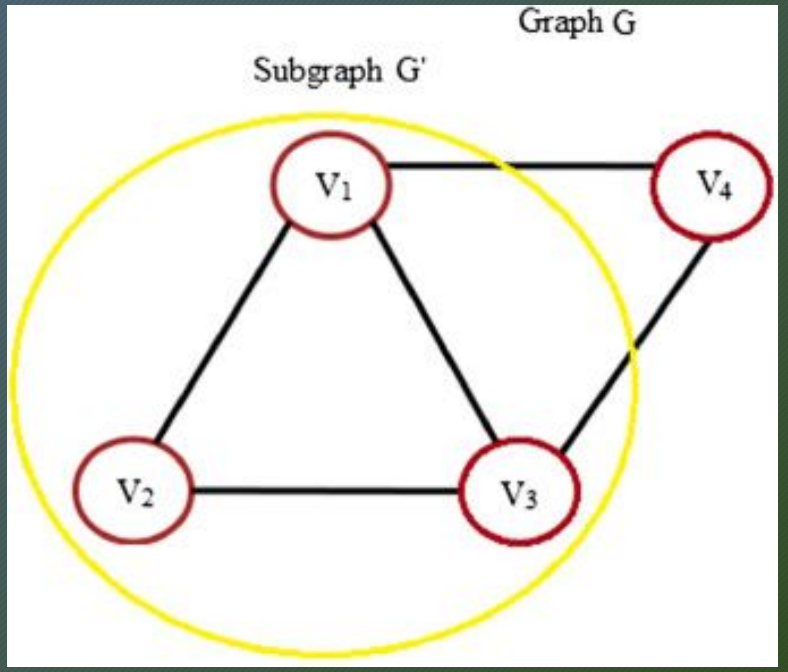
# Subgraphs

- A subgraph G’ of graph G i.e., ($G' \subset G$) is a graph, each of whose vertices ($V' \subset V$) and edges ($E' \subset E$)

