Image Transformation

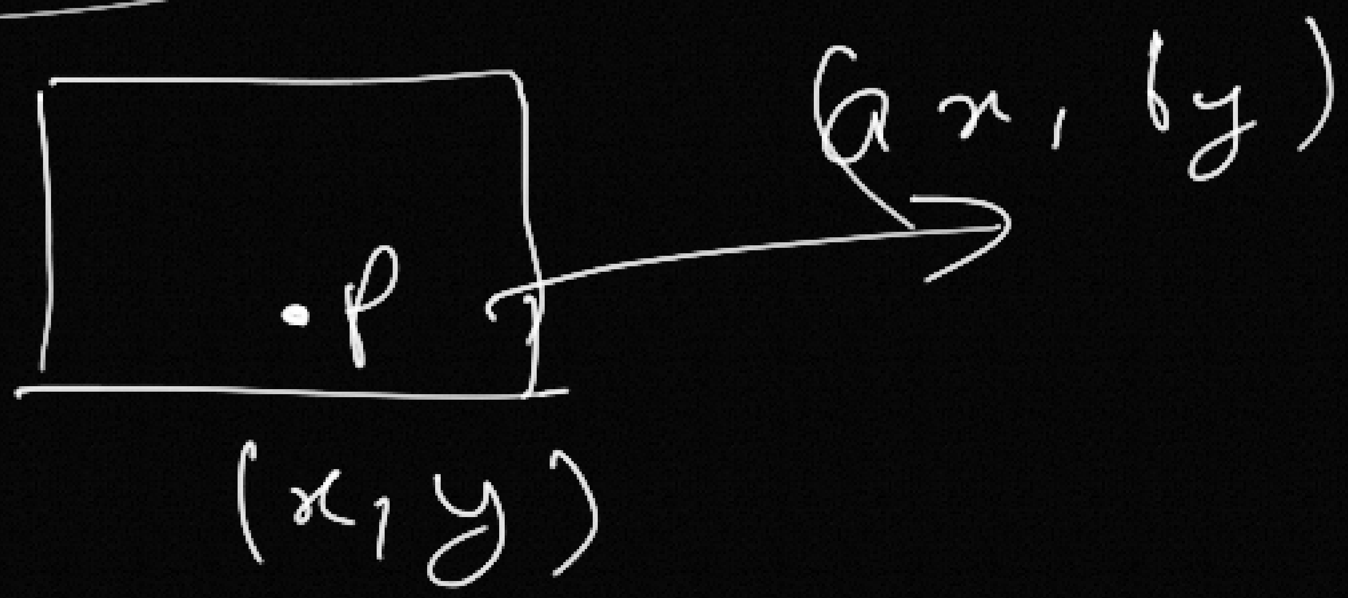
$$I_A \to \boxed{T_r} \longrightarrow I_B$$

