



DeepLearning.AI

# Derivatives and Optimization

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## Machine learning motivation

# Machine Learning Motivation

# Machine Learning Motivation



# Machine Learning Motivation



Number of Bedrooms

# Machine Learning Motivation



1



2

Number of Bedrooms

# Machine Learning Motivation



1

\$150,000



2

\$250,000

Number of Bedrooms

# Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has



1

\$150,000



2

\$250,000

Number of Bedrooms

# Machine Learning Motivation

**Predict the price of a house using the number of bedrooms it has**

# Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms
1
2
3
5
6
7
8
9
10

# Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms	Price of House
1	\$150,000
2	\$250,000
3	\$350,000
5	\$600,000
6	\$650,000
7	\$750,000
8	\$800,000
9	??
10	\$1,050,000

# Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

Number of Bedrooms	Price of House
1	\$150,000
2	\$250,000
3	\$350,000
5	\$600,000
6	\$650,000
7	\$750,000
8	\$800,000
9	??
10	\$1,050,000

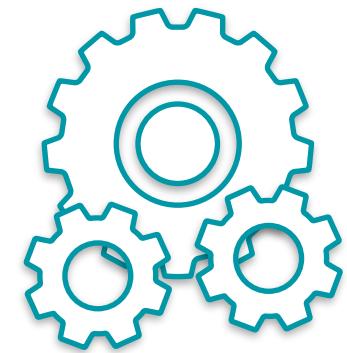
9 ???

# Machine Learning Motivation

Predict the price of a house using the number of bedrooms it has

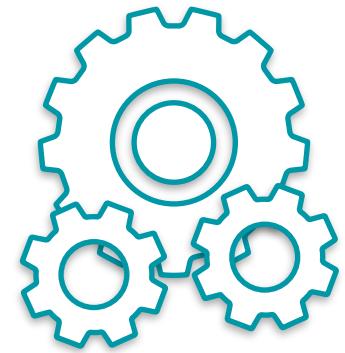
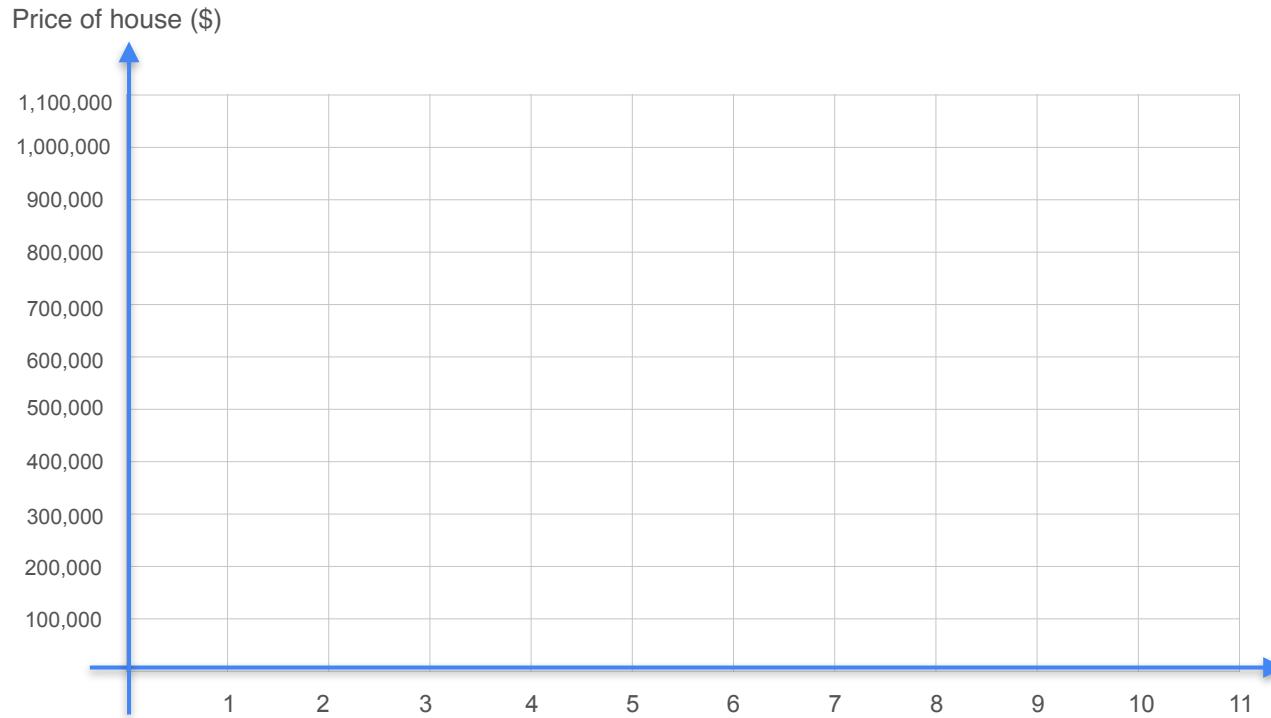
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Machine Learning  
Model

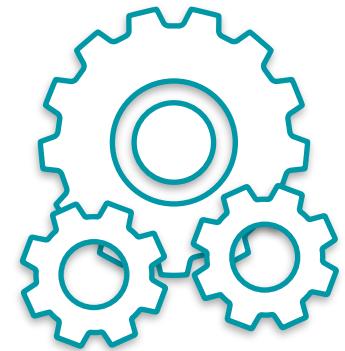
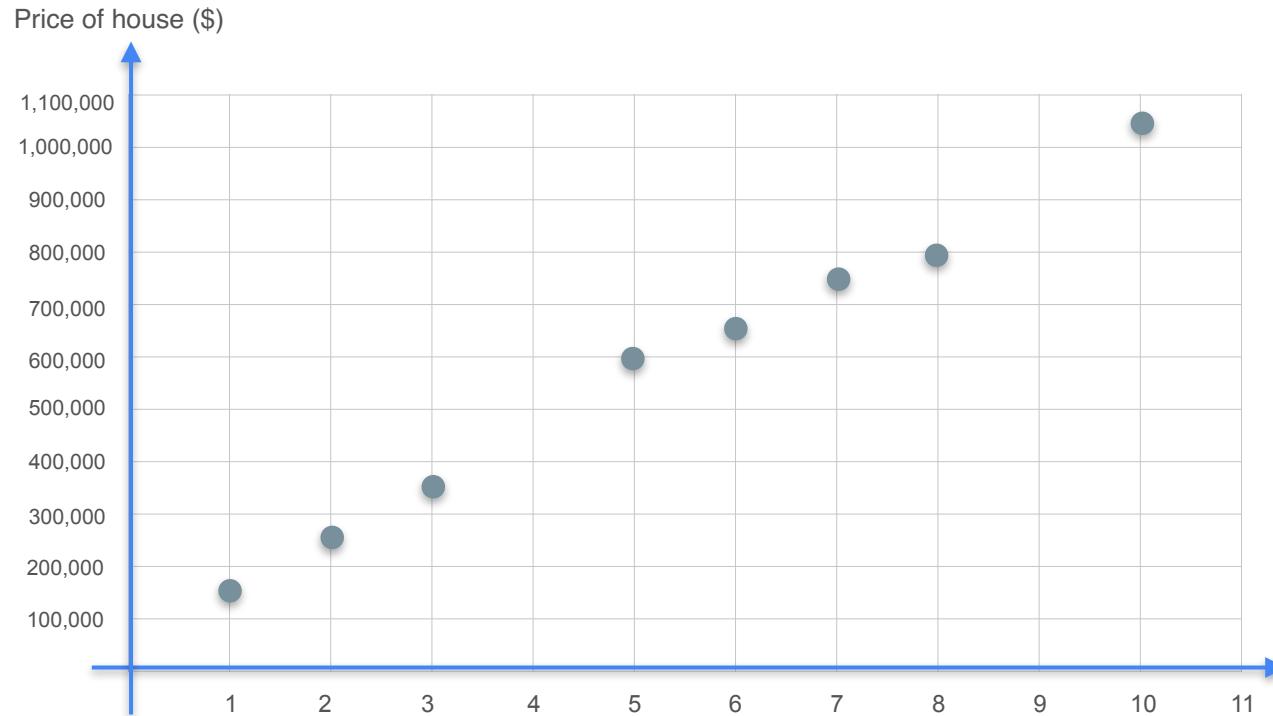
# Machine Learning Motivation



Machine Learning  
Model

*Number of  
Bedrooms*

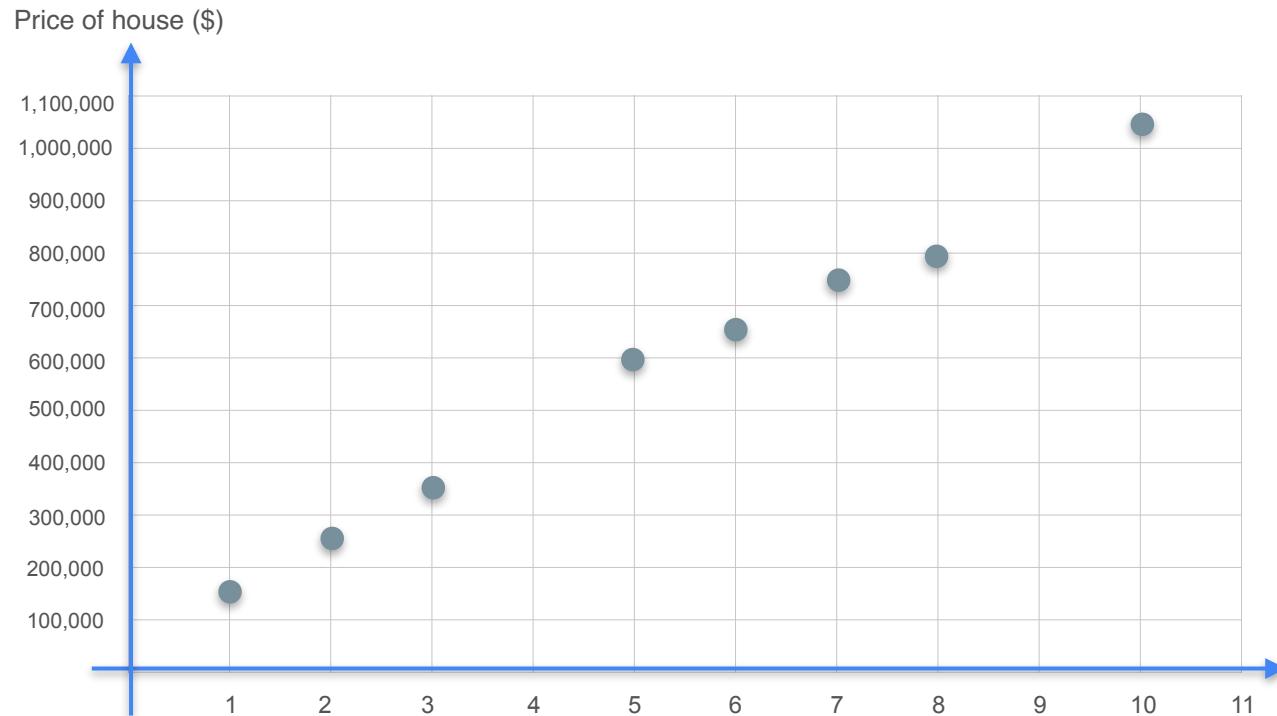
# Machine Learning Motivation



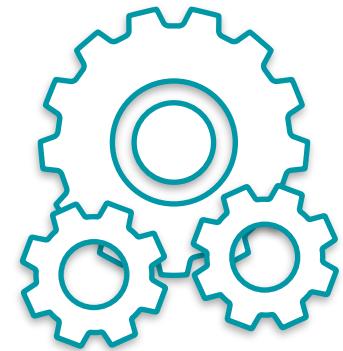
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



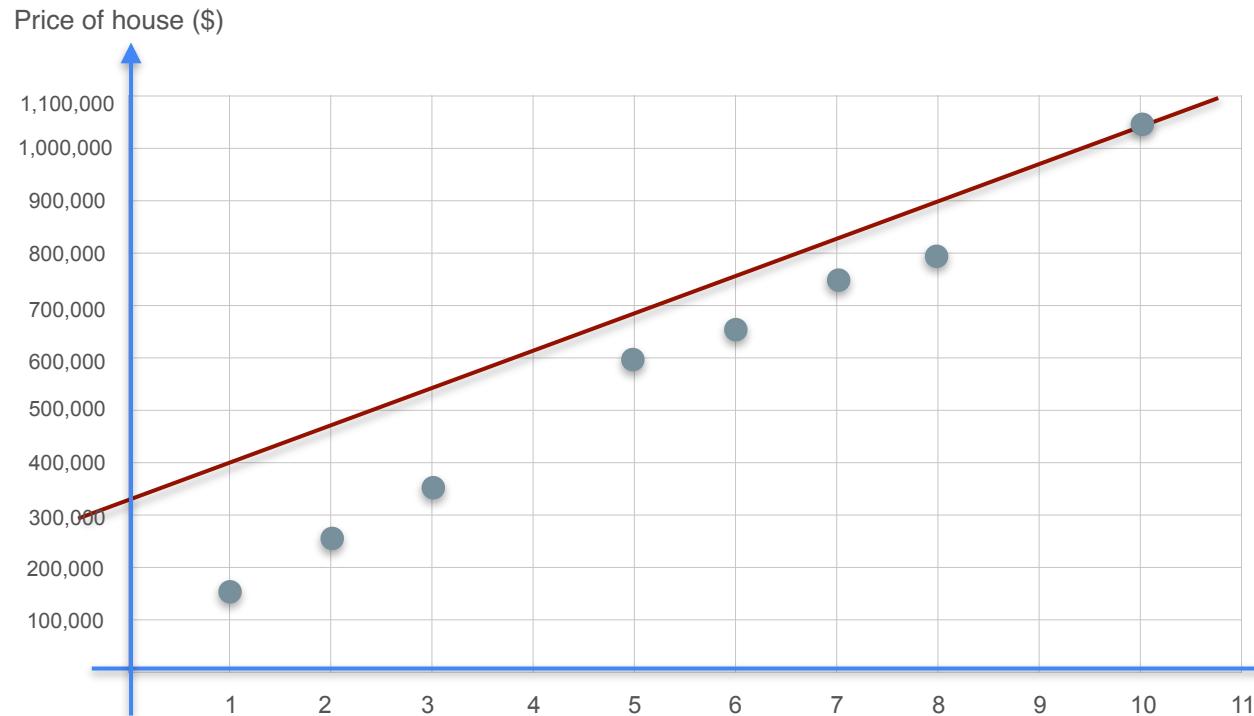
Model Training



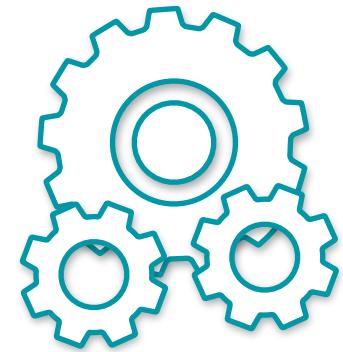
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



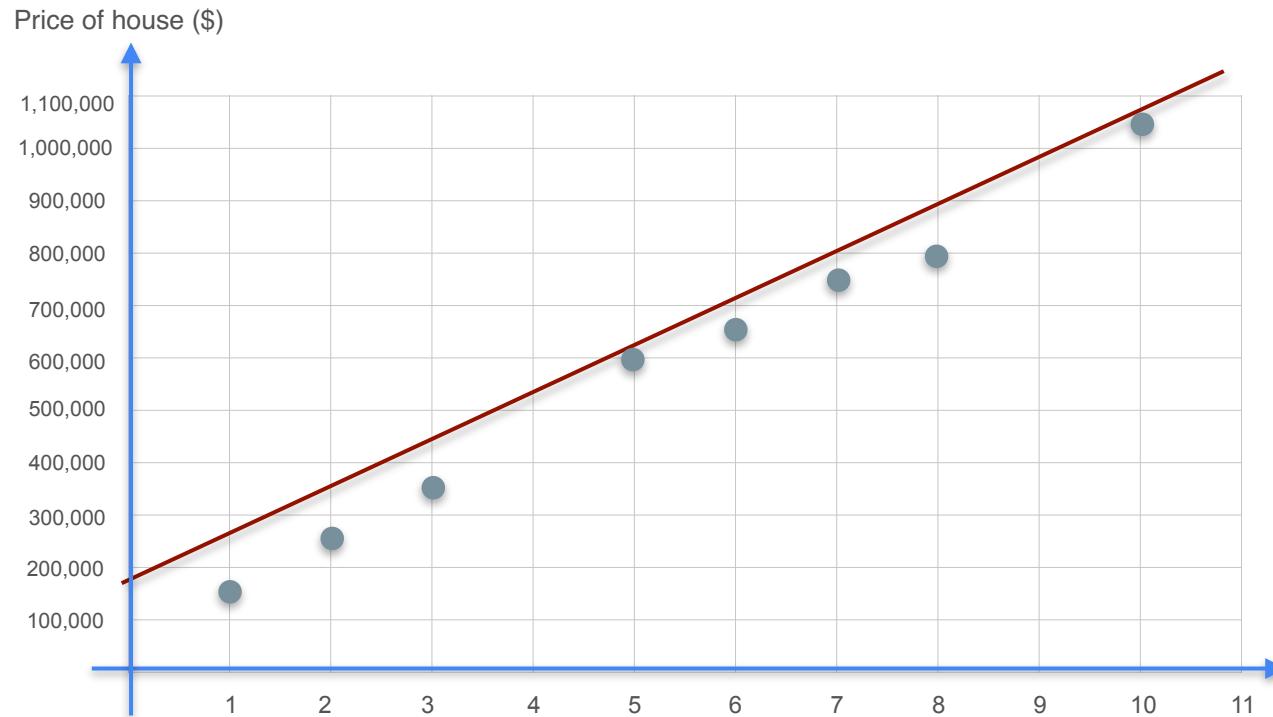
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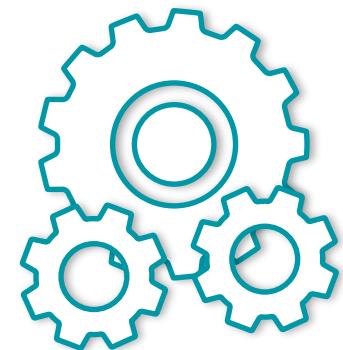
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



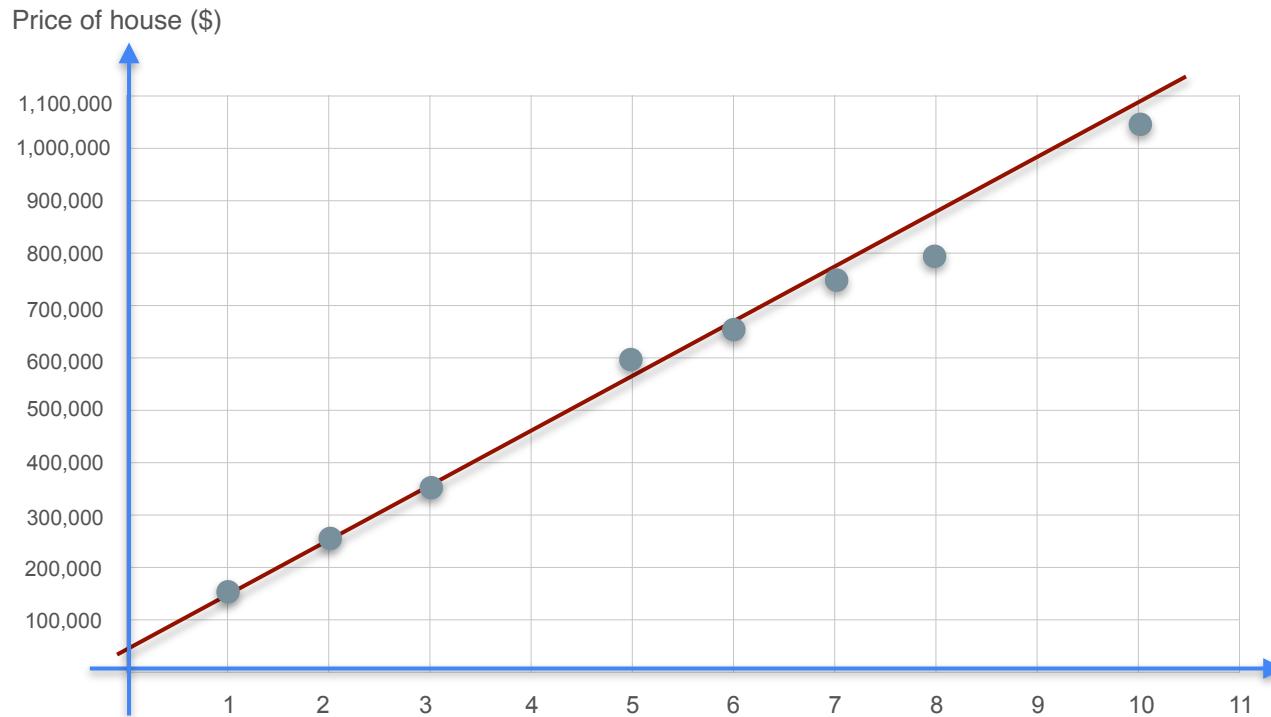
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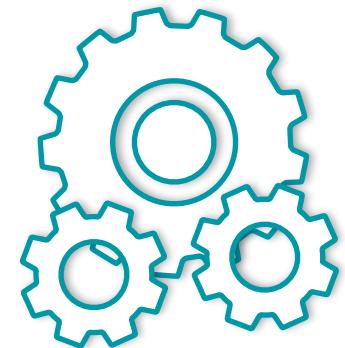
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



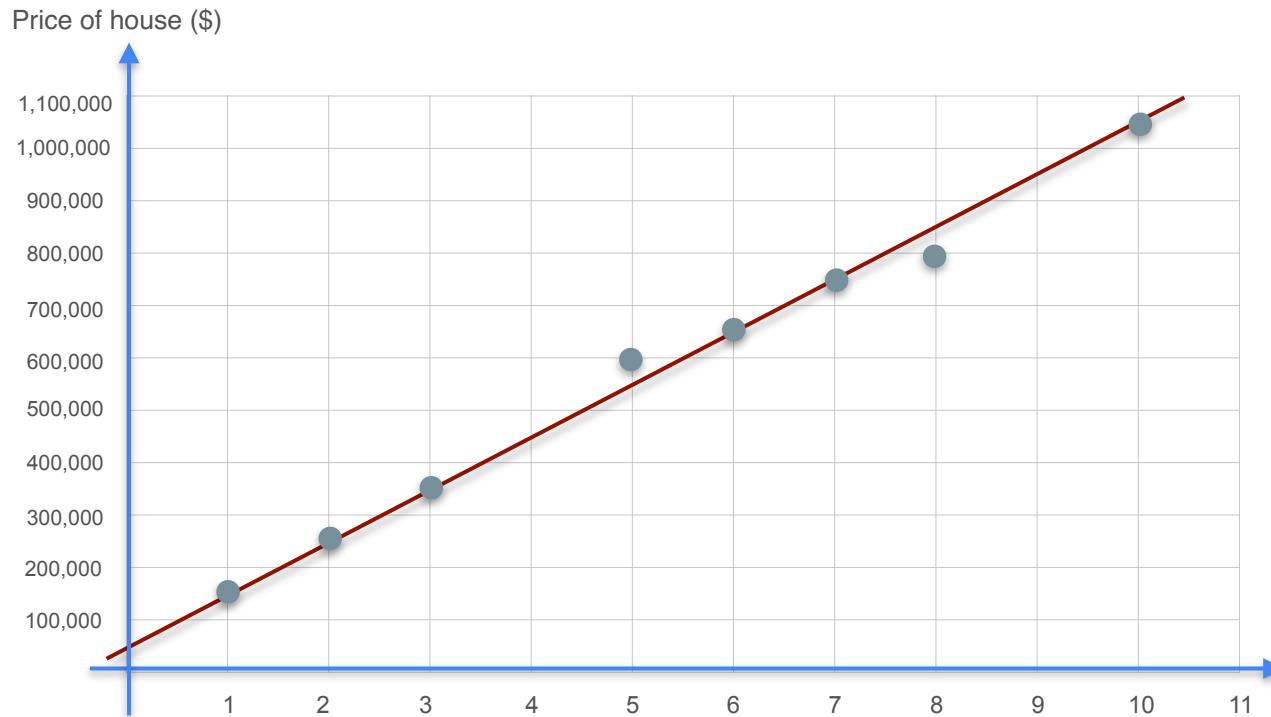
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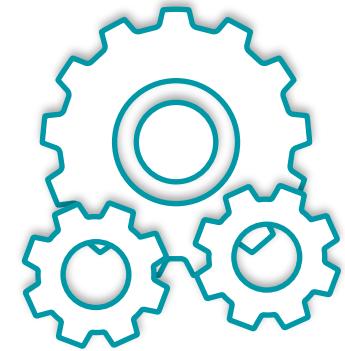
Machine Learning  
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# Machine Learning Motivation



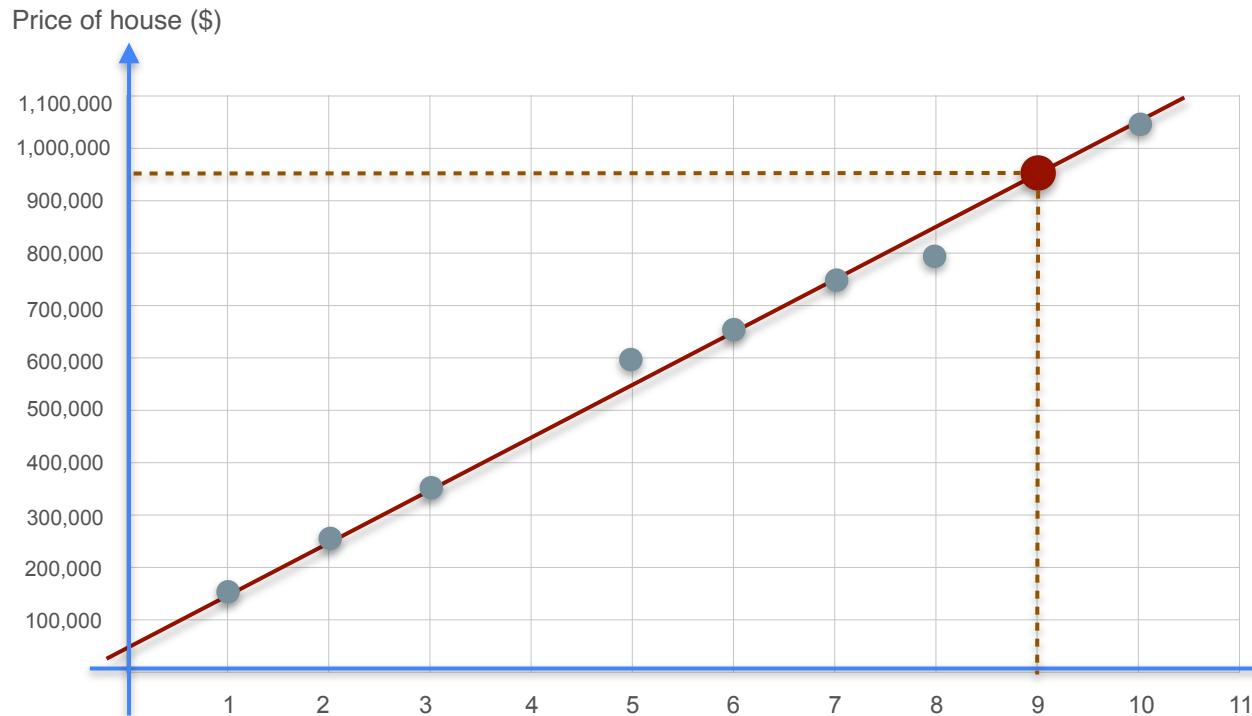
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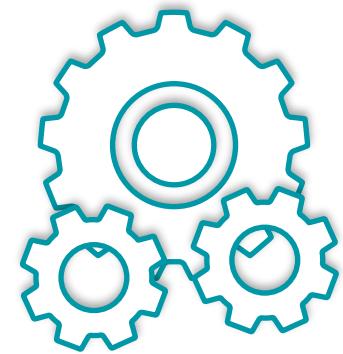
Machine Learning  
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*Number of  
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# Machine Learning Motivation



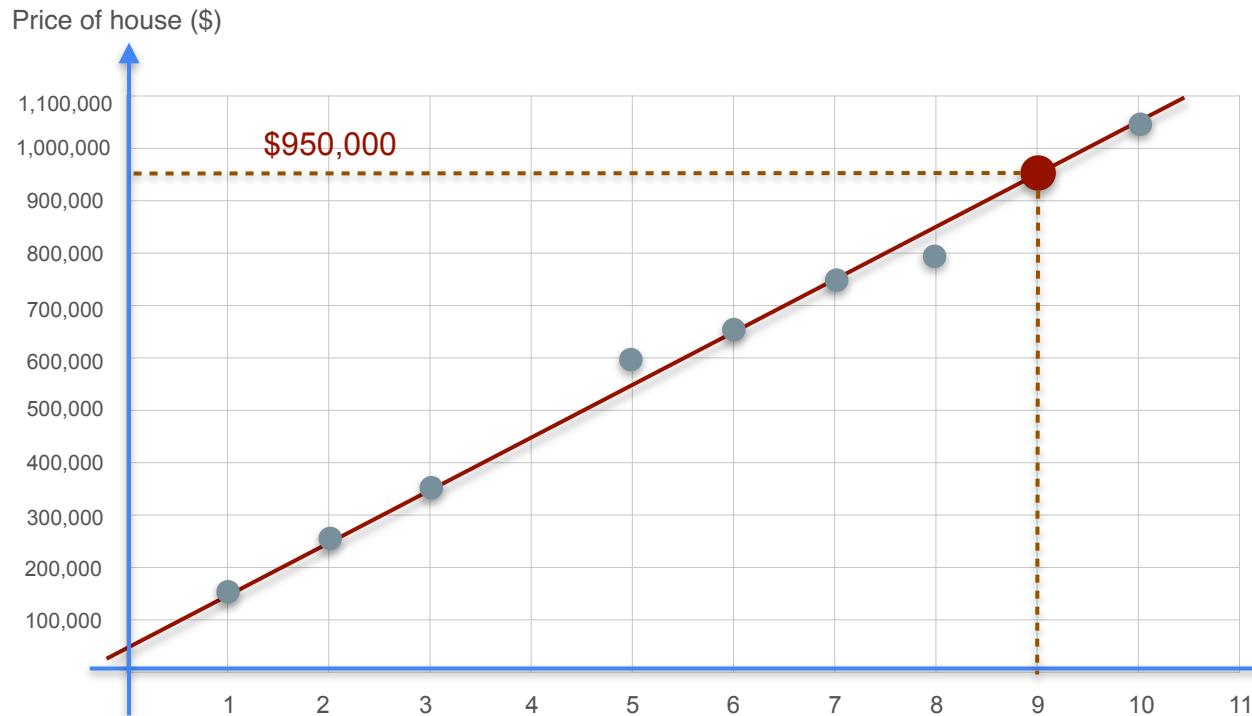
Model Training



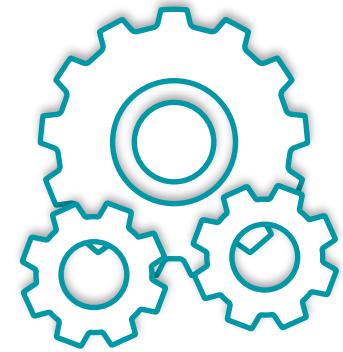
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



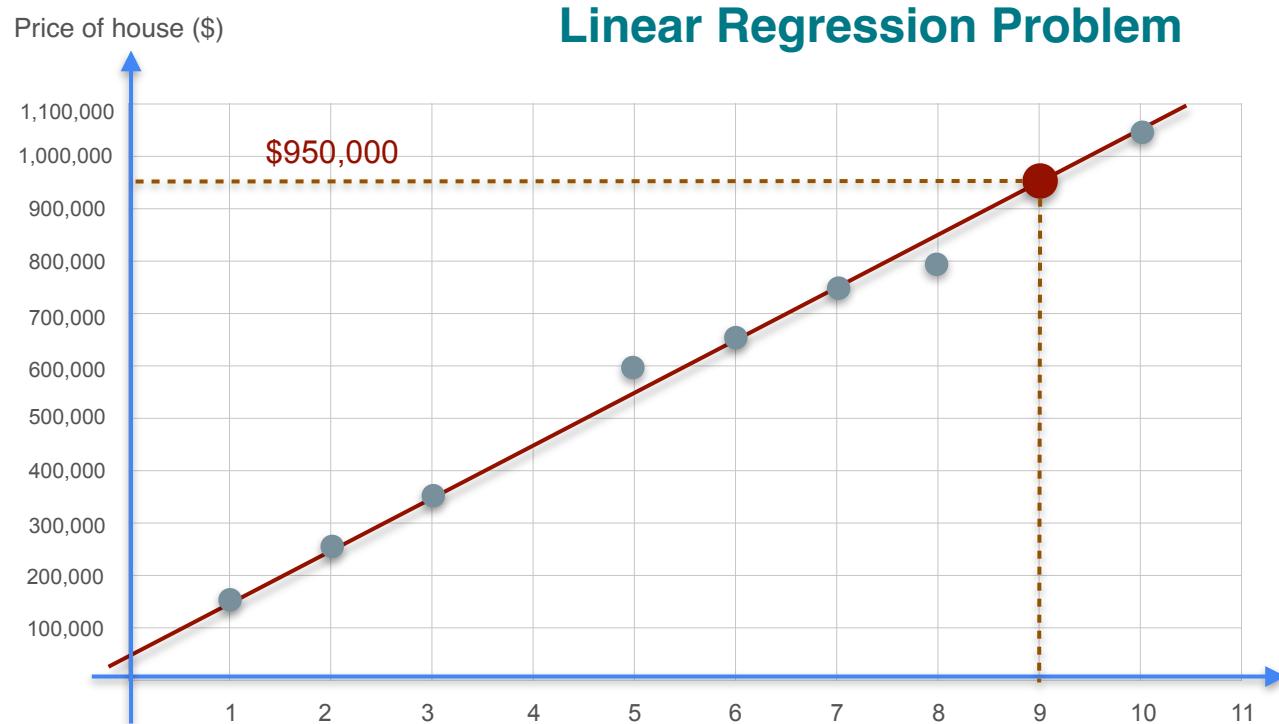
Model Training



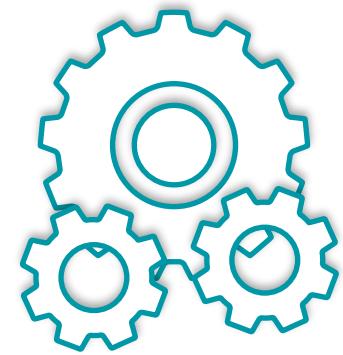
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation



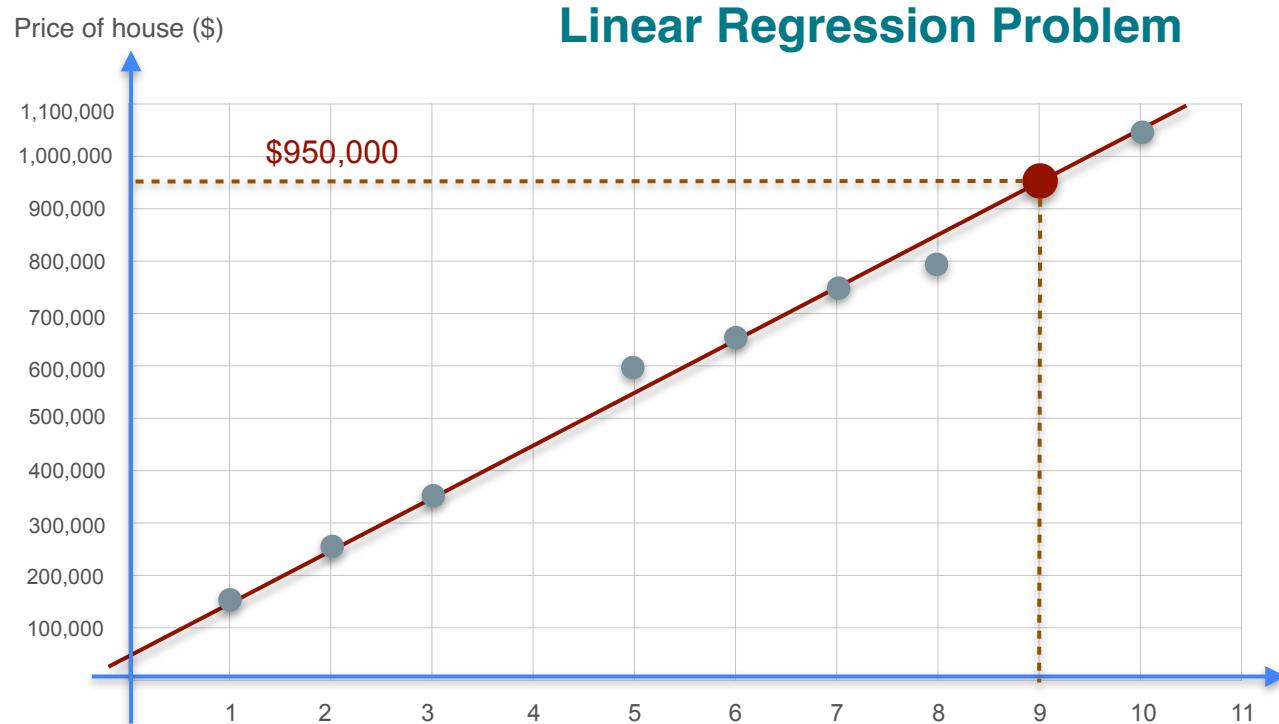
Model Training



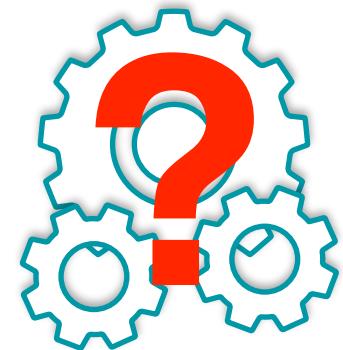
Machine Learning  
Model

*Number of  
Bedrooms*

# Machine Learning Motivation

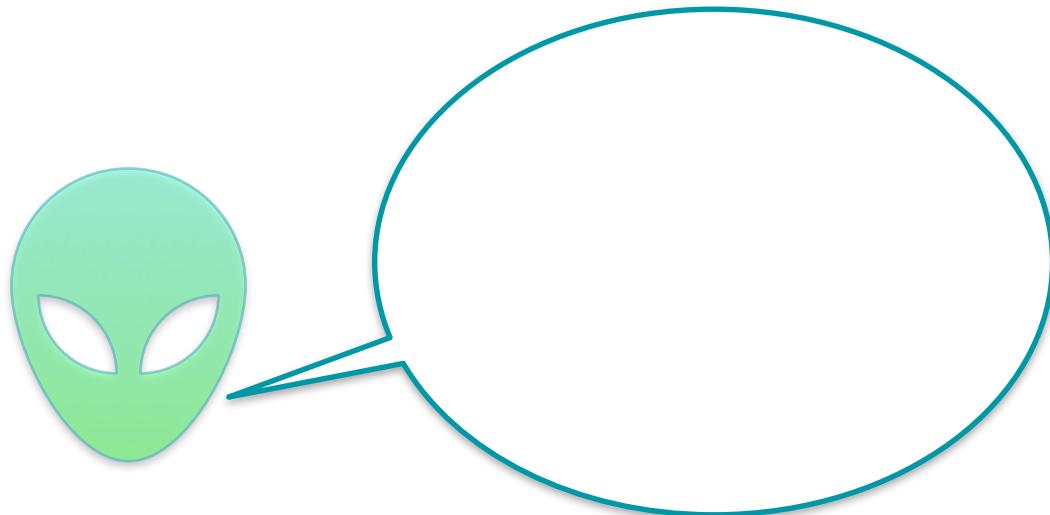


Model Training

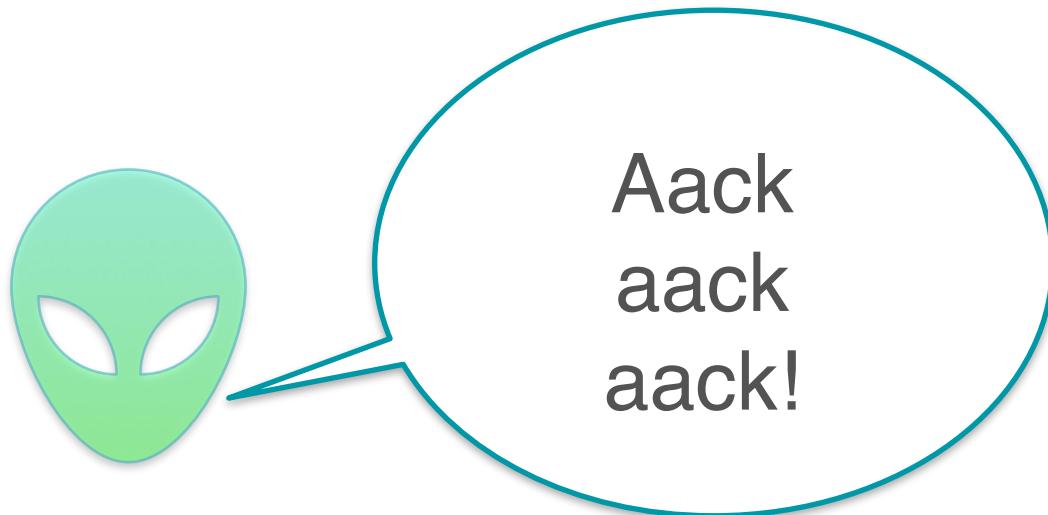


Machine Learning  
Model

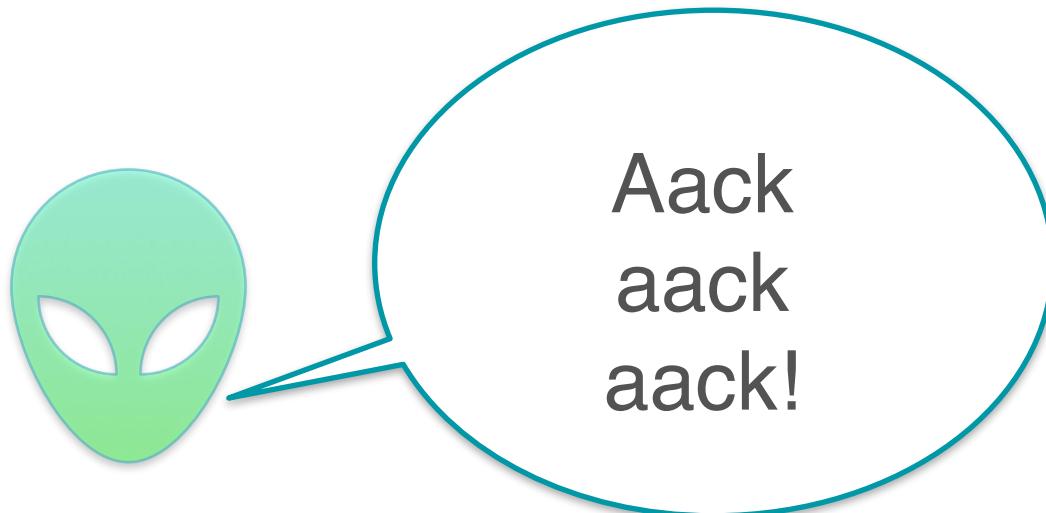
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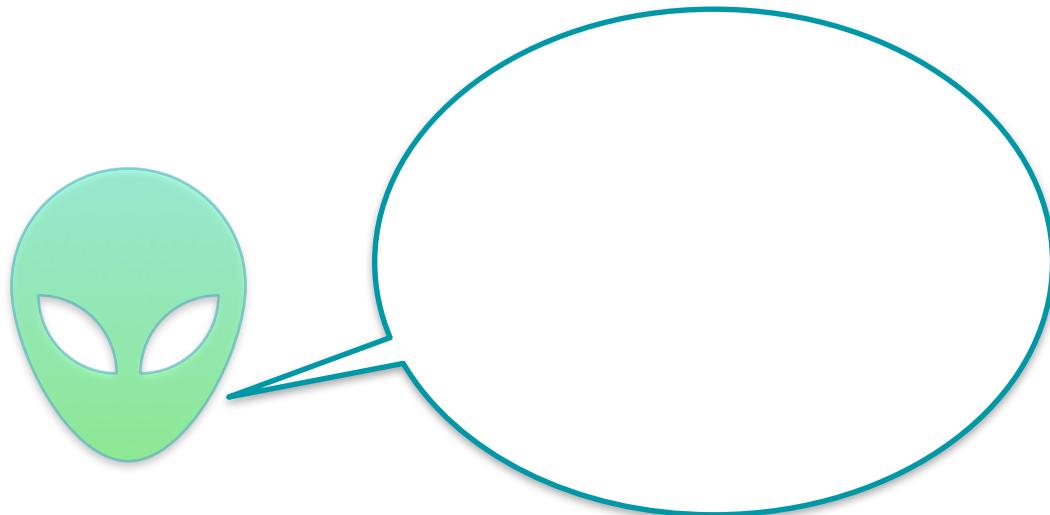
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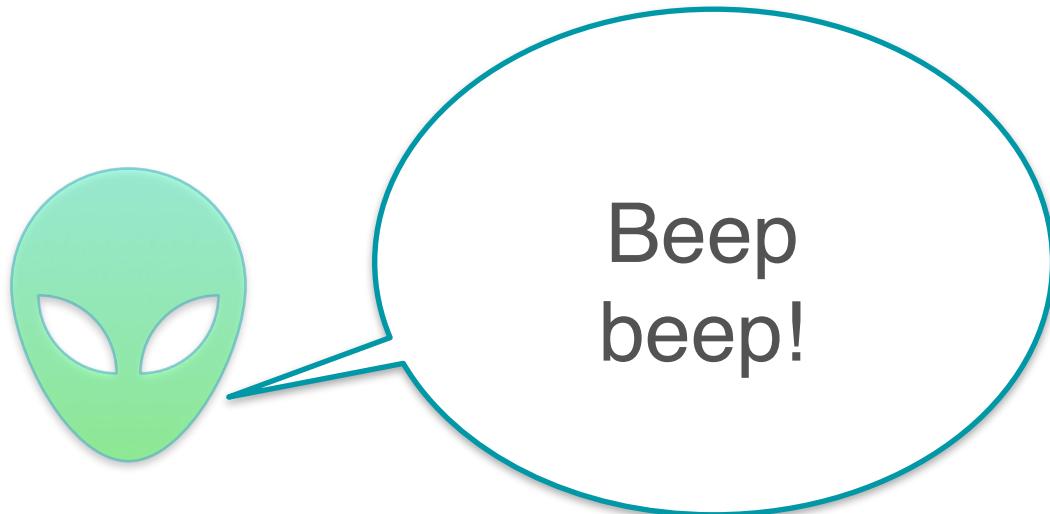
# Machine Learning Motivation



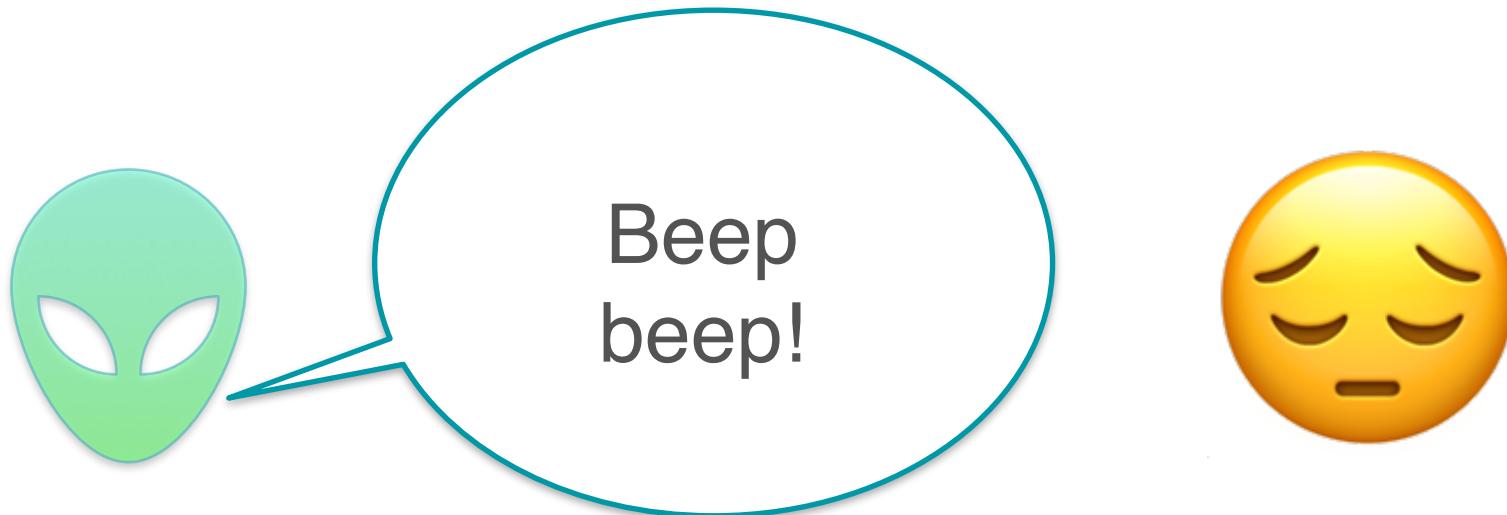
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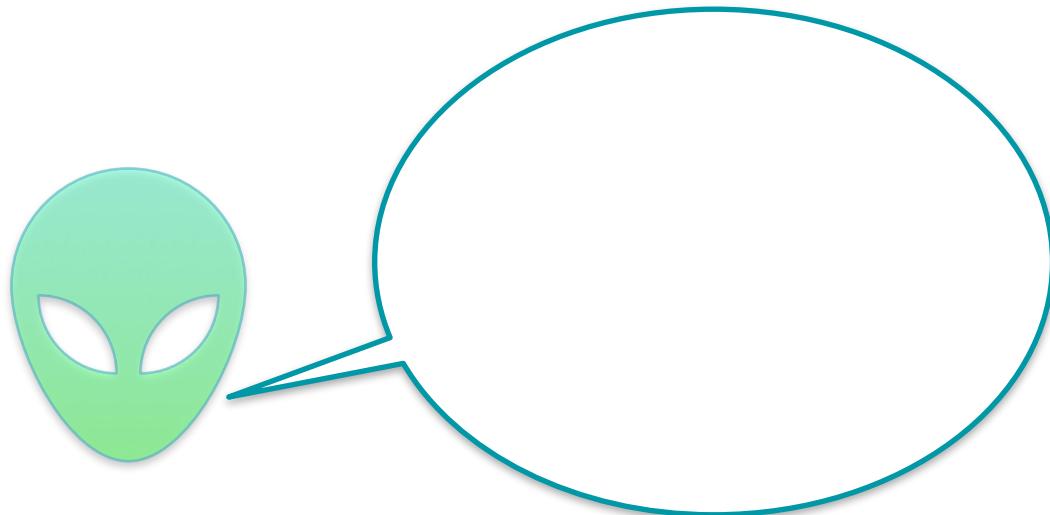
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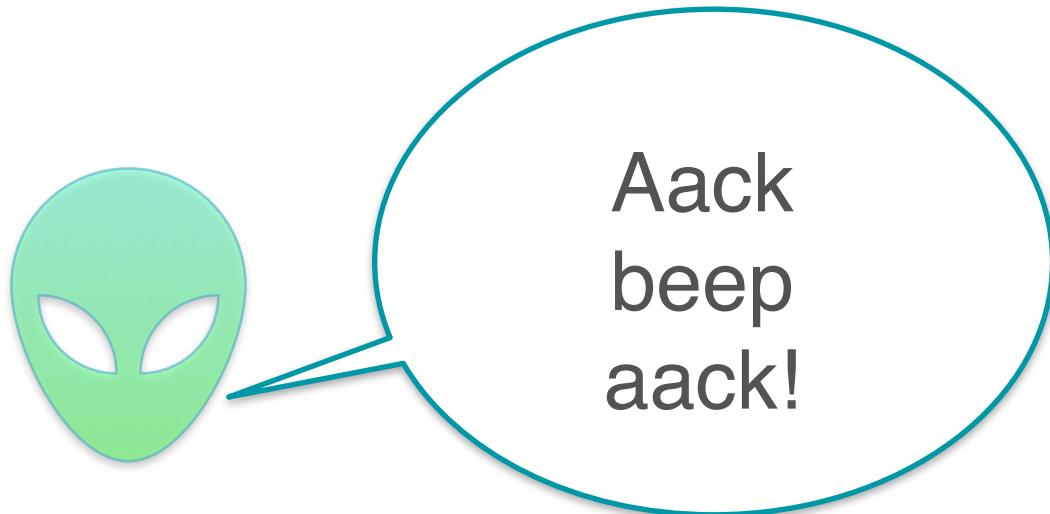
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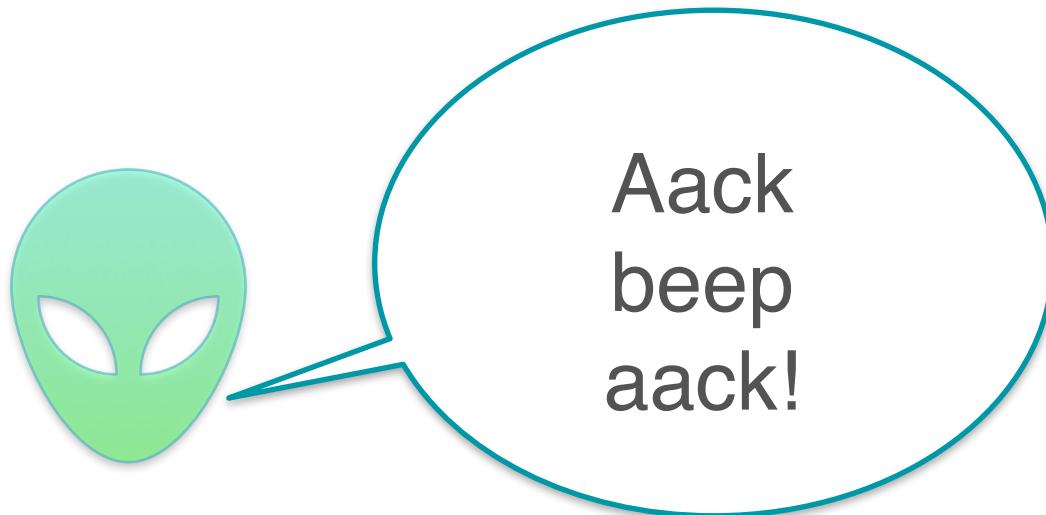
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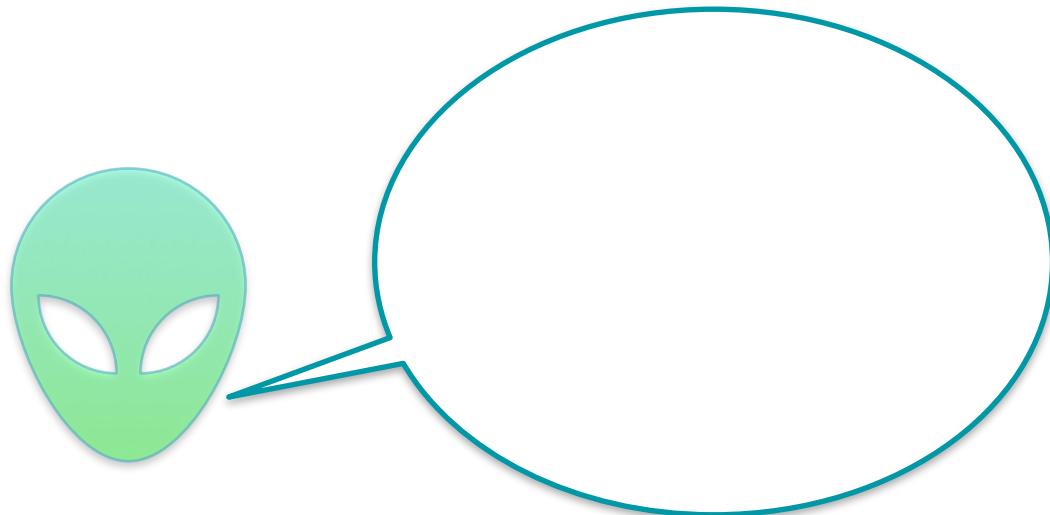
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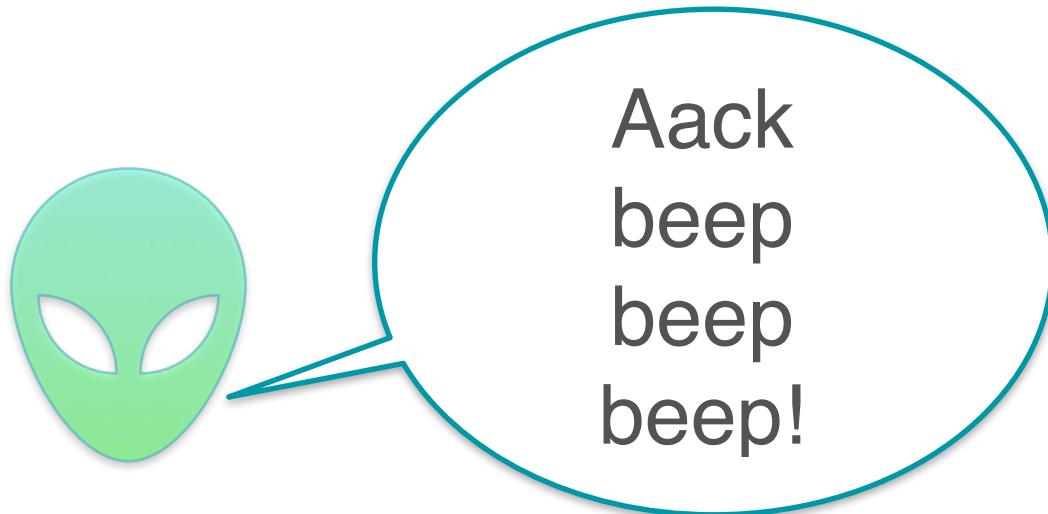
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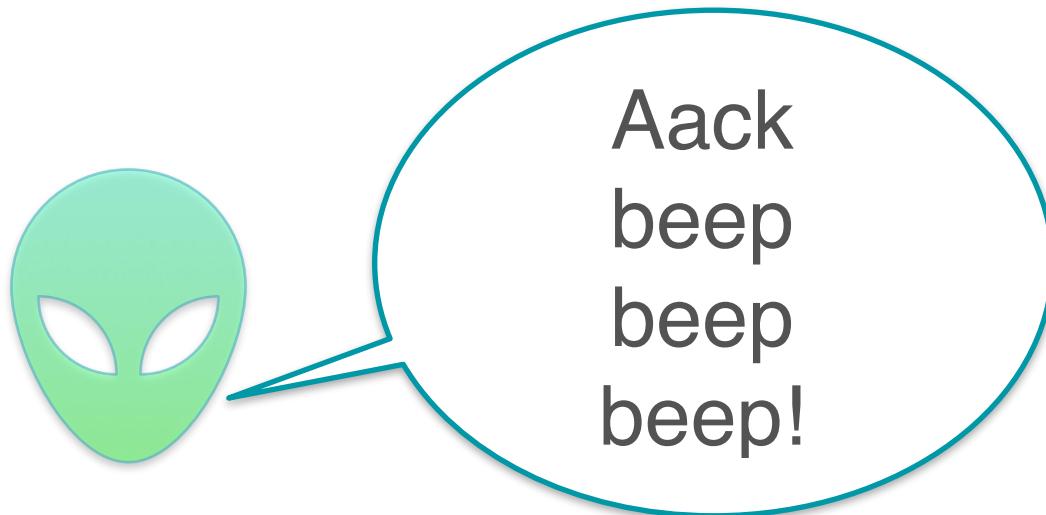
# Machine Learning Motivation



# Machine Learning Motivation



# Machine Learning Motivation



# Machine Learning Motivation



# Machine Learning Motivation



*Sentence*

*Aack aack aack!*

*Beep beep!*

*Aack beep aack!*

*Aack beep beep  
beep!*

# Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>
<i>Aack aack aack!</i>	3	0

*Beep beep!*      0      2

<i>Aack beep aack!</i>	2	1
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*Aack beep beep  
beep!*      1      3

# Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

*Beep beep!*      0      2      😞

<i>Aack beep aack!</i>	2	1	😊
------------------------	---	---	---

*Aack beep beep  
beep!*      1      3      😞

# Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

**A classification problem**

*Beep beep!*      0      2      😞

<i>Aack beep aack!</i>	2	1	😊
------------------------	---	---	---

*Aack beep beep  
beep!*      1      3      😞

# Machine Learning Motivation



<i>Sentence</i>	<i>Aack</i>	<i>Beep</i>	<i>Mood</i>
<i>Aack aack aack!</i>	3	0	😊

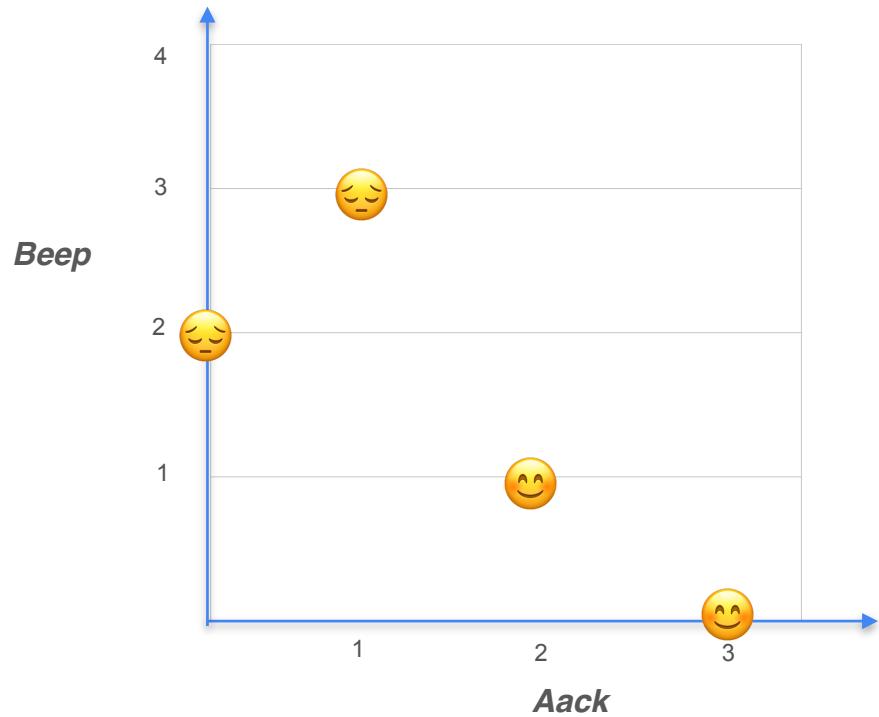
**A classification problem**

<i>Beep beep!</i>	0	2	😴
<i>Aack beep aack!</i>	2	1	😊

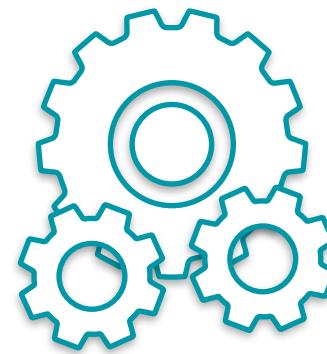
**Sentiment analysis**

<i>Aack beep beep beep!</i>	1	3	😴
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# Machine Learning Motivation

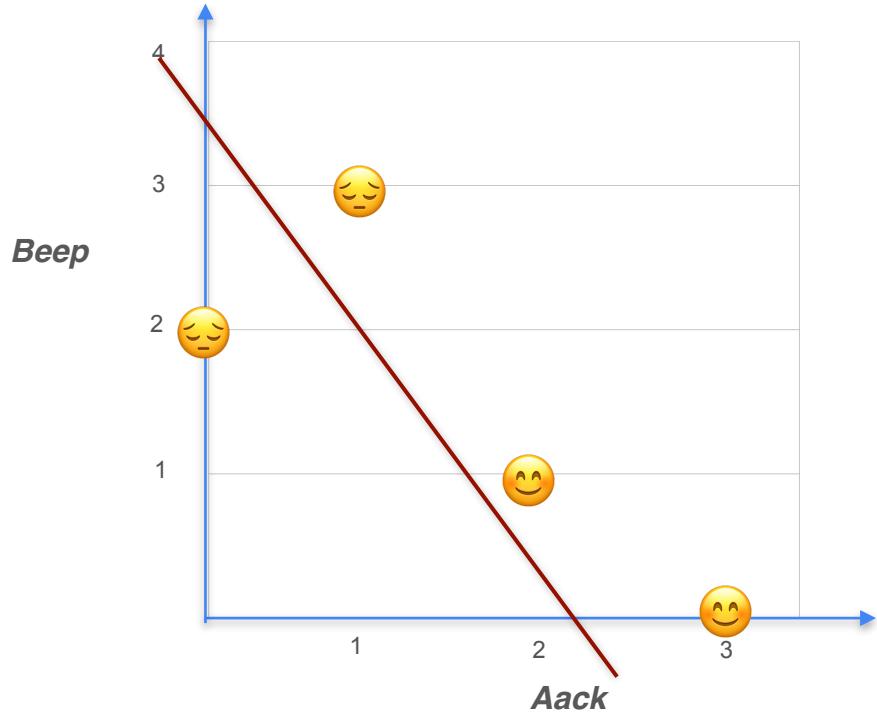


Model Training

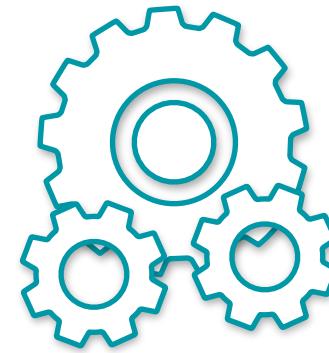


Machine Learning  
Model

# Machine Learning Motivation

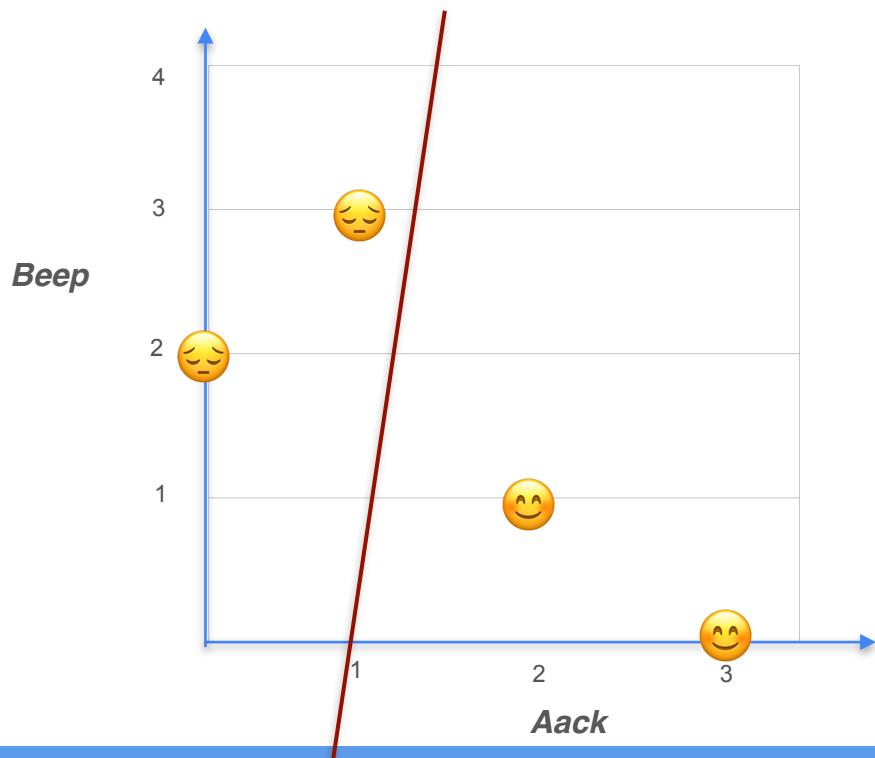


Model Training

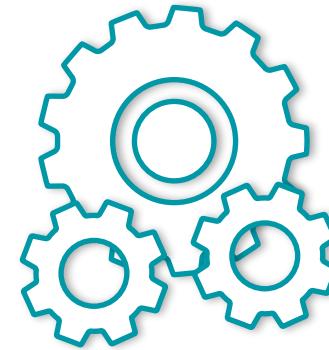


Machine Learning  
Model

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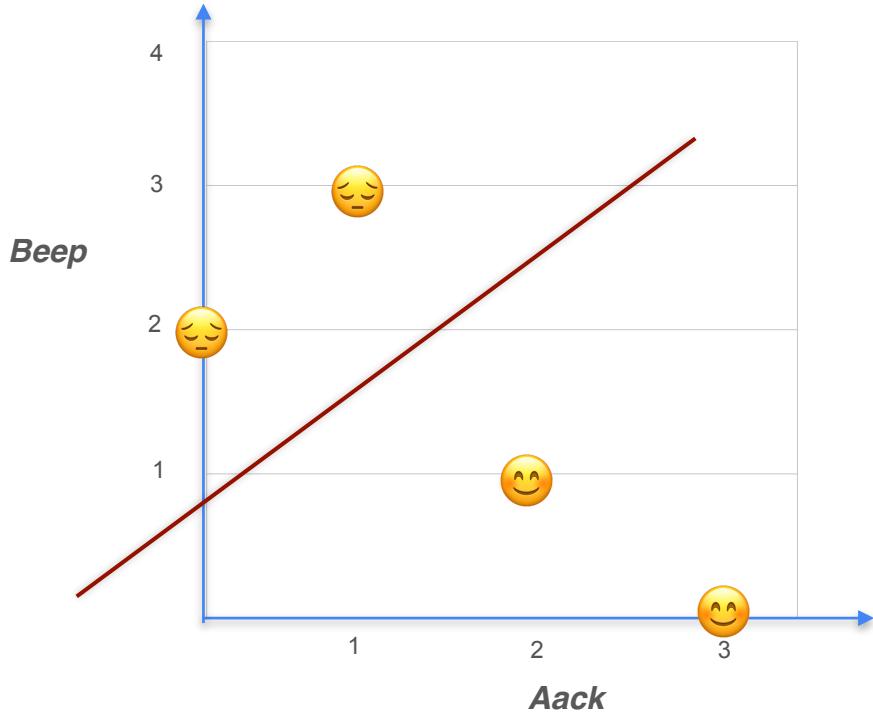


Model Training

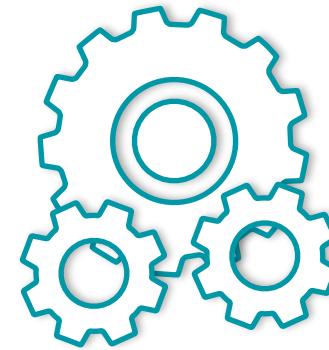


Machine Learning  
Model

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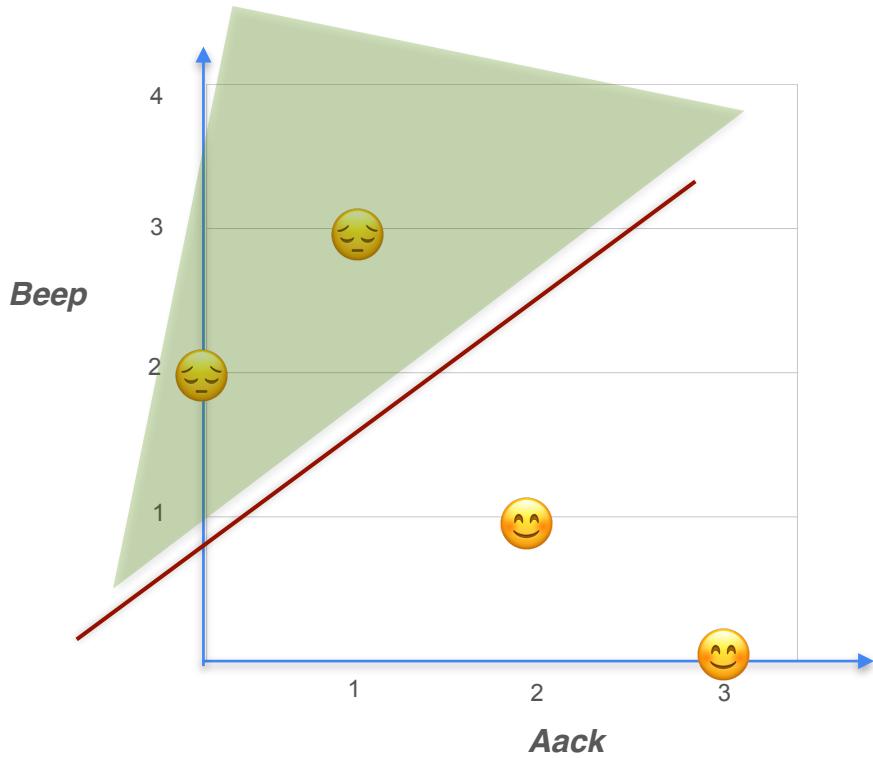


Model Training

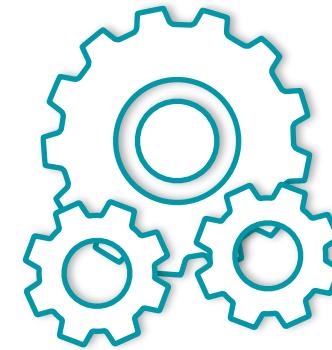


Machine Learning  
Model

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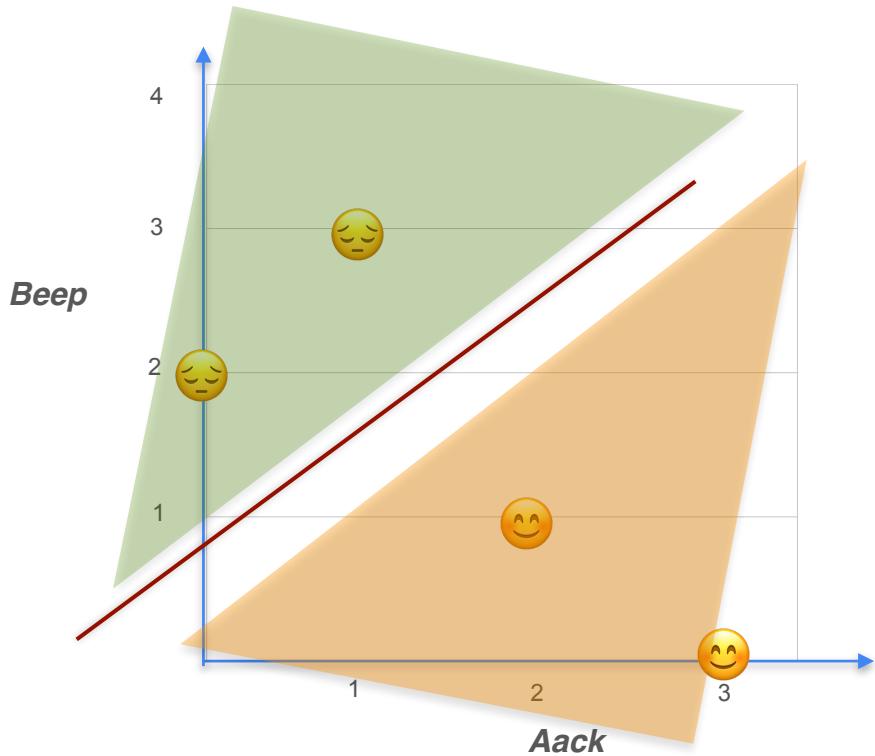


Model Training

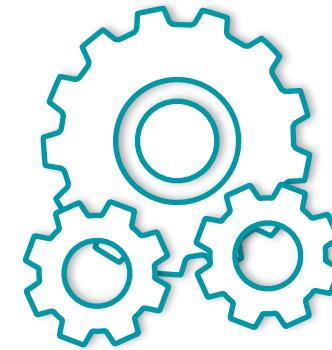


Machine Learning Model

# Machine Learning Motivation

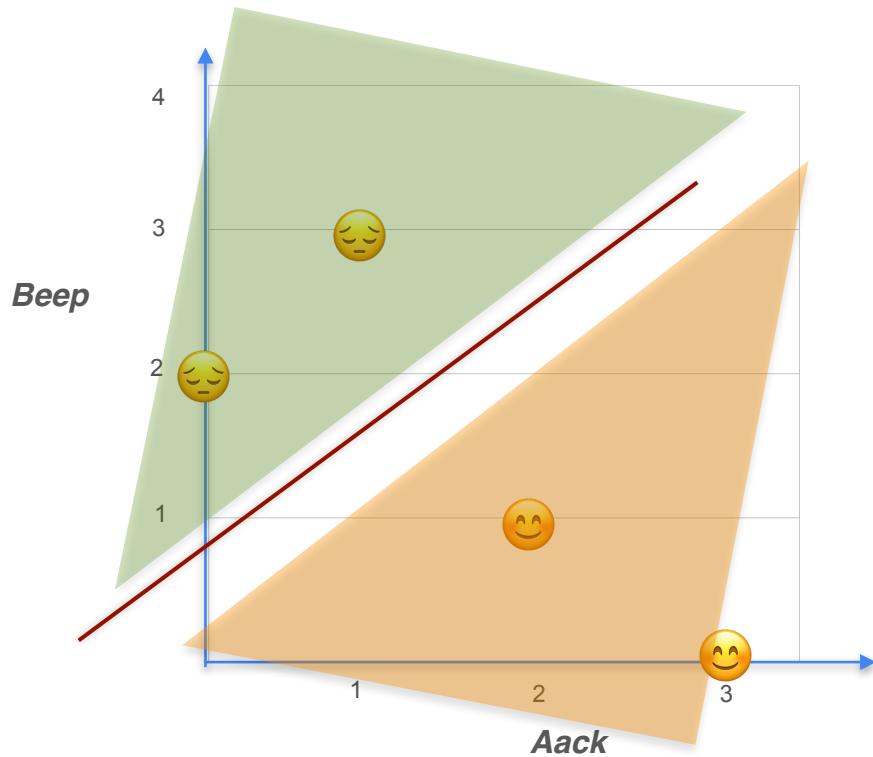


Model Training

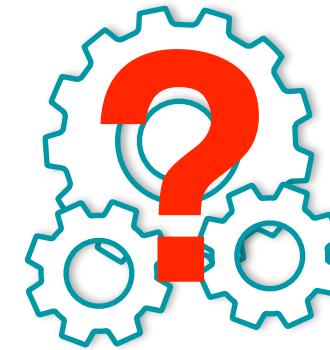


Machine Learning  
Model

# Machine Learning Motivation

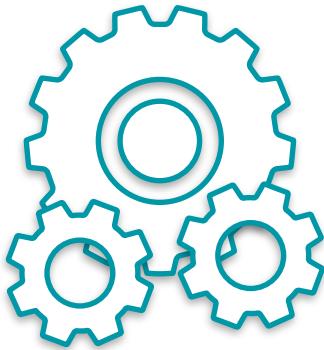


Model Training



Machine Learning  
Model

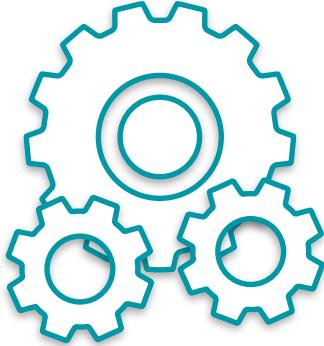
# Machine Learning Motivation



Machine Learning  
Model

# Machine Learning Motivation

Maths concepts used in training a model



Machine Learning  
Model

# Machine Learning Motivation

## Maths concepts used in training a model

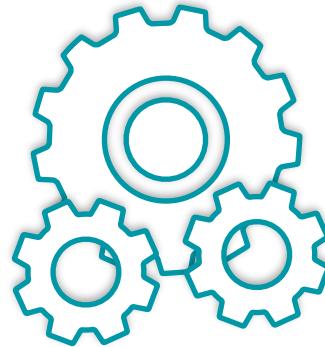
Gradients

Derivatives

Optimization

Loss and Cost functions

Gradient Descent



Machine Learning  
Model

# Machine Learning Motivation

## Maths concepts used in training a model

Gradients

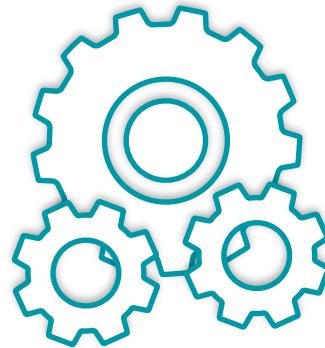
Derivatives

Optimization

Loss and Cost functions

Gradient Descent

Linear Regression



Machine Learning  
Model

# Machine Learning Motivation

## Maths concepts used in training a model

Gradients

Derivatives

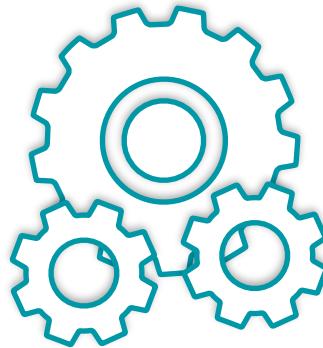
Optimization

Loss and Cost functions

Gradient Descent

Linear Regression

Classification



Machine Learning  
Model

# Machine Learning Motivation

## Maths concepts used in training a model

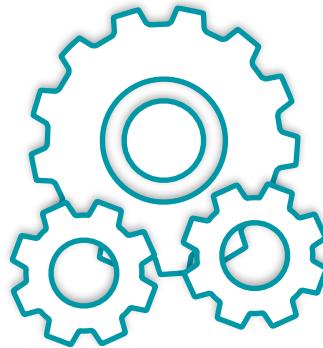
Gradients

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Optimization

Loss and Cost functions

Gradient Descent



Machine Learning  
Model

Linear Regression

Classification

Neural Networks



DeepLearning.AI

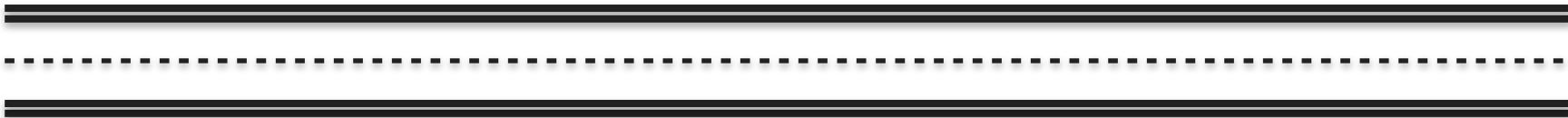
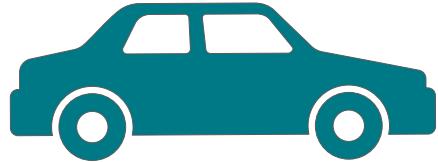
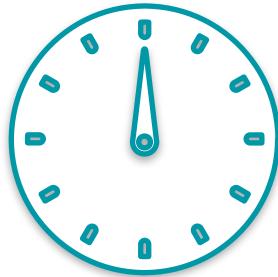
# Derivatives and Optimization

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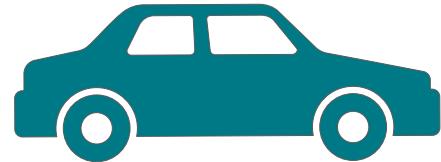
## Introduction to derivatives

# Introduction to Derivatives

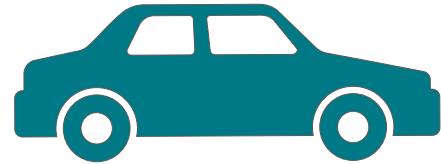
# Introduction to Derivatives



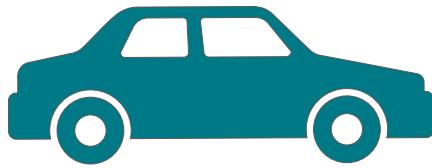
# Introduction to Derivatives



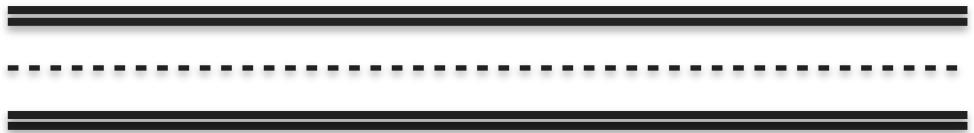
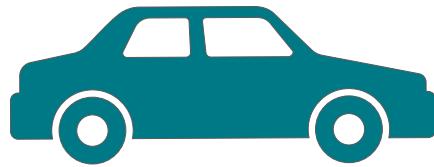
# Introduction to Derivatives



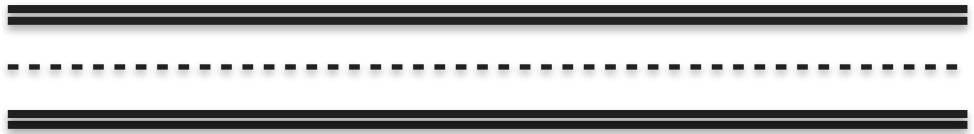
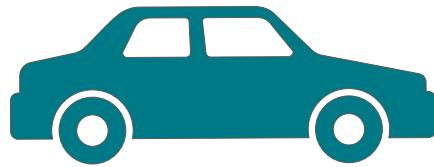
# Introduction to Derivatives



# Introduction to Derivatives

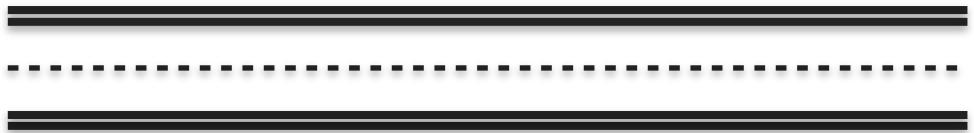
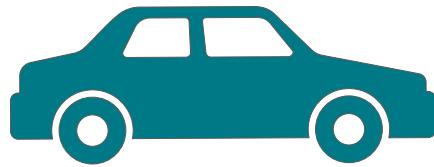
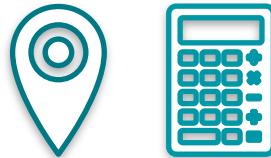


# Introduction to Derivatives



# Introduction to Derivatives

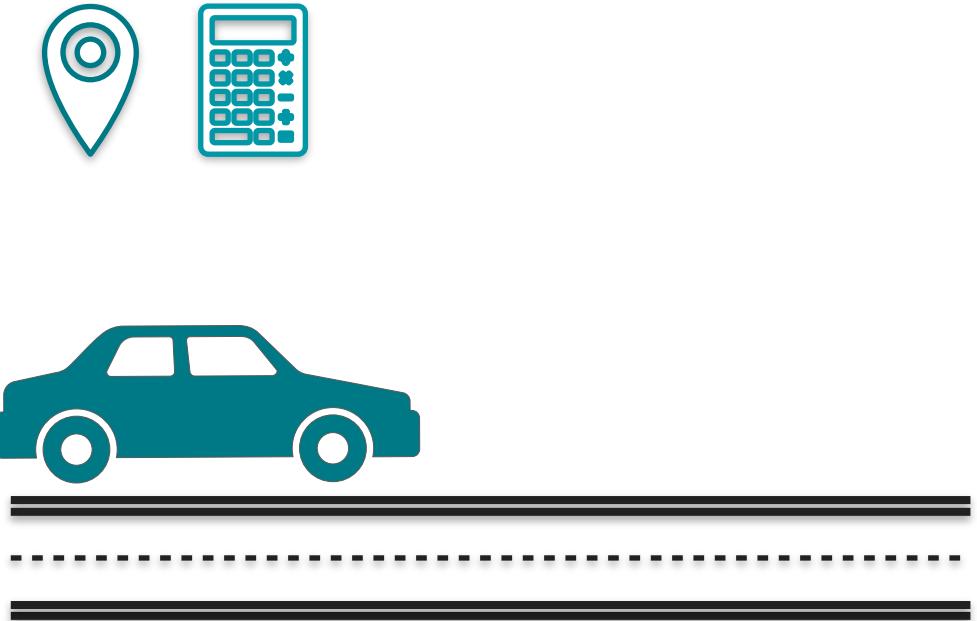
Every 5 seconds



# Introduction to Derivatives

Every 5 seconds

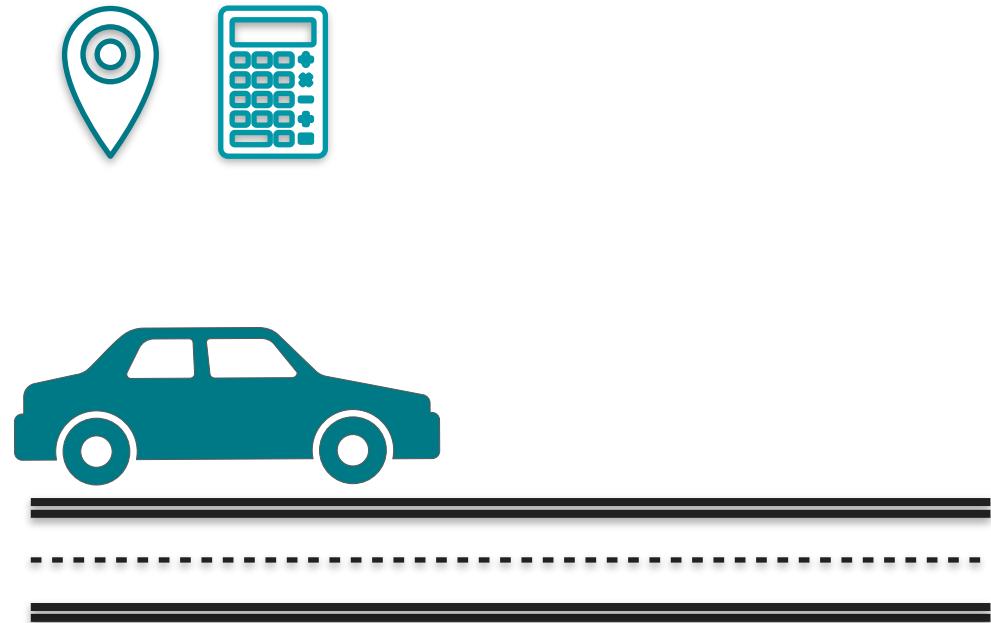
t (seconds)
0
5
10
15
20
25
30
35
40
45
50
55
60



# Introduction to Derivatives

Every 5 seconds

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



# Introduction to Derivatives

Every 5 seconds

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



Is the car moving at a constant speed?

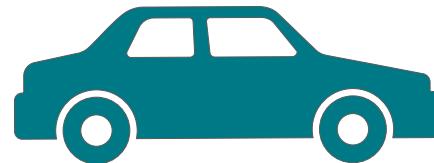
# Introduction to Derivatives

Every 5 seconds

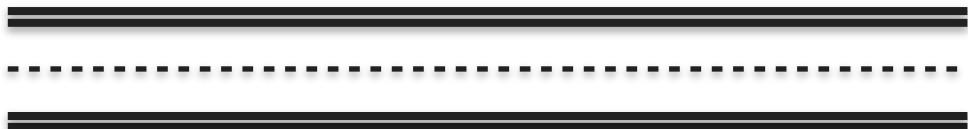
t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000



Is the car moving at a constant speed?



**Hint:** Look at the distance values in the table



# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds:  $202 - 122 = 80$  meters

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds:  $202 - 122 = 80$  meters

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds:  $202 - 122 = 80$  meters

15 - 20 seconds:  $265 - 202 = 63$  meters

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

10 - 15 seconds:  $202 - 122 = 80$  meters

15 - 20 seconds:  $265 - 202 = 63$  meters

**Not traveling at constant speed**

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly  **$t = 12.5 \text{ seconds}$** ?

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	265
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly  **$t = 12.5 \text{ seconds}$** ?

Hint: velocity = distance traveled / time taken

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly  **$t = 12.5 \text{ seconds}$** ?

# Introduction to Derivatives

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

Can you use the information presented to determine your velocity at exactly  $t = 12.5 \text{ seconds}$ ?

# Quiz : Slope of a Line 1

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

# Quiz : Slope of a Line 1

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

What was the average velocity of the car on the interval from 10 to 15 seconds?

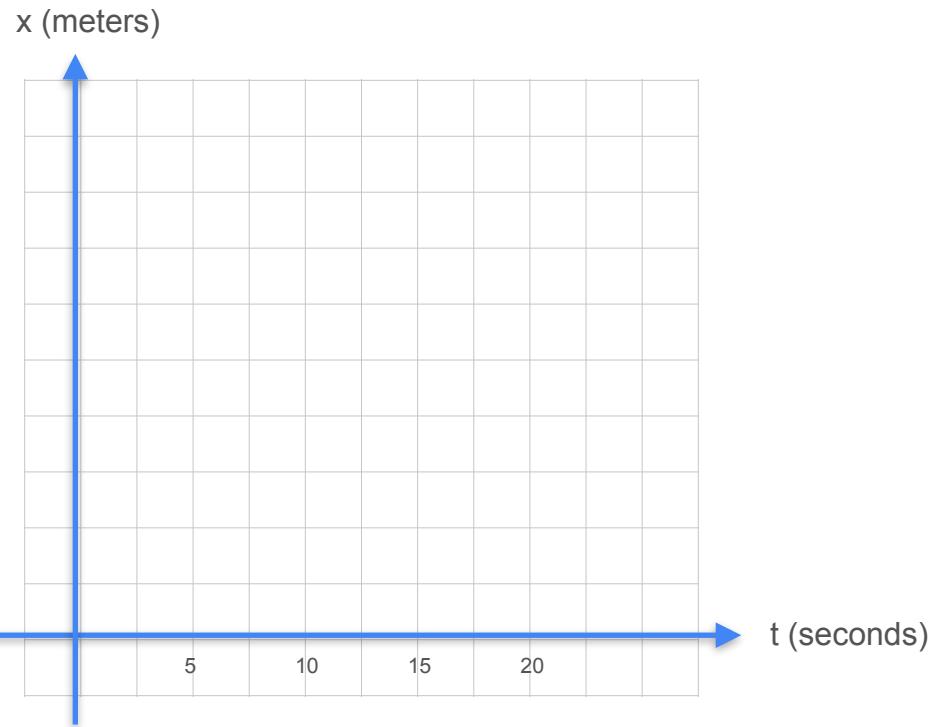
# Quiz : Slope of a Line 1 - Solution

t (seconds)	x (meters)
0	0
5	36
10	122
15	202
20	275
25	351
30	441
35	551
40	591
45	716
50	816
55	900
60	1000

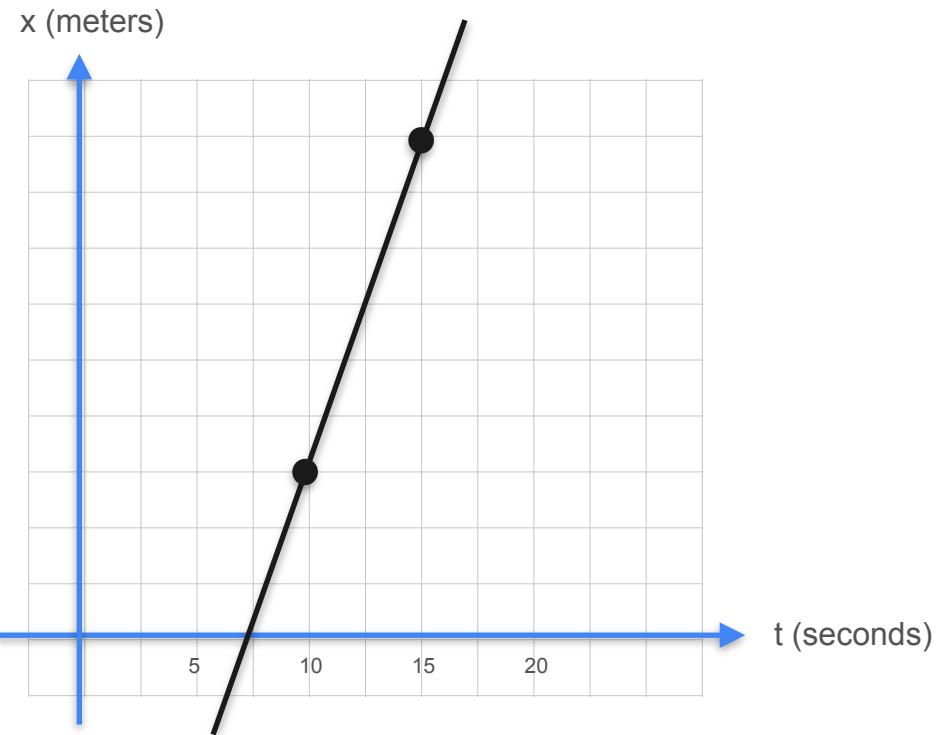
What was the average velocity of the car on the interval from 10 to 15 seconds?

# Calculating the Slope

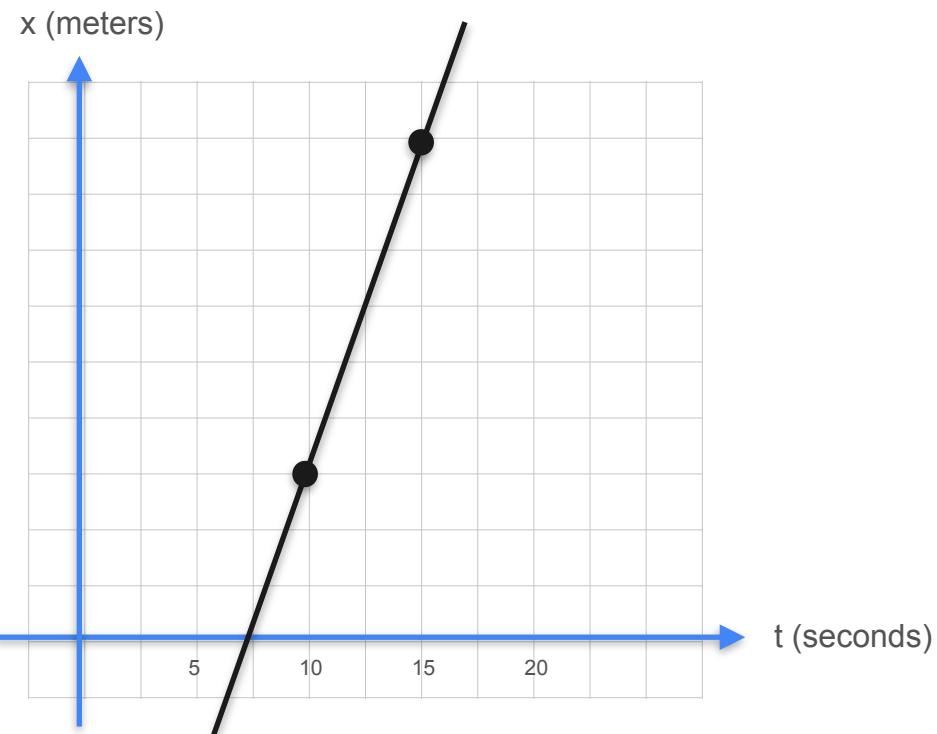
# Calculating the Slope



# Calculating the Slope

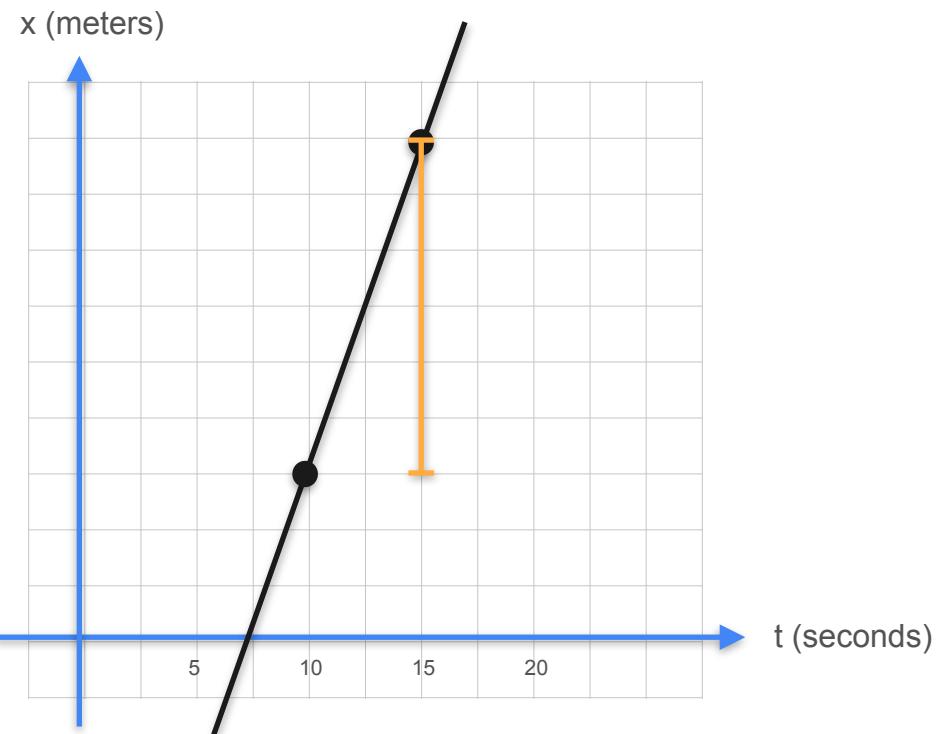


# Calculating the Slope



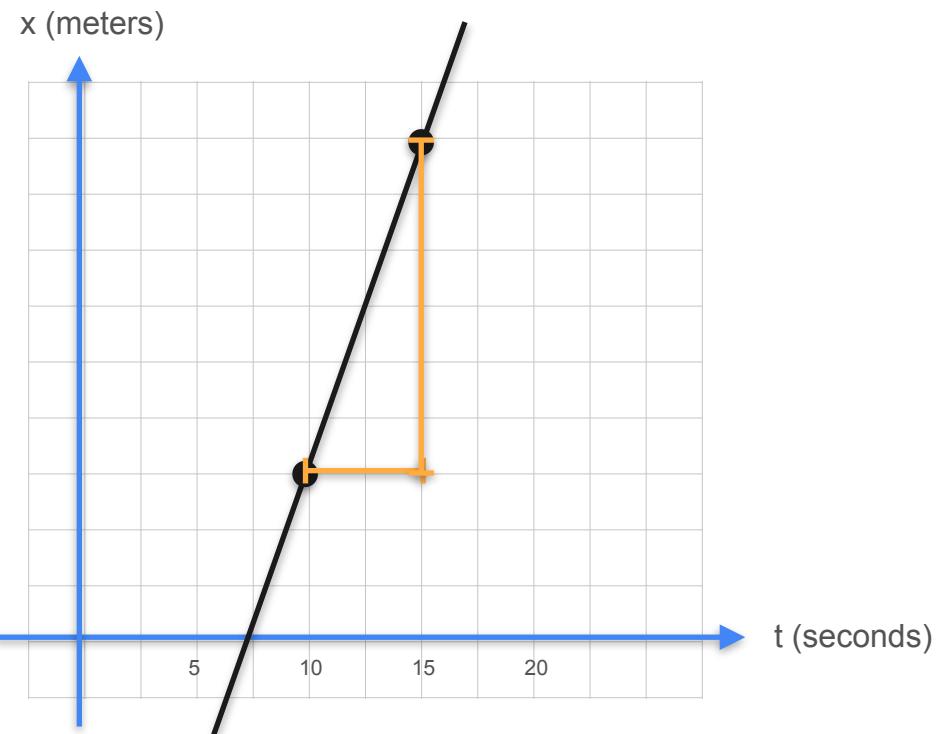
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

# Calculating the Slope



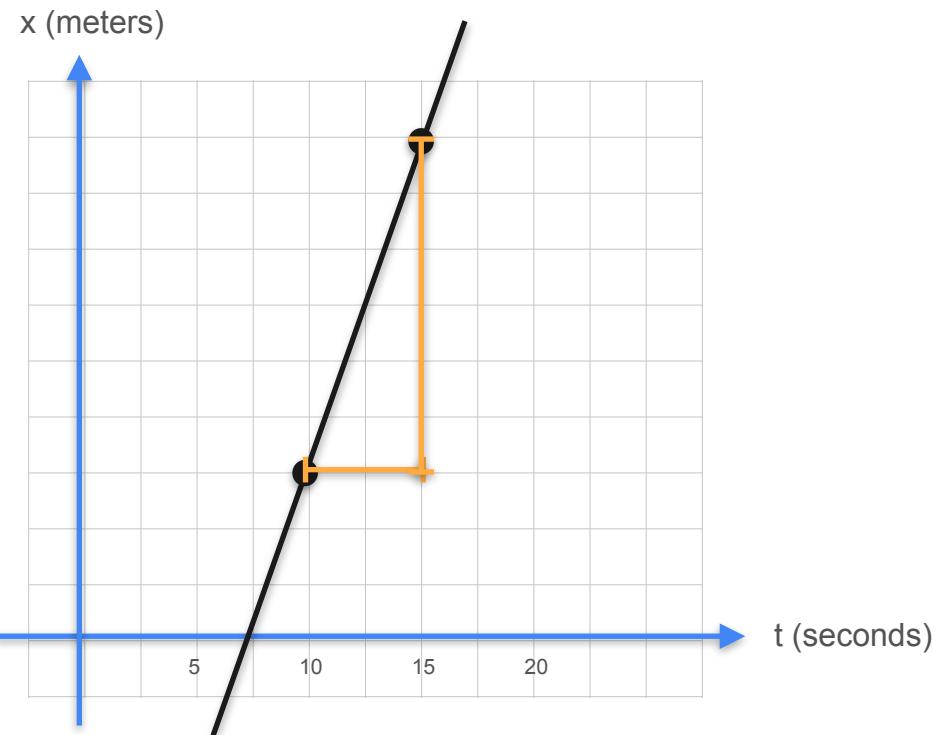
$$slope = \frac{rise}{run}$$

# Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

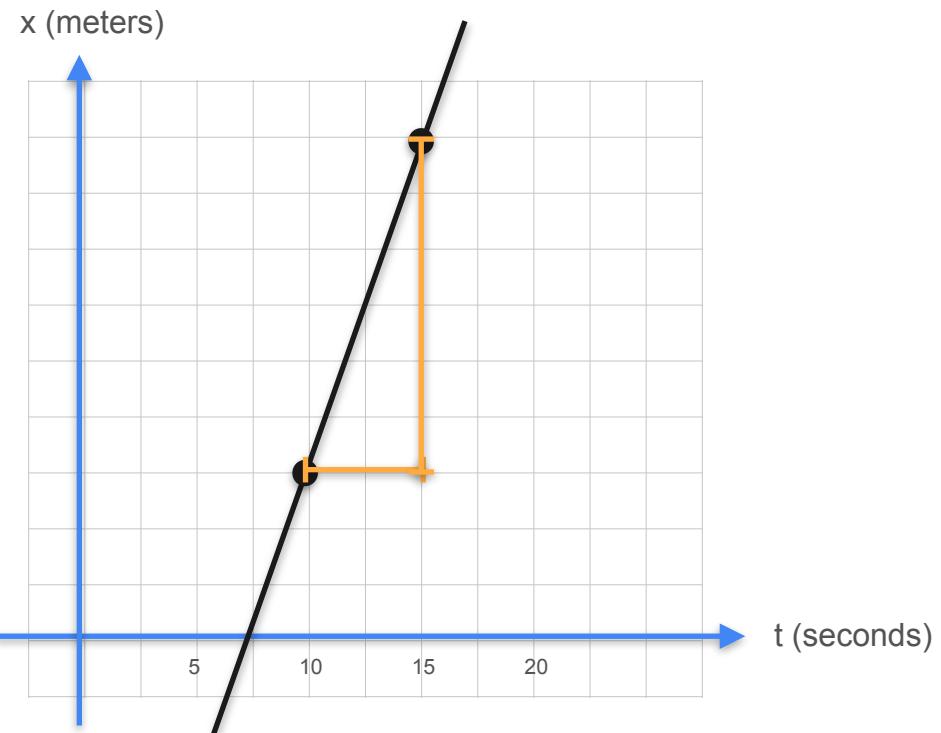

# Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

# Calculating the Slope



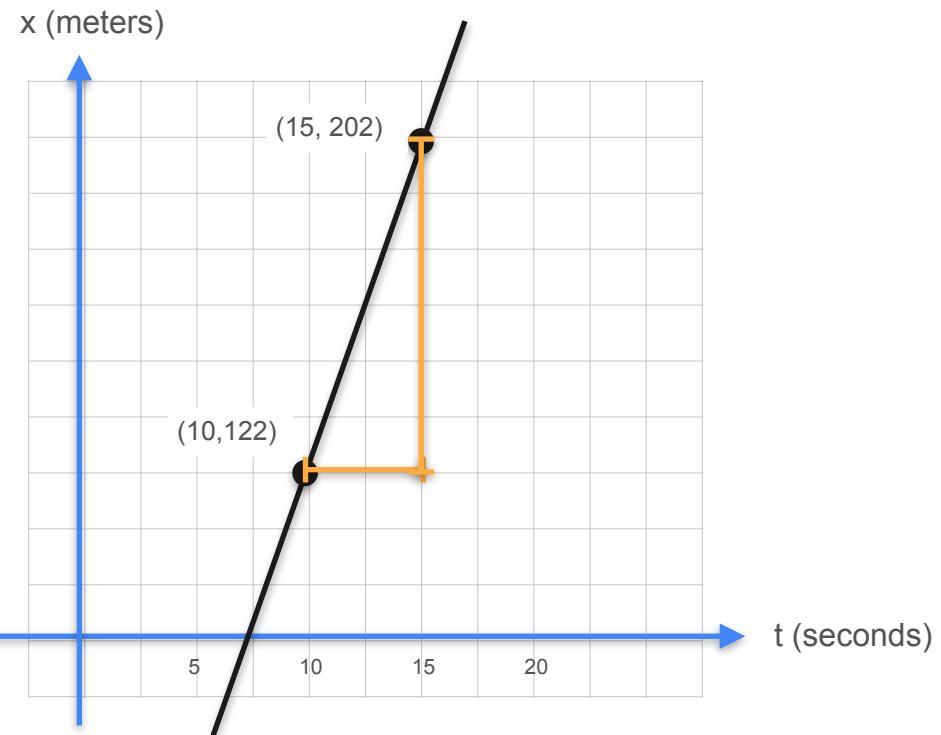
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

A diagram showing a right-angled triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run".

