

Introduction to Graph Theory

Why Graphs?

Alexander Golovnev

Outline

What is a Graph?

Graph Examples

Graph Applications

Graphs

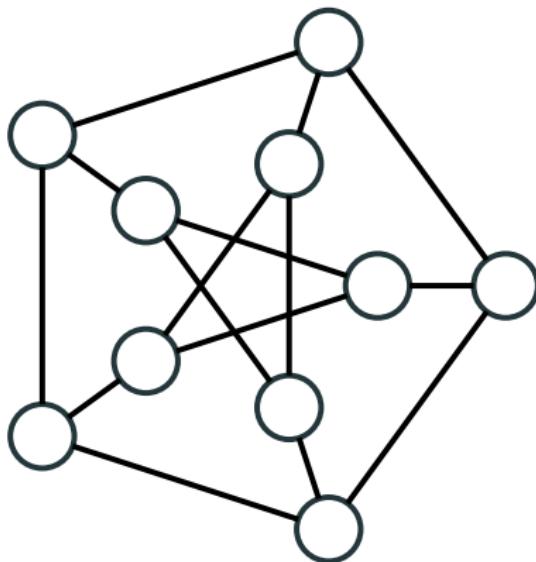
- A set of Objects

Graphs

- A set of Objects
- Relations between pairs of objects

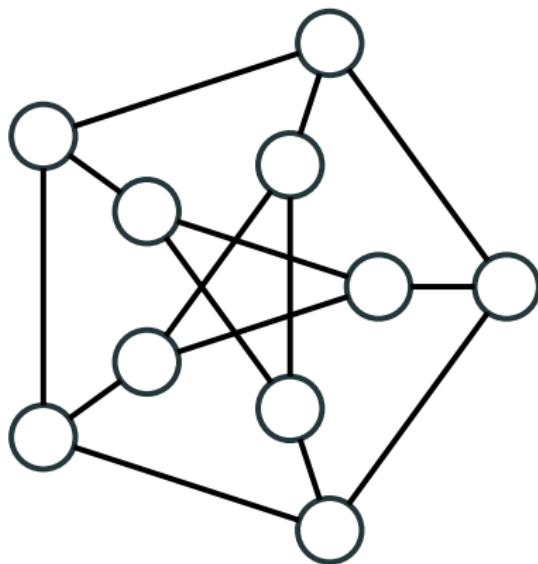
Graphs

- A set of **Objects**
- **Relations** between pairs of objects



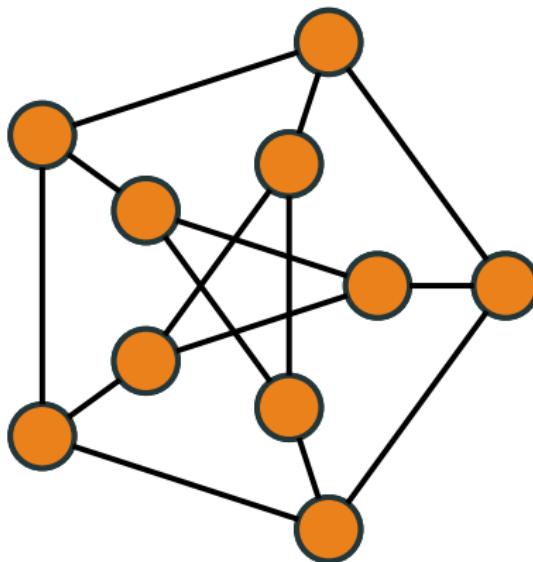
Definition

- A **Graph** $G = (V, E)$



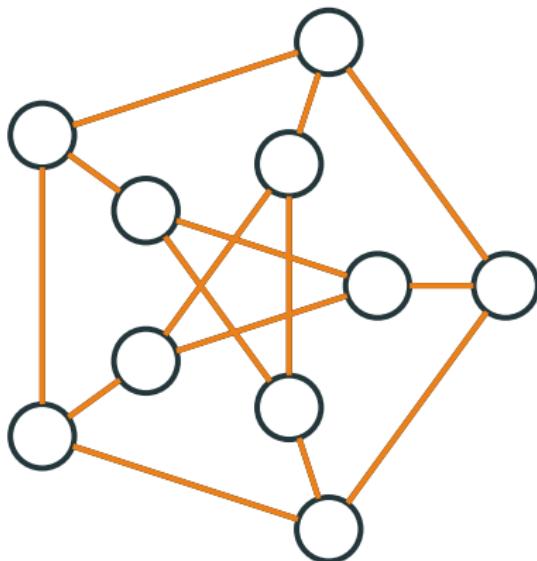
Definition

- A **Graph** $G = (V, E)$
- A set V of **Vertices/Nodes**



Definition

- A **Graph** $G = (V, E)$
- A set V of **Vertices/Nodes**
- A set E of **Edges**



Vocabulary

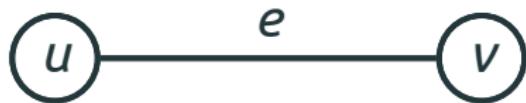


Vocabulary



- We can name individual vertices and edges

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Vocabulary



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- e Connects u and v

Vocabulary



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Vocabulary



- We can name individual vertices and edges
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- u and v are End Points of e
- u and e are Incident
- u and v are Adjacent
- u and v are Neighbors

Drawing a Graph

Objects: {A,B,C,D}

Relations: {{A,C},{D,A},{B,D},{C,B}}

Drawing a Graph

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(B)

(A)

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(B)

(C)

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Drawing a Graph

Objects: {A,B,C,D}

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B

C

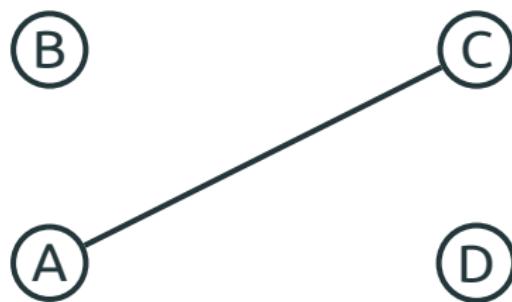
A

D

Drawing a Graph

Objects: {A,B,C,D}

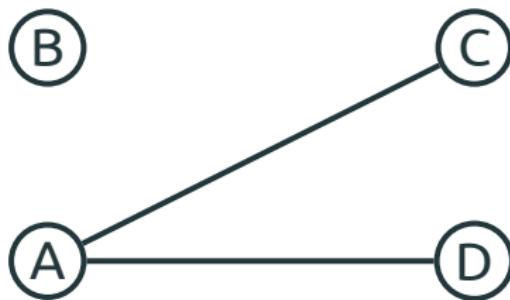
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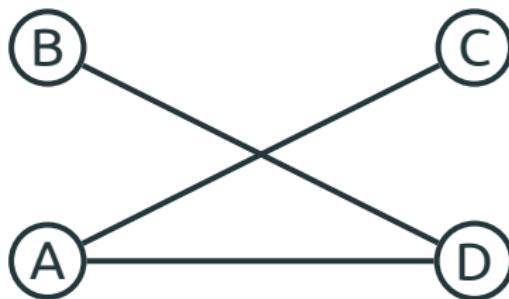
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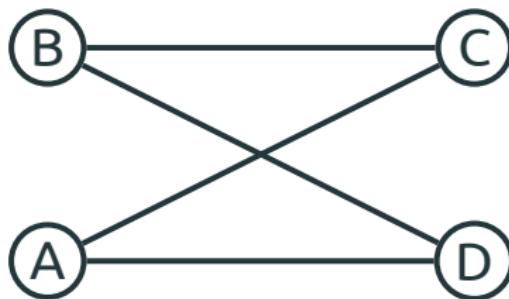
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Drawing a Graph

