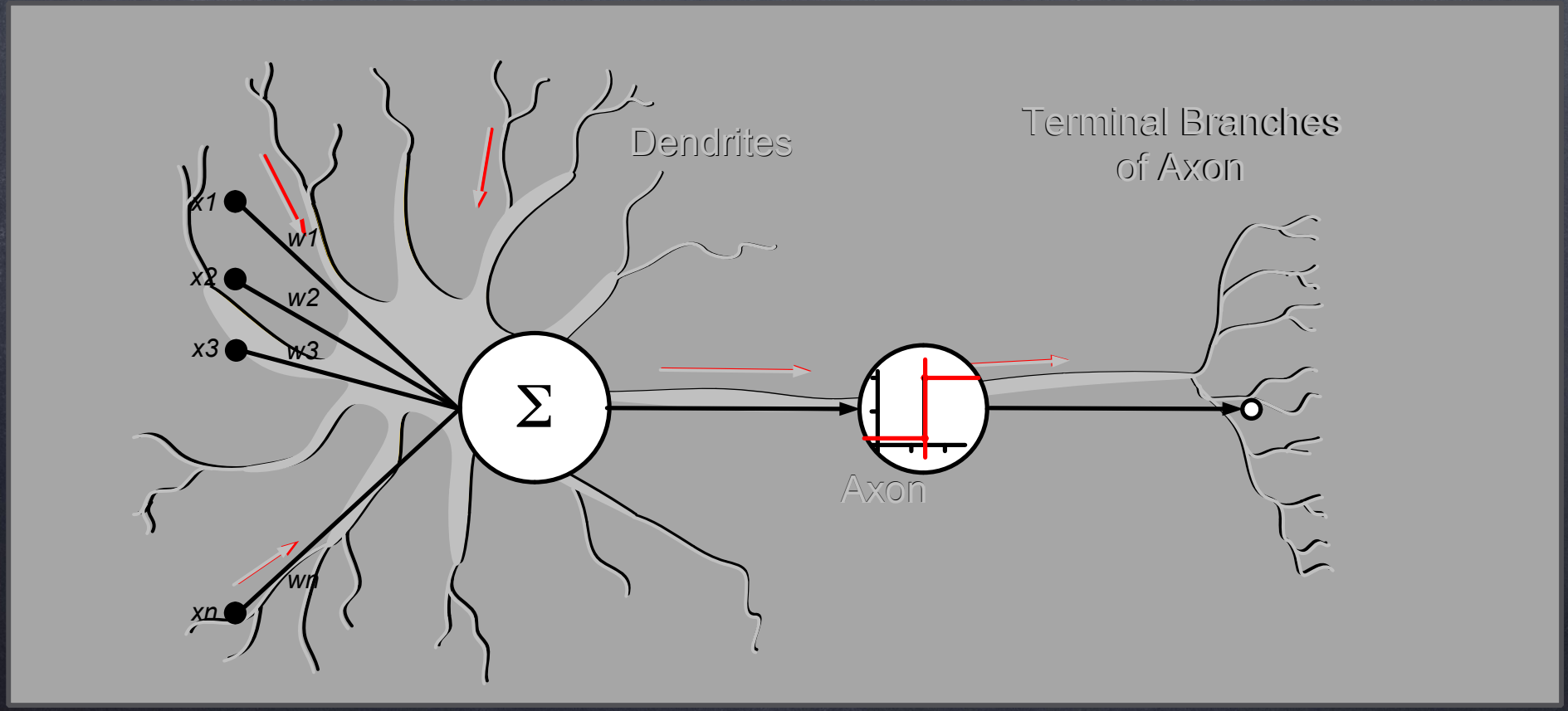
Deep Learning

Mayank Vatsa

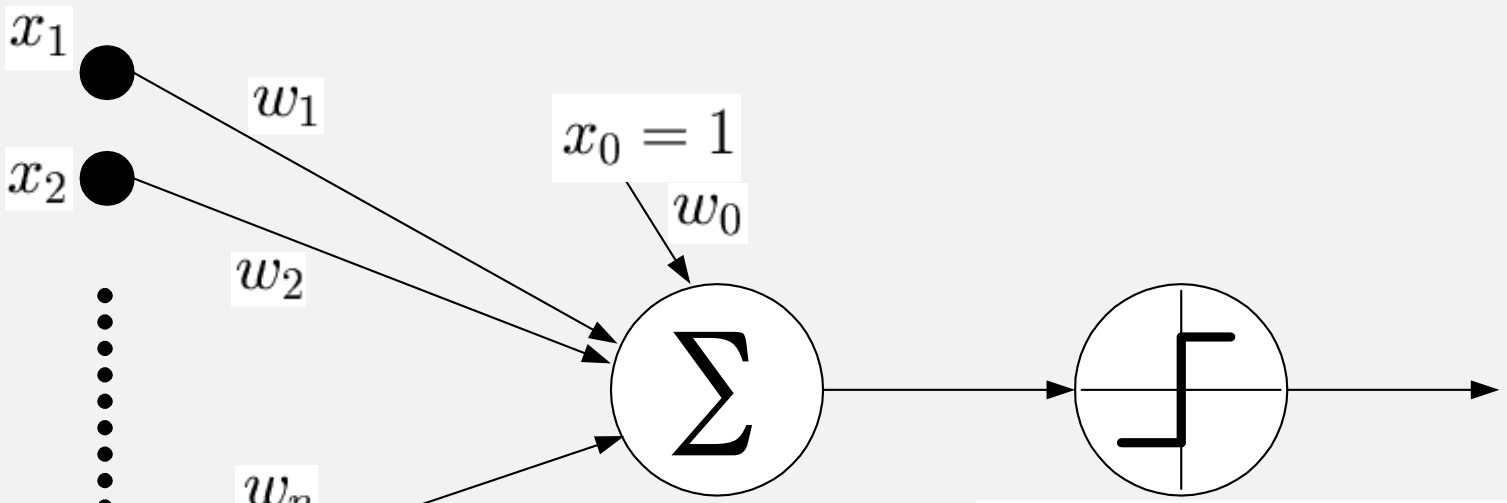
A Simple Artificial Neuron

- Perceptron



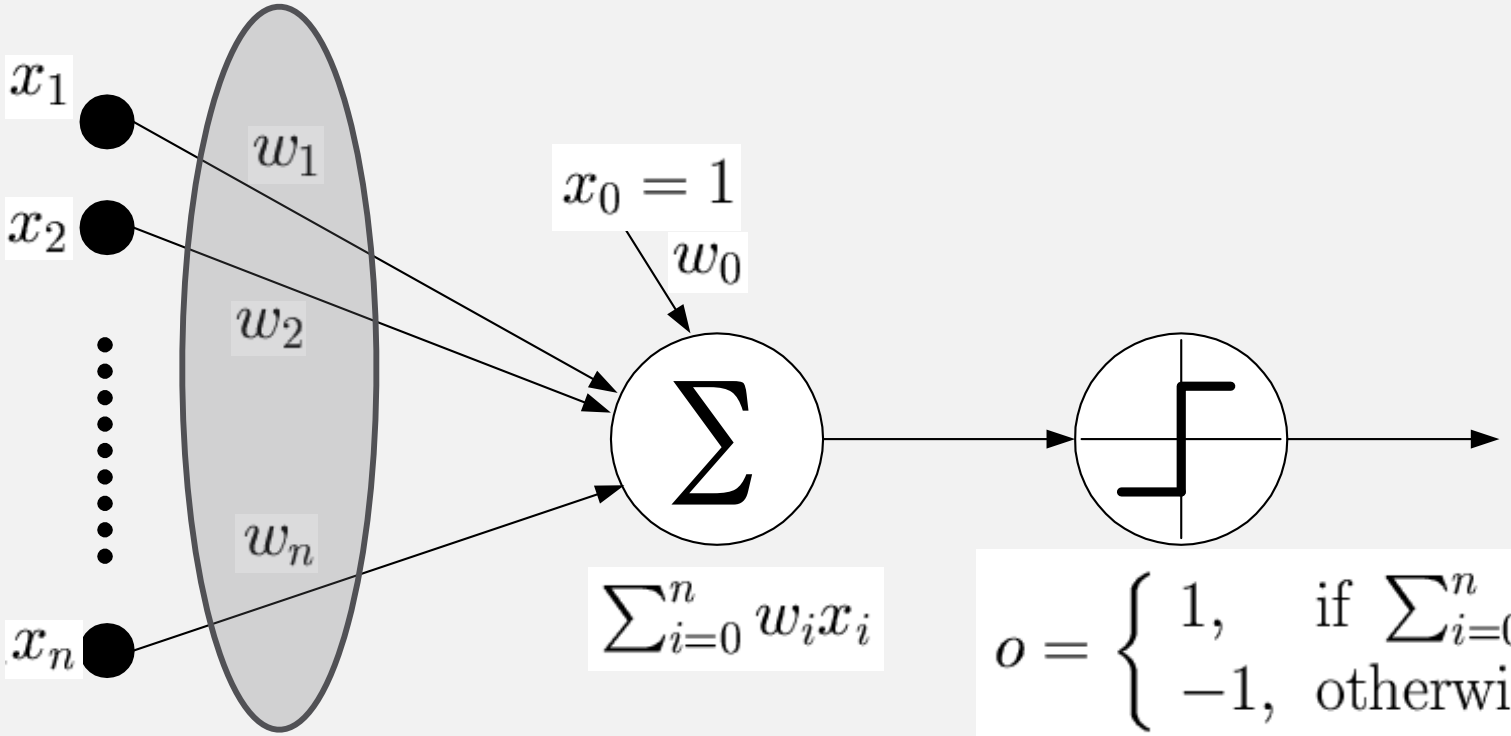
Perceptron

- The neuron calculates a weighted sum of inputs and compares it to a threshold. If the sum is higher than the threshold, the output is set to 1, otherwise to -1.



$$o = \begin{cases} 1, & \text{if } \sum_{i=0}^{n} w_i x_i > 0, \\ -1, & \text{otherwise.} \end{cases}$$

What is here to Learn?

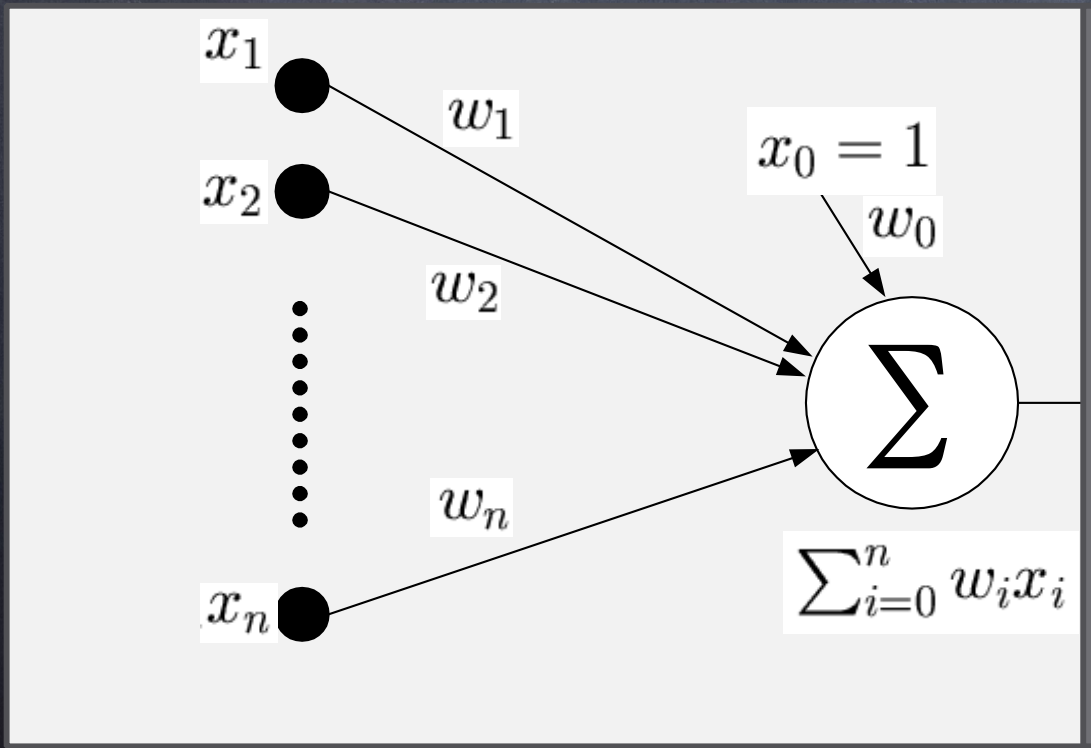


The Perceptron Training Rule

Gradient Descent and Delta Rule

- The delta training rule is best understood by considering the task of training an *unthresholded* perceptron; that is, a *linear unit* for which the output *o* is given by

$$o(\vec{x}) = \vec{w} \cdot \vec{x}$$



Gradient Descent and Delta Rule

In order to derive a weight learning rule for linear units, let us begin by specifying a measure for the training error of a hypothesis (weight vector), relative to the training examples.

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

Derivation GDR

The vector derivative is called the gradient of E $\vec{w}$ with respect to

$$\nabla E(\vec{w}) \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots, \frac{\partial E}{\partial w_n}\right]$$

The gradient specifies the direction that produces the steepest increase in E. The negative of this vector therefore gives the direction of steepest decrease. The training rule for gradient descent is

$$\vec{w} \leftarrow \vec{w} + \Delta\vec{w} \quad \text{where} \quad \Delta\vec{w} = -\eta \nabla E(\vec{w})$$

Derivation of GDR …

The vector of $\frac{\partial E}{\partial w_i}$ derivatives that form the gradient can be obtained by differentiating E

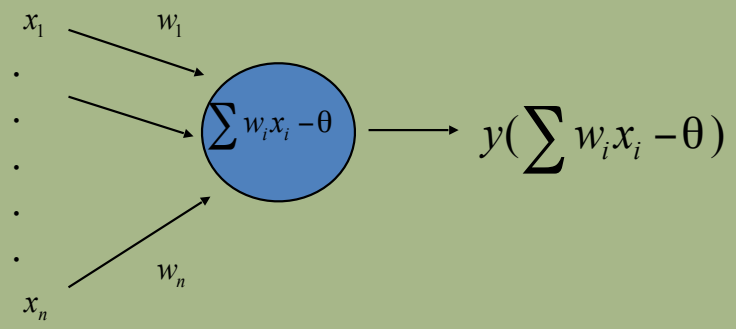
$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i}\frac{1}{2}\sum_{d\in D}(t_d - o_d)^2 \\
&= \frac{1}{2}\sum_{d\in D}\frac{\partial}{\partial w_i}(t_d - o_d)^2 \\
&= \frac{1}{2}\sum_{d\in D}2(t_d - o_d)\frac{\partial}{\partial w_i}(t_d - o_d) \\
&= \sum_{d\in D}(t_d - o_d)\frac{\partial}{\partial w_i}(t_d - \vec{w}\cdot\vec{x}_d) \\
\frac{\partial E}{\partial w_i} &= \sum_{d\in D}(t_d - o_d)(-x_{id})
\end{aligned}$$