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

# Calculating the Slope

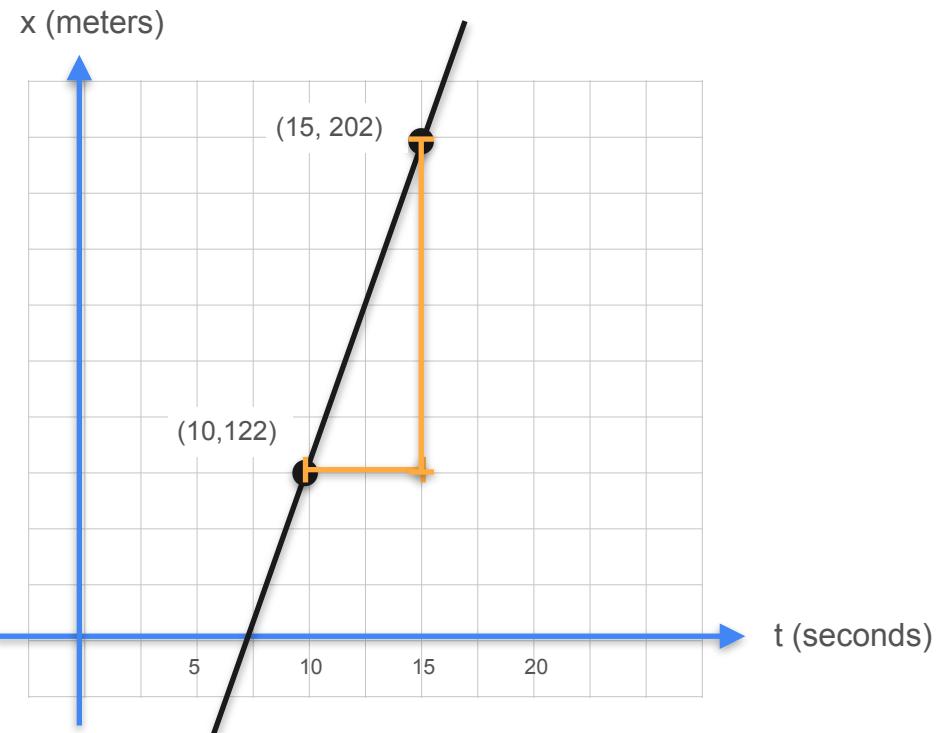


$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

# Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

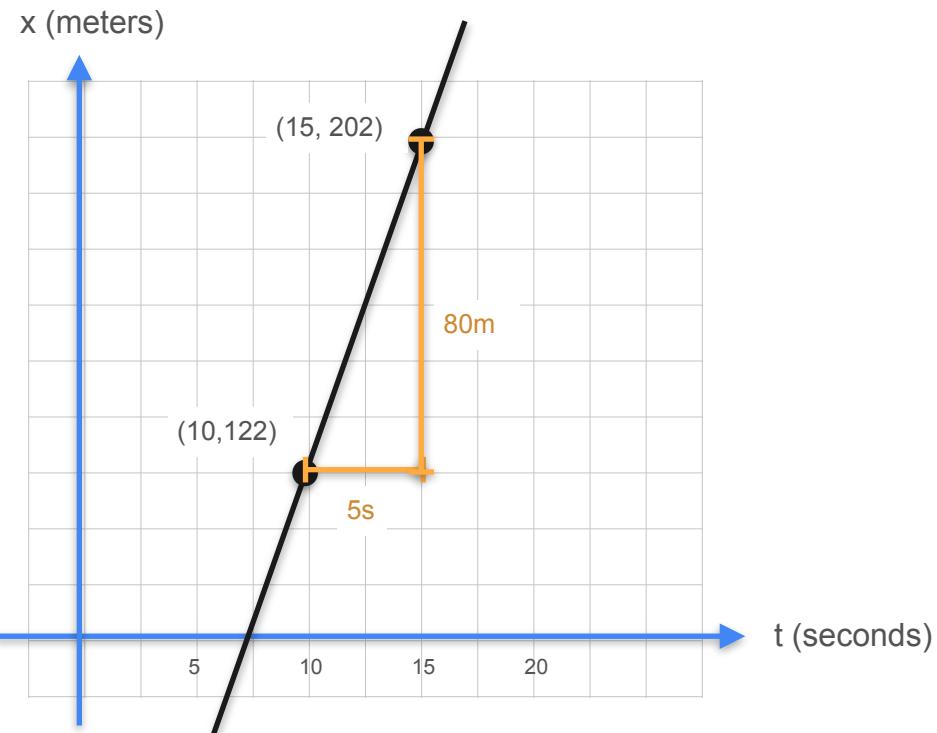
A diagram showing a right triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run".

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

# Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

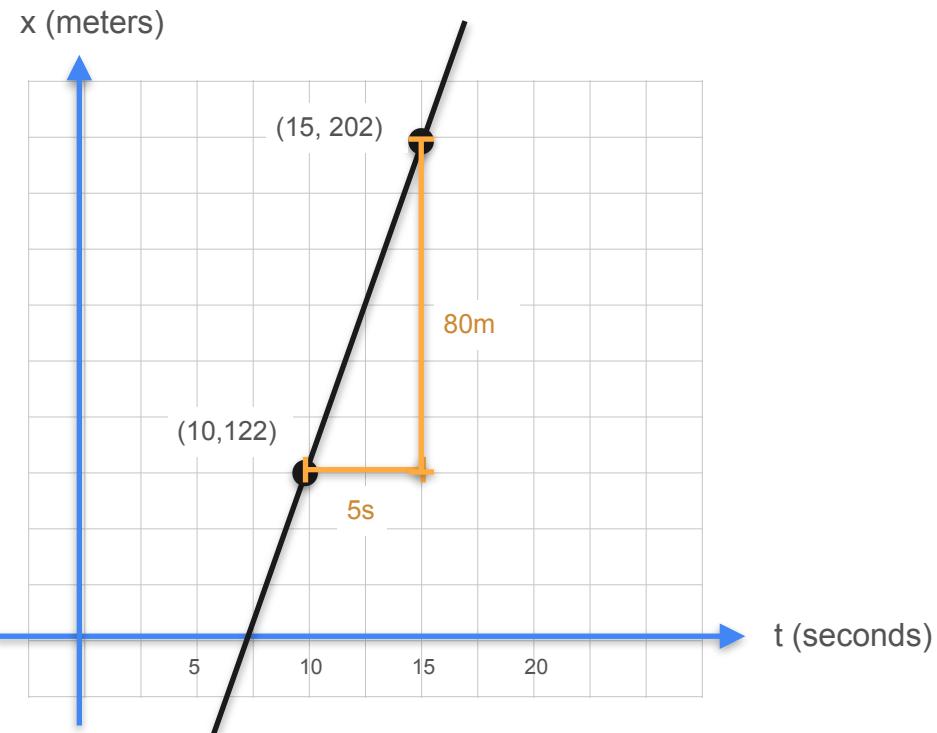
A diagram illustrating the slope formula. It shows a right triangle with a vertical leg labeled "rise" and a horizontal leg labeled "run". Blue arrows point from the labels to the respective sides of the triangle.

$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

# Calculating the Slope



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

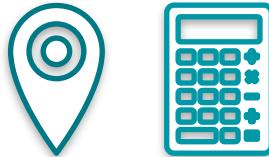
$$\text{slope} = \frac{\text{change in distance}(\Delta x)}{\text{change in time}(\Delta t)}$$

$$\text{slope} = \frac{x(15) - x(10)}{t(15) - t(10)}$$

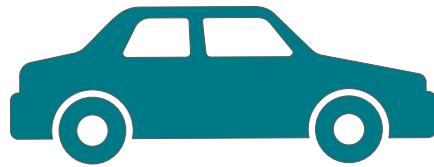
$$\text{slope} = \frac{202m - 122m}{15s - 10s}$$

$$\text{slope} = \frac{80m}{5s} = 16m/s$$

# Introduction to Derivatives

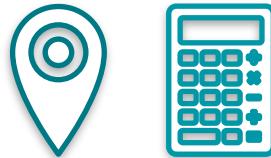


Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

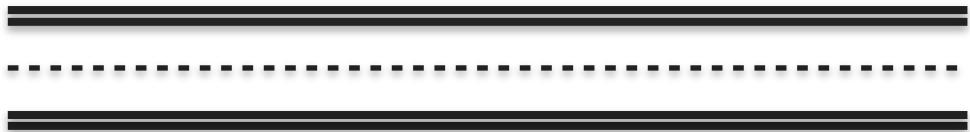
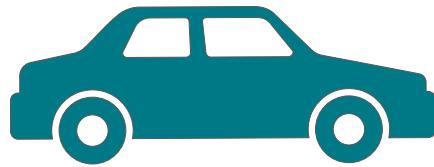


# Introduction to Derivatives

Every second



Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?



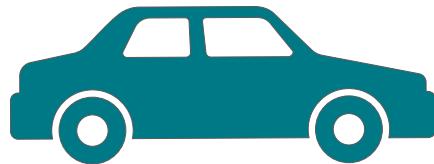
# Introduction to Derivatives

Every second

t (s)
10
11
12
13
14
15
16
17
18
19
20



Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?



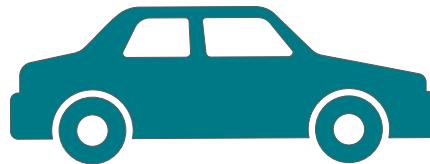
# Introduction to Derivatives

Every second

t (s)	x(m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265



Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?



# Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

# Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

# Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

# Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

$$\text{slope} = \frac{170m - 155m}{1s}$$

# Quiz : Slope of a Line 2

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202
16	218
17	234
18	250
19	265
20	265

Can you find a good estimate of the velocity of the car at time  $t = 12.5$  seconds using this data?

$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(13) - x(12)}{13s - 12s}$$

$$\text{slope} = \frac{170m - 155m}{1s}$$

$$\text{slope} = 15m/s$$



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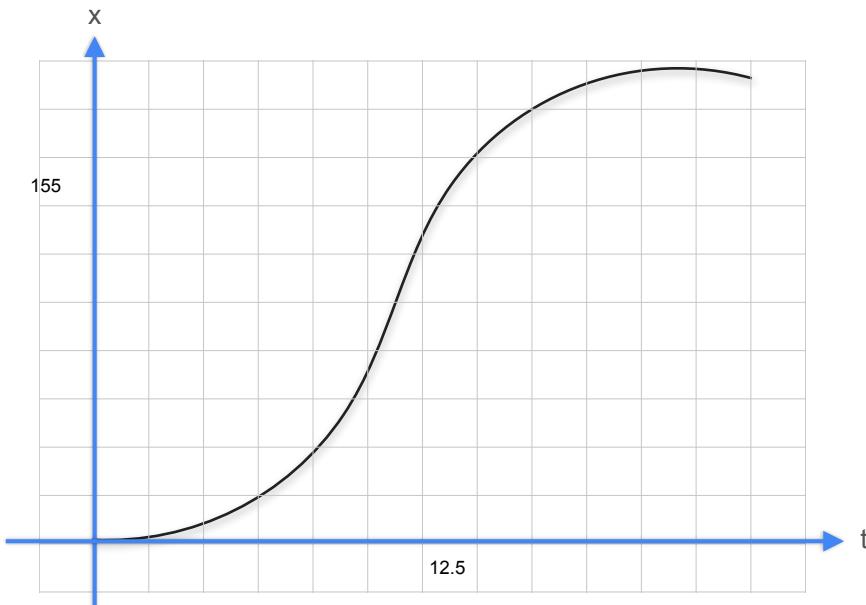
# Derivatives and Optimization

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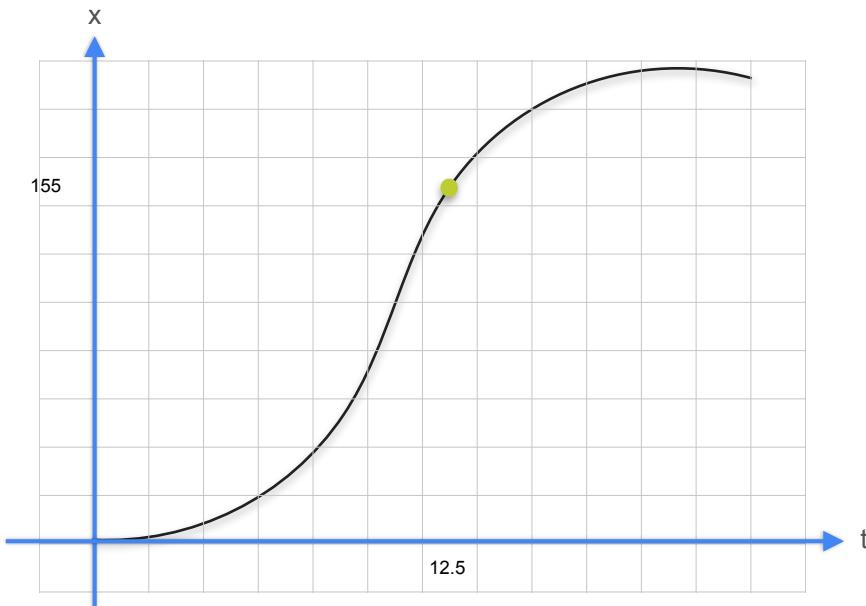
## Derivatives and tangents

# The derivative

# The derivative

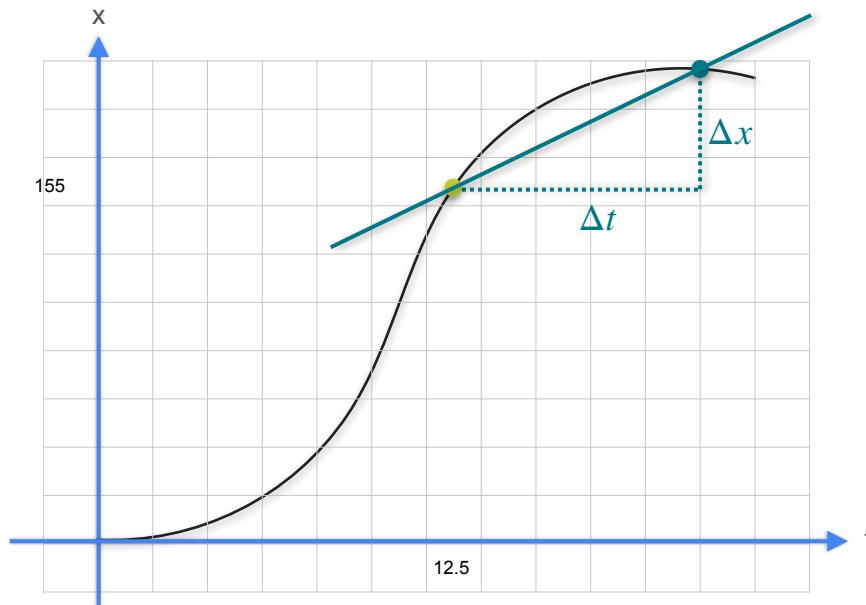


# The derivative



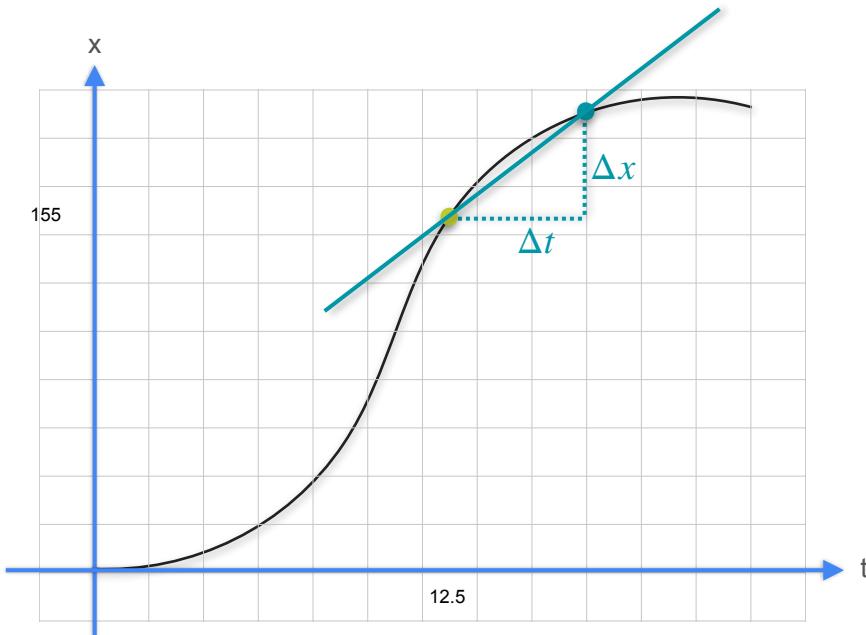
# The derivative

$$\frac{\Delta x}{\Delta t}$$



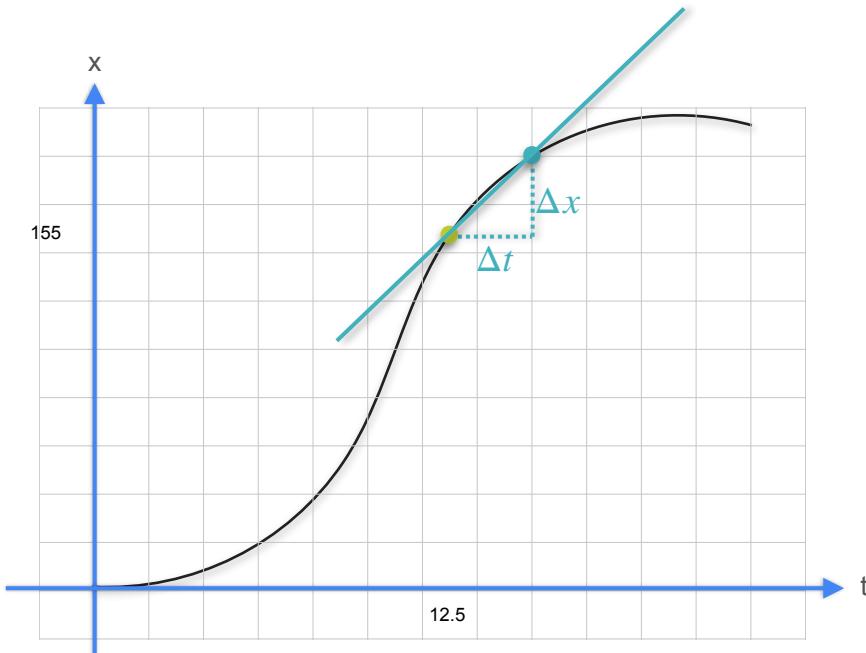
# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



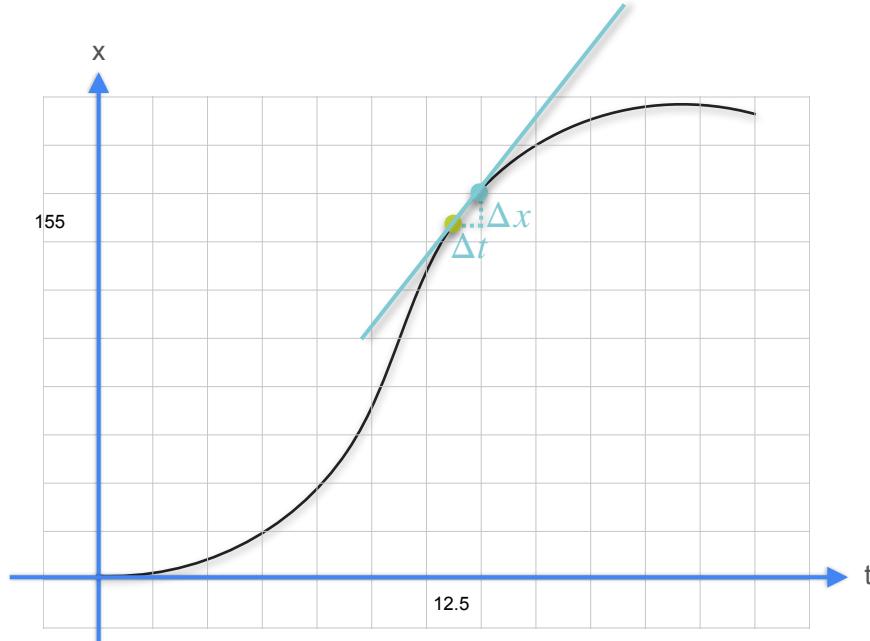
# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



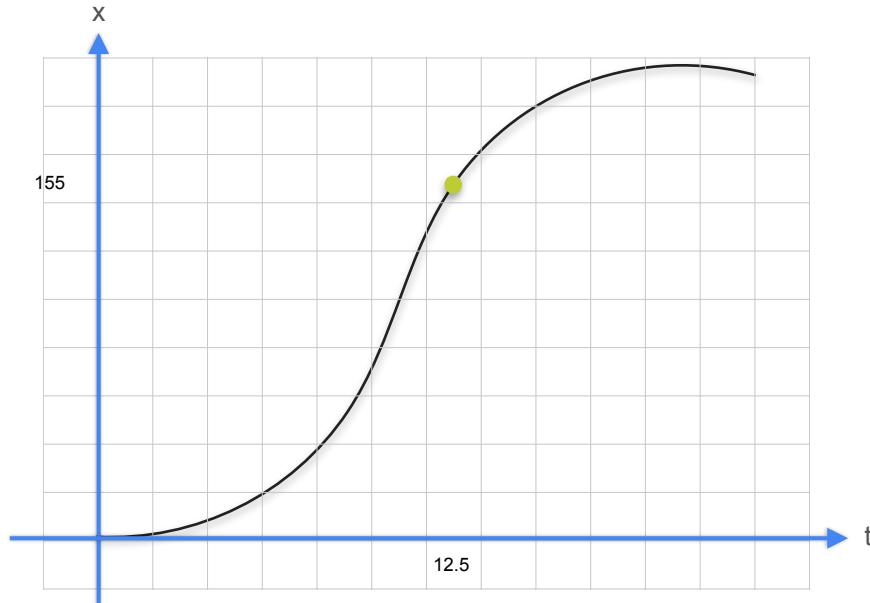
# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t}$$



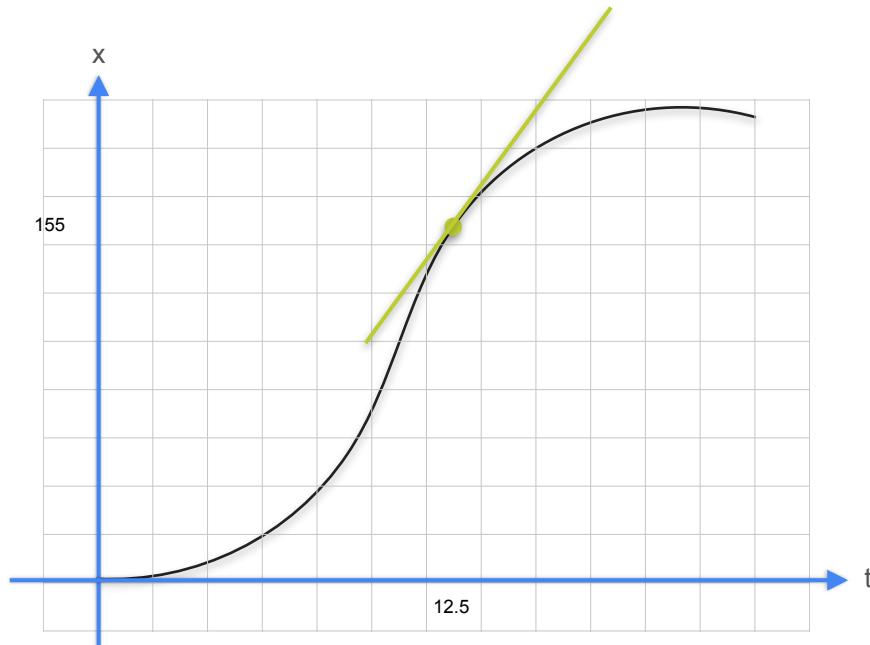
# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



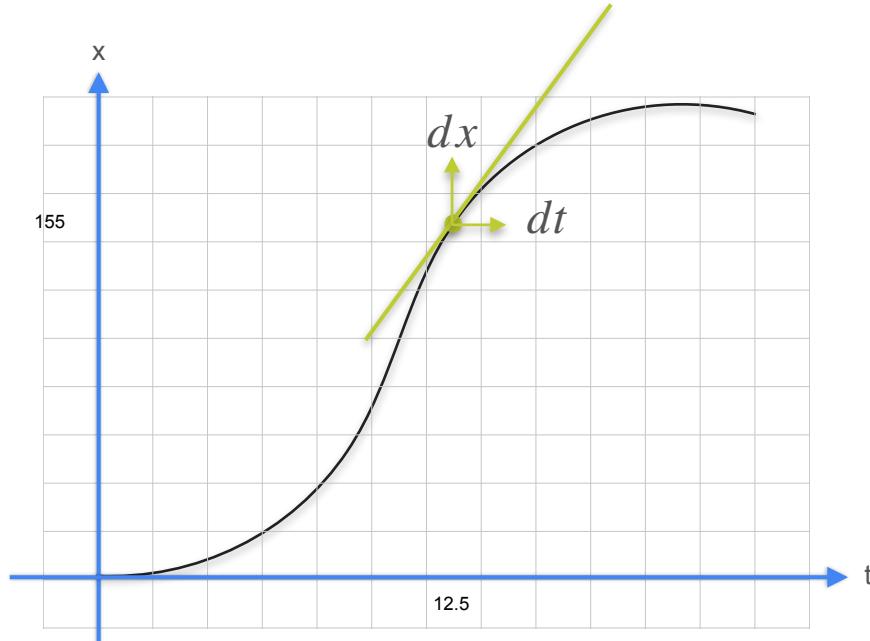
# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



# The derivative

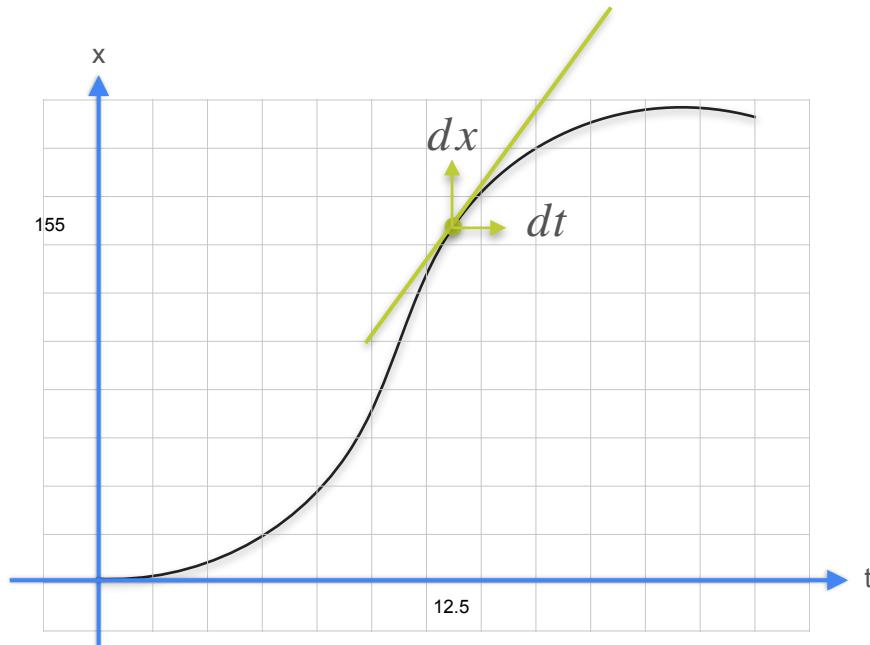
$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$



# The derivative

$$\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \frac{dx}{dt}$$

Derivative





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# Derivatives and Optimization

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## Slopes, maxima, and minima

# Zero Slope

t (s)	x (m)
19	265
20	265

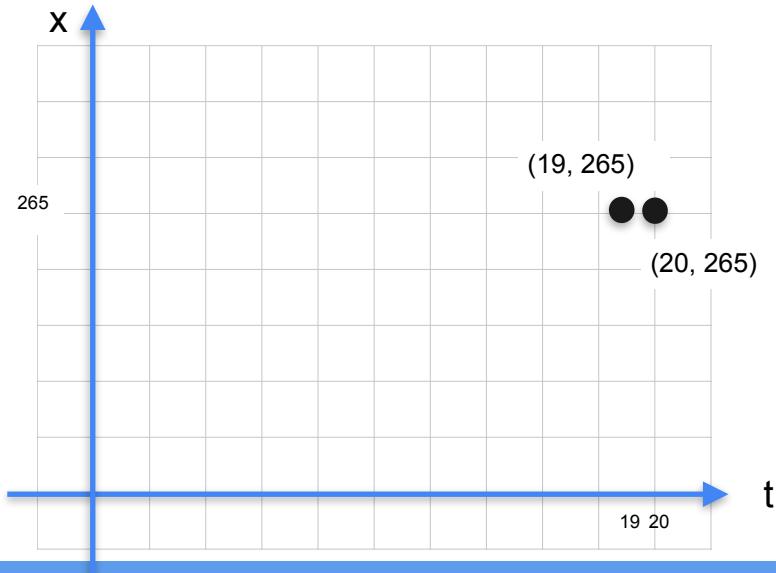
# Zero Slope

t (s)	x (m)
19	265
20	265



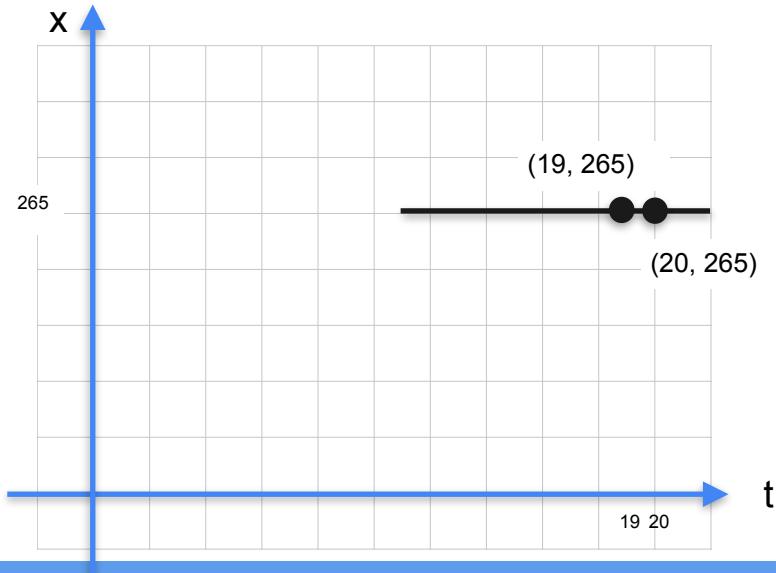
# Zero Slope

$t$ (s)	$x$ (m)
19	265
20	265



# Zero Slope

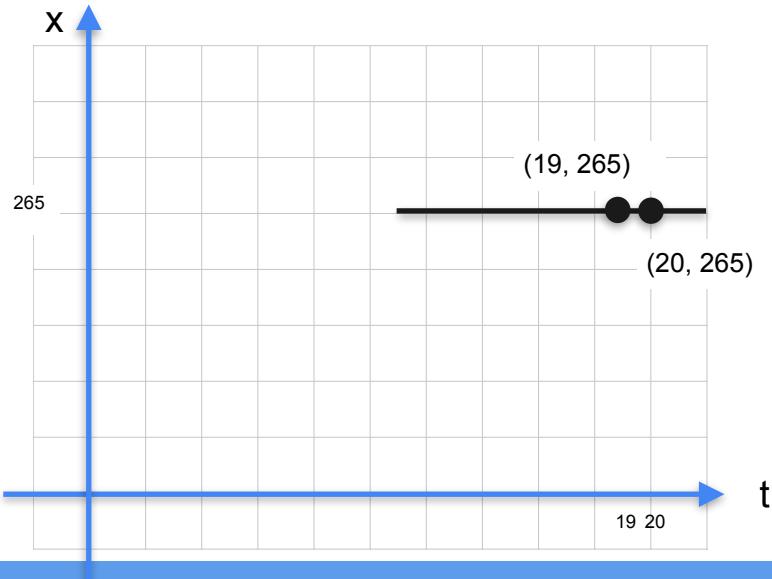
$t$ (s)	$x$ (m)
19	265
20	265



# Zero Slope

t (s)	x (m)
19	265
20	265

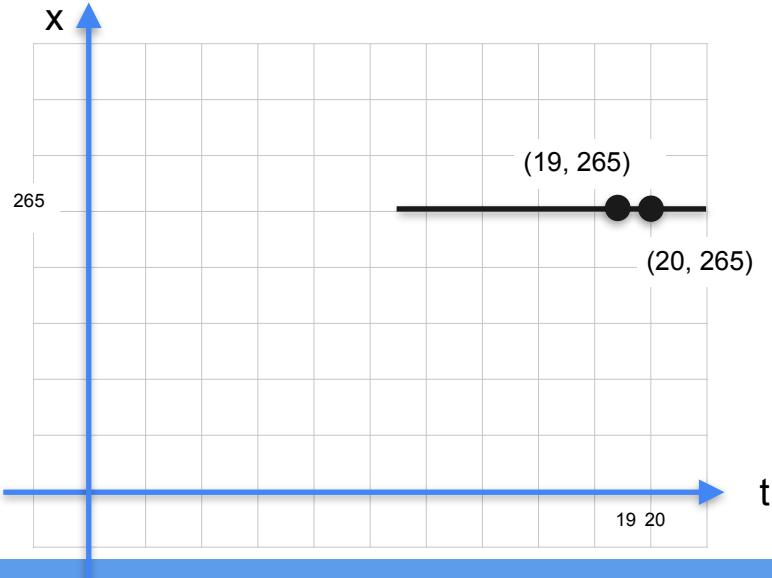
$$slope = \frac{\Delta x}{\Delta t}$$



# Zero Slope

t (s)	x (m)
19	265
20	265

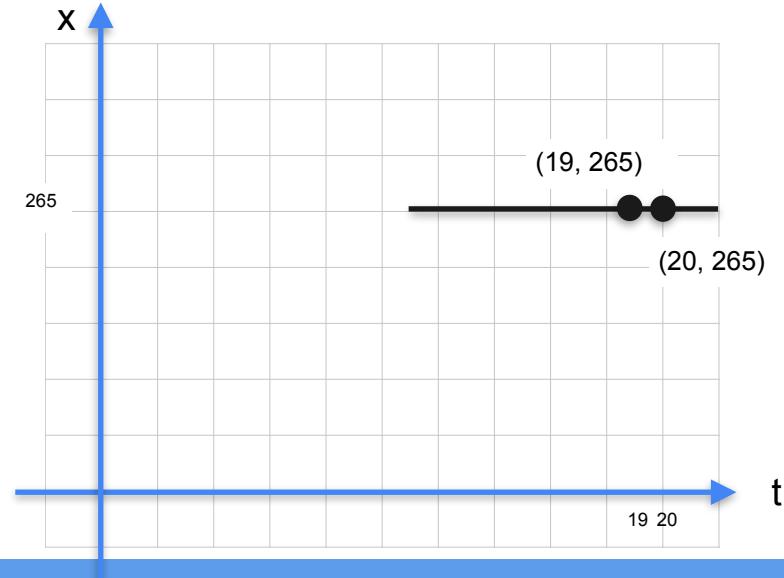
$$slope = \frac{\Delta x}{\Delta t}$$



$$slope = \frac{x(20) - x(19)}{20 - 19}$$

# Zero Slope

t (s)	x (m)
19	265
20	265



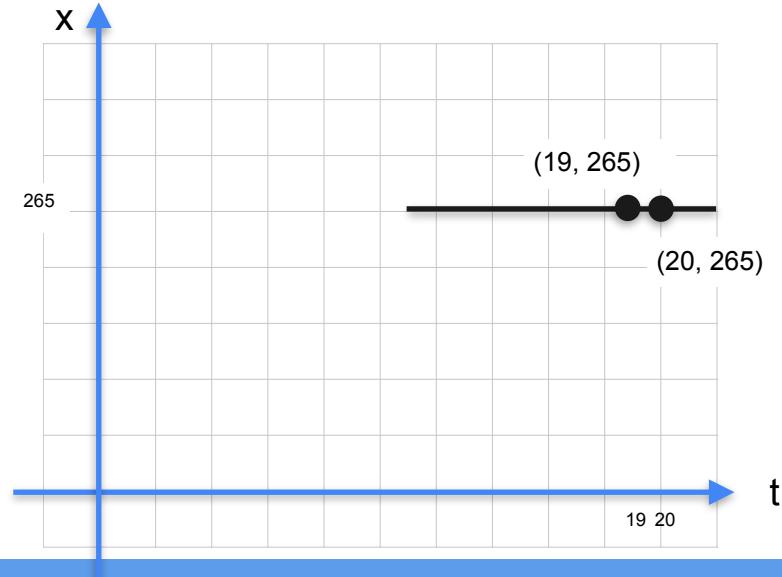
$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(20) - x(19)}{20 - 19}$$

$$\text{slope} = \frac{265m - 265m}{1s}$$

# Zero Slope

t (s)	x (m)
19	265
20	265



$$slope = \frac{\Delta x}{\Delta t}$$

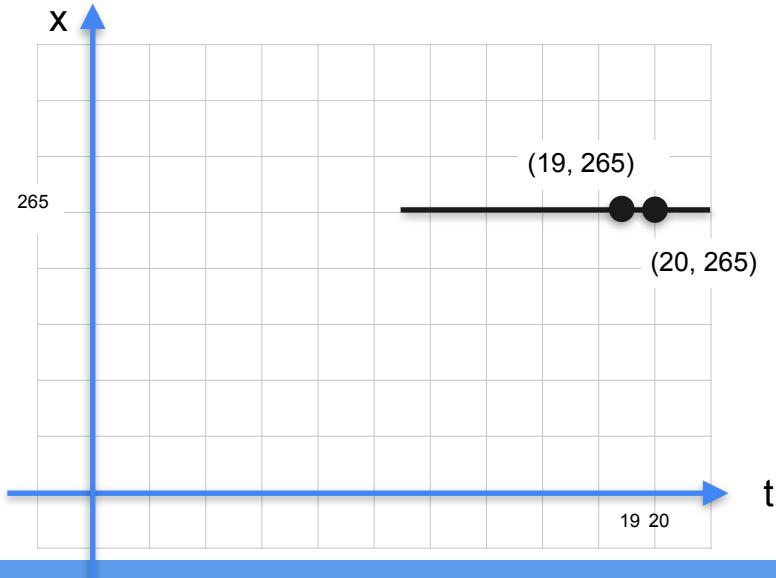
$$slope = \frac{x(20) - x(19)}{20 - 19}$$

$$slope = \frac{265m - 265m}{1s}$$

$$slope = \frac{0}{1}$$

# Zero Slope

t (s)	x (m)
19	265
20	265



$$\text{slope} = \frac{\Delta x}{\Delta t}$$

$$\text{slope} = \frac{x(20) - x(19)}{20 - 19}$$

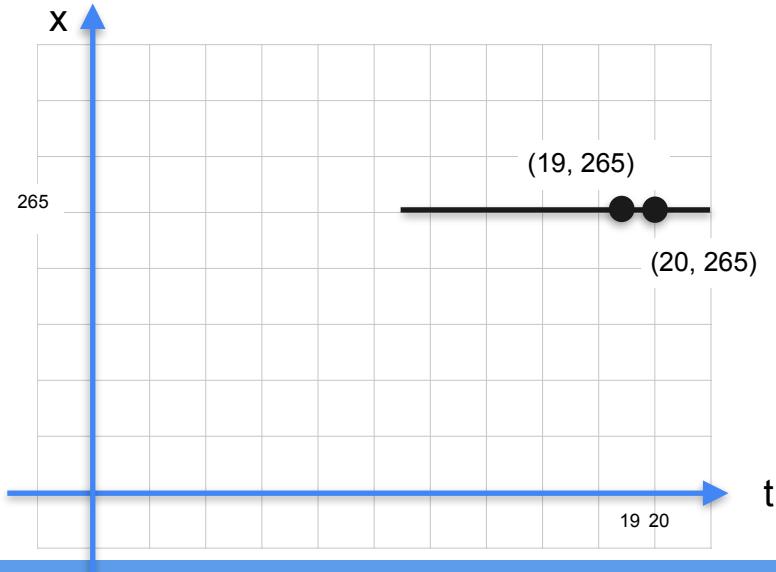
$$\text{slope} = \frac{265m - 265m}{1s}$$

$$\text{slope} = \frac{0}{1}$$

No rise in distance

# Zero Slope

t (s)	x (m)
19	265
20	265



$$\text{slope} = \frac{\Delta x}{\Delta t}$$

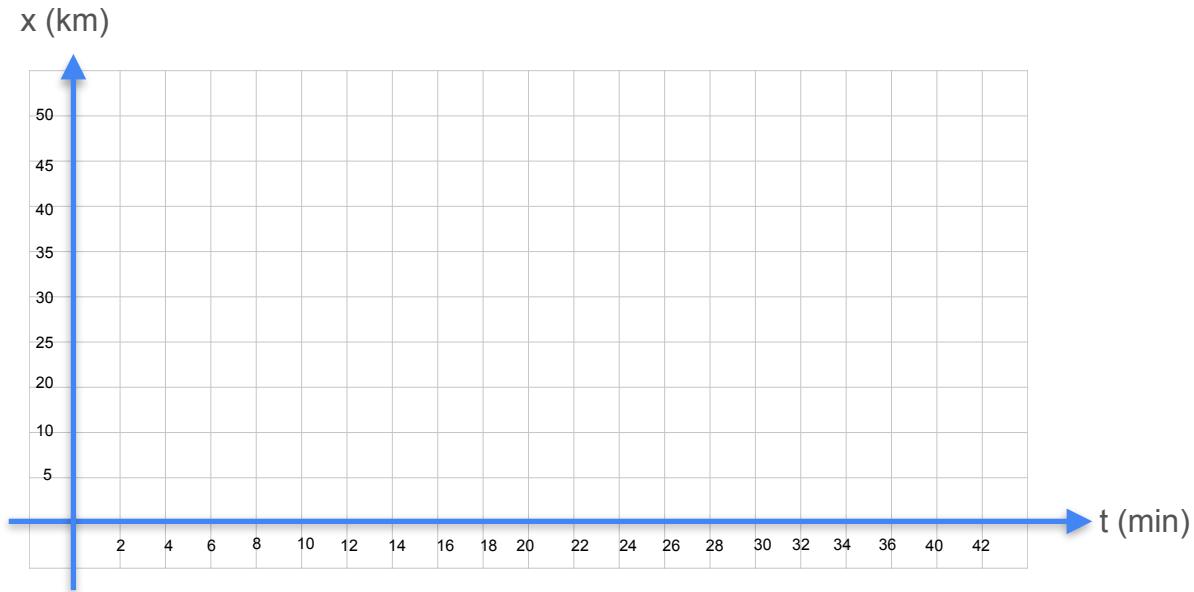
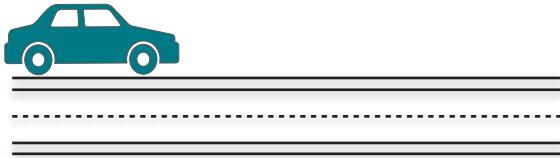
$$\text{slope} = \frac{x(20) - x(19)}{20 - 19}$$

$$\text{slope} = \frac{265m - 265m}{1s}$$

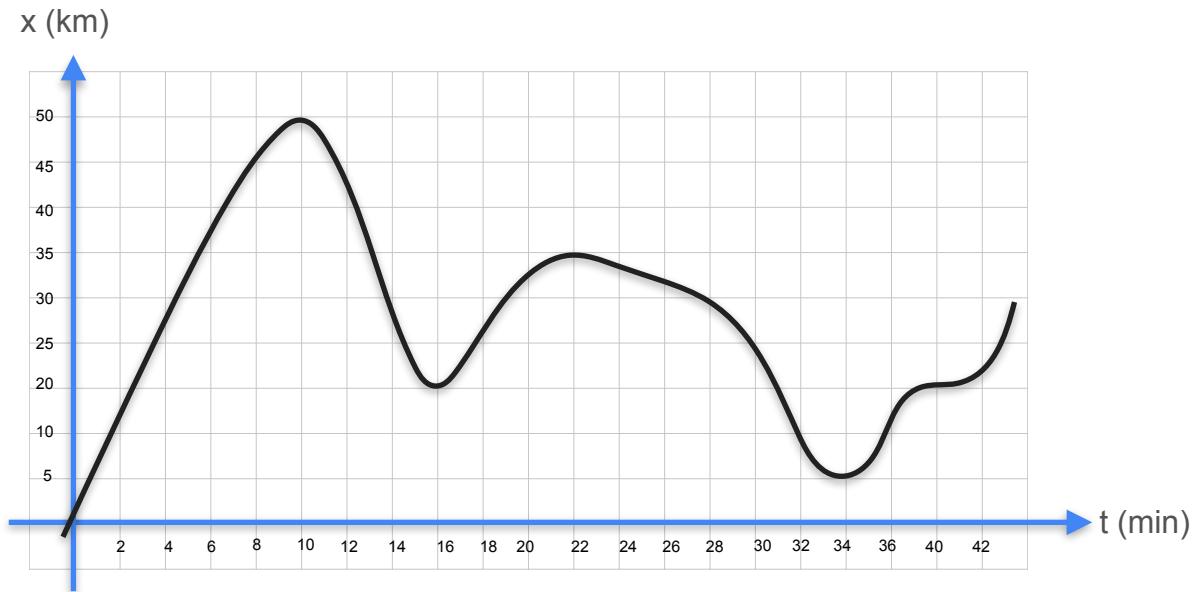
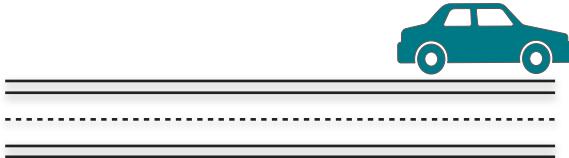
$$\text{slope} = \frac{0}{1} \quad \text{No rise in distance}$$

$$\text{slope} = 0 \text{ m/s}$$

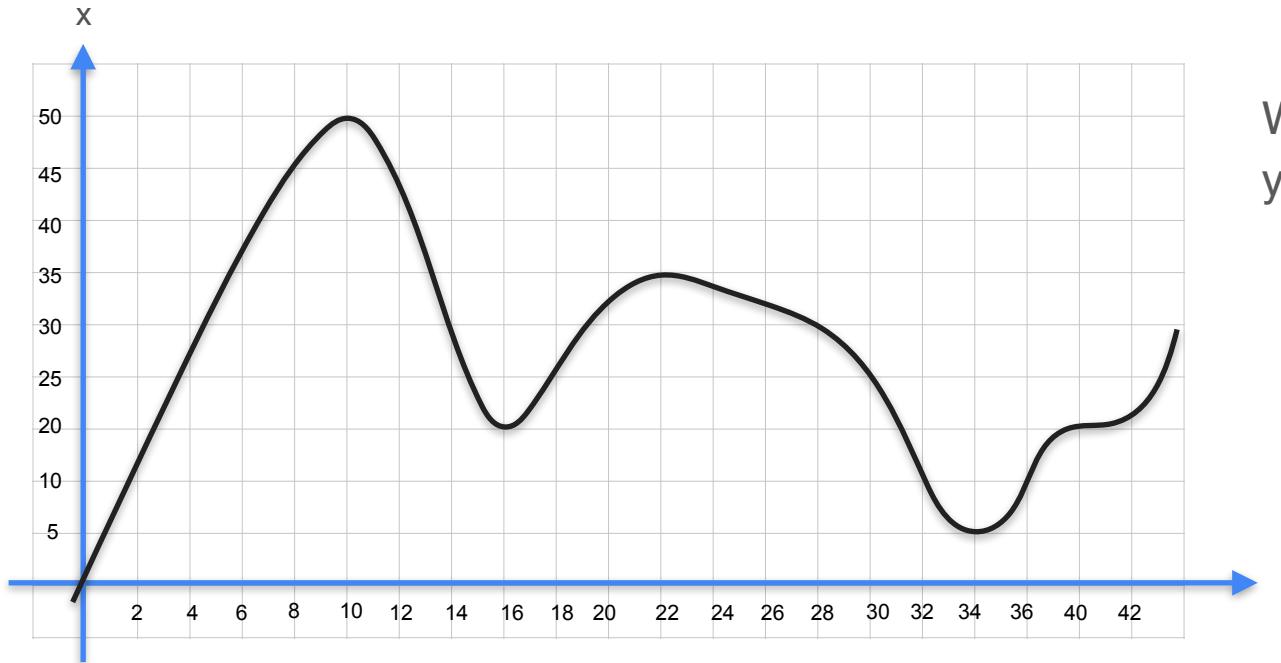
# Minima and Maxima



# Minima and Maxima

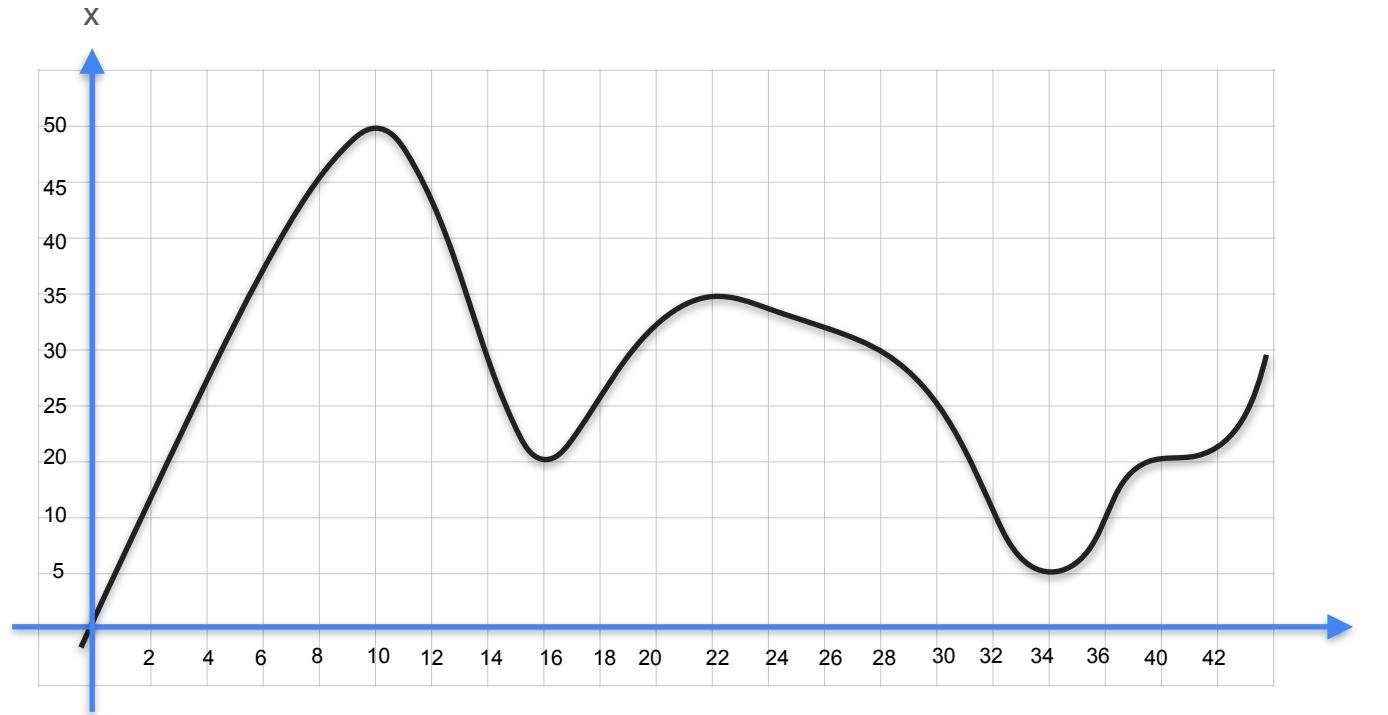


# Quiz : Minima and Maxima 1

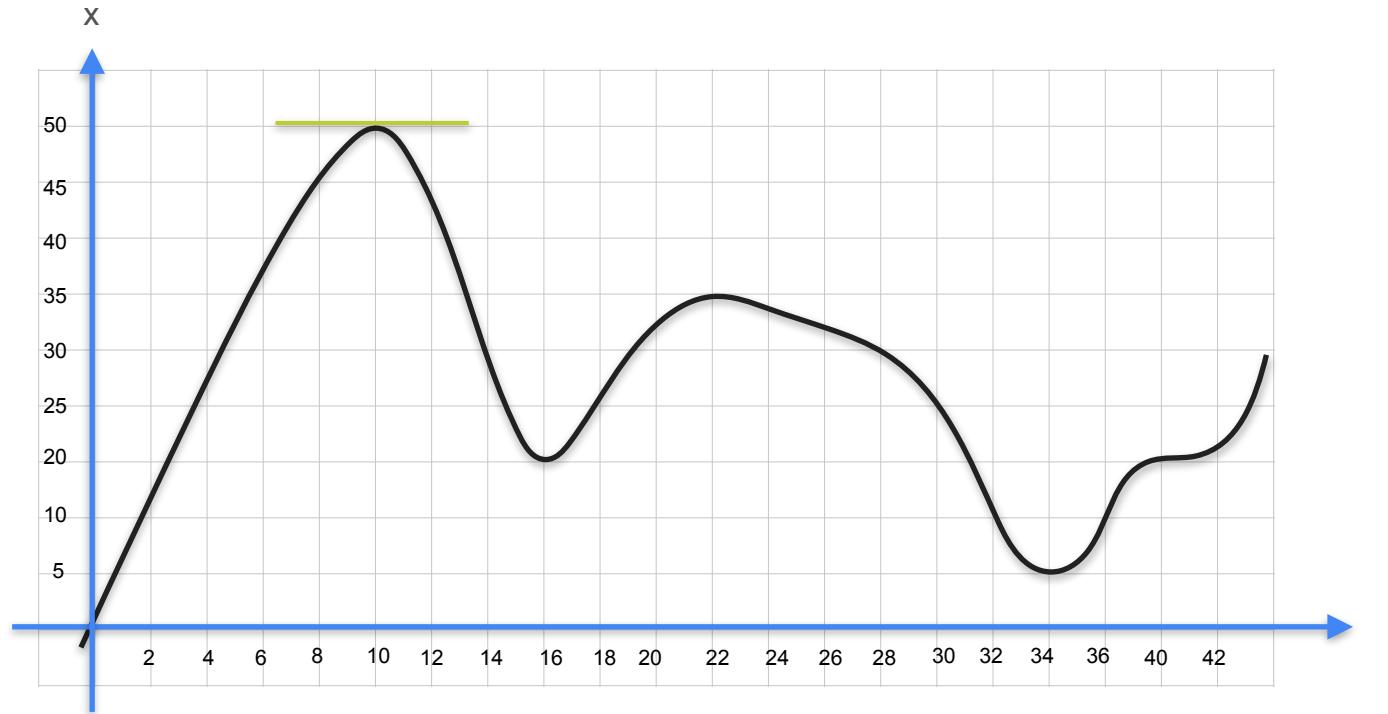


Where was the velocity of your car zero?

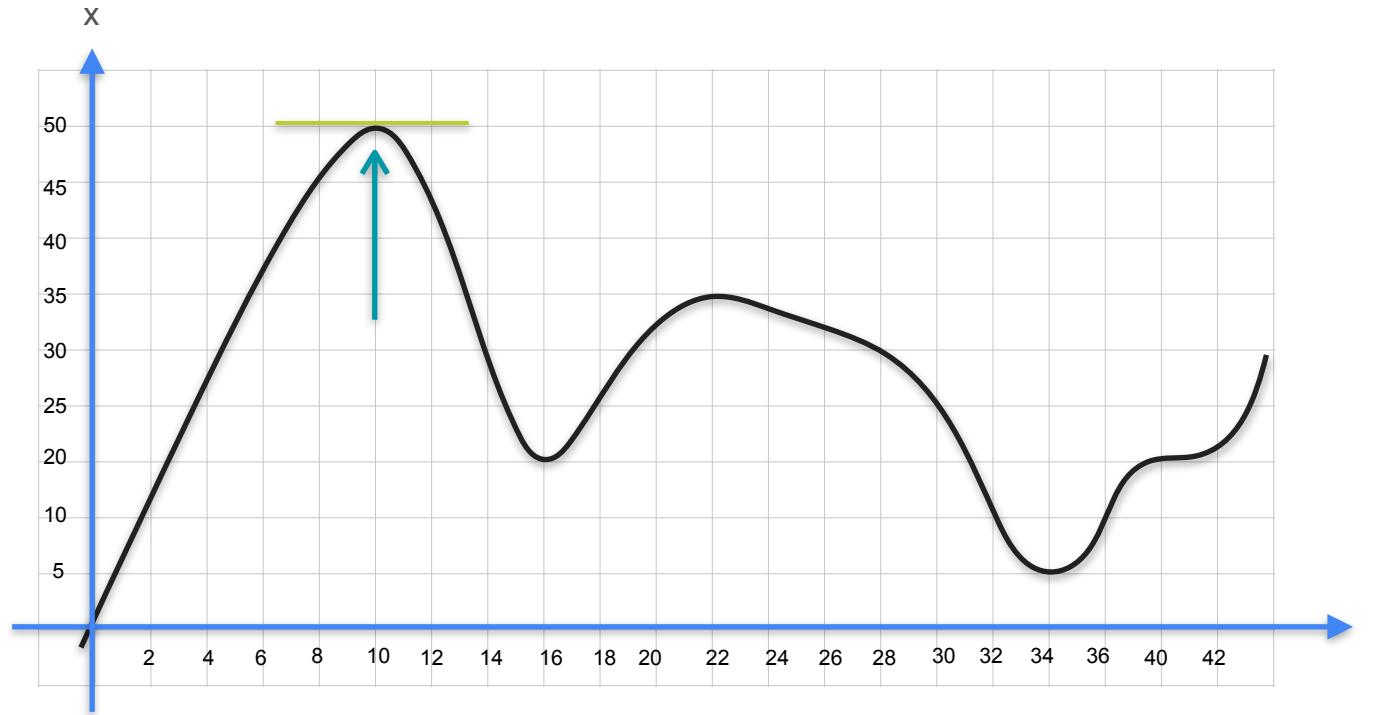
# Quiz : Minima and Maxima 1 - Solution



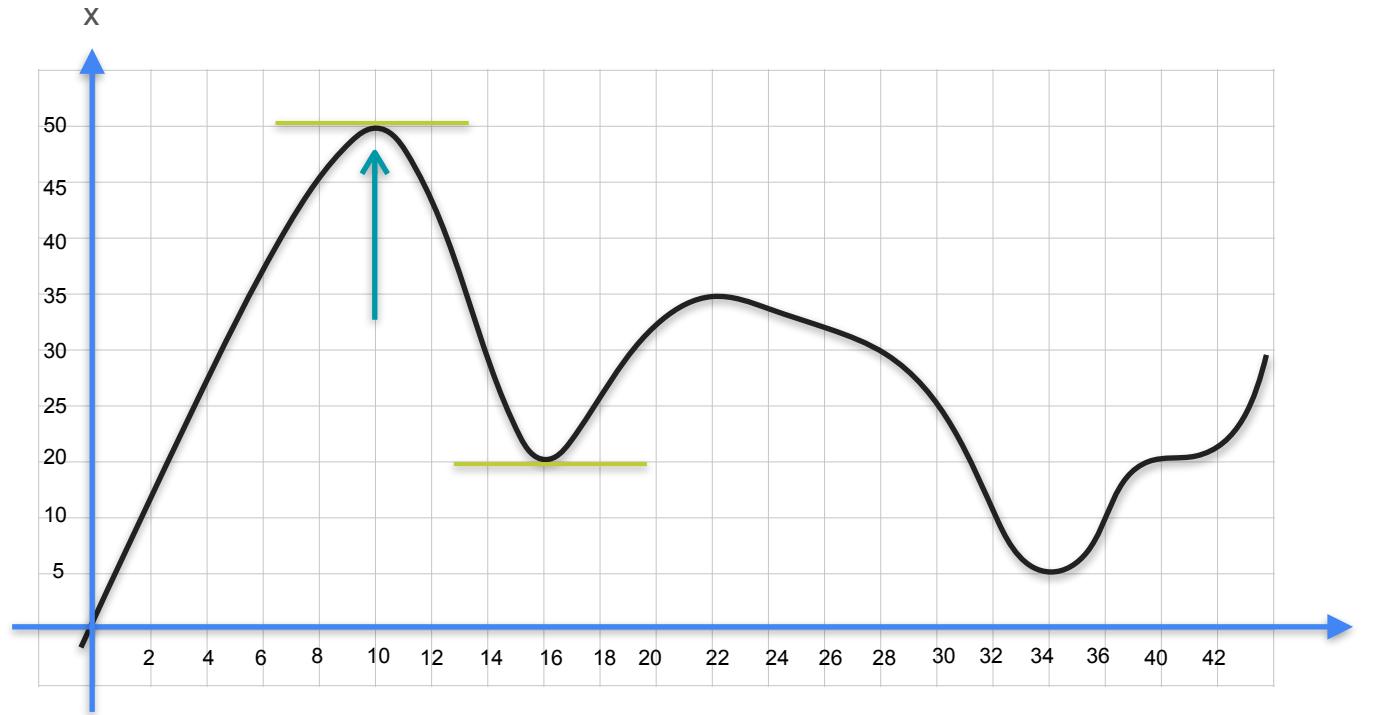
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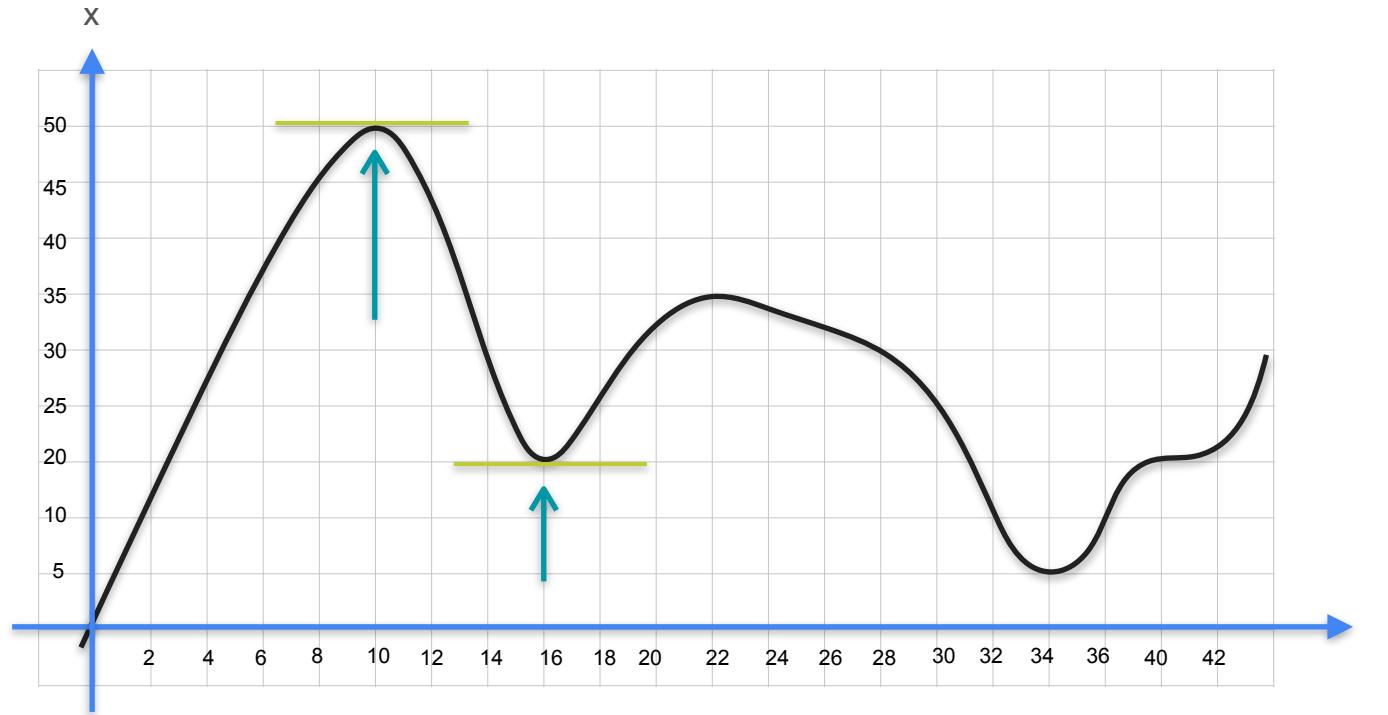
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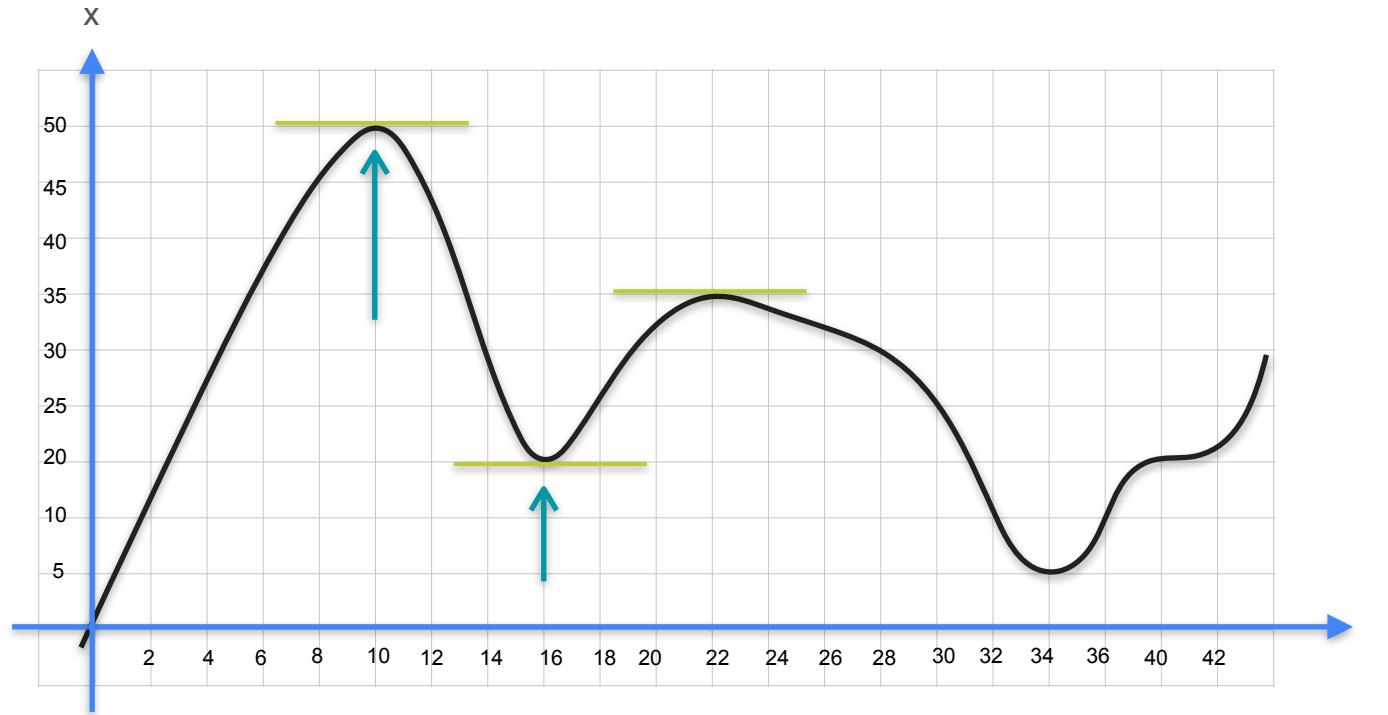
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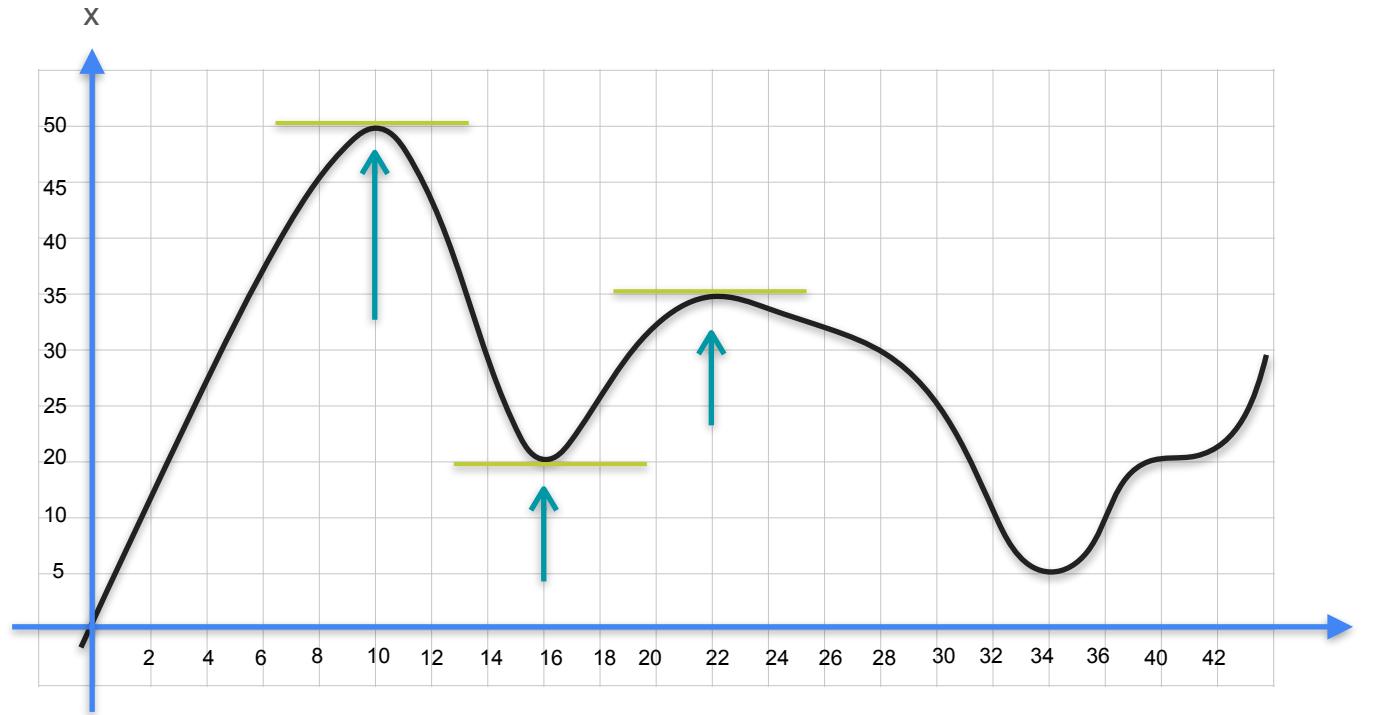
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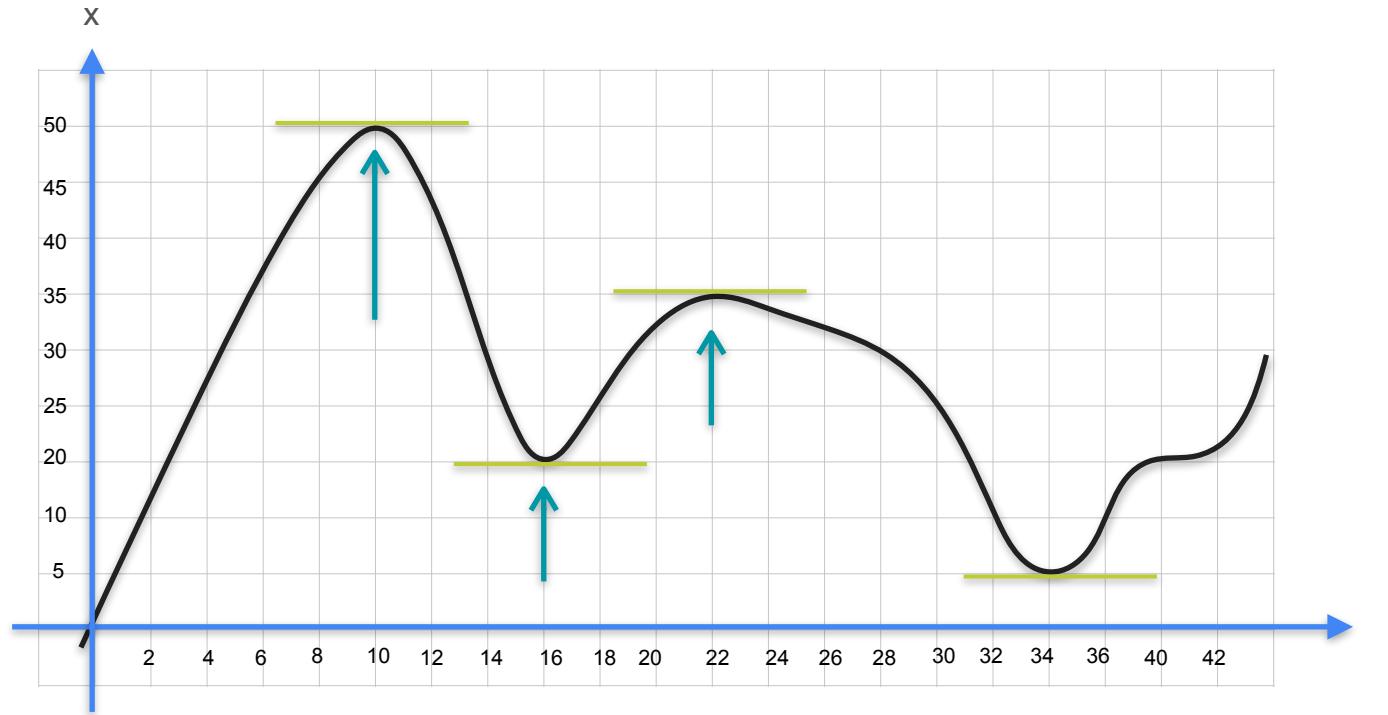
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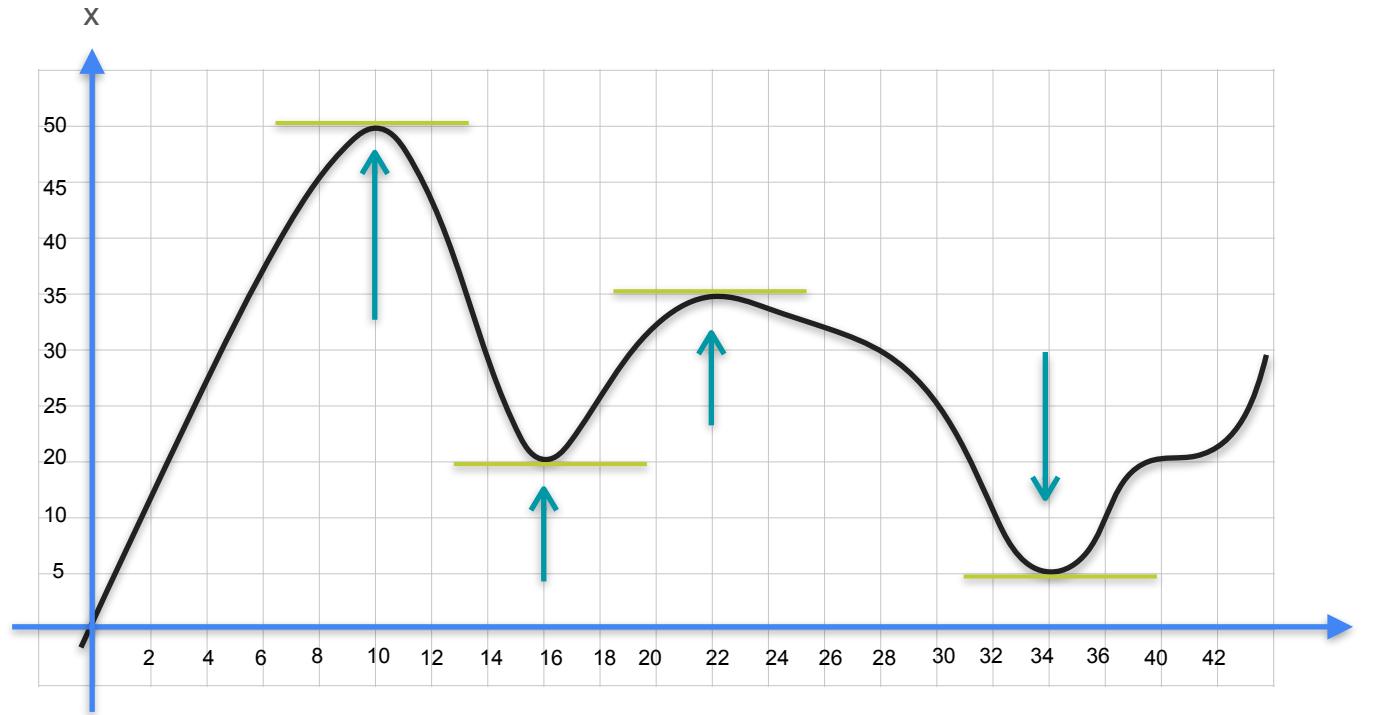
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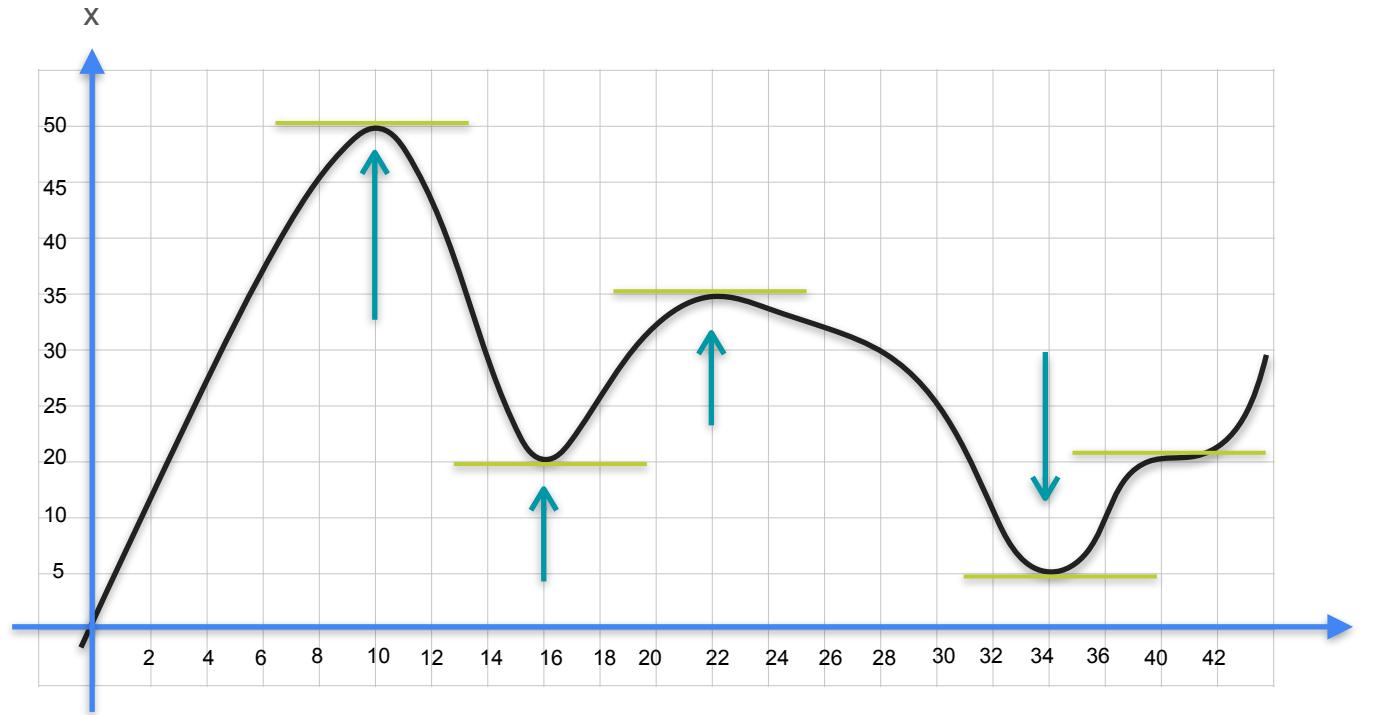
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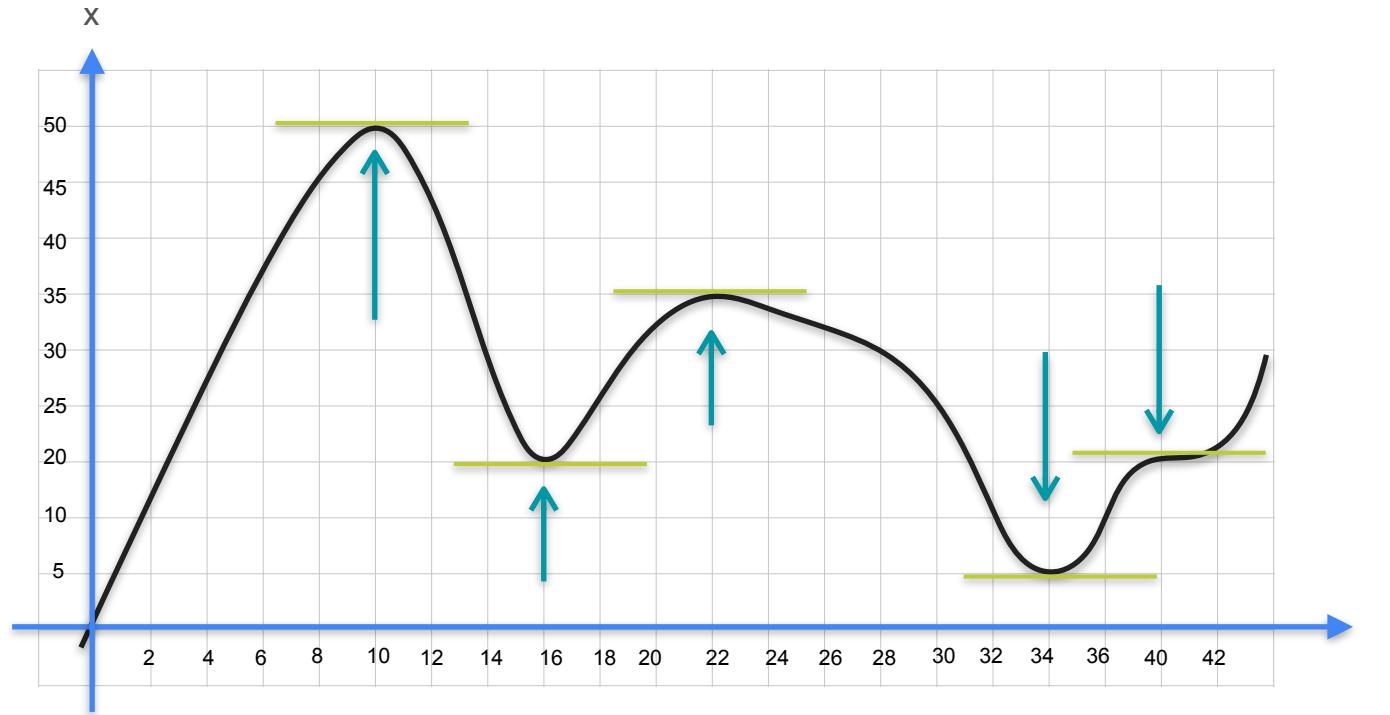
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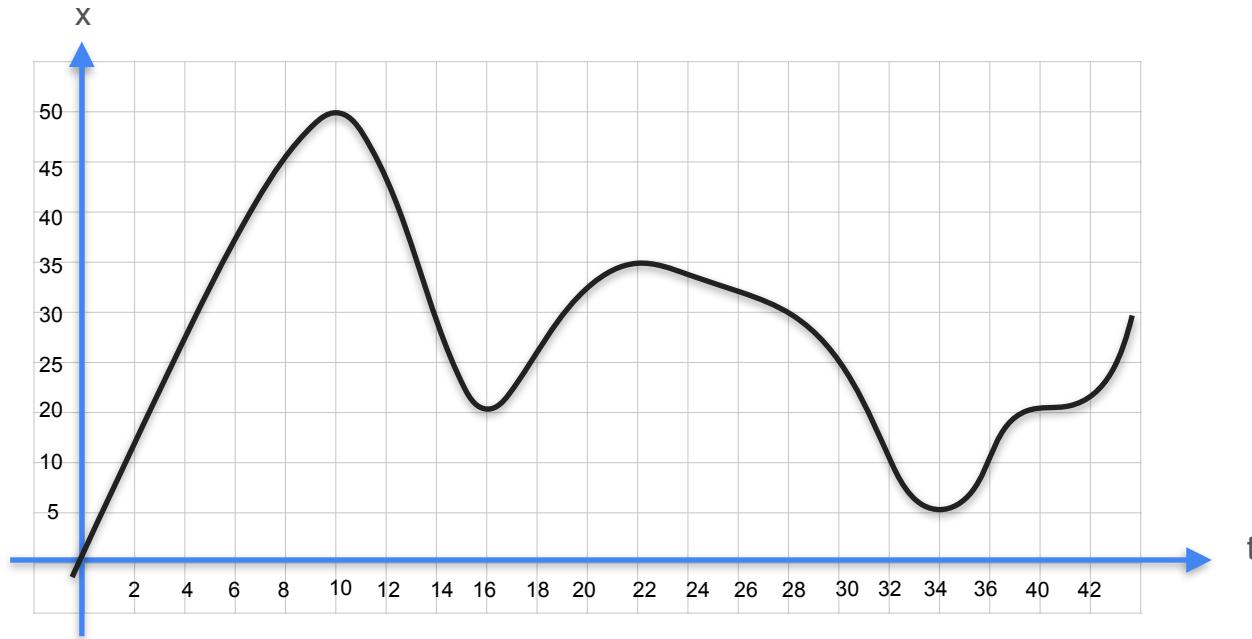
# Quiz : Minima and Maxima 1 - Solution



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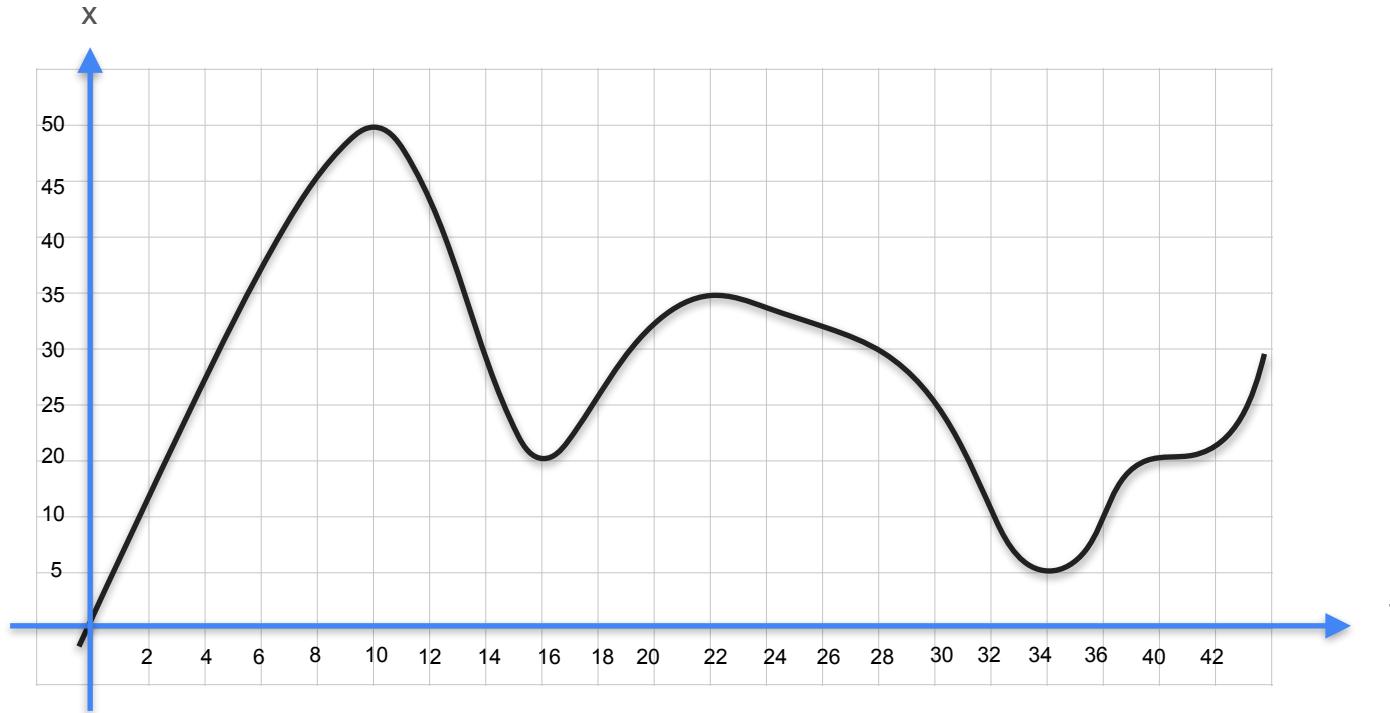


# Quiz : Concept of Derivatives 3

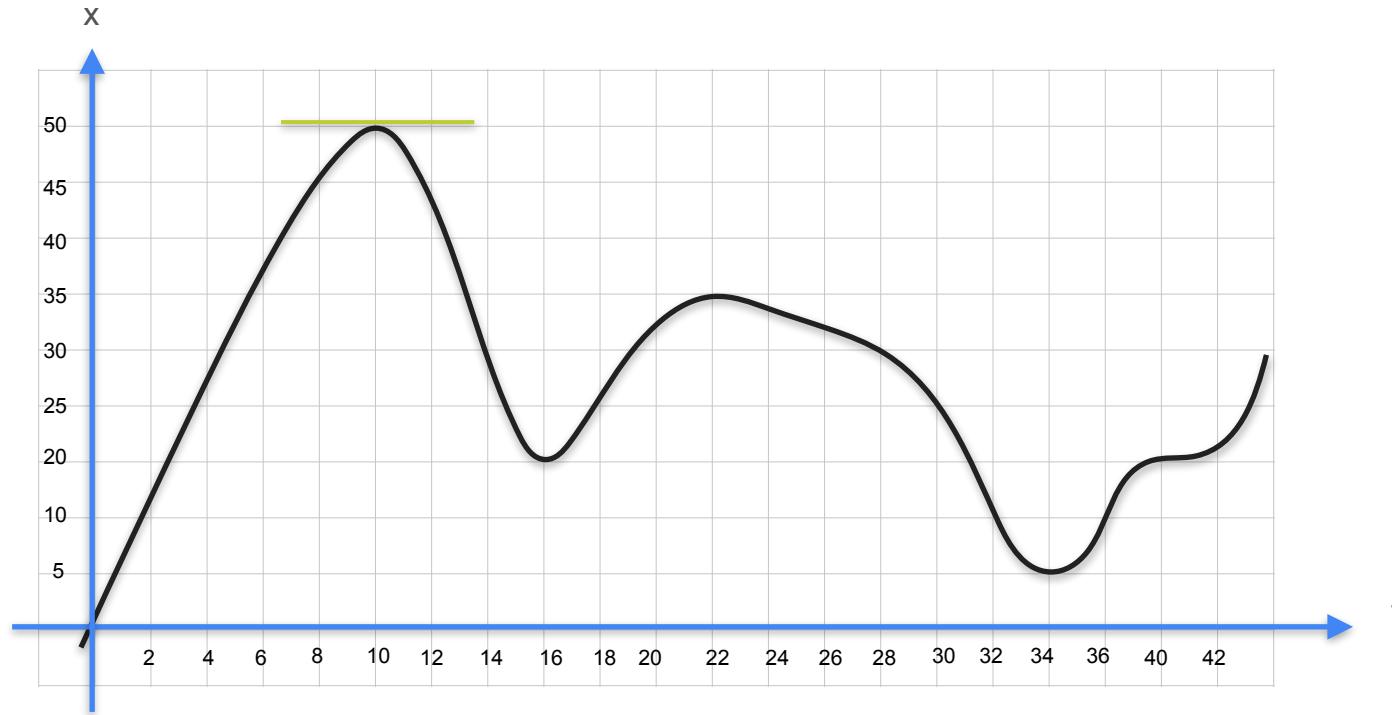


At what time was the car farthest from its starting point?

