

Biological Vision and Applications

Module 03-05: Conditional independence

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Joint probability

Recap

$$N = 80$$

	a_1	a_2
b_1	25	10
b_2	5	40

- Joint probability:
 $P(a_1 b_1) = \frac{|a_1 b_1|}{N} = \frac{25}{80} \approx 0.31$
- Marginal probability:
 $P(a_1) = \frac{|a_1|}{N} = \frac{30}{80} \approx 0.38$
 - ▶ $|a_1| = |a_1 b_1| + |a_1 b_2| = 25 + 5 = 30$
- Conditional probability:
 $P(a_1 | b_1) = \frac{|a_1 b_1|}{|b_1|} = \frac{25}{35} \approx 0.71$
 - ▶ $|b_1| = |a_1 b_1| + |a_2 b_1| = 25 + 10 = 35$

Conditional Independence

- The variable A is conditionally independent of B , iff
 - ▶ the states of A does not depend on the states of B
- Formally $\forall i = 1..m, j = 1..n : P(a_i | b_j) = P(a_i)$
- Now $P(a_i | b_j) = \frac{P(a_i b_j)}{P(b_j)}$
- Substituting, condition for “conditional independence” in symmetric form
 - ▶ $\forall i, j : P(a_i b_j) = P(a_i).P(b_j)$
- **Conditional independence is symmetric**
 - ▶ $CondInd(A, B) \Leftrightarrow CondInd(B, A)$

Exercise: Prove that if $\forall i, j : P(a_i | b_j) = P(a_i)$, then $\forall i, j : P(b_j | a_i) = P(b_j)$

Conditional Independence

Continued

$$N = 80$$

	a_1	a_2
b_1	25	10
b_2	5	40

- Are the variables A and B conditionally independent ?

- ▶ $P(a_1) = \frac{30}{80} \approx 0.38$
- ▶ $P(a_1 | b_1) = \frac{25}{35} \approx 0.71$
- ▶ $P(a_1) \neq P(a_1 | b_1)$
 - ▶ A and B are not conditionally independent

Case of three variables

Place

- o Tamil Nadu
- o Kashmir

Fruit

- o Banana
- o Apple

Color

- o Red
- o Green
- o Yellow

$$P(B | T) = 0.8$$
$$P(A | T) = 0.2$$

$$P(B | K) = 0.1$$
$$P(A | K) = 0.9$$

$$P(R | B) = 0.1$$
$$P(G | B) = 0.4$$
$$P(Y | B) = 0.5$$

$$P(R | A) = 0.6$$
$$P(G | A) = 0.2$$
$$P(Y | A) = 0.2$$

- Place not specified: $P(Y) = ?$
 - ▶ $P(B) = 0.7 \times 0.8 + 0.3 \times 0.1 = 0.59$
 - ▶ $P(A) = 0.7 \times 0.2 + 0.3 \times 0.9 = 0.41$
 - ▶ $P(Y) = 0.59 \times 0.5 + 0.41 \times 0.2 = 0.377$
- Place specified as Kashmir: $P(Y | K) = ?$
 - ▶ $P(B) = 0.1$
 - ▶ $P(A) = 0.9$
 - ▶ $P(Y) = 0.1 \times 0.5 + 0.9 \times 0.2 = 0.23$
- $P(Y) \neq P(Y | K)$
 - ▶ Place and Color are not conditionally independent

Case of three variables

contd.

Place

- o Tamil Nadu
- o Kashmir

Fruit

- o Banana
- o Apple

Color

- o Red
- o Green
- o Yellow

$$P(B | T) = 0.8$$
$$P(A | T) = 0.2$$

$$P(B | K) = 0.1$$
$$P(A | K) = 0.9$$

$$P(R | B) = 0.1$$
$$P(G | B) = 0.4$$
$$P(Y | B) = 0.5$$

$$P(R | A) = 0.6$$
$$P(G | A) = 0.2$$
$$P(Y | A) = 0.2$$

- Given that Fruit = Banana
- Place not specified
 - ▶ $P(Y) = P(Y | B) = 0.5$
- Place specified as Kashmir:
 - ▶ $P(Y | K, B) = P(Y | B) = 0.5$
 - ▶ ... similarly, for TN, for other colors, for apple
- Given F , P and C are conditionally independent

Quiz 03-05

End of Module 03-05