

Intrinsic Matrix:

- The intrinsic matrix is a **3x3 matrix** that represents the **internal parameters** of the camera, such as the **focal length, aspect ratio, and principal point**.
- The intrinsic matrix is **invertible, which means that we can recover the camera's parameters from the matrix**.
- The determinant of the intrinsic matrix is equal to the product of the diagonal elements. This product is also known as the camera's "effective focal length."
- The intrinsic matrix is typically normalized such that the last element of the last row is equal to 1.

Extrinsic Matrix:

- The extrinsic matrix is a **3x4 matrix** that represents the external parameters of the camera, such as the **camera's position and orientation in the world**.
- The extrinsic matrix is **not invertible, which means that we cannot directly recover the camera's position and orientation from the matrix**.
- The first three columns of the extrinsic matrix represent the **rotation matrix** that transforms the 3D scene to the camera's coordinate system.
- The last column of the extrinsic matrix represents the camera's position in the world coordinate system**.
- The determinant of the 3x3 rotation matrix in the extrinsic matrix is always equal to 1, which means that the matrix represents a valid rotation.

Overall, the intrinsic and extrinsic matrices are essential mathematical constructs in computer vision and computer graphics that help us model the camera's parameters and transform 3D scenes to 2D images.

Intrinsic Matrix:

- The **rank of the intrinsic matrix is 2 or 3**, depending on the camera's calibration. If the camera is calibrated using a **planar object, the rank is 2**. Otherwise, if the camera is calibrated using a **non-planar object, the rank is 3**.
- The **degree of freedom of the intrinsic matrix is 4**, corresponding to the 4 internal parameters of the camera (focal length, aspect ratio, and principal point coordinates).

Extrinsic Matrix:

- The rank of the extrinsic matrix is 3**, since it is a **3x4 matrix** and the last column is not included in the rank calculation.
- The **degree of freedom of the extrinsic matrix is 6**, corresponding to the 6 external parameters of the camera (**3 for position and 3 for orientation**).

Calibration Matrix:

In computer vision, a calibration matrix (also known as camera matrix or intrinsic matrix) is a **3x3 matrix** that describes the intrinsic parameters of a camera, such as focal length, image centre, and lens distortion. Here are some mathematical properties of calibration matrix:

- A calibration matrix can be **estimated from a set of 3D points and their corresponding 2D image points using methods such as direct linear transform or iterative refinement**.

- b. A calibration matrix can be decomposed into its constituent parameters, such as focal length, skew, aspect ratio, and image centre.
- c. A calibration matrix is invertible, which means that it can be used to project 3D points onto the image plane.
- d. The rank of a calibration matrix is 2 or 3, depending on whether the camera has skew or not. If the camera has no skew, the rank is 2 because the last row is a linear combination of the first two rows. If the camera has skew, the rank is 3.
- e. The degree of freedom of a calibration matrix is 4 or 5, depending on whether the camera has skew or not. The four independent parameters are the focal length, aspect ratio, and image center coordinates. If the camera has skew, an additional parameter (the skew angle) is included, making the degree of freedom 5.

Homography Matrix:

In computer vision and image processing, a homography matrix is a 3x3 matrix that describes a projective transformation between two 2D image planes. Here are some mathematical properties of homography matrix:

- a. A homography matrix is invertible, which means that we can recover the original image coordinates from the transformed coordinates.
- b. A homography matrix can be used to represent various types of image transformations, such as rotation, translation, scaling, skewing, and perspective distortion.
- c. A homography matrix can be estimated using at least 4 correspondences between the points in the two images.
- d. The determinant of a homography matrix is not necessarily equal to 1, unlike the rotation matrix. However, the determinant is proportional to the scale factor of the transformation.
- e. The rank of a homography matrix is 2, since the matrix is a 3x3 matrix and the last row is a linear combination of the first two rows.
- f. The degree of freedom of a homography matrix is 8, corresponding to the 8 independent elements in the matrix.

Projection Matrix:

- a. A projection matrix is a combination of the intrinsic and extrinsic matrices, which represent the internal and external parameters of the camera, respectively.
- b. A projection matrix is typically represented in homogeneous coordinates, where the last row of the matrix is [0, 0, 0, 1].
- c. The projection matrix is invertible if the intrinsic matrix is invertible, which means that we can recover the 3D world coordinates from the 2D image coordinates.
- d. The projection matrix maps a 3D point in the world coordinate system to a 2D point in the image plane, using perspective projection.
- e. The rank of a projection matrix is 3, since it is a 3x4 matrix and the last column is not included in the rank calculation.
- f. The degree of freedom of a projection matrix is 11, corresponding to the 11 independent elements in the matrix. However, the intrinsic and extrinsic matrices have their own degrees of freedom (4 for intrinsic and 6 for extrinsic), which reduces the total degree of freedom to $11 - 4 - 6 = 1$. This means that the projection matrix can be uniquely determined given the camera's parameters.

Essential Matrix:

In computer vision, an essential matrix is a 3×3 matrix that relates corresponding points in two views of a scene taken by two cameras, up to an unknown scale factor. Here are some mathematical properties of essential matrix:

- An essential matrix can be estimated from a set of corresponding points in two views using methods such as normalized eight-point algorithm, RANSAC, or robust fitting.
- An essential matrix is a special case of a fundamental matrix that assumes that the two cameras have a known intrinsic calibration matrix.
- An essential matrix satisfies the epipolar constraint, which means that the corresponding points in two views lie on epipolar lines that intersect at the epipole.
- The rank of an essential matrix is 2, since it is a 3×3 matrix and the last eigenvalue is zero.
- The degree of freedom of an essential matrix is 5, corresponding to the 5 independent elements in the matrix. However, the essential matrix is only defined up to a scale factor, which reduces the degree of freedom to 4. This means that an essential matrix can be uniquely determined given 5 or more corresponding points.

Fundamental Matrix:

In computer vision, a fundamental matrix is a 3×3 matrix that relates corresponding points in two views of a scene taken by two cameras. Here are some mathematical properties of fundamental matrix:

- A fundamental matrix can be estimated from a set of corresponding points in two views using methods such as normalized eight-point algorithm, RANSAC, or robust fitting.
- A fundamental matrix is invariant to projective transformations, which means that it can be used to recover the 3D structure of a scene.
- A fundamental matrix satisfies the epipolar constraint, which means that the corresponding points in two views lie on epipolar lines that intersect at the epipole.
- The rank of a fundamental matrix is 2, since it is a 3×3 matrix and the last row is a linear combination of the first two rows.
- The degree of freedom of a fundamental matrix is 7, corresponding to the 7 independent elements in the matrix. However, the fundamental matrix is only defined up to a scale factor, which reduces the degree of freedom to $7-1=6$. This means that a fundamental matrix can be uniquely determined given 7 or more corresponding points.