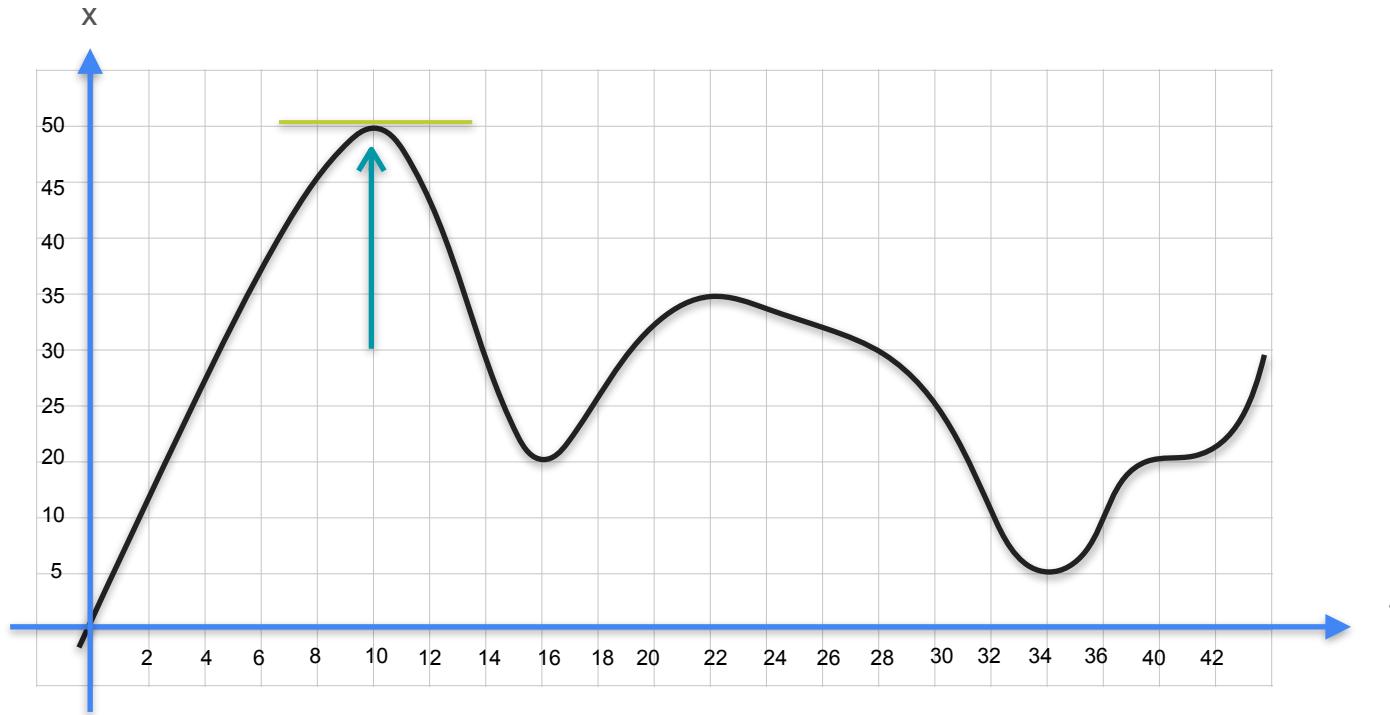
# Quiz : Maxima & Minima - Solution



# Quiz : Maxima & Minima - Solution



# Quiz : Maxima & Minima - Solution





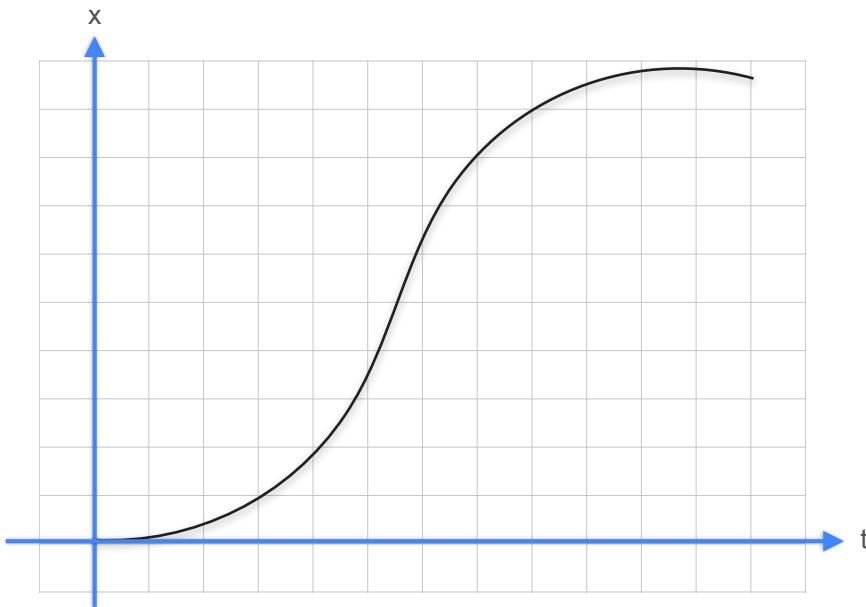
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# Derivatives and Optimization

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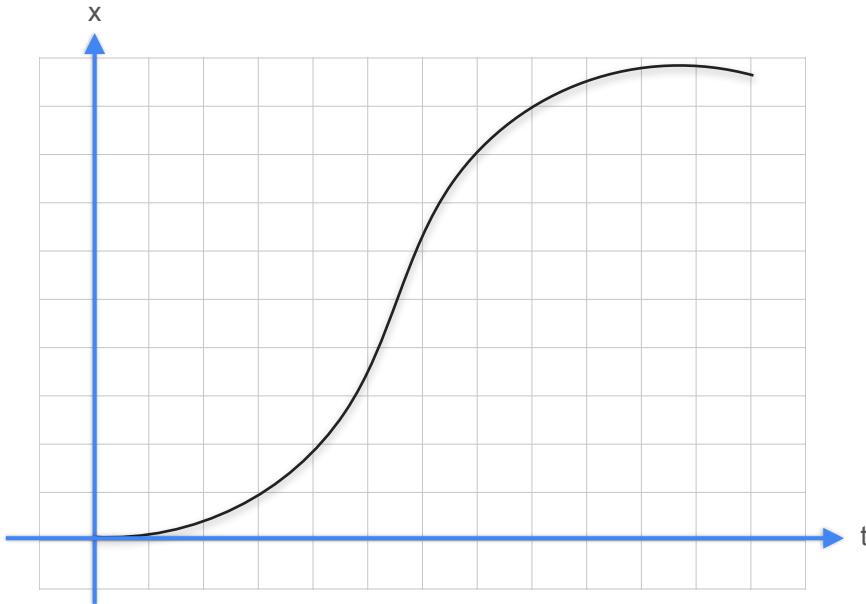
## Derivative notation

# Derivatives



# Derivatives

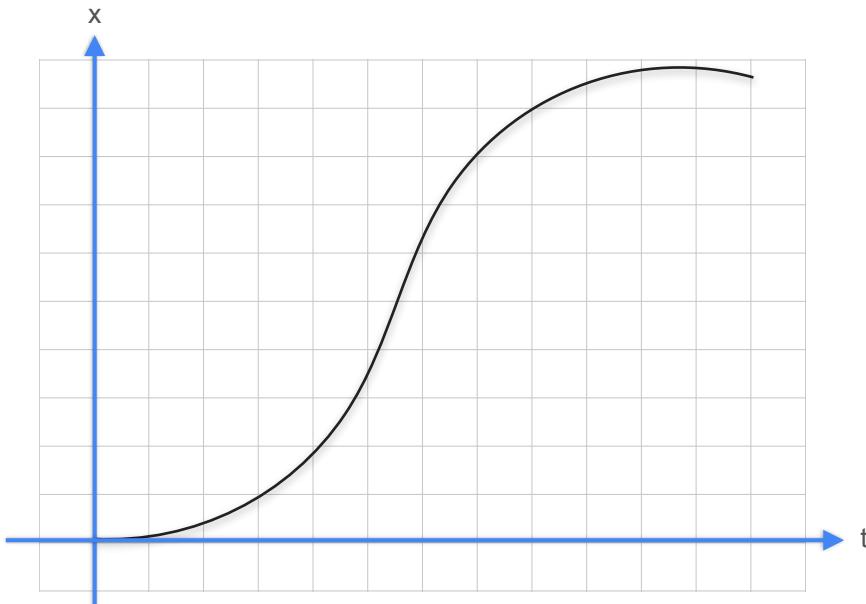
$$\text{slope} = \frac{\text{change in distance}}{\text{change in time}}$$



# Derivatives

$$\text{slope} = \frac{\text{change in distance}}{\text{change in time}}$$

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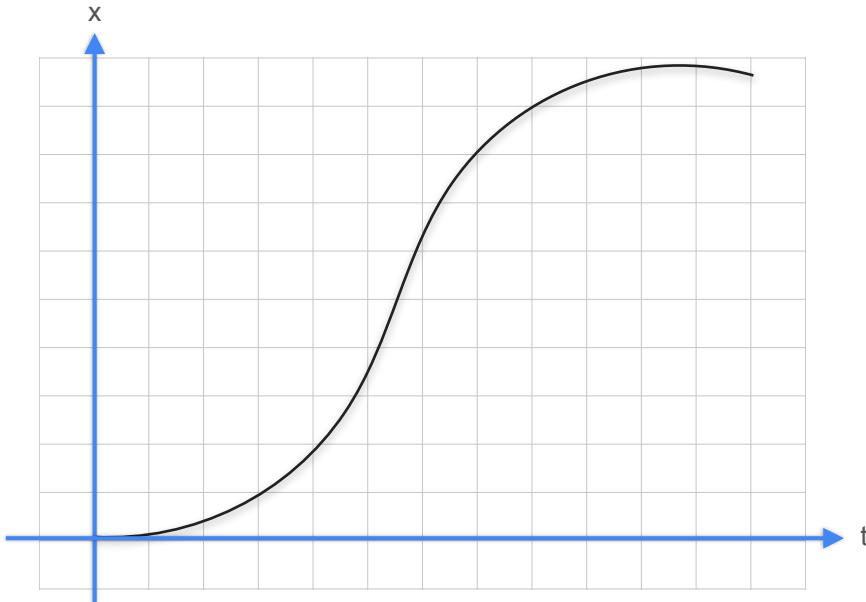


# Derivatives

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$$\text{slope at a point} = \frac{dx}{dt}$$

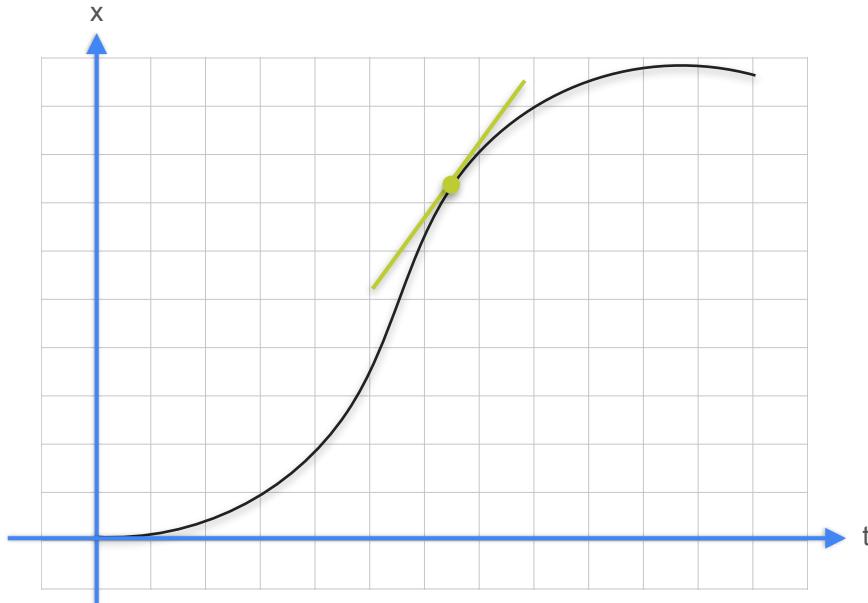


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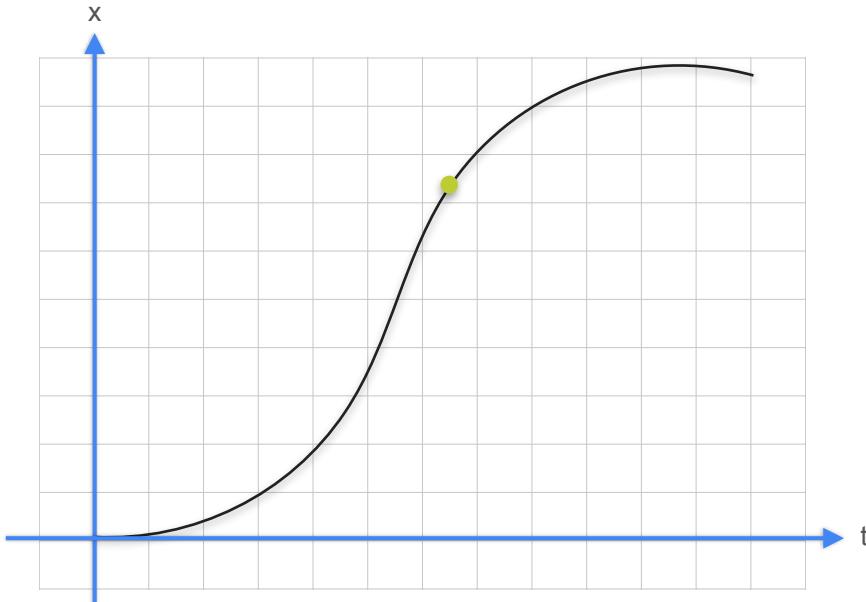


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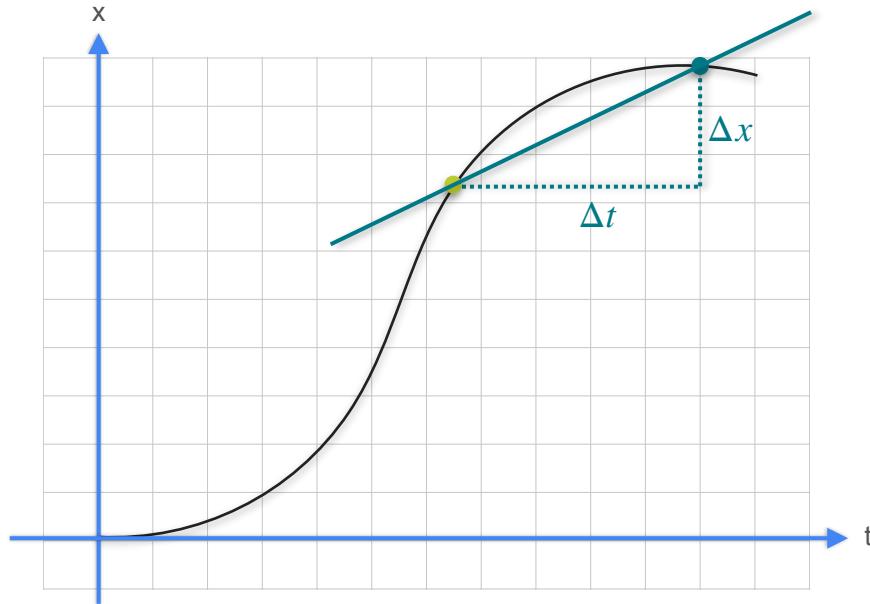


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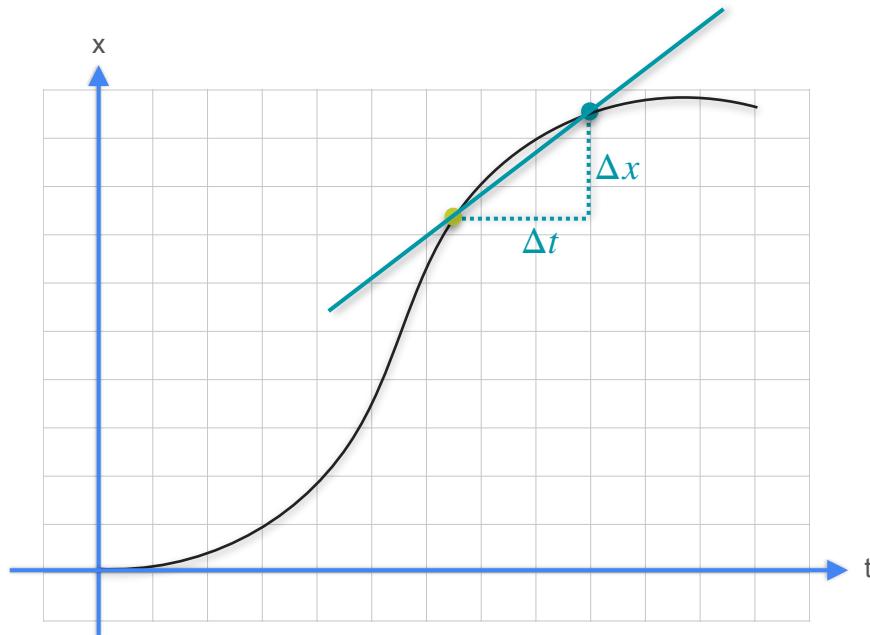


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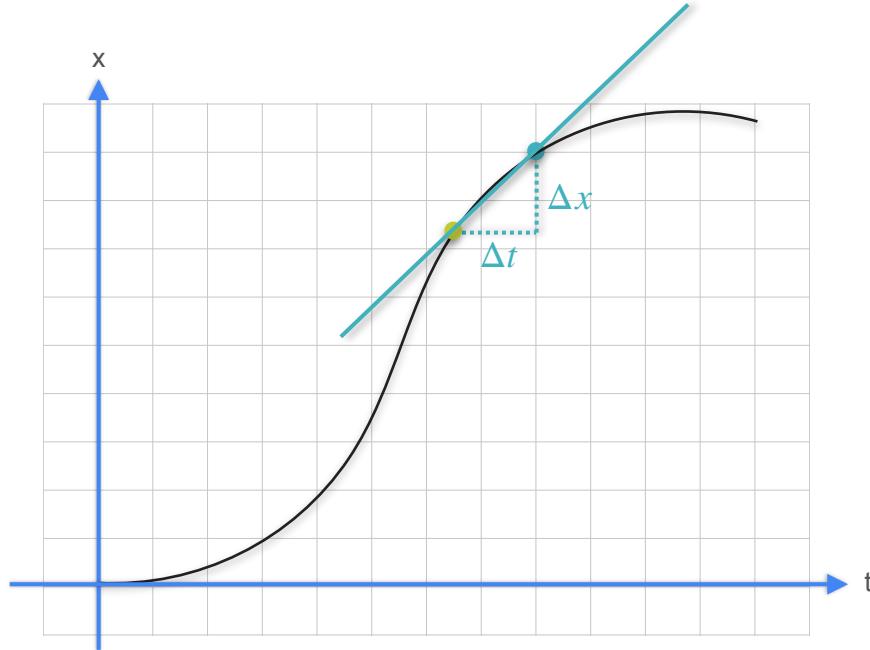


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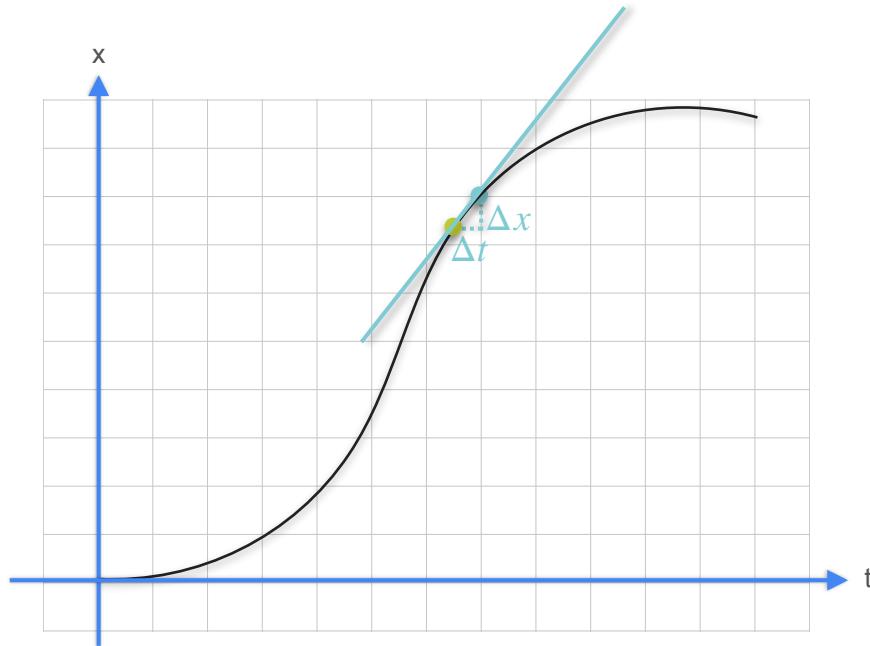


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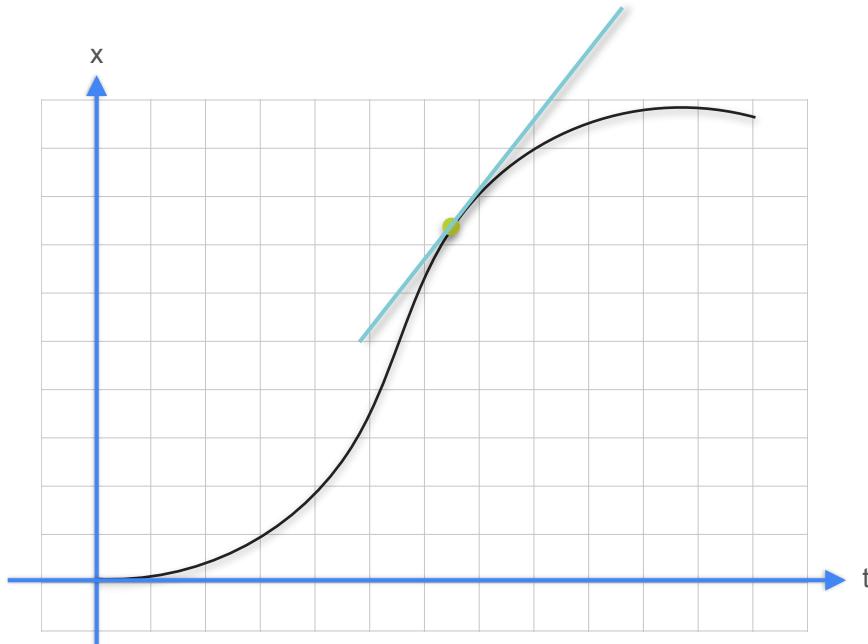


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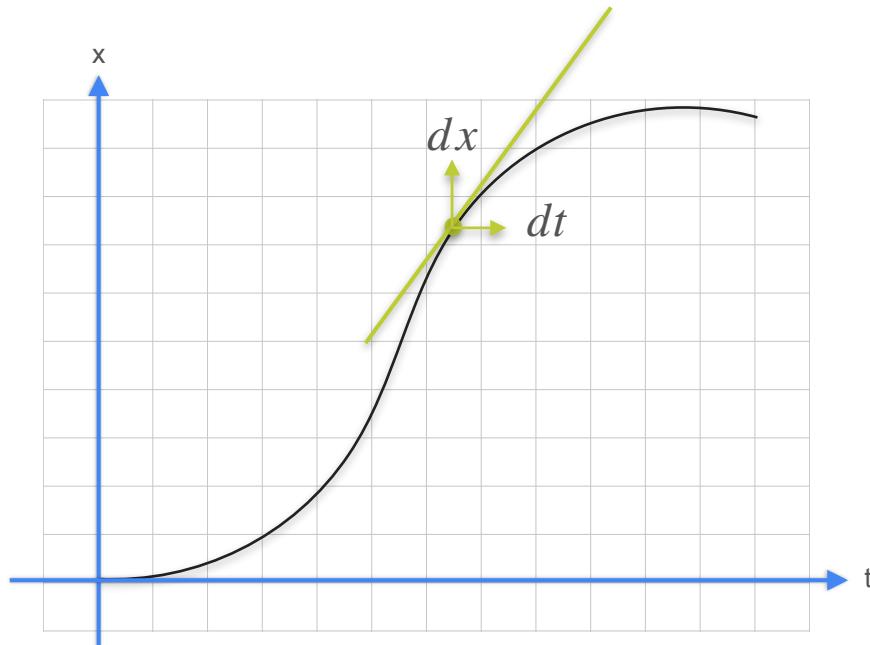


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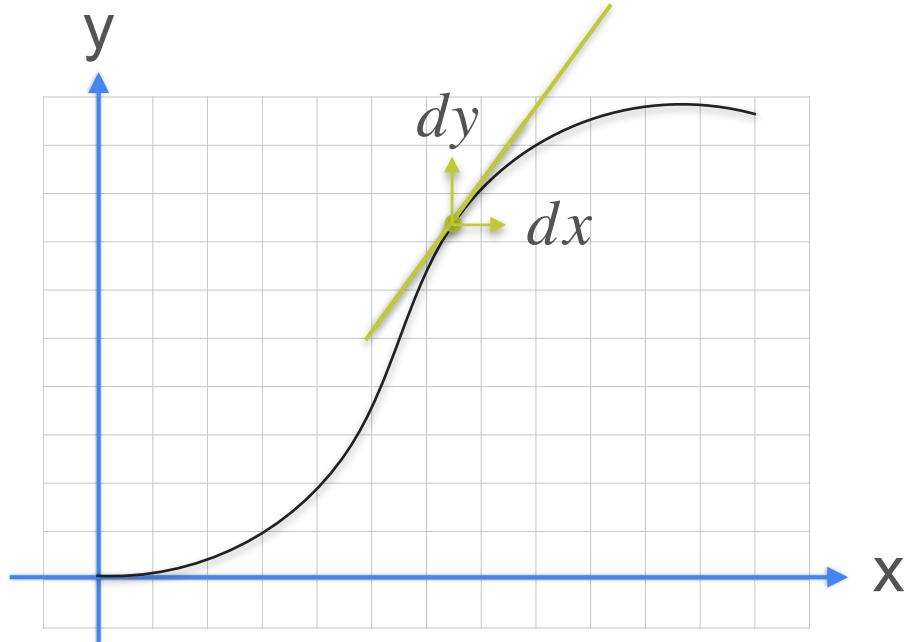


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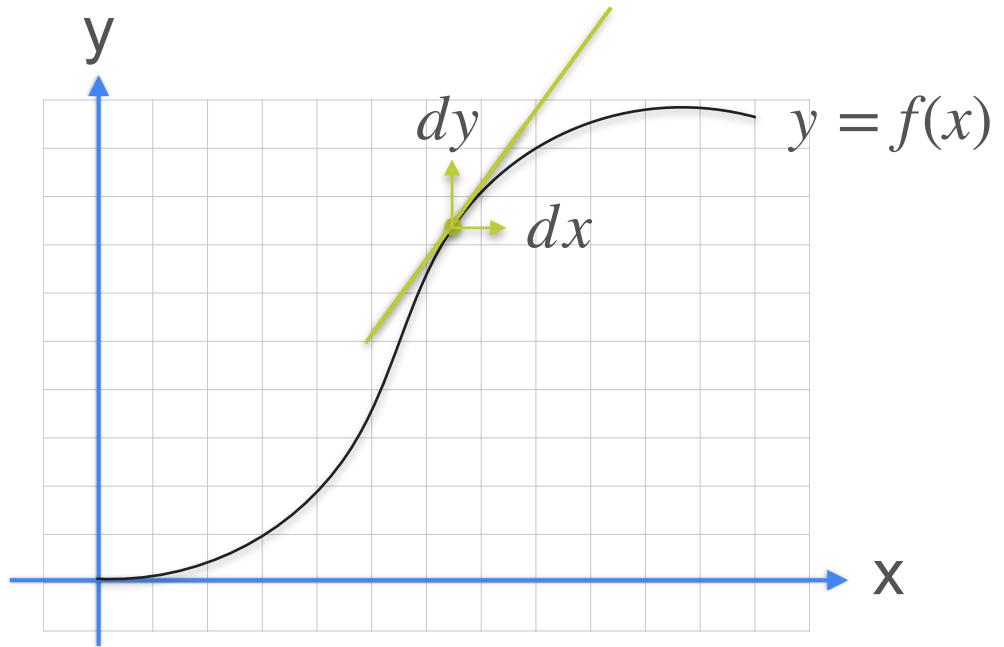
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\text{slope at a point} = \frac{dy}{dx}$$

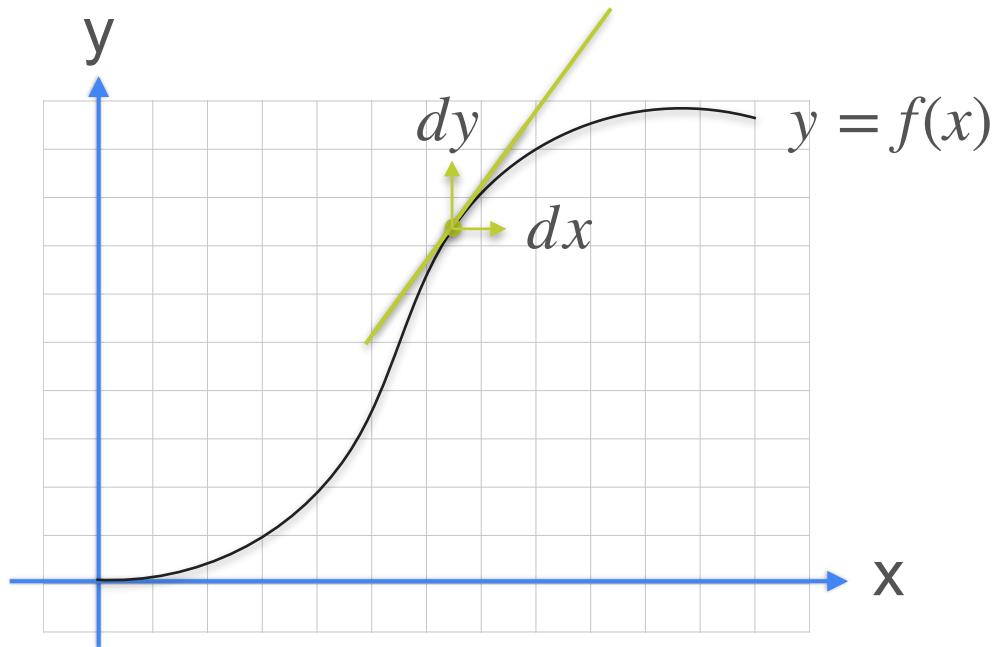


# Derivatives



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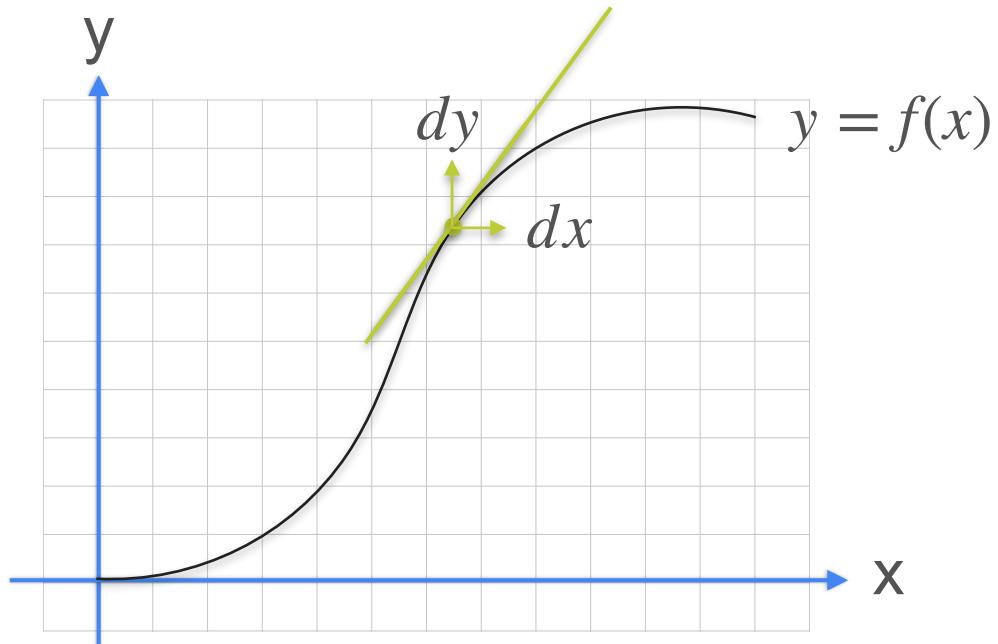
$$y = f(x)$$



# Derivatives

$$y = f(x)$$

Derivative of  $f$  is expressed as:

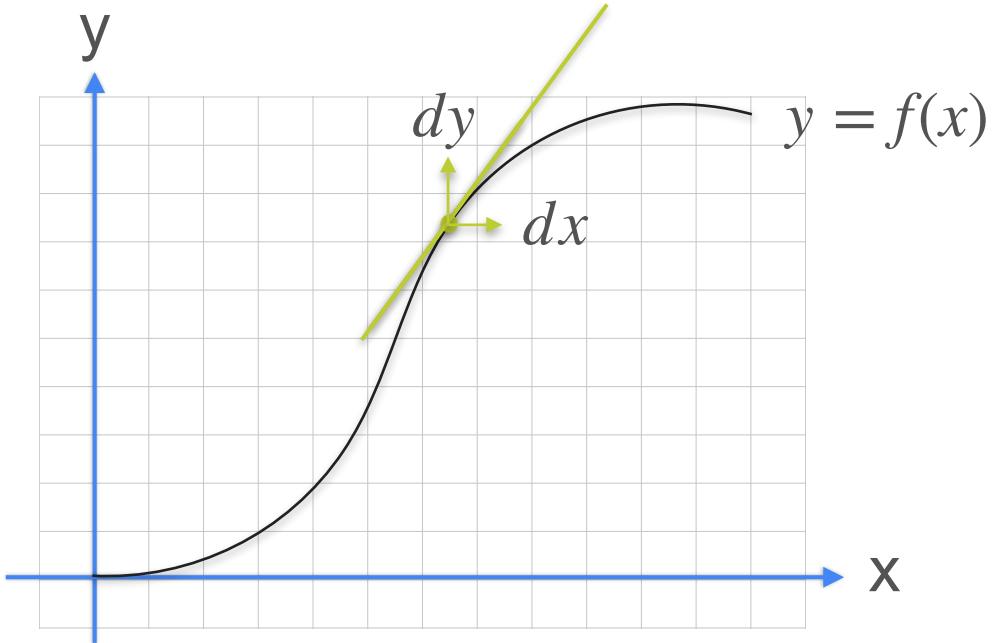


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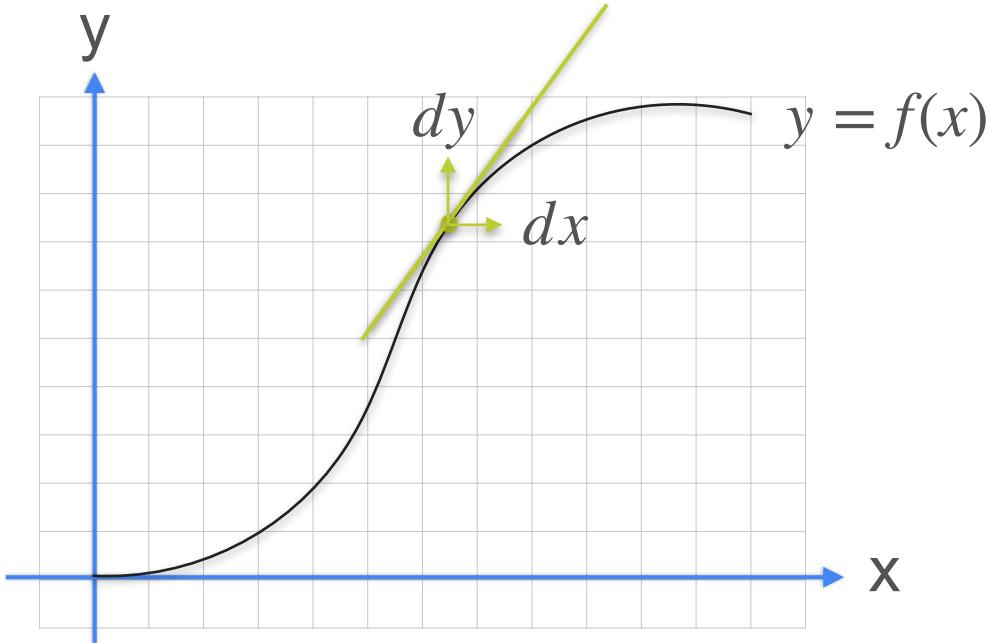


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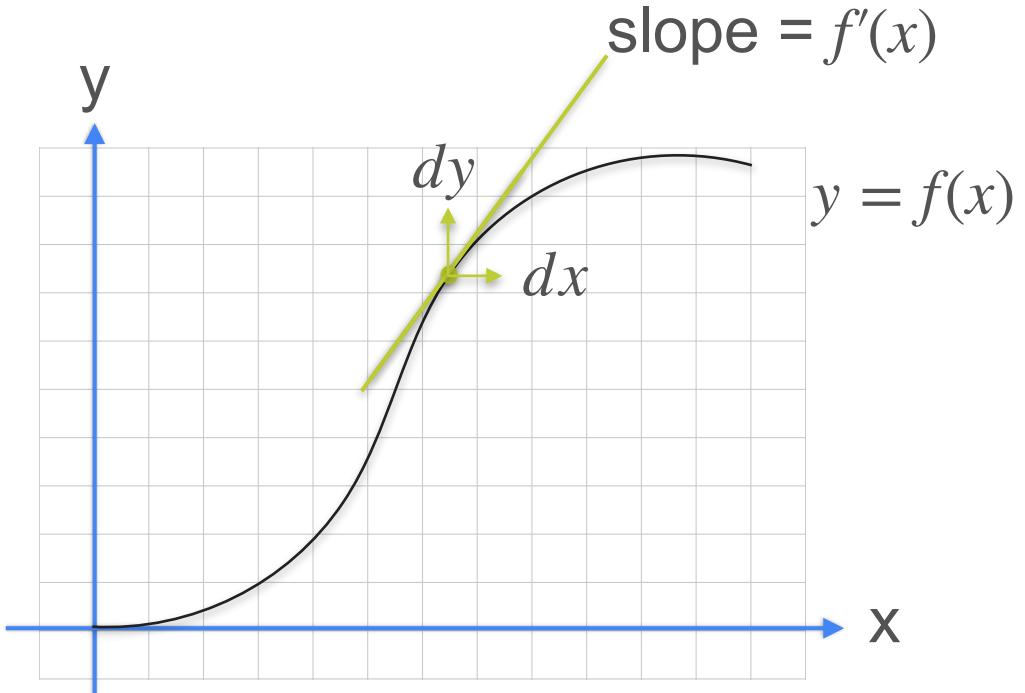
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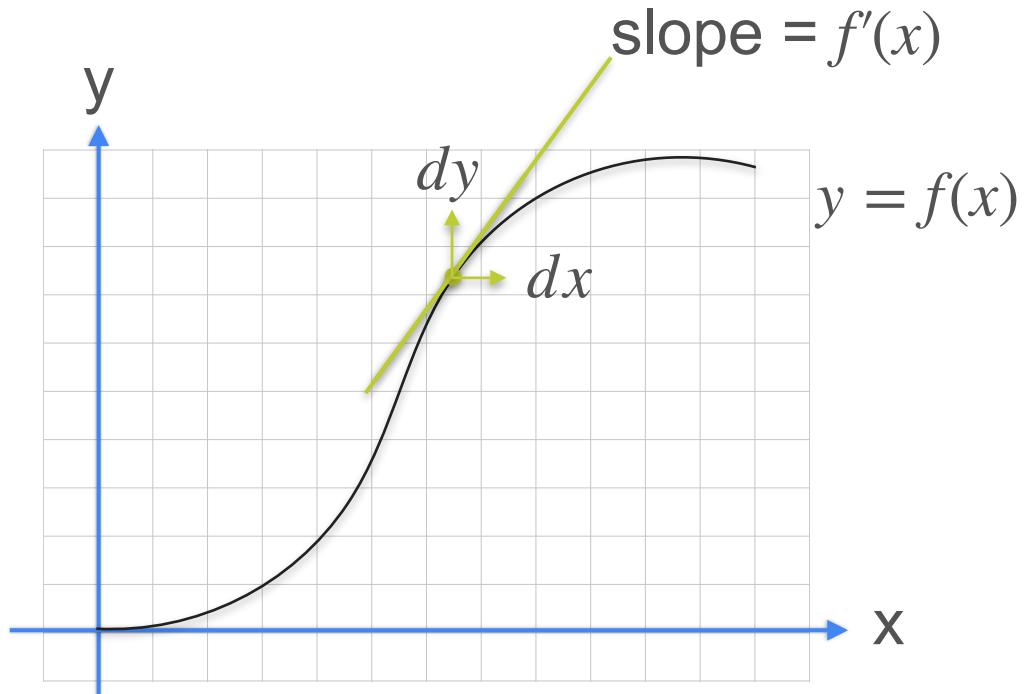


# Derivatives: Lagrange's and Leibniz's Notation



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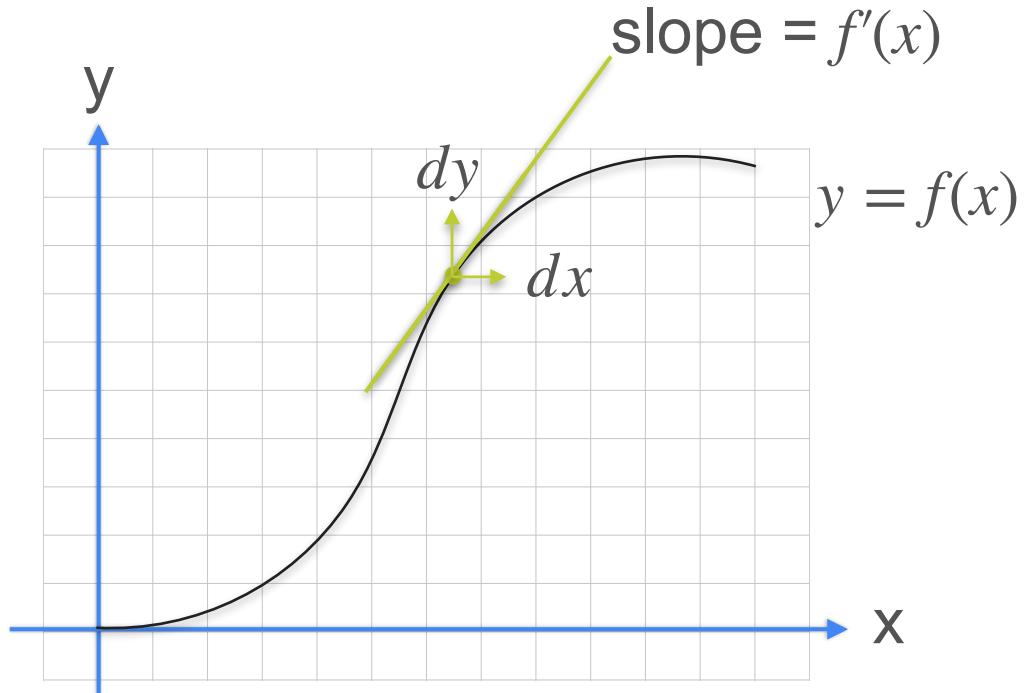
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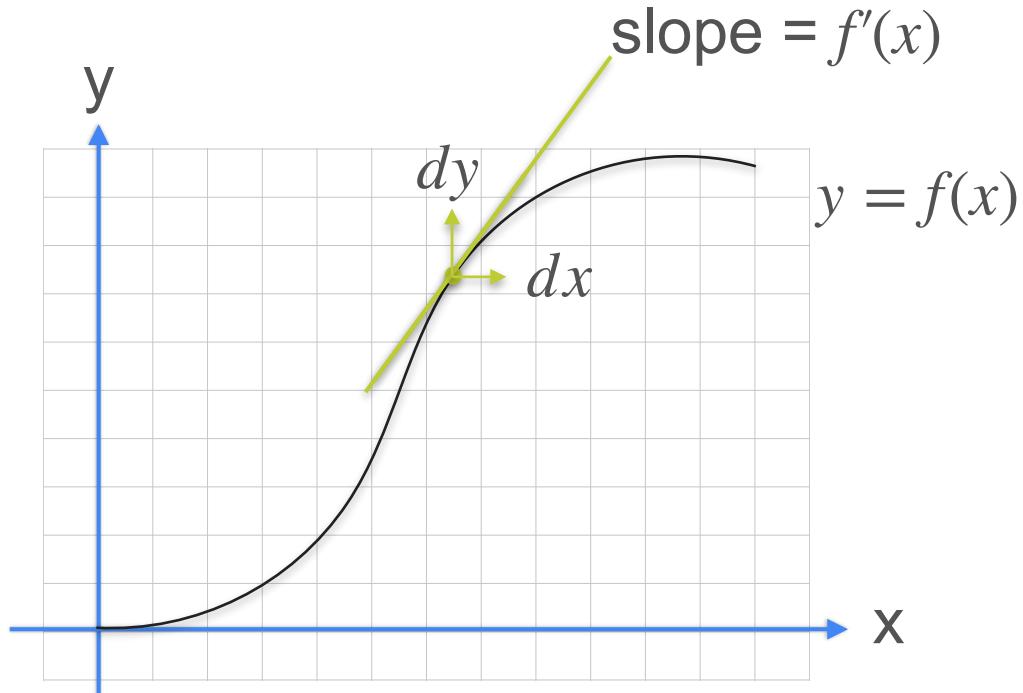


# Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

Derivative of  $f$  is expressed as:

$$f'(x)$$

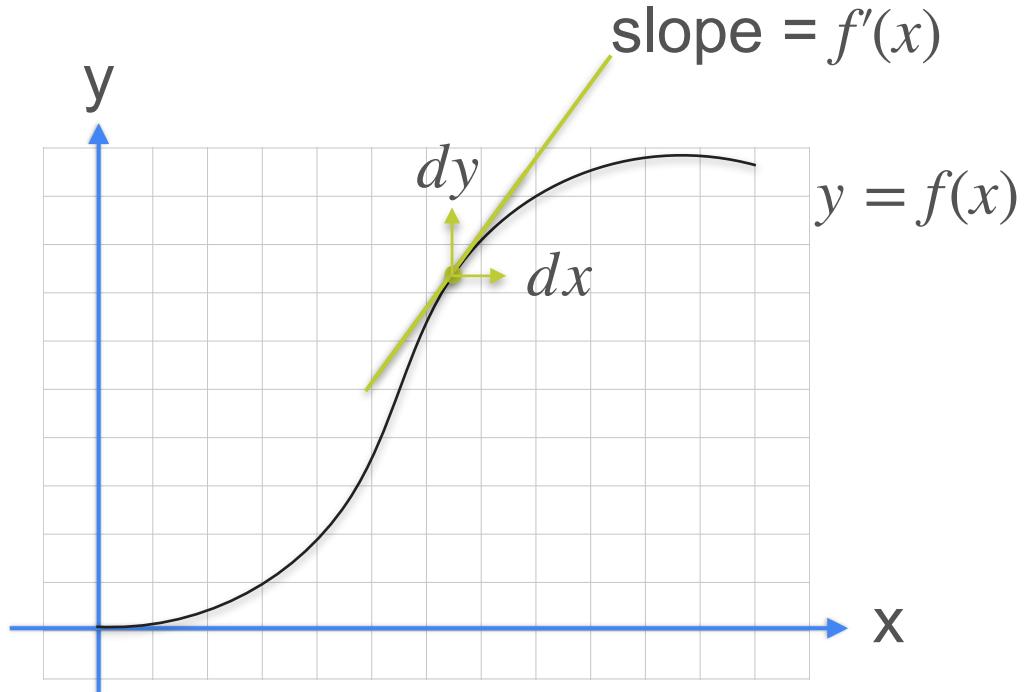


# Derivatives: Lagrange's and Leibniz's Notation

$$y = f(x)$$

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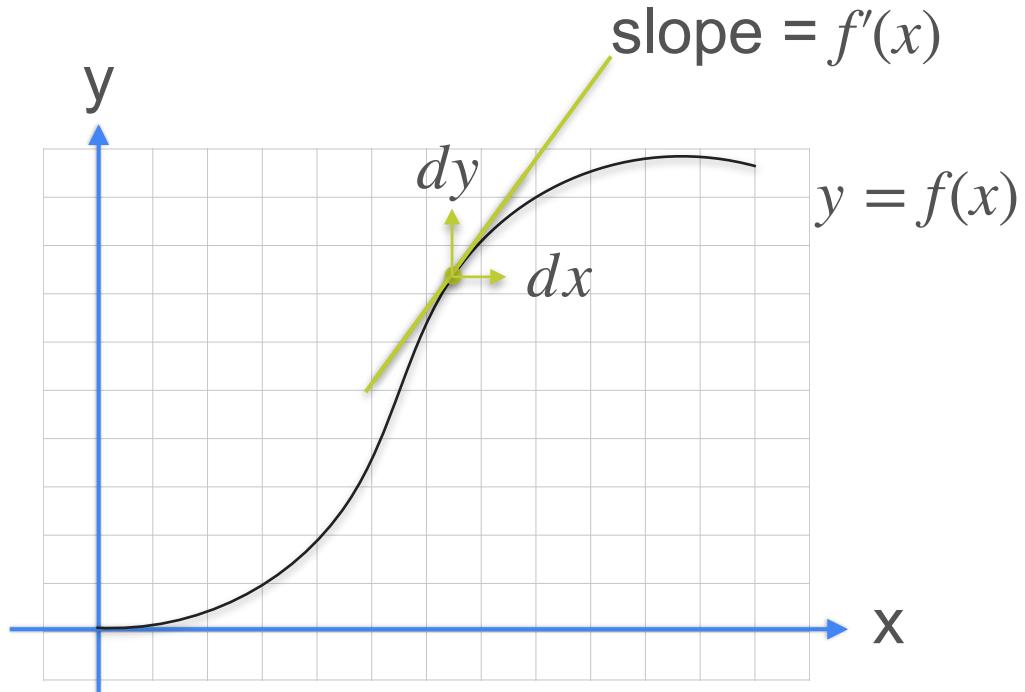
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$$\frac{dy}{dx}$$



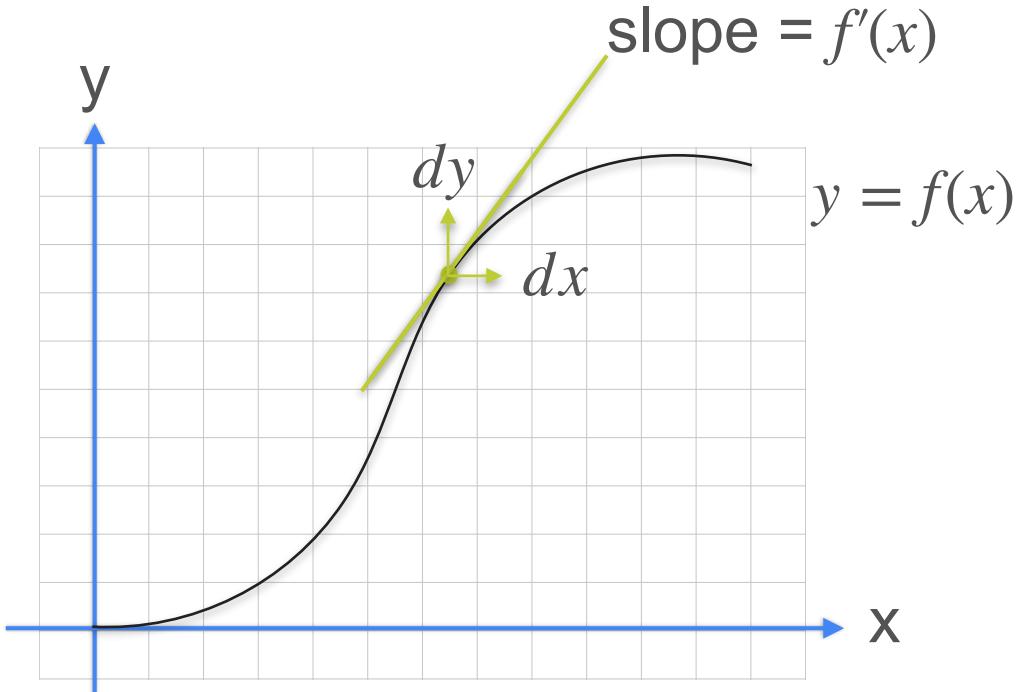
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$$\frac{dy}{dx} = \frac{d}{dx}f(x)$$



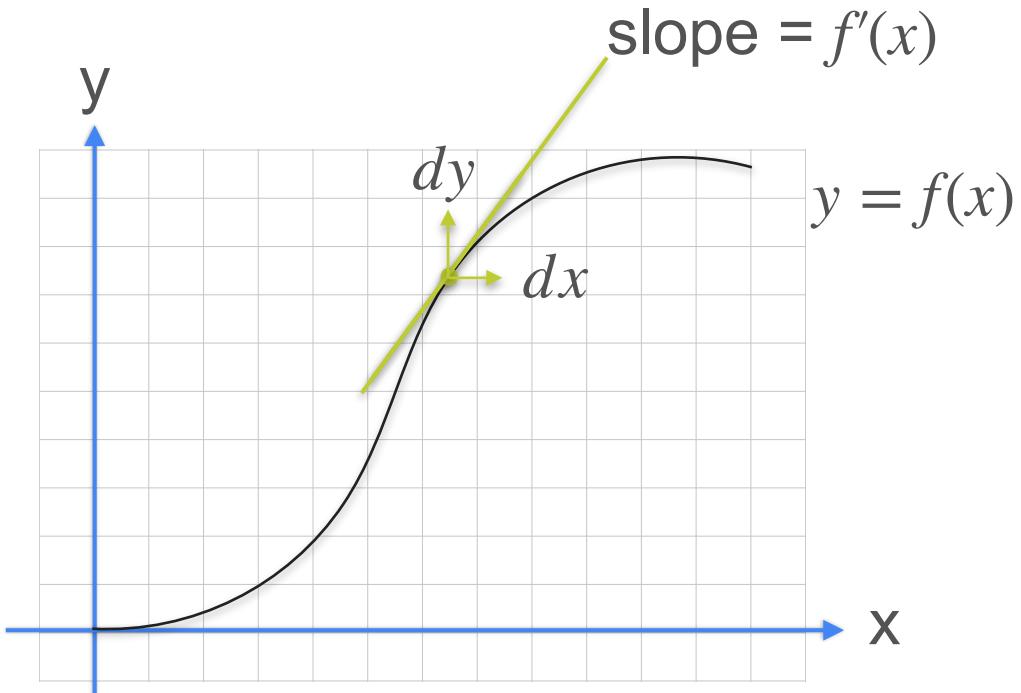
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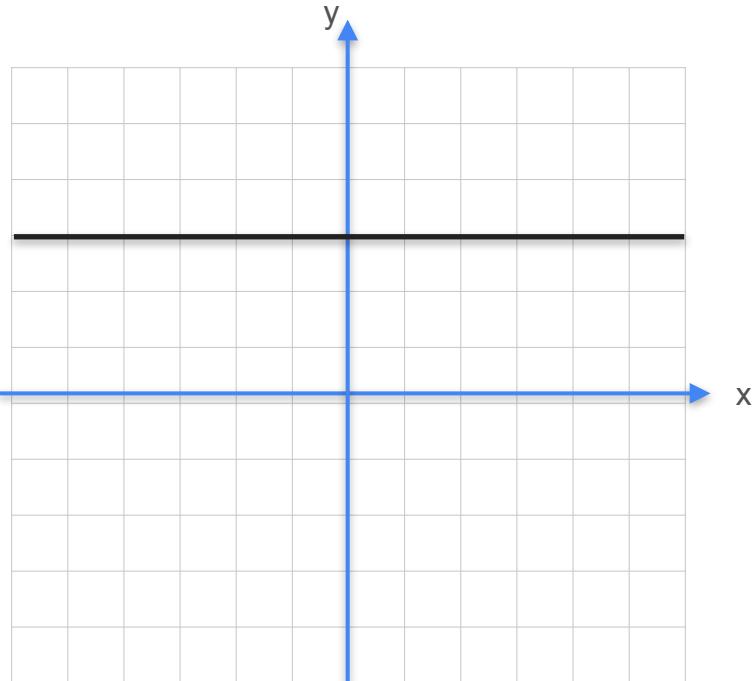
# Derivatives and Optimization

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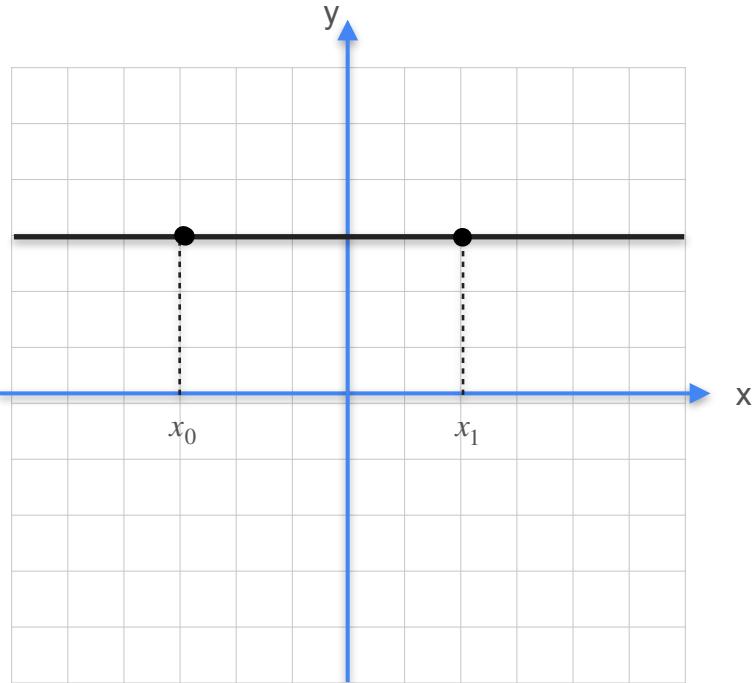
**Some common derivatives:  
Lines**

# Derivative of a Constant

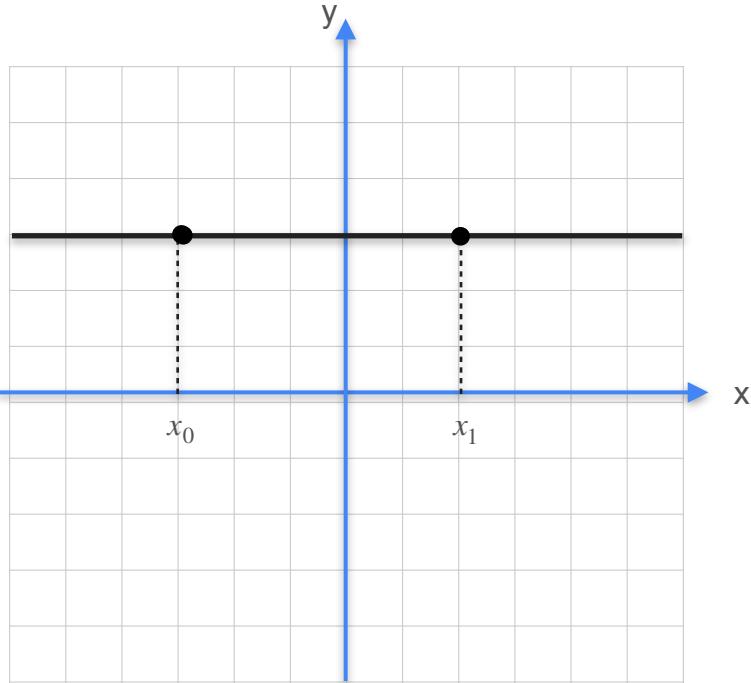
# Derivative of a Constant



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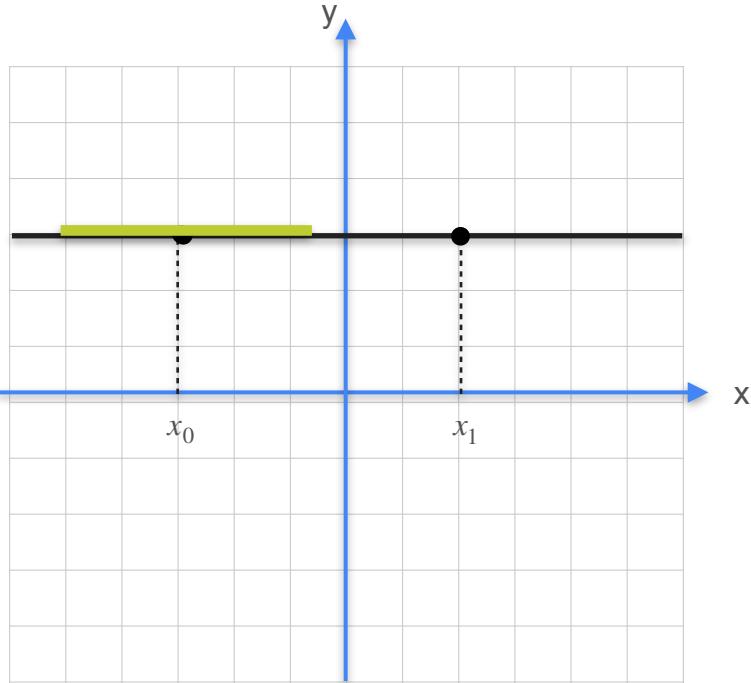


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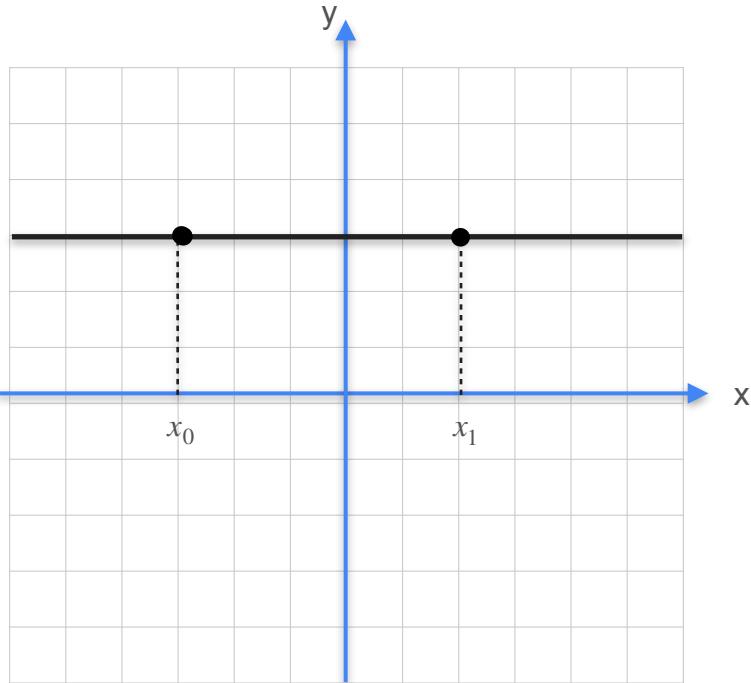
$$y = f(x) = c$$

# Derivative of a Constant



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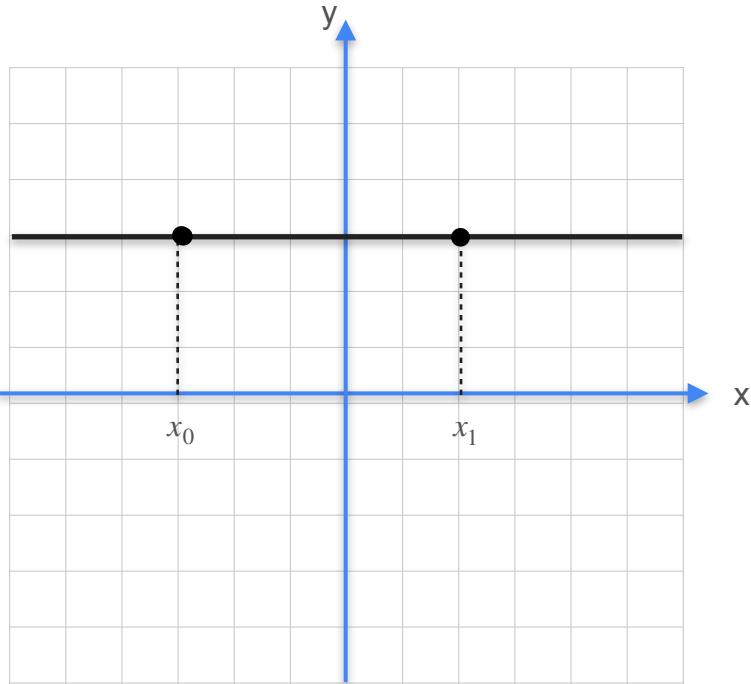
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$$y = f(x) = c$$

Slope?

# Derivative of a Constant

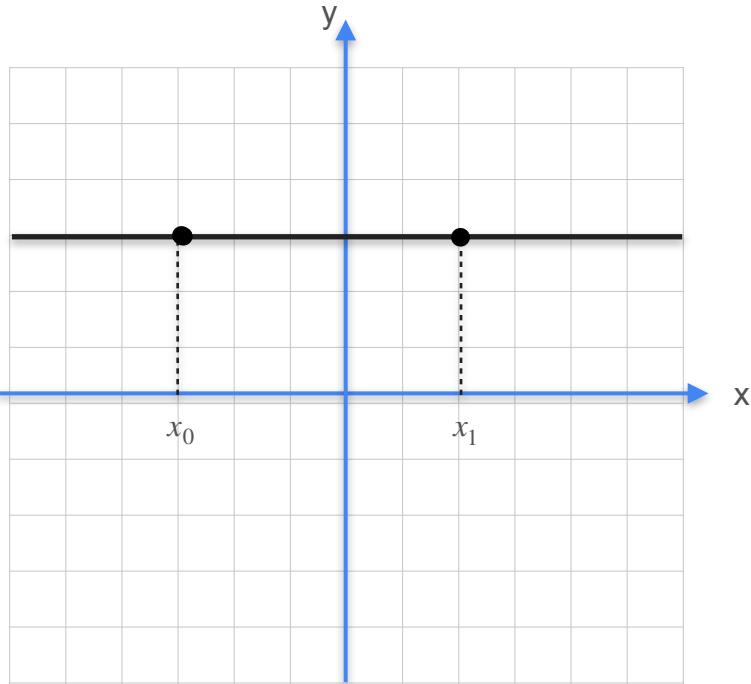


$$y = f(x) = c$$

Slope?

$$\frac{\Delta y}{\Delta x}$$

# Derivative of a Constant

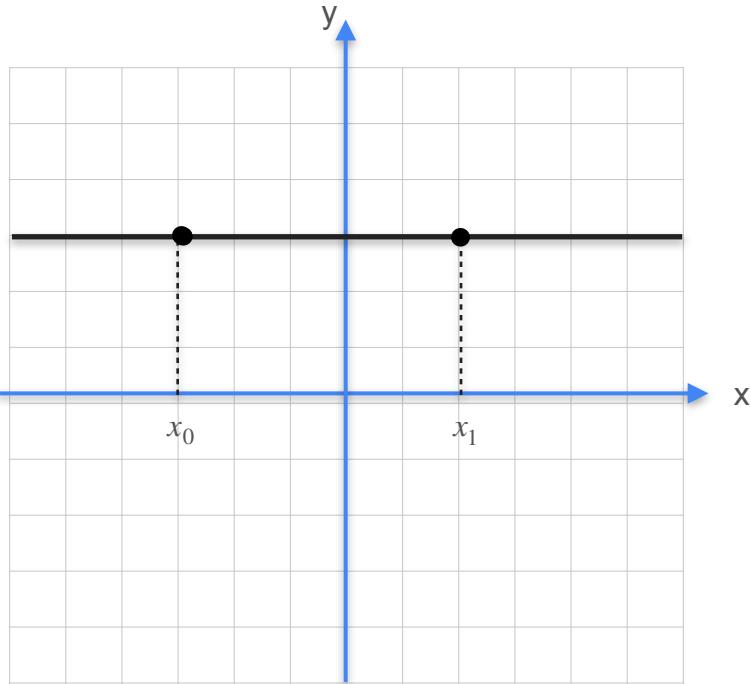


$$y = f(x) = c$$

Slope?

$$\frac{\Delta y}{\Delta x} = \frac{c - c}{x_1 - x_0} = 0$$

# Derivative of a Constant



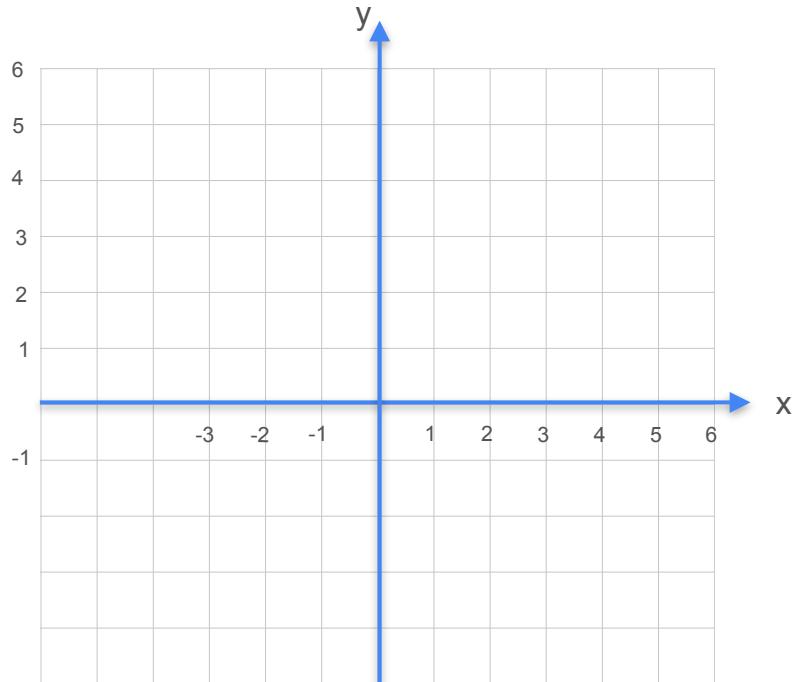
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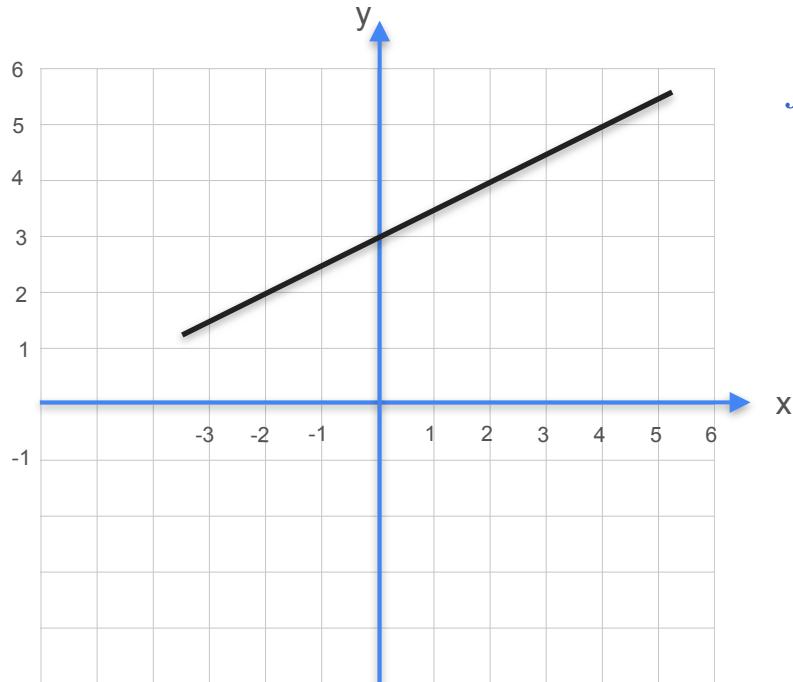
$$\frac{\Delta y}{\Delta x} = \frac{c - c}{x_1 - x_0} = 0 \rightarrow f'(x) = 0$$

# Derivative of a Line

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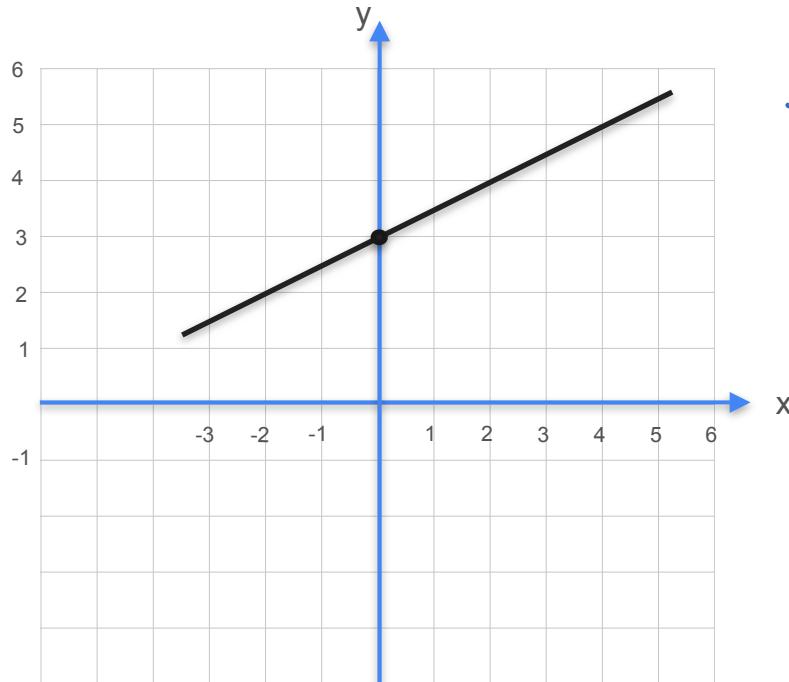


# Derivative of a Line



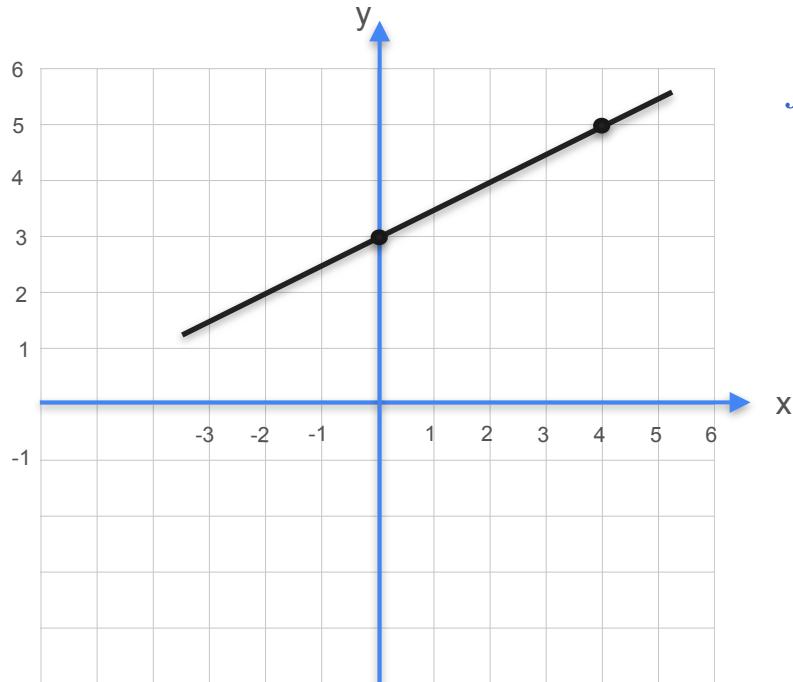
$$f(x) = ax + b$$

# Derivative of a Line



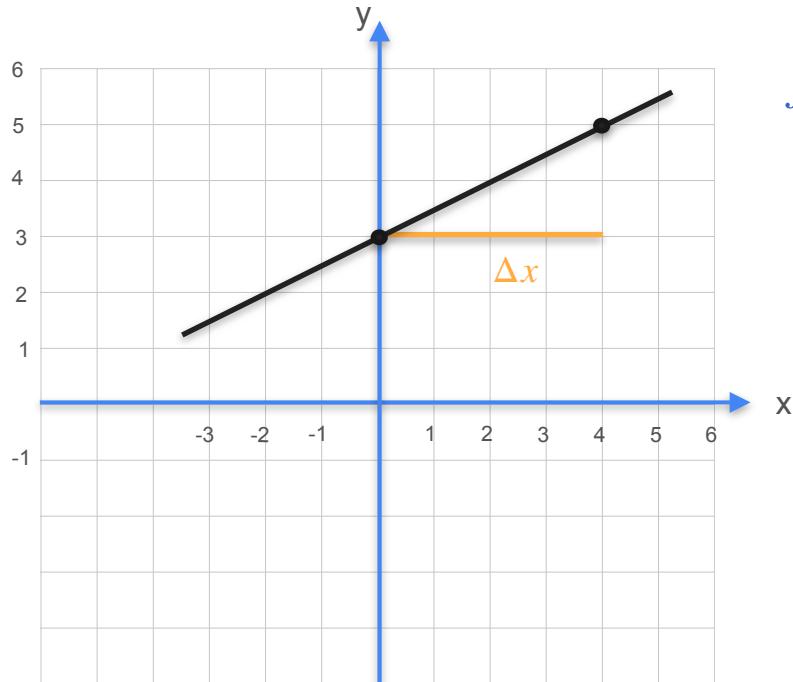
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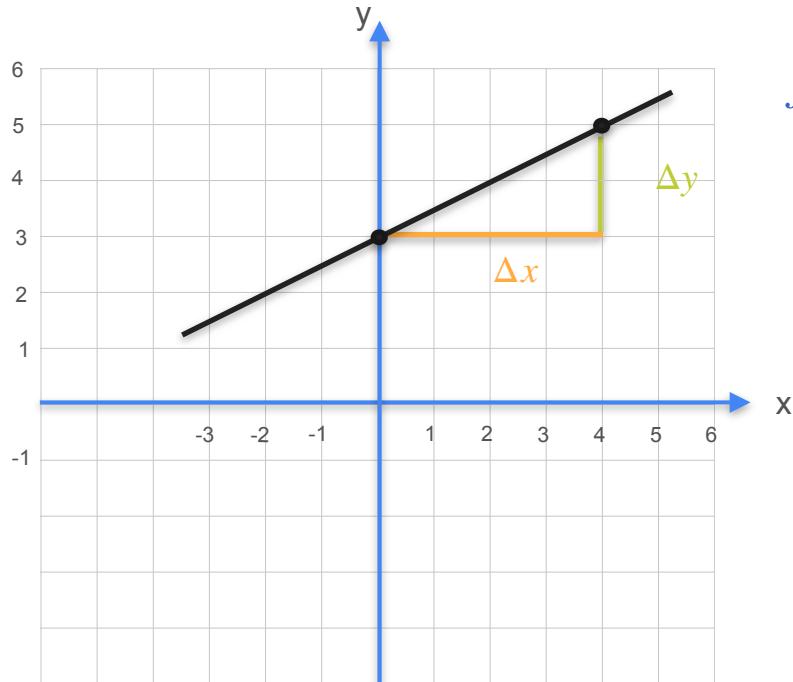
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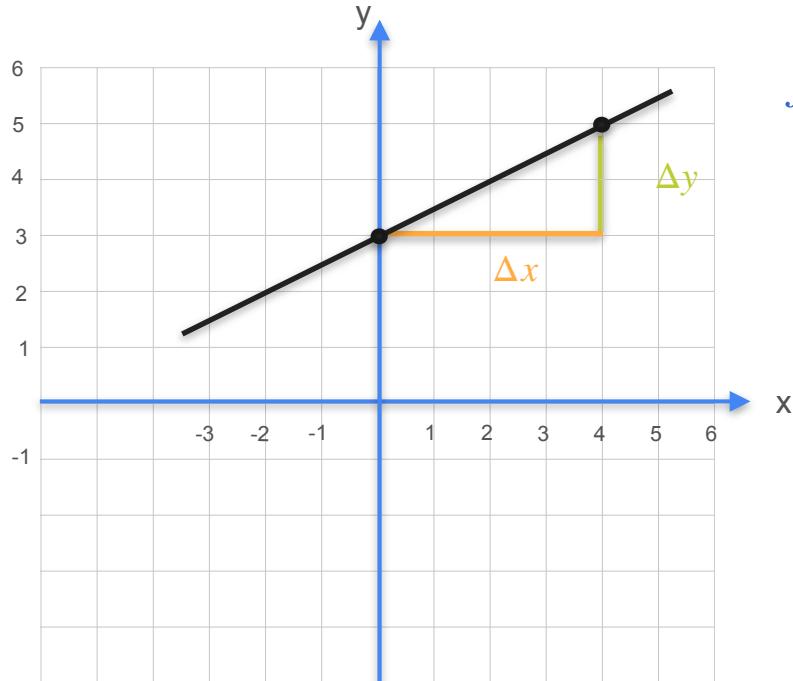
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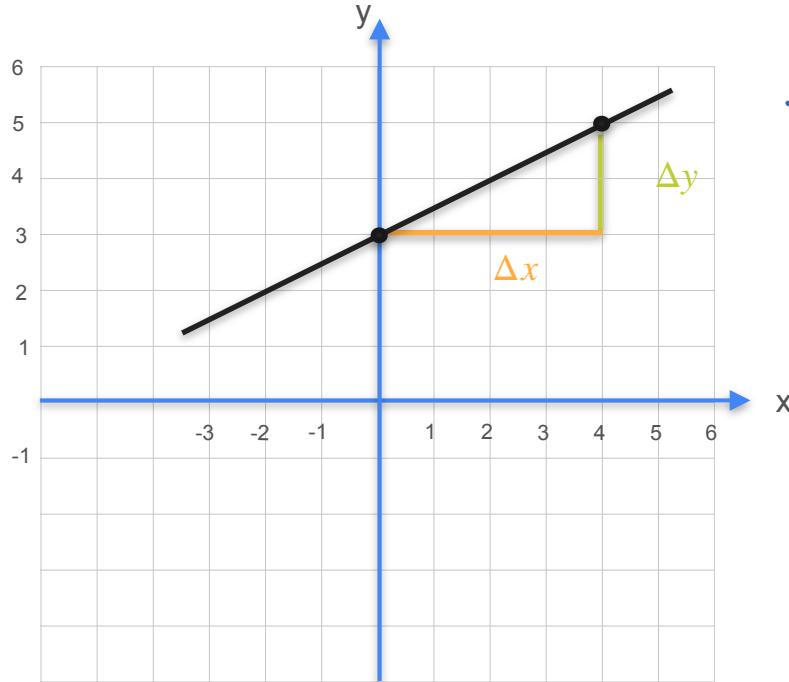
# Derivative of a Line



$$f(x) = ax + b$$

$$\frac{\Delta y}{\Delta x}$$

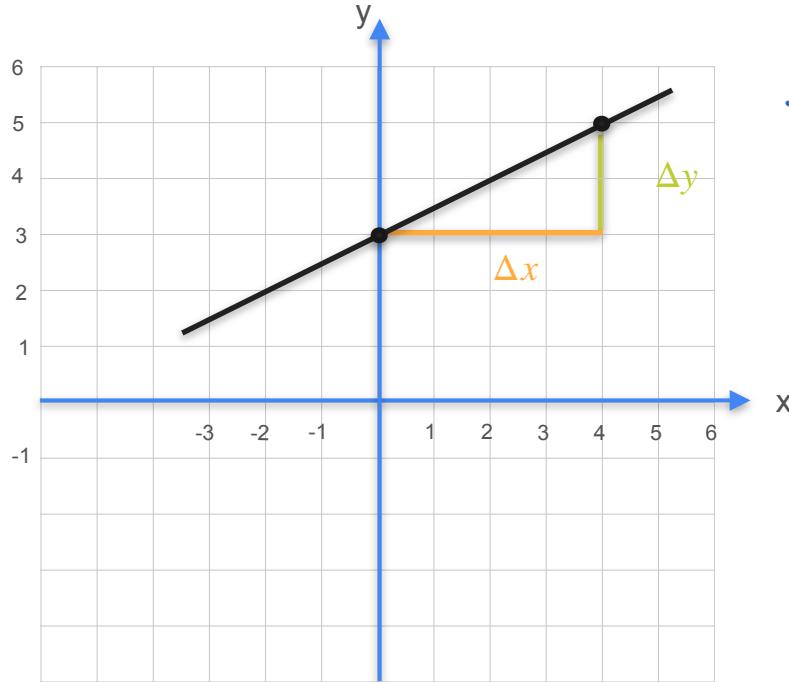
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$$f(x) = ax + b$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

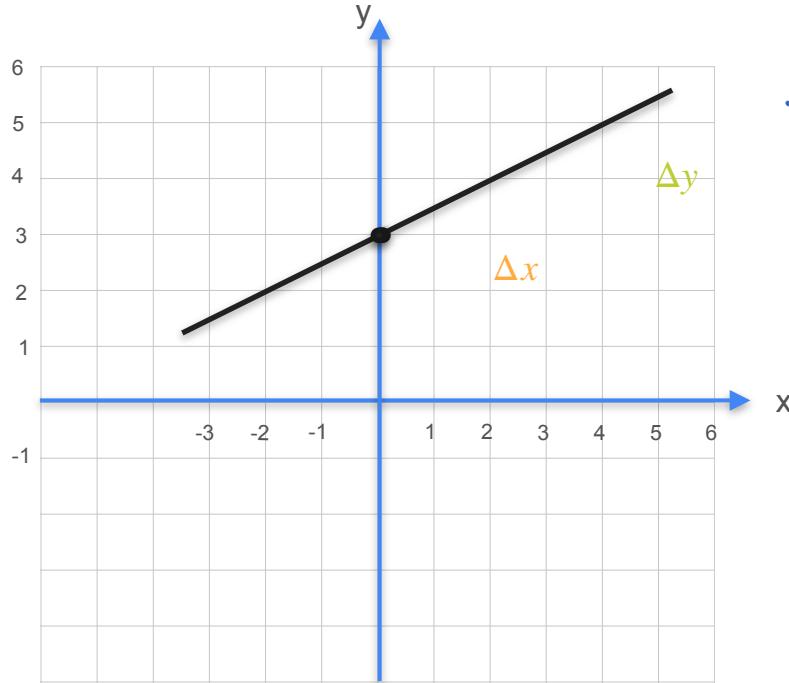
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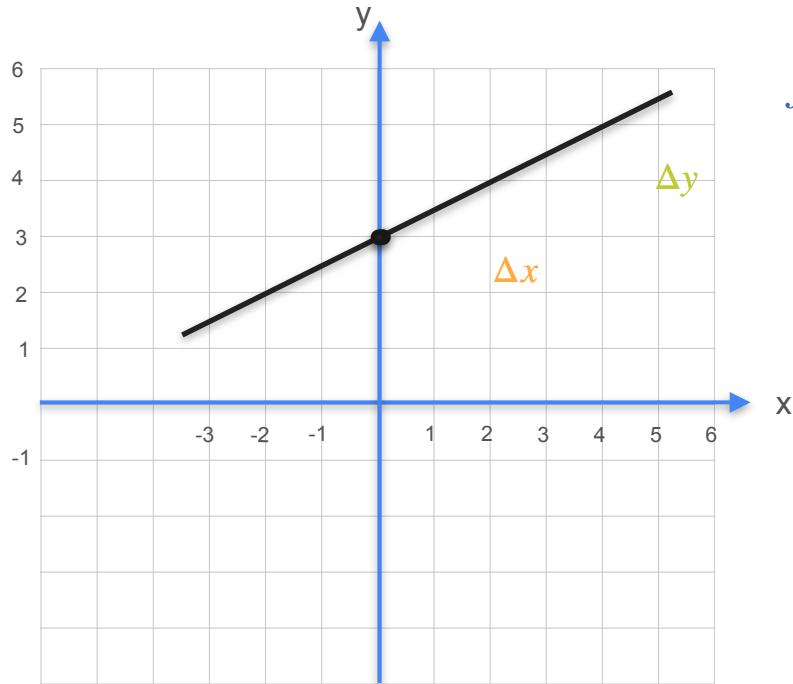
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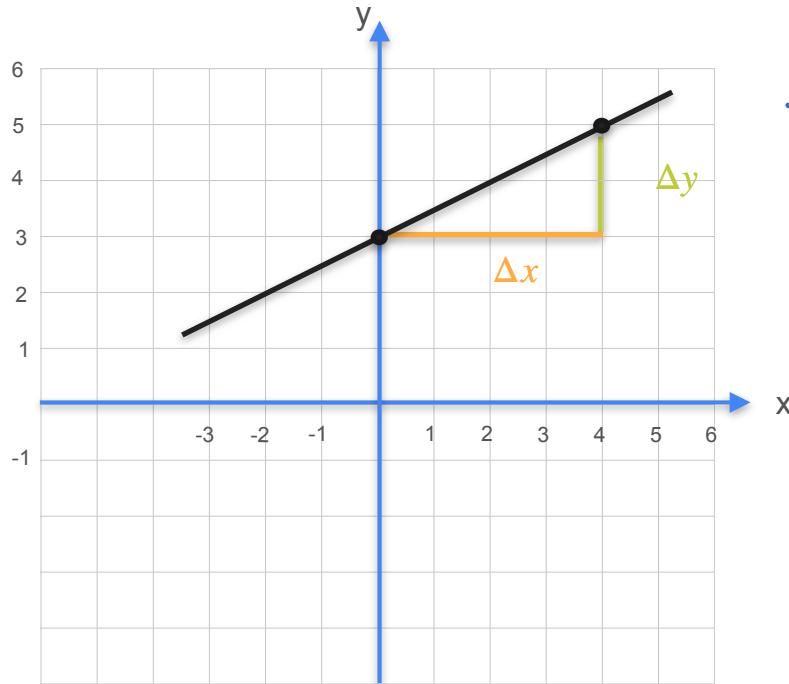
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$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

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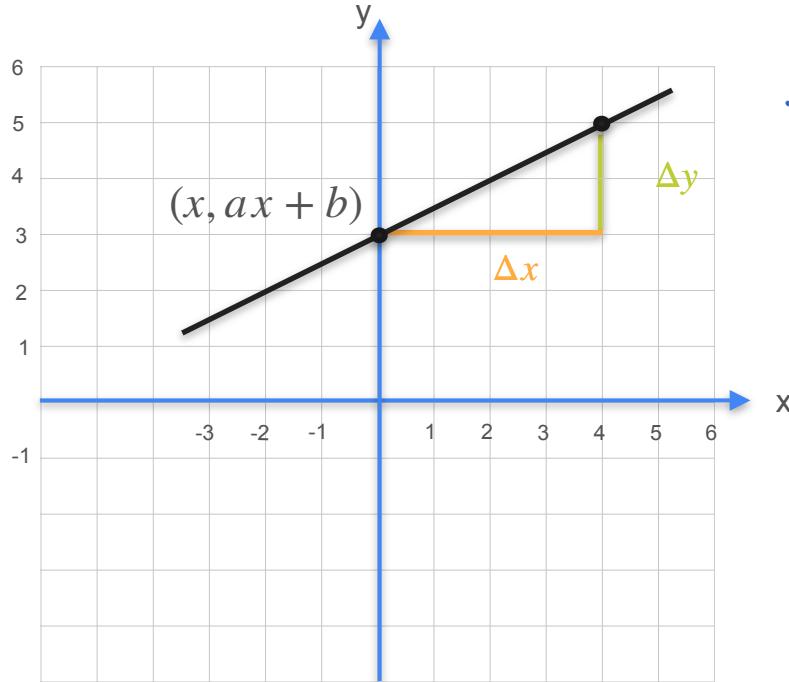
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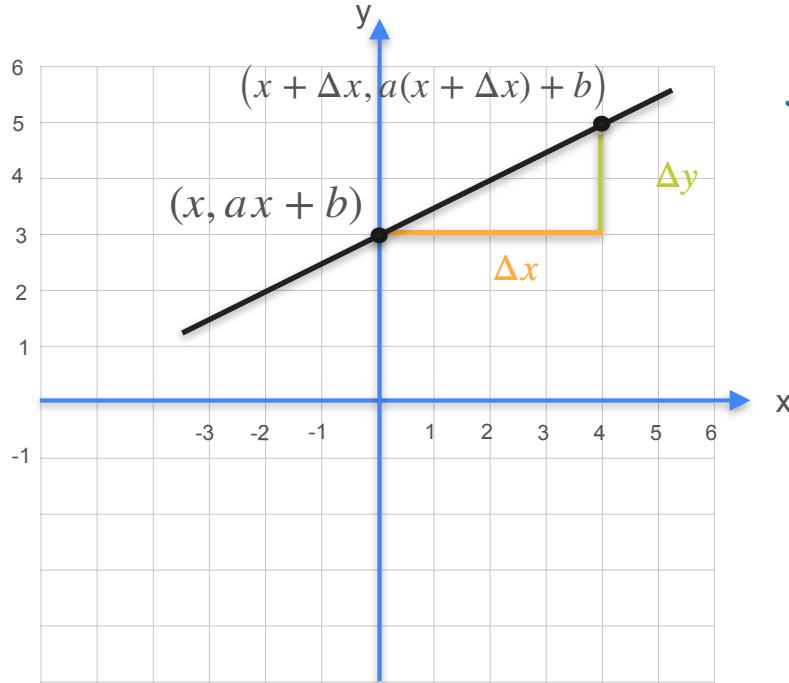
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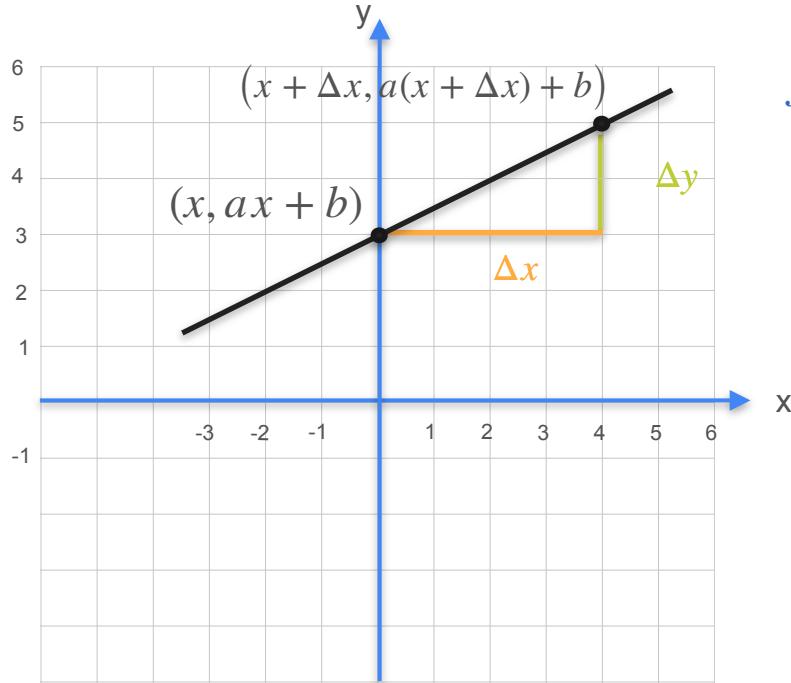
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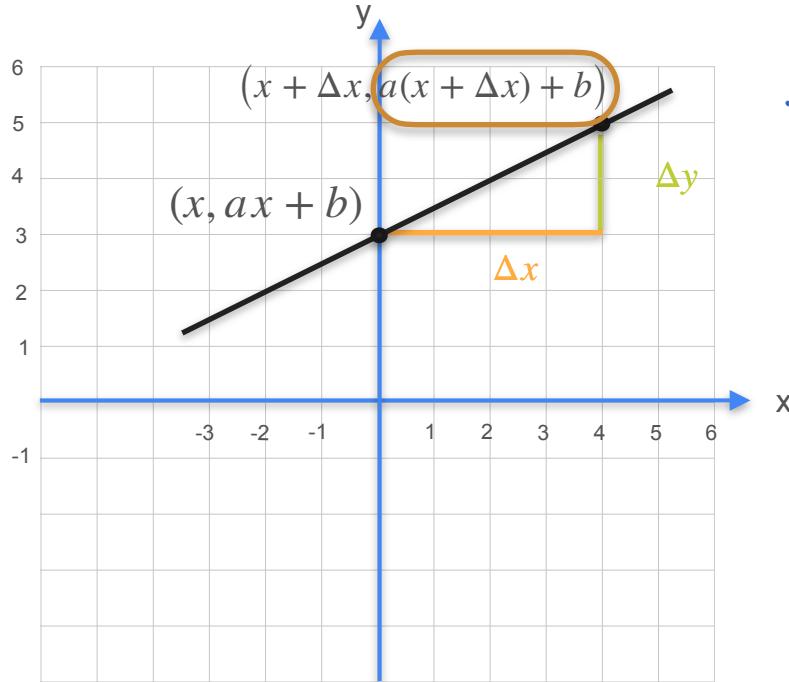


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$$\frac{\Delta y}{\Delta x}$$

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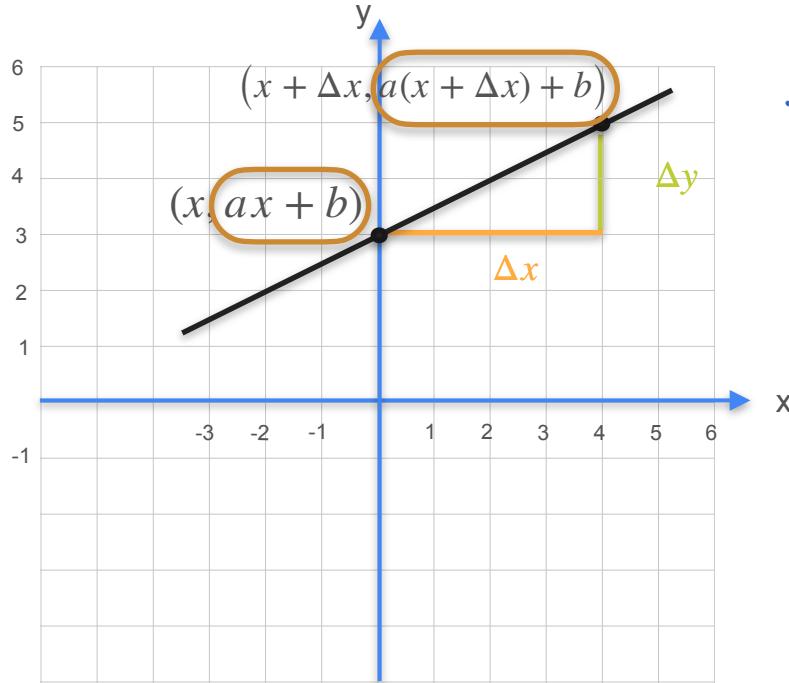


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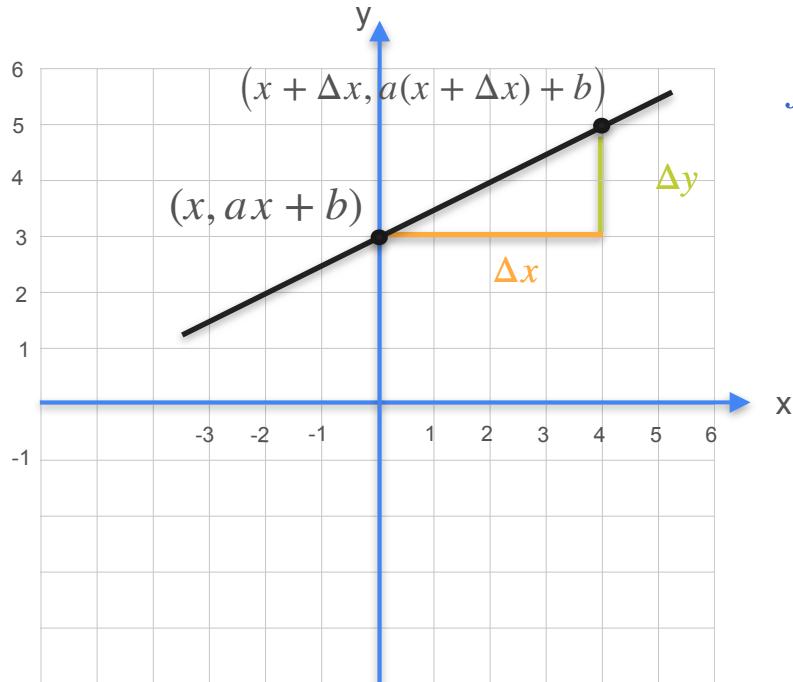


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x}$$

# Derivative of a Line

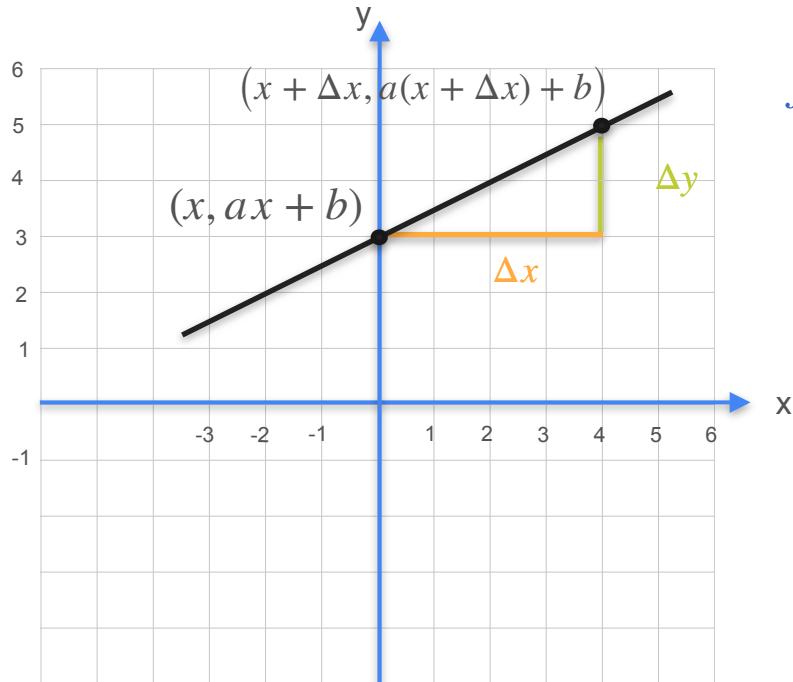


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

# Derivative of a Line

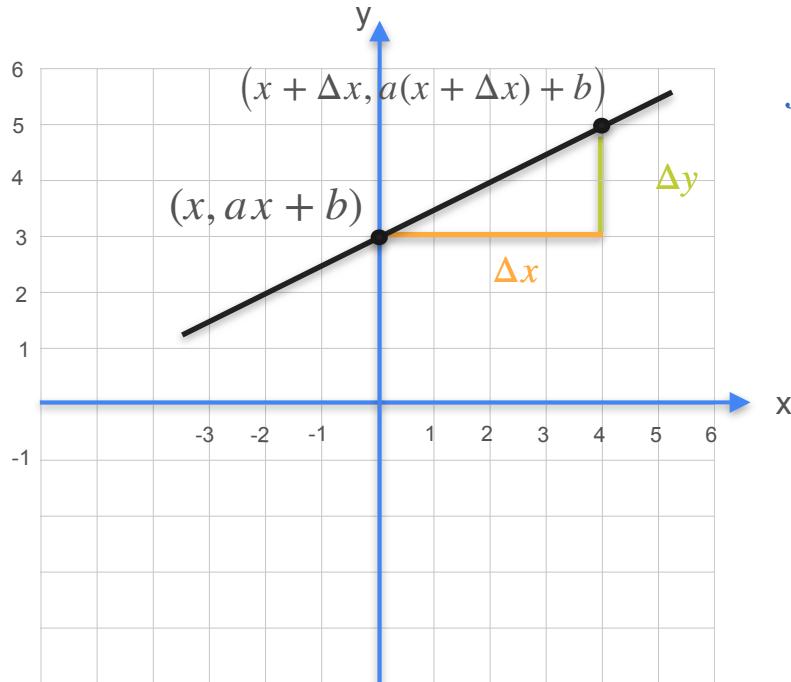


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

# Derivative of a Line

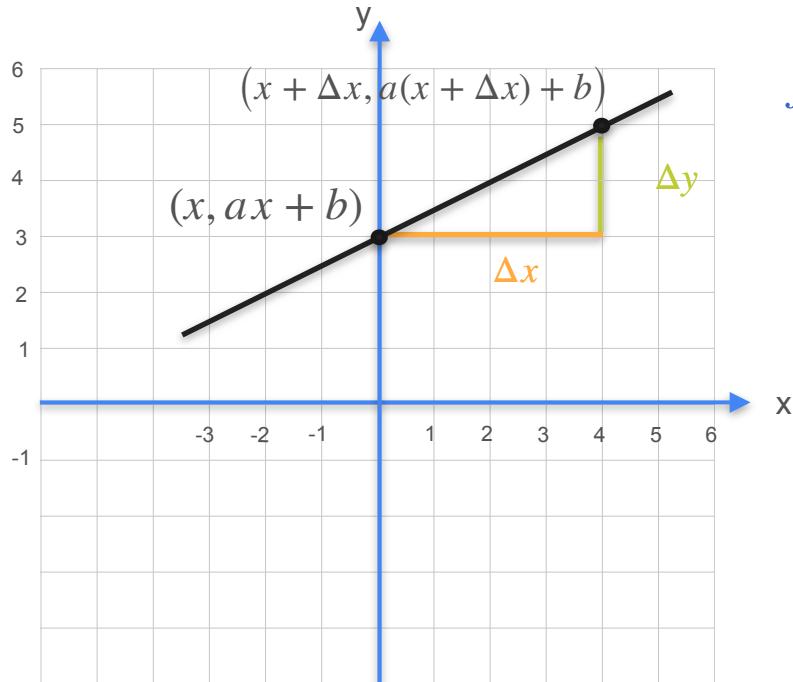


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

# Derivative of a Line



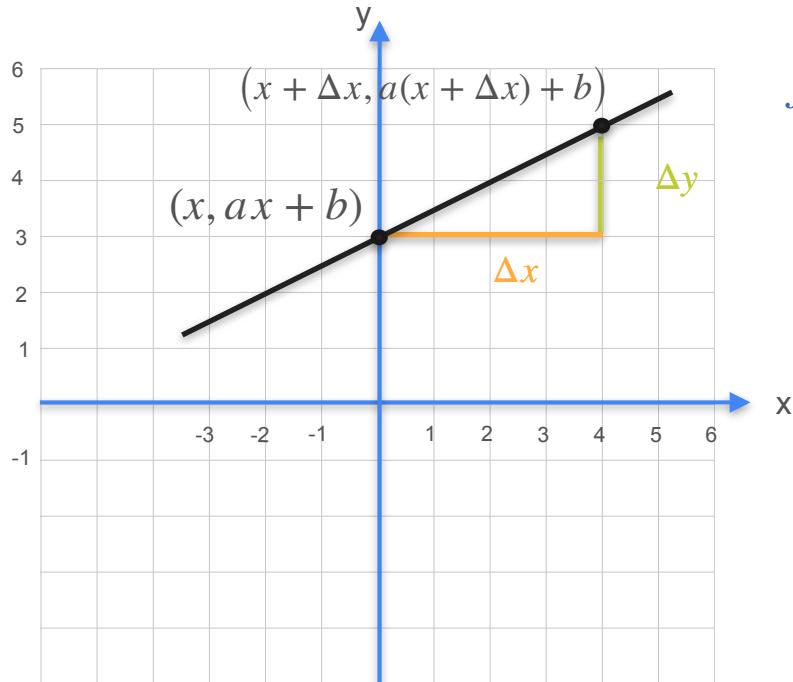
$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

$$= a \frac{\Delta x}{\Delta x}$$

# Derivative of a Line

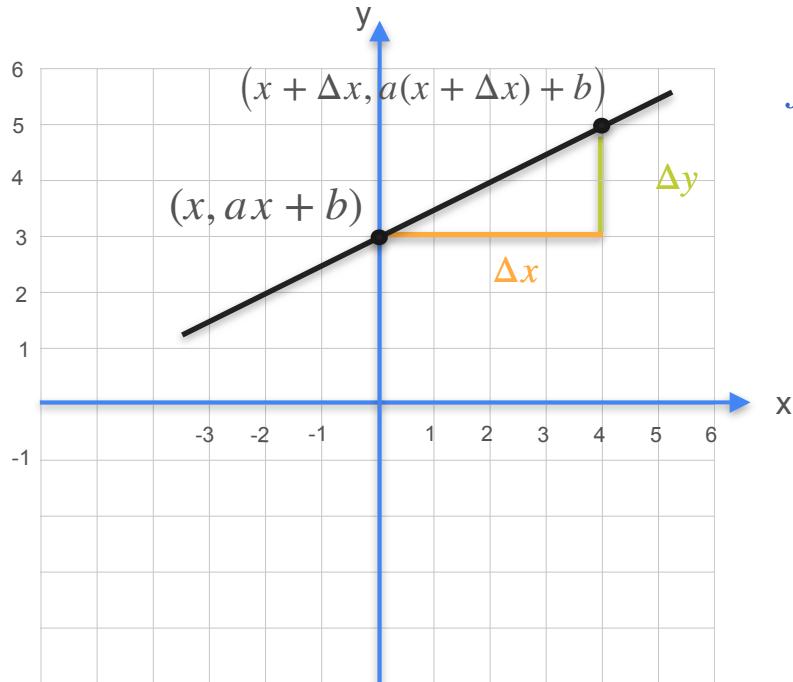


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\Delta x}{\Delta x}\end{aligned}$$

# Derivative of a Line

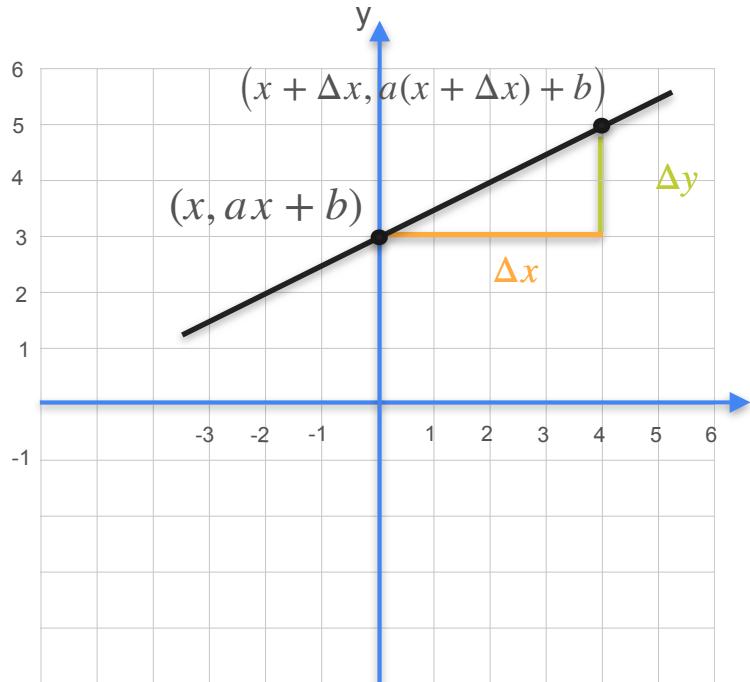


$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\Delta x}{\Delta x} = a\end{aligned}$$

# Derivative of a Line



$$f(x) = ax + b \quad \rightarrow \quad f'(x) = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\begin{aligned} x \frac{df}{dx} &= \frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x} \\ &= a \frac{\Delta x}{\Delta x} = \boxed{a} \end{aligned}$$



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# Derivatives and Optimization

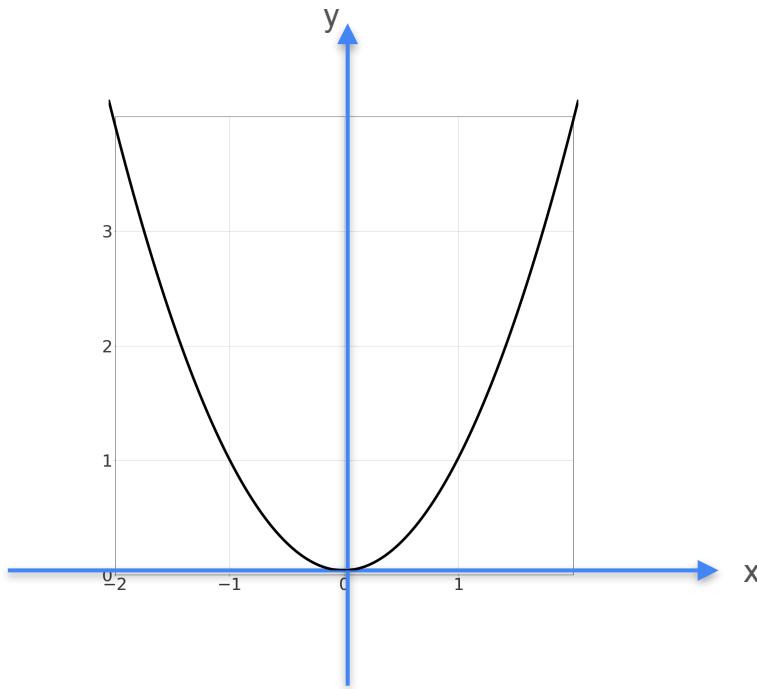
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**Some common derivatives:  
Quadratics**

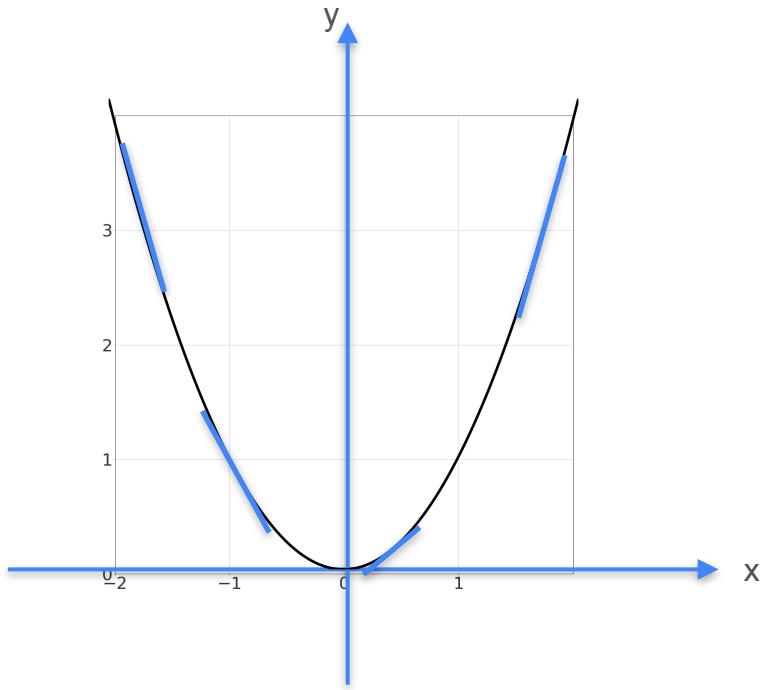
# Derivative of Quadratic Functions

# Derivative of Quadratic Functions

Quadratics:  $y = f(x) = x^2$



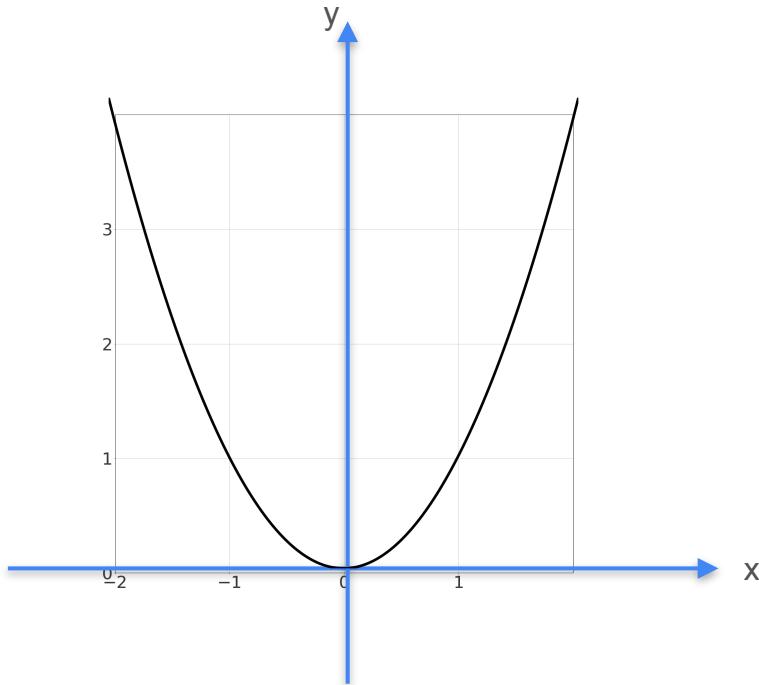
# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x}$

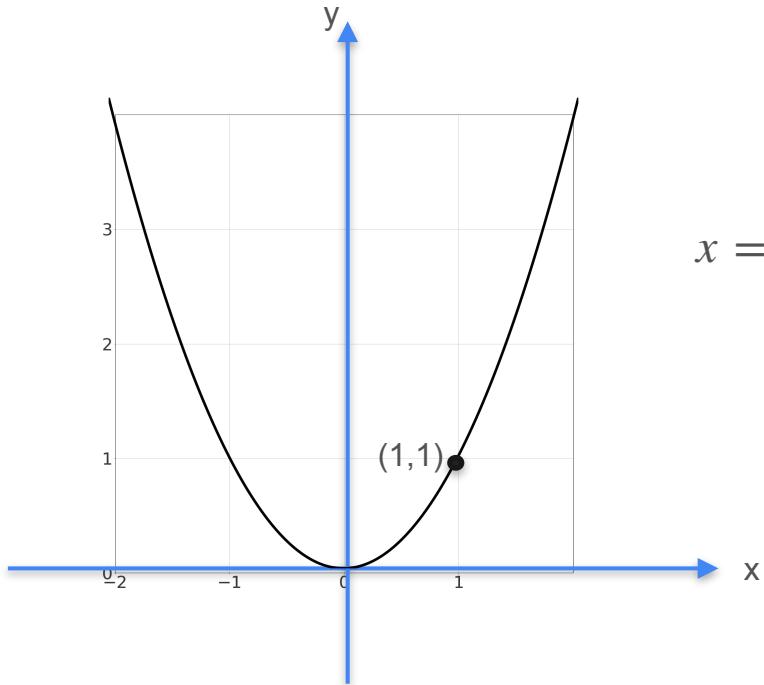
# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

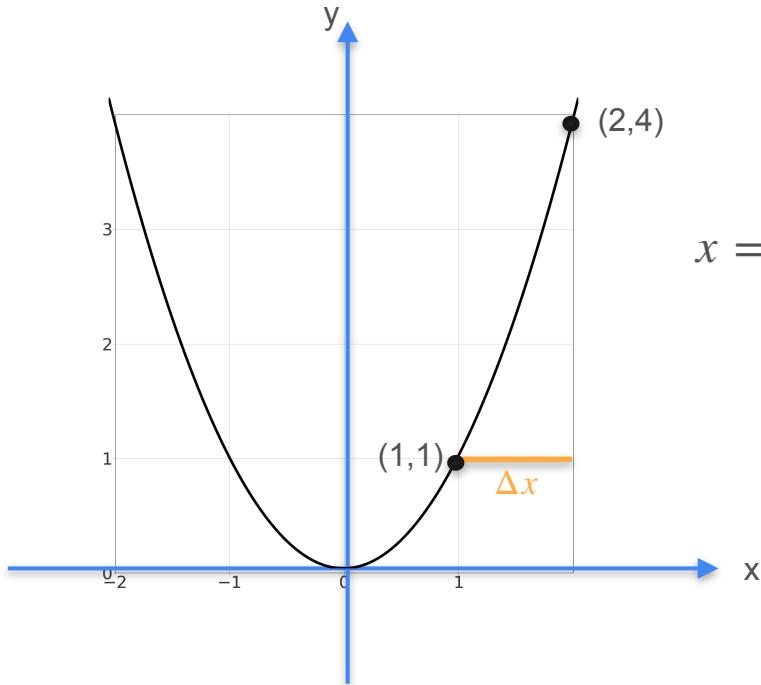
# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

# Derivative of Quadratic Functions



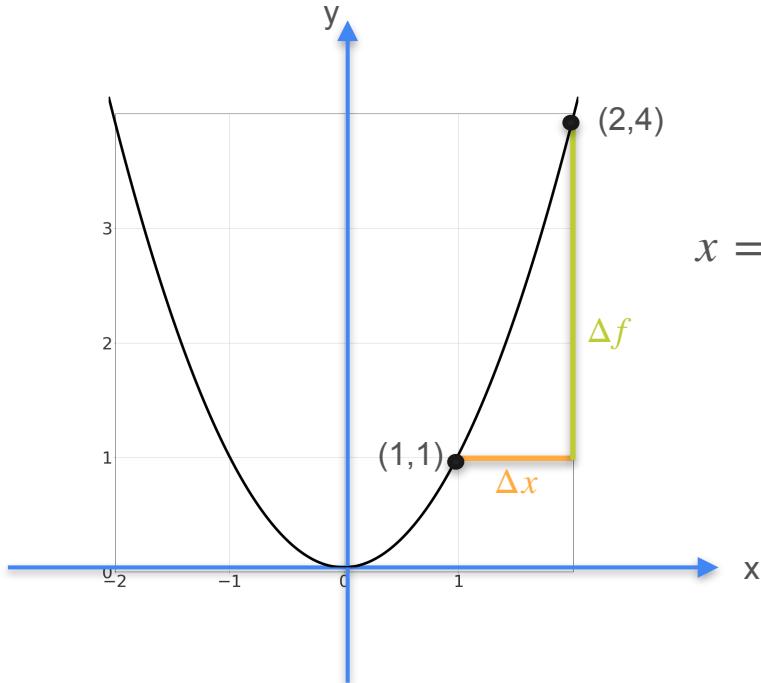
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$$x = 1$$

