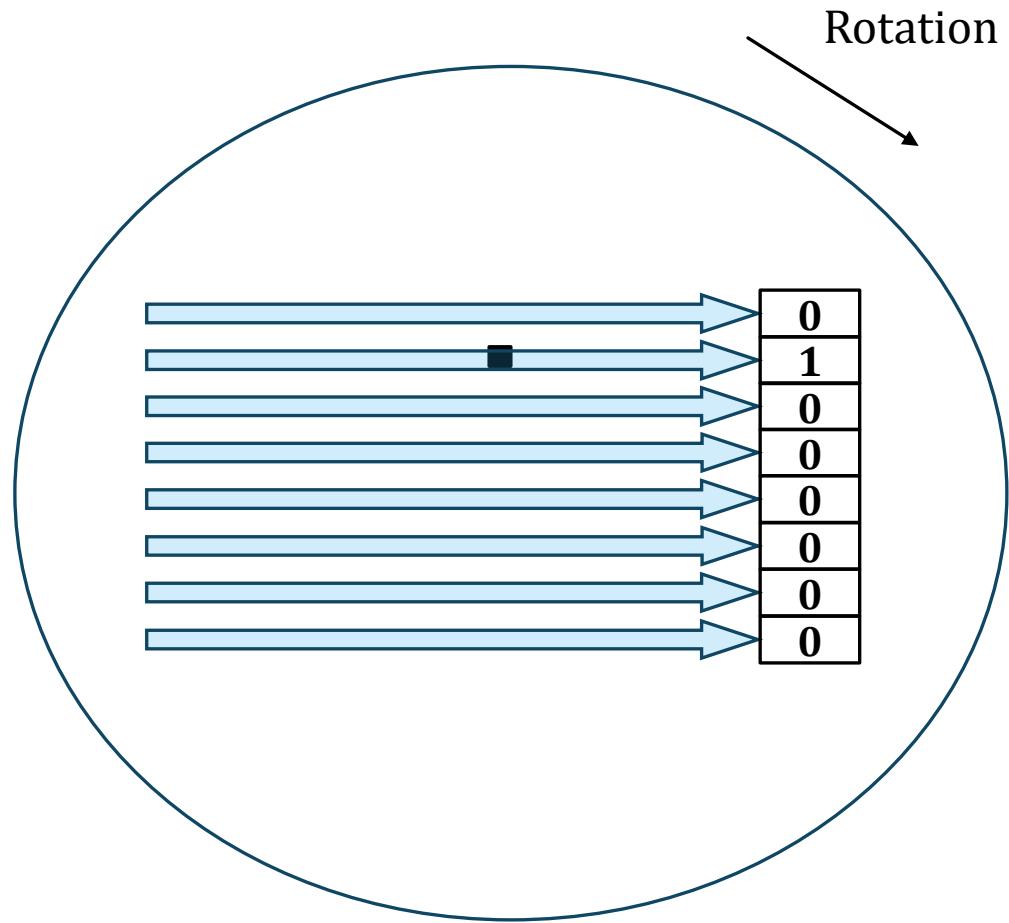


0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0



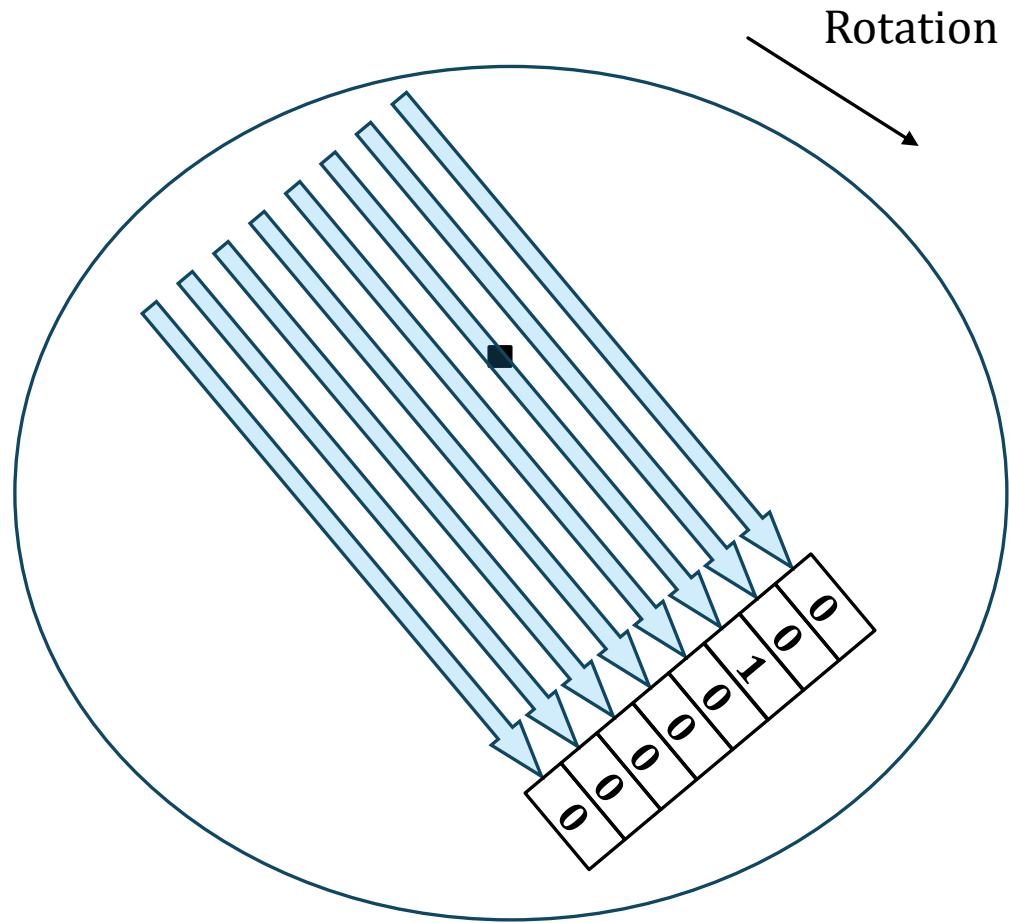
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0

Let's see once again



0	1	0	0	0	0	0	0
---	---	---	---	---	---	---	---

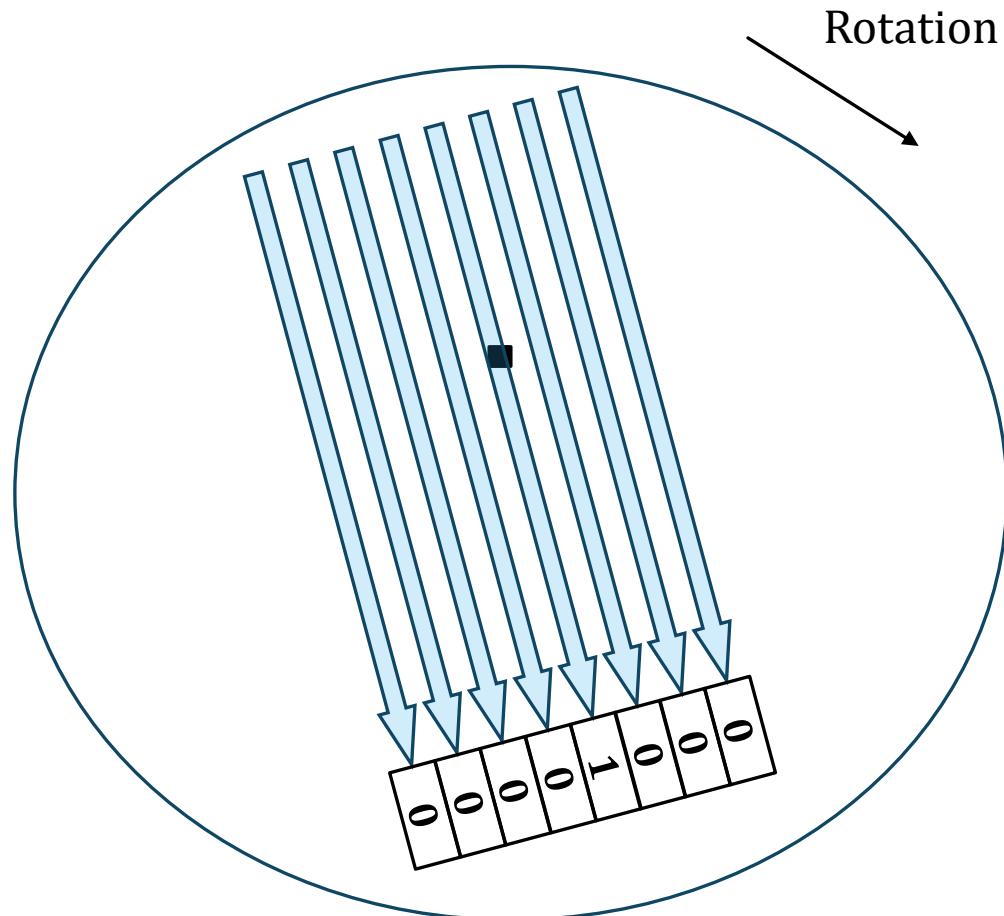
**$\Theta = 0$**



0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0

$\Theta = 0$

$\Theta = 30^\circ$

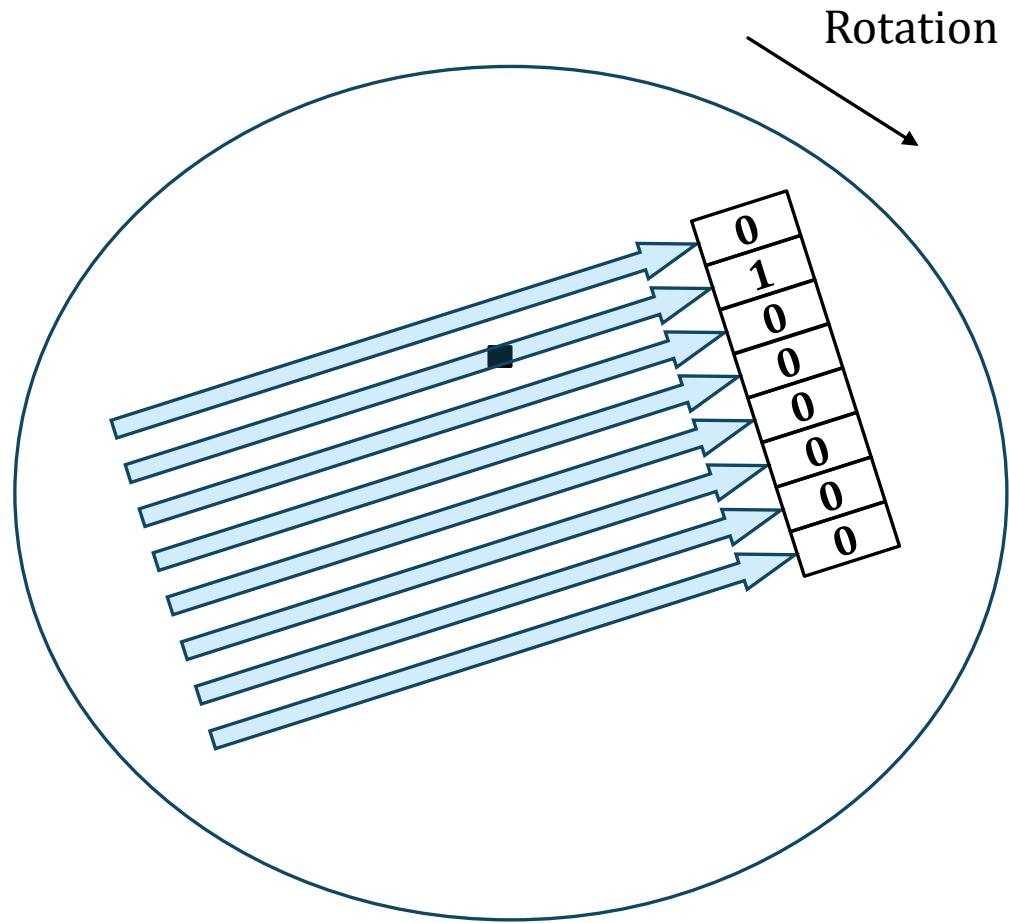


0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0

$\Theta = 0$

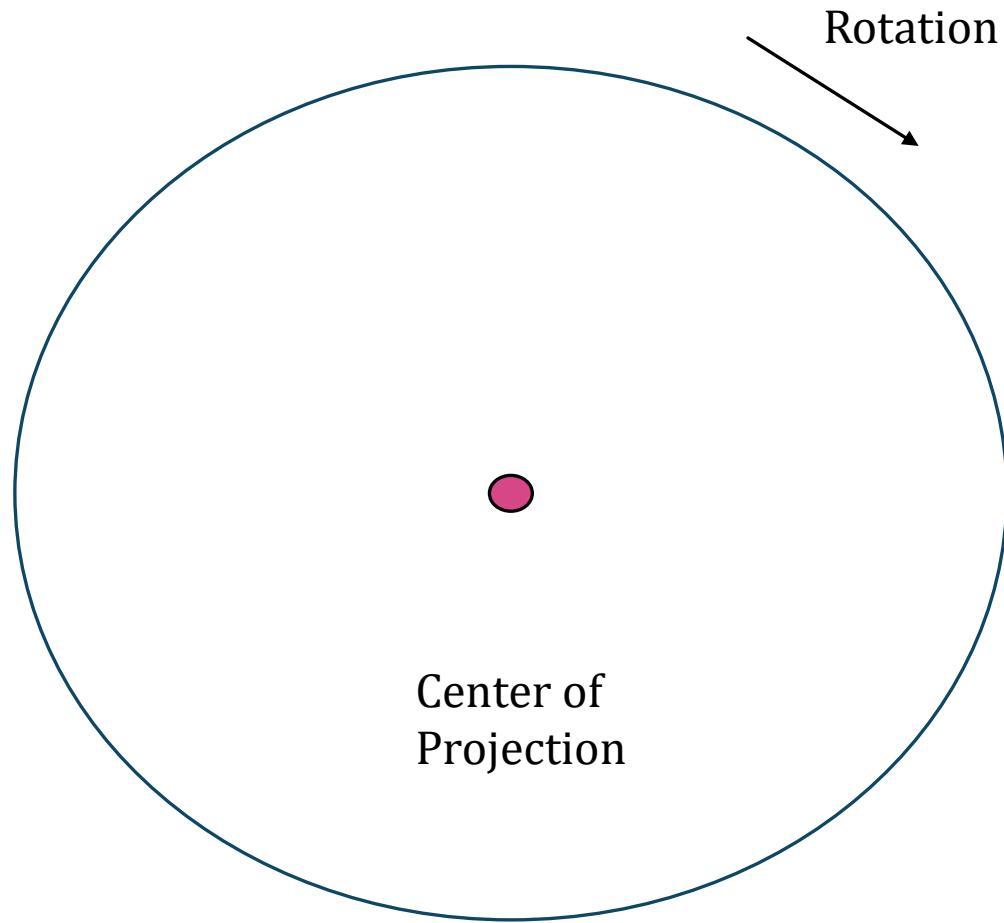
$\Theta = 30^\circ$

$\Theta = 45^\circ$



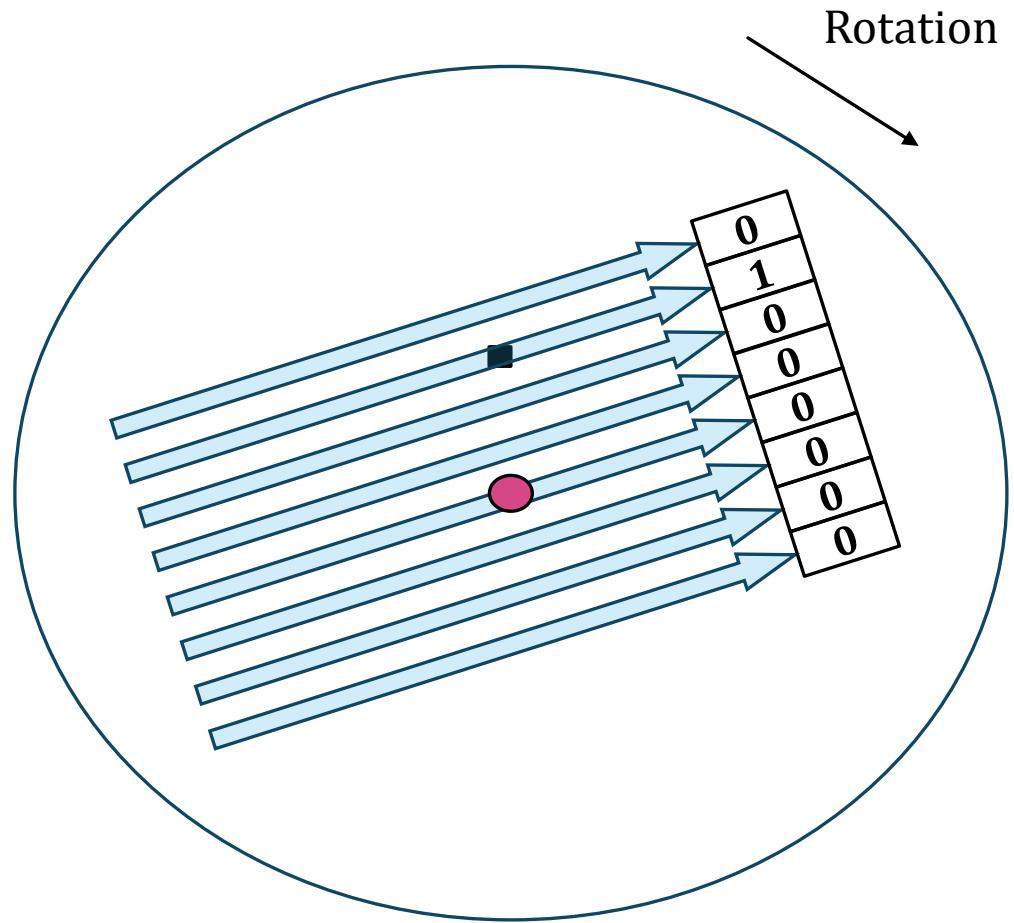
$\Theta$

0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0



$\Theta$

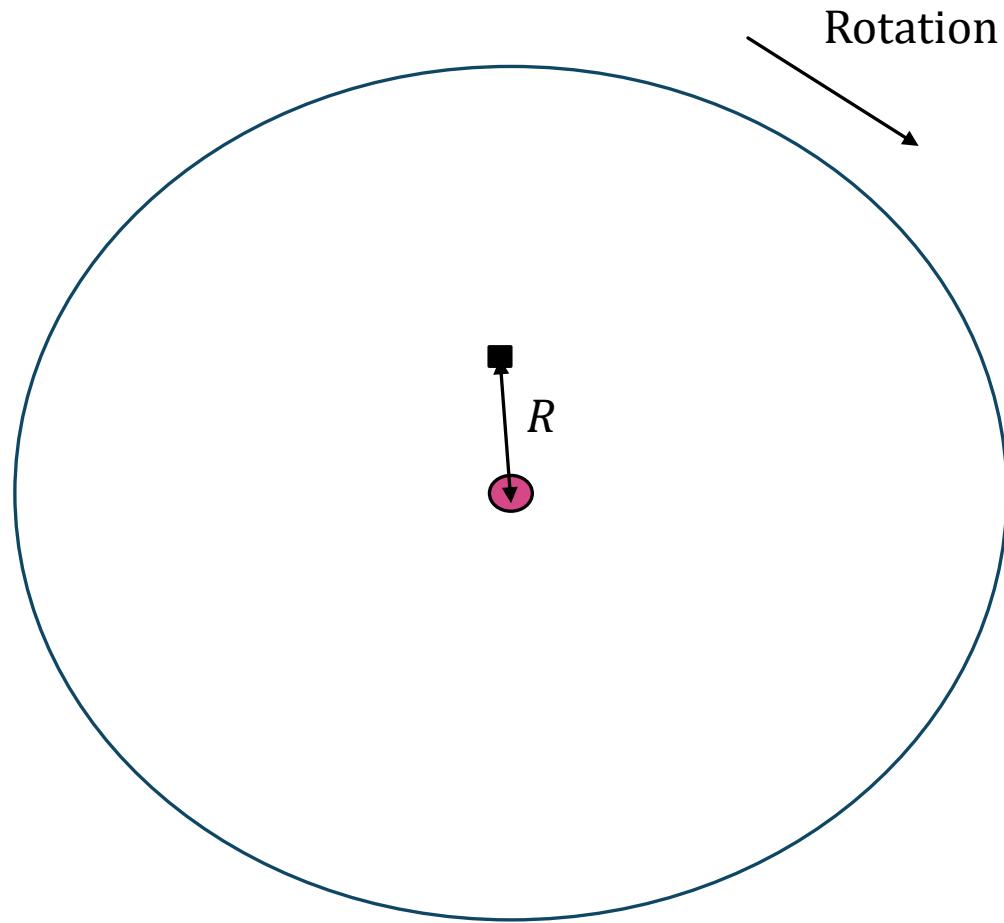
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0



0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0

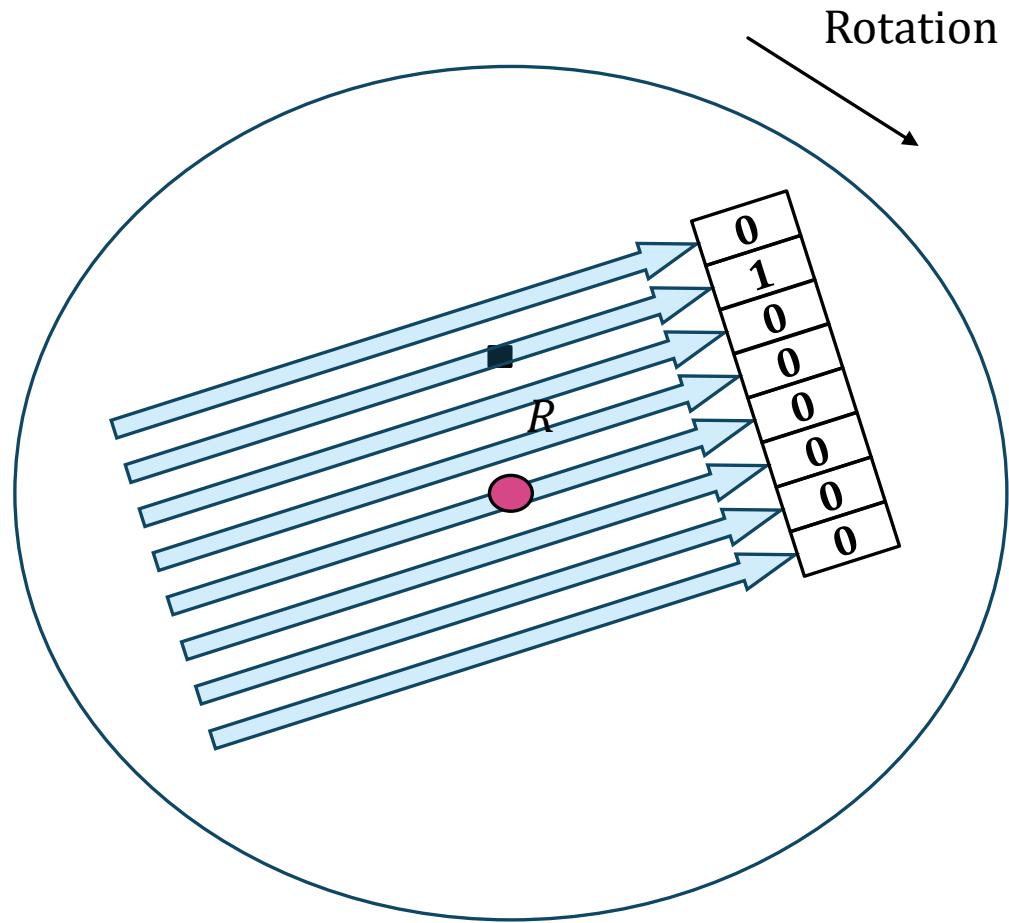
$\Theta$





$\Theta$

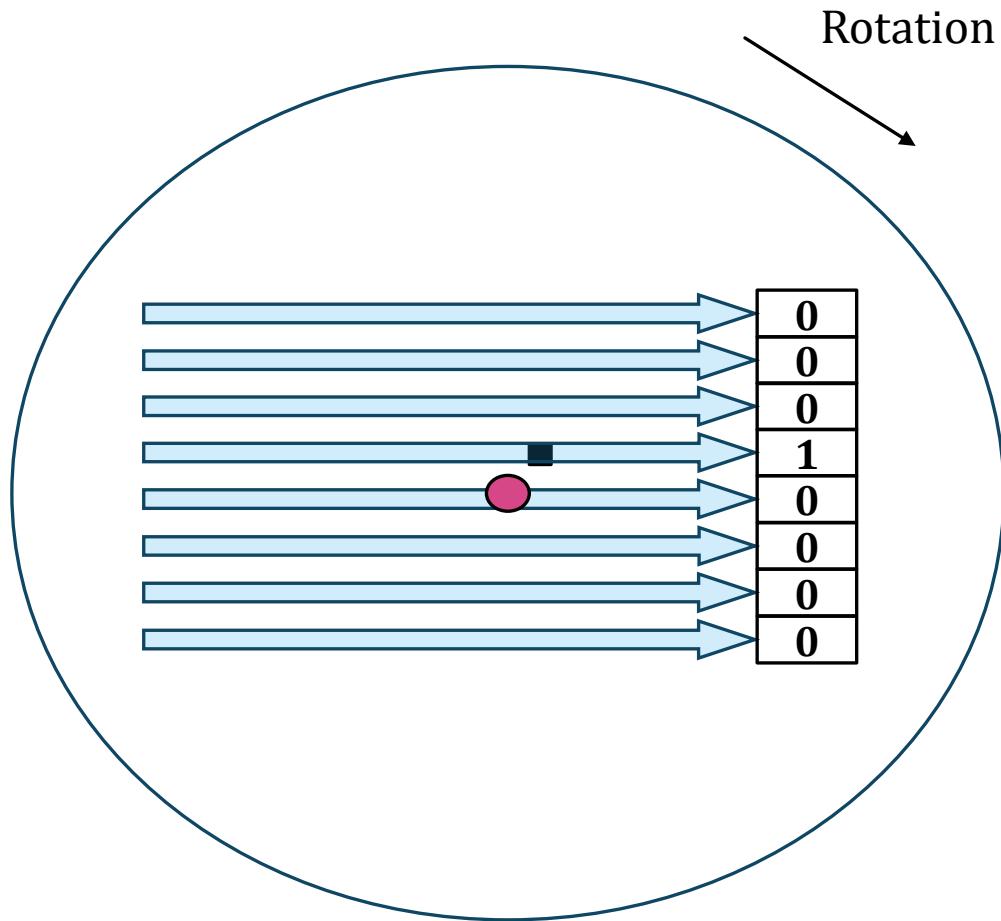
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0



$\Theta$

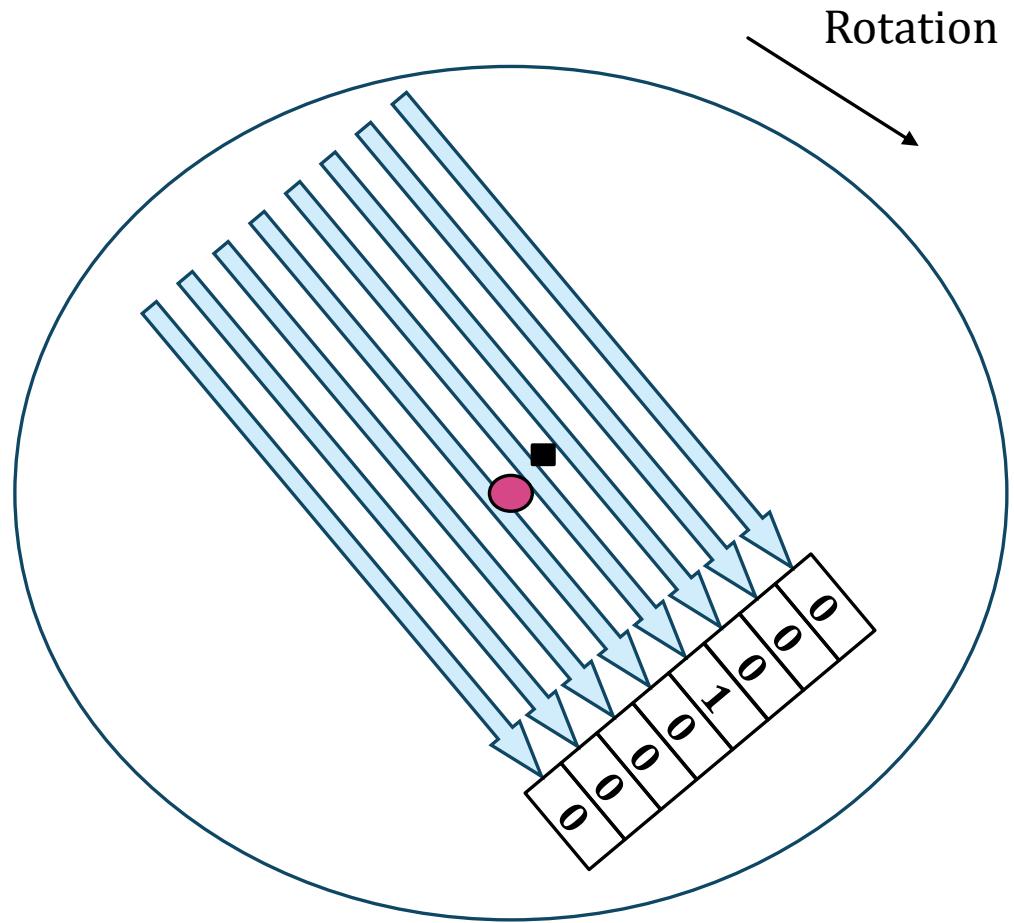
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0

If distance is smaller

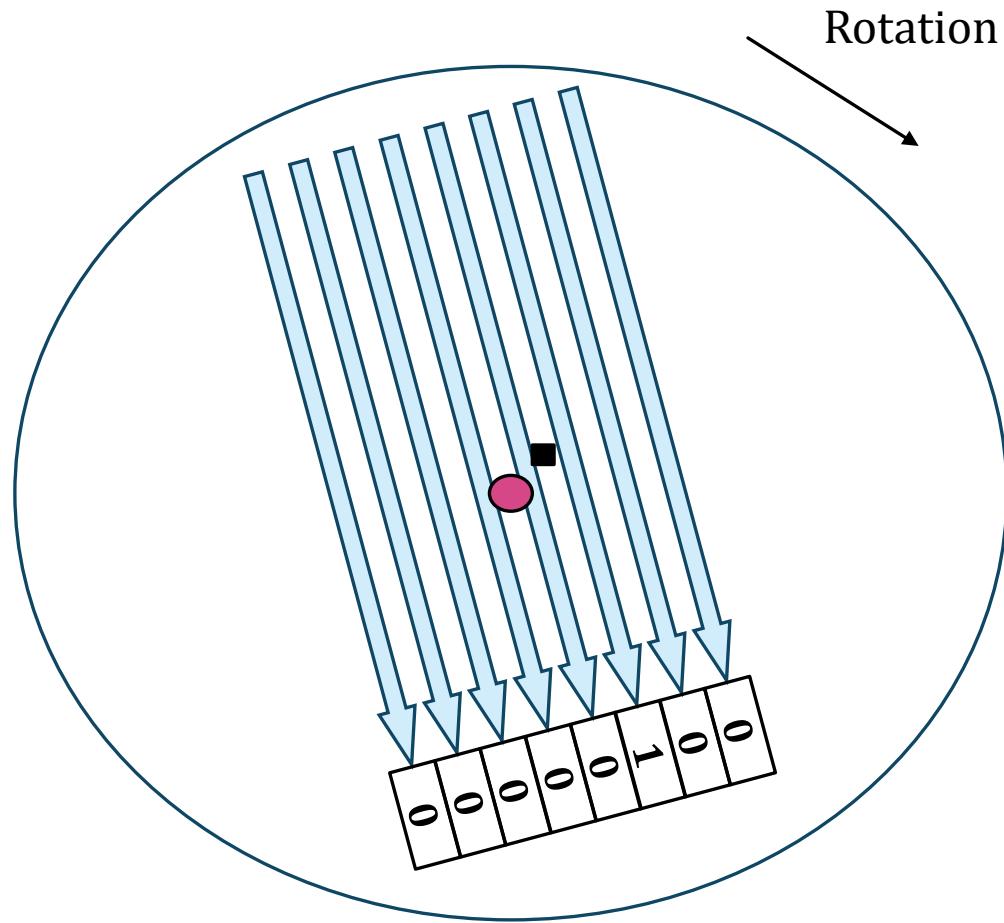


0	0	0	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---

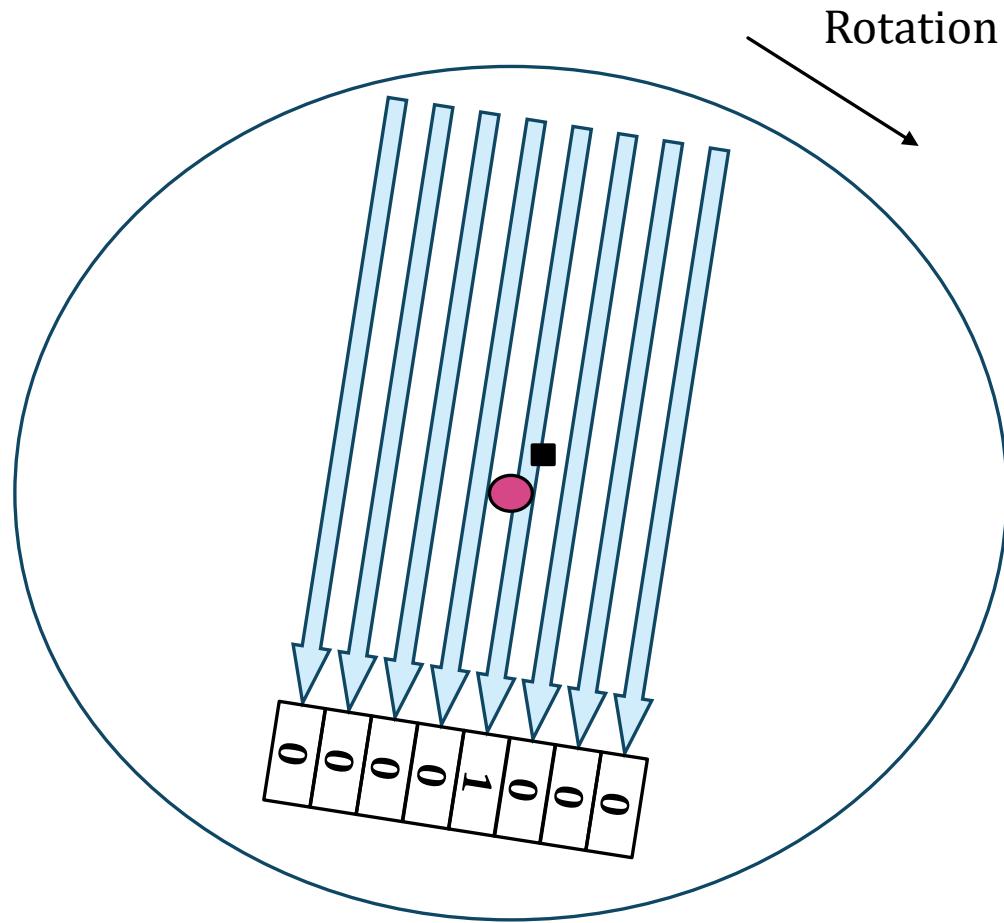
**$\Theta = 0$**



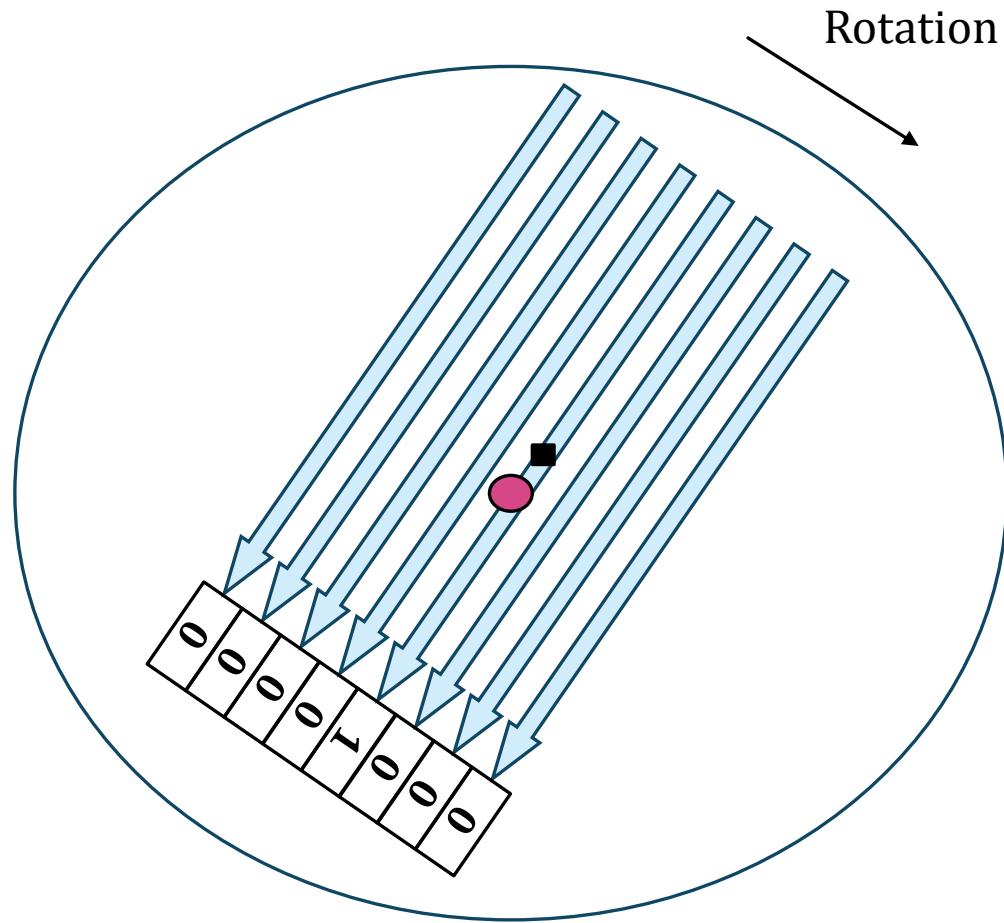
0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0



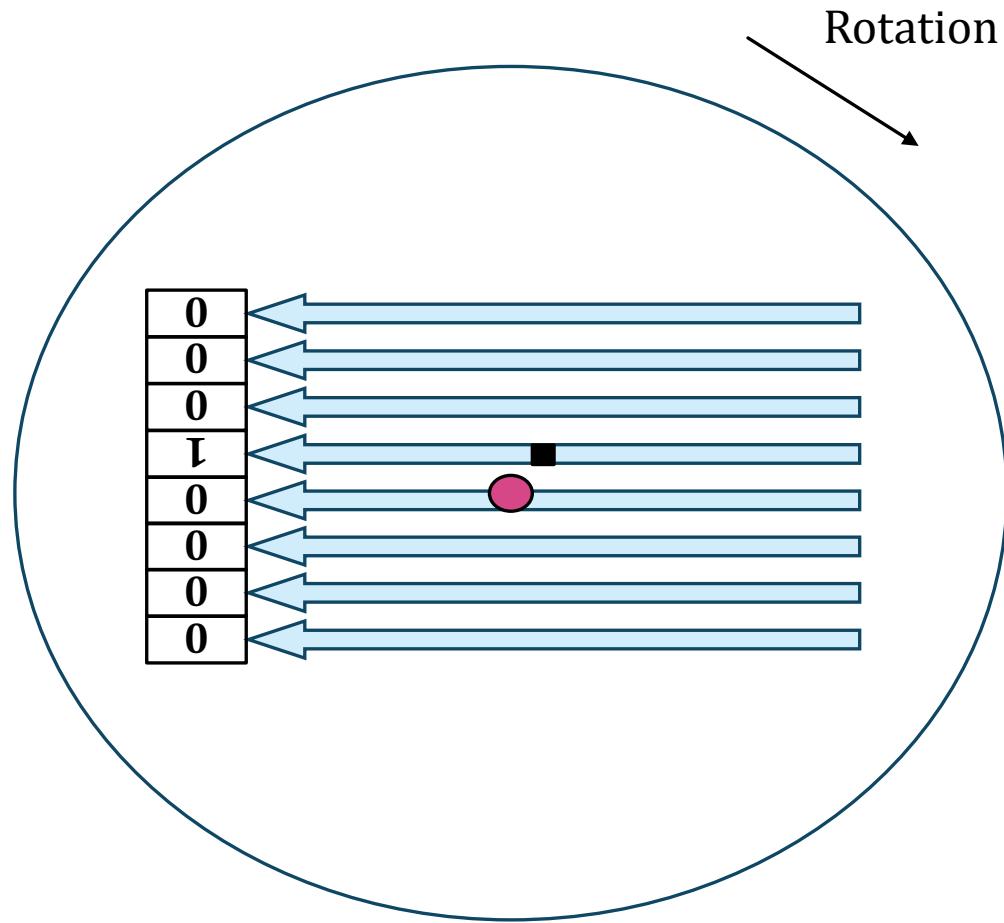
0	0	0	<b>1</b>	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0
0	0	<b>1</b>	0	0	0	0	0



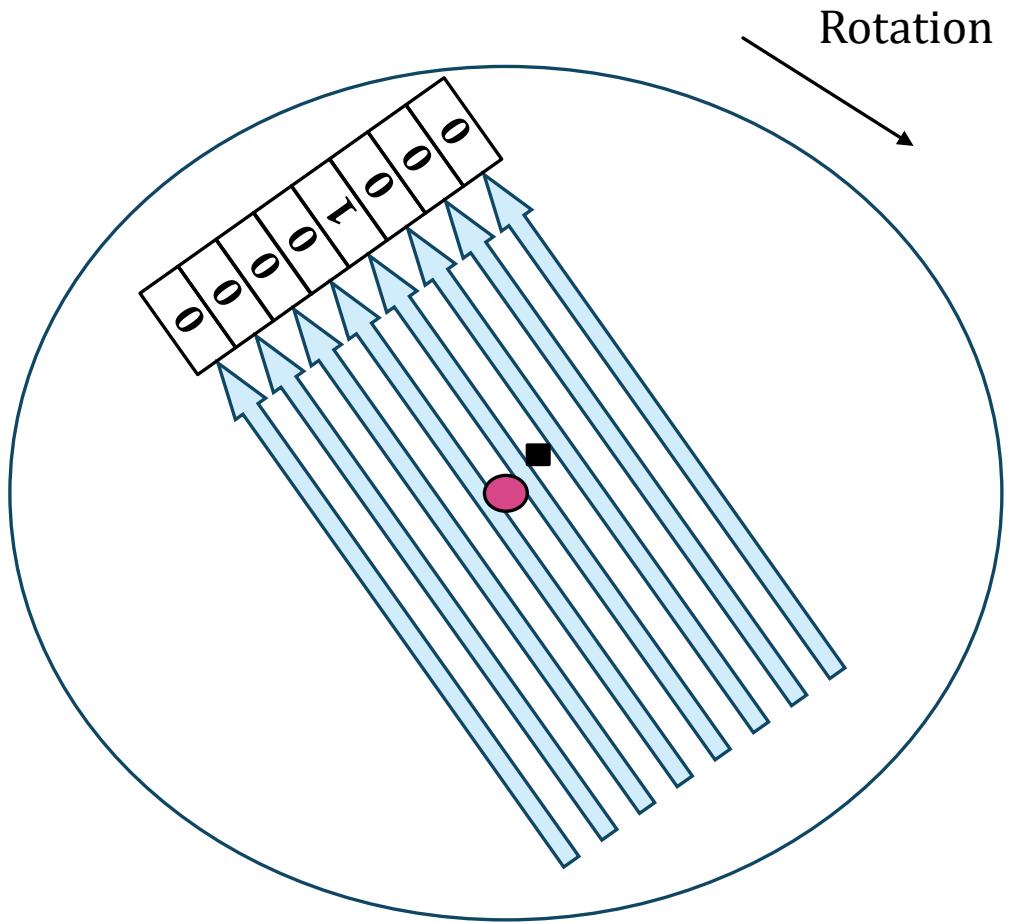
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	<b>1</b>	0	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0



