

Intro to

Computer Vision

Image filtering

filter

ϕ

filter

I

image

ϕ

filter

$$I \xrightarrow{\hspace{1cm}} \phi \xrightarrow{\hspace{1cm}} \phi(I)$$

image

image

Image

NOISE

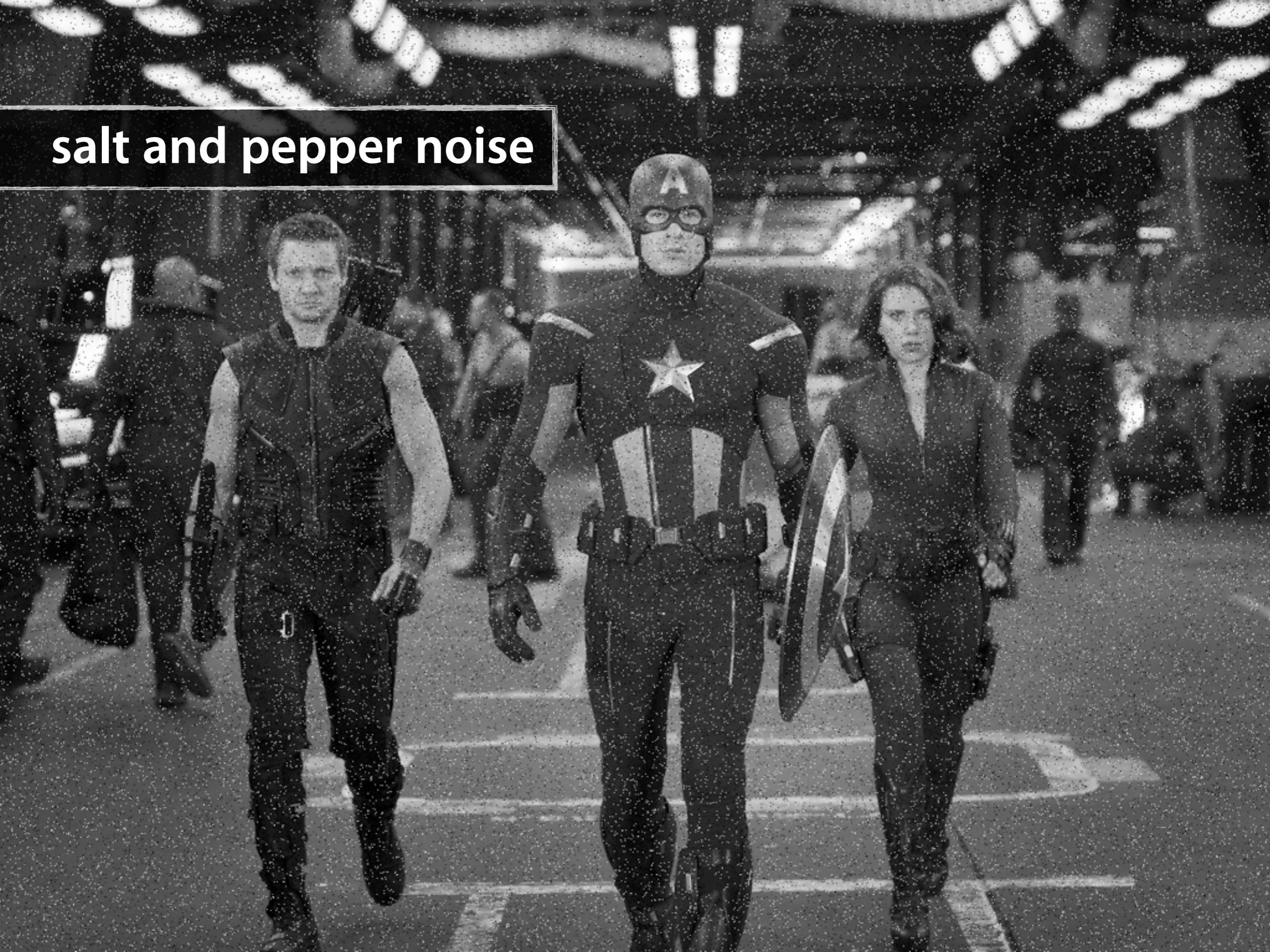
Image

NOISE

original



salt and pepper noise



Gaussian noise



image
noise

$$I(x, y) = I_{\text{ideal}}(x, y) + \eta(x, y)$$

image
noise

$$I(x, y) = I_{\text{ideal}}(x, y) + \eta(x, y)$$

where

$$\eta(x, y) \sim \mathcal{N}(\mu = 0, \sigma)$$

image
noise

$$I(x, y) = I_{\text{ideal}}(x, y) + \eta(x, y)$$

where

$$\eta(x, y) \sim \mathcal{N}(\mu = 0, \sigma)$$

noise is assumed **independent and identically distributed**

image
noise

$$I(x, y) = I_{\text{ideal}}(x, y) + \eta(x, y)$$

where

$$\eta(x, y) \sim \mathcal{N}(\mu = 0, \sigma)$$

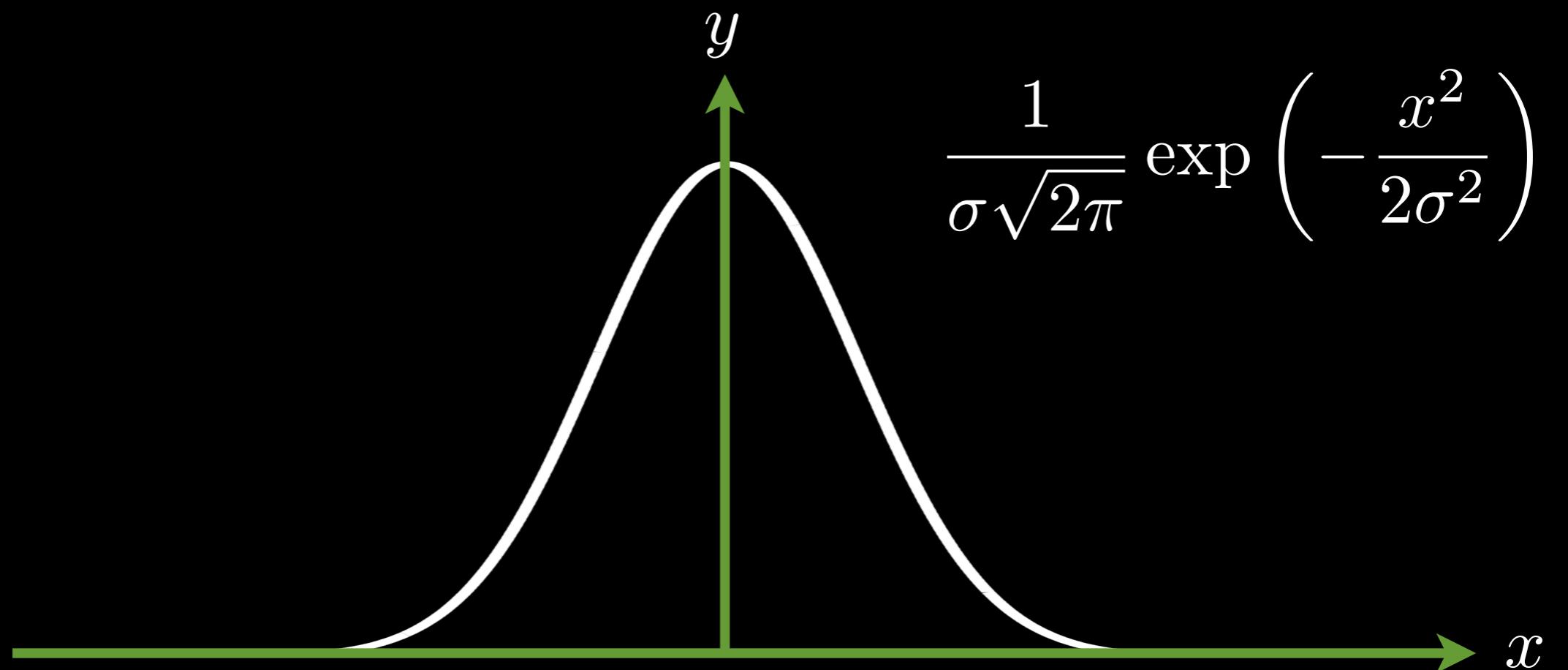
noise is assumed **independent and identically distributed**

I.I.D.

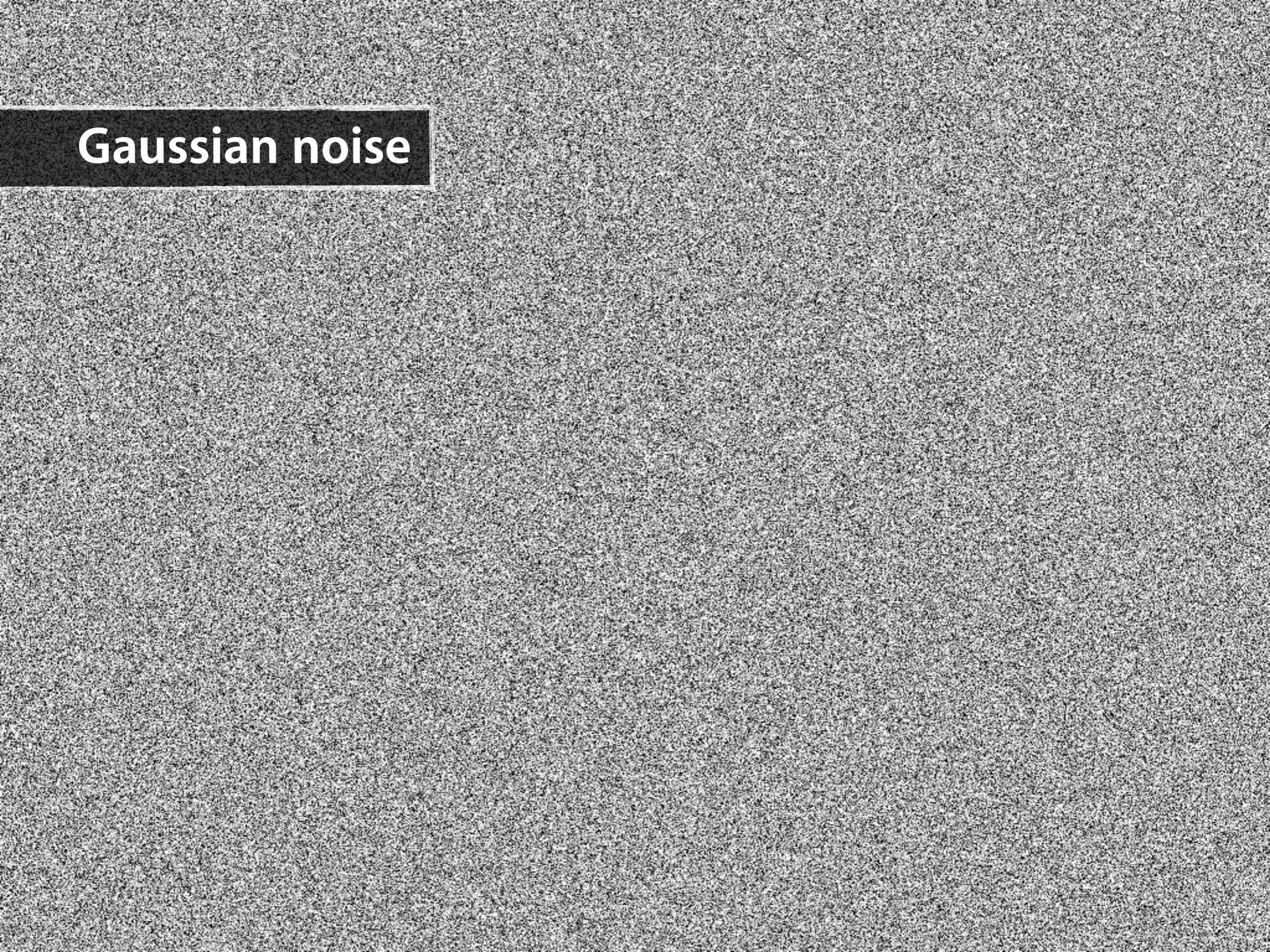
$$I(x, y) = I_{\text{ideal}}(x, y) + \eta(x, y)$$

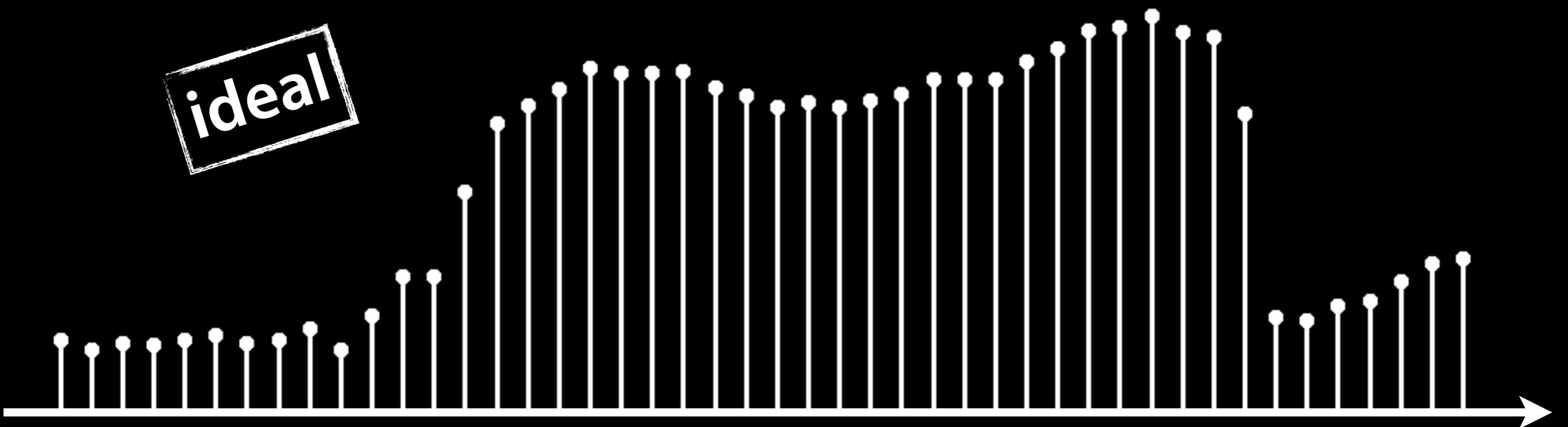
where

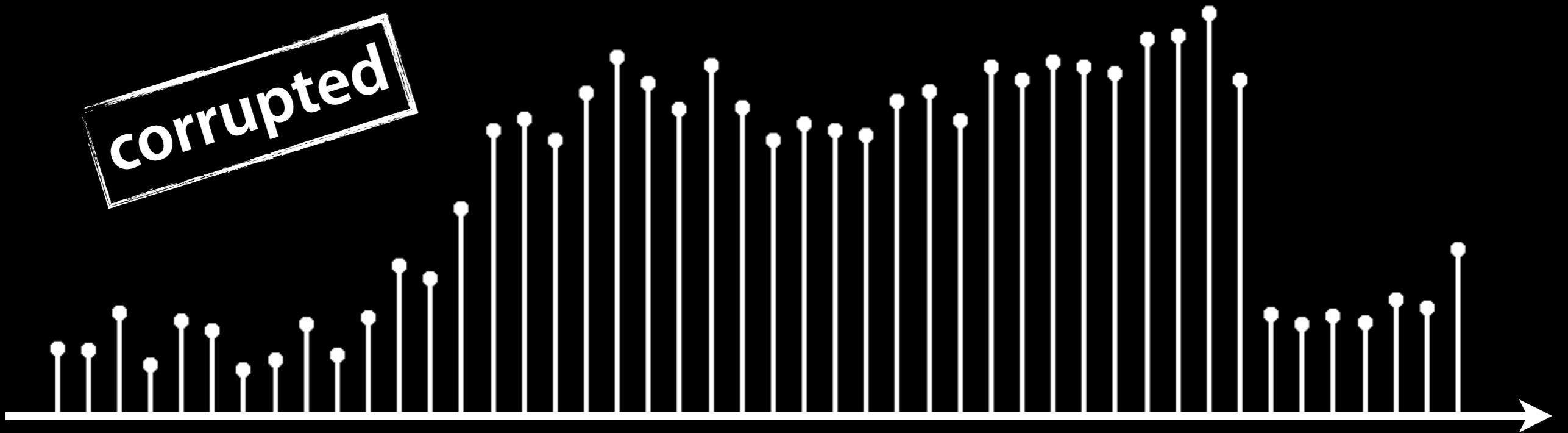
$$\eta(x, y) \sim \mathcal{N}(\mu = 0, \sigma)$$