$\Delta x$	1.0
------------	-----

# Derivative of Quadratic Functions



$$x = 1$$

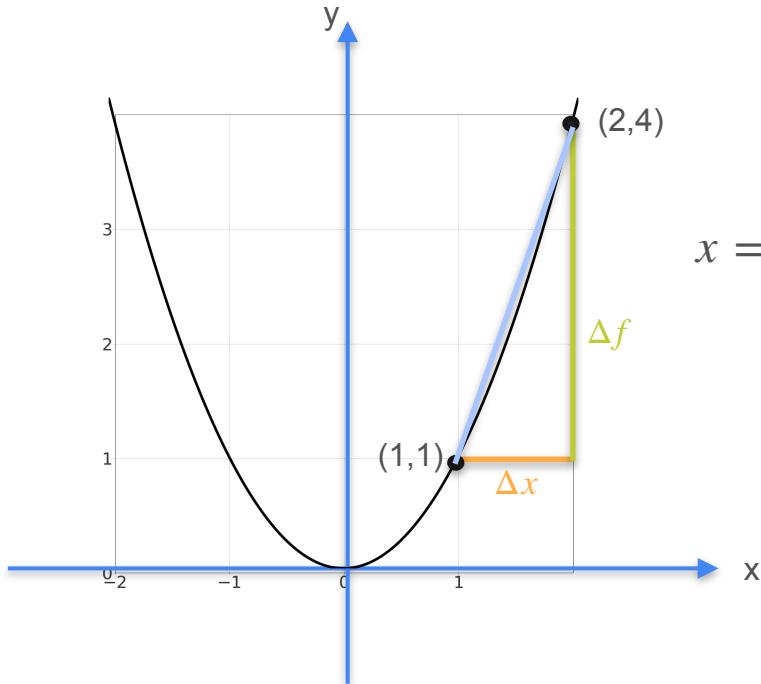
$\Delta x$	1.0
$\Delta f$	3

$$(1 + 1)^2 - 1^2 = 4 - 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

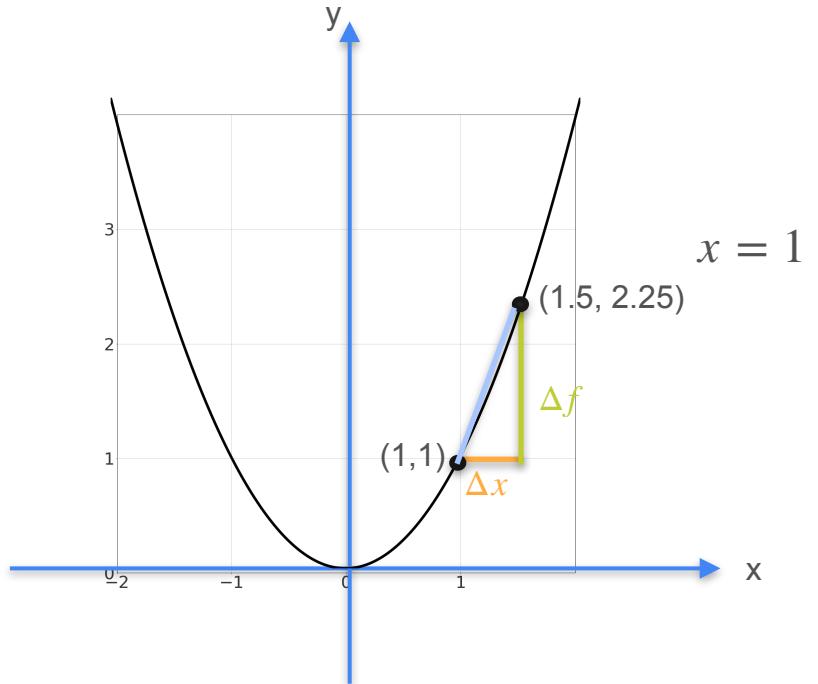
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	3
Slope	3

$$(1 + 1)^2 - 1^2 = 4 - 1$$

$$\frac{3}{1}$$

# Derivative of Quadratic Functions

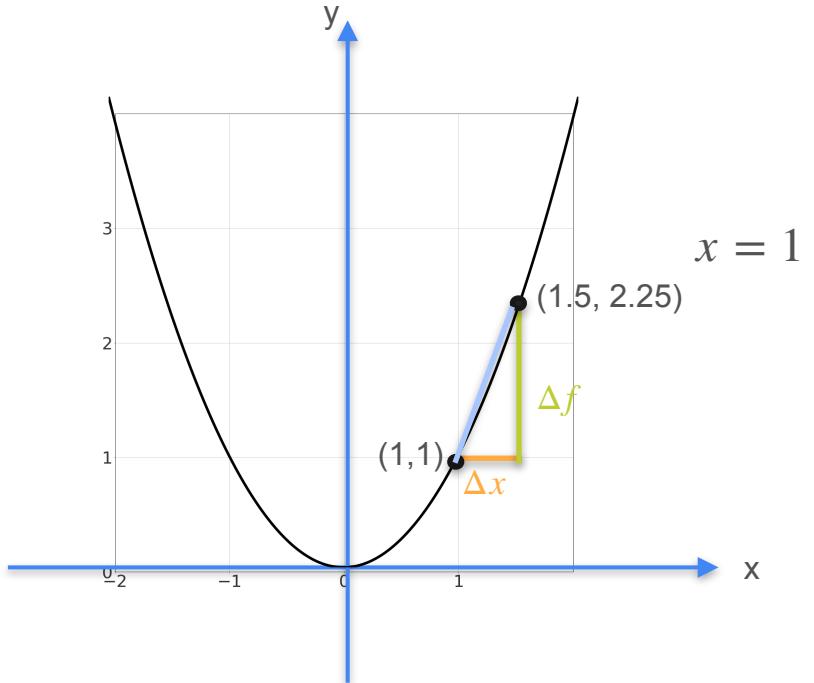


Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	3	
Slope	3	

# Derivative of Quadratic Functions



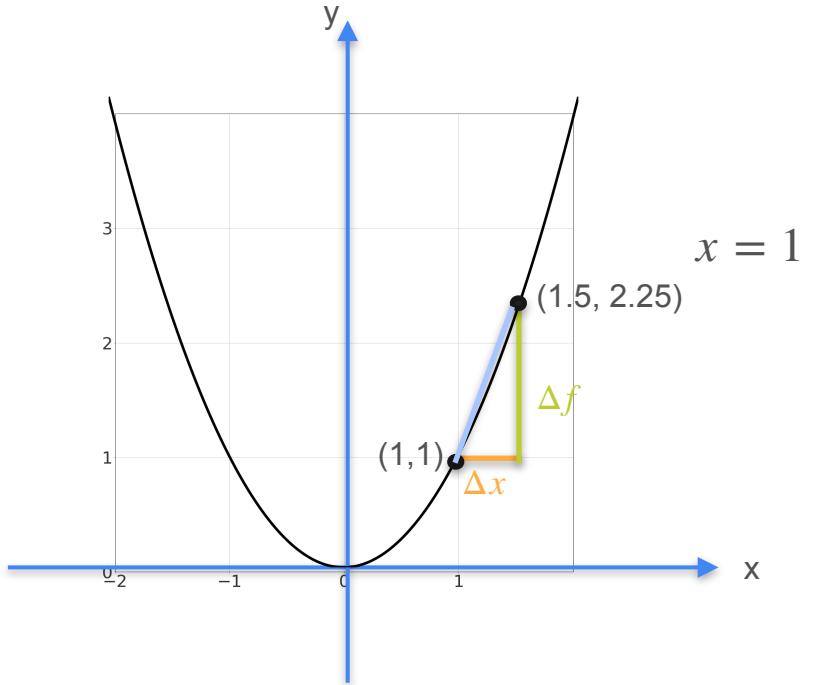
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	$1/2$
$\Delta f$	3	1.25
Slope	3	

$$(1 + 0.5)^2 - 1^2 = 2.25 - 1$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

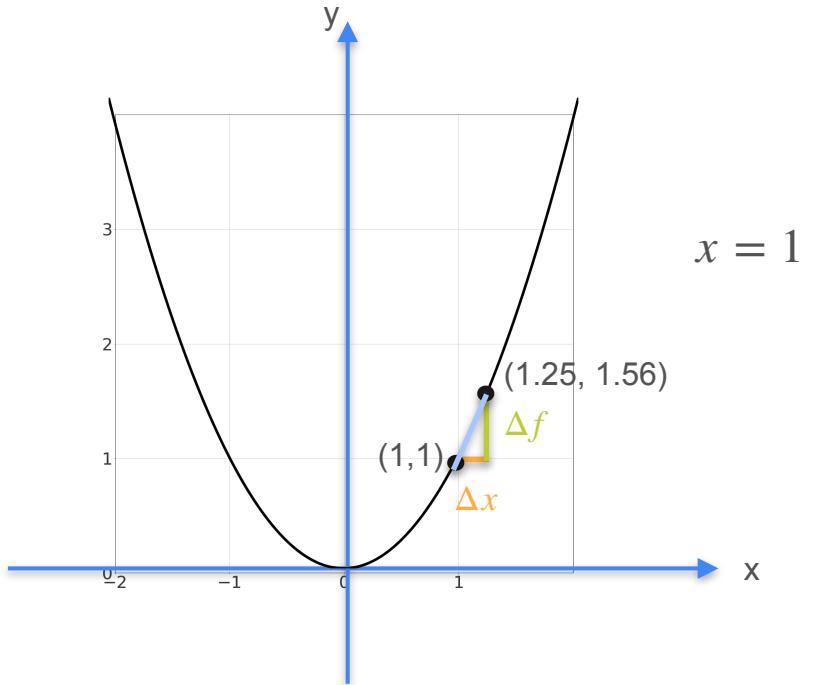
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	$1/2$
$\Delta f$	3	1.25
Slope	3	2.5

$$(1 + 0.5)^2 - 1^2 = 2.25 - 1$$

$$\frac{1.25}{0.5}$$

# Derivative of Quadratic Functions

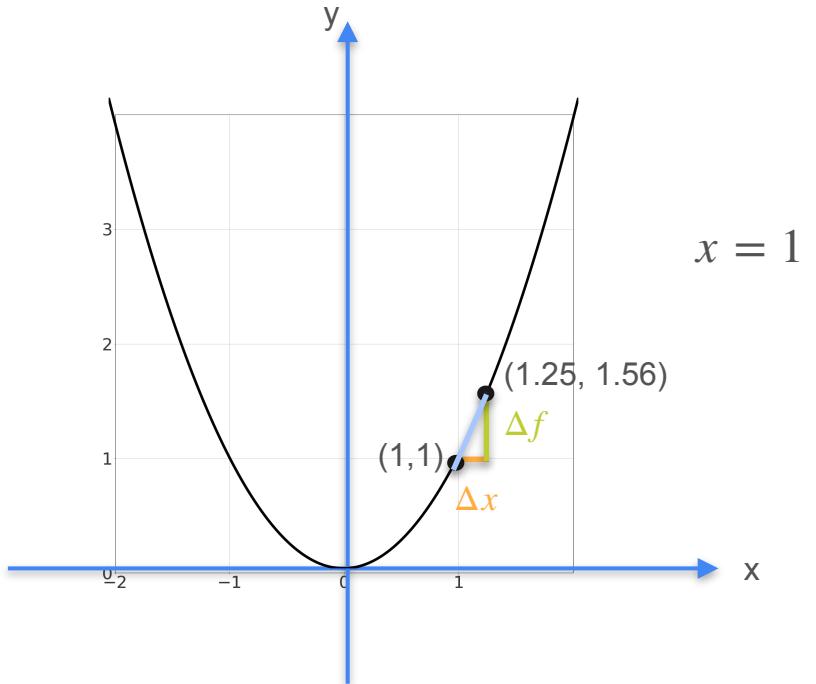


Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	3	1.25	
Slope	3	2.5	

# Derivative of Quadratic Functions



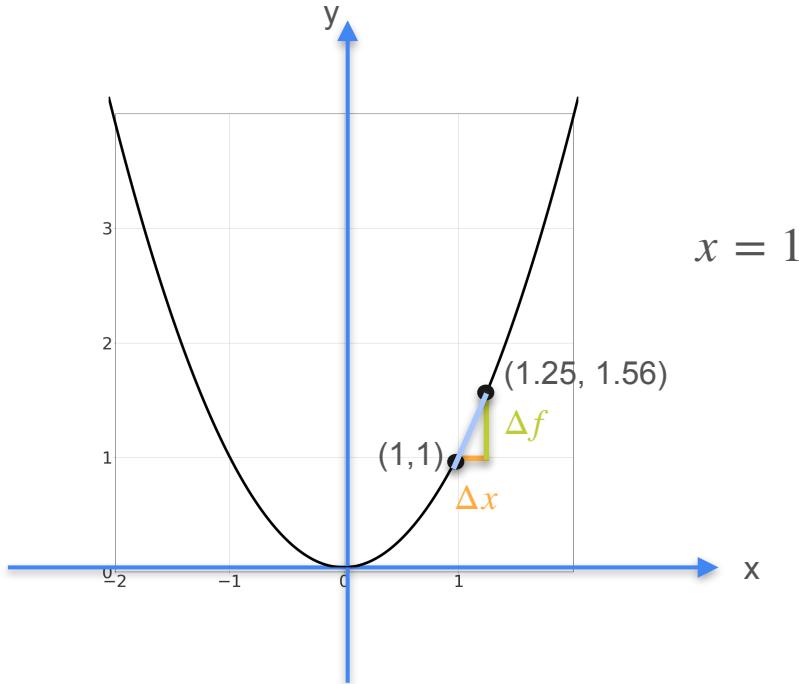
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	3	1.25	0.562
Slope	3	2.5	

$$(1 + 0.25)^2 - 1^2 = 1.56 - 1$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

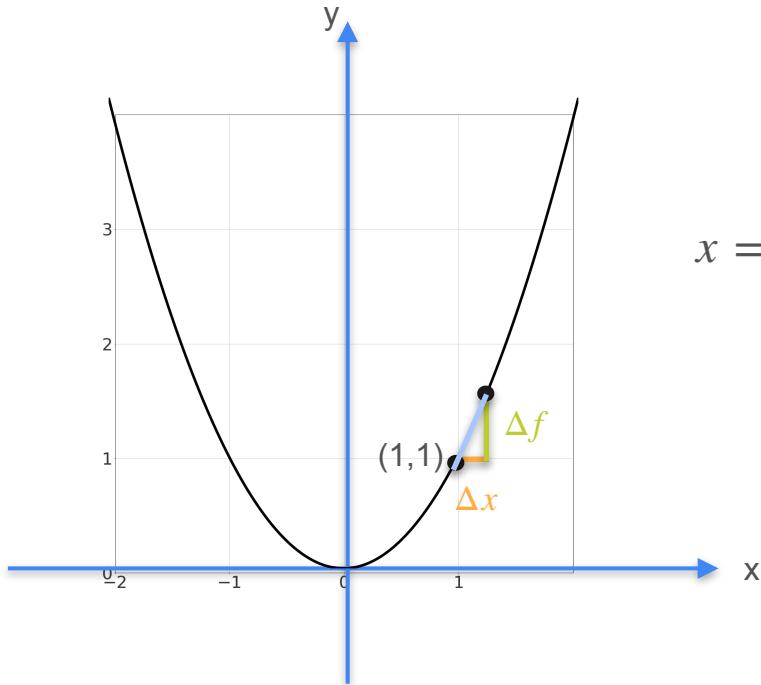
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	3	1.25	0.562
Slope	3	2.5	2.25

$$(1 + 0.25)^2 - 1^2 = 1.56 - 1$$

$$\frac{0.56}{0.25}$$

# Derivative of Quadratic Functions



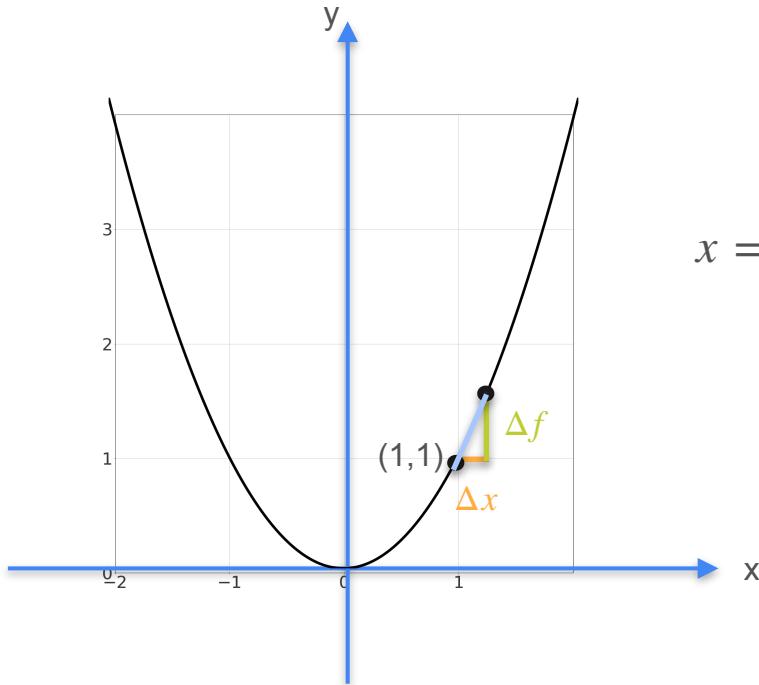
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	3	1.25	0.562
Slope	3	2.5	2.25

# Derivative of Quadratic Functions



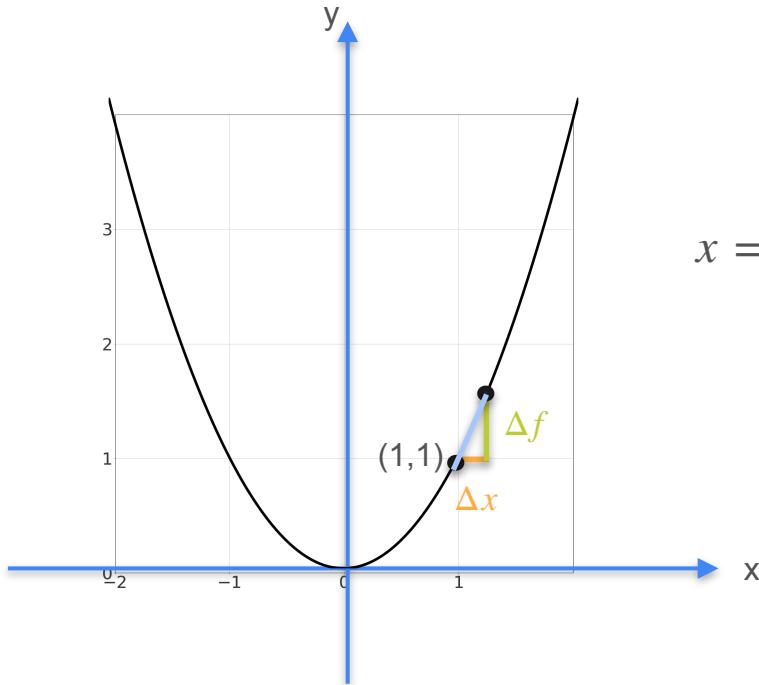
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	3	1.25	0.562	0.265	
Slope	3	2.5	2.25	2.125	

# Derivative of Quadratic Functions



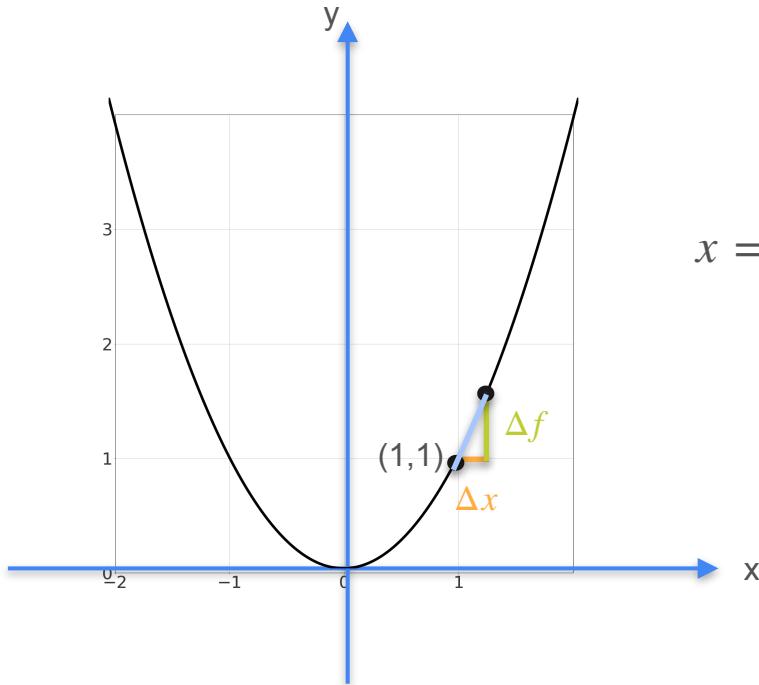
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	3	1.25	0.562	0.265	0.128
Slope	3	2.5	2.25	2.125	

# Derivative of Quadratic Functions



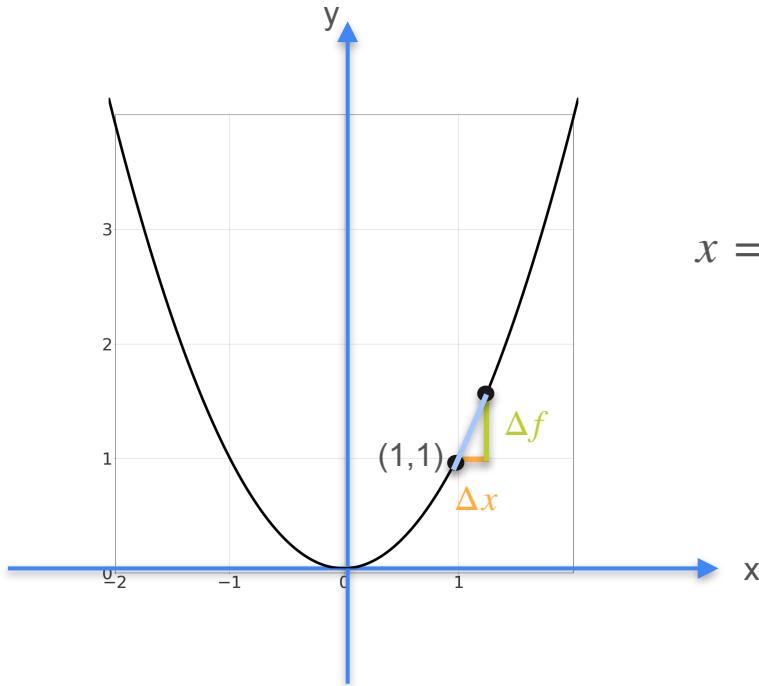
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	3	1.25	0.562	0.265	0.128
Slope	3	2.5	2.25	2.125	2.065

# Derivative of Quadratic Functions



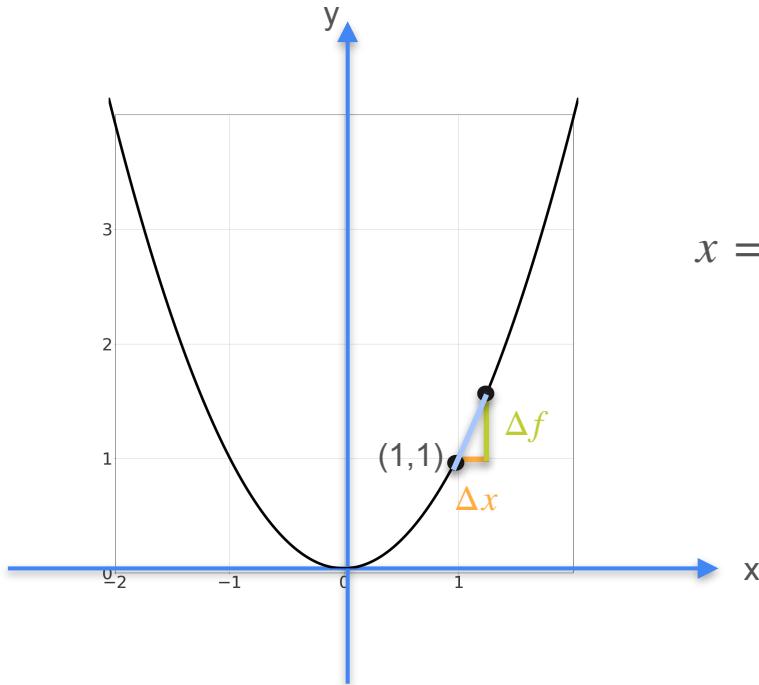
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3	1.25	0.562	0.265	0.128	
Slope	3	2.5	2.25	2.125	2.065	

# Derivative of Quadratic Functions



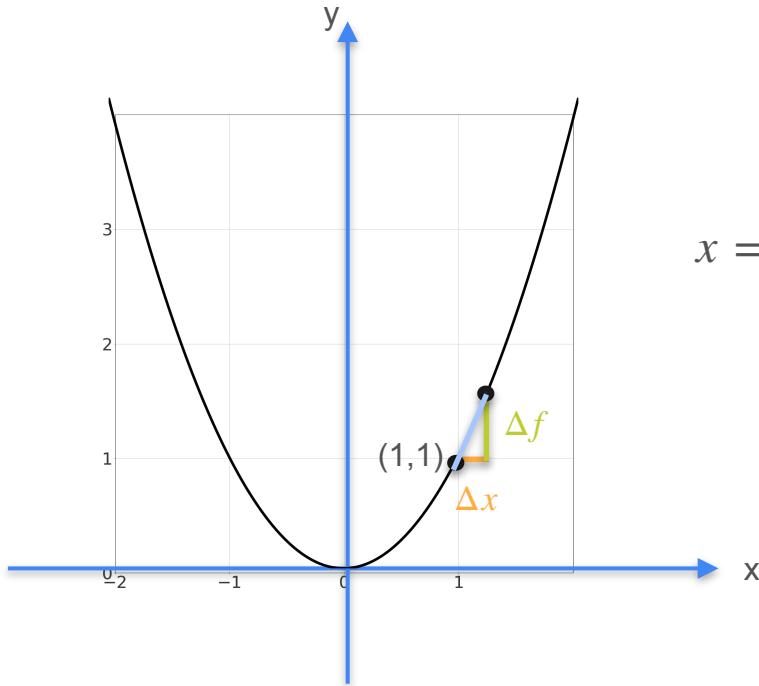
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	

# Derivative of Quadratic Functions



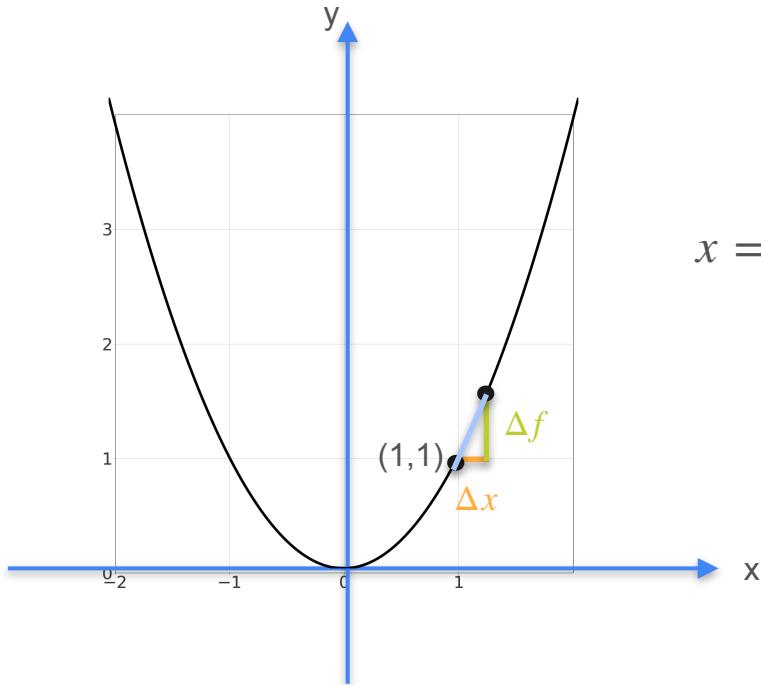
$$x = 1$$

Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

# Derivative of Quadratic Functions



$$x = 1$$

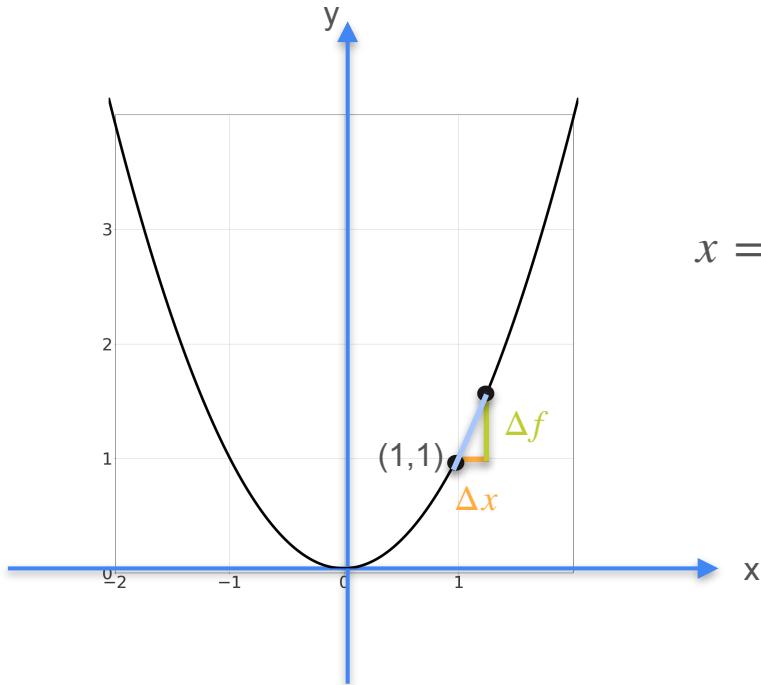
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

$$f'(1) = \frac{d}{dx} f(1) = 2$$

# Derivative of Quadratic Functions



$$x = 1$$

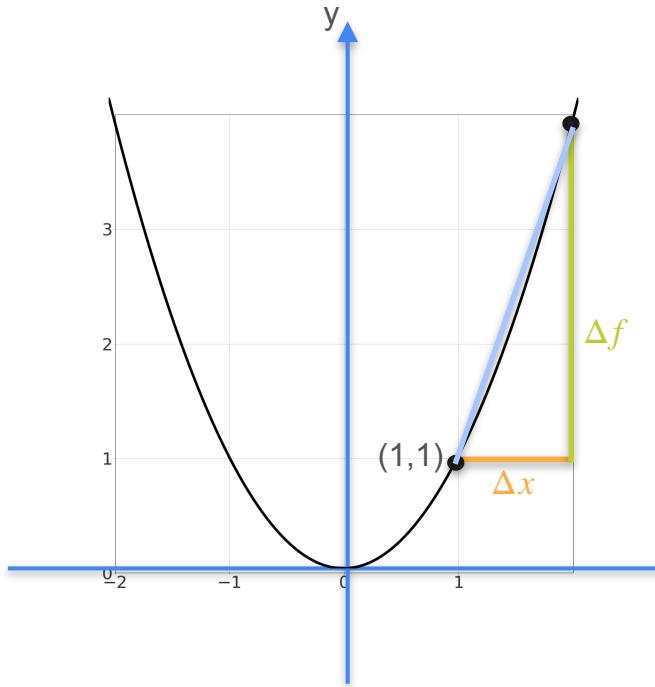
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3	1.25	0.562	0.265	0.128	0.002
Slope	3	2.5	2.25	2.125	2.065	2.001

$$f'(1) = \frac{d}{dx} f(1) = 2 = 2 \times 1$$

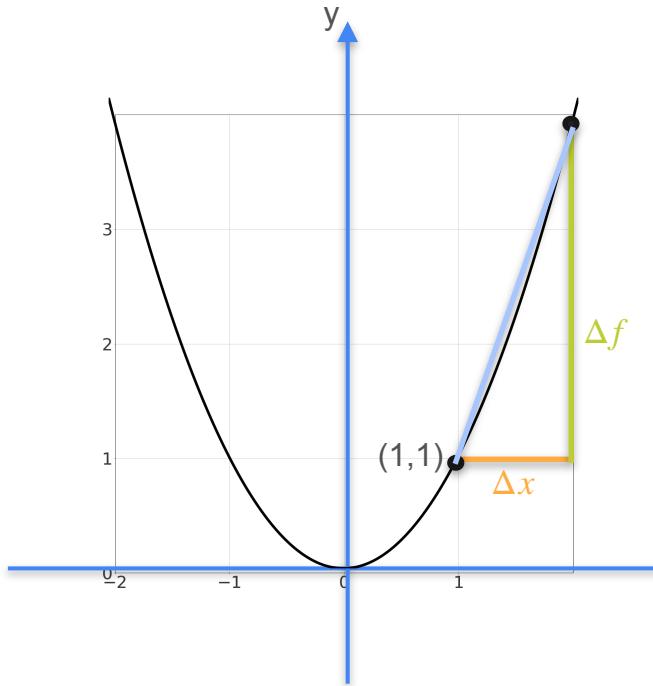
# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

# Derivative of Quadratic Functions

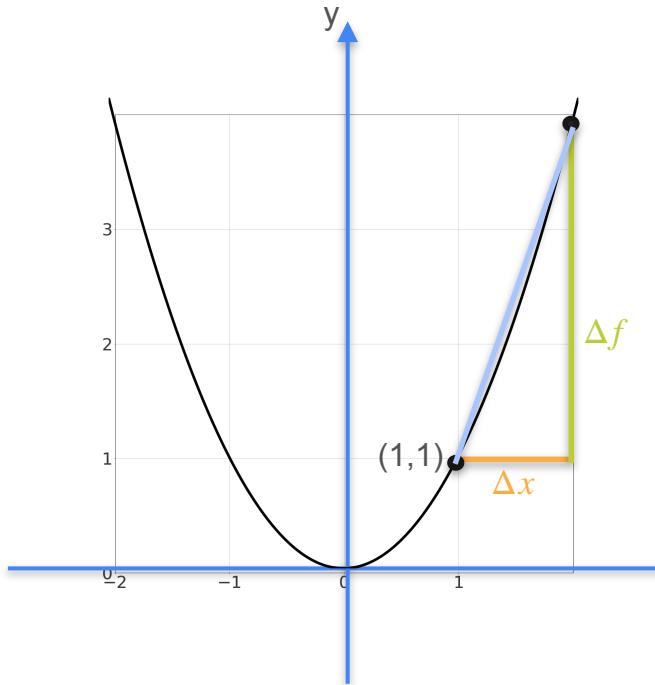


Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x}$$

# Derivative of Quadratic Functions

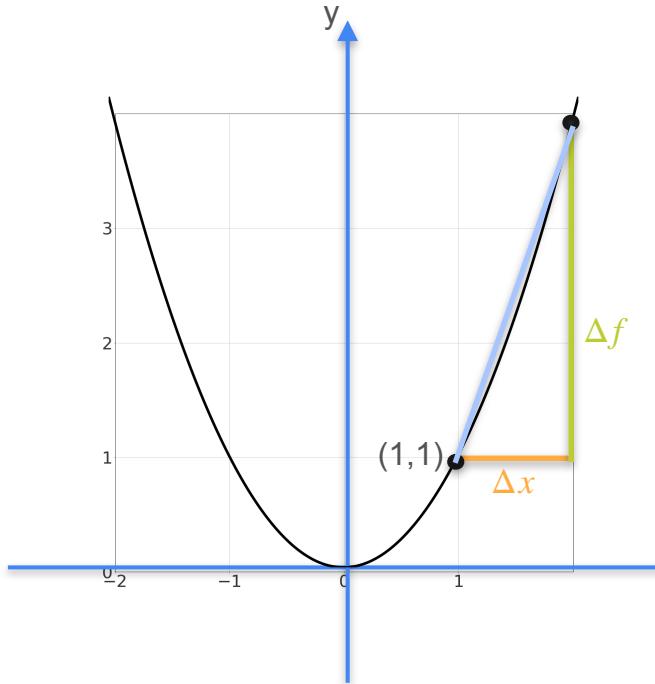


Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

# Derivative of Quadratic Functions



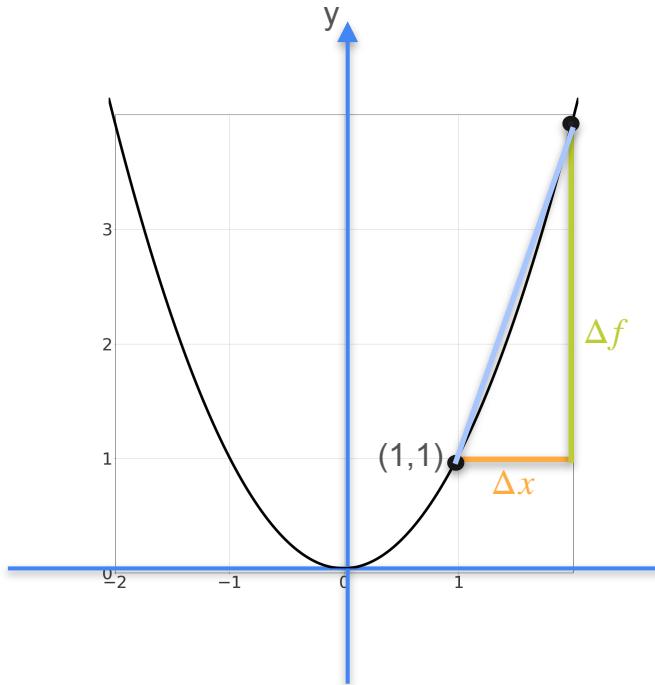
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

# Derivative of Quadratic Functions



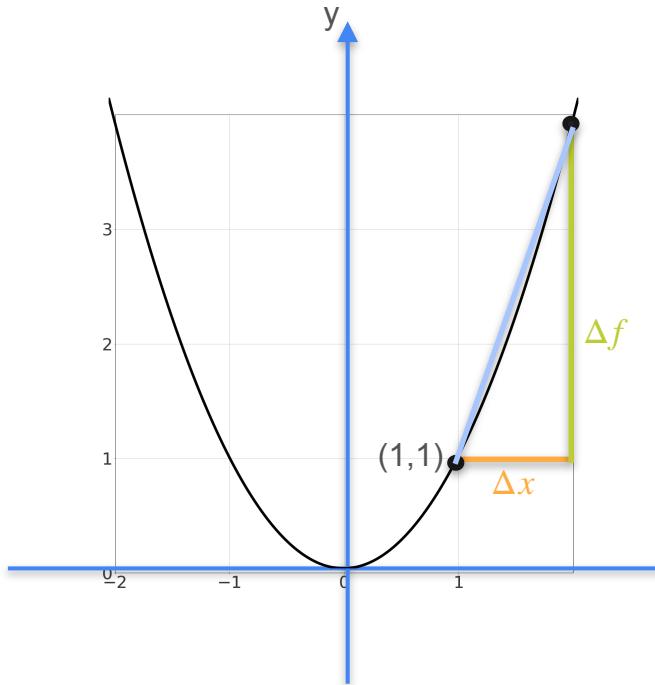
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

# Derivative of Quadratic Functions



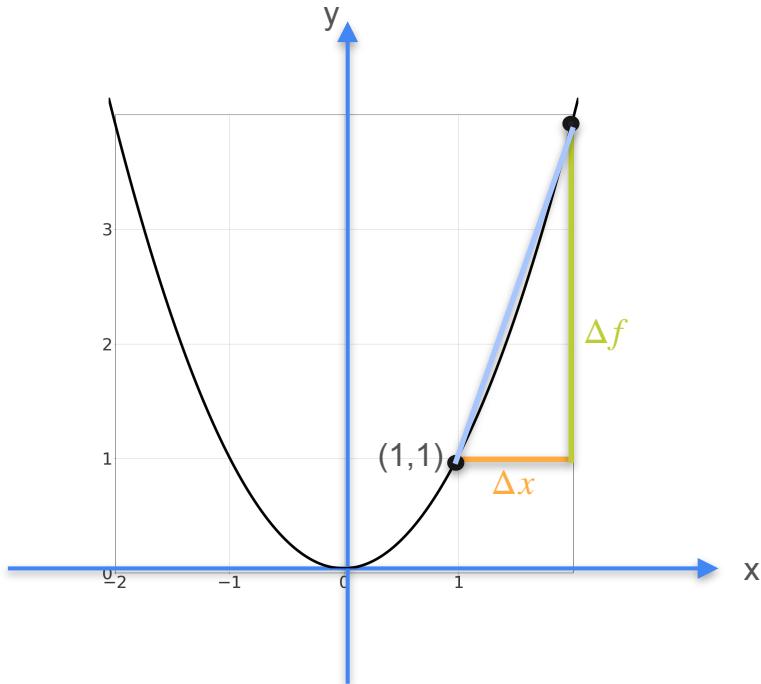
Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

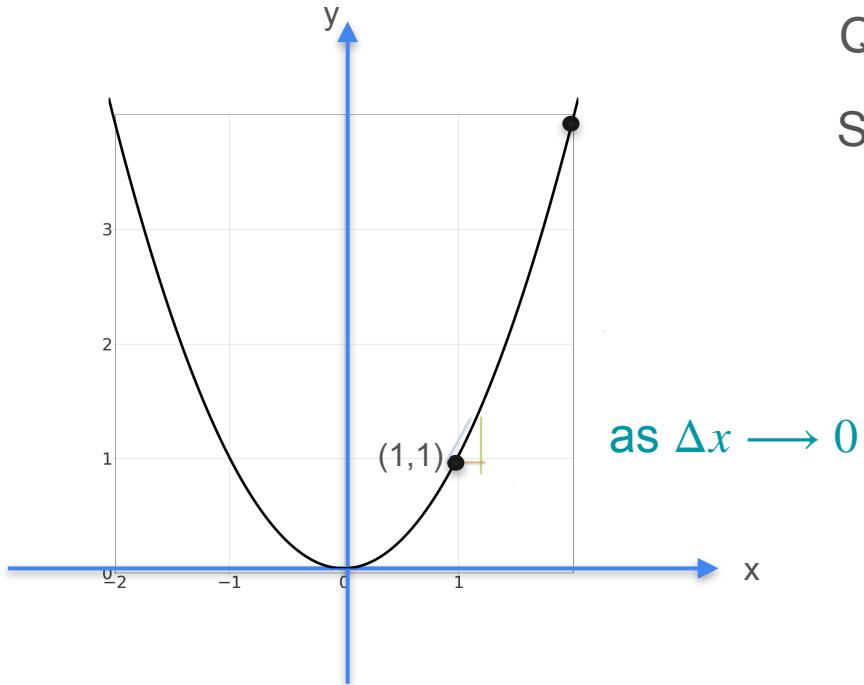
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \Delta x$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

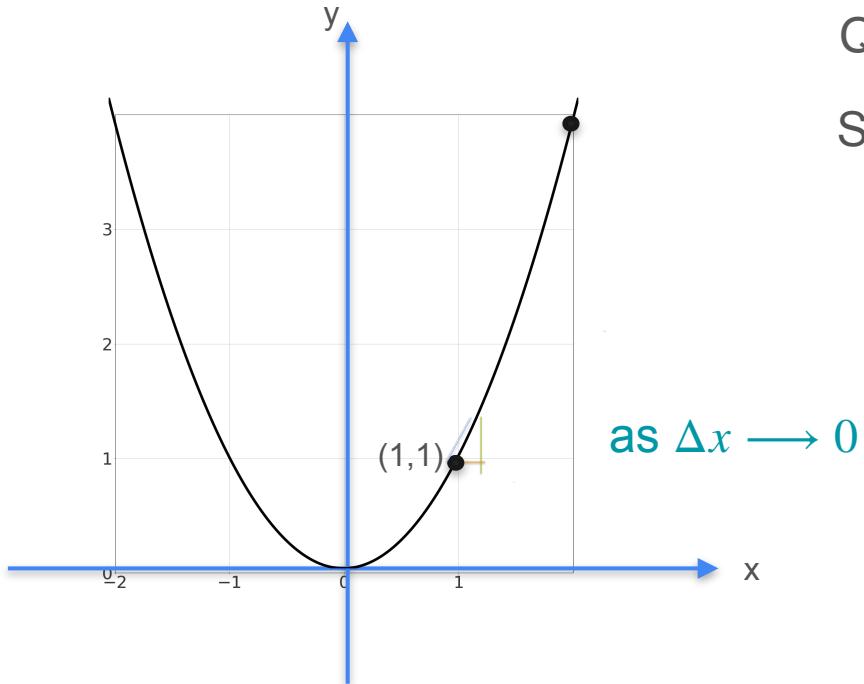
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \Delta x$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

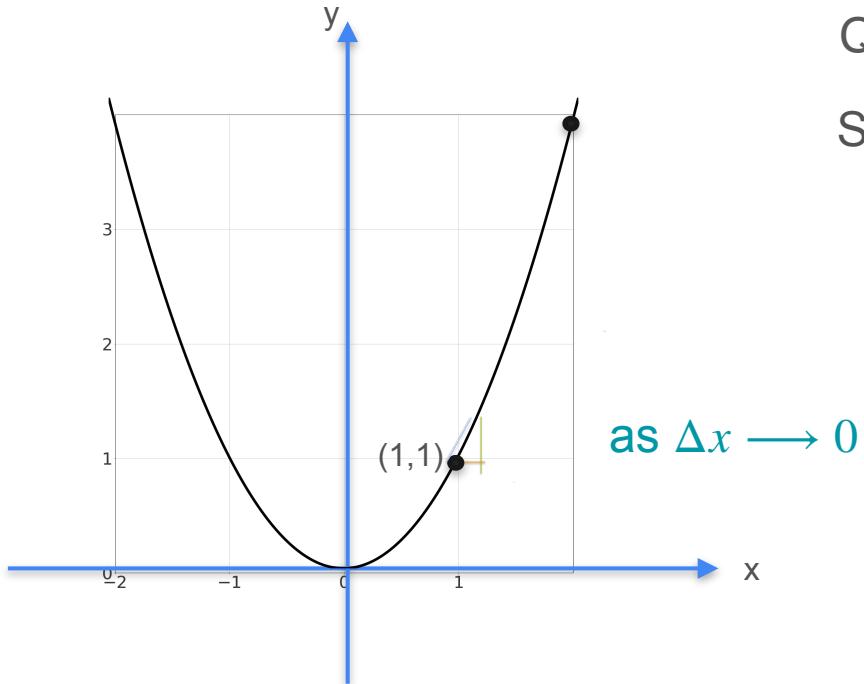
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \boxed{\Delta x} \xrightarrow{0}$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

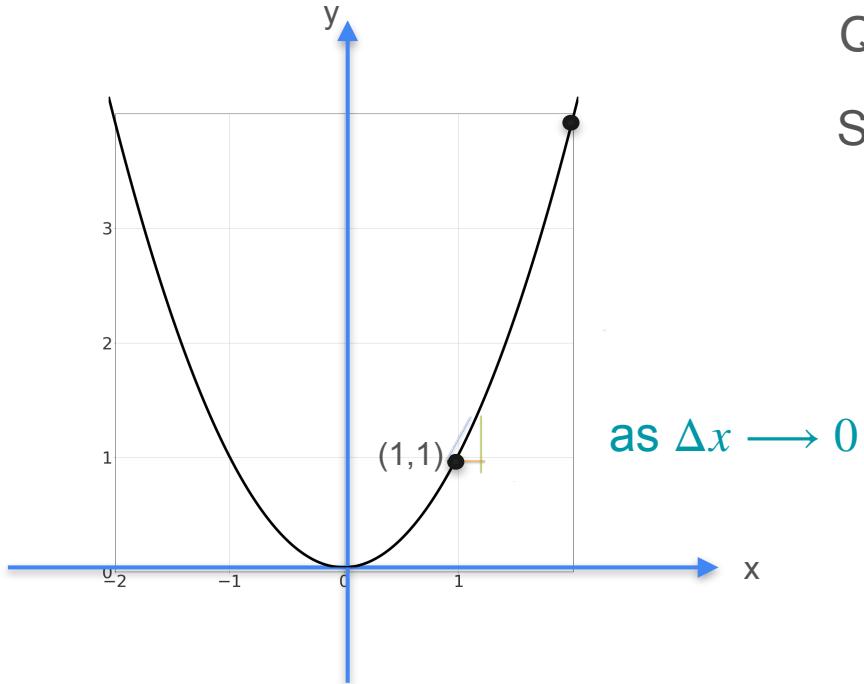
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= 2x + \boxed{\Delta x} \xrightarrow{0}$$

# Derivative of Quadratic Functions

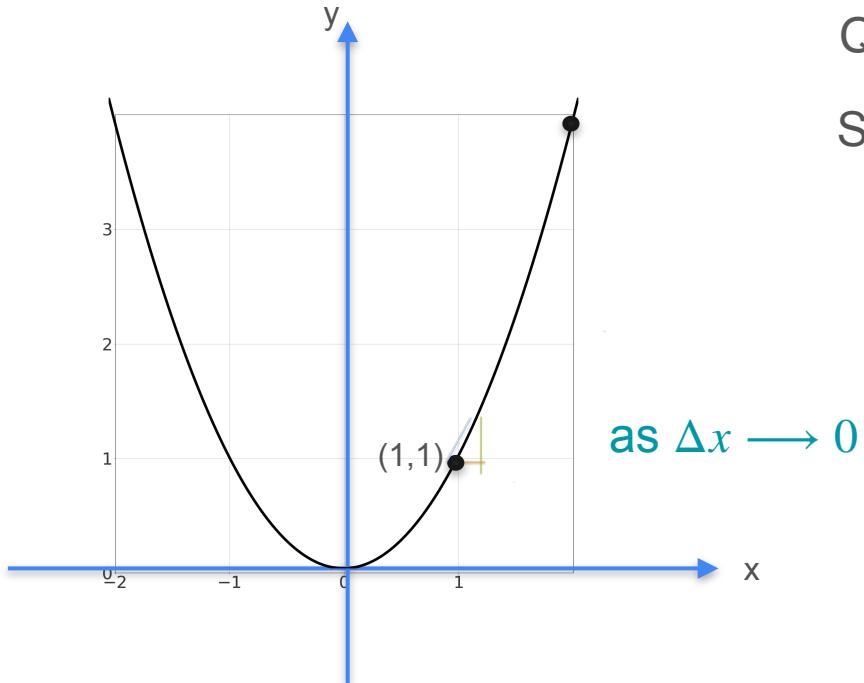


Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= 2x + \boxed{\Delta x} \xrightarrow[0]{\Delta x}\end{aligned}$$

# Derivative of Quadratic Functions



Quadratics:  $y = f(x) = x^2$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= 2x + \Delta x\end{aligned}$$

$$f(x) = x^2 \rightarrow f'(x) = 2x$$



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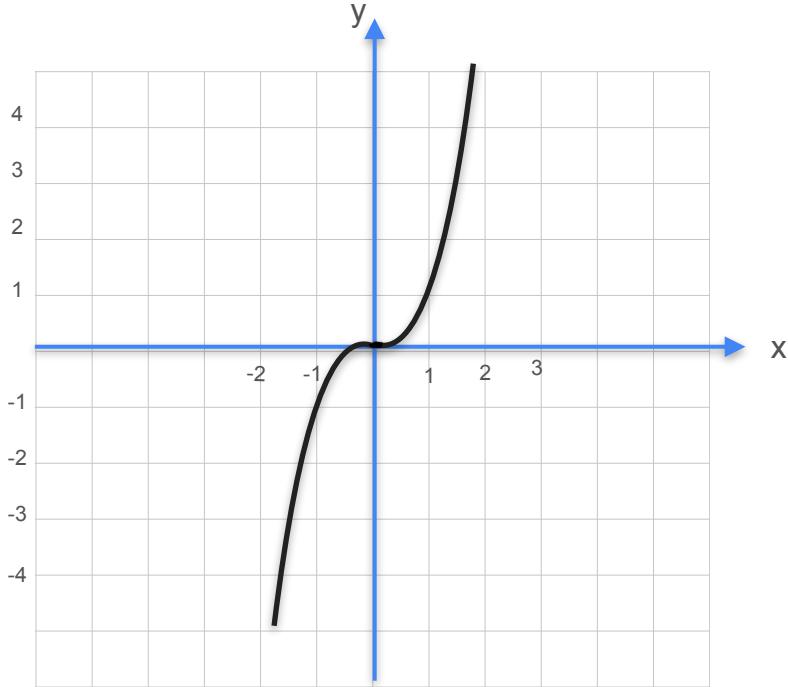
# Derivatives and Optimization

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**Some common derivatives:  
Higher degree polynomials**

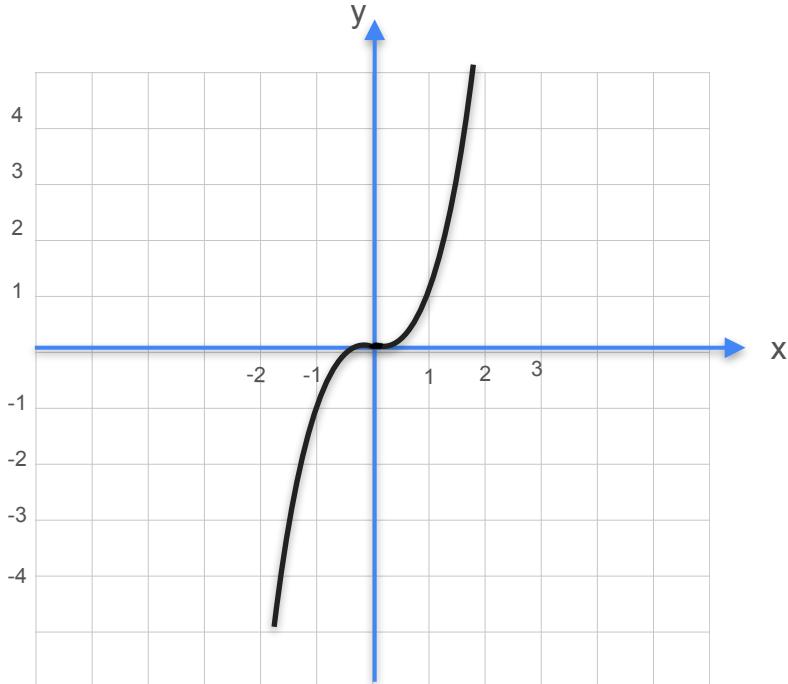
# Derivative of Cubic Functions

# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

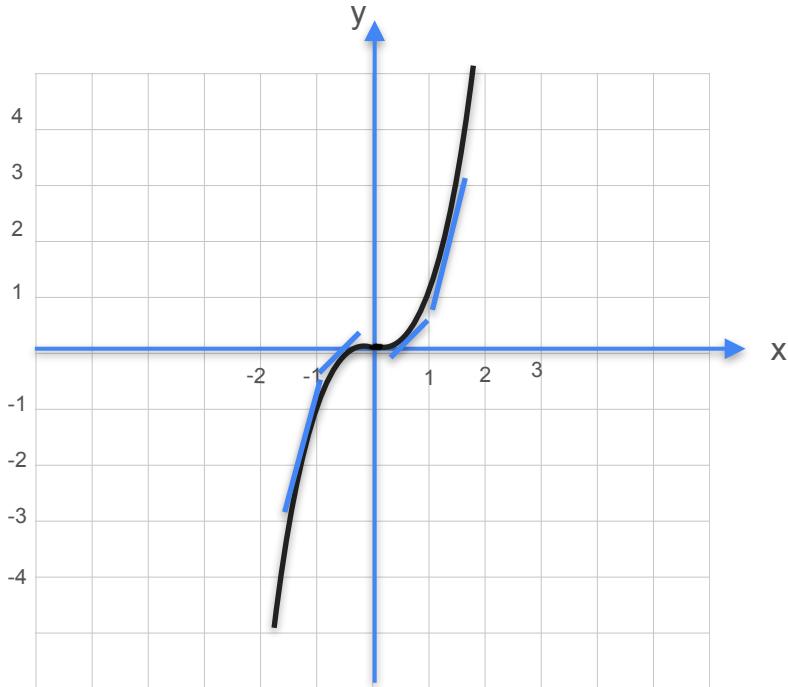
# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x}$

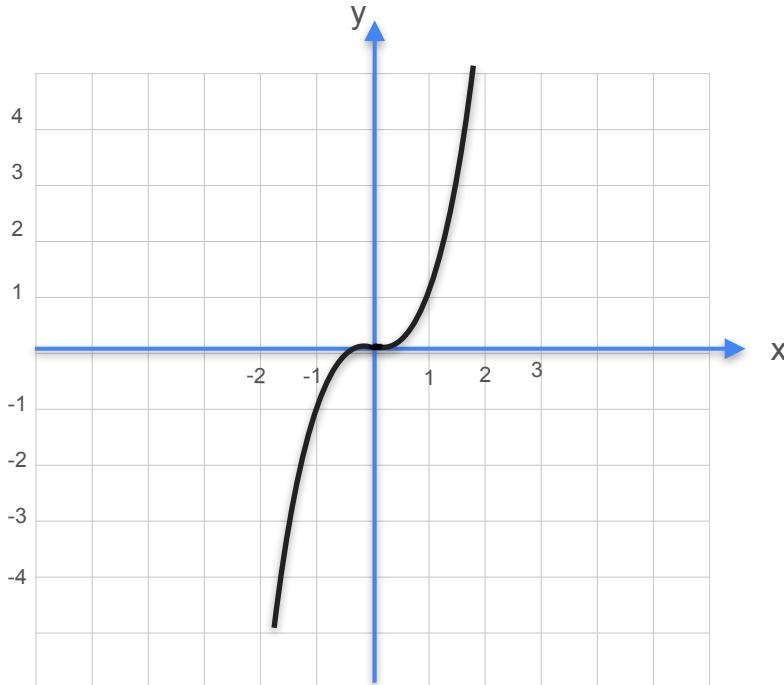
# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x}$

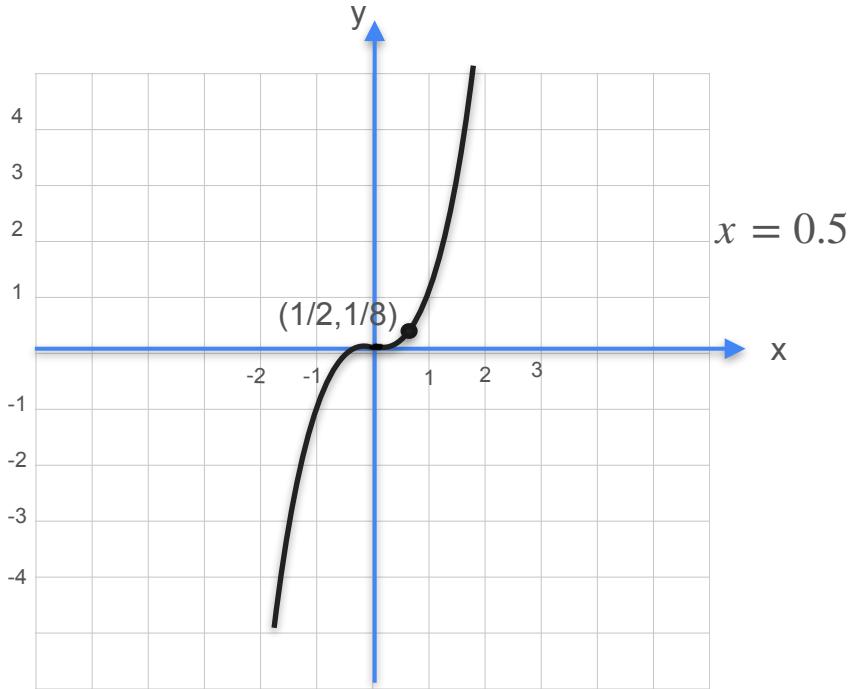
# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

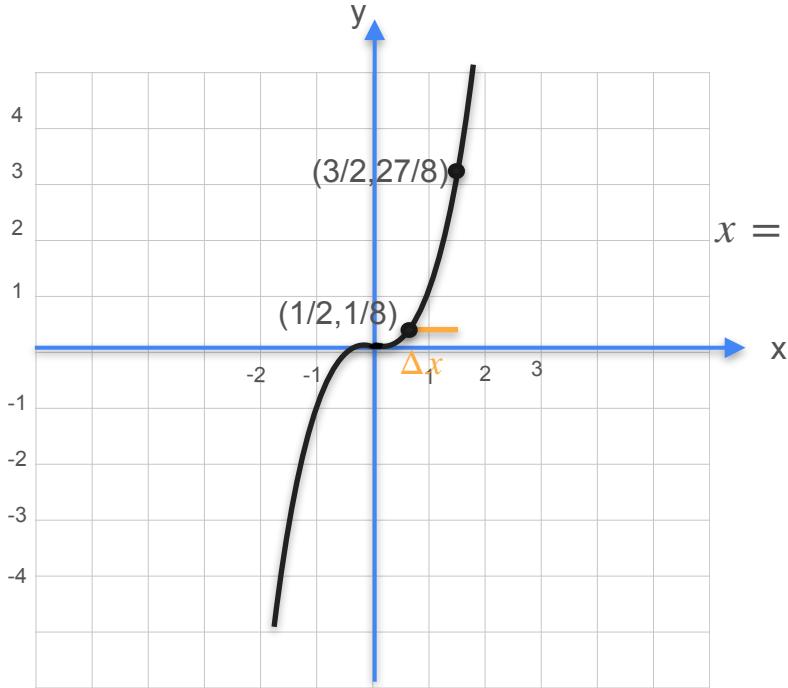
# Derivative of Cubic Functions



$$\text{Cubic: } y = f(x) = x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

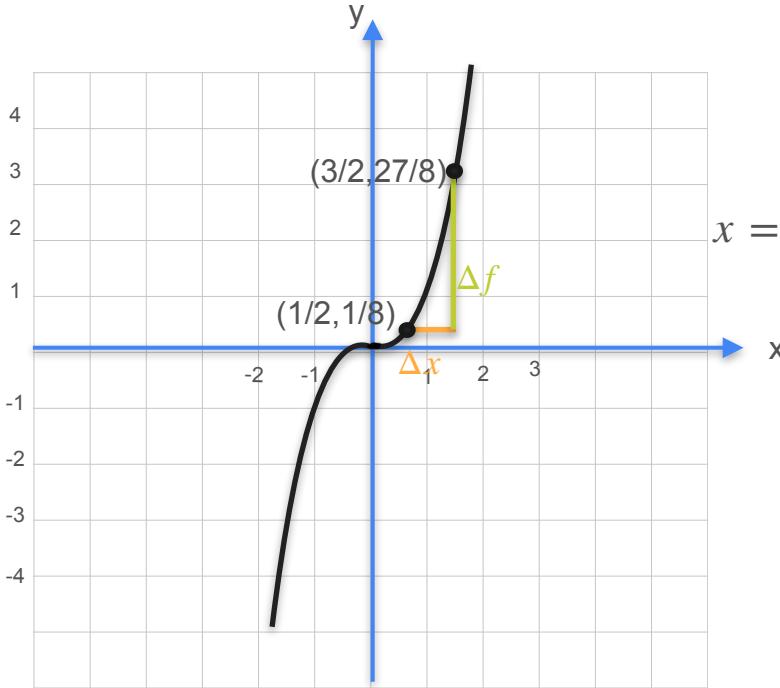
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

$\Delta x$

1.0

# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

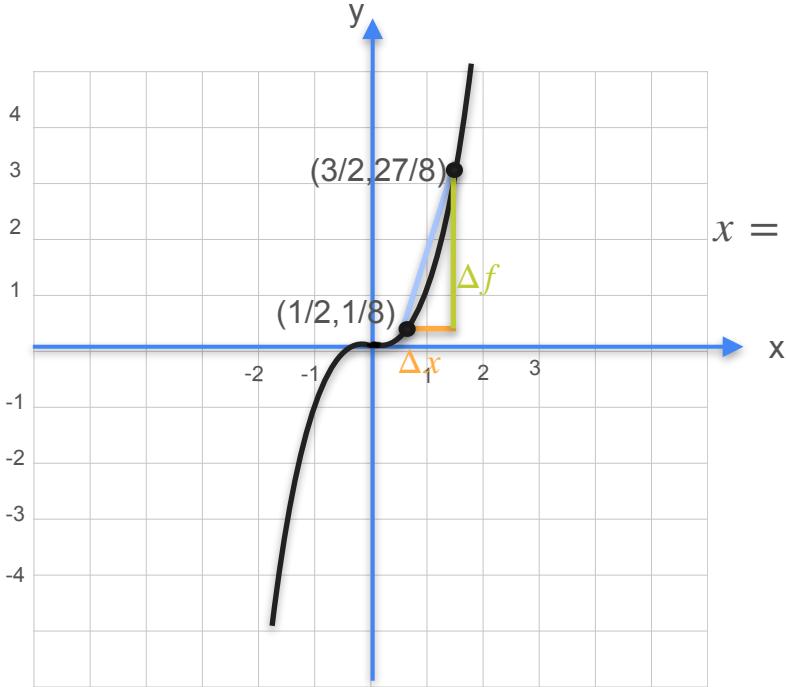
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$$x = 0.5$$

$\Delta x$	1.0
$\Delta f$	3.25

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$

# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

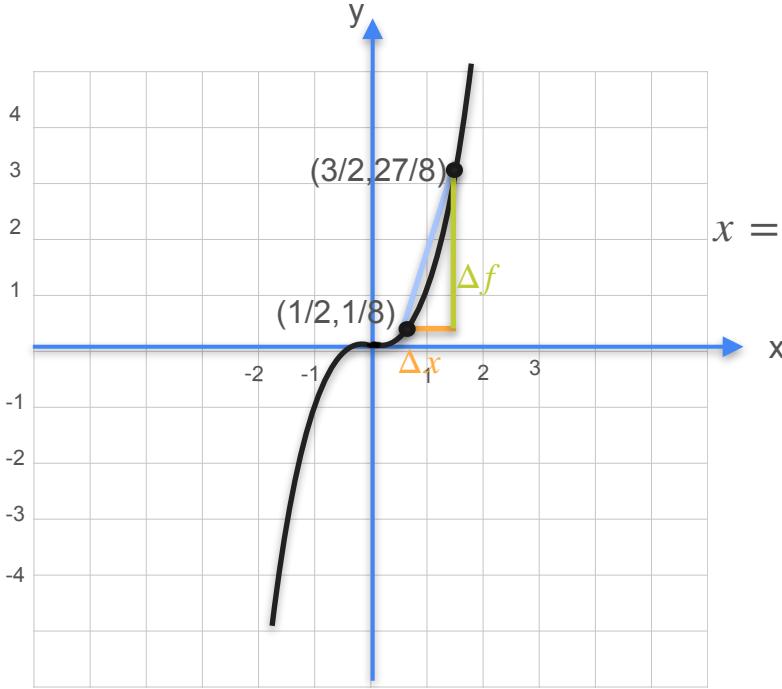
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$$x = 0.5$$

$\Delta x$	1.0
$\Delta f$	3.25
Slope	

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$

# Derivative of Cubic Functions



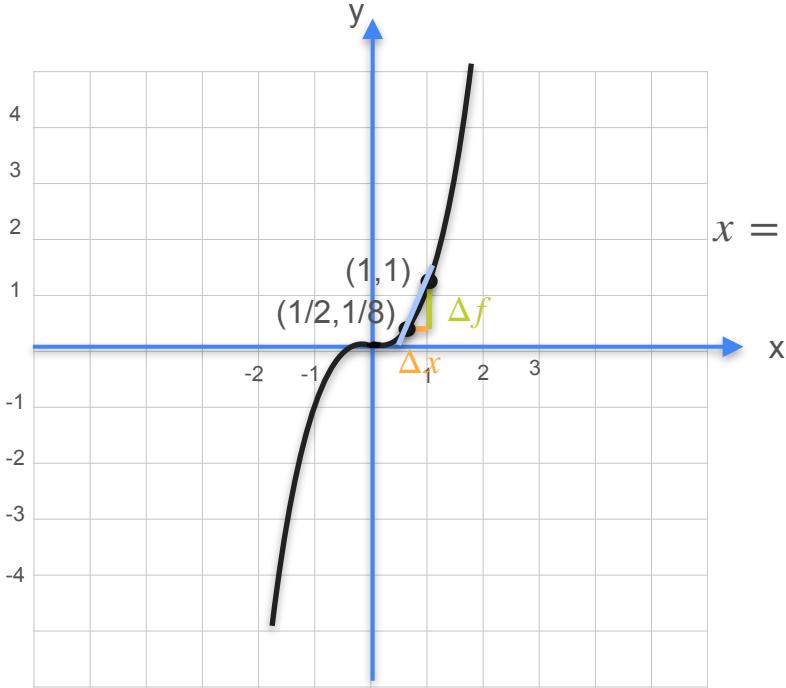
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	3.25
Slope	3.25

$$\left(\frac{1}{2} + 1\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{27}{8} - \frac{1}{8}$$
$$\frac{3.25}{1}$$

# Derivative of Cubic Functions



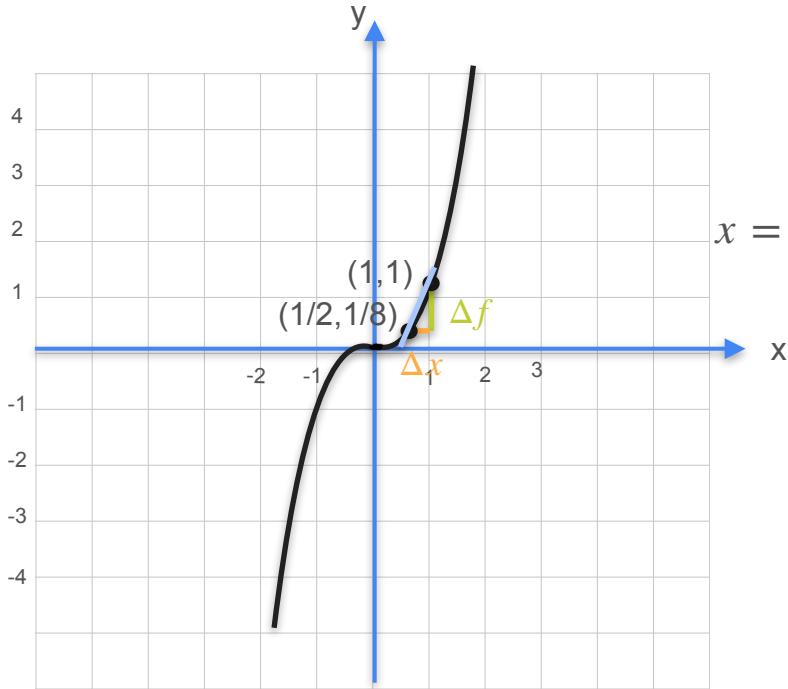
$$x = 0.5$$

Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	3.25
Slope	3.25

# Derivative of Cubic Functions



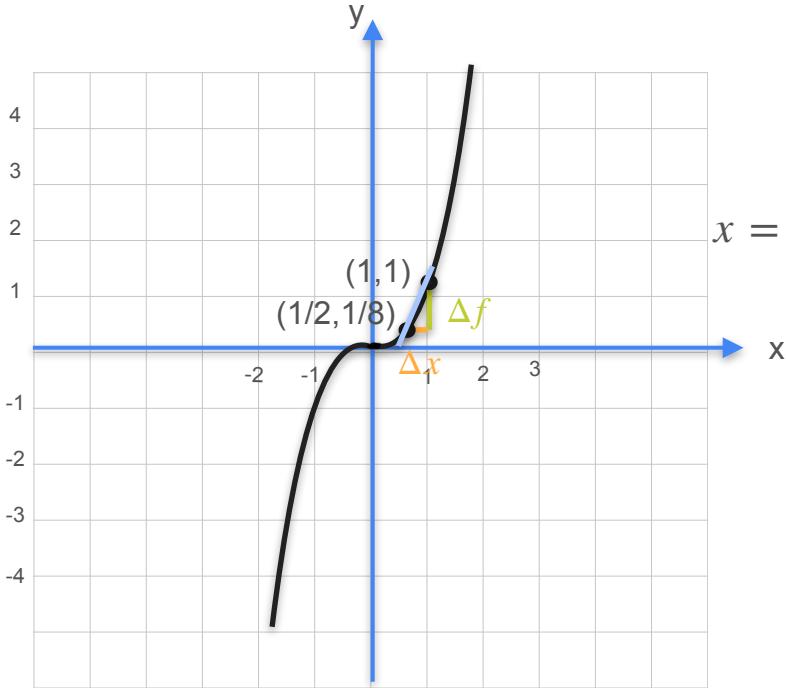
$$x = 0.5$$

Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	3.25	
Slope	3.25	

# Derivative of Cubic Functions



$$x = 0.5$$

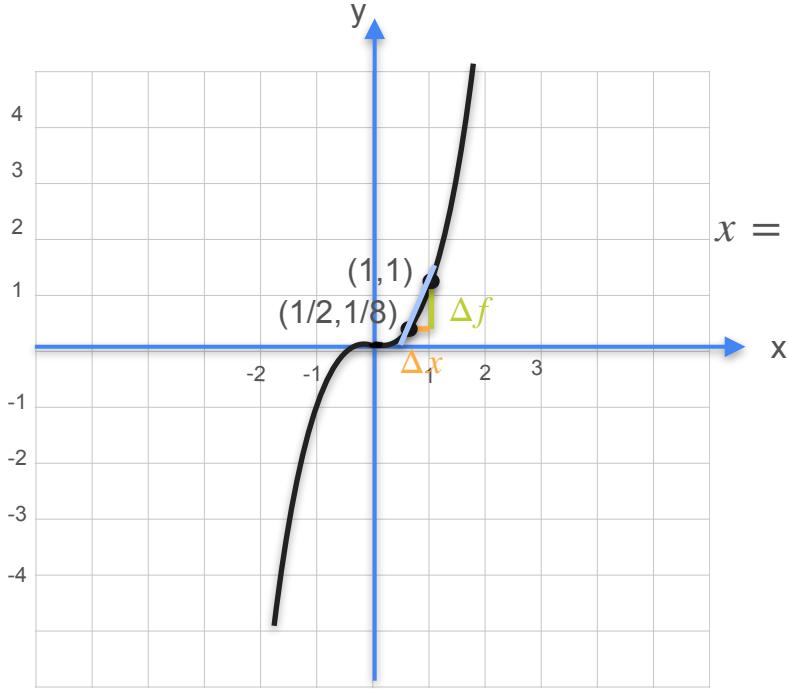
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	3.25	0.86
Slope	3.25	

$$\left(\frac{1}{2} + \frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8}$$

# Derivative of Cubic Functions



$$x = 0.5$$

Cubic:  $y = f(x) = x^3$

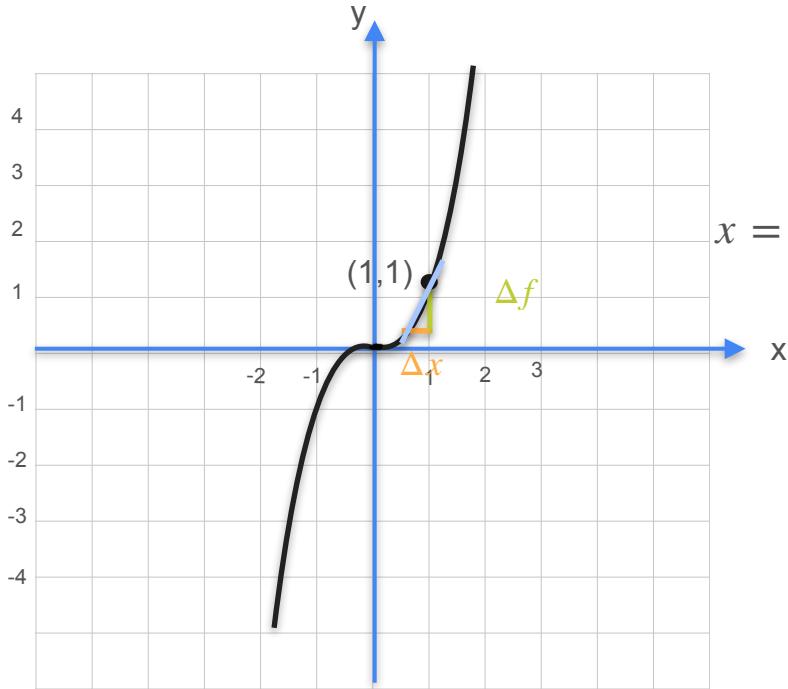
Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	3.25	0.86
Slope	3.25	1.75

$$\left(\frac{1}{2} + \frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8}$$

$$\frac{0.86}{0.5}$$

# Derivative of Cubic Functions

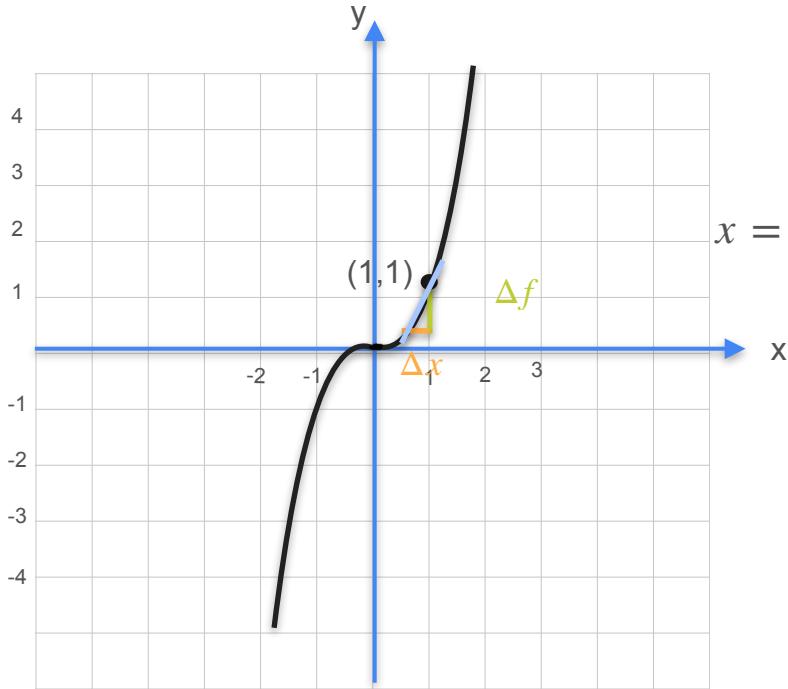


Cubic:  $y = f(x) = x^3$

Slope: 
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

$\Delta x$	1.0	1/2
$\Delta f$	3.25	0.86
Slope	3.25	1.75

# Derivative of Cubic Functions

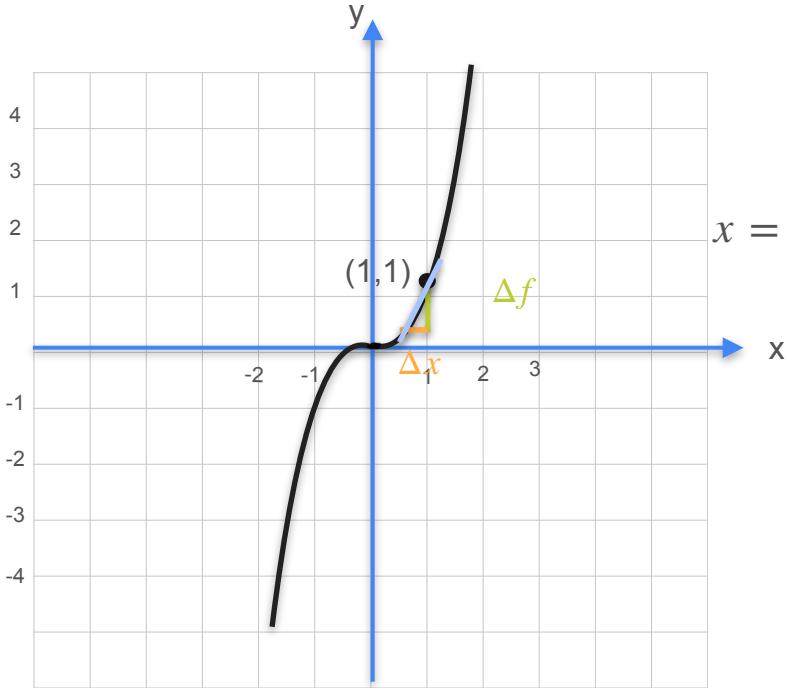


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$
$\Delta f$	3.25	0.86	
Slope	3.25	1.75	

# Derivative of Cubic Functions

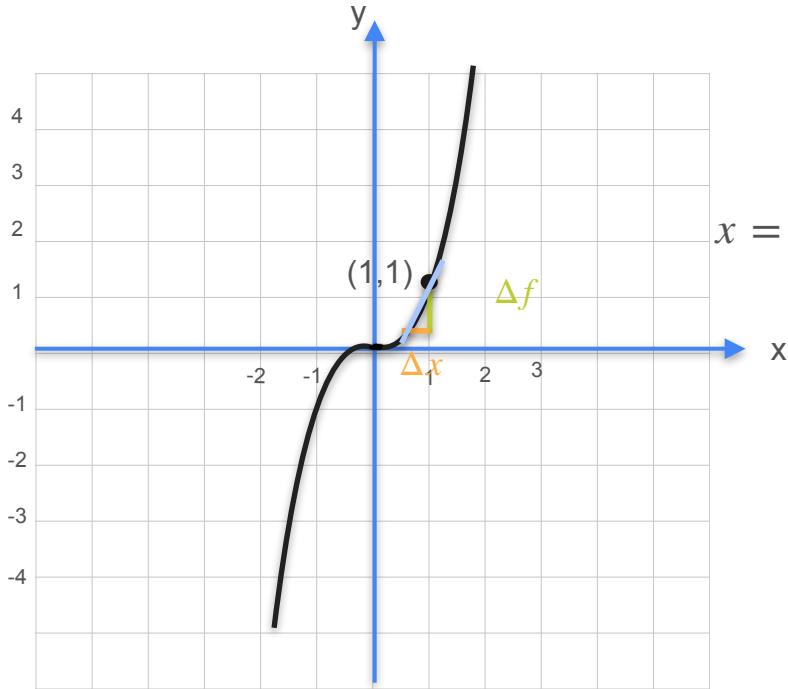


Cubic:  $y = f(x) = x^3$

Slope: 
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

$\Delta x$	1.0	$1/2$	$1/4$
$\Delta f$	3.25	0.86	0.30
Slope	3.25	1.75	

# Derivative of Cubic Functions

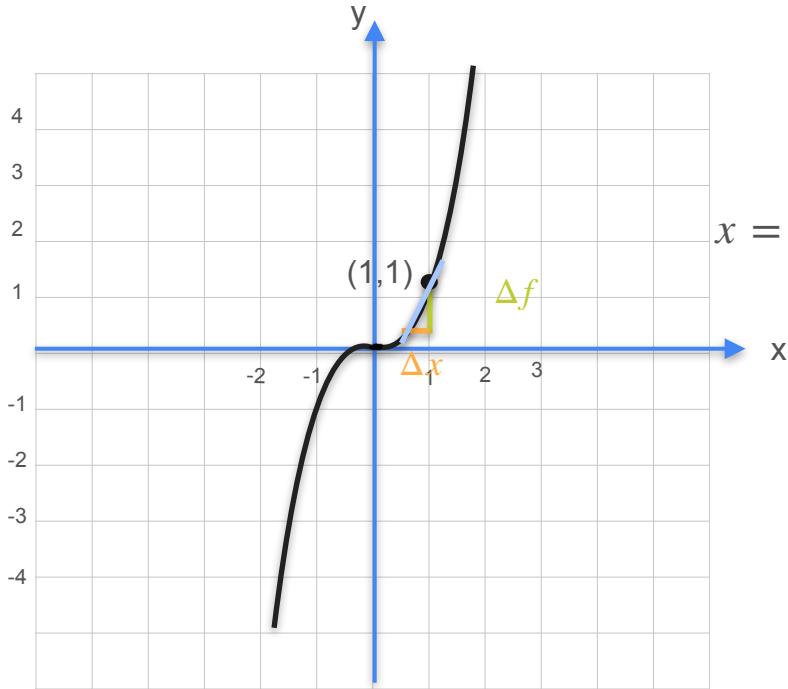


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$
$\Delta f$	3.25	0.86	0.30
Slope	3.25	1.75	1.188

# Derivative of Cubic Functions

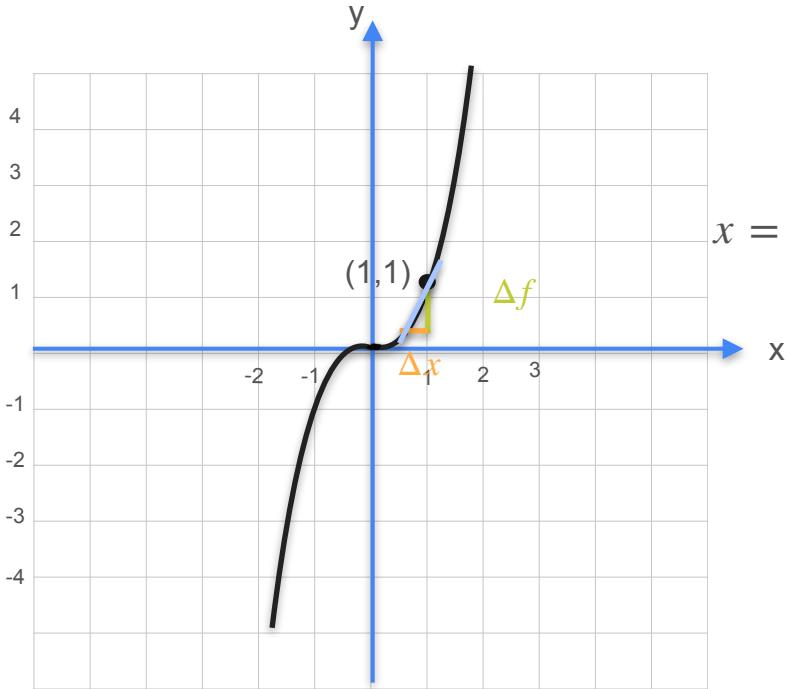


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$
$\Delta f$	3.25	0.86	0.30	
Slope	3.25	1.75	1.188	

# Derivative of Cubic Functions

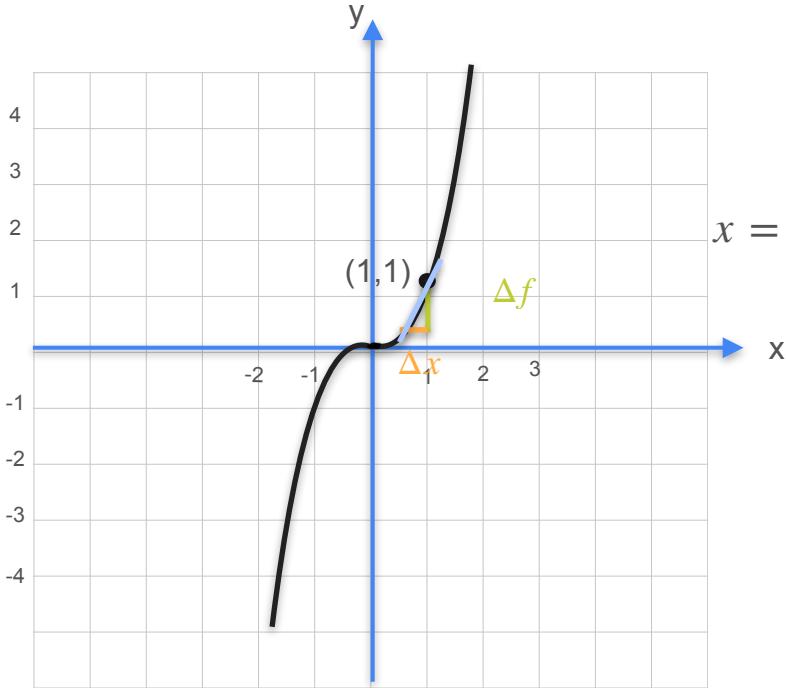


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$
$\Delta f$	3.25	0.86	0.30	0.12
Slope	3.25	1.75	1.188	

# Derivative of Cubic Functions

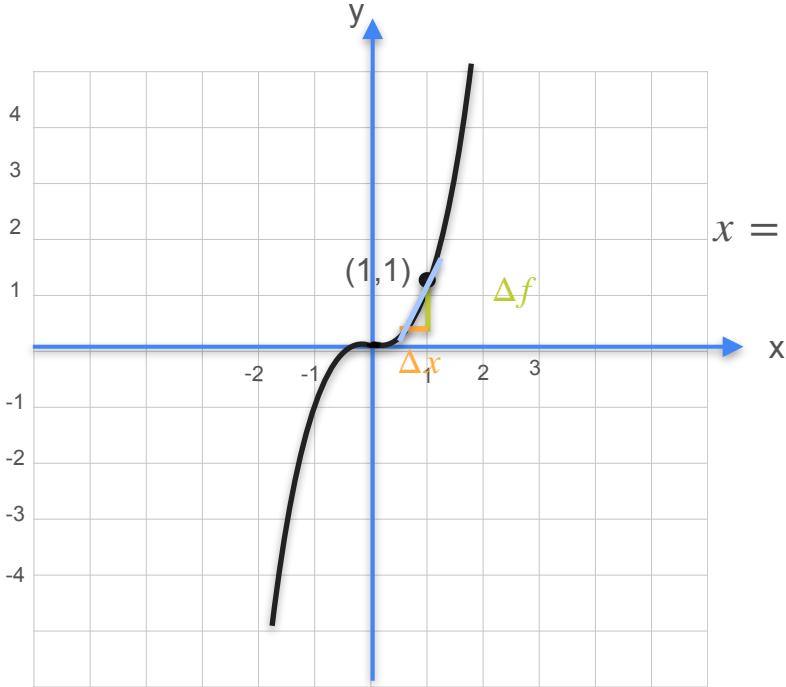


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$
$\Delta f$	3.25	0.86	0.30	0.12
Slope	3.25	1.75	1.188	0.95

# Derivative of Cubic Functions

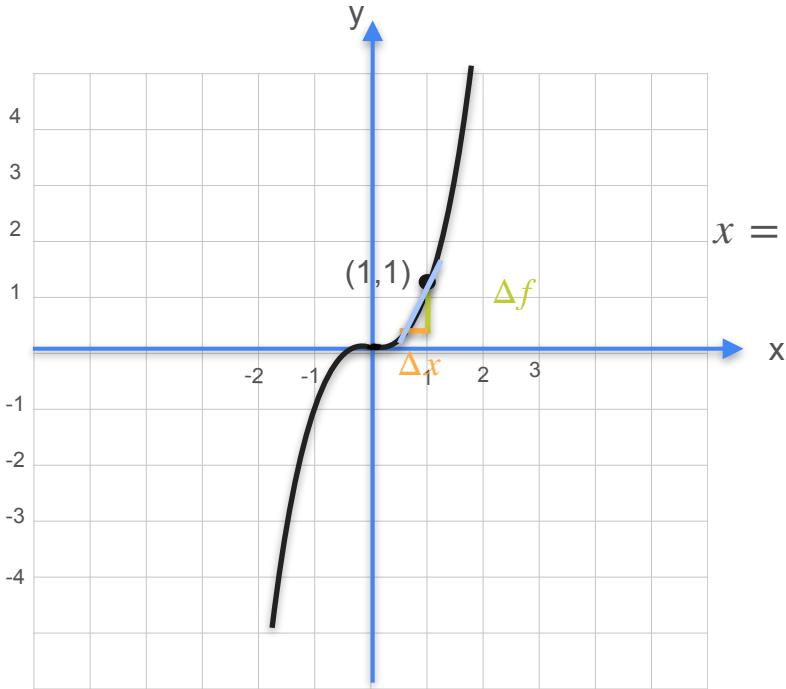


$$\text{Cubic: } y = f(x) = x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$
$\Delta f$	3.25	0.86	0.30	0.12	
Slope	3.25	1.75	1.188	0.95	

# Derivative of Cubic Functions

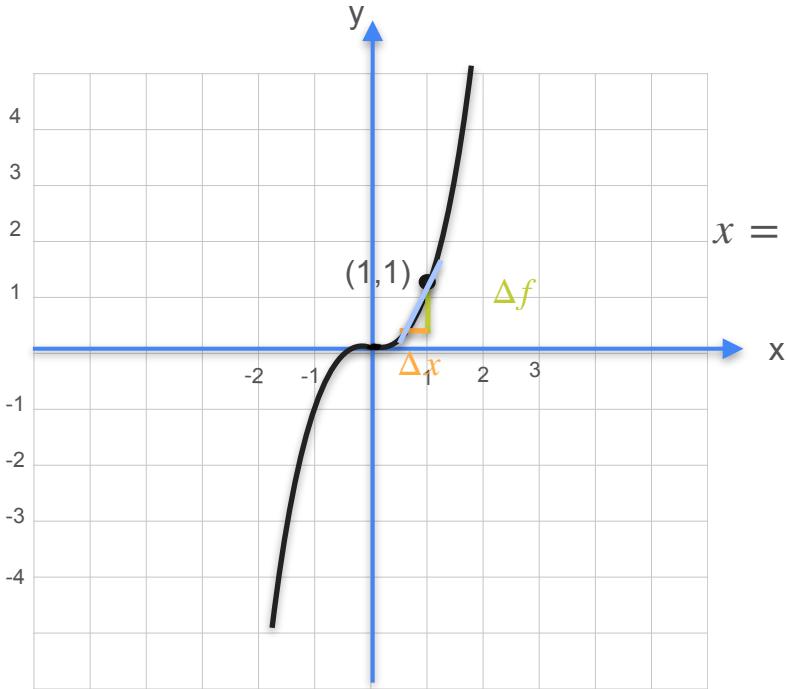


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$
$\Delta f$	3.25	0.86	0.30	0.12	0.05
Slope	3.25	1.75	1.188	0.95	

# Derivative of Cubic Functions

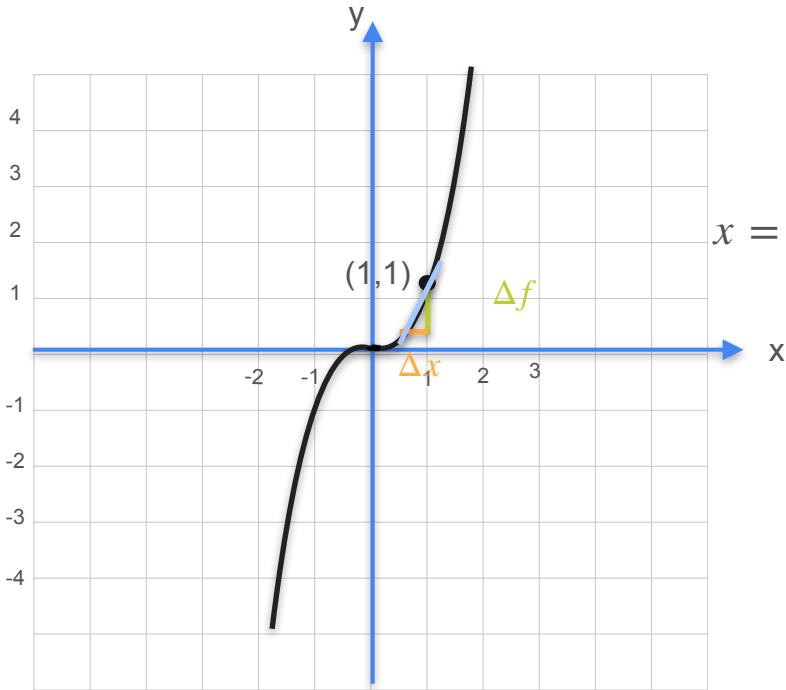


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$
$\Delta f$	3.25	0.86	0.30	0.12	0.05
Slope	3.25	1.75	1.188	0.95	0.85

# Derivative of Cubic Functions

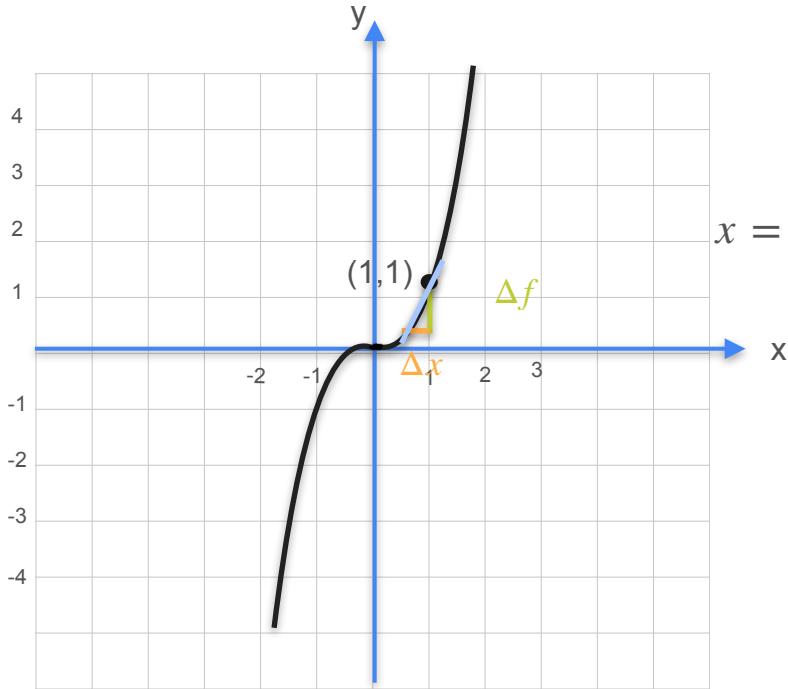


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
$\Delta f$	3.25	0.86	0.30	0.12	0.05	
Slope	3.25	1.75	1.188	0.95	0.85	

# Derivative of Cubic Functions

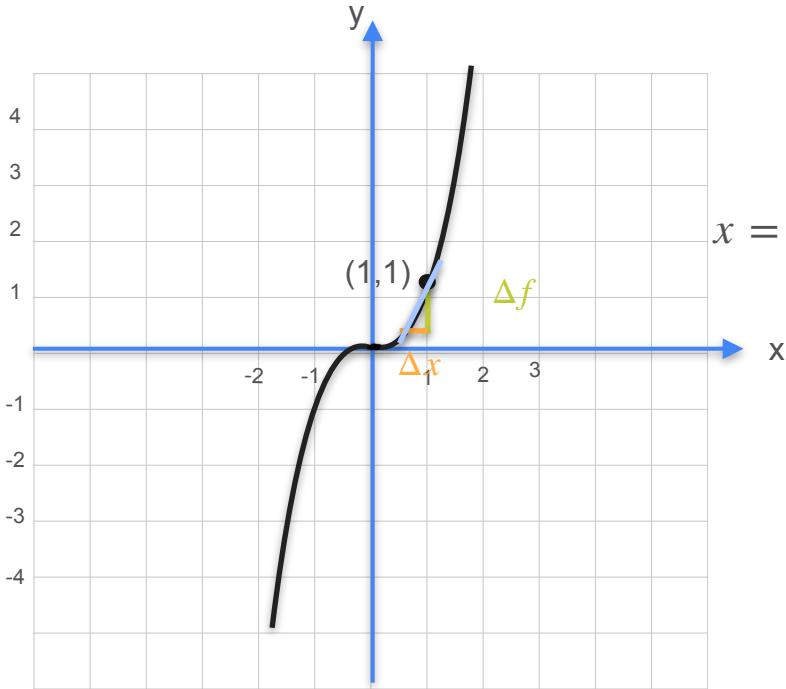


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
$\Delta f$	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	

# Derivative of Cubic Functions

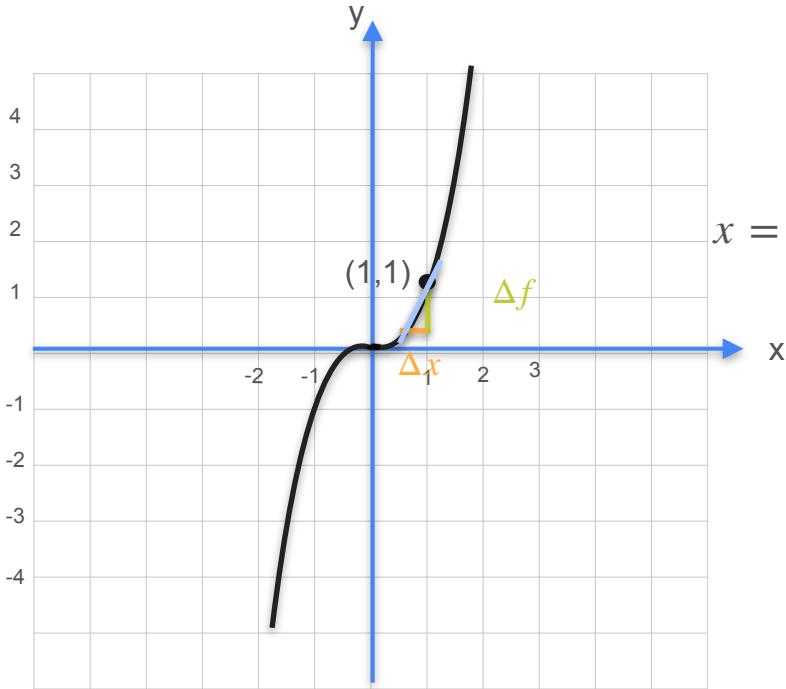


Cubic:  $y = f(x) = x^3$

Slope: 
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$$

$\Delta x$	1.0	$1/2$	$1/4$	$1/8$	$1/16$	$1/1000$
$\Delta f$	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

# Derivative of Cubic Functions



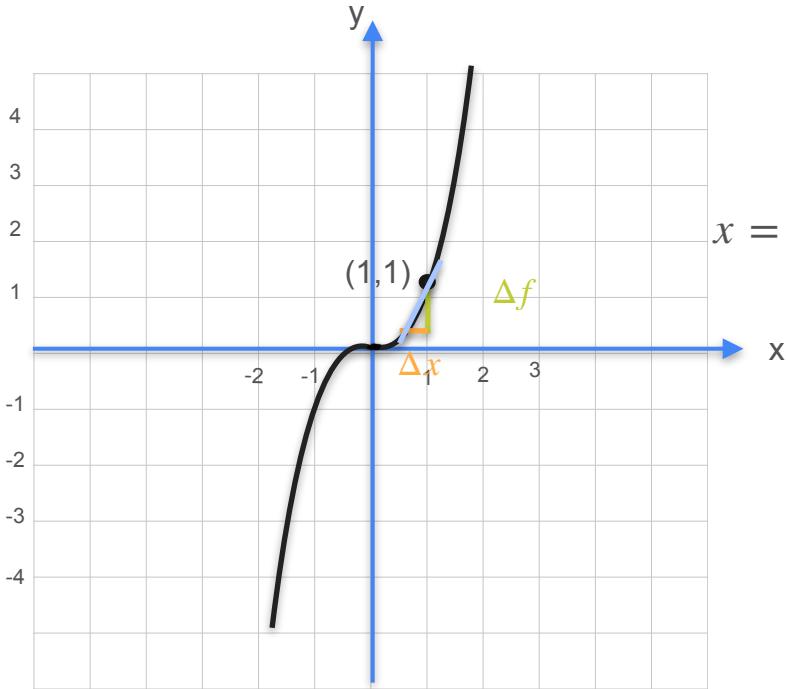
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

$$f'(0.5) = \frac{d}{dx} f(0.5) = 0.75 = 3 \times 0.5^2$$

# Derivative of Cubic Functions



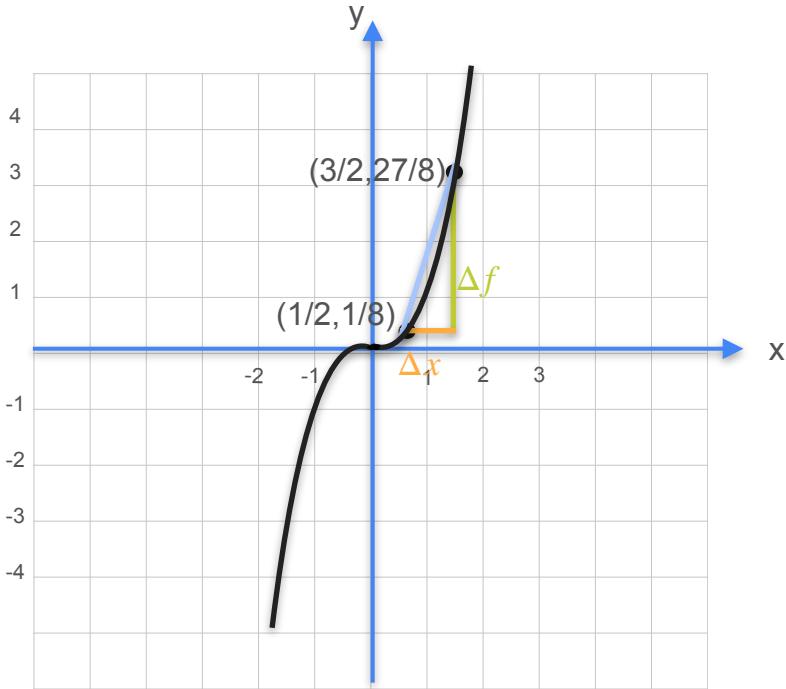
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

$$f'(0.5) = \frac{d}{dx} f(0.5) = 0.75 = 3 \times 0.5^2 = 3 \times 0.5^2$$

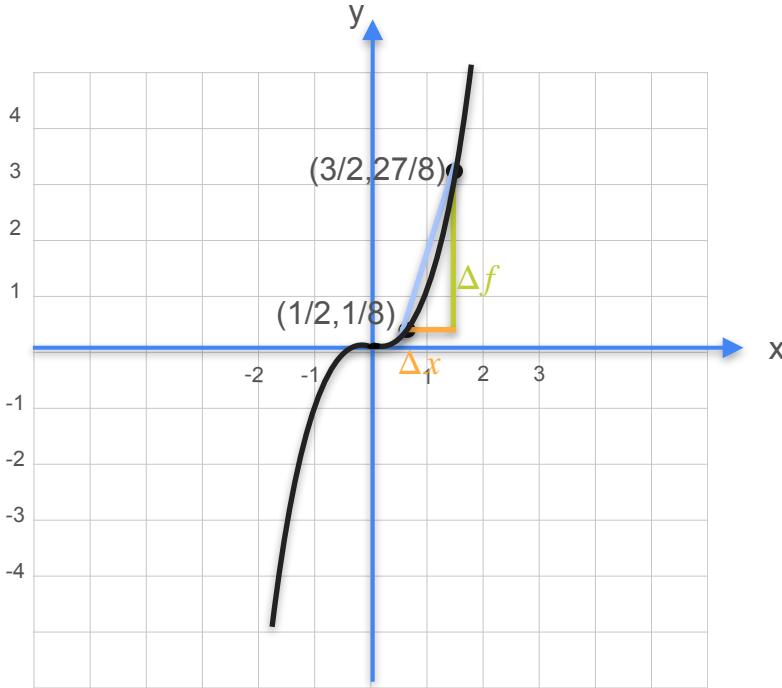
# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

Slope: 
$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

# Derivative of Cubic Functions

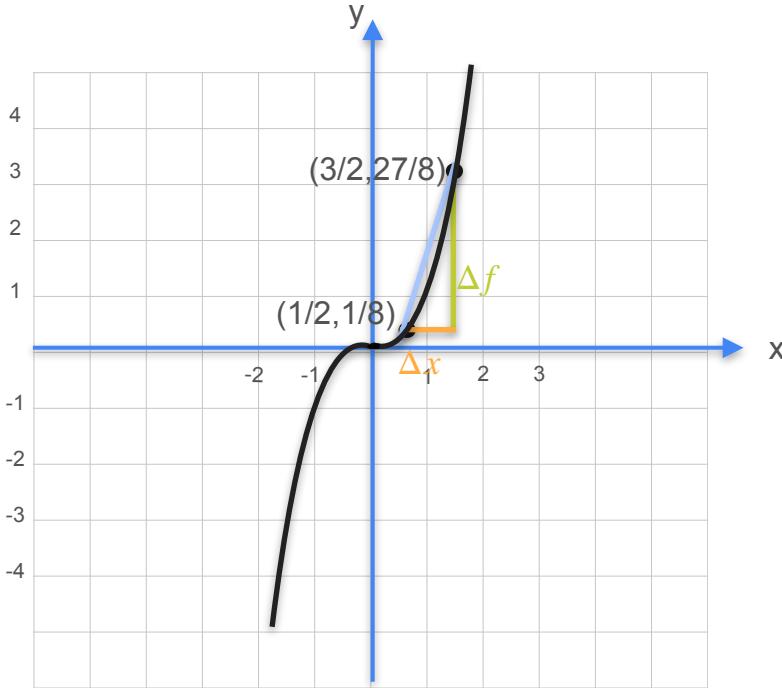


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x}$$

# Derivative of Cubic Functions

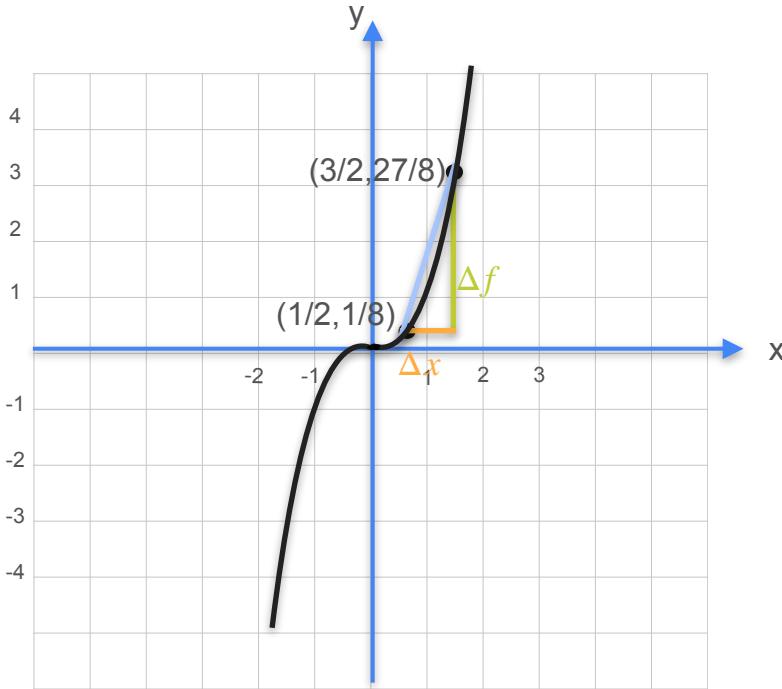


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

# Derivative of Cubic Functions

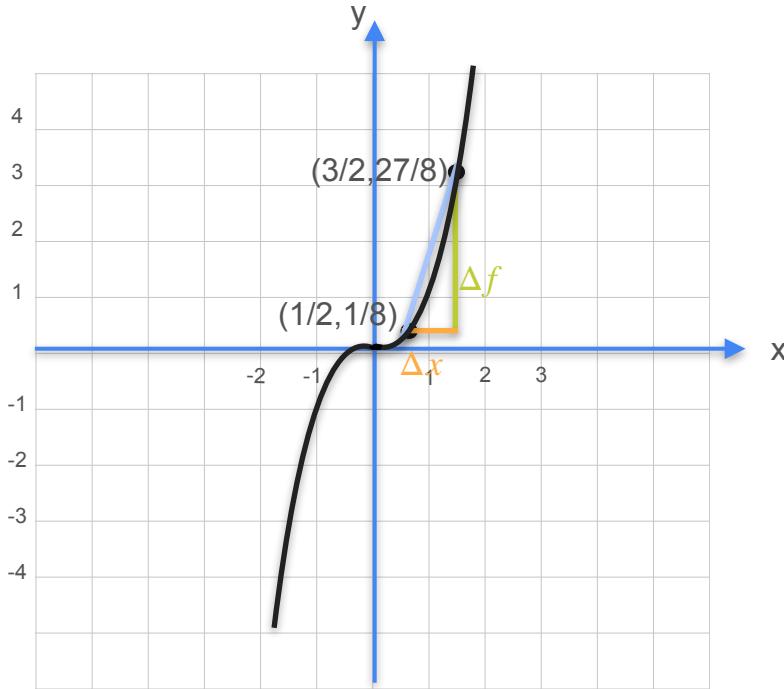


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

# Derivative of Cubic Functions



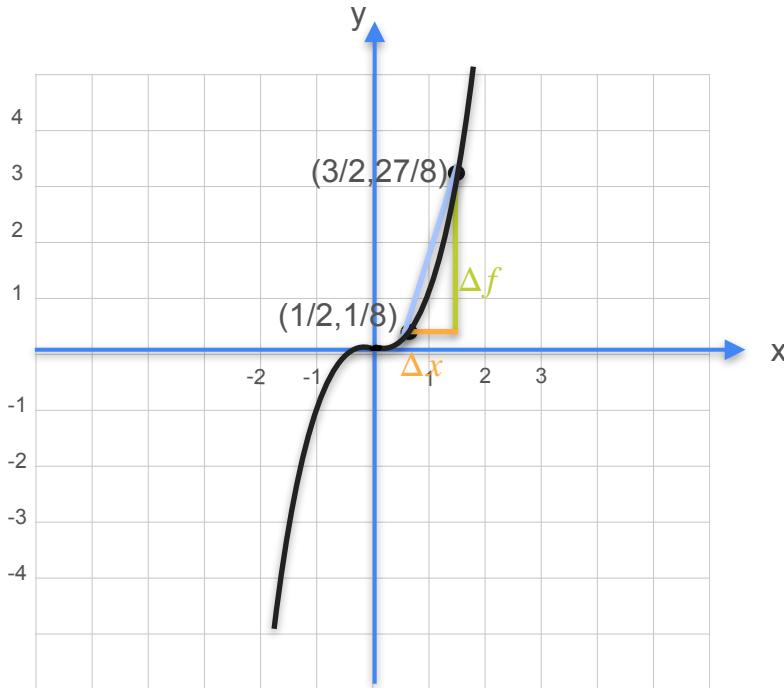
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