The weight update rule for standard gradient descent can be summarized as

$$\Delta w_i = -\eta\frac{\partial E}{\partial w_i}$$

$$\Delta w_i = \eta\sum_{d\in D}(t_d - o_d)x_{id}$$

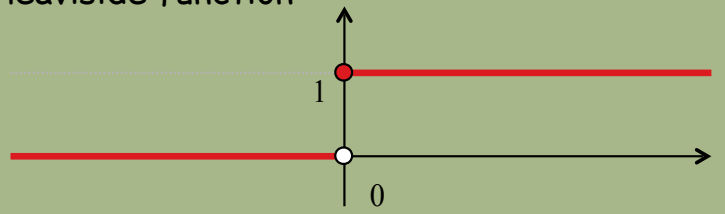
Activating function



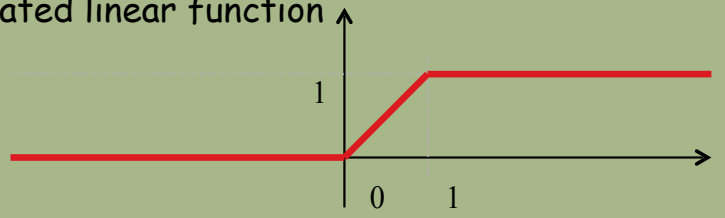
$$y(\xi) = \frac{1}{1+e^{-\xi}}$$

$$y(\xi) = \frac{1-e^{-\xi}}{1+e^{-\xi}}$$

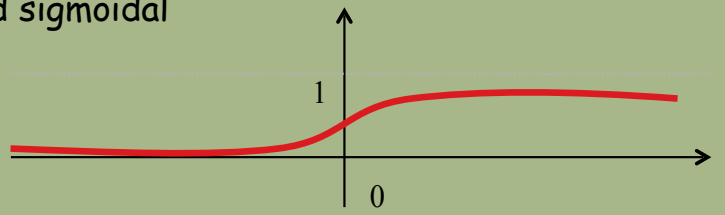
Heaviside function



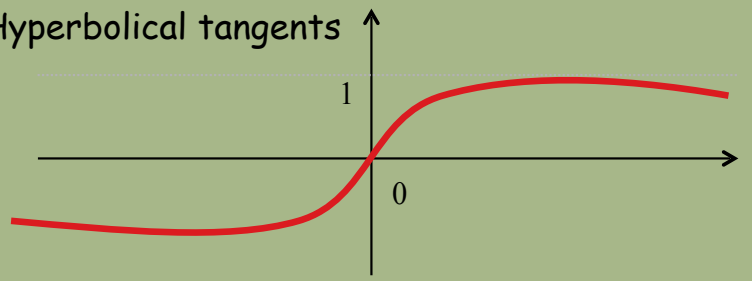
Saturated linear function



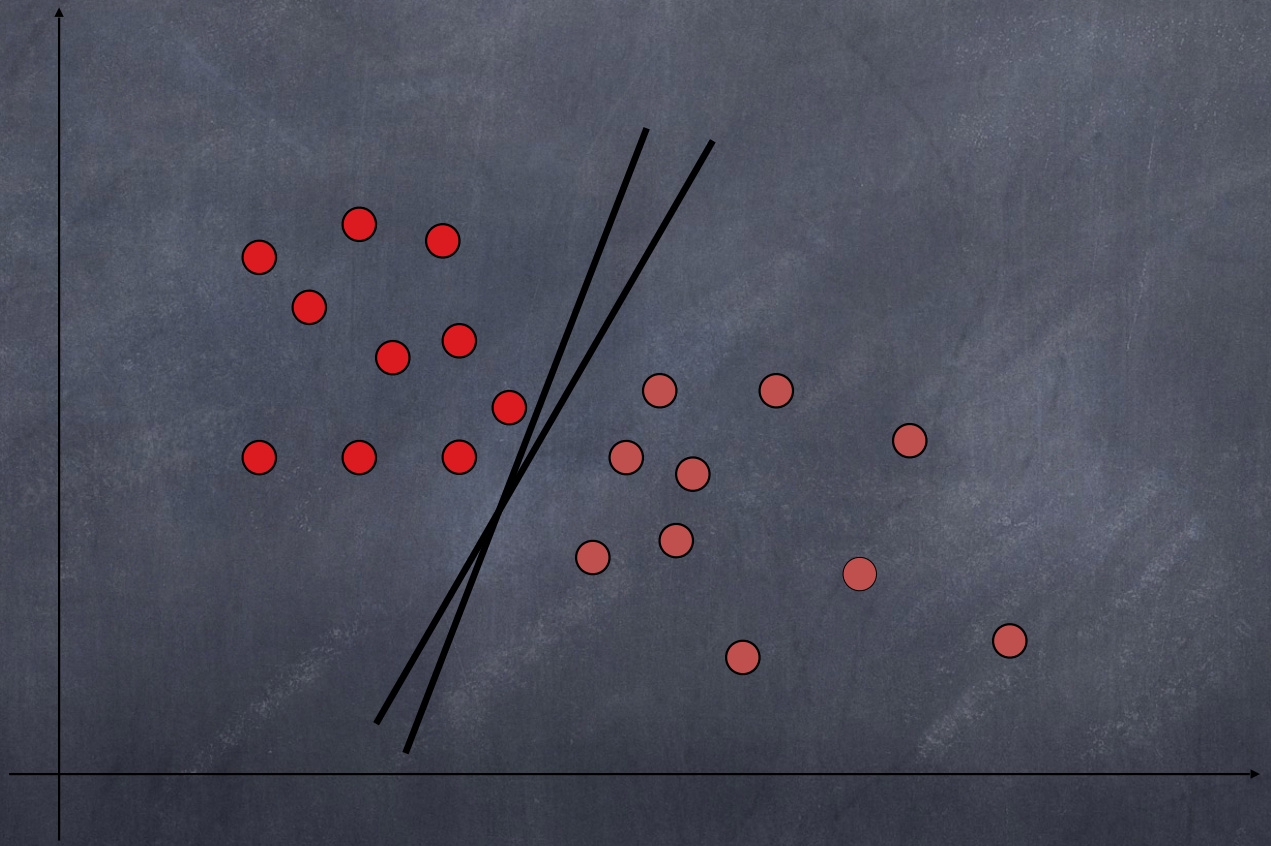
Standard sigmoidal function



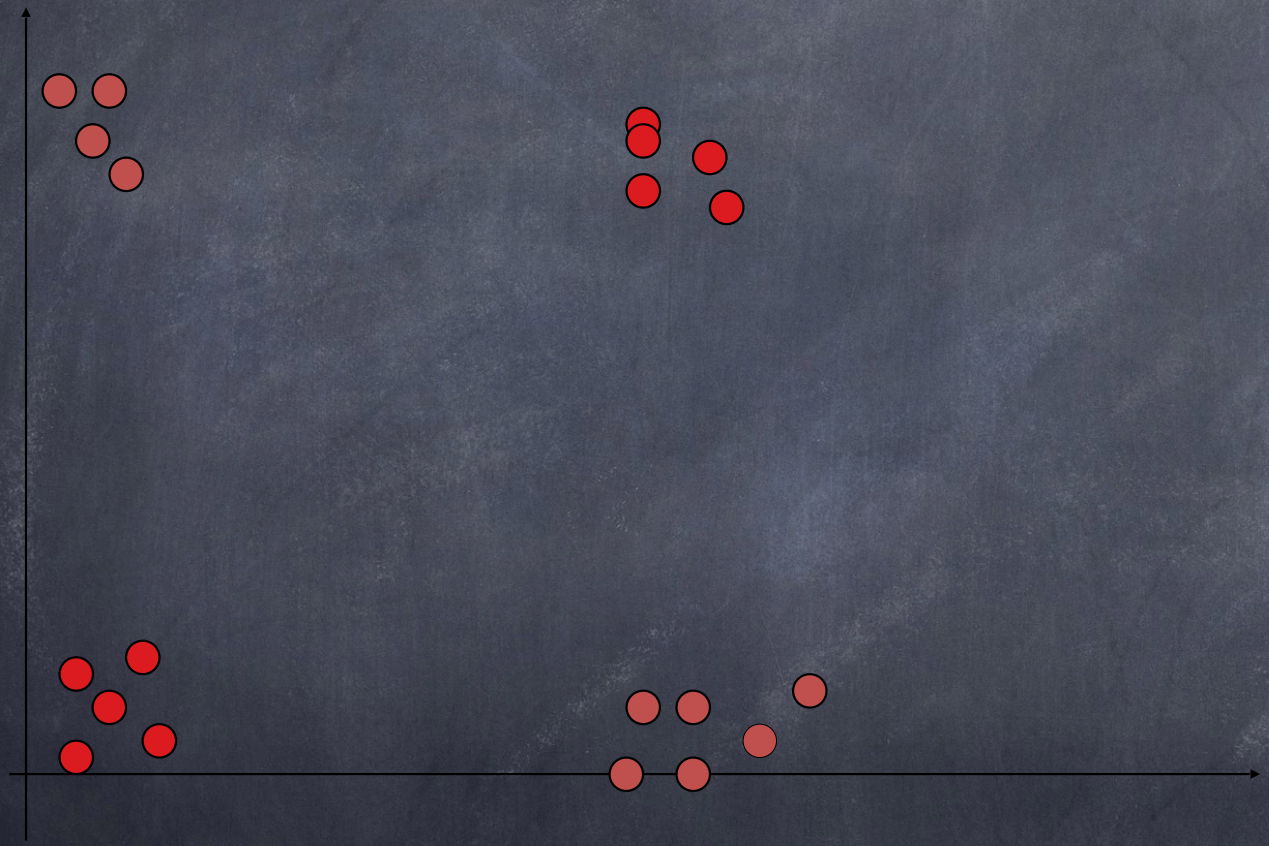
Hyperbolical tangents



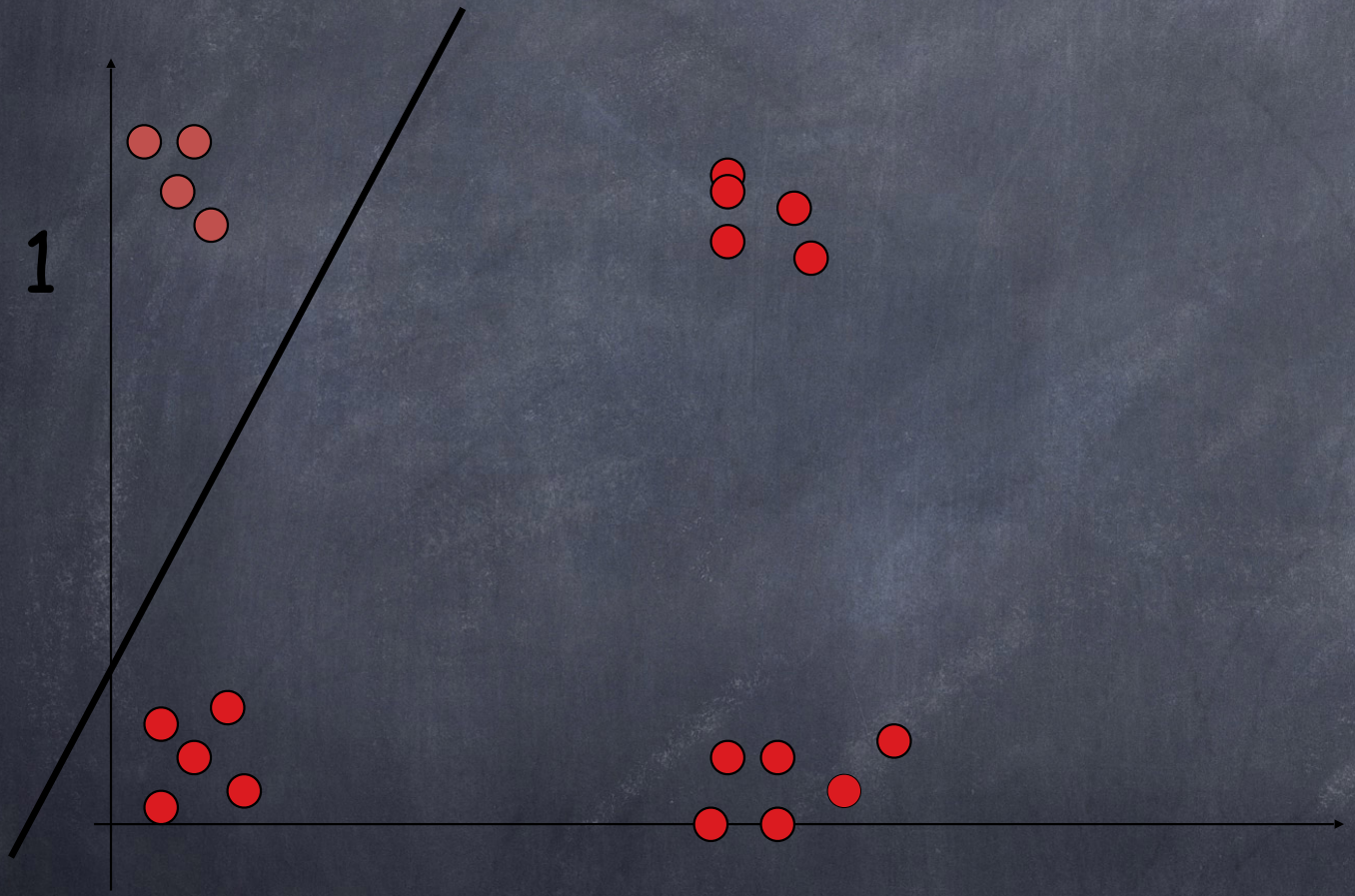
AB problem



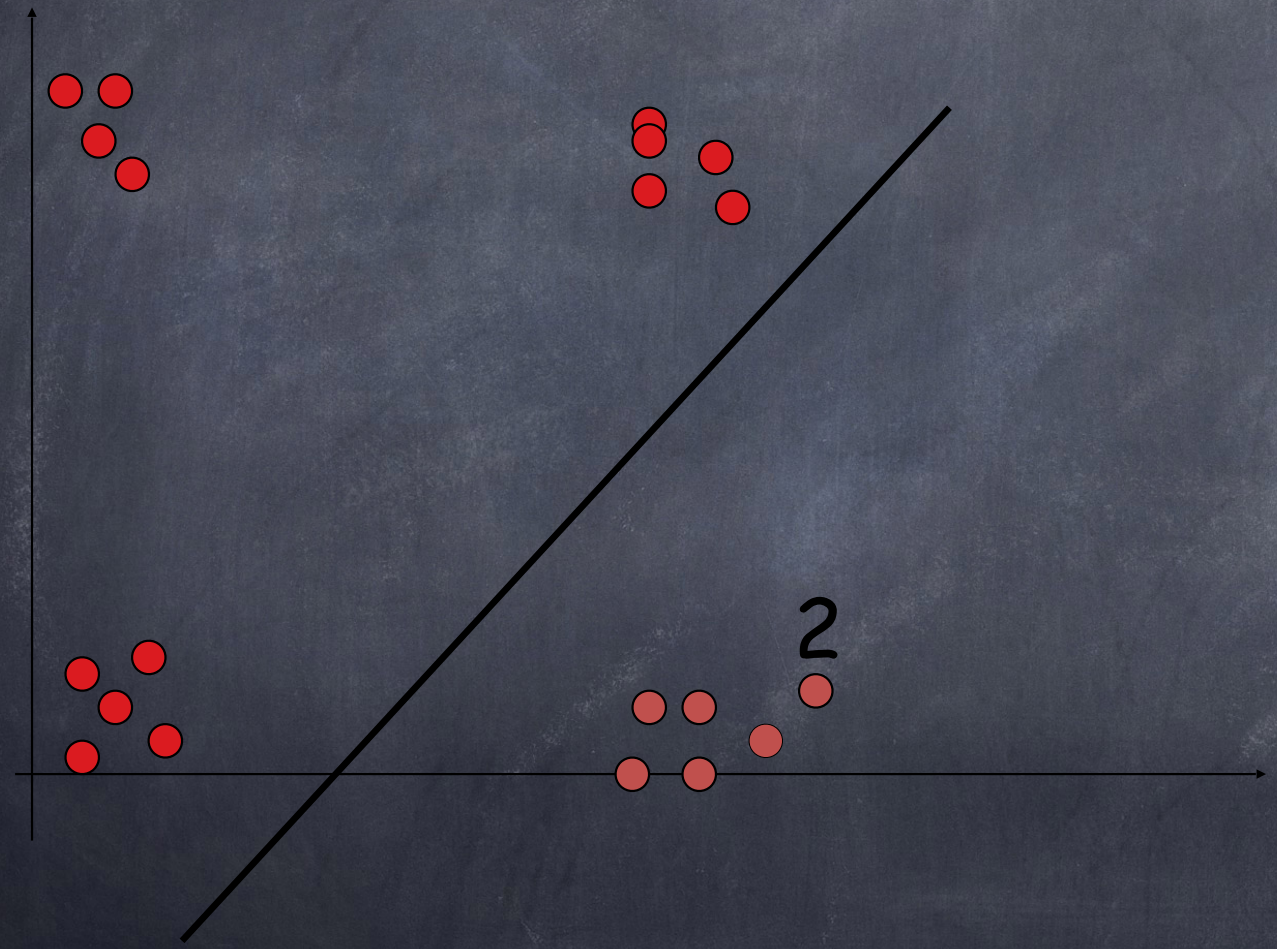
XOR problem



XOR problem



XOR problem



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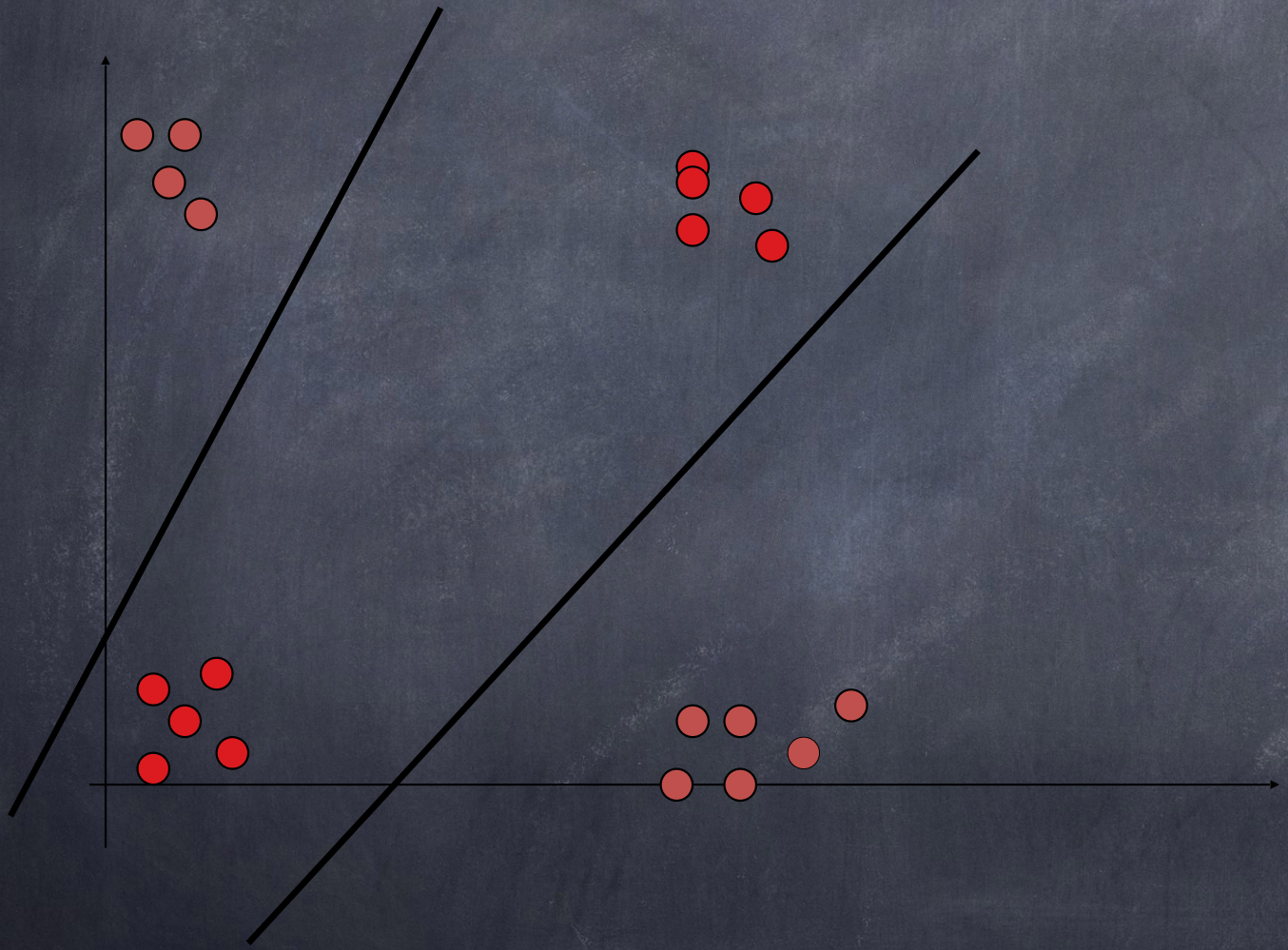
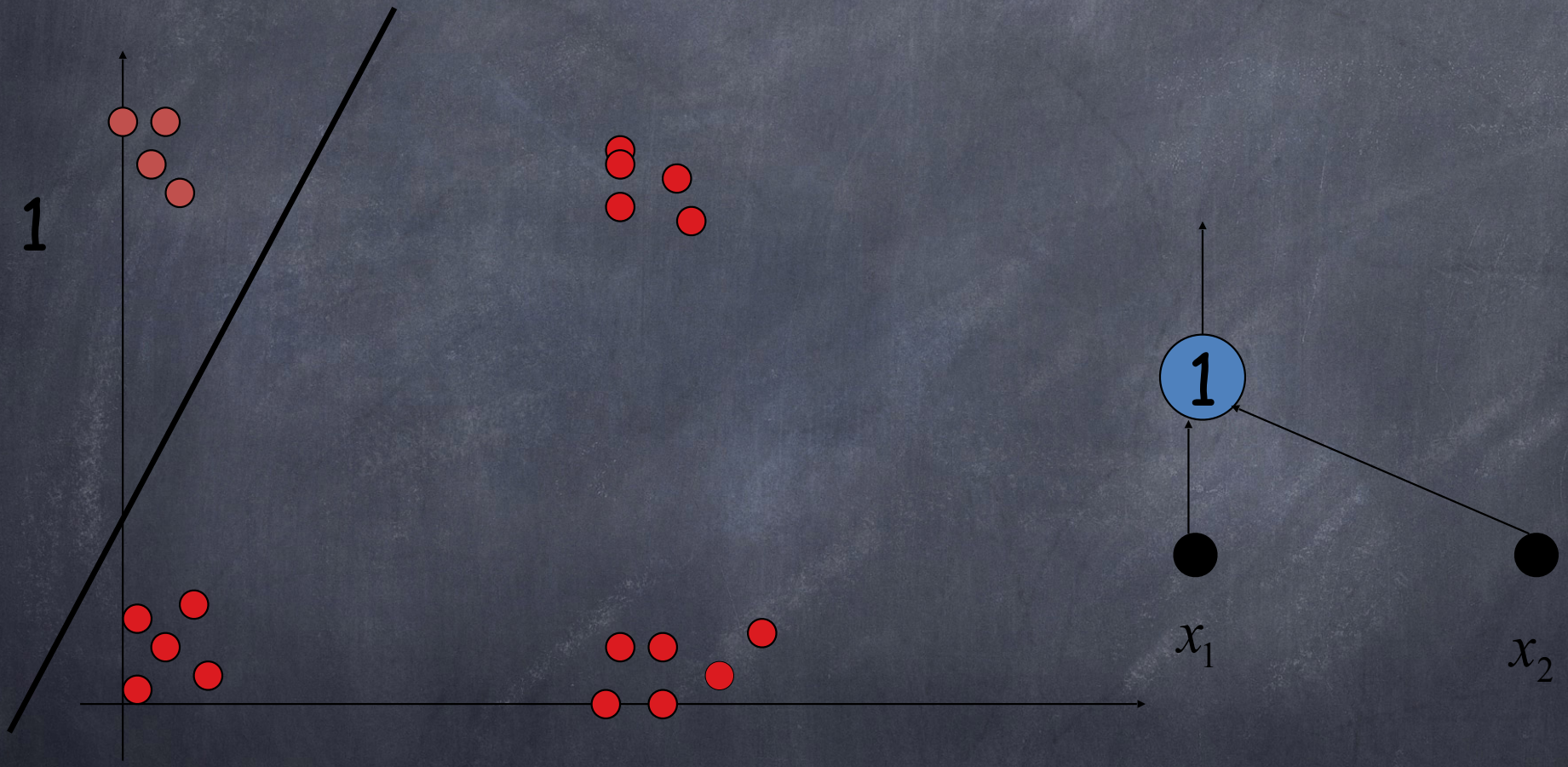
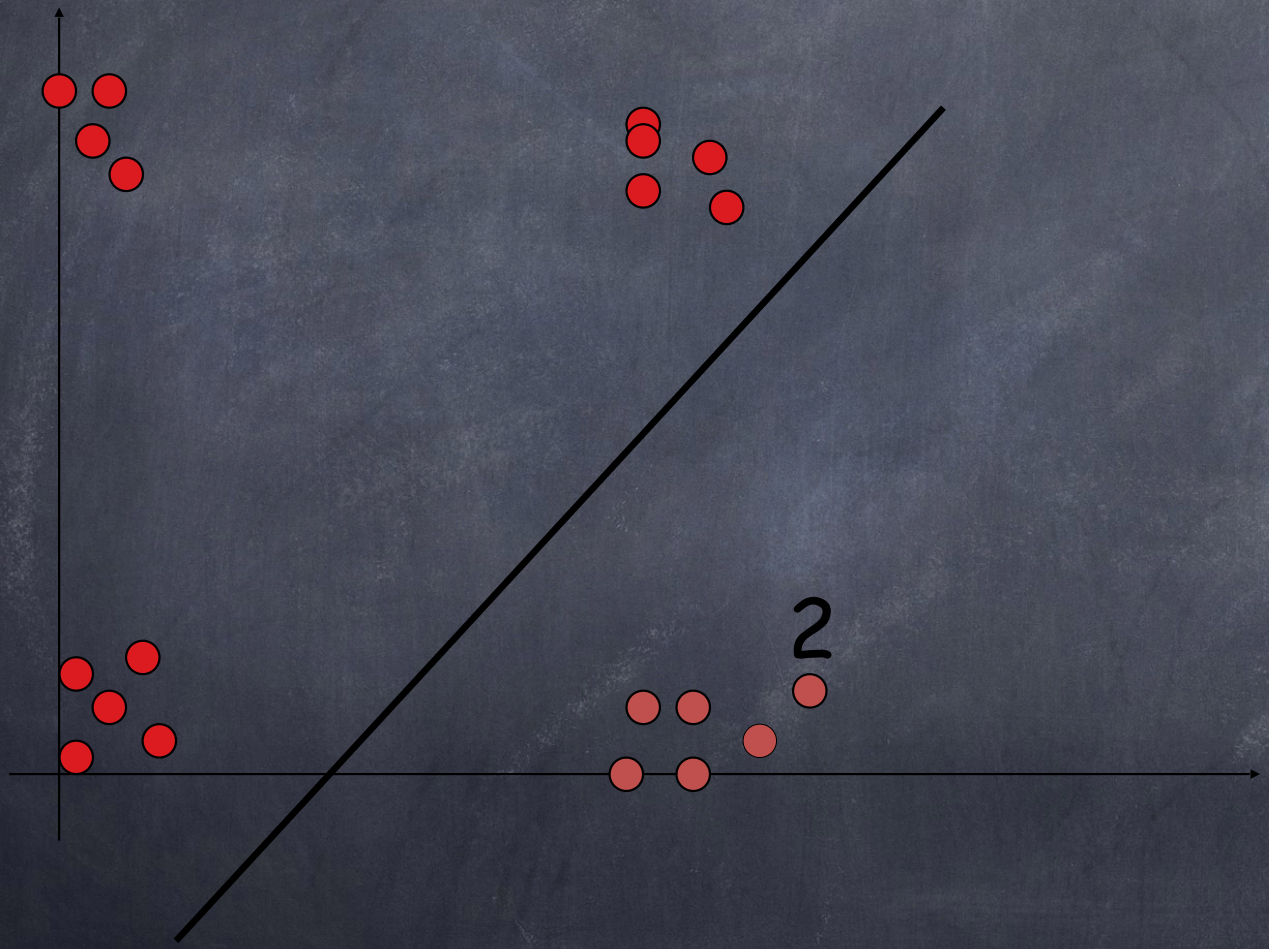
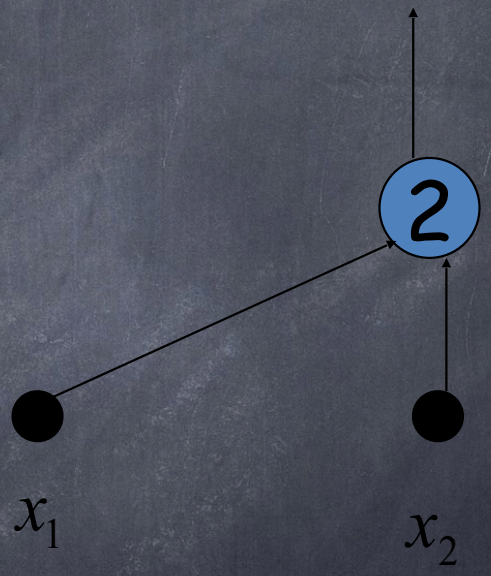


