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SP20-BSE-085
Section A

Q 1a)

~~Set~~ $S = \{t+1, t-1\}$,

$L(t+1) = t-1 = 0\,(t+1) + 1(t-1)$

$L(t-1) = 2t+1 = \frac{3}{2}(t+1) + \frac{1}{2}(t-1)$

So $[L(t+1)]_S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $[L(t-1)]_S = \begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix}$

Matrix L with respect to S is

$$A = \begin{bmatrix} 0 & \frac{3}{2} \\ 1 & \frac{1}{2} \end{bmatrix}$$

b) $L(2t+3) = L\left(\frac{5}{2}(t+1) - \frac{1}{2}(t-1)\right)$

$= \frac{5}{2}L(t+1) - \frac{1}{2}L(t-1)$

$= \frac{5}{2}(t-1) - \frac{1}{2}(2t+1)$

$= \frac{1}{2}(3t-6)$

$2t+3 = \frac{5}{2}(t+1) - \frac{1}{2}(t-1)$

Scanned with Ca

So $[2t+3]_S = \begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{bmatrix}$

$$[L(2t+3)]_S = A \begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{3}{2} \\ 1 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} \\ \frac{9}{4} \end{bmatrix}$$

So,

$$2t+3 = -\frac{3}{4}(t+1) + \frac{9}{4}(t-1)$$

$$= \frac{1}{4}(6t-12) = \frac{1}{2}(3t-6)$$

c)

$$L(at+b) = L\left(\frac{a+b}{2}(t+1) + \frac{a-b}{2}(t-1)\right)$$

$$= \frac{a+b}{2} L(t+1) + \frac{a-b}{2} L(t-1)$$

$$= \frac{a+b}{2}(t-1) + \frac{a-b}{2}(2t+1)$$

$$= \frac{1}{2}\{(3a-b)t - 2b\}$$

Q2a)

$L: M_{22} \to M_{22}$

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix}$$

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 0M_{22}$$

$$\begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a+b=0,\ b+c=0,\ a+d=0,$
$b+d=0$

$a = b = c + d = 0$

So, $\operatorname{Ker}(L) = \operatorname{Ker} L = \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right)$

Basis of $\operatorname{Ker} L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q2 b)

$$\text{Range } L = \left\{ \begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix} \right\} =$$

$$a \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$+ d \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \}$$

$$\text{Span}(\text{Range} L) = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

We know $\dim(\text{Ker} L) + \dim(\text{Range} L)$

$= \dim(\text{range} L) = 4 - 0 = 4$

Basis of range L =

$$\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

Q3)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Finding Matrices P_1, P_2
For which $P_1 \neq 0$, $P_2 \neq 0$

The similar matrices will be given by $P_1^{-1} A P_1$ and $P_2^{-1} A P_2$

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_2^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow P_2^{-1} A P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$P_2^{-1} A P_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

Q4 a)

$$x + y - z = 0$$
$$2x + y + 2z = 0$$

let $z = t$

$$x + y = t$$
$$2x \pm y = \pm 2t$$
$$-x = 3t$$
$$x = -3t$$

Put in 1

$$-3t + y - t = 0$$
$$y = 4t$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3t \\ 4t \\ t \end{bmatrix} = t\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$$\text{Basis} = \left\{ \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \right\}$$

let $u_1 = v_1 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$

orthonormal basis

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{26}} \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

b)

$$Q_{T \leftarrow S} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$V_1 = a_1 w_1 + a_2 w_2$$
$$= 2w_1 + (-1) w_2$$
$$= 2t - (t-1)$$
$$= 2t - t + 1$$
$$V_1 = t+1$$

$$V_2 = a_1 w_1 + a_2 w_2$$
$$= 3w_1 + 2w_2$$
$$= 3t + 2(t-1)$$
$$= 3t + 2t - 2$$
$$= 5t - 2$$