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Drawing a Graph

Objects: {A,B,C,D}

Relations: {{A,C},{D,A},{B,D},{C,B}}



Drawing a Graph

Objects: {A,B,C,D}

Relations: {{C,A},{D,A},{B,D},{C,B}}



Directed Graph



- It is often convenient to consider **Directed Edges (Arcs)**

Directed Graph



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Directed Graph



- It is often convenient to consider **Directed Edges (Arcs)**
- They describe **asymmetric** relations
- There is a flight from A to B, but not the other way around
- Such a graph is called **Directed**

Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(A,C),(D,A),(B,D),(C,B)}



Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(A,C),(D,A),(B,D),(C,B)}



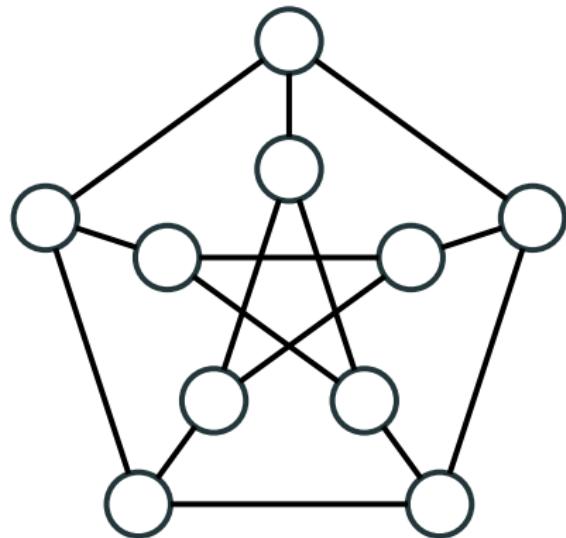
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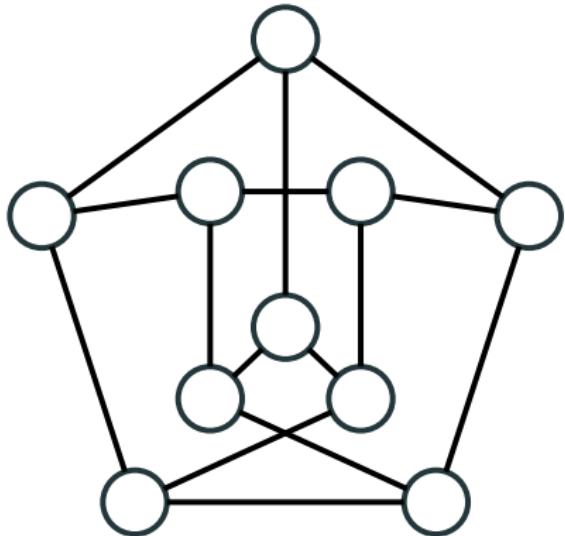
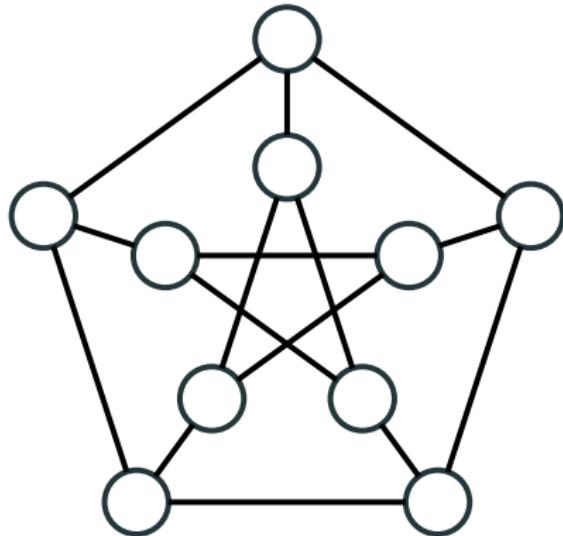


Many Ways to Draw a Graph



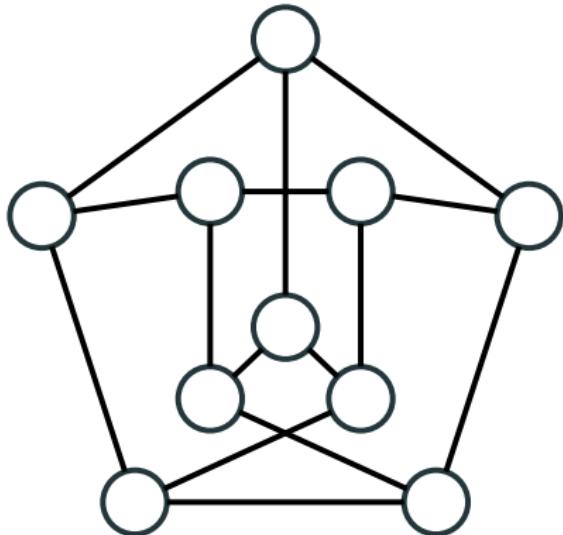
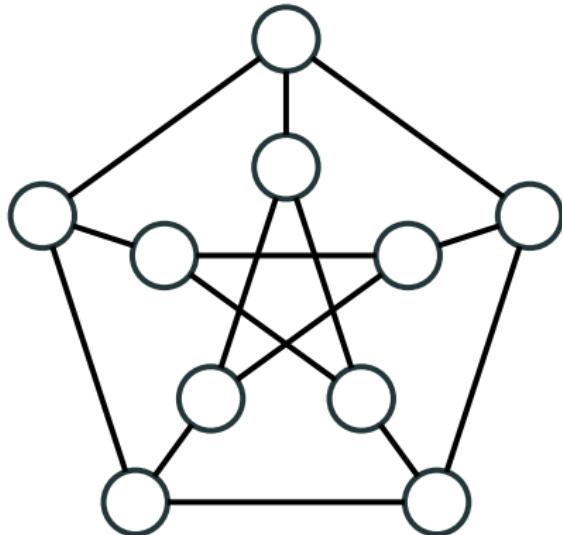
Many Ways to Draw a Graph

Are these graphs the same?



