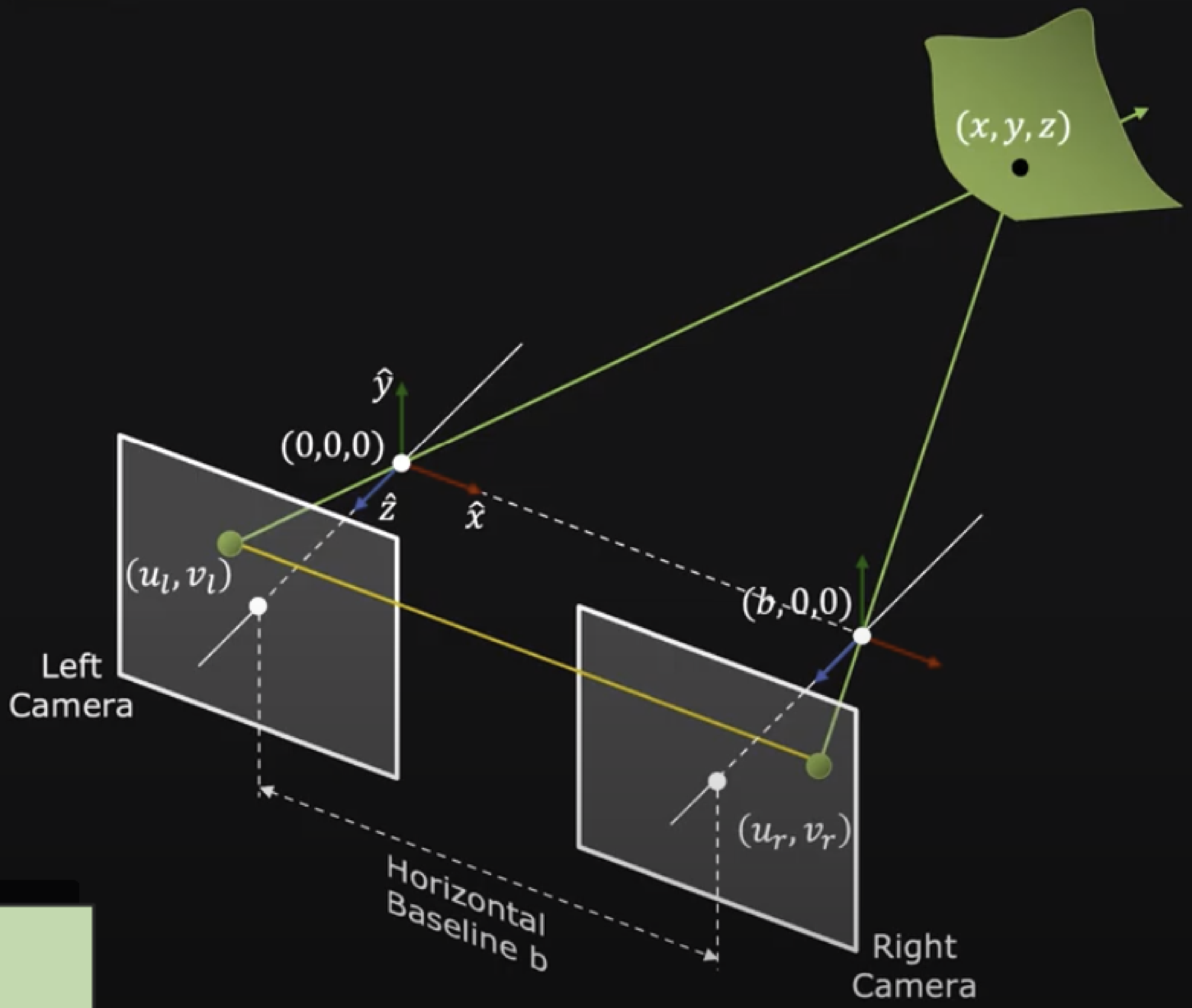
Uncalibrated Stereo

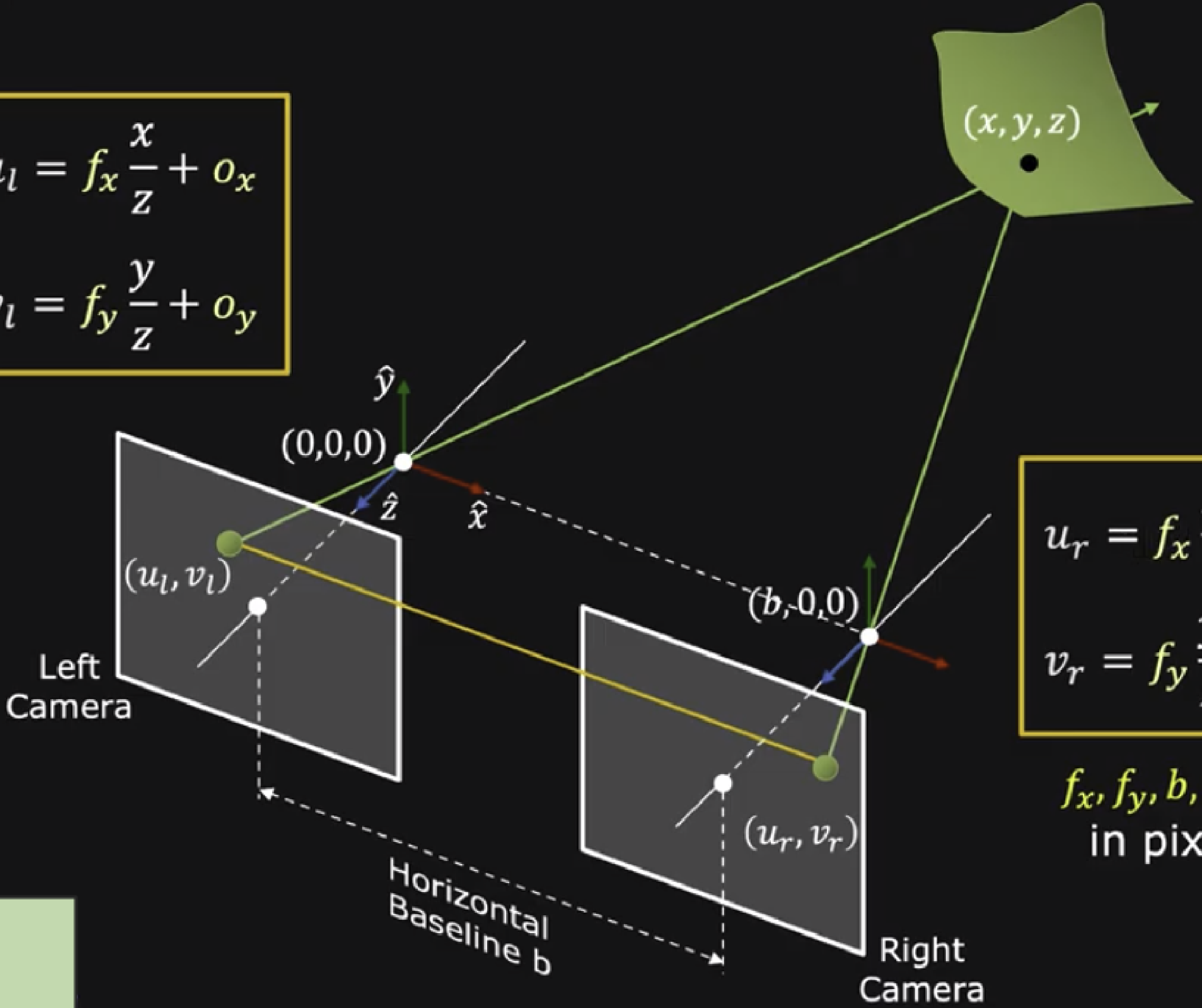
Simple (Calibrated) Stereo



Simple (Calibrated) Stereo

$$u_l = f_x \frac{x}{z} + o_x$$

$$v_l = f_y \frac{y}{z} + o_y$$



$$u_r = f_x \frac{x - b}{z} + o_x$$

$$v_r = f_y \frac{y}{z} + o_y$$

f_x, f_y, b, o_x, o_y are in pixel units.