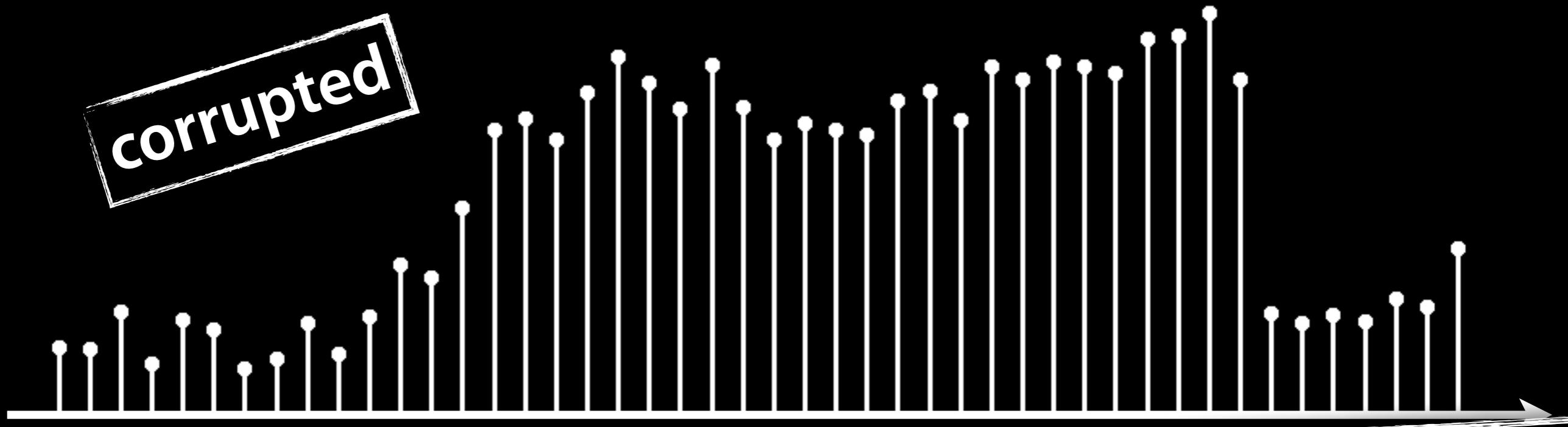
Gaussian noise





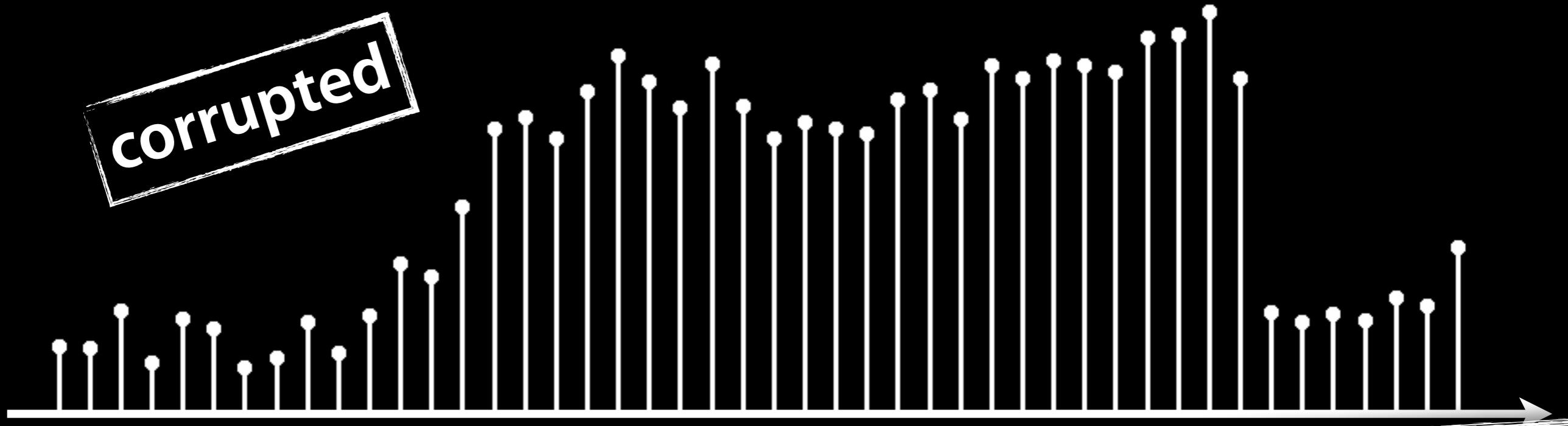


corrupted

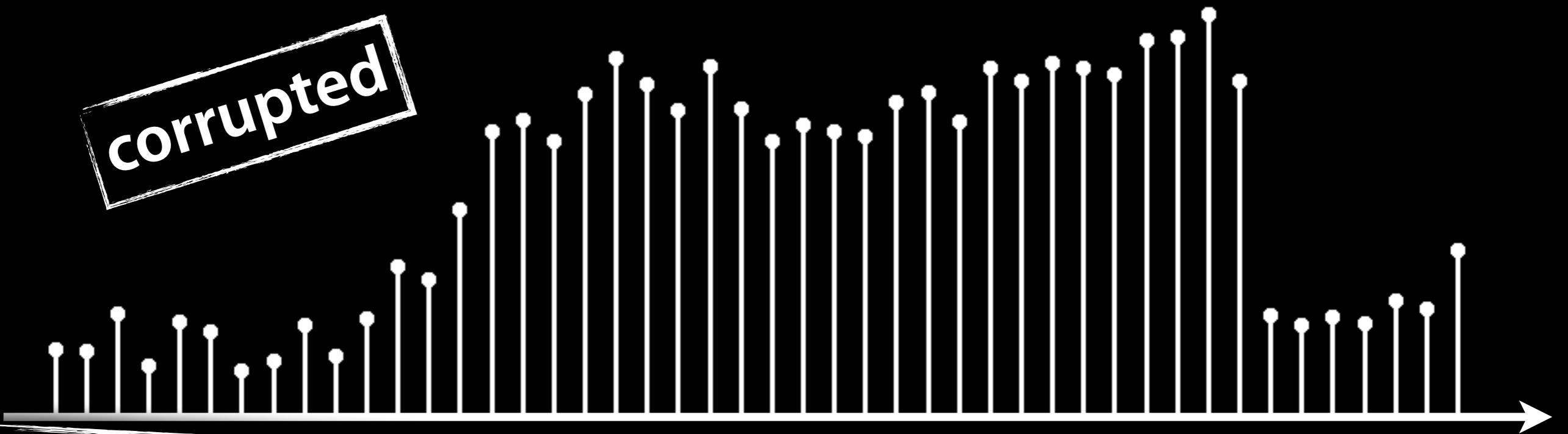


How can we remove the noise?

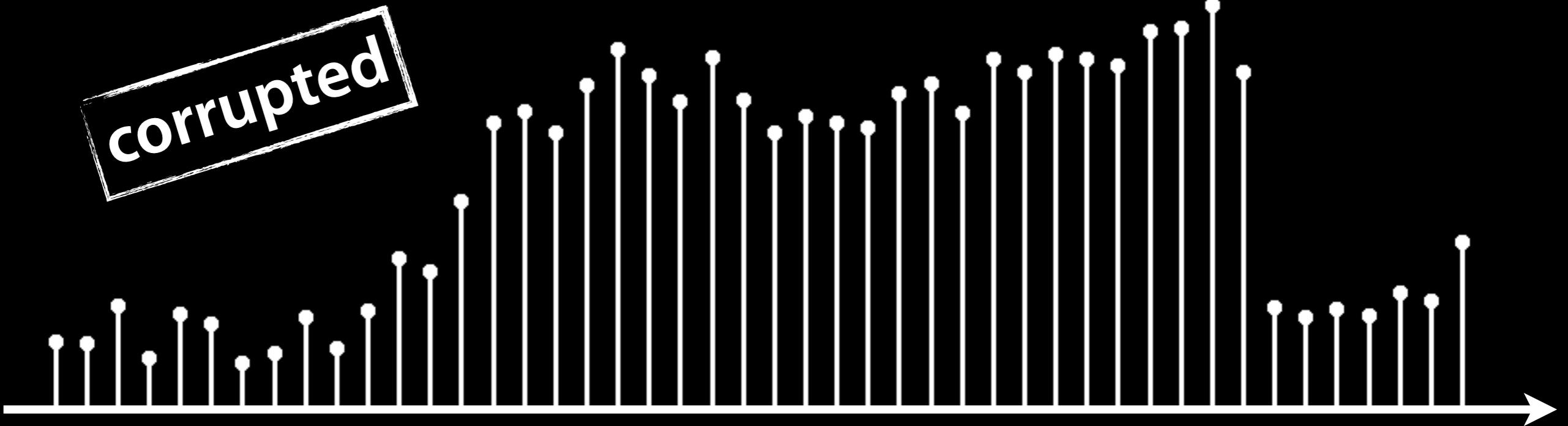
corrupted



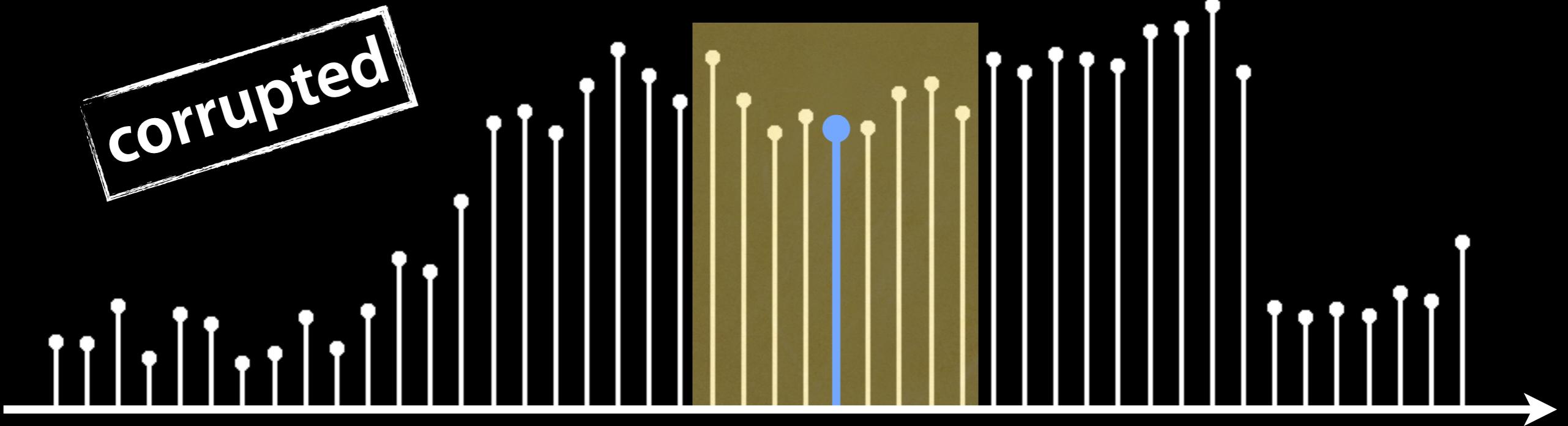
How can we remove the noise?



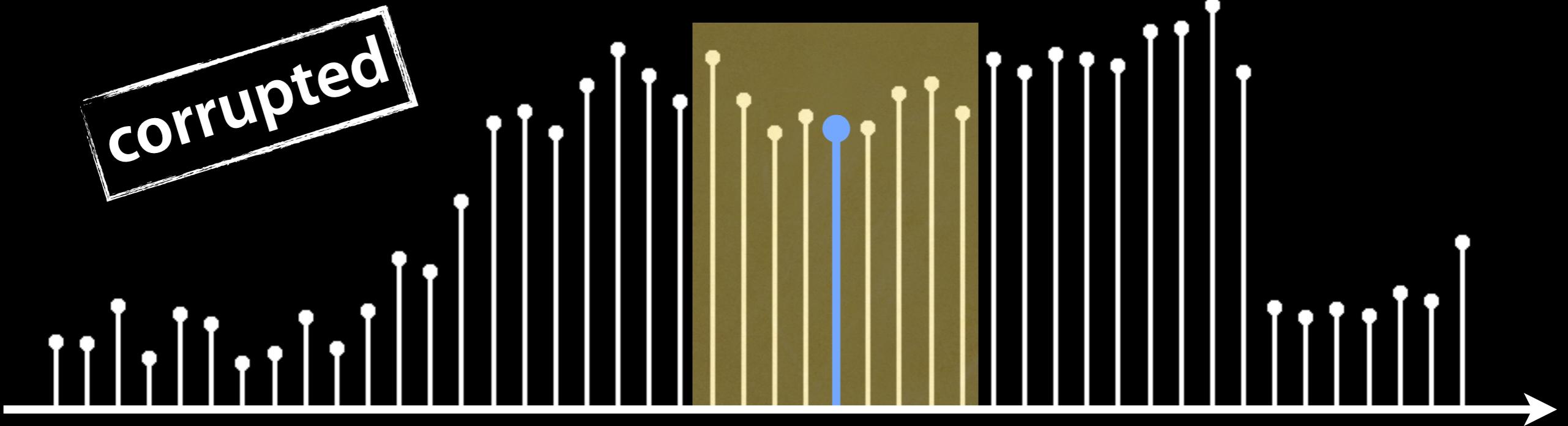
assume noise is I.I.D. and pixel neighbours are similar