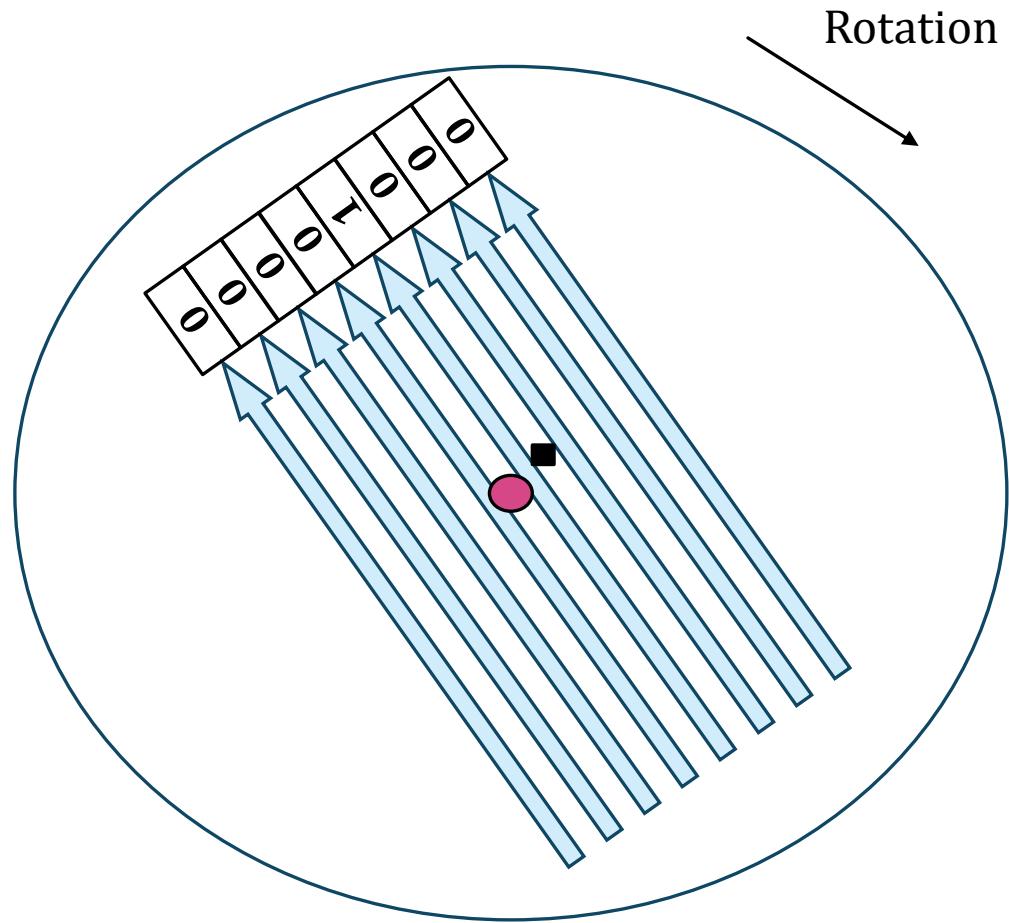
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	<b>1</b>	0	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0



0	0	0	<b>1</b>	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0
0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0
0	0	0	0	<b>1</b>	0	0	0

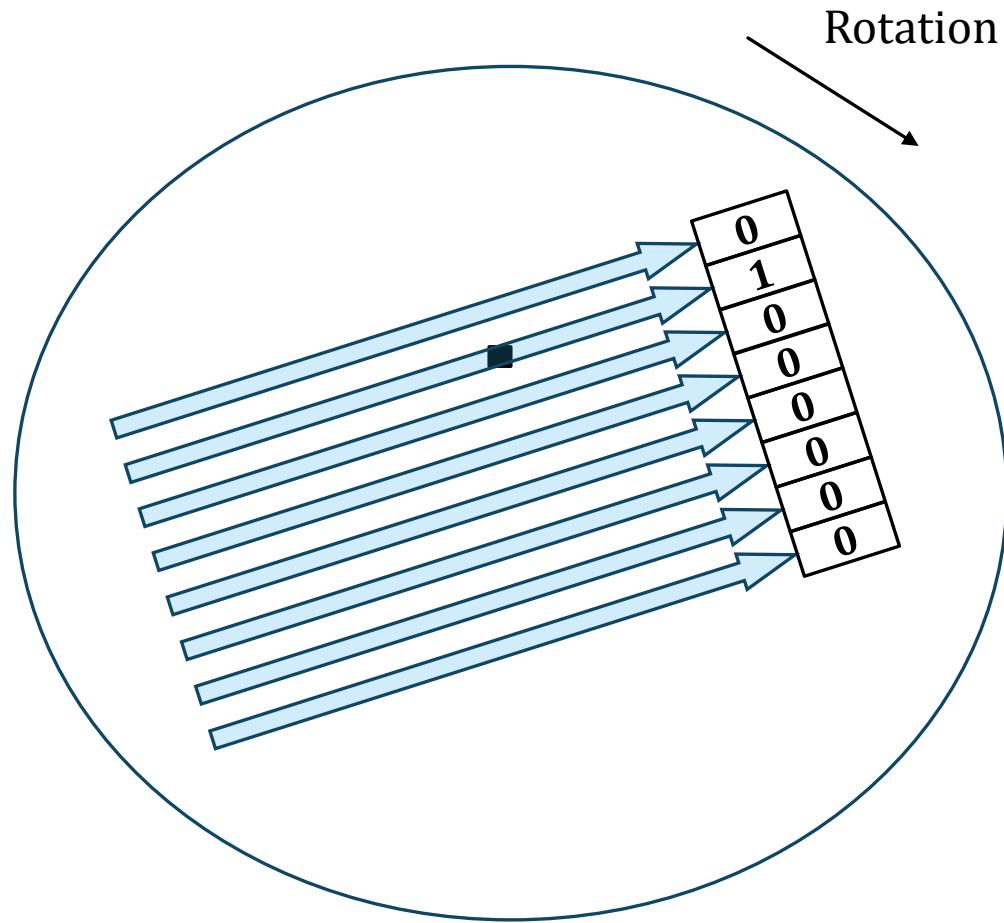


0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0



The closer an object from the center of rotation, the smaller is the amplitude

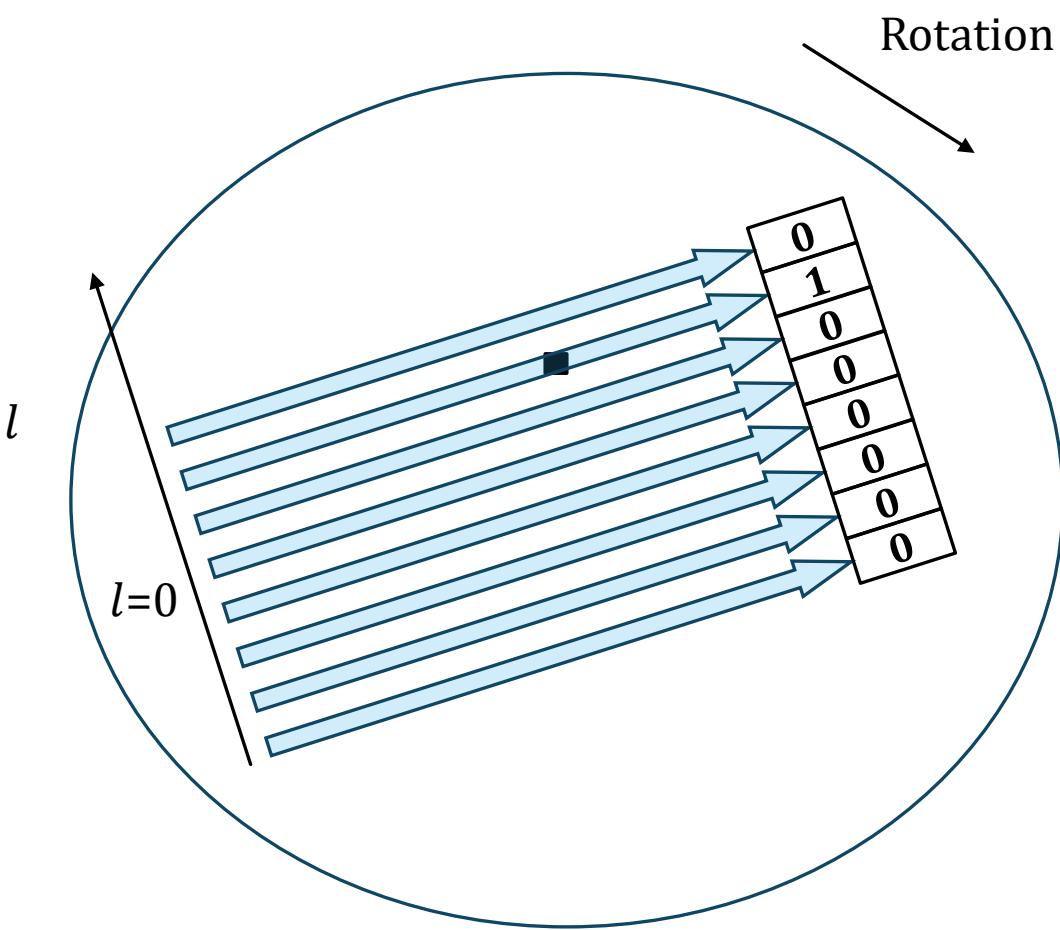
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	<b>1</b>	0	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0
0	0	0	0	<b>1</b>	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0	0



l

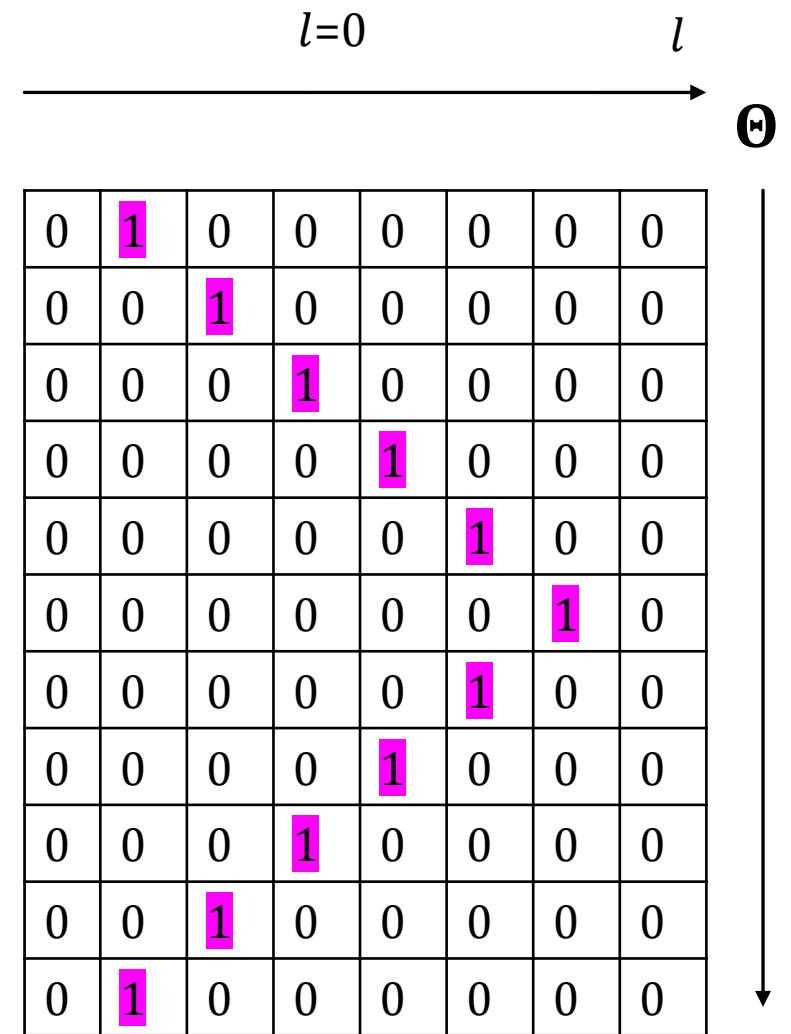
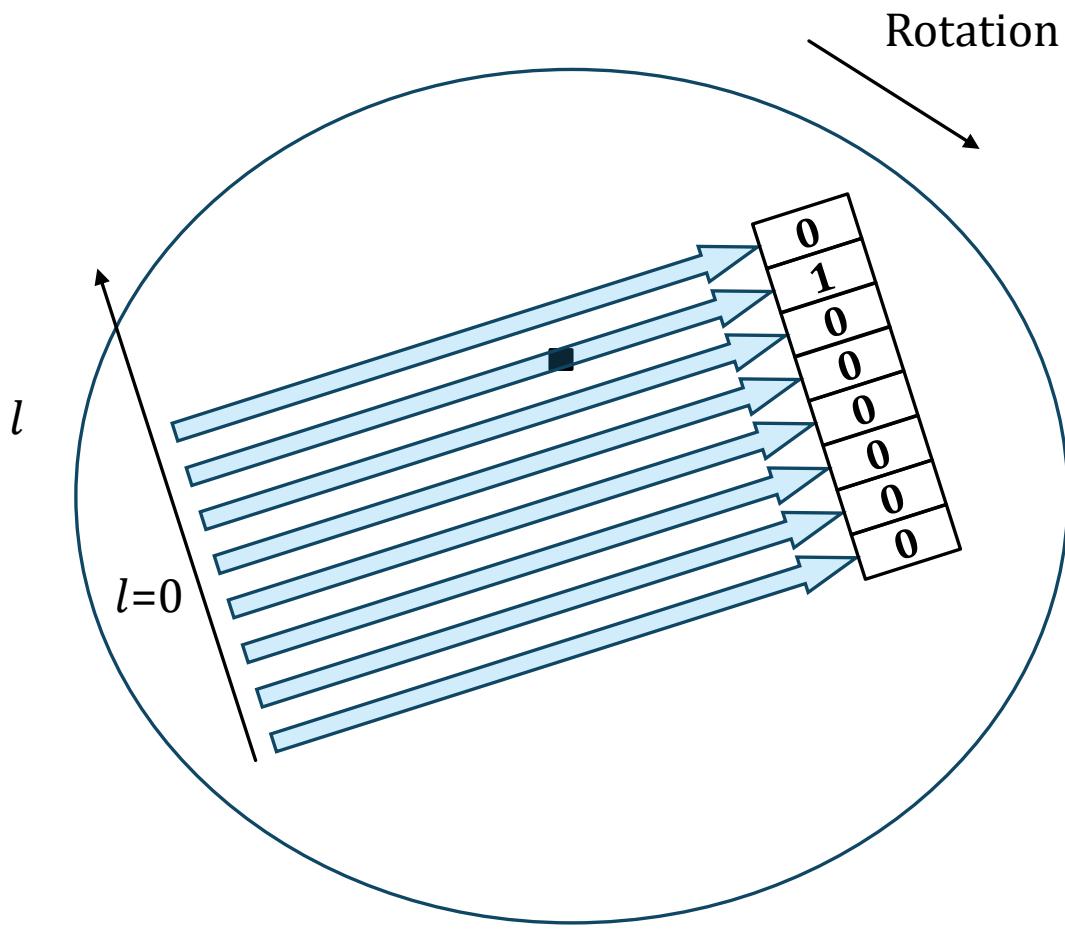
θ

0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0



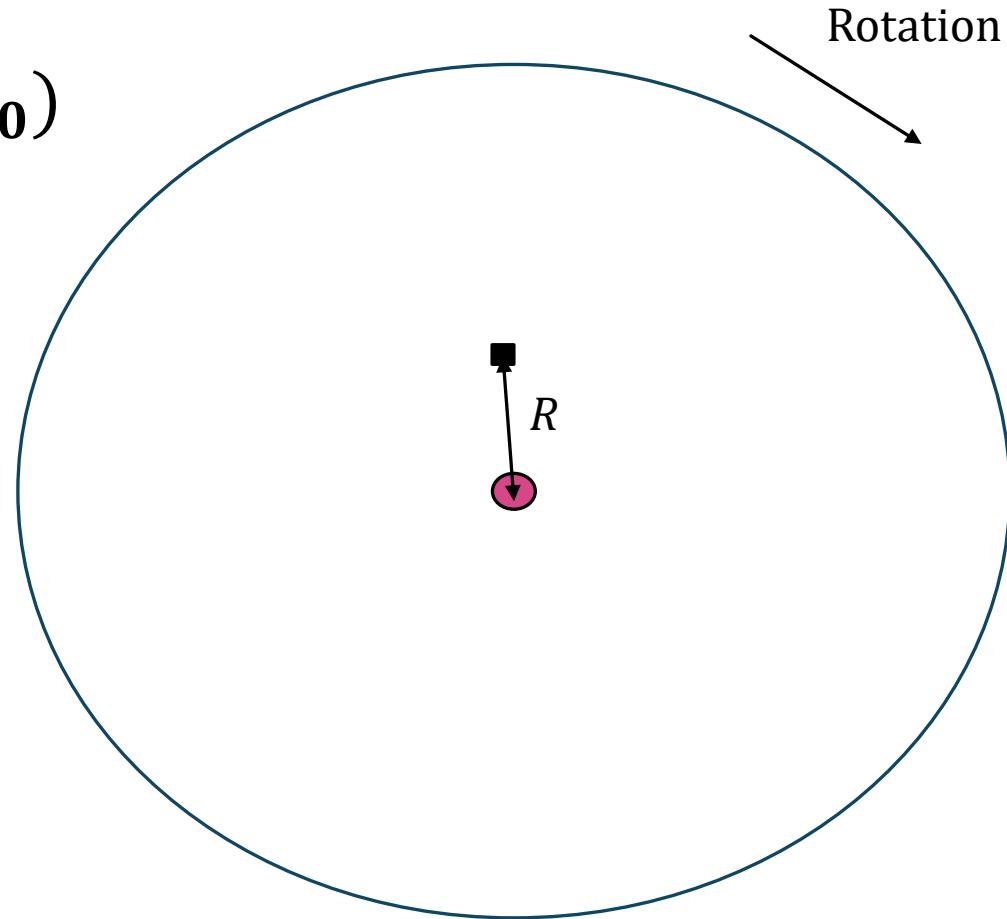
$l=0$	$l$	$\Theta$
0	1	0
0	0	1
0	0	0
0	0	1
0	0	0
0	0	1
0	0	0
0	0	1
0	1	0

# Sinogram



# Sinogram

$$R \sin(\theta + \theta_0)$$



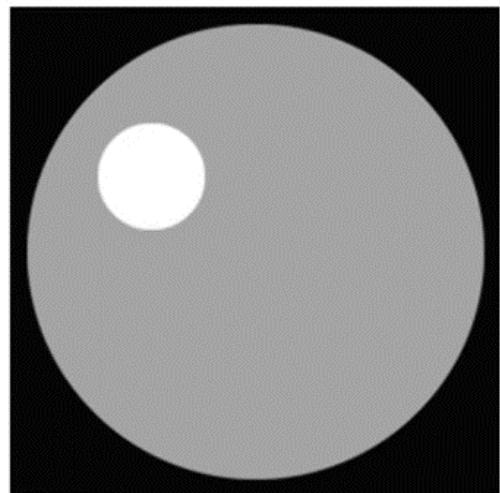
$\Theta$

↓

0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0

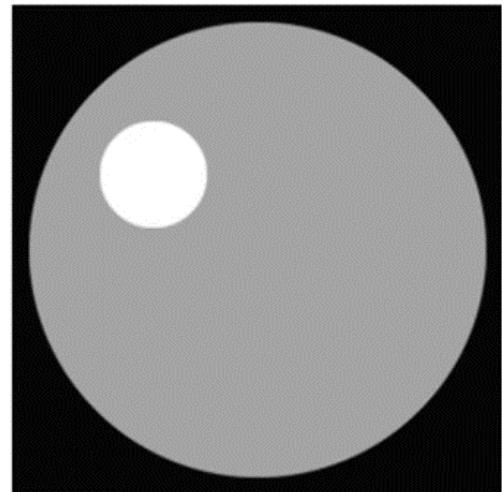
# Sinogram

---

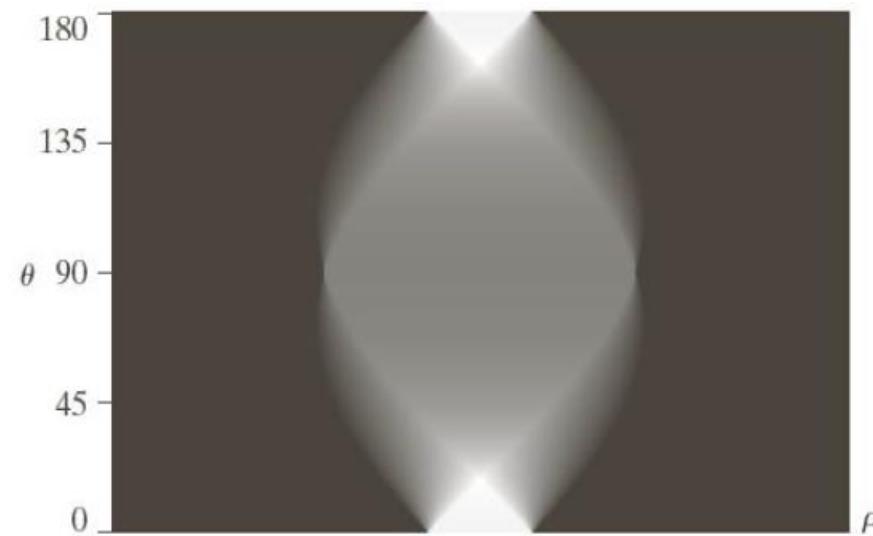
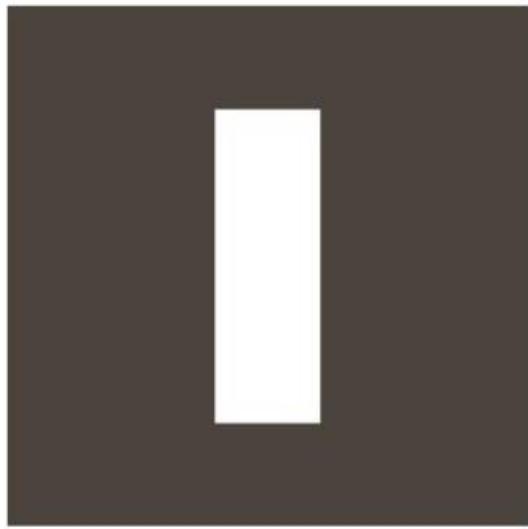


# Sinogram

---



Radom Transform is the transformation  
of any object into its sinogram



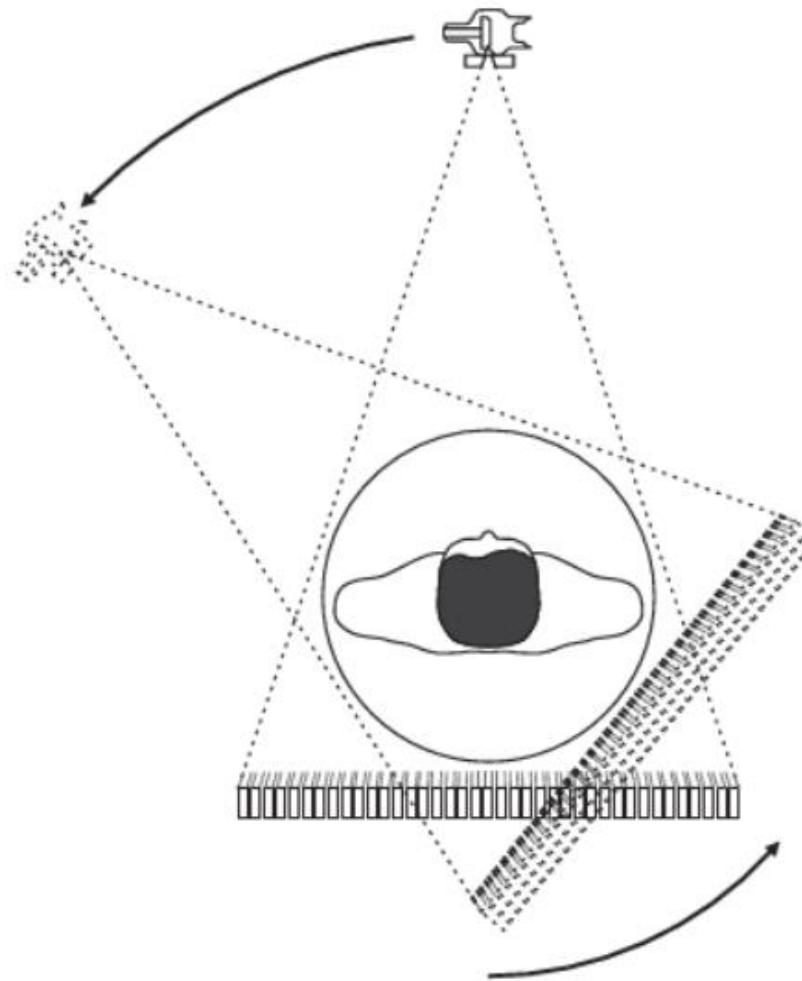
a b  
c d

## Sinogram

It is very difficult to interpret sinograms

**FIGURE 5.39** Two images and their sinograms (Radon transforms). Each row of a sinogram is a projection along the corresponding angle on the vertical axis. Image (c) is called the *Shepp-Logan phantom*. In its original form, the contrast of the phantom is quite low. It is shown enhanced here to facilitate viewing.

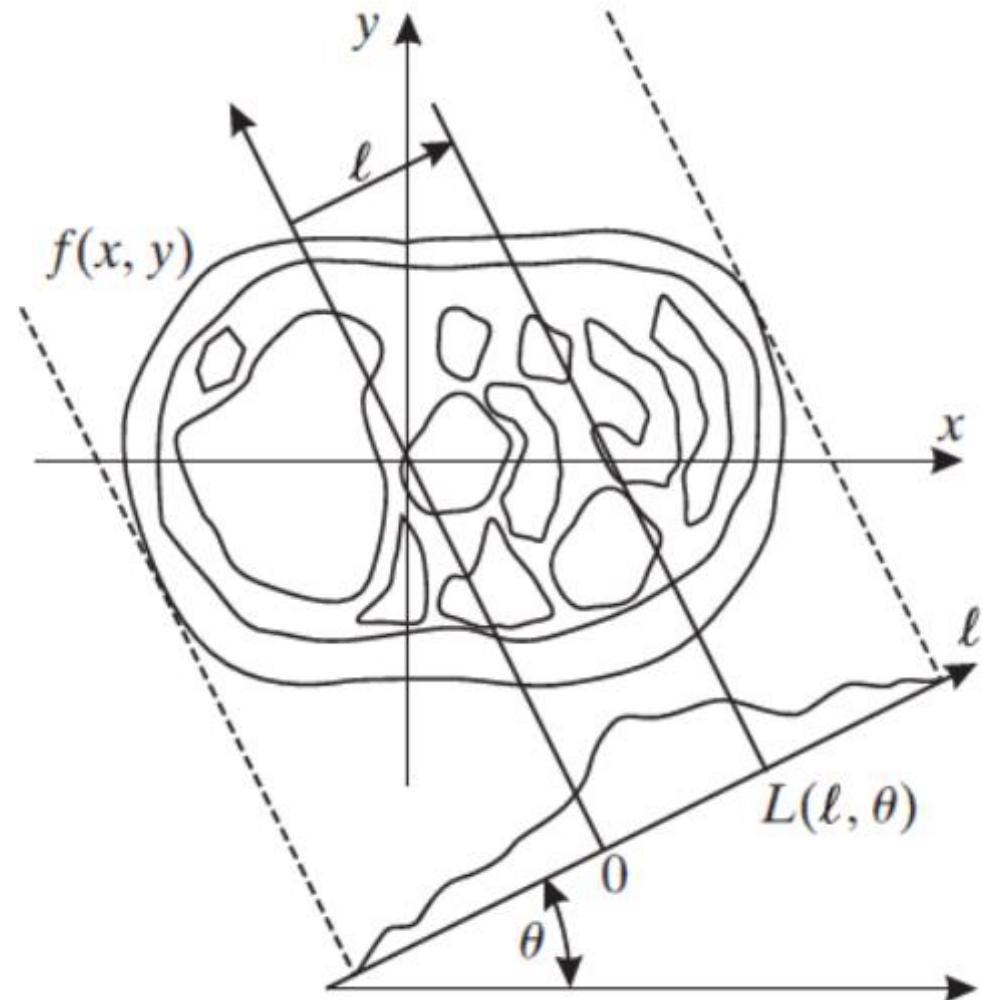
# CT Scanner



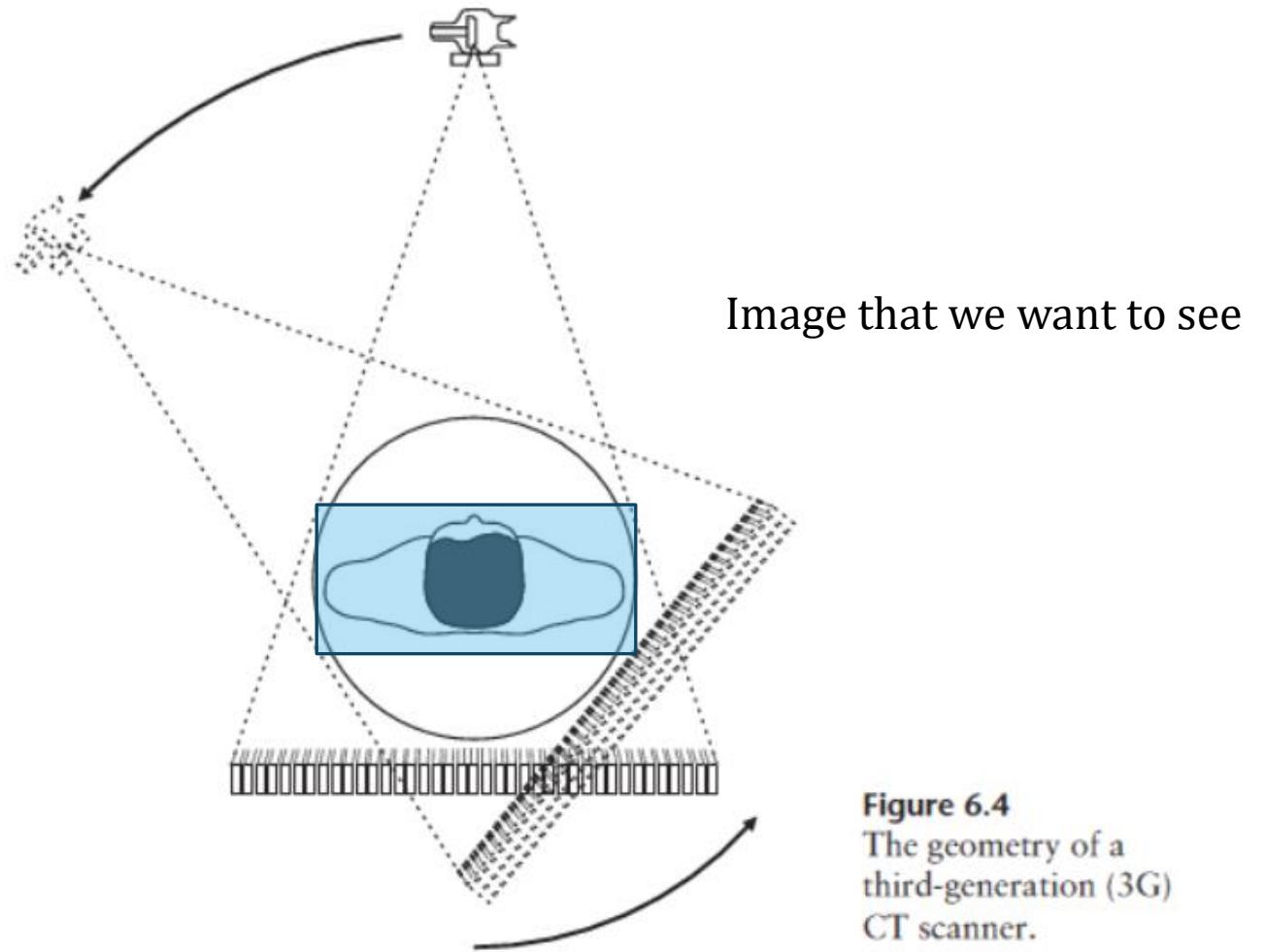
**Figure 6.4**  
The geometry of a  
third-generation (3G)  
CT scanner.

# Projection

Parallel rays

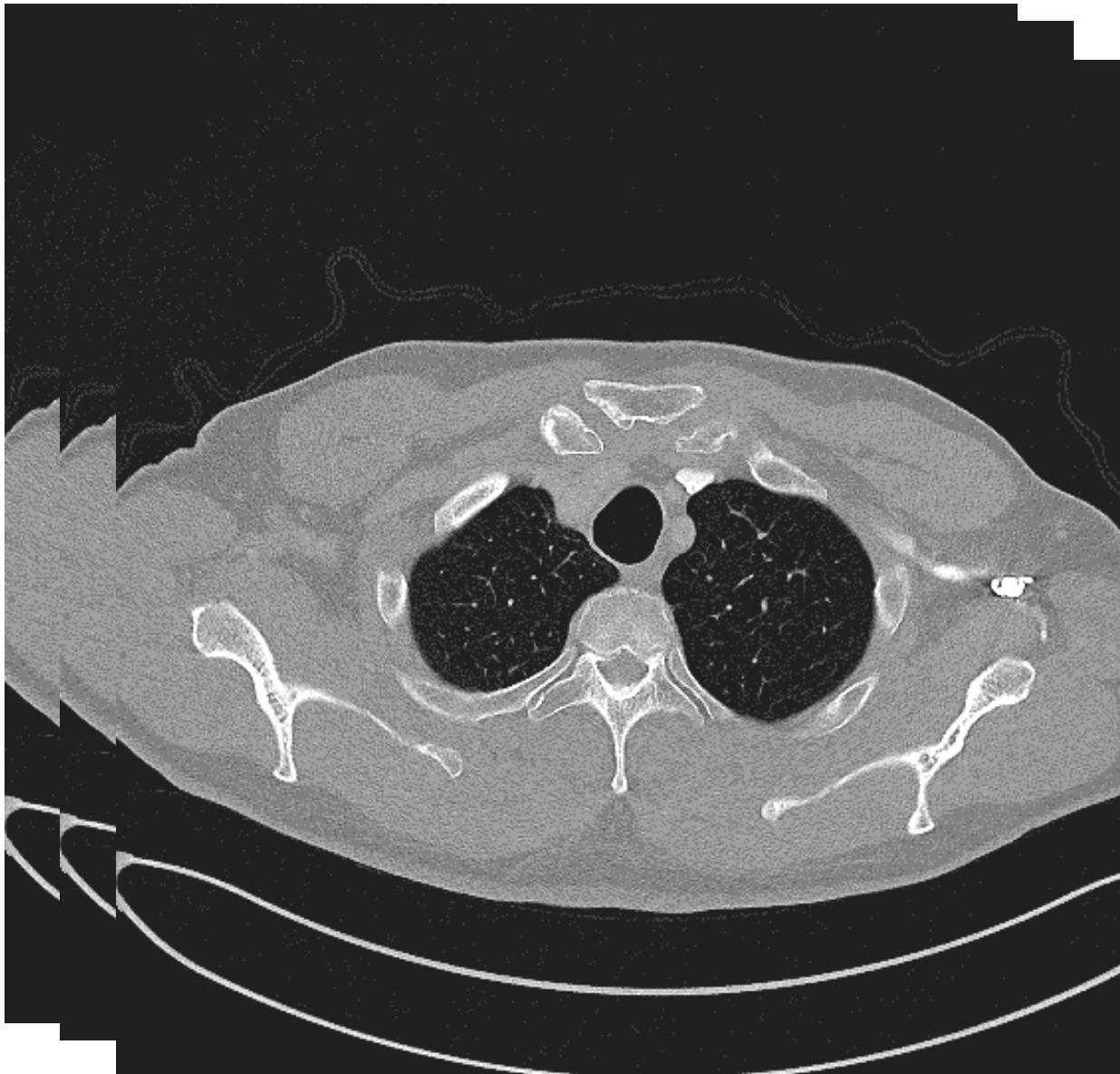


# CT Scanner



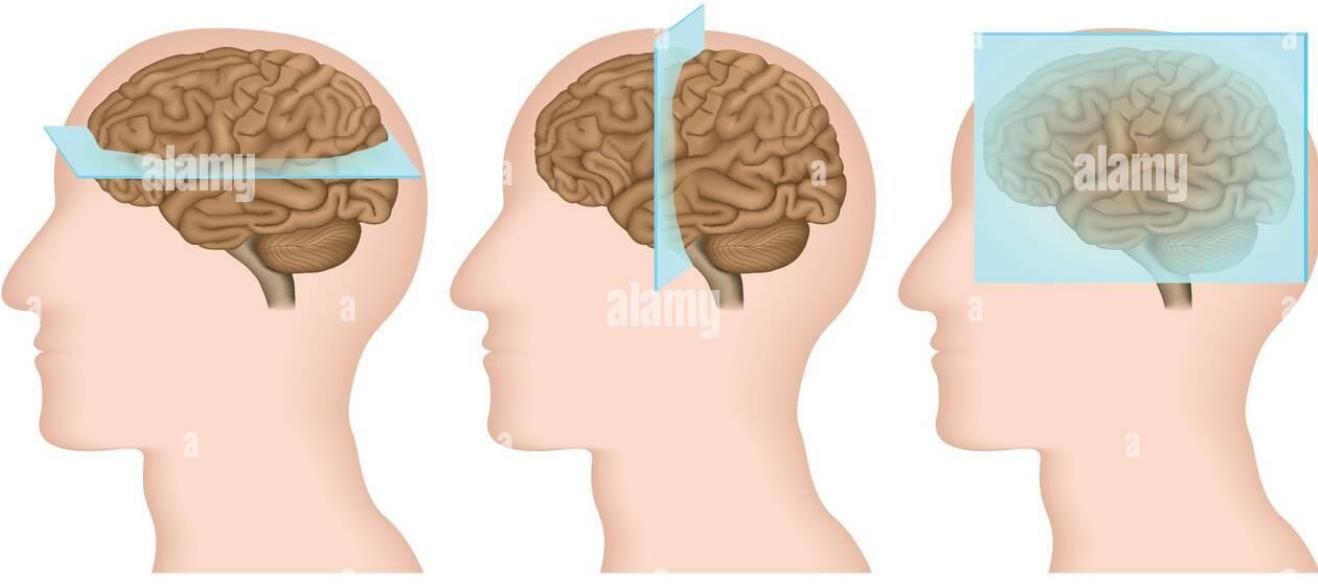


Slices stacked together  
provides 3D information



# Planes of Body

## Anatomical Terminology



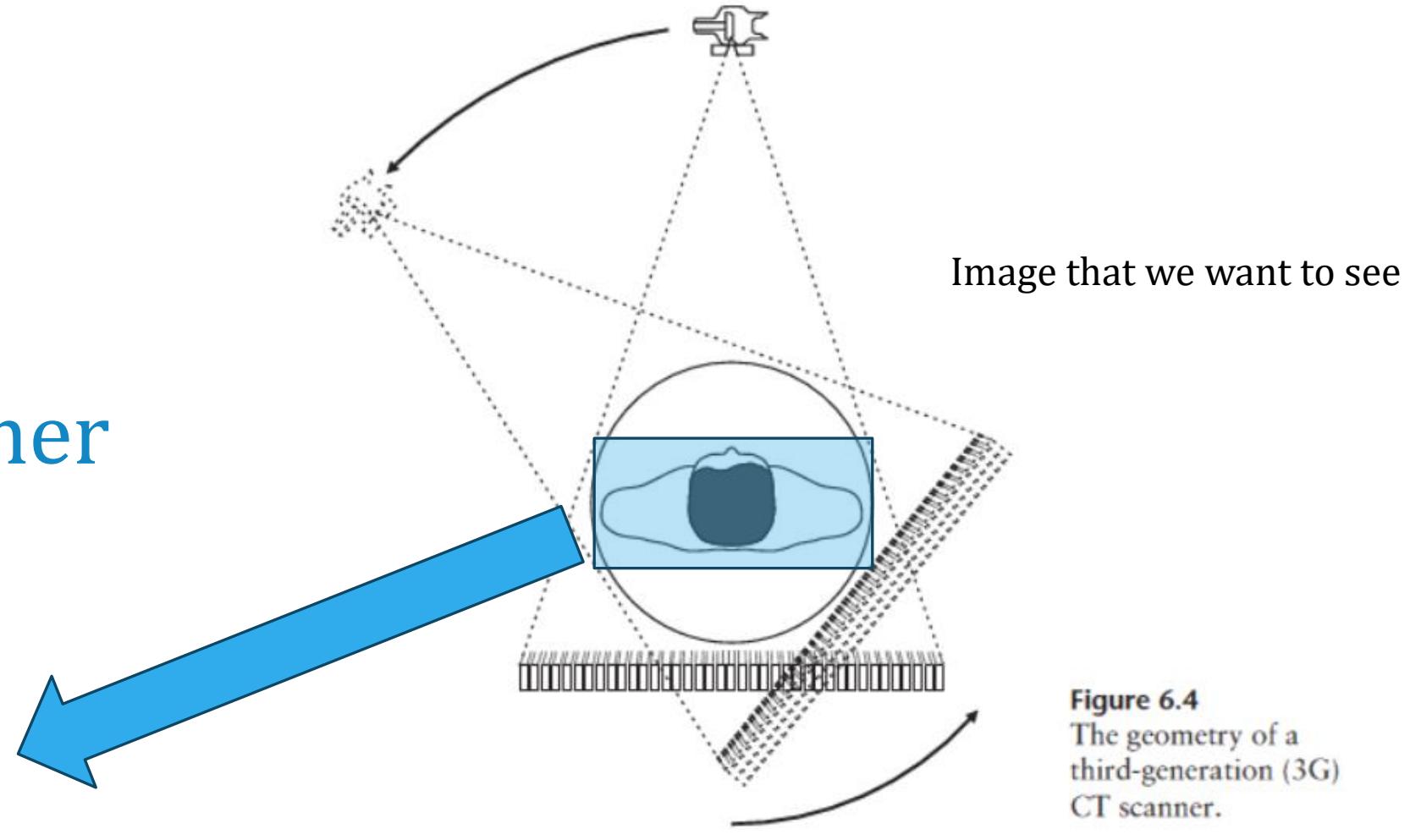
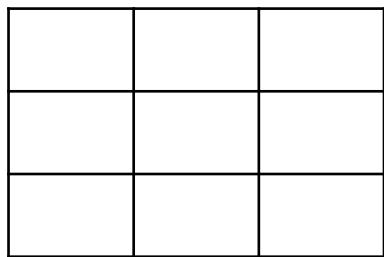
Transverse Plane

Axial Plane

Coronal Plane

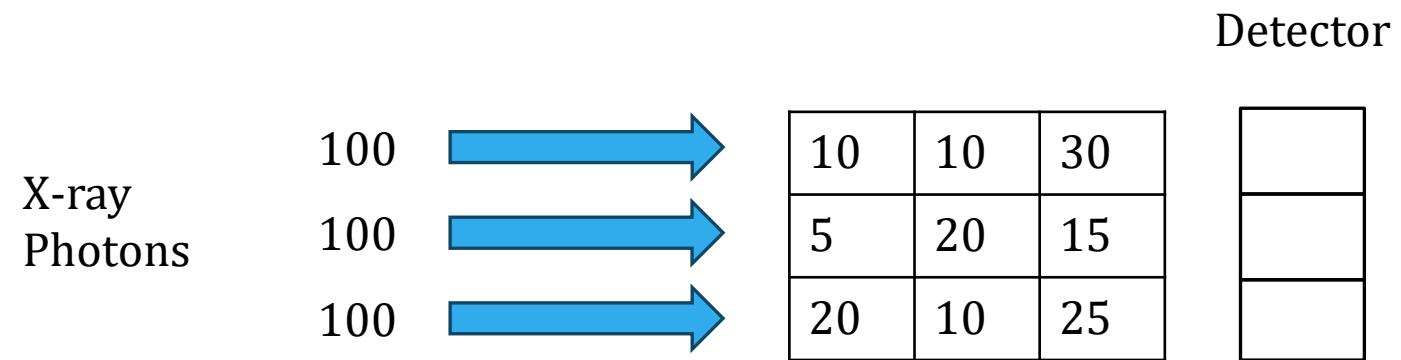
Sagittal Plane

# CT Scanner

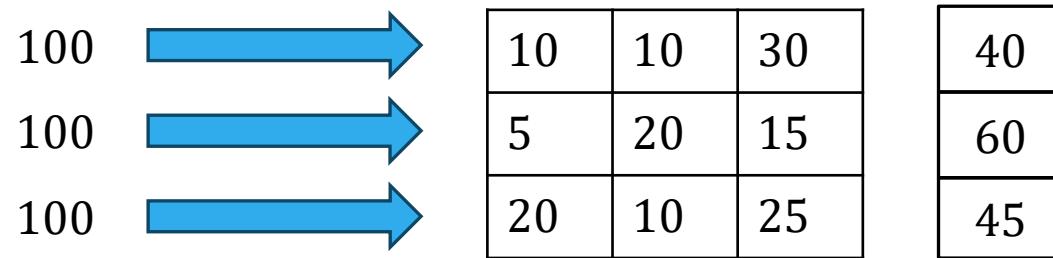


**Figure 6.4**  
The geometry of a  
third-generation (3G)  
CT scanner.

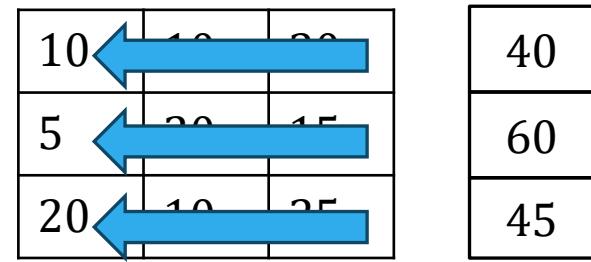
# Projection



# Forward Projection

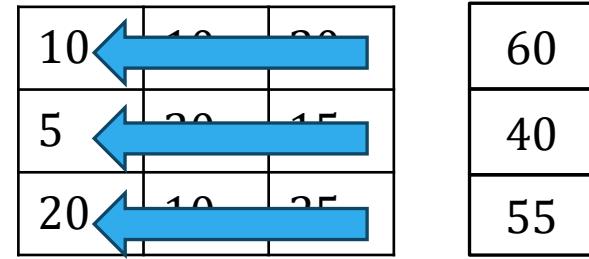


# Back Projection



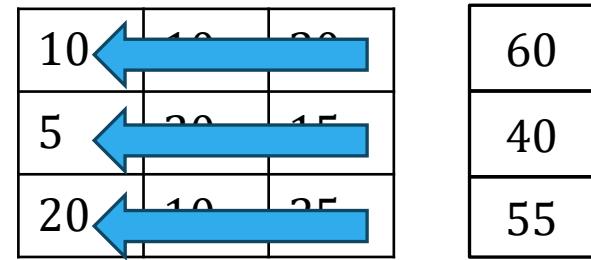
# Back Projection

Photons absorbed  
along the straight line  
of the x-ray



# Back Projection

Photons absorbed  
along the straight line  
of the x-ray

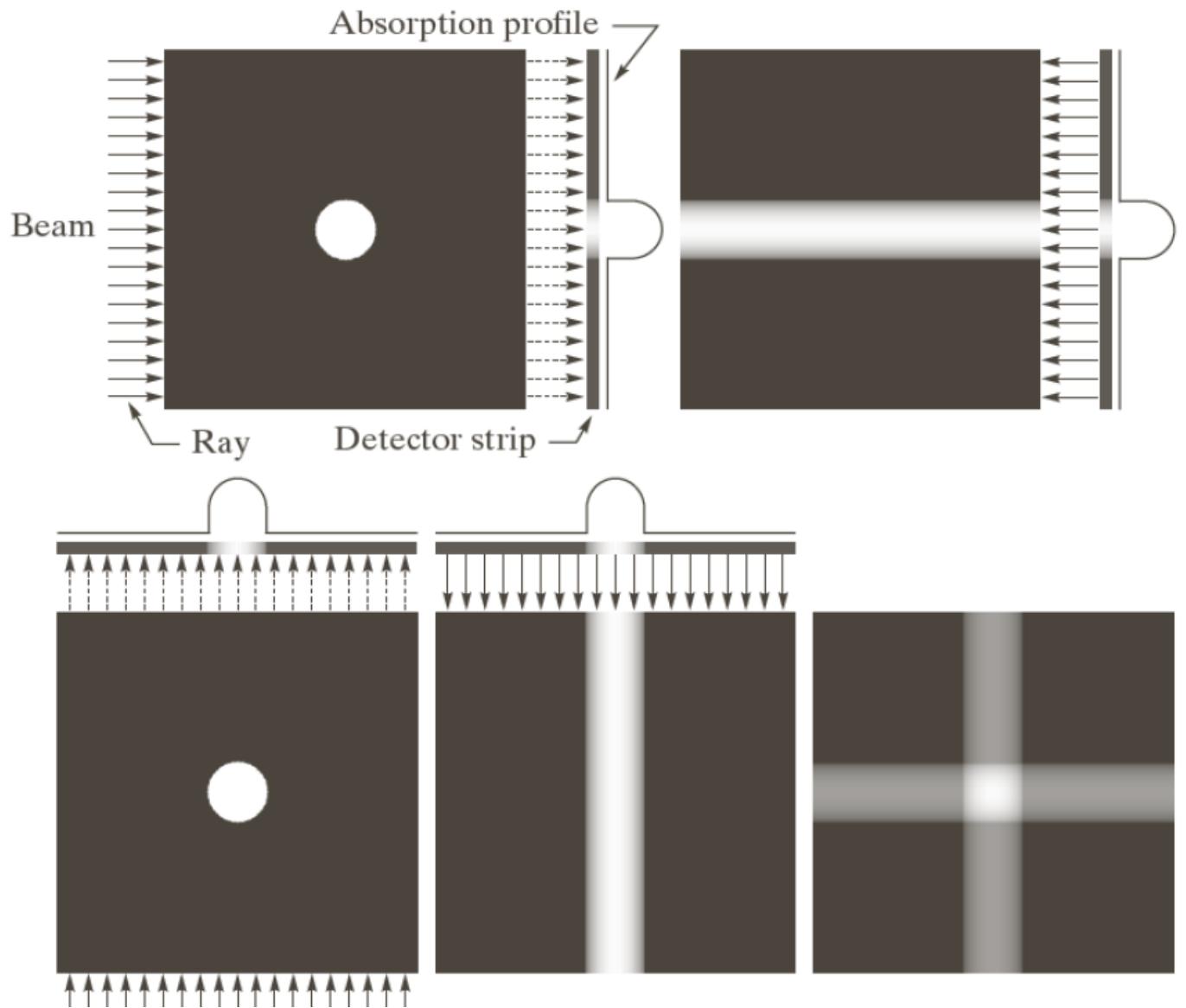


Fill all the pixels in a  
straight line along an  
x-ray with the  
number of photons  
absorbed in that line

