Dynamic TAO: Bittensor Improvement Template 1 (BIT001)

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Abstract

In this paper, we introduce 'Dynamic TAO' as a pivotal enhancement to the Bittensor token system framework. Dynamic TAO represents a strategic refinement in Bittensor's token allocation process. Our primary objective with this proposal is to more accurately reward value-add subnets through an innovative, open market, decentralized approach. This approach is essential to address and counteract current potential issues such as cronyism, apathy, and monopolistic tendencies of root network TAO validators who exert significant influence over network emissions. Ultimately, this development aims at a more profound decentralization within the blockchain, empowering all TAO holders to engage in informed speculation on subnets and actively participate in the judicious allocation of Bittensor resources. This methodology promises a more dynamic, fair, and efficient progression of the network, aligning with our vision of a decentralized future.

Upward economic mobility in society, whereby new participants can come to own relatively large shares of the overall pie through hard work, is essential to the creation of wealth.[1]. At the same time, the need to maintain rule-bound property rights is essential to the functioning of society and the efficiency of market behaviour. Token economic systems like Bittensor share this quality since they tokenize a shared value system ('the pie') that must be able to incentivize new players who can continuously grow said pie, while maintaining the assurance to earlier participants that value is earned, remains earned and stays undiluted. To strike a balance between these two aims, we construct a distinction that arises between the TAO on the coldkey balance, and the Dynamic TAO that token holders fluidly stake throughout the ecosystem as they speculate on subnets. We call this separation TAO and Dynamic TAO because the latter holds the characteristic that it is represented exclusively as TAO, but temporally dynamic in terms of its instantaneous price relative to TAO.

This proposal protects the immutable property rights and respective bandwidth allocation to token holders while facilitating the advantage that Dynamic TAO rewards active builders and contributors to the ecosystem at a higher rate than those that passively hold. We propose this change to maximally incentivize valuable subnet development through a more liquid and dynamic flow of funds through any TAO holder. This flow will represent, at equilibrium, the perception of value for each subnet by all TAO owners. Currently, this flow of emission distribution is solely controlled by the root validators, a slow-moving group that is not as motivated to allocate funds efficiently. Dynamic TAO is a perfect extension to the thesis of the original Bittensor whitepaper, which spoke of the need for efficient markets to solve issues of resource allocation in AI. Specifically, the solution translates staked TAO (used within subnet incentive mechanisms) into a unique, non-fungible token whose exchange rate in TAO fluctuates based on demand. Subnets that attract demand by adding value to the ecosystem appreciate, whereas those that do not, depreciate. This allows for a dynamic market-driven evaluation in which TAO inflation is not exclusively allocated from the root network. The design protects the immutability of TAO's emission schedule while extending it with greater powers to extract digital commodities. Subnet creators and miners can be more accurately rewarded for their efforts, active TAO holders are better rewarded for participating in the liquid flow of emissions,

[1]"For life and death are one, even as the river and the sea are one." - Lao Tzu

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potential for economic centralization is drastically reduced, and governmental cronyism is eliminated from the original design.

1 Background

Bittensor uses incentives to extract digital commodities through a process of emitting a distribution of newly minted TAO every block into a set of incentive structures called subnets. This distribution, called the emission vector [1], is $E \rightarrow [S_1, S_{2,} ..., S_n]$. Within each subnet, miners[2] are rewarded by this emission in such a way that validators[3] can attain that created value [1].

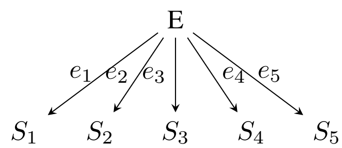

Figure 1: Emission of newly minted token vector E through subnet incentive mechanisms.

The Bittensor network currently uses a weighted consensus voting system derived from the original Bittensor white paper called Yuma Consensus V1 (YC1) as the means to determine the proportion of each $e_i \in E$. YC1 takes as input weights W set by peers within Bittensor's 'Root Network', and measures trust T and consensus C as a protection against adversarial actors who simply vote for themselves. However, the system breaks down when honest participants cannot form a strong consensus – a quality of the root network, where varying viewpoints on what Bittensor requires and who should build it, diverge.

$$\mathbf{E} = W^T S \cdot \sigma(C^T S) = R \cdot T \tag{1}$$

Yuma Consensus V1

Furthermore, because of the extreme power centralization of the Root Network we cannot be sure this system elicits the truth nor the best outcome for Bittensor as a whole. For instance, YC1 does not penalize the misallocation of emission to low-value subnets, thus opening up the potential for self-interest, cronyism, or apathy to leak into the determination of the emission vector. Even with properly aligned incentives, no matter how capable, the small number of root participants cannot compete with equilibrium market dynamics, especially as the number of active subnets on Bittensor increases. Dynamic TAO seeks to fix this.

