HW2

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习题 1.3

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(1)

$\lim\limits_{\substack{x\to 3\\ y\to 0}} \frac{\ln(x+\sin y)}{\sqrt{x^2+y^2}} = \frac{\ln 3}{3}$

(2)

$\frac{x+y}{x^2+xy+y^2} = (\frac{1}{x}+\frac{1}{y})(\frac{1}{\frac{x}{y}+\frac{y}{x}+1})$
因为 $\frac{x}{y}+\frac{y}{x} \geq 2$ 或 $\frac{x}{y}+\frac{y}{x} \leq -2$
则有 $|\frac{x}{y}+\frac{y}{x}+1| \geq 1$
故 $0 \leq |\frac{x+y}{x^2+xy+y^2}| \leq |\frac{1}{x}+\frac{1}{y}| \to 0\ (x\to\infty, y\to\infty)$
故 $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \frac{x+y}{x^2+xy+y^2} = 0$

(3)

$0 \leq (x^2+y^2)e^{y-x} \leq (e^x+e^y)e^{y-x} = e^y + e^{2y-x} \to 0\ (x\to+\infty, y\to-\infty)$
故 $\lim\limits_{\substack{x\to+\infty\\ y\to-\infty}} (x^2+y^2)e^{y-x} = 0$

(4)

$$\lim_{\substack{x\to\infty\\ y\to\infty}} \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}} = \lim_{t\to\infty} \frac{\ln t}{\sqrt{t}} \xlongequal{\text{洛必达}} \lim_{t\to\infty} \frac{1/t}{1/(2\sqrt{t})} \lim_{t\to\infty} \frac{2}{\sqrt{t}} = 0$$

(5)

$x^2+y^2\ge 2|xy|$

$0\le |(\frac{xy}{x^2+y^2})^{x^2}|\le=(\frac{|xy|}{x^2+y^2})^{x^2}\le(\frac{1}{2})^{x^2}\to 0\ (x\to\infty)$

故 $\lim\limits_{\substack{x\to\infty\\y\to\infty}}(\frac{xy}{x^2+y^2})^{x^2}=0$

(6)

$\lim\limits_{\substack{x\to\infty\\y\to\infty}}\frac{x+y}{xy}=\lim\limits_{\substack{x\to\infty\\y\to\infty}}\frac{1}{x}+\lim\limits_{\substack{x\to\infty\\y\to\infty}}\frac{1}{y}=0$

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(3)

两个二次极限都不存在，但是二重极限存在。

$\lim\limits_{y\to 0}\sin\frac{1}{y}$ 不存在，则 $\lim\limits_{y\to 0}(x+y)\sin\frac{1}{x}\sin\frac{1}{y}$ 也不存在。

故 $\lim\limits_{x\to 0}\lim\limits_{y\to 0}(x+y)\sin\frac{1}{x}\sin\frac{1}{y}$ 不存在。

同理，$\lim\limits_{y\to 0}\lim\limits_{x\to 0}(x+y)\sin\frac{1}{x}\sin\frac{1}{y}$ 不存在。

但 $0\le|(x+y)\sin\frac{1}{x}\sin\frac{1}{y}|\le|x+y|\to 0\ (x\to 0,y\to 0)$

故二重极限存在，且为 0：$\lim\limits_{\substack{x\to 0\\y\to 0}}(x+y)\sin\frac{1}{x}\sin\frac{1}{y}=0$

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(3)

在 $(0,0)$ 连续。

$0\le\frac{x^2y^2}{(x^2+y^2)^{\frac{3}{2}}}\le\frac{x^2y^2}{(2xy)^{\frac{3}{2}}}=\frac{\sqrt{2xy}}{4}\to 0\ (x\to 0,y\to 0)$

故 $\lim\limits_{\substack{x\to 0\\y\to 0}}\frac{x^2y^2}{(x^2+y^2)^{\frac{3}{2}}}=0=f(0,0)$

故在 $(0,0)$ 连续。

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(1)

$\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$
故为 2 阶无穷小

(2)