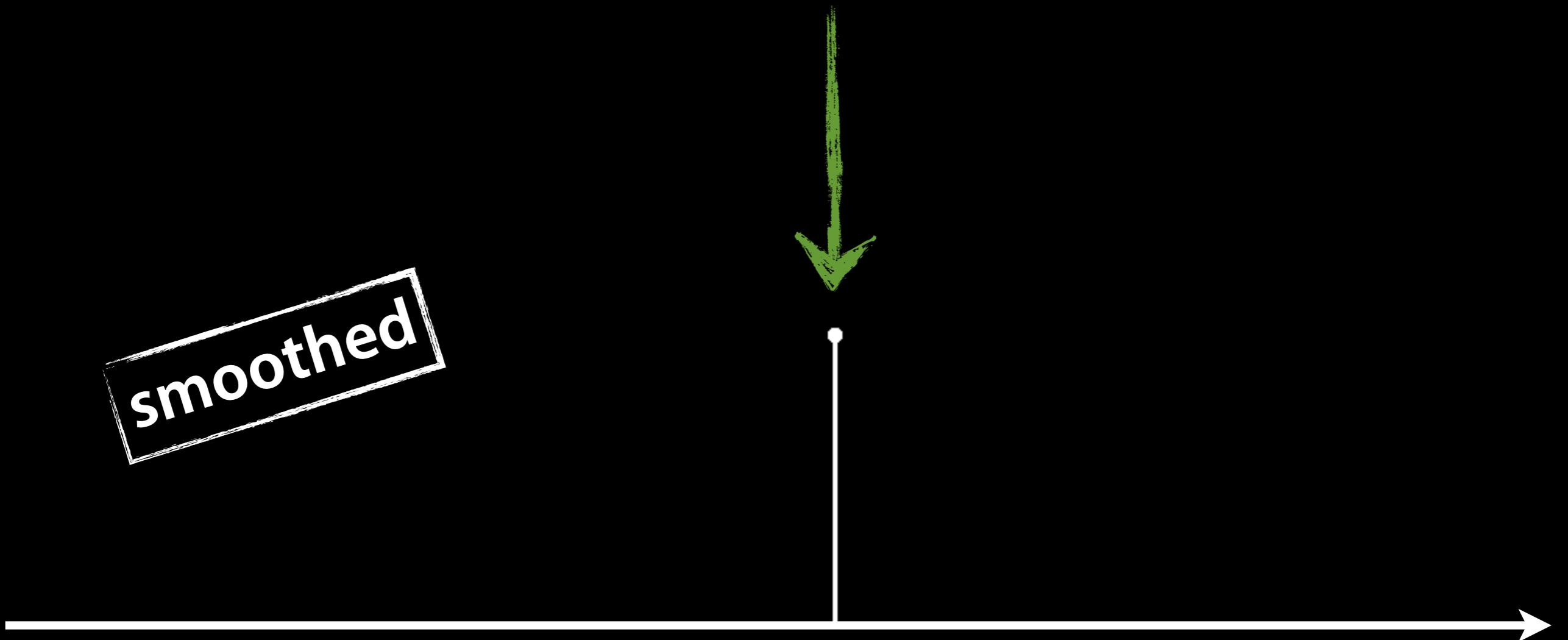
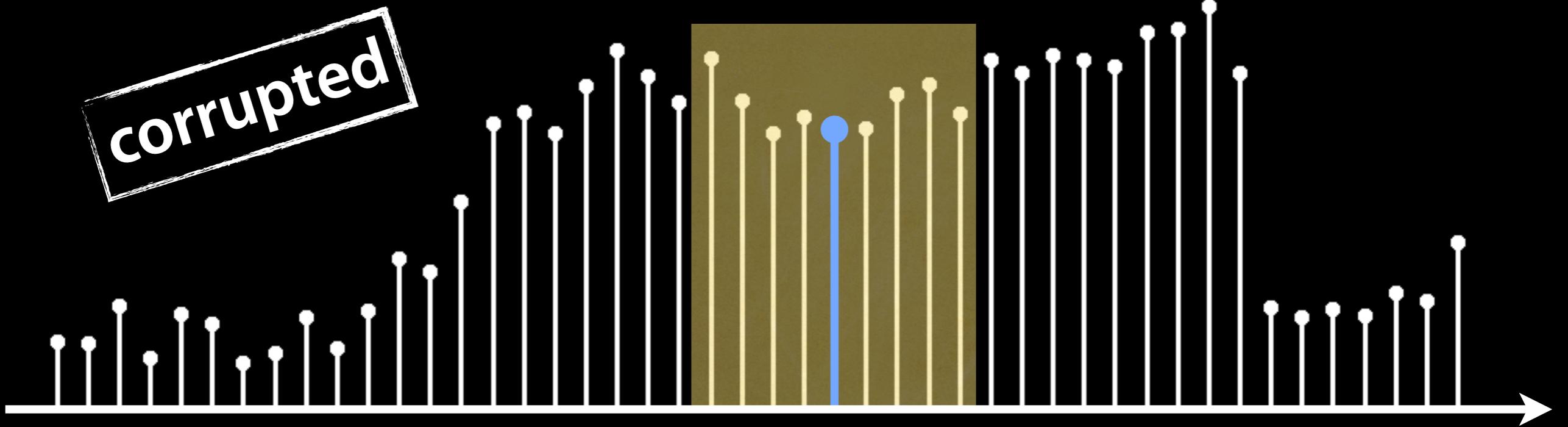
Let's replace each pixel with an average of its neighbours.

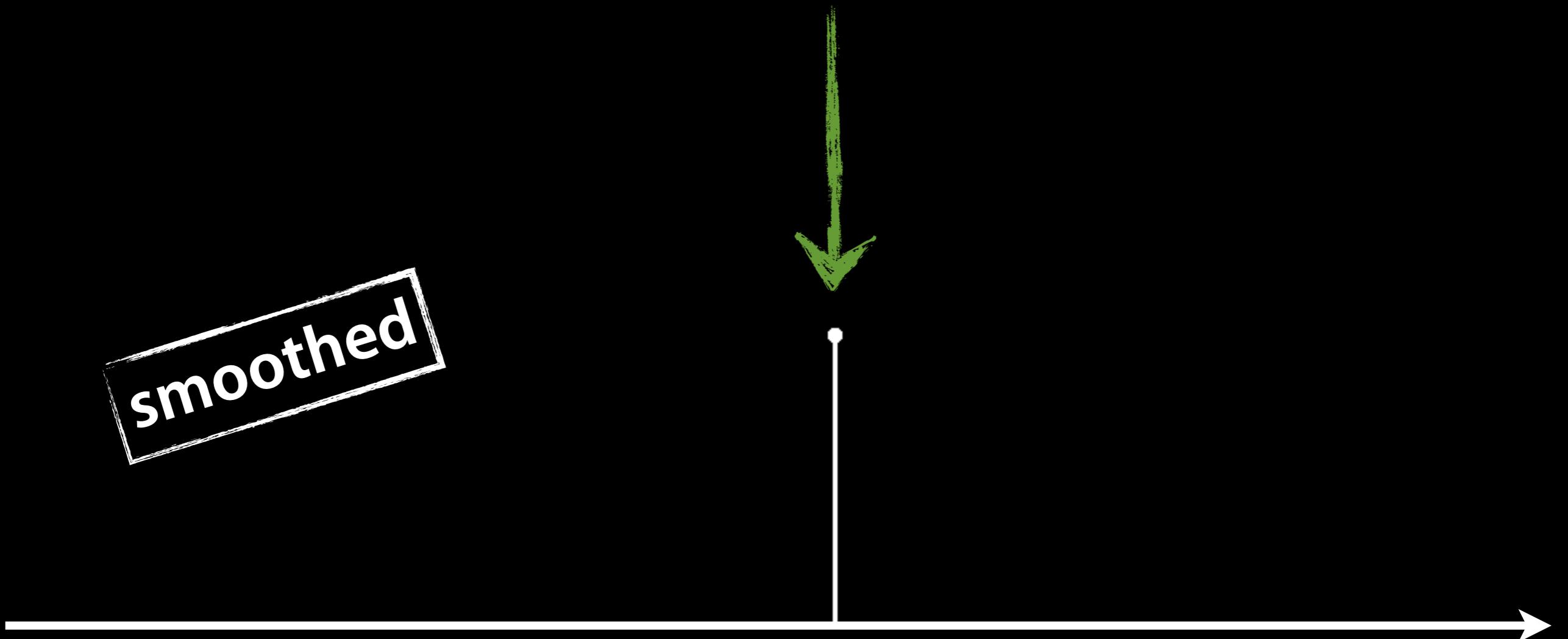
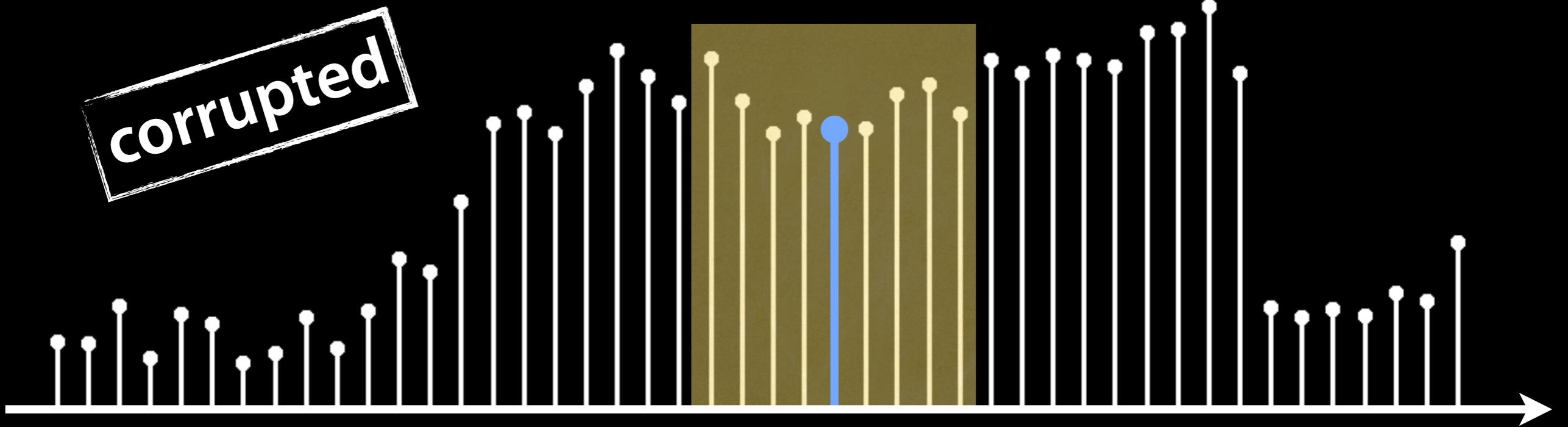


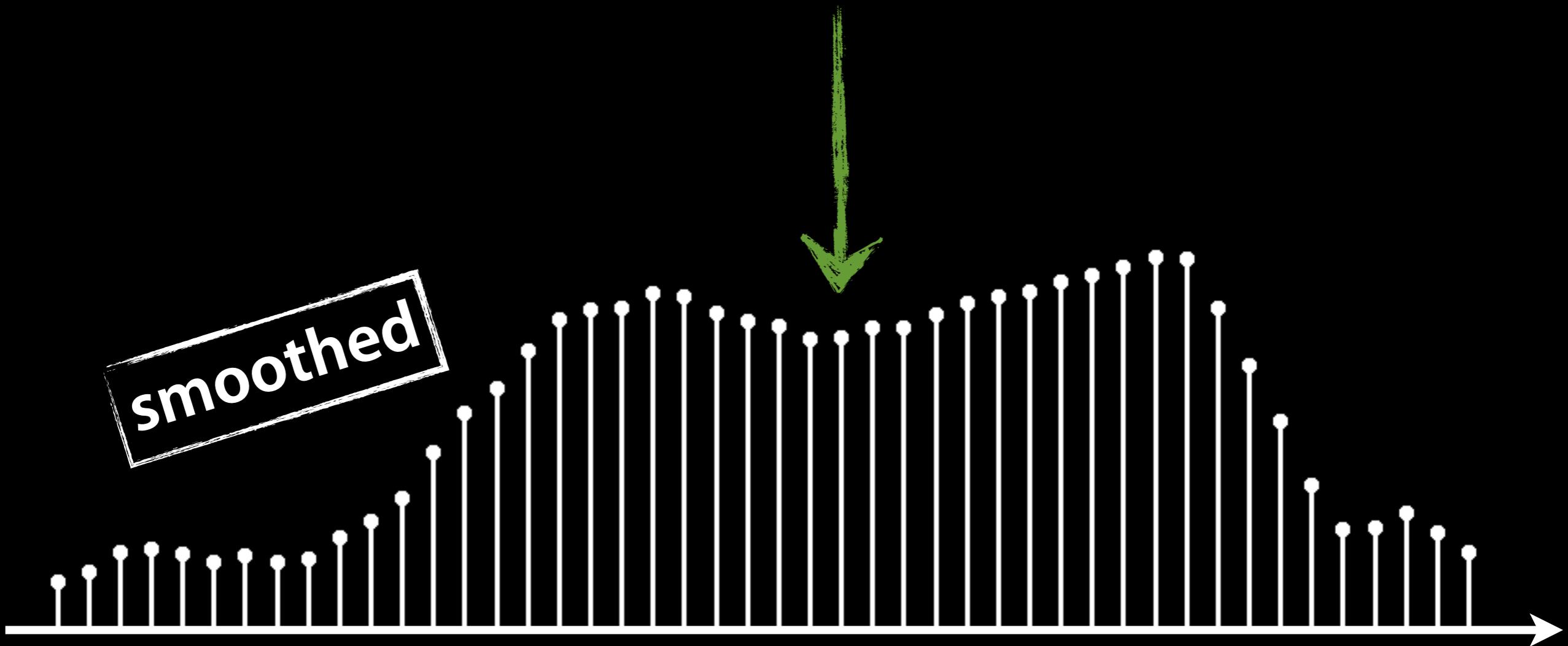
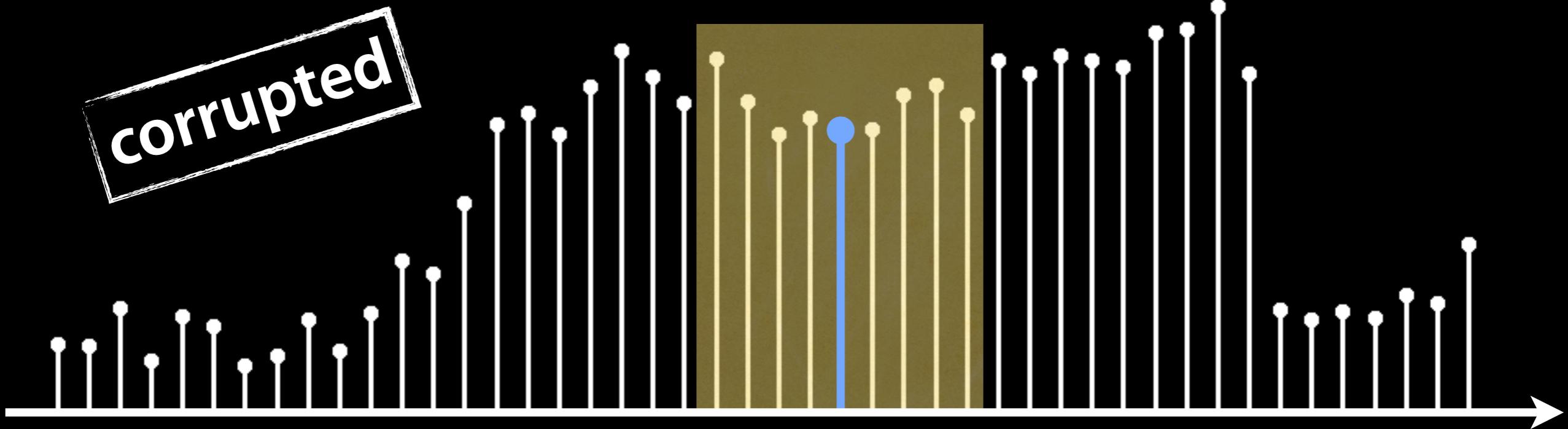
Let's replace each pixel with an average of its neighbours.