A solution requires that the emission vector be computed by eliciting market participation from the widest number of participants. We achieve this by computing it via a set of token exchange pools: $P = [P_1(t, \alpha), P_2(t, \beta)...P_{26}(t, \omega)]$ which TAO holders stake in and out of to receive the dynamic token equivalent for that subnet. The pools measure the market equilibrium prices for each dynamic token, based on the rate of staking and unstaking, allowing us to calculate an emission vector based on each subnet's steady state price:

$$\mathbf{E} = softmax(P_{price}) \tag{2}$$

Emission vector E is computed by applying an activation function over the current price from each pool.

[2]Those that produce value in the form of computational elements like intelligence.

[3]Those who hold TAO and validate the work done by miners.

2 Pools

We define Dynamic TAO as α_i which is the subnet token[4] used for consensus and mining within subnet i. For each dynamic token the chain implements a liquidity pool $P = [P_0, ..., P_n]$, one for each subnet, such that each pool $P_i(\tau|\alpha_i)$ facilitates liquidity between τ and the subnet specific Dynamic TAO α_i. TAO can only be exchanged to and from each α by staking in and out of the subnet which enforces that the dynamic tokens are entirely gated by TAO and merely represent a non-fungible share of TAO held in the reserve of each pool.

$$\underbrace{\tau}_{\text{TAO held on balance}} \leftrightarrow \underbrace{P_i(\tau|\alpha_i)}_{\text{Subnet i specific swapping pool}} \leftrightarrow \underbrace{\alpha_i}_{\text{Dynamic TAO represented as stake in subnet i}} \tag{3}$$

Staking TAO into a subnet initializes a purchase of the subnet's dynamic token.

Users interact with Dynamic TAO through Bittensor's staking operation which triggers the pools, either introducing new TAO and withdrawing the dynamic token or vice versa. This can be executed by the chain itself requiring no intermediaries. Since all pools maintain a constant factor $\tau \times \alpha = k$, every exchange through the pool affects the relative exchange rate of the dynamic token with slippage quadratic in reserve supply and given by $\frac{k}{\tau_{reserve} + \tau_{added}}$ for τ to α or vice versa.

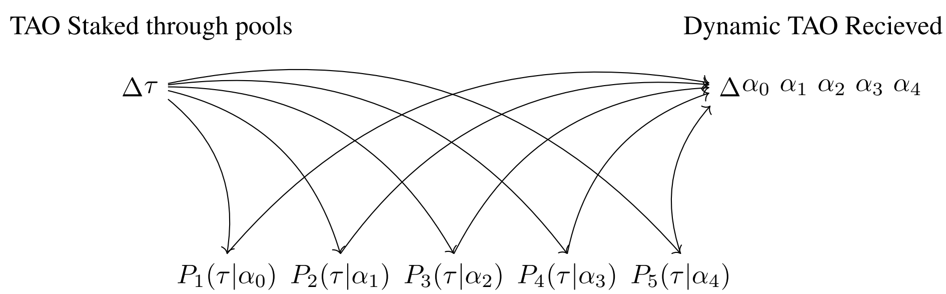

Figure 2: Dynamic TAO and TAO convert through the Uniswap pools via stake and unstake operations injecting the TAO and returning α or vice versa based on Uniswaps $\tau \times \alpha = k$ constant factor.

As a consequence of the manner in which the each pool's exchange rate changes through staking operations, the amount of TAO returnable for each unit of TAO staked is dynamic, hence the name.

3 Coinbase Injection

The problem with typical liquidity pools is that they require substantial reserves to facilitate low friction transfers. The known solution to this problem is to allow outside individuals to introduce liquidity into the pools at specified price intervals. However, this would requires exterior actors to make the system run.

To mitigate this, our proposal uses the chain Coinbase [5] to inject token liquidity directly, each block, into each pool from both sides. By design, 50 percent of each Dynamic Token α_i is added to the pool with the remainder of α_i inflation distributed through the subnet's validation mechanism. Dynamic tokens' tokenomics mirrors that of TAO with the exception of a double capped supply. We propose this to increase tokens in reserves for increased liquidity and decreased slippage when swapping through the pools. Thus, it will be 42 million capped supply (with 21 million going into the pool as reserve) and 21 million distributed as defined by the consensus mechanism. Therefore, the price of each subnet token at equilibrium will be $\frac{1}{n_{\text{subnets}}}$, assuming all n subnets have equal valuation.

[4]In practice we use greek letters to define dynamic tokens, but for simplicity we just use α_i in this paper.
[5]Bitcoin refers to the place where tokens are created on the chain, not the company.

