HW11

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习题 6.2

2

$$
\begin{aligned}
\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} S(x)\mathrm{d}x &= \sum_{n=1}^{\infty} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{2^n} \tan \frac{x}{2^n} \mathrm{d}x \\
&= \sum_{n=1}^{\infty} \int_{\frac{\pi}{6\cdot 2^n}}^{\frac{\pi}{6\cdot 2^{n-1}}} \tan t \mathrm{d}t \\
&= \sum_{n=1}^{\infty} \left(\ln\cos\left(\frac{\pi}{6\cdot 2^n}\right) - \ln\cos\left(\frac{\pi}{6\cdot 2^{n-1}}\right) \right) \\
&= \lim_{n\to\infty} \ln \frac{\cos \frac{\pi}{6\cdot 2^n}}{\cos \frac{\pi}{3}} \\
&= \ln \frac{\sqrt{3}}{2}
\end{aligned}
$$

4

$f(x) = \sum\limits_{n=1}^{\infty} \frac{n}{x^n} \leq \sum\limits_{n=1}^{\infty} \frac{1}{\left(\sqrt{x}\right)^n}$，故 $f(x)$ 一致收敛，而又有 $\forall n > 1$, $u_n(x)$ 连续
故 $f(x)$ 连续

习题 6.3

1

(1)

$q = \lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{n^n}} = 0, R = \frac{1}{q} = +\infty$ 故收敛域为 $\mathbb{R}$

(3)

令 $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \frac{x^3}{2} = 1 \Rightarrow x = \pm\sqrt[3]{2}$
故收敛半径 $R = \sqrt[3]{2}$
若 $x = \sqrt[3]{2}$