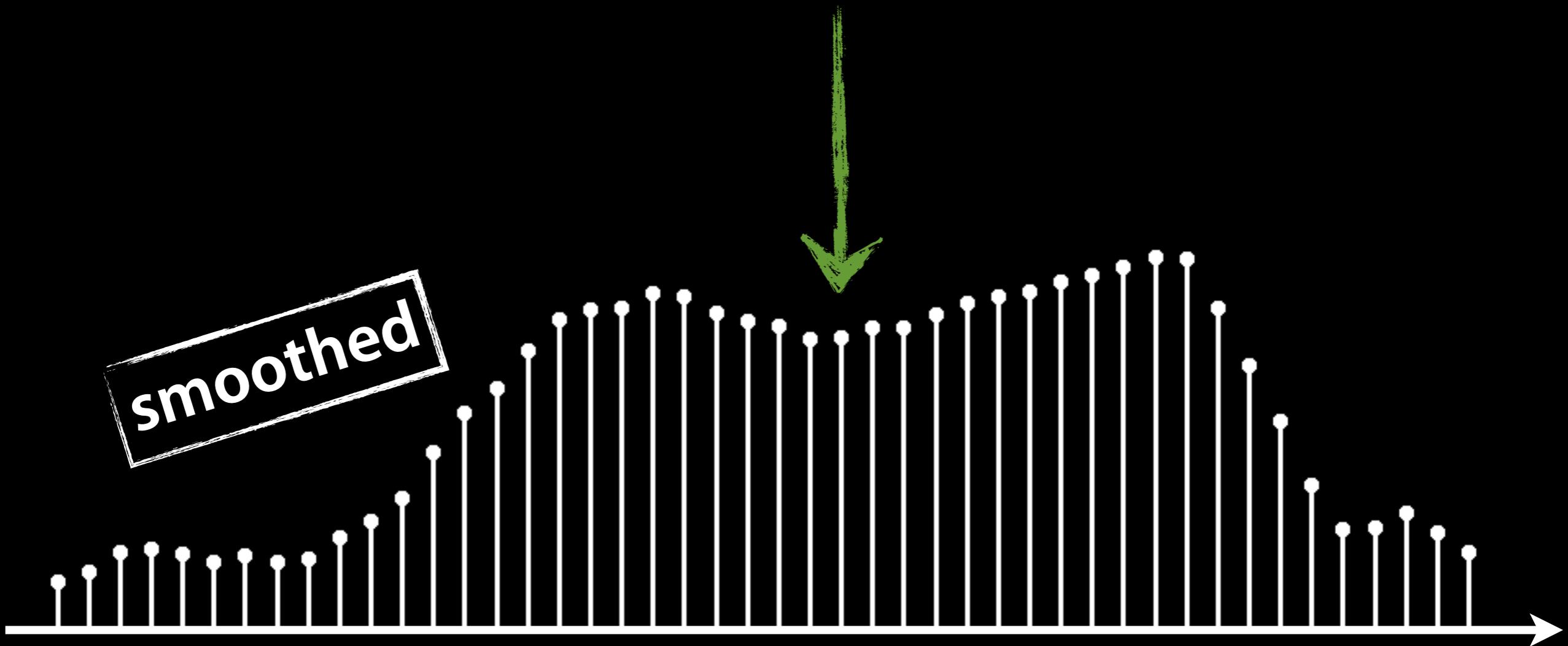
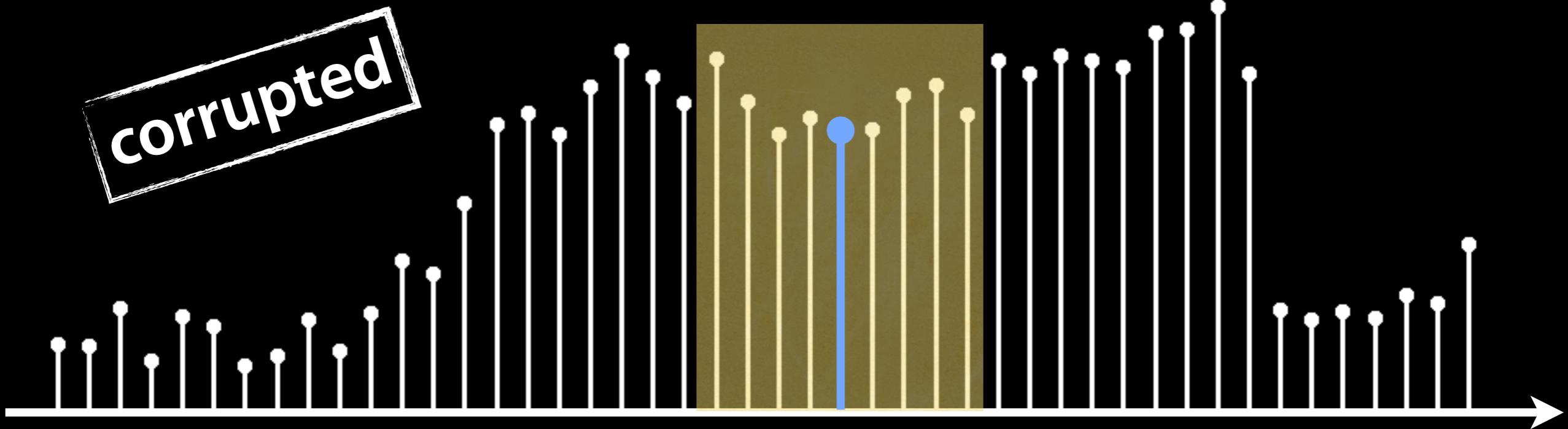


Let's replace each pixel with an average of its neighbours.

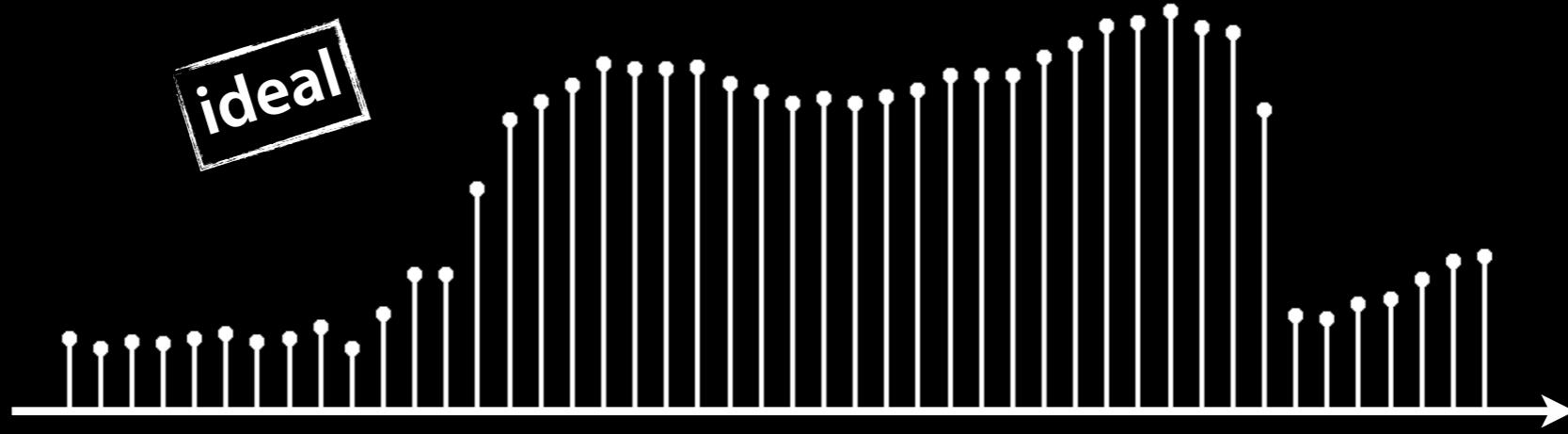




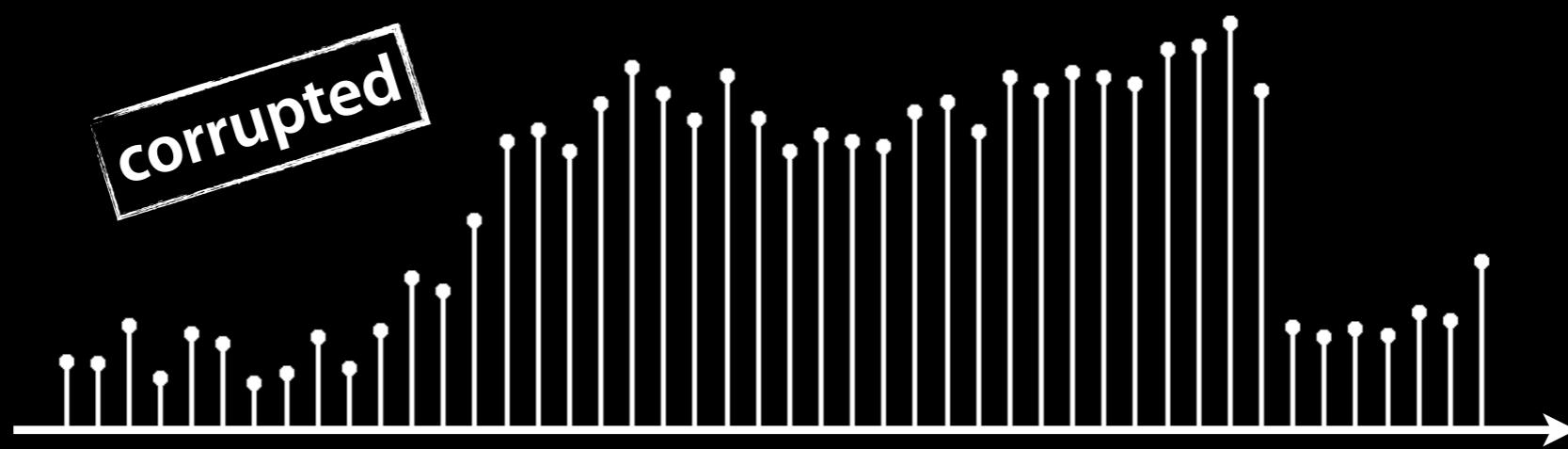




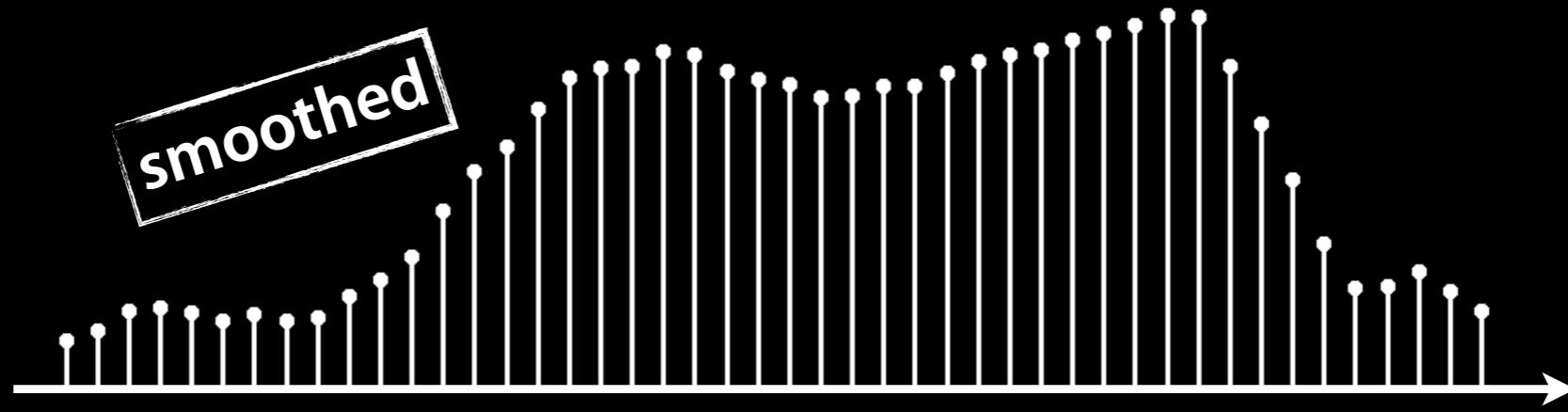
ideal



corrupted



smoothed



moving 2D
average

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

input

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

input

smoothed

moving 2D average

input

A 10x10 grid of black squares, arranged in 10 rows and 10 columns, separated by white borders.

smoothed

moving 2D average

input

A 10x10 grid of black squares with a white border. A single square at the top-left corner is highlighted with a green double-lined border.

smoothed

moving 2D average

input

A 10x10 grid of black squares. The square located at the top-left corner, which corresponds to the coordinates (1,1), is highlighted with a green border. All other squares in the grid are solid black.

smoothed

moving 2D average

input

A 10x10 grid of black squares. The square at position (1,1) is highlighted with a green border and contains the number 0.

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

input

			0							

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

input

0	10										

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

input

			0	10							

smoothed

moving 2D average

input

A 10x10 grid of black squares with white borders. A green square with a white double-lined border is centered at the intersection of the 5th row and 5th column. In the top-left corner of the grid, there are two dark gray squares containing the numbers '0' and '10' respectively.

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

input

			0	10	20						

smoothed

moving 2D average

input

0 10 20

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

input

			0	10	20				

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

input

			0	10	20	30			

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

input

0	10	20	30						

smoothed

moving 2D average

input

A 10x10 grid of black squares. In the top row, the first four squares contain the numbers 0, 10, 20, and 30 respectively. The square containing 30 is highlighted with a green border.

smoothed

moving 2D average

input

A 10x10 grid of black squares. Four squares are highlighted with different shades of gray: a dark gray square at (0,0), a medium gray square at (0,1), a light gray square at (0,2), and a lightest gray square with a green border at (0,3). The numbers 0, 10, 20, and 30 are printed in white on their respective squares.

smoothed

moving 2D average

input

A 10x10 grid of black squares. The first four columns contain the numbers 0, 10, 20, and 30 respectively, each centered in its column. The fifth column contains the number 30, which is also centered but is highlighted by a green square background.

smoothed

moving 2D
average

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

input

			0	10	20	30	30	30	20	10
			0	20	40	60	60	60	40	20
			0	30	60	90	90	90	60	30
			0	30	50	80	80	90	60	30
			0	30	50	80	80	90	60	30
			0	20	30	50	50	60	40	20
			10	20	30	30	30	30	20	10
			10	10	10	0	0	0	0	0

smoothed

Let the averaging window size be $(2K + 1) \times (2K + 1)$

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$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

output

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

input

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

loop over pixels in
neighbourhood

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

weight

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \frac{1}{(2K + 1)^2} \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v]$$

How can **DIFFERENT WEIGHTS** be included based on neighbourhood position?