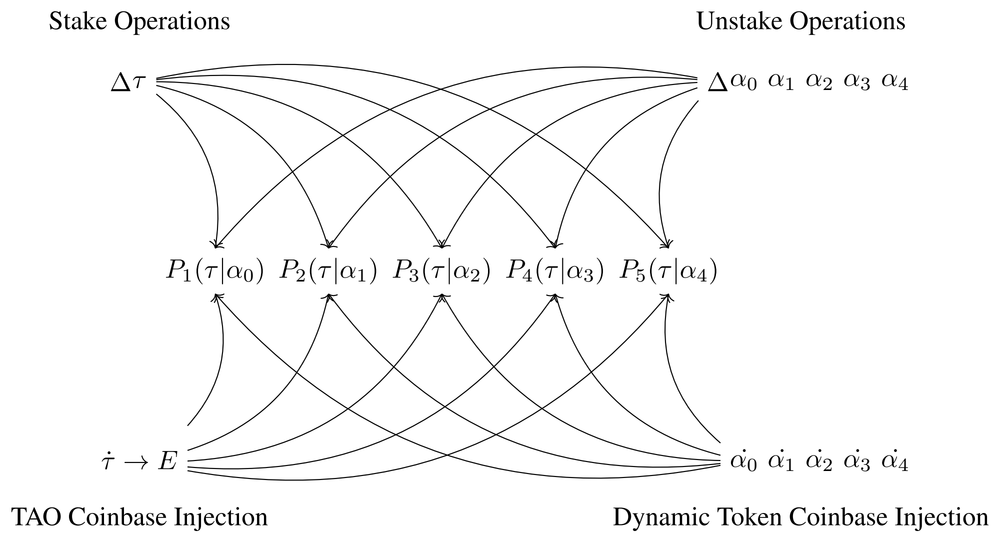

Figure 3: Liquidity in each pool is determined by 4 inflow and outflow rates, the first two of which are determined by market participants: 1) $\Delta\tau$ the rate of staking, and and 2) $\Delta\alpha$ the unstaking rate. The second two are determined by the chain: 3) $\dot{\tau}$ the Coinbase injection, and 4) $\dot{\alpha}$ the dynamic token Coinbase injection.[6] The emission vector determines $E = \dot{\tau}$ the rate of $\dot{\tau}$ injected per block into each pool while dynamic token inflation $\dot{\alpha}$ remains constant at 50 percent of that tokens inflation.

As time progresses (without exterior exchange) the Coinbase injections determine the pool's steady state price since the reserve ratio converges to $\frac{\dot{\tau}}{\dot{\alpha}}$. This can easily be seen from the fact that the instantaneous price is given by the ratio of reserves, namely:

$$Price_i = \frac{\dot{\tau}}{\dot{\alpha}} \tag{4}$$

In this paper we suggest the calculation of the emission vector E via these price calculations. Whereby every block the chain calculates E and distributes the emission into each pool.

$$\mathbf{E}_i = \frac{\exp(Price_i)}{\sum_i \exp(Price_i)} \tag{5}$$

One subtle consequence of this is that without exterior interaction (staking / unstaking) all subnets converge to exactly $\frac{1}{n}$ TAO per block and $\frac{1}{n}$ price in TAO. The use of the $\exp$ function dampens prices but could be substituted with various others like a shifted $relu$. In practice we will use a moving average of the price calculation to determine the emission vector as to reduce the possibility of manipulated price swings effecting the short term running of Bittensor. The remainder of this paper explores the effects and additional design elements of this token economic adjustment.

4 Subnet Initialization and Destruction

Bittensor uses an adaptive Dutch Auction mechanism whereby entrants bid to replace the lowest-ranked subnet with their own. When the registration / deregistration occurs, the subnet state is cleared and the owner key is replaced with the buyer's, whose purchase price is locked for the duration of the slot.

Our design modifies the registration process to use the lowest price over a 30-day moving average to select the subnet to deregister. 4. When the subnet is deregistered, the dynamic tokens associated with that slot are liquidated through the pool, effectively initiating a sell operation across the entire

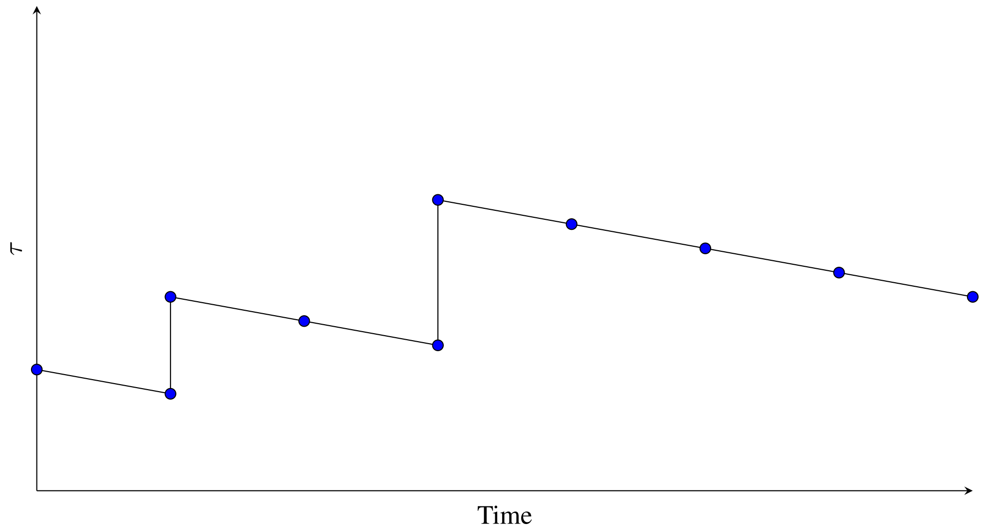

Figure 4: The figure simulates the cost of registering a new subnet. Each doubling corresponds to a subnet registration, with the price tailing off with a fixed half-life such that the mechanism finds an equilibrium price based on demand.

supply. Since each pool's price is given by the ratio $\frac{\tau}{\alpha_{\text{reserve}}}$ the conversion rate between TAO and its Dynamic counterpart creates an immediate loss of $\frac{\alpha_{\text{outstanding}}}{\alpha_{\text{reserve}}}$.

$$S_{deregistered} = \arg\min_i \bar{P}_i^{price} \tag{6}$$

The subnet with the lowest 30 day moving average price is deregistered next.