$$\underline{g(x,y)} = T_r\big(f(x,y)\big)$$

$$g(x,y) = f\big(T_d(x,y)\big)$$

2 X 2 Transformation

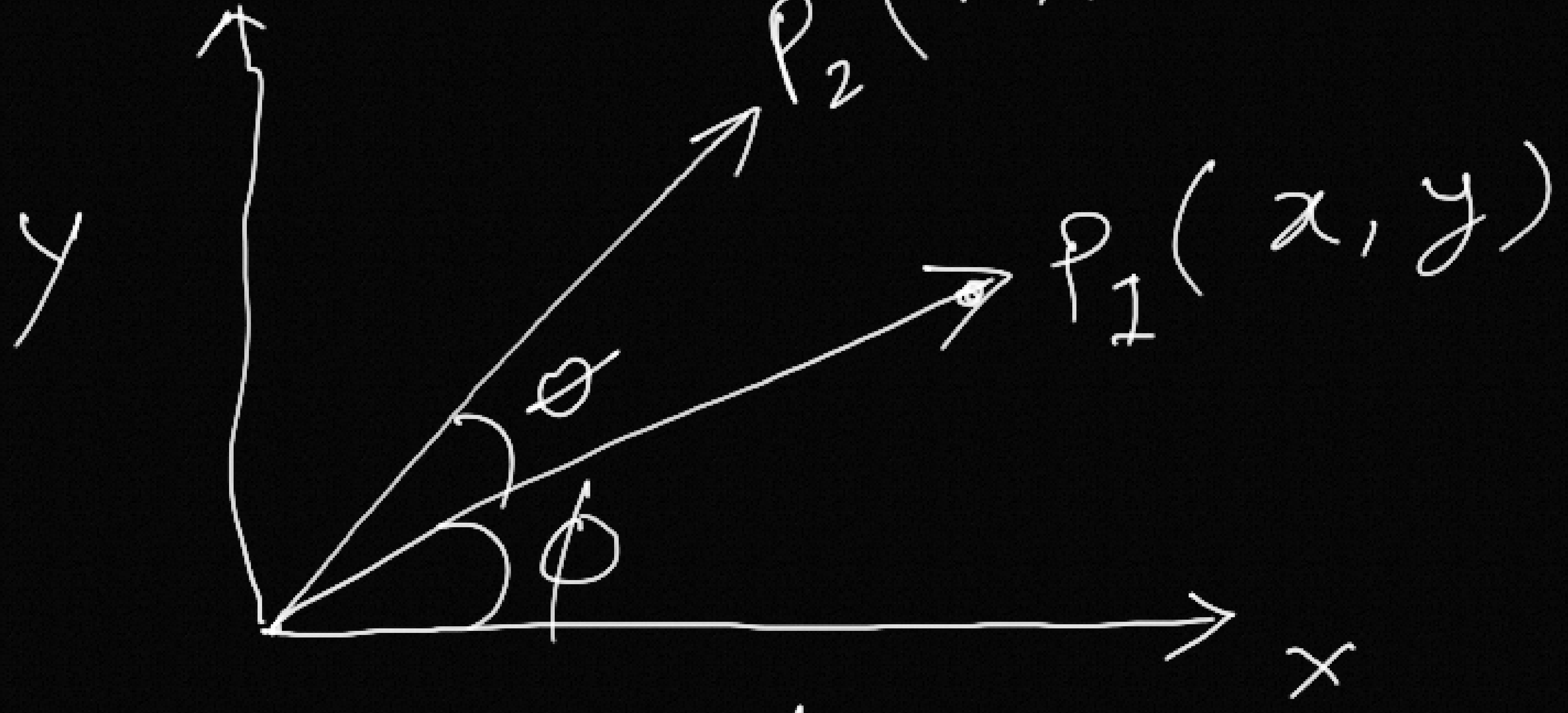
Scaling



$$x' = ax, \quad y' = by$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation



$x = r\cos\phi \,,\quad y = r\sin\phi$

$x' = \underline{r\cos(\phi+\theta)} \,,\quad y' = \underline{r\sin(\phi+\theta)}$

$$x' = r\cos\phi \cdot \cos\theta - \underline{r\sin\phi} \cdot \sin\theta$$

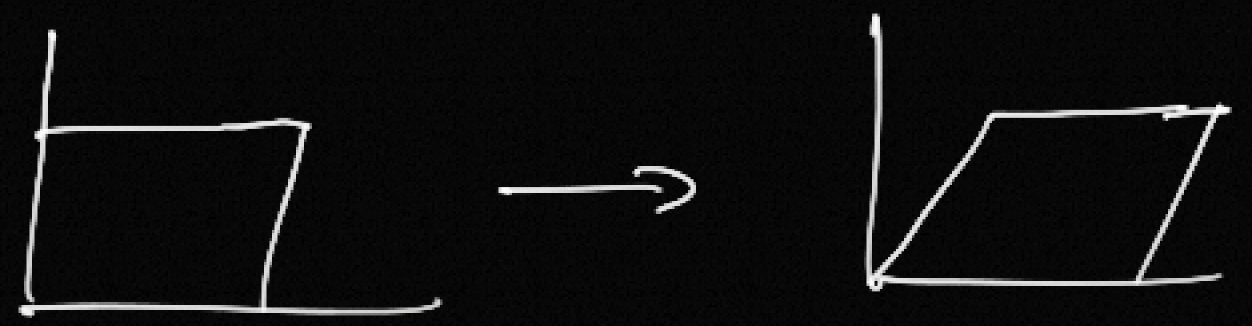
$$y' = r\cos\phi \cdot \sin\theta + r\sin\phi \cdot \cos\theta$$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Horizontal Skew