XOR problem



XOR problem





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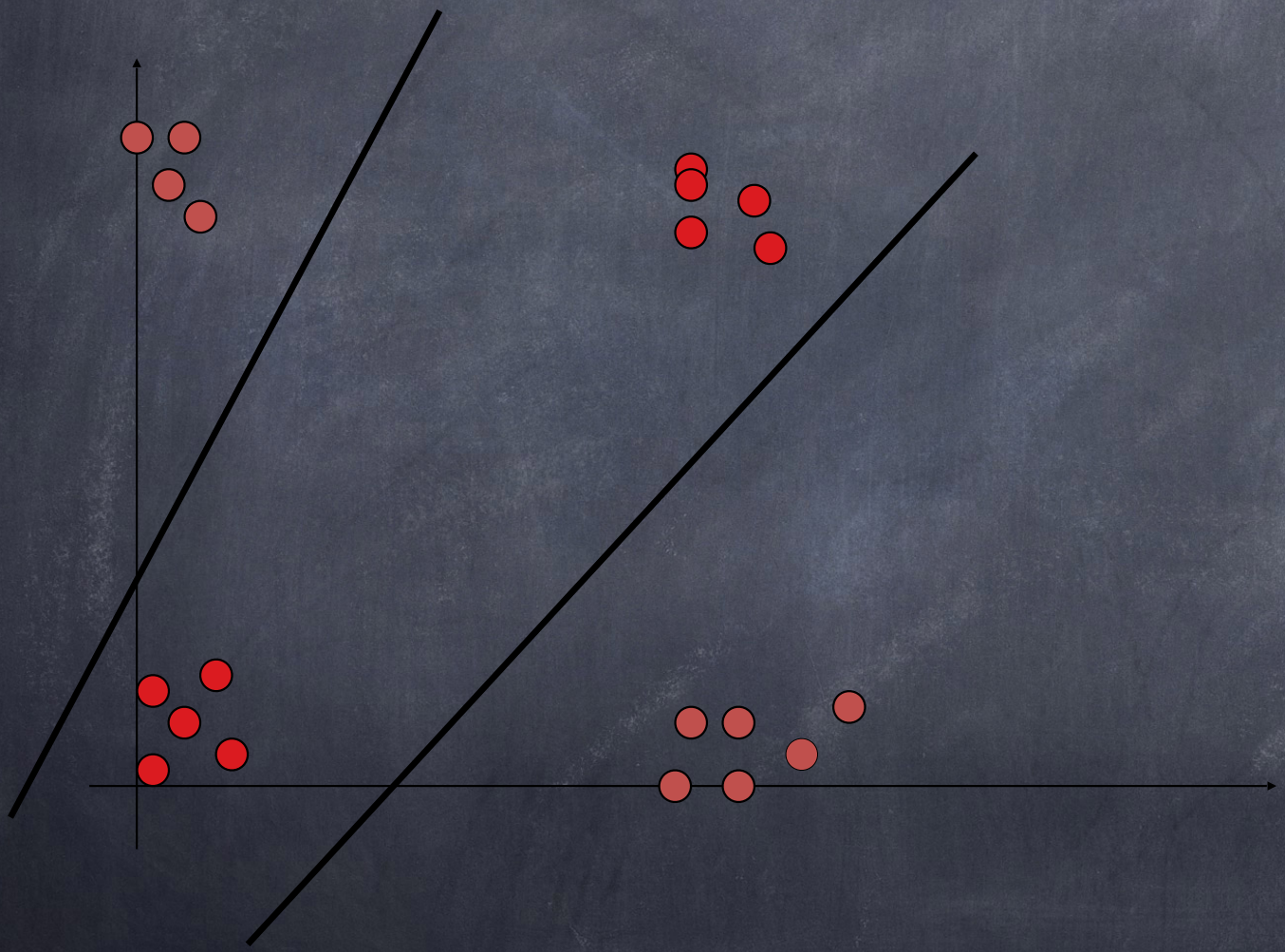
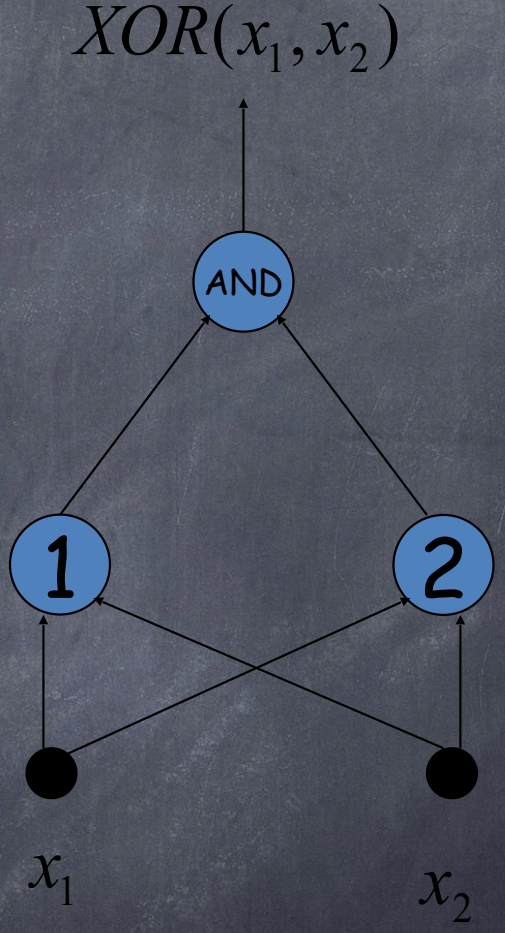
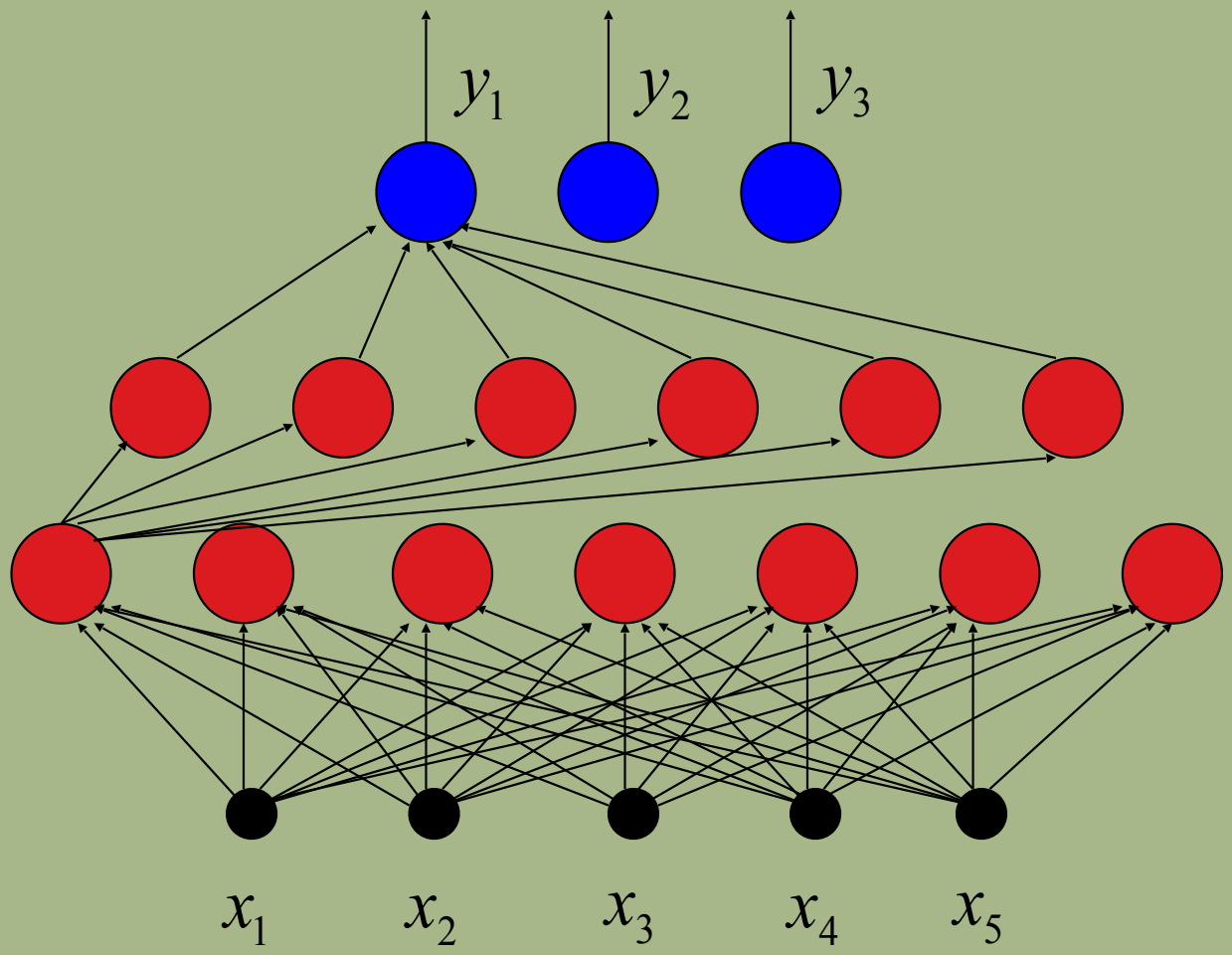




Feed-forward layered network

$$NN : X \rightarrow Y$$



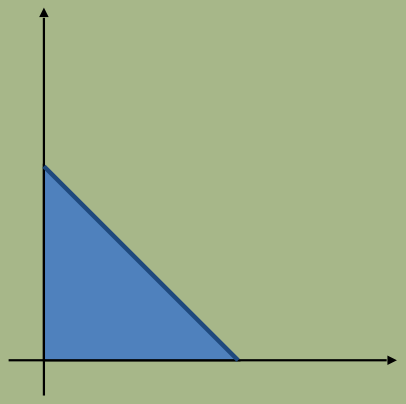
Output layer

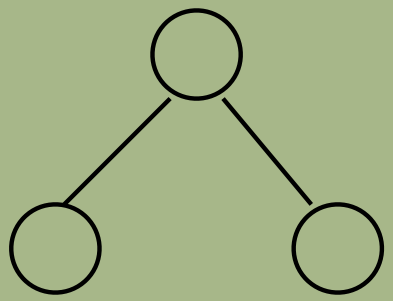
2nd hidden layer

1st hidden layer

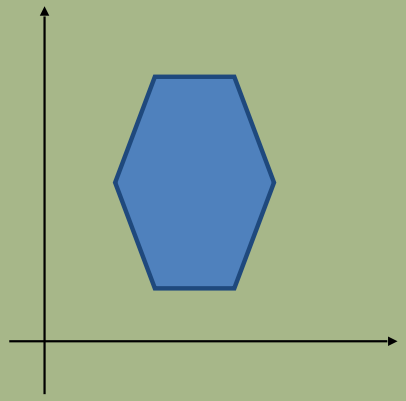
Input layer

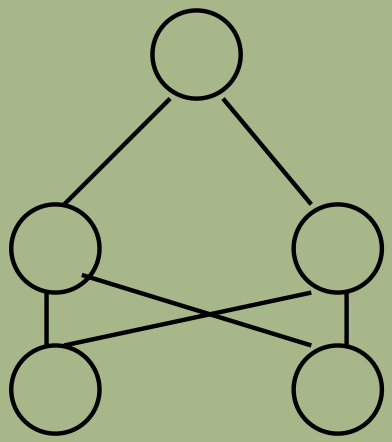
What do each of the layers do?



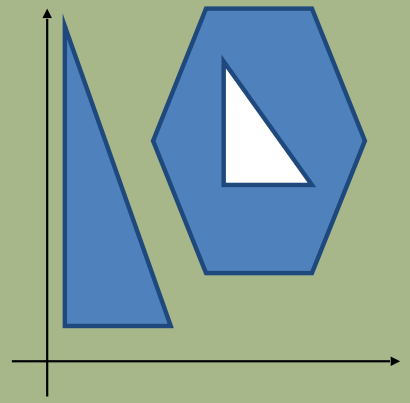


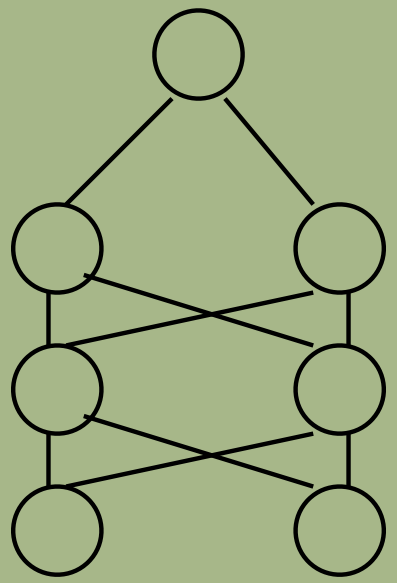
1st layer draws linear boundaries





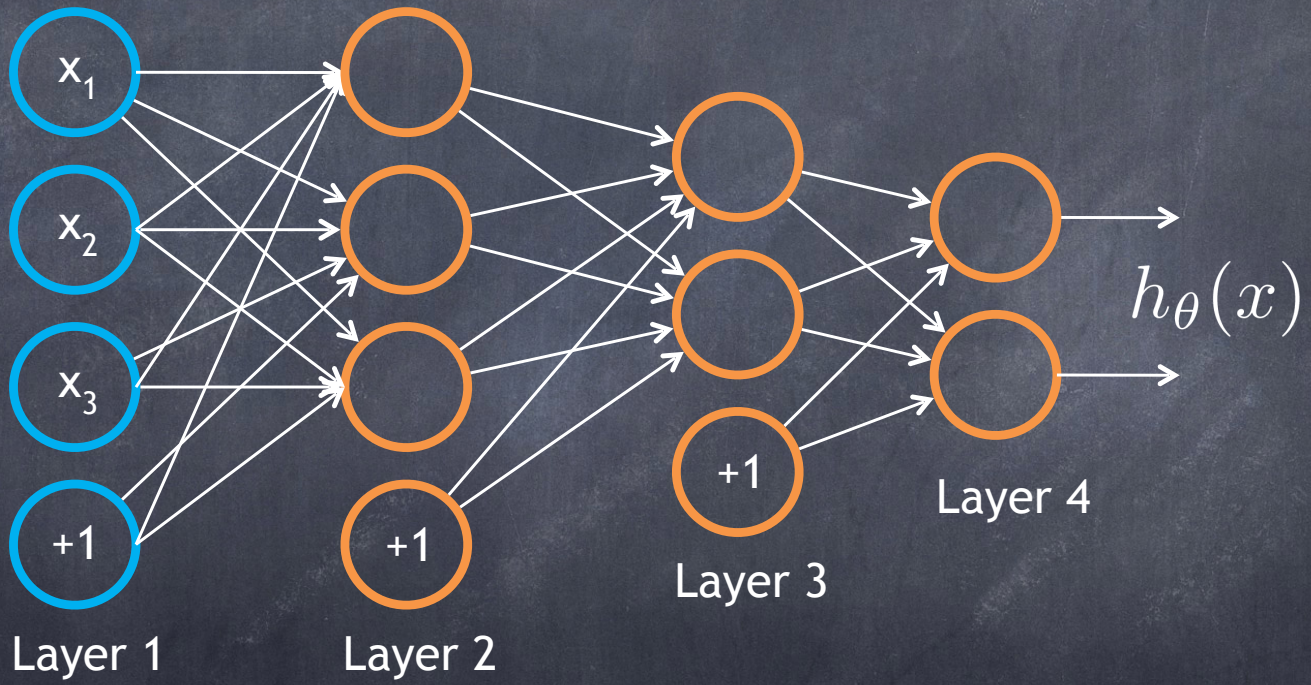
2nd layer combines the boundaries



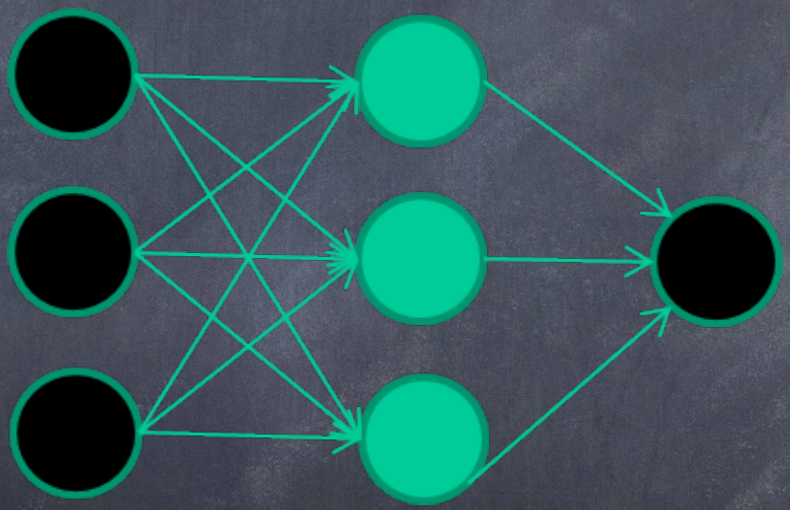


3rd layer can generate arbitrarily complex boundaries

Neural Network



From Single Hidden Layer to Multiple Layers



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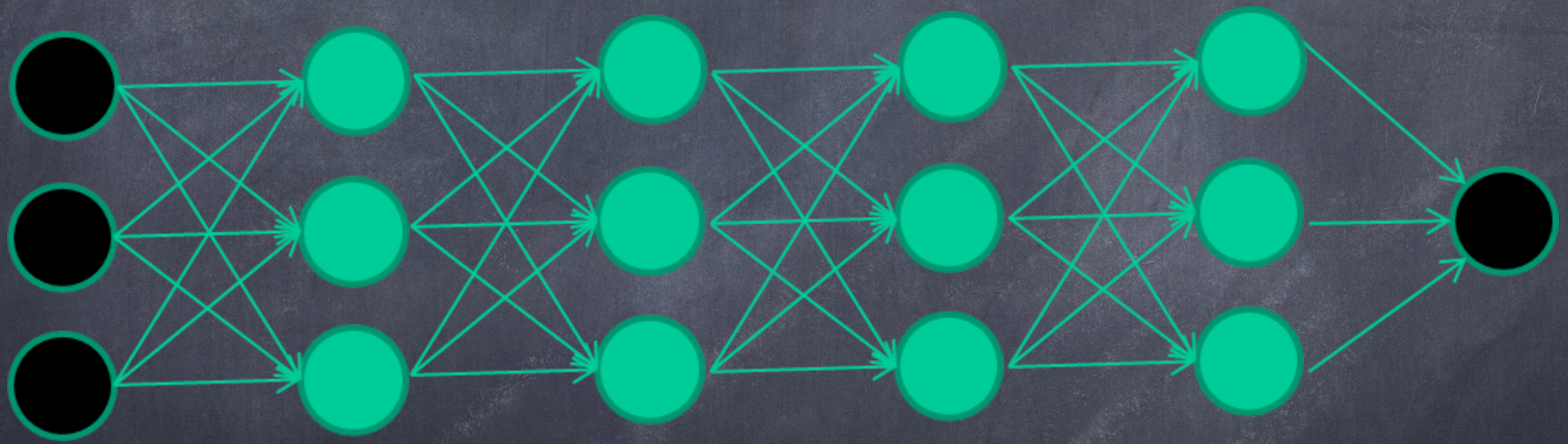
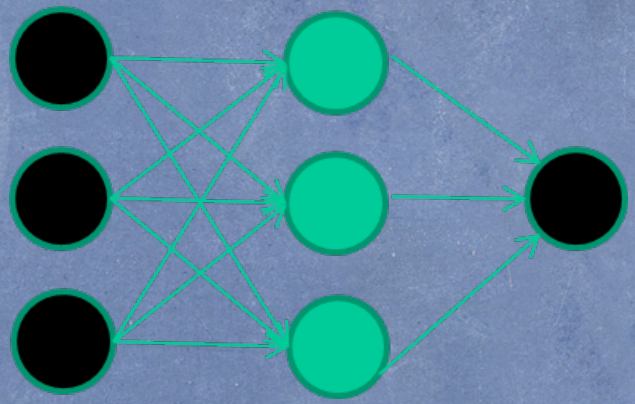


Let Us Train A Simple Network

Training dataset

Fields			class
1.4	2.7	1.9	0
3.8	3.4	3.2	0
6.4	2.8	1.7	1
4.1	0.1	0.2	0

etc ...



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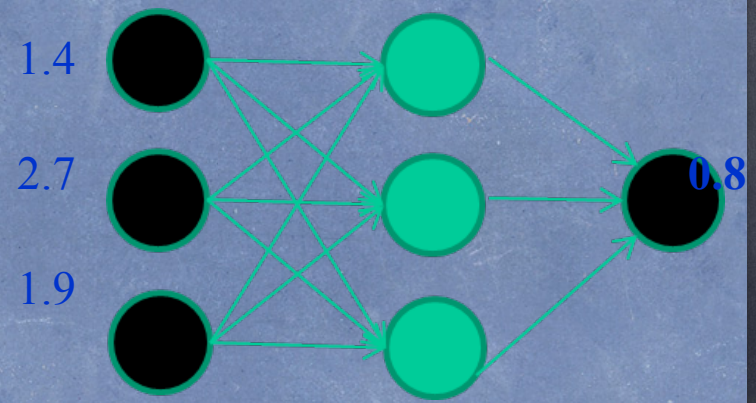
etc ...