For the incoming subnet, rather than lock the TAO for the duration of the subnet, the creator "buys" the initial supply of the tokens at the same rate as the previously deregistered subnet based on the adaptive lock cost as defined above, with 50 percent of those tokens being used to instantiate the pool and the remainder used to bootstrap the consensus mechanism.

$$P(\tau_{\text{locked}} | \tau_{\text{locked}} * \frac{1}{\text{price}}) \tag{7}$$

The subnet creator attains $\tau_{\text{locked}} * \frac{1}{\text{price}}$ of α_i which have an immediate exchange rate through pool of τ_{locked} minus slippage.

As block progression begins, 50 percent of dynamic token inflation is added to the pool, and the remainder is passed through the incentive mechanism to the network participants as usual [1]. As a result of the token appreciation through validation, the initial supply of the subnet token is exponentially more valuable than tokens attained later on. By giving subnet creators a large share of the initial supply, this allows us to potentially remove the usual 18 percent owner fee assigned in Bittensor's previous version.**??**.

5 Consensus Weight

Each of Bittensor's subnets uses a proof-of-stake mechanism called Yuma Consensus V2 (YC2) to ensure that subnets can not be manipulated by a small group of participants. The security of this mechanism is proportional to the economic value of the tokens used.

Two issues arise from the proposal in this paper: First, the use of unique stake accounts per token rather than a global term, bifurcates Bittensor's total market capitalization across 32 subnets, effectively reducing their economic security by the same amount. Second, Bittensor requires a shared sense of consensus weight to facilitate inter-subnet exchange, i.e. where the digital commodities produced in one subnet are accessible to one another.

To alleviate these issues the proposal suggests the use of Global Dynamic TAO, which is a key's total TAO-denominated value across all dynamic tokens. The chain enforces that this term attains 50 percent of consensus weight on all subnets.

$$\underbrace{S_{j v_i}}_{\text{Validator i's stake weight on subnet j}} = \underbrace{\frac{\sum_k P_k(\alpha_i)}{\sum_l \sum_k P_k(\alpha_l)}}_{\text{Normalized Global Dynamic TAO}} + \underbrace{\frac{\alpha_i}{\sum_l \alpha_l}}_{\text{Normalized Subnet Specific Dynamic TAO}} \tag{8}$$

The validator's stake weight on a given subnet is the sum of their total TAO-denominated value across all dynamic tokens and their share of that subnets' tokens.

This solves the above concerns since 1) at least 50 percent of subnet consensus power is reflected by the entire market cap of TAO making it difficult to dominate any single subnet and 2) large holders of Global Dynamic TAO can use their stake weight to bridge protocols and interleave Bittensor's multiple digital commodities.

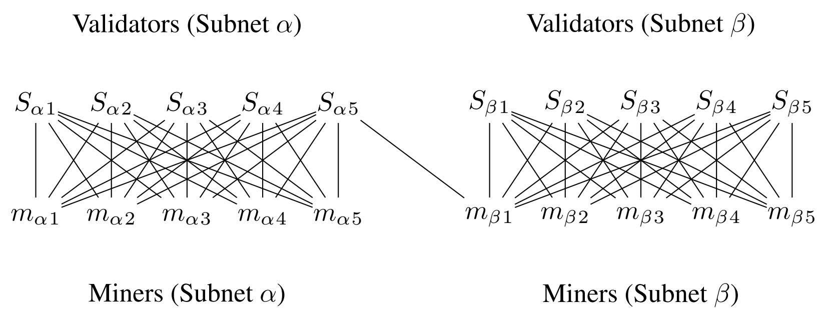

Figure 5: Validators S_5 has non zero stake weight $S_{\alpha 5}$ on subnet α and $S_{\beta 5}$ on subnet β allowing it to make queries cross boundary.

6 Delegation and Governance.

Delegation of TAO holds primary importance for Bittensor's governmental system because it facilitates 1) the selection of Senate members who hold veto power against Bittensor's Triumvirate, 2) it acts as a funding mechanism for teams for contributing to Bittensor and 3) it incentivizes validators to remain active across multiple of Bittensor's incentive mechanisms. Delegation remains conceptually unchanged with Dynamic TAO. Individuals still stake and unstake with validators and dividends are accrued to the nominators based on their choice of validator.

However, since delegates now attain Dynamic TAO delegations rather than TAO, it is now TAO holders that decide the emission allocation. Specifically, delegates cannot swap their delegators' Dynamic TAO across subnets. The power remains in the hands of the individuals.