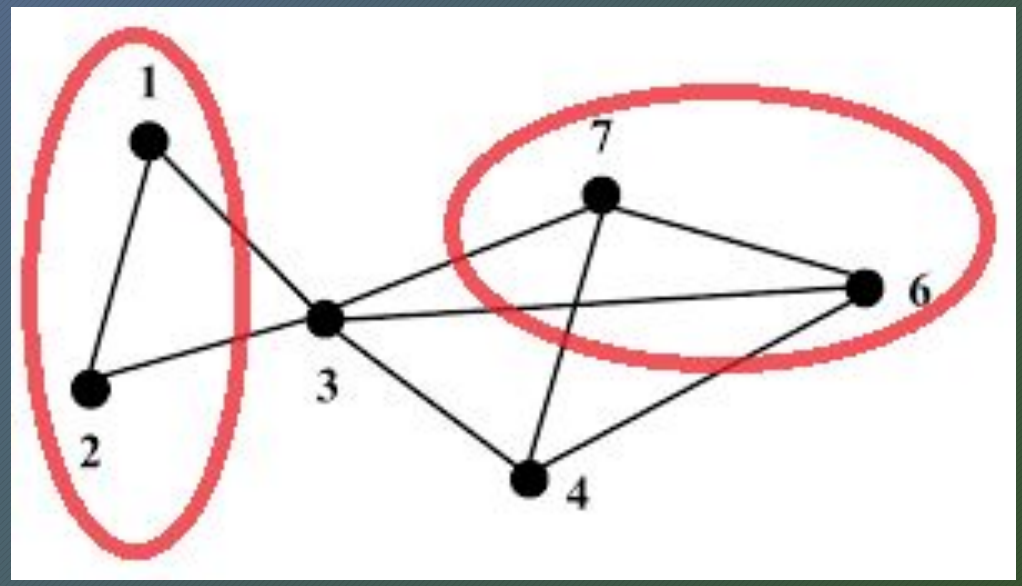
- **Transitivity rule:**

if $G'' \subset G' \subset G$, then $G'' \subset G$



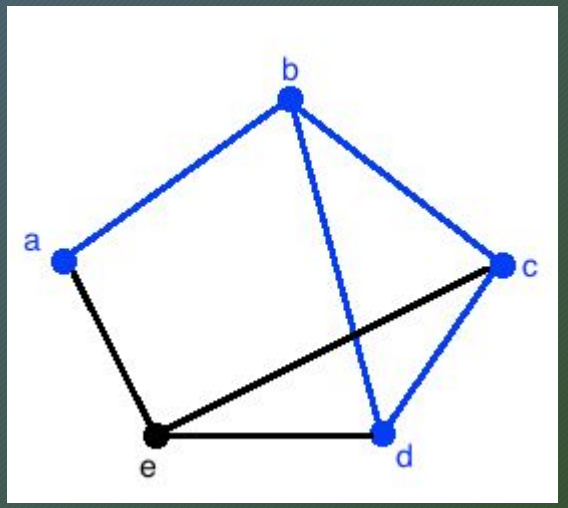
# Edge-disjoint Subgraphs

- Subgraphs G’ and G’’ of graph G are edge disjoint if G’ and G’’ do not have any edge in common

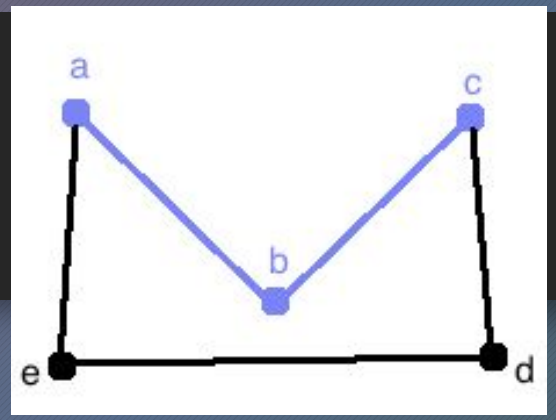


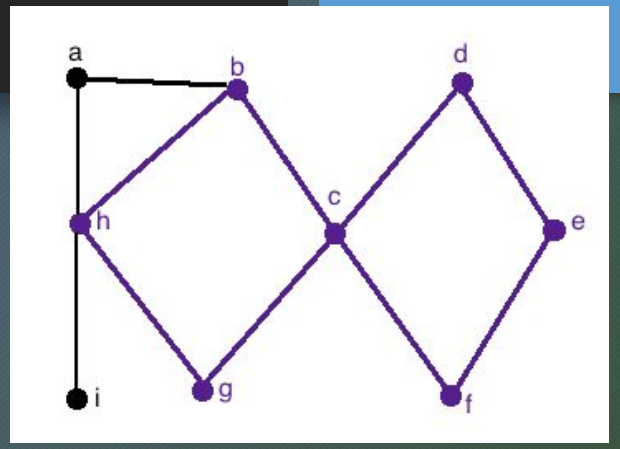
# Walk (repeating edges or vertices is fine)

- A walk in a graph G = (V, E) is a finite sequence of vertices and edges that begins from any vertex $V_0$ and ends at any vertex $V_k$
  - E.g., abcdbcd is a walk
- **Open Walk:** A walk that has different starting and ending vertices i.e., $V_0 \neq V_k$
  - **E.g., abc is an open walk**
- **Closed Walk:** A walk that has the same starting vertex as its ending vertex i.e., $V_0 = V_k$
  - E.g., abcdba is a closed walk