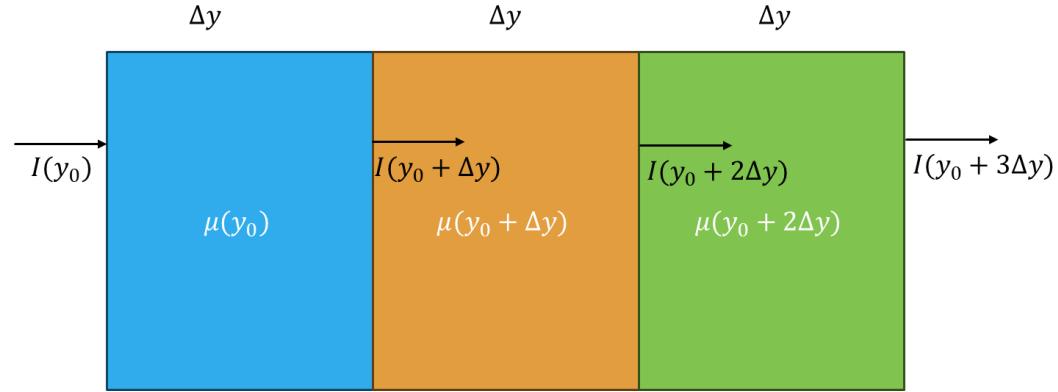
# Back Projection

60	60	60	60
40	40	40	40
55	55	55	55

# Back Projection



# Radon Transform

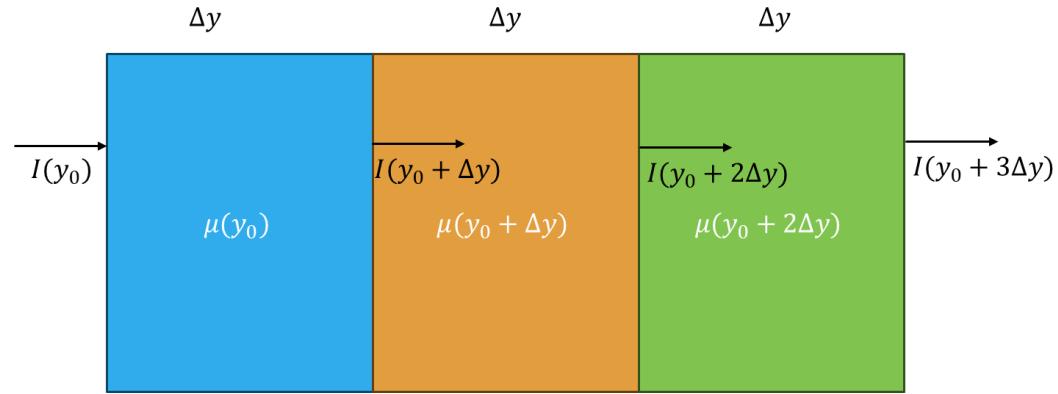


$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

Line integral along the  
line  $x \cos \theta + y \sin \theta = l$

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

# Radon Transform



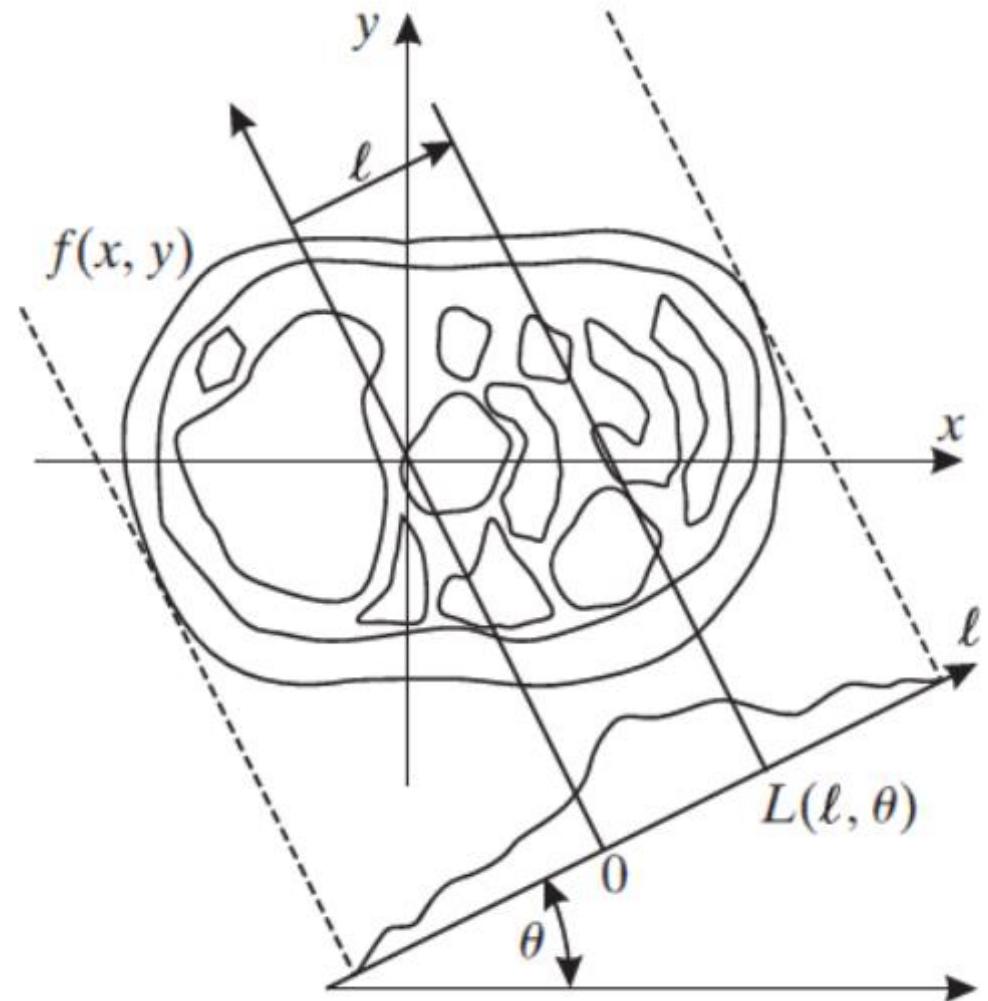
$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

We want to get  $\mu$

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

# Projection

Parallel rays

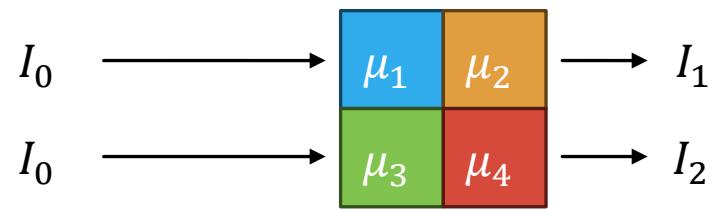


# Reconstruction of Objects

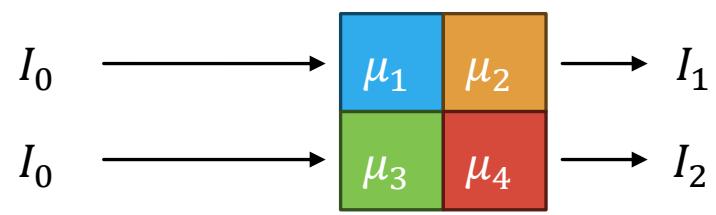
# 2D Objects



# 2D Objects



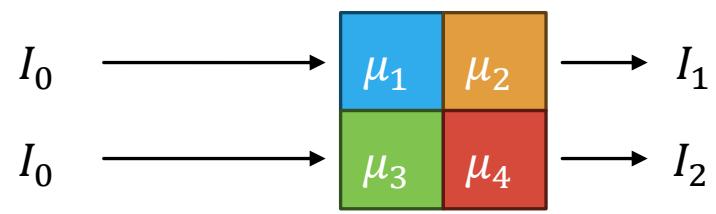
# 2D Objects



$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$

$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

# 2D Objects



$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$

$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

From this, can I find out the various components, the x-ray passing through?

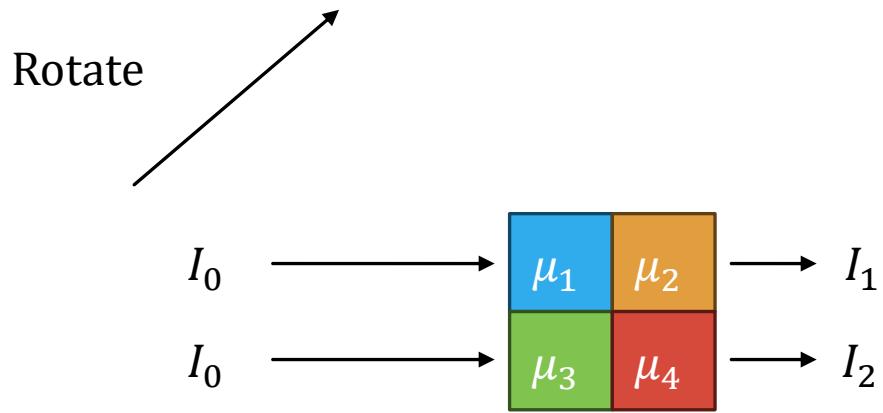
# 2D Objects

$$\begin{array}{ccc} I_0 & \xrightarrow{\hspace{2cm}} & \begin{matrix} \mu_1 & \mu_2 \\ \mu_3 & \mu_4 \end{matrix} & \xrightarrow{\hspace{2cm}} & I_1 \\ I_0 & \xrightarrow{\hspace{2cm}} & \begin{matrix} \mu_1 & \mu_2 \\ \mu_3 & \mu_4 \end{matrix} & \xrightarrow{\hspace{2cm}} & I_2 \end{array}$$
$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$
$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

From this, can I find out the various components, the x-ray passing through?

No. Because I have two equations and four unknowns. So, infinite possibilities

# 2D Objects

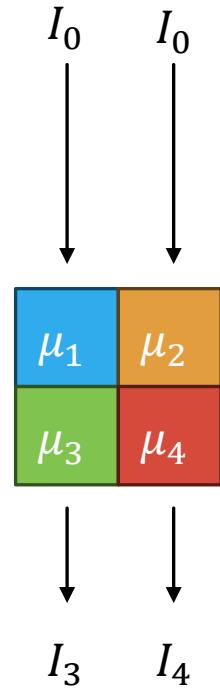


$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$

$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

So, we take projections from another view

# 2D Objects



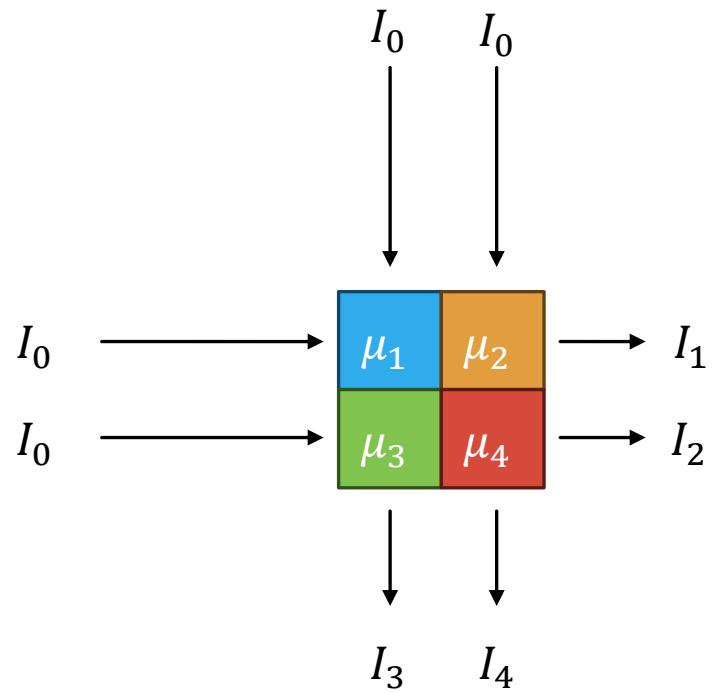
$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$

$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

$$I_3 = I_0 e^{-(\mu_1 \Delta w + \mu_3 \Delta w)}$$

$$I_4 = I_0 e^{-(\mu_2 \Delta w + \mu_4 \Delta w)}$$

# 2D Objects



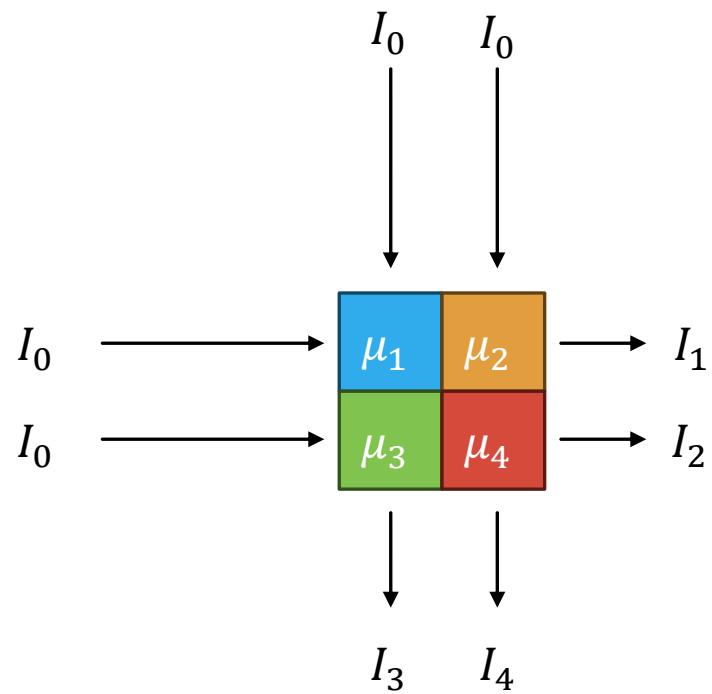
$$I_1 = I_0 e^{-(\mu_1 \Delta w + \mu_2 \Delta w)}$$

$$I_2 = I_0 e^{-(\mu_3 \Delta w + \mu_4 \Delta w)}$$

$$I_3 = I_0 e^{-(\mu_1 \Delta w + \mu_3 \Delta w)}$$

$$I_4 = I_0 e^{-(\mu_2 \Delta w + \mu_4 \Delta w)}$$

# 2D Objects



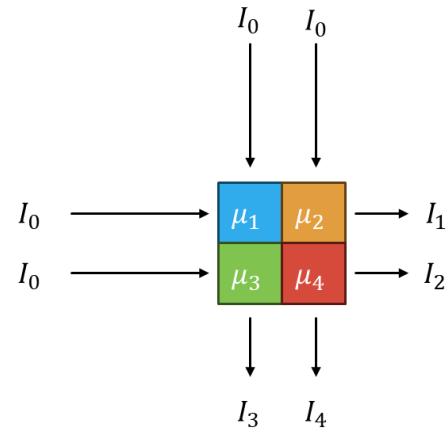
$$-\mu_1 \Delta w - \mu_2 \Delta w = \ln \frac{I_1}{I_0}$$

$$-\mu_3 \Delta w - \mu_4 \Delta w = \ln \frac{I_2}{I_0}$$

$$-\mu_1 \Delta w - \mu_3 \Delta w = \ln \frac{I_3}{I_0}$$

$$-\mu_2 \Delta w - \mu_4 \Delta w = \ln \frac{I_4}{I_0}$$

# 2D Objects



We can solve using matrix inversion. But too complex for larger objects

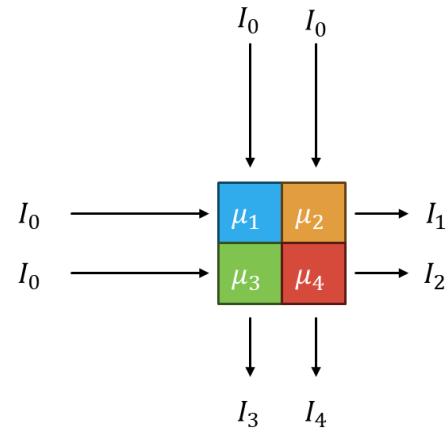
$$-\mu_1 \Delta w - \mu_2 \Delta w = \ln \frac{I_1}{I_0}$$

$$-\mu_3 \Delta w - \mu_4 \Delta w = \ln \frac{I_2}{I_0}$$

$$-\mu_1 \Delta w - \mu_3 \Delta w = \ln \frac{I_3}{I_0}$$

$$-\mu_2 \Delta w - \mu_4 \Delta w = \ln \frac{I_4}{I_0}$$

# 2D Objects



In general, we can solve for and  $n \times n$  matrix if we have  $n^2$  equations

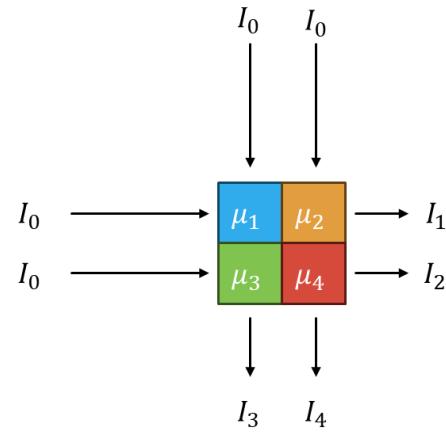
$$-\mu_1 \Delta w - \mu_2 \Delta w = \ln \frac{I_1}{I_0}$$

$$-\mu_3 \Delta w - \mu_4 \Delta w = \ln \frac{I_2}{I_0}$$

$$-\mu_1 \Delta w - \mu_3 \Delta w = \ln \frac{I_3}{I_0}$$

$$-\mu_2 \Delta w - \mu_4 \Delta w = \ln \frac{I_4}{I_0}$$

# 2D Objects



If I can find out the value of  $\mu_i$ s, I can understand the composition of the object

Computationally not efficient

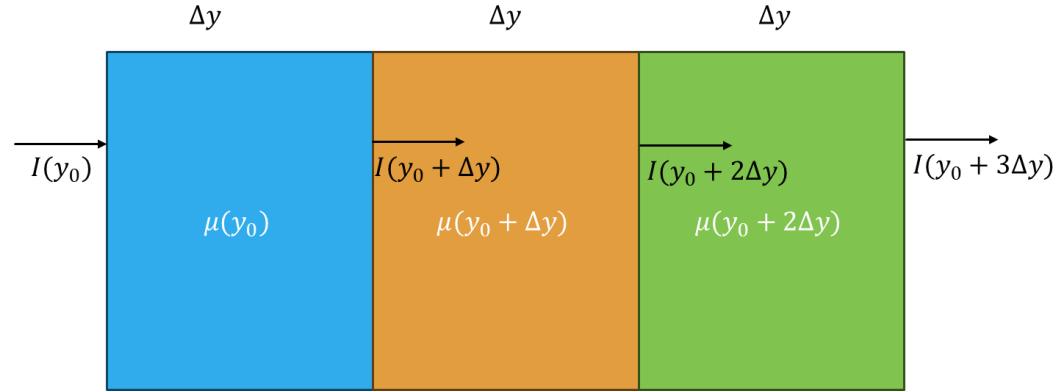
$$-\mu_1 \Delta w - \mu_2 \Delta w = \ln \frac{I_1}{I_0}$$

$$-\mu_3 \Delta w - \mu_4 \Delta w = \ln \frac{I_2}{I_0}$$

$$-\mu_1 \Delta w - \mu_3 \Delta w = \ln \frac{I_3}{I_0}$$

$$-\mu_2 \Delta w - \mu_4 \Delta w = \ln \frac{I_4}{I_0}$$

# Radon Transform



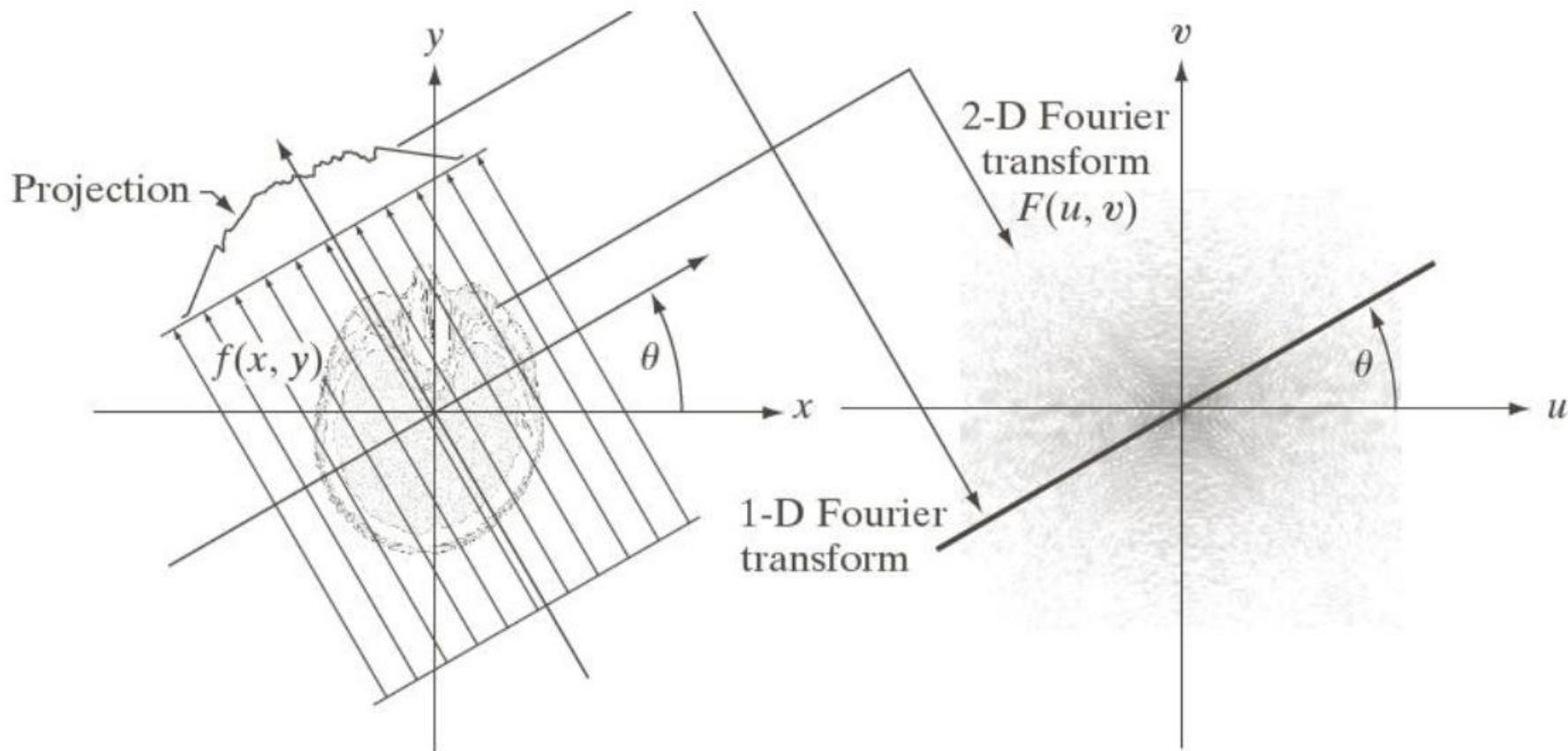
$$I_{out} = I(y_0) e^{- \int_{source}^{detector} \mu(x, y') dy'}$$

We want to get  $\mu$

But from one projection, we can't solve through inverse transform since there would be infinite possibilities

$$g(\ell, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) \delta(x \cos \theta + y \sin \theta - \ell) dx dy$$

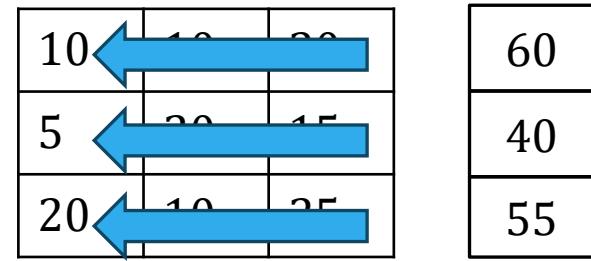
# Fourier Slice Theorem



# Computationally-efficient Reconstruction: Analytical Reconstruction

# Back Projection

Photons absorbed  
along the straight line  
of the x-ray

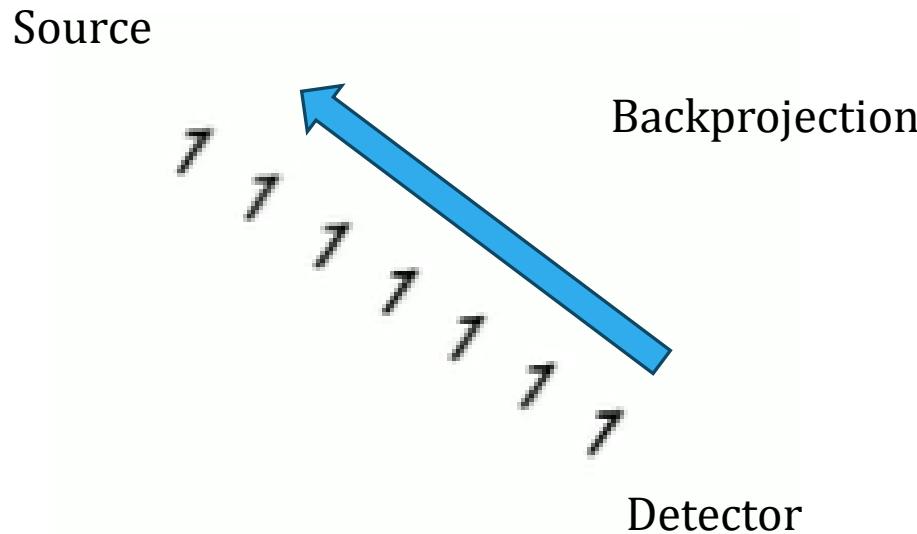


Fill all the pixels in a  
straight line along an  
x-ray with the  
number of photons  
absorbed in that line

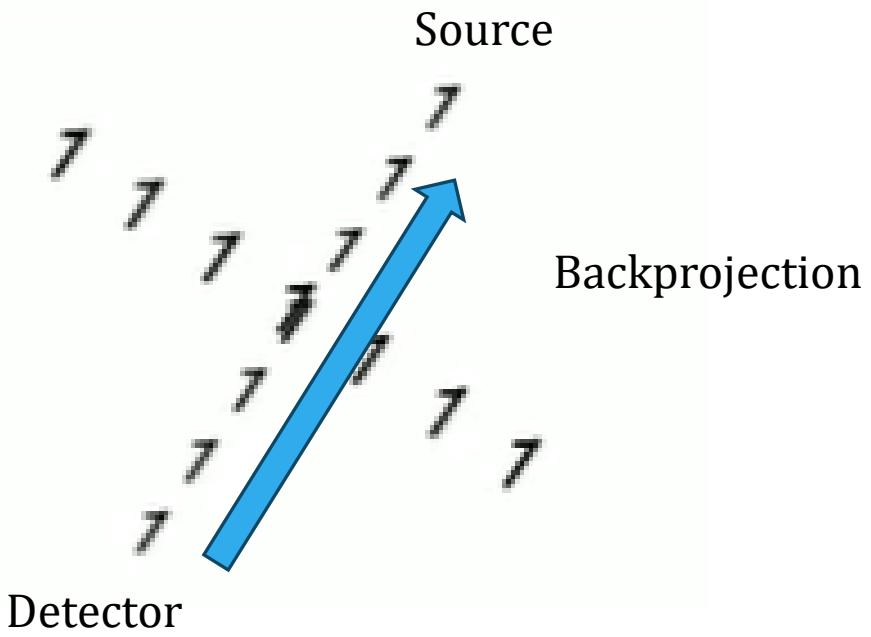
# Back Projection

60	60	60	60
40	40	40	40
55	55	55	55

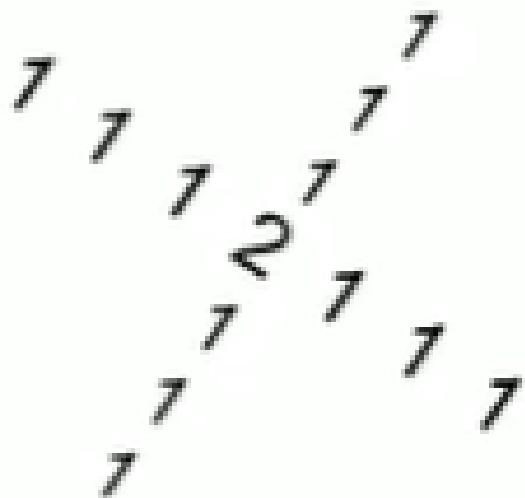
# Backprojection Reconstruction



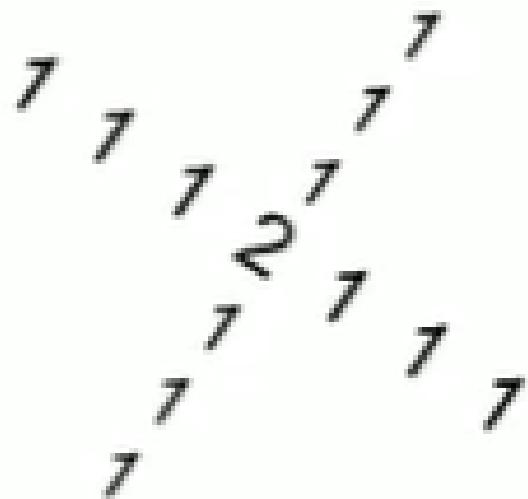
# Backprojection Reconstruction



# Backprojection Reconstruction



# Backprojection Reconstruction



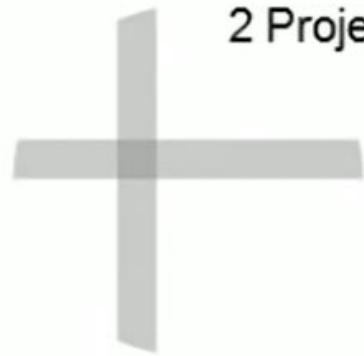
Backprojection

$$b_\theta(x, y) = g(x \cos \theta + y \sin \theta, \theta)$$

Backprojection summation image

$$f_b(x, y) = \int_0^\pi b_\theta(x, y) d\theta$$

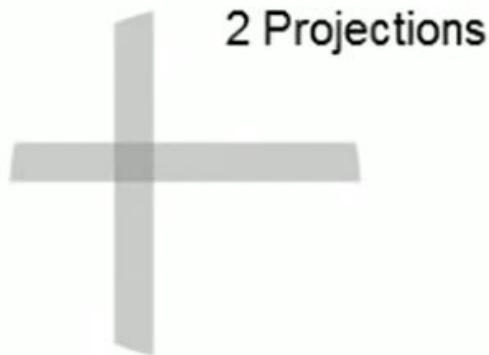
# Backprojection Reconstruction



2 Projections

of a point

# Backprojection Reconstruction

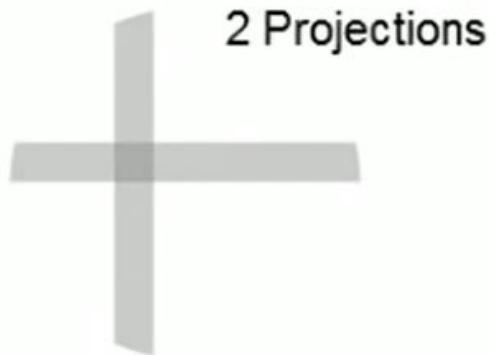


2 Projections

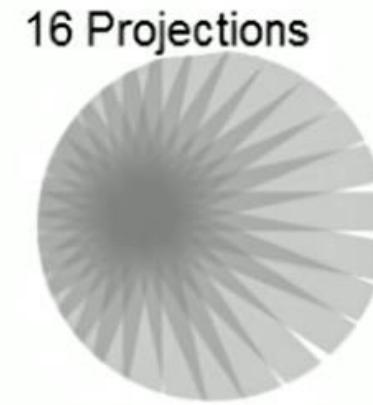
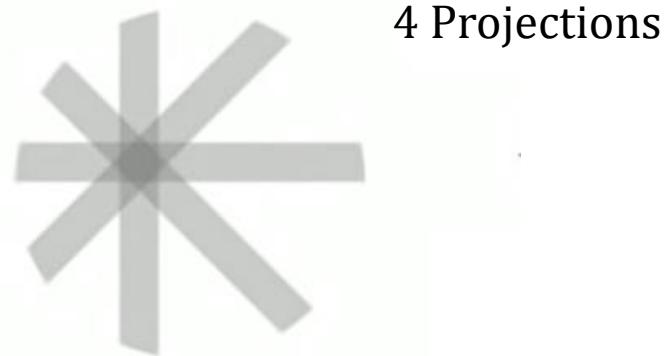
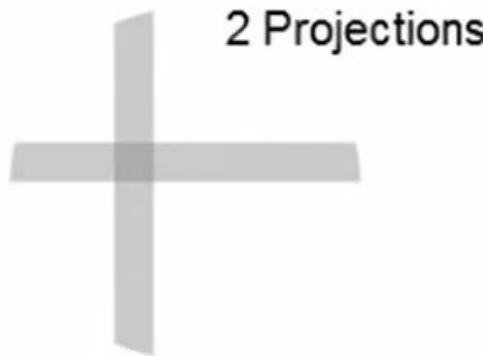


4 Projections

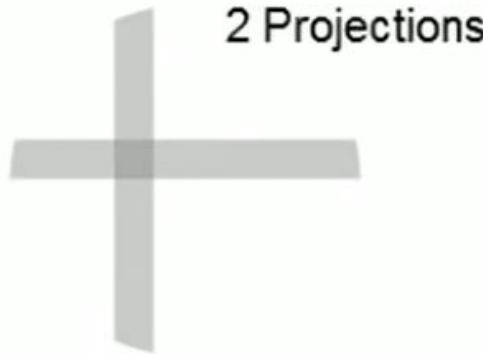
# Backprojection Reconstruction



# Backprojection Reconstruction



# Backprojection Reconstruction



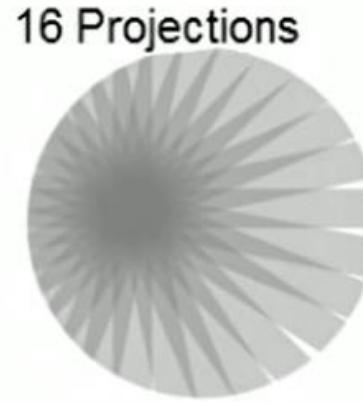
2 Projections



4 Projections



8 Projections



16 Projections

Does not preserve the shape

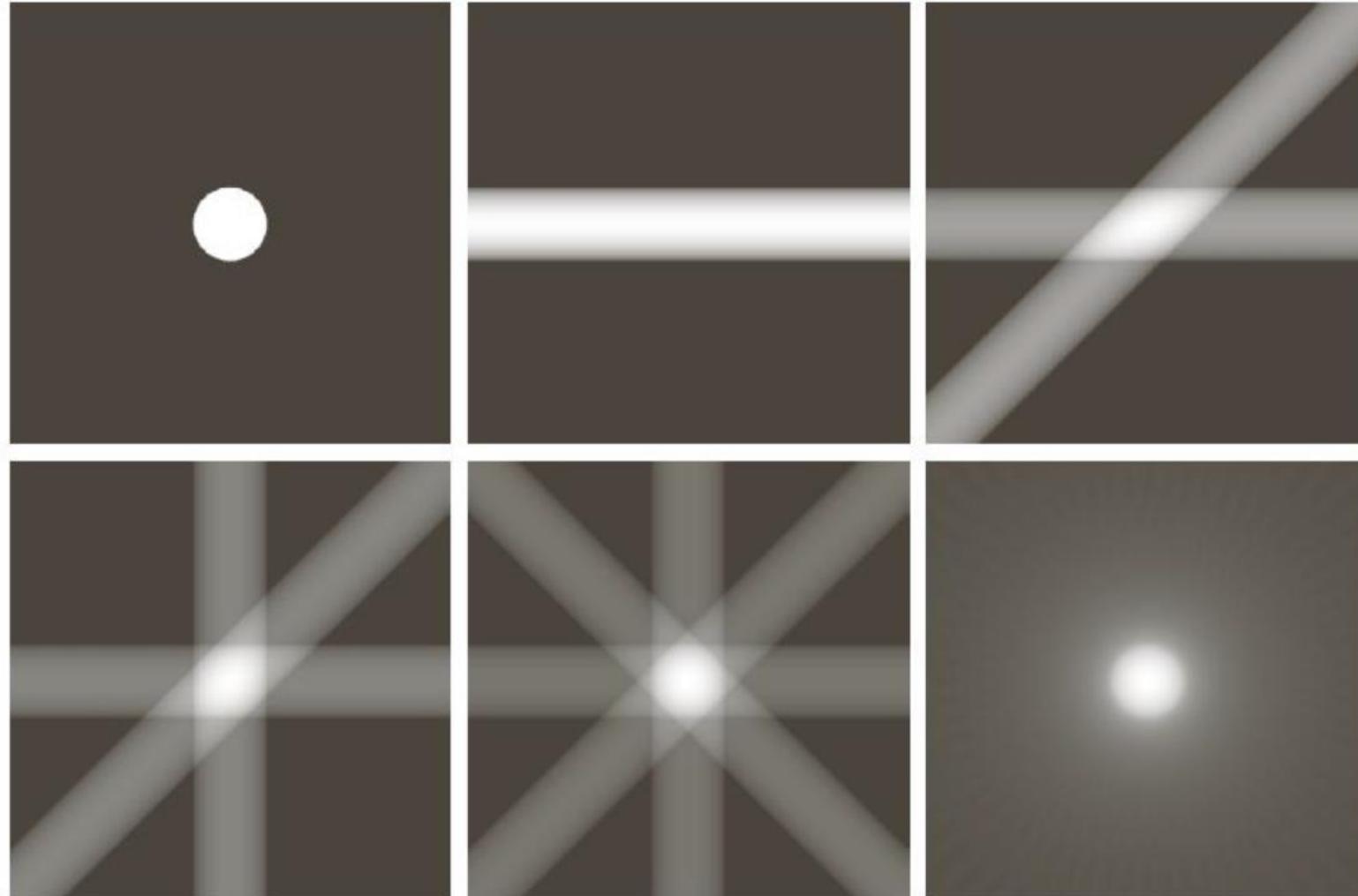
Blurring effect due to under-represented high frequencies

# Backprojection Reconstruction

a	b	c
d	e	f

**FIGURE 5.33**

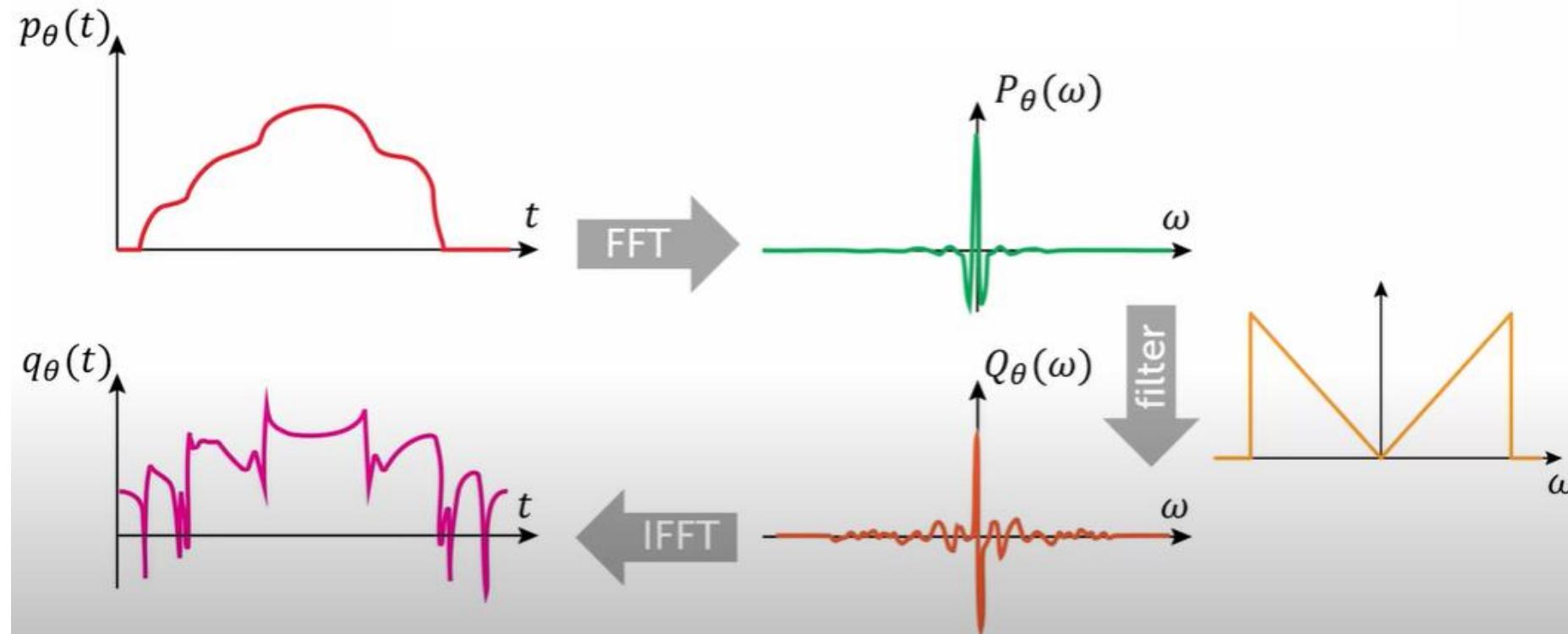
(a) Same as Fig. 5.32(a).  
(b)–(e) Reconstruction using 1, 2, 3, and 4 backprojections  $45^\circ$  apart.  
(f) Reconstruction with 32 backprojections  $5.625^\circ$  apart (note the blurring).