$$x' = x + m_x y$$

$$y' = y$$

Mirror

$$x' = -x$$

$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Properties :-

(i) Origin $\rightarrow$ Origin

(ii) line $\rightarrow$ line

(iii) Parallel $\rightarrow$ par

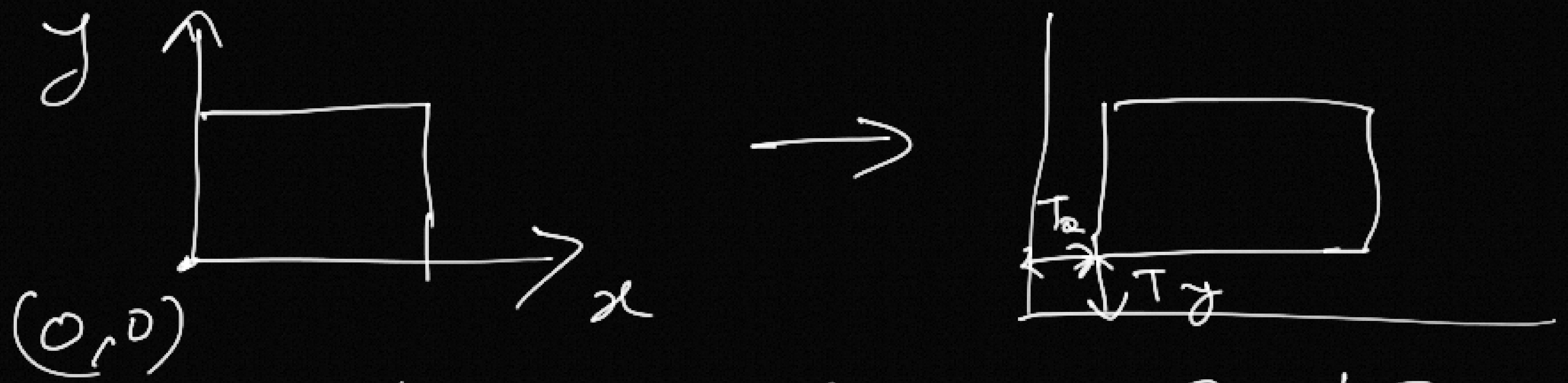
(iv) Closed operation

$$P_2 = T_{21} P_1$$

$$P_3 = T_{32} P_2$$

$$P_3 = T_{31} P_1$$

Translation



$$x' = x + t_x$$
$$y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogenous coordinate system

Homo. Cordinate System :-

of 2D point $\begin{bmatrix} x \\ y \end{bmatrix}$ is a 3D point

$\left[\hat{x}, \hat{y}, \hat{z}\right]$ s.t.

$$x = \frac{\hat{x}}{\hat{z}}, \quad y = \frac{\hat{y}}{\hat{z}}$$

$$P \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \hat{z}x \\ \hat{z}y \\ \hat{z} \end{bmatrix} \equiv \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} \equiv \hat{P}$$