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v] G[u, v]$$

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \sum_{u=-K}^K \sum_{v=-K}^K F[x + u, y + v] G[u, v]$$

mask, kernel or filter

Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x + u, y + v] G[u, v]$$

This is called **cross correlation, denoted $H = G \otimes F$**

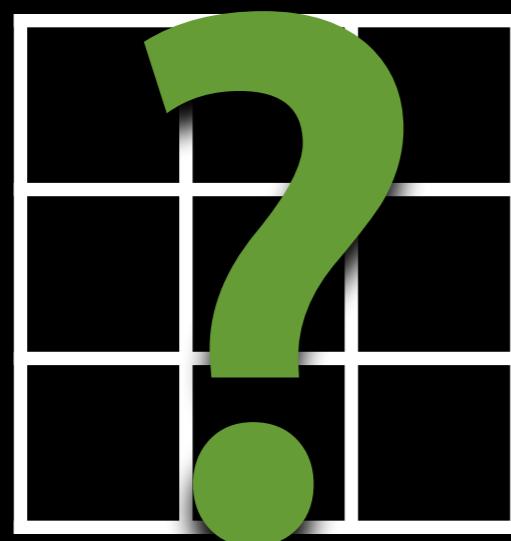
Let the averaging window size be $(2K + 1) \times (2K + 1)$

$$H[x, y] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} F[x + u, y + v] G[u, v]$$

This is called **cross correlation**, denoted $H = G \otimes F$

Replace each pixel with a linear combination of its neighbours

average
filtering



$G[x, y]$

\otimes

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

average
filtering

1	1	1
1	1	1
1	1	1

$$G[x, y]$$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$F[x, y]$$

average
filtering

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \otimes G[x, y]$$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$F[x, y]$$

original



original

box filter

original



smooth by averaging

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 0 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F[x, y]$$

$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

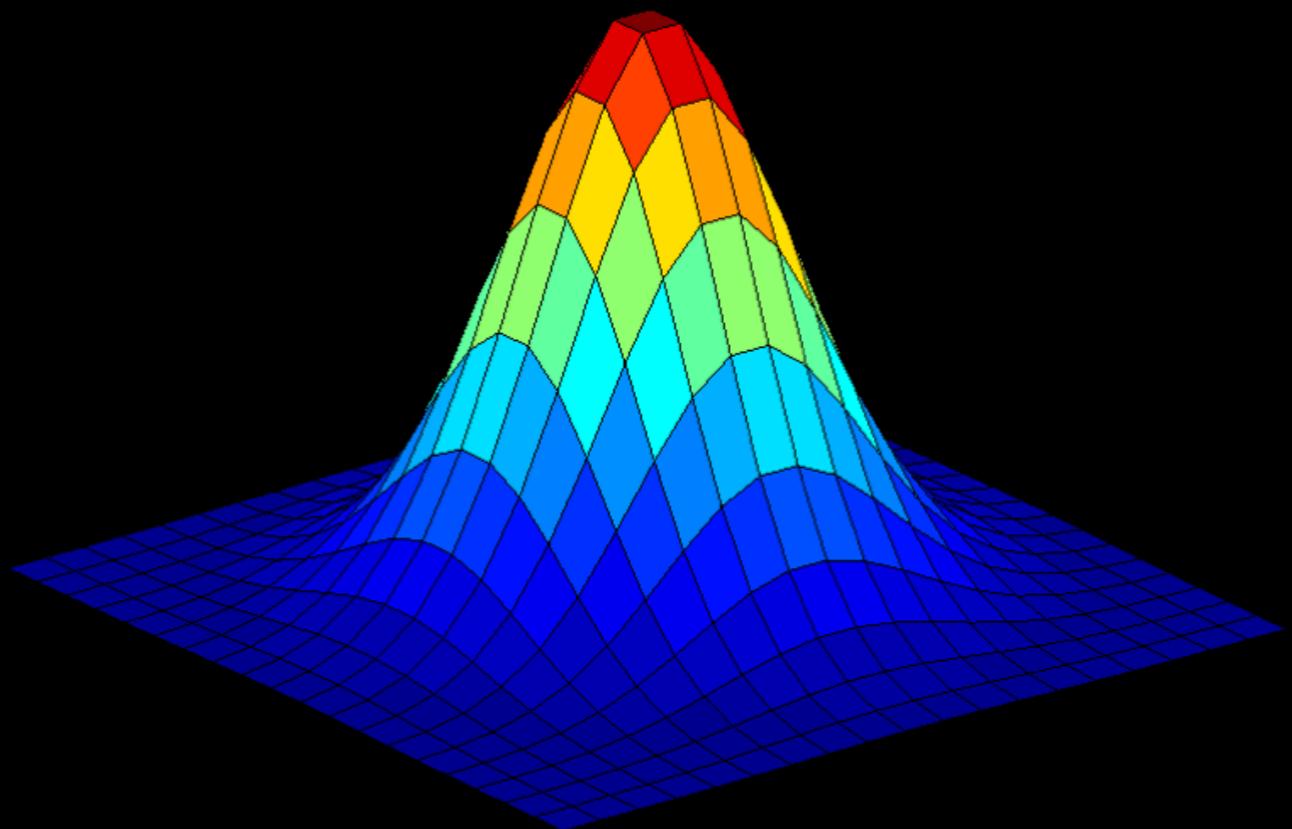
 $G[x, y]$ 

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

 $F[x, y]$

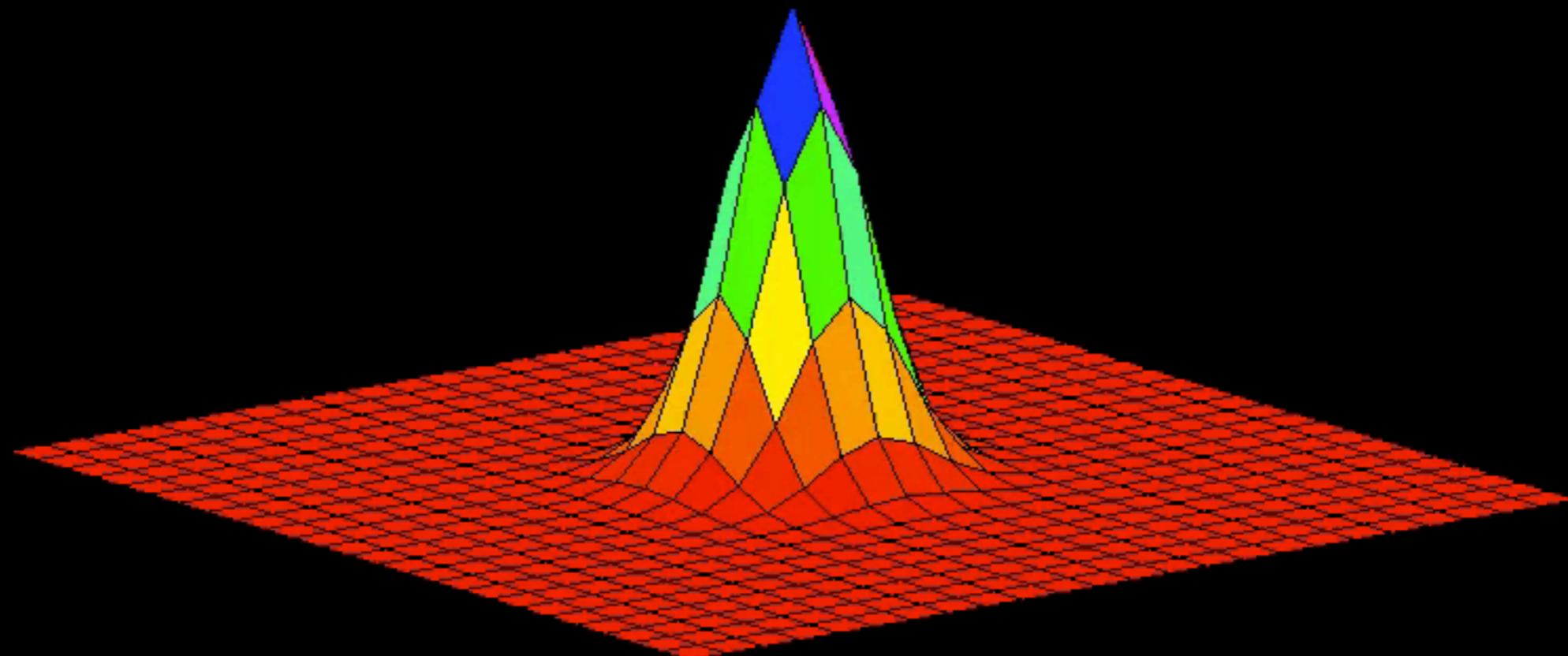
Gaussian
filtering

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \approx G[x, y]$$

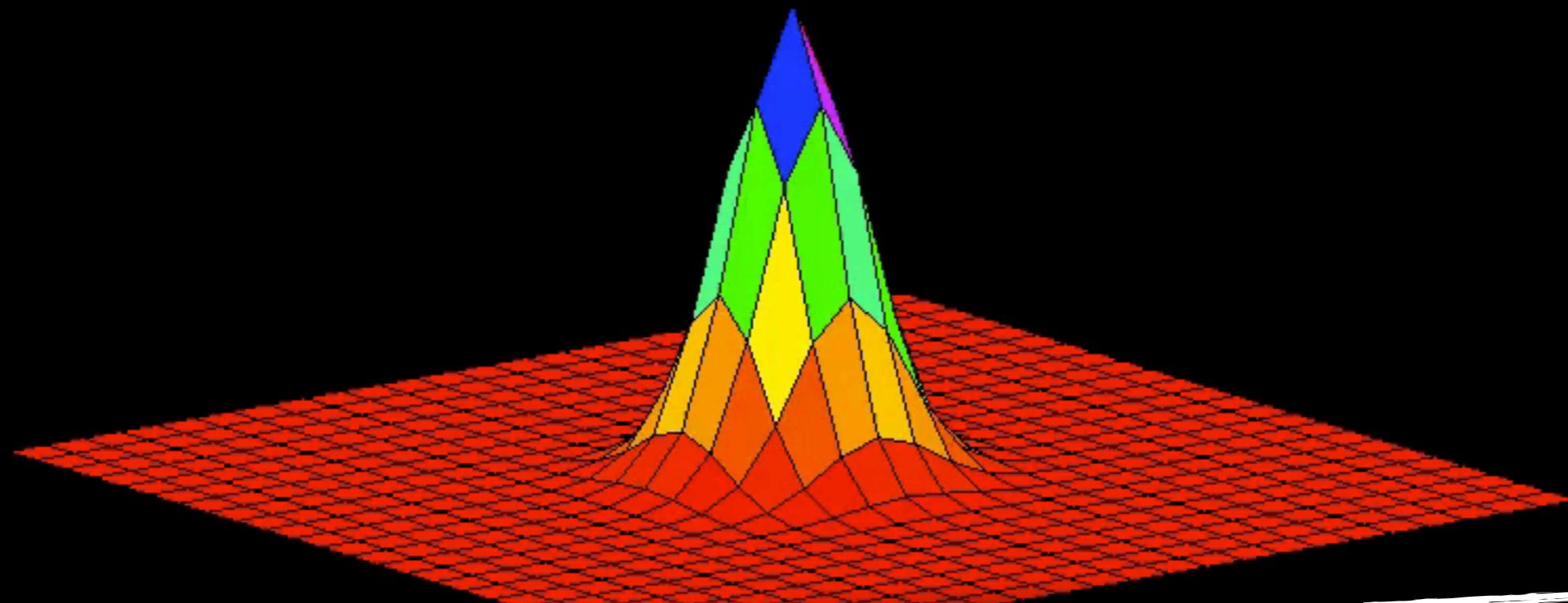


$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



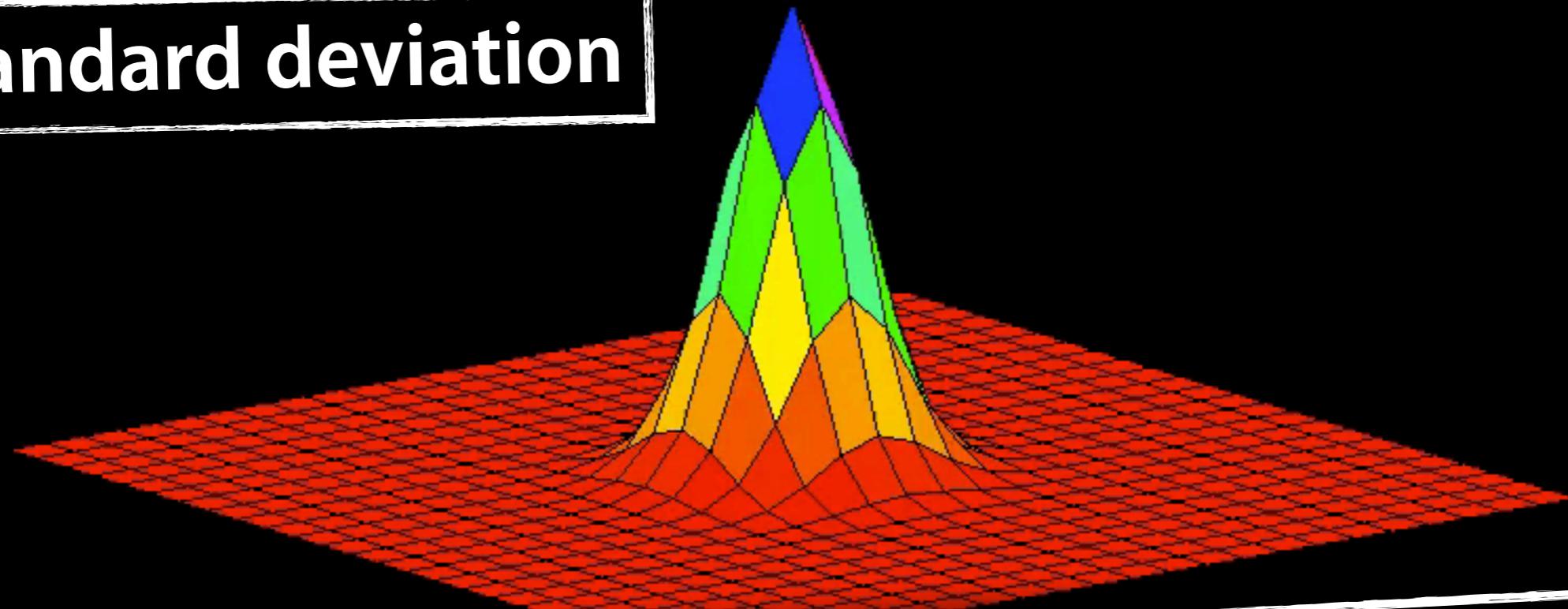
$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



How does the Gaussian vary with σ ?

