lecture 9:-

FA → TG → RE → FA

Kleene theorem III

"Every RE can be represented by an FA".

$$\underbrace{(a+b)^{*}(a+b)^{*}}_{\substack{r_1 \\ FA_1}}\ \underbrace{(a)(b+a)^{*}}_{\substack{r_2 \\ FA_2}}\ \underbrace{------}_{\substack{r_3 \\ FA_3}}\ \underbrace{------}_{\substack{r_4 \\ FA_4}}$$

- How to Combine.
  - Union, Sum, +.
  - Concatenation.
  - Closure, *

- Union, Sum, +.

  r_1, r_2, then $r_3 = r_1 + r_2$ is also RE.

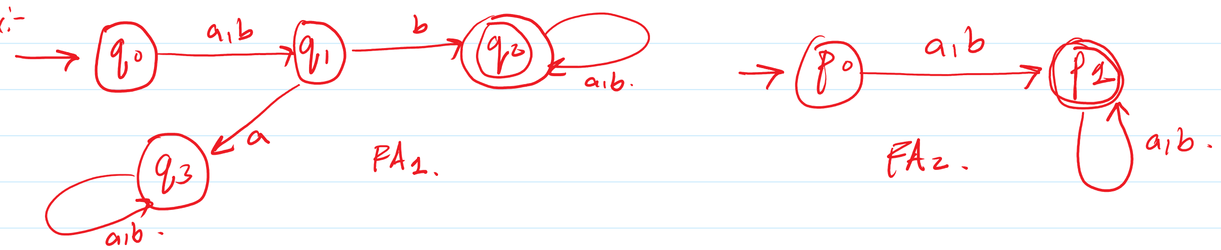
  FA_1, FA_2 $FA_3 = FA_1 + FA_2$

Algorithm :-

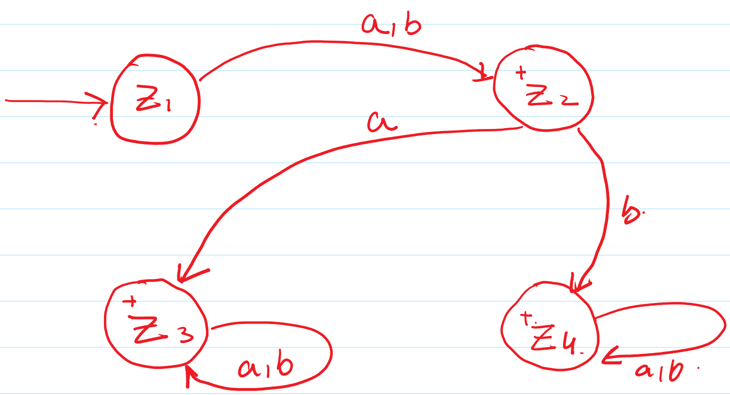
1- Start by taking both FA's initial State. & traverse on the respective inputs.

2- During the process. Any state encountered final, the resultant state will be final.

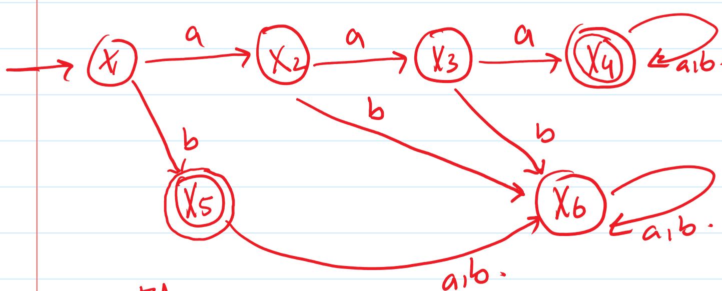
Ex:-



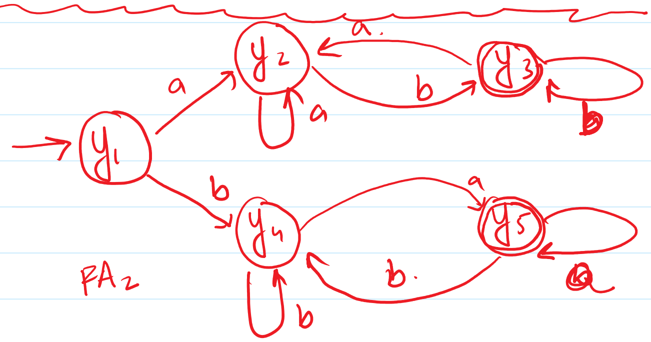
Old State	New states. transition at a	transition at b.
$z_1 \equiv (q_0, p_0)$	$z_2^+ \equiv (q_1, p_1)$	$z_2^+ \equiv (q_1, p_1)$
$z_2^+ \equiv (q_1, p_1)$.	$z_3^+ \equiv (q_3, p_1)$	$z_4^+ \equiv (q_2, p_1)$
$z_3^+ \equiv (q_3, p_1)$.	$z_3^+ \equiv (q_3, p_1)$	$z_3^+ \equiv (q_3, p_1)$
$z_4^+ \equiv (q_2, p_1)$.	$z_4^+ \equiv (q_2, p_1)$.	$z_4^+ \equiv (q_2, p_1)$



$FA_3 = FA_1 + FA_2$.