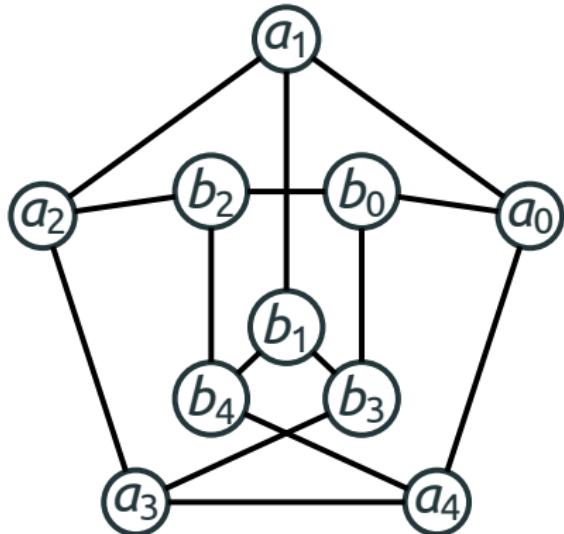
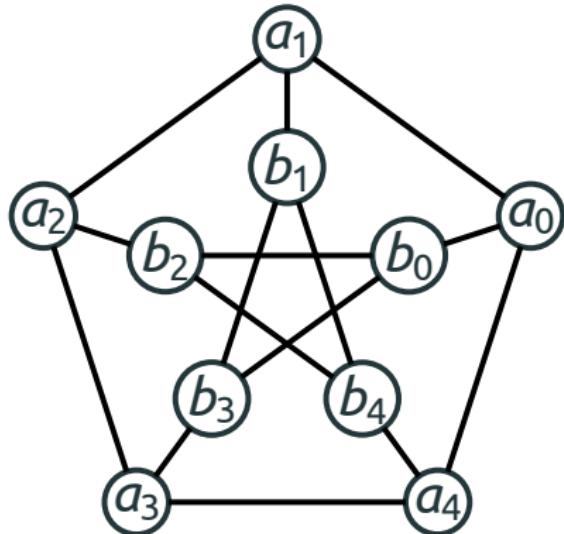
Many Ways to Draw a Graph

10 vertices and 15 edges?



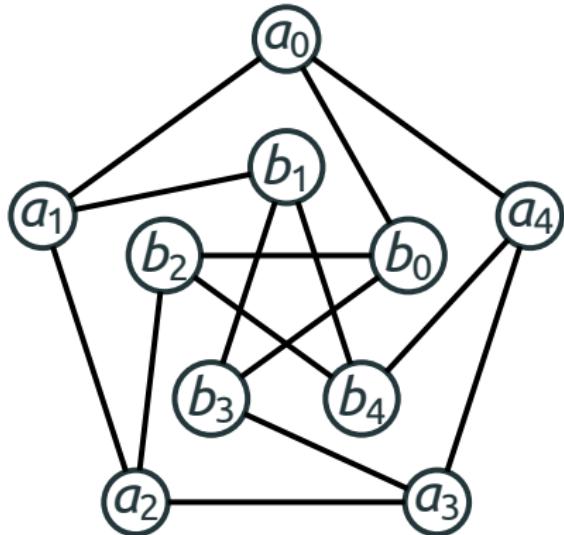
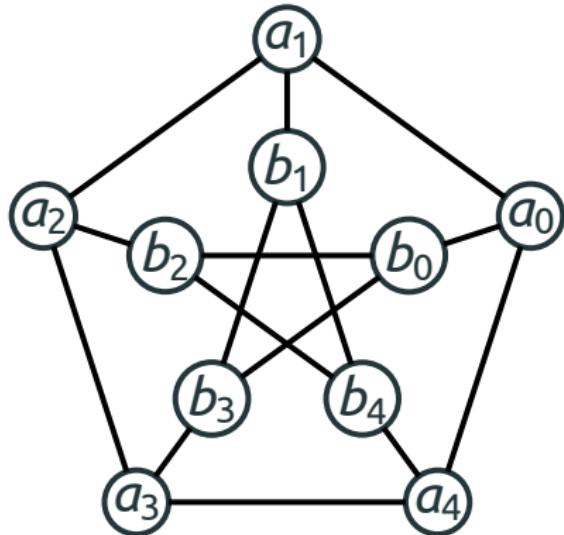
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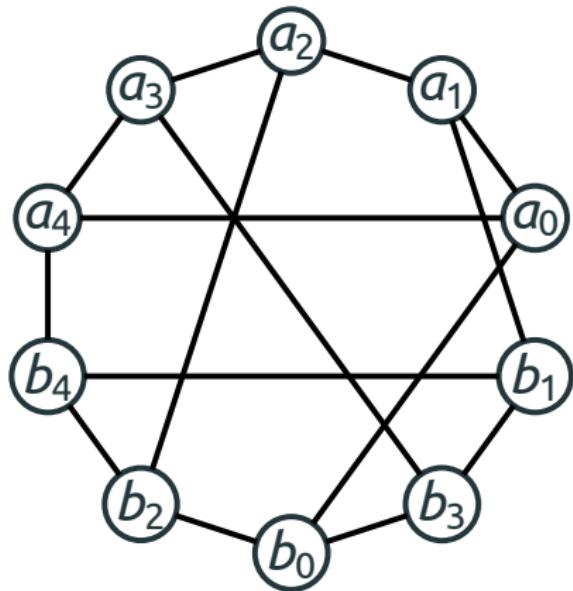
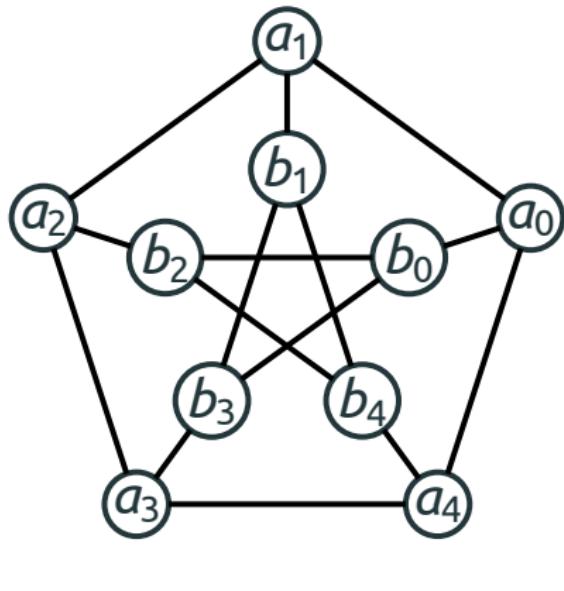
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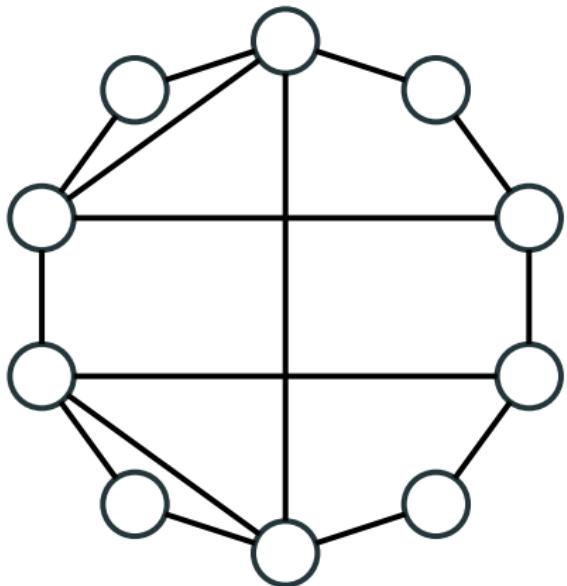
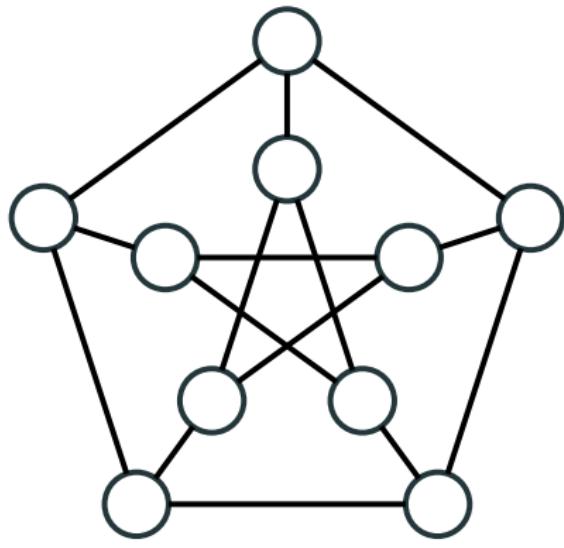