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

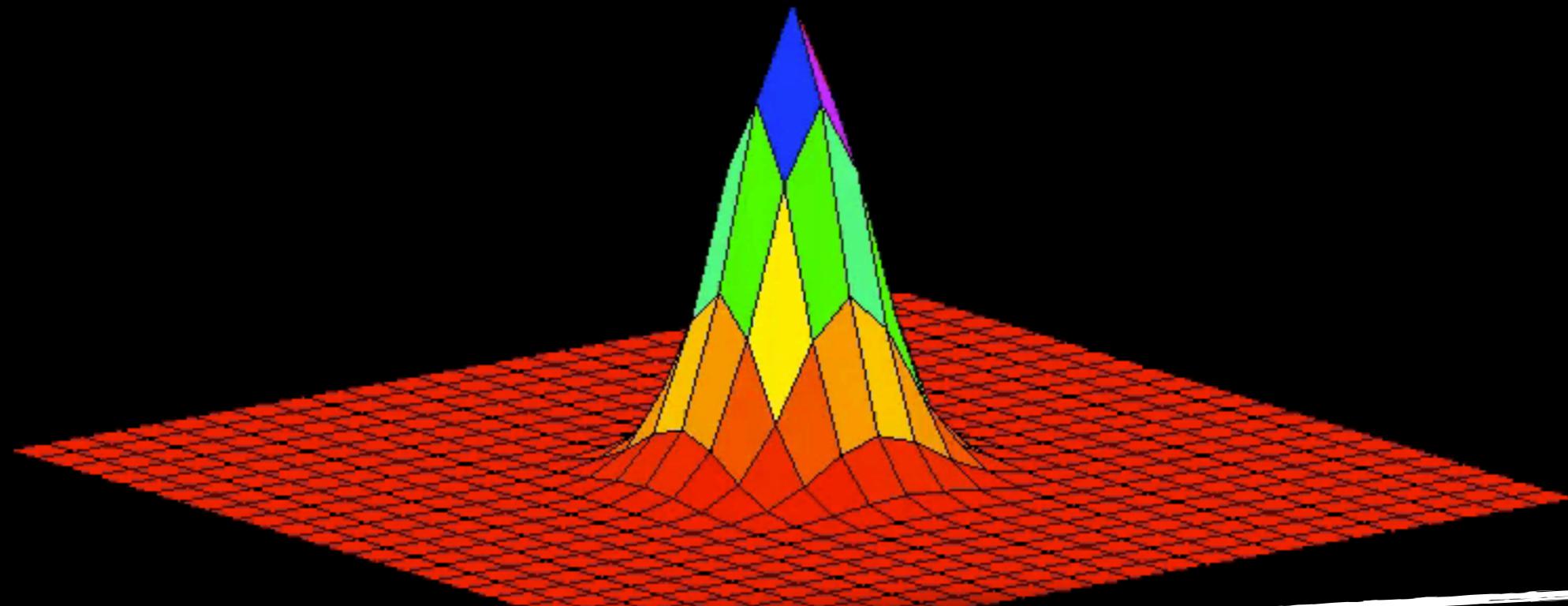
standard deviation



How does the Gaussian vary with σ ?

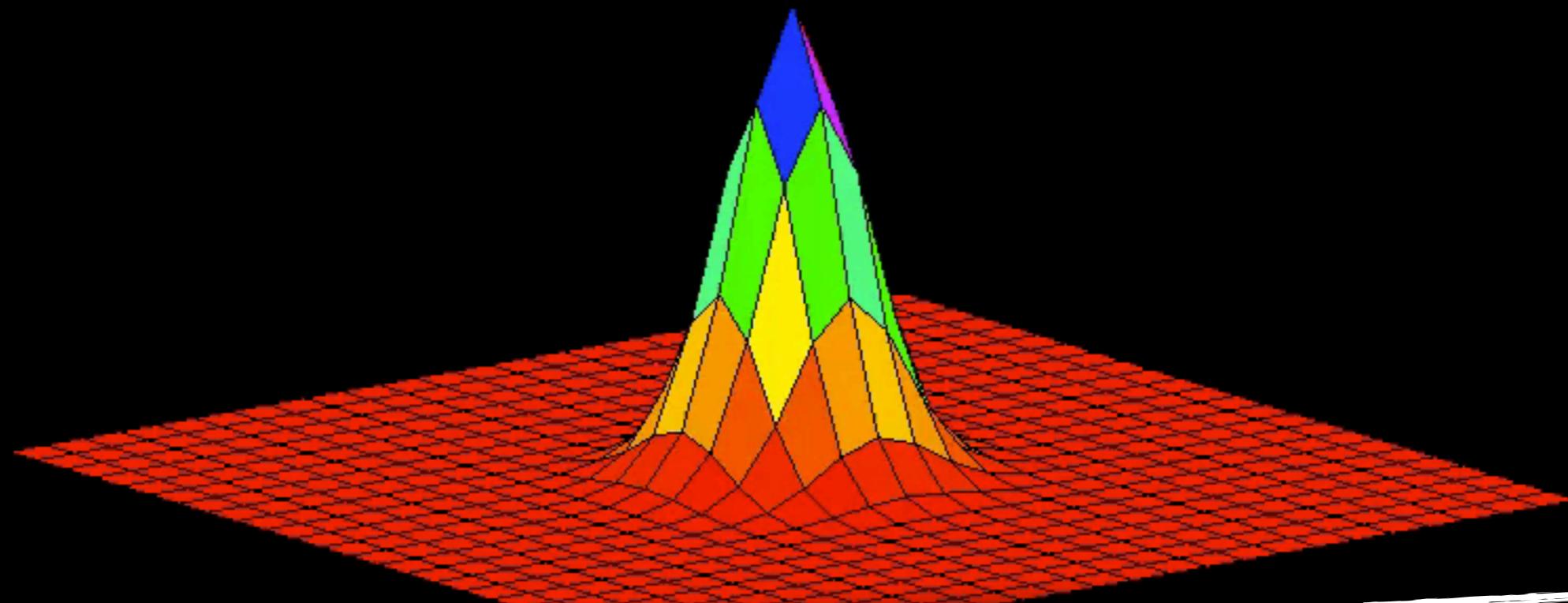
normalization

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



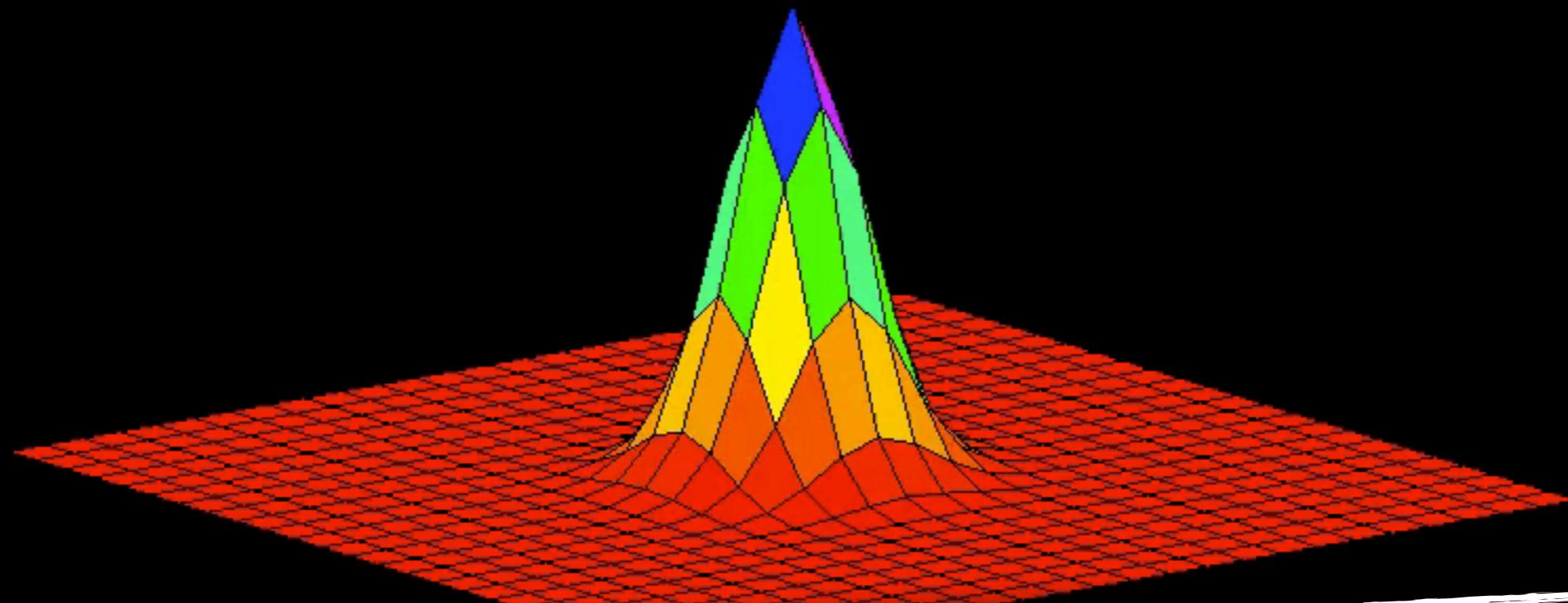
How does the Gaussian vary with σ ?

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



How does the Gaussian vary with σ ?

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

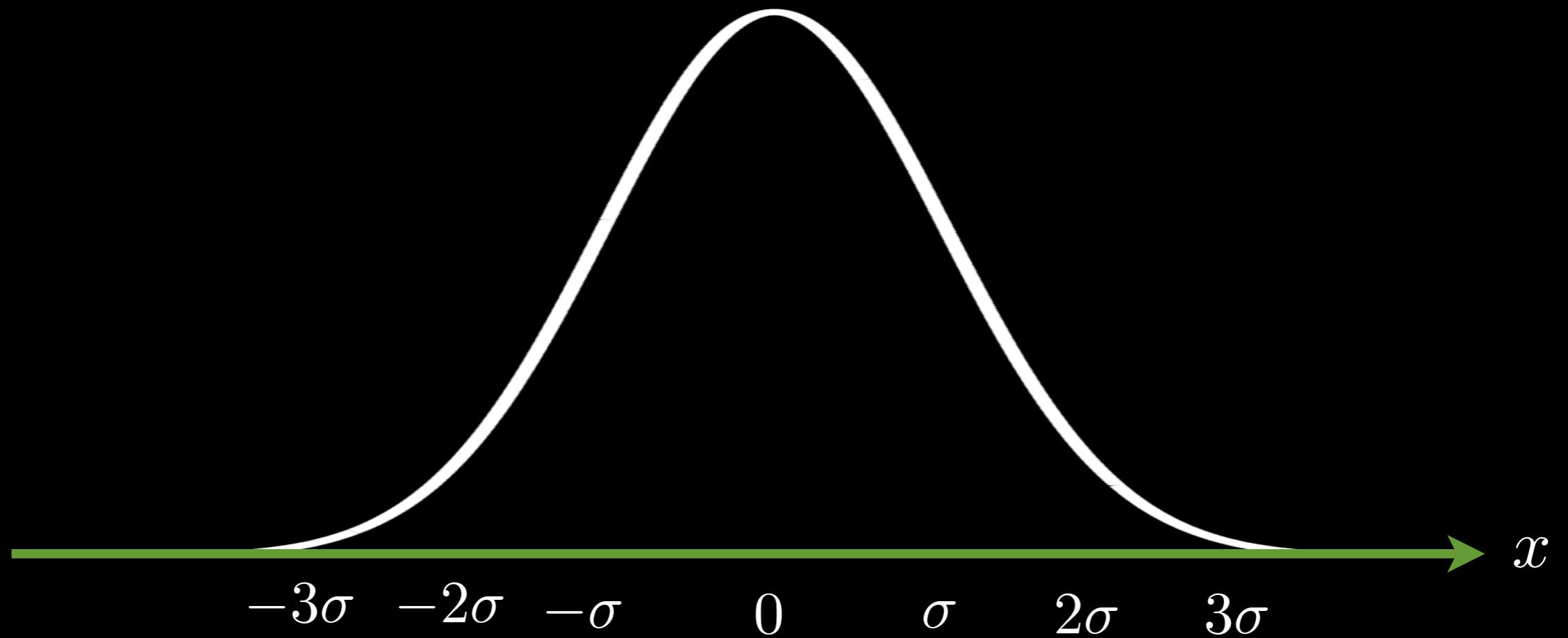


How does the Gaussian vary with σ ?

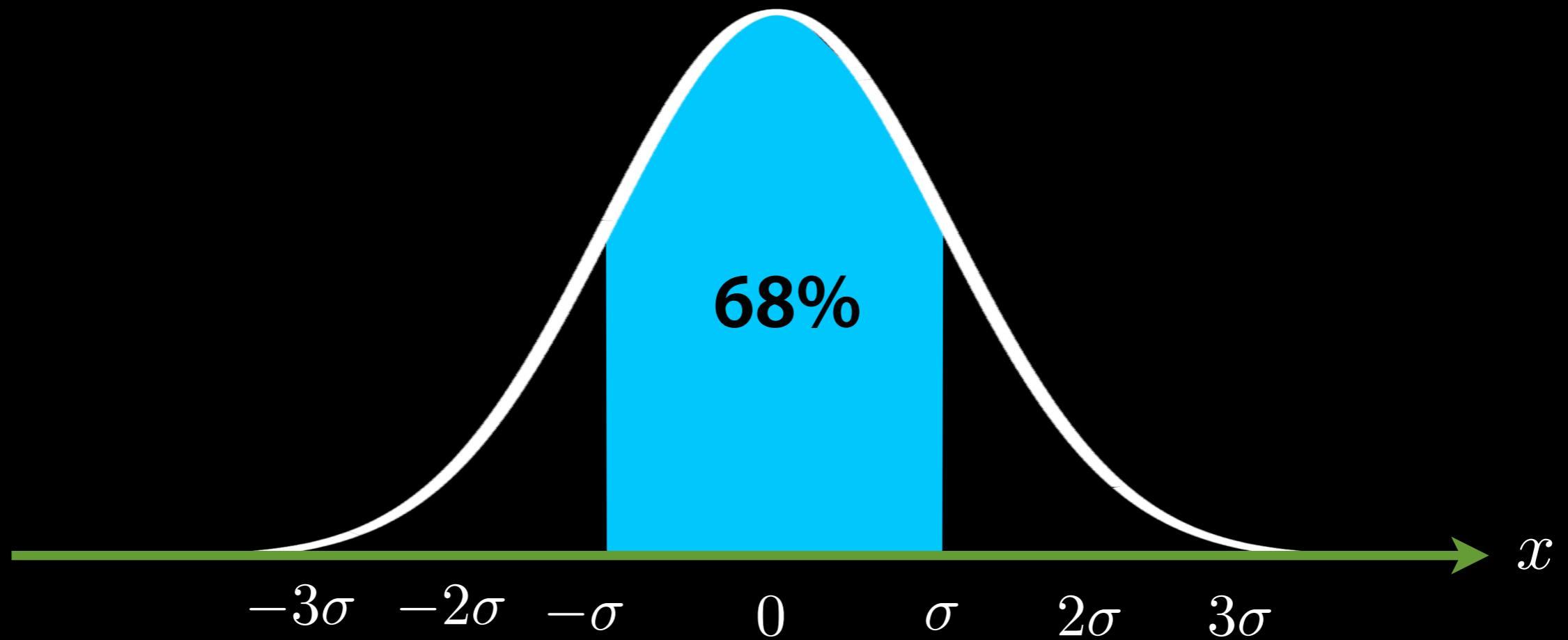
Gaussian filters have **infinite support**

