

We are all connected!

"We are caught in an inescapable network of mutuality, tied in a single garment of destiny. Whatever affects one directly, affects all indirectly." Martin Luther King Jr. (January 15, 1929 – April 4, 1968)



Networks

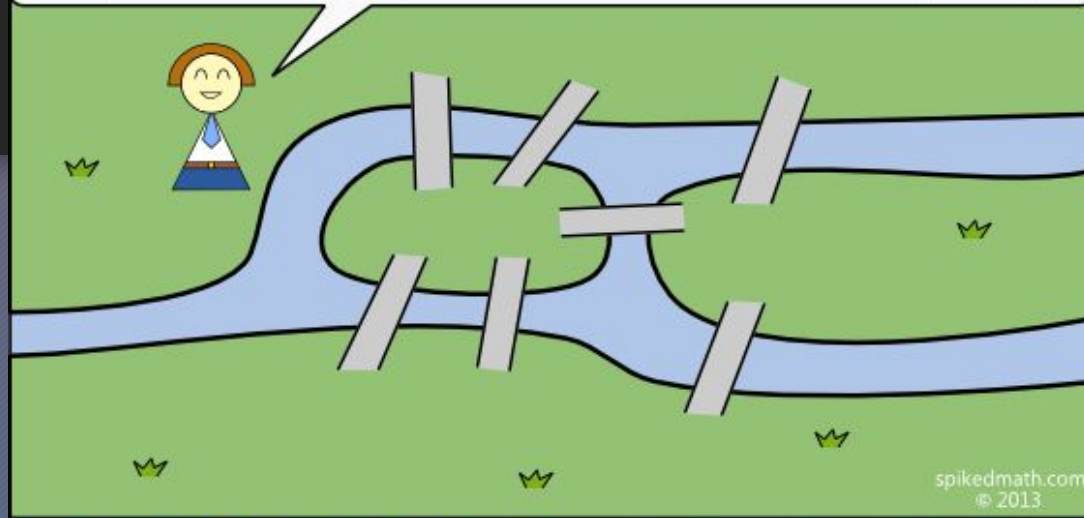
- Many real world systems can be viewed as a network i.e., collection of nodes inter-linked to one another
- Node: A unit, can be a person, organization, computer etc.
- Interaction: relationship, friendship, colleagues... (directed or not)
- World Wide Web (WWW)
 - 2.4 billion users, 50 billion web pages, 600 million web servers, 20 million DNS servers ...

Konigsberg's Bridges

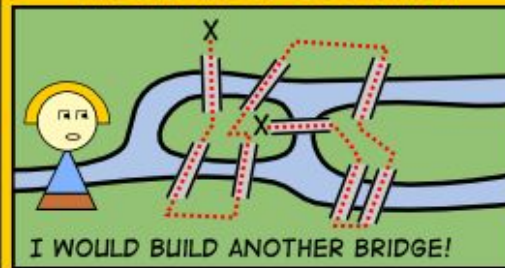
- Solved by Leonhard Euler solved it in 1736, using a graph
- Became the pioneering work in graph theory

The Seven Bridges of Königsberg

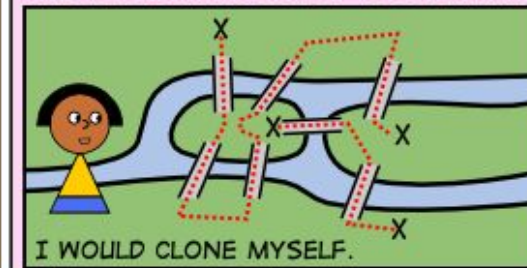
Below is the city of Königsberg with four land masses and seven bridges connecting the various land masses. Can you find a walk through the city of Königsberg that crosses each bridge exactly once? You may start at any land mass you wish but may only travel between land masses by using a bridge.