The effect of this is paramount to the way in which Bittensor is governed since it breaks the relationship between the selection of subnets and the delegates, putting this power back in the hands of a much more liquid and horizontally scaled market dynamic. Further more, this change facilitates the creation of a completely separate political entity within the Bittensor ecosystem, which we propose to call 'owners'.

Distinct from, but related to, the design changes discussed in this paper, we suggest the creation of a 2/3 political governance system involving three distinct parties: 1) The Senate, composed of TAO

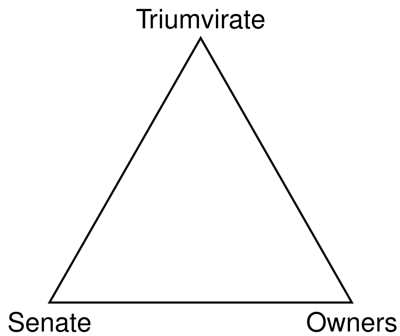

Figure 6: Three independent entities govern Bittensor via a 2/3 multi-signature style vota and veto structure.

whales selected through the delegation process; 2) The Subnet Owners, selected by the open market; and 3) The Triumvirate (known sudo key holders).

7 Example

The following outlines a sequence of events to showcase the proposal.

1. The chain initiates the Dynamic TAO change via a chain upgrade.
2. All currently active subnets are initialized with an amount of Dynamic TAO equivalent to their previous TAO lock, with 50 percent allocated to the pool and the remainder to their owner key.
3. Subnet pool reserves are bootstrapped by the initial owner balance and also by the gradual movement of staked TAO into subnet tokens through exchange.
4. The chain emission of TAO is distributed based on initially determined prices, and injects dynamic tokens at a rate of 7200 per day.
5. Time progresses until a subnet is deregistered. At this point, the tokens in the pool are swapped from the respective dynamic token into TAO at a price determined by the instantaneous P_k at the time of deregistration.
6. The new owner is allocated their fair portion of the new dynamic token based on the competitive lock TAO quantity and 30-day moving average price of the previous subnet.

8 Analysis

8.1 Value Analysis

The free floating nature of the each dynamic token's price measured in TAO opens up edge cases where the total market capitalization of dynamic tokens exceeds that of TAO outstanding. The reasonable question to ask is whether this change will remove demand for TAO in exchange for the subnet tokens themselves.

We investigate this question theoretically based the demand theory of value which posits:

> The price of a good is determined by the interaction of supply and demand in a market.

We start with the base case where there is only a single subnet token gated by the staking operation of TAO. We simplify the terms into three items 1) Dollars A 2) TAO gating token B and 3) Dynamic tokens C_i.

8.1.1 Base Case

Hypothesis: The value of B is greater than or equal to the value of C_0 when both are priced in A. Given: B can only be purchased by units of A and C_0 can only be purchased by units of B.

$$\underbrace{A}_{\text{USD Dollar Demand}} \leftrightarrow \underbrace{B}_{\text{TAO Gating token}} \leftrightarrow \underbrace{P_0(B|C_0)}_{\text{Swapping Pool}} \leftrightarrow \underbrace{C_0}_{\text{Dynamic token only accessible via B.}} \tag{9}$$

Proof: By the demand theory of value, the value of B in terms of A is determined by its supply and demand from A. The value of C_0 is dependent on its demand from B. Since C_0 can only be acquired through B, all demand for C_0 must first pass through B, thus its demand is inherently capped by the demand for B. Furthermore, if the value of C were to exceed the value of B, it would imply that C_0 is more valuable than the means (B) required to obtain it, which is an economic inconsistency.

8.1.2 Inductive Case

Hypothesis: The value of B is greater than the value of $\sum_i (C_i) = [C_0, C_1, ...C_n]$ when all are gated by B. Given that C_i can only be purchased by units of B.

Proof: For $n = 1$, we only have the item C_0 which can be purchased with B. As we have already established, the value of B must be greater than C_0 when priced in A, because B's value includes its own intrinsic value plus its utility in acquiring C_0. Therefore, the base case holds.

Inductive Step: Assume that the statement is true for n items, i.e., the value of B is greater than each of $C_1, C_2, \ldots, C_n$ when priced in A. Now, introduce a new item C_{n+1} which can also only be purchased with B.

With the introduction of C_{n+1}, the demand for B increases because it is now required to purchase $n + 1$ items instead of just n. This increased demand for B, as the sole means of obtaining C_{n+1} (and the other C items), should increase its value.

The value of C_{n+1}, like the other C items, is capped by the value of B. Since B's value has increased due to the added demand from C_{n+1}, and since C_{n+1} cannot have a value exceeding the means to acquire it (B), the value of B remains greater than C_{n+1}.

The trend observed from C_1 to C_n continues with C_{n+1}. The value of B must exceed that of C_{n+1} to prevent economic anomalies, like disproportionate arbitrage opportunities or the illogical situation where the means (B) is less valuable than the end (any C_i).

Since the statement is true for the base case and the inductive step holds, it follows that for any number of items $C_1, C_2, \ldots, C_n$ purchasable only with B, the value of B will always be greater than each of these items when priced in A.