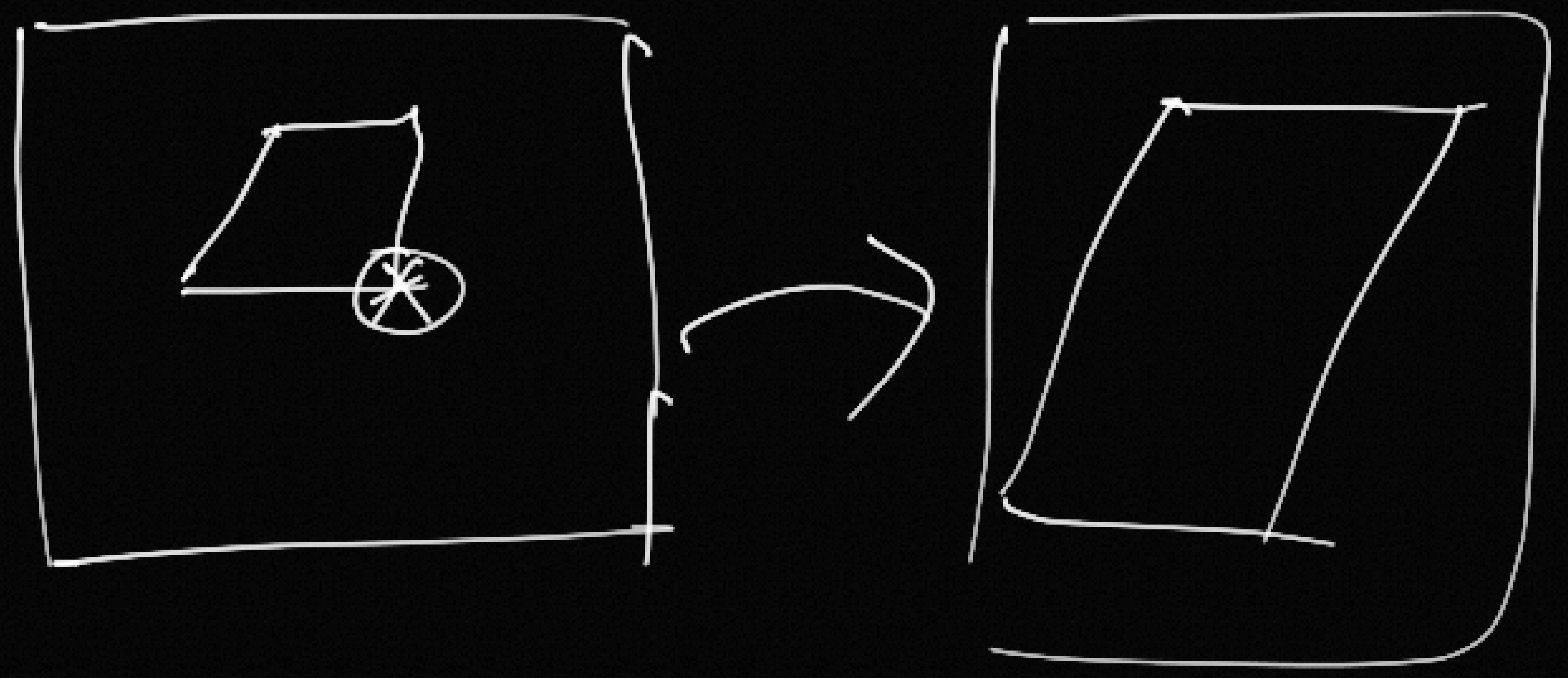
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