FA_1



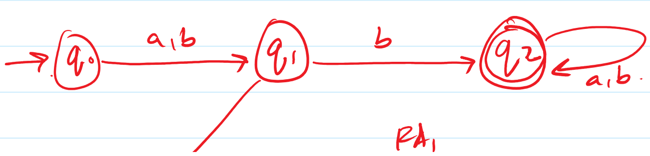
FA_2

old states	transition 'a'	transition 'b'
$Z_1 \equiv (x_1, y_1)$	$Z_2 \equiv (x_2, y_2)$	$Z_3^+ \equiv (x_5, y_4)$
$Z_2 \equiv (x_2, y_2)$	$Z_4 \equiv (x_3, y_2)$	$Z_5^+ \equiv (x_6, y_3)$
$Z_3^+ \equiv (x_5, y_4)$	$Z_6^+ \equiv (x_6, y_5)$	$Z_7 \equiv (x_6, y_4)$
$Z_4 \equiv (x_3, y_2)$	$Z_8^+ \equiv (x_4, y_2)$	$Z_5^+ \equiv (x_6, y_3)$
$Z_5^+ \equiv (x_6, y_3)$	$Z_9 = (x_6, y_2)$	$Z_5^+ = (x_6, y_3)$
$\vdots$	$\vdots$	$\vdots$

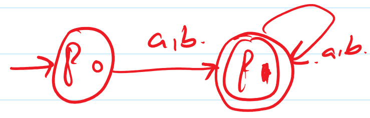
Concatenation :- $r_3 = r_1 r_2$

$FA_3 = FA_1 FA_2$

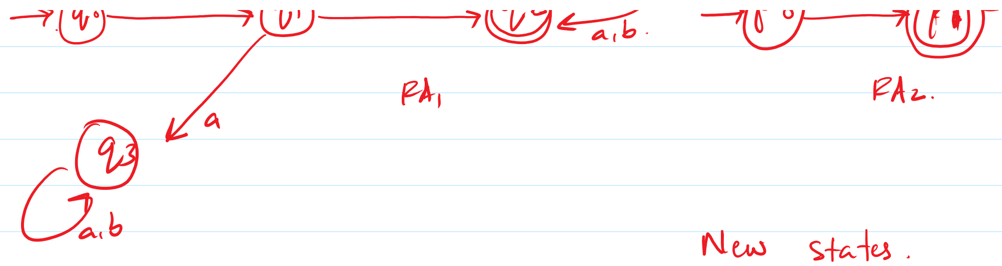
1- Start by taking FA_1 & traverse its states.
2- Initial State = FA_1 Initial State.
3- During process, any state encountered final the resultant state will be final, the second FA's initial will be concatenated with the final of FA_1.



FA_1



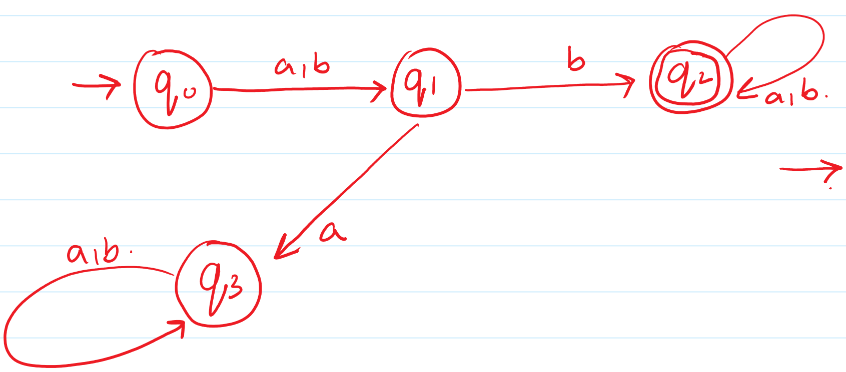
FA_2.

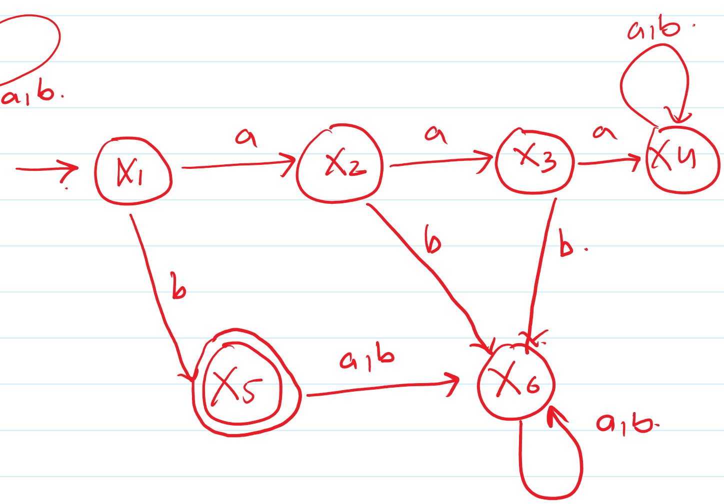


New states.

old States	transition 'a'	transition 'b'
$z_1 \equiv q_0$	$z_2 \equiv q_1$	$z_2 \equiv q_1$
$z_2 \equiv q_1$	$z_3 \equiv q_3$	$z_4^+ \equiv (q_2, p_0)$
$z_3 \equiv q_3$	$z_3 \equiv q_3$	$z_3 \equiv q_3$.
$z_4^+ \equiv (q_2, p_0)$.	$z_5^+ \equiv (q_2, p_0, p_1)$	$z_5^+ \equiv (q_2, p_0, p_1)$
$z_5^+ \equiv (q_2, p_0, p_1)$.	$\equiv (q_2, p_0, p_1, p_1)$. $z_5^+ \equiv (q_2, p_0, p_1)$	$\equiv (q_2, p_0, p_1, p_1)$ $z_5^+ \equiv (q_2, p_0, p_1)$

Draw FA. (HW)





$r_1 r_2 \neq r_2 r_1$

$r_1 + r_2 = r_2 + r_1$