# Filtered Backprojection

- Filter the low frequencies
  - High pass filter
- Detected signal -> FFT -> HPF -> IFFT -> Reconstruction

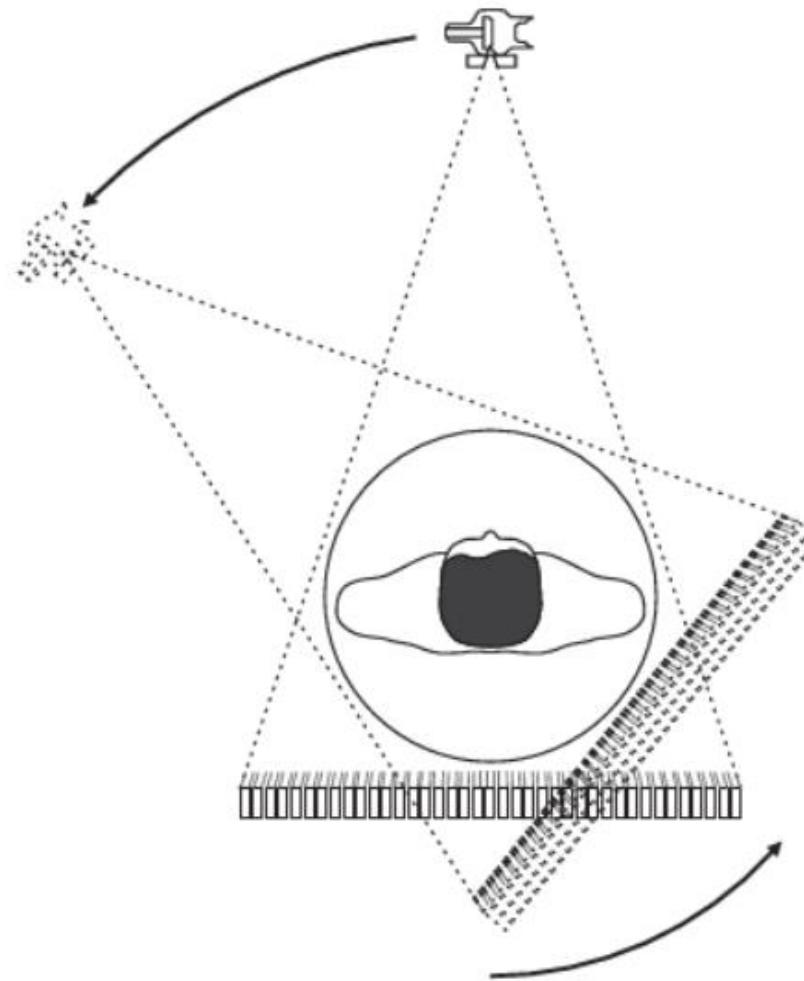
# Filtered Backprojection



# CT Scanner



# CT Scanner

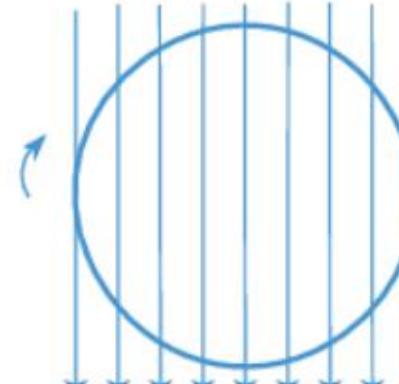


**Figure 6.4**  
The geometry of a  
third-generation (3G)  
CT scanner.

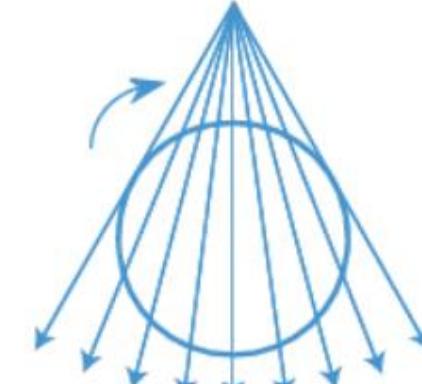
# CT Slices



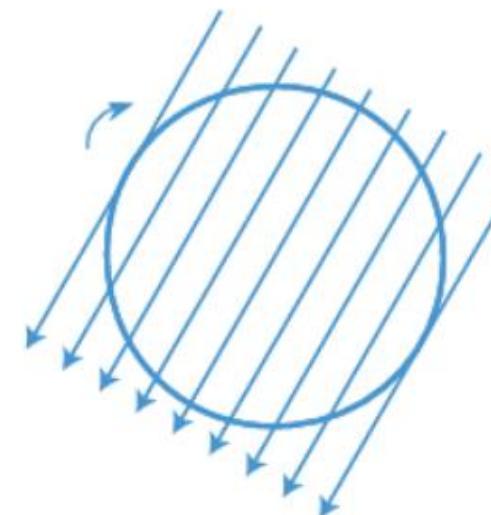
## Beam Type



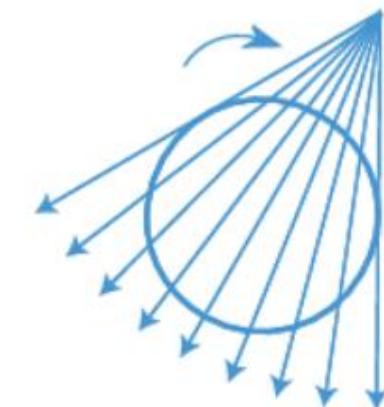
(a)



(b)



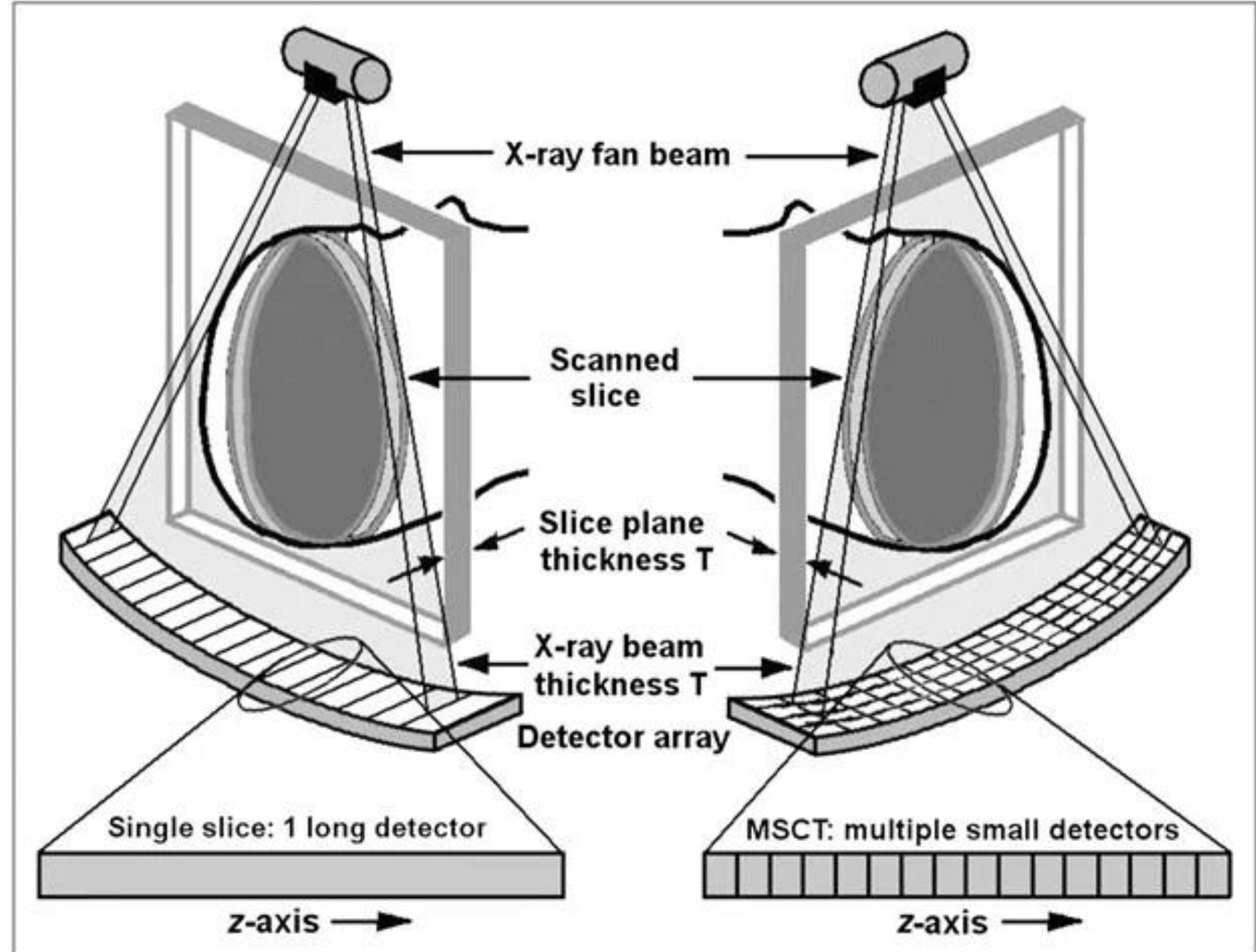
(c)



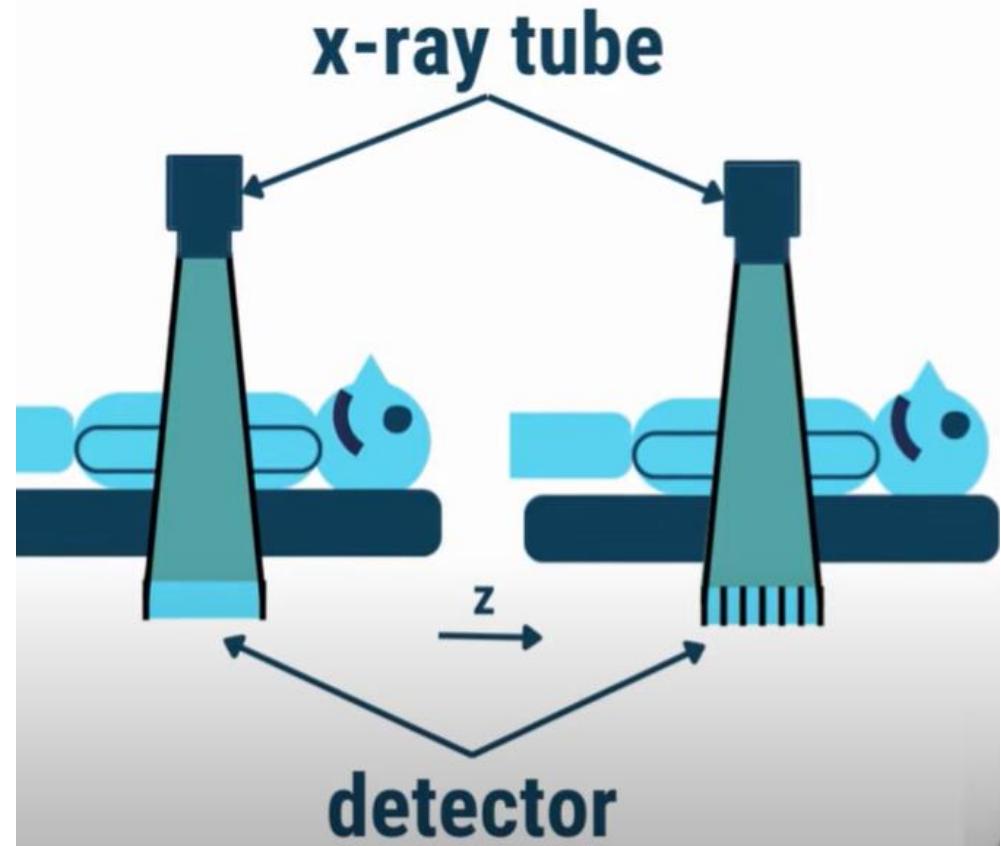
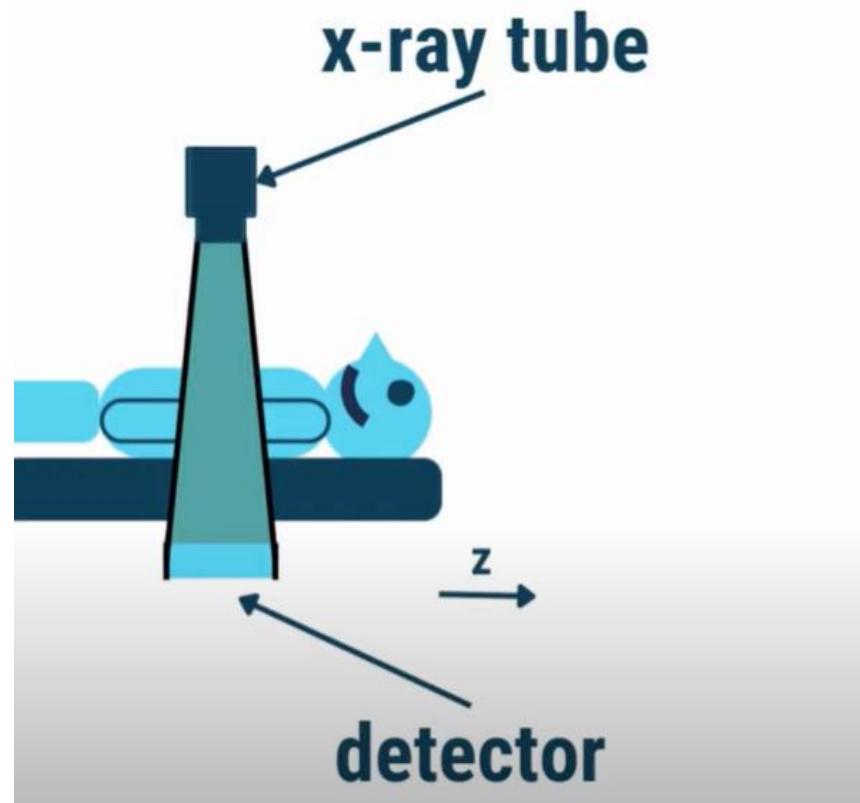
(d)

**Figure 3.2** Basic scanning procedure in CT. A set of lines is scanned covering the entire field of view: (a) parallel-beam geometry and (b) fan-beam geometry. This process is repeated for a large number of angles (c and d).

# CT Slices



# Single Slice vs Multi Slice CT



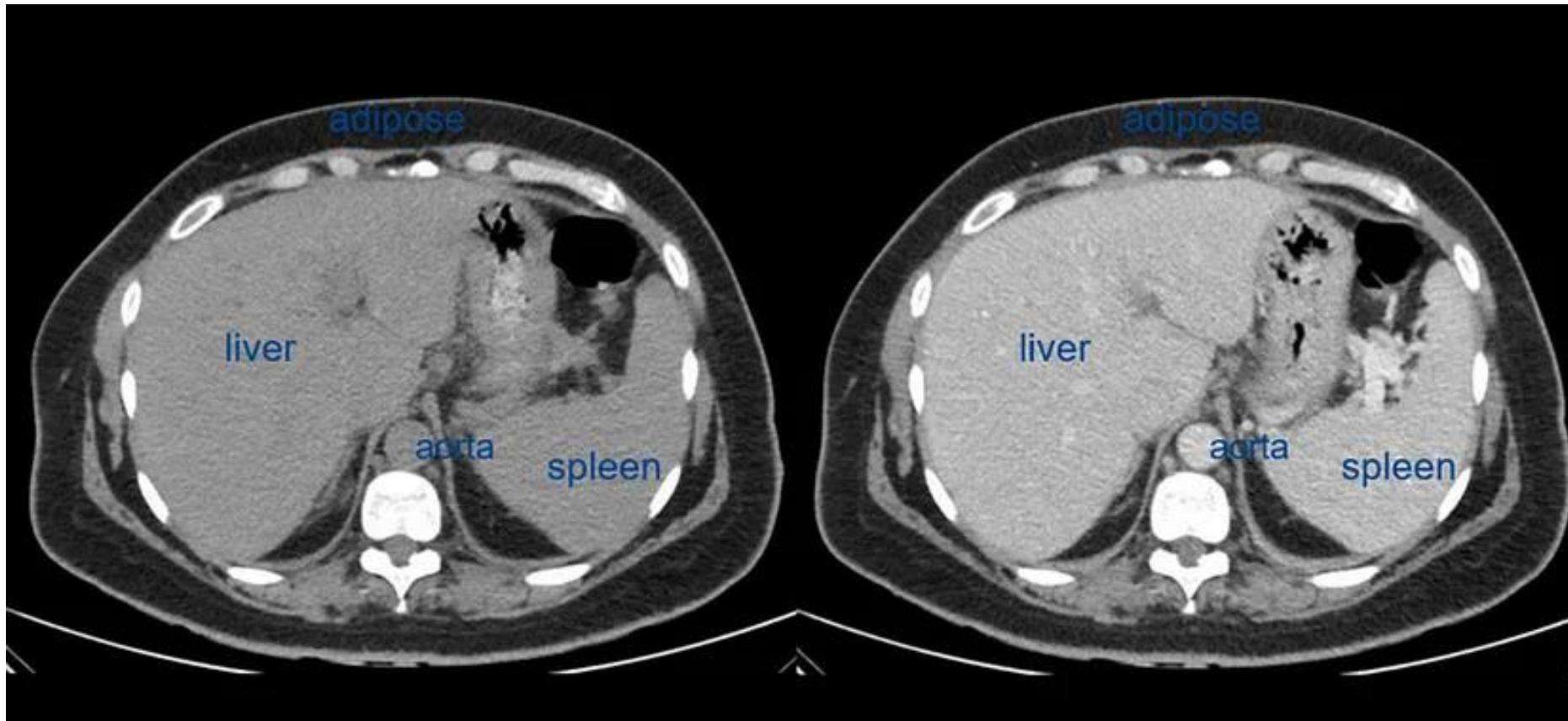
# CT Slices



# Generations of CT

<https://howradiologyworks.com/ctgenerations/>

# Contrast Enhanced CT



Contrast agent: Iodine or Barium (heavy metal)

# Use of CT

- A CT scan is a diagnostic imaging: images of the inside of the body
- Detailed images of any part of the body: bones, muscles, organs and blood vessels
- CT scans can also be used for fluid or tissue biopsies, or as part of preparation for surgery or treatment
- CT scans are frequently done with and without contrast agent to improve the radiologist's ability to find any abnormalities

# Magnetic Resonance Imaging

- Based on the principle of nuclear magnetic resonance (NMR)
  - Discovered in 1945
  - First NMR image: Paul C. Lauterbur (1973)
  - Mathematical theory of fast scanning and reconstruction: Peter Mansfield (1974)
- Non-ionizing
- Suitable for soft tissue and functional imaging

# Spin of Elementary Particles

- An intrinsic property
- Actually does not spin around their axes
  - But shows properties which is equivalent to that of spinning charge
- Elementary particles have spin
  - Quarks, electrons
- Due to the spin property, elementary particles have spin angular momentum

# Spin of Elementary Particles

- Spin of the nucleus
  - Vector sum of spins of constituent proton and neutron
- Inside nucleus
  - Proton-proton pair cancels each other's spin
  - Neutron-neutron pair cancels each other's spin
- If there is any unpaired proton or neutron
  - Contributes to the spin of the nucleus
    - Vector sum of spins of constituent unpaired proton and neutron

# Spin of Nucleus

- If there is any unpaired proton or neutron
  - Nucleus will have some net spin and net angular momentum
- Even proton, even neutron
  - Spin=0, spin angular momentum=0
- Odd proton, even neutron (or opposite)
  - Spin=non zero, spin angular momentum: non-zero
  - Due to the unpaired proton or neutron (whichever is odd in number)
- Odd proton, odd neutron (or opposite)
  - Spin=non zero, spin angular momentum: non-zero
  - Due to the unpaired protons and neutrons

# Spin of Nucleus

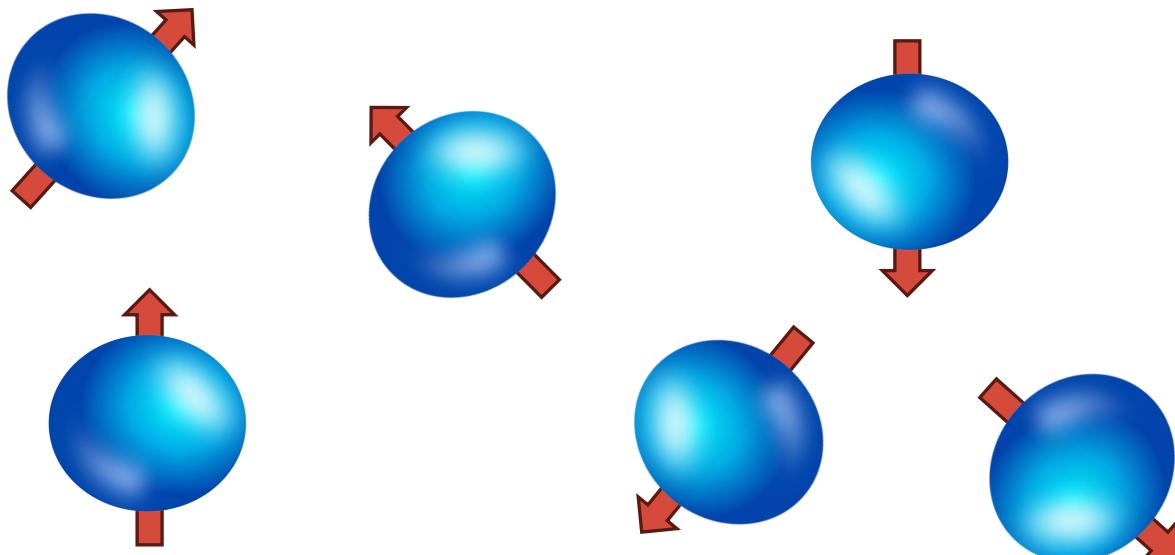
Nucleus	Spin	$\frac{\gamma}{2\pi}$ (MHz/T)
$^1_1H$	$\frac{1}{2}$	42.57
$^2_1H$	1	6.54
$^{12}_6C$	0	
$^{13}_6C$	$\frac{1}{2}$	10.71
$^{14}_7N$	1	3.08
$^{15}_7N$	$\frac{1}{2}$	-4.31
$^{16}_8O$	0	
$^{17}_8O$	$\frac{5}{2}$	-5.77
$^{31}_{15}P$	$\frac{1}{2}$	17.23
$^{33}_{16}S$	$\frac{3}{2}$	3.27
$^{43}_{21}Ca$	$\frac{7}{2}$	-2.86

# Effect of External Magnetic Field



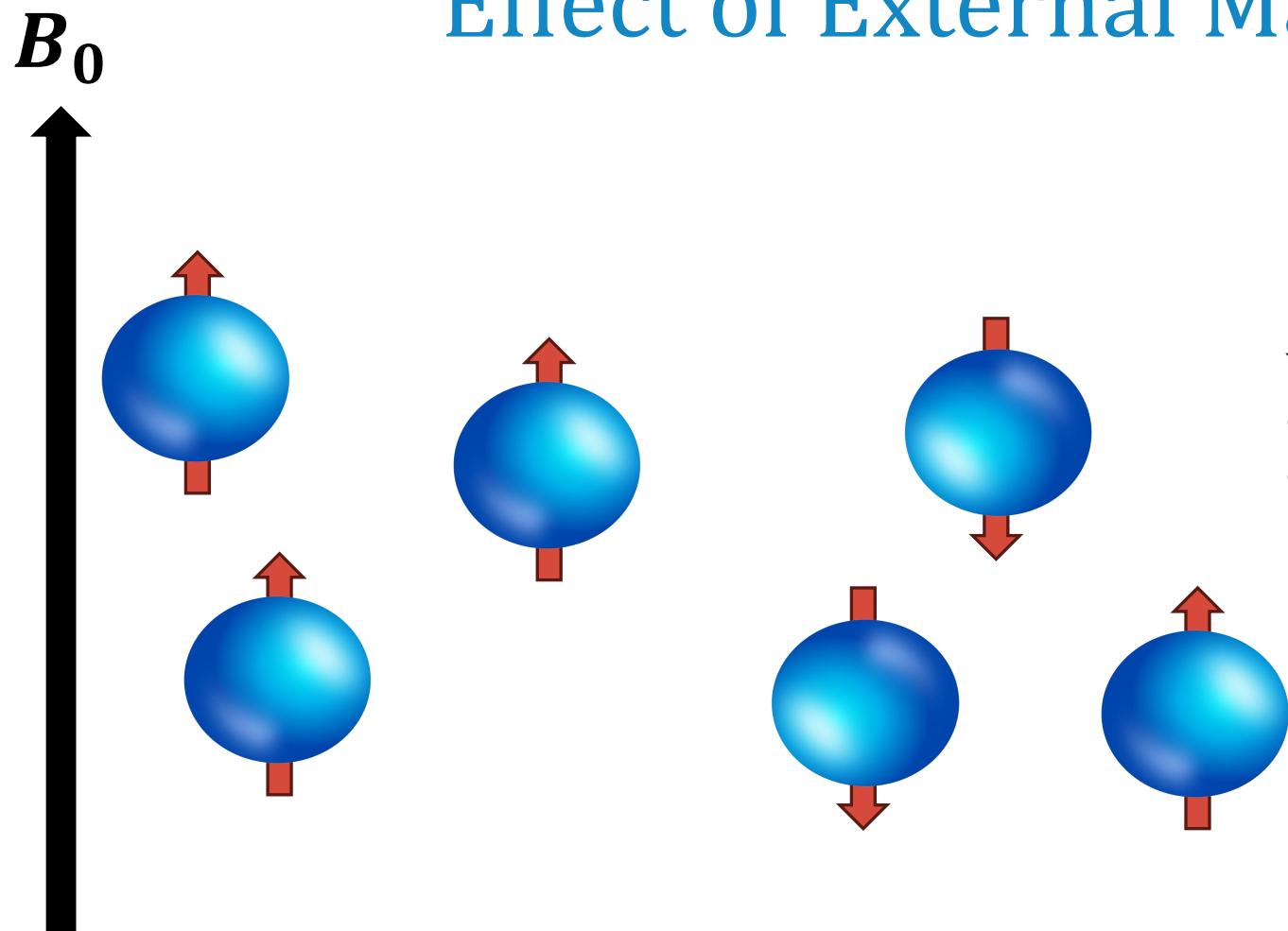
A nucleus may act like a magnet

# Effect of External Magnetic Field



Usually, they are randomly oriented, cancelling each other's spins

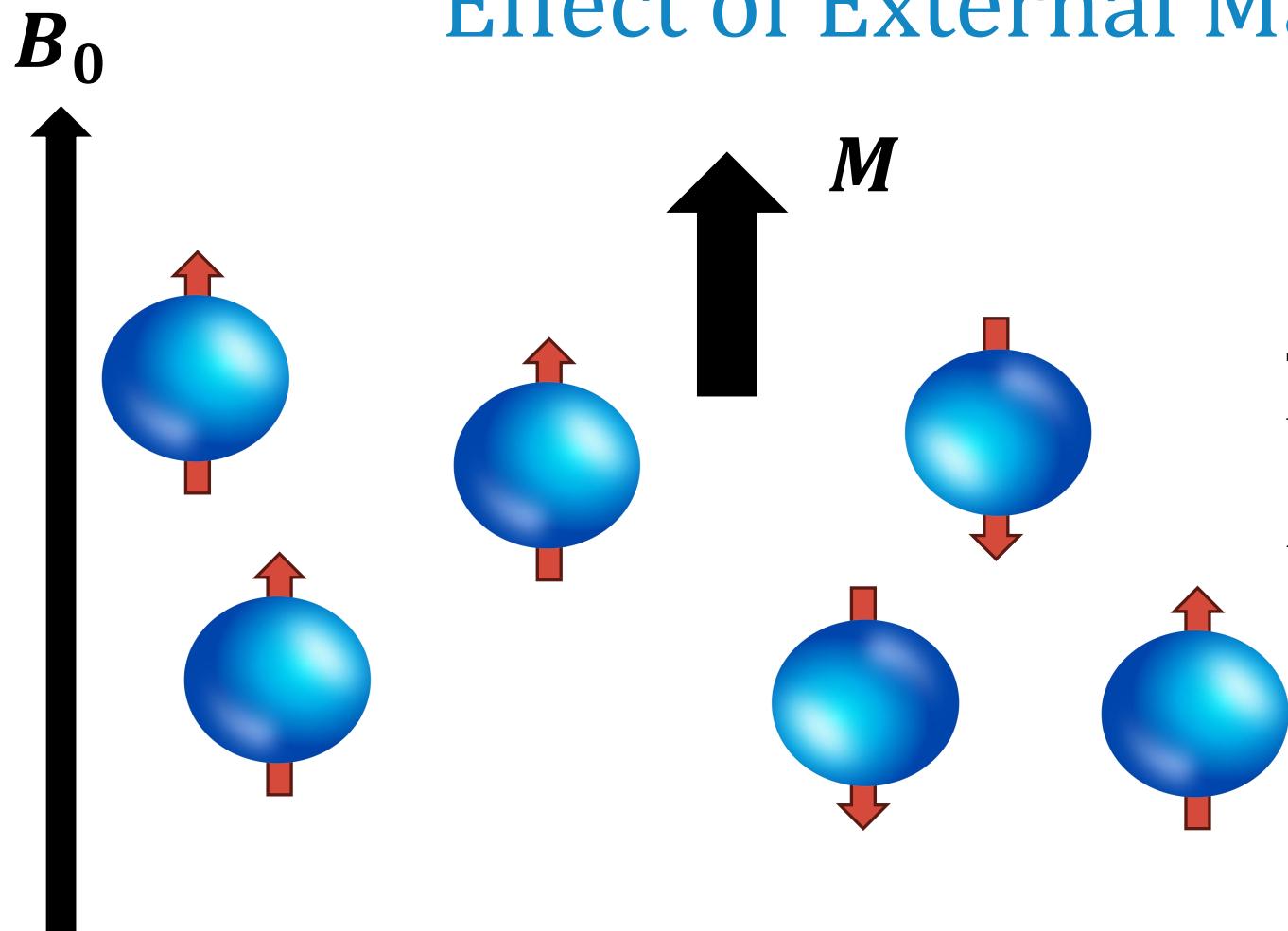
# Effect of External Magnetic Field



When an external magnetic field is applied

- Most nuclei align towards the field (up)
- Some (with higher energy) aligns opposite to the field

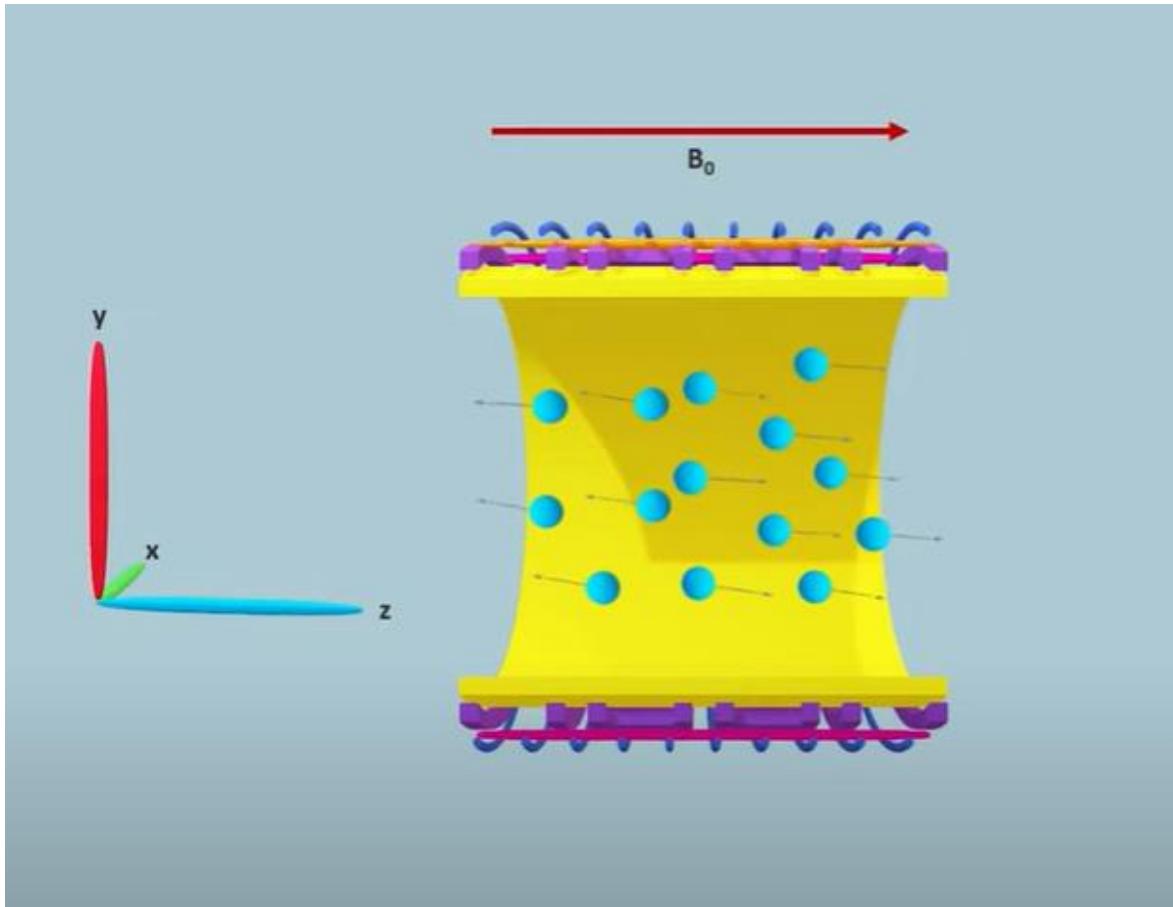
# Effect of External Magnetic Field



There will be a net magnetization vector  $M$  towards  $B_0$

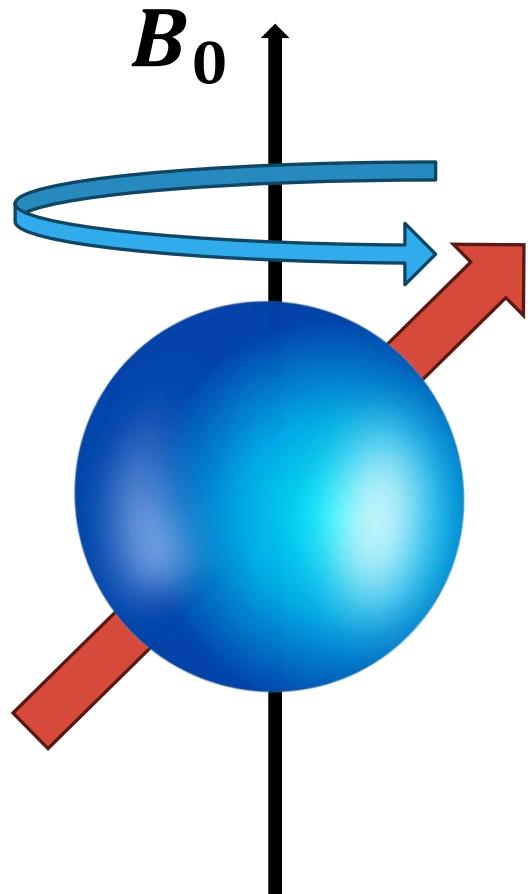
$M$  depends on number of spins (in a voxel)

# Effect of External Magnetic Field



There will be a net magnetization vector  $M$  towards  $B_0$

# Effect of External Magnetic Field



The nuclei will spin wrt  $B_0$  axis at an angle  $\theta$

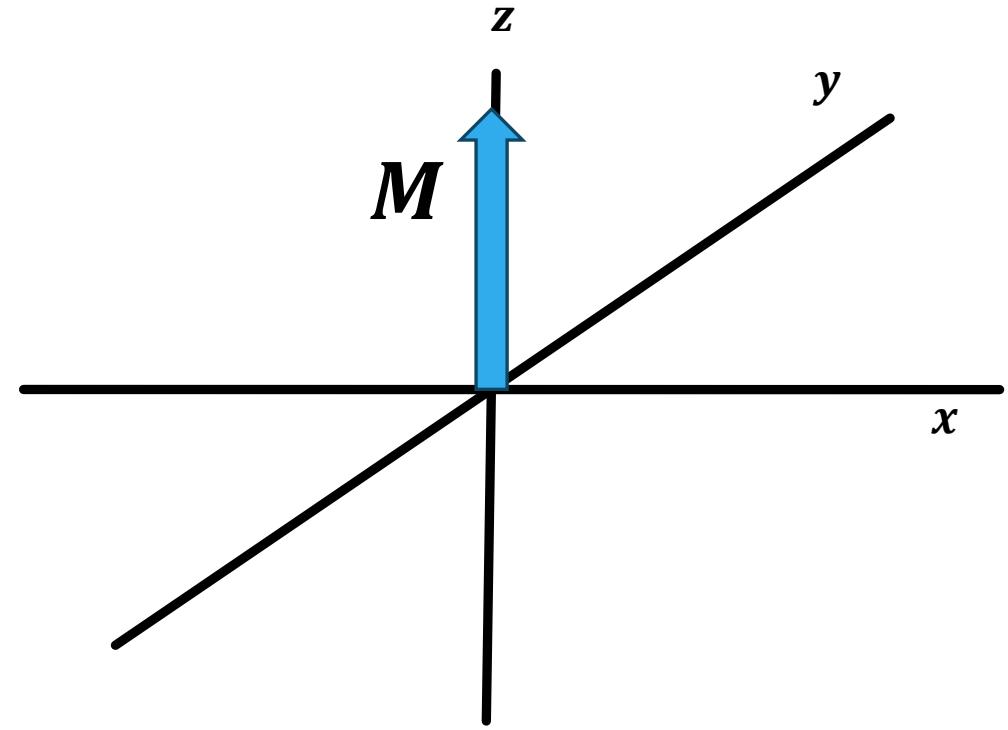
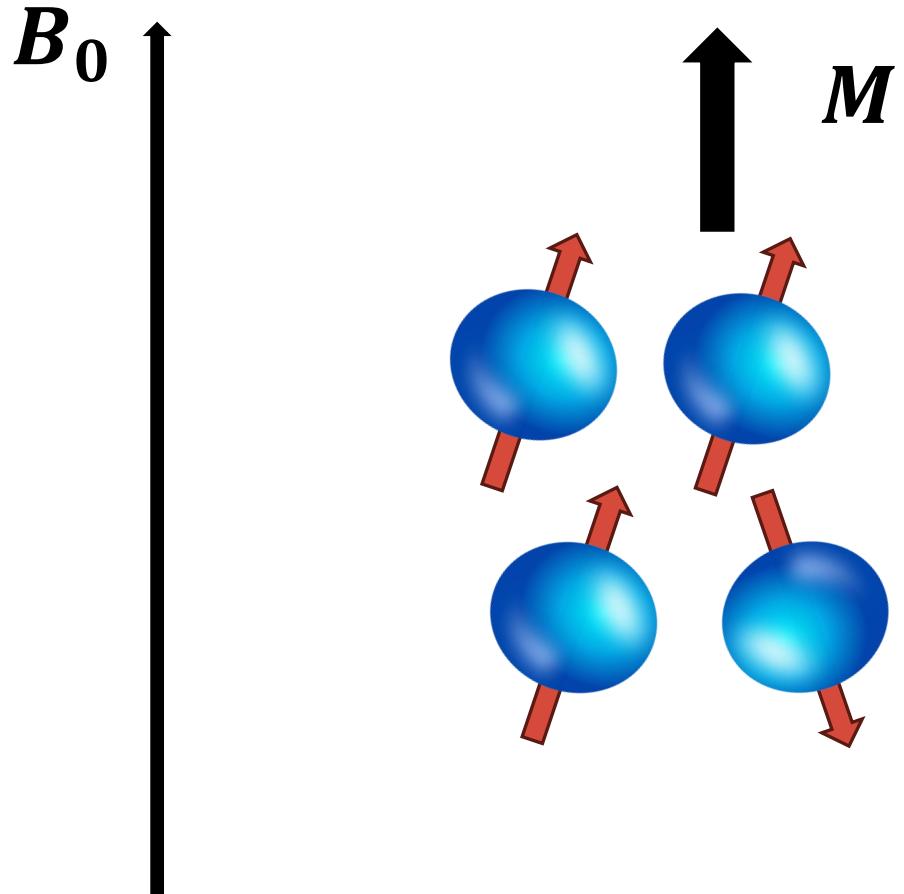
This is called precession

Larmor frequency -> precession rate

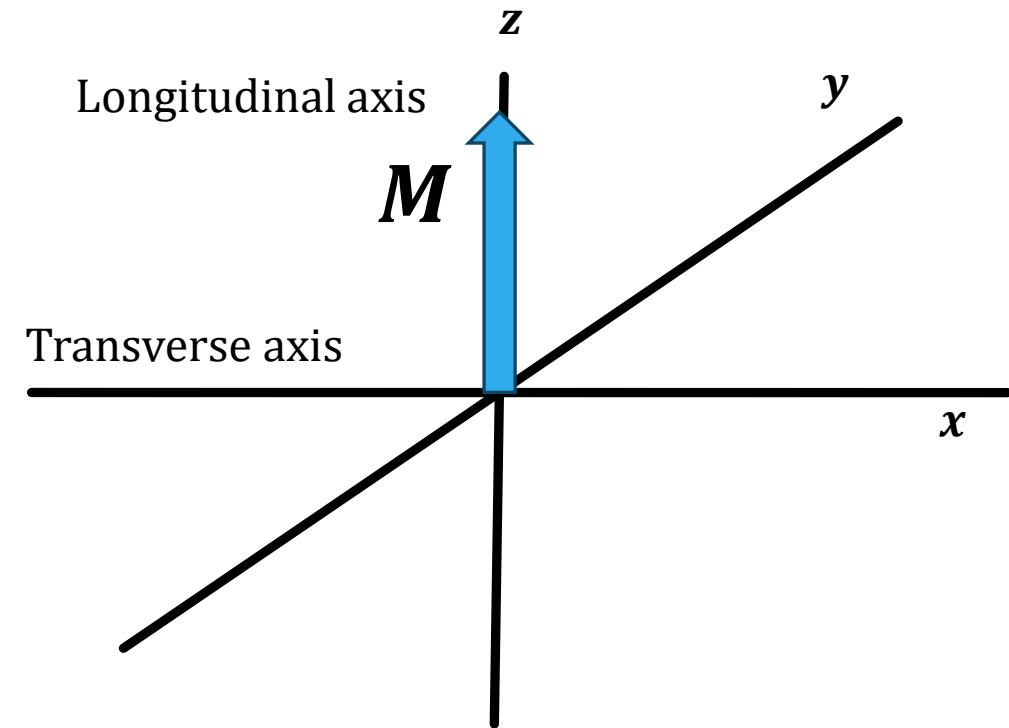
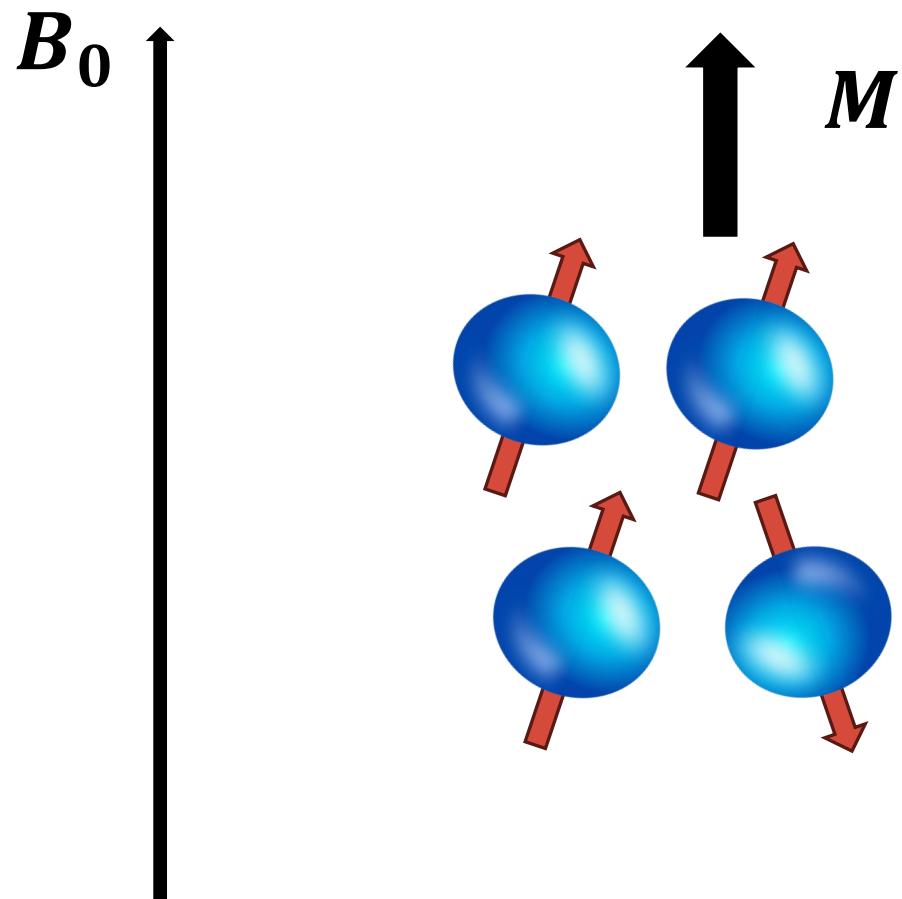
$$\omega_0 = \gamma B_0$$