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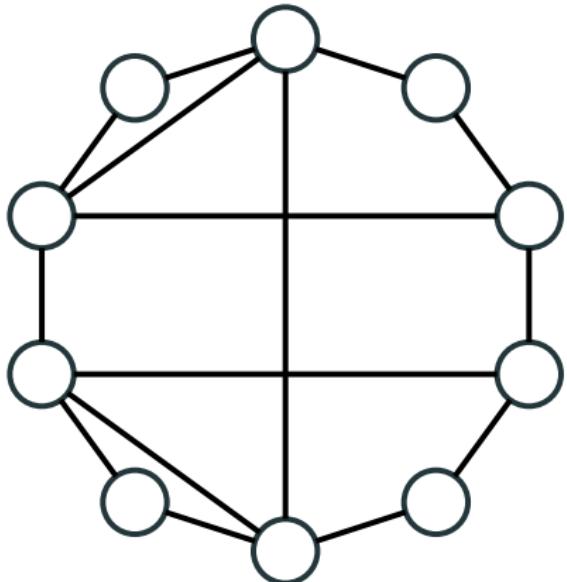
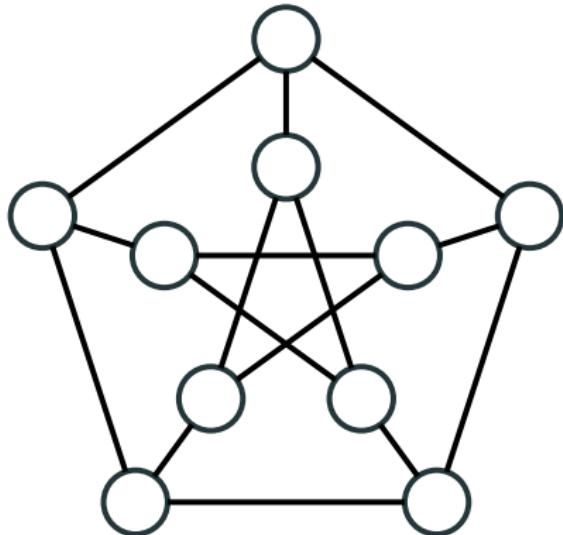


Are These Graphs the Same?



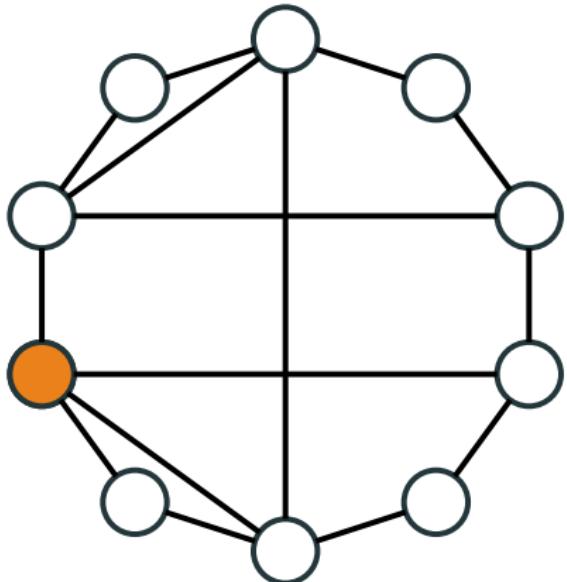
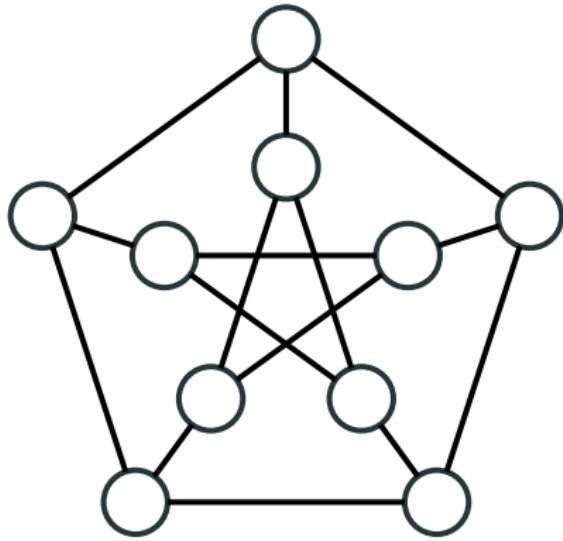
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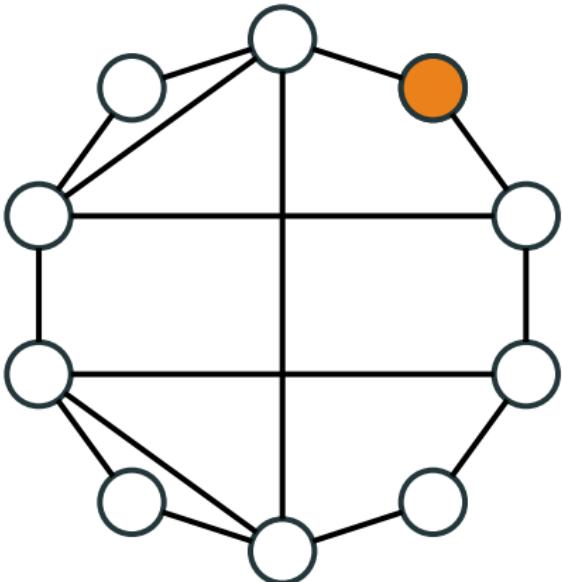
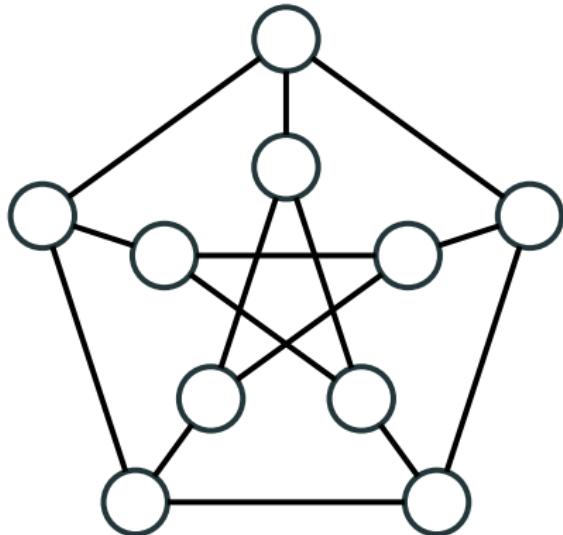
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Are These Graphs the Same?