Discrete filters use **finite kernels**

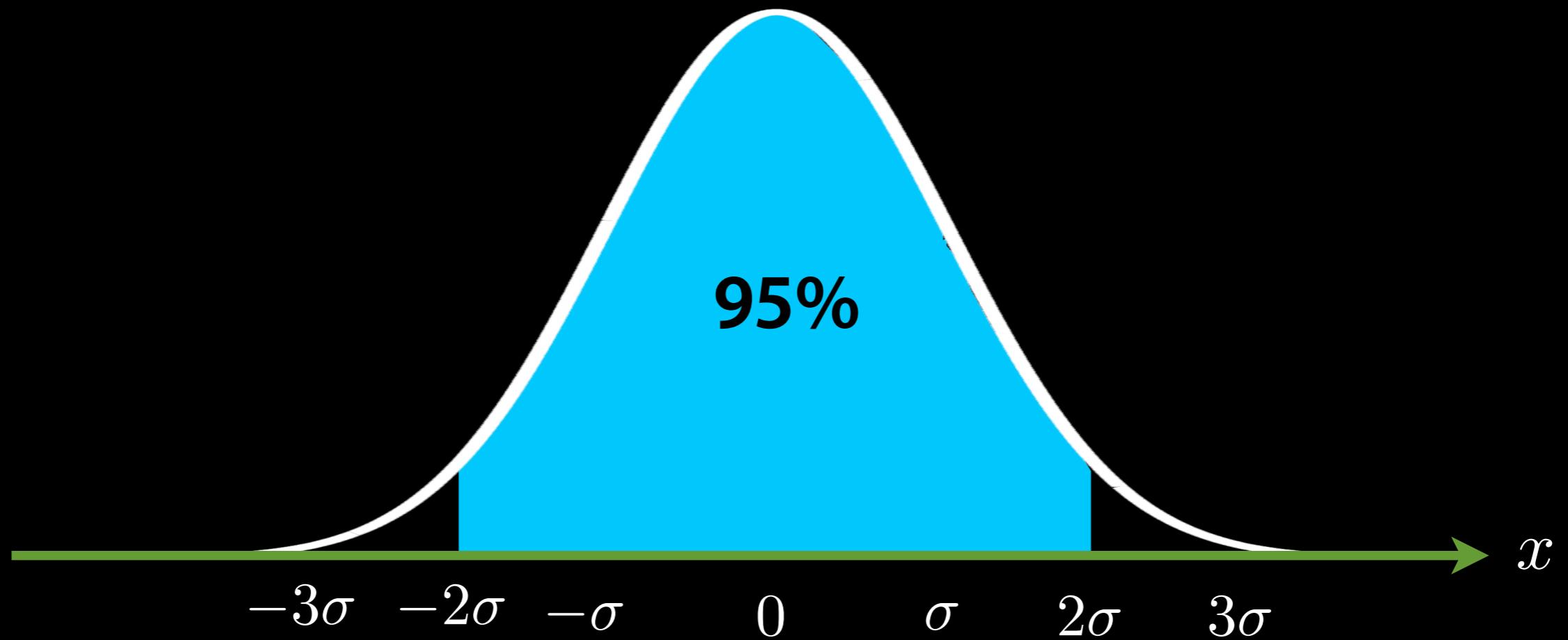
three-sigma
rule



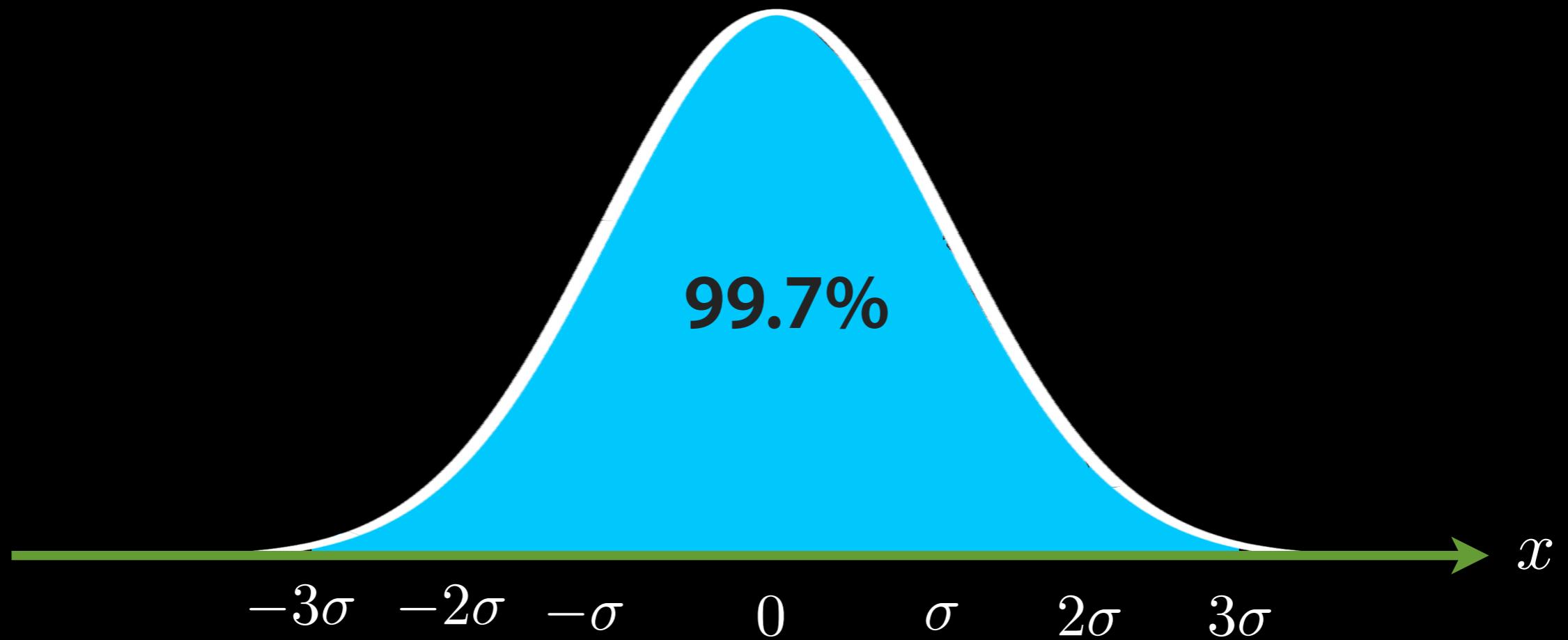
three-sigma
rule



three-sigma
rule



three-sigma
rule



original



original

Gaussian filter

original





smooth by averaging

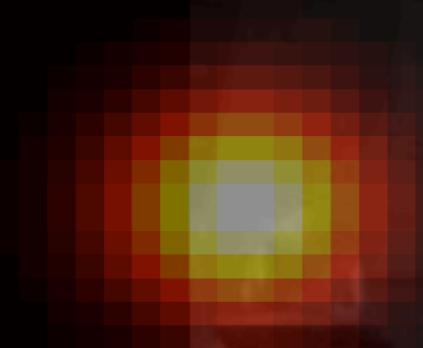


boundary
issues

boundary
issues



boundary
issues



boundary
issues



clip filtering



3	3	2	1	0
0	0	1	3	1
3	1	2	2	3
2	0	0	2	2
2	0	0	0	0

0	0	0	0	0	0	0
0	3	3	2	1	0	0
0	0	0	1	3	1	0
0	3	1	2	2	3	0
0	2	0	0	2	2	0
0	2	0	0	0	0	0
0	0	0	0	0	0	0

boundary
issues



clip filtering

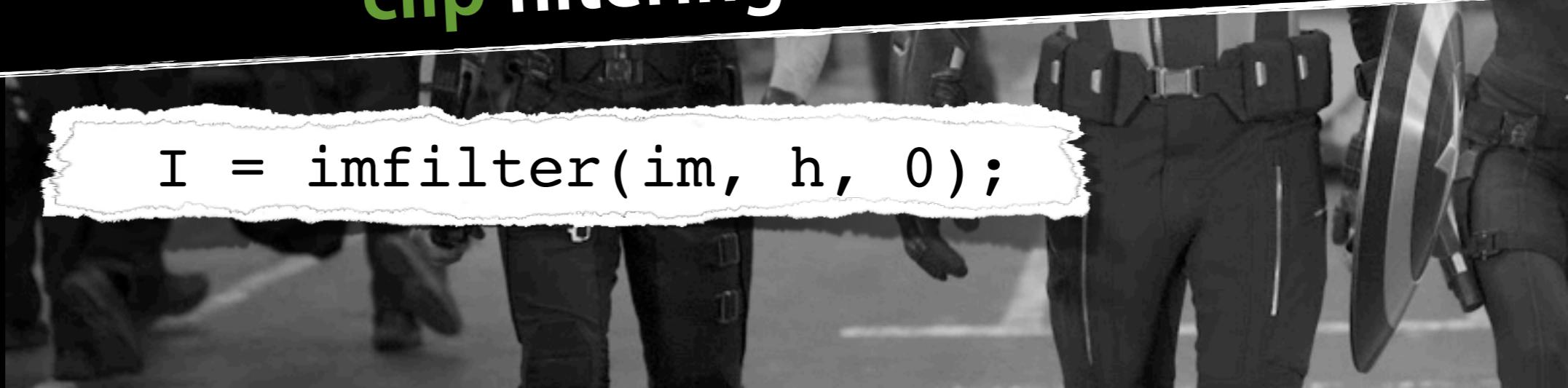


boundary
issues



clip filtering

```
I = imfilter(im, h, 0);
```



boundary
issues

circular filter

boundary
issues



circular filter

```
I = imfilter(im, h, 'circular');
```



boundary
issues

replicate filtering



boundary
issues

replicate filtering

```
I = imfilter(im, h, 'replicate');
```



boundary
issues

symmetric filtering

boundary
issues



symmetric filtering

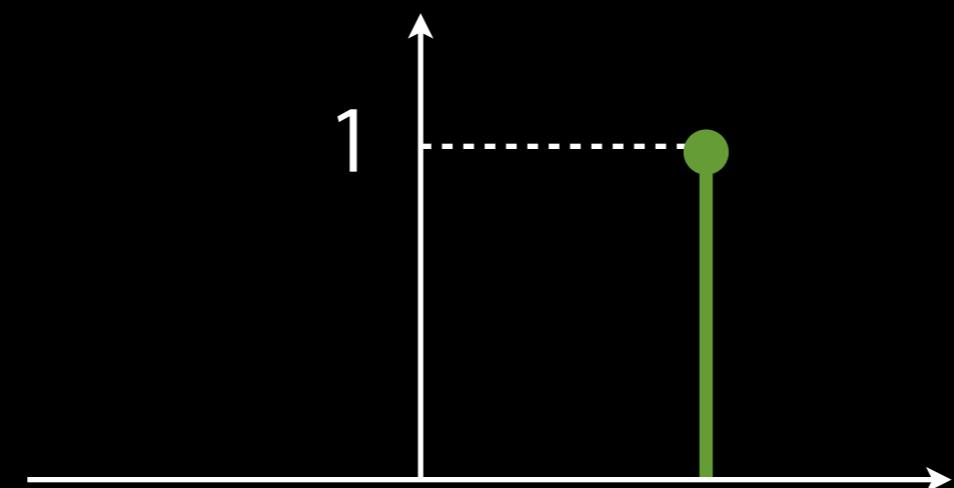
```
I = imfilter(im, h, 'symmetric');
```

Convolution

Convolution

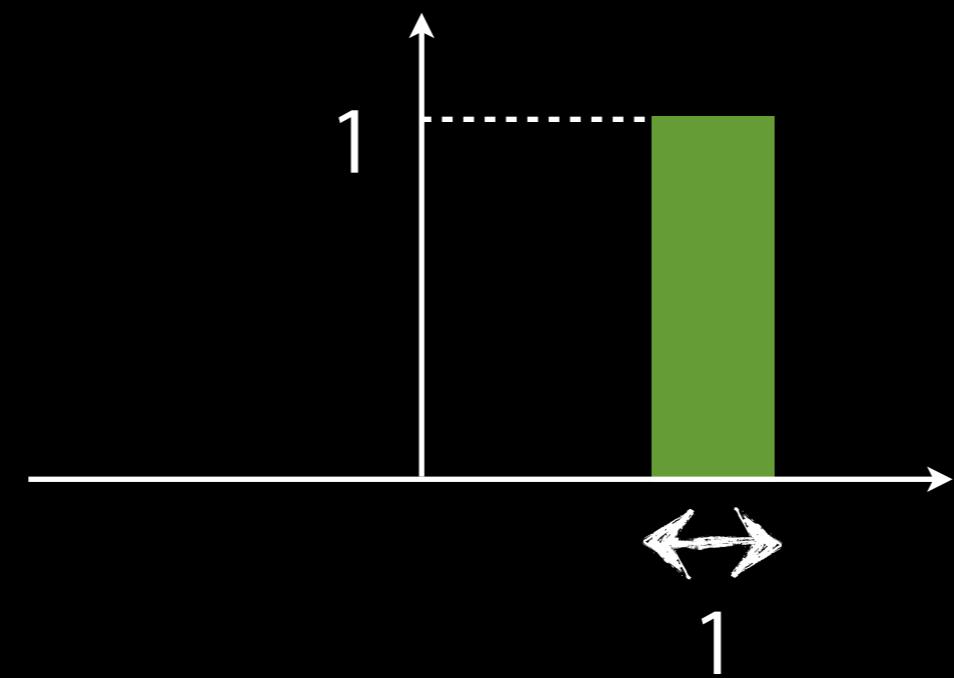
discrete
impulse

*discrete
impulse*



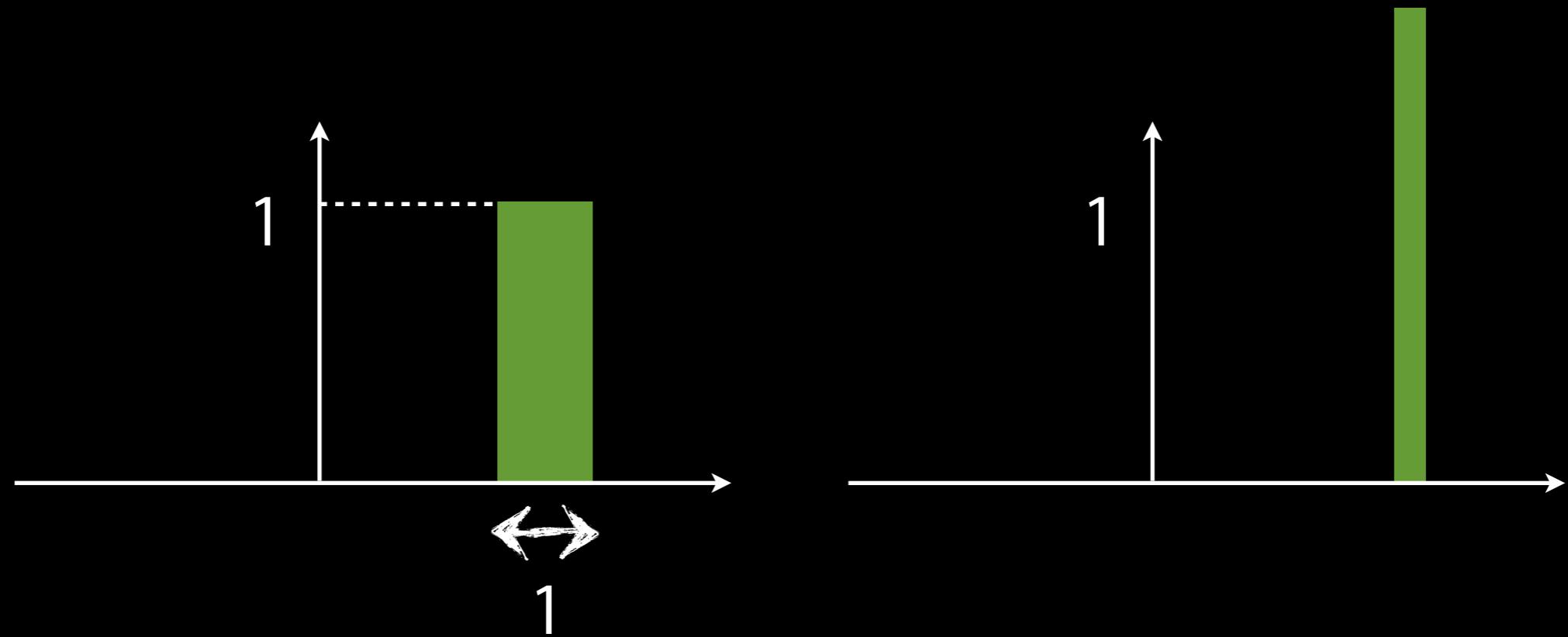
A function with a value of 1 at a single location

continuous
impulse



A function that is very narrow and very tall such that at the limit it has a unit area

continuous
impulse



A function that is very narrow and very tall such
that at the limit it has a unit area

filtering
an impulse

$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline \text{?} & & & & & & \\ \hline \end{array}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & & & & & & \\ \hline \end{array}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{1} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \end{matrix} & = & \begin{matrix} \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} & \text{0} \\ \text{0} & & & & & & \end{matrix} \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \end{matrix} & = & \begin{matrix} \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \square & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix} \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates a convolution operation for filtering an impulse. It shows three matrices: $G[x, y]$, $F[x, y]$, and $H[x, y]$. The input matrix $G[x, y]$ is a 3x3 grid with elements labeled $a, b, c, d, e, f, g, h, i$. The filter matrix $F[x, y]$ is a 7x7 grid with a central value of 1 and all other values being 0. The resulting output matrix $H[x, y]$ is a 7x7 grid where only the element at position (4,4) is 1, while all other elements are 0. This demonstrates how the filter is applied to the input to remove or reduce the impulse.

