Resolution Refutation in Proposition Logic

Resolution Refutation in PL

- Resolution refutation: Another simple method to prove a formula by contradiction.
- Here negation of goal is added to given set of clauses.
 - If there is a refutation in new set using resolution principle then goal is proved
- During resolution we need to identify two clauses,
 - one with positive atom (P) and other with negative atom (~ P) for the application of resolution rule.
- Resolution is based on modus ponen inference rule.

Disjunctive & Conjunctive Normal Forms

- Disjunctive Normal Form (DNF): A formula in the form $(L_{11} \ \Lambda \ \ \Lambda \ L_{1n} \) \ V \ \ V \ (L_{m1} \ \Lambda \ \ \Lambda \ L_{mk} \)$, where all L_{ii} are literals.
 - Disjunctive Normal Form is disjunction of conjunctions.
- Conjunctive Normal Form (CNF): A formula in the form (L_{11} V V L_{1n}) Λ Λ (L_{p1} V V L_{pm}), where all L_{ii} are literals.
 - CNF is conjunction of disjunctions or
 - CNF is conjunction of clauses
- Clause: It is a formula of the form (L₁V ... V L_m), where each L_k is a positive or negative atom.

Conversion of a Formula to its CNF

- Each PL formula can be converted into its equivalent CNF.
- Use following equivalence laws:

$$\begin{array}{cccccc}
- & P \rightarrow Q & \cong & \sim P & V & Q \\
- & P \leftrightarrow Q & \cong & (P \rightarrow Q) & \Lambda & (Q \rightarrow P)
\end{array}$$

Double Negation

(De Morgan's law)

$$- \sim (P \land Q) \cong \sim P \lor \sim Q$$
$$- \sim (P \lor Q) \cong \sim P \land \sim Q$$

(Distributive law)

$$- P V (Q \Lambda R) \cong (P V Q) \Lambda (P V R)$$

Resolvent of Clauses

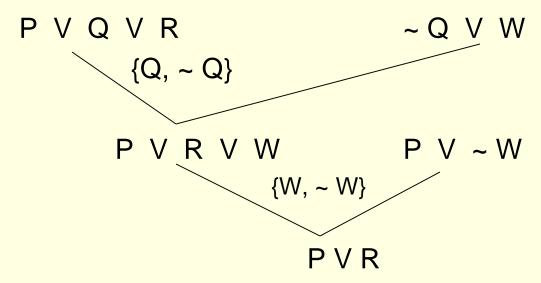
- If two clauses C₁ and C₂ contain a complementary pair of literals {L, ~L},
 - then these clauses may be resolved together by deleting L from C_1 and \sim L from C_2 and constructing a new clause by the disjunction of the remaining literals in C_1 and C_2 .
- The new clause thus generated is called resolvent of C₁ and C₂.
 - Here C1 and C2 are called parents of resolved clause.
- Inverted binary tree is generated with the last node (root) of the binary tree to be a resolvent.
 - This is also called resolution tree.

Example

Find resolvent of the following clauses:

$$- C_1 = P V Q V R; C_2 = - Q V W; C_3 = P V - W$$

Inverted Resolution Tree



Resolvent(C1,C2, C3) = P V R

Logical Consequence

- Theorem1: If C is a resolvent of two clauses C₁ and C₂, then C is a logical consequence of {C₁, C₂}.
 - A deduction of an empty clause (or resolvent as contradiction) from a set S of clauses is called a resolution refutation of S.
- Theorem2: Let S be a set of clauses. A clause C is a logical consequence of S iff the set S'= S ∪ {~ C} is unsatisfiable.
 - In other words, C is a logical consequence of a given set S iff an empty clause is deduced from the set S'.

Example

- Show that C V D is a logical consequence of
 - S ={AVB, ~ AVD, C V~ B} using resolution refutation principle.
- First we will add negation of logical consequence
 - i.e., \sim (C V D) \cong \sim C Λ \sim D to the set S.
 - Get S' = $\{A \lor B, \sim A \lor D, C \lor \sim B, \sim C, \sim D\}.$
- Now show that S' is unsatisfiable by deriving contradiction using resolution principle.