10 vertices and 15 edges?



Graph Drawing is Beautiful!



*Graph drawing is the best possible field I can think of:
It merges aesthetics, mathematical beauty and wonderful algorithms.
It therefore provides a harmonic balance between the left and right brain parts.*

Donald E. Knuth

Outline

What is a Graph?

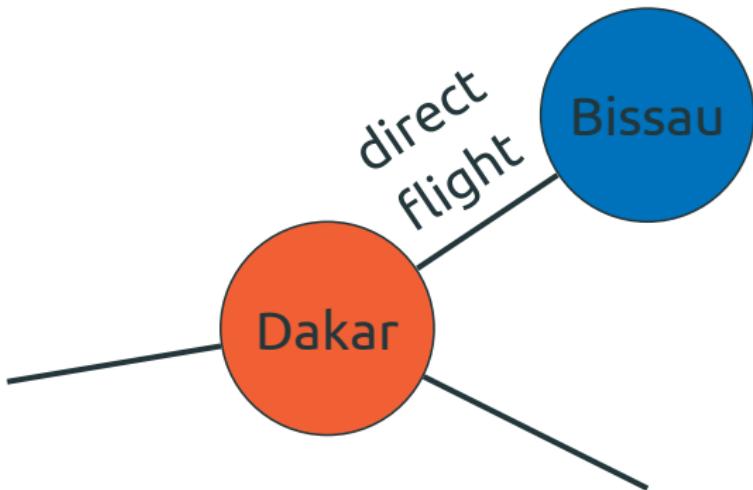
Graph Examples

Graph Applications

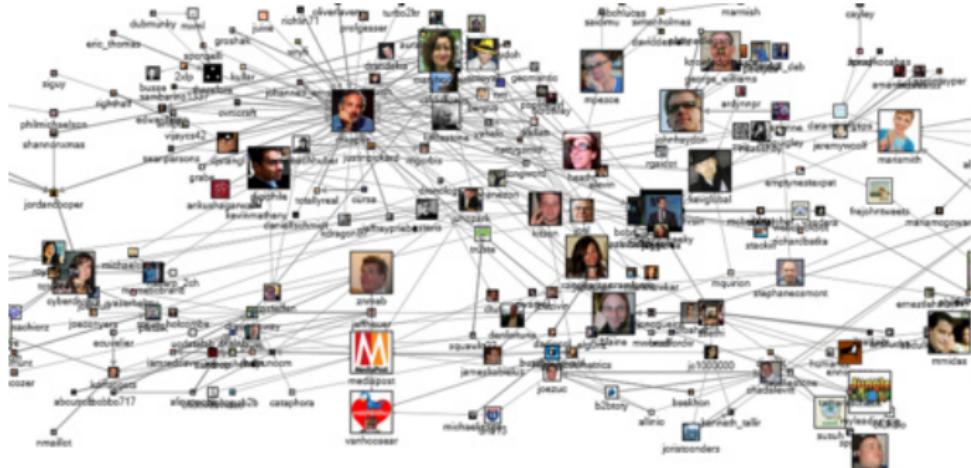
Airlines Graph



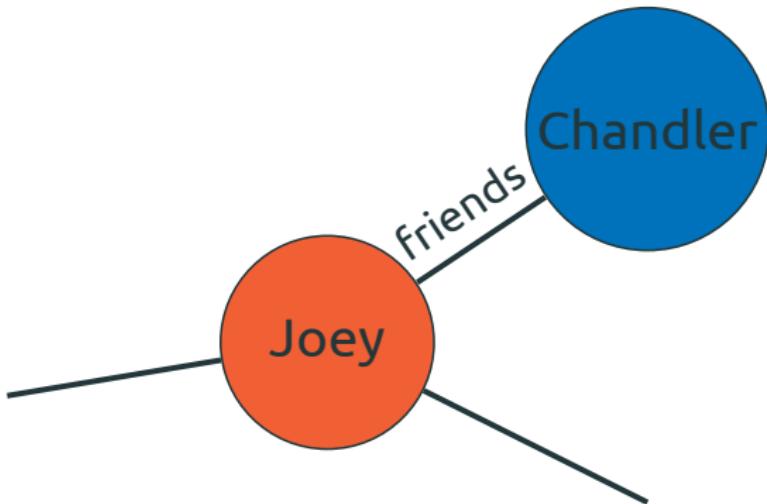
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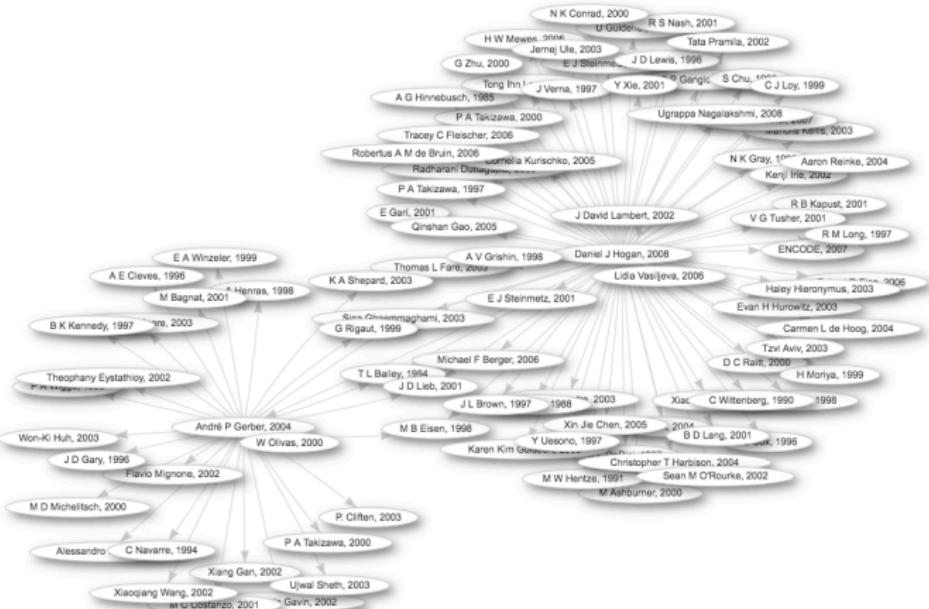
Facebook Graph