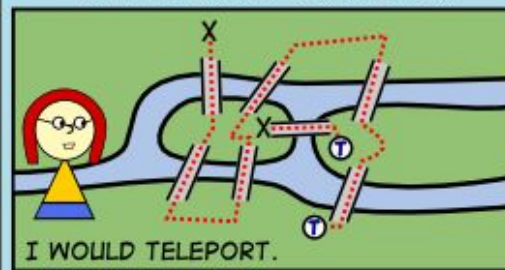
THE ENGINEER'S SOLUTION



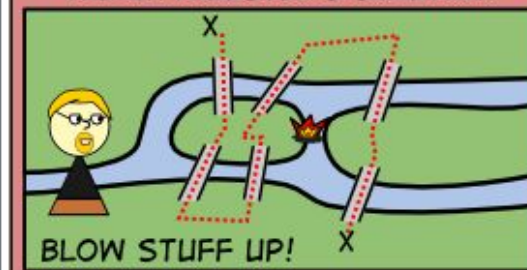
THE BIOTECHNOLOGIST'S SOLUTION



THE PHYSICIST'S SOLUTION



THE MYTHBUSTER'S SOLUTION

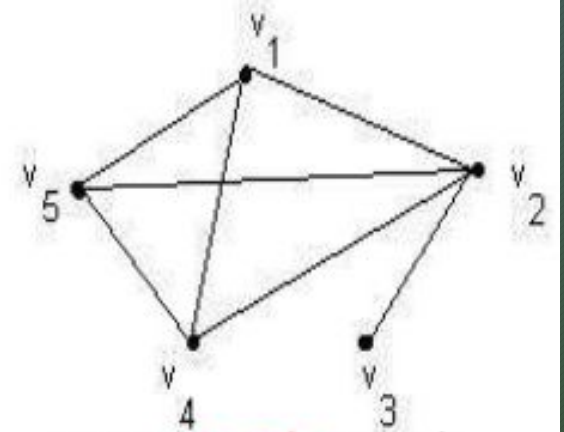


Simple (General) Graphs

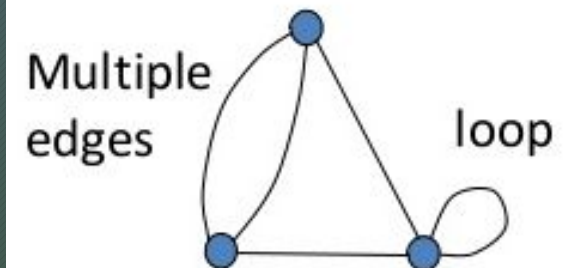
- $G = (V, E)$ is a graph consisting of V and E
- $V = \{v_1, v_2, v_3, \dots, v_N\}$ is a set of objects called vertices
- $E = \{e_1, e_2, e_3, \dots, e_k\}$ is a set of links called edges
- Such that an edge e_{ij} exists between two vertices (v_i, v_j)

But have no

- Self edges i.e., the a vertex sharing an edge with itself e_{aa}
- Some graphs may allow multiple edges between the same vertices!



It is a **simple** graph.

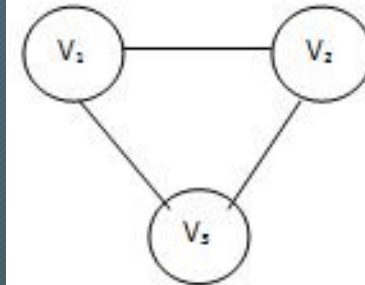


It is **not simple** graph.

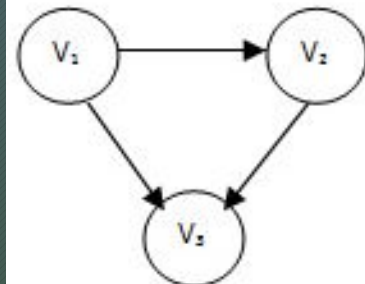
Directions in Graphs

- Graphs may be
 - Unidirectional (Directed graphs)
- or
- Bidirectional (Undirected graphs)