# Derivative of Cubic Functions

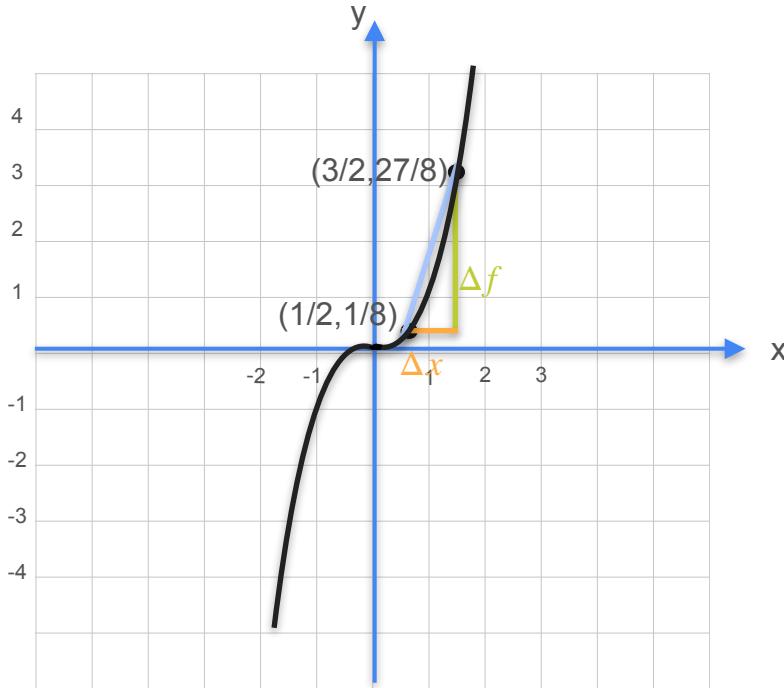


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

# Derivative of Cubic Functions



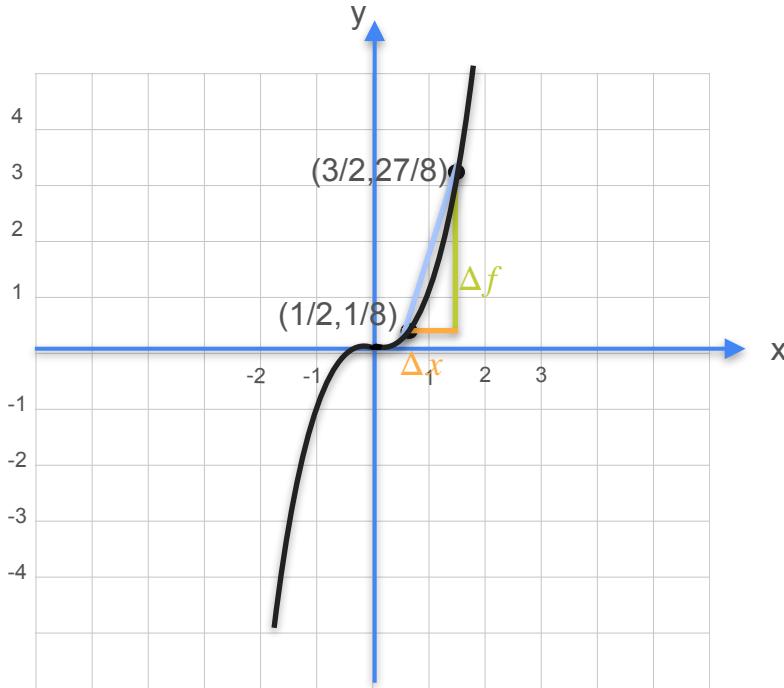
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}$$

# Derivative of Cubic Functions



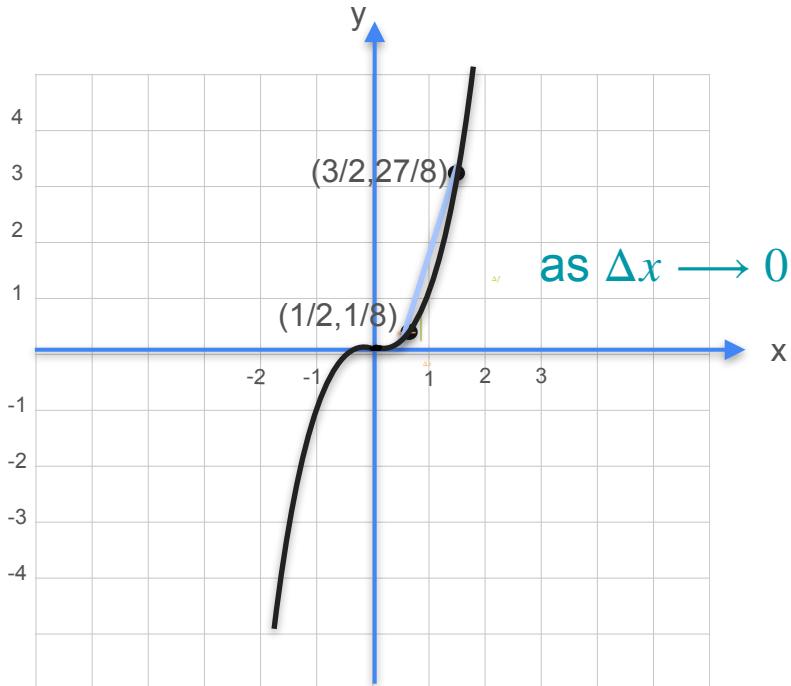
Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{\Delta f}{\Delta x} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x}\end{aligned}$$

$$= 3x\Delta x + 3x^2 + \Delta x^2$$

# Derivative of Cubic Functions

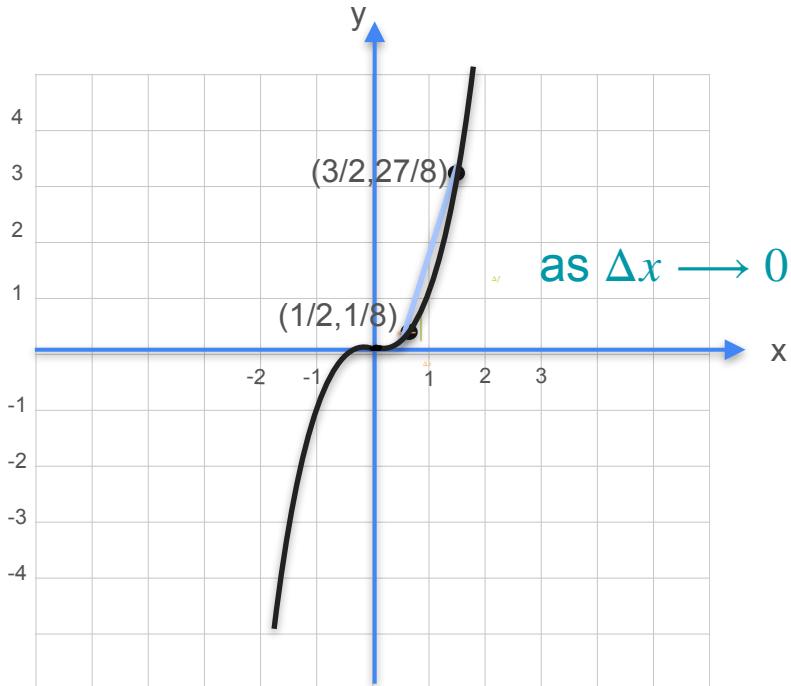


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{\Delta f}{\Delta x} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

# Derivative of Cubic Functions

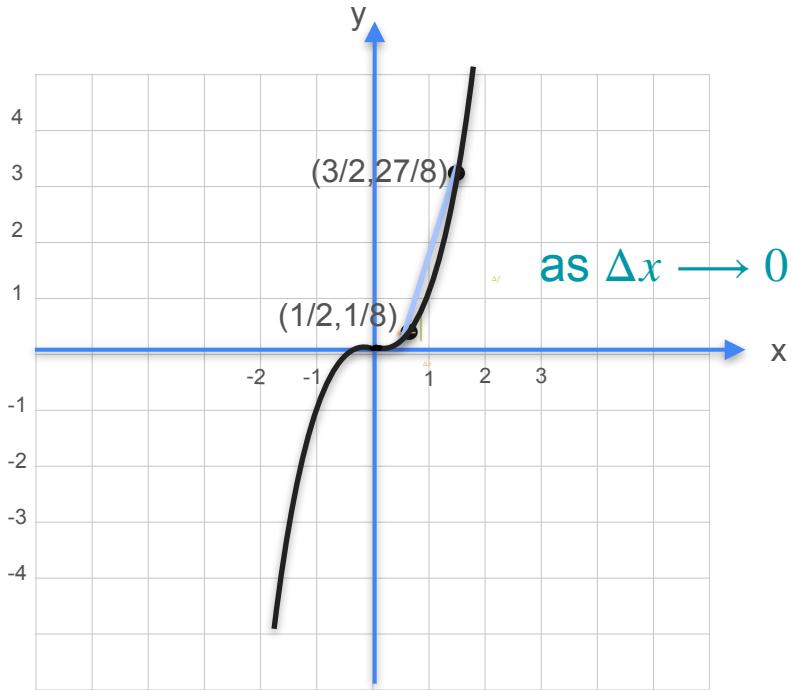


Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

# Derivative of Cubic Functions



Cubic:  $y = f(x) = x^3$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\begin{aligned}\frac{df}{dx} &= \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2\Delta x + (\Delta x)^3 - x^3}{\Delta x} \\ &= 3x\Delta x + 3x^2 + \Delta x^2\end{aligned}$$

$$f(x) = x^3 \rightarrow f'(x) = 3x^2$$



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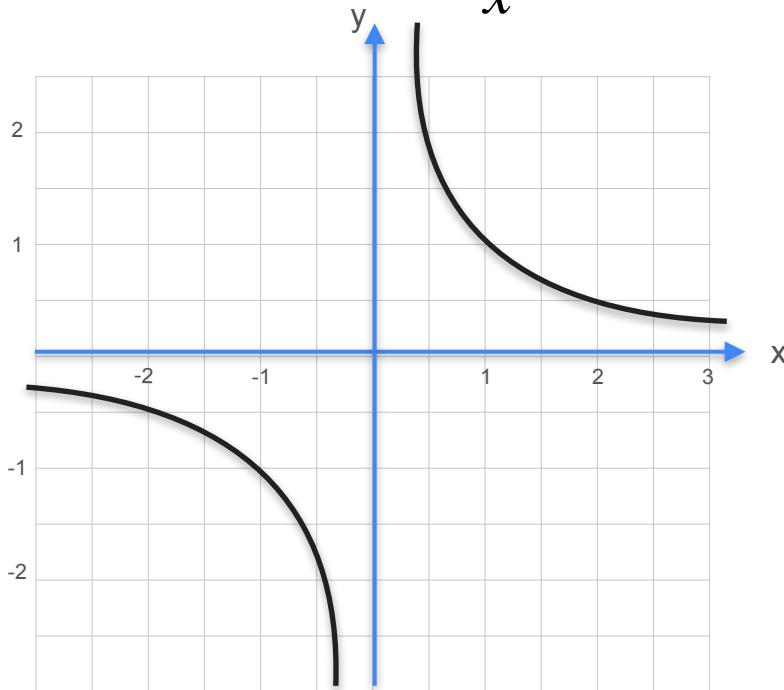
# Derivatives and Optimization

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**Some common derivatives:  
Other power functions**

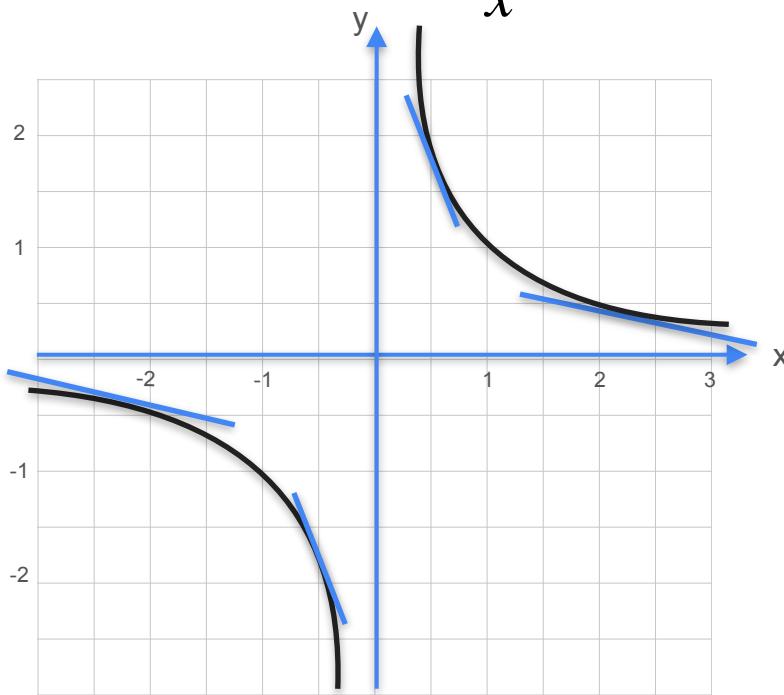
# Derivative of $\frac{1}{x}$

# Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

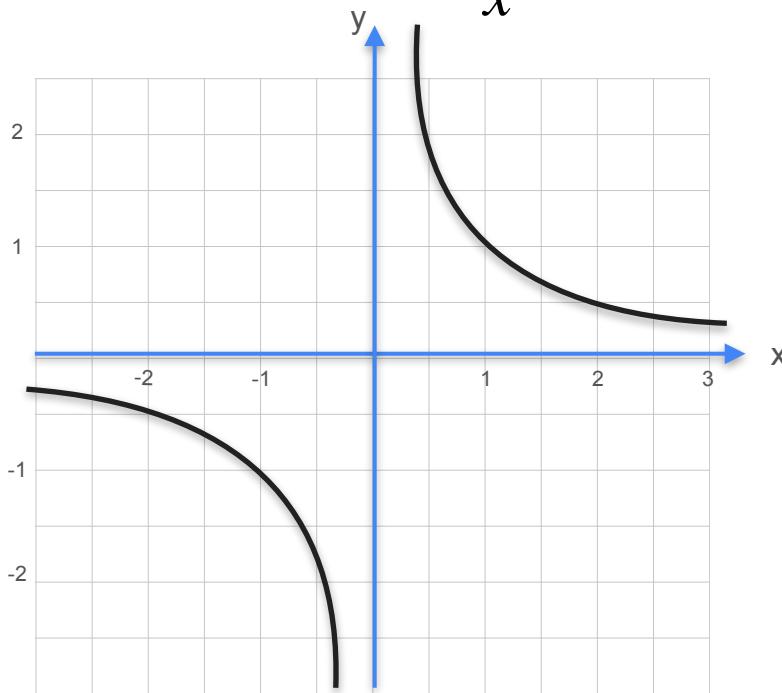
# Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope:  $\frac{\Delta f}{\Delta x}$

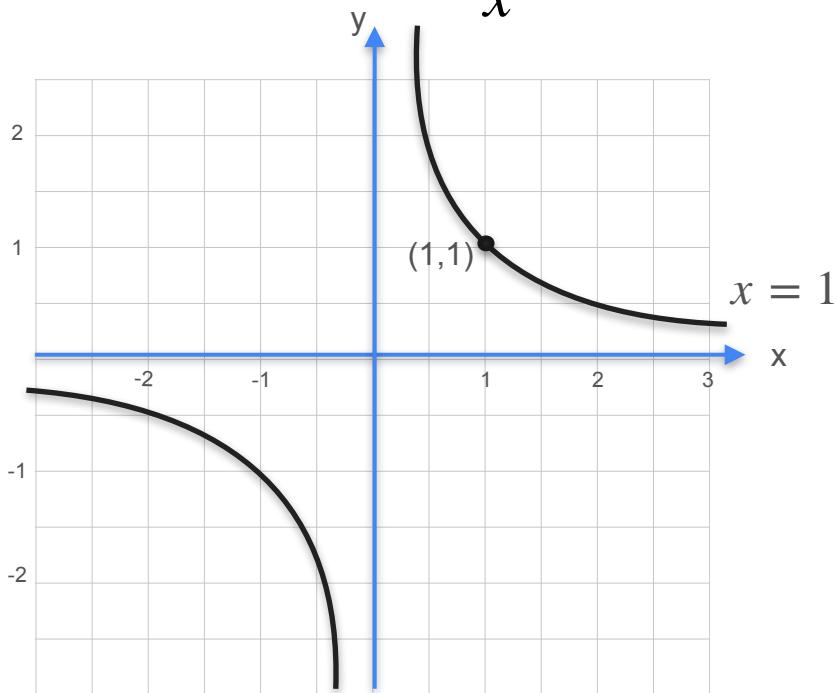
# Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

# Derivative of $\frac{1}{x}$

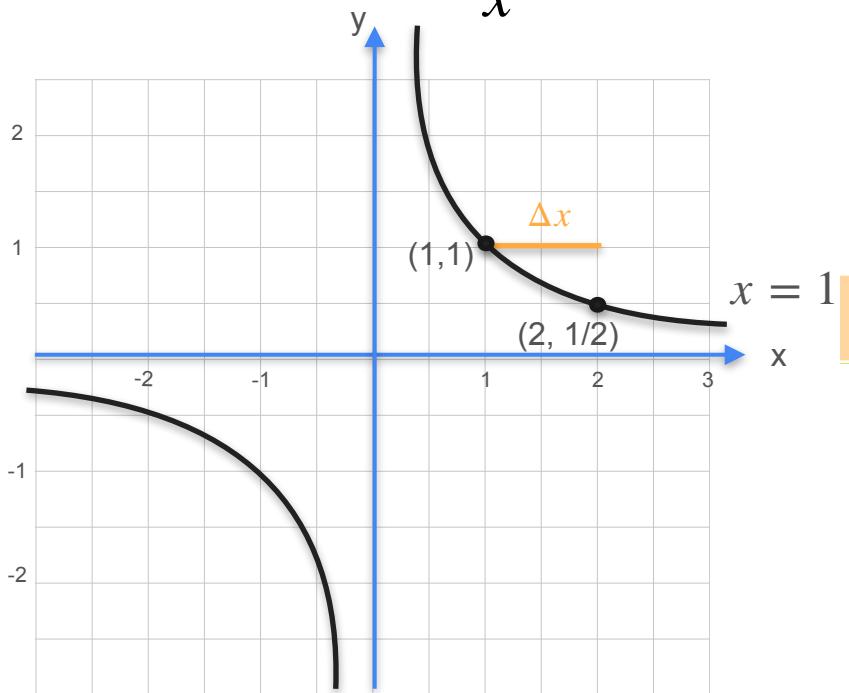


Slope:

$$y = f(x) = x^{-1} = \frac{1}{x}$$
$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$$

.

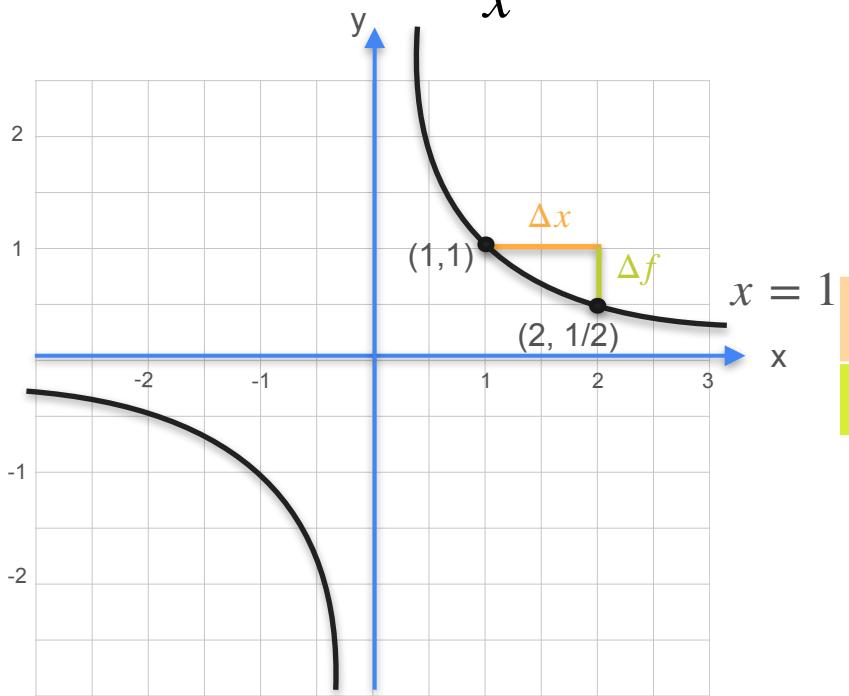
# Derivative of $\frac{1}{x}$



$$y = f(x) = x^{-1} = \frac{1}{x}$$
$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$$

$\Delta x$       1.0

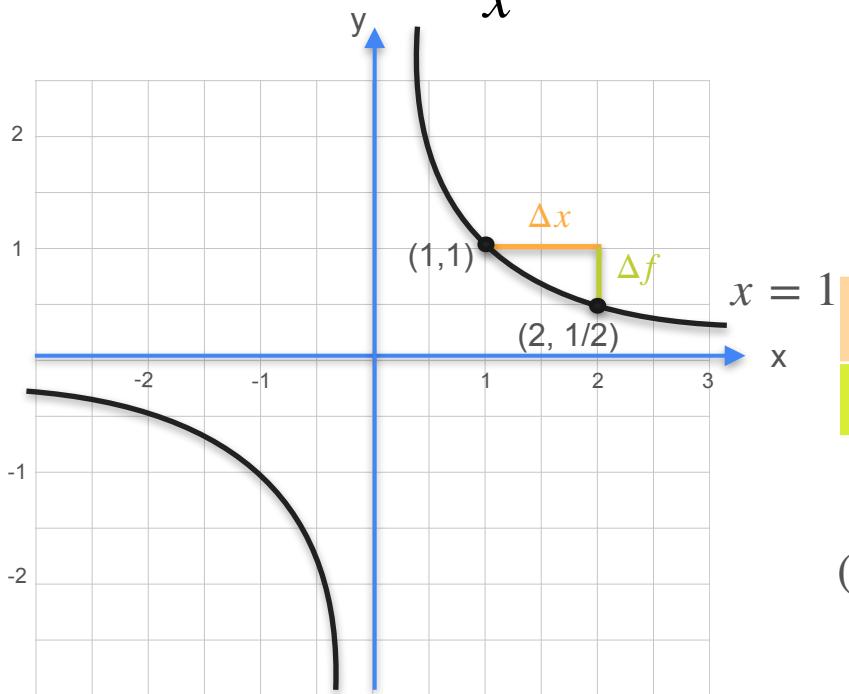
# Derivative of $\frac{1}{x}$



Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	

# Derivative of $\frac{1}{x}$



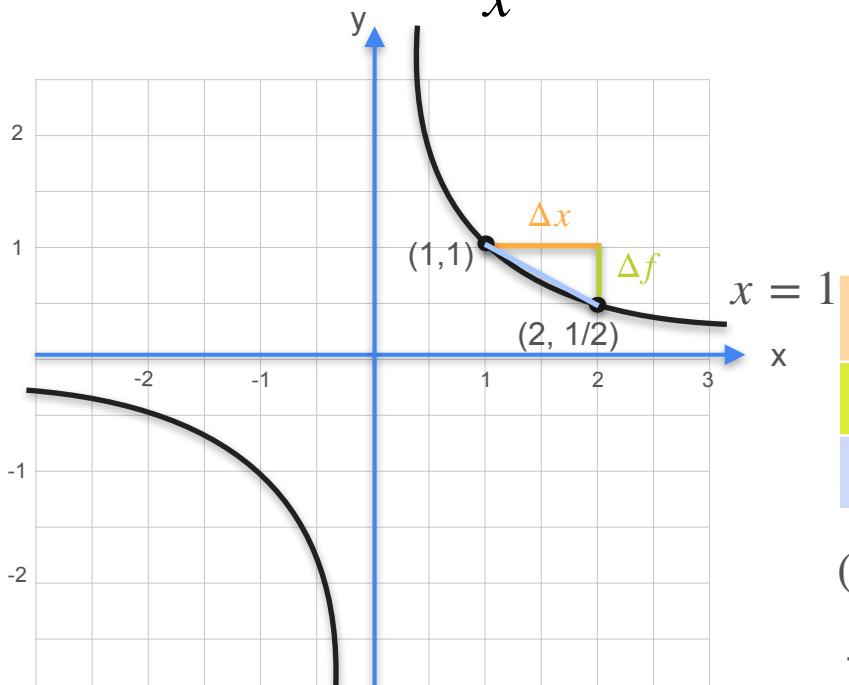
$$y = f(x) = x^{-1} = \frac{1}{x}$$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	-0.5

$$(1 + 1)^{-1} - 1^{-1} = \frac{1}{2} - 1$$

# Derivative of $\frac{1}{x}$

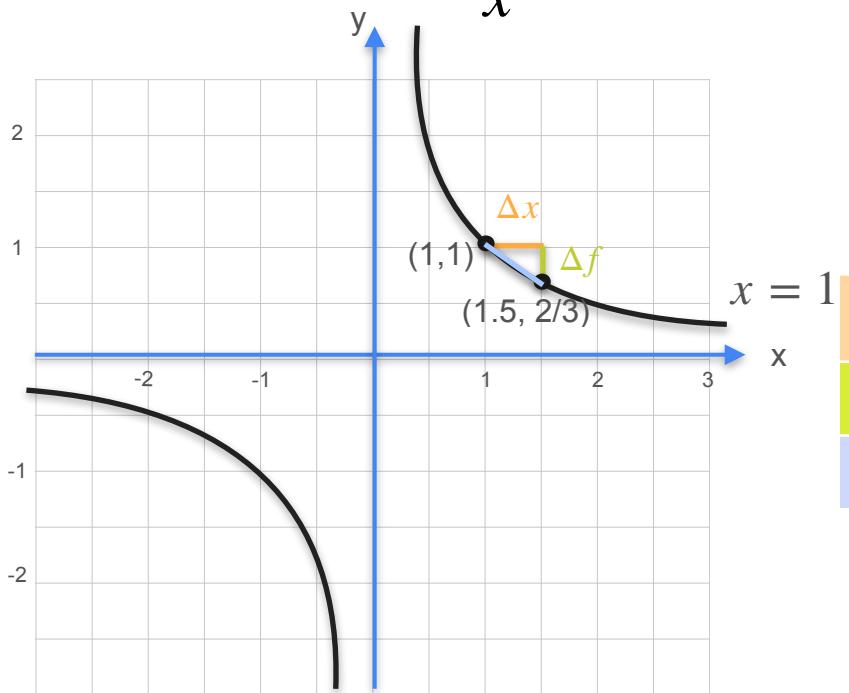


Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	-0.5
Slope	-0.5

$$\begin{aligned}(1 + 1)^{-1} - 1^{-1} &= \frac{1}{2} - 1 \\ -0.5 \\ \hline 1\end{aligned}$$

# Derivative of $\frac{1}{x}$

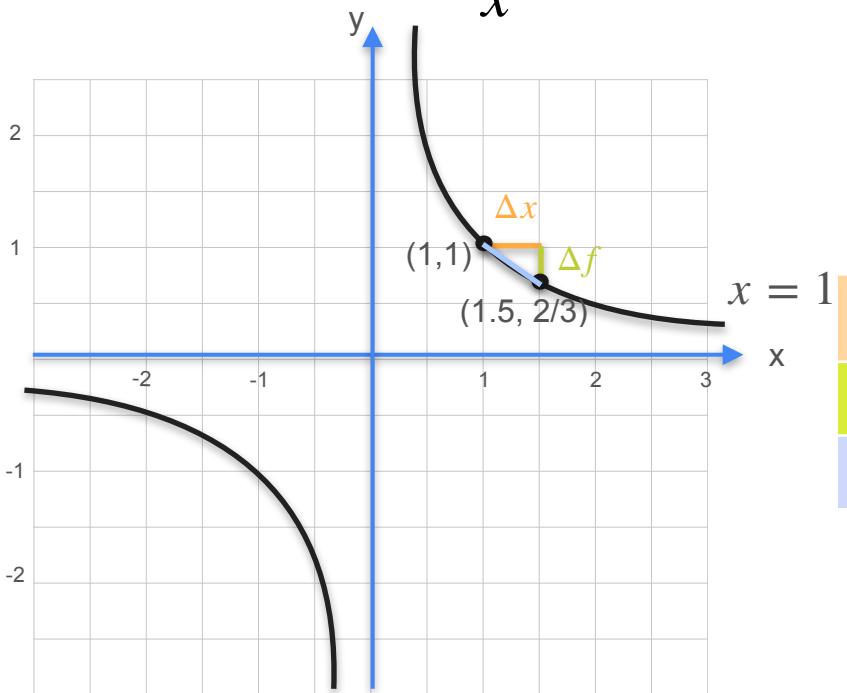


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	-0.5
Slope	-0.5

# Derivative of $\frac{1}{x}$

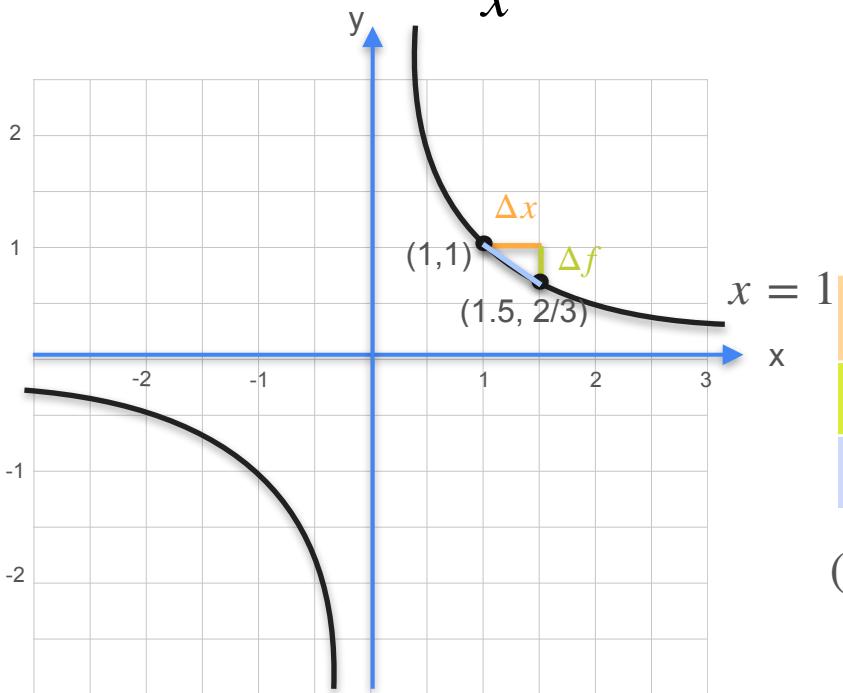


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	$1/2$
$\Delta f$	-0.5	
Slope	-0.5	

# Derivative of $\frac{1}{x}$



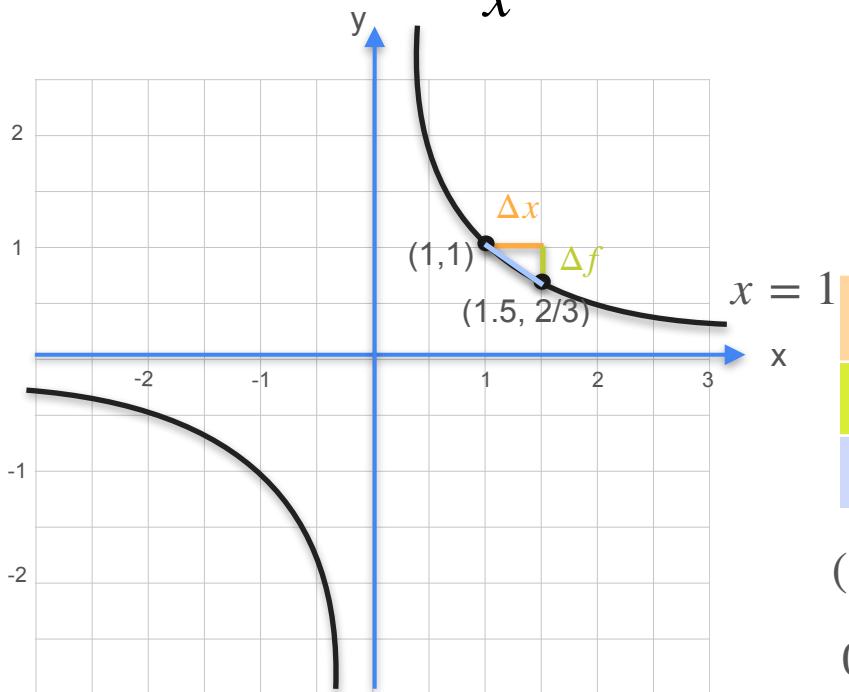
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	-0.5	-0.33
Slope	-0.5	

$$(1 + 0.5)^{-1} - 1^{-1} = \frac{2}{3} - 1$$

# Derivative of $\frac{1}{x}$



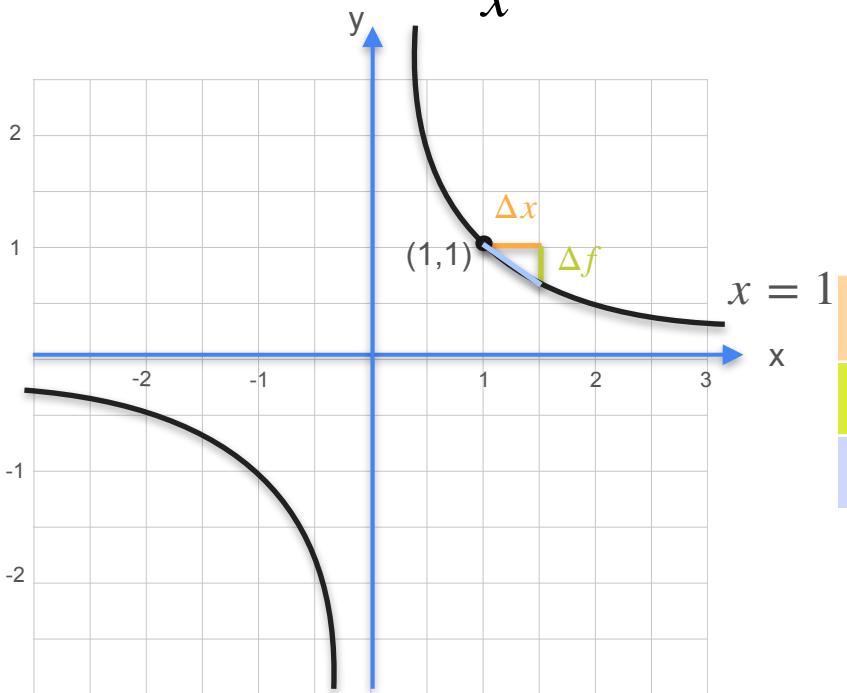
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	-0.5	-0.33
Slope	-0.5	-0.67

$$(1 + 0.5)^{-1} - 1^{-1} = \frac{2}{3} - 1$$
$$\frac{0.33}{0.5}$$

# Derivative of $\frac{1}{x}$

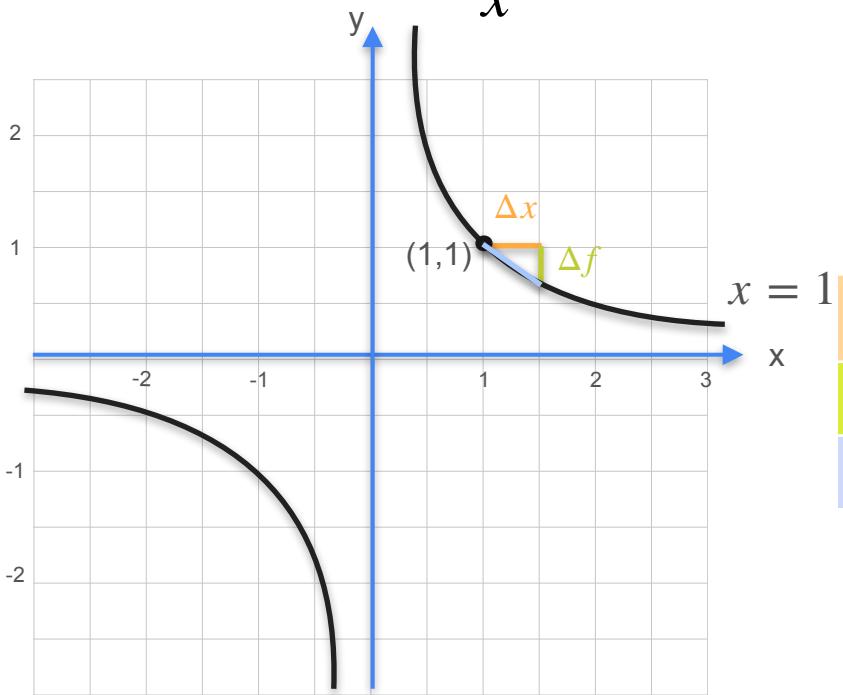


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	$1/2$
$\Delta f$	-0.5	-0.33
Slope	-0.5	-0.67

# Derivative of $\frac{1}{x}$

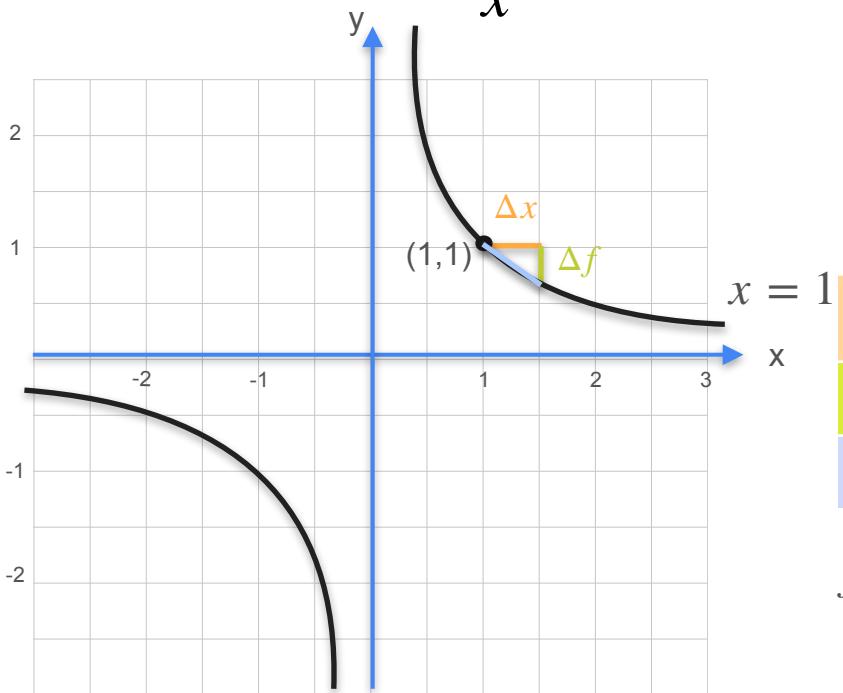


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	-0.5	-0.33	-0.2	-0.11	-0.06	-0.001
Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

# Derivative of $\frac{1}{x}$



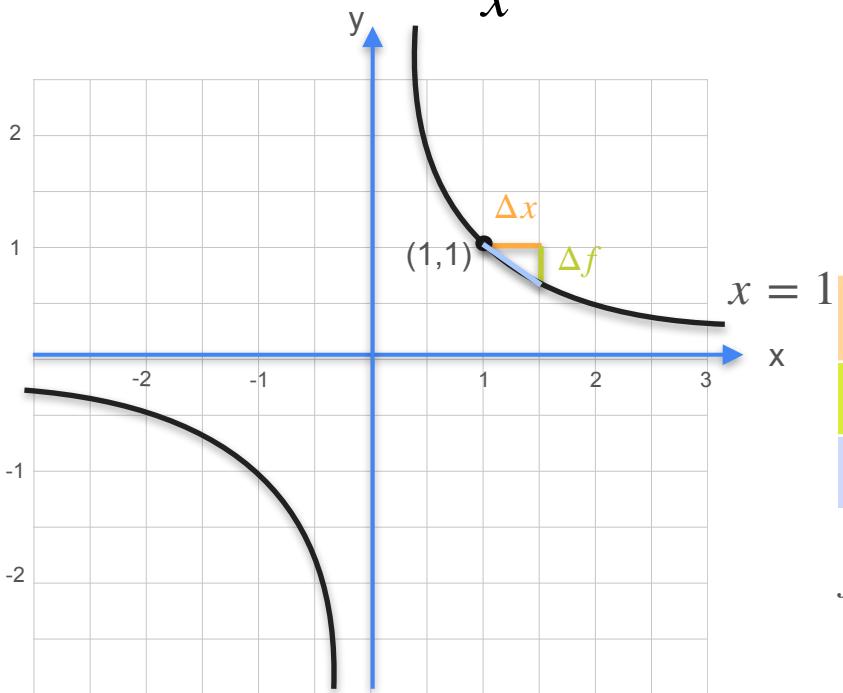
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$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
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Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

$$f'(1) = \frac{d}{dx} f(1) = -1$$

# Derivative of $\frac{1}{x}$



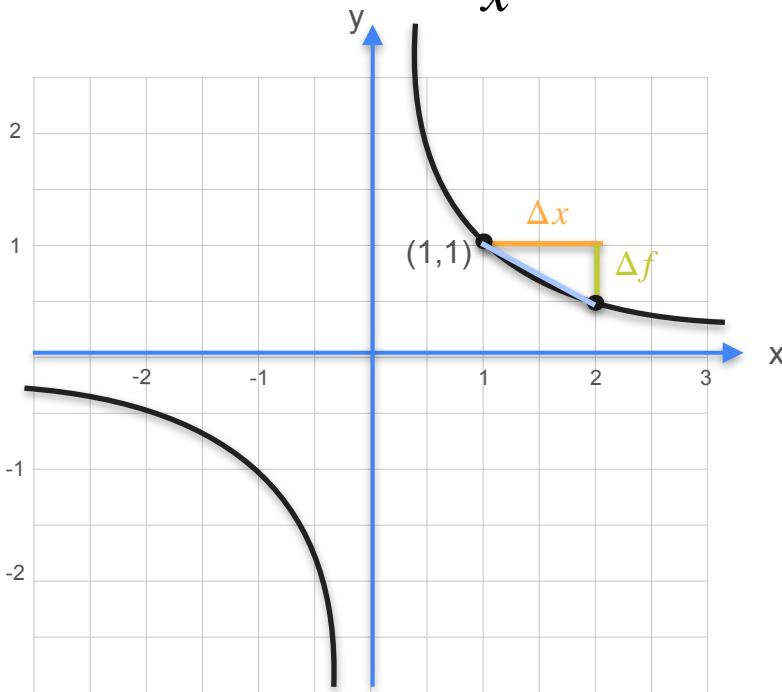
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	-0.5	-0.33	-0.2	-0.11	-0.06	-0.001
Slope	-0.5	-0.67	-0.8	-0.89	-0.94	-0.999

$$f'(1) = \frac{d}{dx} f(1) = -1 = -1 \times 1^2$$

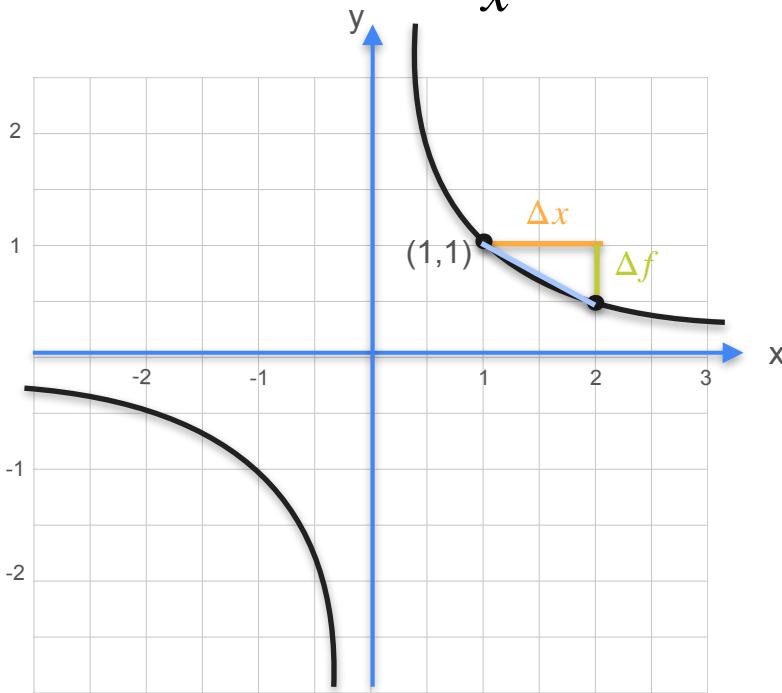
# Derivative of $\frac{1}{x}$



Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

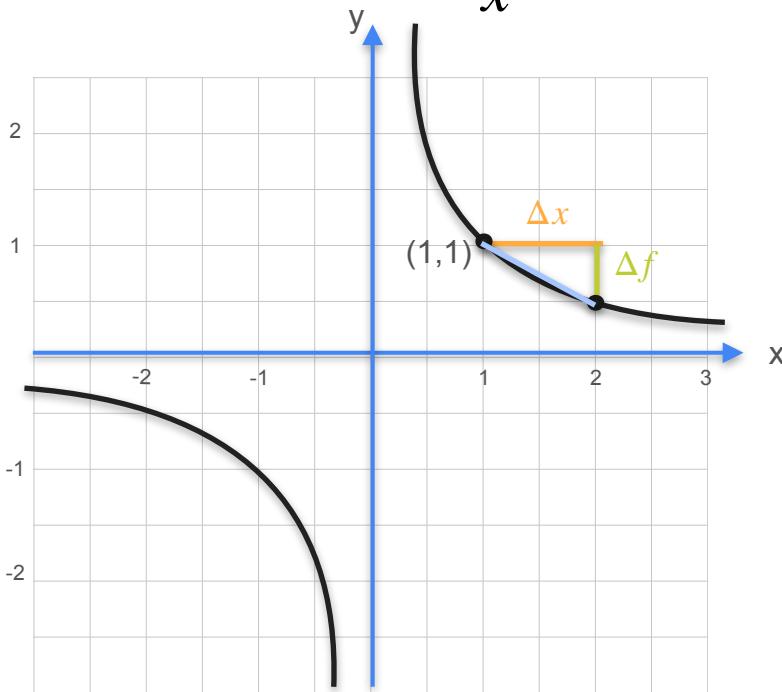
# Derivative of $\frac{1}{x}$



Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

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# Derivative of $\frac{1}{x}$

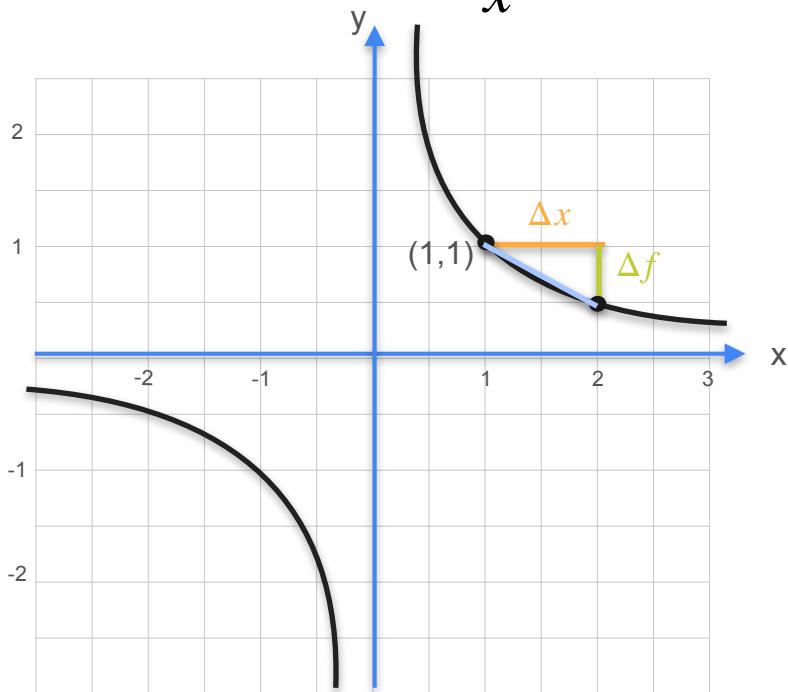


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

# Derivative of $\frac{1}{x}$



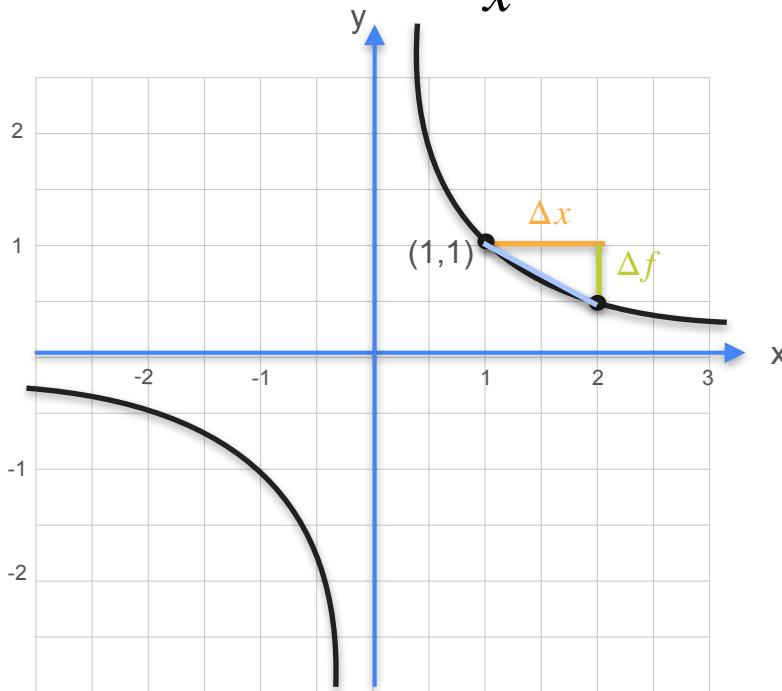
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x}$$

# Derivative of $\frac{1}{x}$



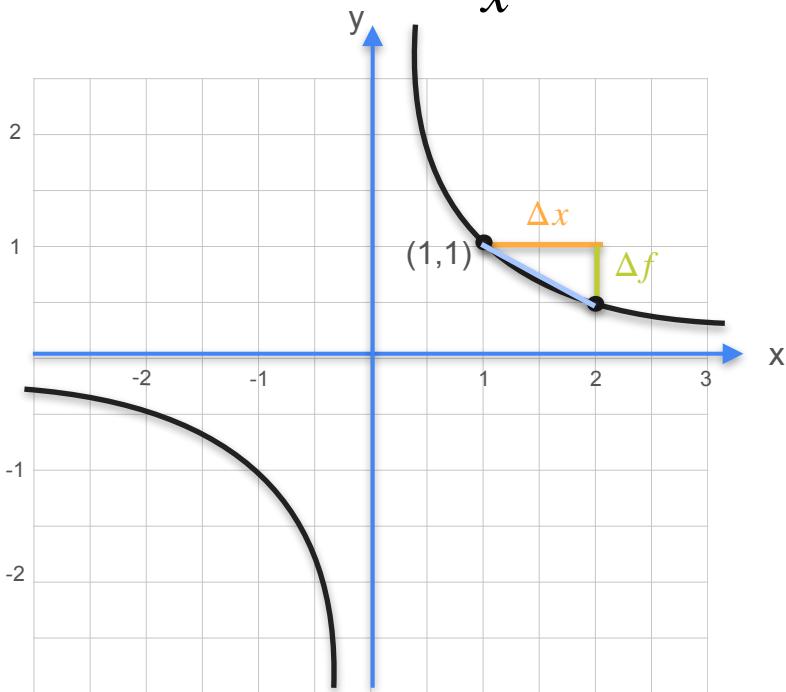
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x}$$

# Derivative of $\frac{1}{x}$



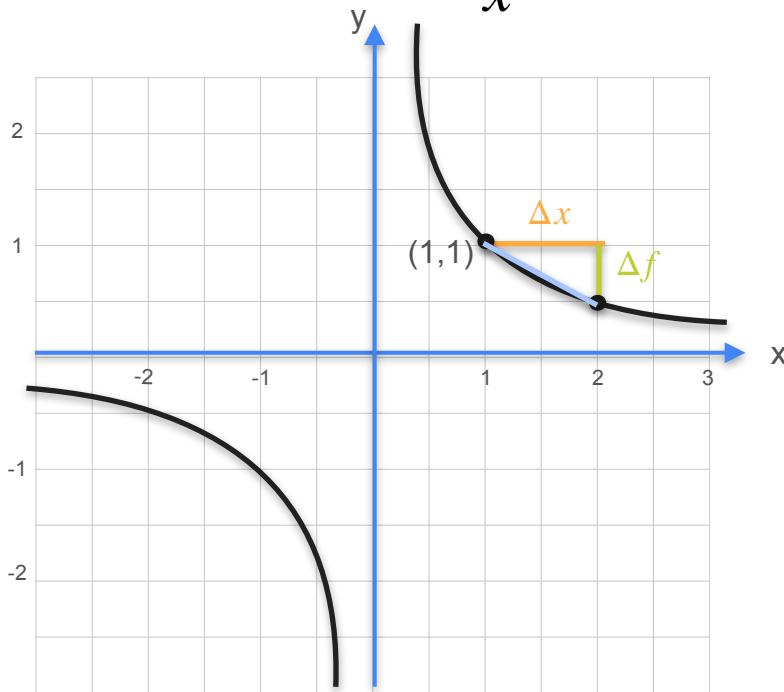
Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope: 
$$\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

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# Derivative of $\frac{1}{x}$



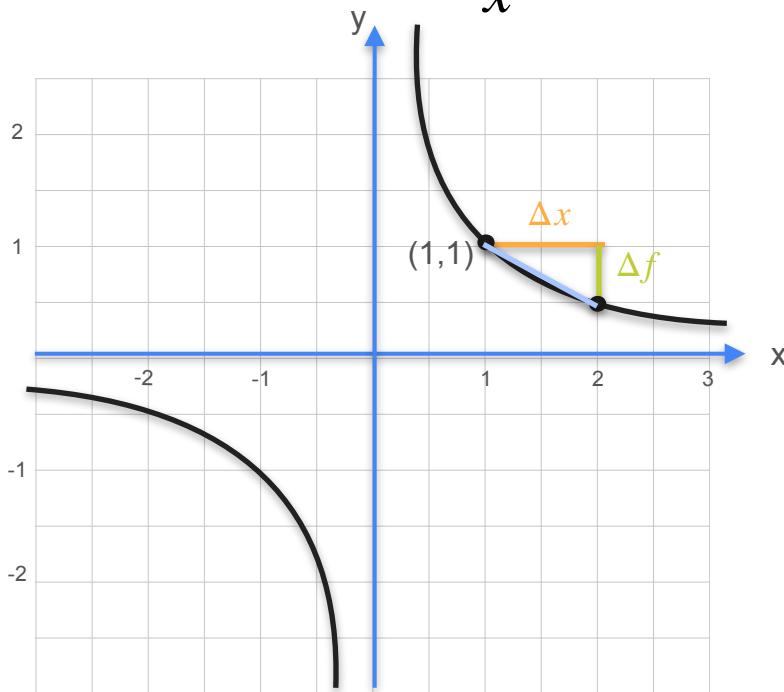
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# Derivative of $\frac{1}{x}$



Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

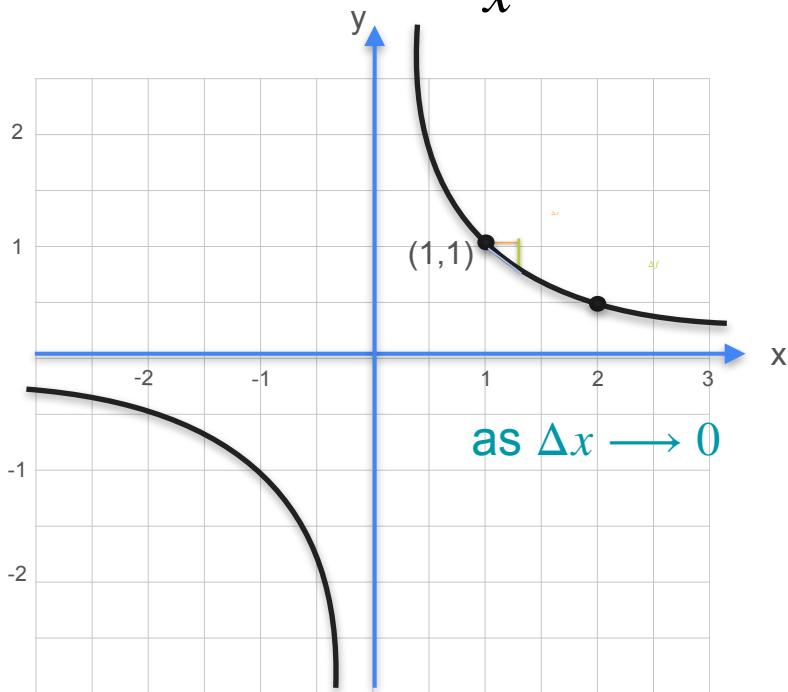
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$= \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x}$$

$$= -\frac{1}{x^2 + x\Delta x}$$

# Derivative of $\frac{1}{x}$



Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

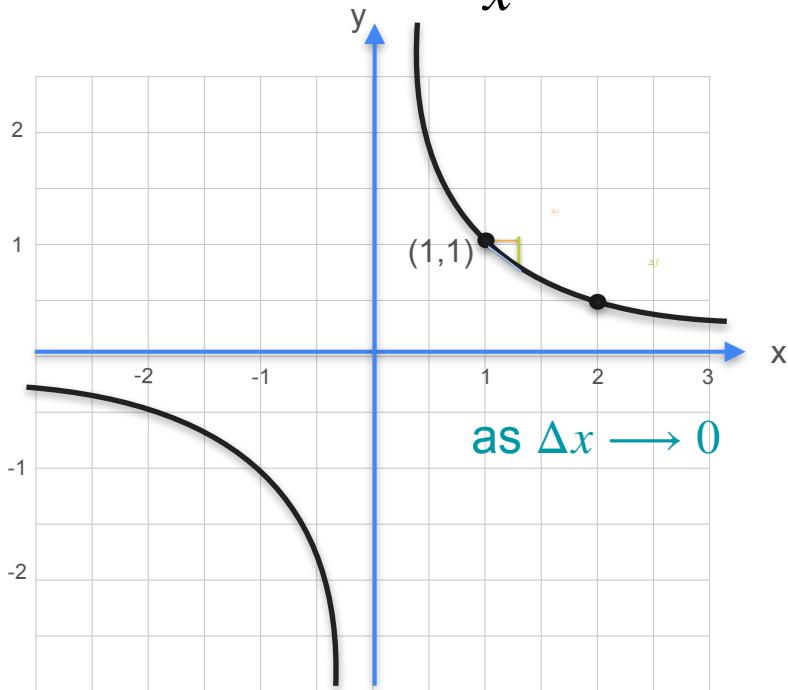
Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

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# Derivative of $\frac{1}{x}$

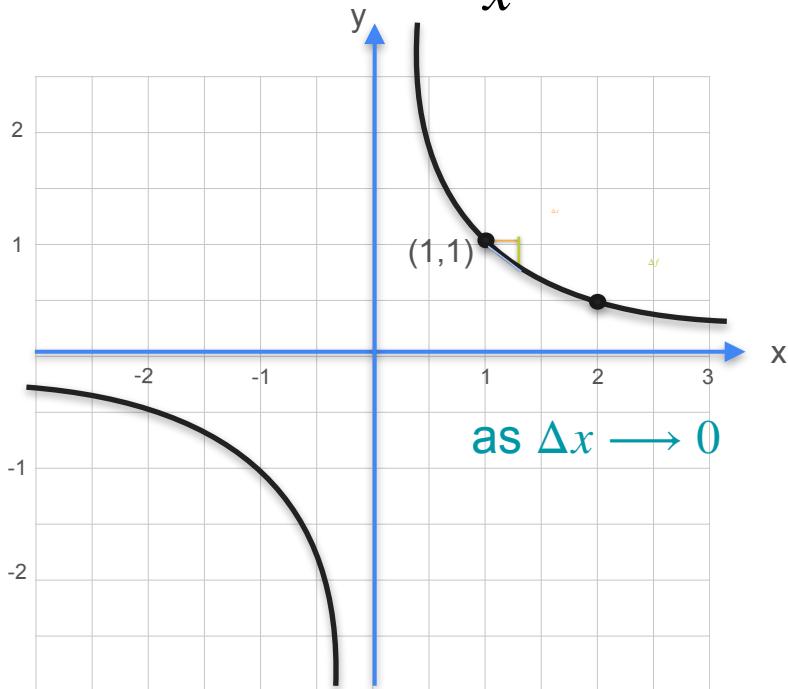


Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

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# Derivative of $\frac{1}{x}$



Inverse:  $y = f(x) = x^{-1} = \frac{1}{x}$

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$$f(x) = x^{-1} \rightarrow f'(x) = -x^{-2}$$

# Derivative of Power Functions

# Derivative of Power Functions

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

# Derivative of Power Functions

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^{\boxed{2}}$$

$$f(x) = x^{\boxed{3}}$$

$$f(x) = x^{\boxed{-1}}$$

$$f'(x) = 2x^1$$

$$f'(x) = 3x^2$$

$$f'(x) = (-1)x^{-2}$$

# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$

$$f'(x) = 2x^1$$

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f(x) = x^{-1}$$

$$f'(x) = (-1)x^{-2}$$



# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$

$$f(x) = x^3$$
$$f'(x) = 3x^2$$

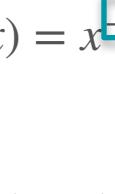
$$f(x) = x^{-1}$$
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# Derivative of Power Functions

Can you spot the pattern?

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Can you spot the pattern?

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# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$

-1

$$f(x) = x^3$$
$$f'(x) = 3x^2$$

-1

$$f(x) = x^{-1}$$
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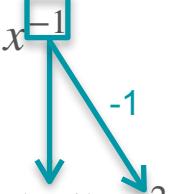
-1

# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$


$$f(x) = x^3$$
$$f'(x) = 3x^2$$


$$f(x) = x^{-1}$$
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# Derivative of Power Functions

Can you spot the pattern?

$$f(x) = x^2$$
$$f'(x) = 2x^1$$


$$f(x) = x^3$$
$$f'(x) = 3x^2$$


$$f(x) = x^{-1}$$
$$f'(x) = (-1)x^{-2}$$


$$f(x) = x^n \quad \rightarrow \quad f'(x) = \frac{d}{dx} f(x) = nx^{n-1}$$



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# Derivatives and Optimization

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**The inverse function and its derivative**

# Inverse Function

What's an inverse?

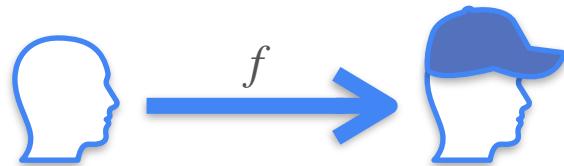
# Inverse Function

What's an inverse?



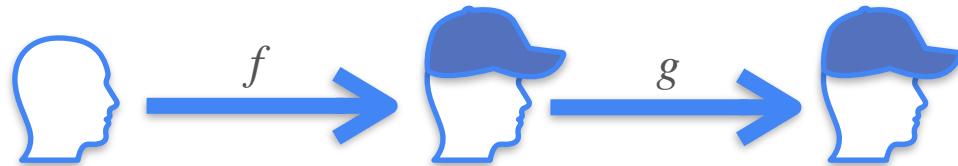
# Inverse Function

What's an inverse?



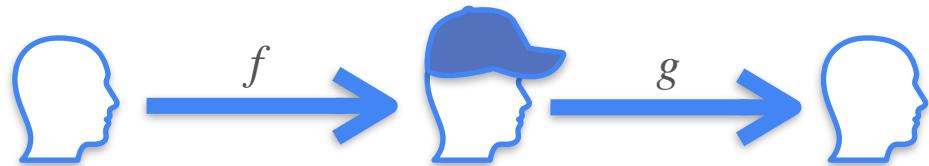
# Inverse Function

What's an inverse?



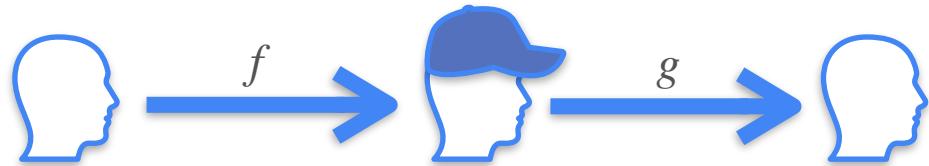
# Inverse Function

What's an inverse?



# Inverse Function

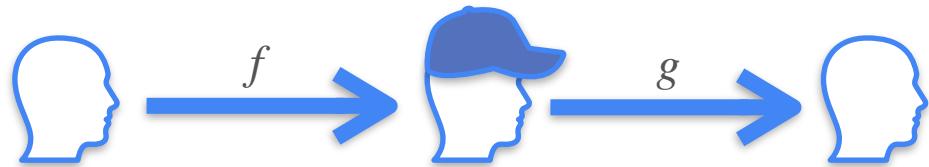
What's an inverse?



$x$

# Inverse Function

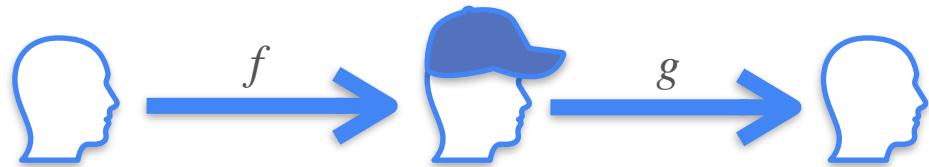
What's an inverse?



$$x \xrightarrow{f} x^2$$

# Inverse Function

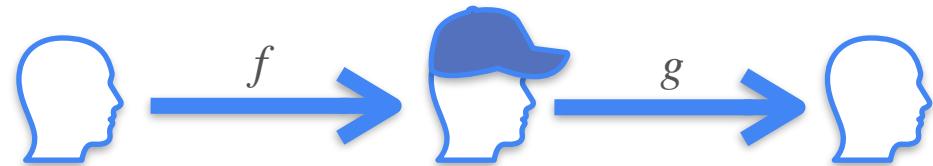
What's an inverse?



$$x \xrightarrow{f} x^2 \xrightarrow{g} x$$

# Inverse Function

What's an inverse?

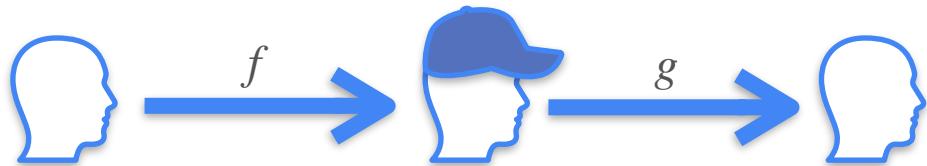


$g(x)$  and  $f(x)$  are  
inverses

$$x \xrightarrow{f} x^2 \xrightarrow{g} x$$

# Inverse Function

What's an inverse?



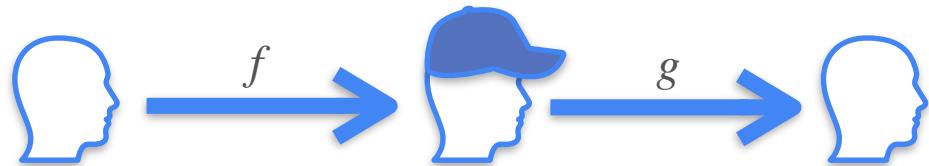
$g(x)$  and  $f(x)$  are  
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$$g(x) = f^{-1}(x)$$



# Inverse Function

What's an inverse?



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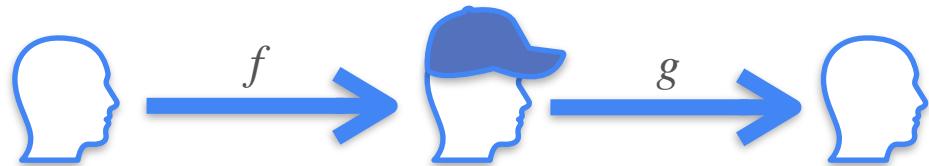
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$$g(f(x)) = x$$



# Inverse Function

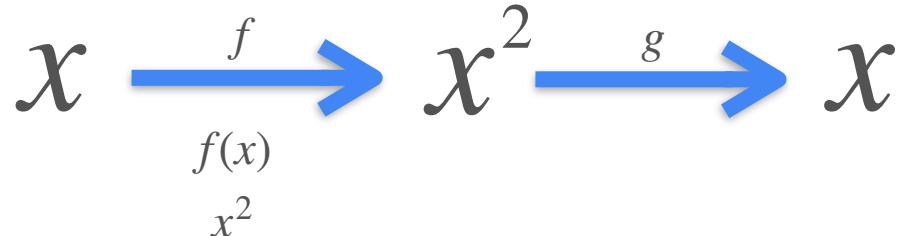
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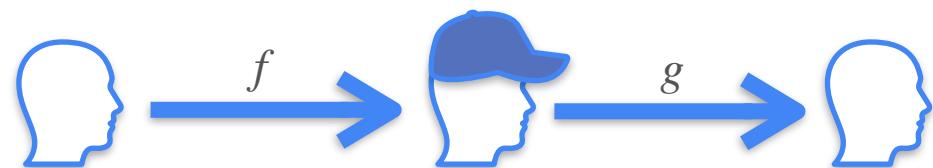
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# Inverse Function

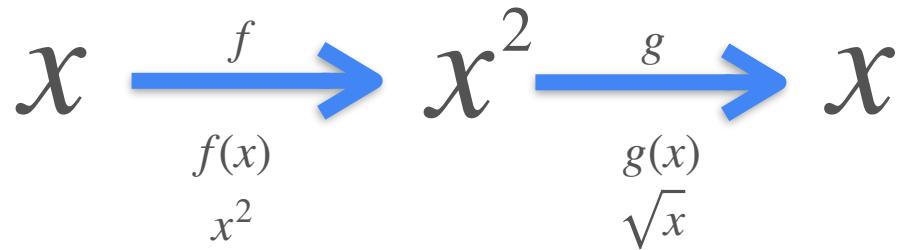
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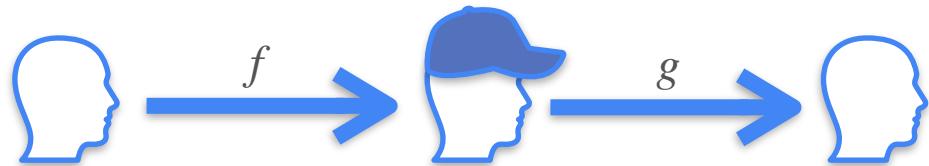
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# Inverse Function

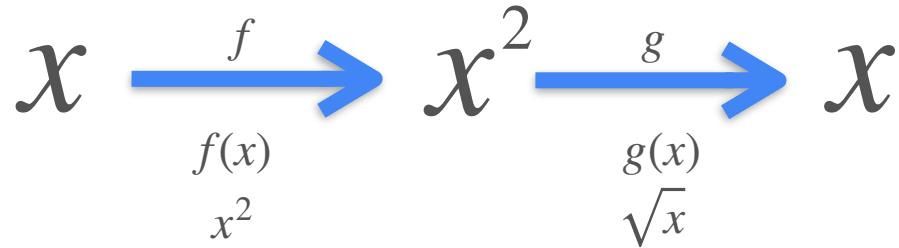
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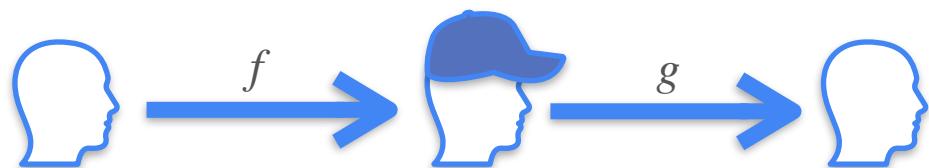
$$g(f(x)) = x$$



$$\sqrt{x^2}$$

# Inverse Function

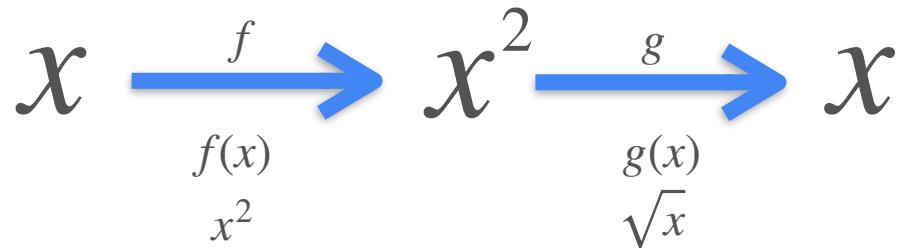
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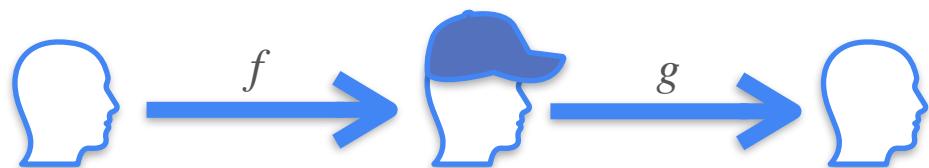
$$g(f(x)) = x$$



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# Inverse Function

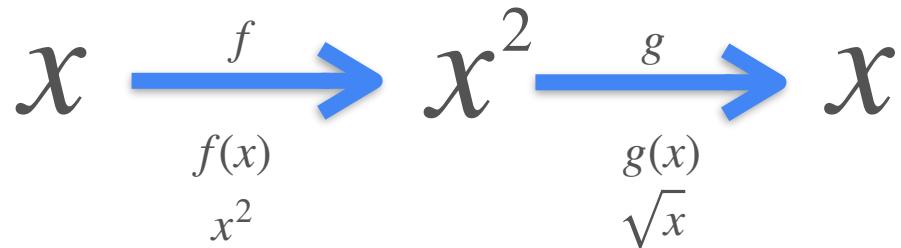
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$g(x)$  and  $f(x)$  are  
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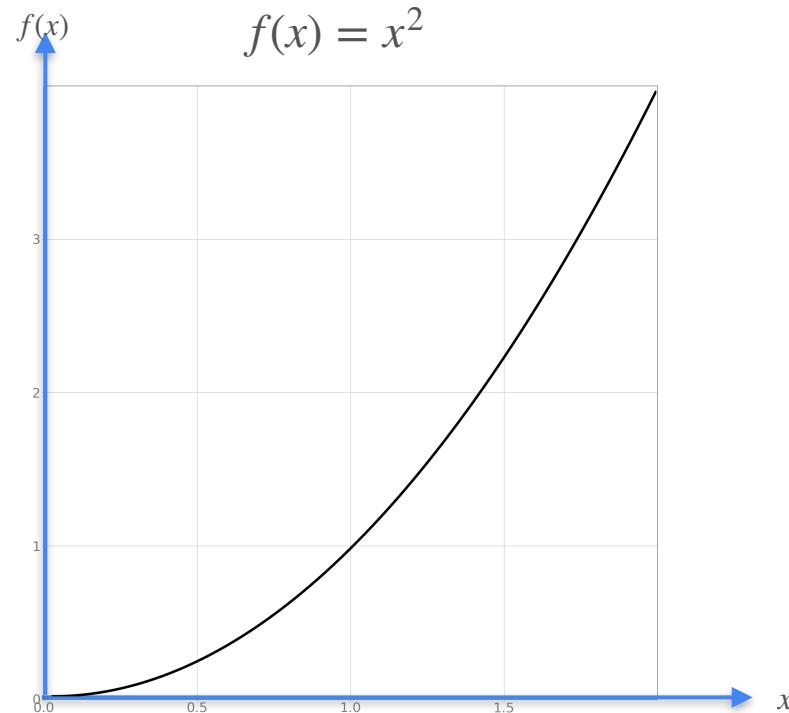
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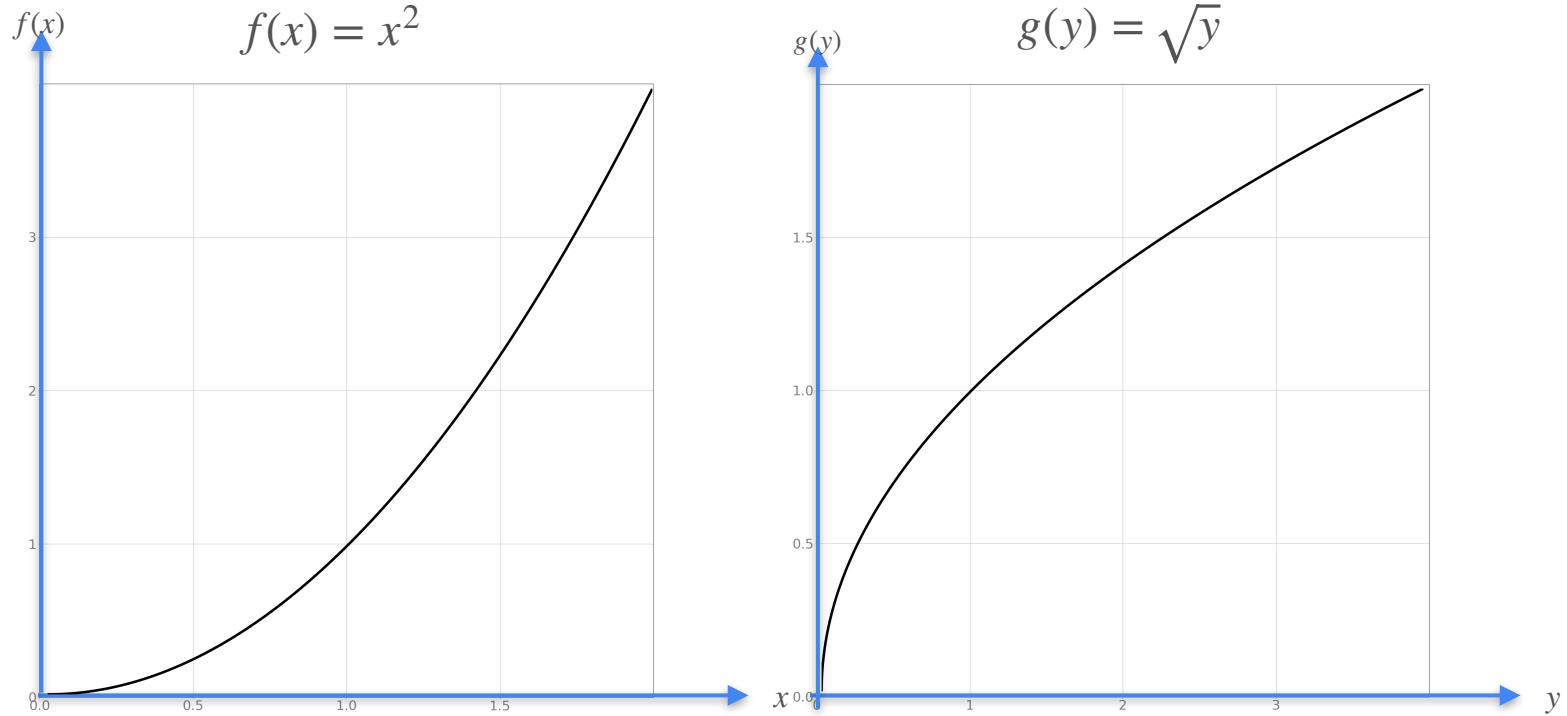
$$\sqrt{x^2} = x \quad \text{for } x > 0$$

# Derivative of the Inverse

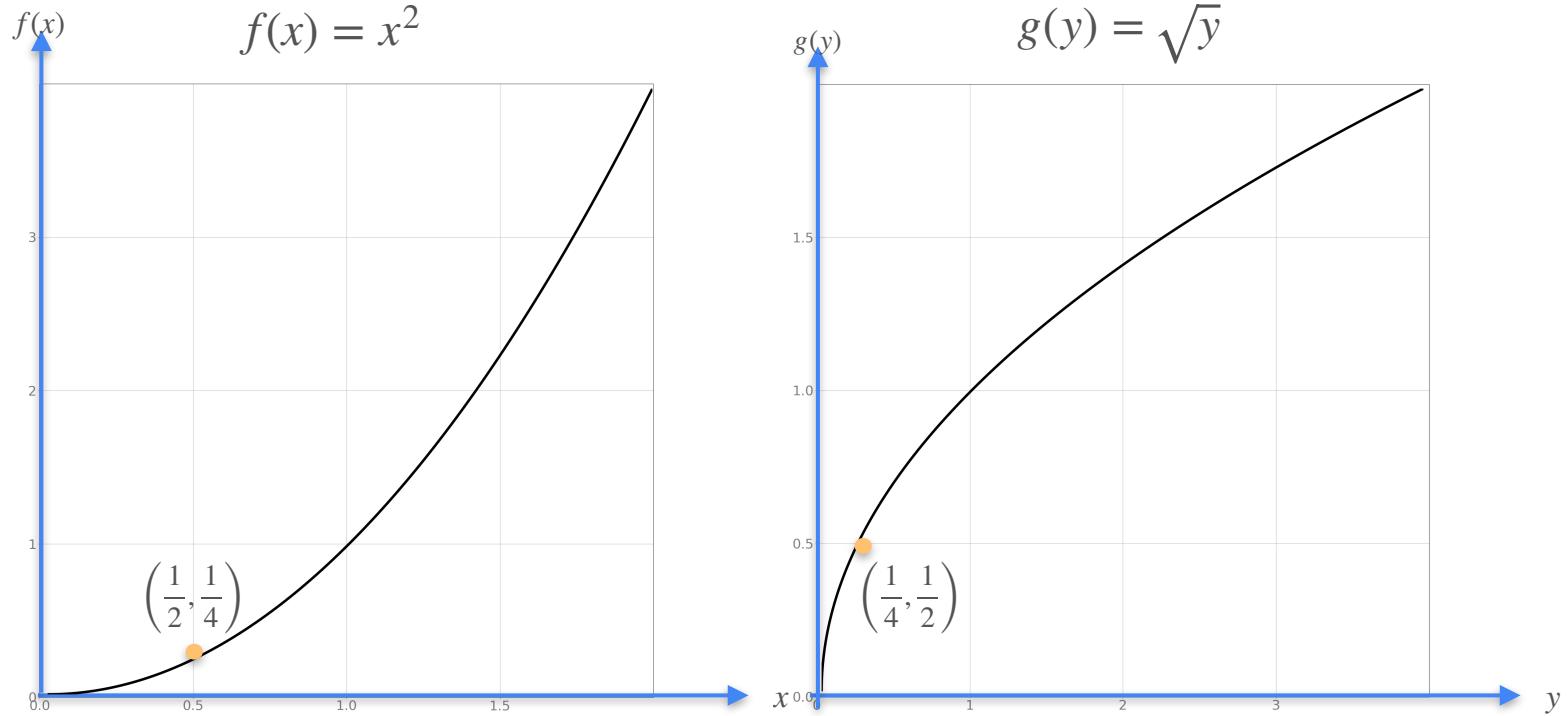
# Derivative of the Inverse



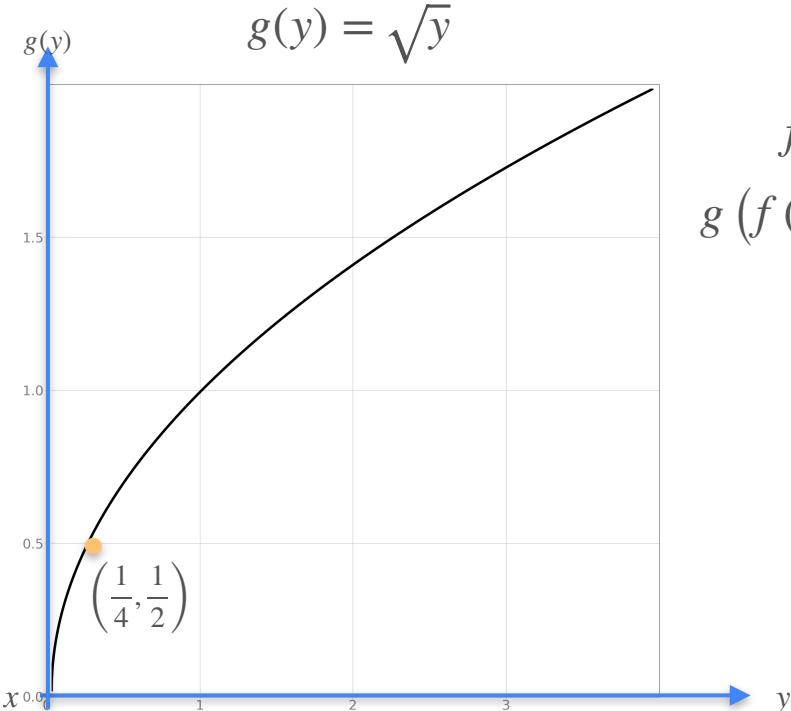
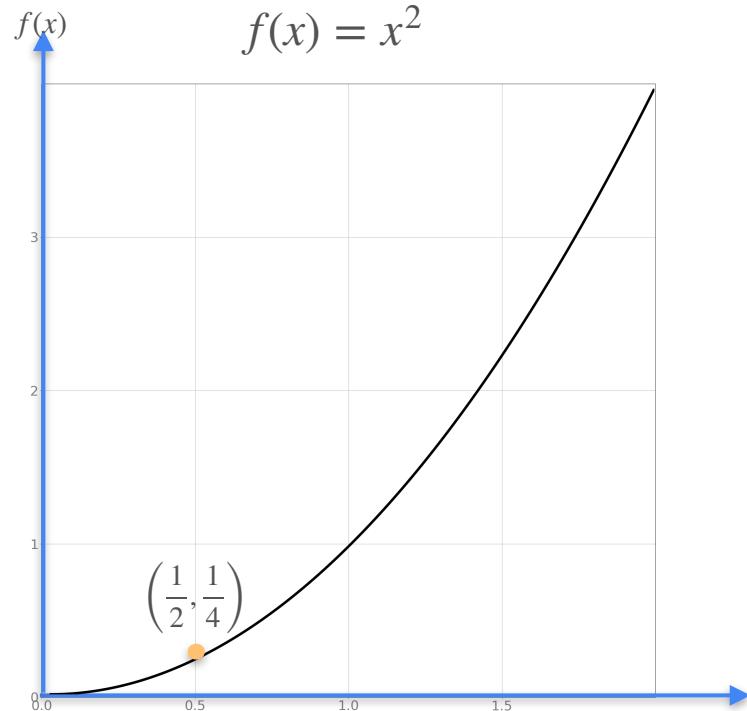
# Derivative of the Inverse



# Derivative of the Inverse



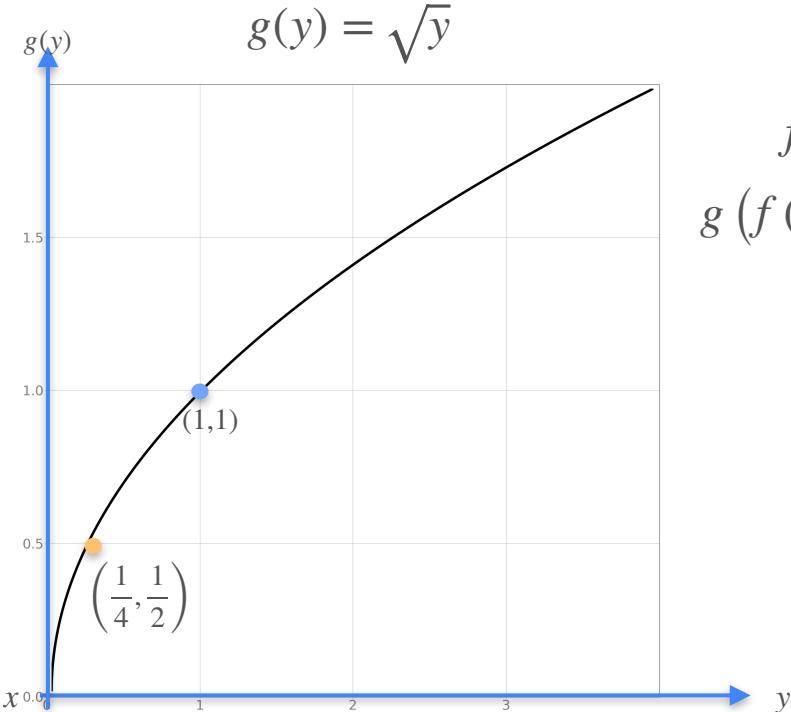
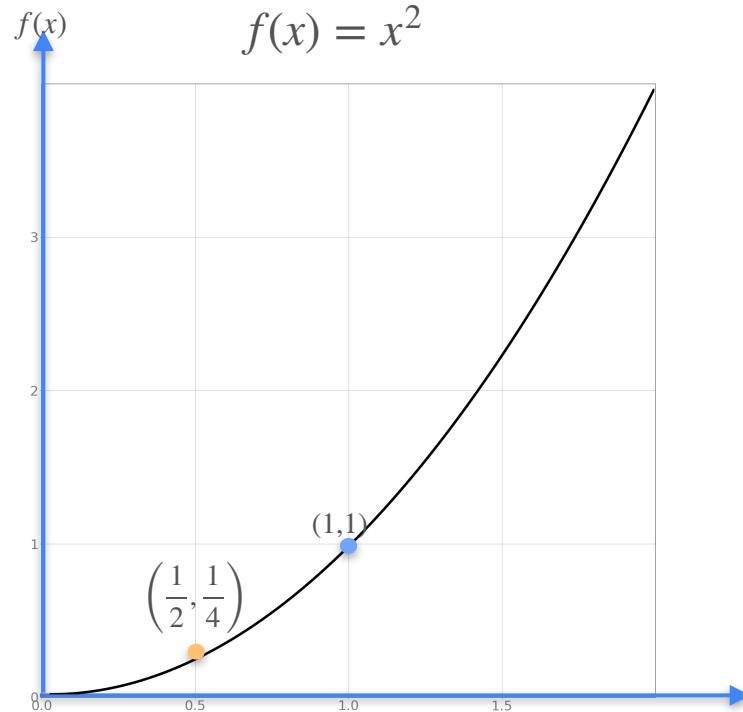
# Derivative of the Inverse



$$f(1/2) = 1/4$$

$$\begin{aligned}g(f(1/2)) &= g(1/4) \\&= 1/2\end{aligned}$$

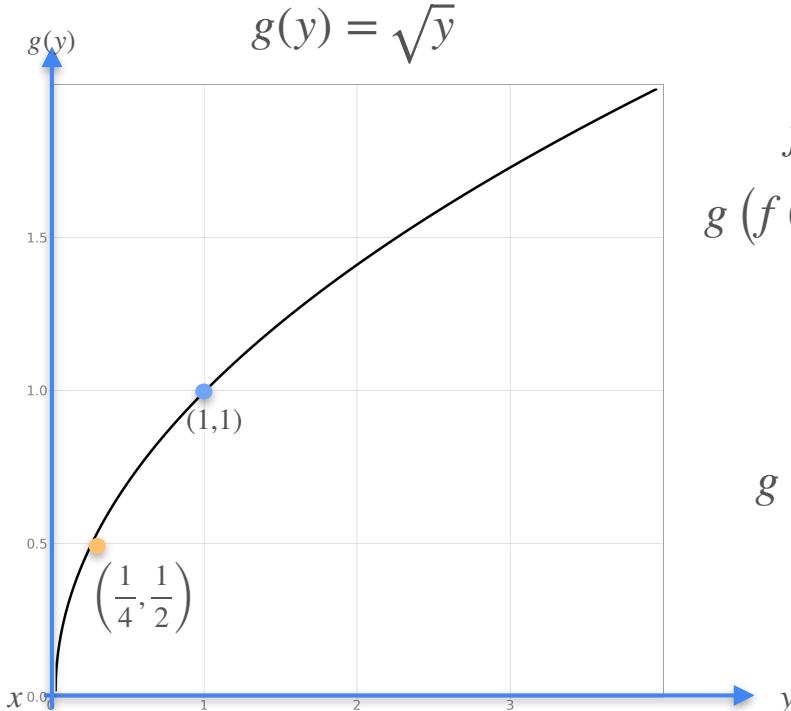
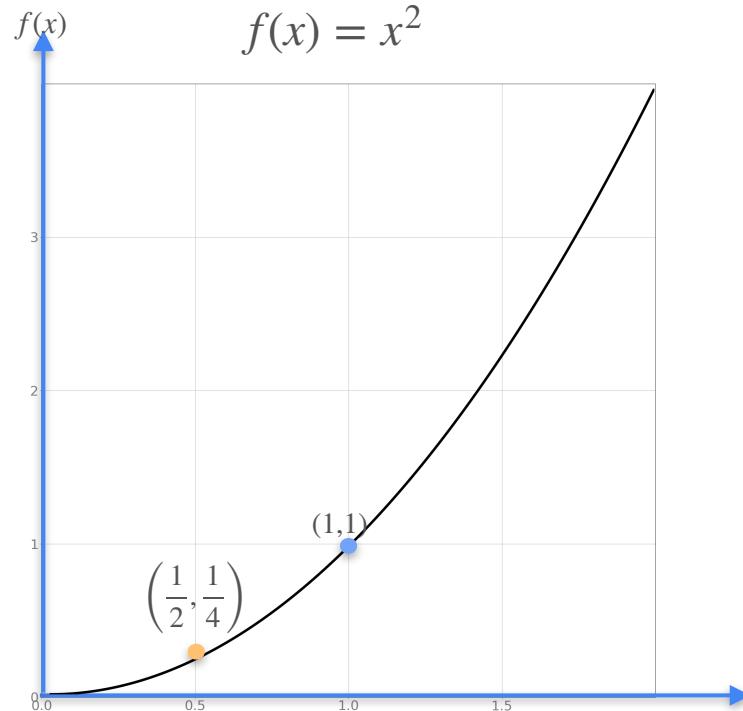
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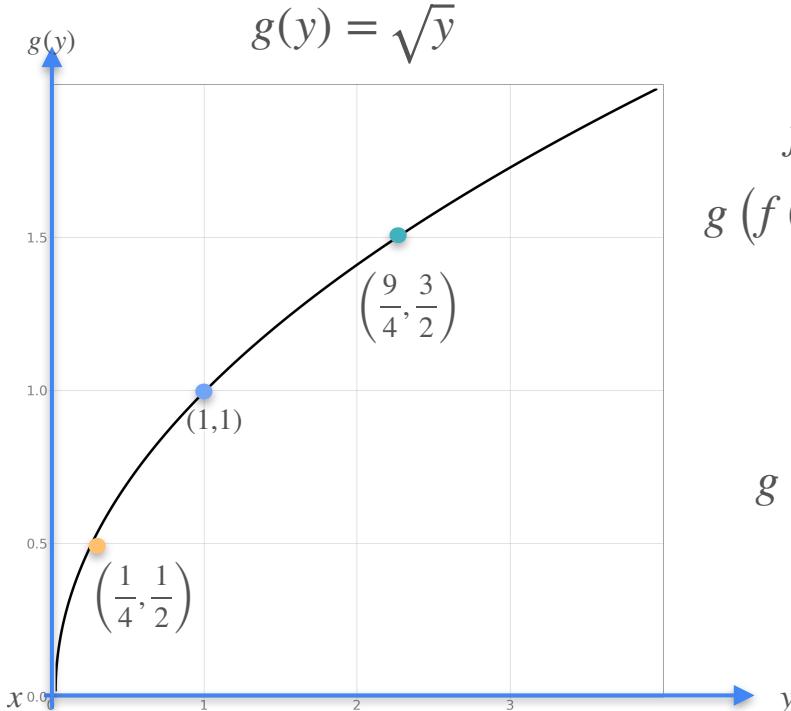
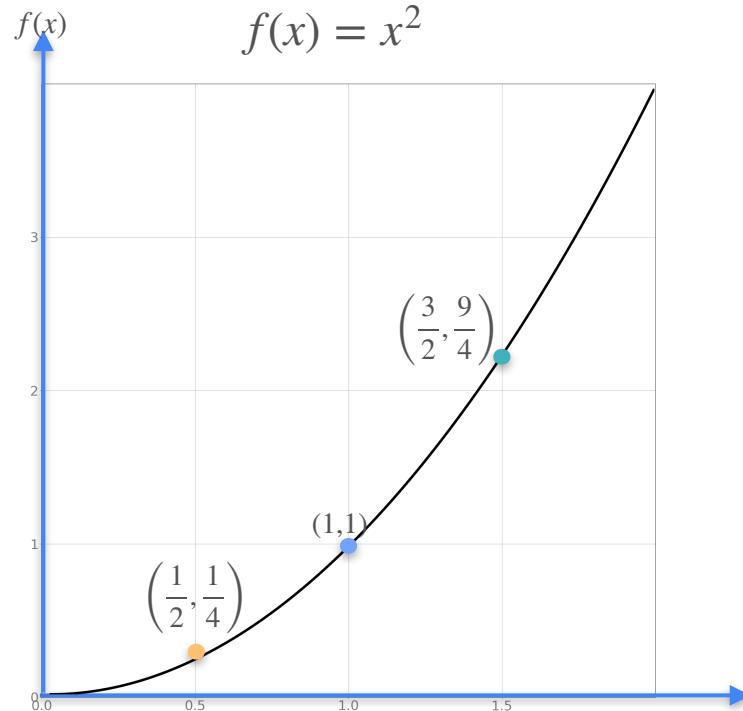
$$f(1/2) = 1/4$$