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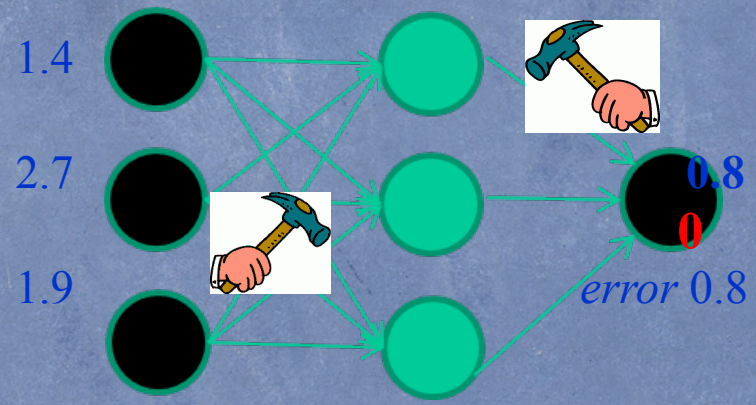

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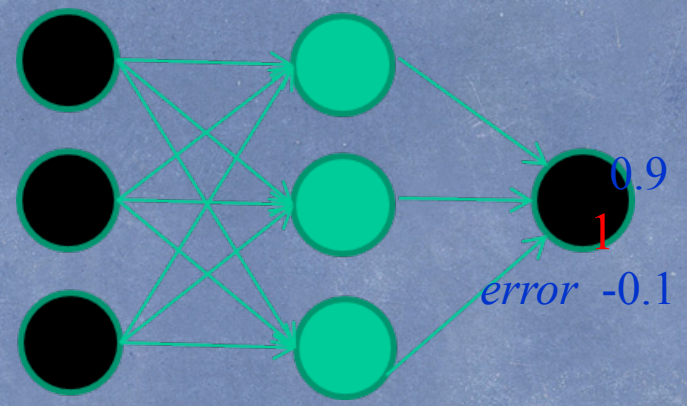
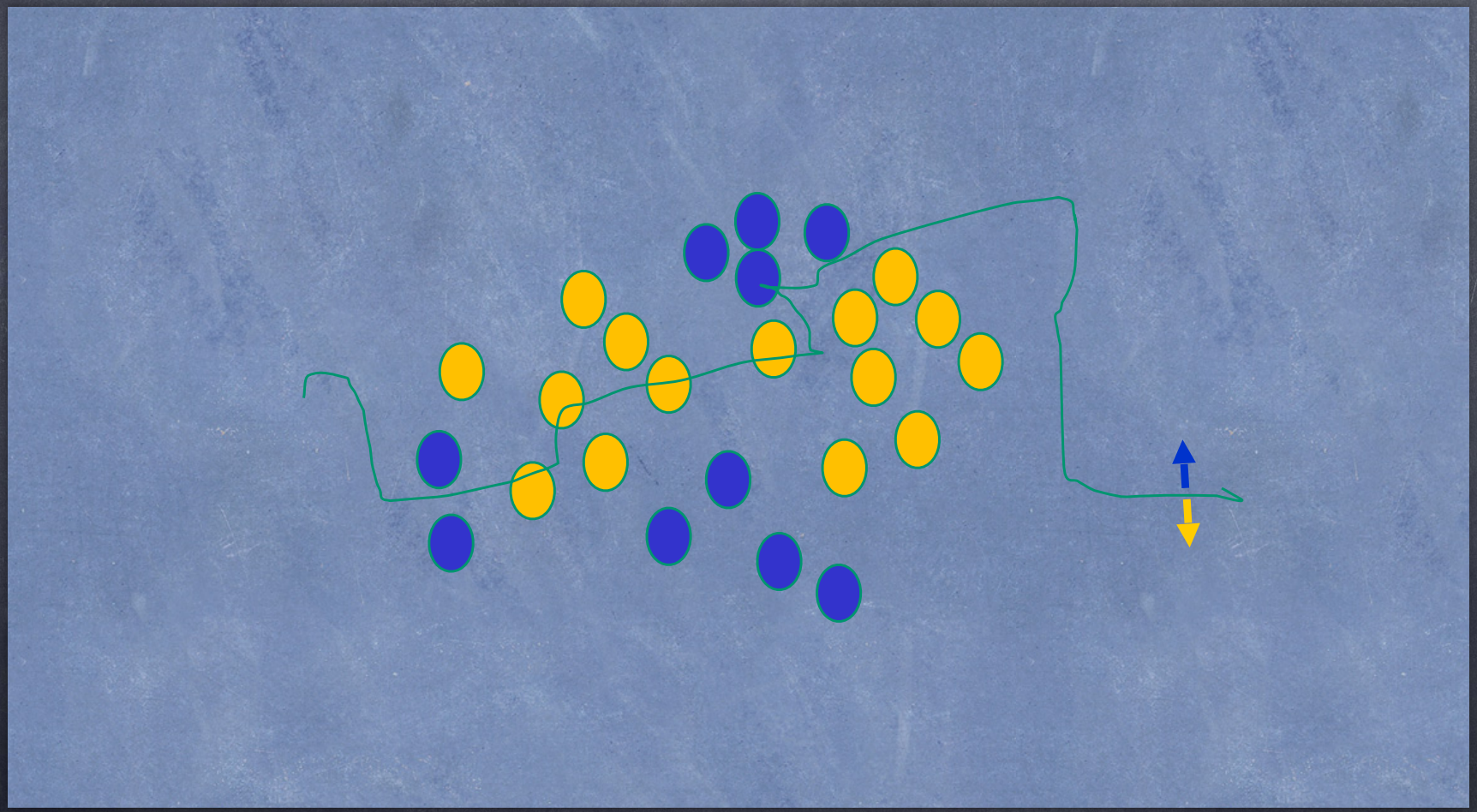


What is happening at the backend?



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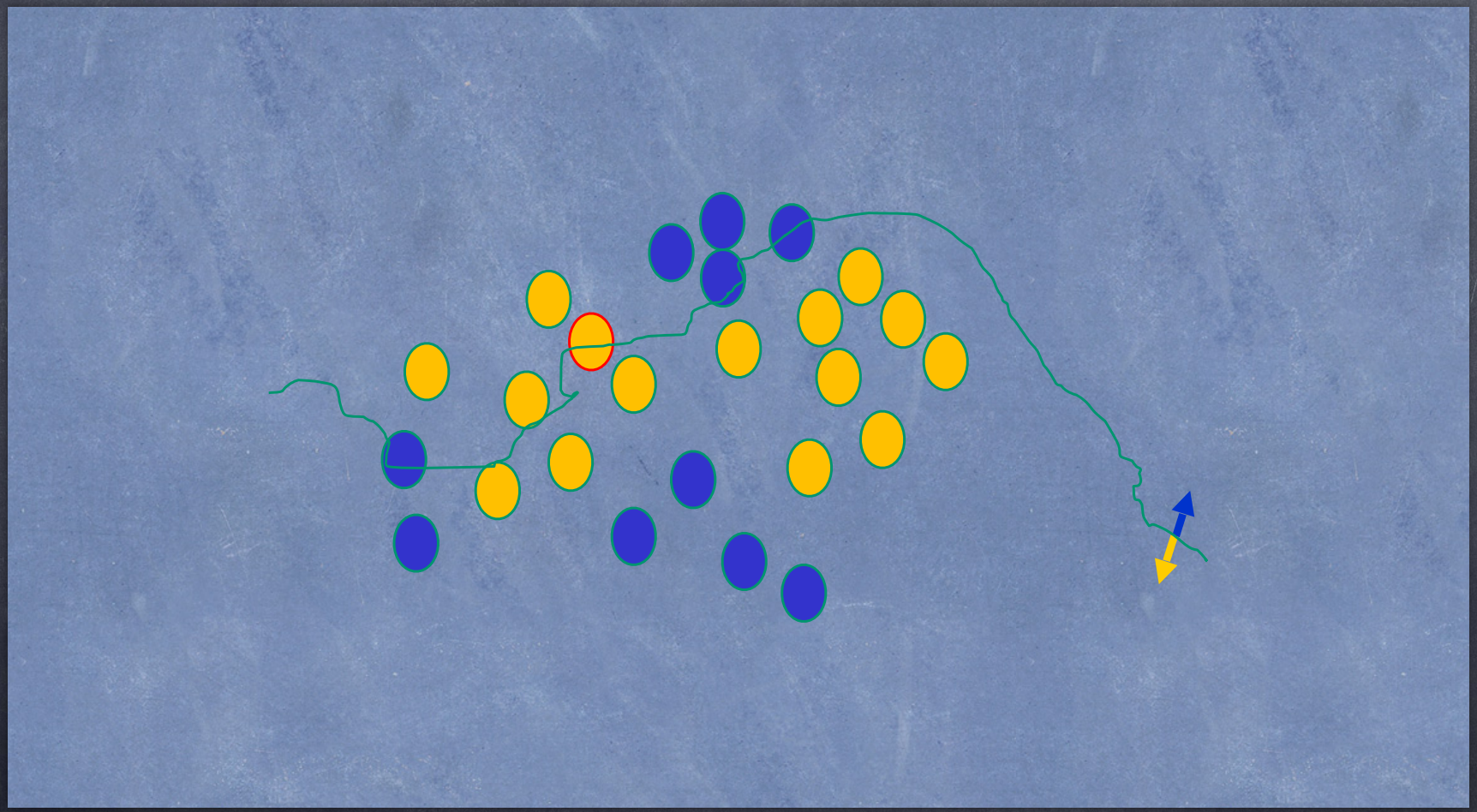

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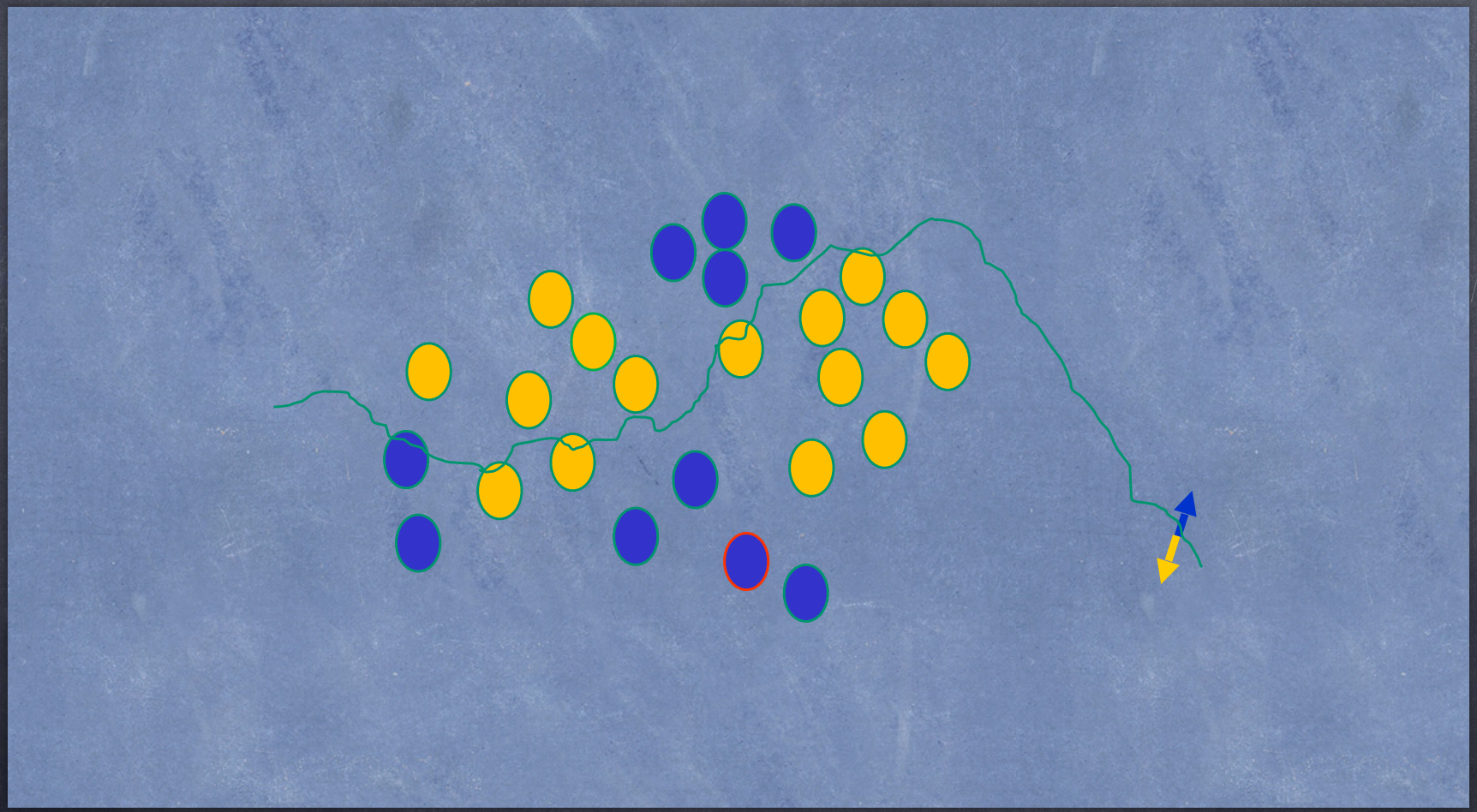

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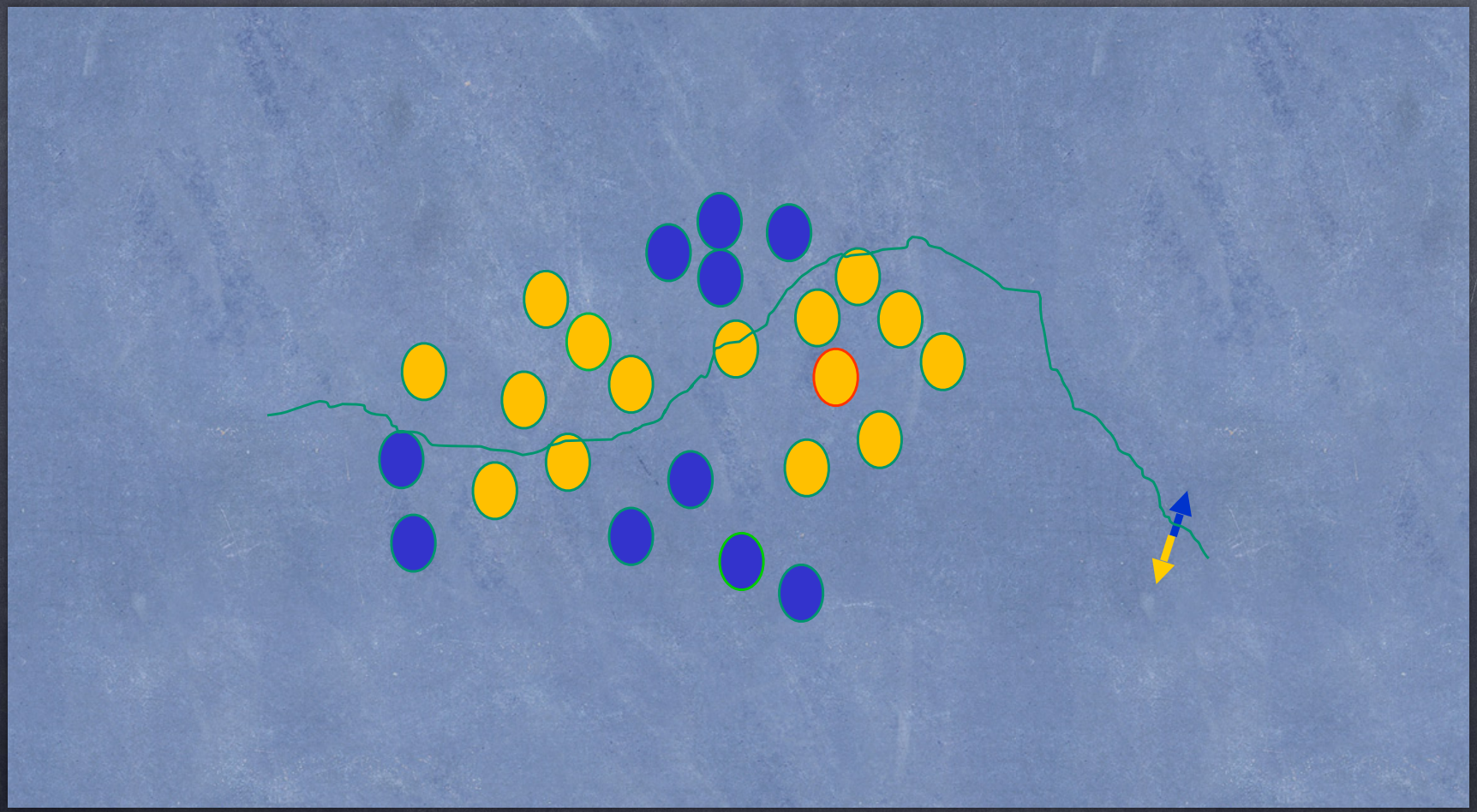

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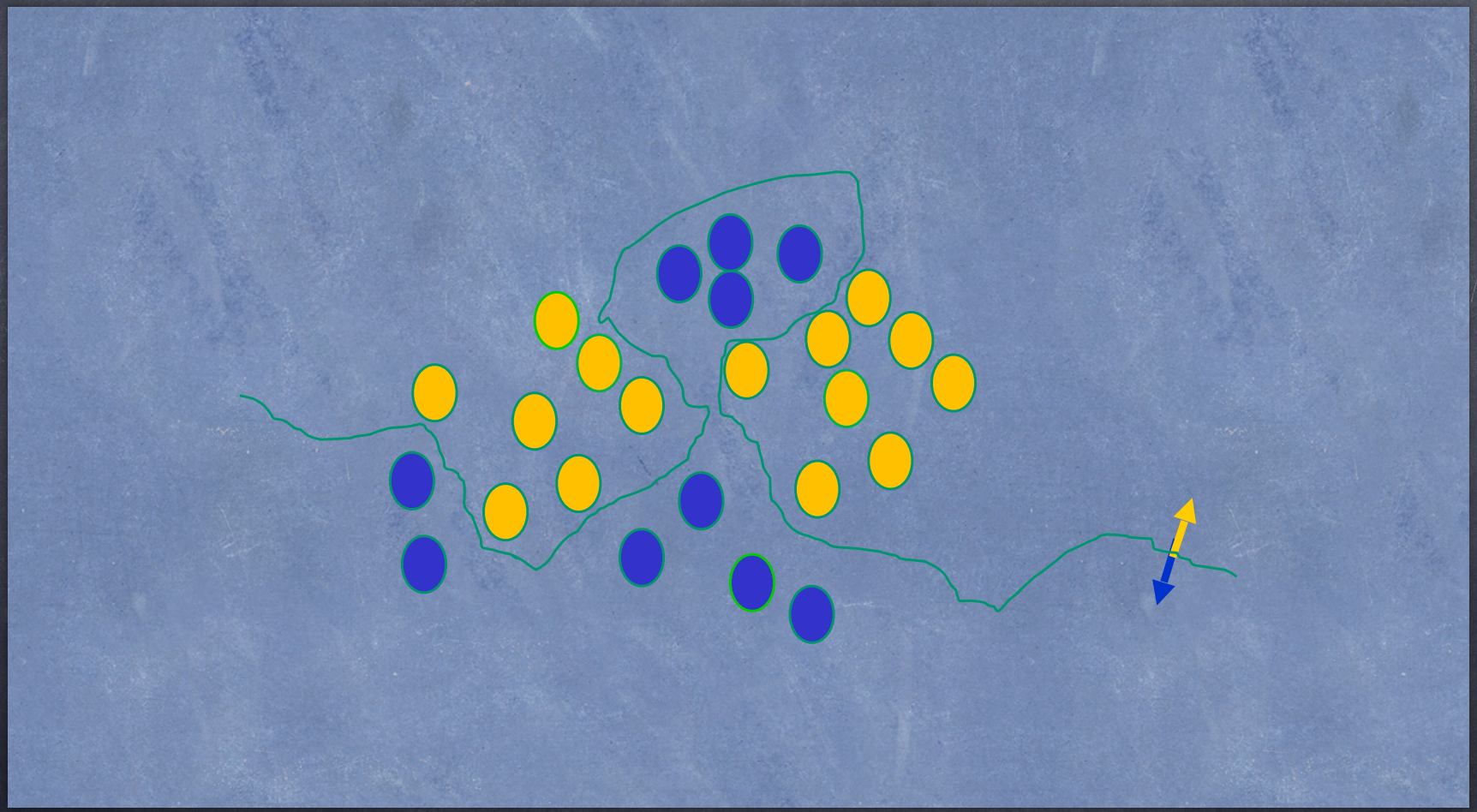
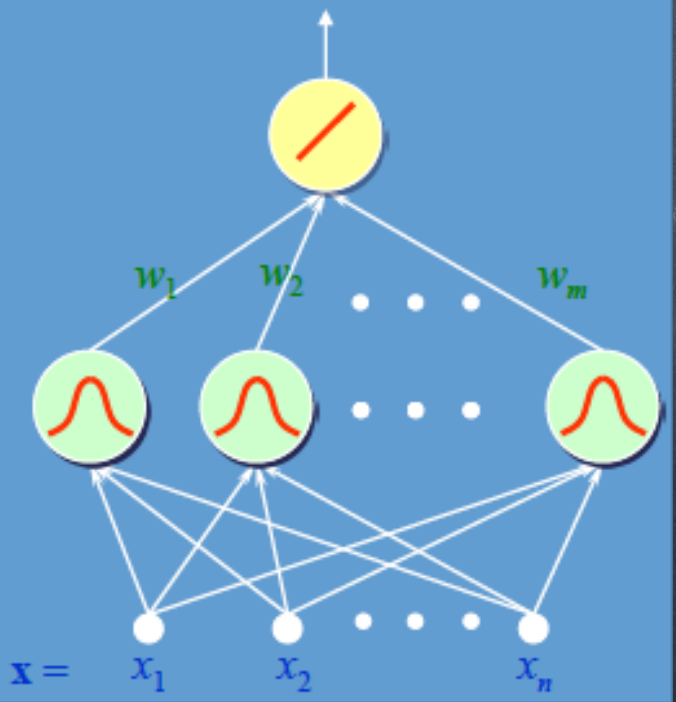


