IIT Jodhpur

Biological Vision and Applications
Module 03-03: Bayesian Reasoning for Vision

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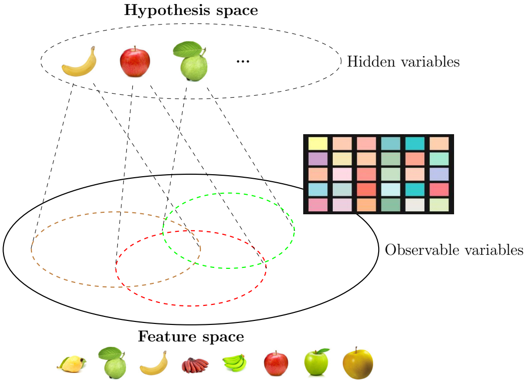
Vision is an inverted problem



- We do not see an object
  - We see the image features caused by the objects
  - We try to find best explanation for the observed image features
- This is an example of abductive reasoning
  - Bayesian reasoning is a probabilistic formulation

Baye's Theorem and Inferencing

Recap

Baye's Theorem:

$$P(A = a_i \mid B = b_j) = \frac{P(B=b_j|A=a_i).P(A=a_i)}{P(B=b_j)}$$

$$P(A \mid B) = \frac{1}{\kappa}.P(B \mid A).P(A)$$

where A and B are stochastic variables: $A = \{a_1, a_2, \ldots, a_m\}$, $B = \{b_1, b_2, \ldots, b_n\}$

- We try to infer the fruit from its color

Joint Probability Distribution

		Fruits (A)			
		Banana	**Apple**	**Guava**	**Total**
Color (B)	**Red**	0.07	0.1	0.01	*0.18*
	Green	0.21	0.04	0.07	*0.32*
	Yellow	0.42	0.06	0.02	*0.5*
	Total	*0.7*	*0.2*	*0.1*	*1*

$$P(banana \mid yellow) = \frac{0.42}{0.5} = 0.84$$

Using Baye's Theorem:

$$P(banana \mid yellow) = \frac{P(yellow|banana).P(banana)}{P(yellow)}$$
$$= \frac{\frac{0.42}{0.7} * 0.7}{0.5} = 0.84$$

Why Baye's Theorem ?

We do not have a complete knowledge about the world

		Fruits (A)				
		Banana	**Apple**	**...**	**...**	**...**
Color (B)	**Red**	0.1	0.5			
	Green	0.3	0.2			
	Yellow	0.6	0.3			
	Total	*1*	*1*			

$$P(Banana) = 0.7, P(Apple) = 0.2, P(Others) = 0.1$$

$$\underbrace{\boxed{P(banana \mid yellow)}}_{\color{red}{Posterior}} = \frac{1}{\kappa} * \underbrace{\boxed{P(yellow \mid banana).P(banana)}}_{\color{red}{Priors}}$$

$$= \frac{1}{\kappa} * 0.6 * 0.7 = \frac{1}{\kappa} * 0.42$$

How do we get to know the priors and κ ?

- Priors: $P(banana), P(yellow \mid banana)$
  - From external sources / context
  - From prior observations

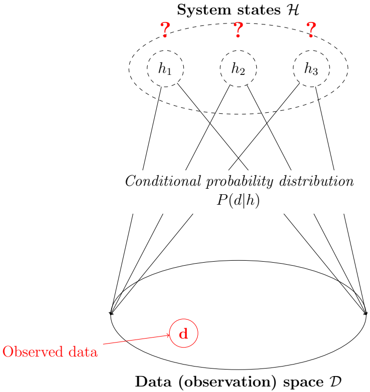
- Proportionality constant κ
  - We do not care
  - We need to find the best explanation
    - $P(banana \mid yellow) = \frac{1}{\kappa} * 0.42$
    - $P(apple \mid yellow) = \frac{1}{\kappa} * 0.06$
    - Banana is a better explanation than apple for observed yellow color

Bayesian Inference

Summary

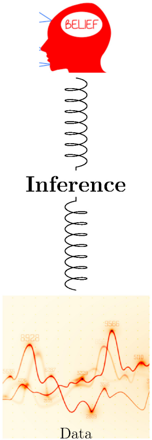
- Hypothesis space: $\mathcal{H} = \{h_1, h_2 \ldots h_m\}$
- Observable space: $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2 \ldots \mathbf{d}_n\}$
- Prior belief: $P(h_1), \ldots$
- Conditional probabilities: $P(\mathbf{d}_1 \mid h_1), \ldots$
- Observed data: $\mathbf{d} \in \mathcal{D}$

- Bayes formula: $P(h_i \mid \mathbf{d}) = \frac{P(\mathbf{d}|h_i).P(h_i)}{P(\mathbf{d})} = \frac{1}{\kappa}.P(\mathbf{d} \mid h_i).P(h_i)$
- Inference by best explanation (abduction):
  - $h^* = \text{argmax}_{h_i \in \mathcal{H}} P(h_i \mid \mathbf{d})$



Belief Revision

Prior belief and evidence



$$\textbf{Baye's Theorem: } P(h_i \mid \mathbf{d}) = \frac{1}{\kappa}.\underbrace{\boxed{P(h_i)}}_{\textit{Prior belief}}.\underbrace{\boxed{P(\mathbf{d} \mid h_i)}}_{\textit{Evidential support}}$$

- Bayesian inference is a synthesis of prior belief and evidence from observation
  - Key advantage over pure data-driven (machine learning) approach
- Challenge:
  - Strong prior belief: Takes lots of evidence to offset it
  - Weak prior belief: Susceptible to noisy data

