Practice problem 4:Optimization in ML

Dr. Md Abu Talhamainuddin Ansary

1. Justify whether d is a descent direction of f at x or not.

   (i) $f(x) = 4x_1^2 + 5x_1x_2 + 2x_2^2 + x_1 - 3x_2, x = (1,1)^T, d = (-1,2)^T$

   (ii) $f(x) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_2x_3 - 2x_1 - 4x_2 - 6x_3\ x = (0,0,0)^T, d = (1,1,0.5)^T$

   (iii) $f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2, x = (2,2)^T, d = (1,1)^T, d = (-1,-2)^T$

   (iv) $f(x) = (x_1 - x_2)^2 + (x_2 - 3)^2, x = (2,2)^T, d = (1,1)^T, d = (-2,-1)^T$.

   (v) $f(x) = x_1^4 + 2x_1^2x_2 + x_2^2 - x_1^2 - 8x_1 - 8x_2\ x = (0.5,0.5)^T, d = (-1,-1)^T$

   (vi) $f(x) = (1 + x_3)(x_1^3 + x_2^2 - 10x_1 - 4x_2), x = (0,0,0)^T, d = (1,2,3)^T$.

2. Find the step length at x for $\min_x\ f(x)$ with exact line search technique/Armijo-Wlofe-backtracking line search technique (using $\beta_1 = 10^{-4}, \beta_2 = 0.9, r = 0.5$) with $d = -\nabla f(x)$.

   (i) $f(x) = 4x_1^2 + 5x_1x_2 + 2x_2^2 + x_1 - 3x_2, x = (2,1)^T$

   (ii) $f(x) = \sum_{i=1}^{2}(x_i - 2)^2, x = (1,1)^T$

   (iii) $f(x) = (x_1 - x_2)^2 + (x_1 - 3)^2, x = (1,2)^T$

   (iv) $f(x) = (x_1 - x_2)^2 + (x_2 - 3)^2\ x = (1,1)^T$

3. Define $f(x) = 2x_1^2 + 3x_2^2 - 4x_1x_2 - 2x_1 + 6x_2$ Perform two iterations of steepest descent with Armijo-Wolfe backtracking line search technique with initial approximation $x^0 = (1,1)^T$. Consider $\beta_1 = 10^{-4}$, $\beta_2 = 0.9, r = 0.5$. Find the value of ${d^1}^T d^0$.