8.1.3 Discussion

The proof above does not rule out the possibility that the market capitalizations of all dynamic tokens exceeds that of TAO. This is a natural possibility based on Uniswap pools, which allow prices to reach infinity. Indeed, this situation is likely, and will arise based on the speculated future value of each token. It does not, however mean that demand has been removed from TAO, on the contrary it will likely fuel ecosystem demand which is gated by TAO.

8.2 Cabal Attack

We consider a situation where a subset of the network stakeholders decides to exploit the system by manipulating the price of dynamic tokens to gain a larger percentage of token emissions. The conflict between the honest subset of participants and the dishonest participants can be determined by the network ownership held by each group. The honest group must attain a higher proportion of ownership to maintain its dominance and protect the network.

We assume that the proportion of the network owned by the honest subset is greater than that of the dishonest subset. Initially, the chain creates two subnets, dividing the network equally between them. We follow the dynamic pool structure as defined earlier. The honest group buys tokens in the 'good' networks honestly and sells tokens in the 'bad' networks. Conversely, the dishonest subset buys 'bad' subnet tokens and sells 'good' subnet tokens. We track Global Dynamic TAO over time.

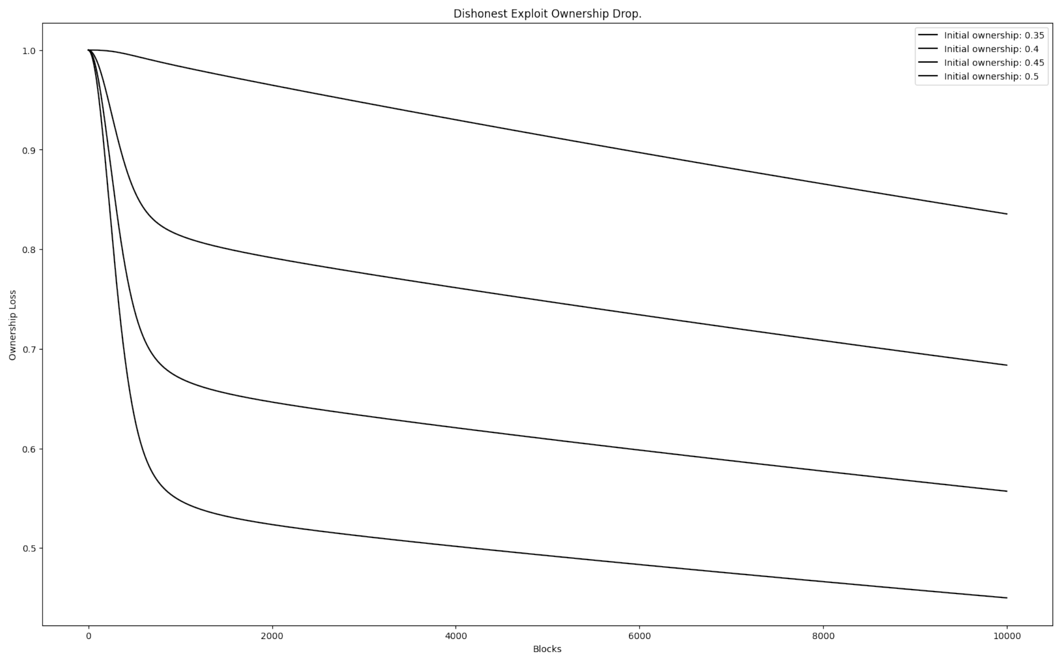

Figure 7: Shows the ownership loss (as percentage of original amount) of the dishonest subset as it performs the greedy purchase strategy suggested above.

As described earlier, the chain progresses daily, emitting τ into the left-hand side of the pools and α into the right-hand side. The honest subset buys TAO with alpha (from the dishonest subnet) and sells TAO for beta (in the honest subnet). In contrast, the dishonest subset buys TAO with beta (from the honest subnet) and sells TAO for alpha (from the dishonest subnet). The imbalance of initial funds means the dishonest subnet is diminished over time. Figure 8.2 shows this loss.

9 Conclusion

The proposal is for an extension to the Bittensor incentive structure which we are calling Dynamic TAO. The design makes a demarcation between TAO that is held on balance and TAO that is staked for the purpose of attaining consensus power or to extracting dividends through validation. The value of Dynamic TAO is captured by the value-holding token TAO while still allowing for dynamism in global share. The primary result is the removal of Bittensor's root network as the primary determining group for subnet emissions. Further more, it introduces the potential for greater economic mobility within Bittensor's token system without dilution and governmental decentralization.

To achieve this aim, the paper showcases the singular importance of computing the emission vector. We showed how a Uniswap pool structure could be used to facilitate its computation through a competitive and speculative mechanism that organically distributed tokens through the pools both from Bittensor's TAO Coinbase, and on the other side from each Dynamic Token Coinbase. Prices for each pool are negotiated by staking (purchase) and unstaking (sale).

