4 Autoregressive models, unit roots and cointegration

Typically, discussion on whether any observed y_t are stationary focuses on whether or not there is a unit root in the AR(1) process:

$$y_t = \rho_0 + \rho_1 y_{t-1} + \varepsilon_t$$

where ρ ≥ 1 indicates a nonstationary process.

Granger and Swanson proposed the stochastic unit root model ("STUR"):

> An introduction to stochastic unit roots (1997) Granger, C. and Swanson, N., *Journal of Econometrics* **80** (1): 35–62.

In this model:

$$y_t = \rho_t y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathscr{N}(0, \sigma_1^2)$$

$$\rho_t = e^{\omega_t}$$

$$\omega_t = \varphi_0 + \varphi_1 \omega_{t-1} + \eta_t \qquad \eta_t \sim \mathscr{N}(0, \sigma_2^2)$$

$$= \mu_\omega + \varphi_1(\omega_{t-1} - \mu_\omega) + \eta_t$$

where

$$\mu_\omega = \frac{\varphi_0}{1 - \varphi_1}$$

> *A Bayesian analysis of stochastic unit root models* (1999) Jones, C. and Marriott, J., *p.* 785–794 in "Bayesian Statistics" (6th Edition), Bernardo, J. M., Berger, J. O., Dawid, A. P. and Smith, A. F. M. (Eds), Oxford University Press, Oxford, England.

consider Bayesian estimation of this model and focus on the value of μ_ω:

$$\mu_\omega < 0 \quad \Leftrightarrow \quad \mathscr{E}(\rho_t) < 1$$

$$\mu_\omega = 0 \quad \Leftrightarrow \quad \mathscr{E}(\rho_t) = 1$$

$$\mu_\omega > 0 \quad \Leftrightarrow \quad \mathscr{E}(\rho_t) > 1$$