Training Neural Network

Training set $\mathcal{T} = \left\{ \left(\mathbf{x}^{(k)}, y^{(k)} \right) \right\}_{k=1}^{p}$

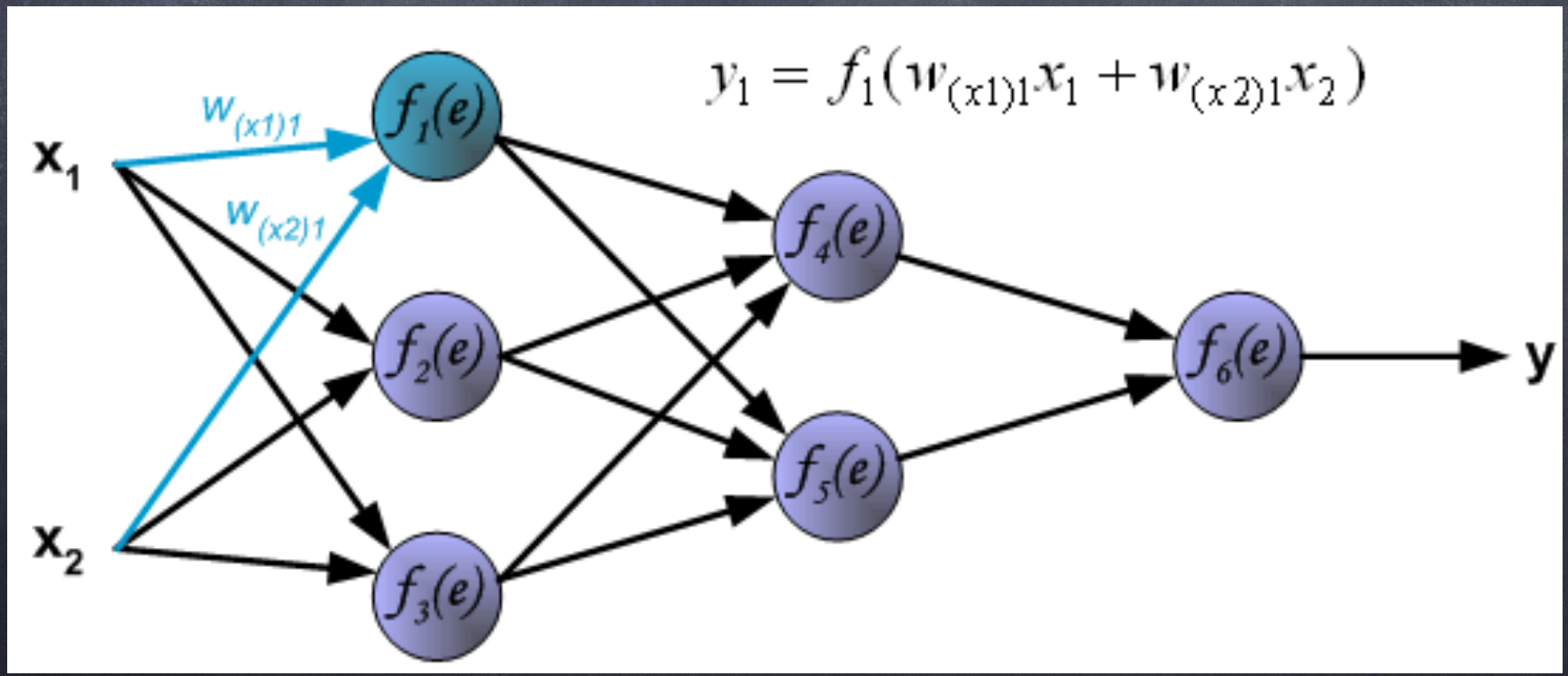
Goal $y^{(k)} \approx f\left(\mathbf{x}^{(k)} \right)$ for all k

$$\min E = \frac{1}{2} \sum_{k=1}^{p} \left[y^{(k)} - f\left(\mathbf{x}^{(k)} \right) \right]^2$$