$$\begin{aligned}g(f(1/2)) &= g(1/4) \\&= 1/2\end{aligned}$$

$$f(1) = 1$$

$$\begin{aligned}g(f(1)) &= g(1) \\&= 1\end{aligned}$$

# Derivative of the Inverse



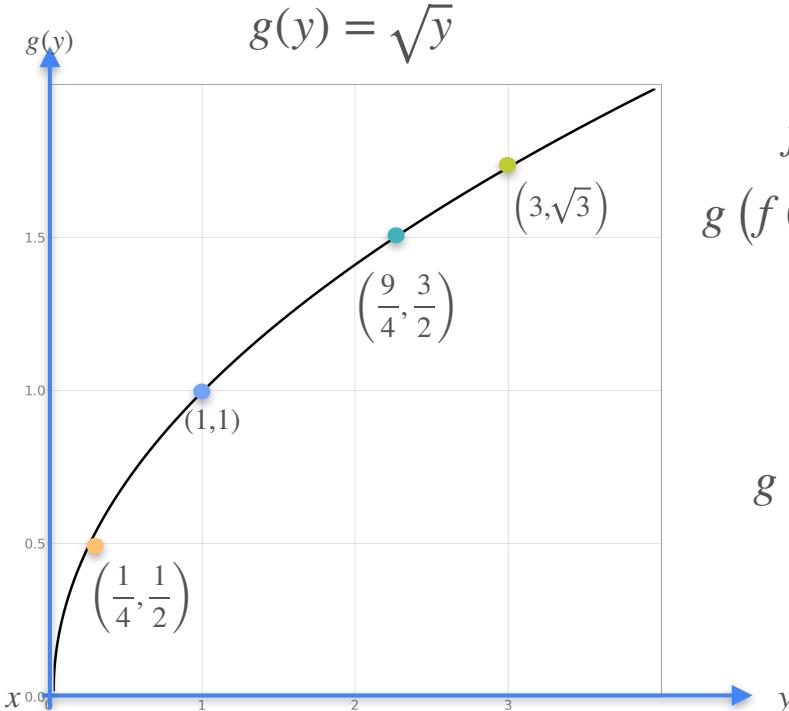
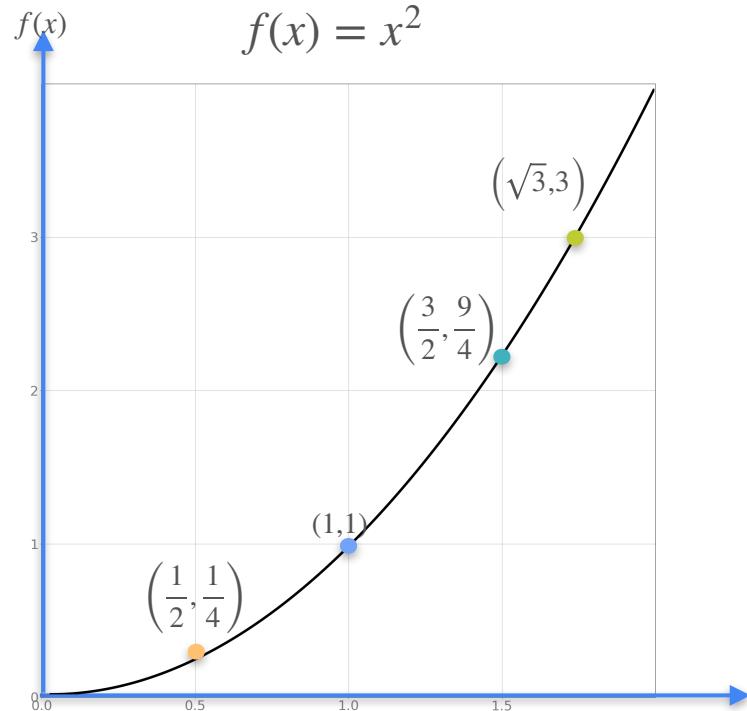
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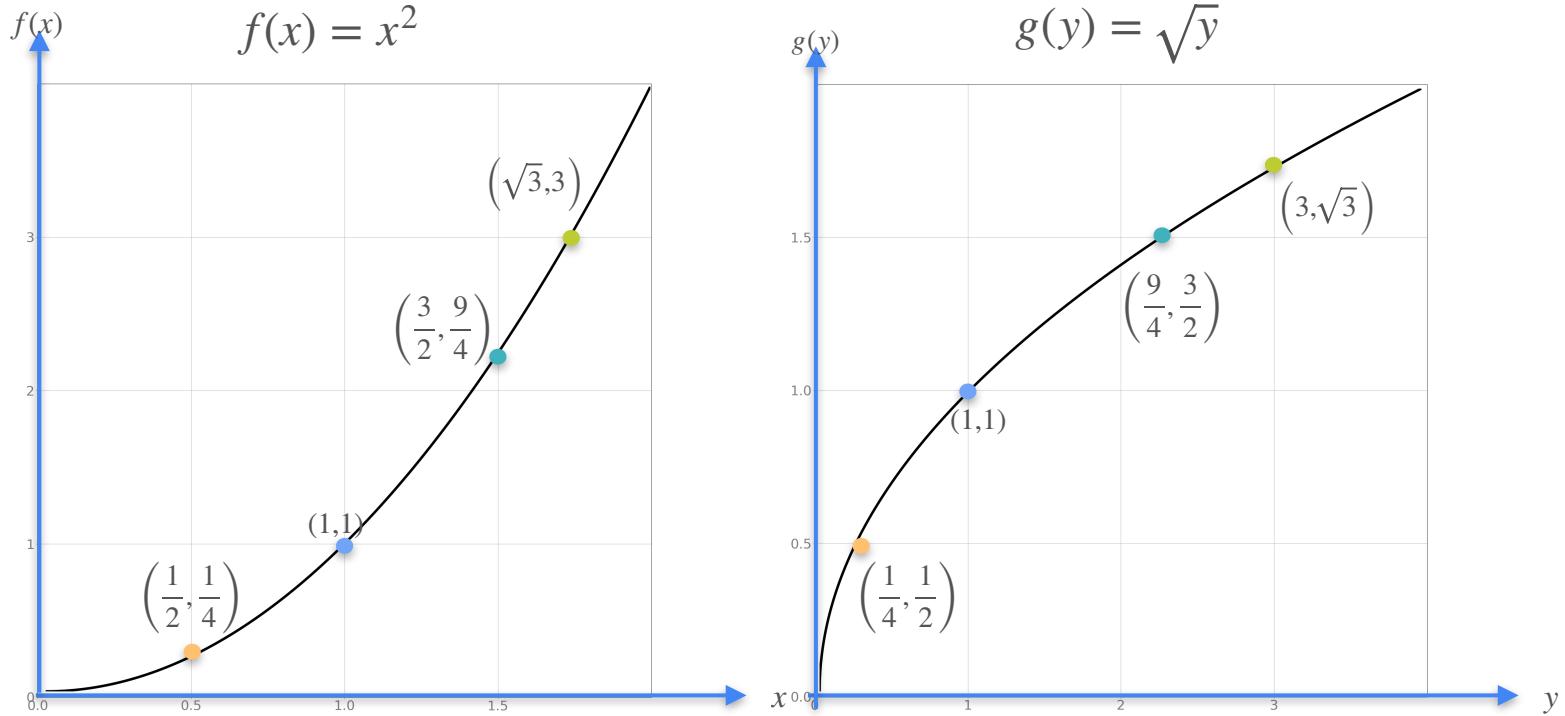
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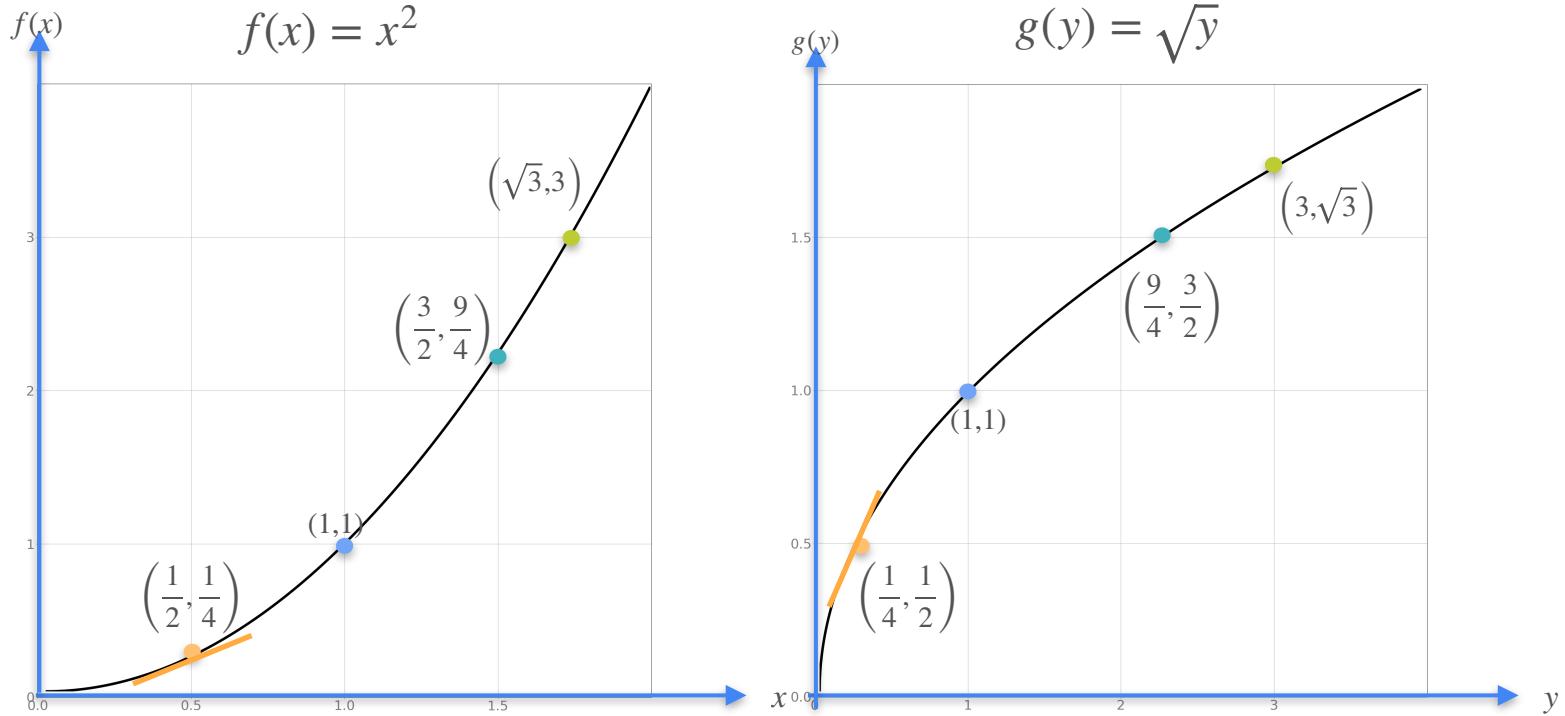
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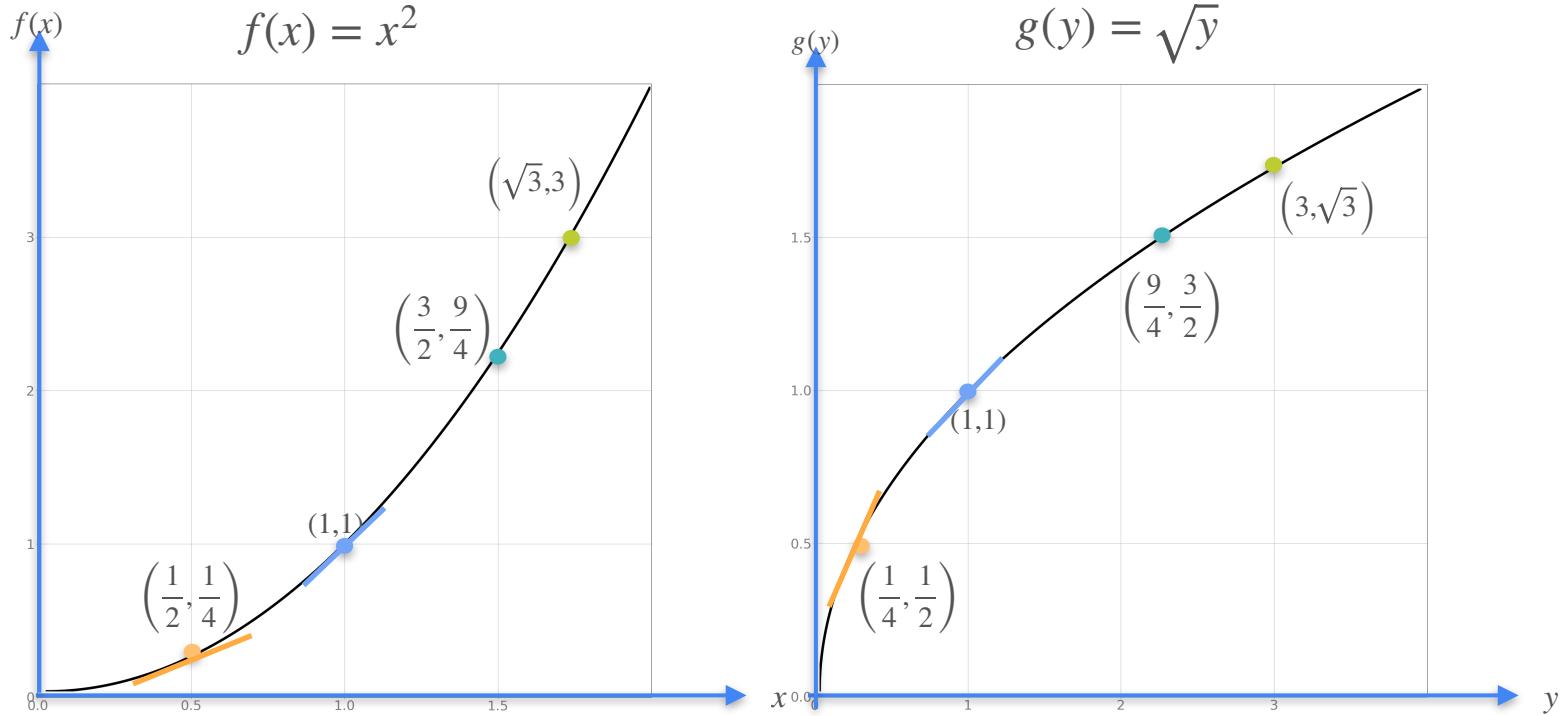
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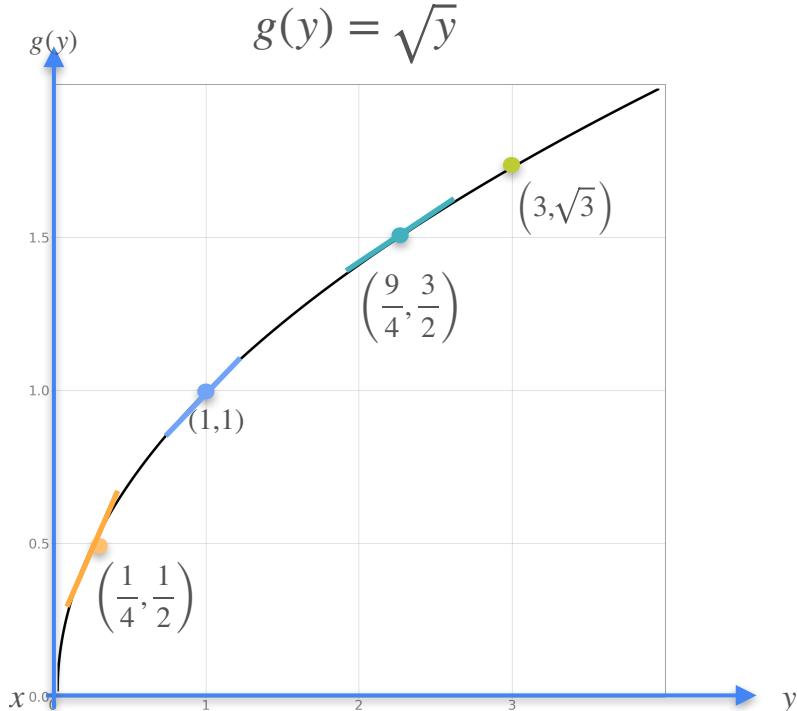
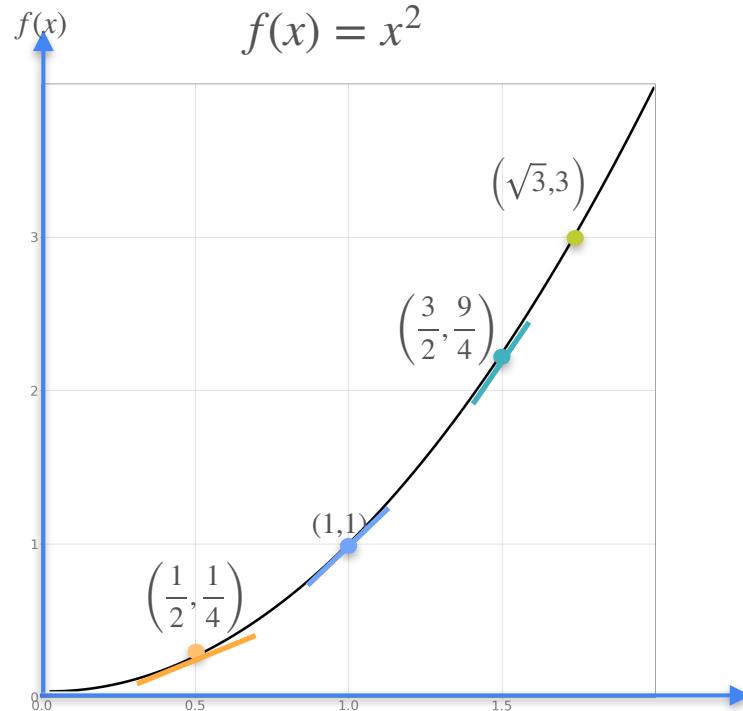
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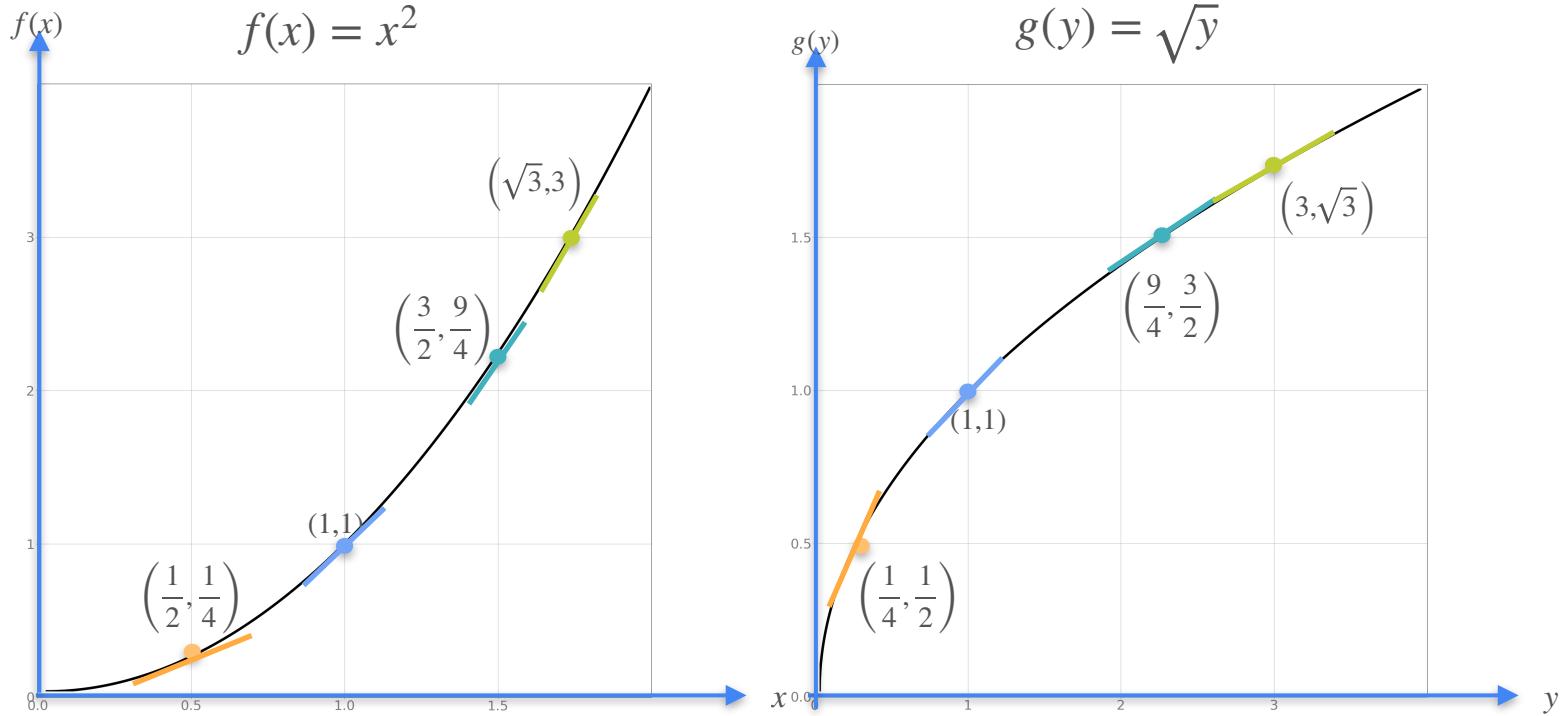
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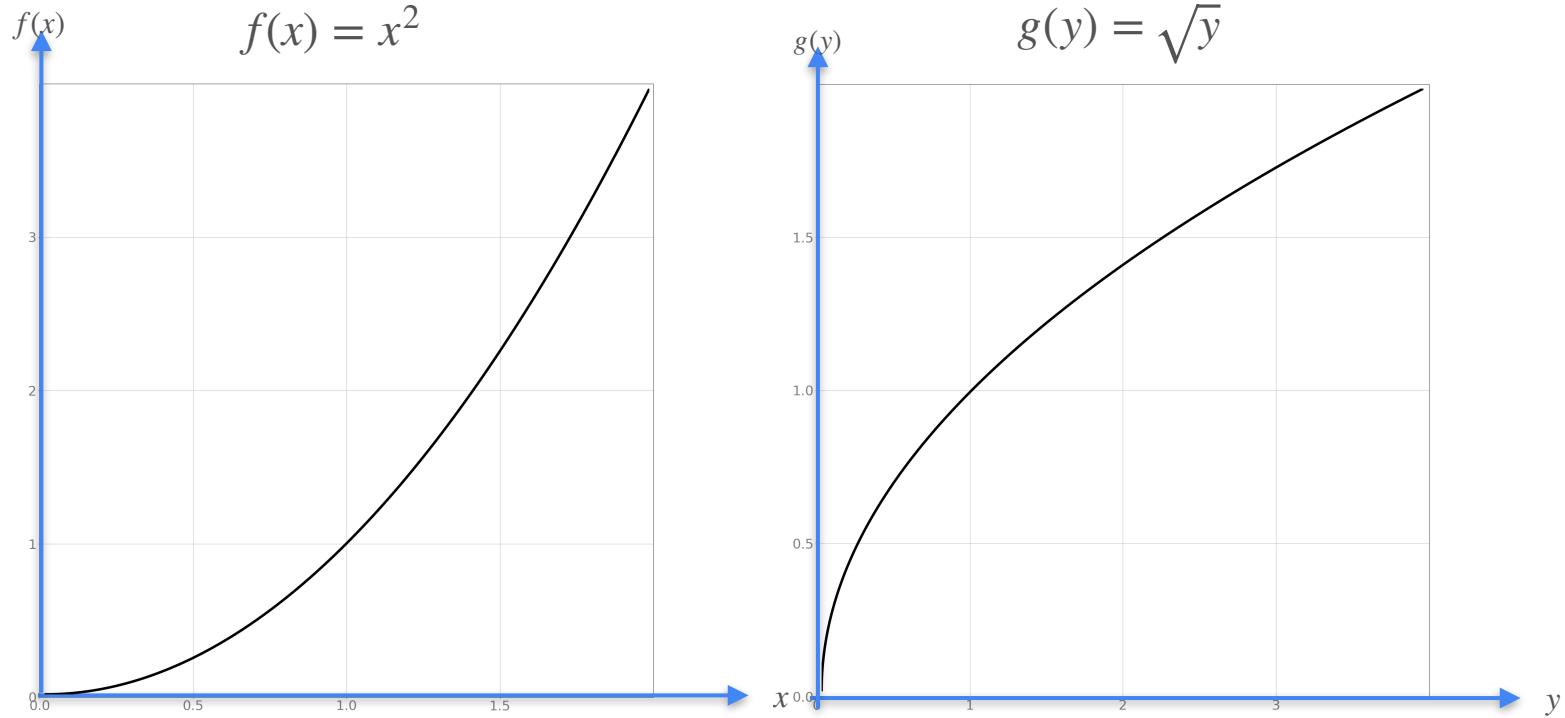
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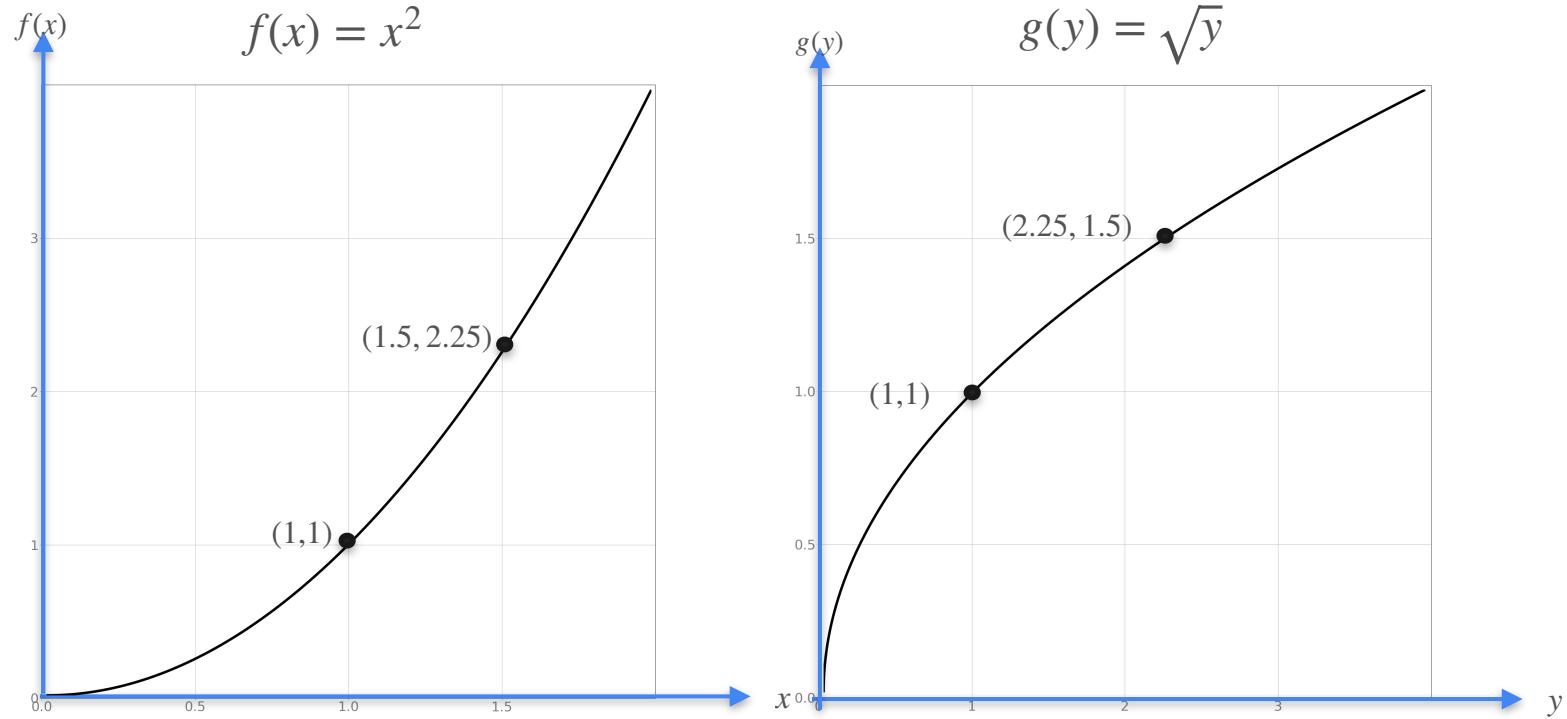
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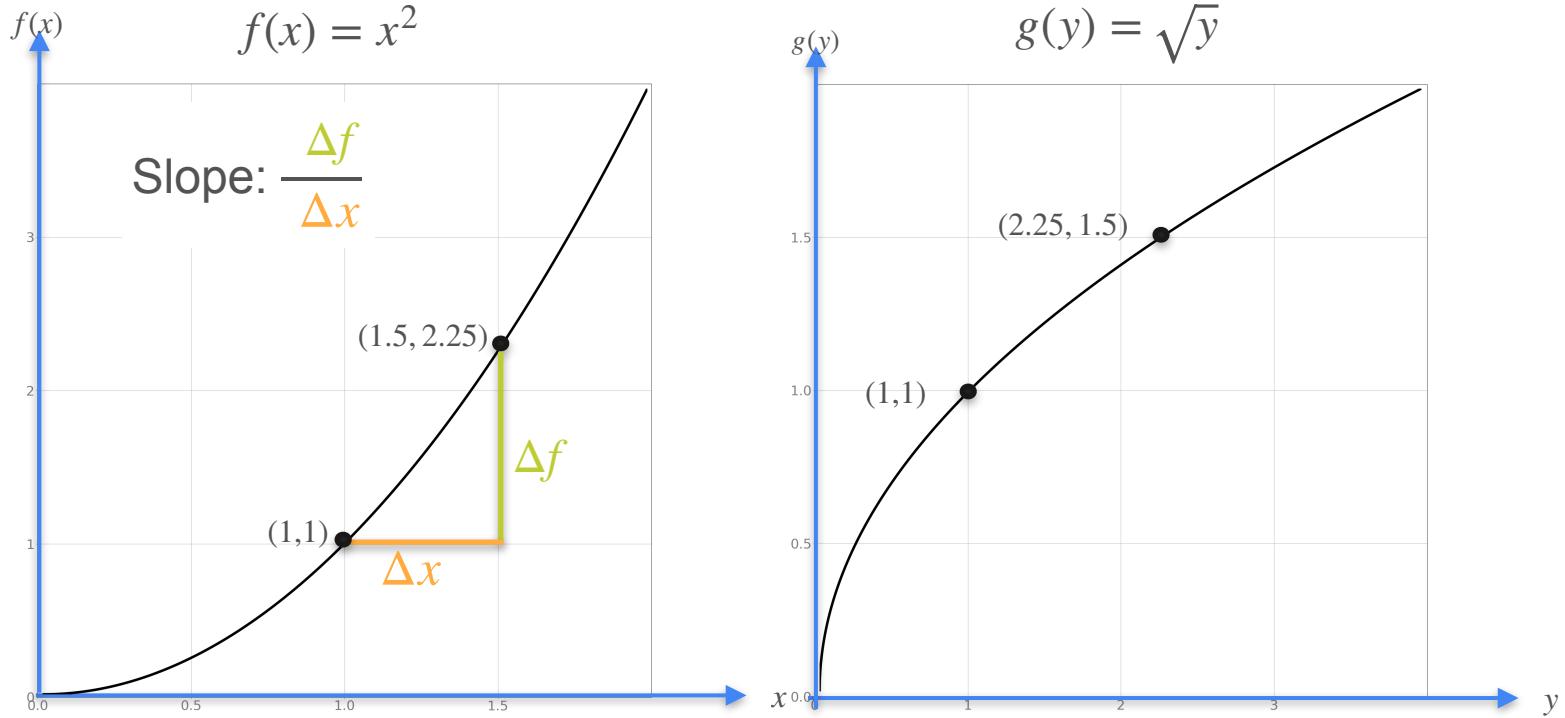
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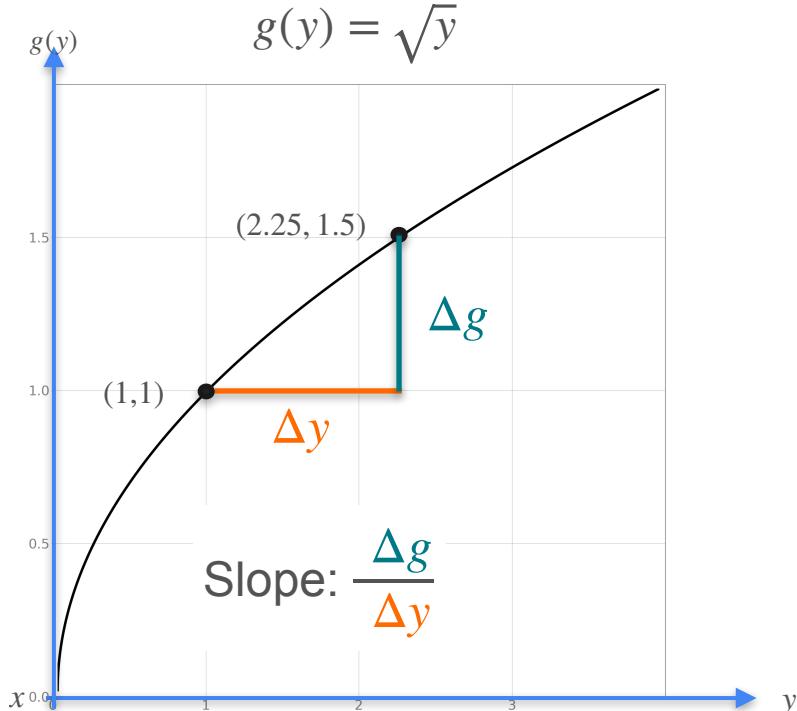
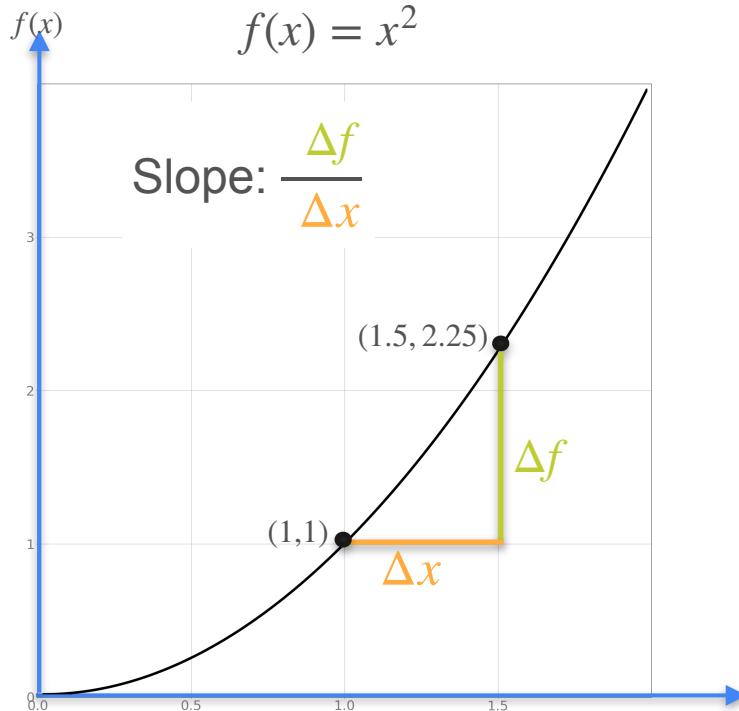
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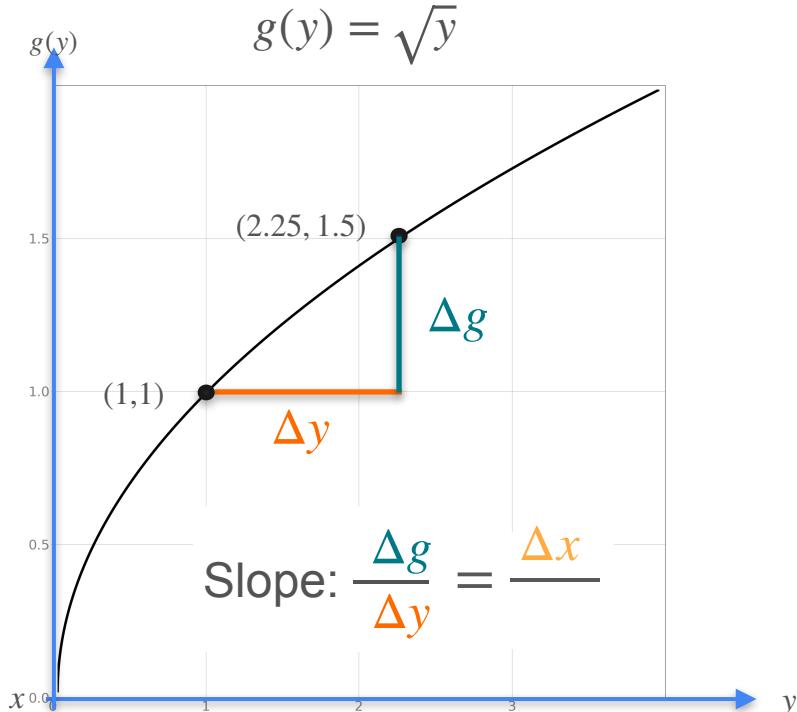
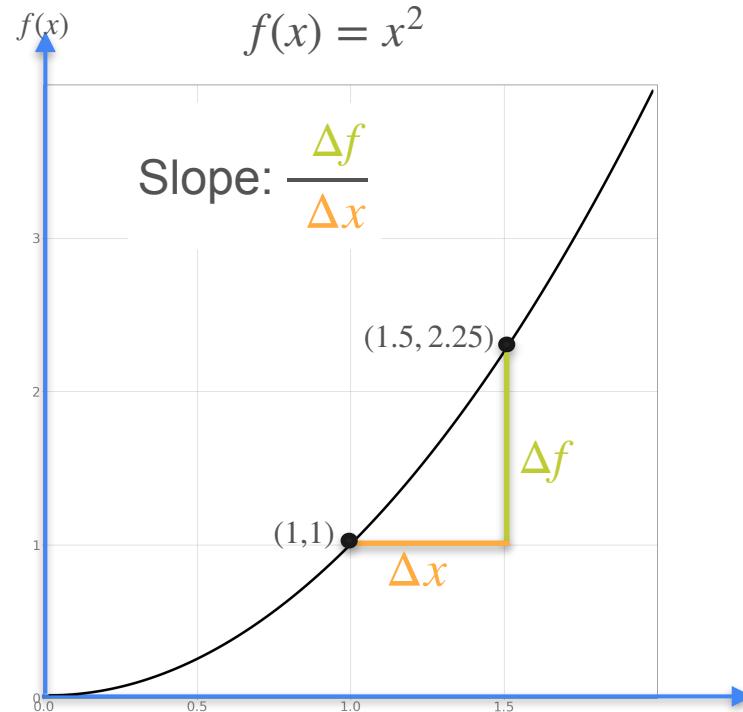
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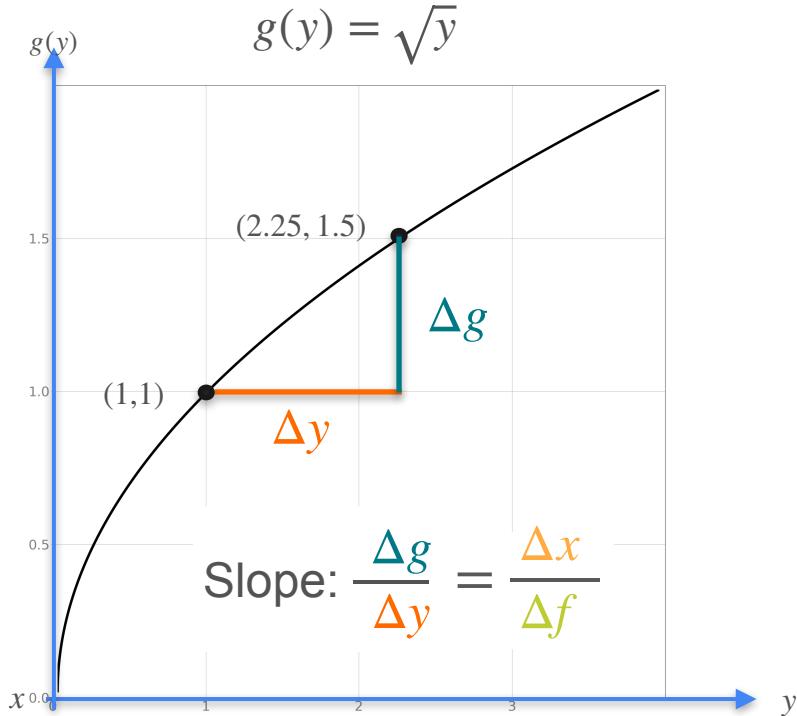
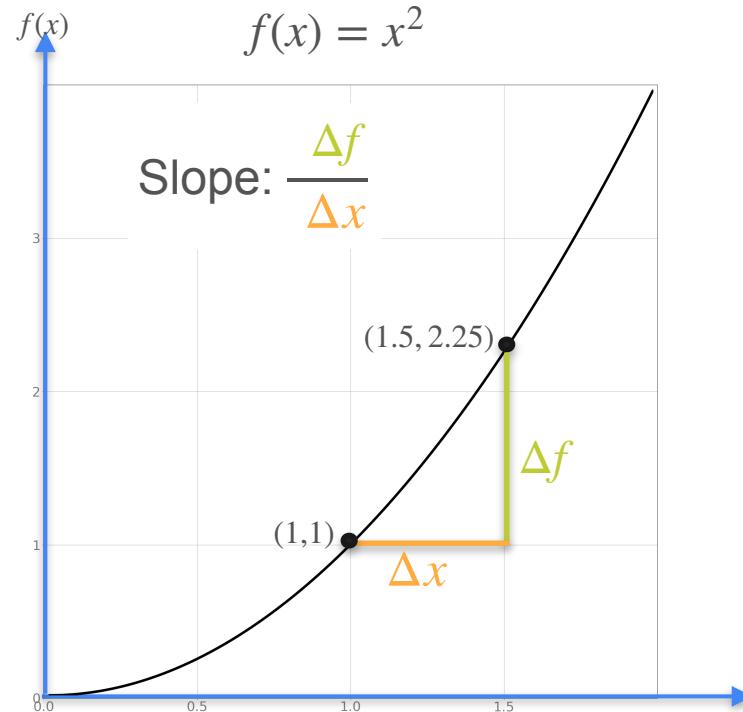
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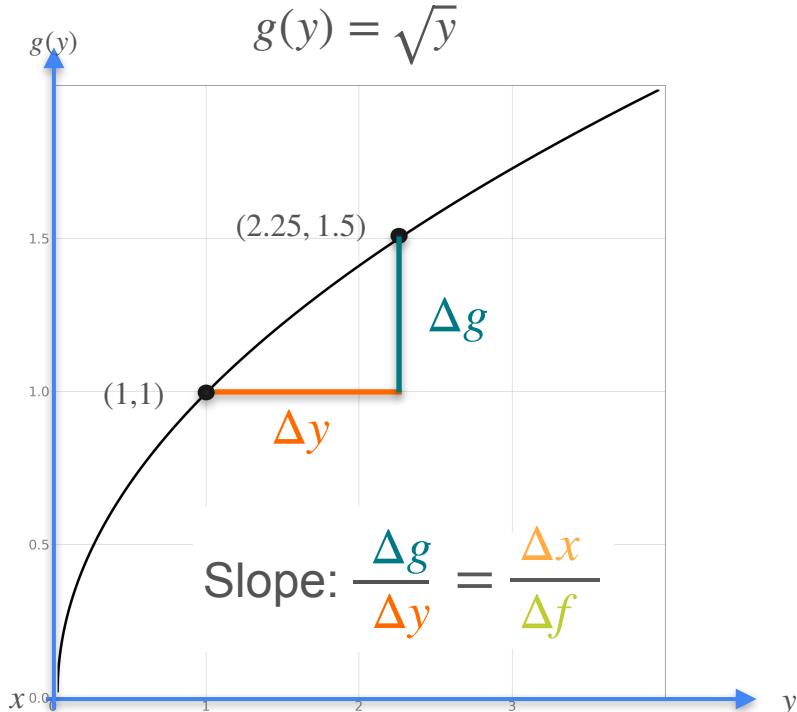
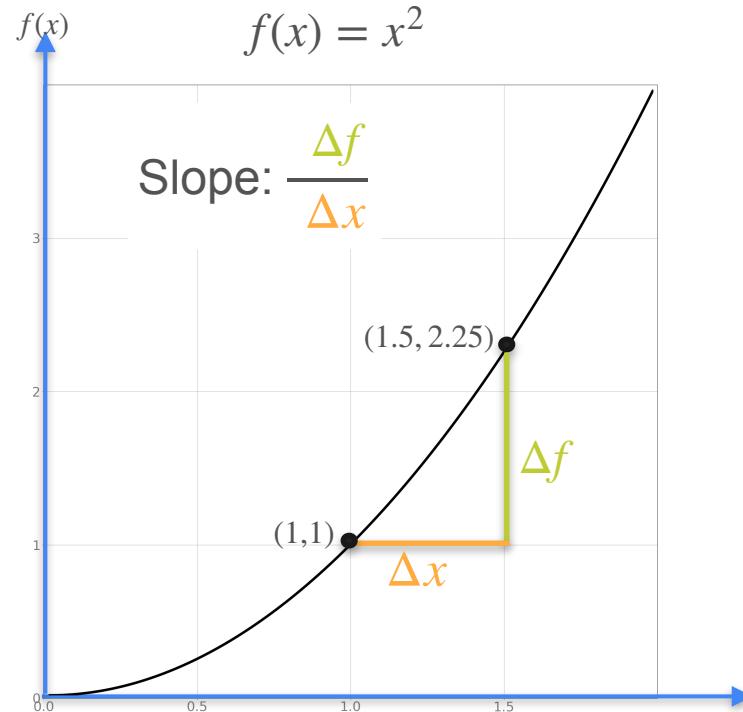
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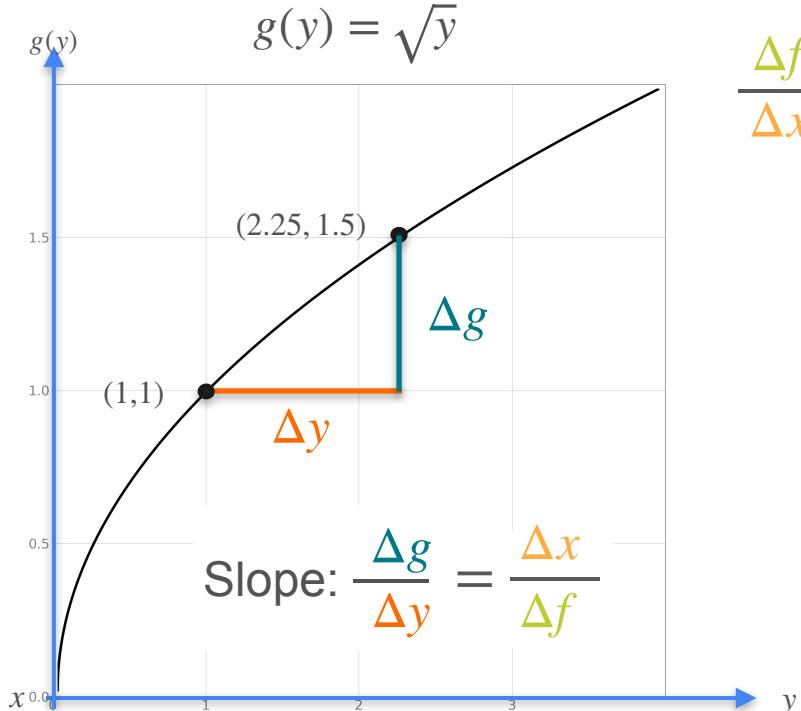
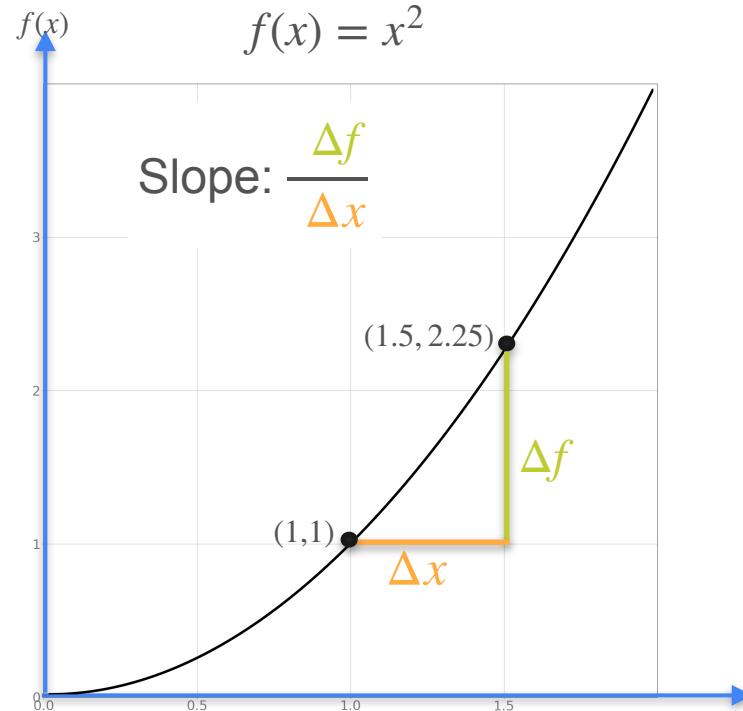
# Derivative of the Inverse



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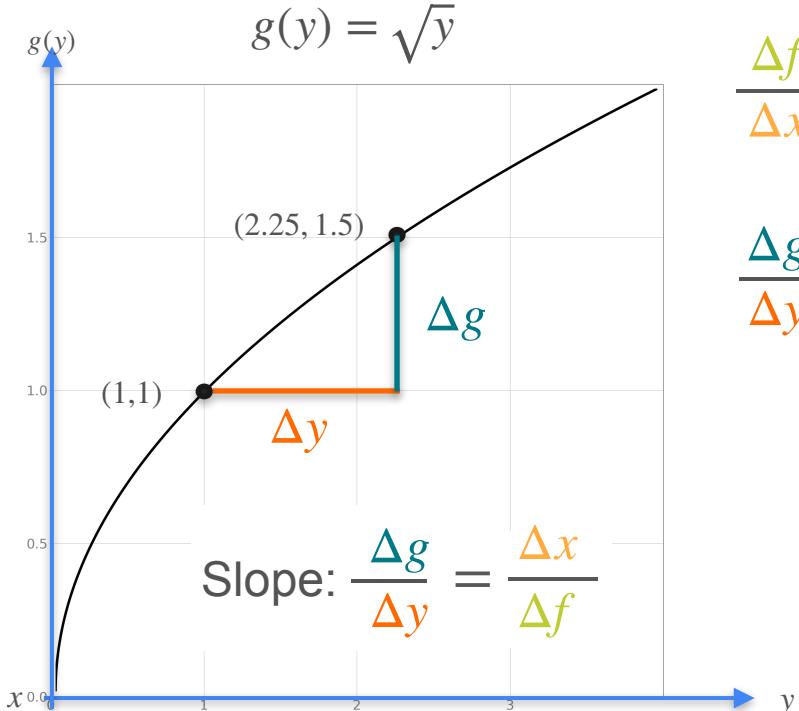
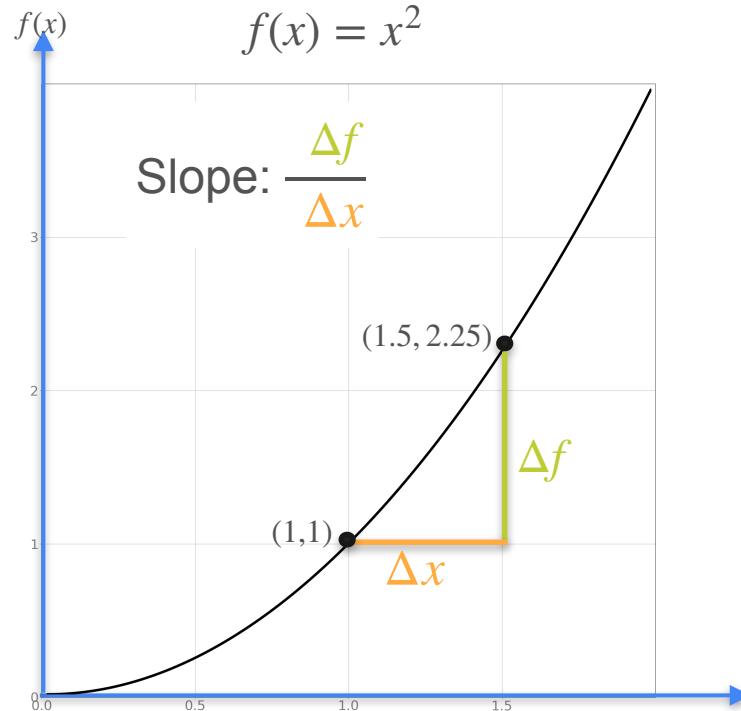


# Derivative of the Inverse



$$\frac{\Delta f}{\Delta x} = f'(x)$$

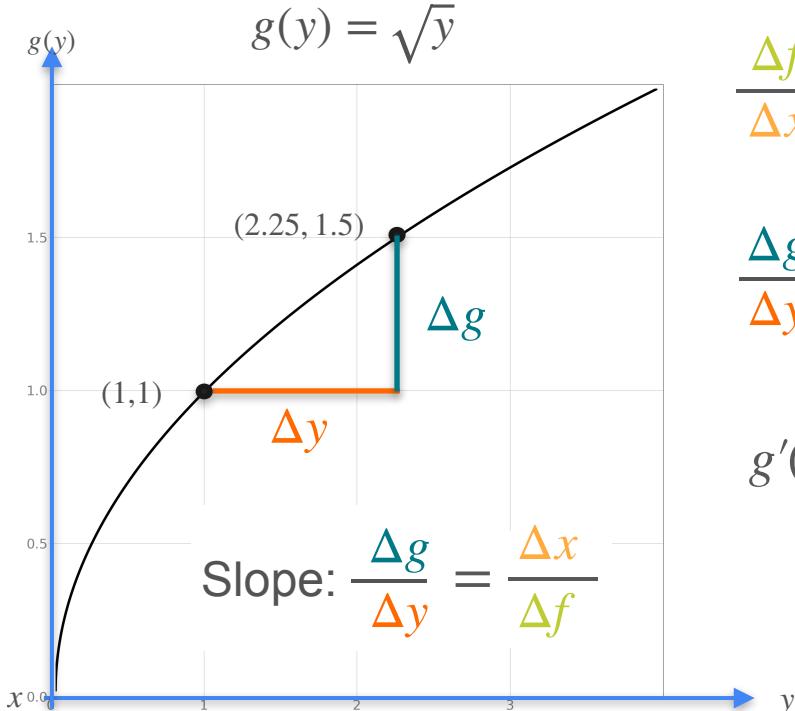
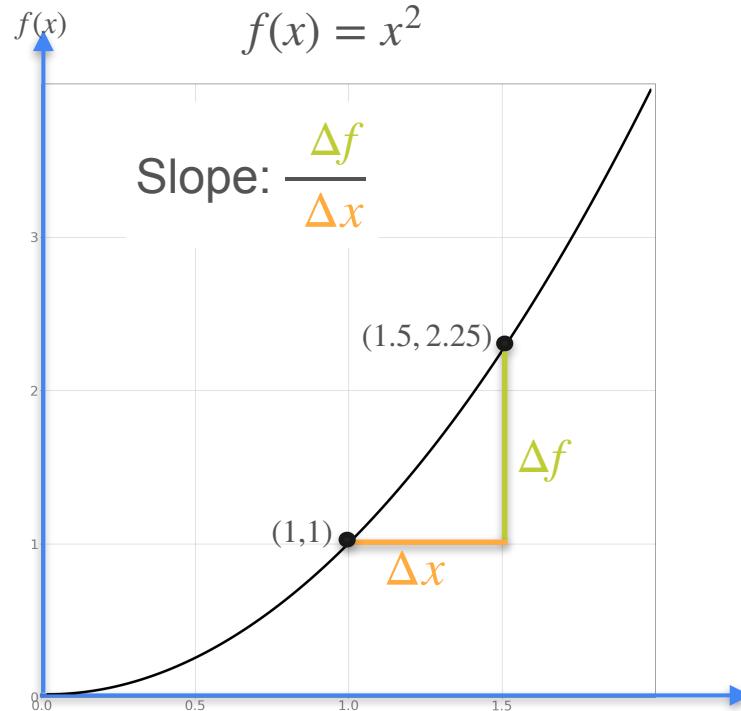
# Derivative of the Inverse



$$\frac{\Delta f}{\Delta x} = f'(x)$$

$$\frac{\Delta g}{\Delta y} = g'(y)$$

# Derivative of the Inverse

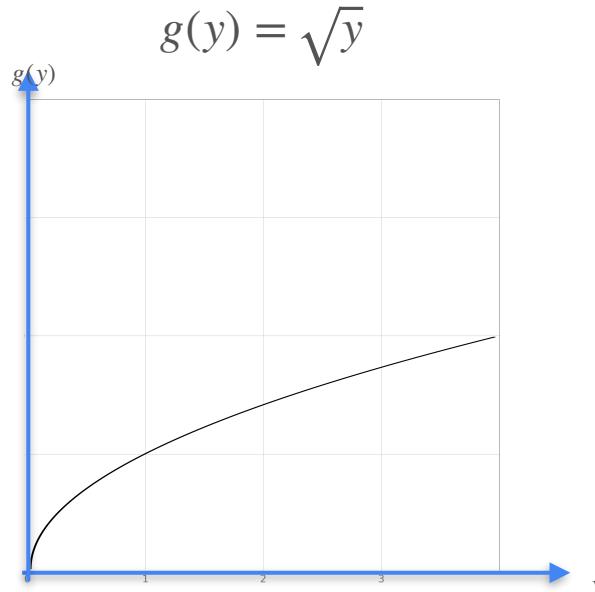
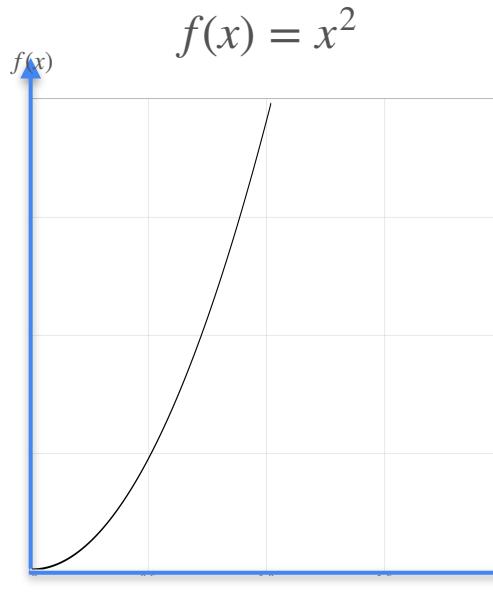


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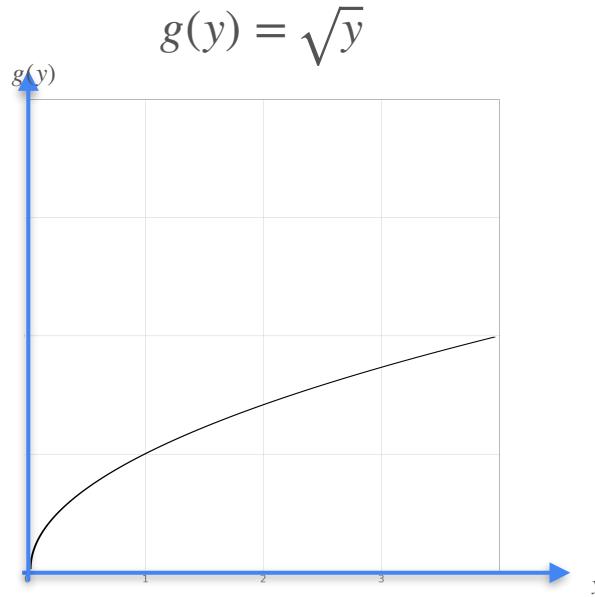
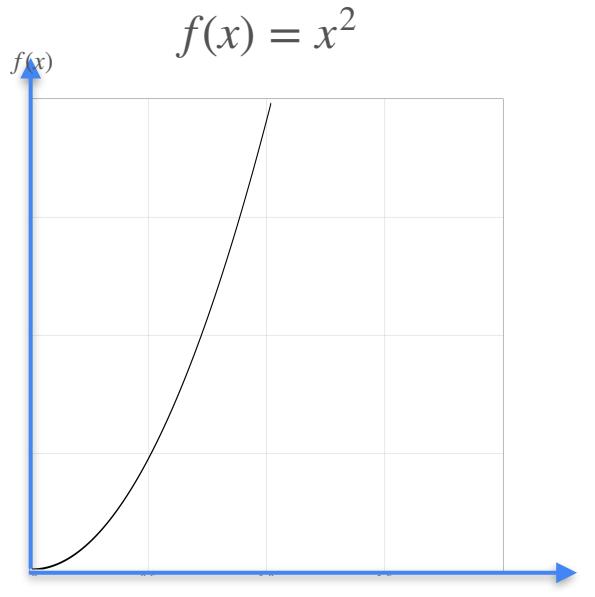
$$g'(y) = \frac{1}{f'(x)}$$

# Derivative of the Inverse



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# Derivative of the Inverse

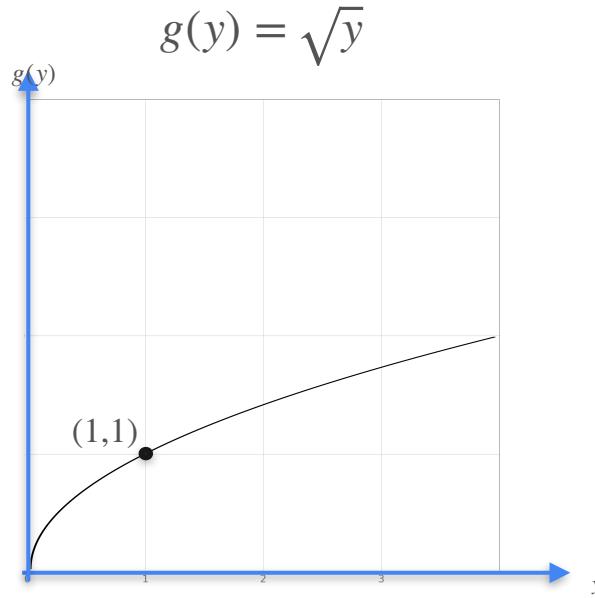
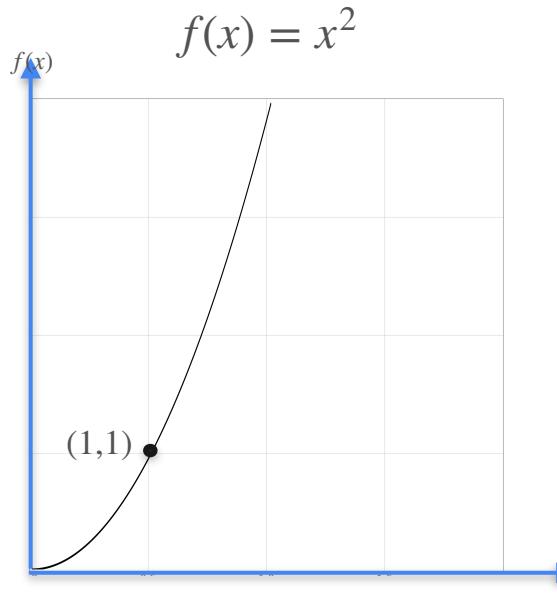


$$g'(y) = \frac{1}{f'(x)}$$

at the point (1,1)

$$f(1) = 1 \quad g(1) = 1$$

# Derivative of the Inverse

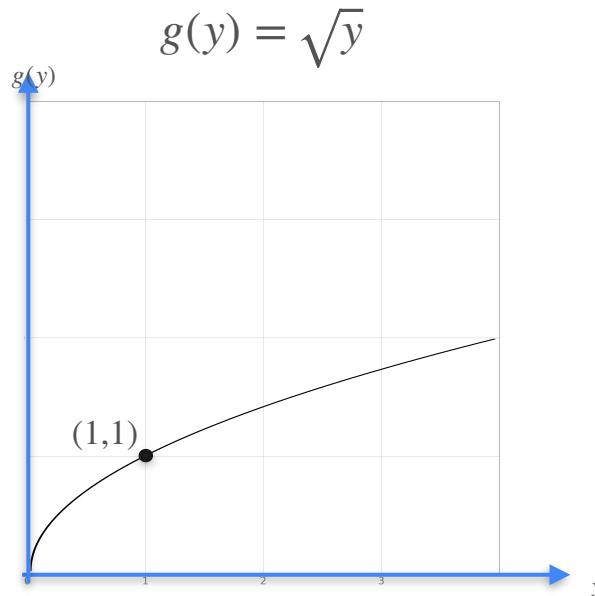
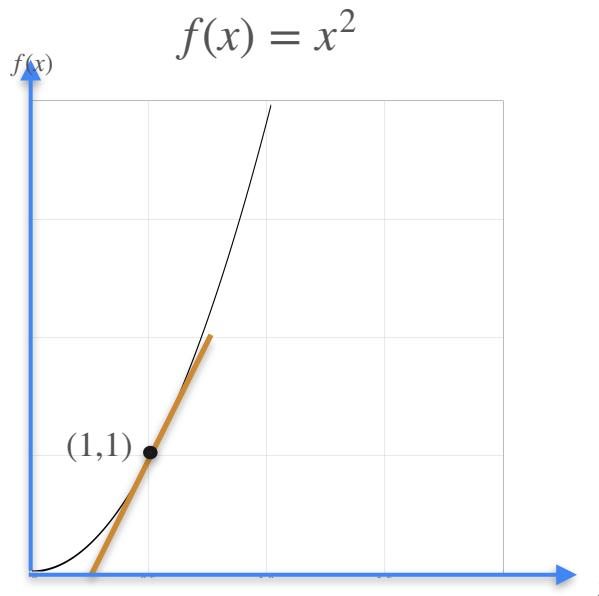


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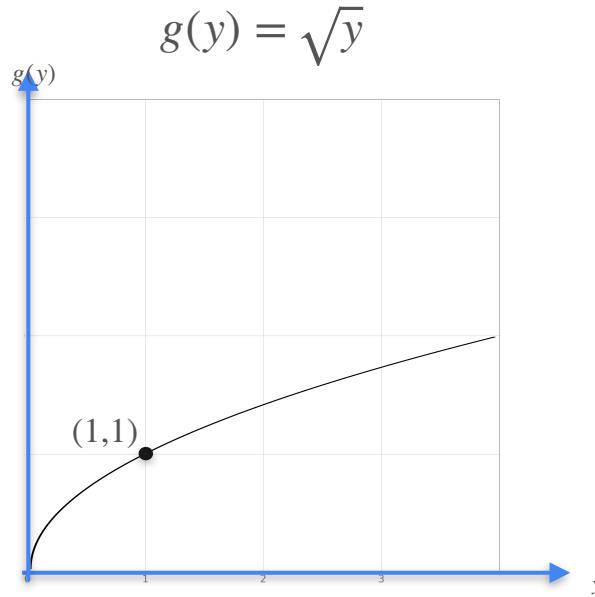
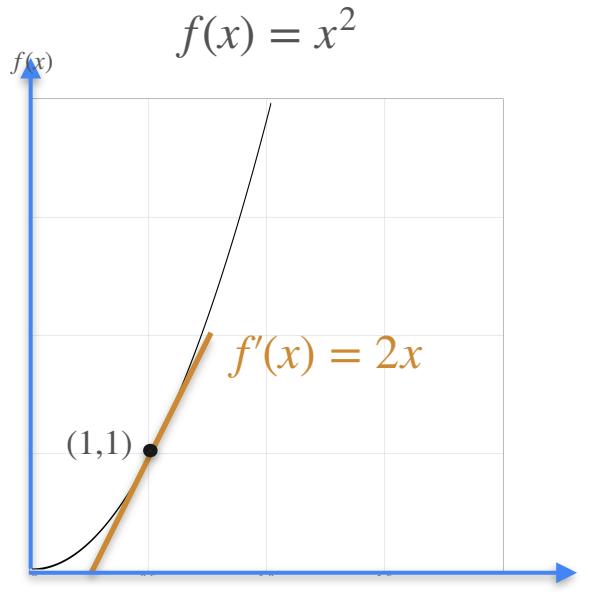


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# Derivative of the Inverse

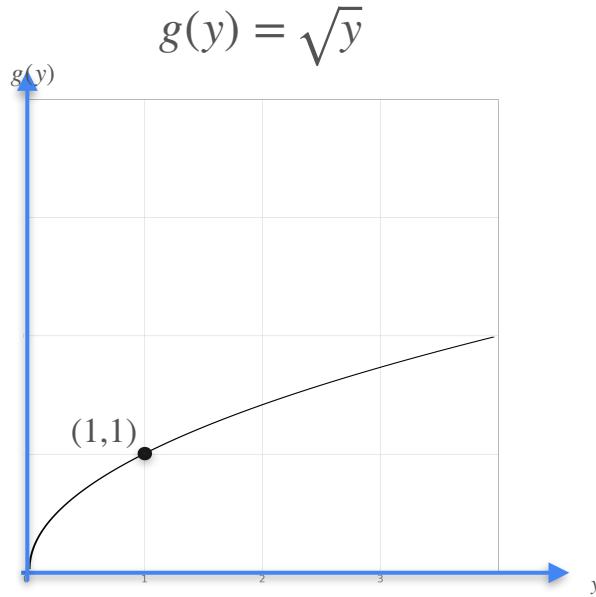
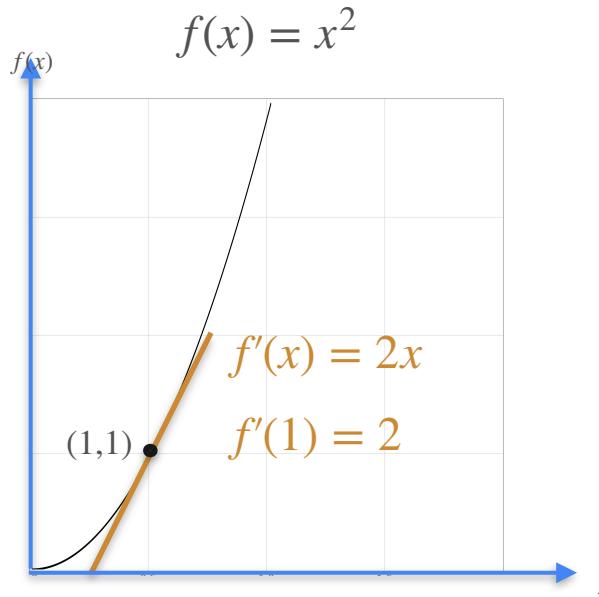


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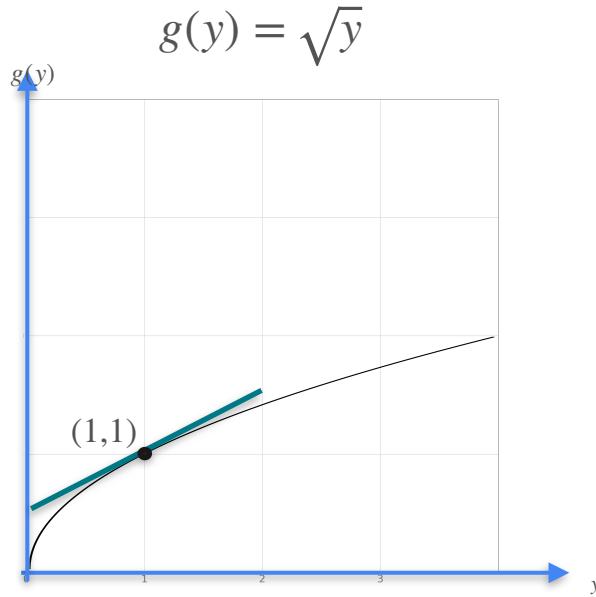
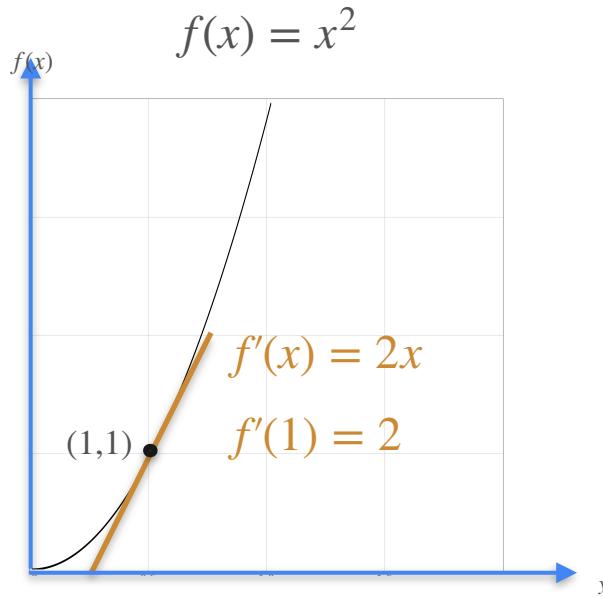


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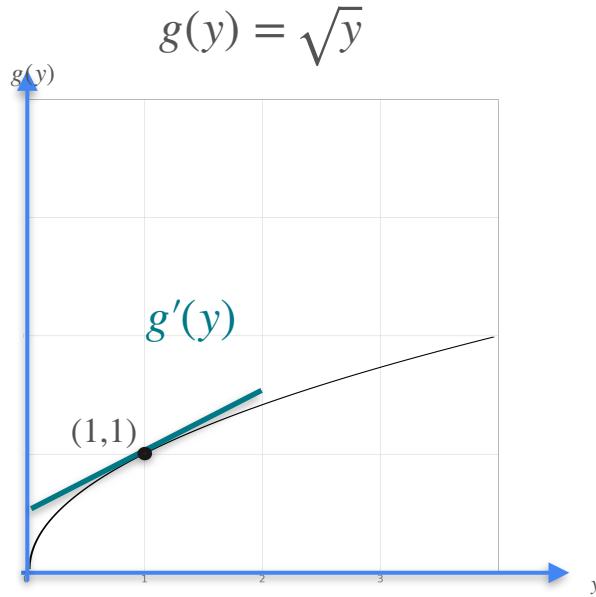
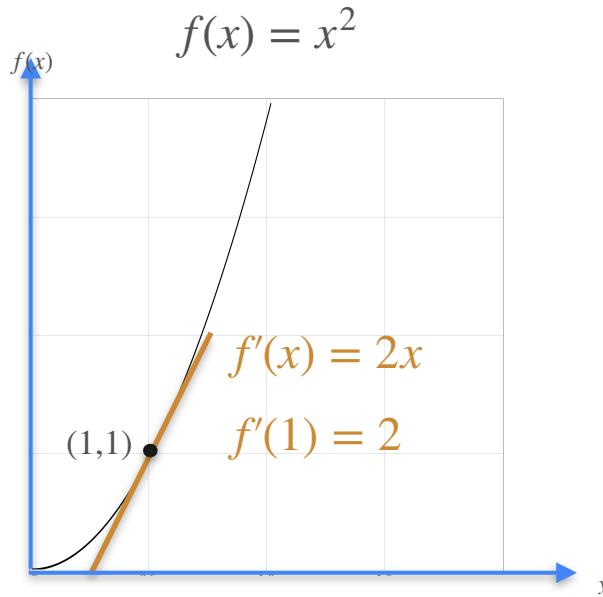


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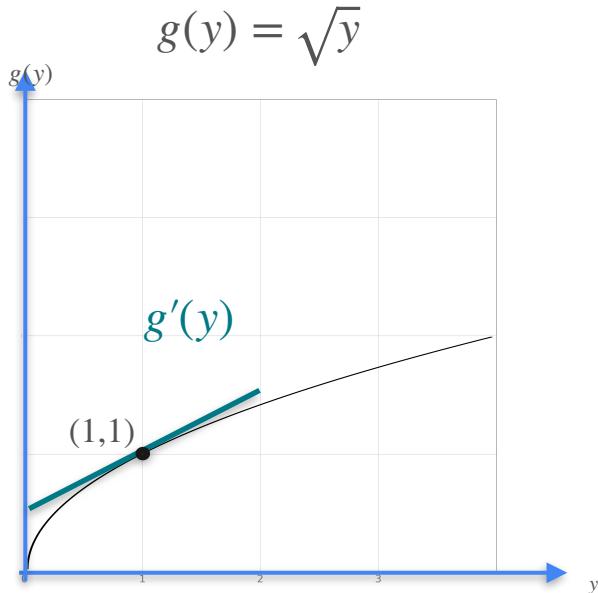
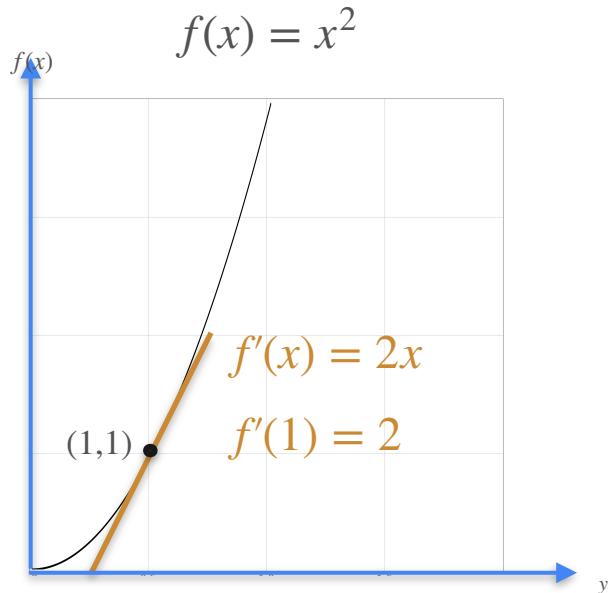


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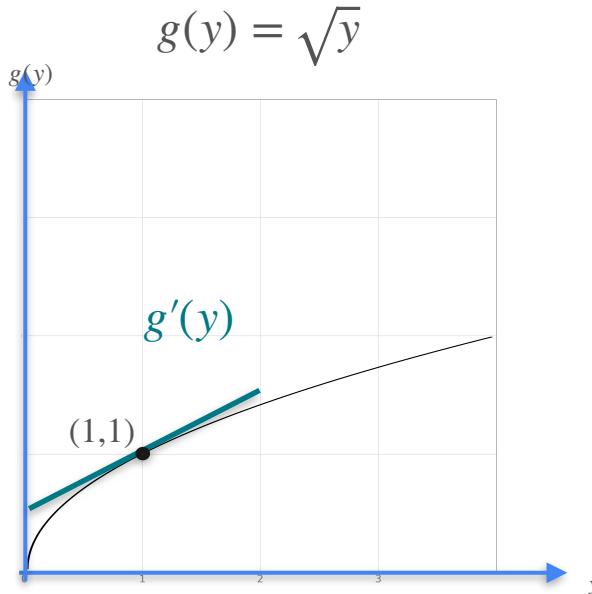
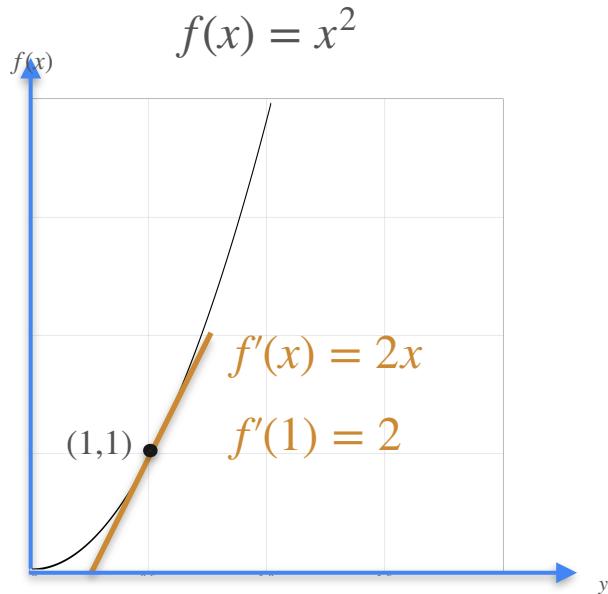
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# Derivative of the Inverse



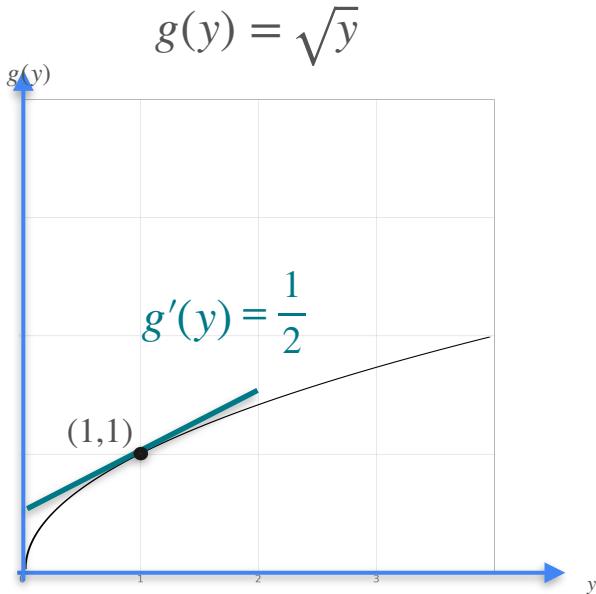
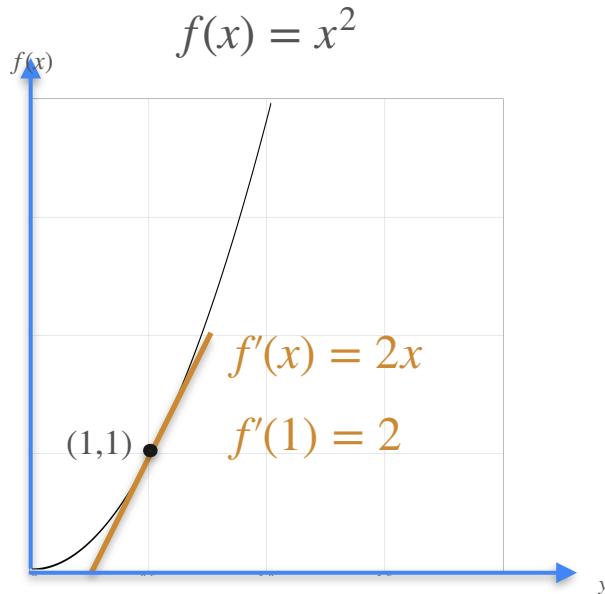
$$g'(y) = \frac{1}{f'(x)}$$

at the point  $(1,1)$

$$f(1) = 1 \qquad g(1) = 1$$

$$g'(1) = \frac{1}{f'(1)} = \frac{1}{2}$$

# Derivative of the Inverse



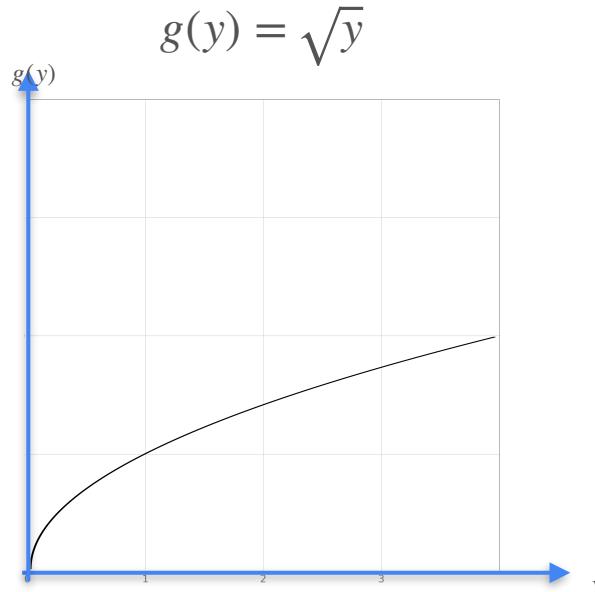
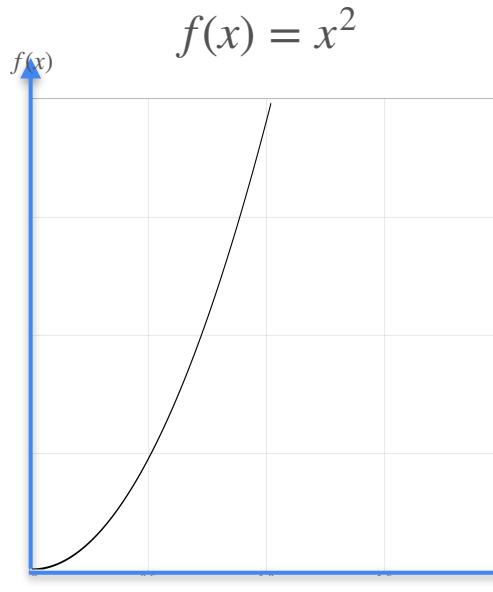
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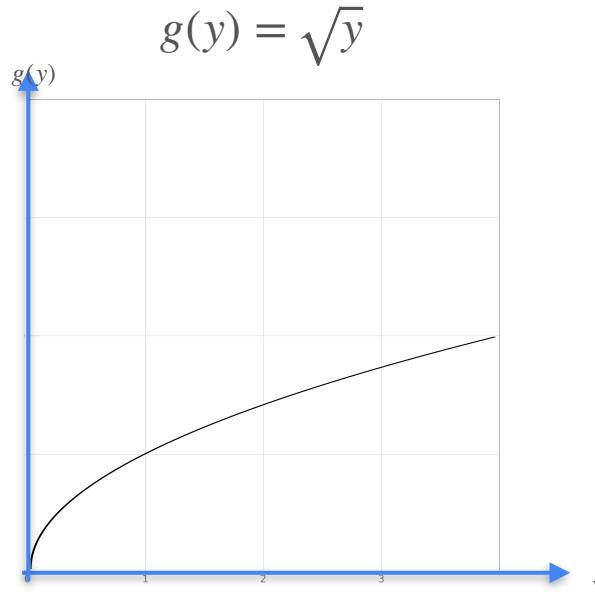
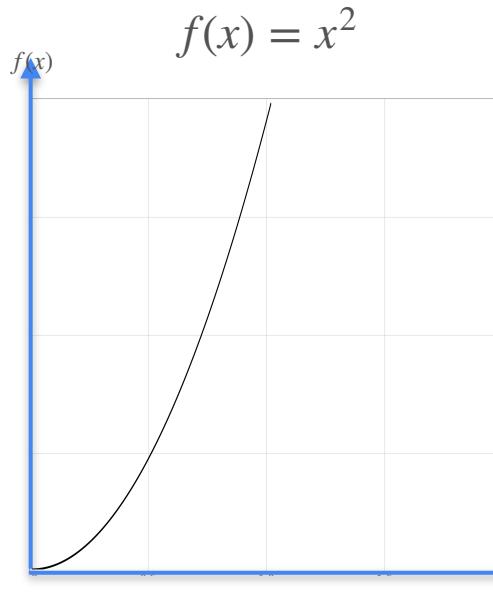
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# Derivative of the Inverse



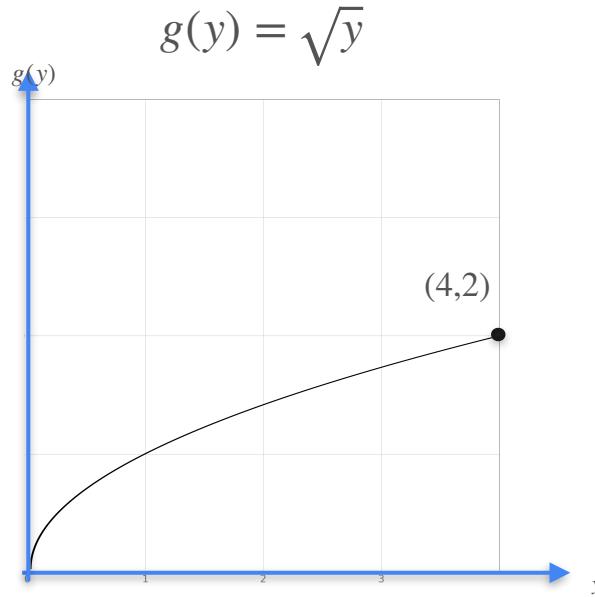
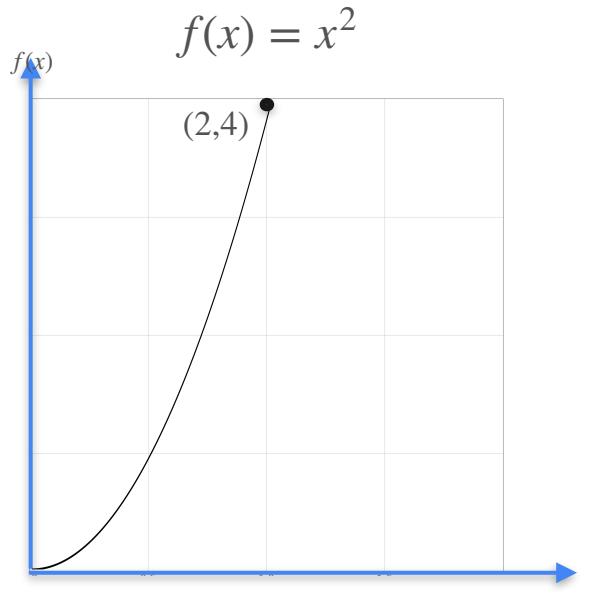
$$g'(y) = \frac{1}{f'(x)}$$

at the point (2,4)

$$f(2) = 4$$

$$g(4) = 2$$

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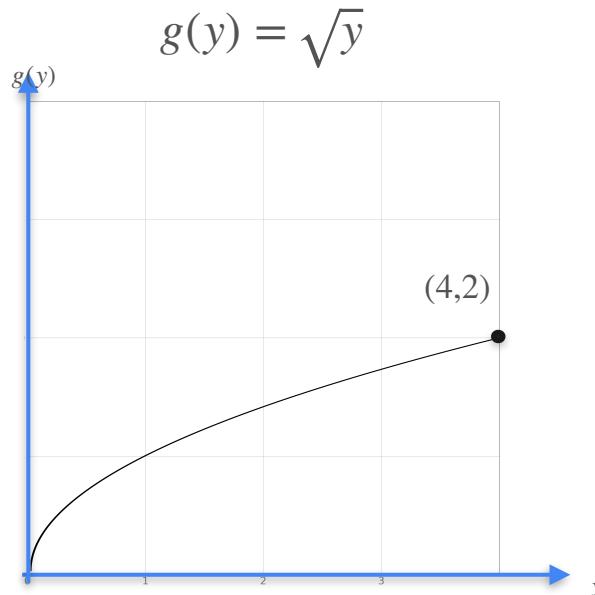
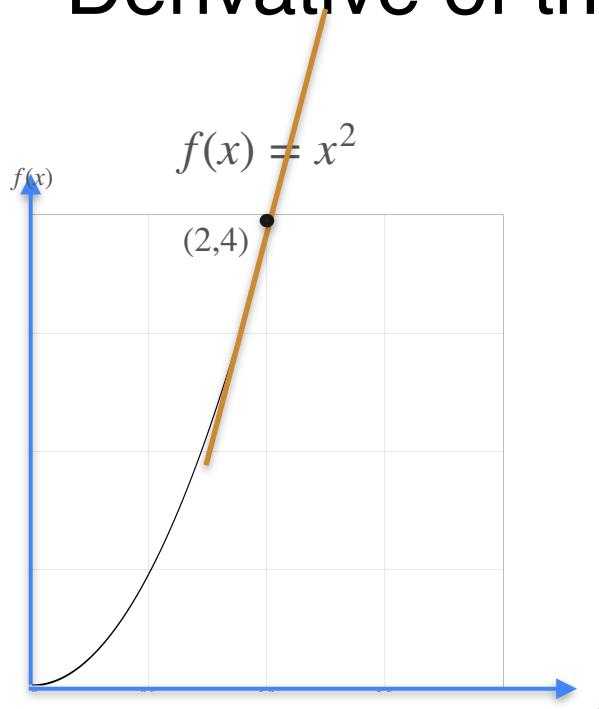
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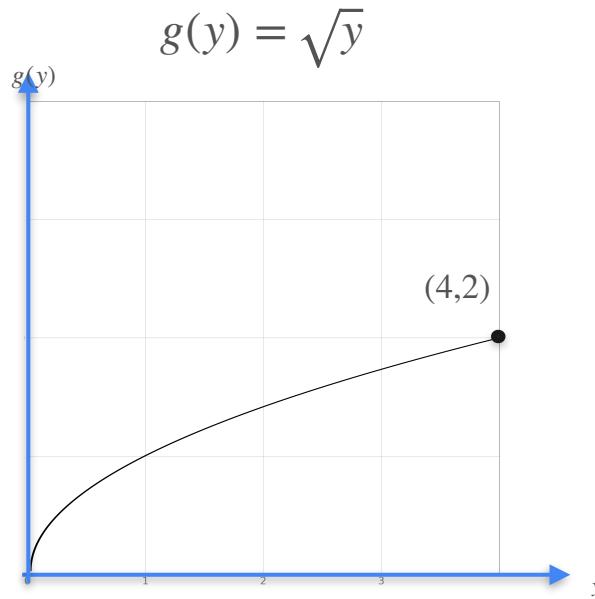
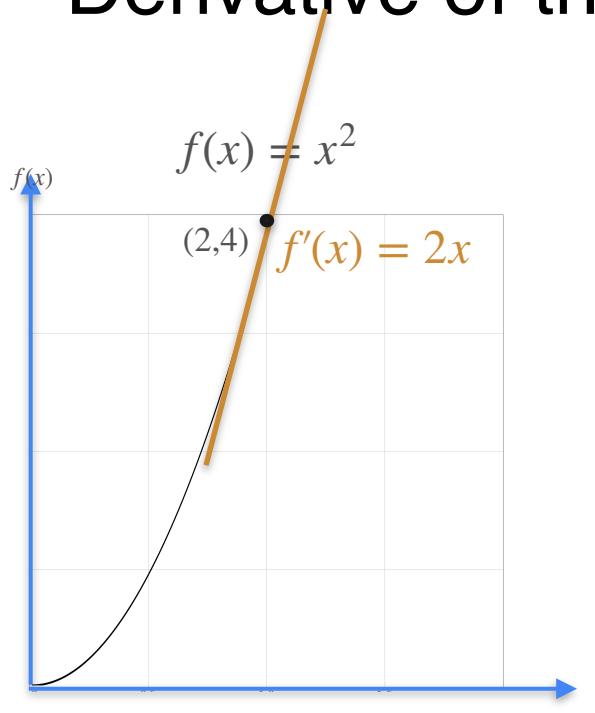
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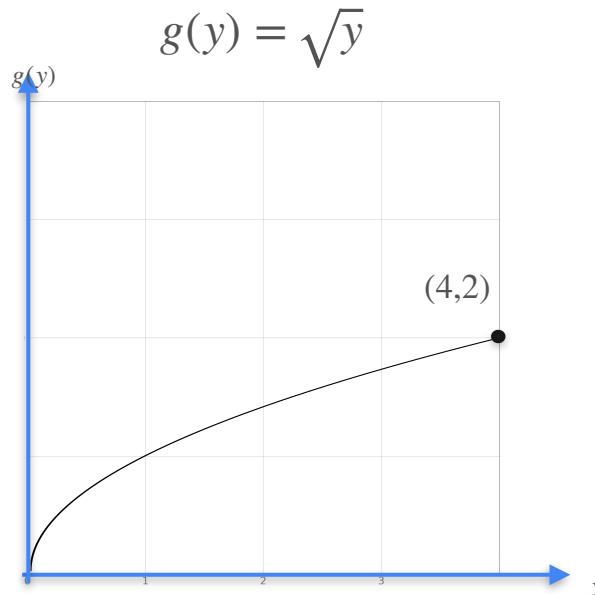
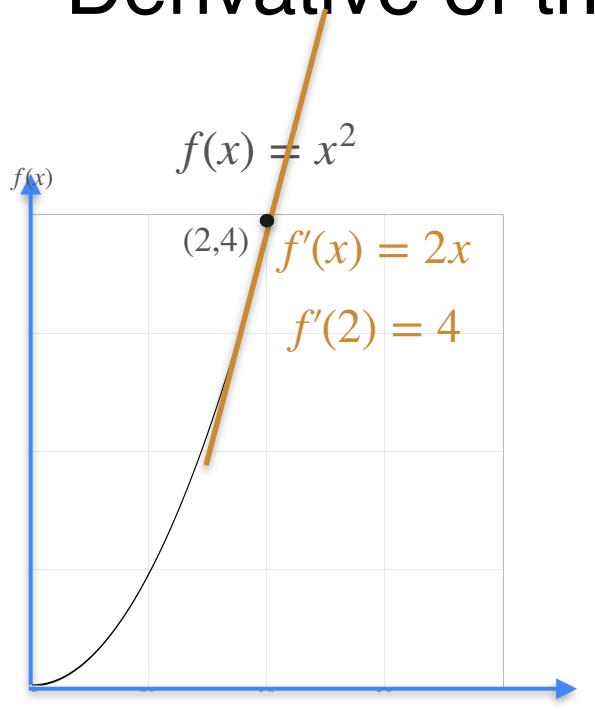


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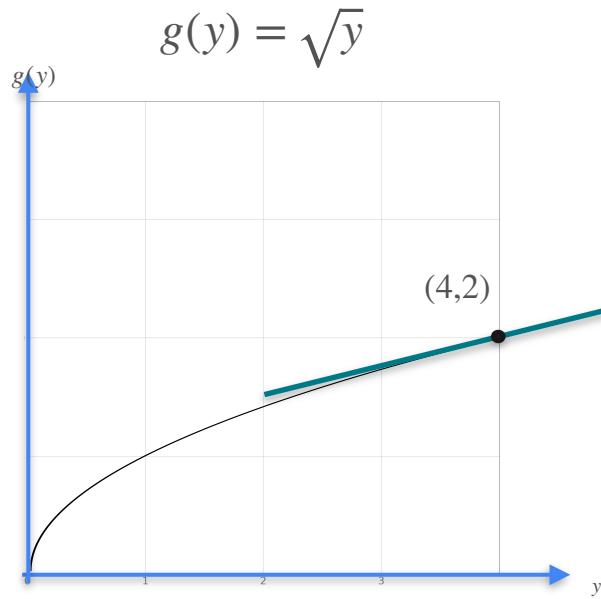
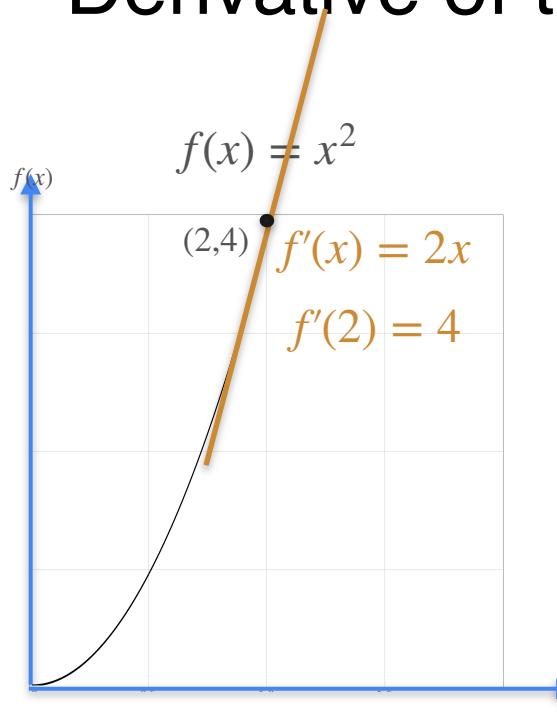
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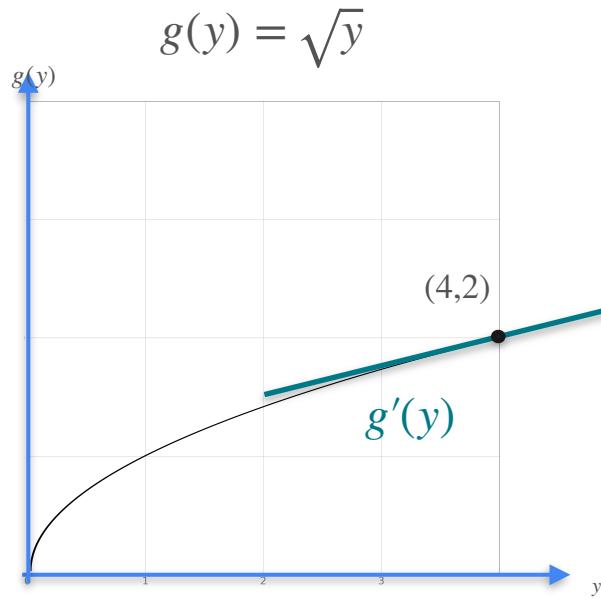
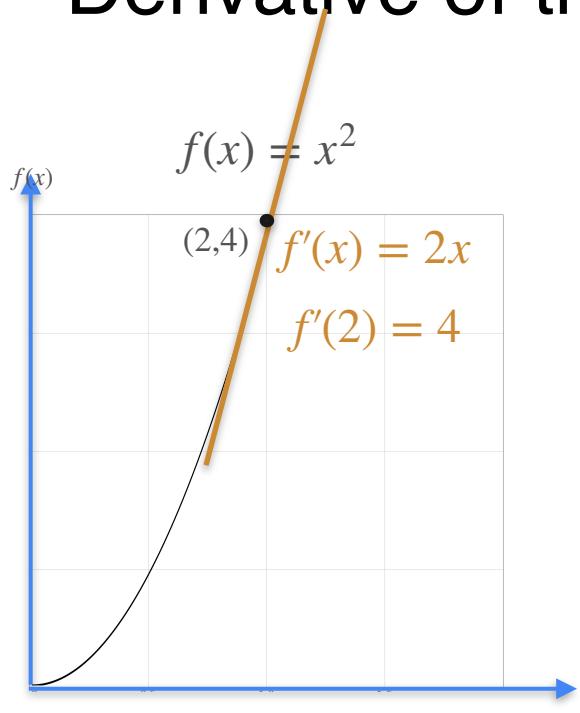


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at the point  $(2,4)$

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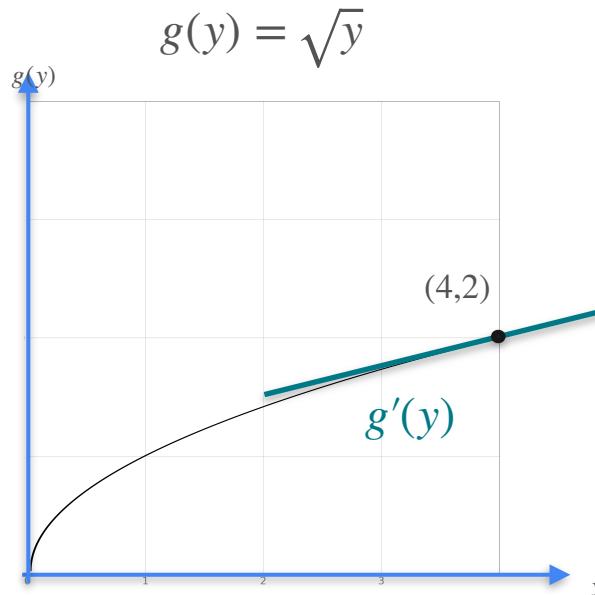
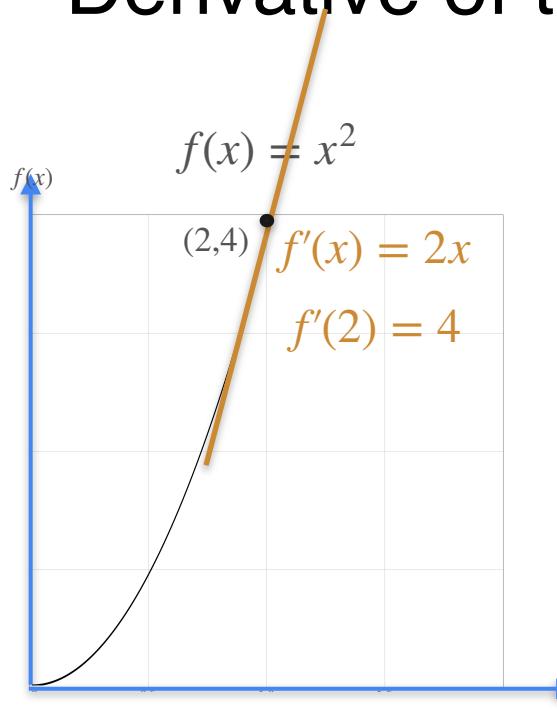
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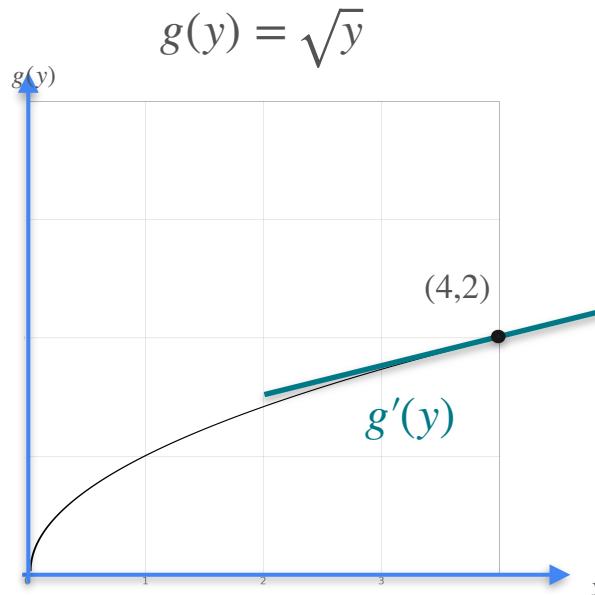
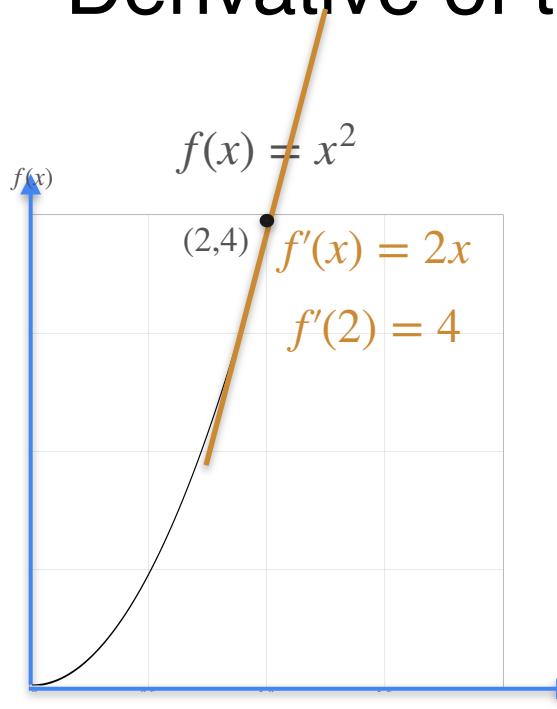
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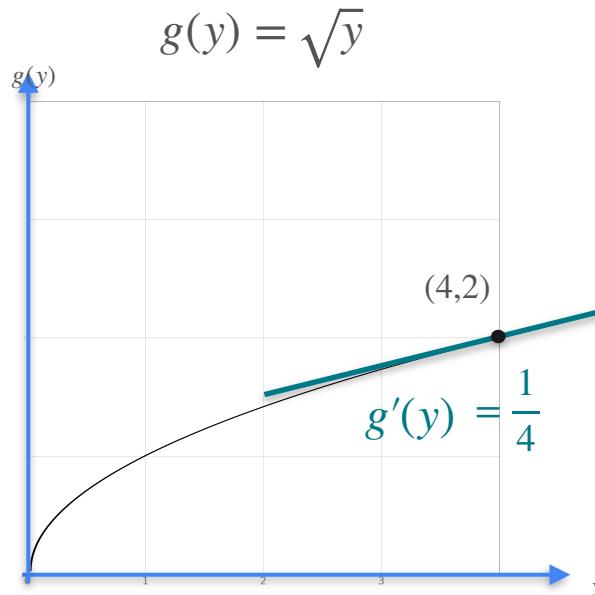
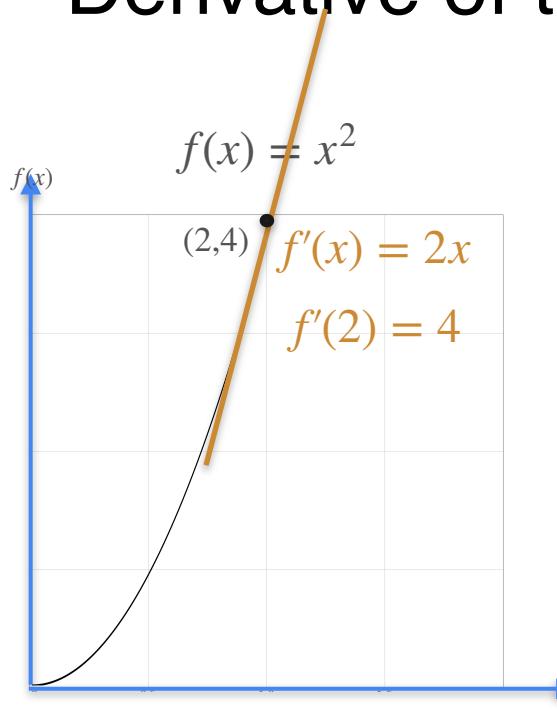


$$g'(y) = \frac{1}{f'(x)}$$

at the point  $(2,4)$

$$f(2) = 4 \quad g(4) = 2$$
$$g'(4) = \frac{1}{f'(2)} = \frac{1}{4}$$

# Derivative of the Inverse



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at the point  $(2,4)$

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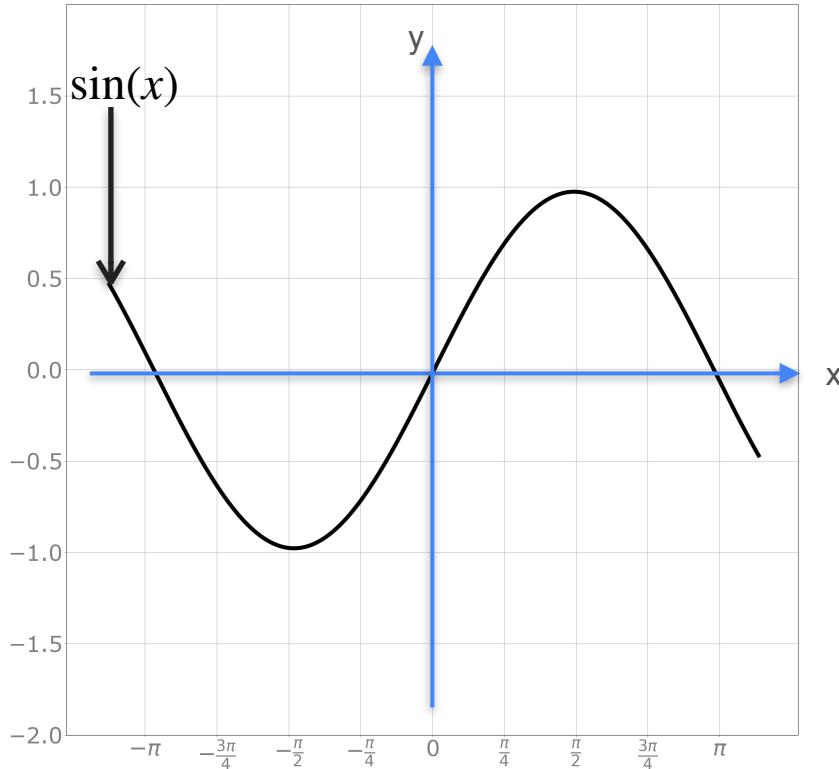
# Derivatives and Optimization

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## Derivative of trigonometric functions

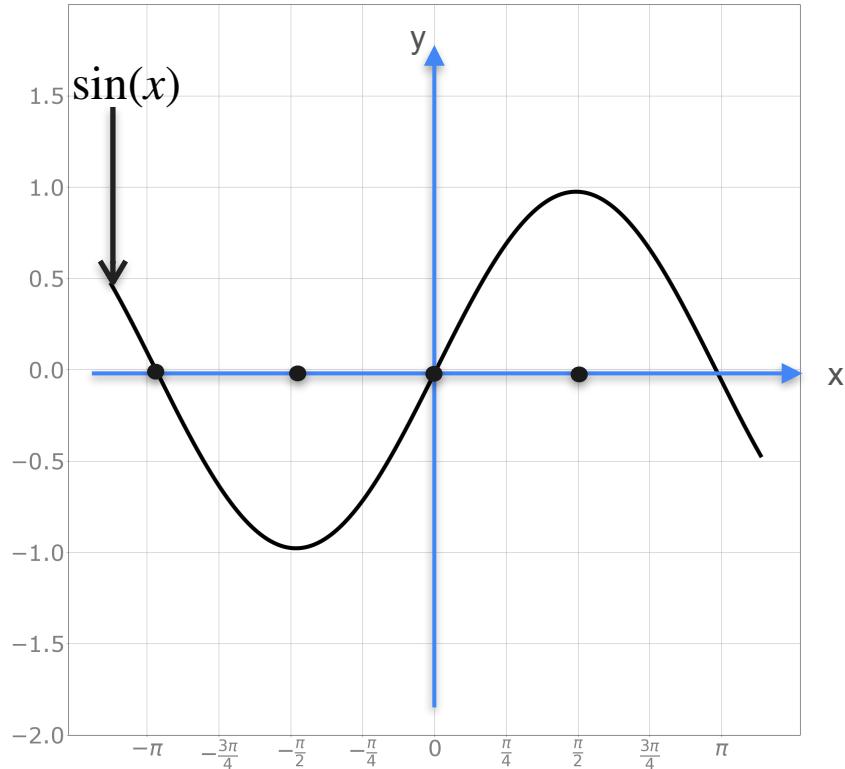
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Sine  $y = f(x) = \sin(x)$

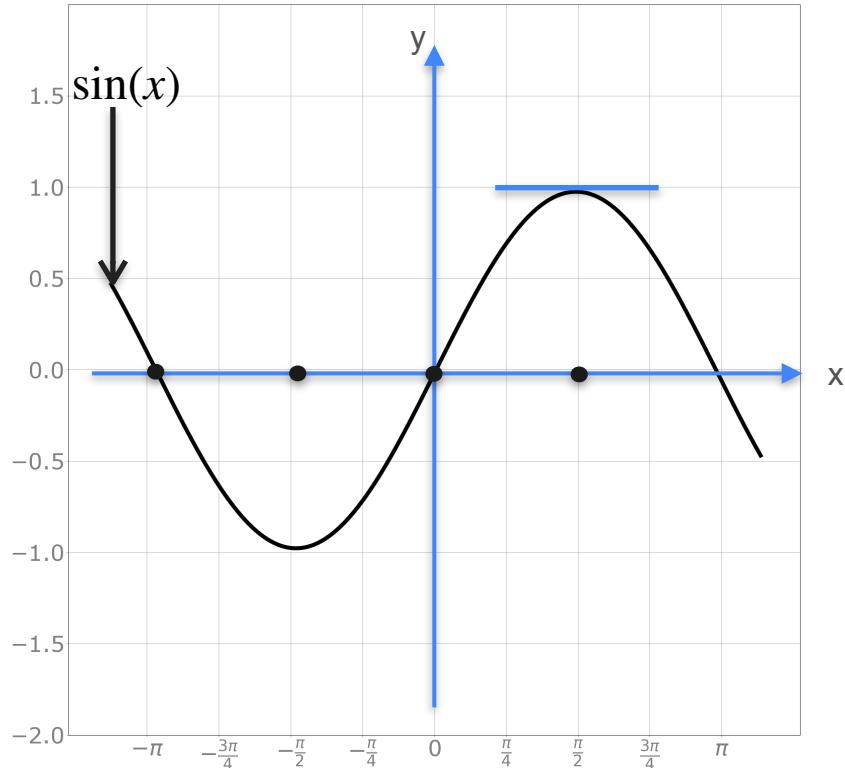
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Sine  $y = f(x) = \sin(x)$

$x$	$\pi/2$	$-\pi/2$	0	$-\pi$
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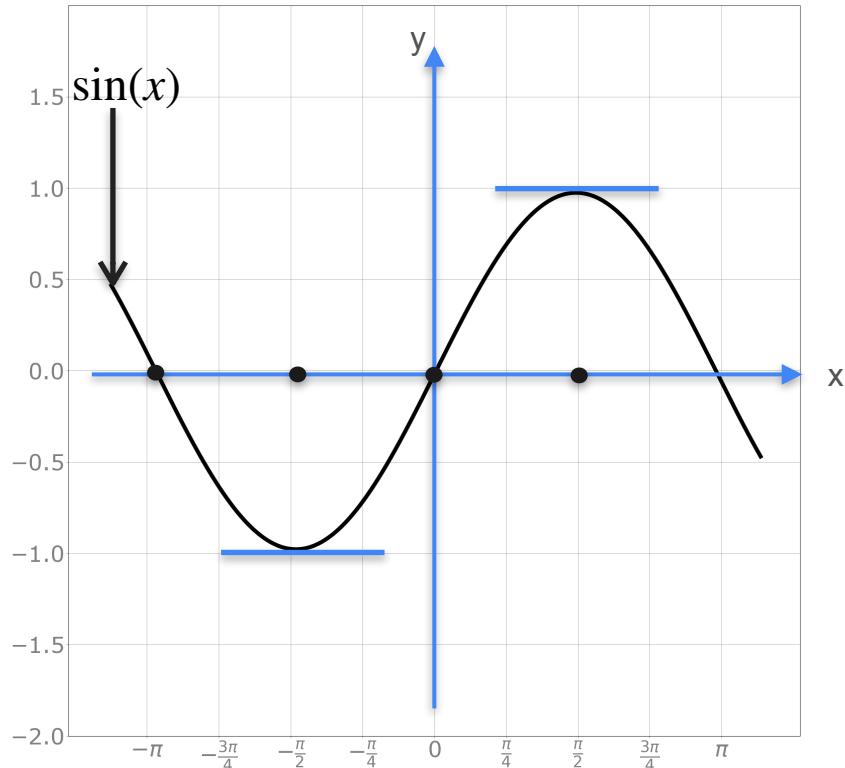
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$x$	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0			