$\gamma$ : Gyromagnetic ratio

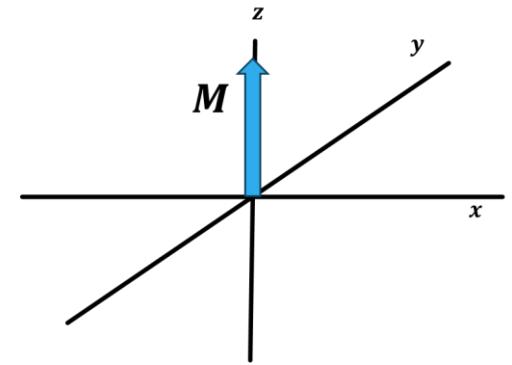
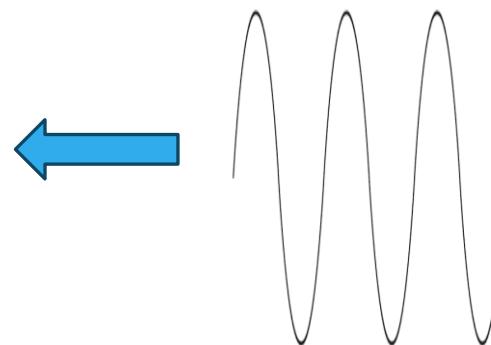
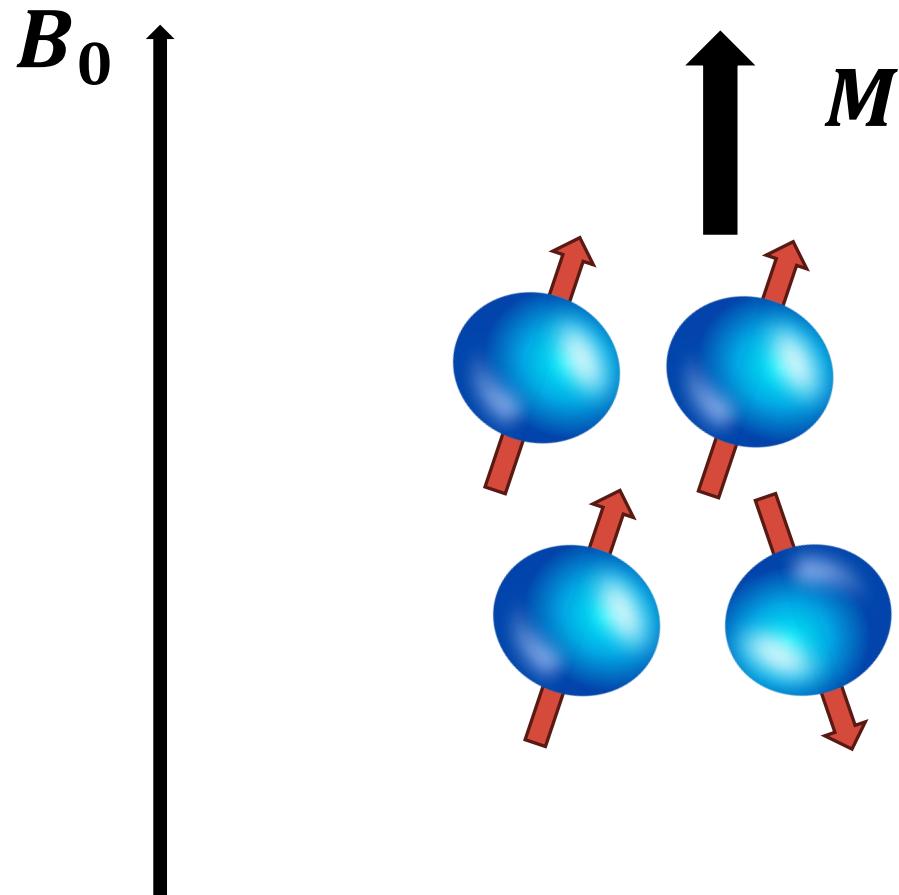
# The Initial Situation



# The Initial Situation

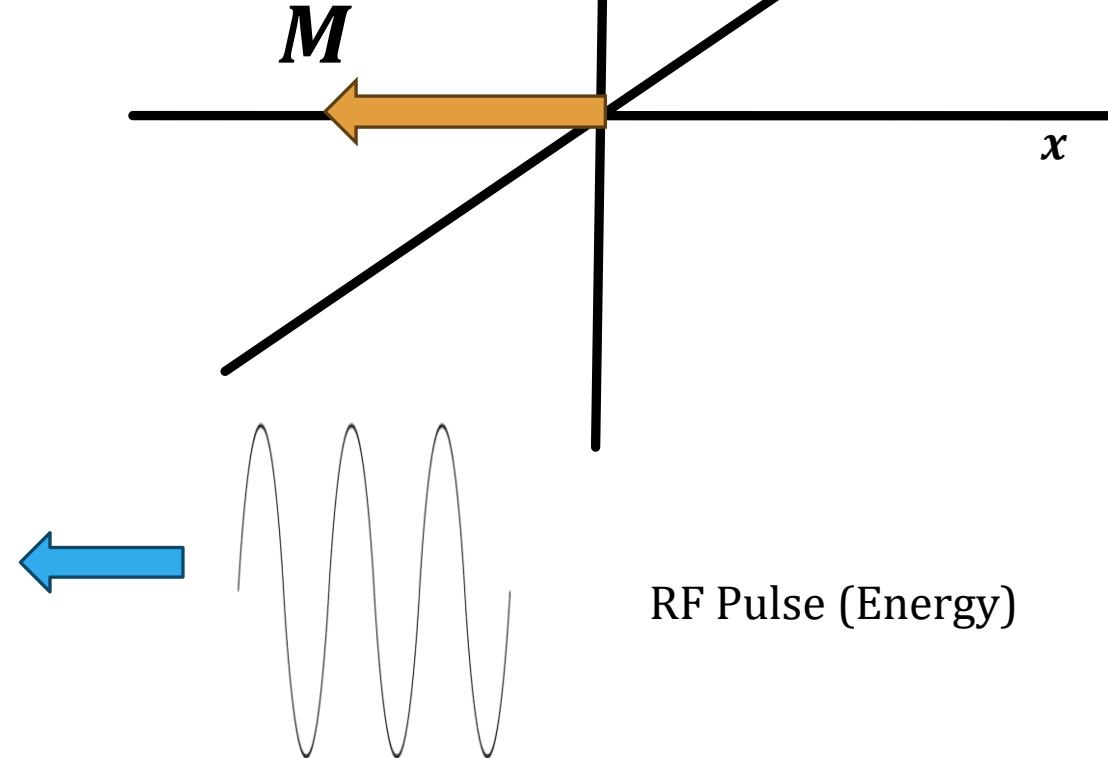
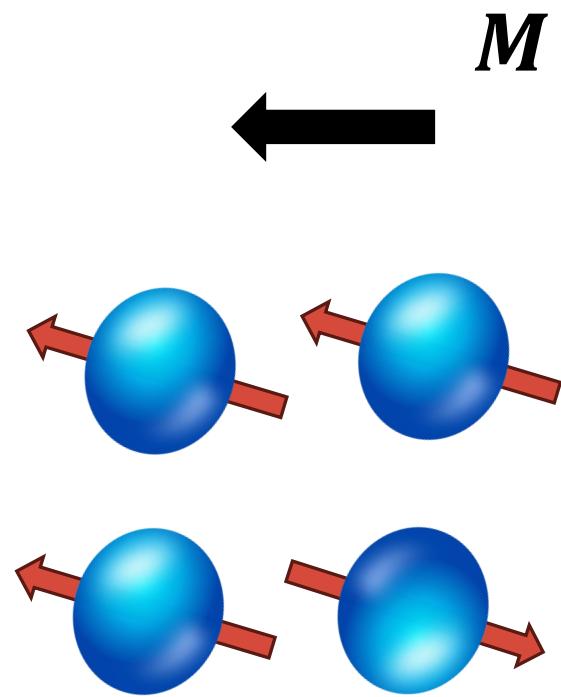


# Application of RF Pulse

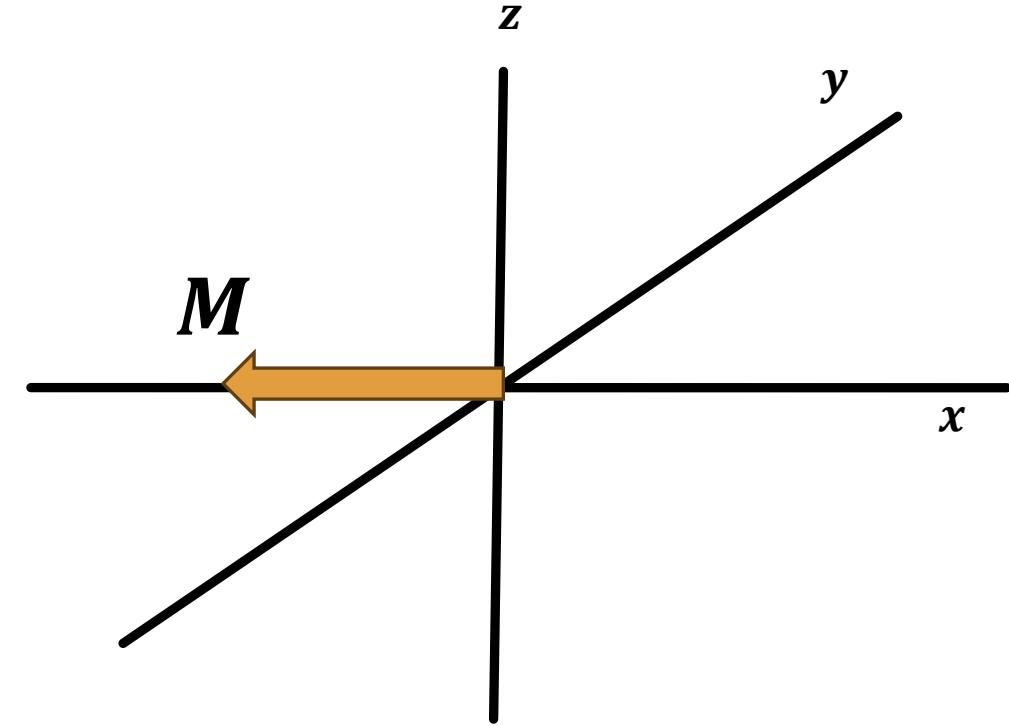
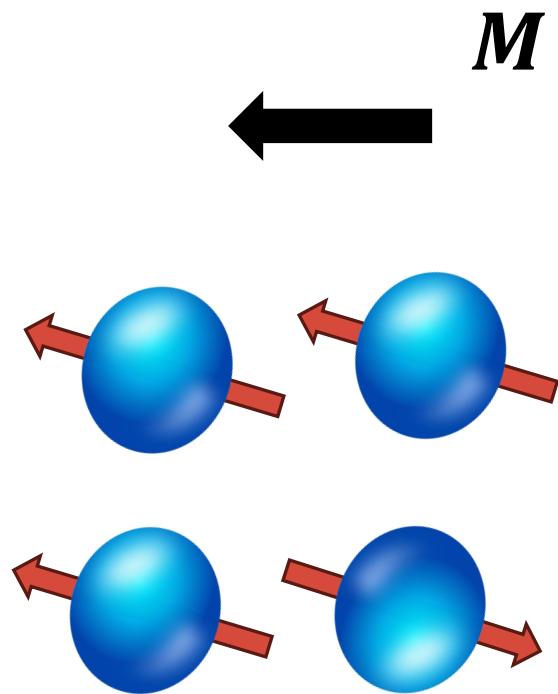


RF Pulse (Energy)

$B_0$

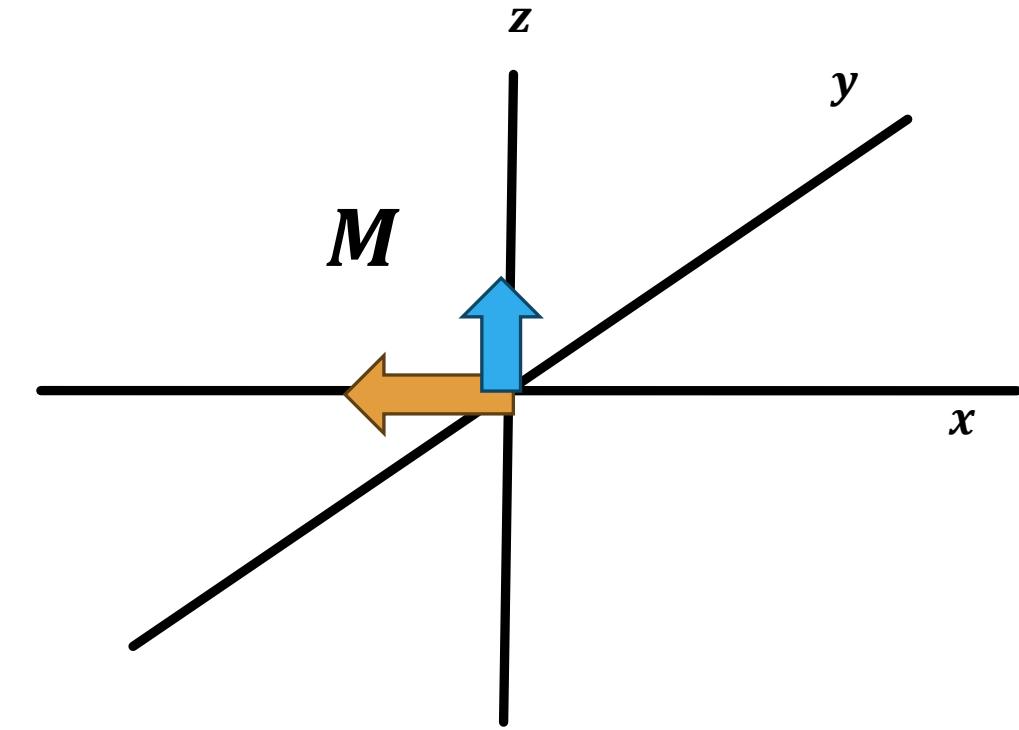
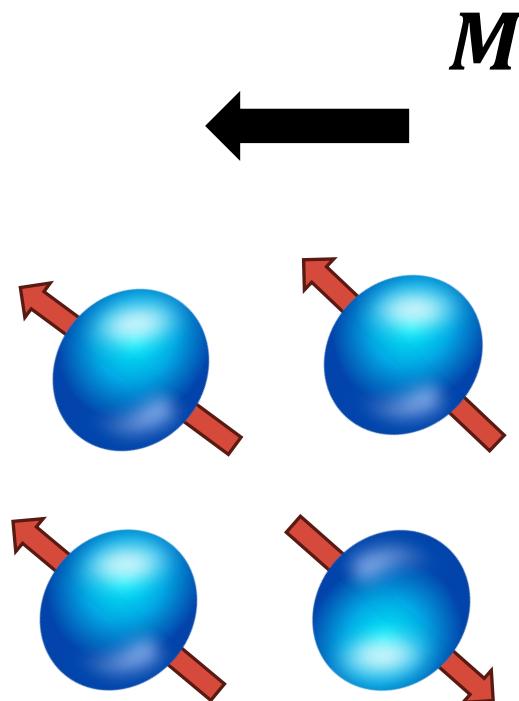


RF Pulse (Energy)

$B_0$ 

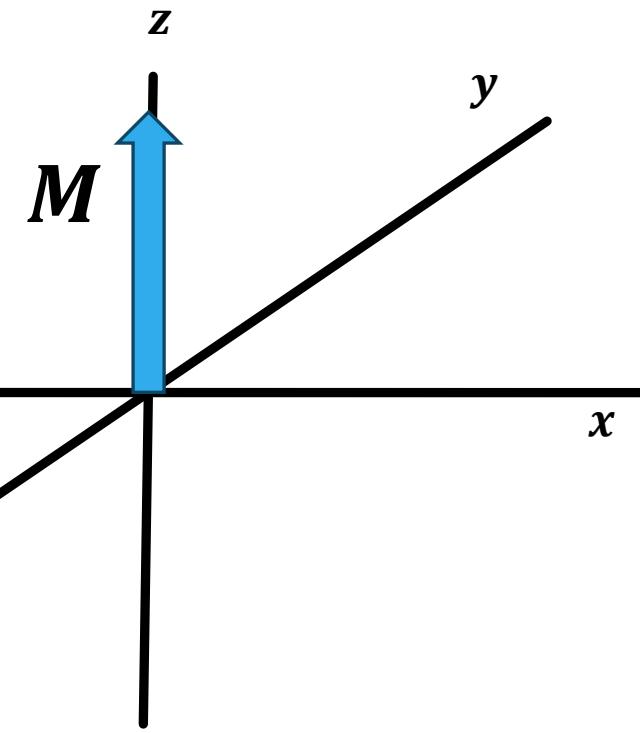
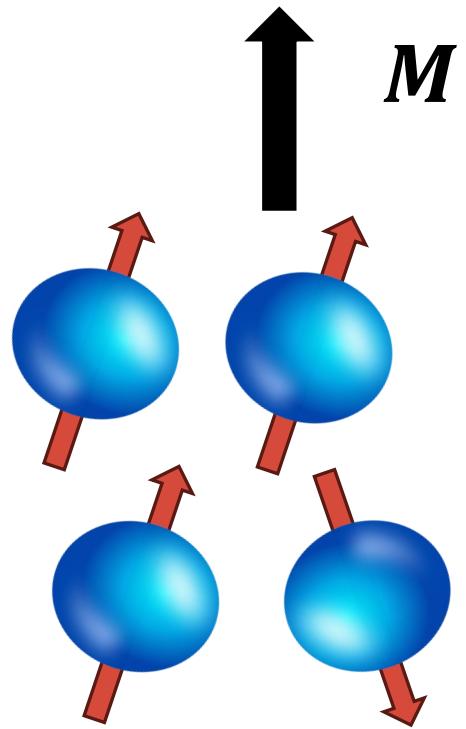
After the RF Pulse is removed,  
nuclei will start coming back to  
the original orientation

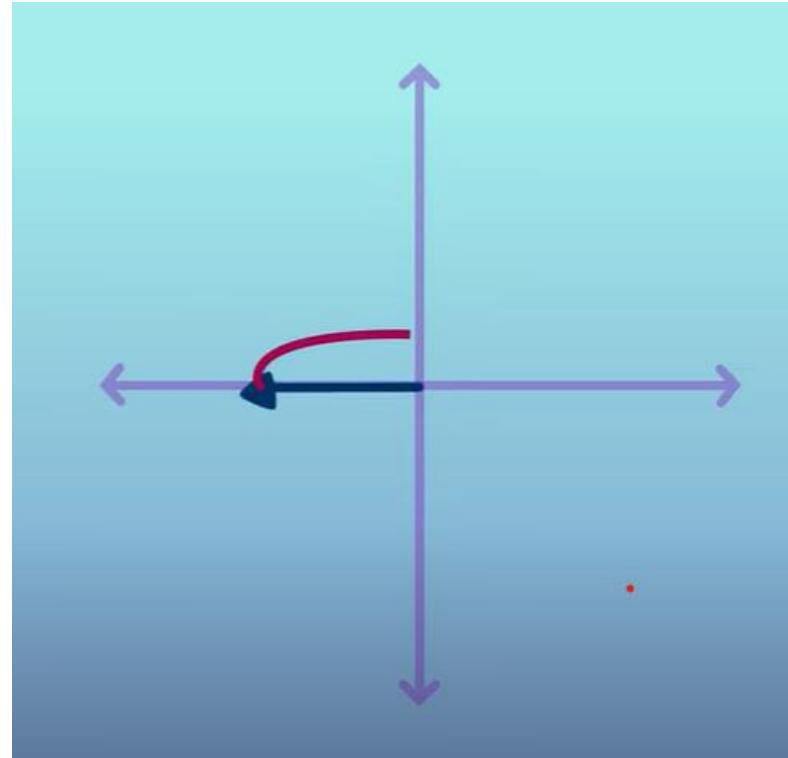
$B_0$

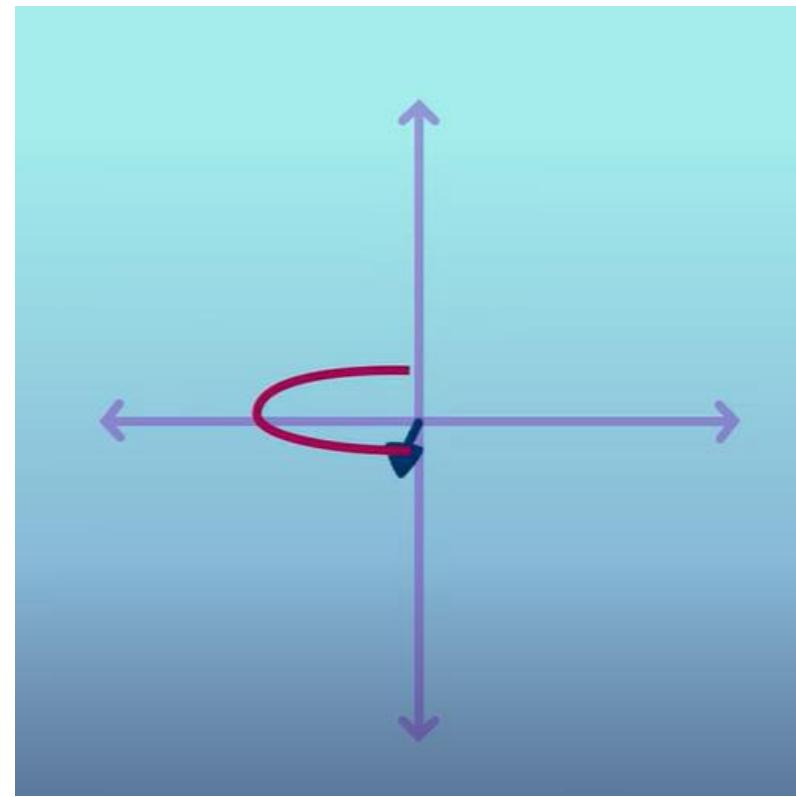


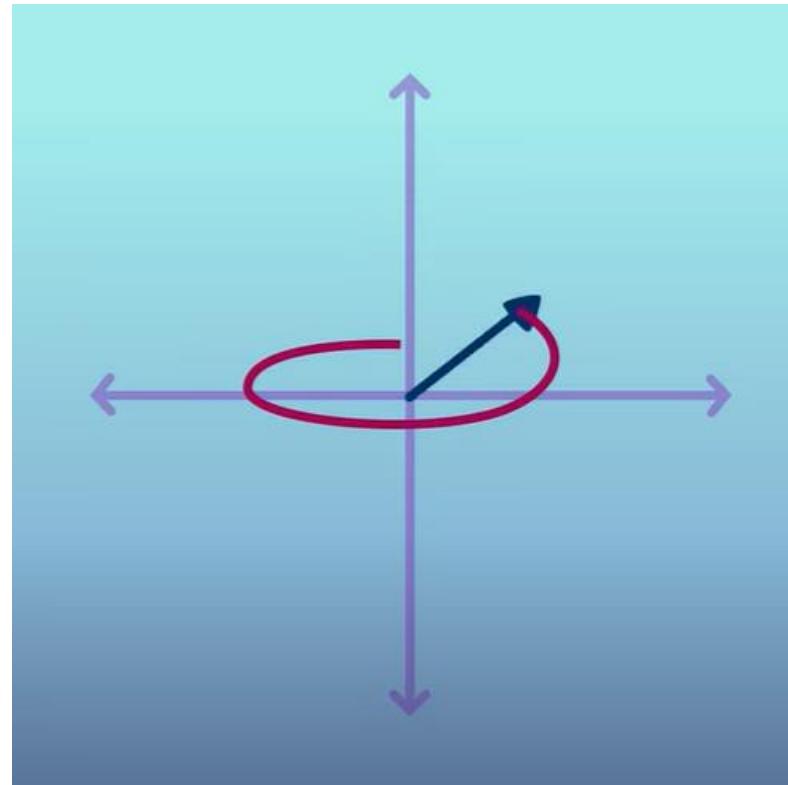
After the RF Pulse is removed,  
nuclei will start coming back to  
the original orientation

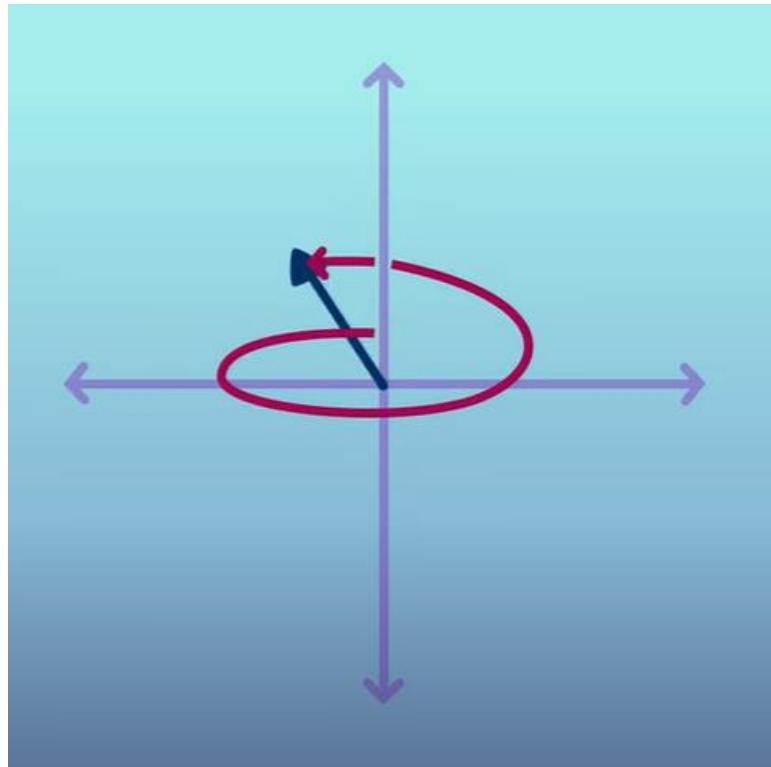
$B_0$

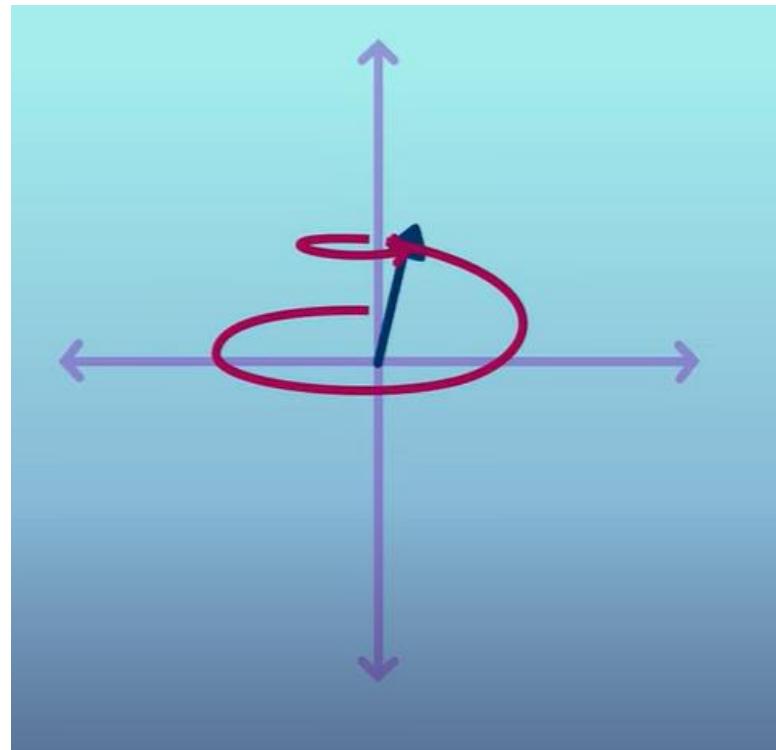


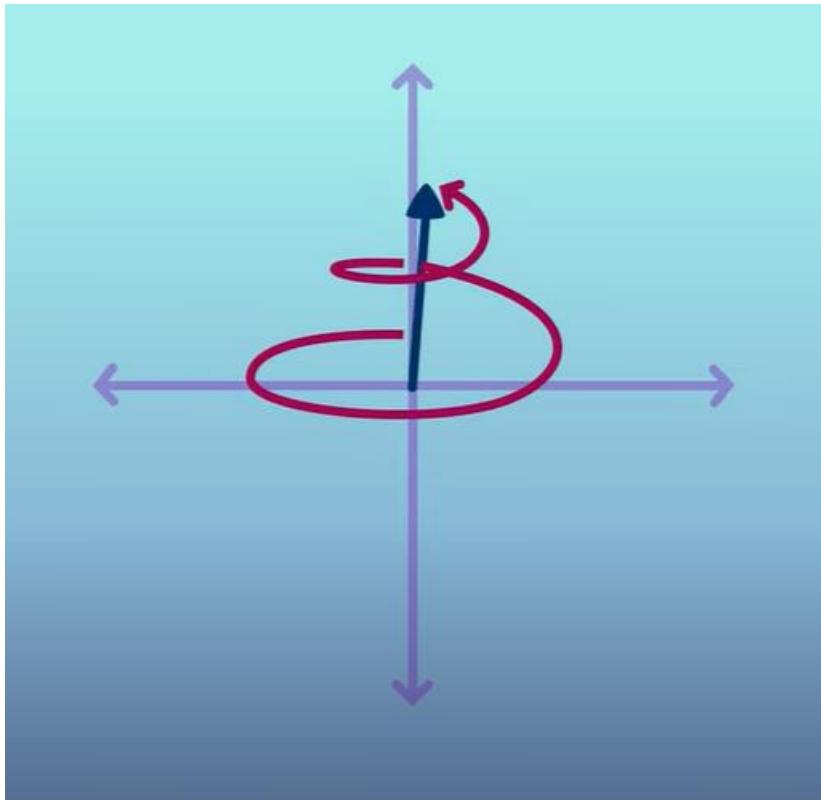


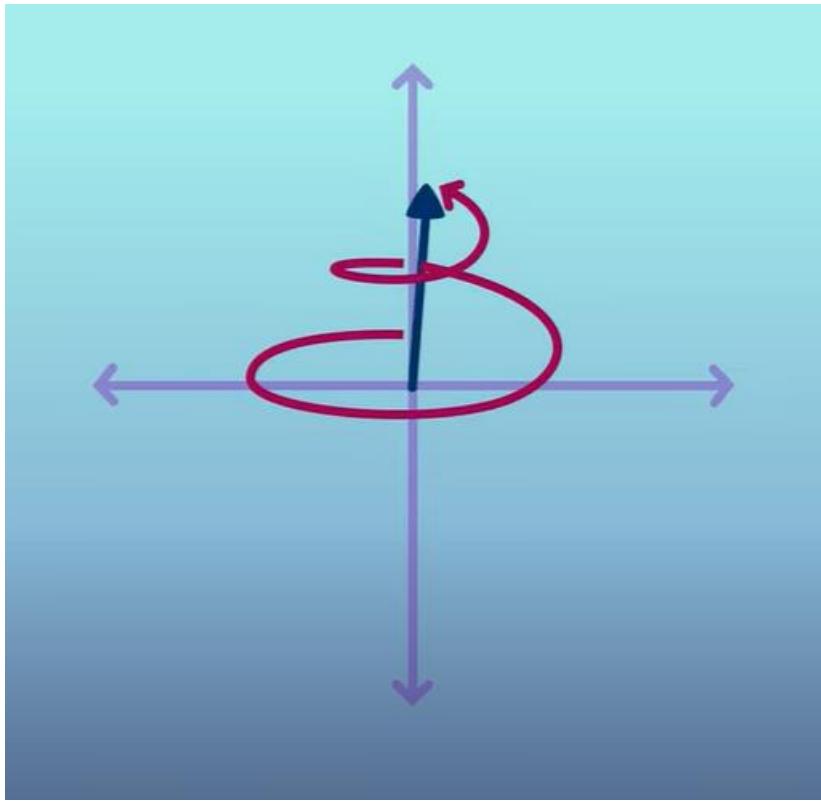






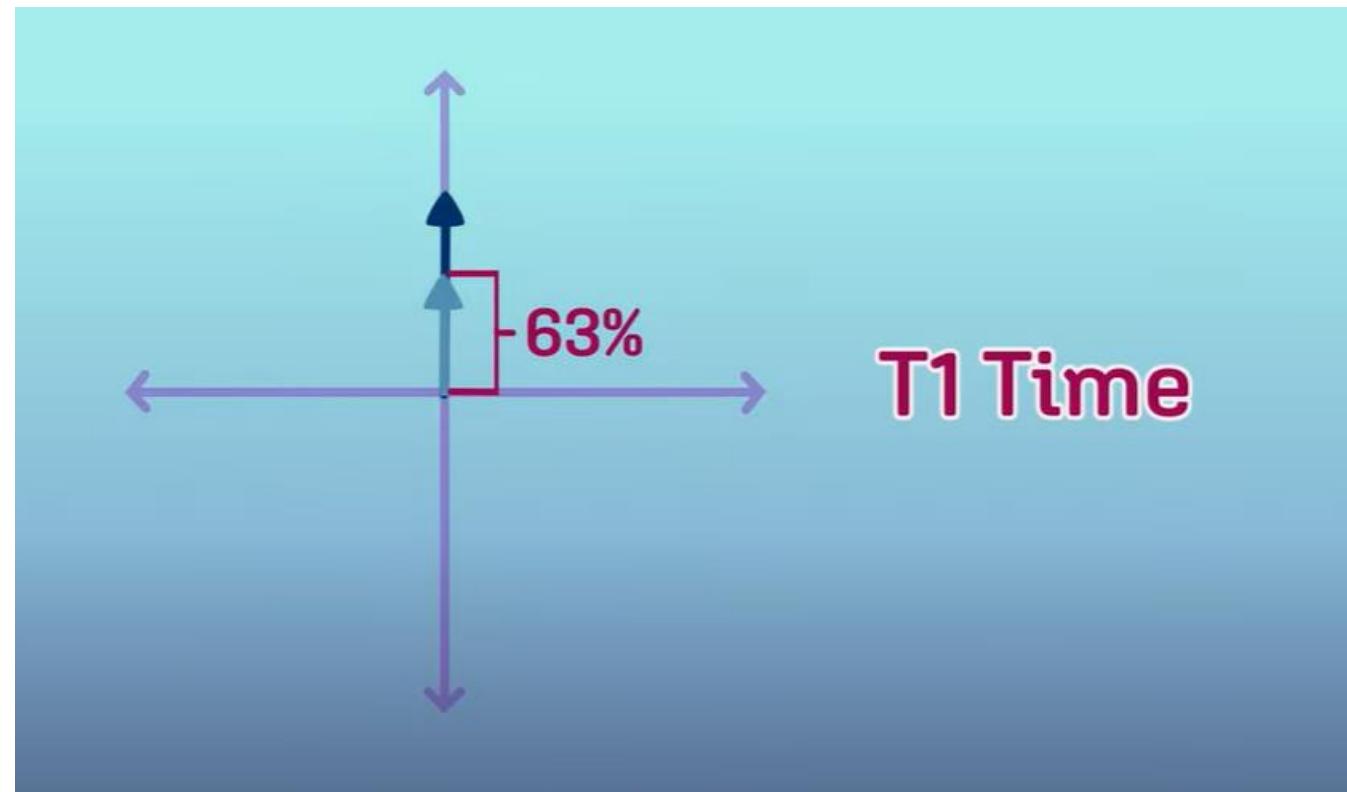




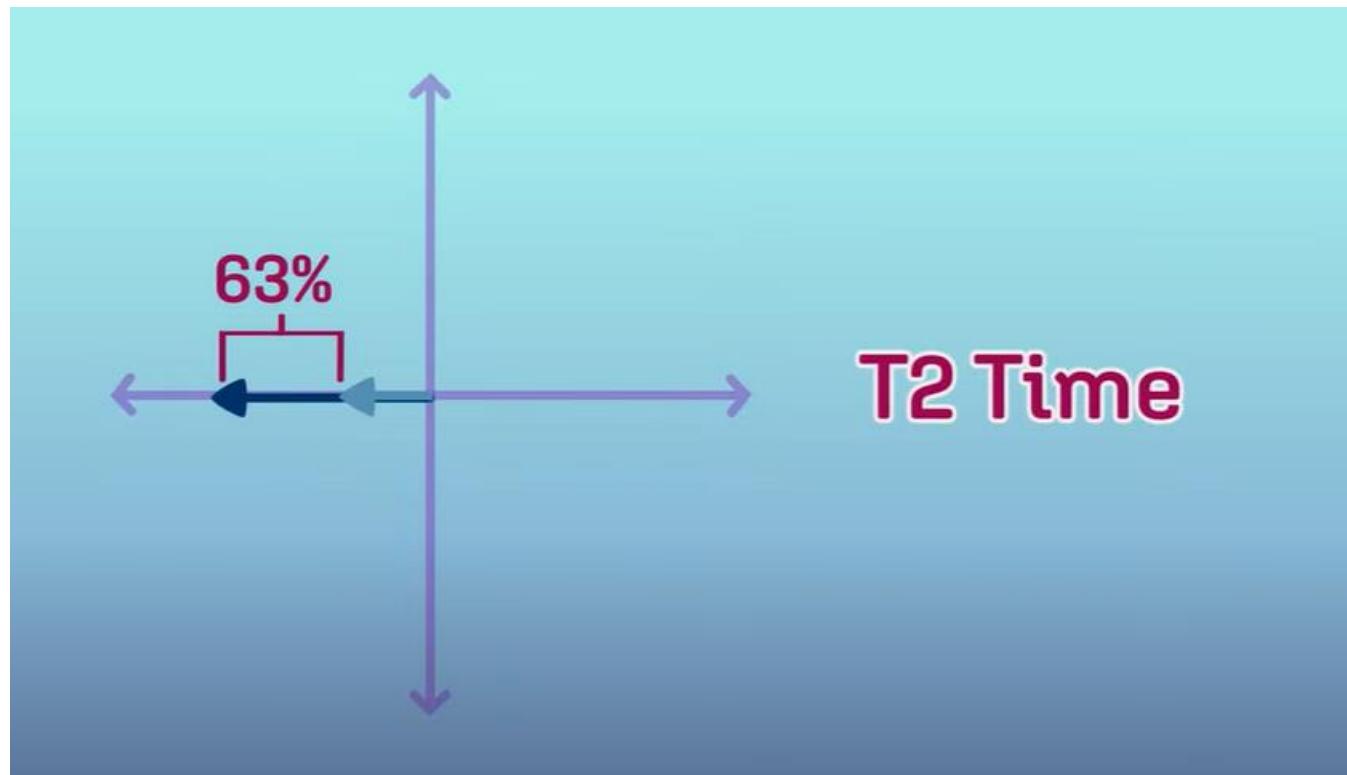


Spin relaxation

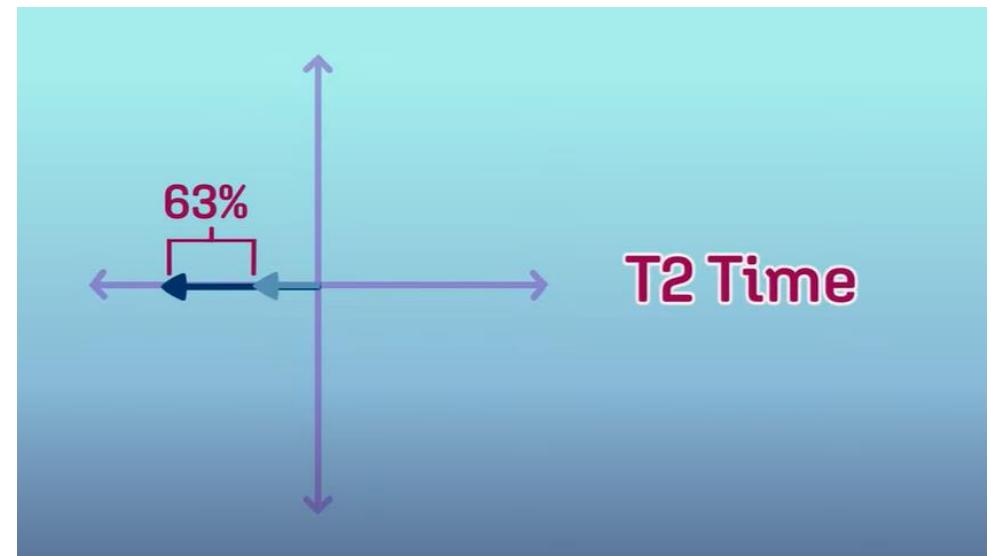
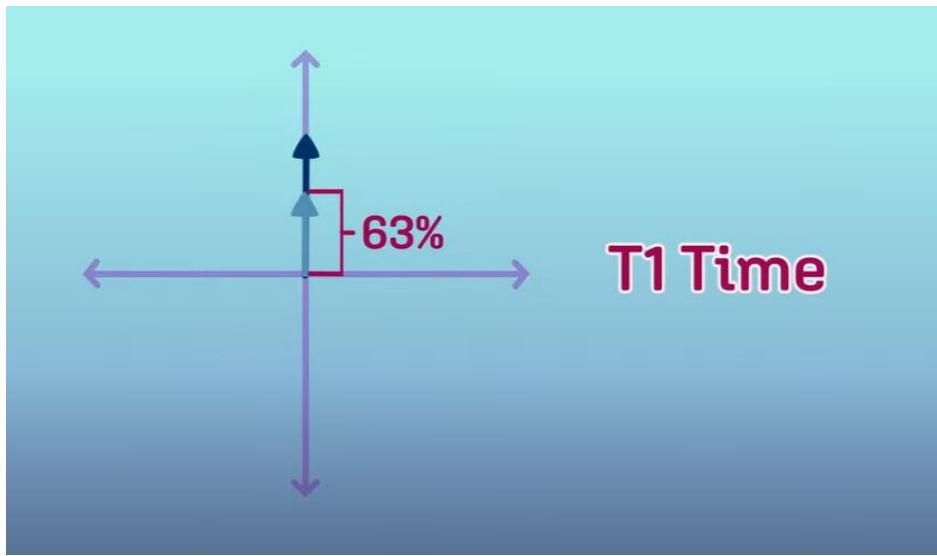
# T1 and T2 Time



# T1 and T2 Time



# T1 and T2 Time



Net magnetization vector and T1 and T2 times  
are unique for each tissue type

# Spin Relaxation Times

Tissue	T1 (msec)	T2 (msec)
Water/CSF	4000	2000
Gray matter	900	90
Muscle	900	50
Liver	500	40
Fat	250	70
Tendon	400	5
Proteins	250	0.1- 1.0
Ice	5000	0.001

Approximate values for 1.5T

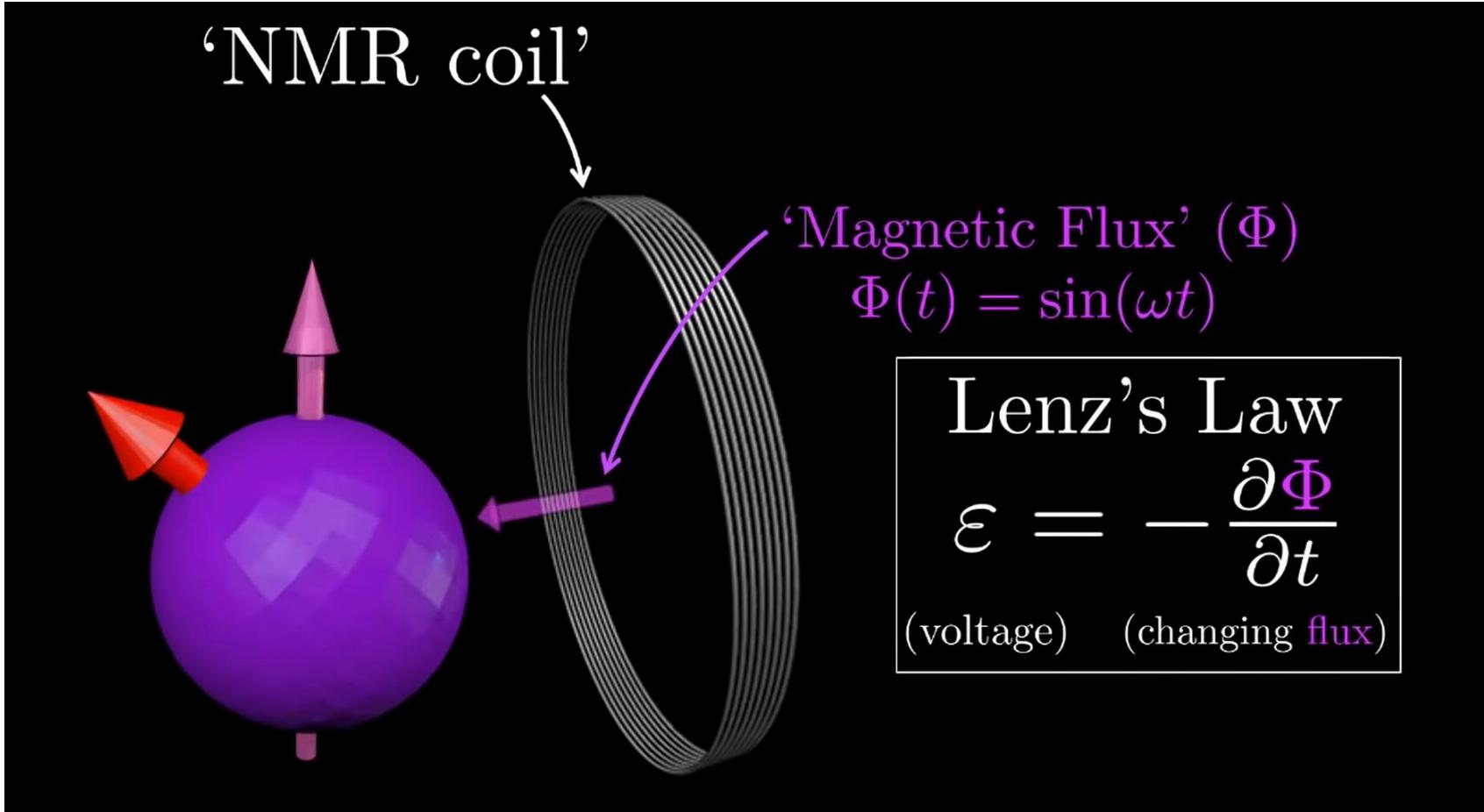
# Spin Relaxation Times

Tissue	T1 (msec)	T2 (msec)
Water/CSF	4000	2000
Gray matter	900	90
Muscle	900	50
Liver	500	40
Fat	250	70
Tendon	400	5
Proteins	250	0.1- 1.0
Ice	5000	0.001