Odds and log-Odds

$$\textbf{odds}(banana, apple \mid yellow) = \frac{P(banana|yellow)}{P(apple|yellow)}$$
$$= \frac{\frac{1}{\kappa} * 0.42}{\frac{1}{\kappa} * 0.06} = 7$$

$$\textbf{logodds}(banana, apple \mid yellow) = log\frac{P(banana|yellow)}{P(apple|yellow))}$$
$$= log(0.42) - log(0.06)$$
$$\approx (-0.38) - (-1.22) = 0.84$$

* log base assumed to be 10

- Useful for comparing the plausibility of pairs of concepts

Composite data

Data item $\mathbf{d}$ may be composite: $\mathbf{d} = (d_1, d_2, \ldots, d_n)$

		Fruits (A)				
		Banana	Apple	...	...	...
Color (B)	Red	0.1	0.5			
	Green	0.3	0.2			
	Yellow	0.6	0.3			
	Total	*1*	*1*			

		Fruits (A)				
		Banana	Apple	...	...	...
Shape	Long	0.8	0.3			
	Round	0.2	0.7			
	Total	*1*	*1*			

$$P(Banana) = 0.7, P(Apple) = 0.2, P(Others) = 0.1$$

- $\mathcal{D} = \{(Red, Long), (Red, Round), \ldots\}$
  - Conditionals: $P(Red, Long \mid Banana), \ldots$
  - Combinatorial explosion of data space makes modeling difficult
  - Data becomes sparse: there may be little data available for some rare combinations
- Assuming conditional independence of features

$$P(\mathbf{d} \mid h_i) = P(d_1 \mid h_i).P(d_2 \mid h_i).\ldots.P(d_n \mid h_i)$$

$$P(h_i \mid \mathbf{d}) = \frac{1}{\kappa}.P(h_i).\prod_{k=1}^{n} P(d_k \mid h_i)$$

$$\mathbf{logodds}(h_i, h_j \mid \mathbf{d}) = P(h_i) - P(h_j) + \sum_{k=1}^{n}(P(d_k \mid h_i) - P(d_k \mid h_j))$$

Advantages of modeling with Elementary data items

- Easier to model the statistical dependency of a hypothesis h_i with an elementary data item d_k than the composite d
  - Model size is additive, rather than combinatorial
  - Statistically more dependable
- Robust inference can be made with a subset of observations
  - Robust against missing / erroneous observations
  - Generally, it is possible to use a few discriminatory data elements

Example: Robust inference

- To recognize the object as a car, you need not consider all visual features of a car
  - Robust against occlusions, etc.

- You can reconstruct the contour of the occluded part of the image

- Can it be done with deductive reasoning?

Incremental belief update

- $P(h_i \mid \mathbf{d}) = \frac{1}{\kappa}.P(h_i).P(\mathbf{d} \mid h_i)$
- Assume that $\mathbf{d} = d_1, d_2, d_3, \ldots$ represents a data stream (possibly infinite)
- After d_1 arrives
  - Posterior: $P(h_i \mid d_1) = \frac{1}{\kappa_1}.P(h_i).P(d_1 \mid h_i)$
  - This posterior becomes the prior for the second observation
- After d_2 arrives
  - Posterior: $P(h_i \mid d_1, d_2) = \frac{1}{\kappa_2}.P(h_i \mid d_1).P(d_2 \mid h_i) = \frac{1}{\kappa_{12}}.P(h_i).P(d_1 \mid h_i).P(d_2 \mid h_i)$
  - This posterior becomes the prior for the third observation
- ... and so on
- System updates it's belief incrementally
  - Does sequence matter ?
- In practice, it may be possible to infer even before all data arrives

Example

		Fruits (A)				
		Banana	Apple	...	...	...
Color (B)	Red	0.1	0.5			
	Green	0.3	0.2			
	Yellow	0.6	0.3			
	Total	*1*	*1*			

		Fruits (A)				
		Banana	Apple	...	...	...
Shape	Long	0.8	0.3			
	Round	0.2	0.7			
	Total	*1*	*1*			

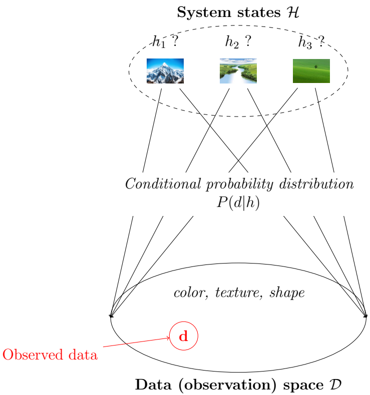
$$P(Banana) = 0.7, P(Apple) = 0.2, P(Others) = 0.1$$

- We see a green and long fruit.
  - Is it a banana or an apple ?
- Solution sketch:
  - Start with prior beliefs $P(banana)$ and $P(apple)$
  - "Observe" features in any order
    - Revise posterior beliefs for the fruits progressively
  - Check whichever is higher

- Alternatively, use **odds**() / **logodds**()

Emergent knowledge

- We observe d
  - Visual patterns: color, texture, shape
- We infer h
  - Semantic concepts: mountain, river, greenery
- The inferred entities are of <u>different kind</u> than the observed entities
- New knowledge is created
- Paradigm applicable to higher layers of cognition also



Limitation of Bayesian reasoning

- We cannot infer an entity unless we have a model for it
  - The fruit was green and round. Was it really a guava ?
- A way to cope up for new concepts
  - Assume uniform probability distribution to begin with
  - Learn with experience
- Results are good only if
  - Prior belief is good
  - Model (conditionals) is good
  - Data (observation) is good
- Robust against imperfect priors / models / noisy data
  - We need best explanation, not accurate probability values

Does human mind follow Bayesian reasoning ?

- Both Yes & No

EdPuzzle: Cognitive bias

Quiz

Quiz 03-03

End of Module 03-03