HW3

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习题 1.4

5

$V=\frac{\pi h}{3}(R^2+Rr+r^2)$
$\mathrm{d}V=\frac{\pi}{3}(\frac{\partial V}{\partial h}\mathrm{d}h+\frac{\partial V}{\partial R}\mathrm{d}R+\frac{\partial V}{\partial r}\mathrm{d}r)=$
$\frac{\pi}{3}((R^2+Rr+r^2)\mathrm{d}h)+h(2R+r)\mathrm{d}R+h(2r+R)\mathrm{d}r$
$\mathrm{d}V\Big|_{(40,30,20)}=\frac{\pi}{3}(1900\mathrm{d}h+3200\mathrm{d}R+2800\mathrm{d}r)$
代入 $\Delta h=0.2,\Delta R=0.3,\Delta r=0.4$，可得体积增量近似值为：
$\Delta V\approx 2576(\mathrm{cm}^3)$

11

(1)

$\mathrm{grad}z(0,\frac{\pi}{2})=(-\sin(x+y),-\sin(x+y))\Big|_{(0,\frac{\pi}{2})}=(-1,-1)$
$\frac{\partial z}{\partial l}(P_0)=(-1,-1)\cdot(\frac{3}{5},\frac{-4}{5})=\frac{1}{5}$

(3)

$\frac{\partial z}{\partial x_i}\Big|_{(1,1,\cdots,1)}=2\sum_{i=1}^{n}x_i\Big|_{(1,1,\cdots,1)}=2n$
故 $\frac{\partial z}{\partial l}(P_0)=2n(1,1,\cdots,1)\cdot\frac{(-1,-1,\cdots,-1)}{\sqrt{n}}=-2n^{\frac{3}{2}}$

12

(1)

$\mathrm{grad}u = (\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}})$

(3)

$\mathrm{grad}u = (1, 1, \cdots, 1)$

13

考虑单位向量 $\boldsymbol{I}$
$\mathrm{grad}u(P_0) = (2x - y - z, 2y - x + z, 2z - x + y)\Big|_{(P_0)} = (0, 2, 2)$
则 $\frac{\partial u}{\partial \boldsymbol{I}} = (0, 2, 2) \cdot \boldsymbol{I}$
则当 $\boldsymbol{I}$ 和 $(0, 2, 2)$ 同向即 $\boldsymbol{I} = (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ 时，方向导数取最大值
$||(0, 2, 2)|| = 2\sqrt{2};$
当 $\boldsymbol{I}$ 和 $(0, 2, 2)$ 反向即 $\boldsymbol{I} = (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ 时，方向导数取小值
$-||(0, 2, 2)|| = -2\sqrt{2};$
当 $\boldsymbol{I}$ 和 $(0, 2, 2)$ 垂直即 $\boldsymbol{I}$ 位于平面 $y + z = 0$ 上时，方向导数为 0

14

(1)

$\frac{\partial u}{\partial x} = -a\sin(2ax - 2by)$
$\frac{\partial u}{\partial y} = b\sin(2ax - 2by)$
$\frac{\partial^2 u}{\partial x^2} = -2a^2\cos(2ax - 2by)$
$\frac{\partial^2 u}{\partial y^2} = -2b^2\cos(2ax - 2by)$
$\frac{\partial^2 u}{\partial x \partial y} = 2ab\cos(2ax - 2by)$

(3)

$\frac{\partial u}{\partial x} = e^{-xy}(1 - xy)$
$\frac{\partial u}{\partial y} = -x^2 e^{-xy}$

$\frac{\partial^2 u}{\partial x^2} = ye^{-xy}(xy-2)$
$\frac{\partial^2 u}{\partial y^2} = x^3 e^{-xy}$
$\frac{\partial^2 u}{\partial x \partial y} = xe^{-xy}(xy-2)$

15

(1)

证明. 令 $v = \frac{\partial u}{\partial y} = \sin(2x-y)$
$\nabla v = (2\cos(2x-y), -y\cos(2x-y))$
$\nabla v \cdot (2,1) = 0$
即 $2\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} = 0$ □

(2)

证明. $\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{\frac{-3}{2}}$
$\frac{\partial^2 u}{\partial x^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(2x^2-y^2-z^2)$
同理，$\frac{\partial^2 u}{\partial y^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(2y^2-x^2-z^2)$
$\frac{\partial^2 u}{\partial z^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(2z^2-x^2-y^2)$
故 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ □

习题 1.5

1

(1)

$\boldsymbol{J}_f = \begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{bmatrix}$
$|\boldsymbol{J}_f| = \frac{x(x+y)}{(x^2+y^2)^{\frac{3}{2}}}$
故当 $x \neq 0$ 且 $x+y \neq 0$ 时，Jacobi 矩阵可逆

(2)

$$\boldsymbol{J}_f = \begin{bmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin y & e^x \cos y \end{bmatrix}$$

$|\boldsymbol{J}_f| = e^{2x} > 0$

故在整个 $\mathbb{R}^2$ 上都可逆