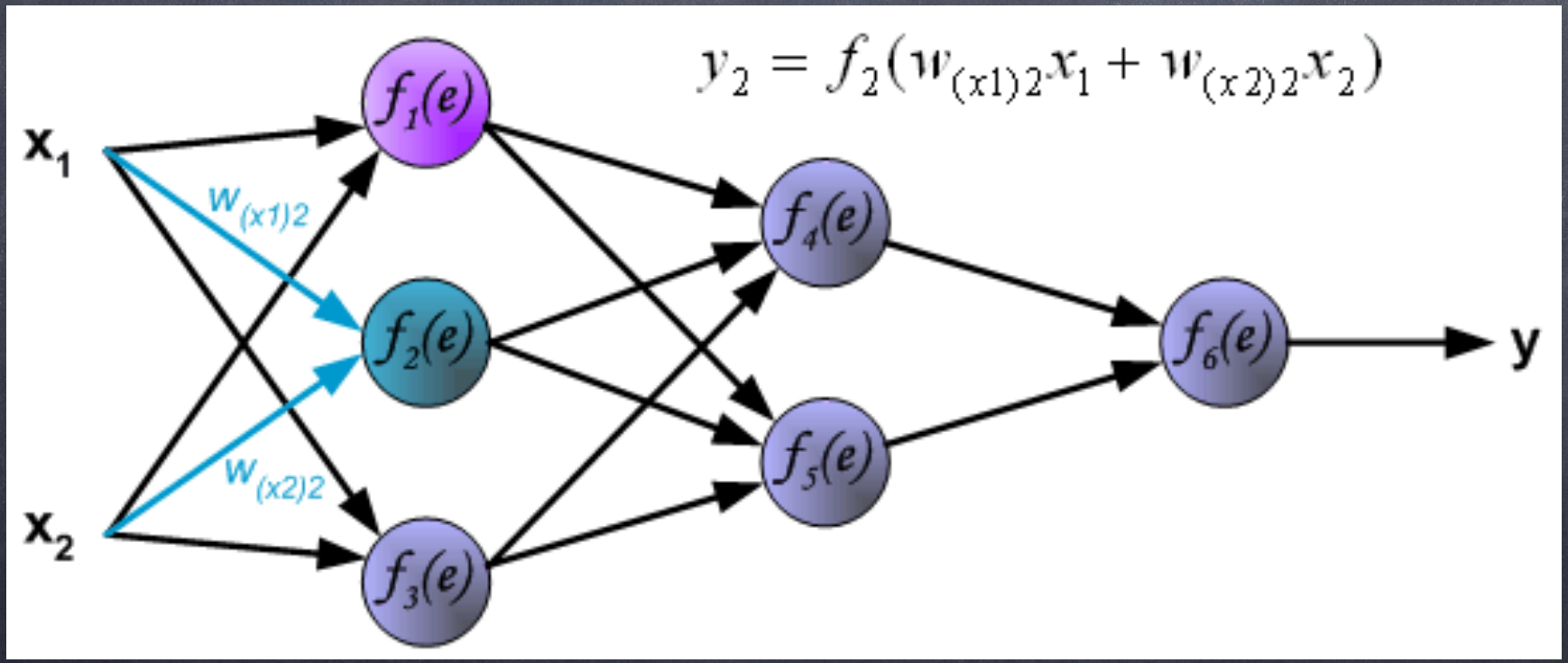


How to Train Neural Network

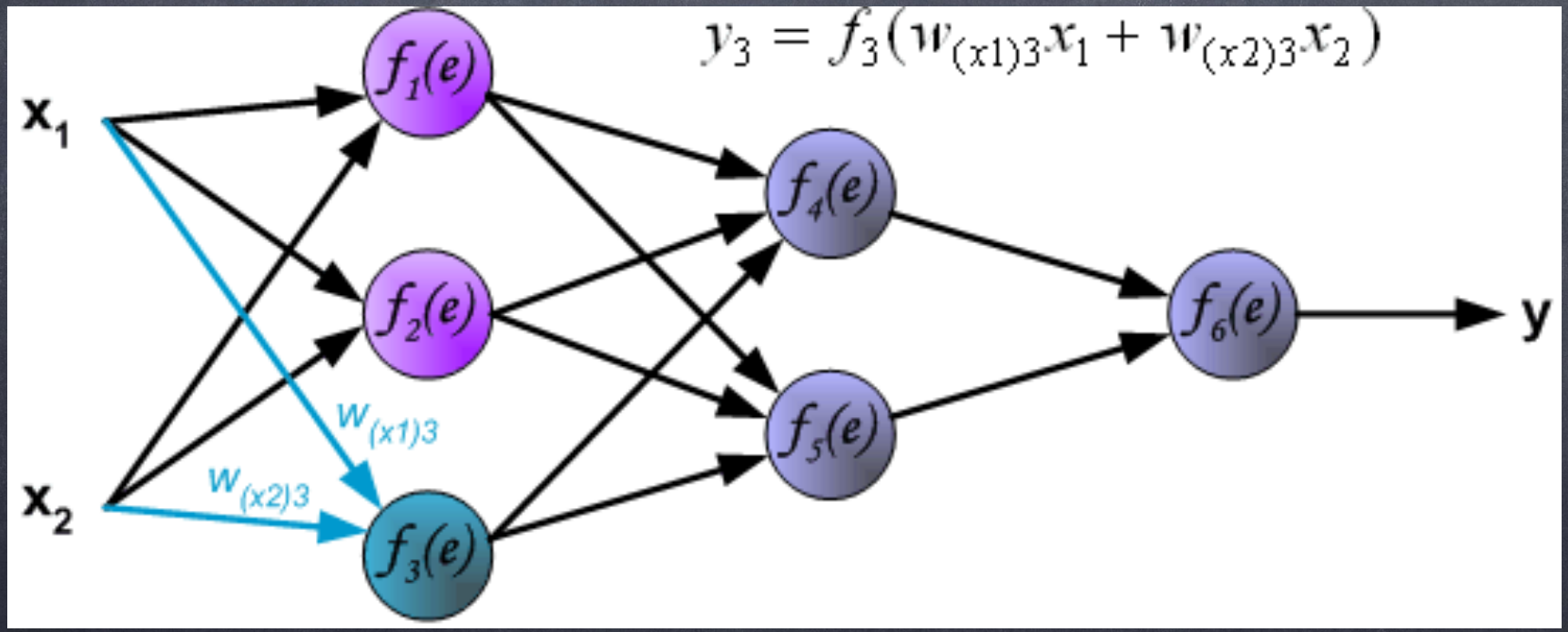
- Back-propagation approach



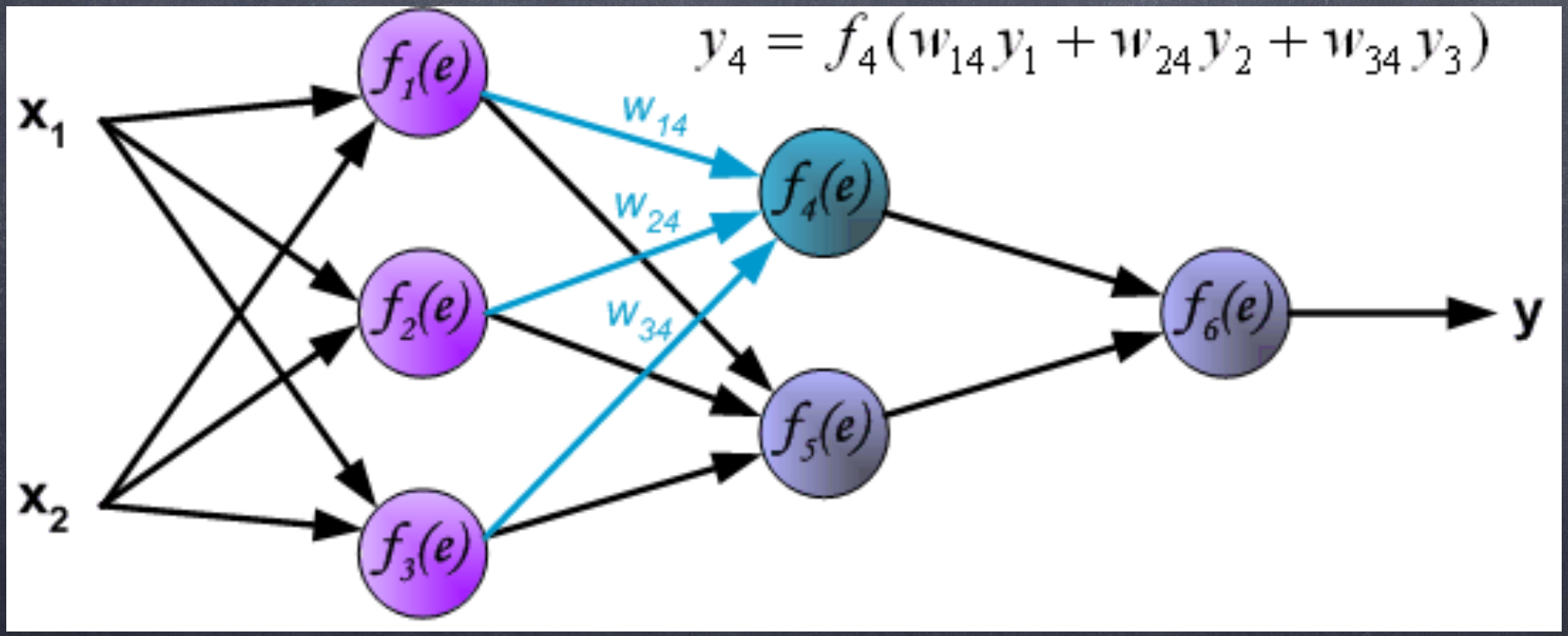
How to Train Neural Network



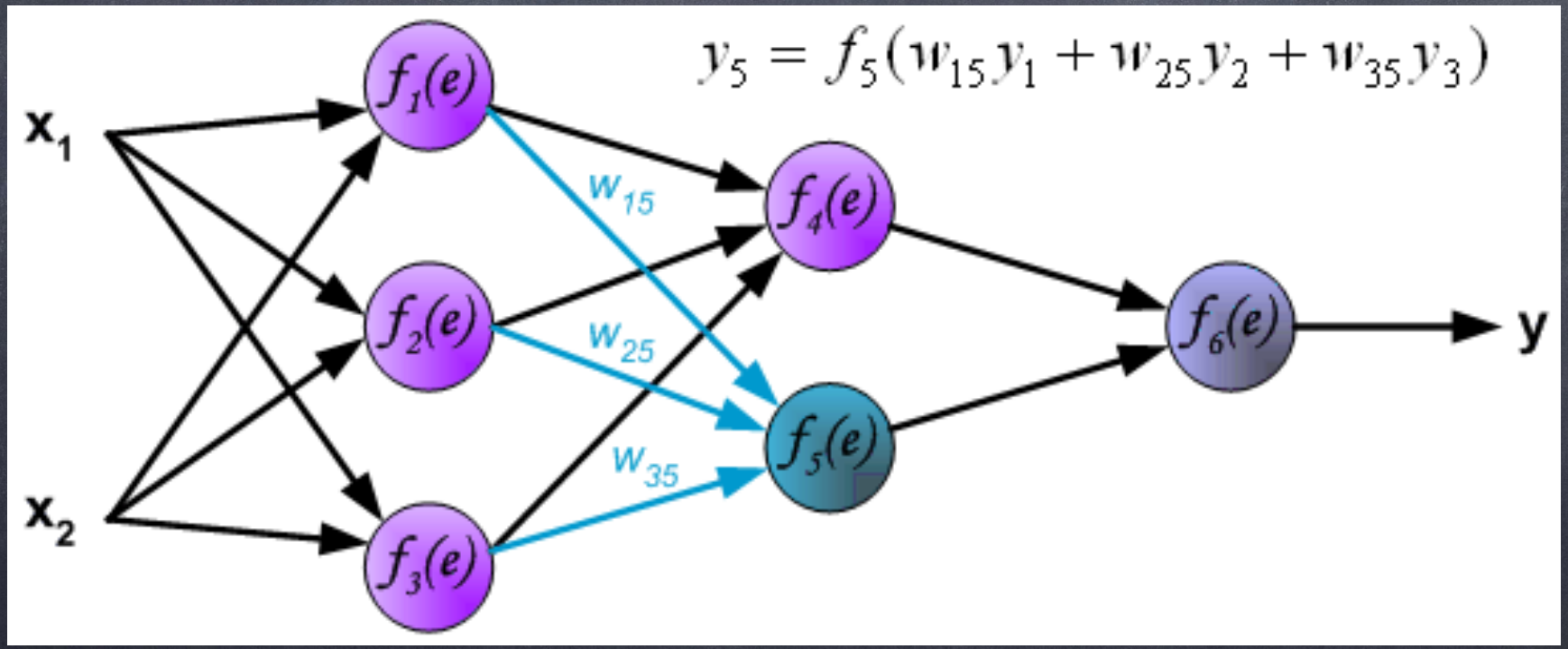
How to Train Neural Network



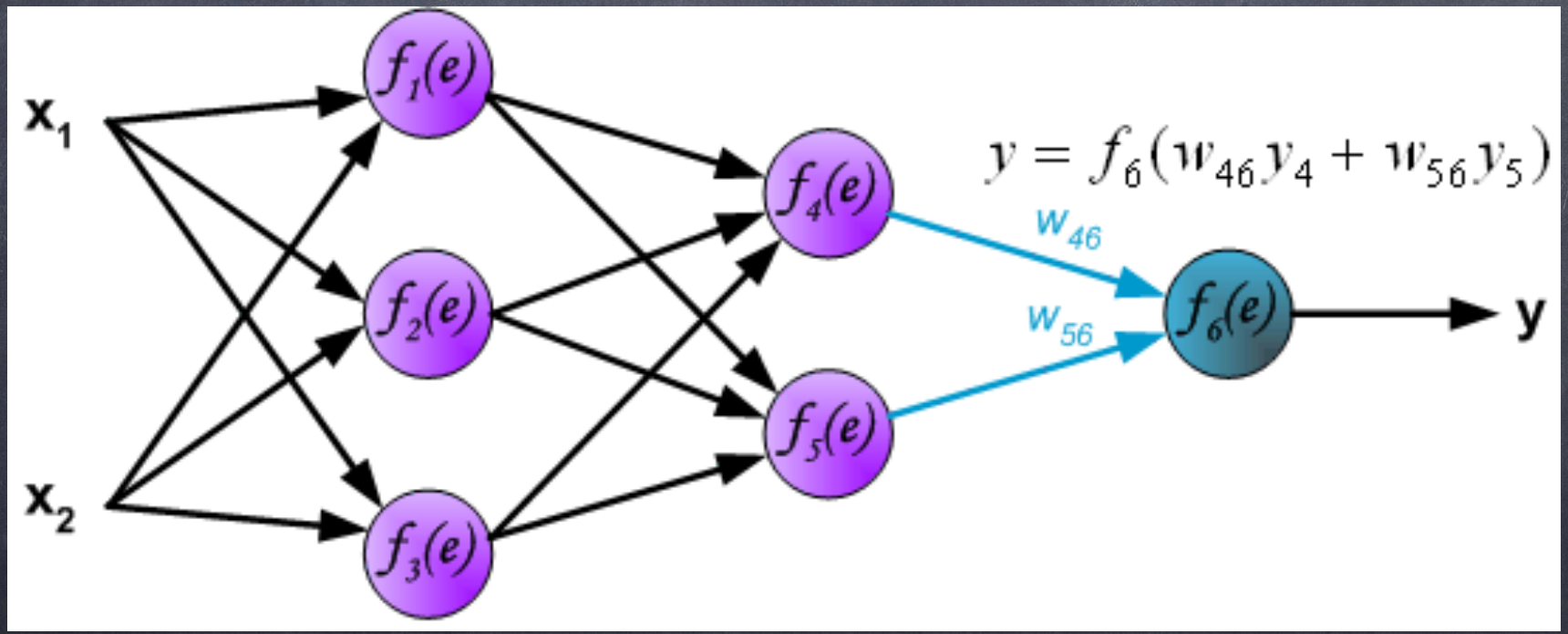
How to Train Neural Network



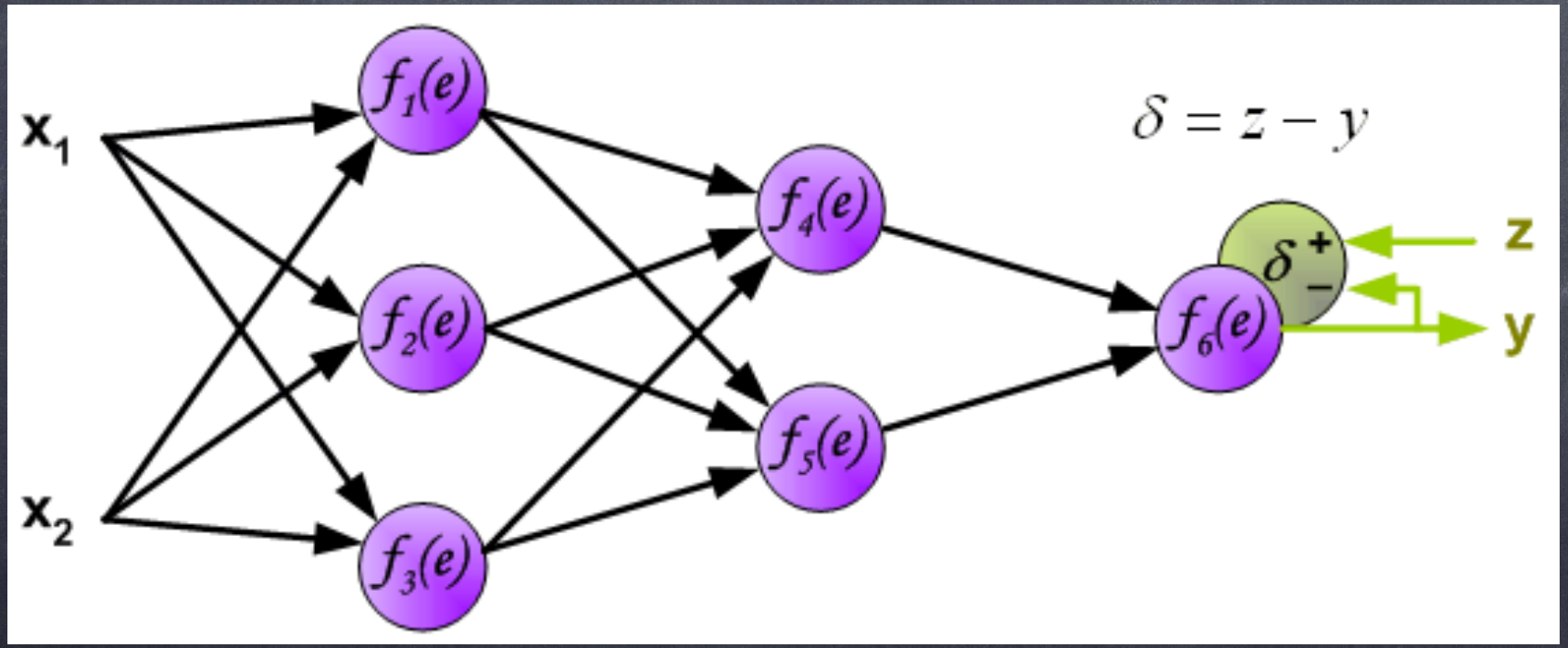
How to Train Neural Network



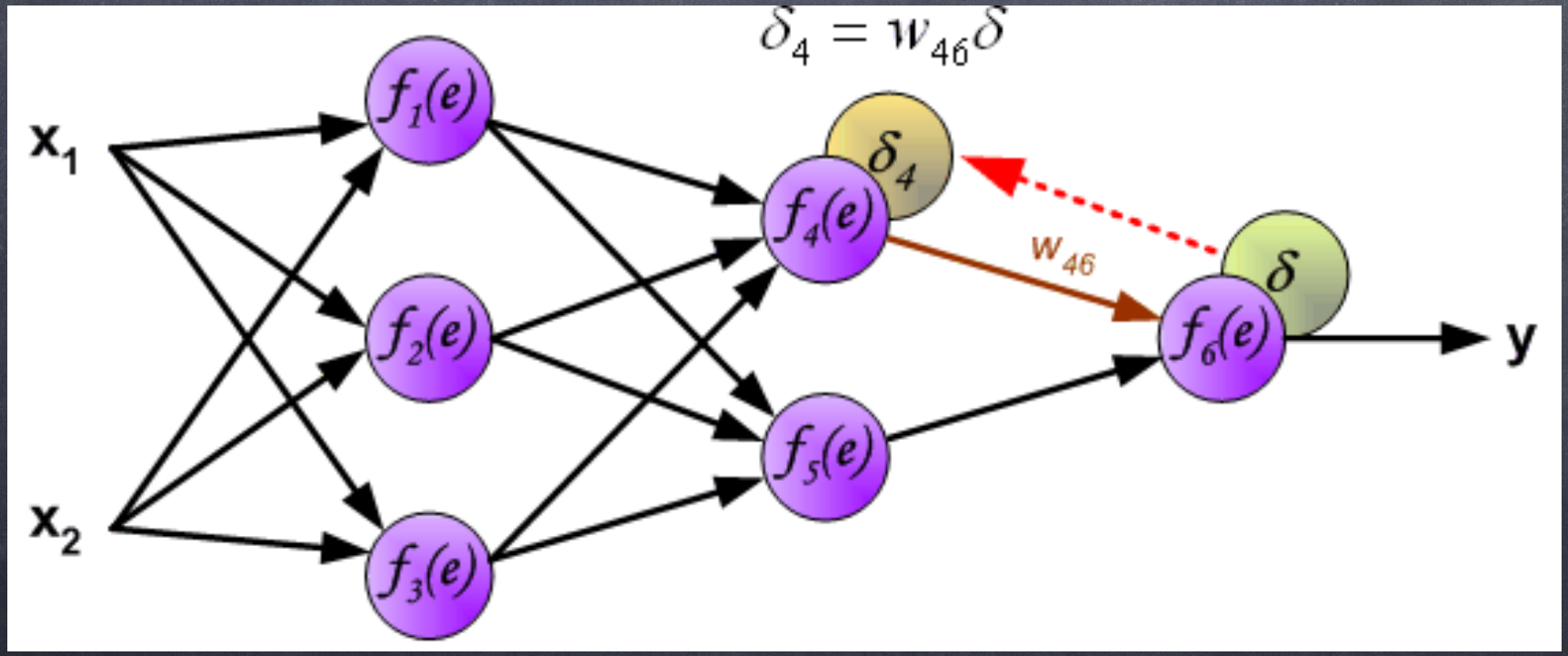
How to Train Neural Network



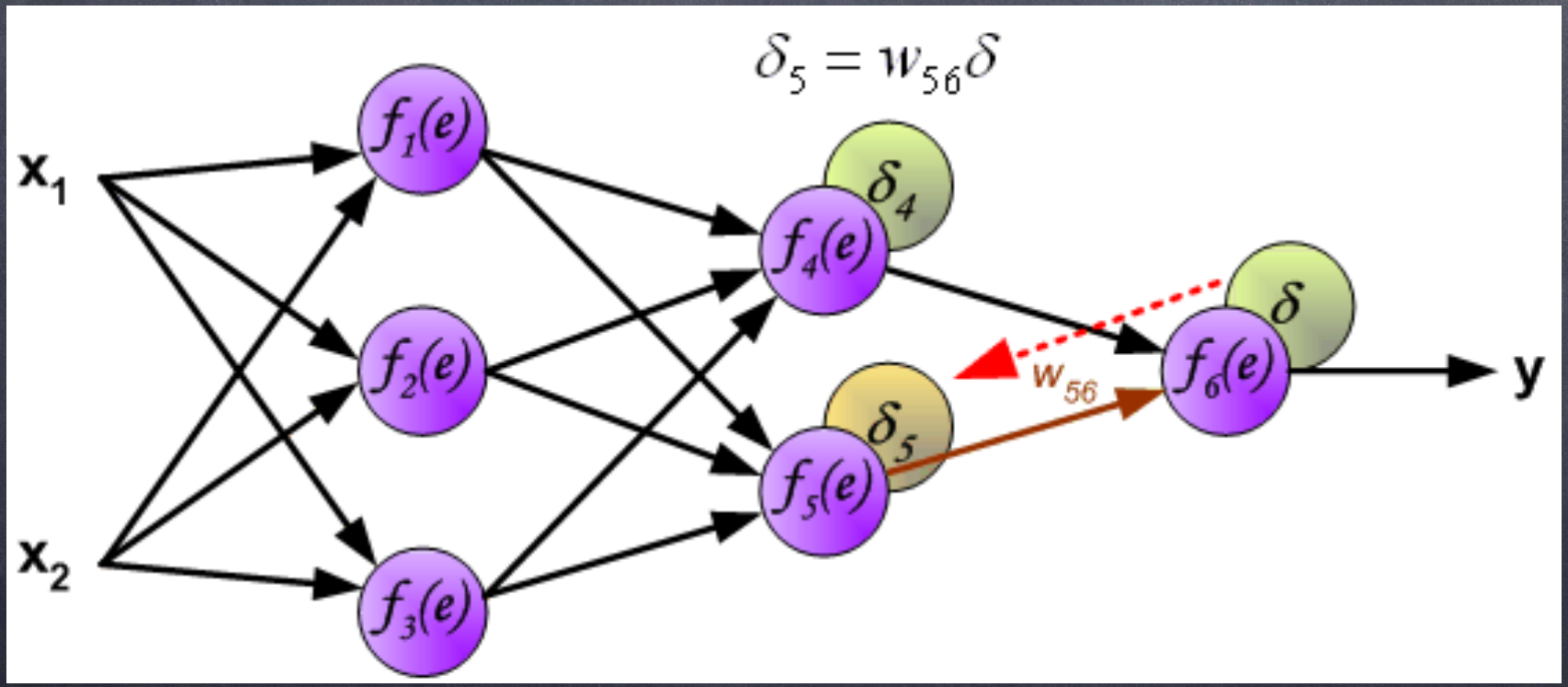
How to Train Neural Network



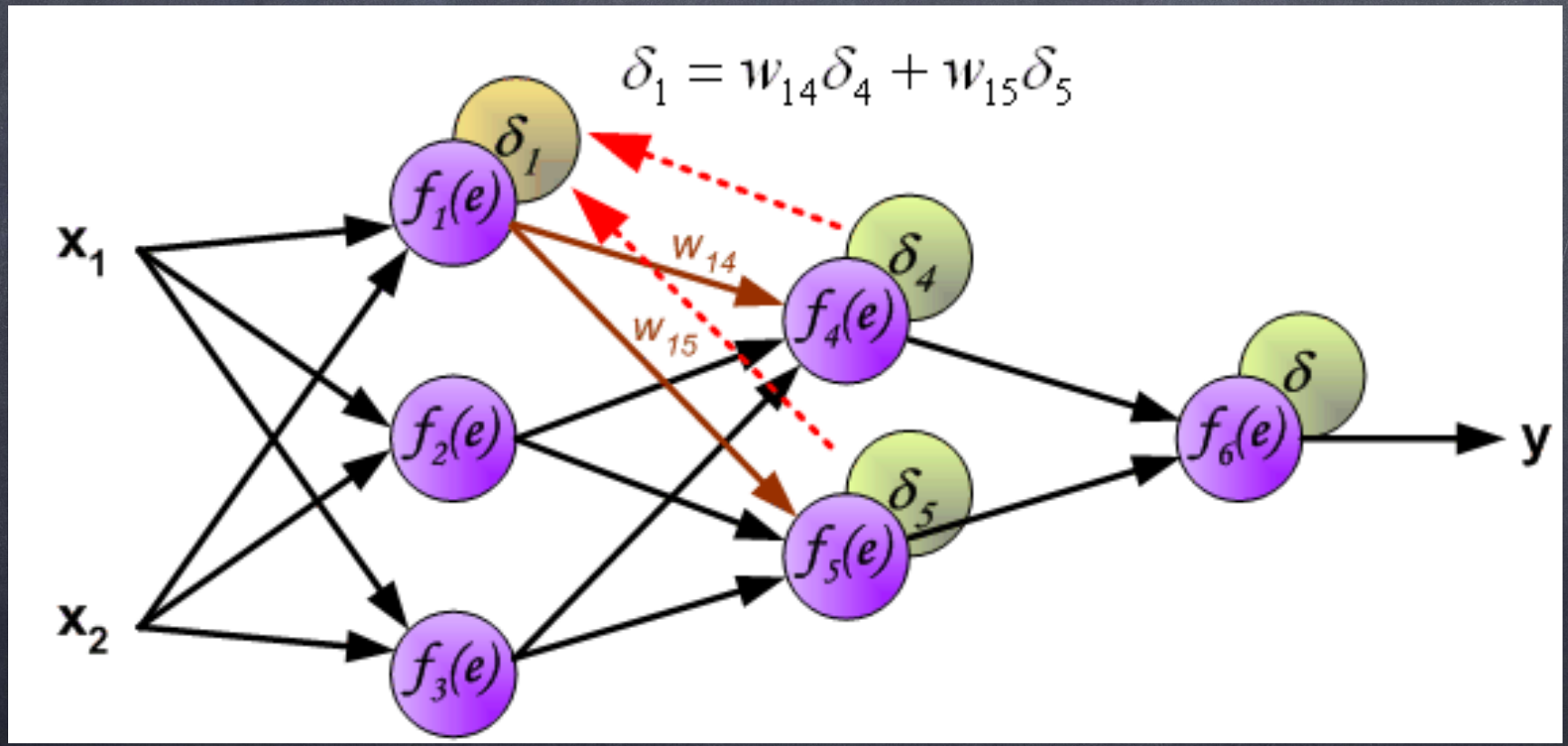
How to Train Neural Network



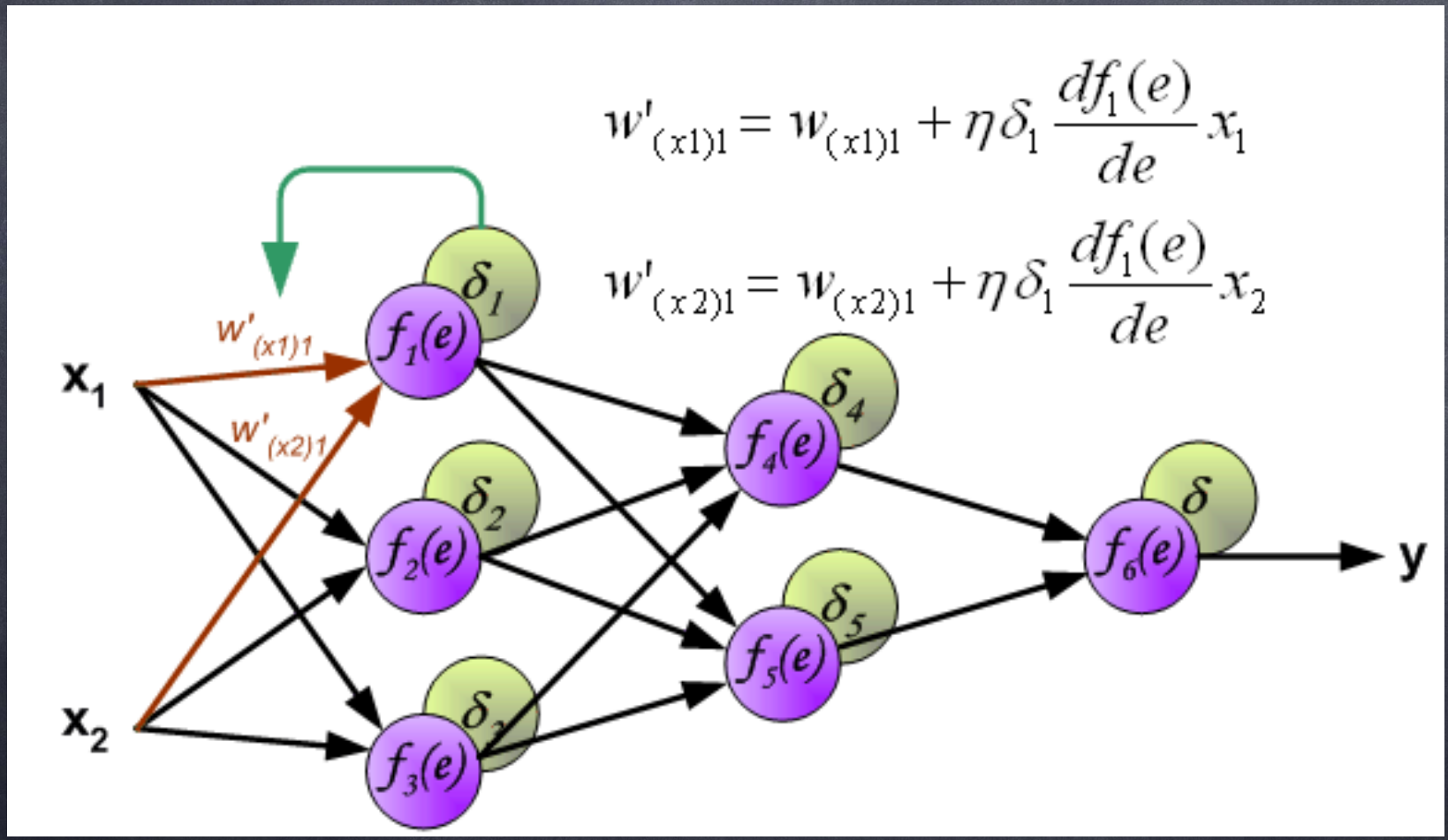
How to Train Neural Network



How to Train Neural Network



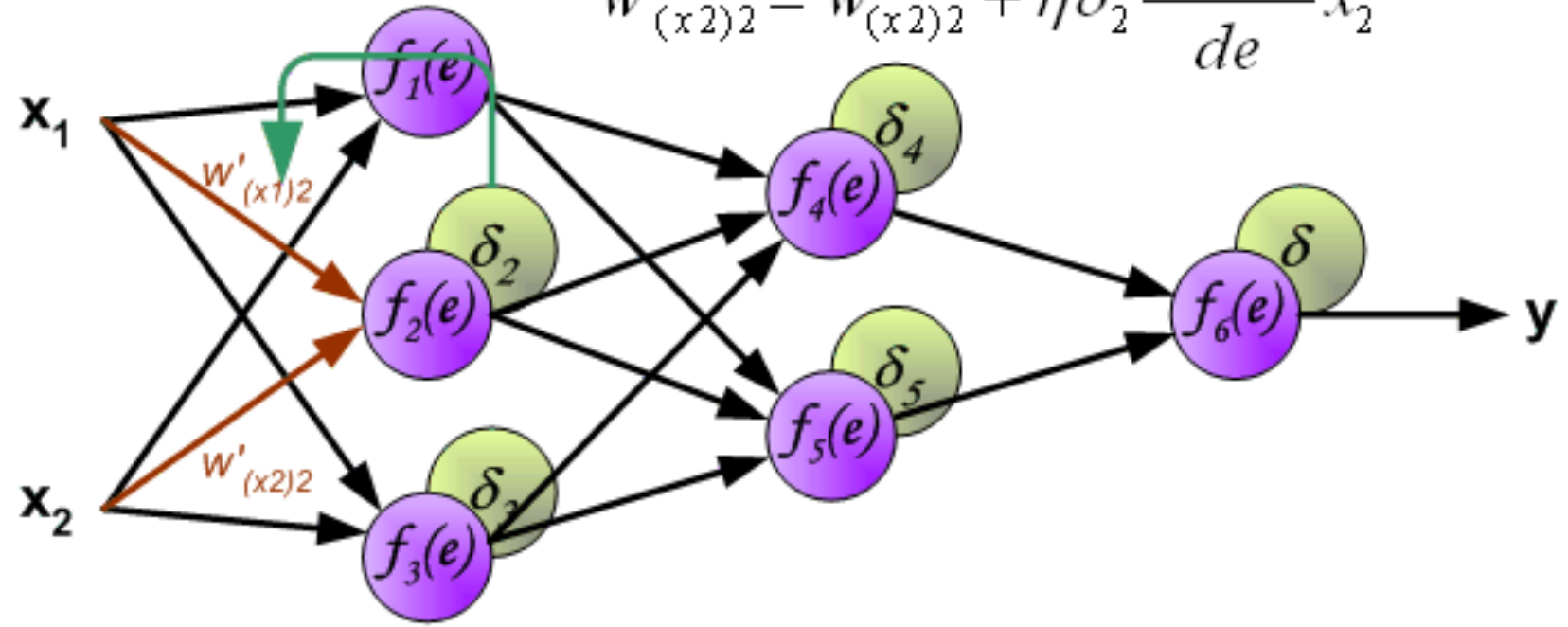
How to Train Neural Network



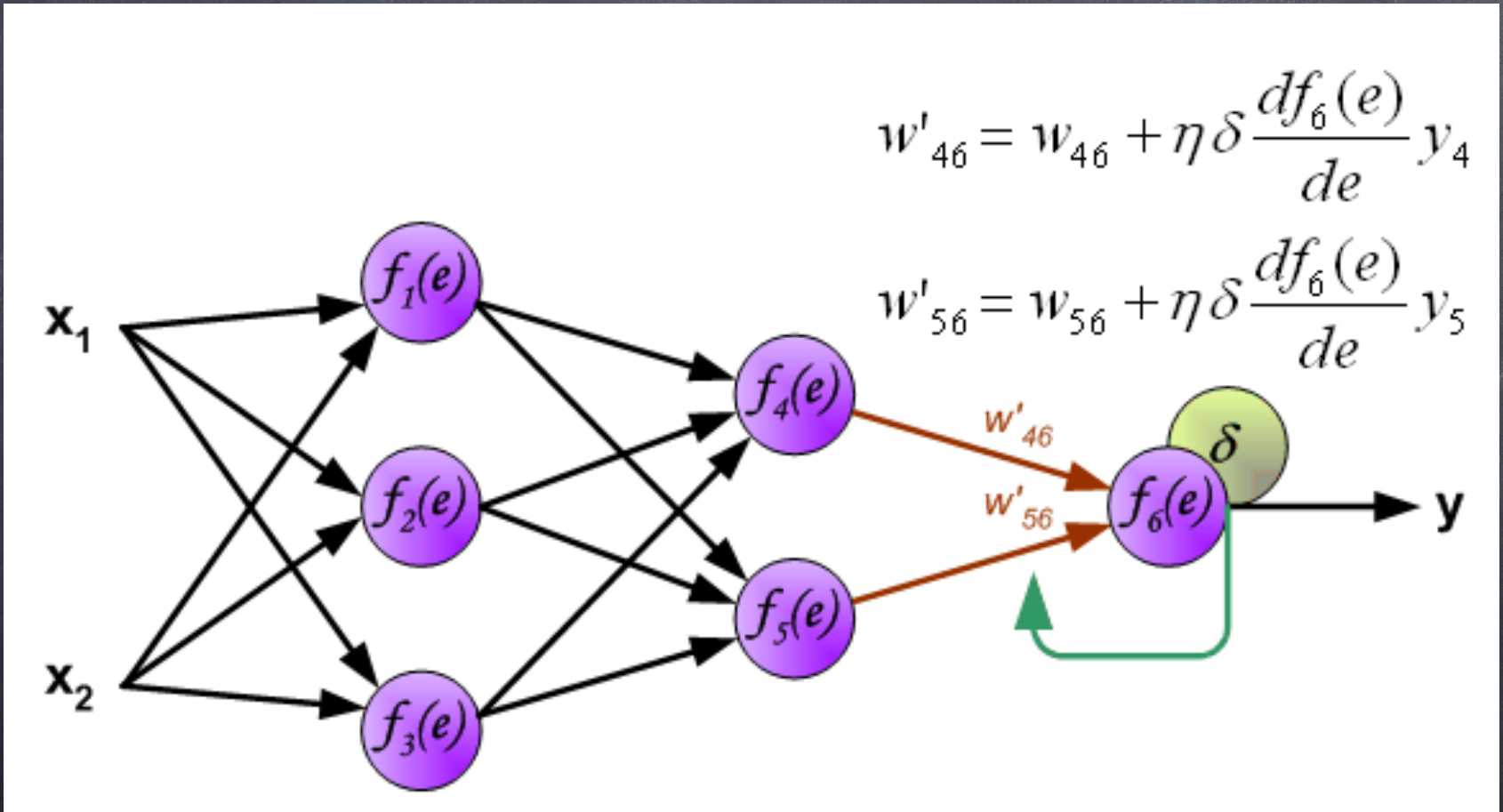
How to Train Neural Network

$$w'_{(x1)2} = w_{(x1)2} + \eta \delta_2 \frac{df_2(e)}{de} x_1$$

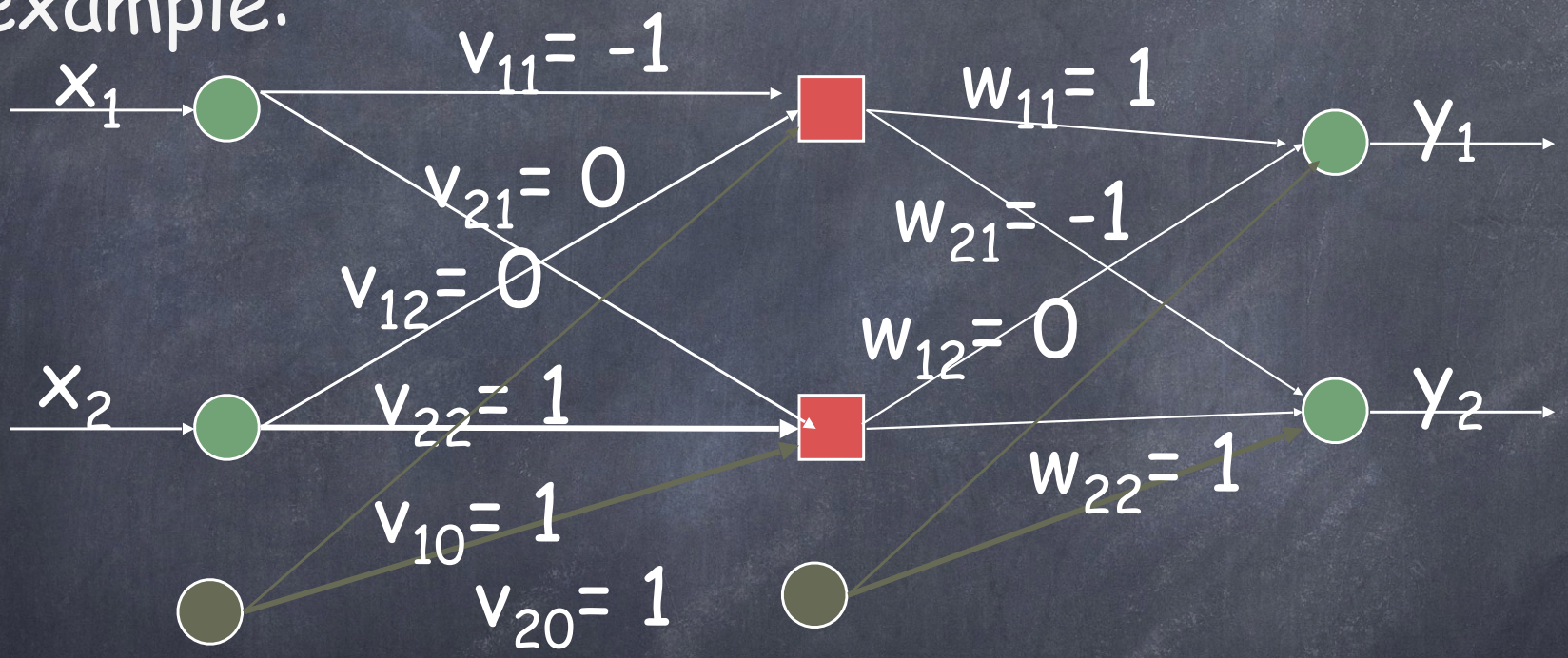
$$w'_{(x2)2} = w_{(x2)2} + \eta \delta_2 \frac{df_2(e)}{de} x_2$$



