Practice set 1:Constrained Optimization

Dr. Md Abu Talhamainuddin Ansary

(A). Solve the following problem using Lagrange multiplier method/substitution of one variable:

(1)

$$\begin{aligned} \max \;\; & x_1^2 + 4x_1x_2 + x_2^2 \\ s.\,t. \;\; & x_1^2 + x_2^2 = 1. \end{aligned}$$

(2)

$$\begin{aligned} \max \;\; & x_1^2 + x_2^2 + 2x_3 \\ s.\,t. \;\; & x_1 + x_2 + x_3 = 6 \\ & -x_1 + x_2 + x_3 = 4 \end{aligned}$$

(3)

$$\begin{aligned} \min \;\; & x_1^2 + 2x_2^2 \\ s.\,t. \;\; & x_1 + x_2 = 2. \end{aligned}$$

(4)

$$\begin{aligned} \min \;\; & (x_1 - 2)^2 + (x_2 - 5)^2 \\ s.\,t. \;\; & -2x_1 + x_2 = 4. \end{aligned}$$

(5)

$$\begin{aligned} \min \;\; & 2x_1^2 + x_2^2 \\ s.\,t. \;\; & 3x_1 + 2x_2 = 6 \end{aligned}$$

rest problems solve using Lagrange multiplier method only

(6)

$$\min \ 2x_1 + 3x_2 + x_3$$
$$s.\,t. \ x_1^2 + x_2^2 = 5$$
$$x_1 + x_3 = 1$$

(7) Show that the rectangular parallelepiped with surface area 64 will have maximum volume if it is a cube.

(8) Show that the rectangle with perimeter $2R$, where R is the last two digits of your roll number will have maximum diagonal if it is a square.

(9) Find the dimensions of a right circular cone of fixed lateral area with minimum volume.

(B). Justify whether the system of inequalities has a non-zero solution or not

(1) $\begin{bmatrix} -4 & 0 \\ 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(2) $\begin{bmatrix} -2 & -2 \\ 4 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & 2 \\ 4 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(C). Justify whether x^* is a Fritz-John point of the following problem or not.

(1)

$$\min \ (x_1 - 4)^2 + (x_2 - 6)^2$$
$$s.\,t. \ x_1 \geq x_1^2$$
$$x_2 \leq 4$$

$x^* = (2, 4)^T$.

(2)

$$\min \left(x_1 - \frac{3}{2}\right)^2 + (x_2 - 5)^2$$
$$-x_1 + x_2 \leq 2$$
$$2x_1 + 3x_2 \leq 11$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$x^* = (1, 3)^T$

(3)

$$\max \;\; x_1 + 3x_2$$
$$2x_1 + 3x_2 \leq 6$$
$$-x_1 + 4x_2 \leq 4$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$x^* = \left(\frac{12}{11}, \frac{14}{11}\right)^T$

(4)

$$\max \;\; (x_1 - 6)^2 + (x_2 - 2)^2$$
$$-x_1 + 2x_2 \leq 4$$
$$3x_1 + 2x_2 \leq 12$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$x^* = (2, 3)^T$

(5)

$$\begin{aligned} \min \ \ x_1^2 + x_2^2 \\ x_1^2 + x_2^2 \leq 5 \\ x_1 + 2x_2 = 4 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{aligned}$$

$x^* = (2,1)^T, (4/5, 8/5)^T$

(6)

$$\begin{aligned} \min \ \ (x_1-3)^2 + (x_2-2)^2 \\ x_1^2 + x_2^2 \leq 5 \\ x_1 + 2x_2 = 4 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{aligned}$$

$x^* = (2,1)^T, (4/5, 8/5)^T$