The entire process  
is repeated several  
times

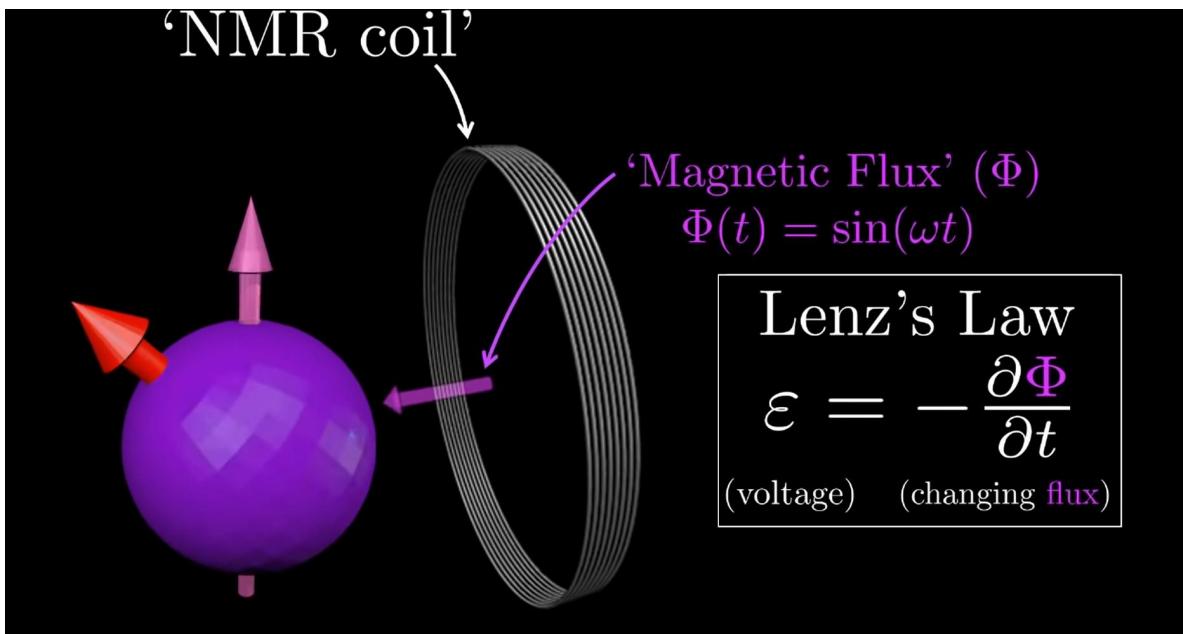
Approximate values for 1.5T

# Detection of Signal

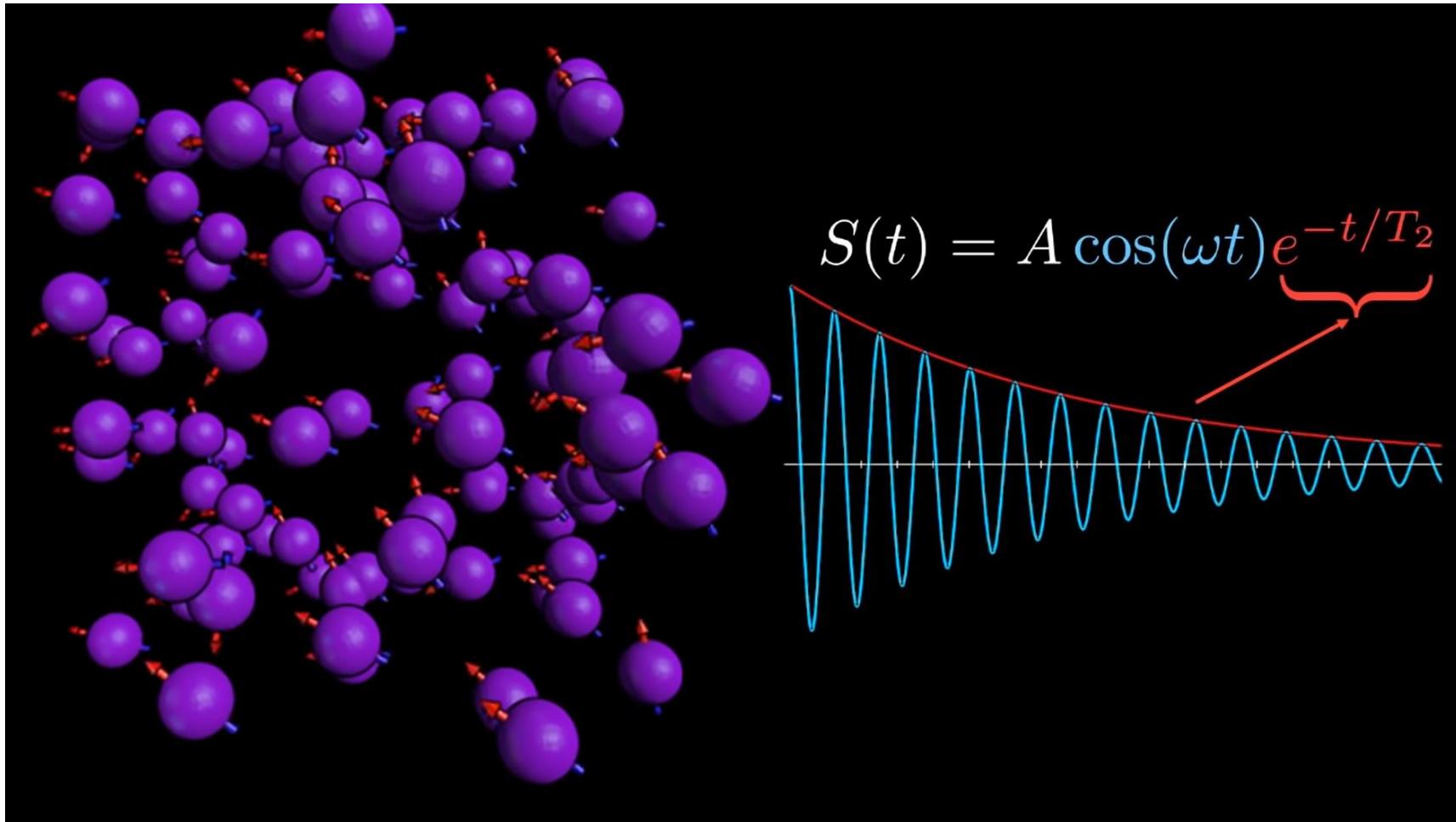


# Detection of Signal

$$s(t) = M_0 \left(1 - e^{-\text{TR}/T_1}\right) e^{-t/T_2}$$

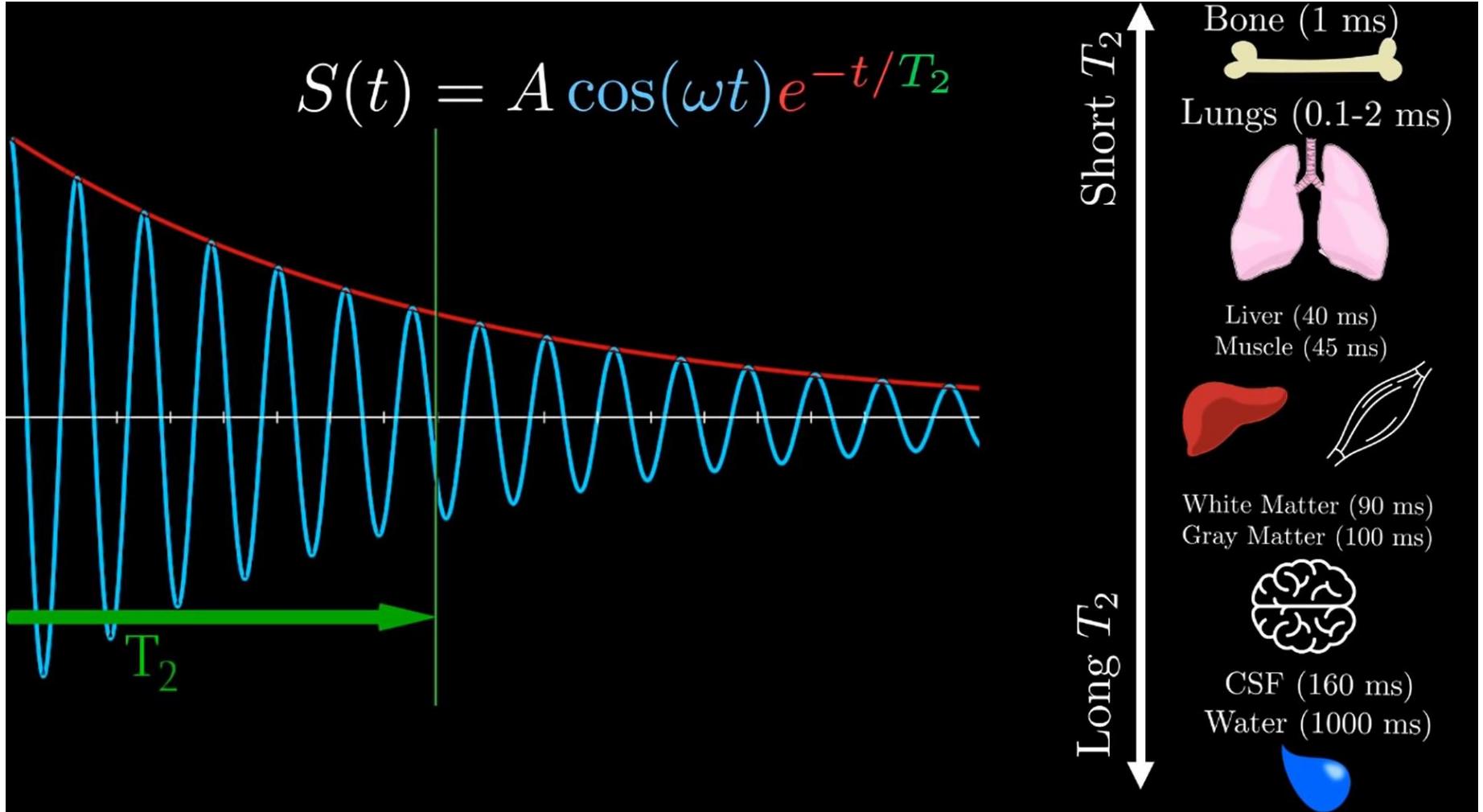


# Detection of Signal



Signal decay

# Detection of Signal



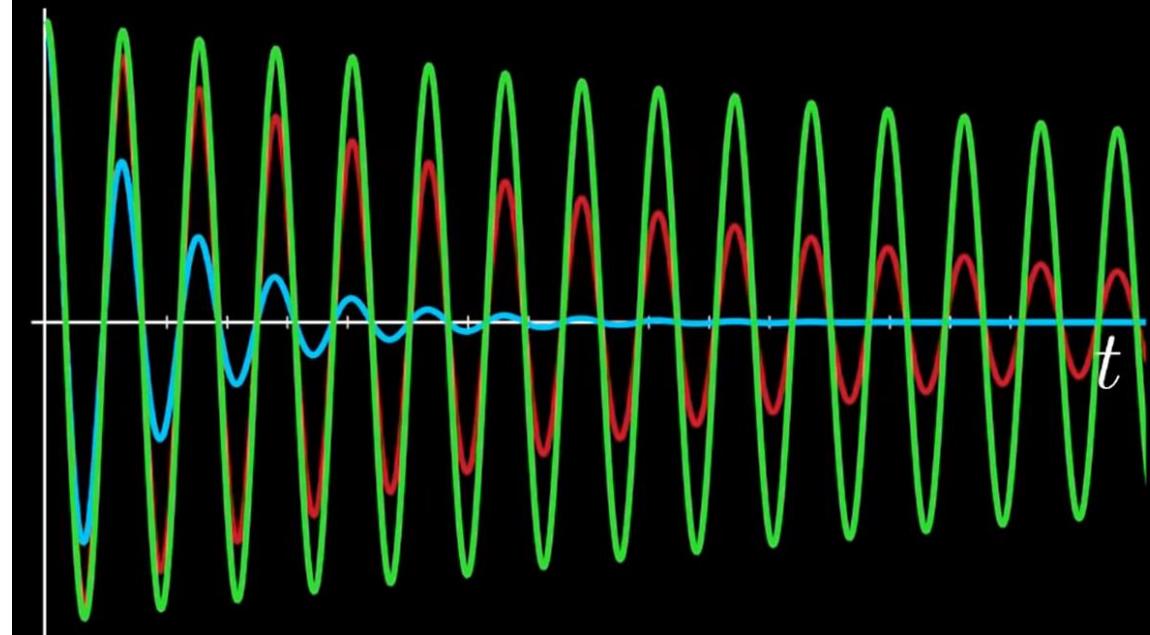
# Detection of Signal

$$S(t) = e^{-t/T_{2s}} \cos(\omega t)$$

$$S(t) = e^{-t/T_{2m}} \cos(\omega t)$$

$$S(t) = e^{-t/T_{2l}} \cos(\omega t)$$

Let  $\omega \rightarrow 0$



Transitioning  
to the rotating  
frame

# Detection of Signal

$$S(t) = e^{-t/T_{2s}}$$

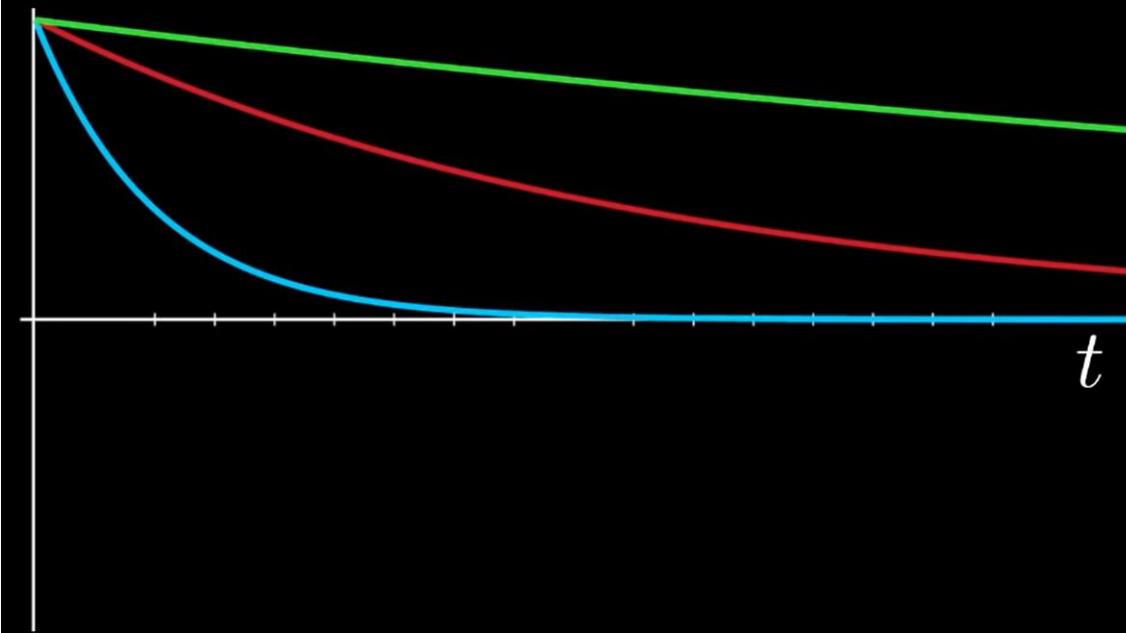
$$S(t) = e^{-t/T_{2m}}$$

$$S(t) = e^{-t/T_{2l}}$$

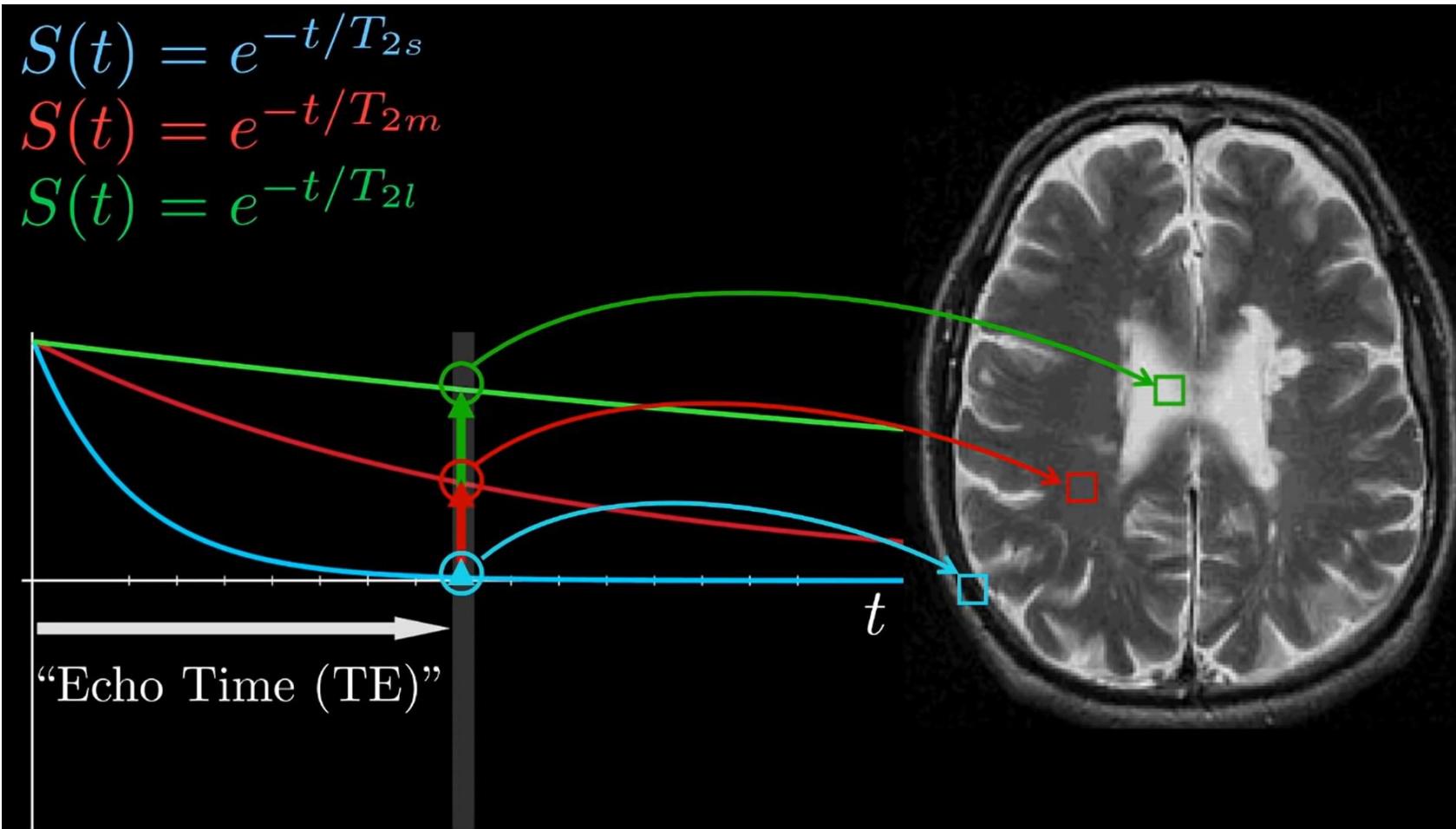
Transition to the ‘Rotating Frame’

Let  $\omega \rightarrow 0$

Transitioning  
to the rotating  
frame



# Detection of Signal



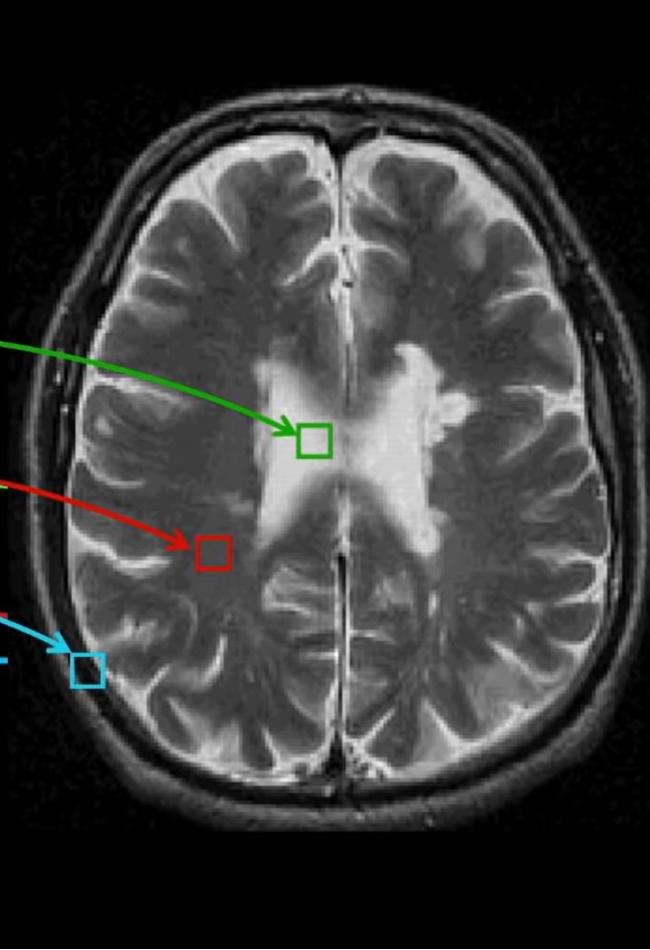
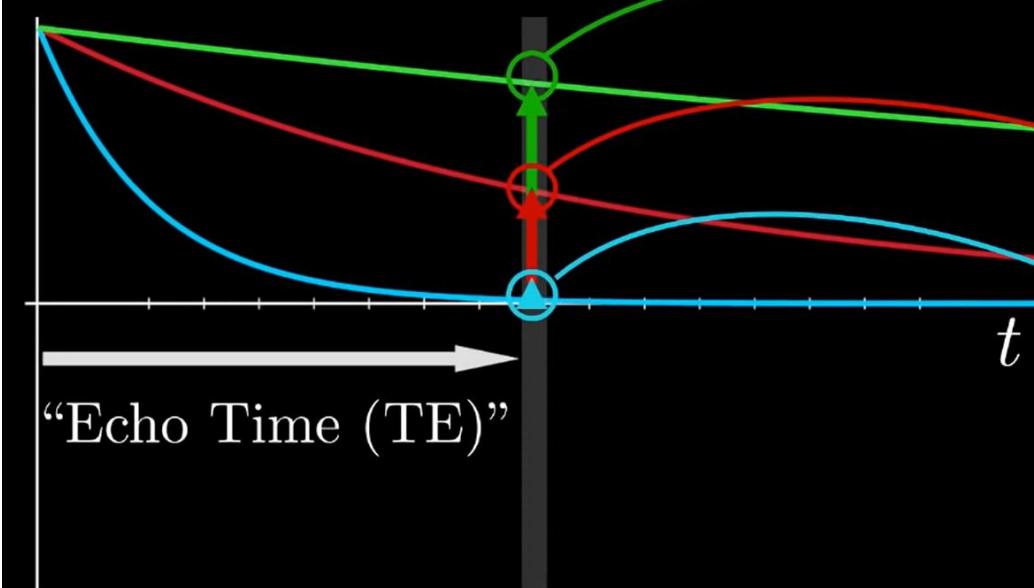
Echo time:  
Chosen by MRI  
technician to  
differentiate  
between the  
tissue the  
radiologist  
wants to see

# Detection of Signal

$$S(t) = e^{-t/T_{2s}}$$

$$S(t) = e^{-t/T_{2m}}$$

$$S(t) = e^{-t/T_{2l}}$$



Echo time: Chosen by MRI technician to differentiate between the tissue the radiologist wants to see

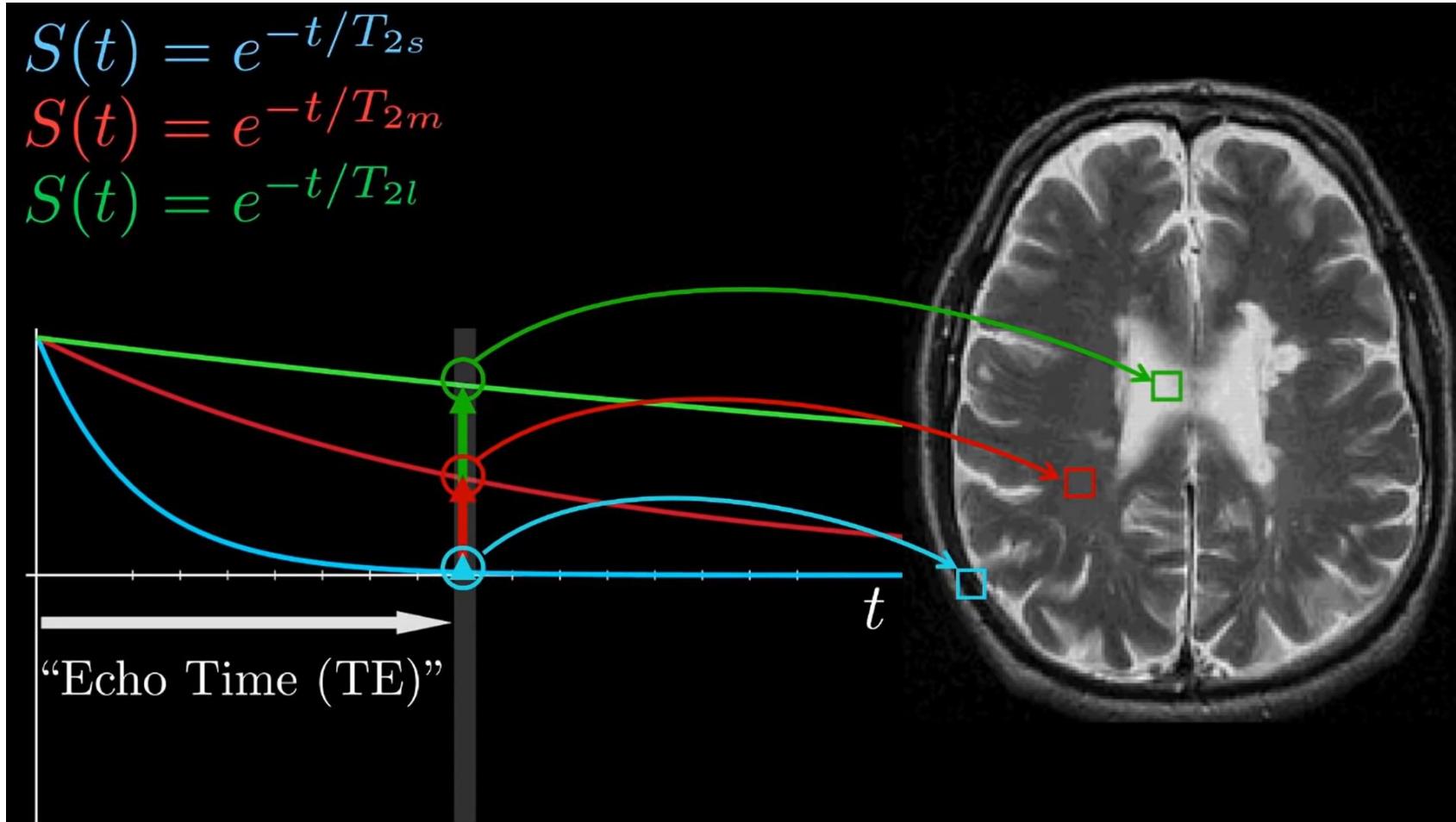
# Detection of Signal

$$S(t) = e^{-t/T_{2s}}$$

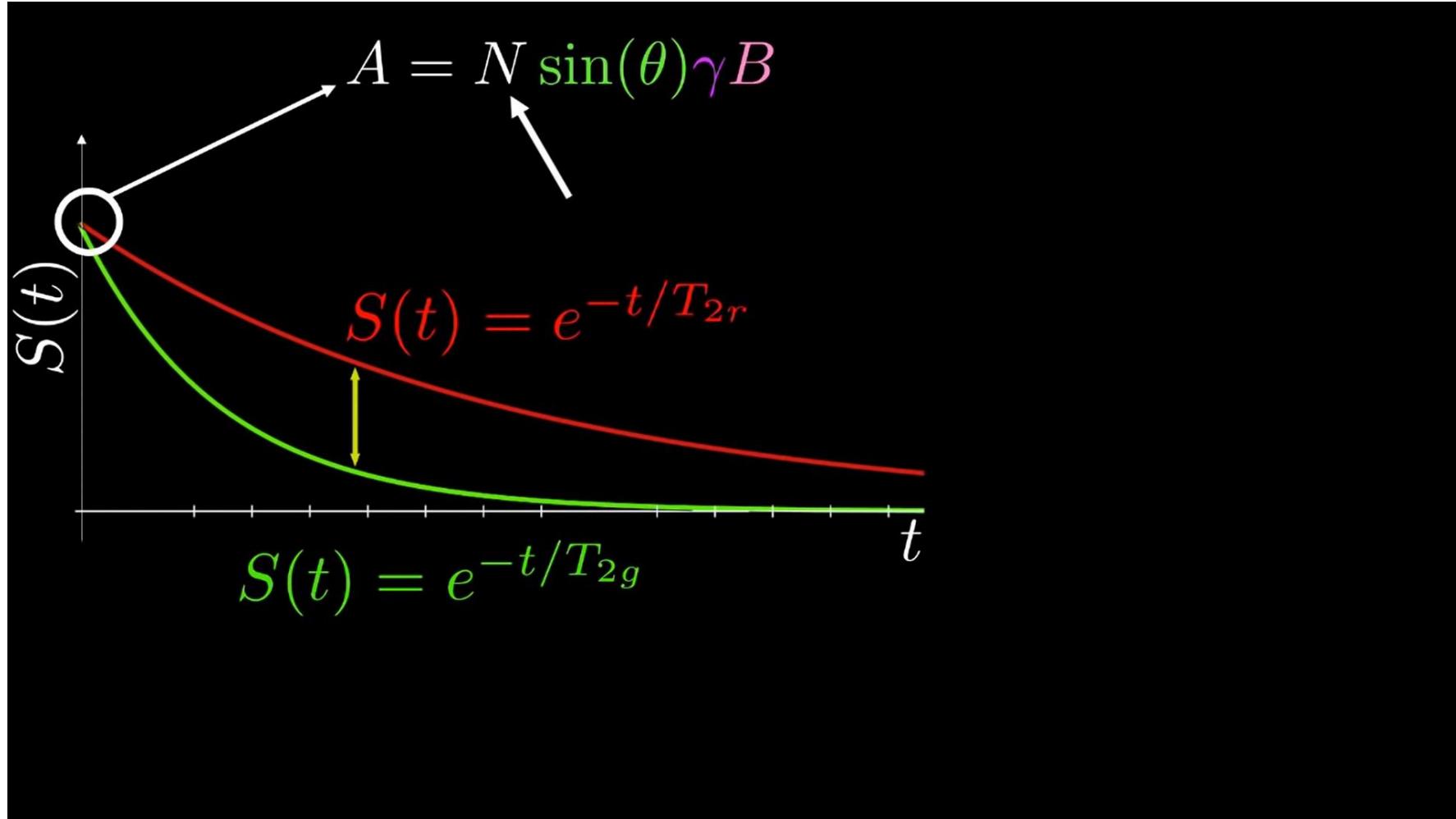
$$S(t) = e^{-t/T_{2m}}$$

$$S(t) = e^{-t/T_{2l}}$$

T2 weighted image

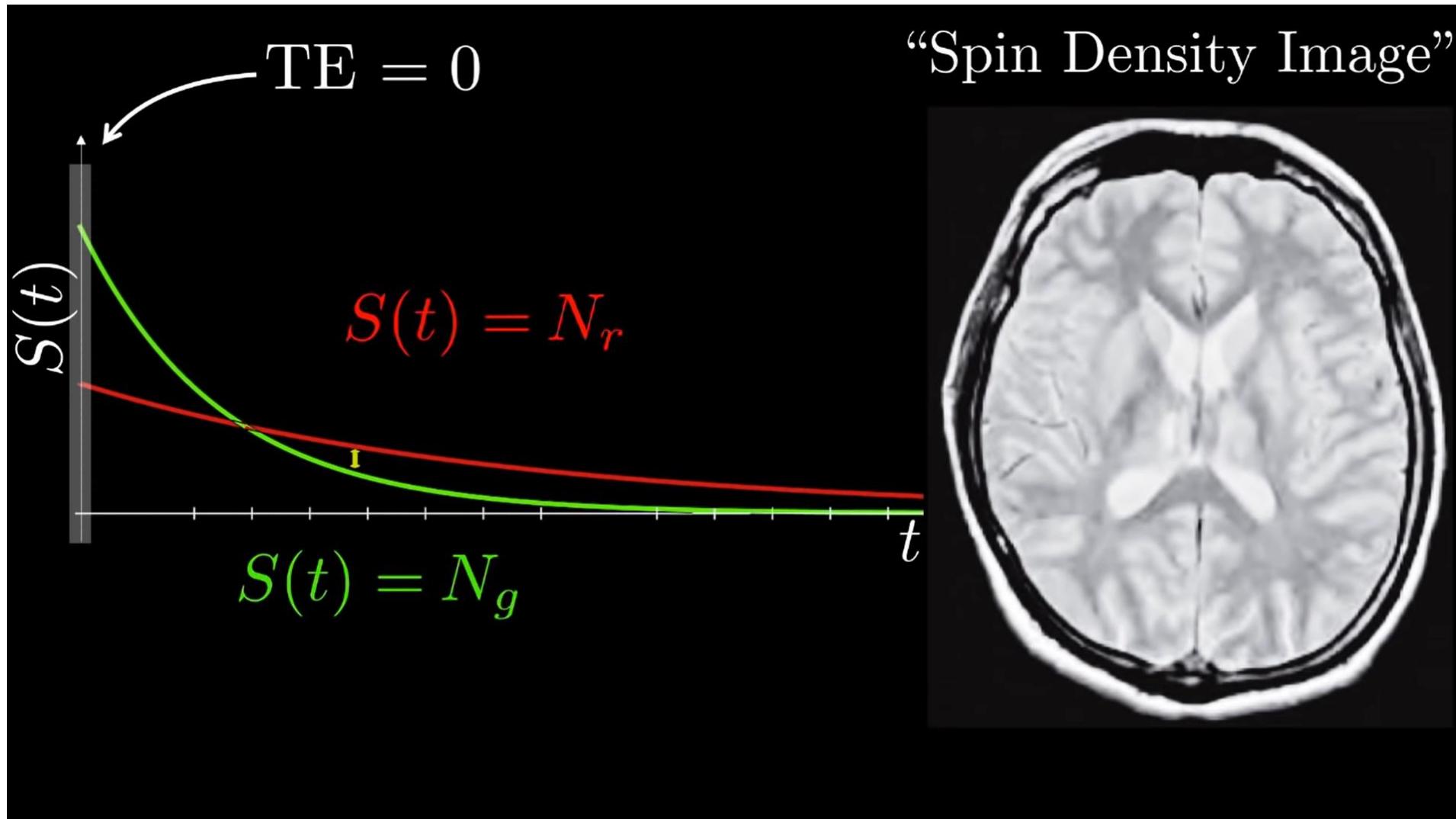


# Detection of Signal

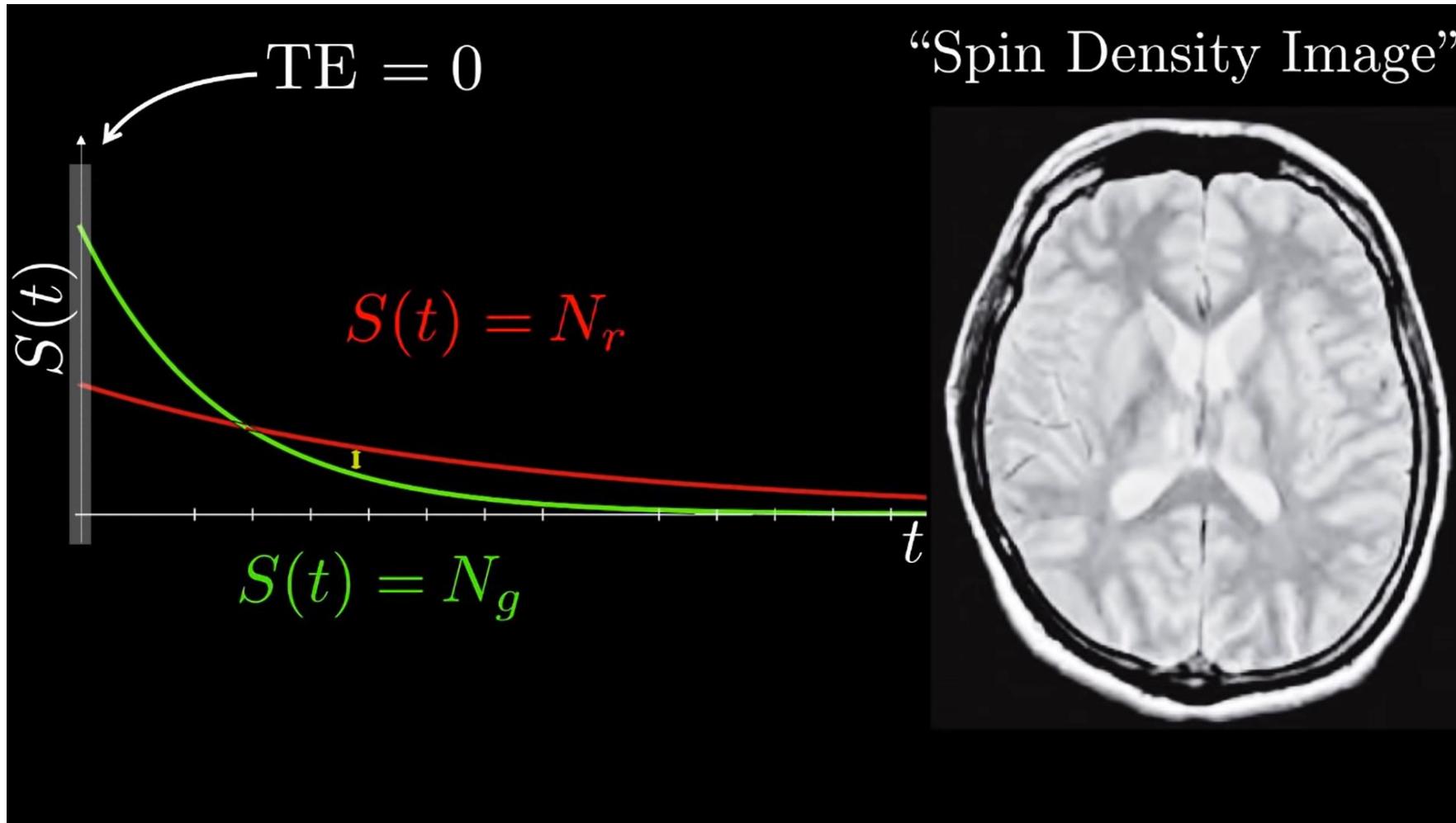


$N$  will actually be different for different tissues

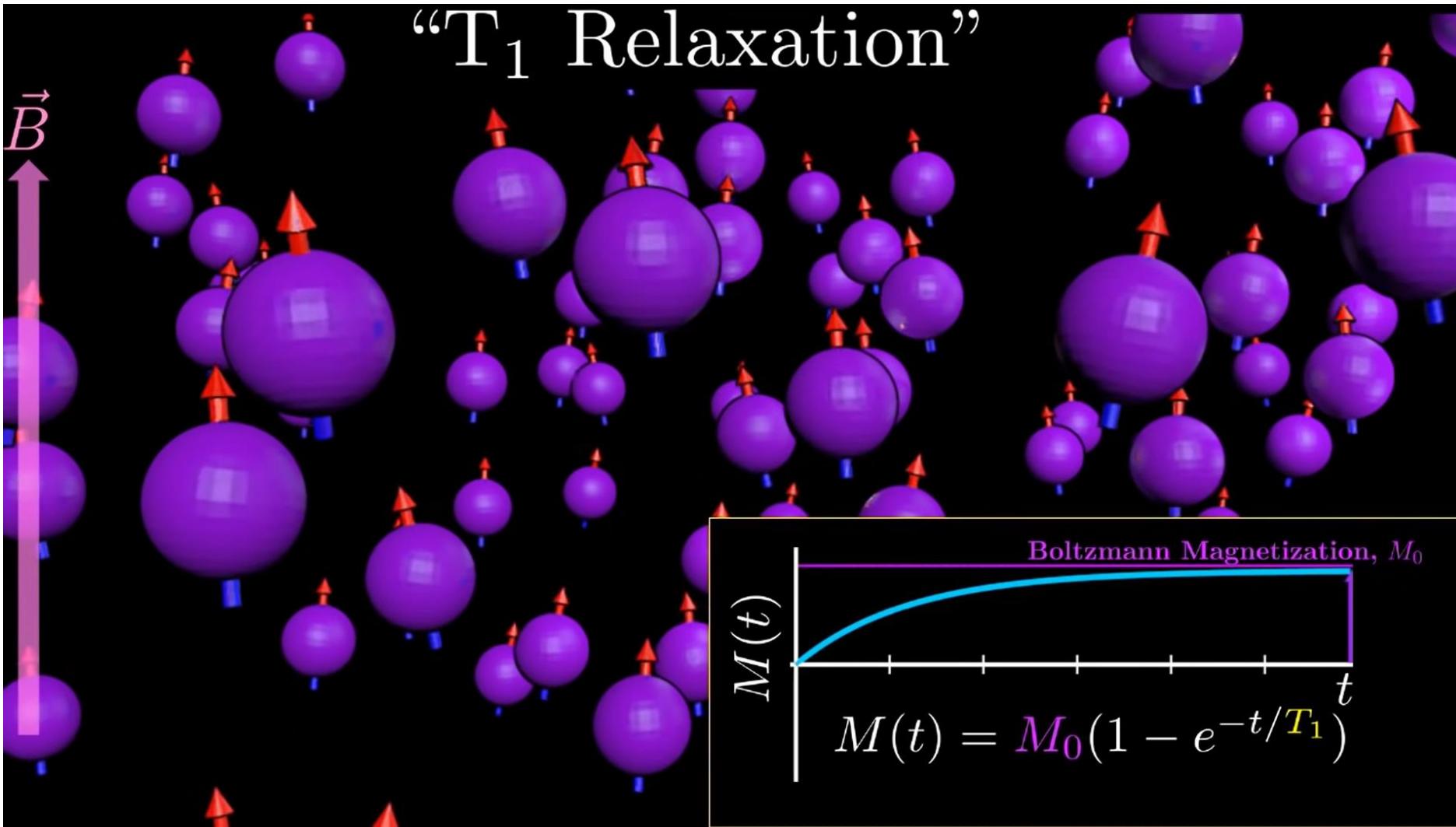
# Detection of Signal



# Detection of Signal



# Detection of Signal



$T_1$  weighting

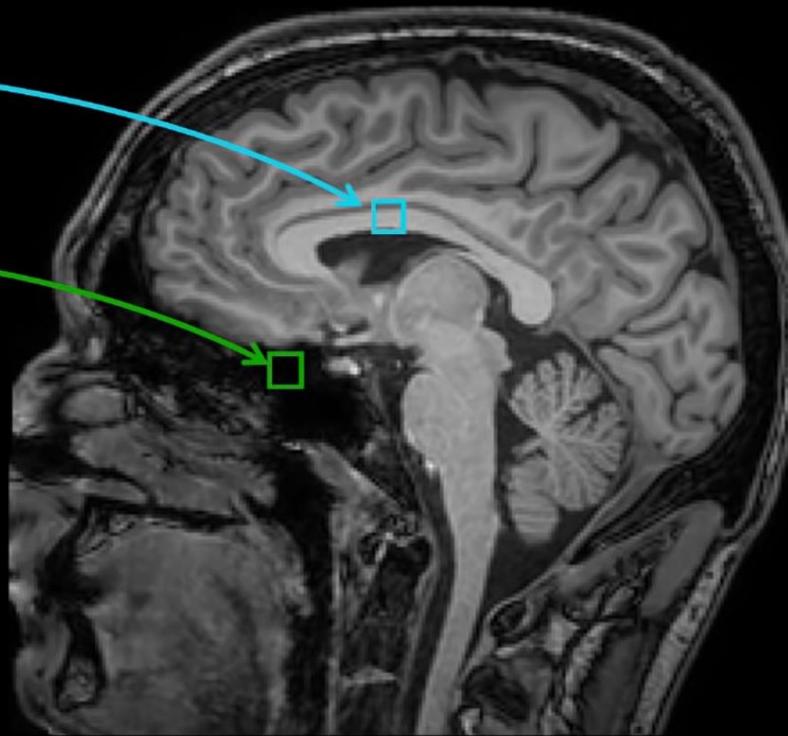
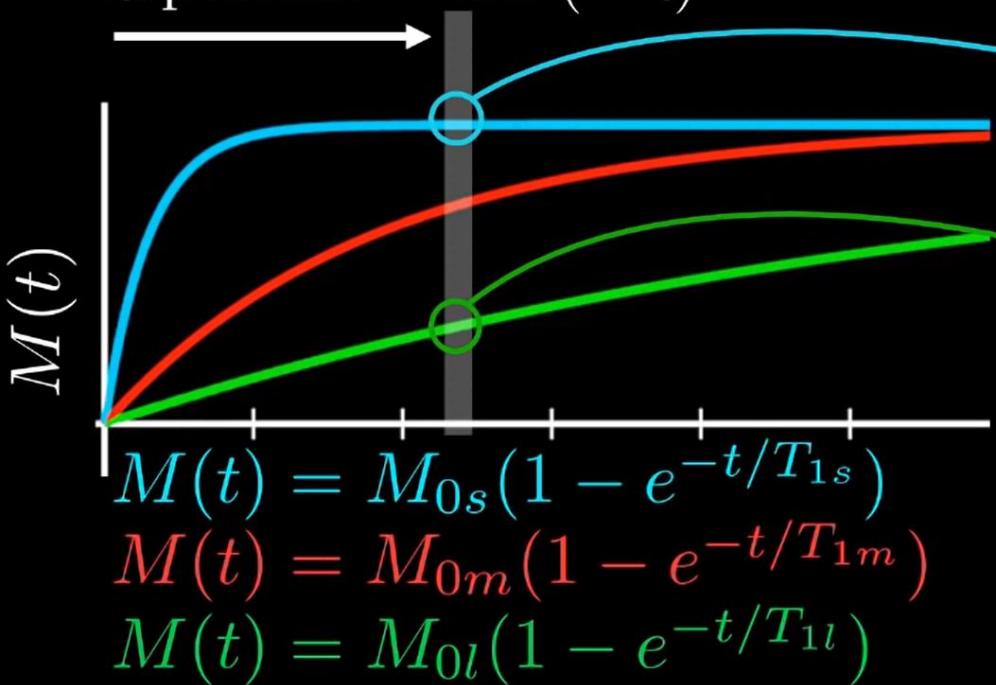
Different tissues  
exhibit different  $T_1$