<u>Scaling</u>

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>Rotation</u>

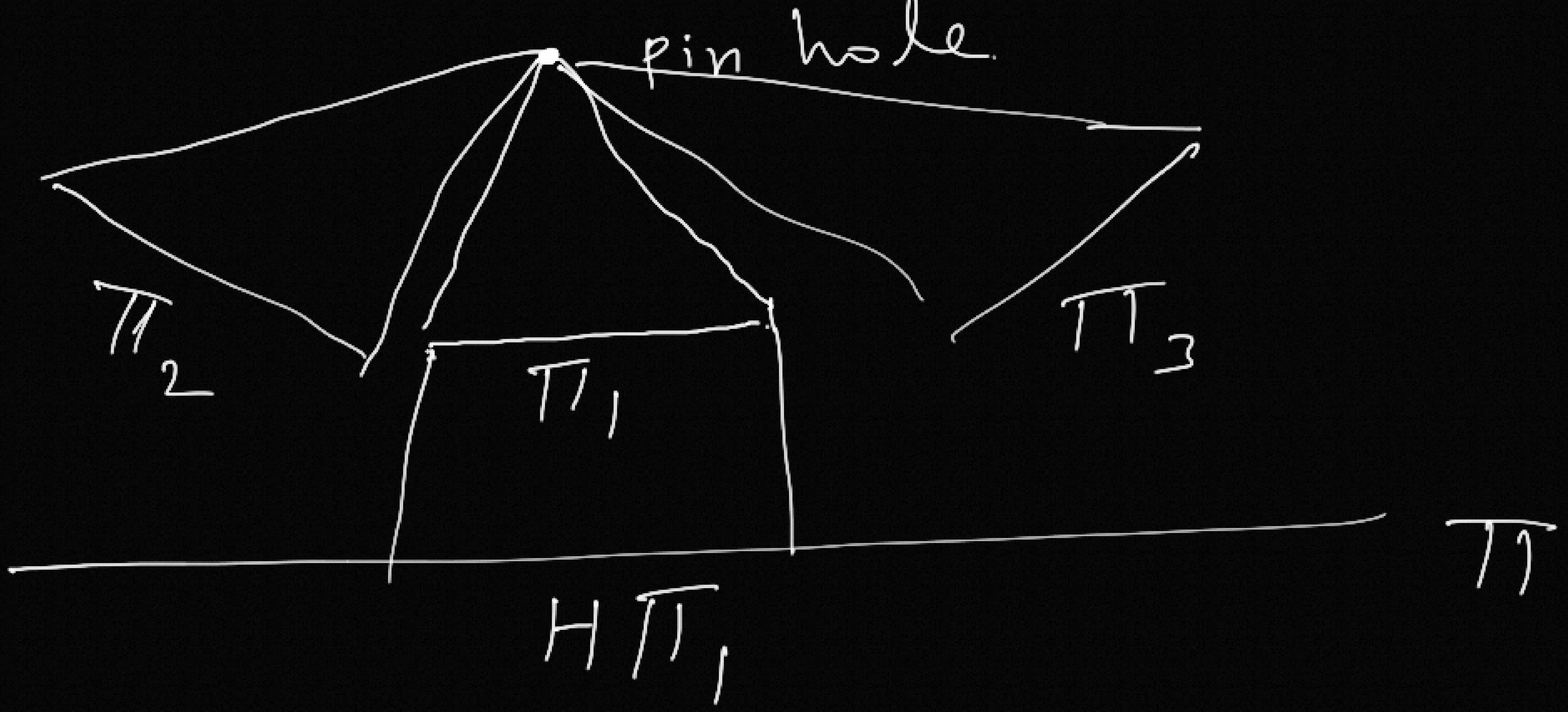
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

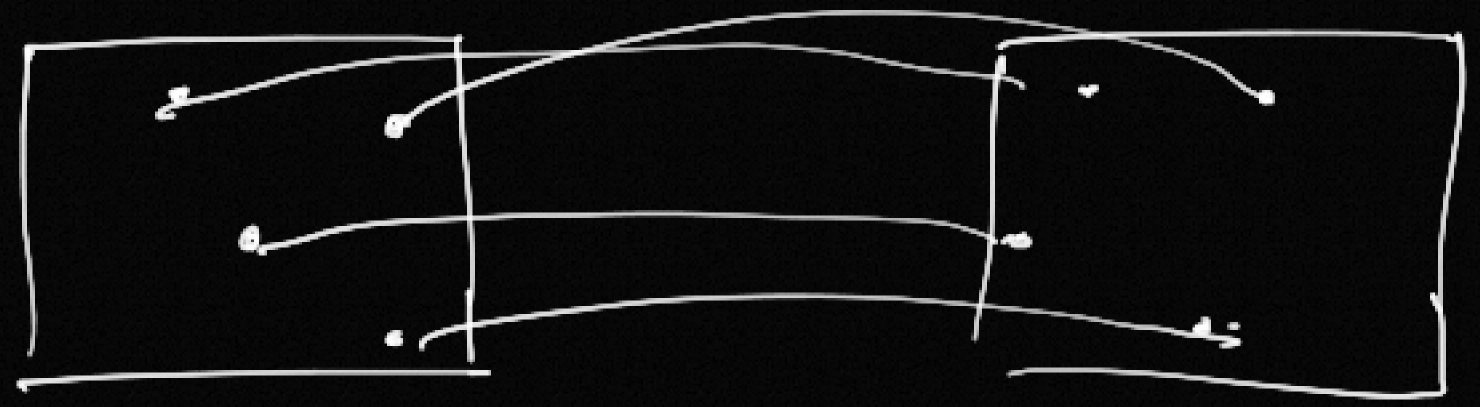
$$\longrightarrow \sqrt{(\Sigma h_{ij})^2} = 1$$

Homography Matrix


pin hole
Π_2
Π_1
Π_3
Π
$H\Pi_1$

Homography

4 points



S D

$$\begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix}$$

$$Ah = 0 \quad \text{s.t.} \quad \|h\|^2 = 1$$