filtering
an impulse

$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolutional filtering of an impulse noise from a 3x3 input image G onto a 7x7 output image H . The input image G has values $a, b, c, d, e, f, g, h, i$ in its 3x3 receptive field. The filter F is a 7x7 matrix where all elements are 0 except for the center element at position (4,4) which is 1. This filter is applied to the input image G using a convolution operation (indicated by the \otimes symbol). The resulting output image H is a 7x7 matrix where every element is 0, indicating that the impulse noise at position (4,4) in the input has been removed.

filtering
an impulse

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} \otimes \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

The diagram illustrates a convolution operation. On the left, a 3x3 input matrix G is shown with elements labeled a through i . To its right is a 7x7 filter matrix F , with the central element being 1 . The multiplication is indicated by a circled cross symbol (\otimes). The resulting output matrix H is shown on the right, where only the element at index $(3,3)$ is labeled i , indicating that the impulse at position $(3,3)$ in the input has been filtered.

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolutional filtering of an impulse noise from a 3x3 input image G onto a 7x7 output image H . The input image G has values $a, b, c, d, e, f, g, h, i$ at positions (1,1) through (3,3). The filter F is a 3x3 kernel with a central value of 1. The resulting output image H shows the impulse at position (3,3) with value i , while all other positions are zero. The operation is indicated by a circled cross symbol (\otimes) between G and F .

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolution process for filtering an impulse. It shows three matrices: $G[x, y]$, $F[x, y]$, and $H[x, y]$. The input matrix G has values $a, b, c, d, e, f, g, h, i$. The filter matrix F is a 3x3 kernel with a central value of 1. The output matrix H is the result of applying the filter to G . A green box highlights the central element of the filter kernel in the F matrix. Another green box highlights the element i in the H matrix, which is the result of the impulse being filtered.

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolution process for filtering an impulse. It shows three matrices: $G[x, y]$, $F[x, y]$, and $H[x, y]$. The input matrix G has values $a, b, c, d, e, f, g, h, i$. The filter matrix F is a 3x3 kernel with a central value of 1. The output matrix H is the result of applying the filter to G . A green box highlights the central element of the filter kernel in the F matrix. Another green box highlights the element i in the H matrix, which is the result of the impulse being filtered.

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolution process for filtering an impulse. It shows three matrices: $G[x, y]$, $F[x, y]$, and $H[x, y]$. The input matrix G is a 3x3 matrix with elements labeled a through i . The filter matrix F is a 3x7 matrix where only the central element is 1, and all other elements are 0. The output matrix H is a 3x7 matrix resulting from the convolution of G and F . The result is that the value i from G is multiplied by the 1 in the center of the filter, resulting in the value i at position (2,3) in the output matrix H . The other elements in H are 0.

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolutional filtering of an impulse noise in a 3x3 input image. The input image $G[x, y]$ is a 3x3 matrix with values a, b, c, d, e, f, g, h, i. The filter $F[x, y]$ is a 3x3 matrix with a central value of 1 and zeros elsewhere. The output image $H[x, y]$ is the result of the convolution, showing the impulse at position (3,3) filtered by the kernel. The value 'i' in the input is replaced by the sum of the kernel elements (which is 1), resulting in 'g' in the output.

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolution process for filtering an impulse. It shows three matrices: $G[x, y]$, $F[x, y]$, and $H[x, y]$. The input matrix G is a 3x3 matrix with elements labeled a through i . The filter matrix F is a 3x7 matrix where only the central element is 1, and all other elements are 0. This 1 is highlighted with a green box. The resulting output matrix H is a 3x7 matrix where only the central element is g , and all other elements are 0. This g is also highlighted with a green box. The operation is indicated by the symbol \otimes .

filtering
an impulse

$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & g & 0 & 0 \\ 0 & 0 & \square & \square & \square & \square & \square \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

filtering
an impulse

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} \otimes \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & g & 0 & 0 \\ 0 & 0 & f & & & & \\ 0 & 0 & & & & & \\ 0 & 0 & & & & & \\ 0 & 0 & & & & & \end{matrix}$$

$G[x, y]$

$F[x, y]$

$H[x, y]$

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$$\begin{matrix} \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} & \otimes & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & h & g & 0 & 0 \\ 0 & 0 & f & & & & \\ \vdots & & & & & & \\ \vdots & & & & & & \end{matrix} \\ G[x, y] & & F[x, y] & & H[x, y] \end{matrix}$$

The diagram illustrates the convolutional operation of filtering an impulse. On the left, a 3x3 input matrix $G[x, y]$ is shown with elements labeled a through i. It is multiplied by a 3x3 filter matrix $F[x, y]$, indicated by the circled \otimes symbol. The filter matrix has a central value of 1. The result of this multiplication is the output matrix $H[x, y]$ on the right. In the output matrix, the element at position (3,3) is labeled 'f', indicating it is the result of the central filter unit. The other elements are 0, representing zero-padded input or boundary conditions.

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$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & i & h & g & 0 & 0 \\ \hline 0 & 0 & f & e & d & 0 & 0 \\ \hline 0 & 0 & c & b & a & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

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$G[x, y]$ $F[x, y]$ $H[x, y]$

Filter output is **REVERSED!!!**

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an impulse

$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

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$H[x, y]$

How would you modify the kernel to avoid the flip?

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How would you modify the kernel to avoid the flip?

Flip the kernel

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$G[x, y]$

$F[x, y]$

$H[x, y]$

How would you modify the kernel to avoid the flip?

Flip the kernel

Let the filter window size be $(2K + 1) \times (2K + 1)$

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This is called **cross correlation, denoted $H = G \otimes F$**

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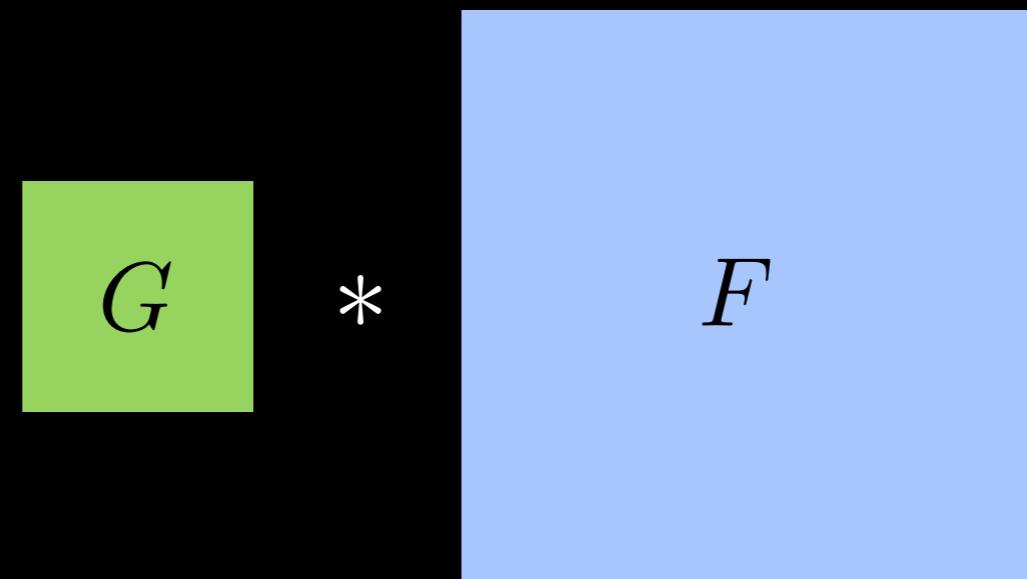
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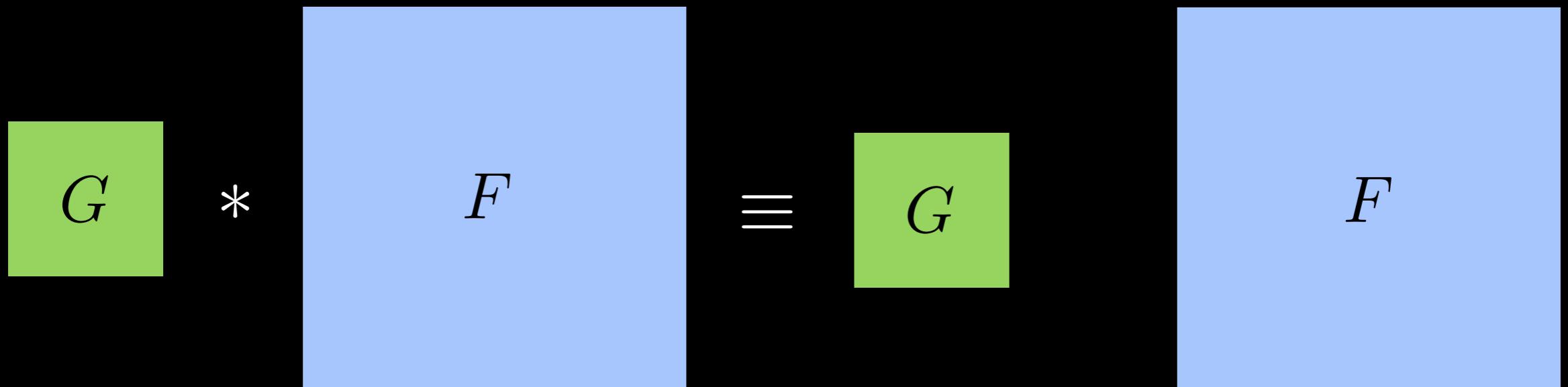
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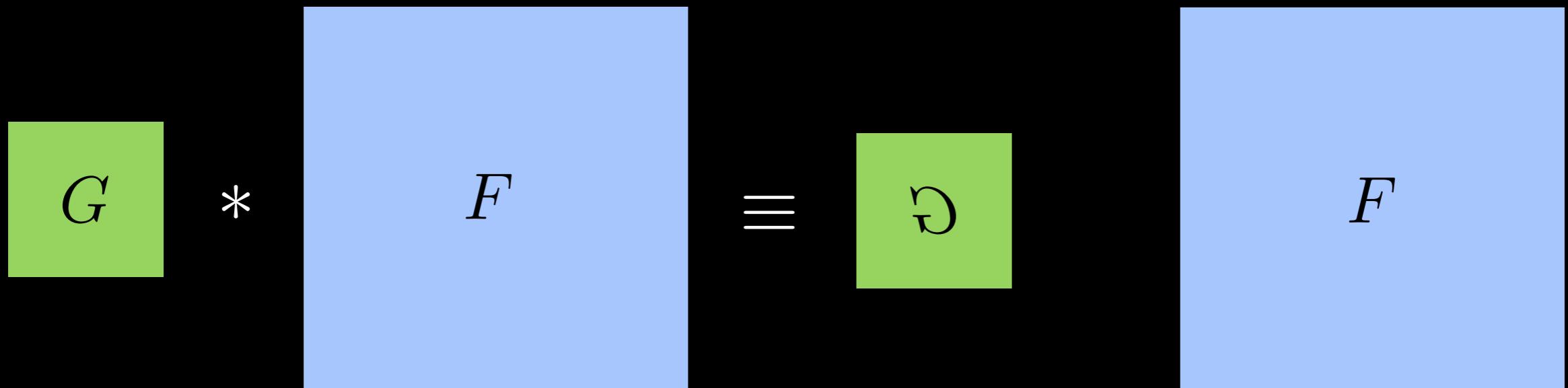
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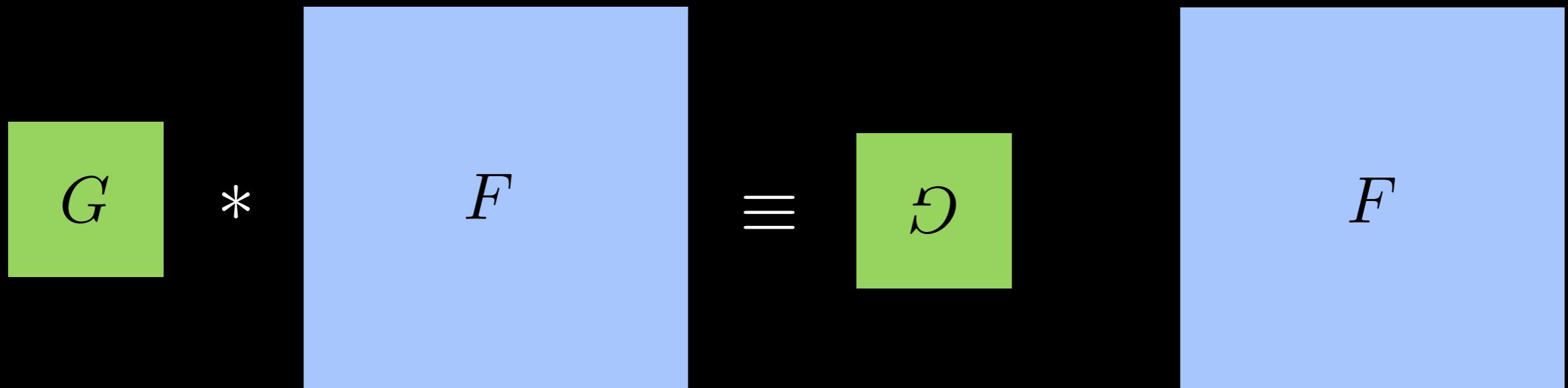
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$$\begin{matrix} G & * & F \end{matrix} \equiv \begin{matrix} \mathcal{G} & \otimes & F \end{matrix}$$