Depth and Disparity

Solving for (x, y, z):

$$x = \frac{b(u_l - o_x)}{(u_l - u_r)} \qquad y = \frac{bf_x(v_l - o_y)}{f_y(u_l - u_r)} \qquad \boxed{z = \frac{bf_x}{(u_l - u_r)}}$$

where $(u_l - u_r)$ is called the Disparity.

Uncalibrated Stereo

Method to estimate 3D structure of a static scene from two arbitrary views.

Topics:

(1) Problem of Uncalibrated Stereo

(2) Epipolar Geometry

(3) Estimating Fundamental Matrix

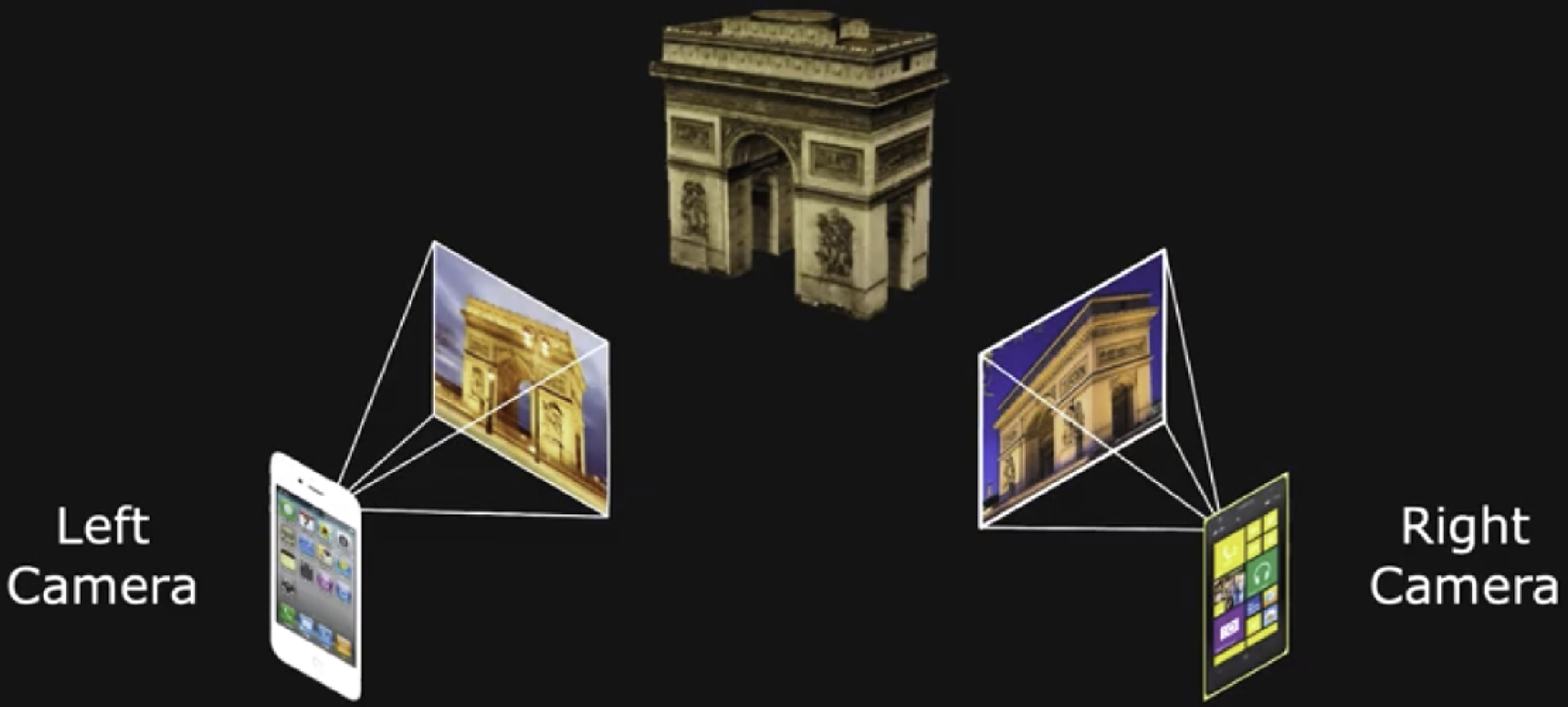
(4) Finding Dense Correspondences

(5) Computing Depth

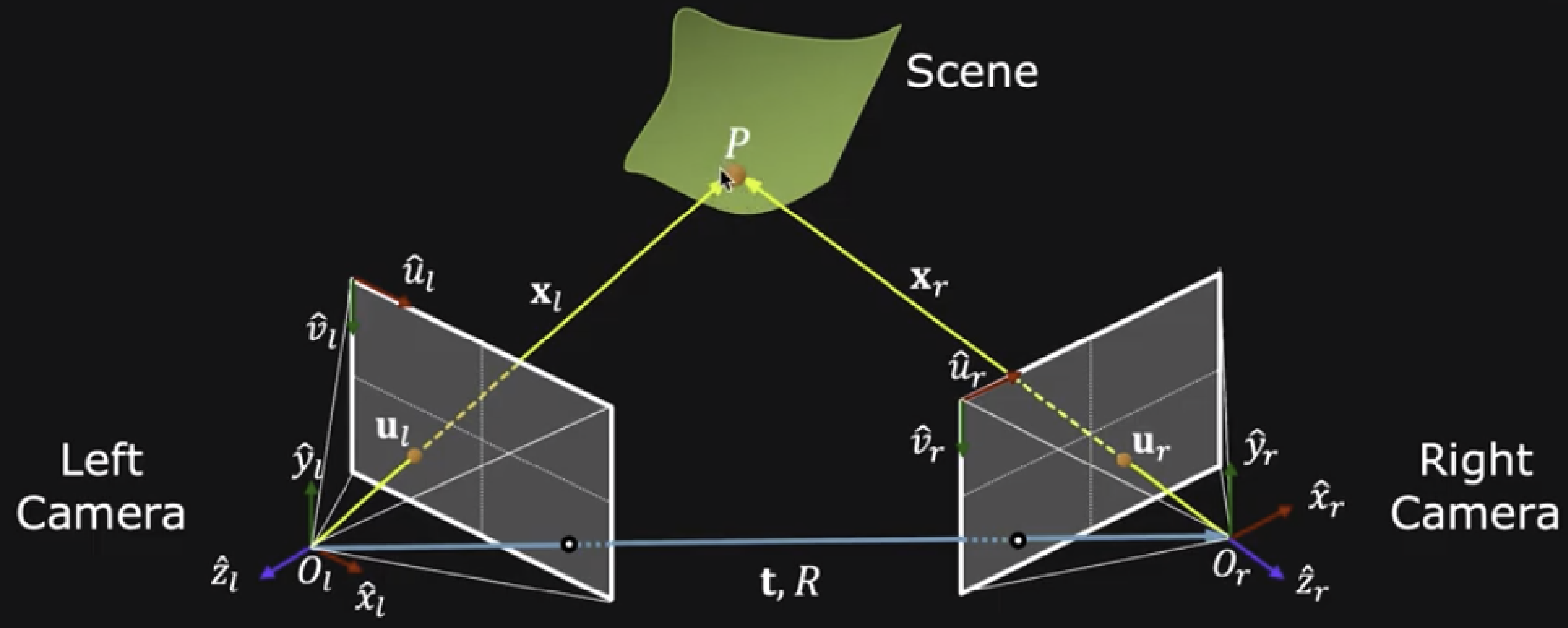
Problem of Uncalibrated Stereo

Uncalibrated Stereo

