lecture 4:-

$L_2$ { It will consist of concatenation of words only when they are not equal }.

$= \{ w_1 \ \& \ w_2 \in L$ iff $w_1 \neq w_2 \}$.

Can such a language exist.

1) $w_1 \in L$ & $w_2 \in L$ $\quad w_1 \neq w_2$

$\rightarrow w_1 w_2 \in L$.

2) $w_1 w_2 \in L$ & $w_1 \in L$ $\quad w_1 w_2 \neq w_1$.

$\rightarrow w_1 w_2 w_1 \in L$.

3) $w_1 w_2 w_1 \in L$ & $w_2 \in L$ $\quad w_1 w_2 w_1 \neq w_2$.

$\rightarrow w_1 w_2 w_1 w_2 \in L$.

$(w_1 w_2)(w_1 w_2)$ $\quad \boxed{w_1 w_2 = w_1 w_2}$.

Against the def.
Hence such a language
Does not exist.

Ways to define a language.

2) Recursive:-

Rust
Types → Recursion.

1- Some basic words are specified for a language.
2- Rules for constructing more words based on 1.
3- No string except those constructed above are allowed.

INTEGER.

1- 1 is in INTEGER.
2- if x is in a then x+1 and x−1 they are in a.
3- No string except those constructed above are in INTEGER.

5 $\in$ INTEGER.

$4 + 1 = 5$
$3 + 1 = 4$

$4 + 1 = 5$
$3 + 1 = 4$
$2 + 1 = 3$
$1 + 1 = 2$
$x$

EVEN

1. 2 is EVEN
2. if x, y is EVEN then x+2 & x−2 also belongs to EVEN.
3. No String except those Constructed above belongs to EVEN.

PALINDROME.

1. a & b are in PALINDROME
2. if x is in PALINDROME, $s\,x\,\text{reverse}(s) \in$ Palindrome

$\Sigma = \{a, b\}$.
a, b.
aaaa = aa + aa.
$x\,\text{rev}(x) \in$ PALINDROME $\quad s \in \Sigma^*$.
aa = a+a

3. No - - - - - - .

aba. = aba.
x = b. $\quad x \in$ Palindrome from 1.
s = a. $\quad \therefore s\,x\,\text{reverse}(s)$.
aba $\in$ Pal 2.

$\underbrace{ab}_{x}\underbrace{ba}_{\text{reverse}(x)}$.

Define Language $\{a^n b^n\}$ $n = 1, 2, 3 \dots$ $\Sigma = \{a, b\}$.

$\{ab, aabb, \underline{aaabbb}, \dots\}$.

1. ab is in $\{a^n b^n\}$.
2. if x is in $\{a^n b^n\}$. Then $a\,x\,b$ is in $\{a^n b^n\}$.
3. - - - - - - - .

aaabbb $\quad x = aabb$.
$= a^2 b^2 = (a^1 b^1)^2$
$= (ab)^2$
$= ab \in \{a^n b^n\}$.

Define a language that ends in a $\Sigma = \{a, b\}$.

1. $a \in L$
2. if $x \in L$ then $s.x \in L$ $\quad s \in \Sigma^*$.
3. - - - - - -

$\Sigma = \{a, b\}$.
$\Sigma^* = \{\Lambda, a, b,$ - - - - - $bba\}$.

$\underbrace{bba}\underbrace{a}_{x} \in L$.

$aa = \underbrace{a}_{s}.a$ $\quad \underbrace{b}_{s}\underbrace{aa} \in L$.

$\underbrace{b}_{s}\underbrace{baa}_{x} \in L$.

Regular Expressions:-

$\Sigma = \{a, b\}$.
$\Sigma^* = \{\Lambda, a, b, aa, ab, ba, bb,$ - - - $\}$.

$\Sigma^+ = \{a, b, aa, ab, ba, bb,$ - - - $\}$.

$\Sigma = \{x\}$ $\quad \Sigma^* = \{\Lambda, x, xx, xxx,$ - - - $\}$.
$\underline{x^*} = \Lambda, x, xx, xxx,$ - - - - -
$\underline{x^+} = x, xx, xxx,$ - - - - - .

$* = 0, 1, 2,$ - - - .
$+ = 1, 2, 3,$ - - - - - .

$(a + b) = a$ or $b$.

$L = \{a, b\}$.
$(a + b)$

$(a+b)(a+b) = aa, ab, ba, bb$.

one Regex = one language.

multiple Regex can be formed for a single language.

$(a+b)^* = \Lambda,$

$(a+b)' = a, b$

$(a+b)^2 = (a+b)(a+b) = aa, ab, ba, bb.$

$(a+b)^3 = (a+b)(a+b)(a+b) = \underline{aaa}, aab, \ldots$

Language = Strings of even length on $\Sigma = \{a, b\}$.

$((a+b)(a+b))^* = \Lambda$

$((a+b)(a+b))^1 = (a+b)(a+b) = \underline{aa, ab, ba, bb.}$

$((a+b)(a+b))^2 = ((a+b)(a+b)(a+b)(a+b)) = aaaa,$

Language = Strings of odd length on $\Sigma = \{a, b\}$.

$((a+b)(a+b))^* (a+b) = \Lambda a, \Lambda b = a, b.$

$((a+b)(a+b))^1 (a+b) = aaa, aab.$

Q1:- $a^* \overset{or}{+} b^* \neq (a+b)^*$ X. $\Sigma^*$

$\{\Lambda, a, aa, aaa, \ldots\} + \{\Lambda, b, bb, bbb \ldots\}.$

Q2:- $(a+b^*)^* \overset{=}{?} (a+b)^*$. ✓

$aba = ?$

$(a+b^*)^3 = (a+b^*)(a+b^*)(a+b^*)$
$= aba$

Ex. $\{ab, bc\} = ab + bc$
$\{abb, bcb\} = (ab+bc)b$

$\{a, b\}^* = (a+b)^*$

$\{ac, c\} = (a+\Lambda)c$

$\{\Lambda, a, b, ab\} = (a+\Lambda)(b+\Lambda).$

Length(2) = $\Sigma = \{a, b\}$. = $\{aa, ab, ba, bb\}$. = $(a+b)(a+b)$.

" (3) = " = $\{aaa, aab ---\}$ = $(a+b)(a+b)(a+b)$.

Begins with a followed by anything $\Sigma = \{a, b\}$.

$a(a+b)^*$

$b(a+b)^*$ begins with b.

All possible string ending in b.

$(a+b)^*b$.

All possible strings containing atleast one a.

$(a+b)^* a (a+b)^*$. = a, aa, ba,

aa, ab,

aaa, aab, baa, bab.

Starting with double aa & ending with double bb.

$aa(a+b)^*bb$

Starting & ending with same letter.

$a(a+b)^*a + b(a+b)^*b$.

Either ending in aa or ending in bb.

$(a+b)^*aa + (a+b)^*bb$.