Following this, we showed (1) how consensus weight is still liquid across the ecosystem both for economic security and for inter-subnet communication and (2) how the proposal introduces true disjoint governance between 3 parties. At the end we prove how demand for TAO is not diluted, and investigate how this system is resistant to collusion from a less than majority stake weight cabal.

References

[1] Y. Rao, "Bittensor: A peer to peer intelligence benchmark," *arXiv preprint arXiv:1804.07461*, 2020.

10 Appendix

10.1 Owner Share

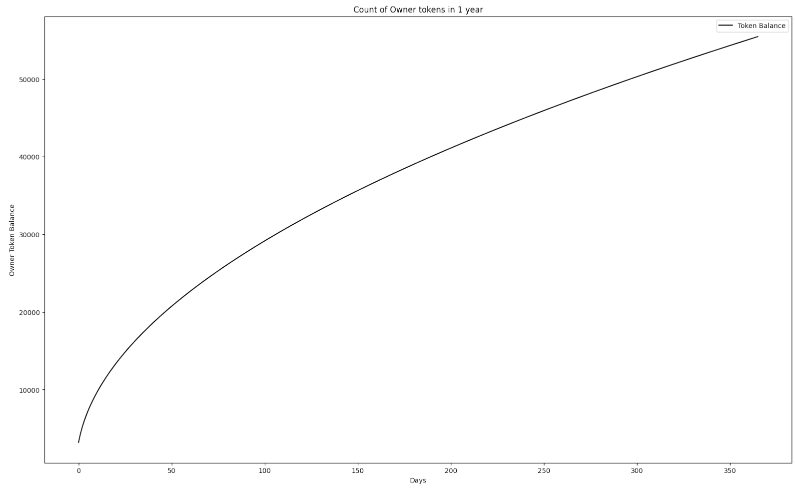

Figure 8: Shows owner's token balance over the first year without mining, simply through holding their own Dynamic Token.

10.2 Value Analysis

Can this design leak value away from TAO? We investigate this concept from a demand theory of value:

> The demand theory of value posits:the price of a good is determined by the interaction of supply and demand in a market.

We investigate this system theoretically, starting with the case that there is only a single subnet token gated by the staking operation of TAO. We simplify the terms into three items 1) Dollars A 2) TAO gating token B and 3) Dynamic tokens C_i.

$$\underbrace{A}_{\text{USD Dollar Demand}} \leftrightarrow \underbrace{B}_{\text{TAO Gating token}} \leftrightarrow \underbrace{P_i(B|C)}_{\text{Swapping Pool}} \leftrightarrow \underbrace{C_0}_{\text{Dynamic token only accessible via B.}}$$

10.2.1 Base Case

Hypothesis: The value of B is greater than the value of C_0 when both are priced in A. Given: B can only be purchased by units of A. C_0 can only be purchased by units of B.

Proof: By the demand theory of value, the value of B in terms of A is determined by its supply and demand from A. The value of C_0 is dependent on its demand from B. Since C_0 can only be acquired through B, all demand for C_0 must first pass through B, thus its demand is inherently capped by the demand for B. Furthermore, if the value of C were to exceed the value of B, it would imply that C_0 is more valuable than the means (B) required to obtain it, which is an economic inconsistency.

As such, under perfect market efficiency if C_0 were valued more than B, it would lead to an unsustainable situation where everyone would prefer to trade B for C_0 directly, ignoring the intrinsic value of B leading to an equilibrium price drop to match this discrepancy.

10.2.2 Inductive Case

Hypothesis: The value of B is greater than the value of $\sum_i(C_i) = [C_0, C_1, ...C_n]$ when all are gated by B. Given that B can only be purchased by units of A. C_i can only be purchased by units of B and the value of B in terms of A is initially set as the supply of B times the price of B in A.

Proof: For $n = 1$, we only have the item C_0 which can be purchased with B. As we have already established, the value of B must be greater than C_0 when priced in A, because B's value includes its own intrinsic value plus its utility in acquiring C_0. Therefore, the base case holds.

Inductive Step: Assume that the statement is true for n items, i.e., the value of B is greater than each of $C_1, C_2, \ldots, C_n$ when priced in A. Now, introduce a new item C_{n+1} which can also only be purchased with B.

With the introduction of C_{n+1}, the demand for B increases because it is now required to purchase $n + 1$ items instead of just n. This increased demand for B, as the sole means of obtaining C_{n+1} (and the other C items), should increase its value.

The value of C_{n+1}, like the other C items, is capped by the value of B. Since B's value has increased due to the added demand from C_{n+1}, and since C_{n+1} cannot have a value exceeding the means to acquire it (B), the value of B remains greater than C_{n+1}.

The trend observed from C_1 to C_n continues with C_{n+1}. The value of B must exceed that of C_{n+1} to prevent economic anomalies, like disproportionate arbitrage opportunities or the illogical situation where the means (B) is less valuable than the end (any C_i).

Since the statement is true for the base case and the inductive step holds, it follows that for any number of items $C_1, C_2, \ldots, C_n$ purchasable only with B, the value of B will always be greater than each of these items when priced in A. This conclusion hinges demand theory of value.