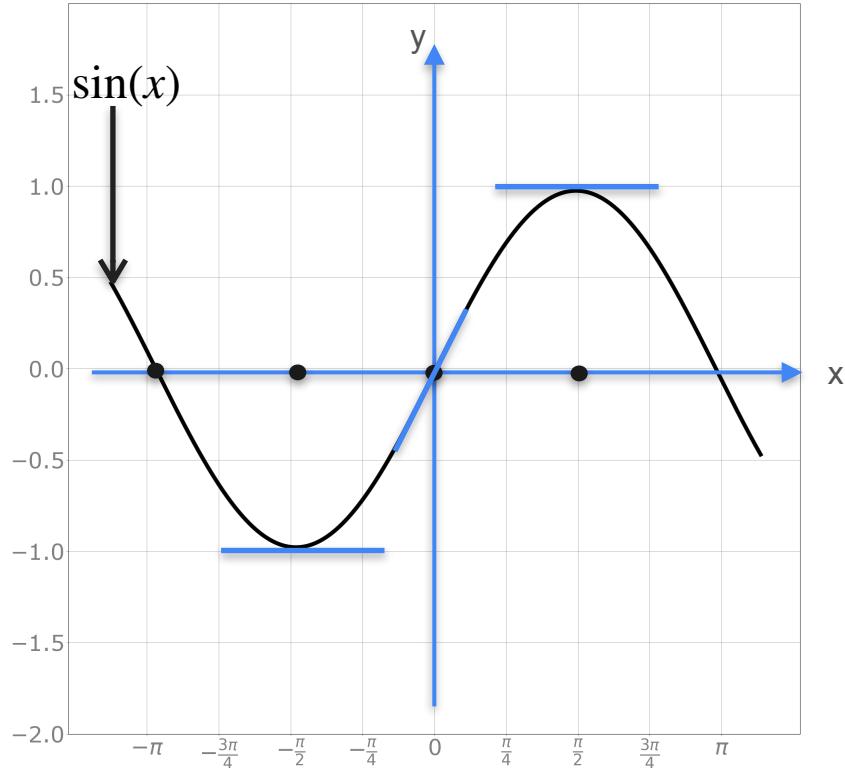
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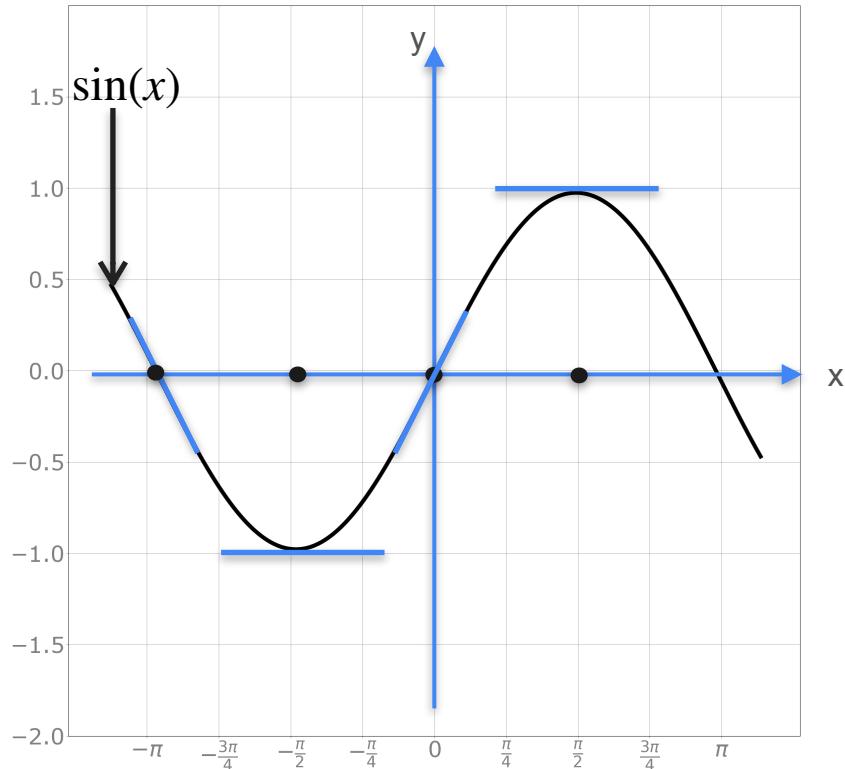
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$x$	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	

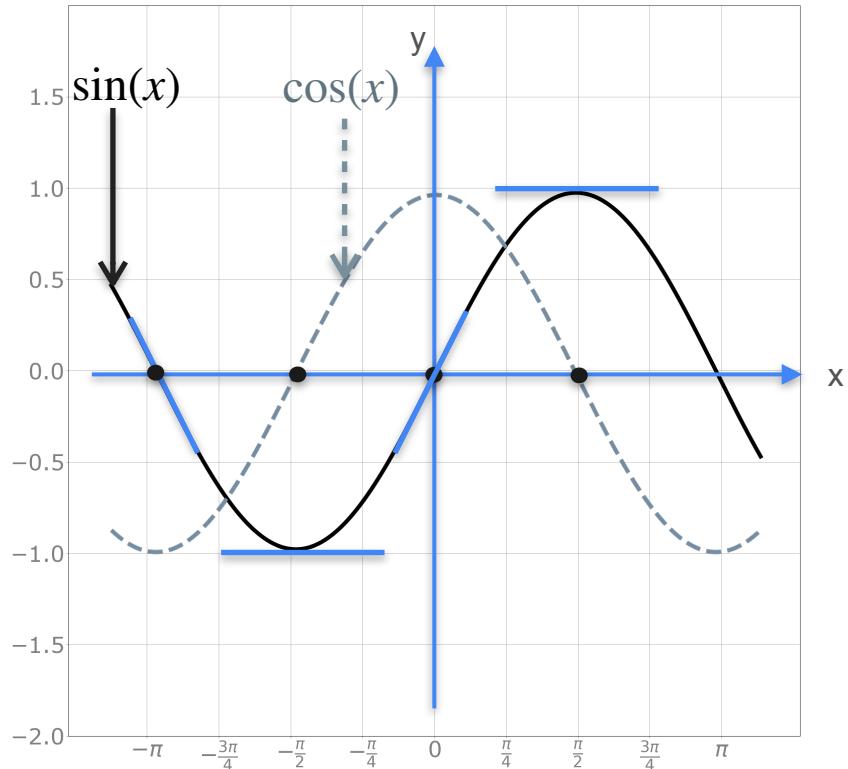
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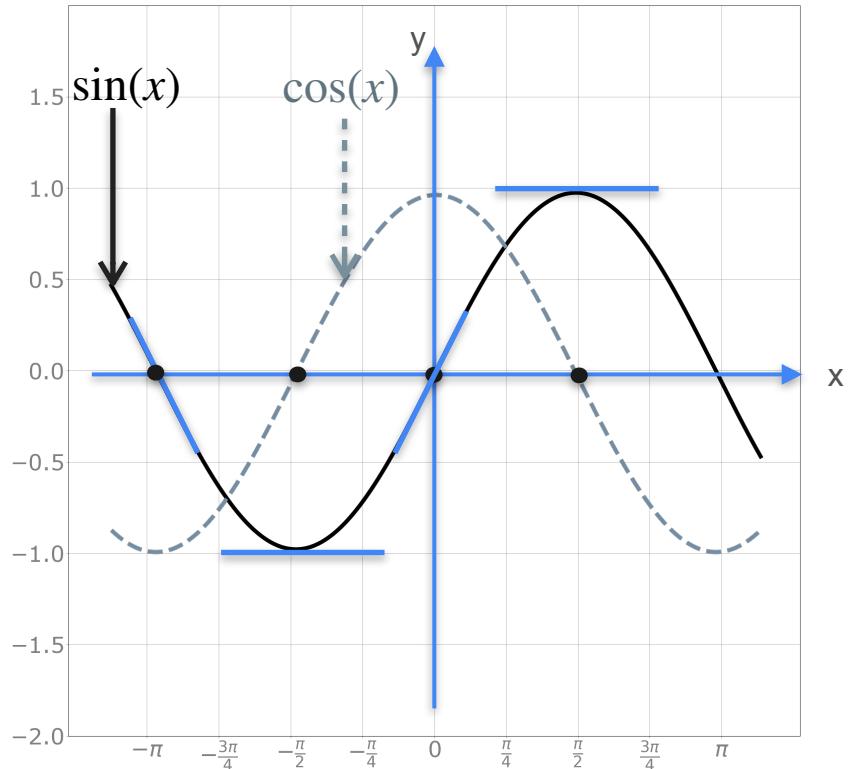
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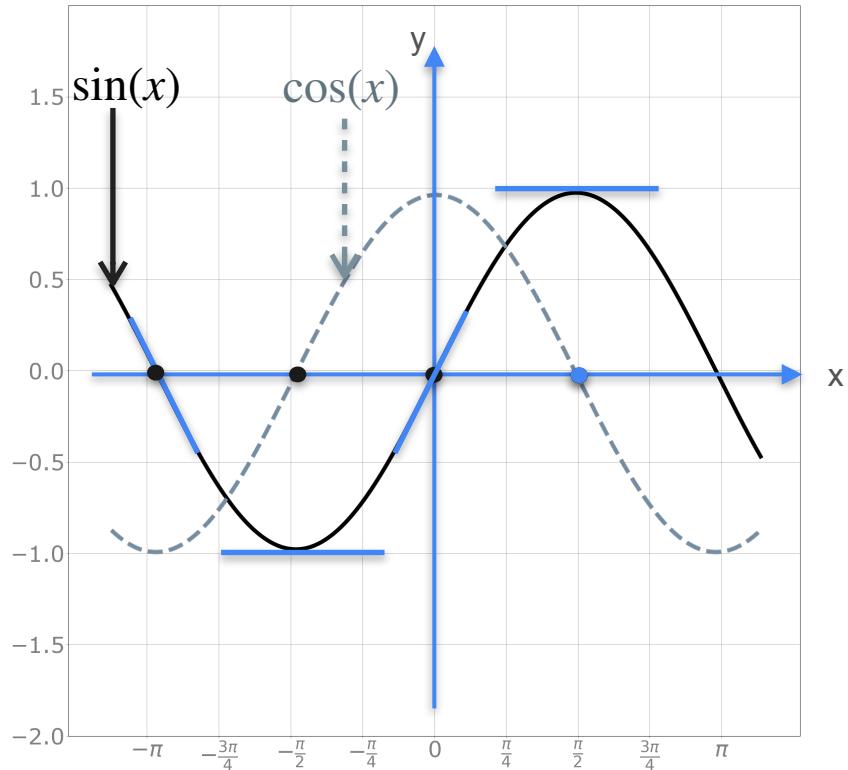
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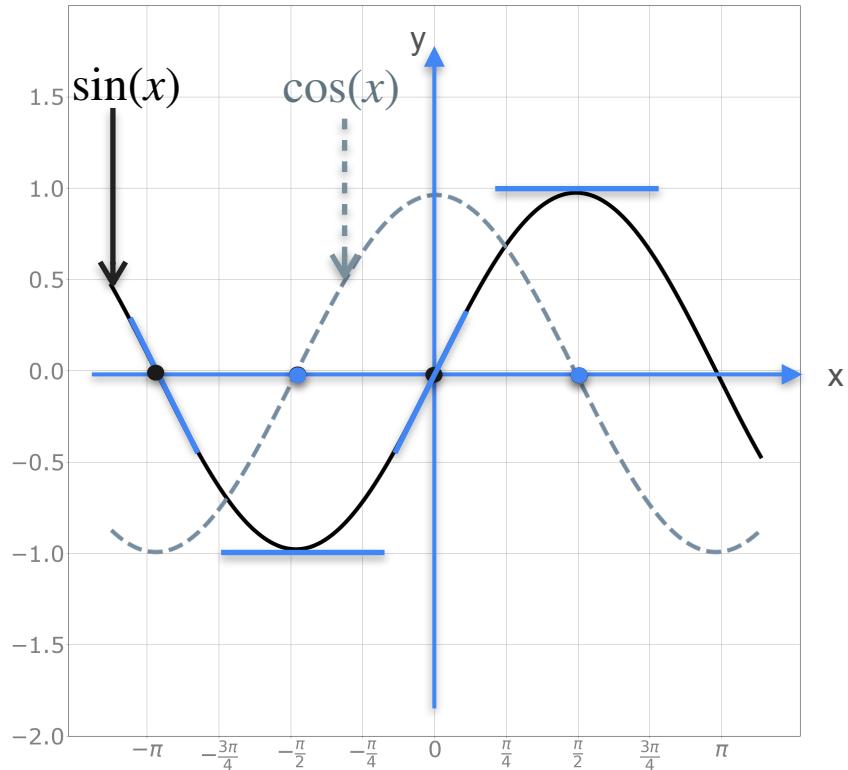
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Slope	0	0	1	-1
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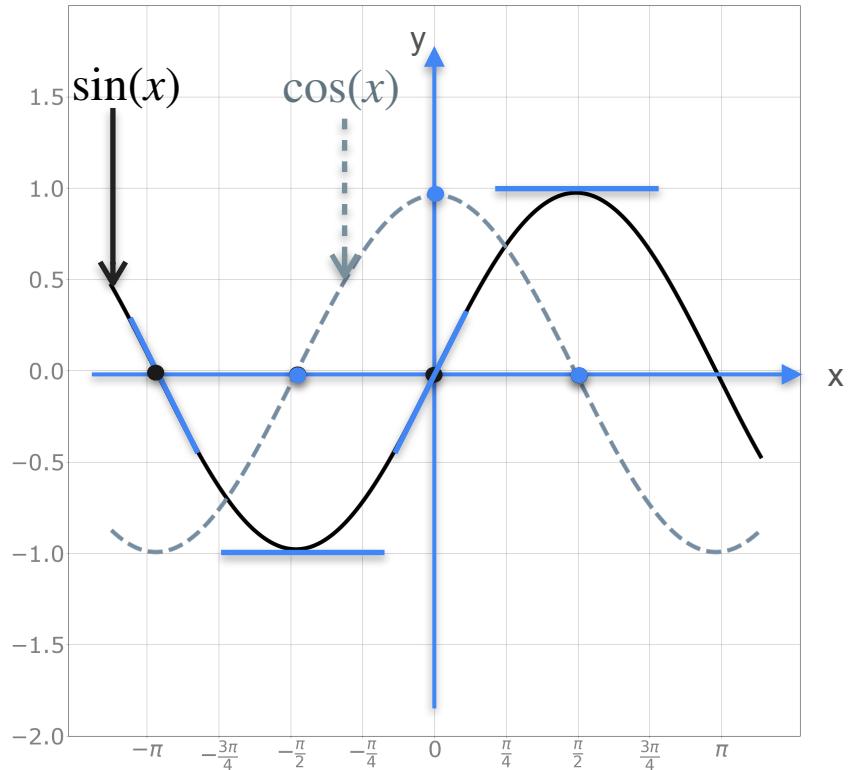
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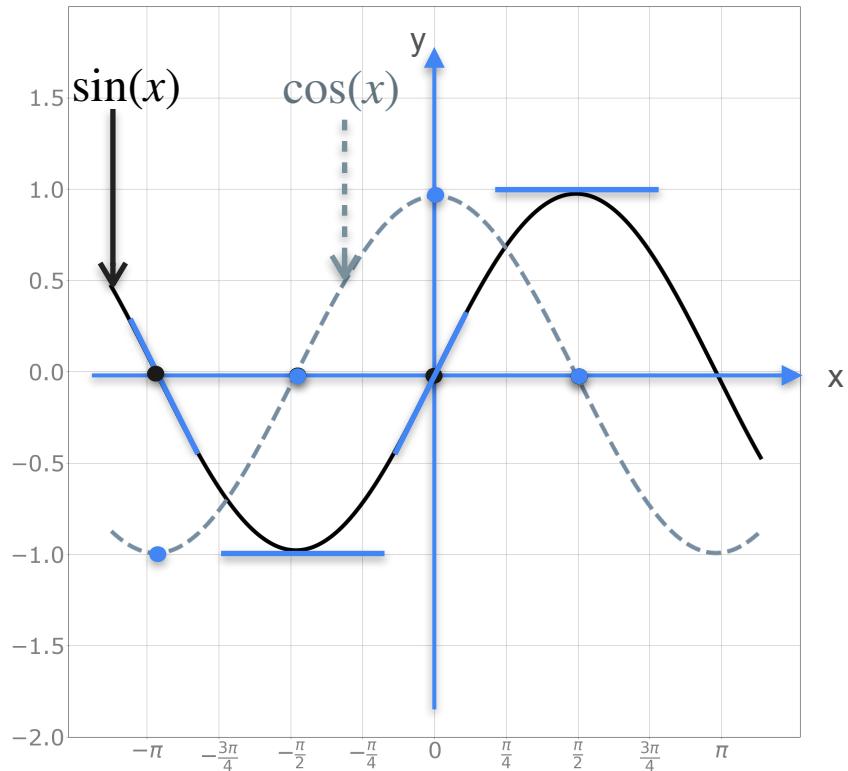
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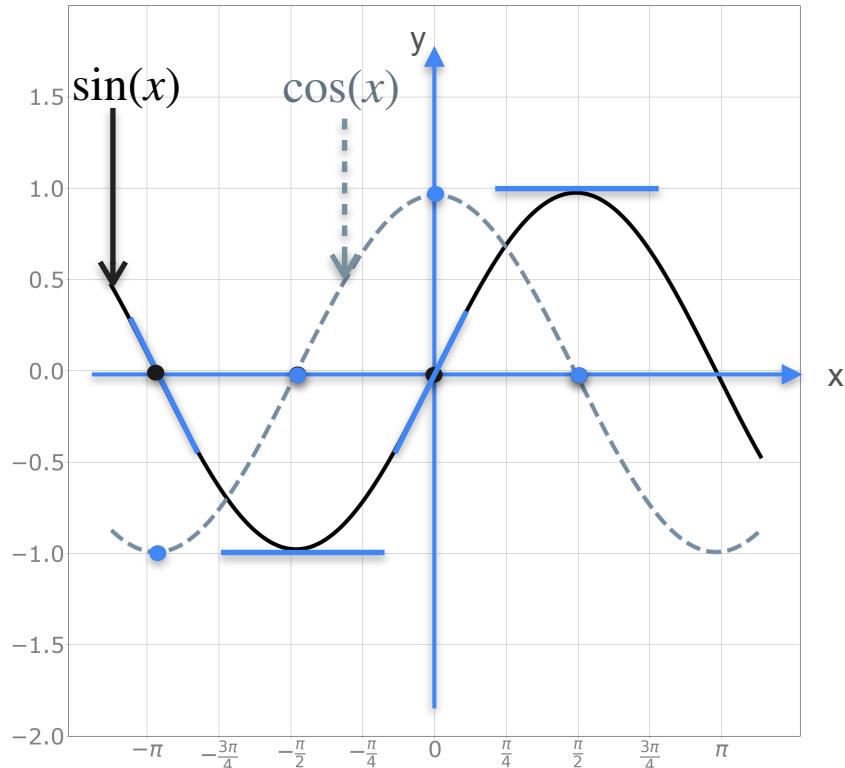
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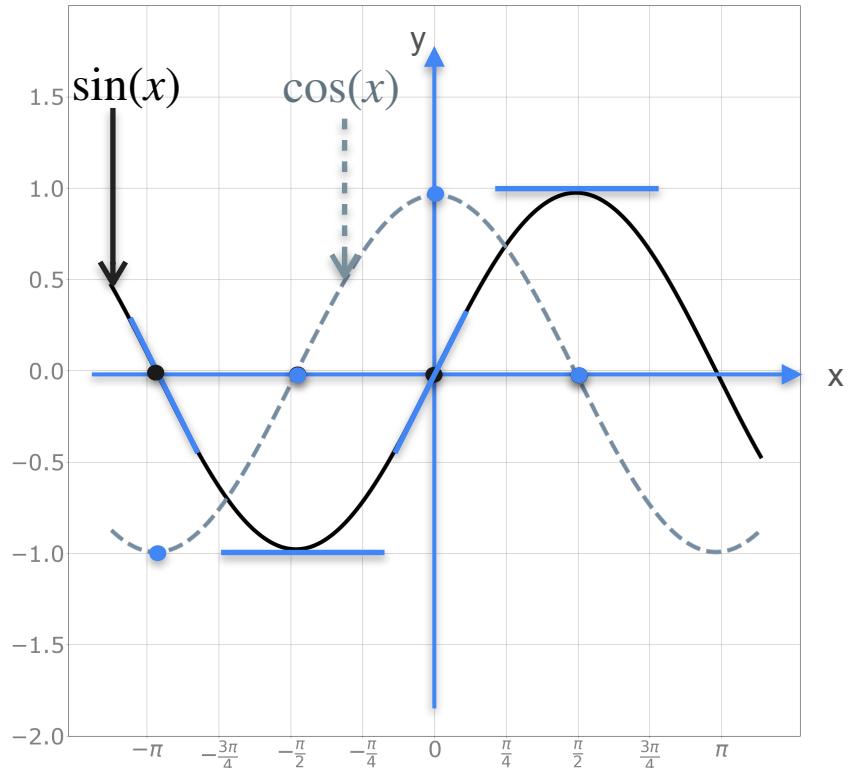


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$$f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$

# Derivative of Trigonometric Functions



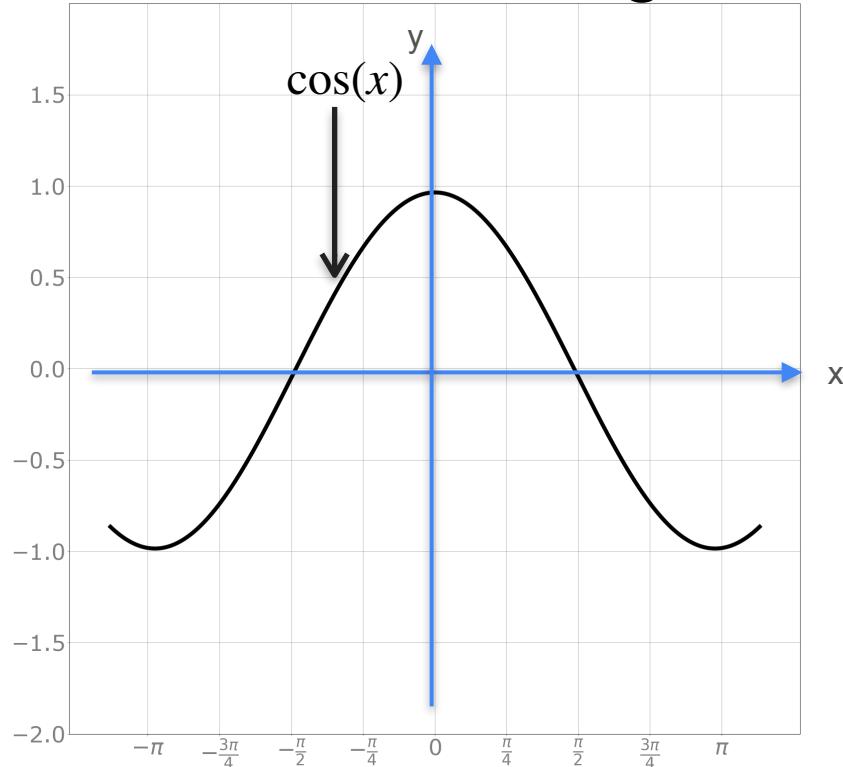
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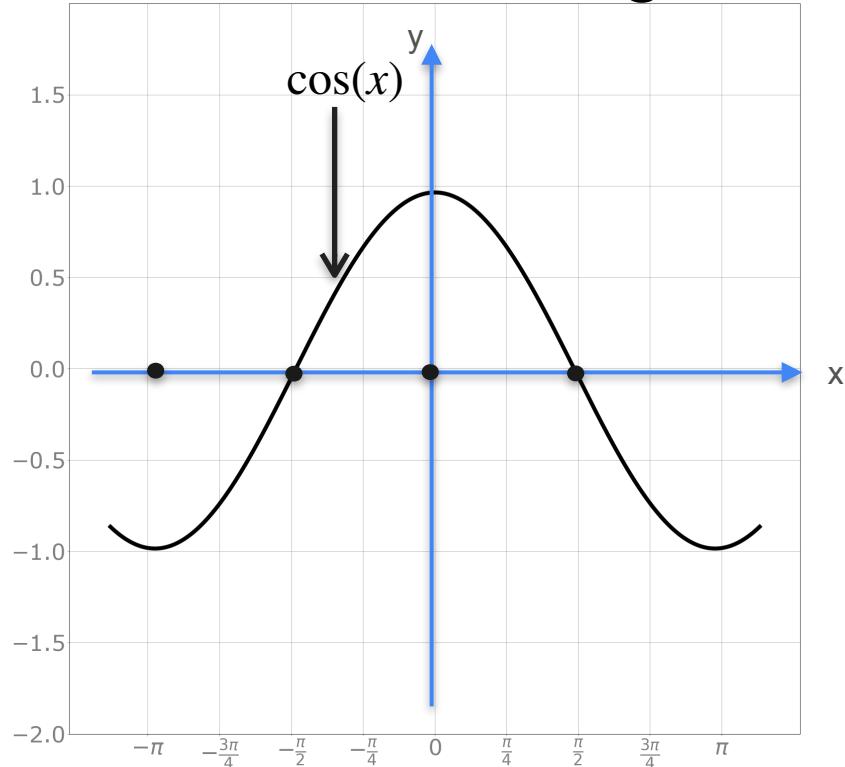
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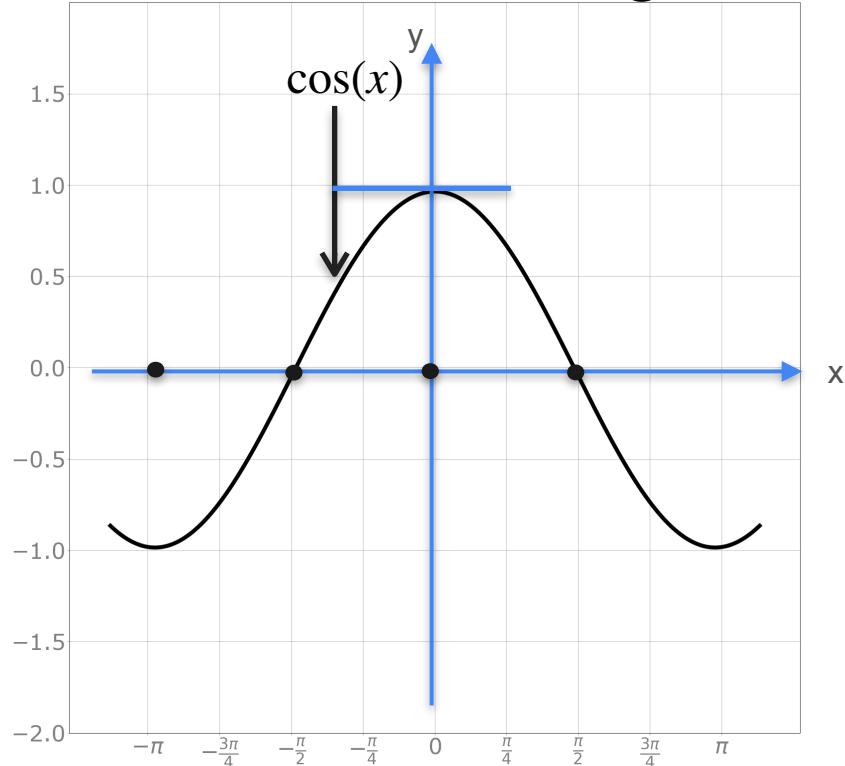
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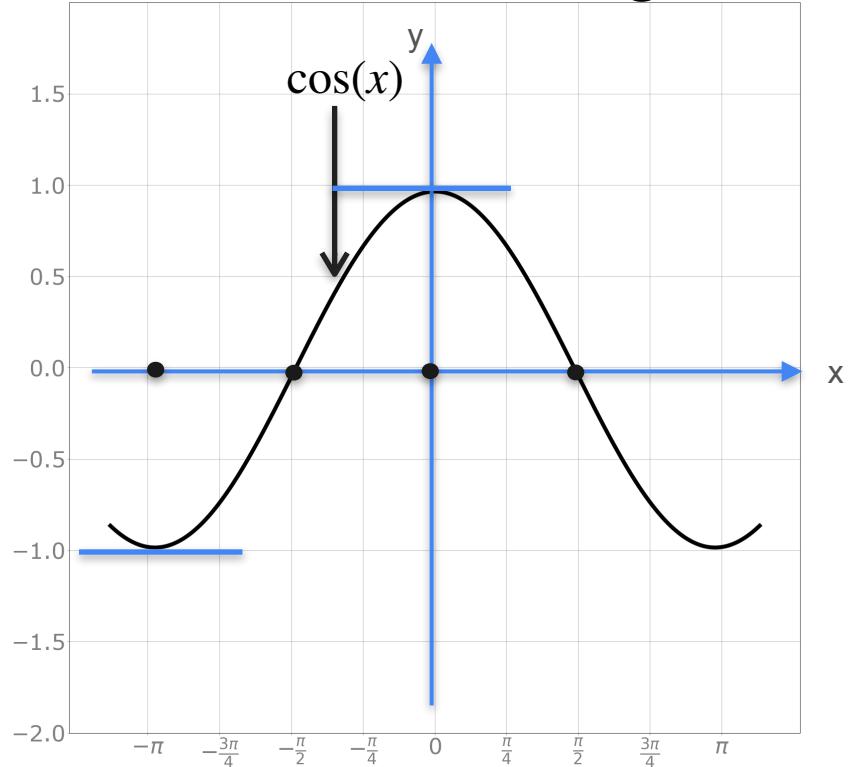
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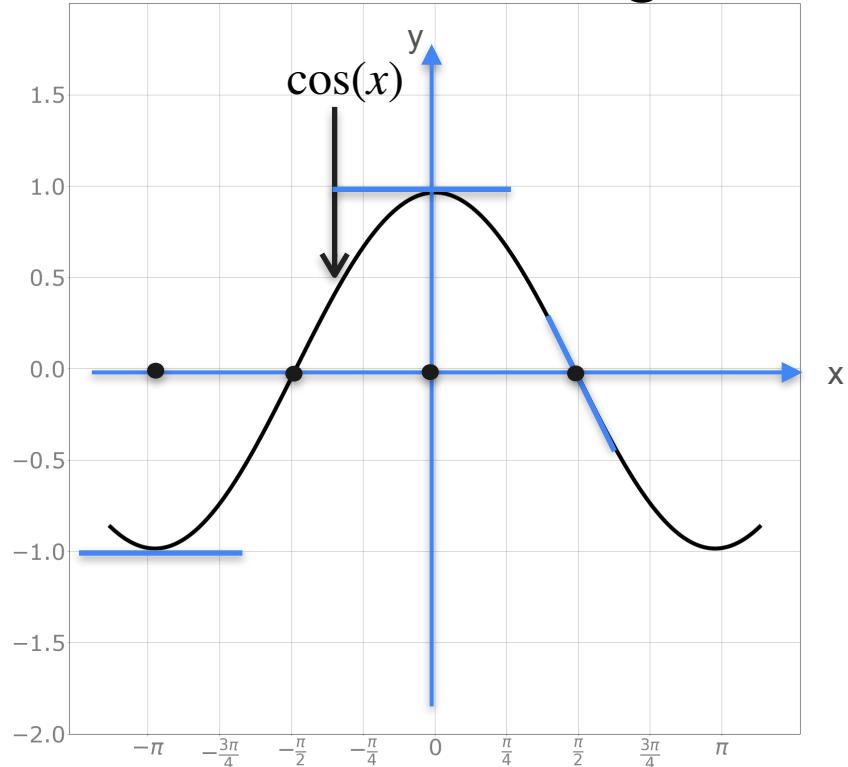
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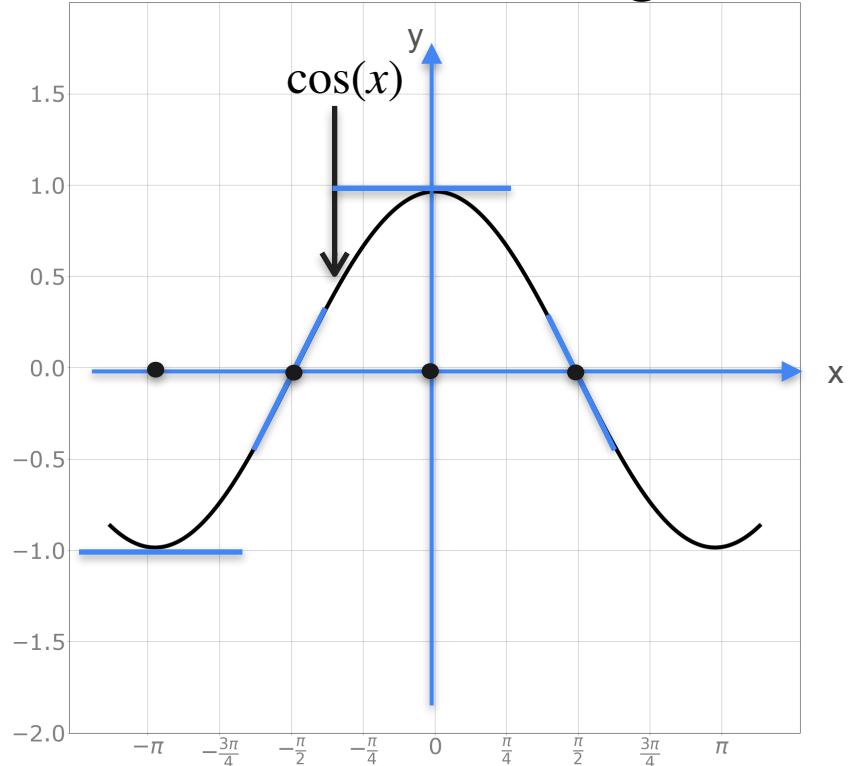
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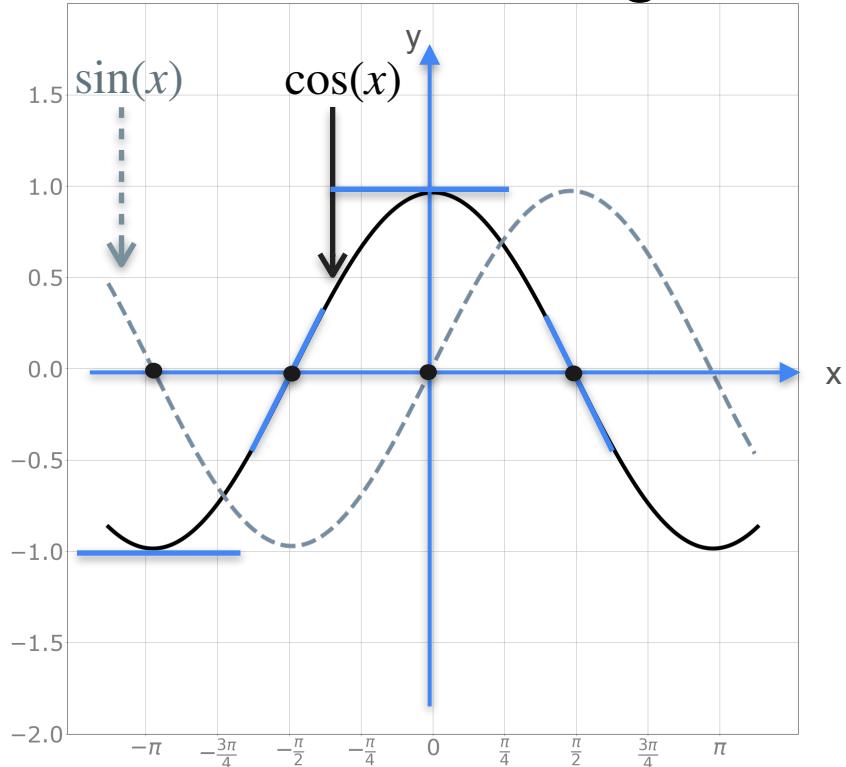
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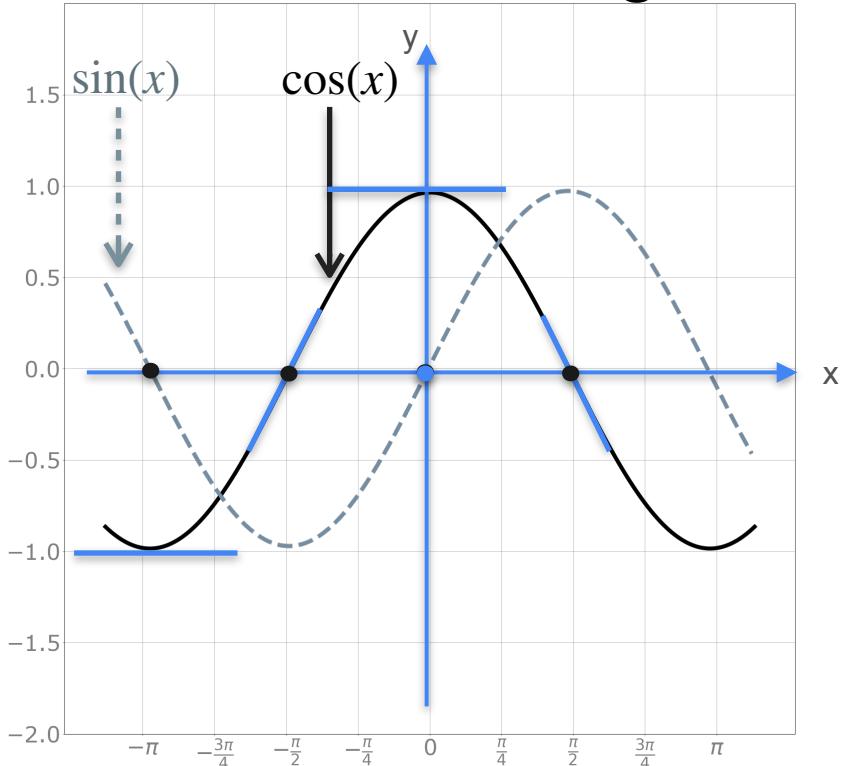
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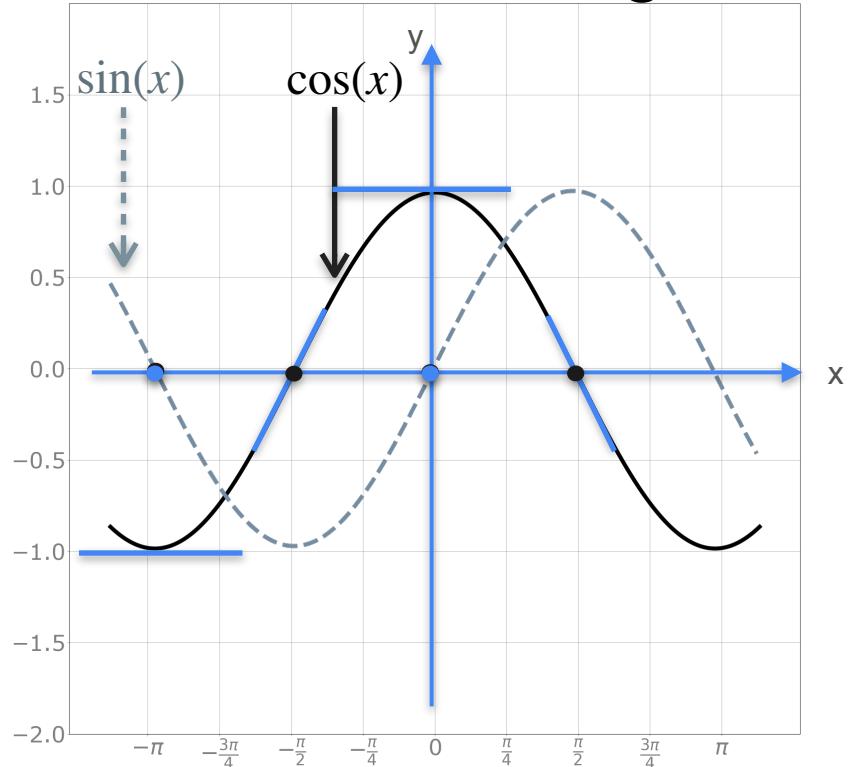
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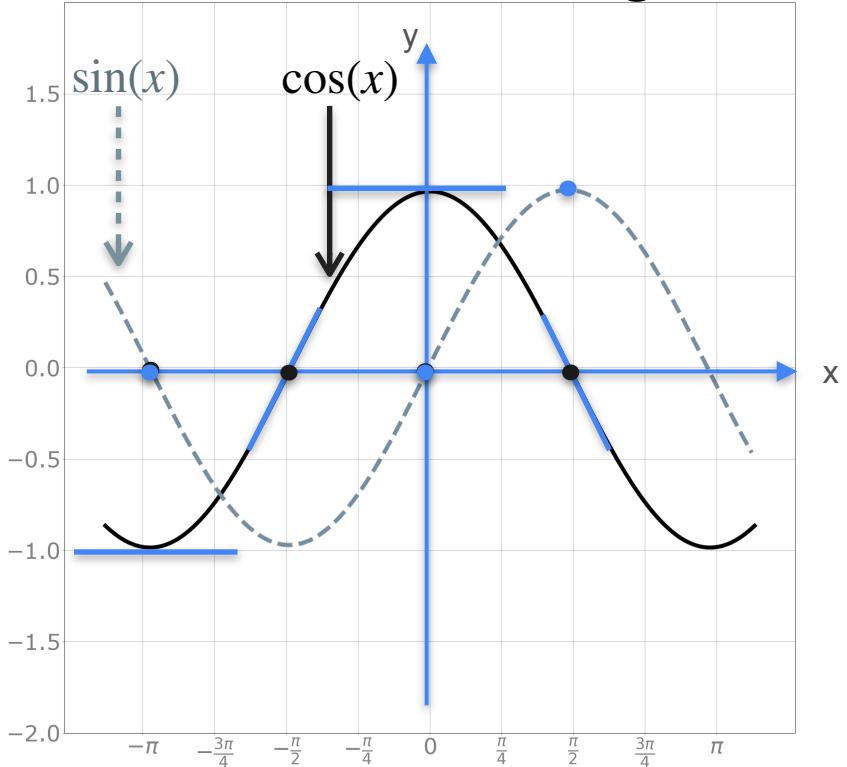
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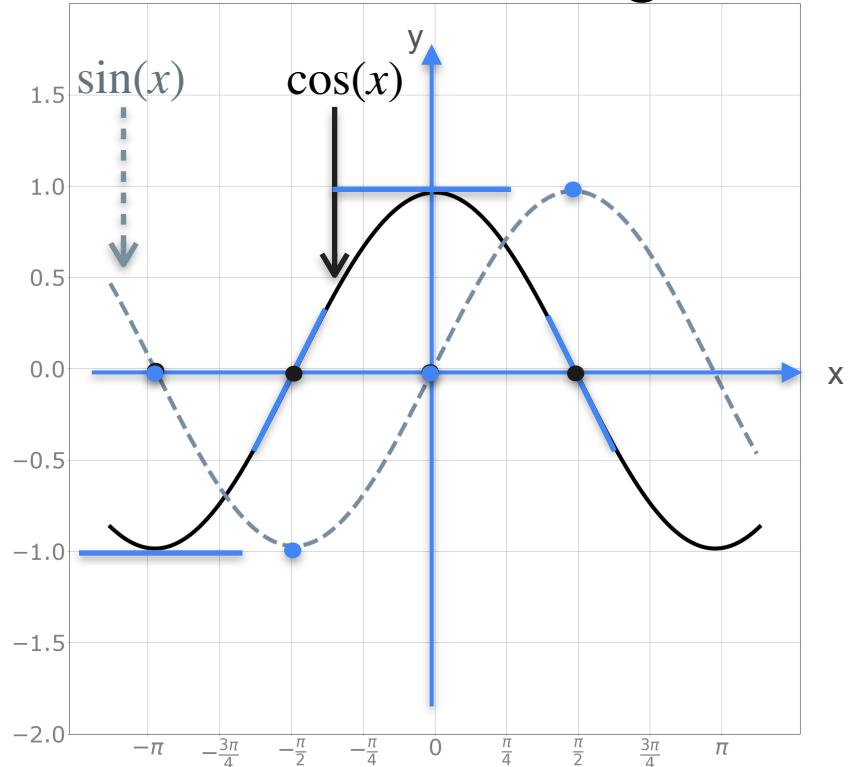
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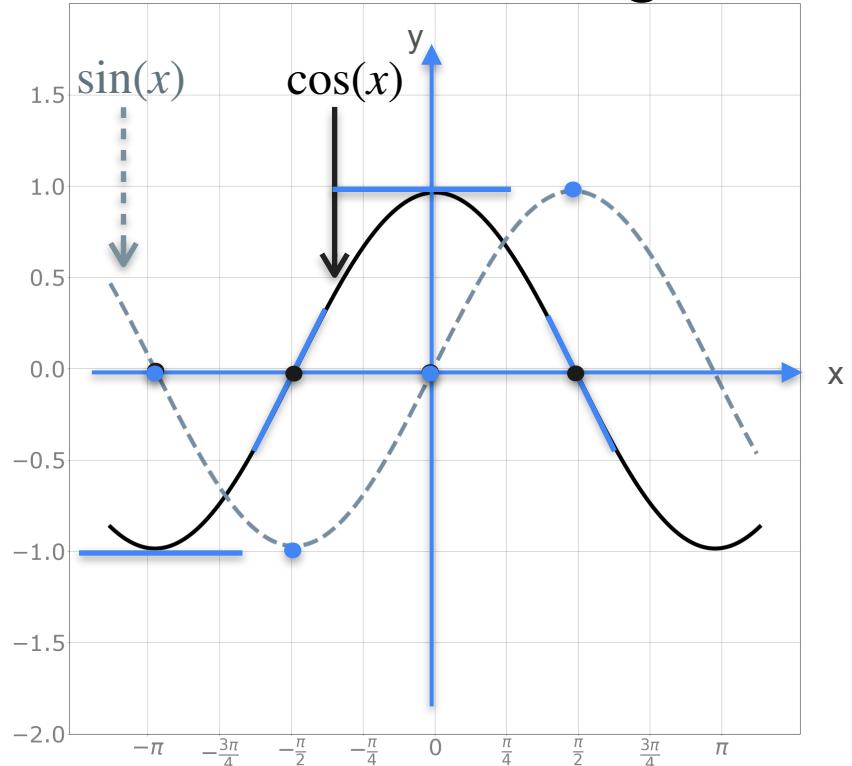
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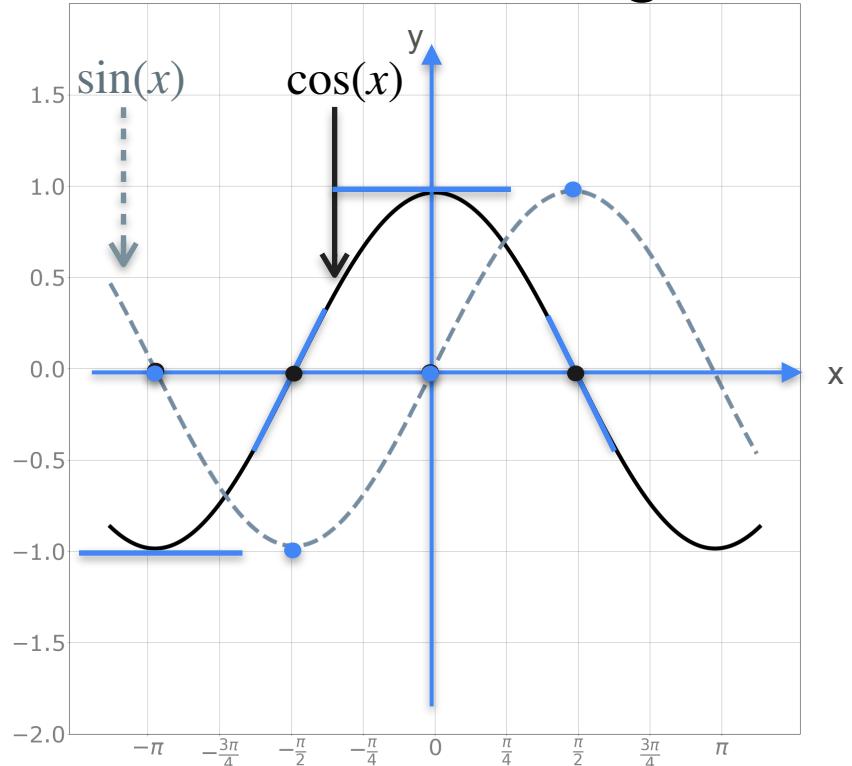


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$$f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$$

# Derivative of Trigonometric Functions



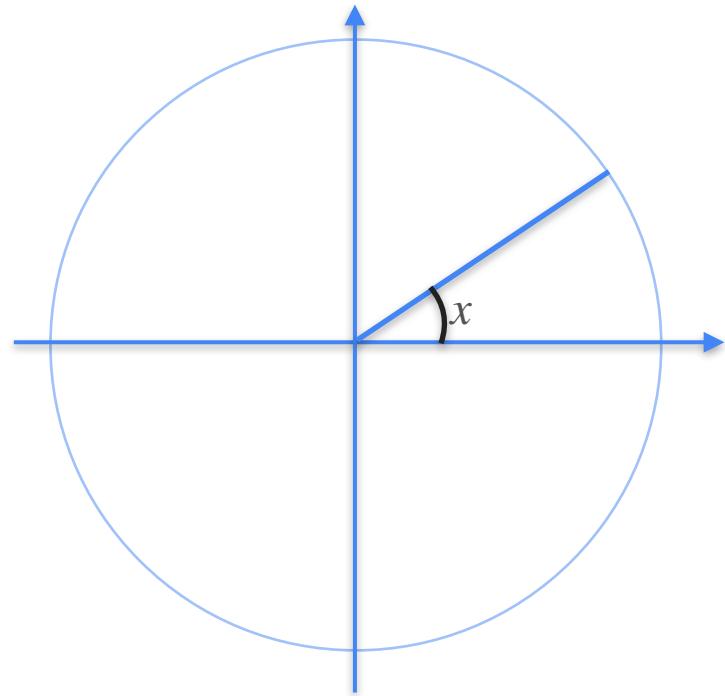
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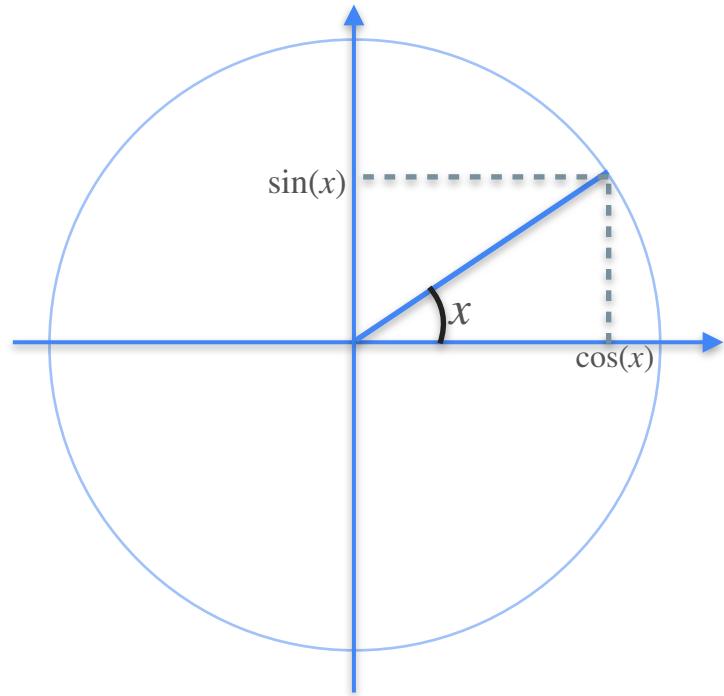
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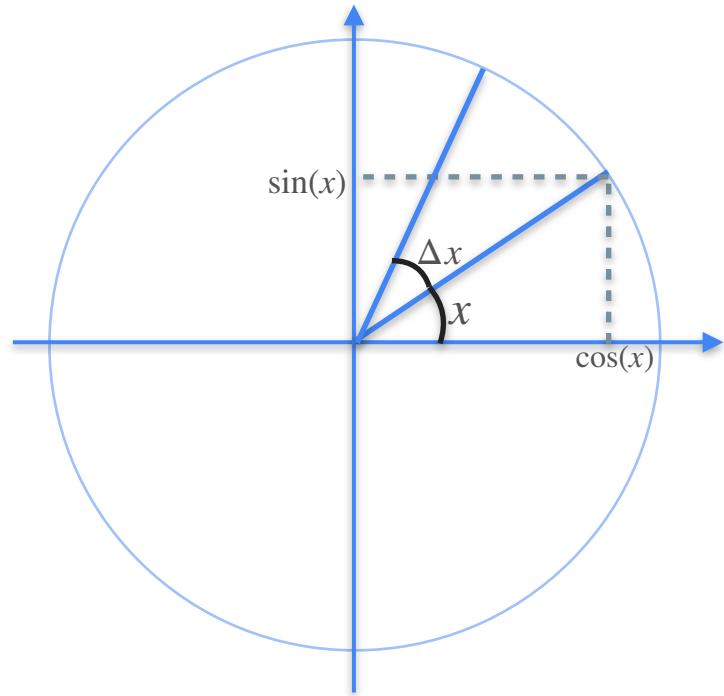
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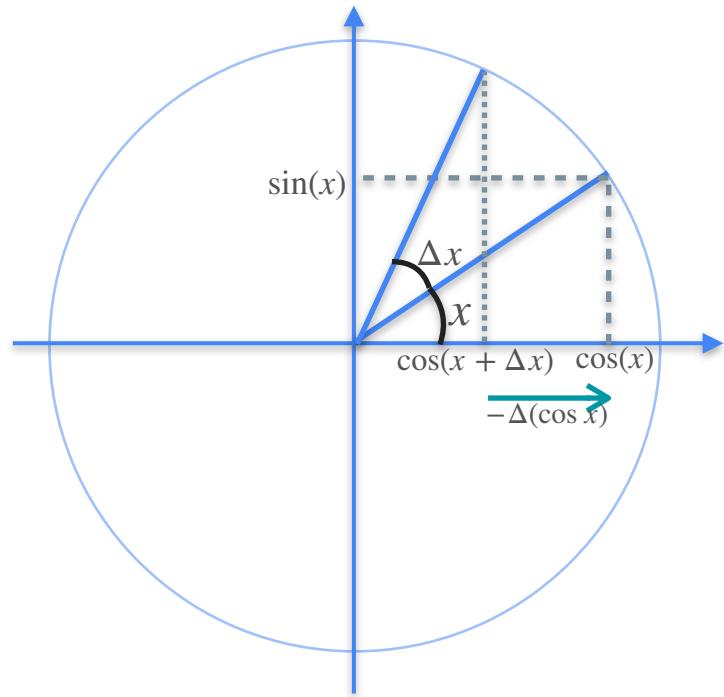
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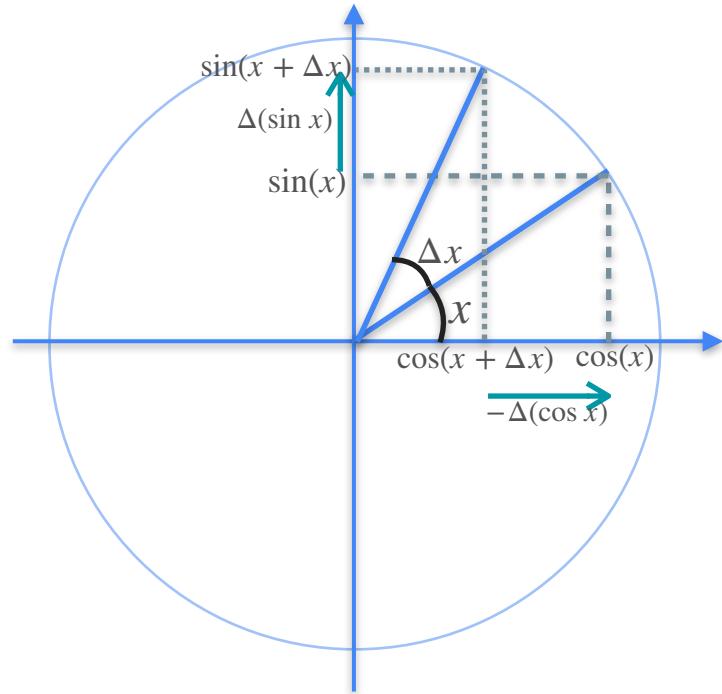
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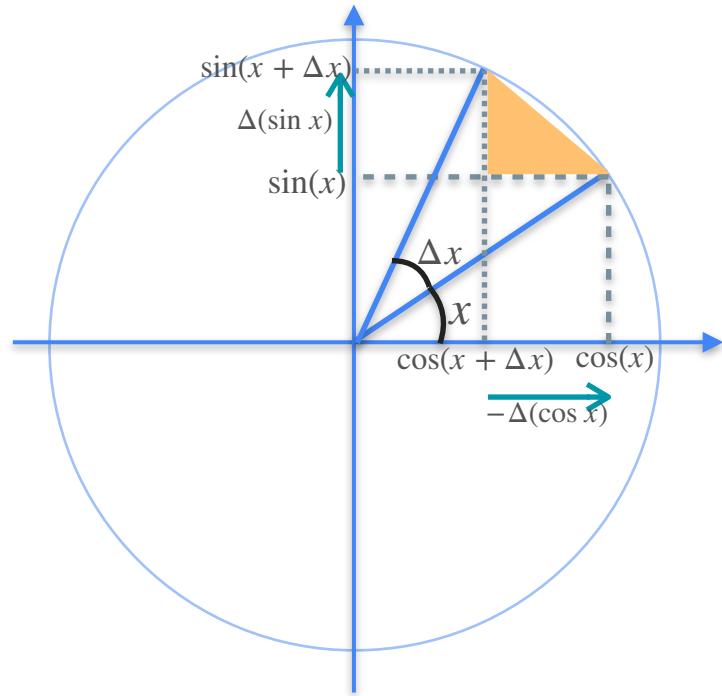
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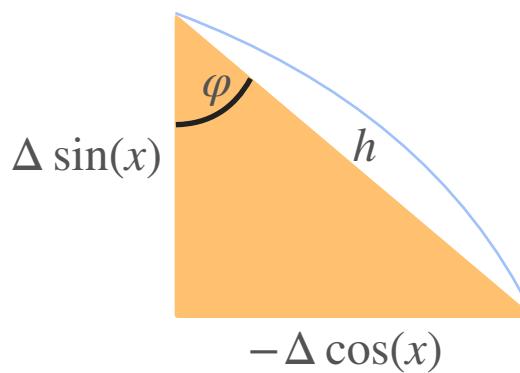
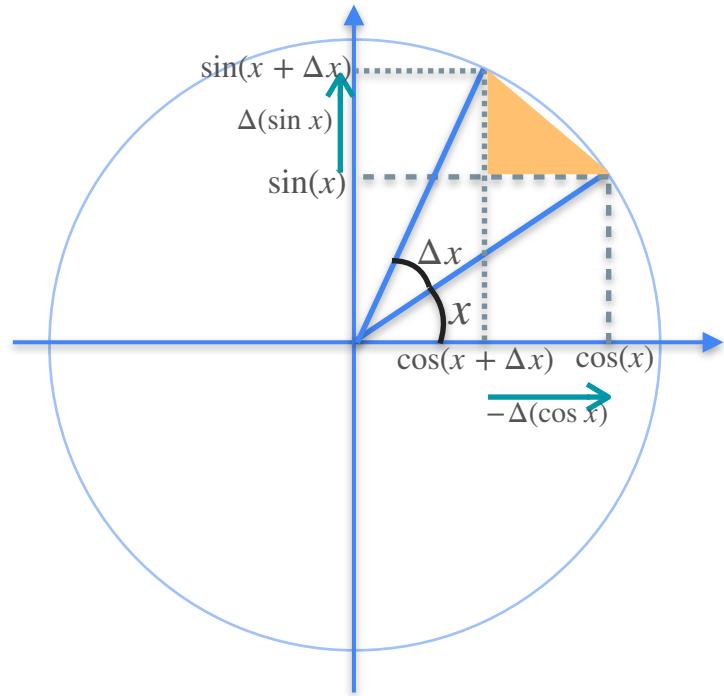
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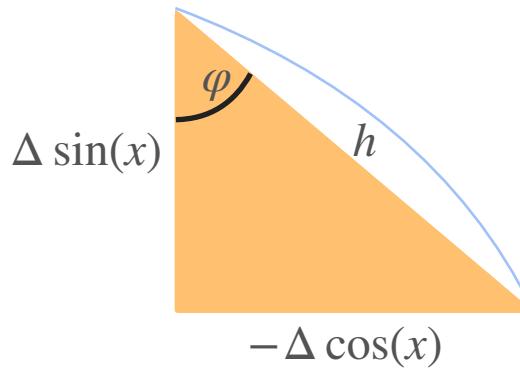
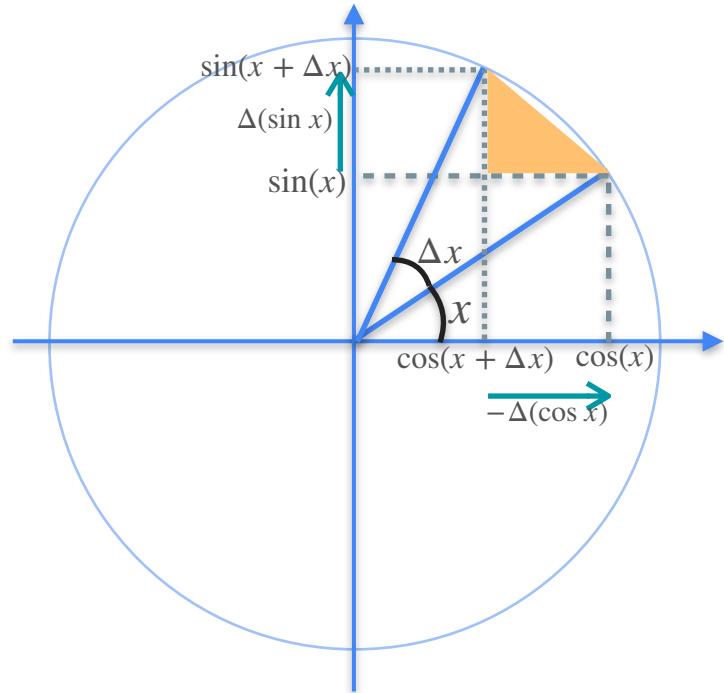
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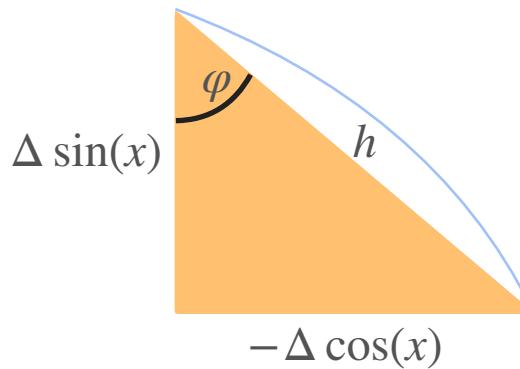
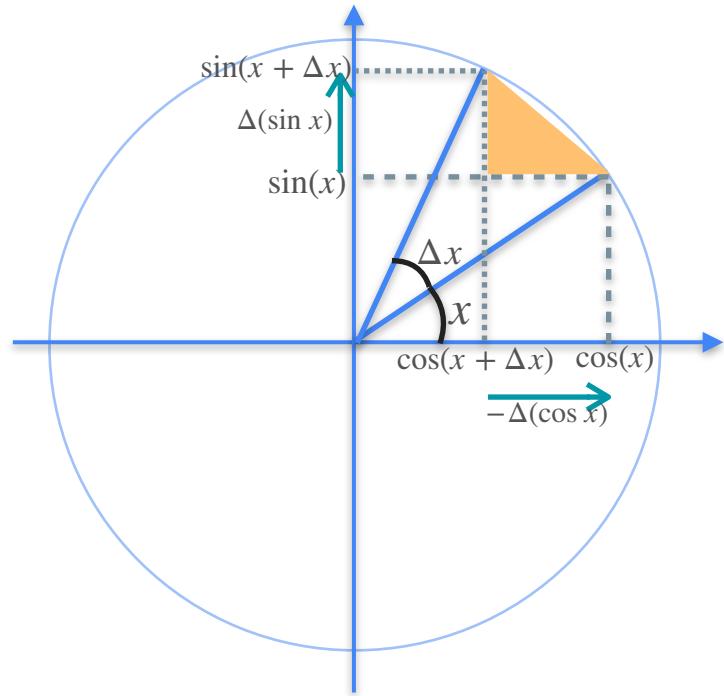


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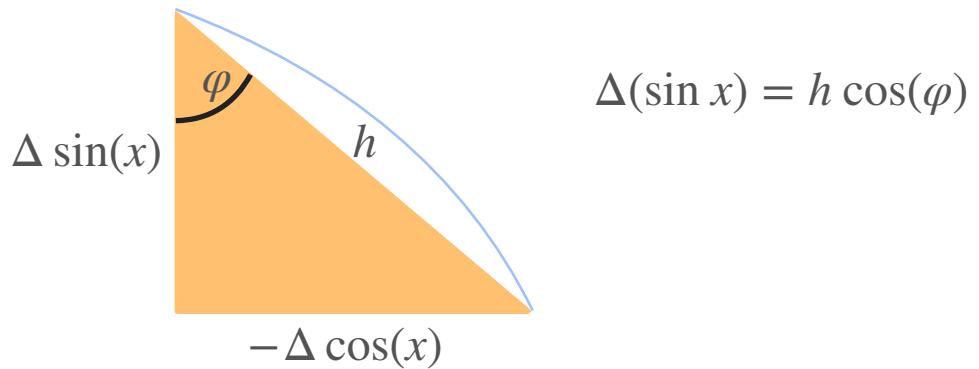
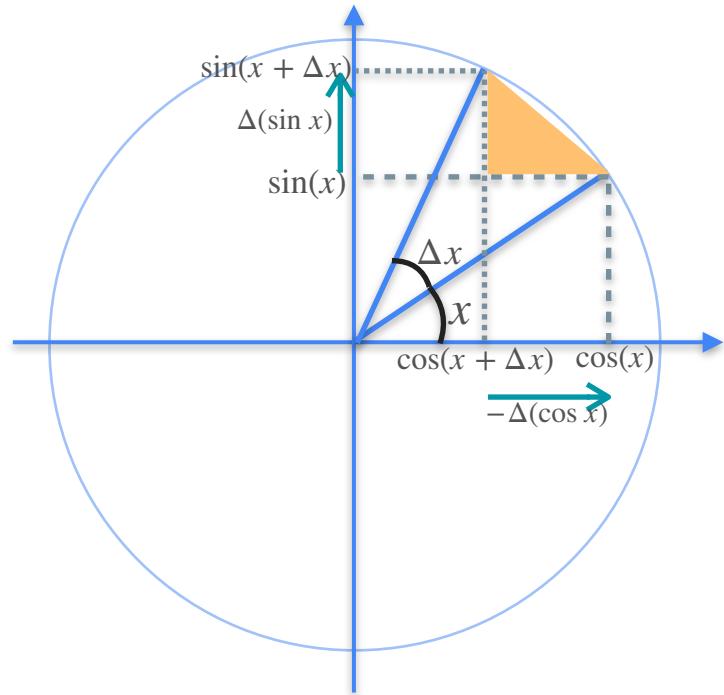
$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}}$$

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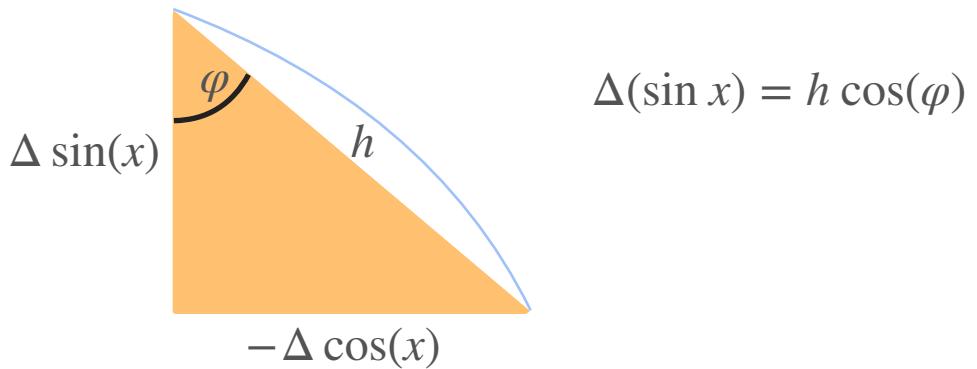
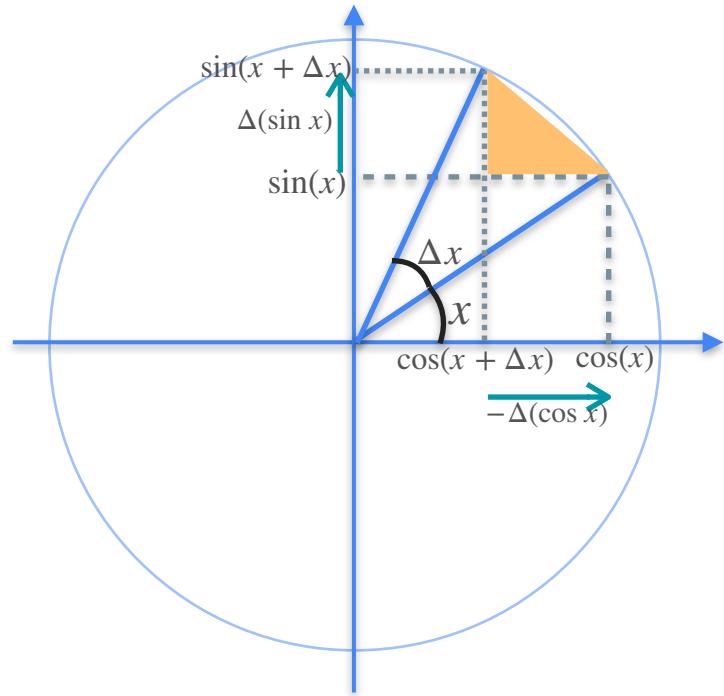
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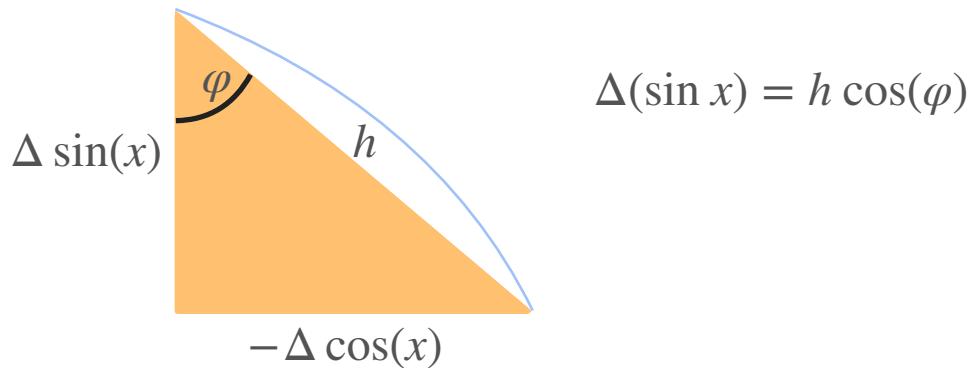
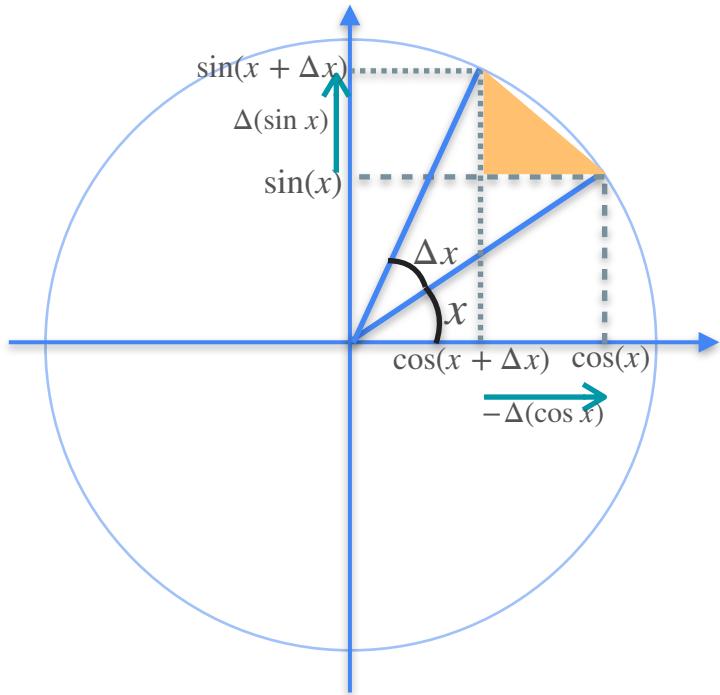
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# Derivative of Trigonometric Functions



$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$
$$\sin(\varphi) = \frac{\text{opp}}{\text{hyp}}$$

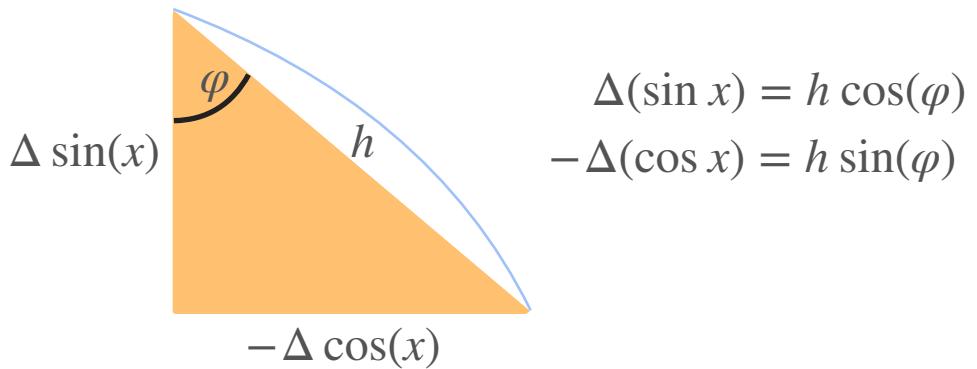
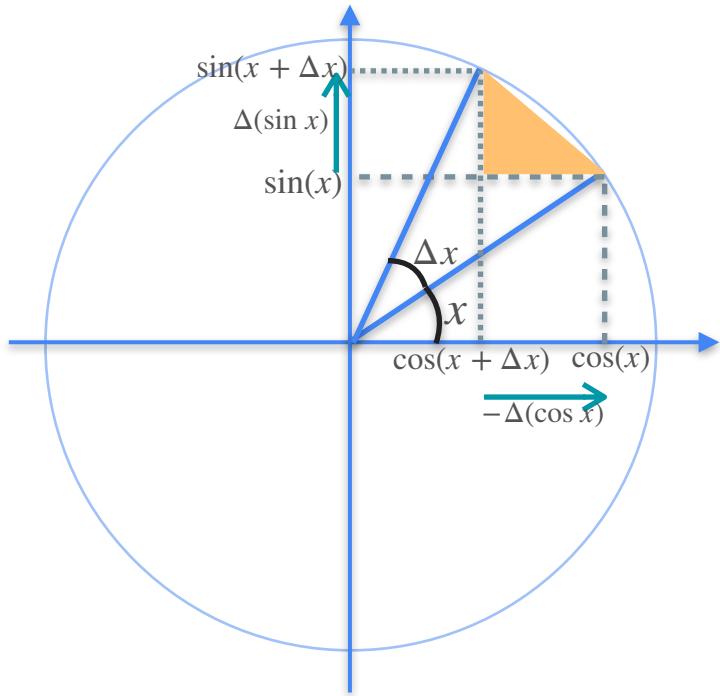
# Derivative of Trigonometric Functions



$$\cos(\varphi) = \frac{\text{adj}}{\text{hyp}} = \frac{\Delta(\sin x)}{h}$$

$$\sin(\varphi) = \frac{\text{opp}}{\text{hyp}} = \frac{-\Delta(\cos x)}{h}$$

# Derivative of Trigonometric Functions

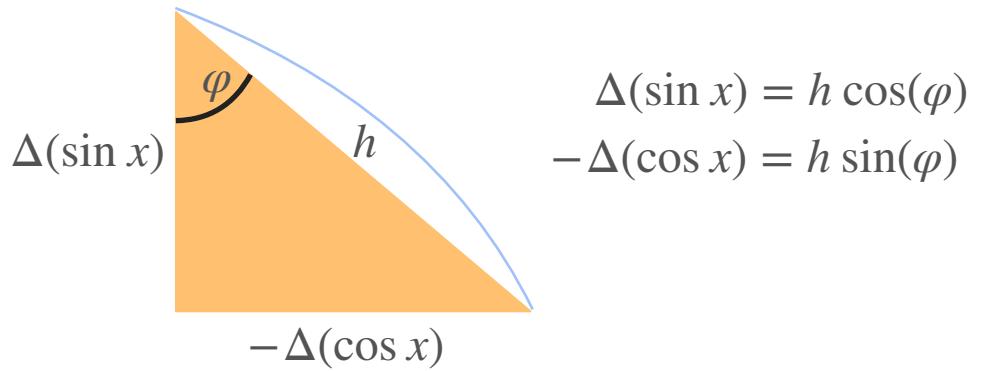


$$\cos(\varphi) = \frac{adj}{hyp} = \frac{\Delta(\sin x)}{h}$$

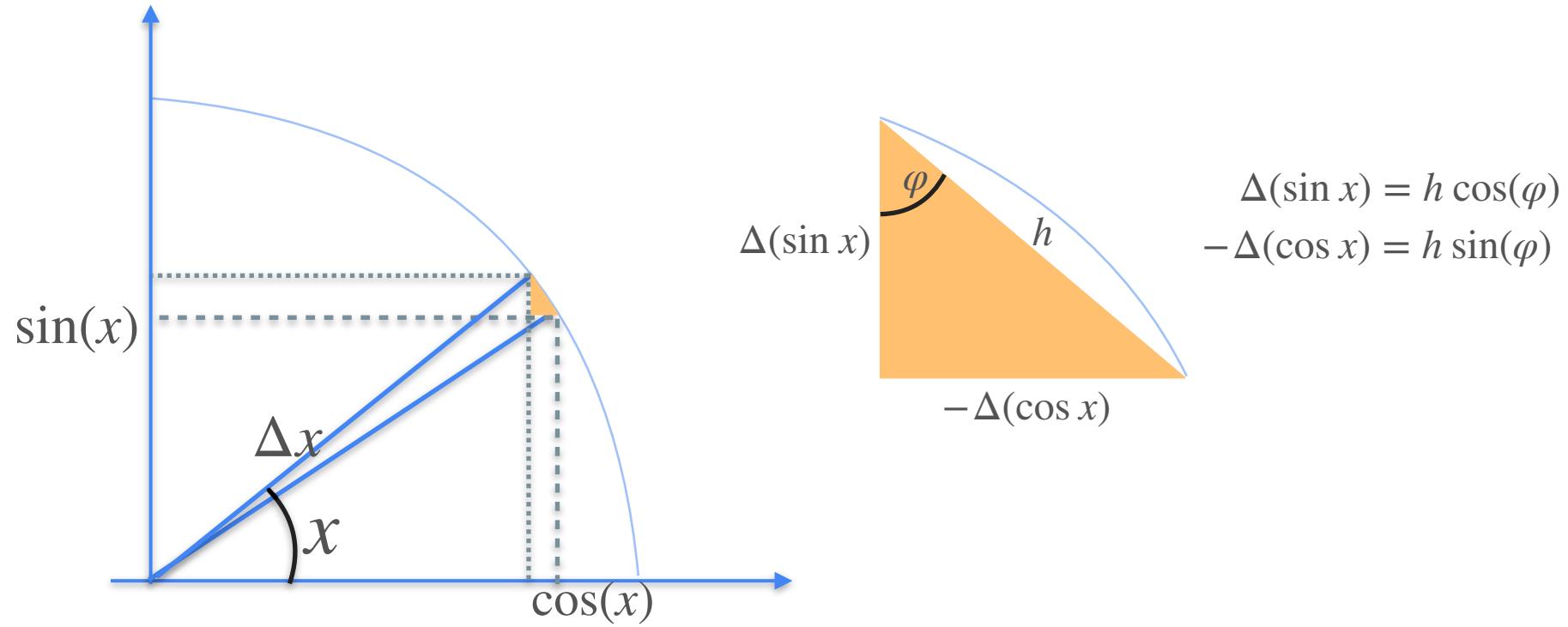
$$\sin(\varphi) = \frac{opp}{hyp} = \frac{-\Delta(\cos x)}{h}$$

$$\begin{aligned}\Delta(\sin x) &= h \cos(\varphi) \\ -\Delta(\cos x) &= h \sin(\varphi)\end{aligned}$$

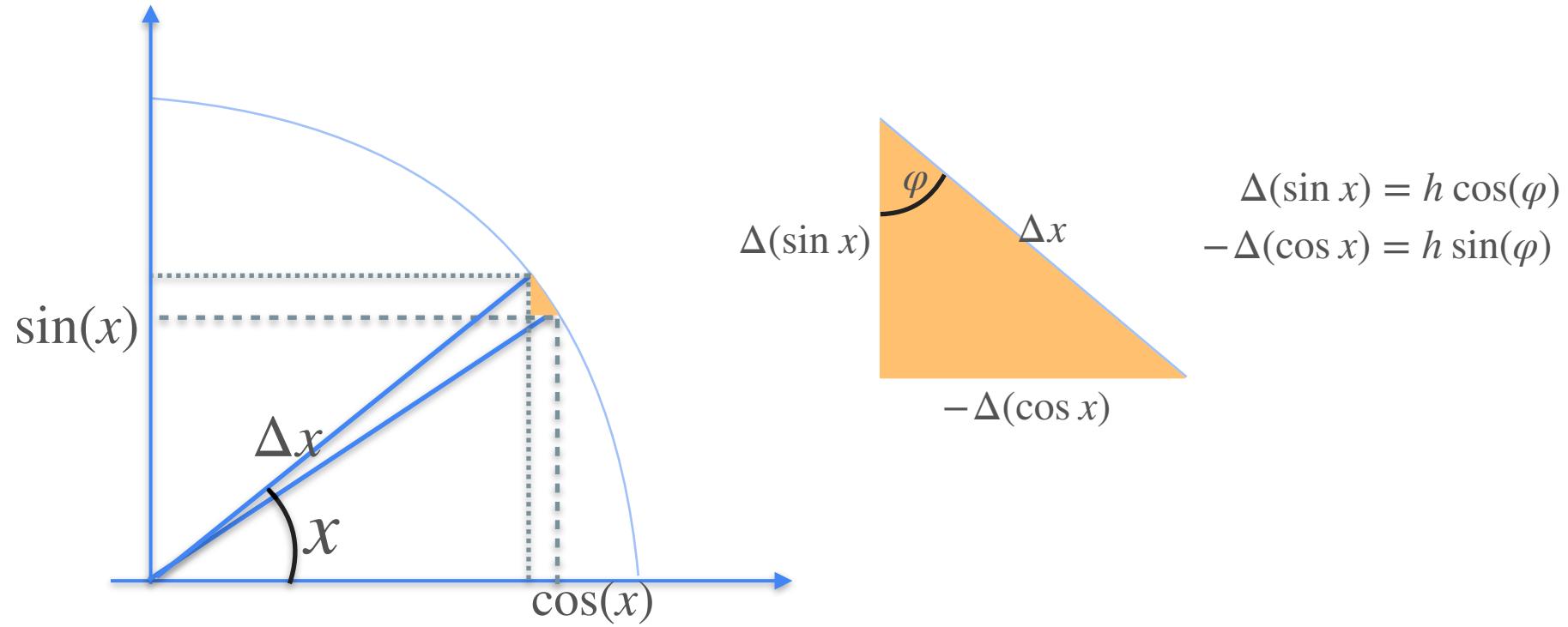
# Derivative of Trigonometric Functions



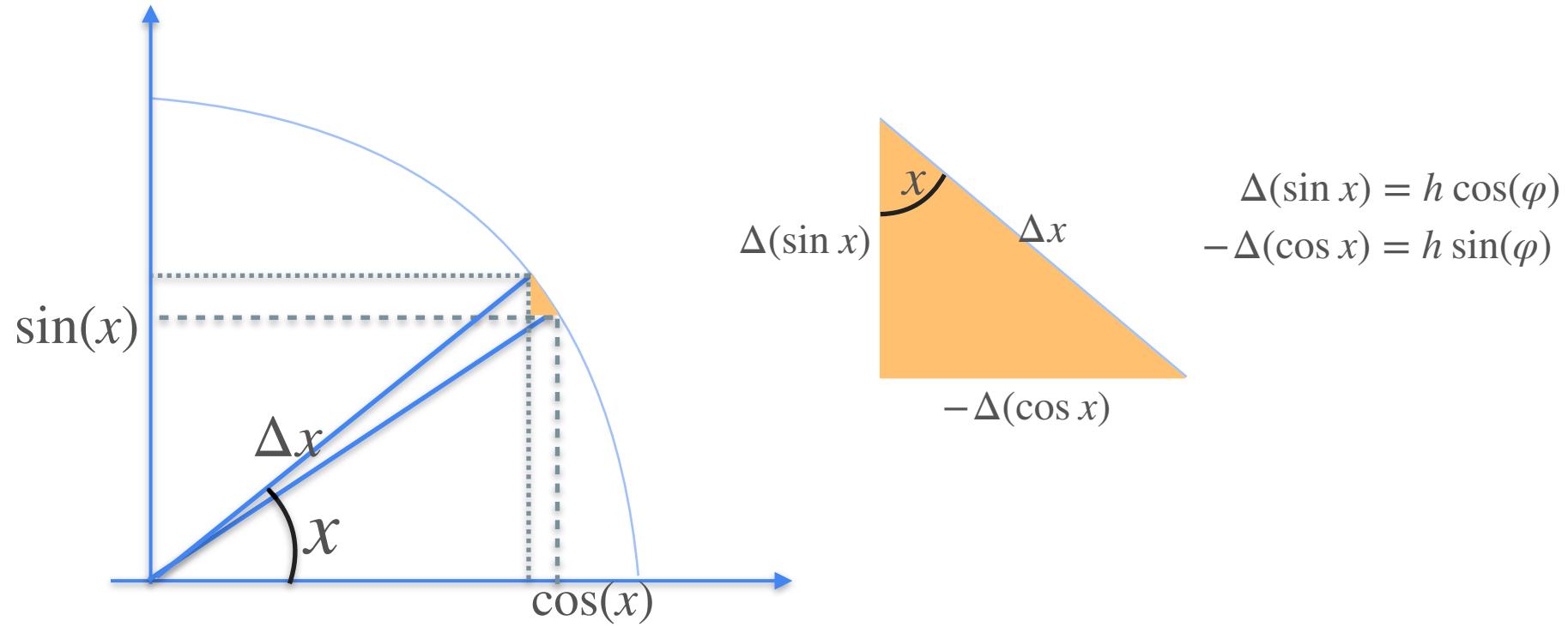
# Derivative of Trigonometric Functions



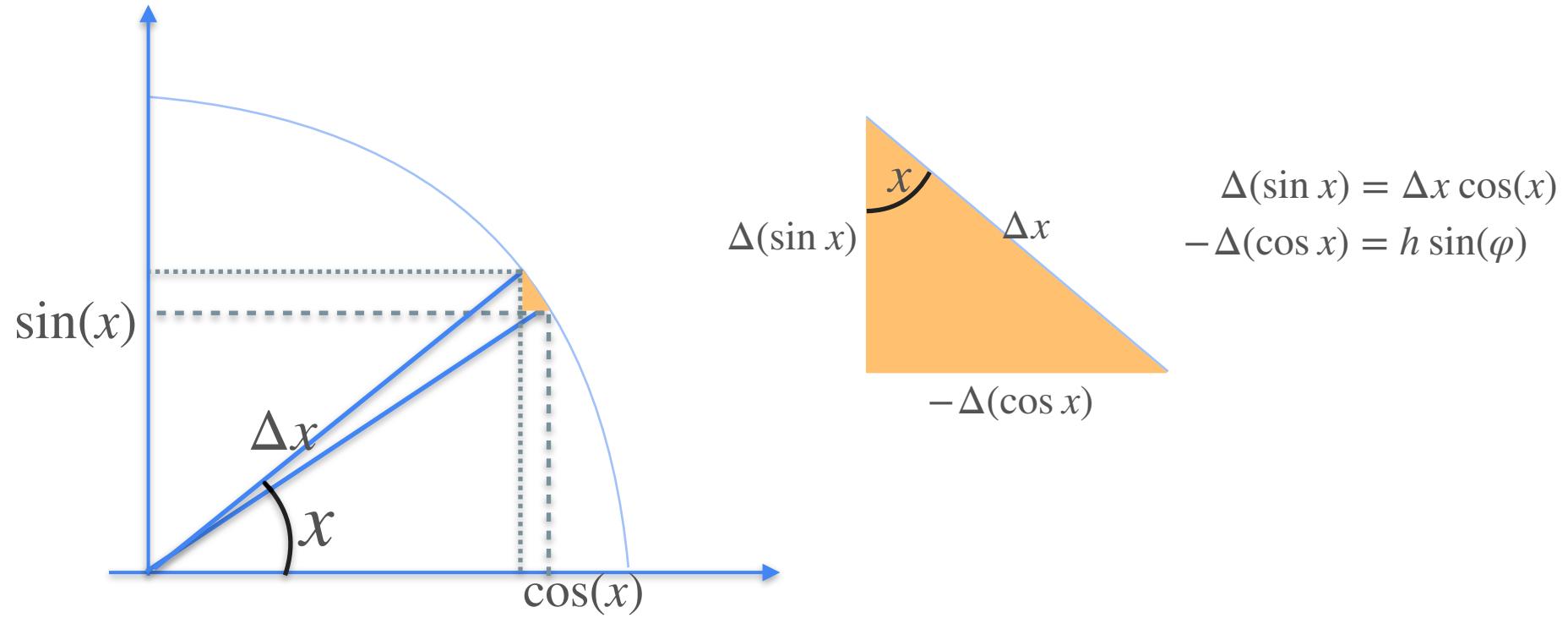
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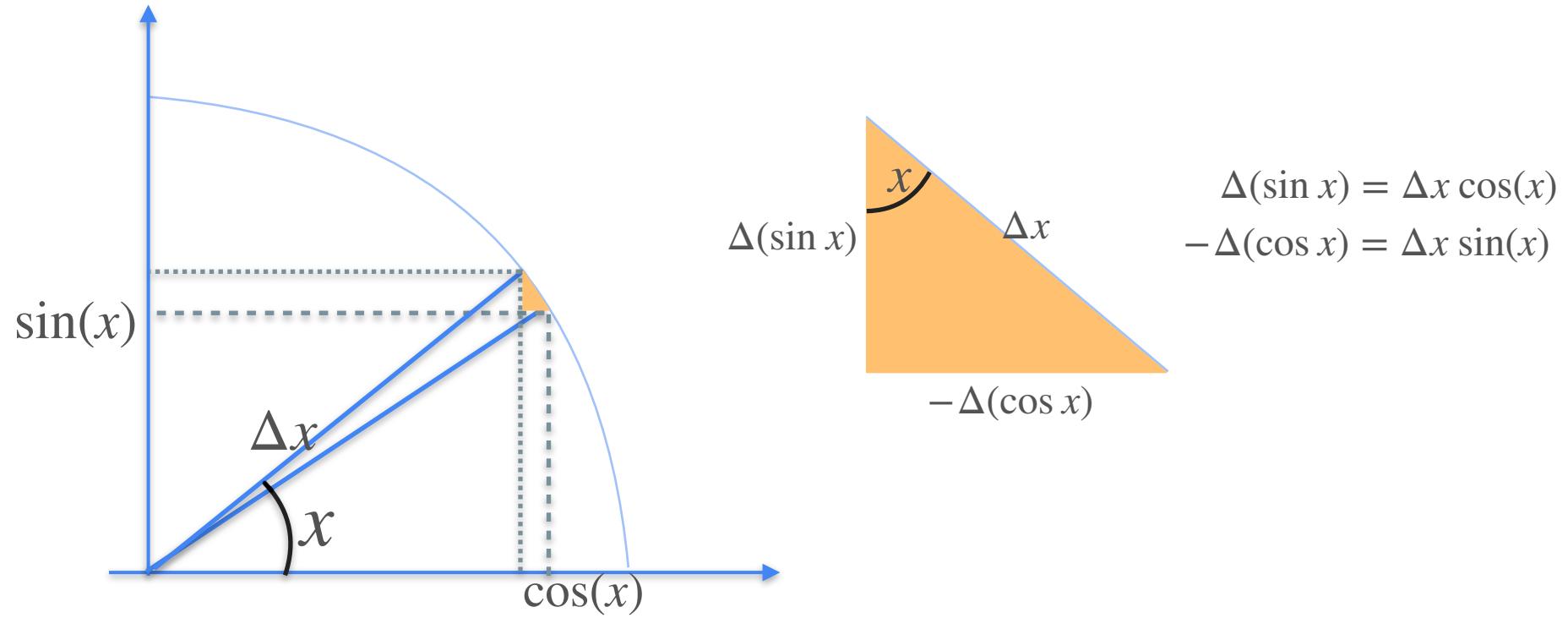
# Derivative of Trigonometric Functions



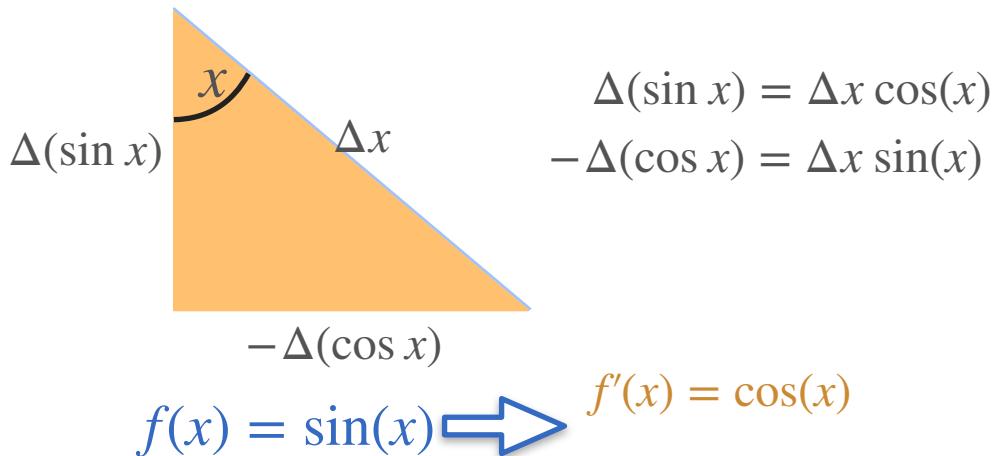
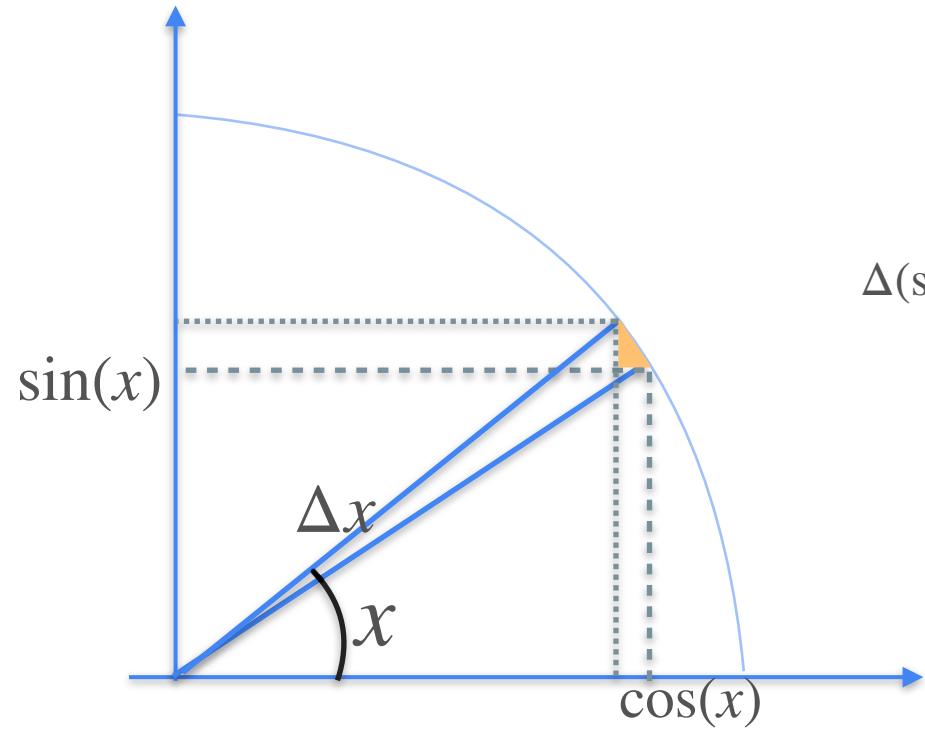
# Derivative of Trigonometric Functions



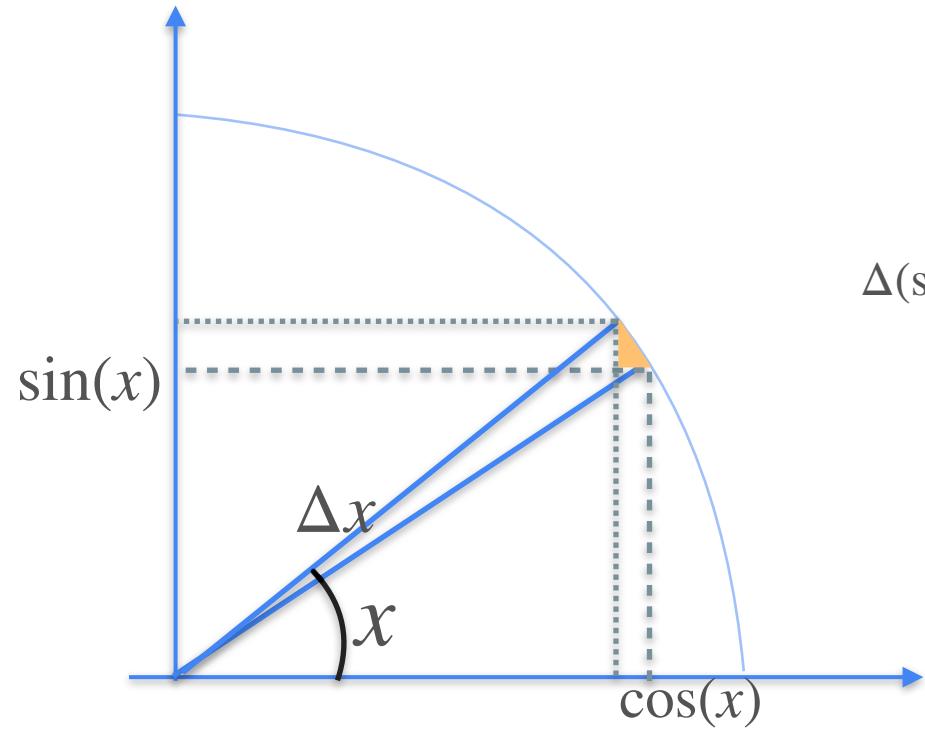
# Derivative of Trigonometric Functions



# Derivative of Trigonometric Functions



# Derivative of Trigonometric Functions



A right triangle is shown with a hypotenuse labeled  $\Delta x$ . The angle at the bottom-left vertex is labeled  $x$ . The vertical leg of the triangle is labeled  $\Delta(\sin x)$  and the horizontal leg is labeled  $-\Delta(\cos x)$ .

$$\Delta(\sin x) = \Delta x \cos(x)$$
$$-\Delta(\cos x) = \Delta x \sin(x)$$
$$f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$$
$$g(x) = \cos(x) \rightarrow g'(x) = -\sin(x)$$



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# Derivatives and Optimization

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## Meaning of the exponential $(e)$

$e = 2.71828182\dots$

$e = 2.71828182\dots$

$$\begin{array}{c} n \\ \left(1 + \frac{1}{n}\right)^n \end{array}$$

$e = 2.71828182\dots$

$n$	1
-----	---

$$\left(1 + \frac{1}{n}\right)^n$$

$$\left(1 + \frac{1}{1}\right)^1$$

$e = 2.71828182\dots$

$n$	1	10
$\left(1 + \frac{1}{n}\right)^n$	2	2.594

$$\left(1 + \frac{1}{10}\right)^{10}$$

$e = 2.71828182\dots$

$n$	1	10	100
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705

$$\left(1 + \frac{1}{100}\right)^{100}$$

$e = 2.71828182\dots$

$n$	1	10	100	1000
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717

$$\left(1 + \frac{1}{1000}\right)^{1000}$$

$e = 2.71828182\dots$

$n$	1	10	100	1000	$\infty$
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	2.718

$e = 2.71828182\dots$

$n$	1	10	100	1000	$\infty$
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	$e$

$$f(x) = e^x$$
$$f'(x) = e^x$$

$e = 2.71828182\dots$



$n$	1	10	100	1000	$\infty$
$\left(1 + \frac{1}{n}\right)^n$	2	2.594	2.705	2.717	$e$

A diagram illustrating the exponential function  $f(x) = e^x$ . It shows two equations in a teal oval:  $f(x) = e^x$  and  $f'(x) = e^x$ . A curved teal arrow points from the oval towards the value  $e$  in the table, indicating that the derivative of the exponential function at  $x=0$  is equal to the function's value at  $x=0$ .

# Choosing a Bank



# Choosing a Bank



**Interests**  
100% every year

# Choosing a Bank



Bank 1

## Interests

100% every year

(all your money once a year)

# Choosing a Bank



## Interests

100% every year

(all your money once a year)

# Choosing a Bank



Bank 1



Bank 2

## Interests

100% every year

(all your money once a year)

## Interests

50% every 6 months

# Choosing a Bank



Bank 1



Bank 2

## Interests

100% every year

(all your money once a year)

## Interests

50% every 6 months

(half of your money twice a year)

# Choosing a Bank



Bank 1

**Interests**  
100% every year  
(all your money once a year)



Bank 2

**Interests**  
50% every 6 months  
(half of your money twice a year)



Bank 3

# Choosing a Bank



Bank 1

**Interests**  
100% every year  
(all your money once a year)



Bank 2

**Interests**  
50% every 6 months  
(half of your money twice a year)



Bank 3

**Interests**  
33.3% every 4 months

# Choosing a Bank



**Interests**  
100% every year

(all your money once a year)



**Interests**  
50% every 6 months

(half of your money twice a year)



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

# Which bank is better?



Bank 1

**Interests**  
100% every year  
(all your money once a year)



Bank 2

**Interests**  
50% every 6 months  
(half of your money twice a year)



Bank 3

**Interests**  
33.3% every 4 months  
(A third of your money three times a year)

# Which bank is better?



Bank 1

**Interests**  
100% every year  
(all your money once a year)



Bank 2

**Interests**  
50% every 6 months  
(half of your money twice a year)



Bank 3

**Interests**  
33.3% every 4 months  
(A third of your money three times a year)

# Which bank is better?



Bank 1

**Interests**  
100% every year

(all your money once a year)



Bank 2

**Interests**  
50% every 6 months

(half of your money twice a year)



Bank 3



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

# You have \$1



Bank 1

**Interests**  
100% every year

(all your money once a year)



Bank 2

**Interests**  
50% every 6 months

(half of your money twice a year)



Bank 3

**Interests**  
33.3% every 4 months

(A third of your money three times a year)

# You have \$1



Bank 1



Bank 2



Bank 3



**Interests**  
100% every year

(all your money once a year)

**Interests**  
50% every 6 months

(half of your money twice a year)

**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	US\$2

# You have \$1



Bank 1



Bank 2



Bank 3



**Interests**  
100% every year

(all your money once a year)

**Interests**  
50% every 6 months

(half of your money twice a year)

**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	US\$2

+100%

# You have \$1



Bank 1

### Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

### Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

### Interests

33.3% every 4 months

(A third of your money three times a year)



# You have \$1



Bank 1

## Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

# You have \$1



Bank 1

## Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?

?



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

# You have \$1



Bank 1

## Interests

100% every year

(all your money once a year)

Now	US\$1
In 1 year	US\$2

+100%



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	US\$1
In 1 year	?



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	US\$1
In 1 year	?