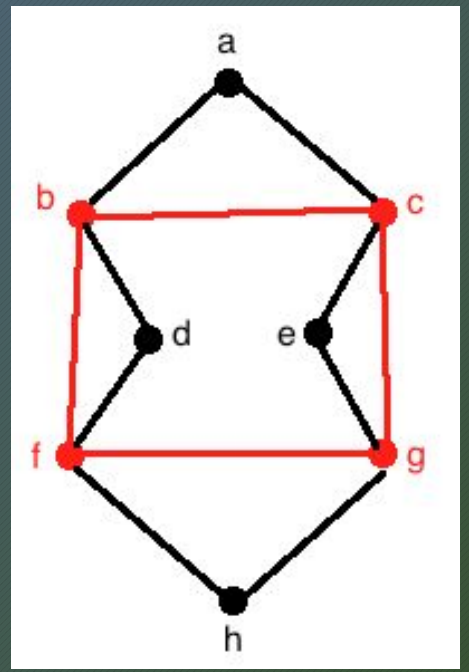


# Trails, Circuits, Cycles



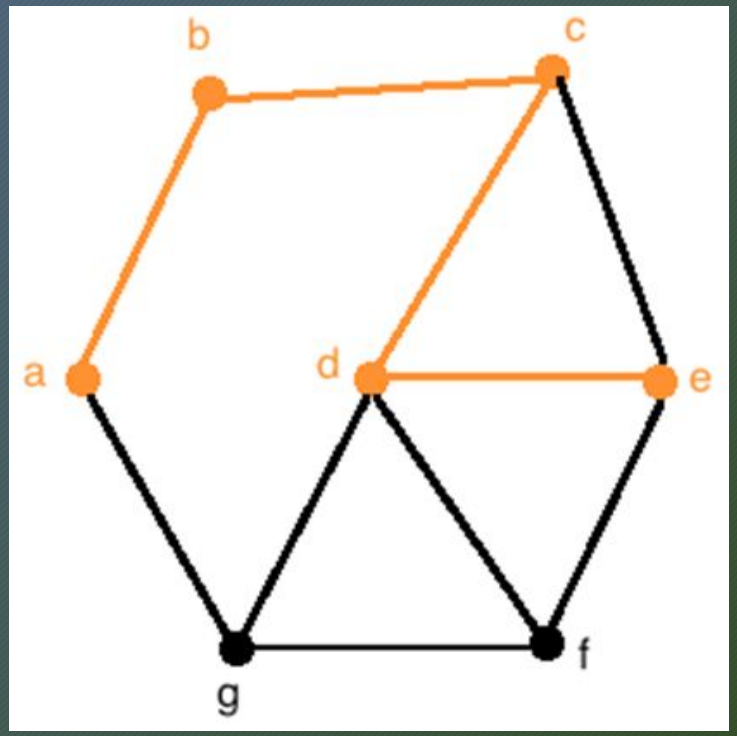




- Trails are walks with no edge repeated
  - E.g., abc is a trail
- Circuit is a closed trail having same starting and ending vertex, while in between vertices may repeat as well
  - E.g., hbcdefcgh
- Cycles are circuits with only one repeated vertex i.e., the starting vertex as its ending vertex
  - E.g., bcgfb



# Paths

- A Path is a trial with no repeated vertex (so it has to be an open trail)
  - E.g., abcde is a path



# Connected / Disconnected Graph

- A graph G is connected if there is at least one path between each pair of vertices in G. Otherwise its disconnected
- Each connected part in a disconnected graph is called a component or a community

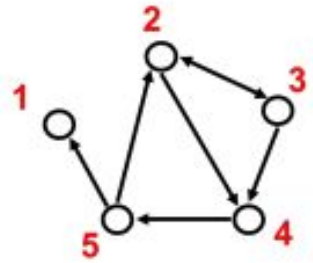
- A simple graph with N vertices and K components can have at most $\frac{(N-K)(N-K+1)}{2}$ edges

# Adjacency, Incidences and Degree

- **Adjacency:**
  - Two vertices $V_1$ and $V_2$ are adjacent if there is an edge joining them
  - A vertex to vertex property
- **Incidence:**
  - **If** $V_1$ and $V_2$ are joined by an edge e, then both $V_1$ and $V_2$ are incident on edge e
  - **An edge to edge or edge to vertex property**
- **Degree:**
  - **Degree of a vertex V in graph G is the number of edges incident with V and is written as deg(V) or d(V)**
  - **A vertex property**

# Adjacency Matrix, Edge List & Adjacency list

- Adjacency matrix




$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

**Edge List**

2, 3
2, 4
3, 2
3, 4
4, 5
5, 2
5, 1

**Adjacency List**

1:
2: 3 4
3: 2 4
4: 5
5: 1 2