10.3 Extended Cabal Analysis

We run the following code to simulate the global dynamic proportion of the honest and dishonest subset as explained in the Analysis section of the paper.

```
import pandas as pd
import matplotlib.pyplot as plt

dishonest = []
honest = []
splits =  [0.35, 0.40, 0.45, 0.50 ]
for split in splits:
    dishonest_i = []
    honest_i = []
    total_initial_supply = 1000000

    initial_honest_subset_tao = split * total_initial_supply
    honest_subset_tao = initial_honest_subset_tao
    honest_subset_alpha = 0
    honest_subset_beta = 0

    initial_dishonest_subset_tao = (1 - split) * total_initial_supply
    dishonest_subset_tao = initial_dishonest_subset_tao
    dishonest_subset_alpha = 0
    dishonest_subset_beta = 0

    initial_tao_pool_supply = 100 # assuming a 100 tao lock cost.
    initial_alpha_pool_supply = initial_tao_pool_supply * 32 # pool gets proportional number of alpha.
    initial_alpha_outstanding = initial_tao_pool_supply * 32 # owner gets proportional number of alpha.
    pool_A = DynamicTAO(initial_tao_pool_supply, initial_alpha_pool_supply, initial_alpha_outstanding, 0.5 )
    pool_B = DynamicTAO(initial_tao_pool_supply, initial_alpha_pool_supply, initial_alpha_outstanding, 0.5 )

    total_blocks = 10000
    history = []
    for day in range( total_blocks ):

        # Inject tao via coinbase based on price.
        pool_A.inject_tao( 1 * (pool_A.alpha_price() / (pool_A.alpha_price() + pool_B.alpha_price()) ))
        pool_B.inject_tao( 1 * (pool_B.alpha_price() / (pool_B.alpha_price() + pool_A.alpha_price()) ))

        # Purchase alpha and beta with tao.
        dishonest_subset_alpha += pool_A.buy_alpha( dishonest_subset_tao / total_blocks )
        honest_subset_beta += pool_B.buy_alpha( honest_subset_tao / total_blocks )

        # Sell alpha and beta for alternative tao.
        dishonest_subset_alpha += pool_A.buy_alpha( pool_B.buy_tao( dishonest_subset_beta ) )
        dishonest_subset_beta = 0
        honest_subset_beta += pool_B.buy_alpha( pool_A.buy_tao( honest_subset_alpha ) )
        honest_subset_alpha = 0

        # Distribute validator emissions from Dynamic TAO.
        dishonest_subset_alpha += 0.5 * 0.25 * ( dishonest_subset_alpha / (dishonest_subset_alpha + honest_subset_alpha) )
        dishonest_subset_beta += 0.5 * 0.25 * ( dishonest_subset_beta / (dishonest_subset_beta + honest_subset_beta) )
        honest_subset_alpha += 0.5 * 0.25 * ( honest_subset_alpha / (dishonest_subset_alpha + honest_subset_alpha) )
        honest_subset_beta += 0.5 * 0.25 * ( honest_subset_beta / (dishonest_subset_beta + honest_subset_beta) )

        # Compute global Dynamic TAO.
        dishonest_global_dynamic_tao = pool_A.test_buy_tao( dishonest_subset_alpha ) + pool_B.test_buy_tao( dishonest_subset_beta )
        honest_global_dynamic_tao = pool_A.test_buy_tao( honest_subset_alpha ) + pool_B.test_buy_tao( honest_subset_beta )

        # Distribute validator emissions from global Dynamic TAO.
        dishonest_subset_alpha += 0.5 * 0.25 * ( dishonest_global_dynamic_tao / (dishonest_global_dynamic_tao + honest_global_dynamic_tao) )
        dishonest_subset_beta += 0.5 * 0.25 * ( dishonest_global_dynamic_tao / (dishonest_global_dynamic_tao + honest_global_dynamic_tao) )
        honest_subset_alpha += 0.5 * 0.25 * ( honest_global_dynamic_tao / (dishonest_global_dynamic_tao + honest_global_dynamic_tao) )
        honest_subset_beta += 0.5 * 0.25 * ( honest_global_dynamic_tao / (dishonest_global_dynamic_tao + honest_global_dynamic_tao) )

        dishonest_i.append( ( dishonest_global_dynamic_tao / honest_global_dynamic_tao ) / ( initial_dishonest_subset_tao / initial_honest_subset_tao ) )
        honest_i.append( ( honest_global_dynamic_tao / dishonest_global_dynamic_tao ) / ( initial_honest_subset_tao / initial_dishonest_subset_tao ) )

    dishonest.append( dishonest_i )
    honest.append( honest_i )

# >>
# block | size of dishonest graph.
# 0 0.9999996859260101
# 1 0.9999841314933972
# 2 0.9999656915000956
# 3 0.9999425065444754
# 4 0.999914158955644
# 5 0.9998804799864087
# 6 0.9998413751972821
# 7 0.9997967800591582
# 8 0.9997466449742356
# 9 0.9996909292336893
# ...
# 996 0.794923831228632
# 997 0.7947713381837173
# 998 0.7946188840985718
# 999 0.794466468953128
```