So, this can be  
another source of  
contrast

# Detection of Signal

“T<sub>1</sub> Relaxation”

“Repetition Time (TR)”

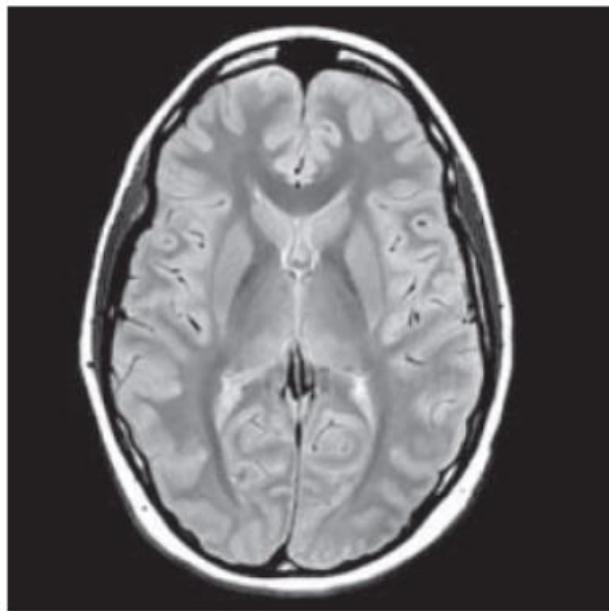


“T<sub>1</sub>-Weighted Image”

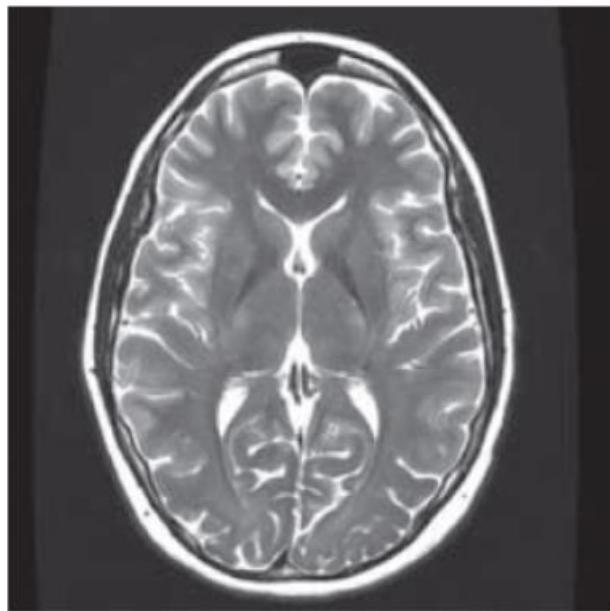
$T_R$ : Repetition time

Chosen by MRI technician to provide the required contrast

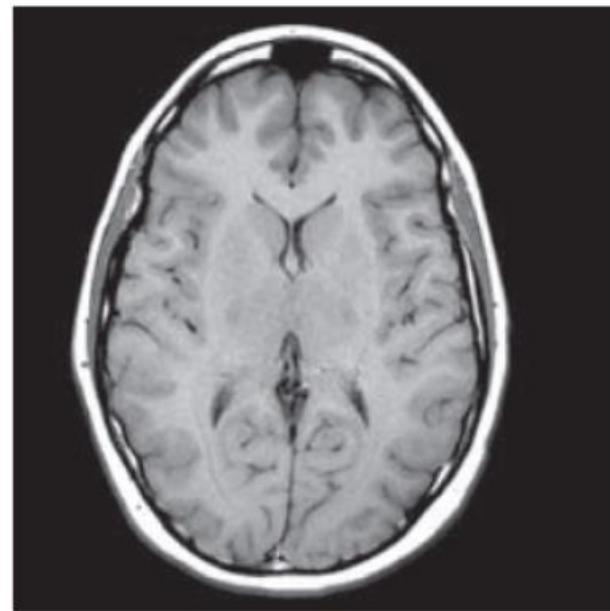
# PD, T1 and T2 Weighted Images



(a)



(b)

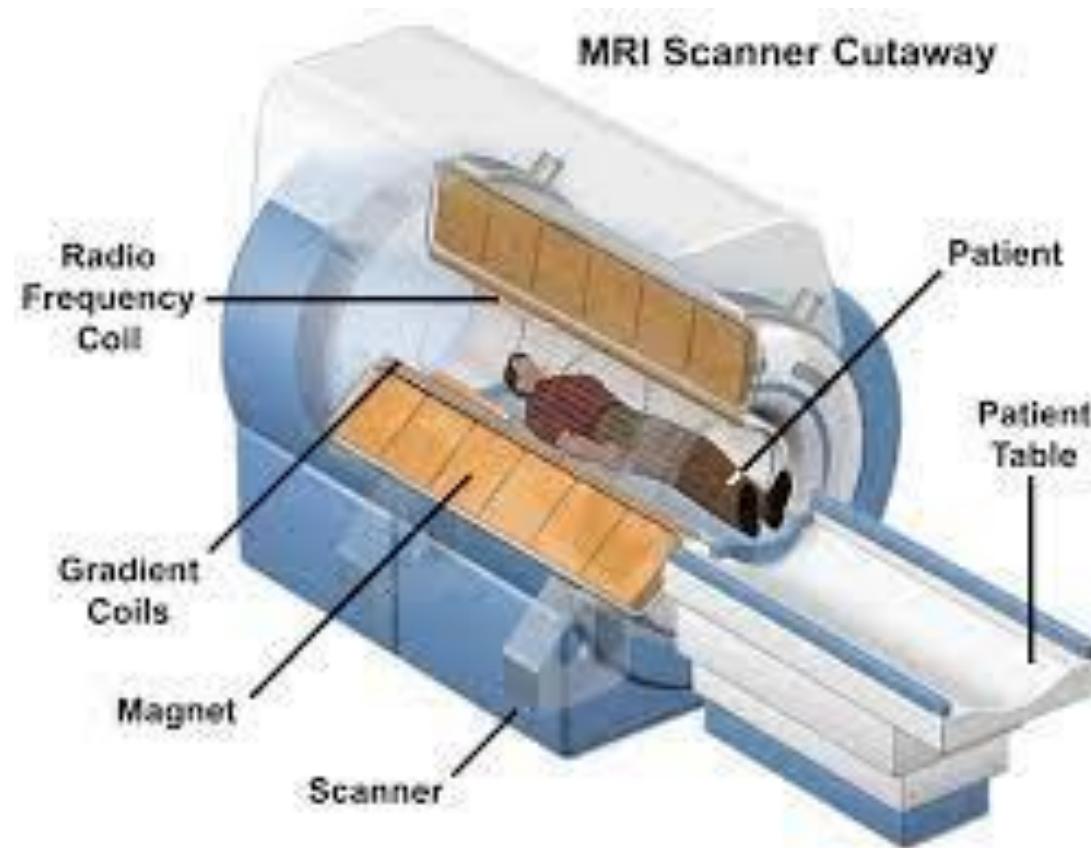


(c)

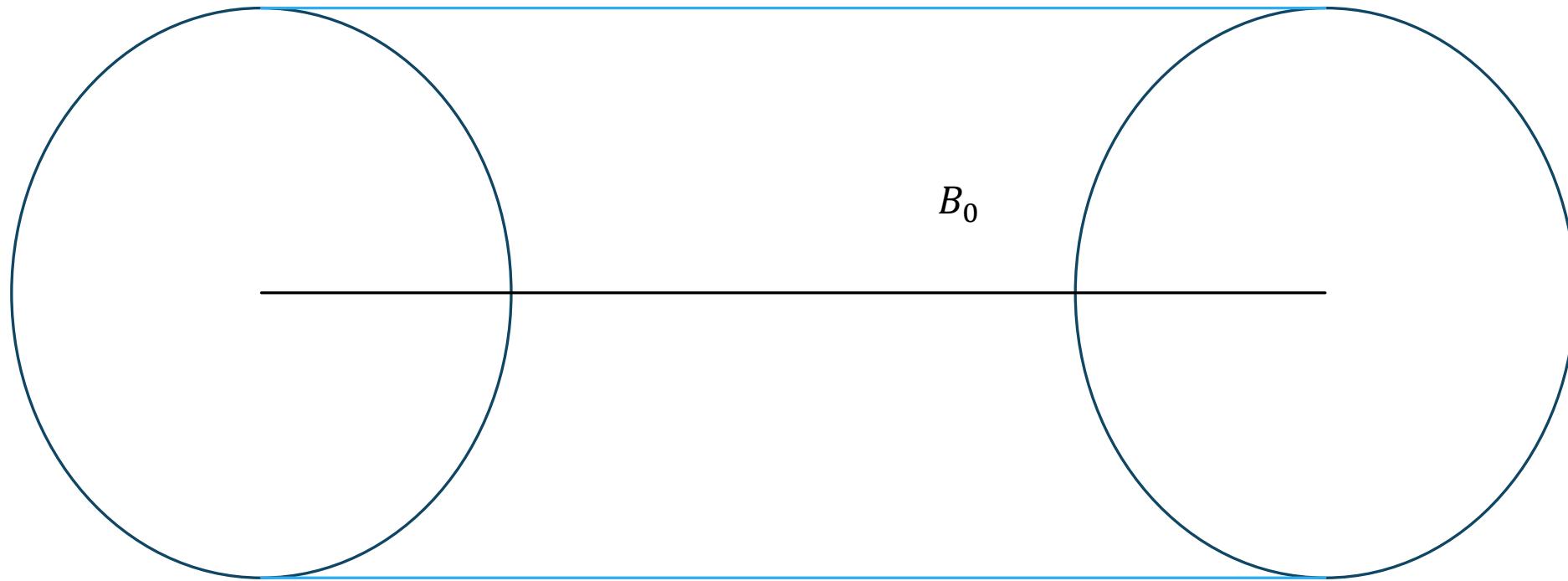
**Figure 12.10**

Three images of the same slice through the skull. Contrast between the tissue types are classified as (a)  $P_D$ -weighted, (b)  $T_2$ -weighted, and (c)  $T_1$ -weighted. Courtesy of GE Healthcare.

# MRI Scanner

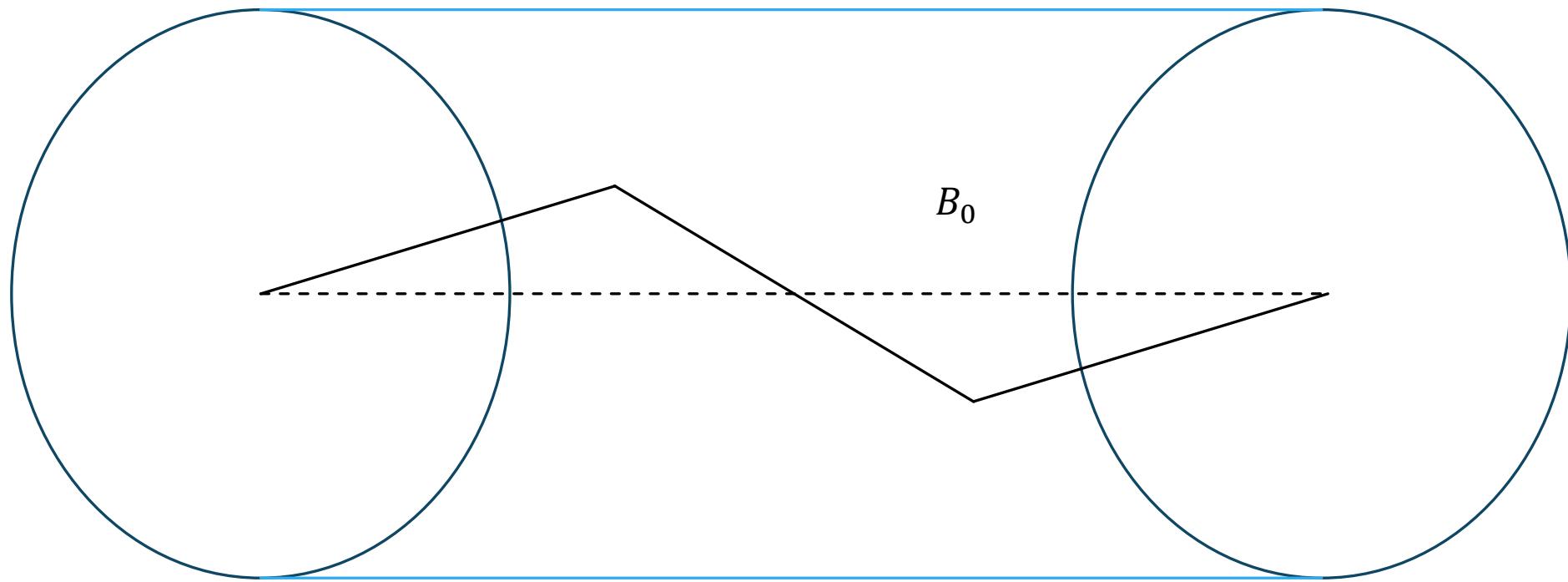


# Slice Selection



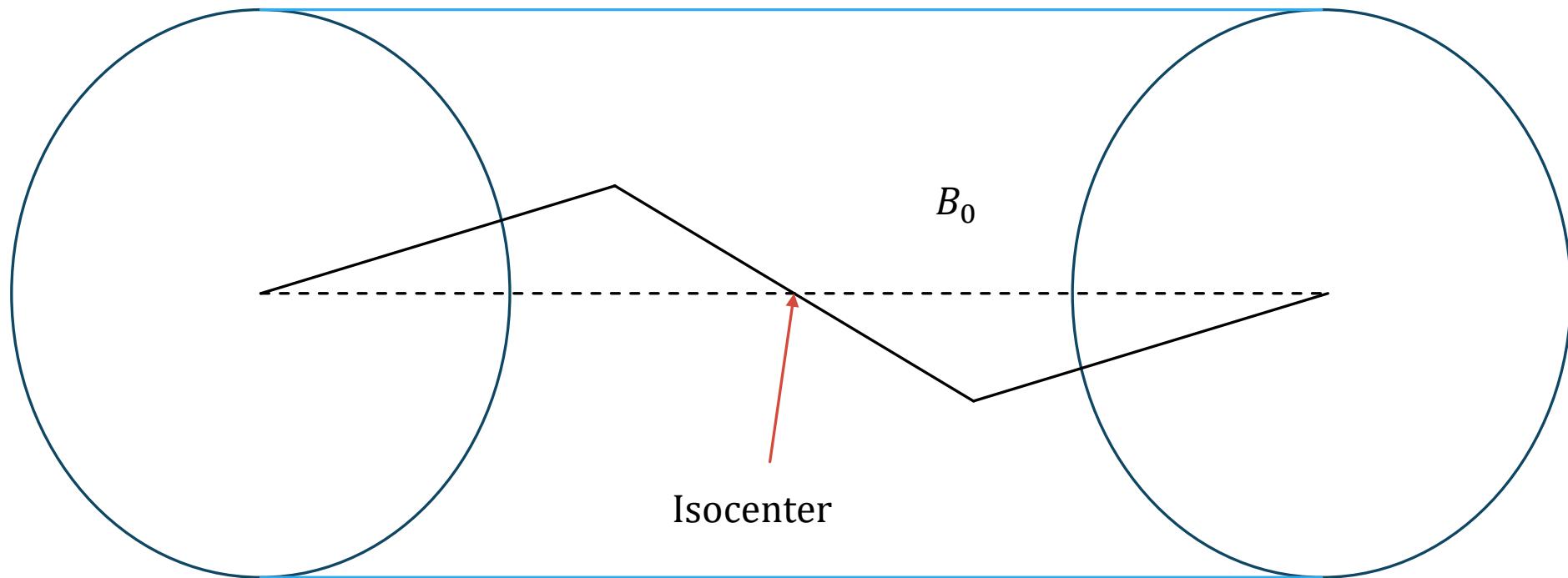
# Slice Selection

Gradient magnetic field



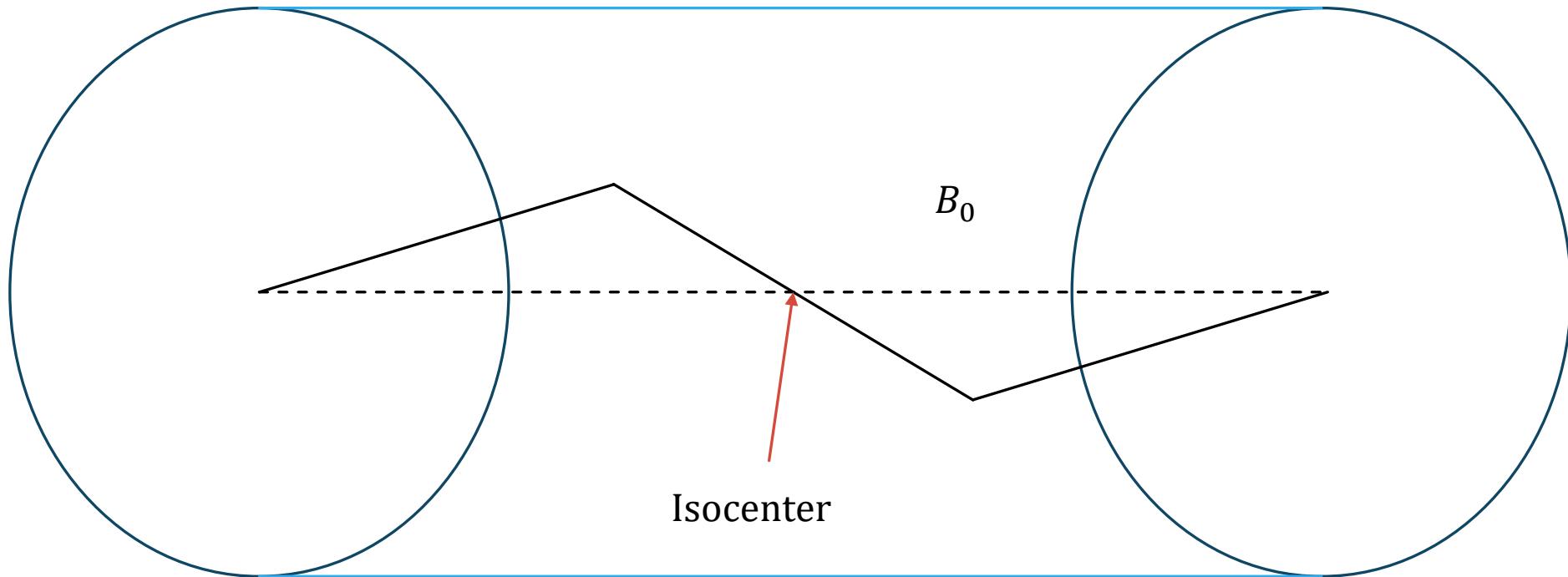
# Slice Selection

Gradient magnetic field



# Slice Selection

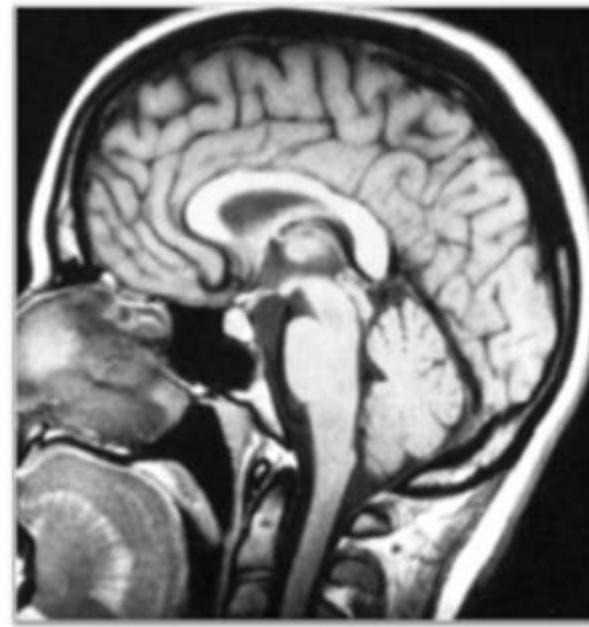
Slice thickness is dependent  
on BW of RF pulse



# k-Space



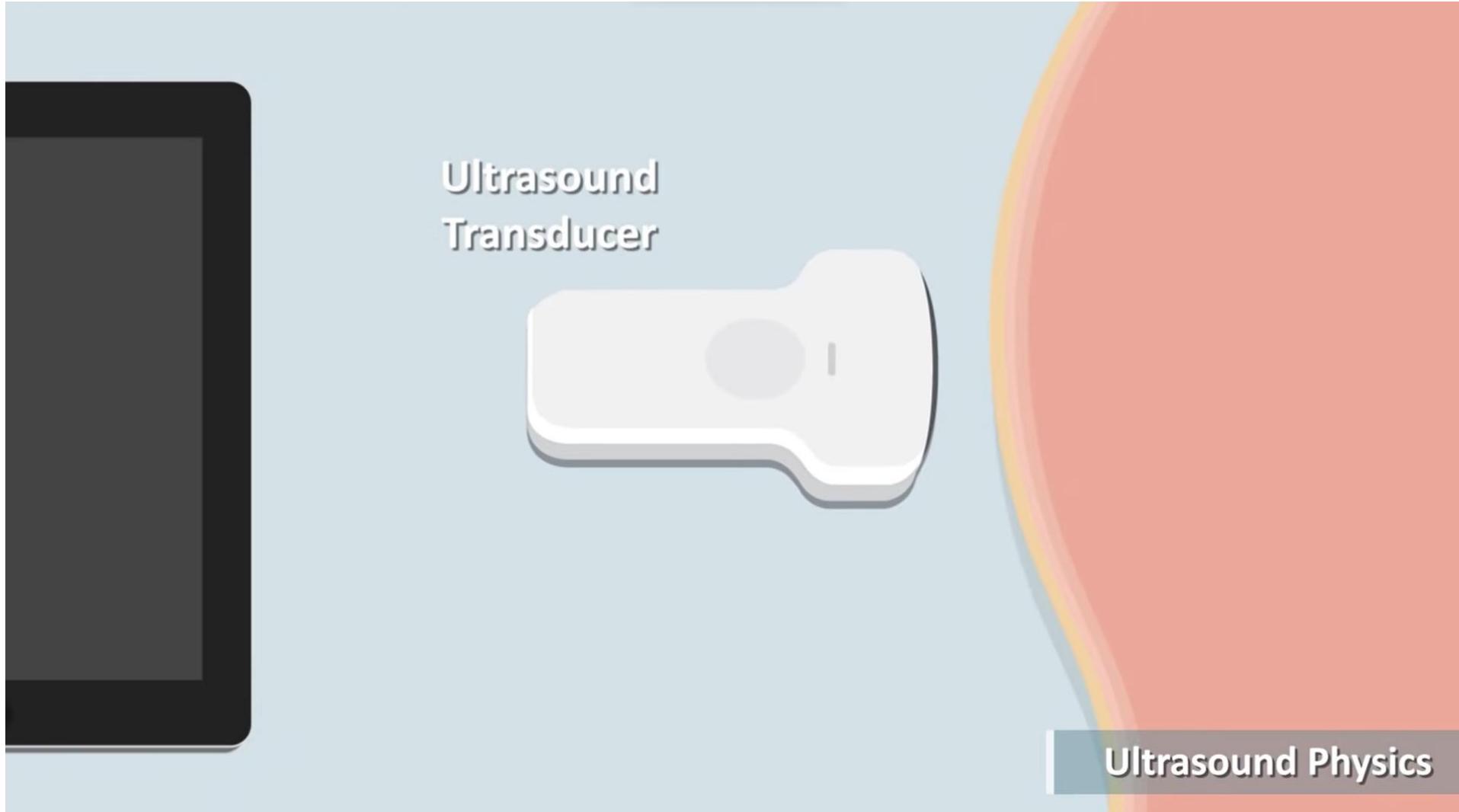
FT  
↔

A pink double-headed arrow symbol positioned between the k-space image and the MR image, indicating the Fourier Transform relationship.

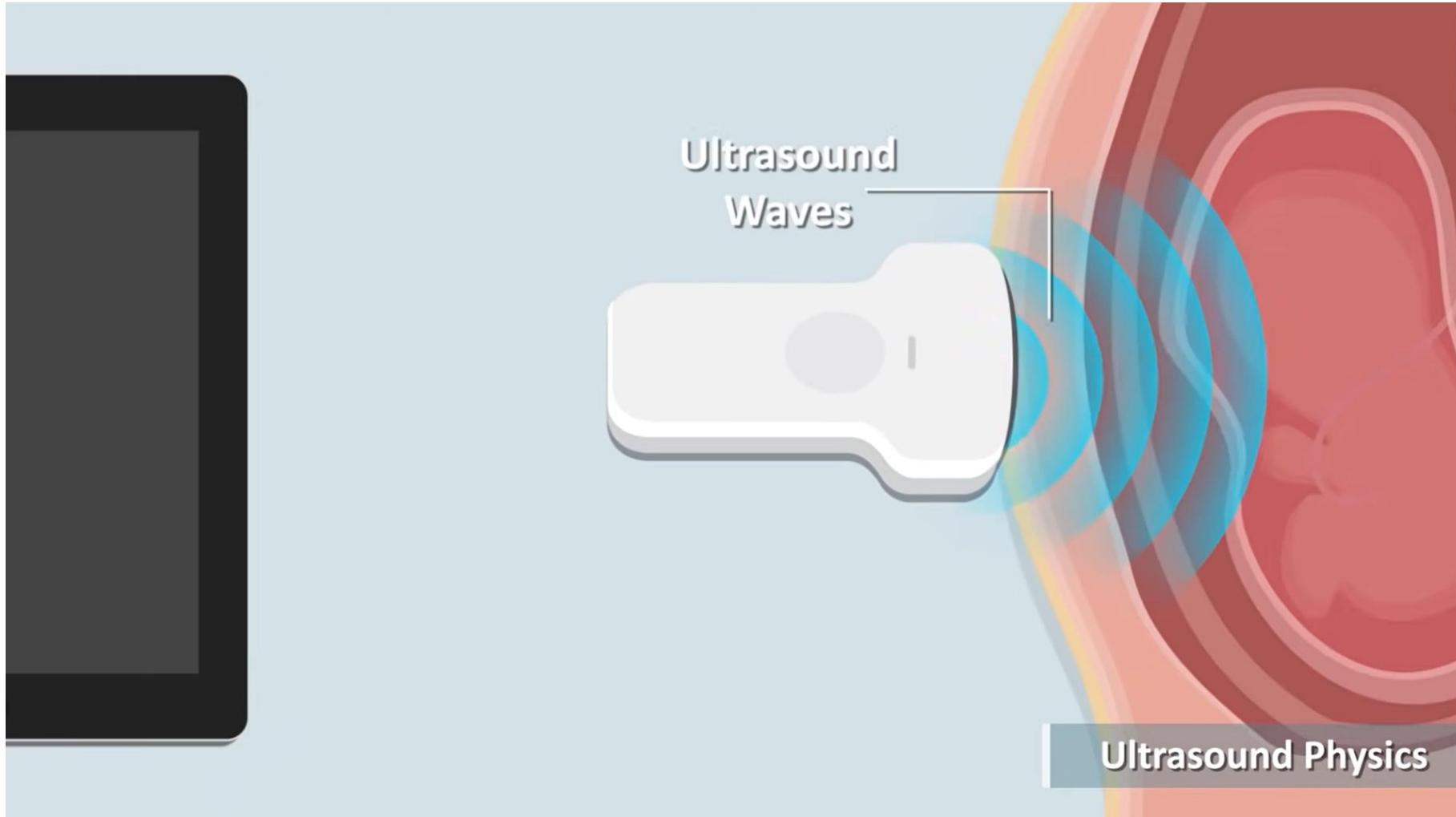
K-space is the Fourier Transform of MR images