Hence $S = \{V_1, V_2\} = \{t+1, 5t-2\}$

Q5a)

$$A^t = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & -7 \\ 5 & 2 & 8 \end{bmatrix}$$

$$\begin{matrix} R_2 + 2R_1 \\ R_3 - 5R_1 \end{matrix} \quad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 7 & -7 \\ 0 & -8 & 8 \end{bmatrix}$$

$$R_2/7 \quad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -8 & 8 \end{bmatrix}$$

$$R_3 + 8R_2 \quad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - 2R_2 \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Taking leading one row in column

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

b) $V = \{e_1(t^2+2t+1) + e_2(t^2-t+2)$
$+ e_3(t^3+2) + e_4(-t^3+t^2-5t+2)\}$

$v = [a \; b \; c \; d]$

$at^3+bt^2+ct+d = (e_3-e_4)t^3 +$
$(e_1+e_2+e_4)t^2 (2e_1-e_2-5e_4)t$
$+ (e_1+2e_2+2e_3+2e_4)$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & a \\ 1 & 1 & 0 & 1 & b \\ 2 & -1 & 0 & -5 & c \\ 1 & 2 & 2 & 2 & d \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & b \\ 0 & 0 & 1 & -1 & a \\ 2 & -1 & 0 & -5 & c \\ 1 & 2 & 2 & 2 & d \end{array}\right]$$

$R_3 - 2R_1$
$R_4 - R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & b \\ 0 & 0 & 1 & -1 & a \\ 0 & -3 & 0 & -1 & -2b+c \\ 0 & 1 & 2 & 1 & -b+d \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & b \\ 0 & -3 & 0 & -1 & -2b+c \\ 0 & 0 & 1 & -1 & a \\ 0 & 1 & 2 & 1 & -b+d \end{array}\right]$$

$R_4 + \frac{1}{3} R_2$
$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & b \\ 0 & -3 & 0 & -1 & -2b+c \\ 0 & 0 & 1 & -1 & a \\ 0 & 0 & 2 & -4/3 & \frac{-5b+c+3a}{3} \end{array}\right]$$

$R_4 - 2R_3$
$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & b \\ 0 & -3 & 0 & -1 & -2b+c \\ 0 & 0 & 1 & -1 & a \\ 0 & 0 & 0 & \frac{2}{3} & \frac{-6a-5b+c+3d}{3} \end{array}\right]$$

$$\frac{2}{3} x_4 = \frac{-6a-5b+c+3d}{3}$$

$$x_4 = \frac{-6a-5b+c+3d}{2}$$

$$x_3 = a + x_4 = a + \left(\frac{-6a-5b+c+3d}{2}\right)$$

$$= \frac{-4a-5b+c+3d}{2}$$

$$-3x_2 = -2b + c + 7x_4$$

$$= -2b + c + 7\left(\frac{-6a - 5b + c + 3d}{2}\right)$$

$$x_2 = \frac{14a + 13b - 3c - 7d}{2}$$

$$x_1 = b - x_2 - x_4 = b - \frac{14a + 13b - 3c - 7d}{2} - \frac{-6a - 5b + c + 3d}{2} = -4a - 3b + c + 2d$$

$$x = \begin{pmatrix} -4a - 3b + c + 2d \\ \frac{14a + 13b - 3c - 7d}{2} \\ \frac{-4a - 5b + c + 3d}{2} \\ \frac{-6a - 5b + c + 3d}{2} \end{pmatrix}$$

S from P3

c) $u \oplus w = w \oplus u$

$2u - w \neq 2w - u$

Not Hold

$\rightarrow (u \oplus w) = u \oplus (v \oplus w)$

$(2u + v) \oplus w = u \oplus (2v - w)$

$2u - 2v - w \neq 2u - 2v + w$

Not Hold

$\rightarrow (u \oplus 0) = 0 \oplus u = u$

$2u - 0 = 2(0) - u$

$2u \neq -u$

Not Hold

Not a vector SPACE