How to Train Neural Network



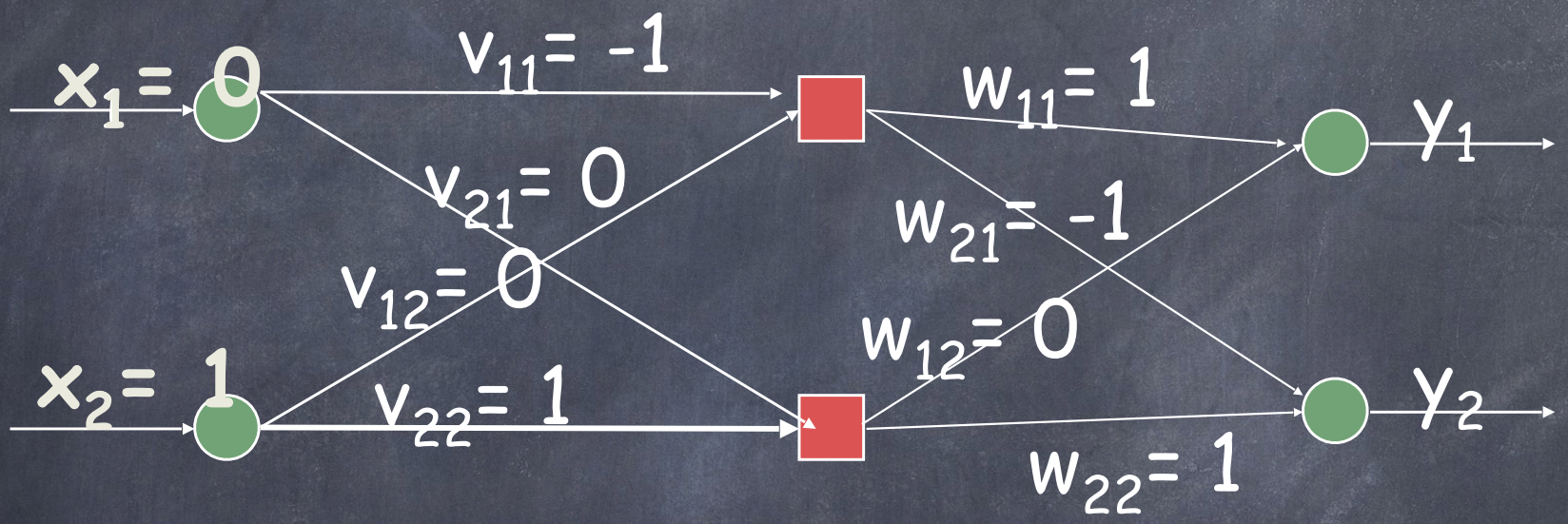
Once weight changes are computed for all units, weights are updated
at the same time (bias included as weights here).
An example:



Use identity activation function (ie g(a) = a)

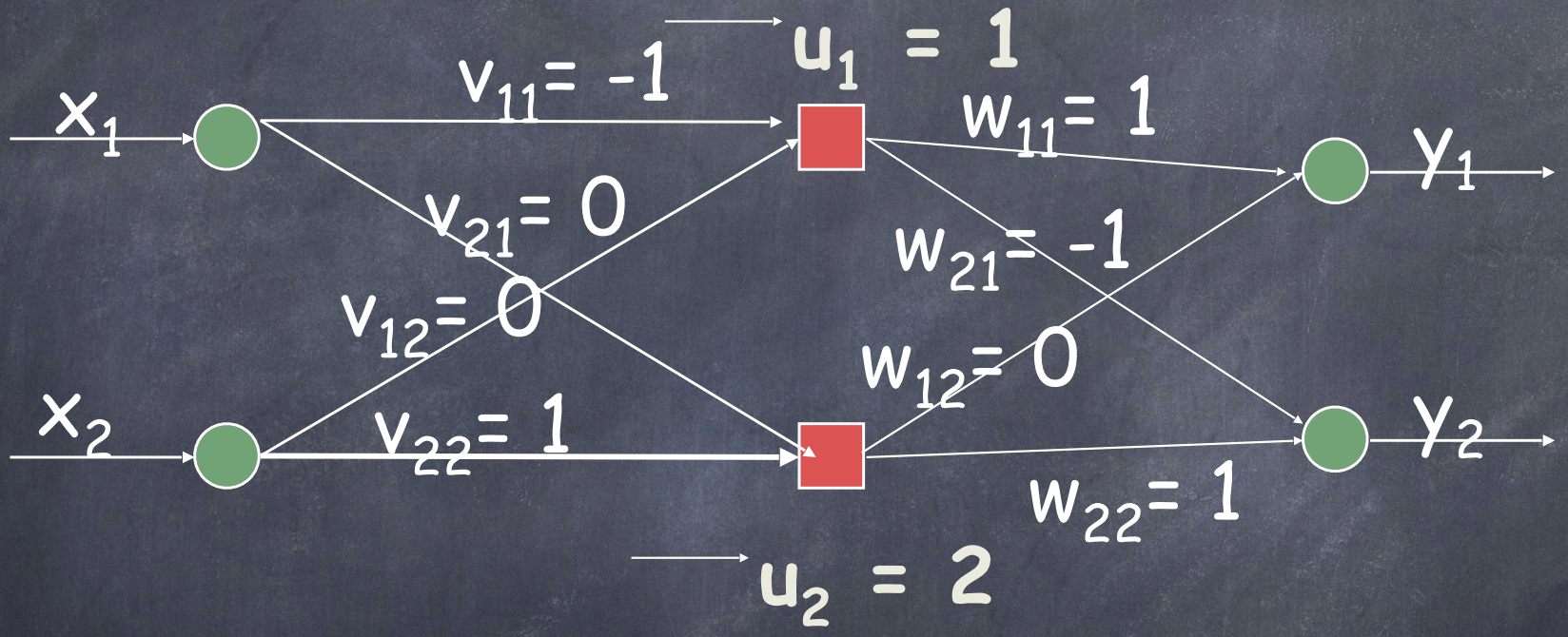
All biases set to 1. Will not draw them for clarity.

Learning rate $\eta = 0.1$



Have input [0 1] with target [1 0].

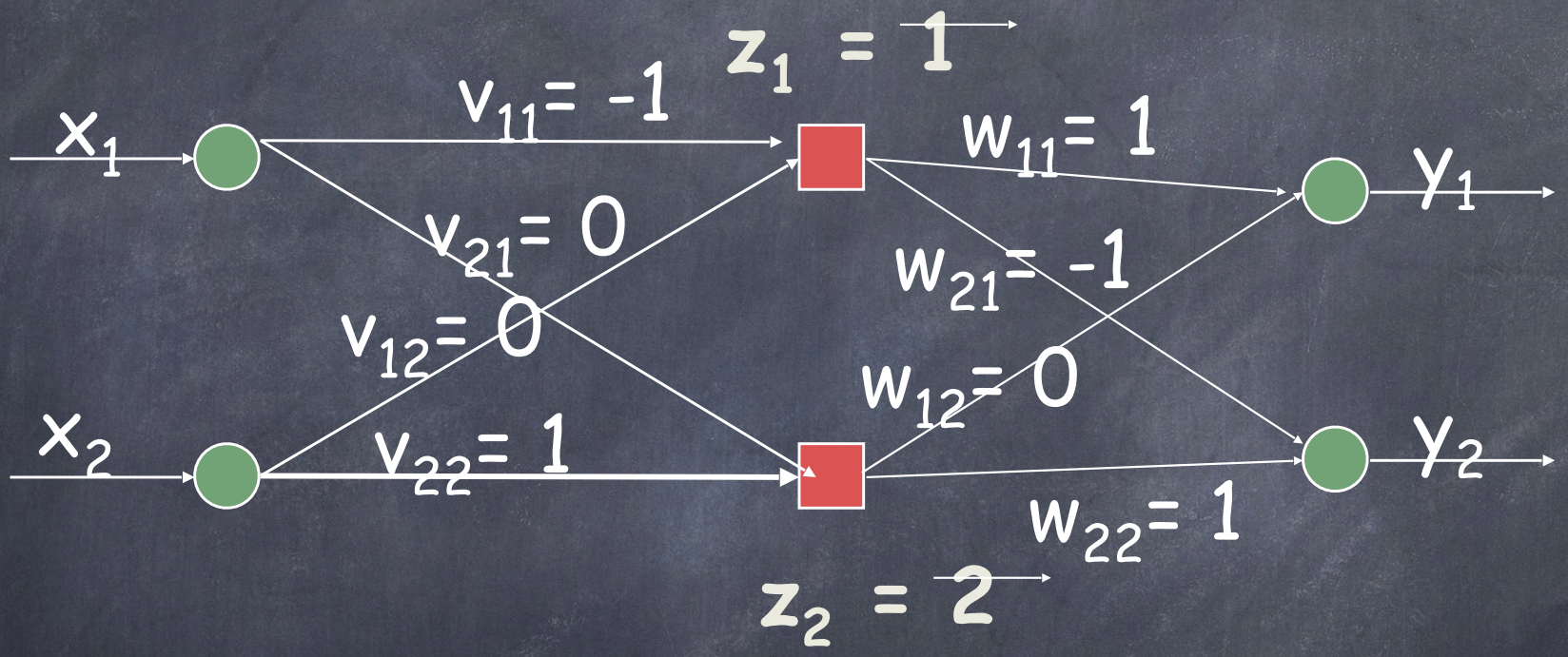
Forward pass. Calculate 1st layer activations:



$u_1 = -1x0 + 0x1 + 1 = 1$

$u_2 = 0x0 + 1x1 + 1 = 2$

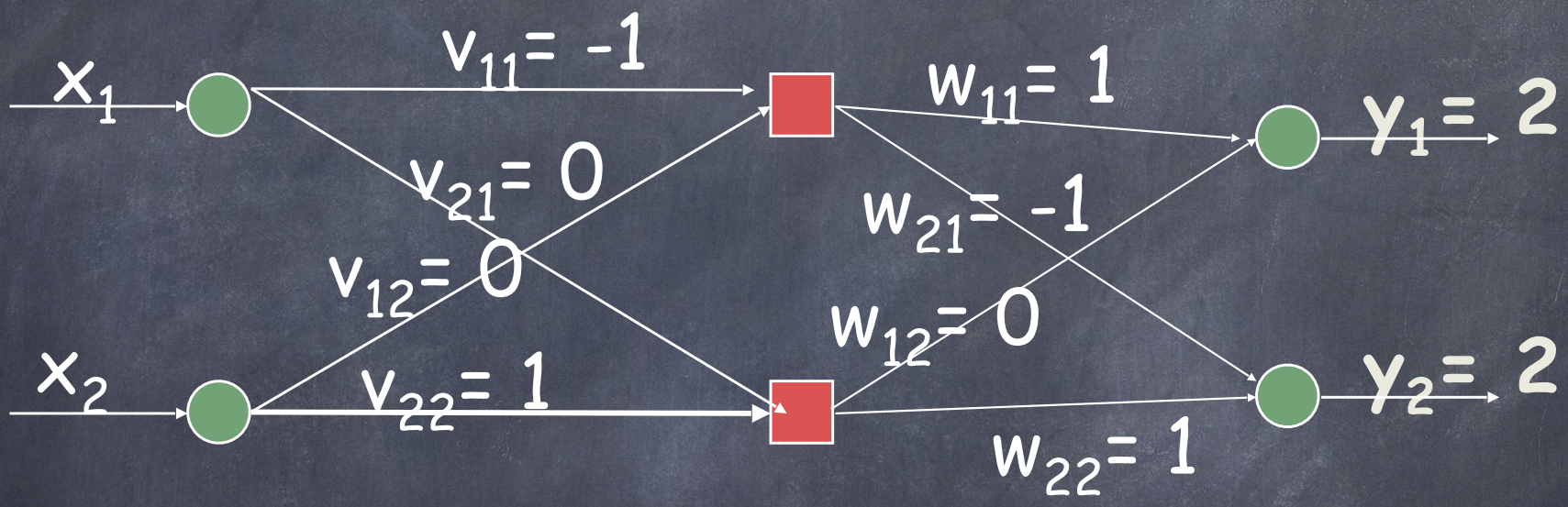
Calculate first layer outputs by passing activations thru activation functions



$z_1 = g(u_1) = 1$

$z_2 = g(u_2) = 2$

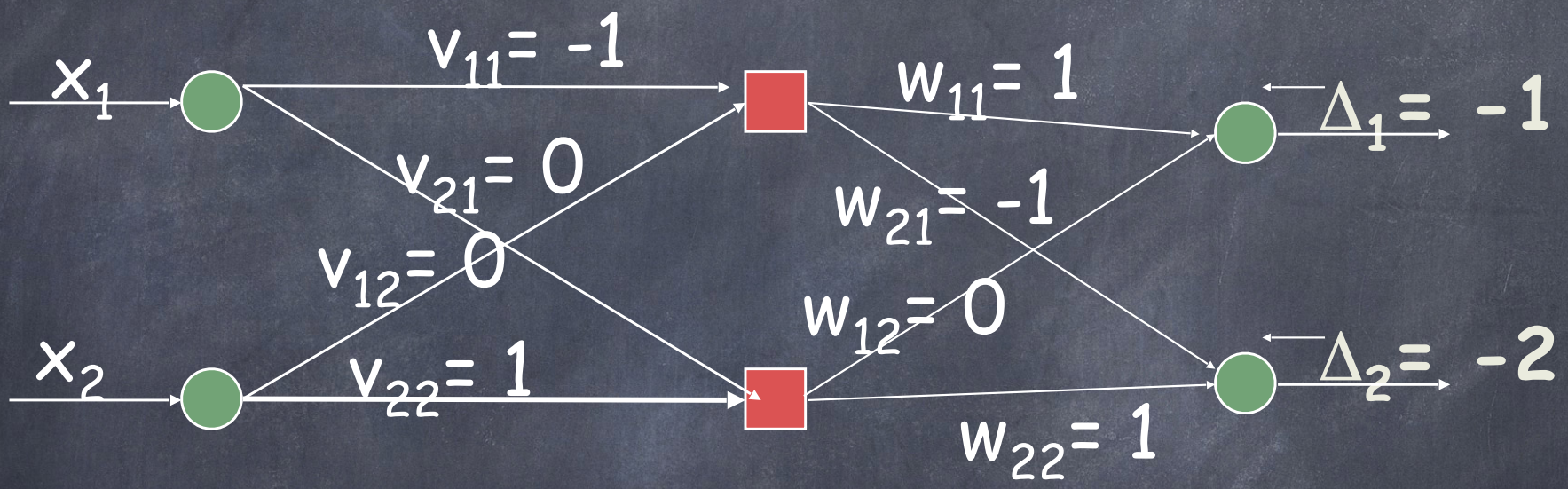
Calculate 2nd layer outputs (weighted sum thru activation functions):



$y_1 = a_1 = 1x1 + 0x2 + 1 = 2$

$y_2 = a_2 = -1x1 + 1x2 + 1 = 2$

Backward pass:



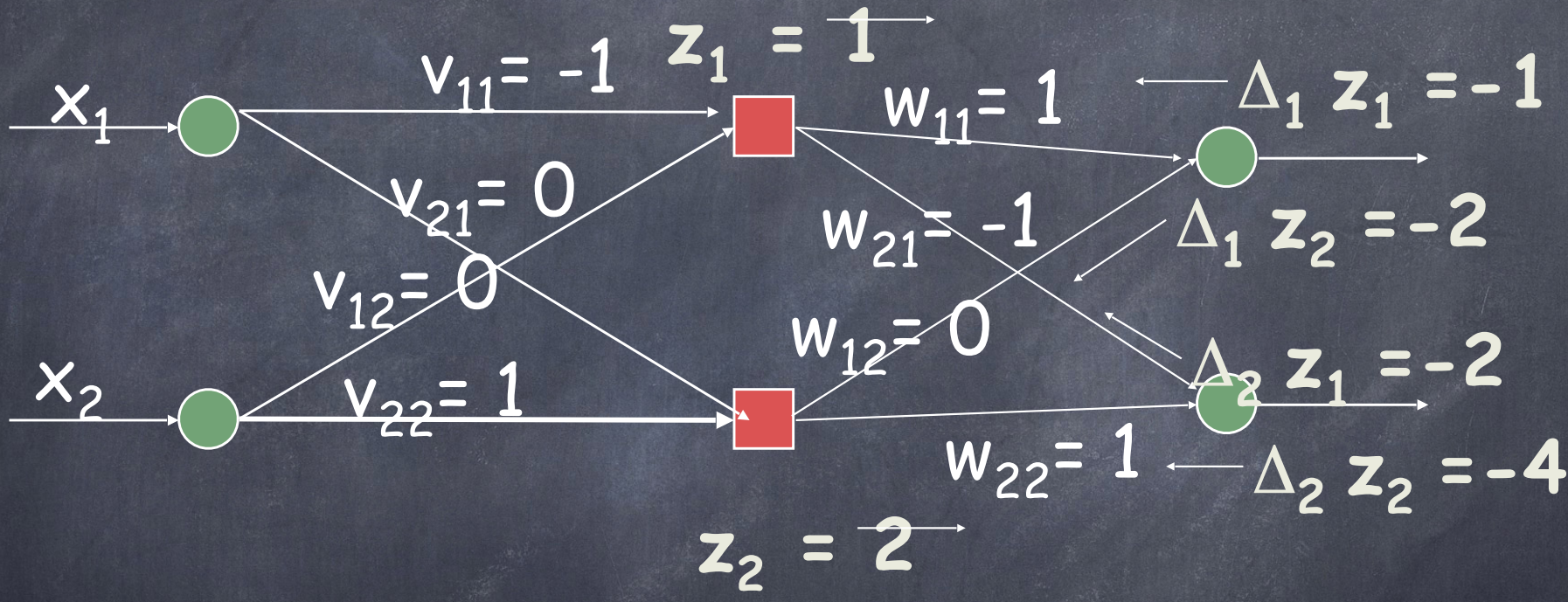
Target =[1, 0] so $d_1 = 1$ and $d_2 = 0$