Compute 3D structure of static scene from two arbitrary views



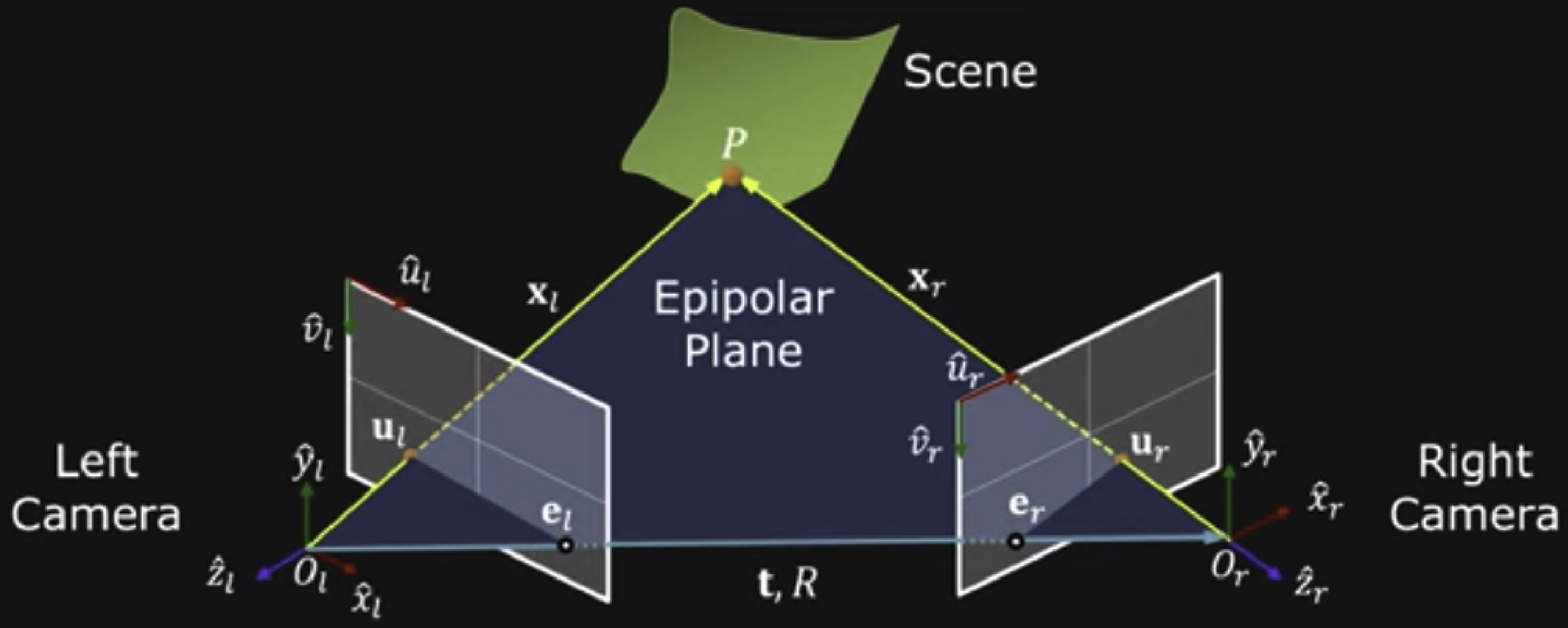
Uncalibrated Stereo



1. Assume Camera Matrix K is known for each camera
2. Find a few Reliable Corresponding Points
3. Find Relative Camera Position $\mathbf{t}$ and Orientation R
4. Find Dense Correspondence
5. Compute Depth using Triangulation