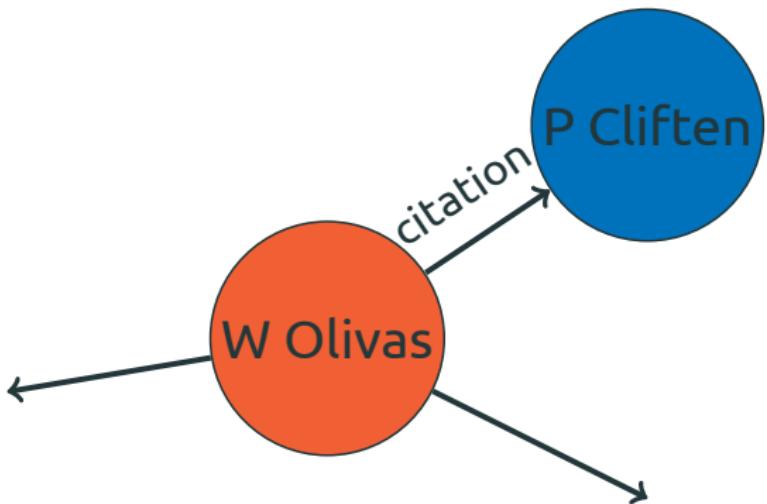
Facebook Graph



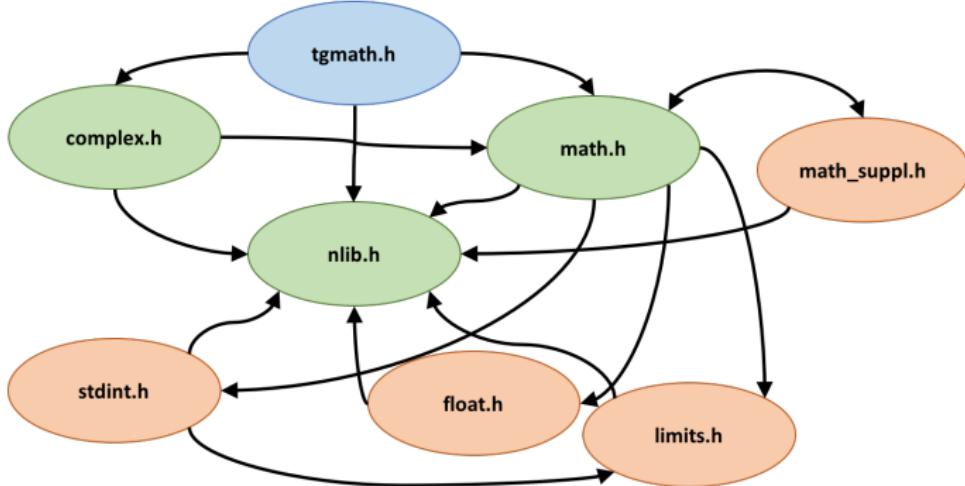
Citation Graph For a Paper



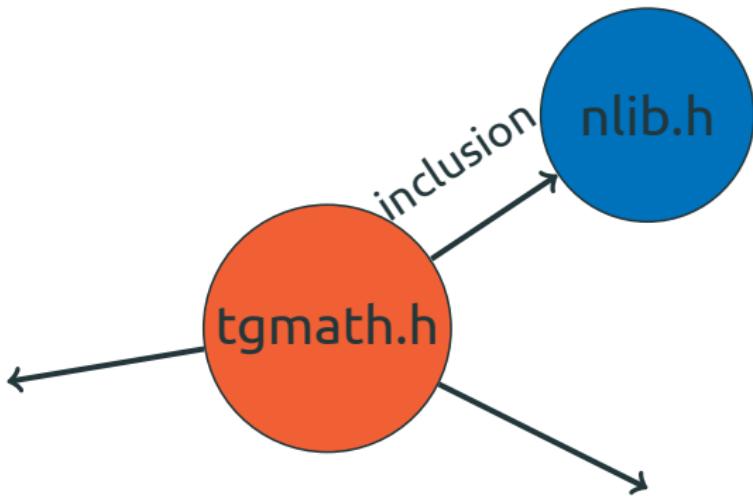
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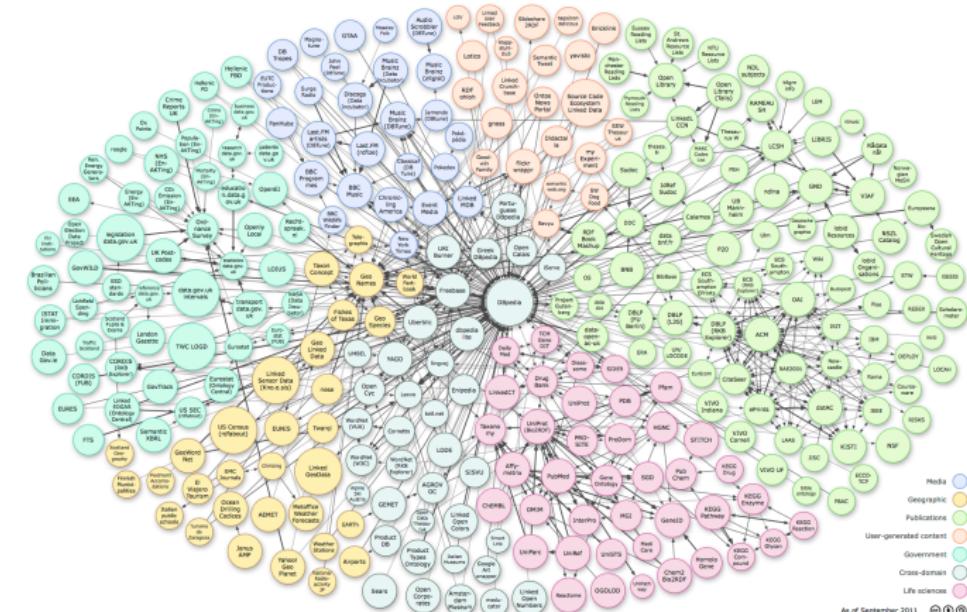
Dependency Graph