Undirected Graph

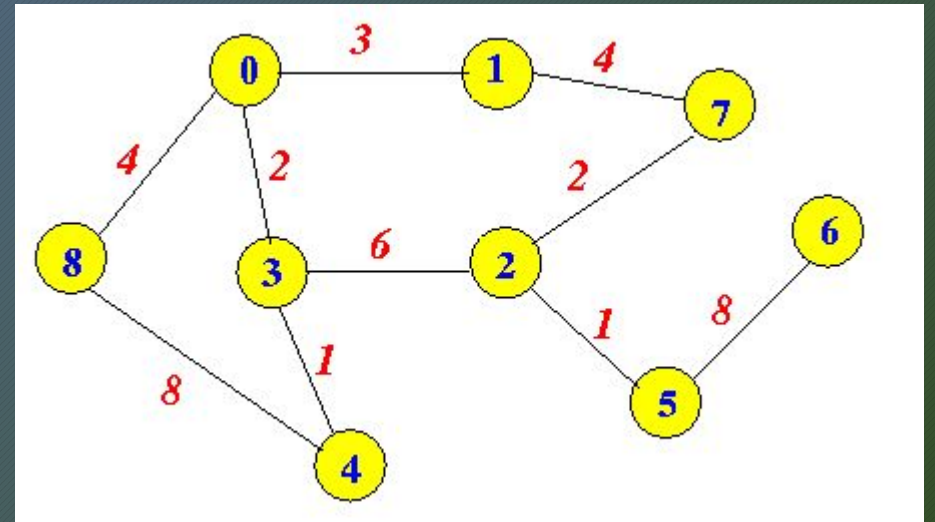


Directed Graph



Weighted Graph

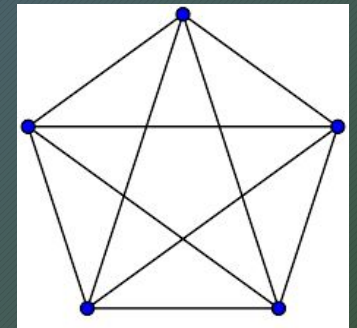
- A **weighted graph** is a graph in which each branch is given a numerical **weight**. A **weighted graph** is therefore a special type of **labeled graph** in which the labels are numbers (which are usually taken to be positive).



Complete Graph

- $G = (V, E)$ where each pair of distinct vertices are adjacent and total number of vertices > 1
- The total edges in the graph, K_N are