So:

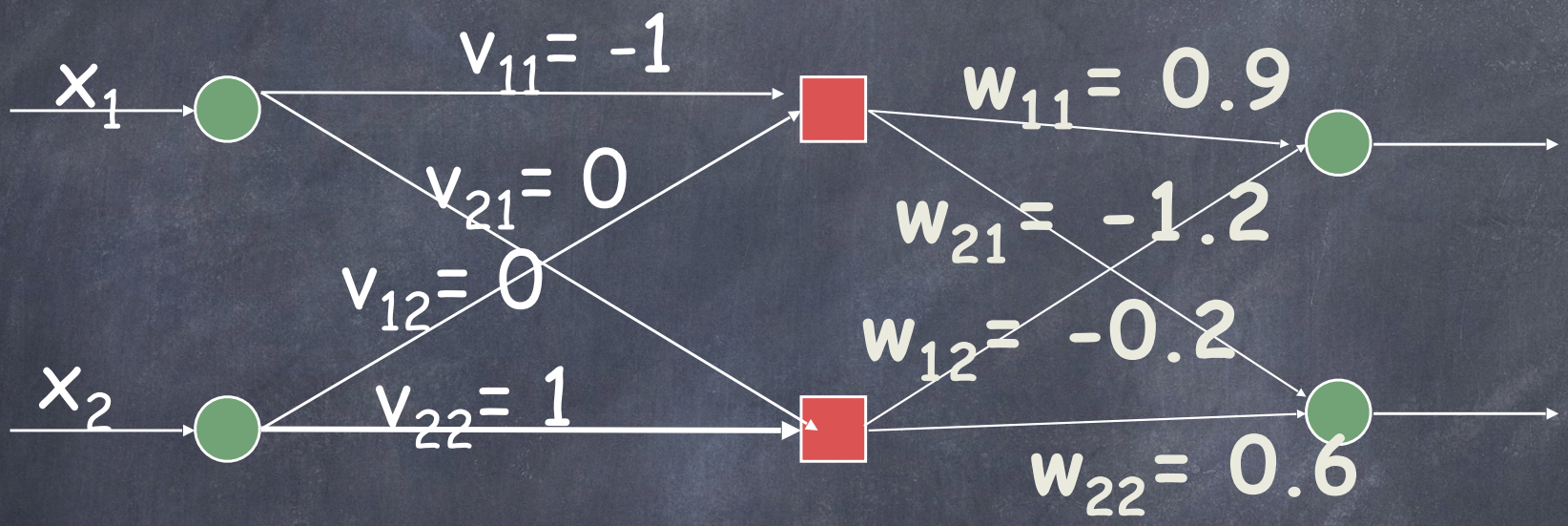
$$\Delta_1 = (d_1 - y_1) = 1 - 2 = -1$$

$$\Delta_2 = (d_2 - y_2) = 0 - 2 = -2$$

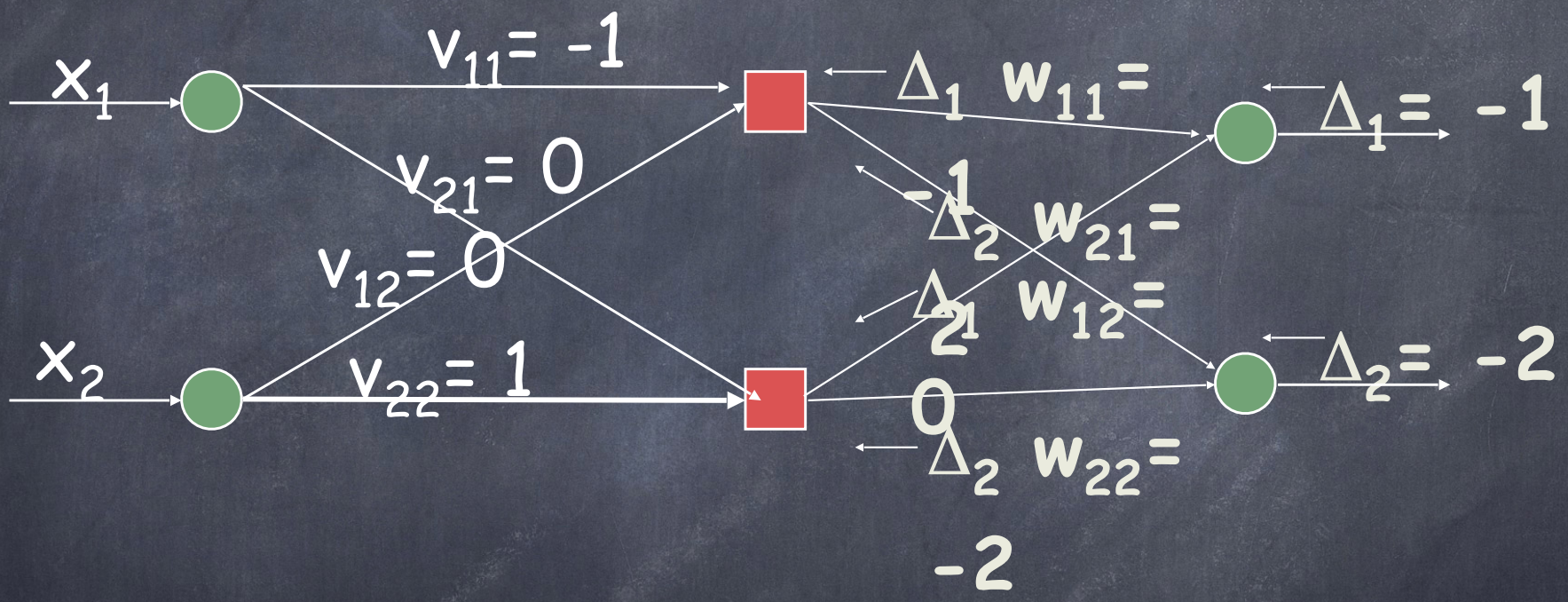
Calculate weight changes for 1st layer (cf perceptron learning):



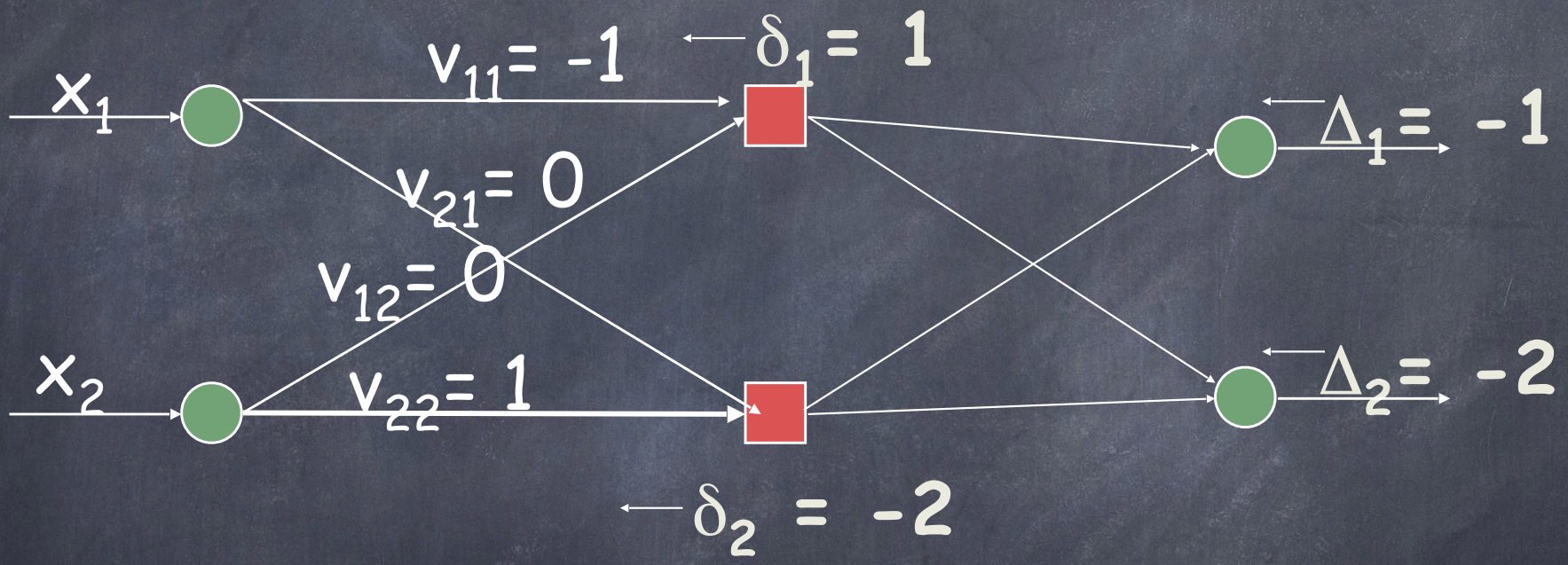
Weight changes will be:



But first must calculate δ's:



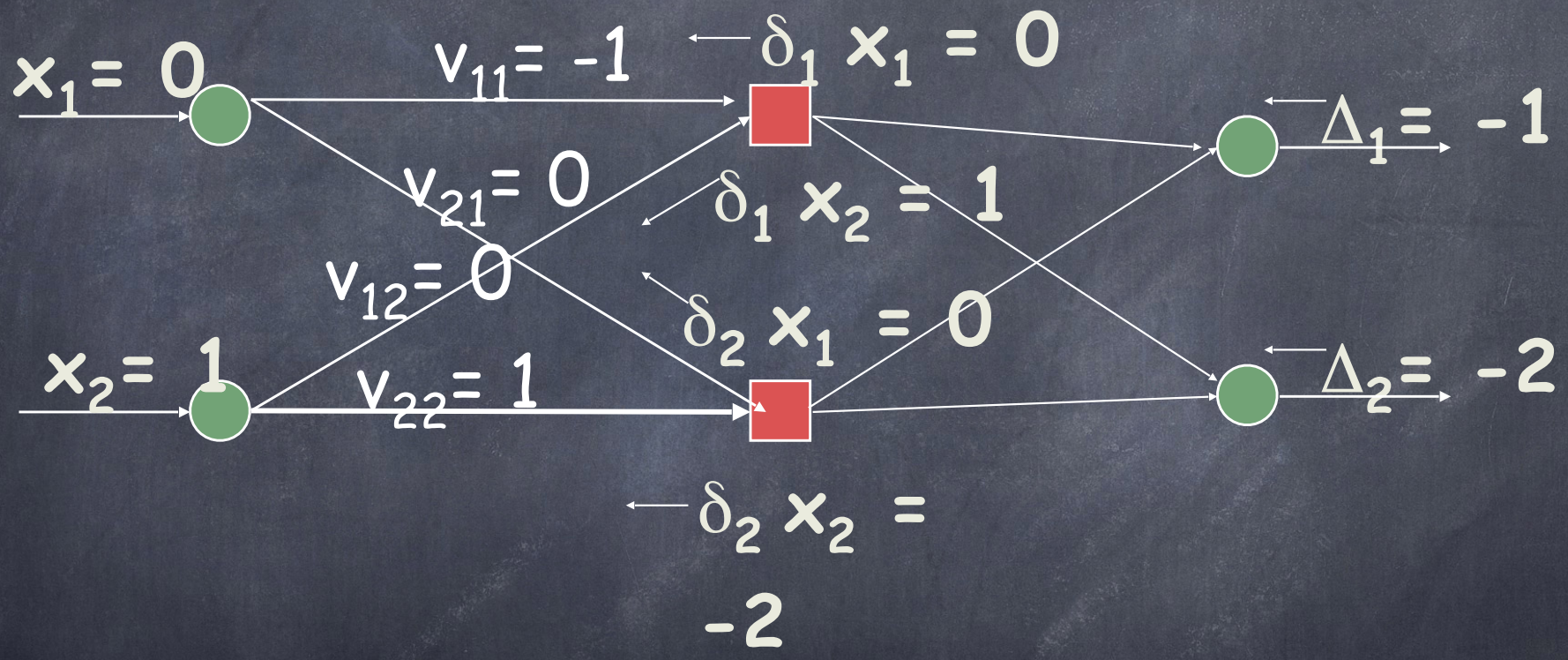
Δ's propagate back:



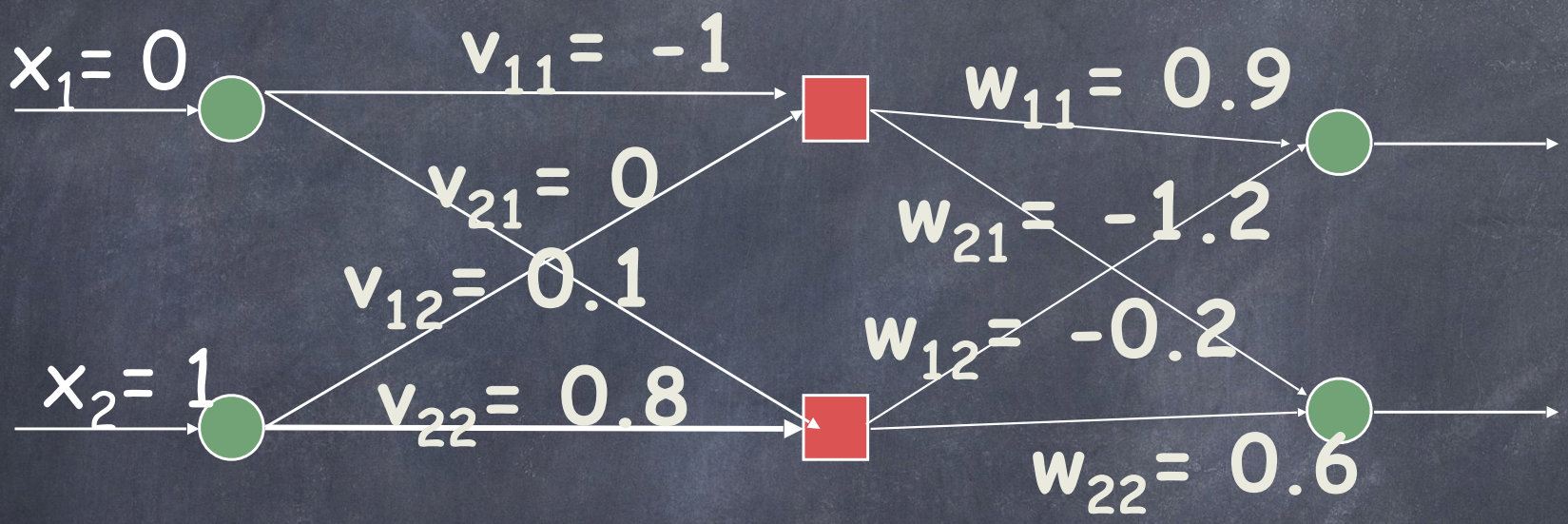
$\delta_1 = -1 + 2 = 1$

$\delta_2 = 0 - 2 = -2$

And are multiplied by inputs:

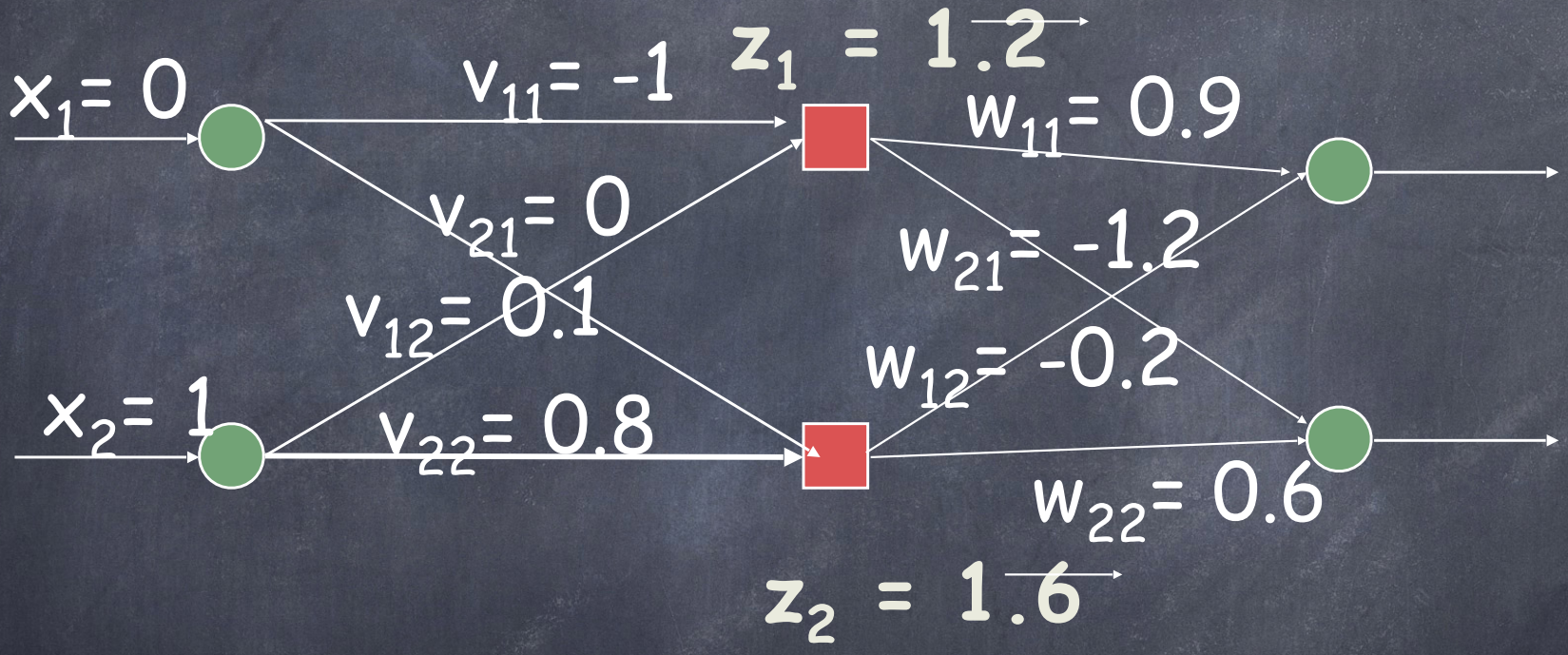


Finally change weights:

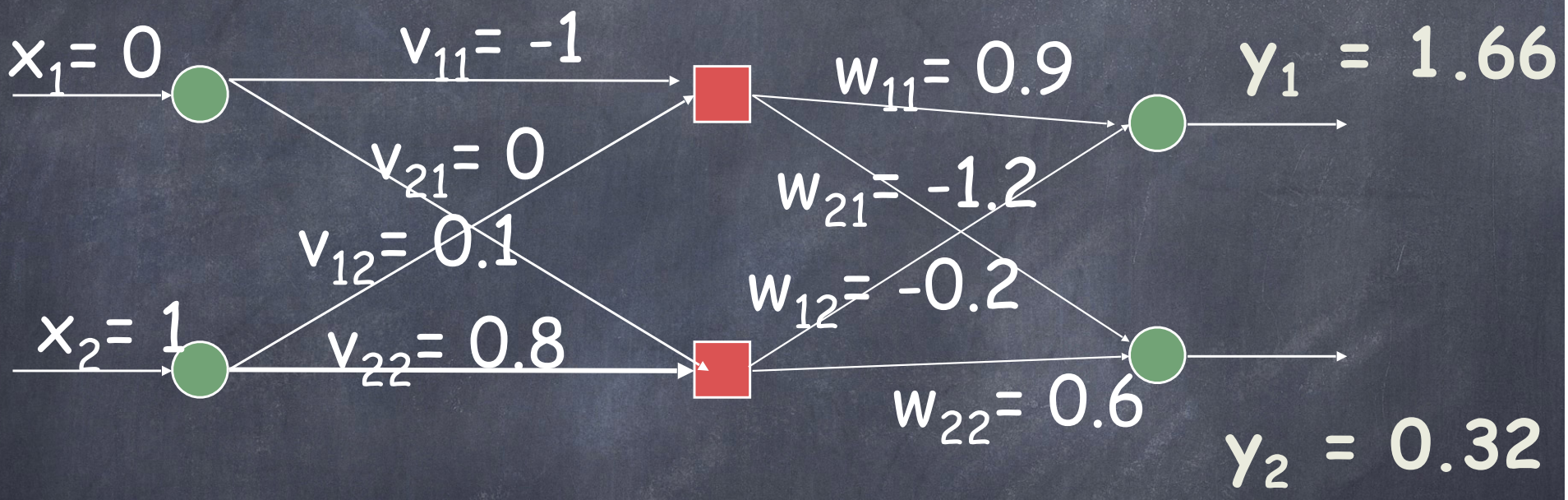


Note that the weights multiplied by the zero input are unchanged as they do not contribute to the error

Now go forward again (would normally use a new input vector):



Now go forward again (would normally use a new input vector):



Outputs now closer to target value [1, 0]