Dependency Graph



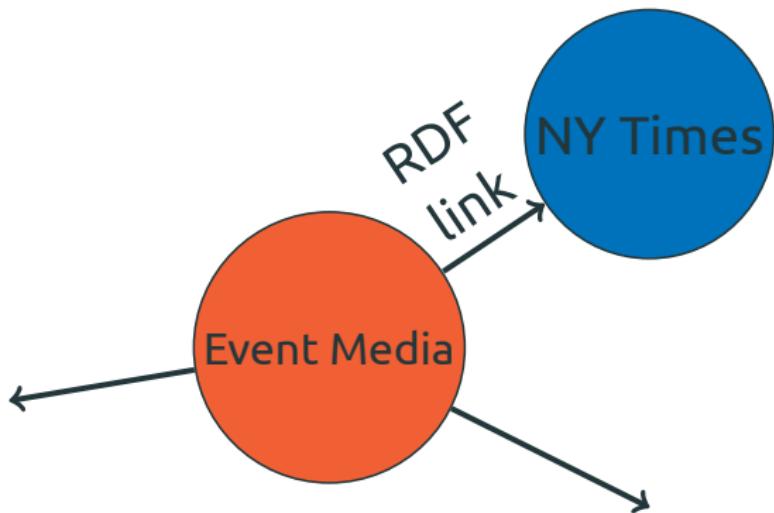
Linked Open Data Diagram



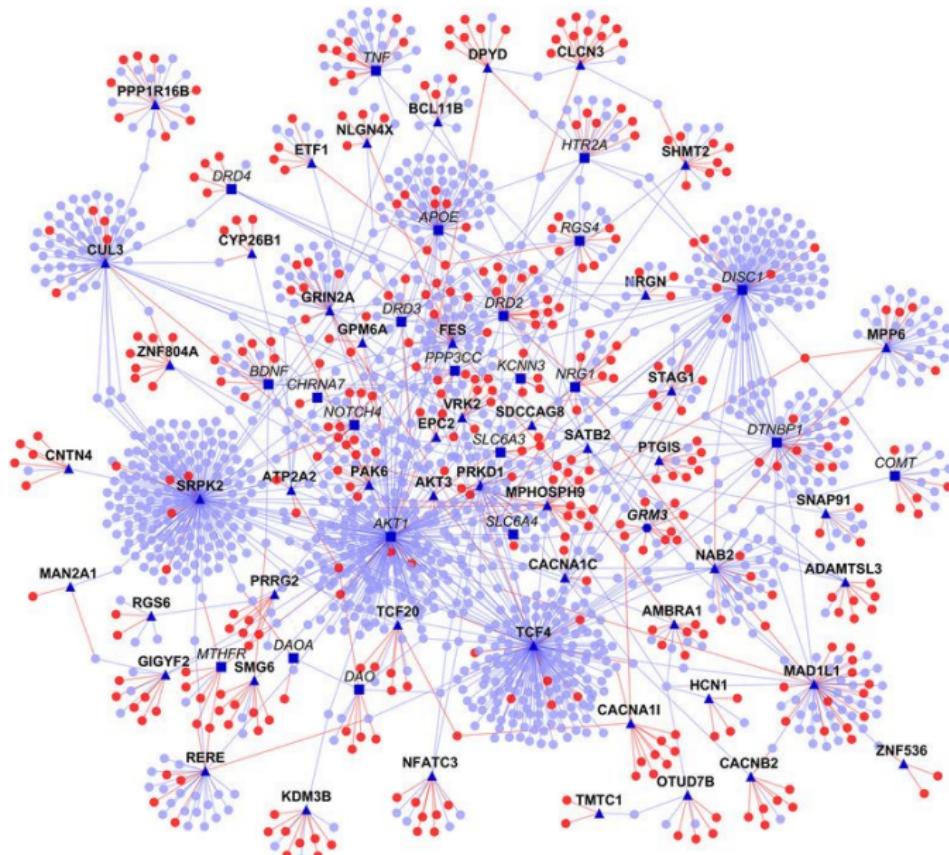
As of September 2011



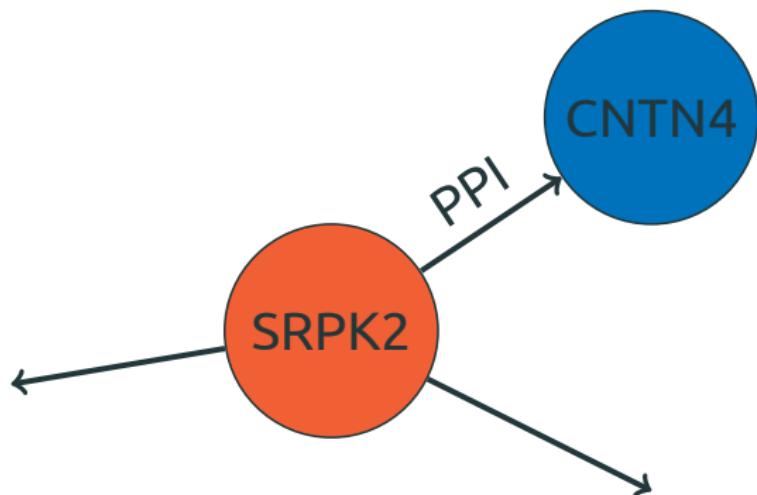
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Schizophrenia Protein–Protein Interaction



Schizophrenia Protein–Protein Interaction (PPI)



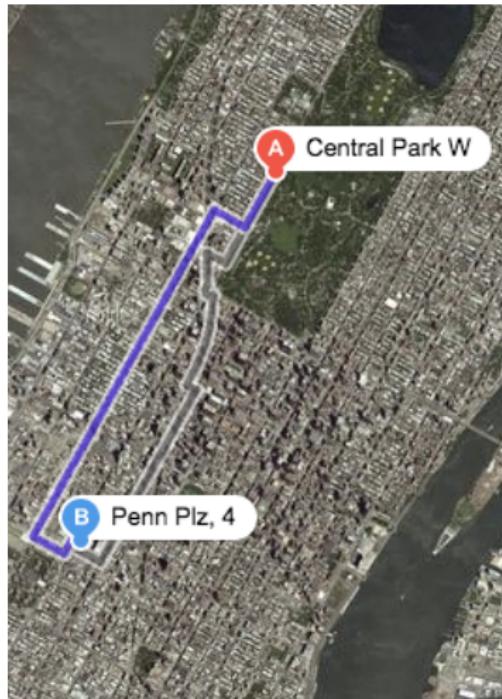
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Navigation



Navigation



PageRank

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PageRank

graph

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About 333,000,000 results (1.94 seconds)

graph¹
/graf/ ⓘ

noun

1. a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.
synonyms: chart, diagram; More

verb

1. plot or trace on a graph.
synonyms: plot, trace, draw up, delineate
"we graphed the new prices"

Translations, word origin, and more definitions

Feedback

Graph - Wikipedia

<https://en.wikipedia.org/wiki/Graph> ▾

Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).