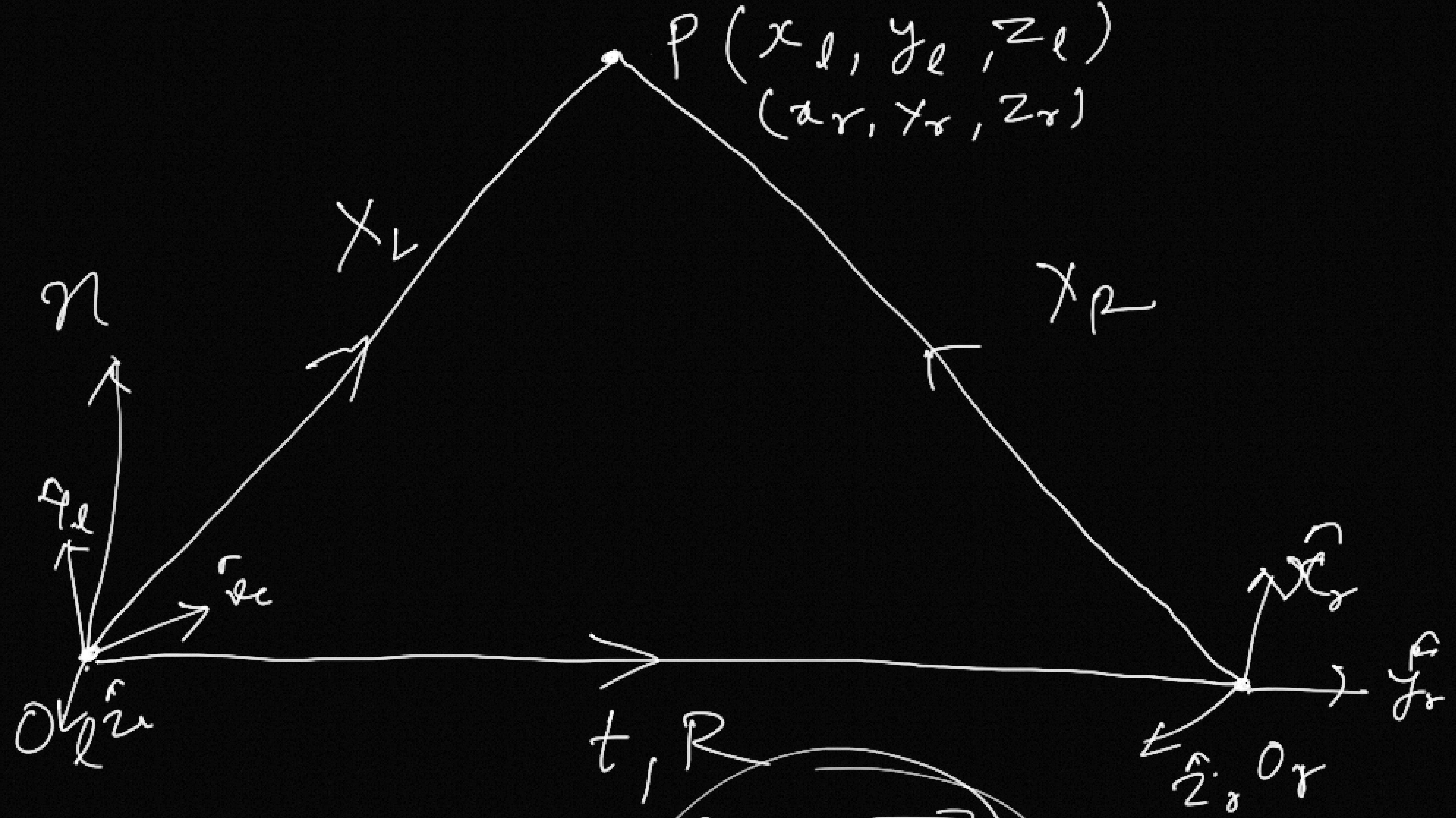
Epipolar Geometry

Epipolar Geometry: Epipolar Plane



Epipolar Plane of Scene Point P: The plane formed by camera origins (O_l and O_r), epipoles ($\mathbf{e}_l$ and $\mathbf{e}_r$) and scene point P.

Every scene point lies on a unique epipolar plane.



$$\vec{n} = \vec{t} \times \vec{X_L}, \quad \vec{X_L} \cdot \underline{\vec{n}} = 0$$

$$\vec{x}_l \cdot \vec{n} = 0$$

$$\Rightarrow \vec{x}_l \cdot \left(\vec{t} \times \vec{x}_l \right) = 0$$

Epipolar Constraint

$$\Rightarrow \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \cdot \left(\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \times \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} \right)$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - b_2 a_3) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$\Rightarrow [x_l \; y_l \; z_l] \begin{bmatrix} t_y z_l - t_z y_l \\ t_z x_l - t_x z_l \\ t_x y_l - t_y x_l \end{bmatrix} = 0$$

$$\Rightarrow \underline{[x_l \; y_l \; z_l]} \underline{\underline{\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}}} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0$$

$a_{ij} = -a_{ji}$

$$\Rightarrow [x_l \; y_l \; z_l] \cdot T_x \cdot \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = 0$$

$$\Rightarrow \vec{X}_L^T \, T_x \, \vec{X}_L = 0$$

$$\vec{X}_l = R\vec{X}_r + \vec{t}$$

$$\begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \left(\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}}_{= 0} \right)$$

$$= 0$$

$$\Rightarrow \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

Skew symmetry

Orthonor

$$\Rightarrow \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} \underbrace{\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}}_{\text{Essential Matrix}} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = 0$$