# Ultrasound

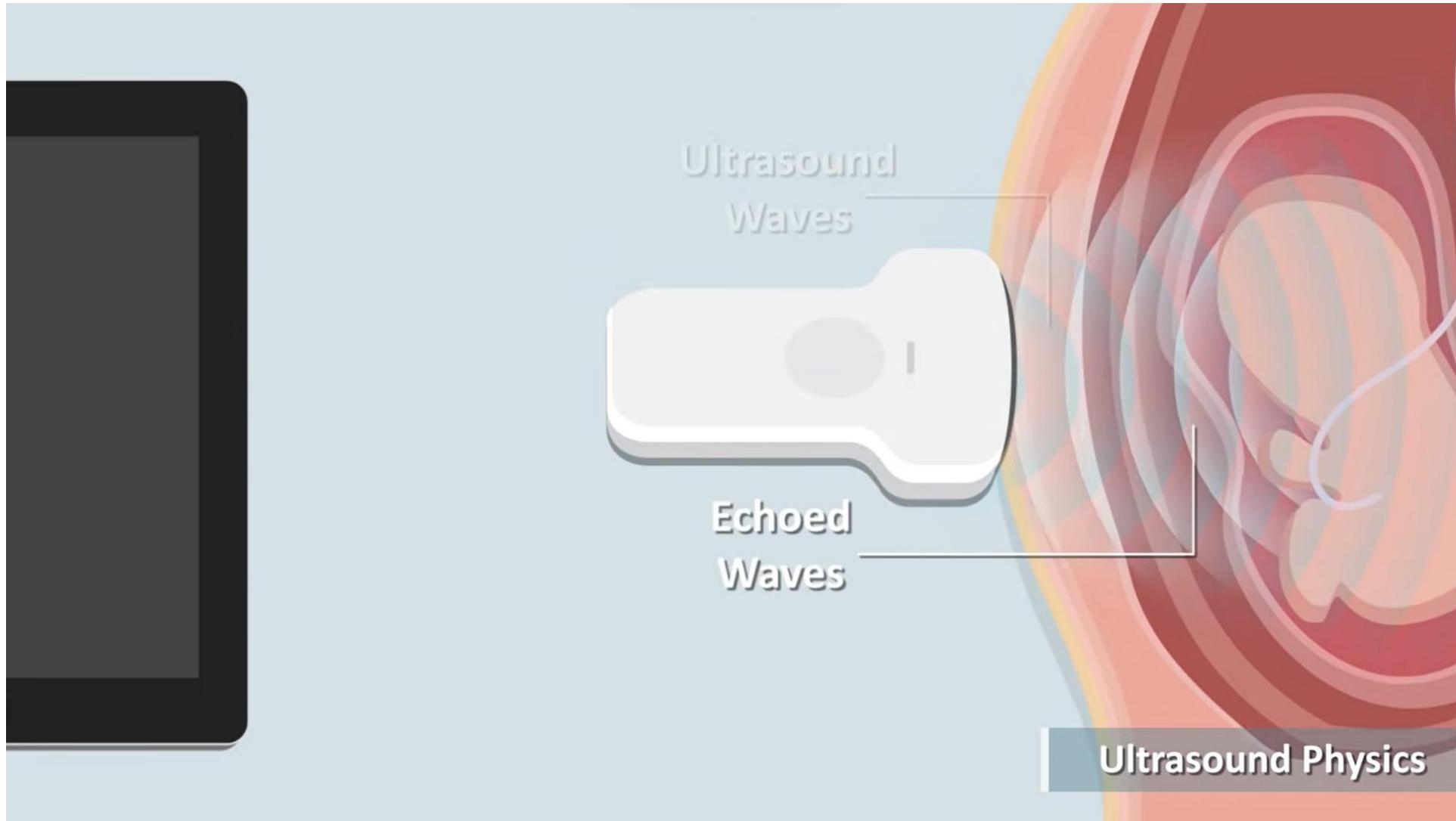
# Ultrasound



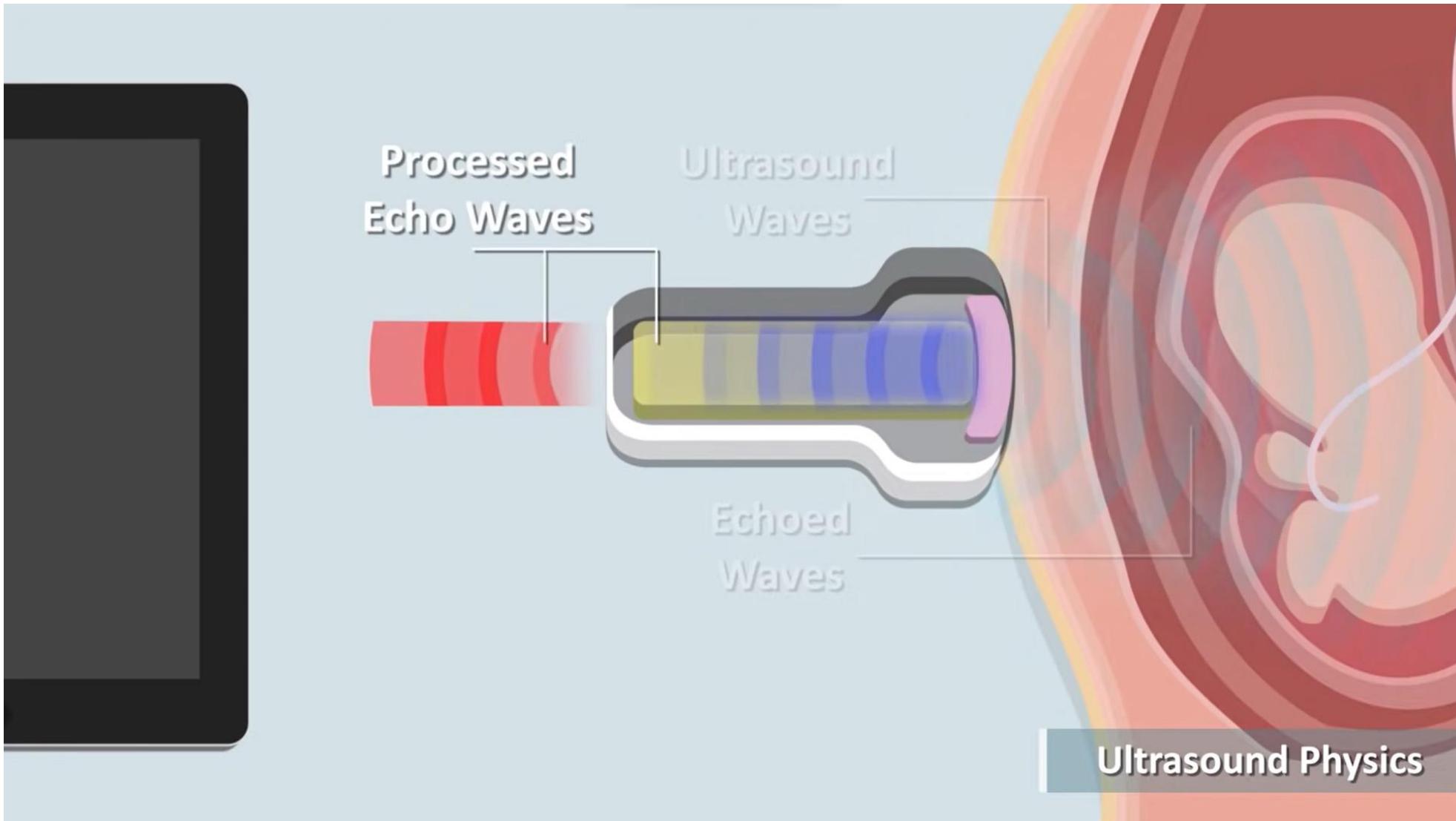
# Ultrasound



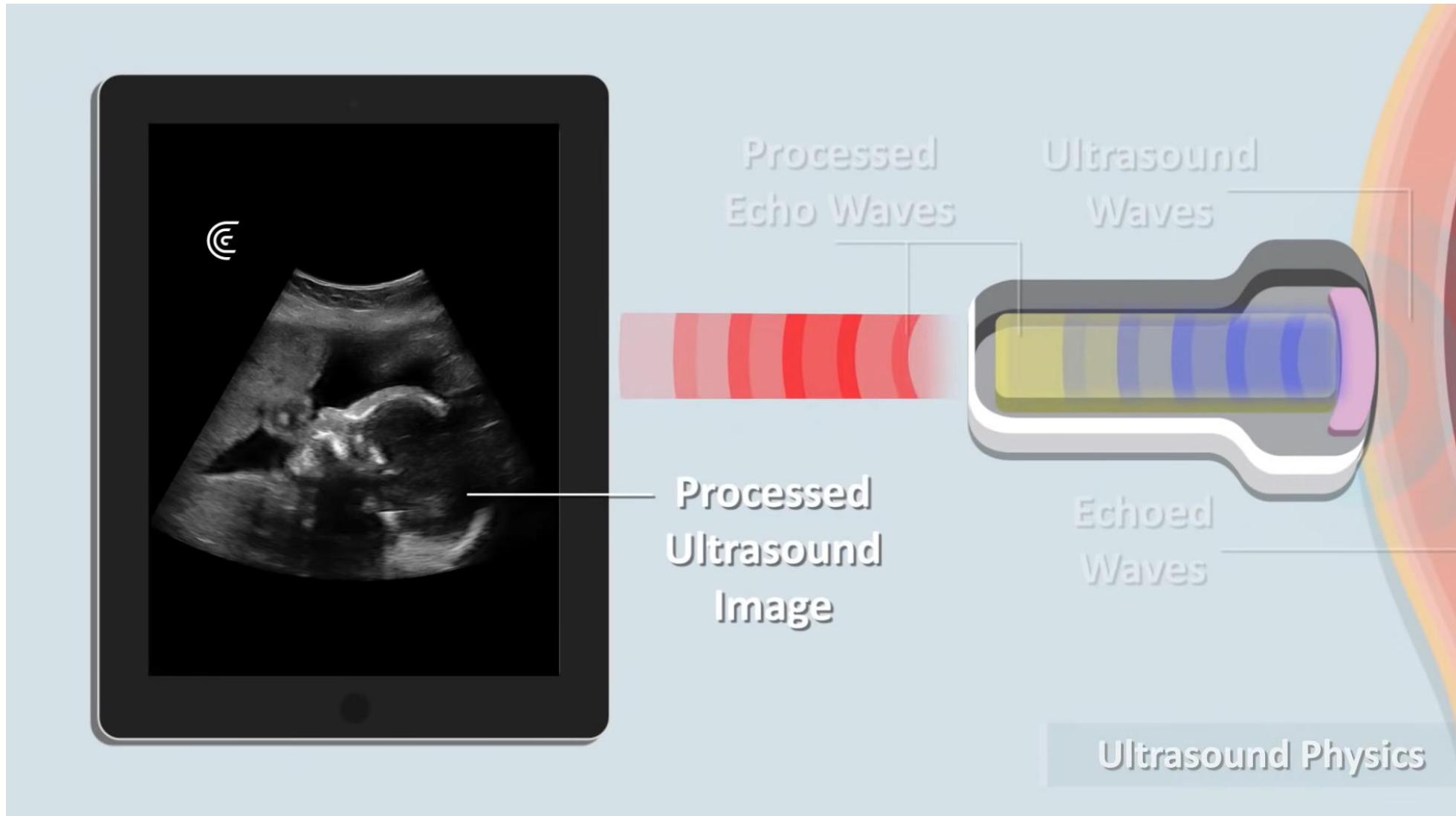
# Ultrasound



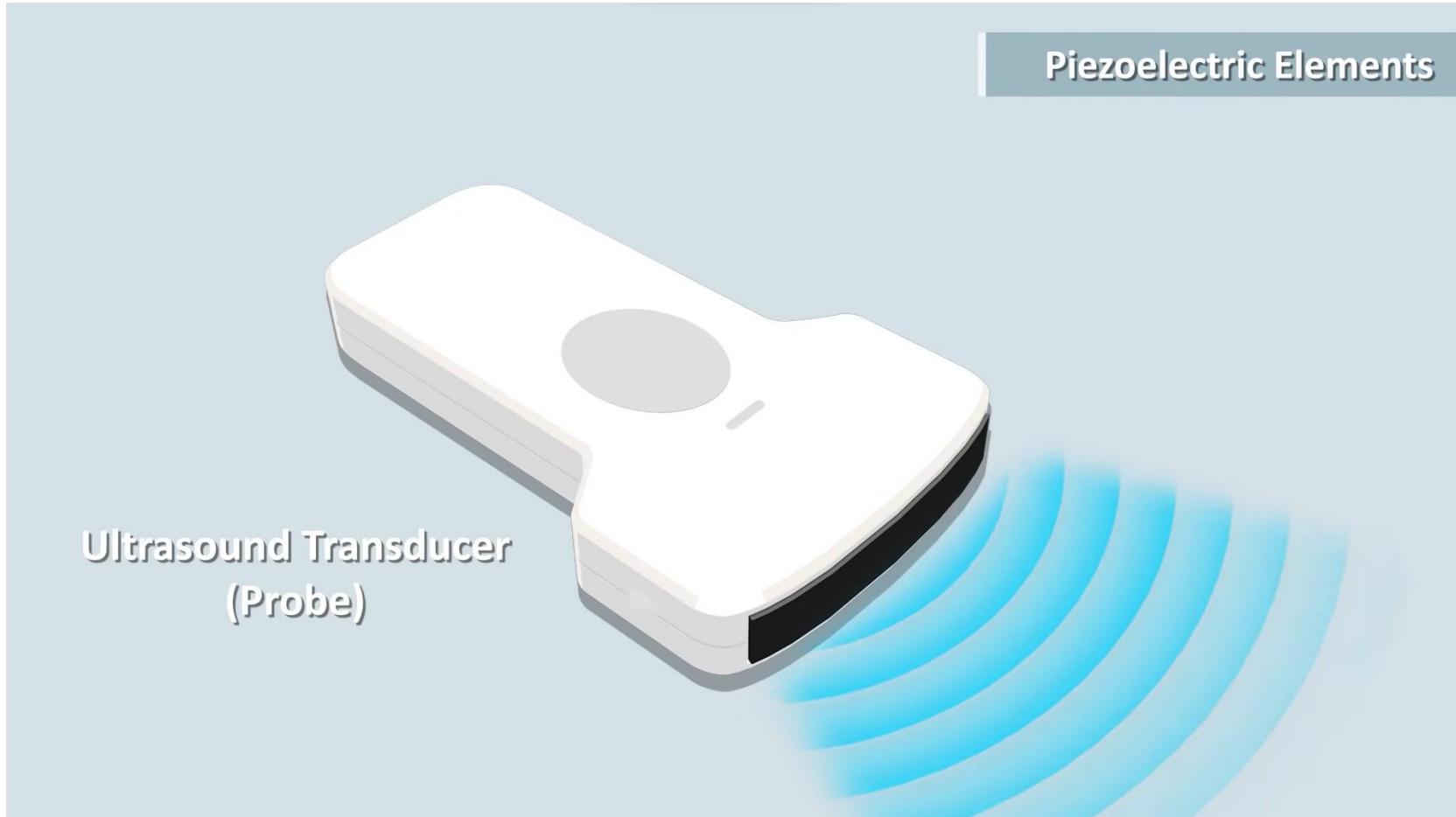
# Ultrasound



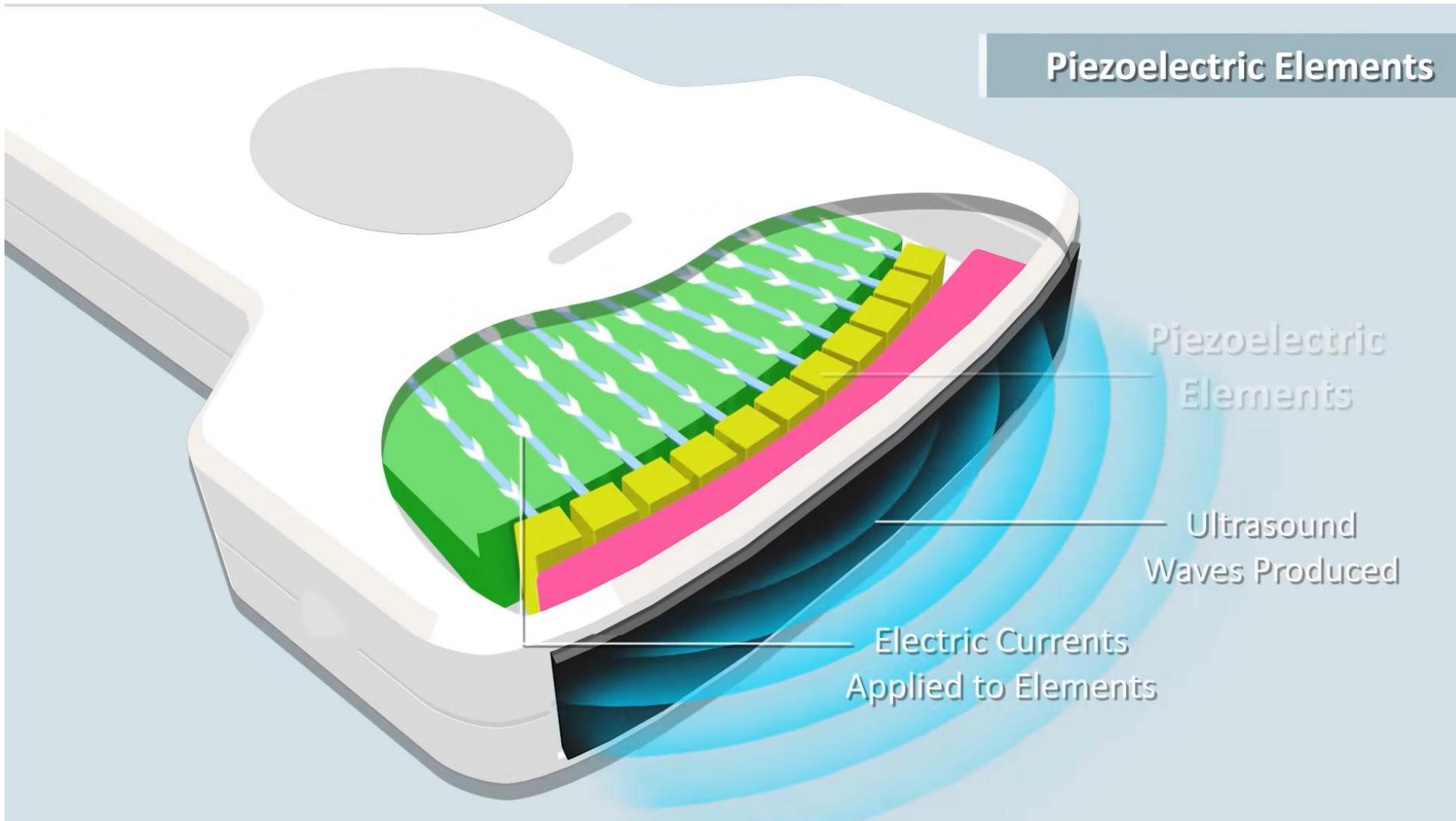
# Ultrasound



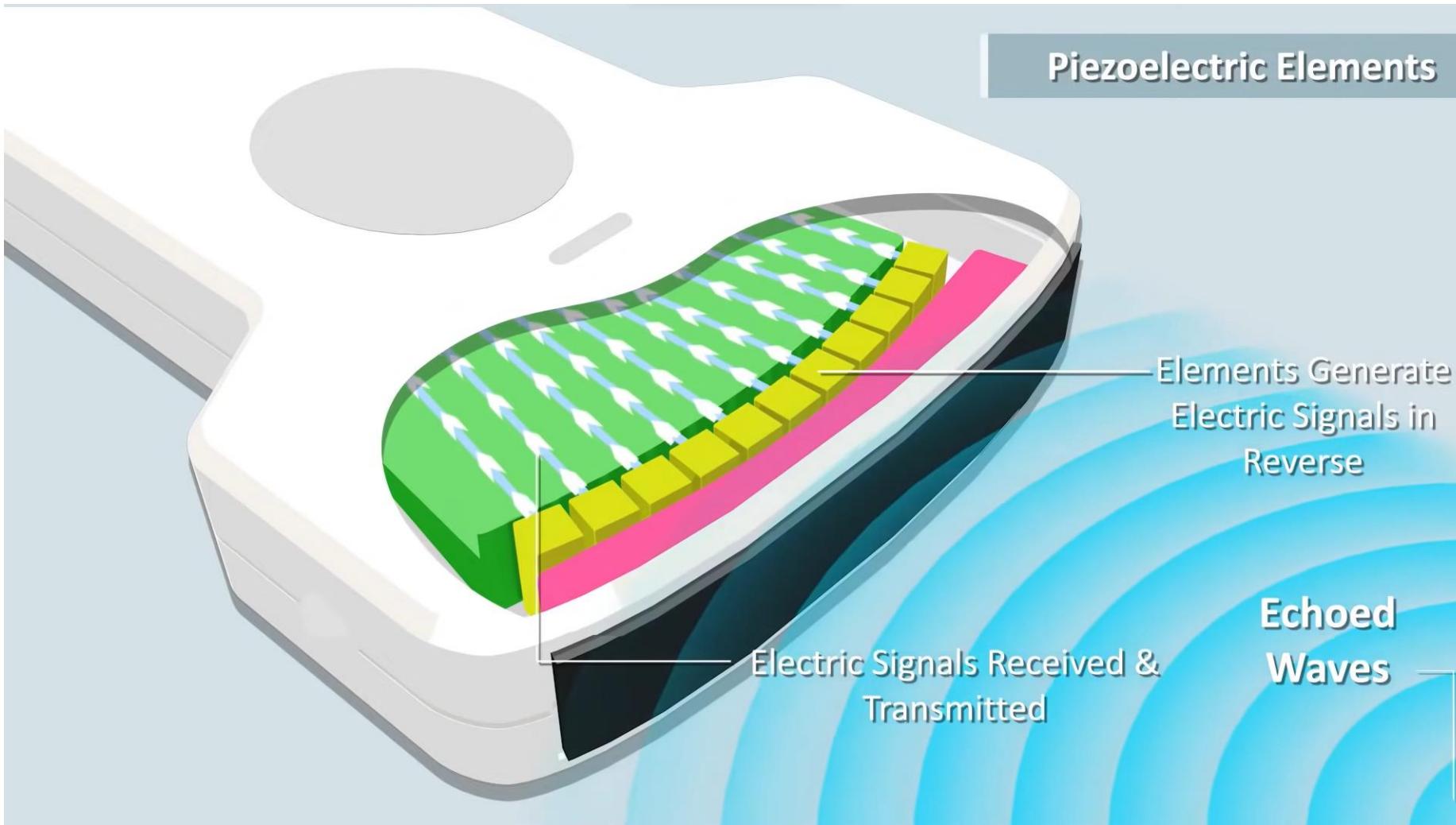
# Ultrasound



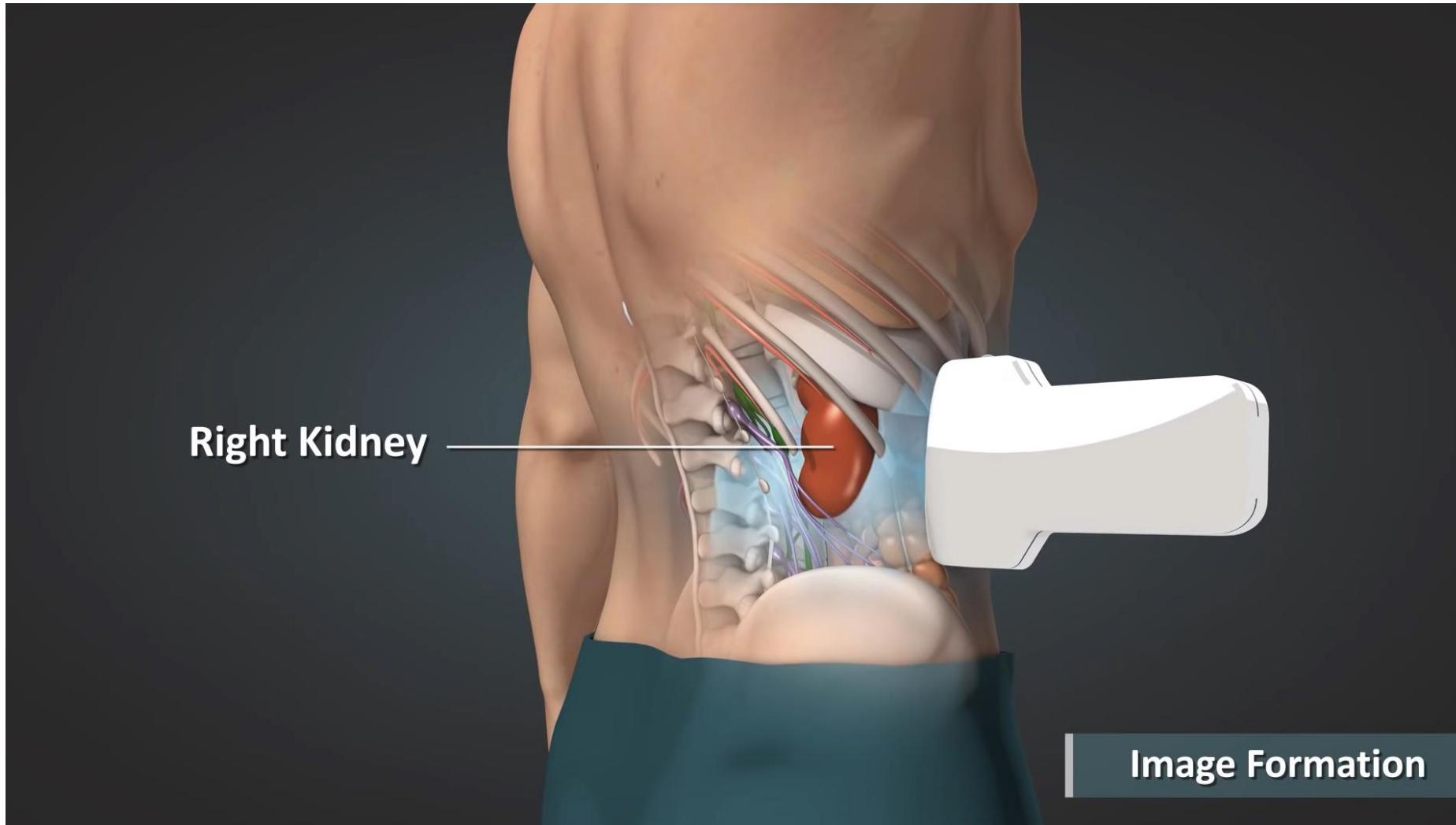
# Ultrasound



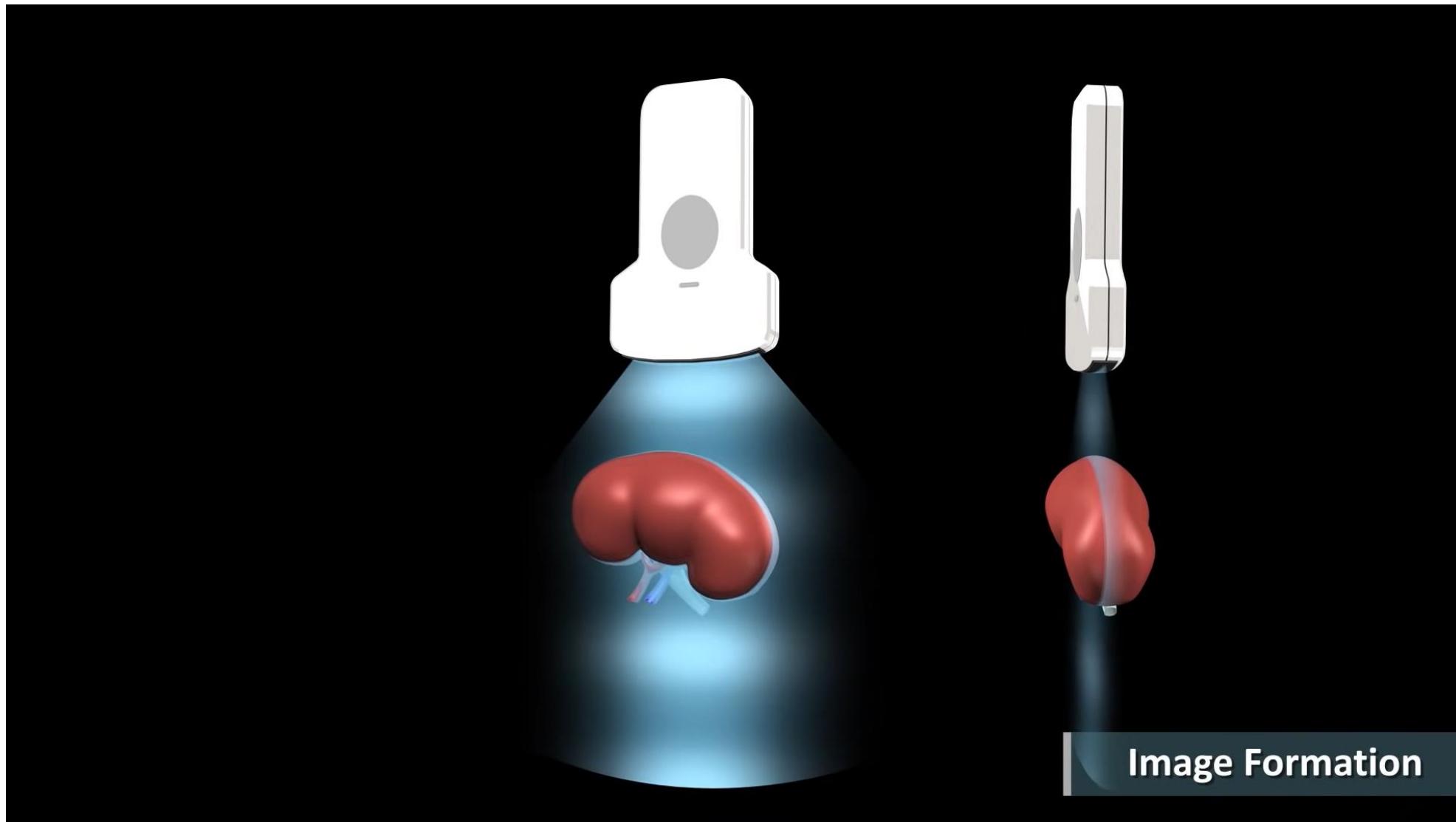
# Ultrasound



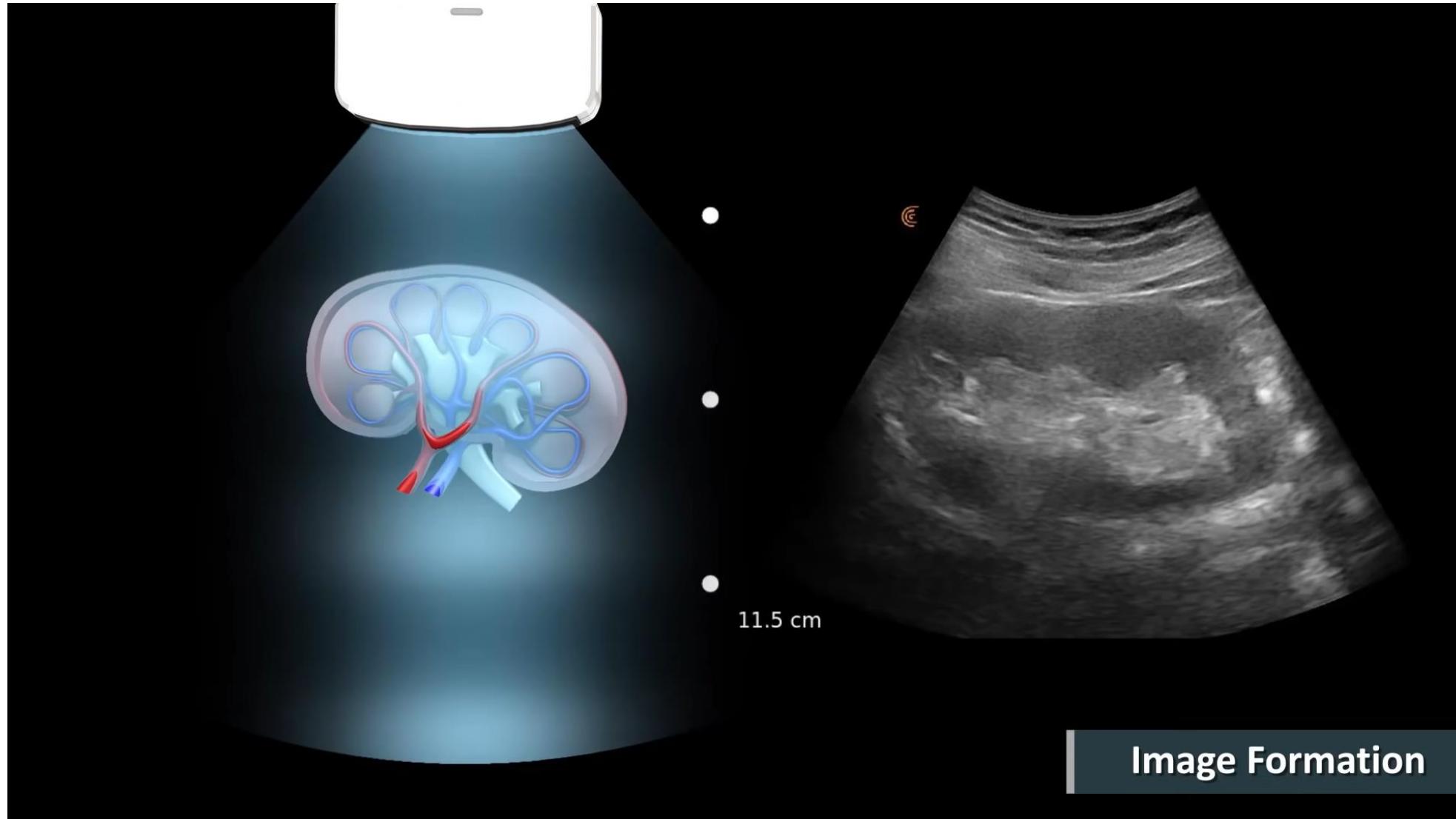
# Ultrasound Image Formation



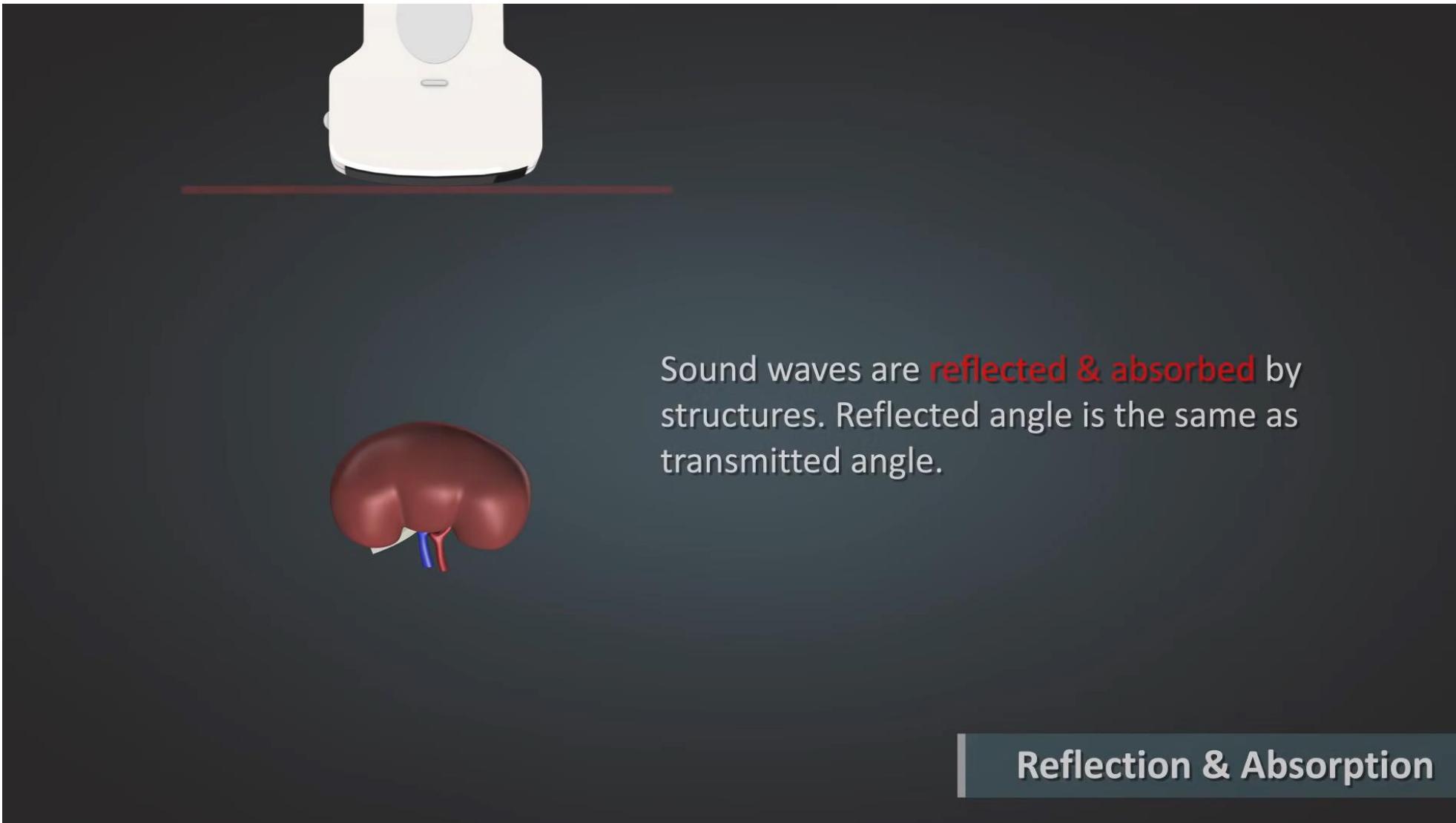
# Ultrasound Image Formation



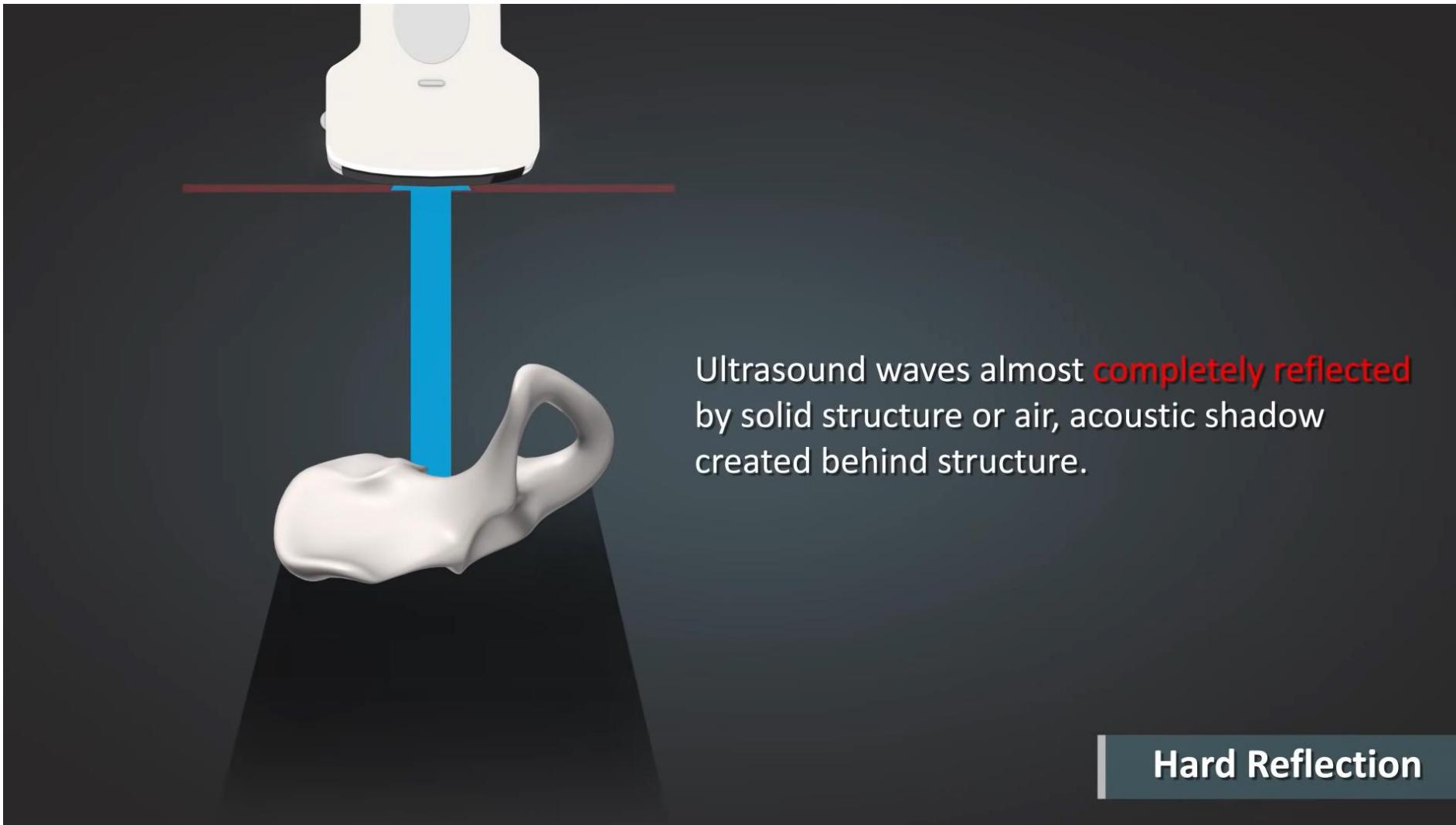
# Ultrasound Image Formation



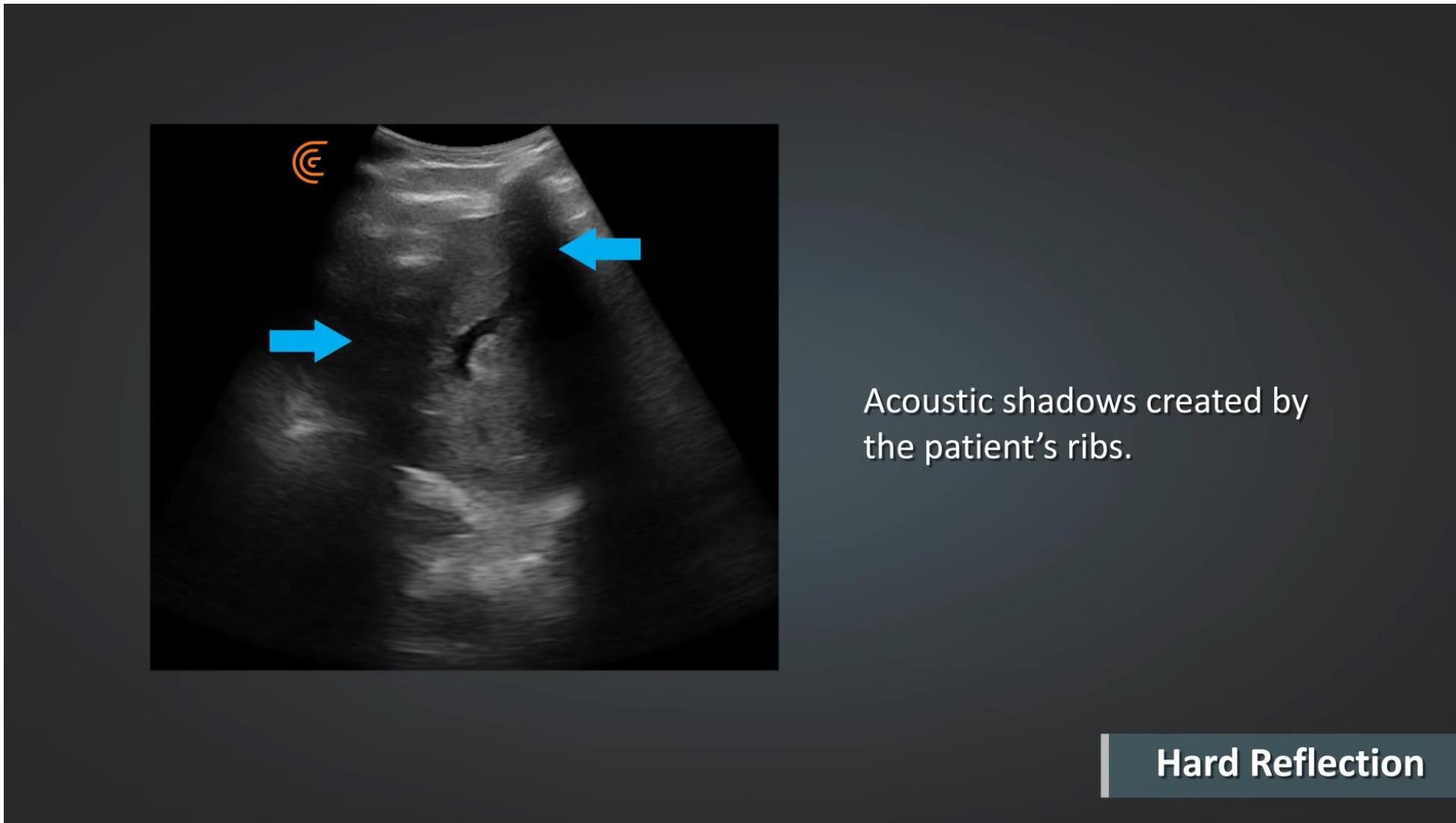
# Ultrasound Image Formation



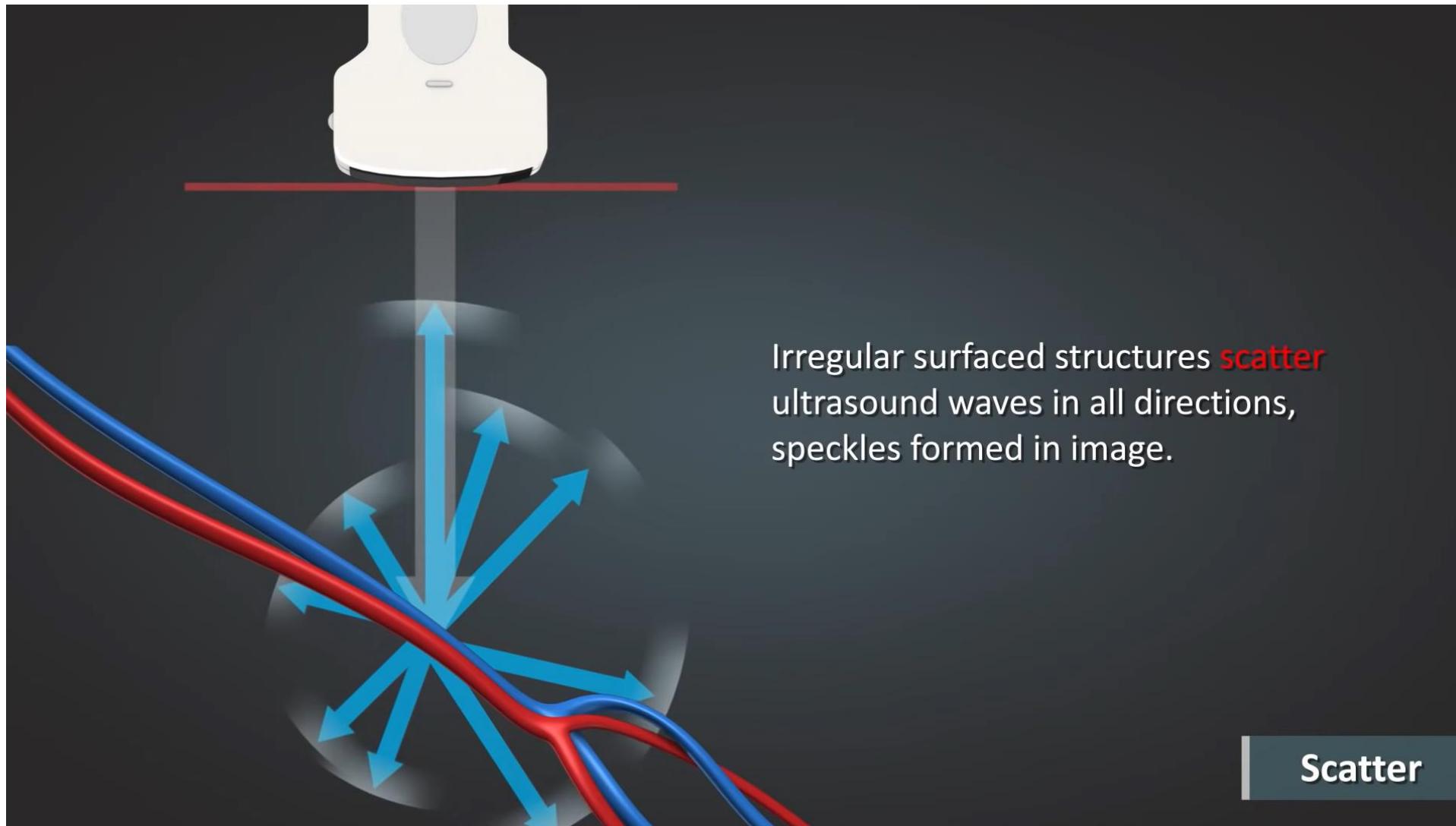
# Ultrasound Image Formation



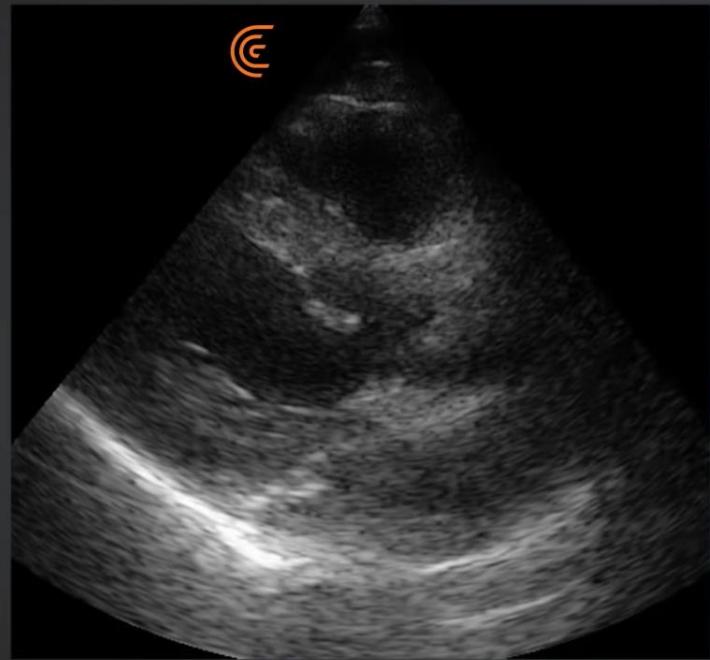
# Ultrasound Image Formation



# Ultrasound Image Formation



# Ultrasound Image Formation

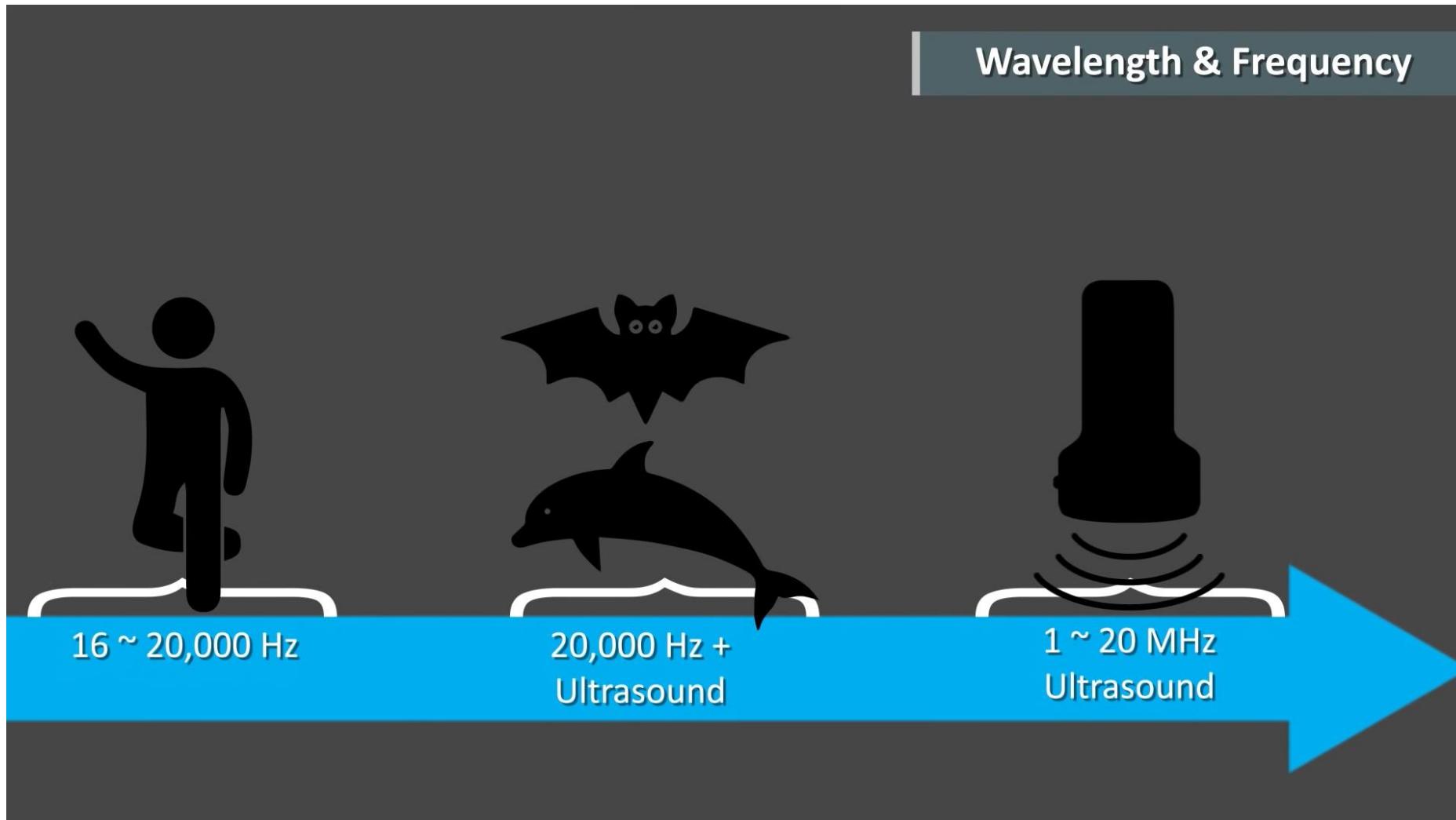


## Speckle

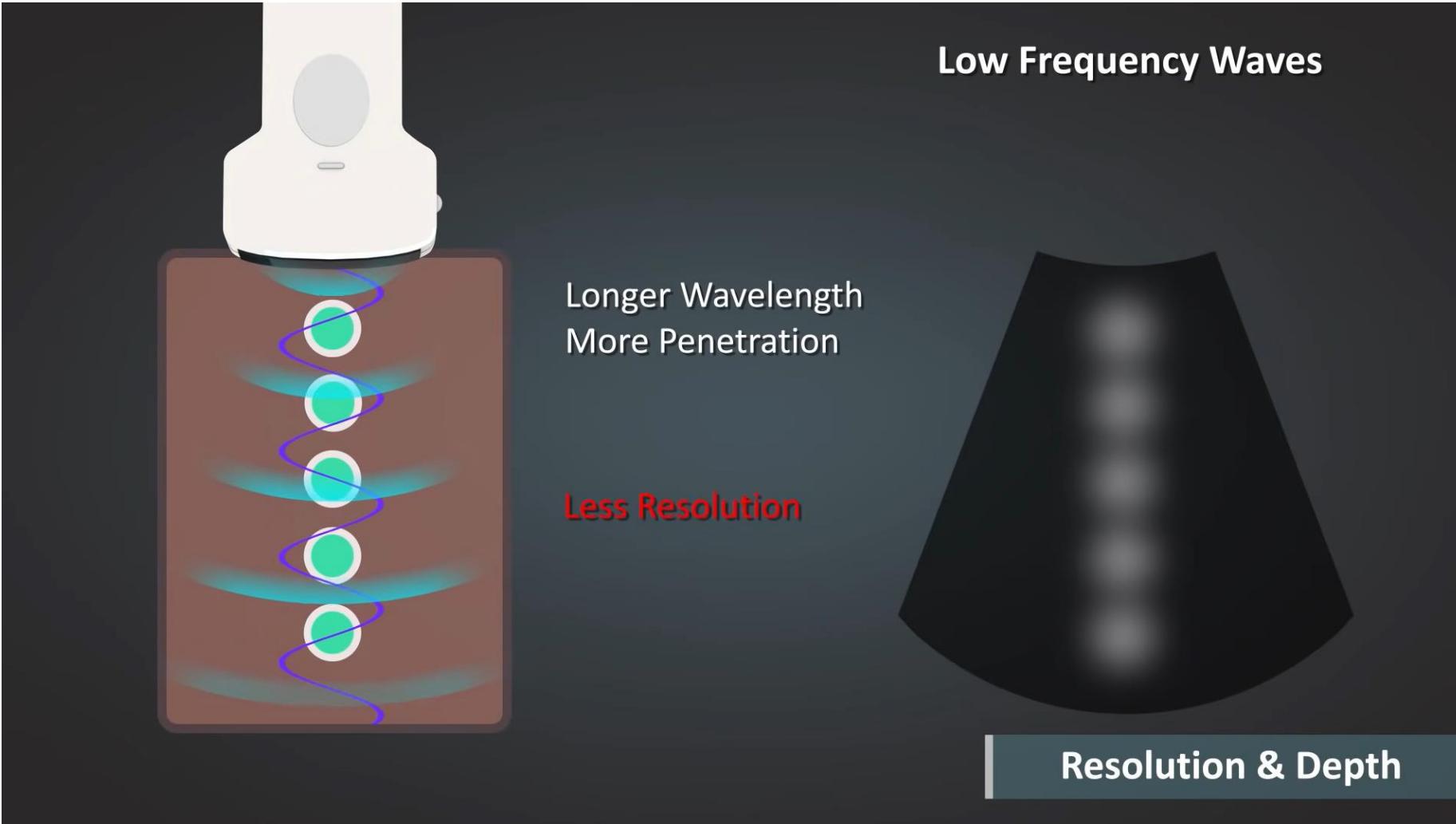
Scatter results in speckle, and produces various types of grainy appearance in ultrasound images.

Scatter

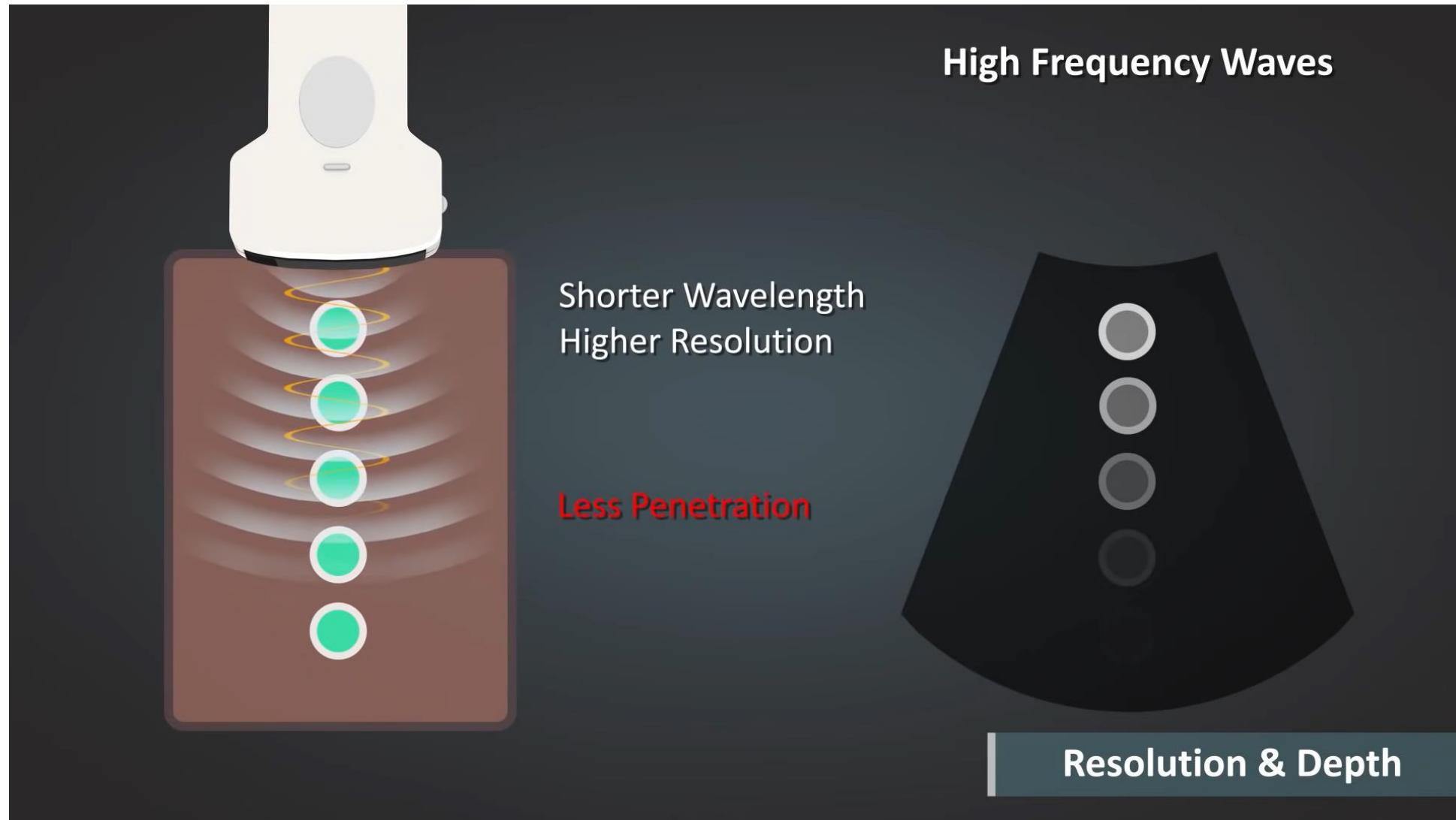
# Ultrasound Image Formation



# Ultrasound Image Formation



# Ultrasound Image Formation



# Ultrasound Image Formation