Graph of a function - Wikipedia

https://en.wikipedia.org/wiki/Graph_of_a_function ▾

PageRank

Graph TV

graphtv.kevininformatics.com/ ▾

Graph Ratings of Your Favorite TV Shows. Visualize IMDb ratings and trends of TV shows by episode.

Have you seen Mad Men, Breaking Bad, or Battlestar ...



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PageRank

A screenshot of a search results page from a search engine. The search term 'graph' is entered in the search bar. Below the search bar, there are navigation links for 'All', 'Images', 'News', 'Books', 'Videos', and 'More'. To the right of these are 'Settings' and 'Tools' buttons. A microphone icon and a magnifying glass icon are also present. Below the search bar, it says 'About 333,000,000 results (1.94 seconds)'. The main content area shows the first search result, which is a definition of 'graph'. It includes the word 'graph' in large bold letters, its pronunciation '/graf/', and its part of speech 'noun'. The definition lists '1. a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.' followed by 'synonyms: chart, diagram; More'. Below this, under 'verb', it lists '1. plot or trace on a graph.' followed by 'synonyms: plot, trace, draw up, delineate' and the quote 'we graphed the new prices'. At the bottom of the result, there is a link to 'Translations, word origin, and more definitions'.

Feedback

[Graph - Wikipedia](#)

<https://en.wikipedia.org/wiki/Graph> ▾

Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).

[Graph of a function - Wikipedia](#)

https://en.wikipedia.org/wiki/Graph_of_a_function ▾

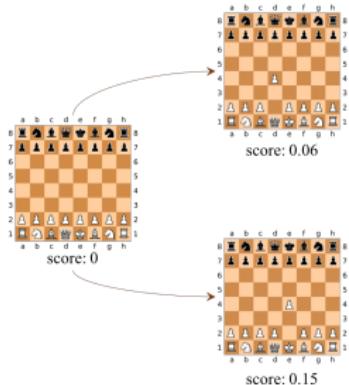
Game Strategies



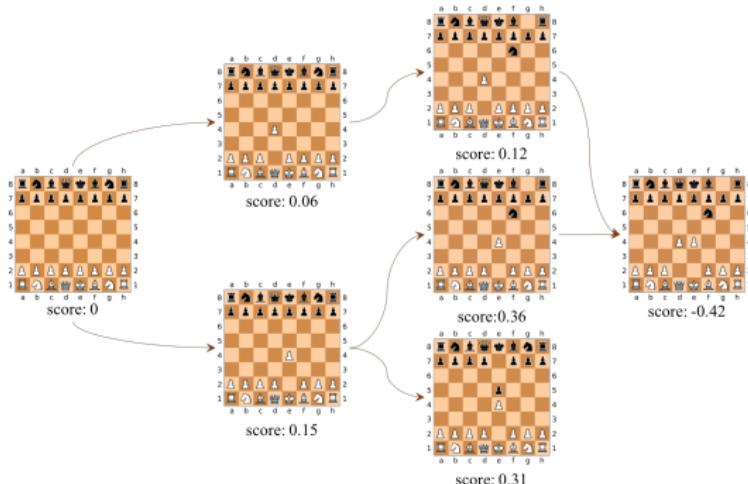
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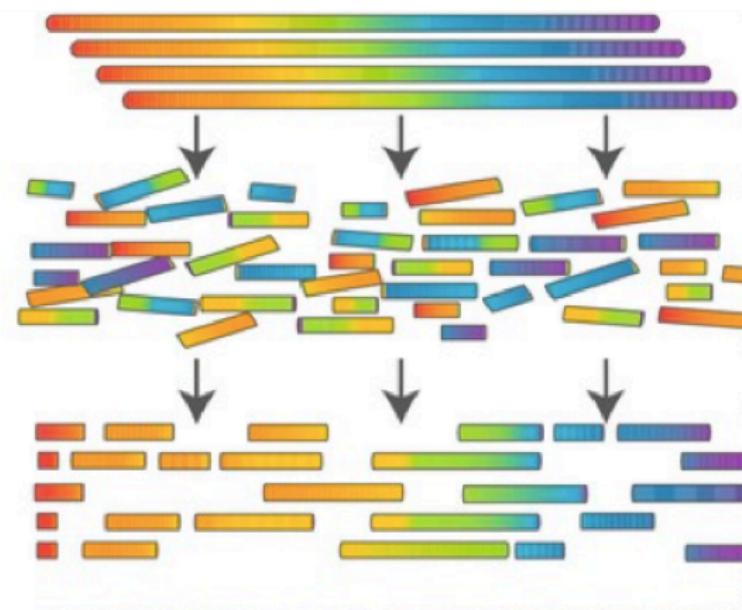
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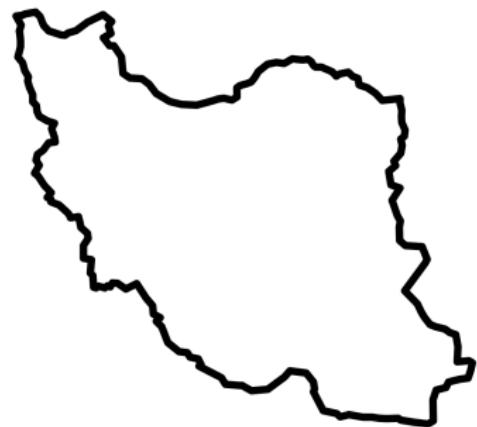
Game Strategies



Genome Assembly



GSM



GSM



GSM

4 Frequency
Ranges

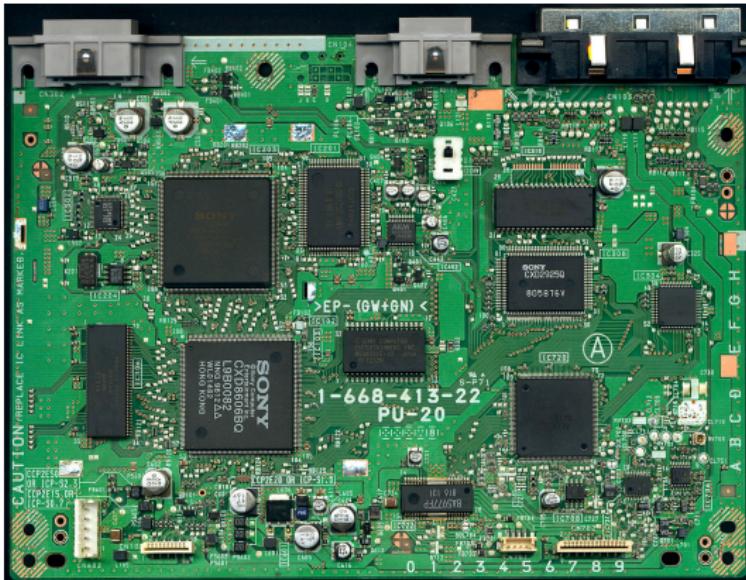


GSM

4 Frequency
Ranges



Computer Chips



Computer Chips