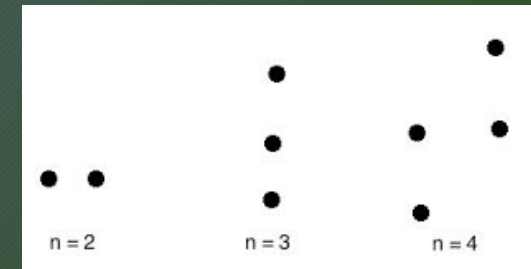
$$K_N = \frac{N(N-1)}{2}$$



Null Graph

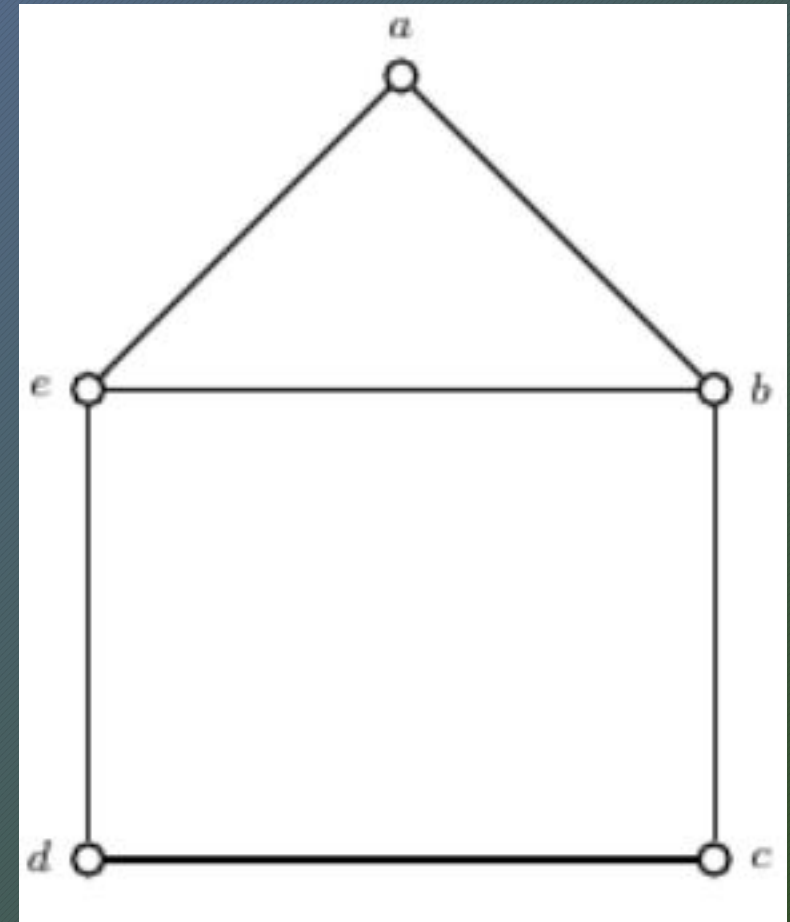
- $G = (V, E)$ where E is a null set

The vertices are there but not linked to one another



Example

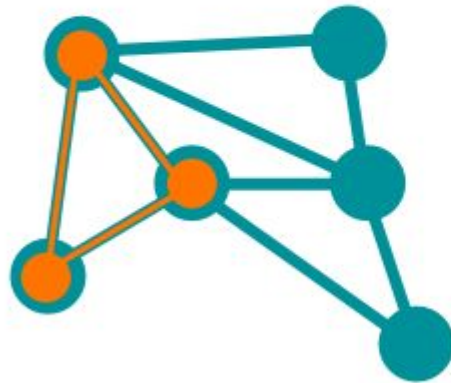
- List all vertices and edges
- List all vertices that are adjacent to each other i.e., sharing a direct edge
- Which vertex has the most number of edges



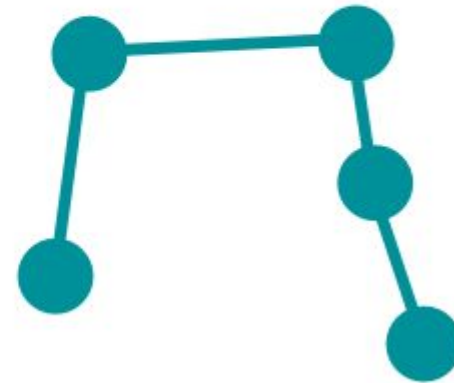
Cyclic or Acyclic Graphs

- A graph having cycles or loops is called cyclic graph
- Graphs without cycles or loops is called acyclic graph

Cyclic Graph

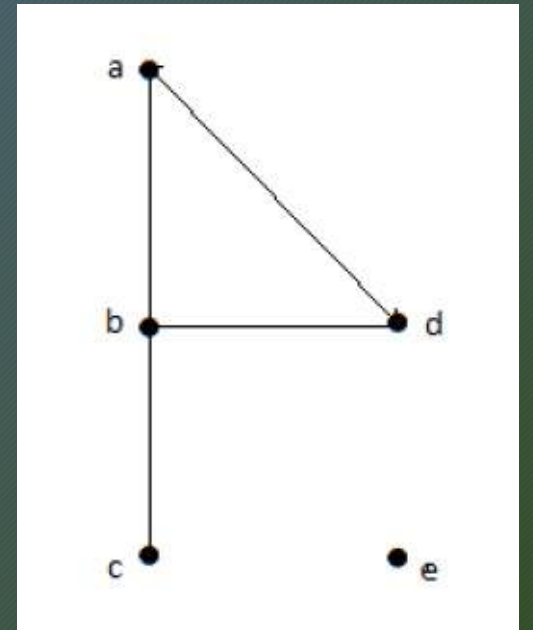


Acyclic Graph



Isolated, Pendant Vertex

- An isolated vertex v is the one having no incident edge, i.e., $d(v) = 0$.
- A vertex v having $d(v) = 1$ is called a pendant vertex.



Theorems

- The sum of degrees of all vertices in a graph G , is twice the number of edges in G ,

$$\sum_{i=1}^N \deg(v_i) = 2e$$

- The number of vertices of odd degree in a graph G , is always even