$\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\ln(1+\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} = 1$
故为 1 阶无穷小

习题 1.4

1

(2)

$\frac{\partial z}{\partial x} = (2\tan(x^2+y^2))(\sec^2(x^2+y^2))(2x) = 4x\tan(x^2+y^2)\sec^2(x^2+y^2)$
$\frac{\partial z}{\partial y} = (2\tan(x^2+y^2))(\sec^2(x^2+y^2))(2y) = 4y\tan(x^2+y^2)\sec^2(x^2+y^2)$

(4)

$\frac{\partial z}{\partial x} = \frac{1}{1+\frac{y^2}{x^4}} \cdot (\frac{-2y}{x^3}) = \frac{-2xy}{x^4+y^2}$
$\frac{\partial z}{\partial x} = \frac{1}{1+\frac{y^2}{x^4}} \cdot \frac{1}{x^2} = \frac{x^2}{x^4+y^2}$

(6)

$\frac{\partial z}{\partial x} = e^{-y} - ye^{-x}$
$\frac{\partial z}{\partial y} = e^{-x} - xe^{-y}$

(8)

$\frac{\partial u}{\partial x} = \frac{-y}{x^2}\cosh\frac{y}{x}$
$\frac{\partial u}{\partial y} = \frac{1}{x}\cosh\frac{y}{x} + z\sinh(yz)$

$\frac{\partial u}{\partial z} = y\sinh(yz)$

2

(2)

$f'_x(0,0) = \lim\limits_{\Delta x \to 0} \frac{f(\Delta x,0)-f(0,0)}{\Delta x} = 0$
$f'_y(0,0) = \lim\limits_{\Delta y \to 0} \frac{f(0,\Delta y)-f(0,0)}{\Delta y} = 0$
考虑 $T(\Delta x, \Delta y) = \frac{f(\Delta x,\Delta y)-f(0,0)-f'_x(0,0)-f'_y(0,0)}{\sqrt{(\Delta x)^2+(\Delta y)^2}} = \frac{2\Delta x\Delta y}{(\Delta x)^2+(\Delta y)^2}$
令 $\Delta y = k\Delta x$
则 $T(\Delta x, \Delta y) = \frac{2k}{k^2+1}$，显然其值随 k 值而异，自然 T 在 (0,0) 处极限不存在
故 $f(x,y)$ 在 $(0,0)$ 处不可微

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(1)

$\left.\frac{\partial u}{\partial x}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = \left.-x(x^2+y^2+z^2)^{-\frac{3}{2}}\cos\frac{1}{\sqrt{x^2+y^2+z^2}}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = -\frac{\sqrt{2}}{2}\cos 1$
$\left.\frac{\partial u}{\partial y}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = \left.-y(x^2+y^2+z^2)^{-\frac{3}{2}}\cos\frac{1}{\sqrt{x^2+y^2+z^2}}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = -\frac{1}{2}\cos 1$
$\left.\frac{\partial u}{\partial z}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = \left.-z(x^2+y^2+z^2)^{-\frac{3}{2}}\cos\frac{1}{\sqrt{x^2+y^2+z^2}}\right|_{(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2})} = \frac{1}{2}\cos 1$
故 $\mathrm{d}u = -\frac{\sqrt{2}}{2}\cos 1\cdot \mathrm{d}x - \frac{1}{2}\cos 1\cdot \mathrm{d}y + \frac{1}{2}\cos 1\cdot \mathrm{d}z$

(3)

$\frac{\partial z}{\partial x} = 2(x+y)$
$\frac{\partial z}{\partial y} = 2(x+y)$
故 $\mathrm{d}z = 2(x+y)\mathrm{d}x + 2(x+y)\mathrm{d}y$

(5)

$\frac{\partial z}{\partial x} = \frac{2y}{(x+y)^2}$

$\frac{\partial z}{\partial x} = \frac{-2x}{(x+y)^2}$

故 $\mathrm{d}z = \frac{2y}{(x+y)^2}\mathrm{d}x - \frac{2x}{(x+y)^2}\mathrm{d}y$