SVD

$$\Rightarrow \underbrace{\begin{bmatrix} x_l & y_l & z_l \end{bmatrix}}_{X_l^T} E_{3\times3} \cdot \underbrace{\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix}}_{X_r} = 0 \quad \text{——} \quad \text{(A)}$$

Perspective projection

$$u_l = f_x^{(l)} \cdot \frac{x_l}{z_l} + O_x^{(l)}, \quad V_l = \underline{\underline{f_y^{(l)}}} \cdot \frac{y_l}{z_l} + O_y^{(l)}$$

$$\Rightarrow z_l u_l = f_x^{(l)} \cdot x_l + O_x^{(l)} z_l, \quad z_l V_l = f_y^{(l)} \cdot y_l + z_l O_y^{(l)}$$

$$z_l \begin{bmatrix} u_l \\ V_l \\ 1 \end{bmatrix} = \begin{bmatrix} z_l u_l \\ z_l V_l \\ z_l \end{bmatrix} = \begin{bmatrix} f_x^{(l)} \cdot x_l + O_x^{(l)} z_l \\ f_y^{(l)} \cdot y_l + z_l O_y^{(l)} \\ z_l \end{bmatrix}$$

$$
= \begin{bmatrix} f_x^{(l)} & 0 & O_x^{(l)} \\ 0 & f_y^{(l)} & O_y^{(l)} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ Z_l \end{bmatrix}
$$

$$
= K_L \vec{X}_L
$$

$$
\left\{ \begin{aligned} \vec{X}_l^{T} &= \begin{bmatrix} x_l & y_l & z_l \end{bmatrix} = \begin{bmatrix} u_l & v_l & 1 \end{bmatrix} z_l K_L^{-1\,T} \\ \vec{X}_r &= \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = K_r^{-1} Z_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \end{aligned} \right\} \; B
$$

$$\Rightarrow \underline{[u_l \ v_l \ 1]} \, z_l \, \underbrace{K_L^{-1T} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} K_R^{-1}}_{} \cdot z_r \underline{\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}} = 0$$

(eq A) and B

$$z_l \neq 0 , z_r \neq 0$$

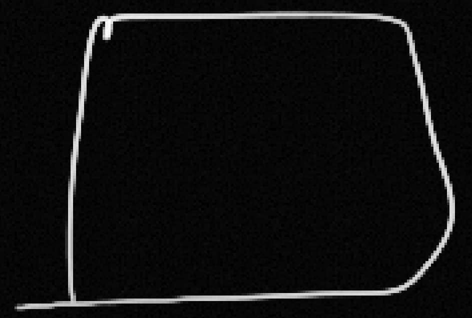
$$\boxed{[u_l \ v_l \ 1] \underbrace{K_L^{-1T} E \ K_R^{-1}}_{F} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0}$$

$$K_L^{-1T} \cdot E \cdot K_R^{-1} = \underline{\underline{F}} \quad \text{(Fundamental Matrix)}$$

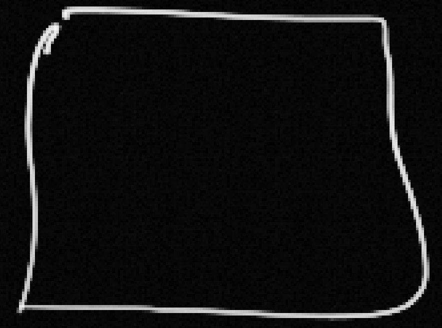
$$E = T_x \times R$$

$$\underbrace{\begin{bmatrix} u_l & v_l & 1 \end{bmatrix}}_{1\times 3} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}_{3\times 3} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}_{3\times 1} = 0$$

Estimating F



Captured by you



Captured by your friend

$$(u_l^{(1)}, v_l^{(1)}) \qquad u_r^{(1)}, v_r^{(1)}$$

$$\vdots$$

$$(u_l^{(m)}, v_l^{(m)}) \qquad (u_r^{(m)}, v_r^{(m)})$$