# How Much Do I Have After 1 Year?



Bank 1

**Interests**

100% every year



Bank 2

**Interests**

50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



**Interests**  
100% every year



Bank 2

**Interests**  
50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



**Interests**  
100% every year

1



Bank 2

**Interests**  
50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



In 1 year



Interests

100% every year

1



Bank 2

Interests

50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests

100% every year

In 1 year



Bank 2

Interests

50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests

100% every year

In 1 year



2



Bank 2

Interests

50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



Interests  
100% every year

1

In 1 year



2  
 $(1 + 1)^1$



Bank 2

Interests  
50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



Interests  
100% every year

1

In 1 year



2  
 $(1 + 1)^1$



Bank 2

Now



Interests  
50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1

Now



1

Interests  
100% every year

In 1 year



<sup>2</sup>  
 $(1 + 1)^1$



Bank 2

Now



1

Interests  
50% every 6 months

# How Much Do I Have After 1 Year?



Now



Bank 1

Interests  
100% every year

1

In 1 year



$2$   
 $(1 + 1)^1$



Now

In 6 months



1

Interests

50% every 6 months

# How Much Do I Have After 1 Year?



Now



Bank 1

Interests  
100% every year

1

In 1 year



<sup>2</sup>  
 $(1 + 1)^1$



Now

In 6 months



1



Interests  
50% every 6 months

# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



Bank 2  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



Bank 2  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$1 + \frac{1}{2}$

# How Much Do I Have After 1 Year?



**Bank 1**  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



**Bank 2**  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$1 + \frac{1}{2}$

In 1 year



# How Much Do I Have After 1 Year?



**Bank 1**  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



**Bank 2**  
Interests  
50% every 6 months

Now



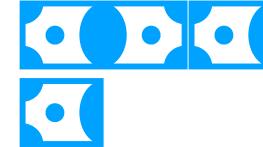
1

In 6 months



$1.5$   
 $1 + \frac{1}{2}$

In 1 year



# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2  
Interests  
50% every 6 months

Now



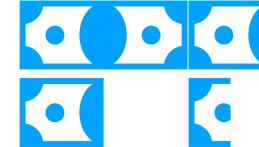
1

In 6 months



$$1.5 \\ 1 + \frac{1}{2}$$

In 1 year



# How Much Do I Have After 1 Year?



**Bank 1**  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



**Bank 2**  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$2$   
 $(1 + 1)^1$



Bank 2  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

$$\left(1 + \frac{1}{2}\right)^2$$

# How Much Do I Have After 1 Year?



Bank 1  
Interests  
100% every year

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$



Bank 2  
Interests  
50% every 6 months

Now



1

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



2.25

$$\left(1 + \frac{1}{2}\right)^2$$

# Compound interest



Bank 1  
Interests  
100% every year

Now



1



Bank 2  
Interests  
50% every 6 months

Now



1

In 1 year



$$2 \\ (1 + 1)^1$$

In 6 months



1.5

$$1 + \frac{1}{2}$$

In 1 year



$$2.25 \\ \left(1 + \frac{1}{2}\right)^2$$

# How Much Do I Have After 1 Year?



Now

In 4 months

In 8 months

In 1 year

Interests

33.3% every 4 months

# How Much Do I Have After 1 Year?



**Interests**  
33.3% every 4 months

Now



1

In 4 months

In 8 months

In 1 year

# How Much Do I Have After 1 Year?



**Interests**  
33.3% every 4 months

	Now	In 4 months	In 8 months	In 1 year
				
1				

# How Much Do I Have After 1 Year?



Now



In 4 months



In 8 months

In 1 year

Interests

33.3% every 4 months

1

# How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
<b>Bank 3</b>				
<b>Interests</b>	1	1.33		
33.3% every 4 months				

# How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
<b>Bank 3</b>				
<b>Interests</b>	1	1.33		
33.3% every 4 months				

# How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3	1	1.33	1.67	2.00
Interests	1	1.33	1.67	2.00
33.3% every 4 months				

# How Much Do I Have After 1 Year?



	Now	In 4 months	In 8 months	In 1 year
Bank 3	1	1.33	1.77	
Interests				

33.3% every 4 months

# How Much Do I Have After 1 Year?



Now



## Bank 3

In 4 months



In 8 months



In 1 year



## Interests

1

1.33

1.77

33.3% every 4 months

# How Much Do I Have After 1 Year?



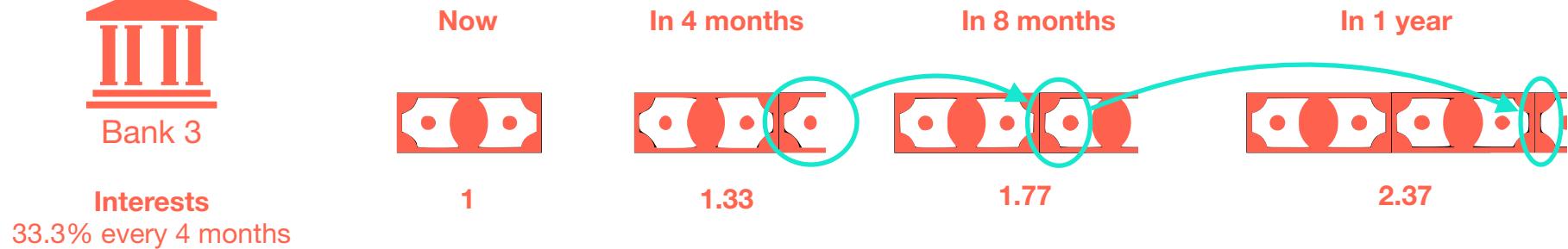
	Now	In 4 months	In 8 months	In 1 year
<b>Bank 3</b>				
<b>Interests</b>	<b>1</b>	<b>1.33</b>	<b>1.77</b>	
33.3% every 4 months				

# How Much Do I Have After 1 Year?

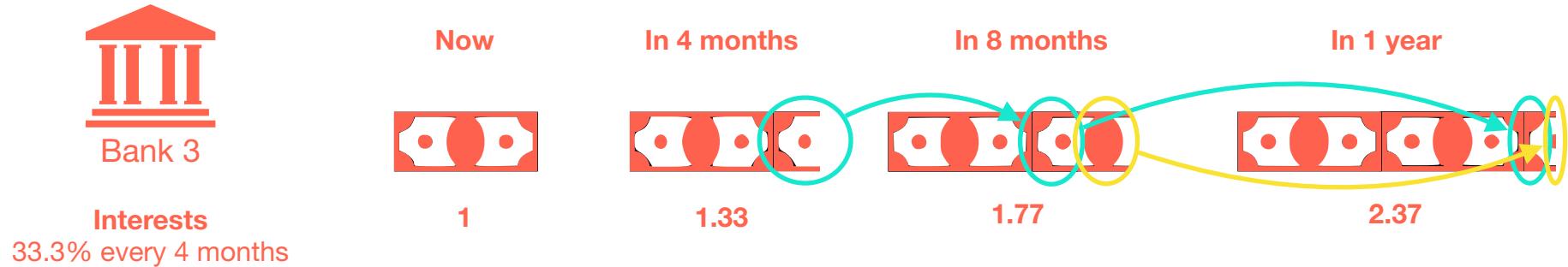


	Now	In 4 months	In 8 months	In 1 year
Bank 3				
Interests	1	1.33	1.77	2.37
33.3% every 4 months				

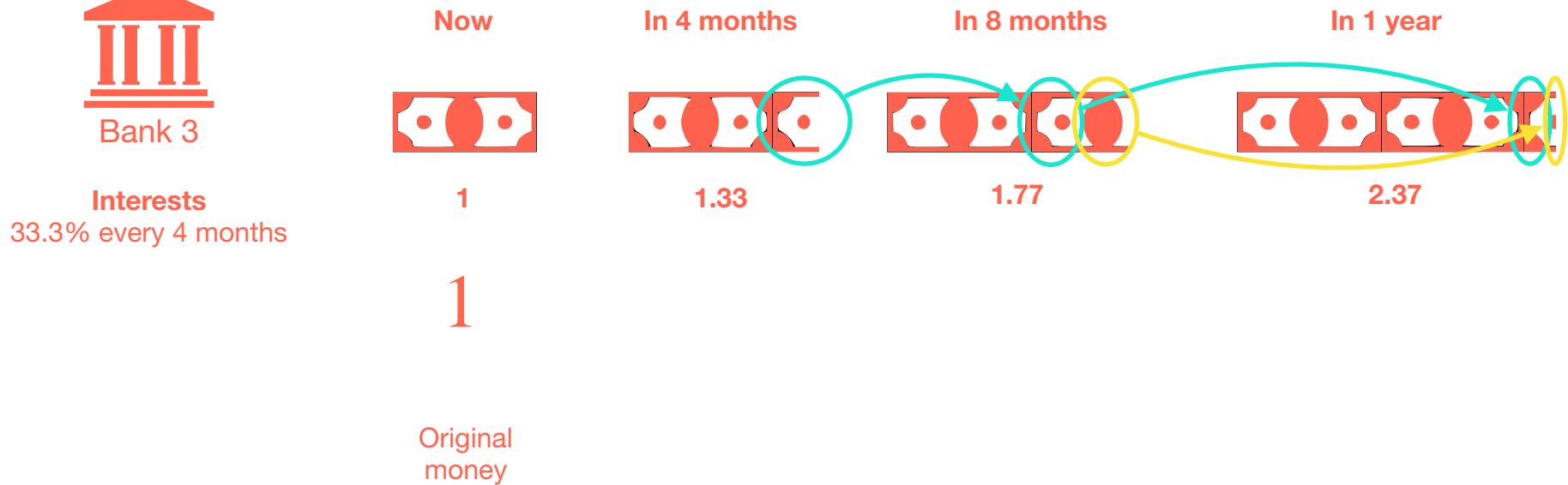
# How Much Do I Have After 1 Year?



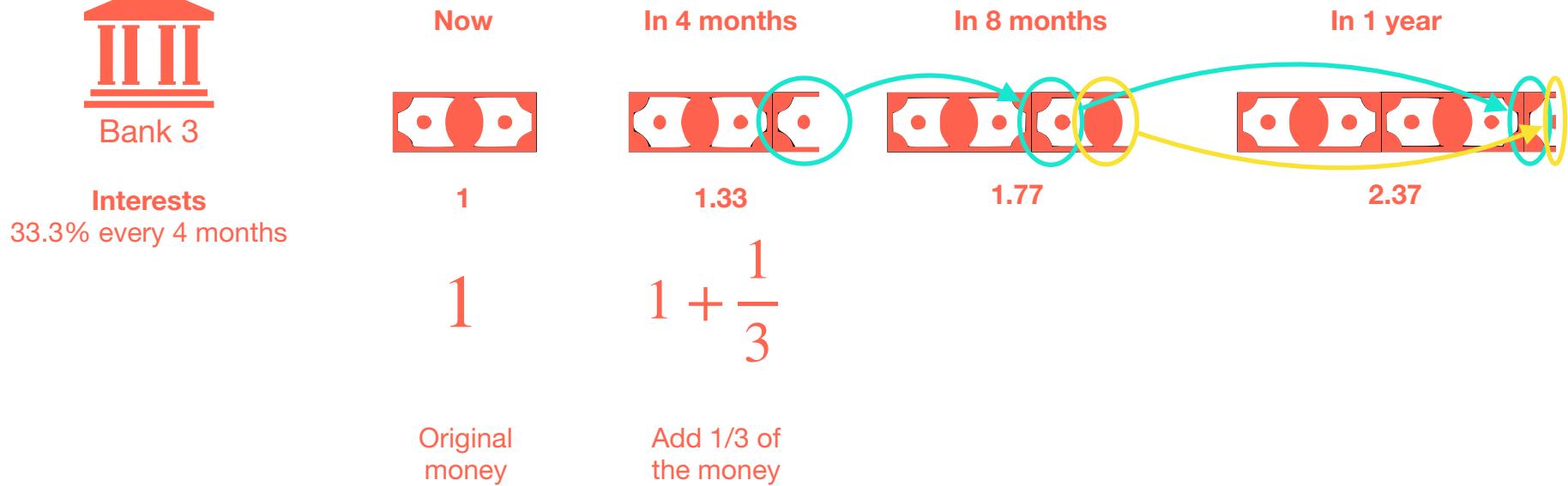
# How Much Do I Have After 1 Year?



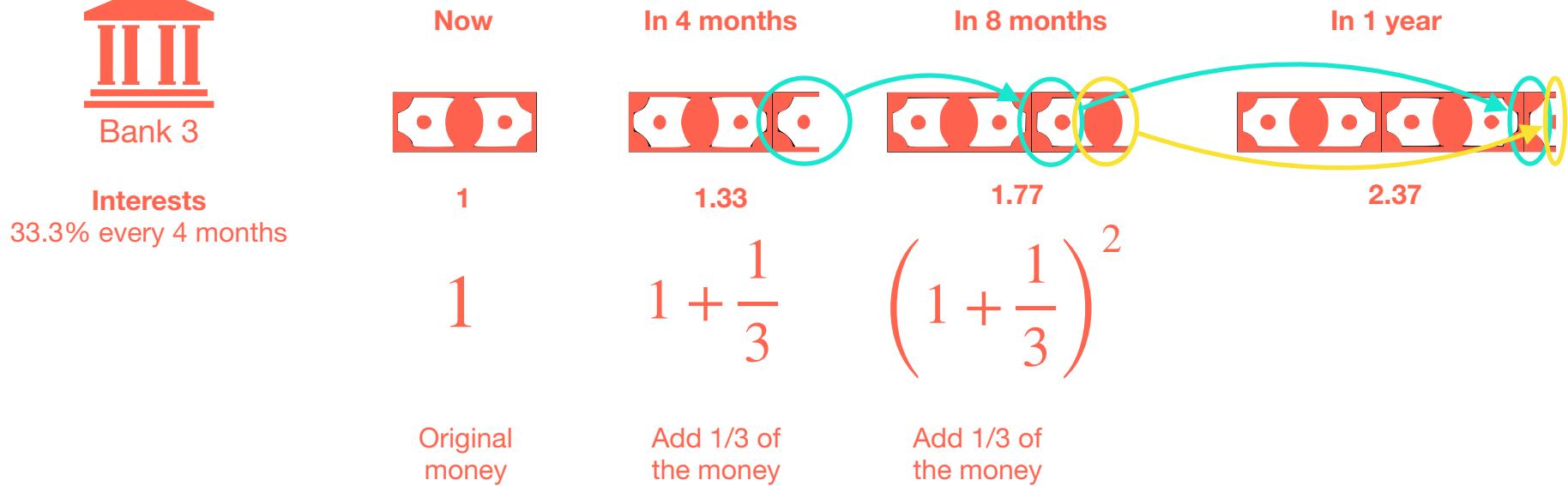
# How Much Do I Have After 1 Year?



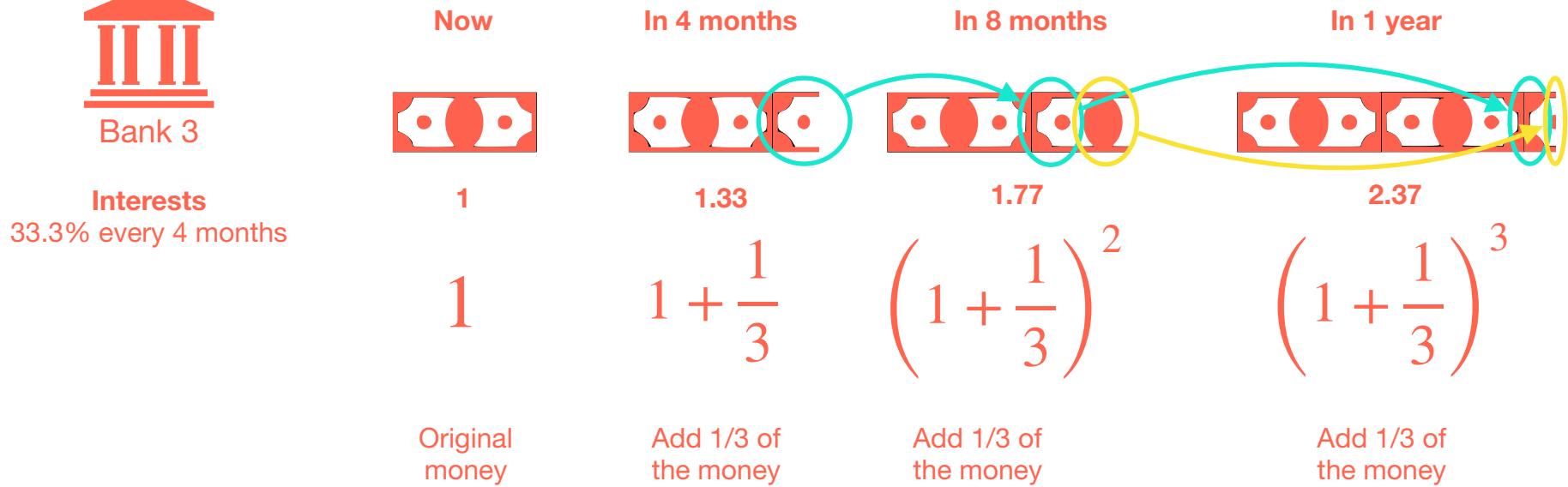
# How Much Do I Have After 1 Year?



# How Much Do I Have After 1 Year?



# How Much Do I Have After 1 Year?



# Which Bank Is Better?



Bank 1

**Interests**

100% every year

(all your money once a year)



Bank 2

**Interests**

50% every 6 months

(half of your money twice a year)



Bank 3

**Interests**

33.3% every 4 months

(A third of your money three times a year)

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)



# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2

↗ +100%



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%  
+100%  
+100%  
+100%



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%  
+100%  
+100%  
+100%



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25

+125%



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%  
+100%  
+100%  
+100%



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%  
+125%  
+125%  
+125%



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%  
+100%  
+100%  
+100%



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%  
+125%  
+125%  
+125%



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37

+137%

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
In 1 year	\$2
In 2	\$4
In 3	\$8
In 4	\$16

+100%  
+100%  
+100%  
+100%



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
In 1 year	\$2.25
In 2	\$5.06
In 3	\$11.39
In 4	\$25.63

+125%  
+125%  
+125%  
+125%



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
In 1 year	\$2.37
In 2	\$5.62
In 3	\$13.32
In 4	\$31.57

+137%  
+137%  
+137%  
+137%

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
...	...



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
...	...



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
...	...



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

$$(1 + 1)^1 = 2$$

# Which Bank Is Better?



Bank 1

## Interests

100% every year

(all your money once a year)

Now	\$1
...	++--

$$(1 + 1)^1 = 2$$



Bank 2

## Interests

50% every 6 months

(half of your money twice a year)

Now	\$1
...	++--

$$\left(1 + \frac{1}{2}\right)^2 = 2.25$$



Bank 3

## Interests

33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	++--

# Which Bank Is Better?



**Interests**  
100% every year

(all your money once a year)

Now	\$1
...	...

$$(1 + 1)^1 = 2$$



**Interests**  
50% every 6 months

(half of your money twice a year)

Now	\$1
...	...

$$\left(1 + \frac{1}{2}\right)^2 = 2.25$$



**Interests**  
33.3% every 4 months

(A third of your money three times a year)

Now	\$1
...	...

$$\left(1 + \frac{1}{3}\right)^3 = 2.37$$

# How Much Do I Have After 1 Year?



Now

In 1 month

In 2 months

...

In 12 months

**Interests**  
1/12 every month

# How Much Do I Have After 1 Year?



**Interests**  
1/12 every month

Now

In 1 month

In 2 months

...

In 12 months

1

Original  
money

# How Much Do I Have After 1 Year?



	Now	In 1 month	In 2 months	...	In 12 months
	1	$1 + \frac{1}{12}$			
	Original money	Add 1/12 of the money			

# How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$		
Original money	Add 1/12 of the money	Add 1/12 of the money		

# How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$	...	
Original money	Add 1/12 of the money	Add 1/12 of the money		

# How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$	...	$\left(1 + \frac{1}{12}\right)^{12}$
Original money	Add 1/12 of the money	Add 1/12 of the money		Add 1/12 of the money

At the end of the year:

# How Much Do I Have After 1 Year?



Now	In 1 month	In 2 months	...	In 12 months
1	$1 + \frac{1}{12}$	$\left(1 + \frac{1}{12}\right)^2$	...	$\left(1 + \frac{1}{12}\right)^{12}$
Original money	Add 1/12 of the money	Add 1/12 of the money		Add 1/12 of the money

At the end of the year: **2.61**

# In General



Bank n

**Interests**  
 $1/n$  every interval

# In General



Bank  $n$

1	2	3	.	.	.	.	.	.	k	.	.	.	.	.	.	.	.	$n-2$	$n-1$	$n$
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-------	-------	-----

Interests

$1/n$  every interval

# In General



**Interests**  
1/n every interval



## In General



**Interests**  
1/n every interval



**Now**

1

## In General



**Interests**  
1/n every interval



**Now**      **In 1 interval**

$$1 + \frac{1}{n}$$

## In General



A horizontal timeline diagram representing a duration of "1 year". The timeline is marked with vertical tick marks at regular intervals. Above the timeline, the text "1 year" is written in black. Below the timeline, there are labels corresponding to the tick marks: "1", "2", "3", followed by three dots, then "k", another three dots, and finally "n-2", "n-1", and "n".

**Interests**  
1/n every interval

### **Now      In 1 interval      In 2 intervals**

$$1 + \frac{1}{n} \left(1 + \frac{1}{n}\right)^2$$

# In General



Interests  
1/n every interval

Now      In 1 interval      In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

$\downarrow$

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

# In General



A horizontal timeline diagram representing a year-long process. The timeline is a blue line with tick marks. Above the line, the text "1 year" is written in blue. Below the timeline, there are 17 numbered boxes labeled 1, 2, 3, ., ., ., ., k, ., ., ., ., ., ., ., n-2, n-1, and n. The first four boxes (1, 2, 3, .) are in red, while the remaining boxes (. through n) are in black.

**Interests**  
1/n every interval

### **Now      In 1 interval      In 2 intervals**

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$



$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money

# In General



Interests  
 $1/n$  every interval

Now      In 1 interval      In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

$\downarrow$

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money      Interest

# In General



Interests  
1/n every interval

Now      In 1 interval      In 2 intervals

$$1 \quad 1 + \frac{1}{n} \quad \left(1 + \frac{1}{n}\right)^2$$

$\downarrow$                      $\nearrow$

$$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$$

Money                  Interest

# In General



Interests  
 $1/n$  every interval

Now	In 1 interval	In 2 intervals	In 3 intervals
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$
	$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$		
	Money	Interest	

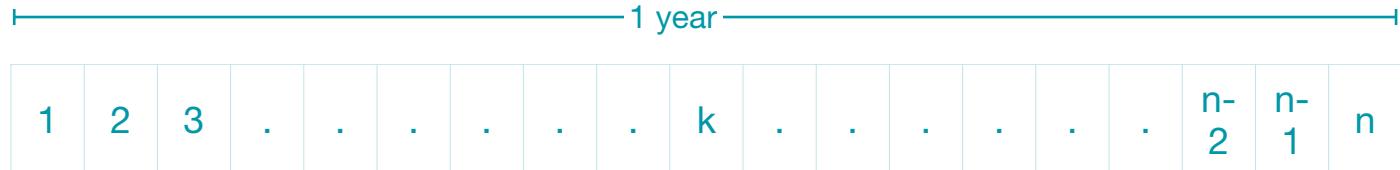
# In General



Interests  
 $1/n$  every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$	
	$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$			
	Money	Interest		

# In General



Interests  
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In $k$ intervals
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$		$\left(1 + \frac{1}{n}\right)^k$

$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$

Money      Interest

# In General



Interests  
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In $k$ intervals	...
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$		$\left(1 + \frac{1}{n}\right)^k$	
	$\left(1 + \frac{1}{n}\right)$ Money	$\frac{1}{n} \left(1 + \frac{1}{n}\right)$ Interest				

# In General



Interests  
1/n every interval

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In k intervals	...	In 1 year (n intervals)
1	$1 + \frac{1}{n}$	$\left(1 + \frac{1}{n}\right)^2$	$\left(1 + \frac{1}{n}\right)^3$	...	$\left(1 + \frac{1}{n}\right)^k$	...	$\left(1 + \frac{1}{n}\right)^n$

$\left(1 + \frac{1}{n}\right) + \frac{1}{n} \left(1 + \frac{1}{n}\right)$

Money      Interest

# In General



**Interests**  
 $1/n$  every interval

# In General



Bank n

**Interests**  
1/n every interval

In 1 year

$$\left(1 + \frac{1}{n}\right)^n$$

# In General



Bank n

**Interests**  
 $1/n$  every interval

**In 1 year**

$$\left( 1 + \frac{1}{n} \right)^n$$

↑  
1/n of your money

# In General



**Interests**  
 $1/n$  every interval

$$\left( 1 + \frac{1}{n} \right)$$

*In 1 year*

*n times*

*1/n of your money*

The diagram shows the mathematical expression for compound interest. A large teal bracket groups the term  $1 + \frac{1}{n}$ . Above this bracket, a pink circle contains the letter  $n$ , with a pink arrow pointing to it from the text "n times". A pink arrow also points upwards from the bottom of the bracket to the text "1/n of your money" at the bottom, indicating the fraction of the principal being added each period.

# A Lot of Banks

# A Lot of Banks



Bank 1



All your money  
1 time a year

# A Lot of Banks



Bank 1



Bank 2

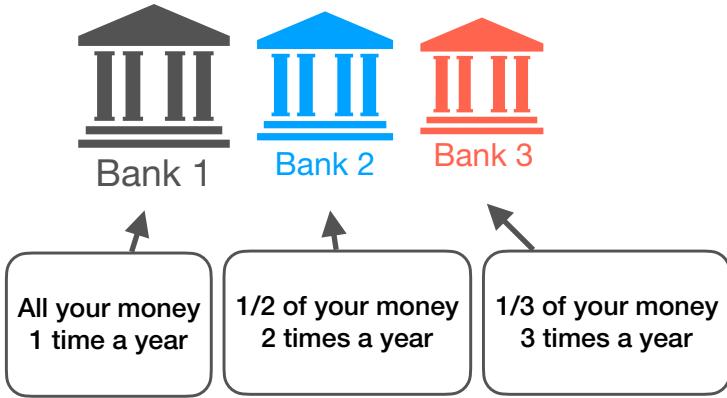


All your money  
1 time a year

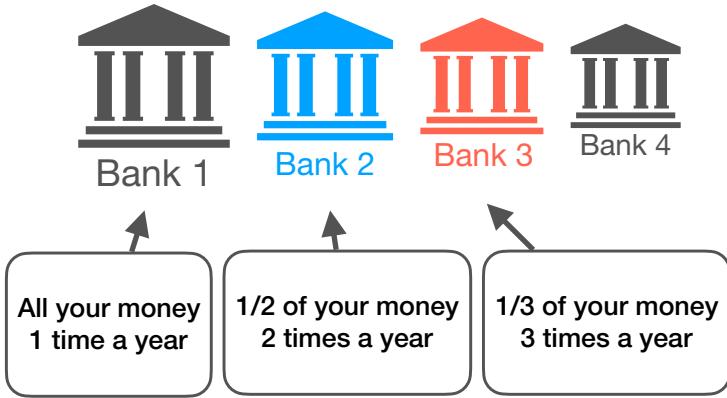


1/2 of your money  
2 times a year

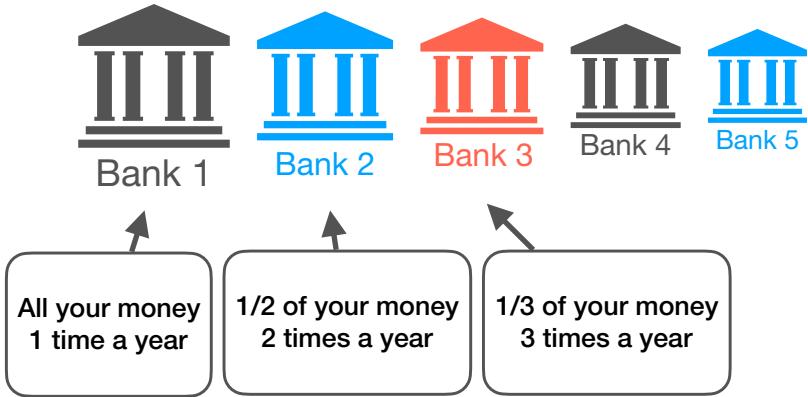
# A Lot of Banks



# A Lot of Banks



# A Lot of Banks



# A Lot of Banks



Bank 1



Bank 2



Bank 3



Bank 4



Bank 5



Bank 6



Bank 7



Bank 8



Bank 9



Bank 10



Bank 11



Bank 12



Bank 13

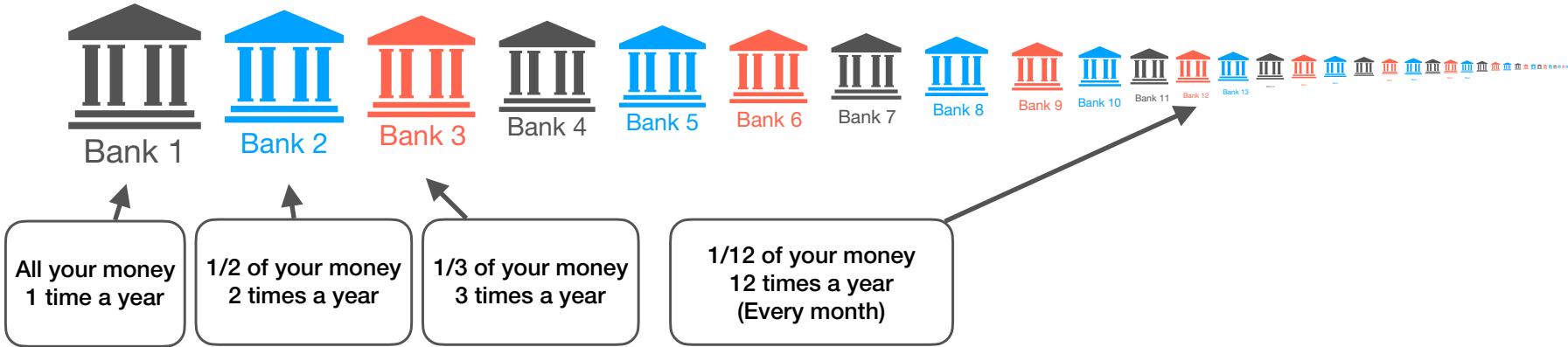
...

All your money  
1 time a year

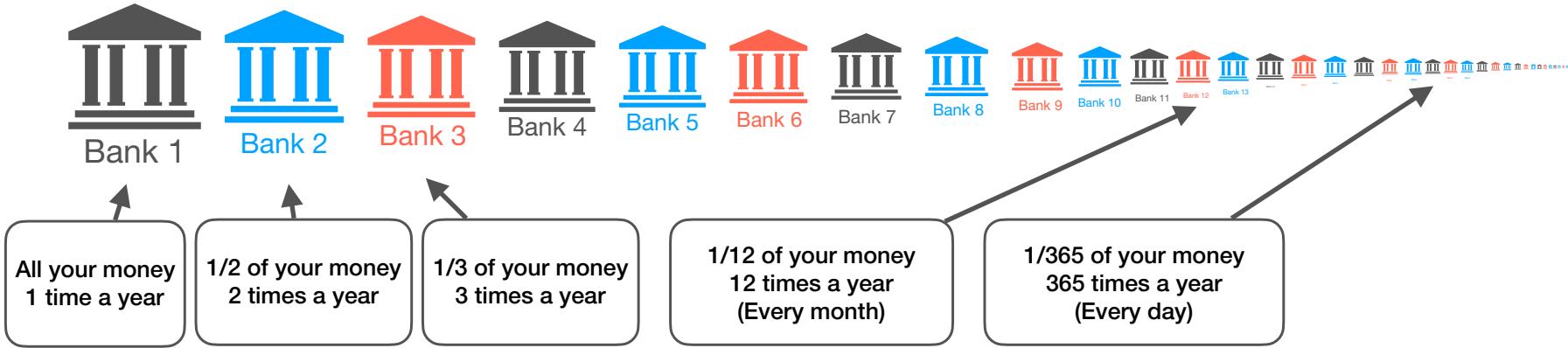
1/2 of your money  
2 times a year

1/3 of your money  
3 times a year

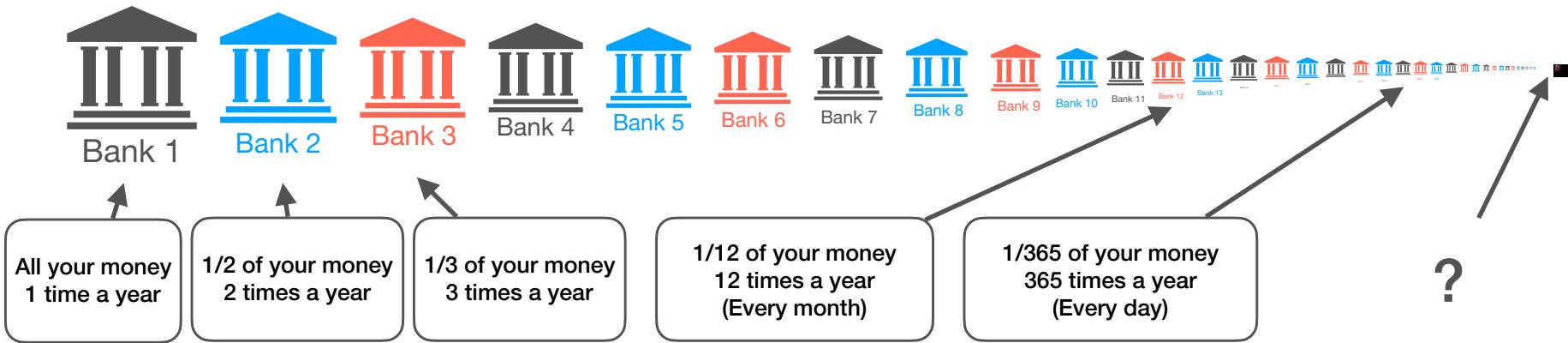
# A Lot of Banks



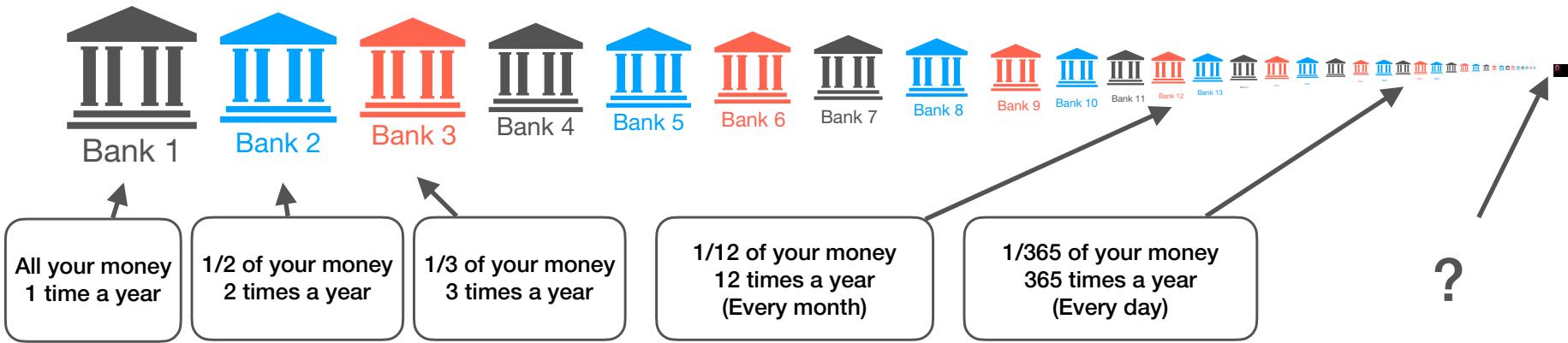
# A Lot of Banks



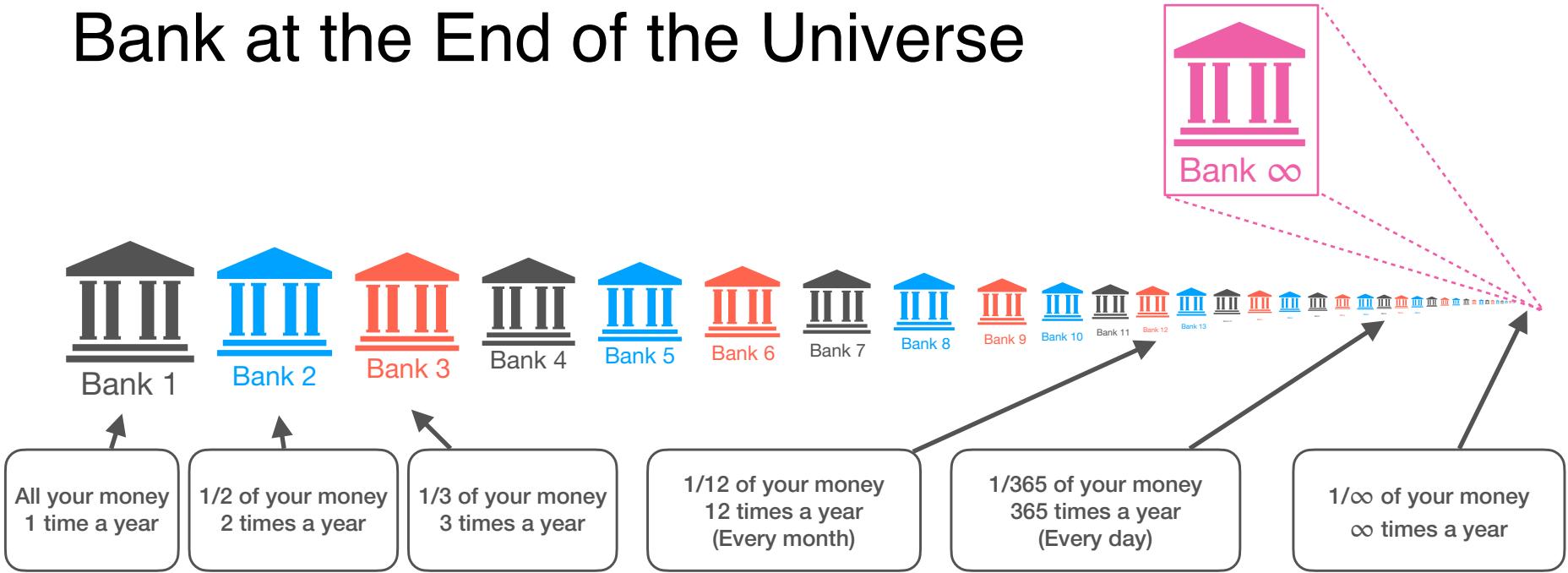
# A Lot of Banks



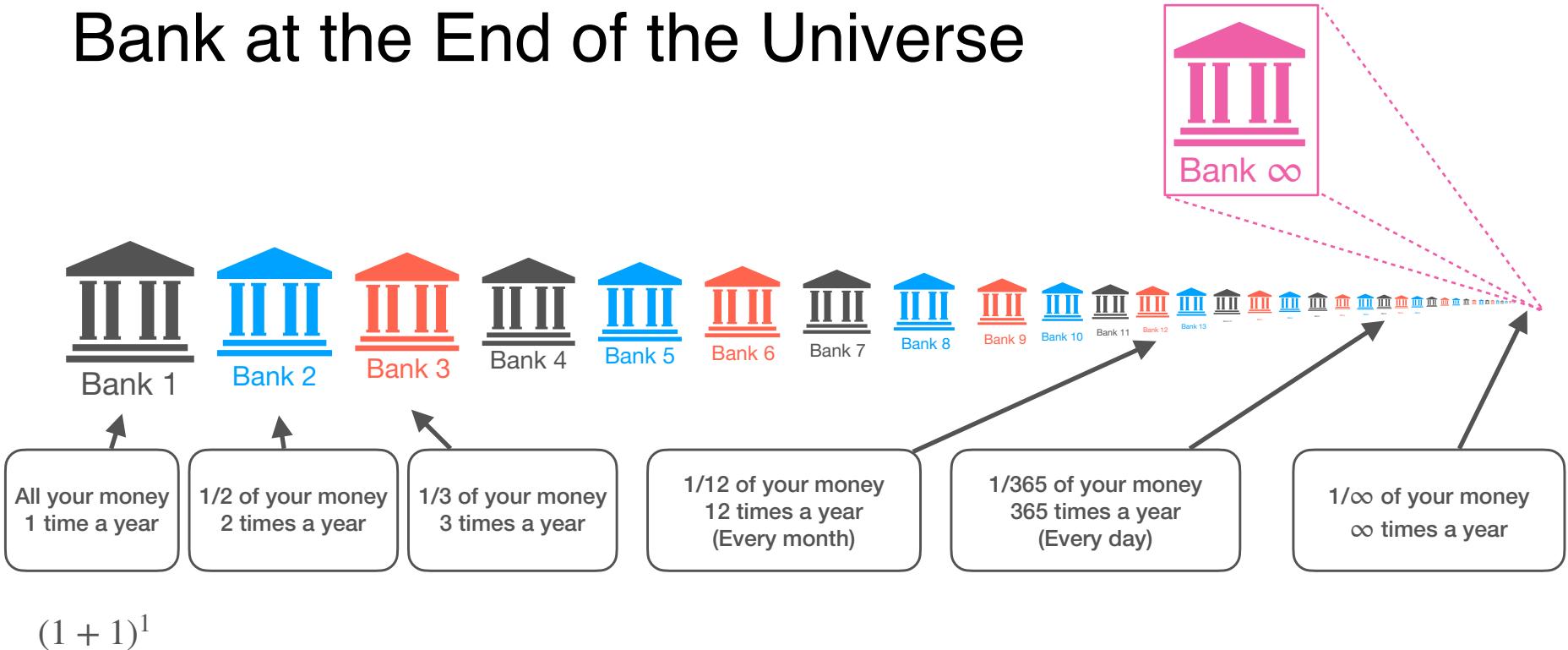
# Bank at the end of the universe



# Bank at the End of the Universe



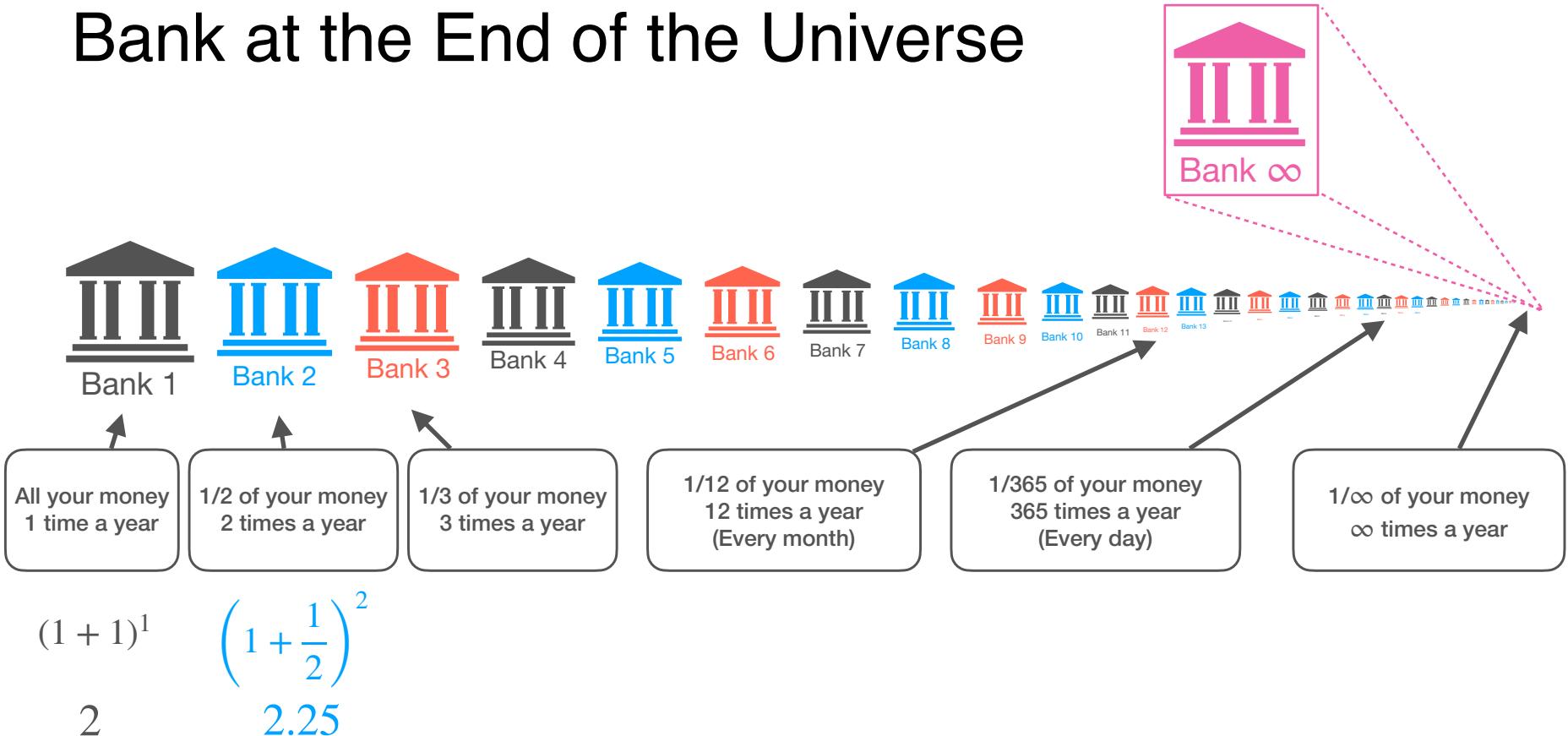
# Bank at the End of the Universe



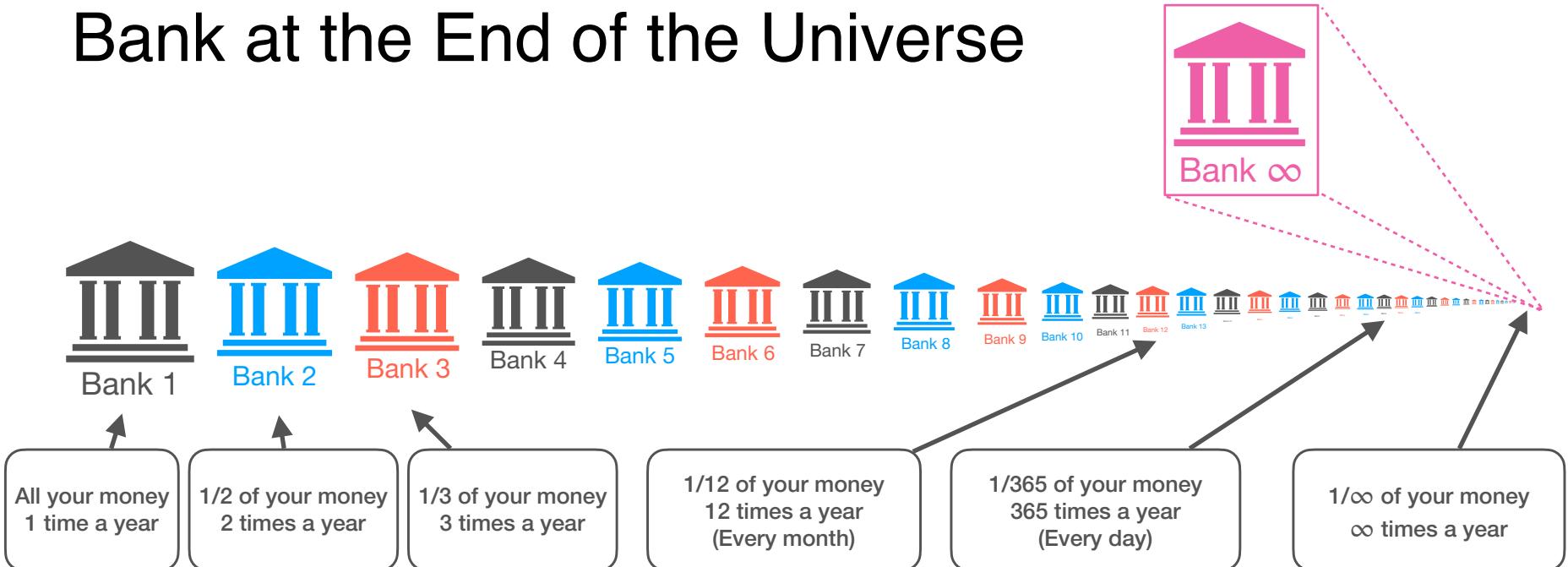
$$(1 + 1)^1$$

2

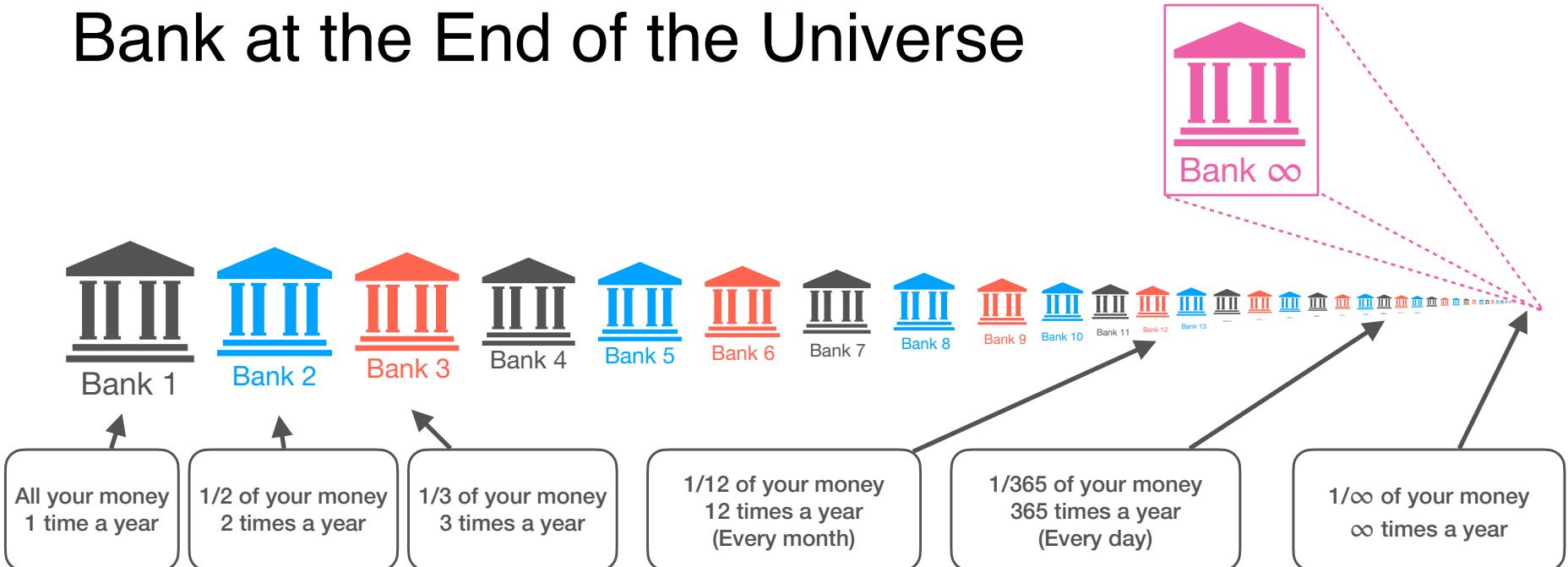
# Bank at the End of the Universe



# Bank at the End of the Universe



# Bank at the End of the Universe



$$(1 + 1)^1$$

$$\left(1 + \frac{1}{2}\right)^2$$

$$\left(1 + \frac{1}{3}\right)^3$$

$$\left(1 + \frac{1}{12}\right)^{12}$$

2

2.25

2.37

2.613

# Bank at the End of the Universe



Bank 1



Bank 2



Bank 3



Bank 4



Bank 5



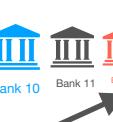
Bank 6



Bank 7



Bank 8



Bank 9



Bank 10



Bank 11



Bank 12



Bank 13

All your money  
1 time a year

1/2 of your money  
2 times a year

1/3 of your money  
3 times a year

1/12 of your money  
12 times a year  
(Every month)

1/365 of your money  
365 times a year  
(Every day)

1/∞ of your money  
∞ times a year

$$(1 + 1)^1$$

2

$$\left(1 + \frac{1}{2}\right)^2$$

2.25

$$\left(1 + \frac{1}{3}\right)^3$$

2.37

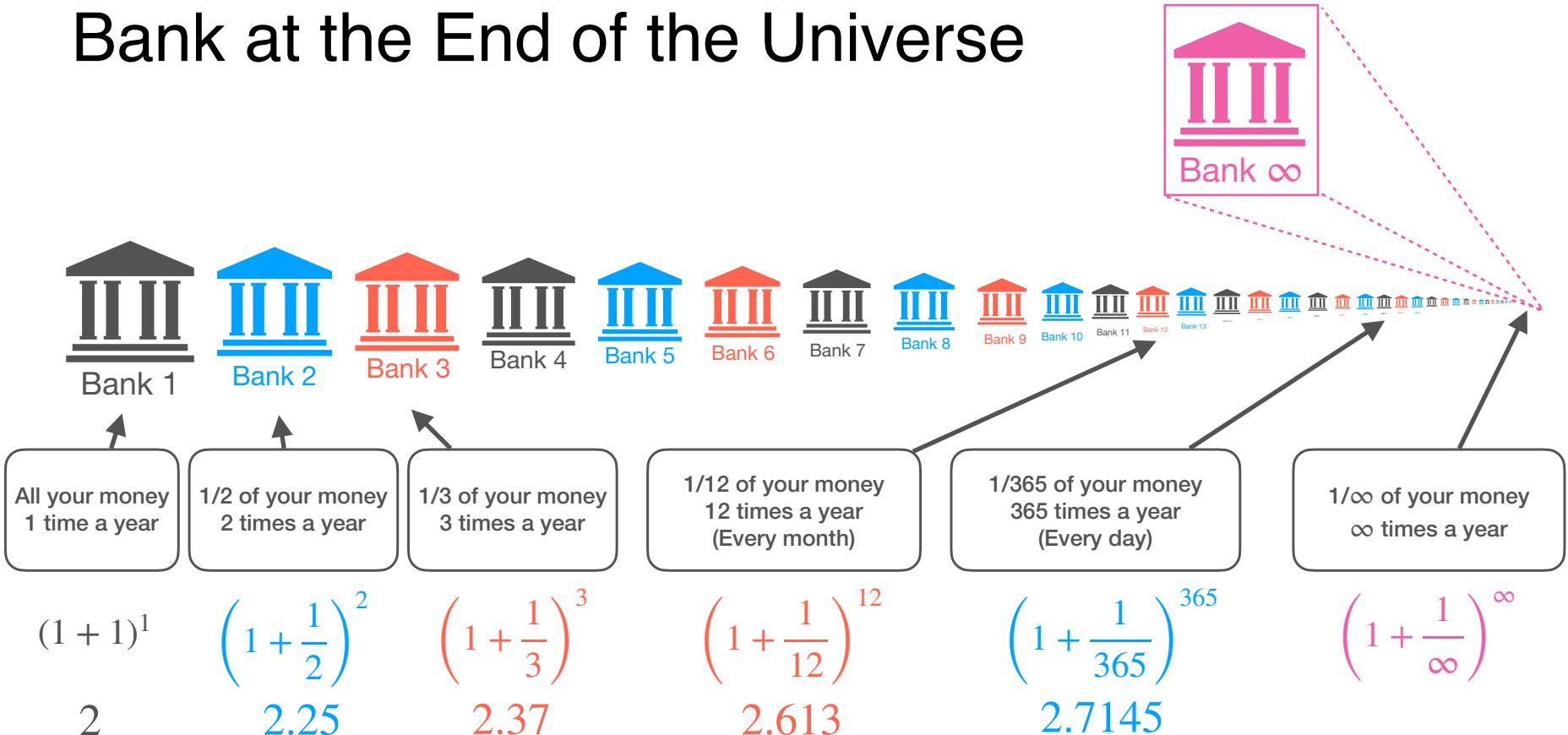
$$\left(1 + \frac{1}{12}\right)^{12}$$

2.613

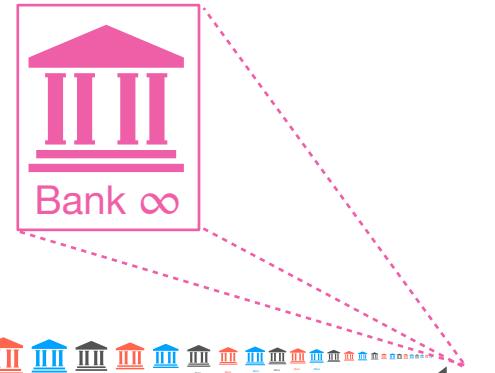
$$\left(1 + \frac{1}{365}\right)^{365}$$

2.7145

# Bank at the End of the Universe



# Bank at the End of the Universe



Bank 1



Bank 2



Bank 3



Bank 4



Bank 5



Bank 6



Bank 7



Bank 8



Bank 9



Bank 10



Bank 11



Bank 12



Bank 13

All your money  
1 time a year

1/2 of your money  
2 times a year

1/3 of your money  
3 times a year

1/12 of your money  
12 times a year  
(Every month)

1/365 of your money  
365 times a year  
(Every day)

1/infinity of your money  
infinity times a year

$$(1 + 1)^1$$

2

$$\left(1 + \frac{1}{2}\right)^2$$

2.25

$$\left(1 + \frac{1}{3}\right)^3$$

2.37

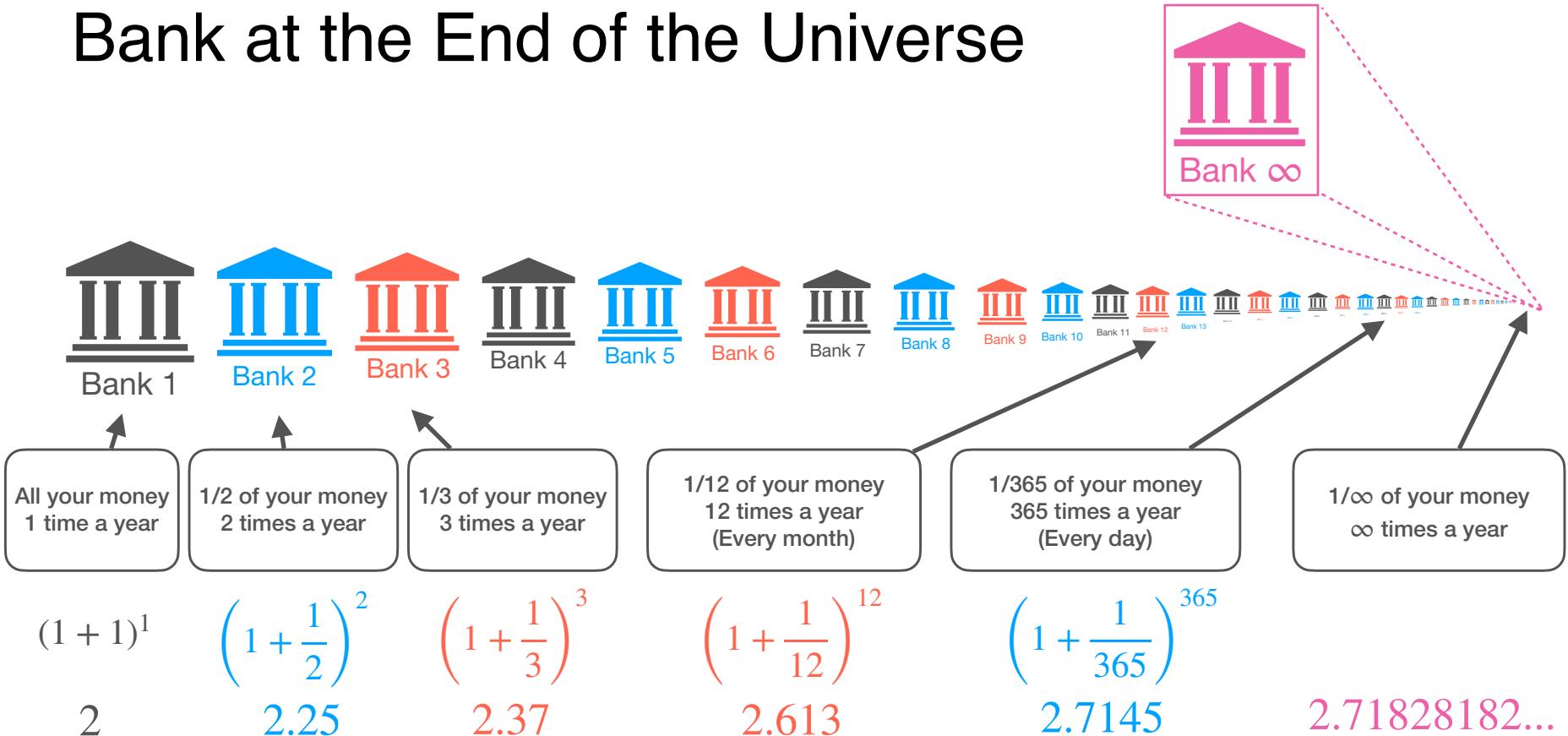
$$\left(1 + \frac{1}{12}\right)^{12}$$

2.613

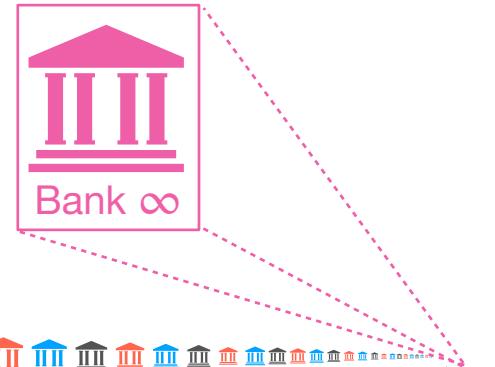
$$\left(1 + \frac{1}{365}\right)^{365}$$

2.7145

# Bank at the End of the Universe



# Bank at the End of the Universe



All your money  
1 time a year

1/2 of your money  
2 times a year

1/3 of your money  
3 times a year

1/12 of your money  
12 times a year  
(Every month)

1/365 of your money  
365 times a year  
(Every day)

1/∞ of your money  
∞ times a year

$$(1 + 1)^1$$

2

$$\left(1 + \frac{1}{2}\right)^2$$

2.25

$$\left(1 + \frac{1}{3}\right)^3$$

2.37

$$\left(1 + \frac{1}{12}\right)^{12}$$

2.613

$$\left(1 + \frac{1}{365}\right)^{365}$$

2.7145

$$e = 2.71828182...$$



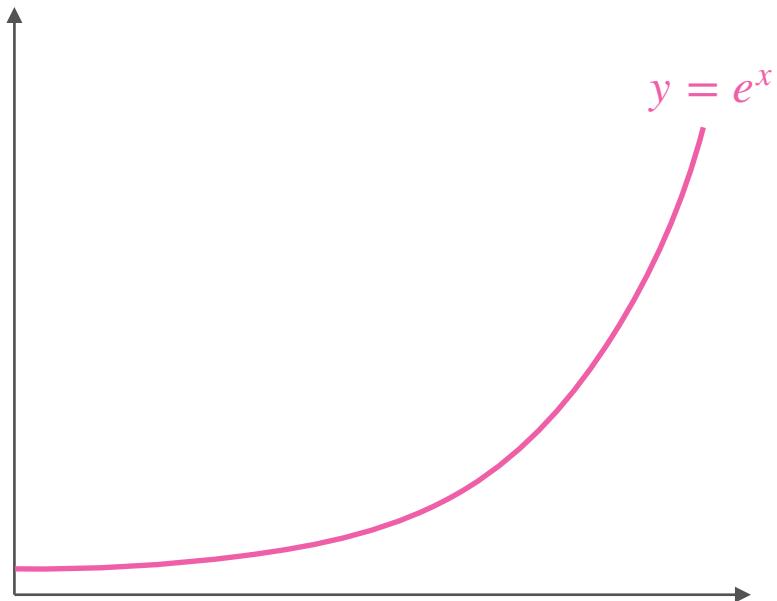
DeepLearning.AI

# Derivatives and Optimization

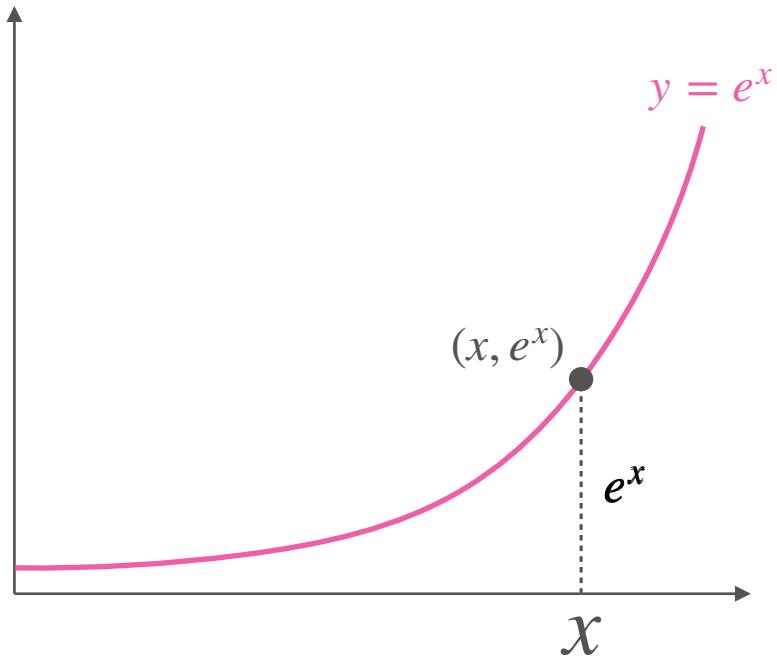
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**The derivative of  $e^x$**

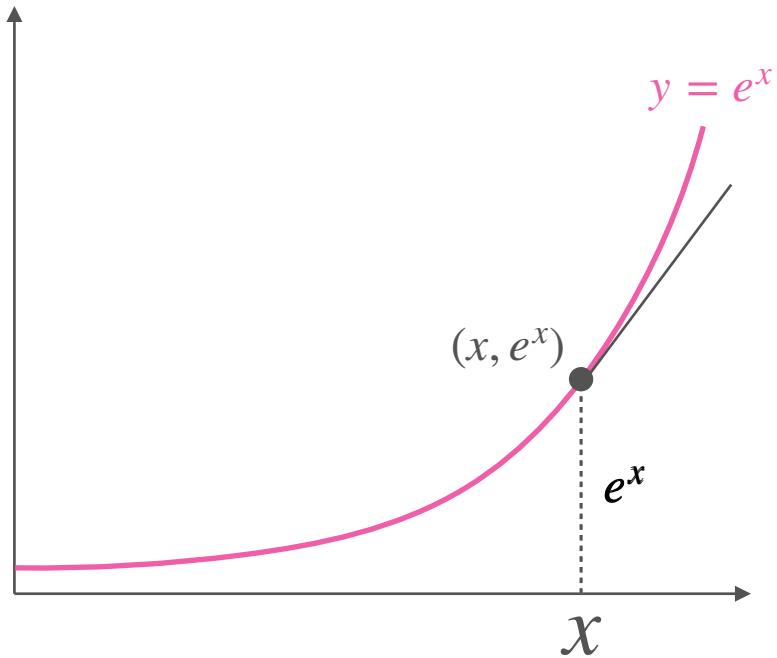
# The Derivative of $e^x$ Is Also $e^x$



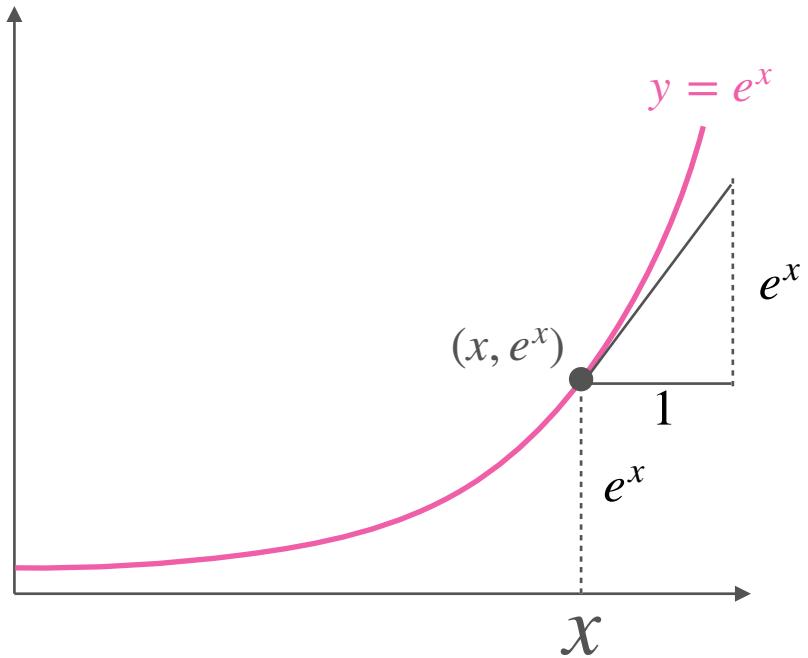
# The Derivative of $e^x$ Is Also $e^x$



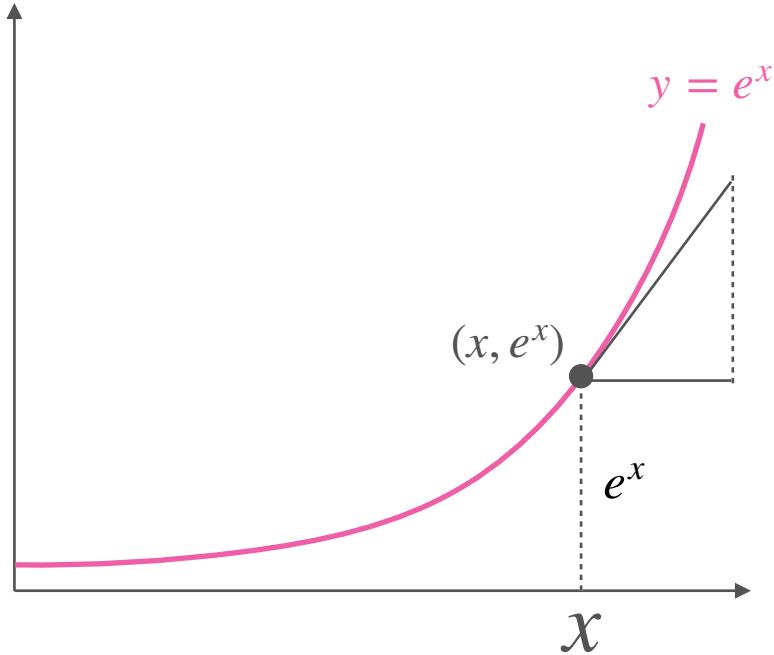
# The Derivative of $e^x$ Is Also $e^x$



# The Derivative of $e^x$ Is Also $e^x$

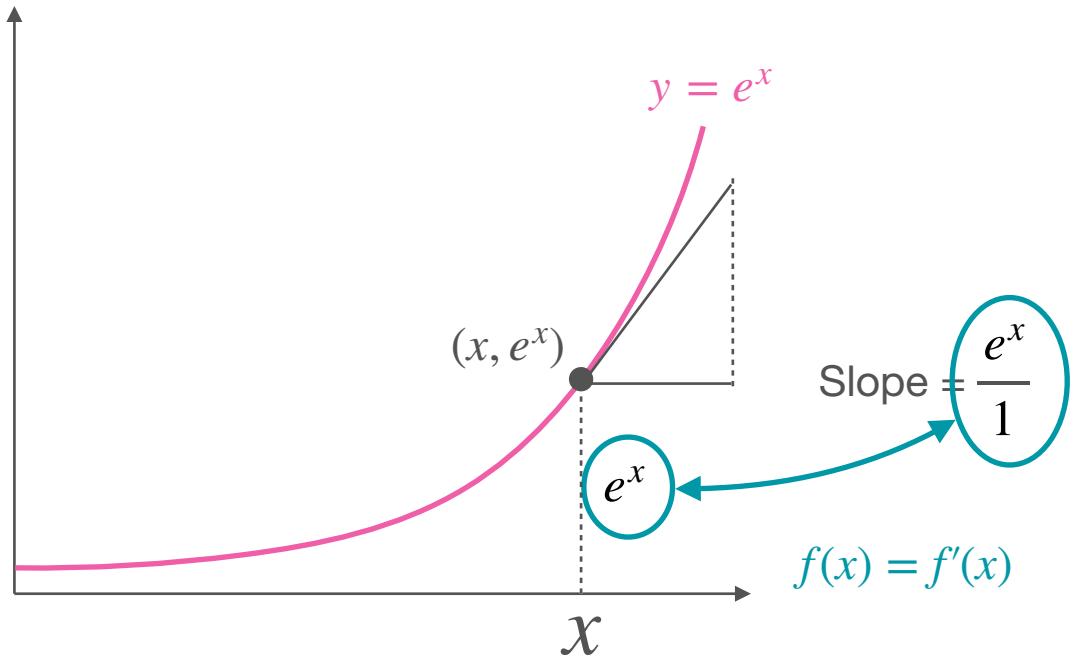


# The Derivative of $e^x$ Is Also $e^x$



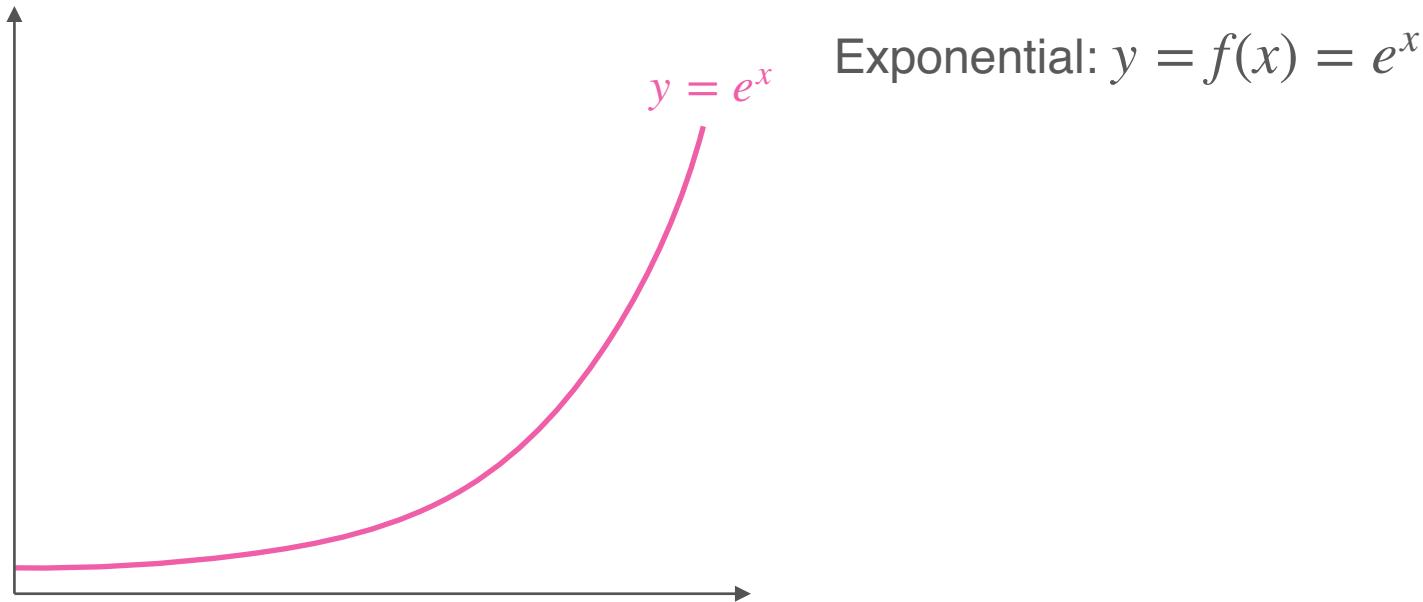
$$\text{Slope} = \frac{e^x}{1}$$

# The Derivative of $e^x$ Is Also $e^x$

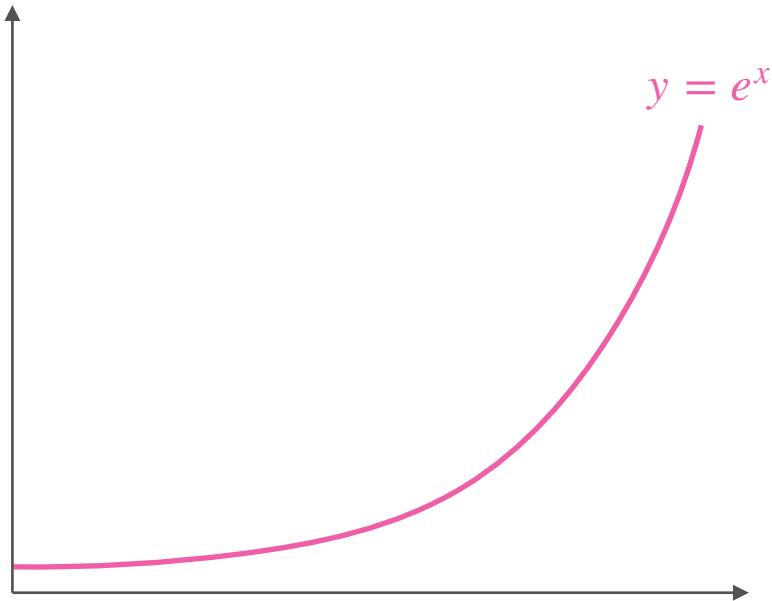


# Derivative of $e^x$

# Derivative of $e^x$



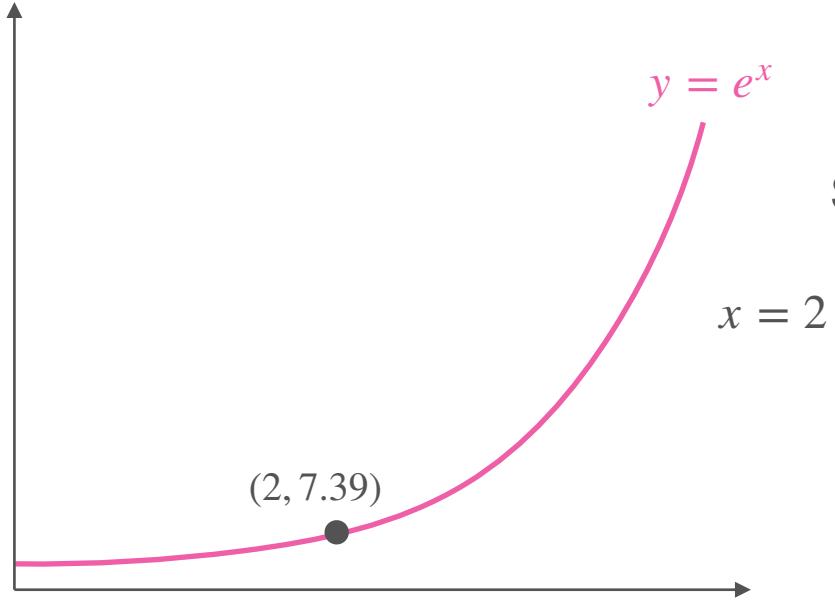
# Derivative of $e^x$



Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

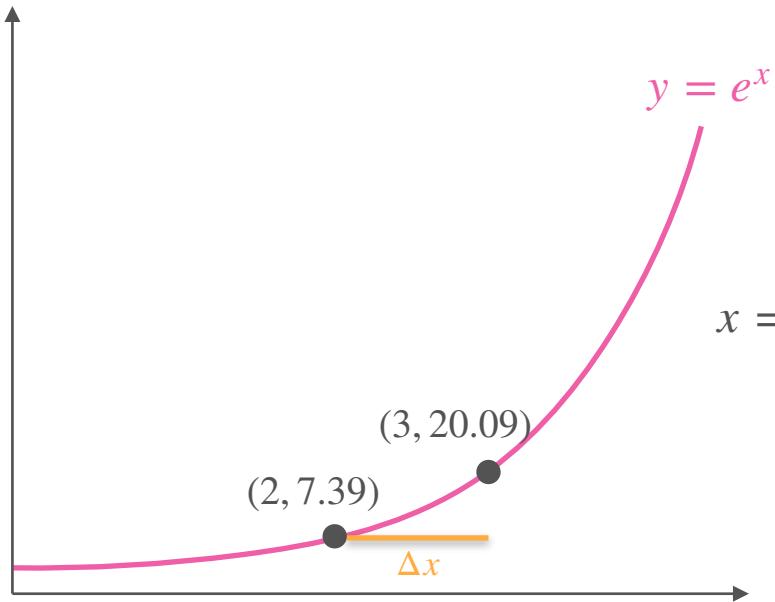
# Derivative of $e^x$



Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

# Derivative of $e^x$

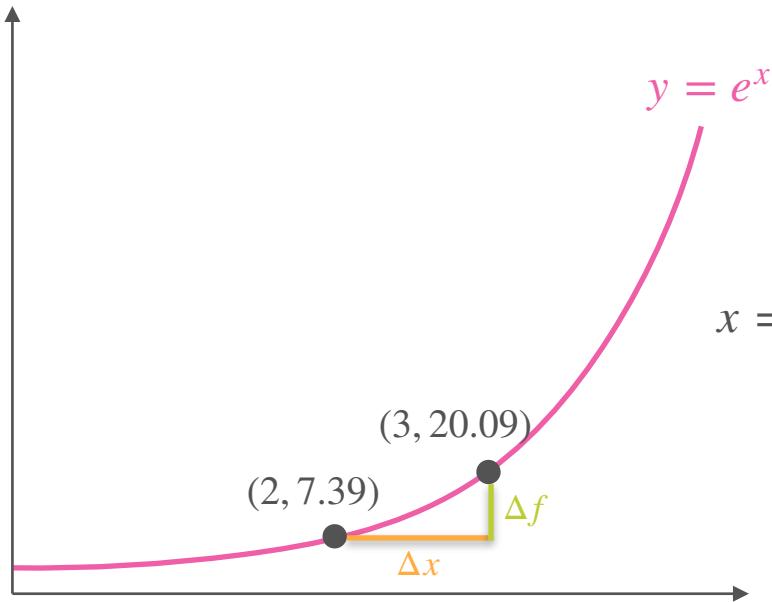


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	$ $	1.0
------------	-----	-----

# Derivative of $e^x$



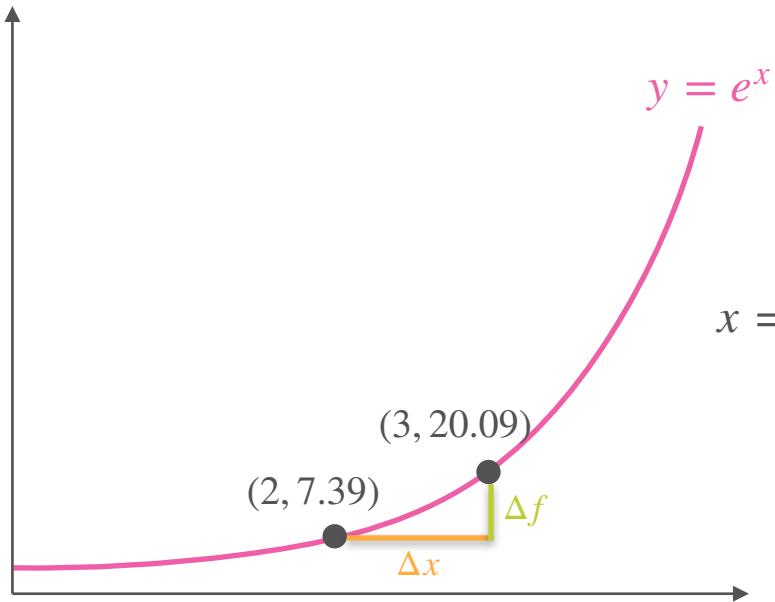
Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	12.70

$$e^{2+1} - e^2 = 20.09 - 7.39$$

# Derivative of $e^x$



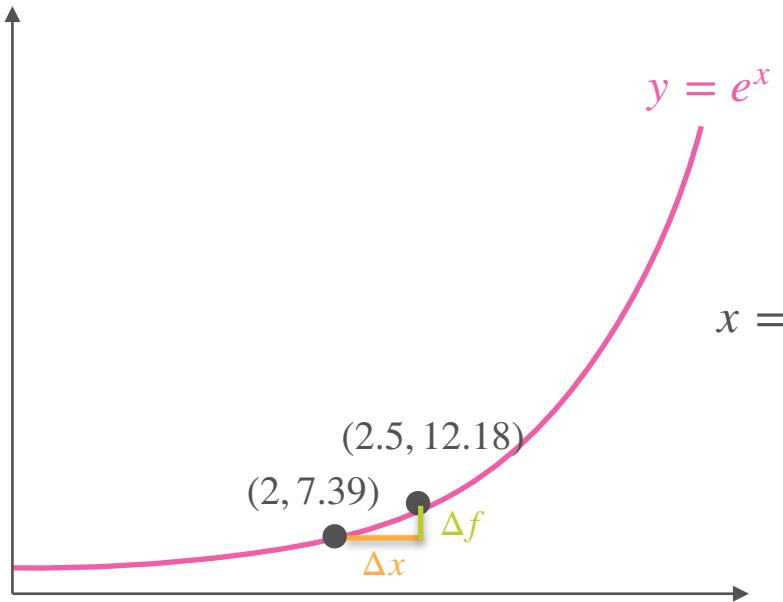
Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0
$\Delta f$	12.70
Slope	12.70

$$\frac{e^{2+1} - e^2}{1} = \frac{20.09 - 7.39}{1}$$

# Derivative of $e^x$

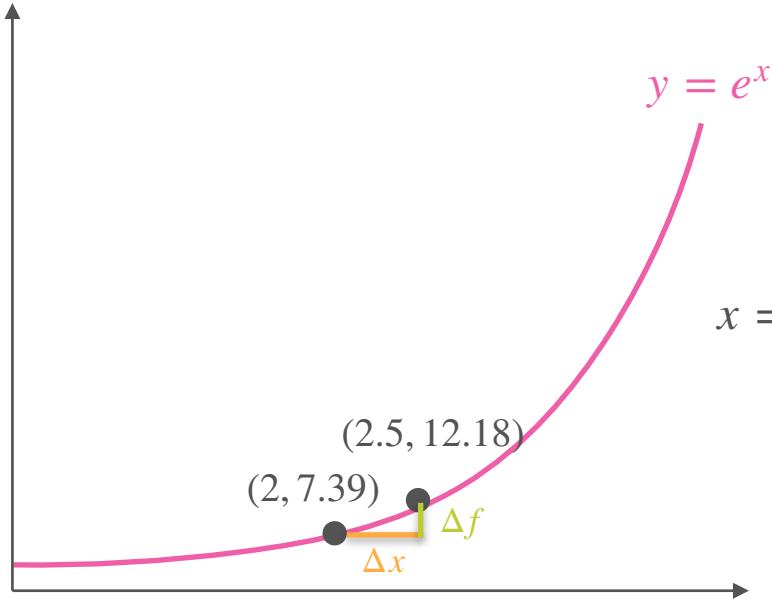


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	12.70	
Slope	12.70	

# Derivative of $e^x$



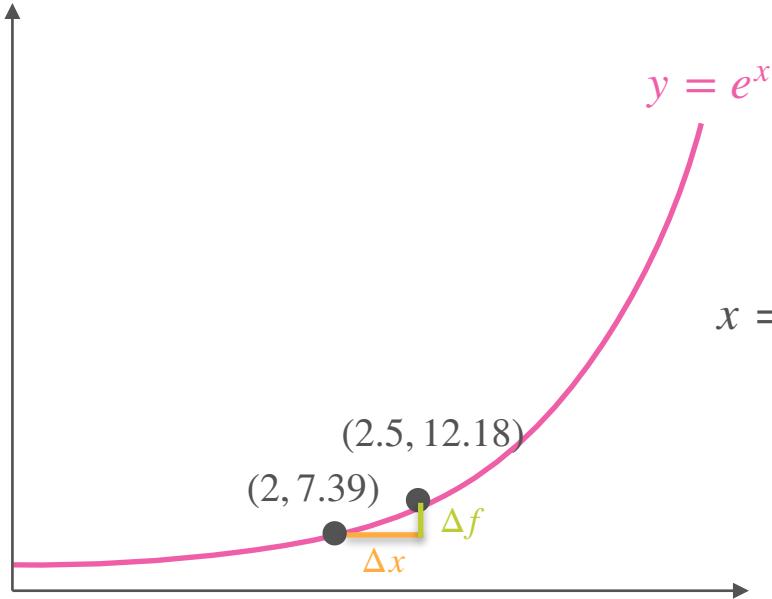
Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	12.70	4.79
Slope	12.70	

$$e^{2+0.5} - e^2 = 12.18 - 7.39$$

# Derivative of $e^x$



Exponential:  $y = f(x) = e^x$

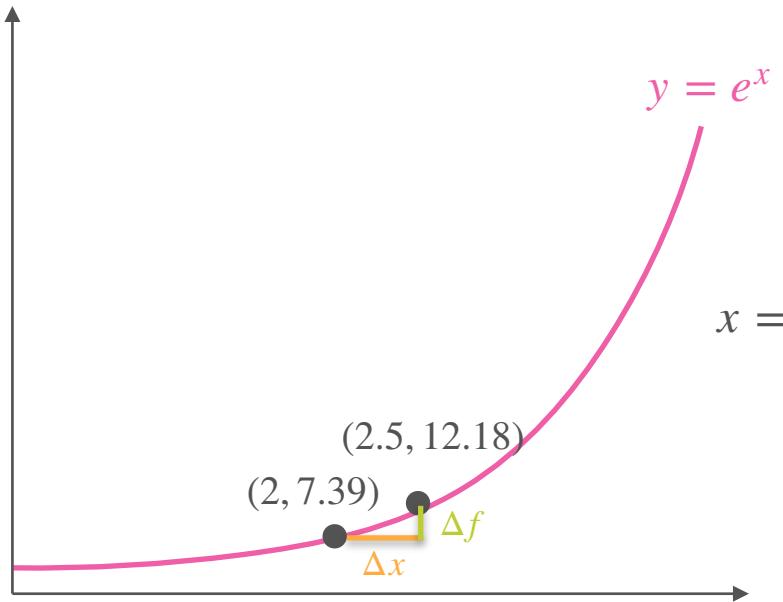
Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2
$\Delta f$	12.70	4.79
Slope	12.70	9.59

$$e^{2+0.5} - e^2 = 12.18 - 7.39$$

$$\frac{4.79}{0.5}$$

# Derivative of $e^x$

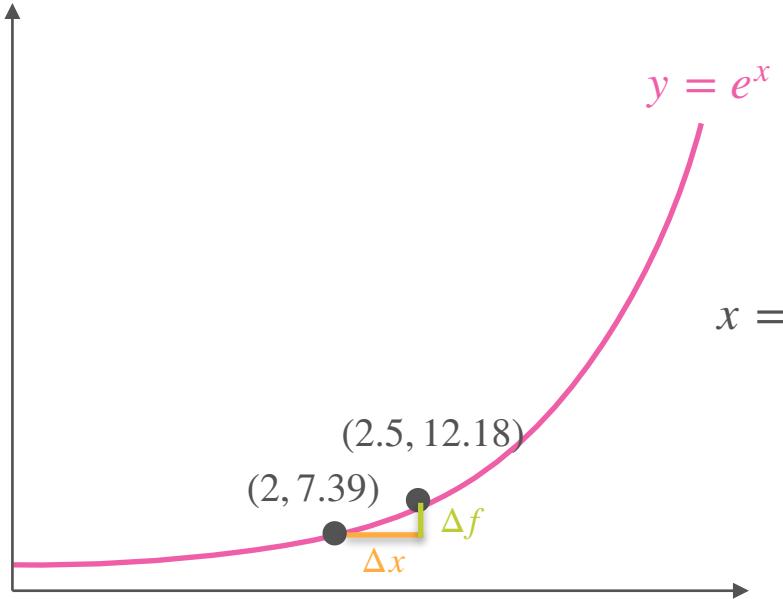


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	12.70	4.79	
Slope	12.70	9.59	

# Derivative of $e^x$

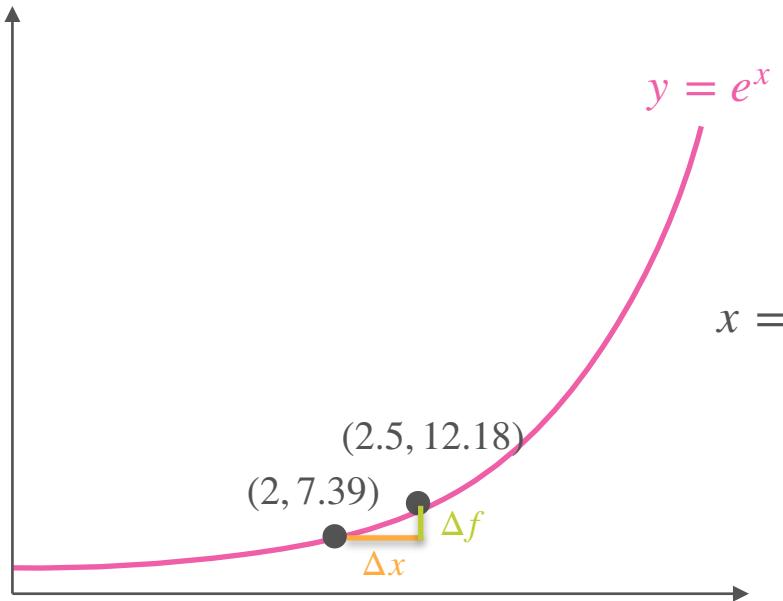


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	12.70	4.79	2.10
Slope	12.70	9.59	

# Derivative of $e^x$

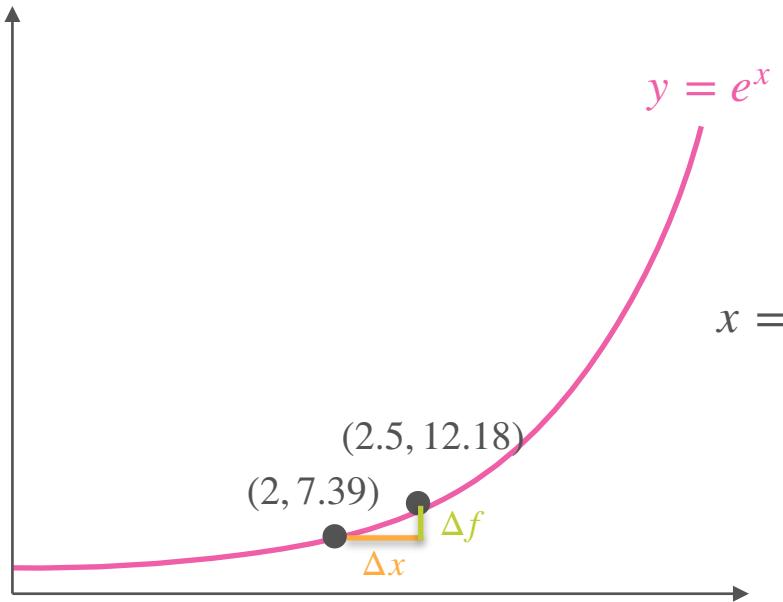


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4
$\Delta f$	12.70	4.79	2.10
Slope	12.70	9.59	8.39

# Derivative of $e^x$

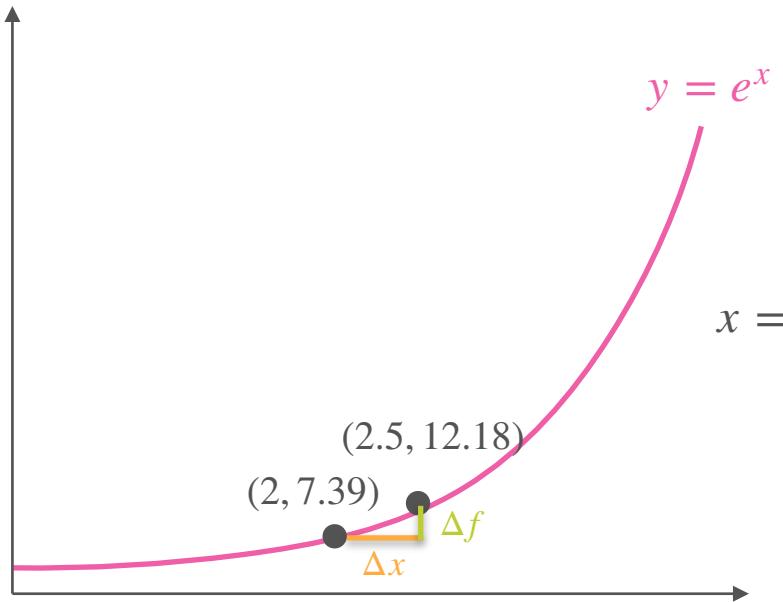


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8
$\Delta f$	12.70	4.79	2.10	
Slope	12.70	9.59	8.39	

# Derivative of $e^x$

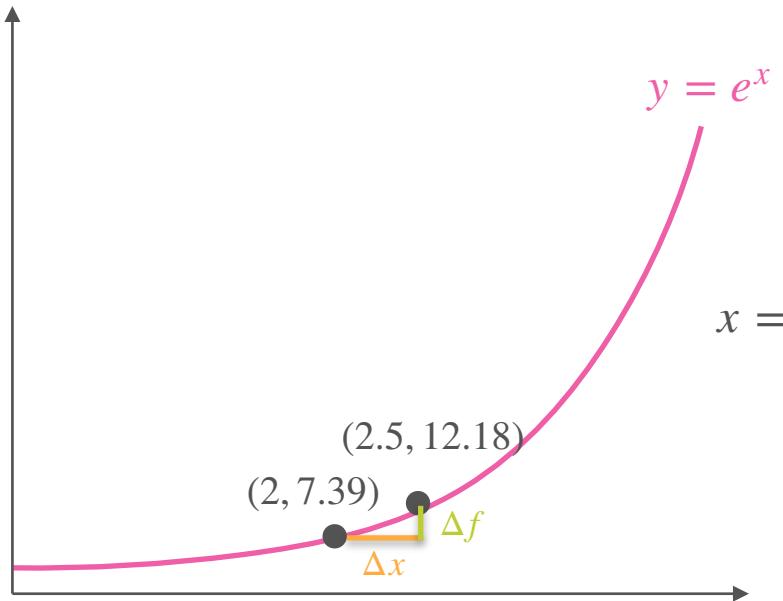


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8
$\Delta f$	12.70	4.79	2.10	0.98
Slope	12.70	9.59	8.39	

# Derivative of $e^x$

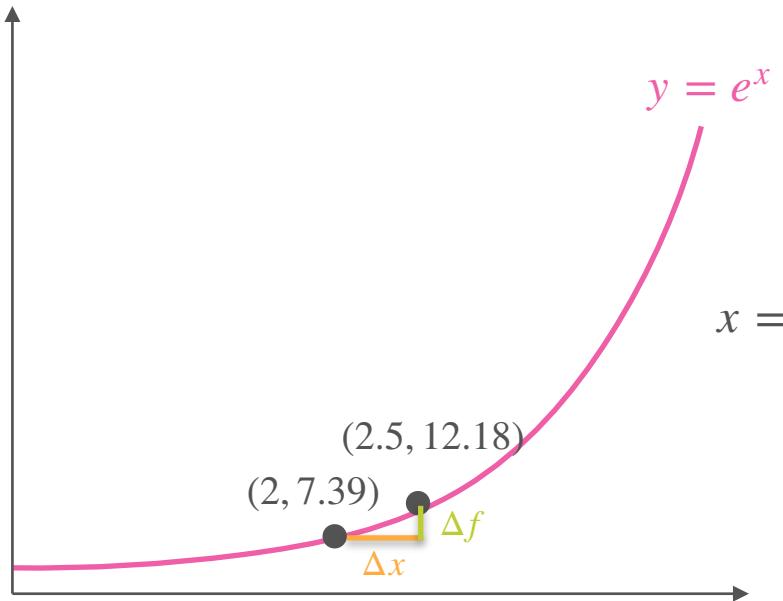


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8
$\Delta f$	12.70	4.79	2.10	0.98
Slope	12.70	9.59	8.39	7.87

# Derivative of $e^x$

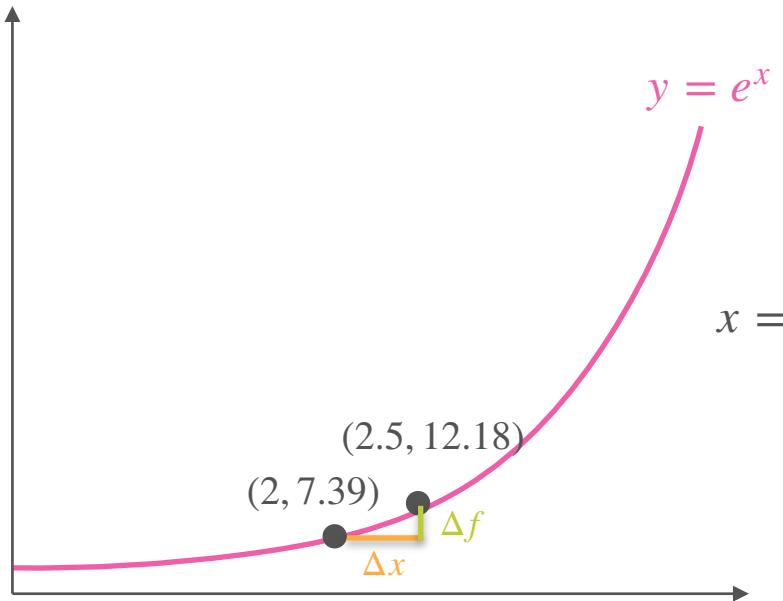


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	12.70	4.79	2.10	0.98	
Slope	12.70	9.59	8.39	7.87	

# Derivative of $e^x$

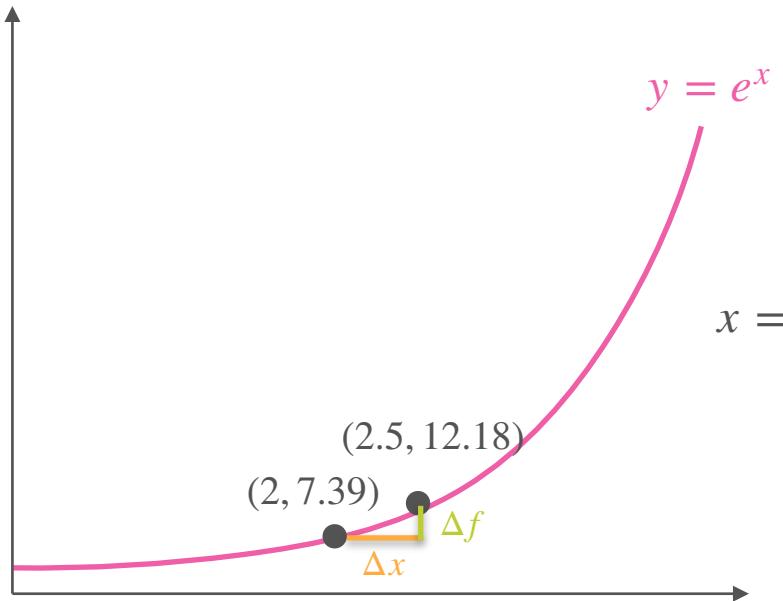


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	12.70	4.79	2.10	0.98	0.48
Slope	12.70	9.59	8.39	7.87	

# Derivative of $e^x$

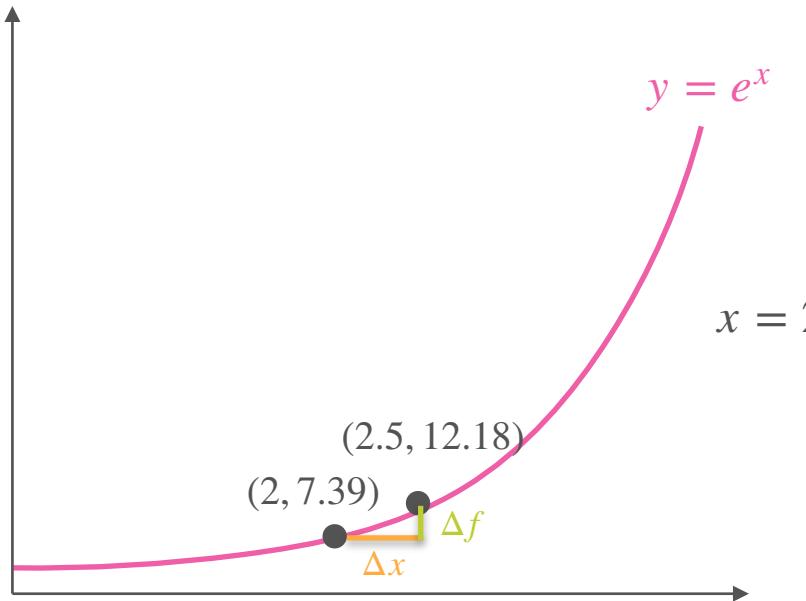


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16
$\Delta f$	12.70	4.79	2.10	0.98	0.48
Slope	12.70	9.59	8.39	7.87	7.62

# Derivative of $e^x$

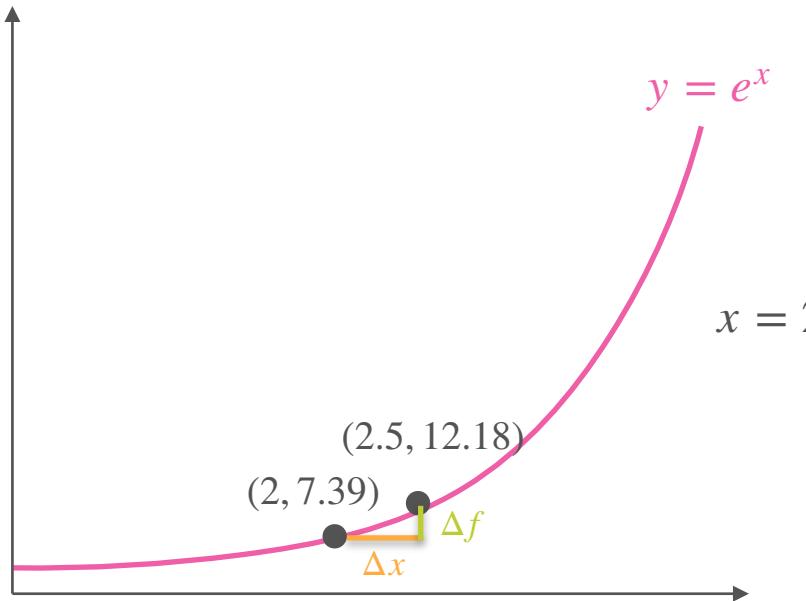


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	12.70	4.79	2.10	0.98	0.48	
Slope	12.70	9.59	8.39	7.87	7.62	

# Derivative of $e^x$

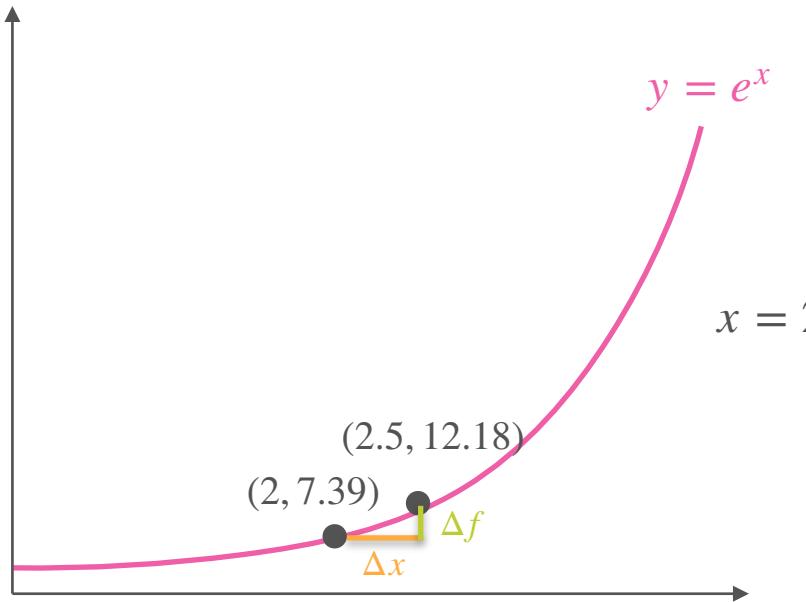


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	

# Derivative of $e^x$

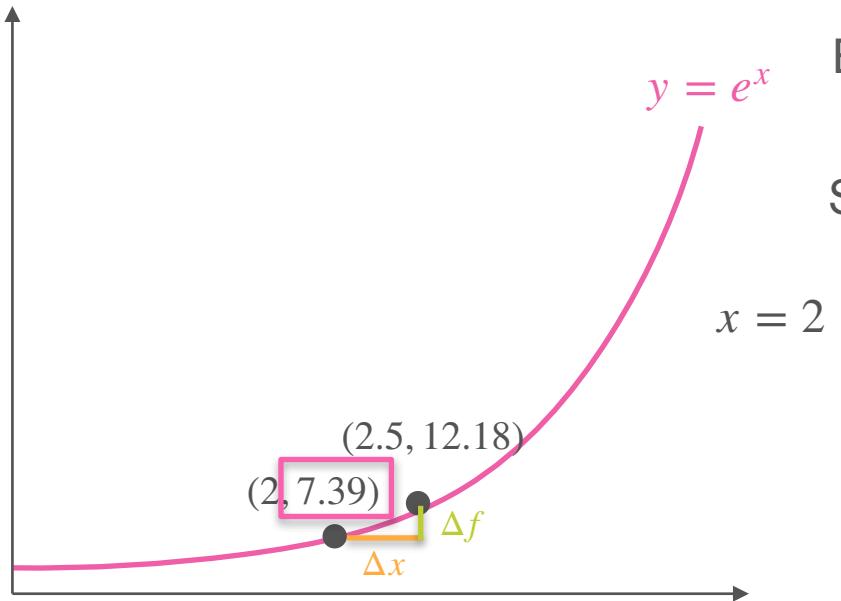


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	7.39

# Derivative of $e^x$

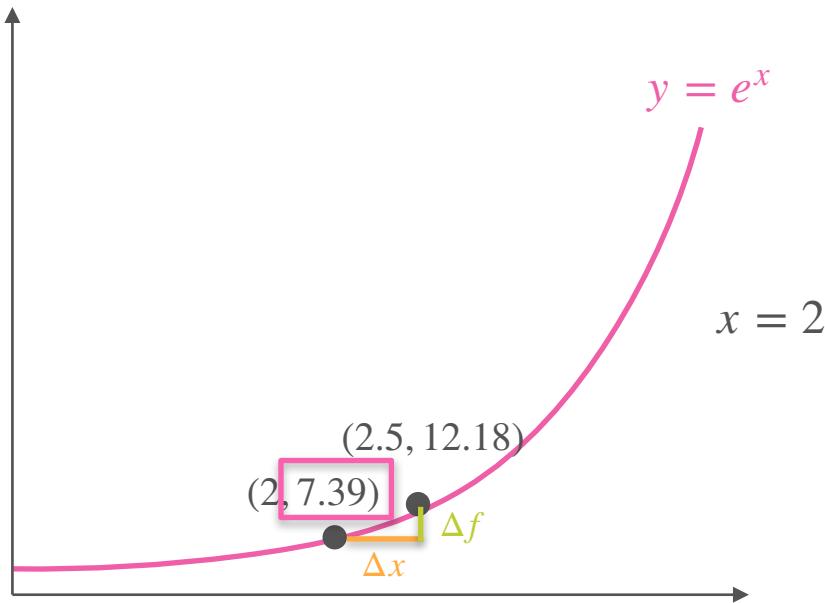


Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	7.39

# Derivative of $e^x$



Exponential:  $y = f(x) = e^x$

Slope:  $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$\Delta x$	1.0	1/2	1/4	1/8	1/16	1/1000
$\Delta f$	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	7.39

$$e^2$$

# (Reading Item) Derivative of $a^x$

$$a^x = e^{x \log(a)}$$

$$\frac{d}{dx} a^x = \log(a) e^{x \log(a)} = \log(a) a^x$$



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# Derivatives and Optimization

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**The derivative of  $\log(x)$**

# Logarithm

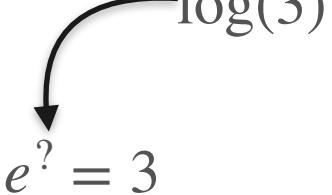
# Logarithm

$$e^? = 3$$

# Logarithm

$$e^? = 3$$

$\log(3)$



# Logarithm

$$\log(3)$$

$$e^? = 3 \qquad \qquad e^? = x$$

# Logarithm

$$\begin{array}{ll} \log(3) & \log(x) \\ \downarrow & \downarrow \\ e^? = 3 & e^? = x \end{array}$$

# Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x$$

# Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x$$

$$f^{-1}(y) = \log(y)$$

# Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x \quad f^{-1}(y) = \log(y)$$

$$e^{\log(x)} = x$$

# Logarithm

$$e^? = 3 \quad \log(3)$$
$$e^? = x \quad \log(x)$$

$$f(x) = e^x \quad f^{-1}(y) = \log(y)$$

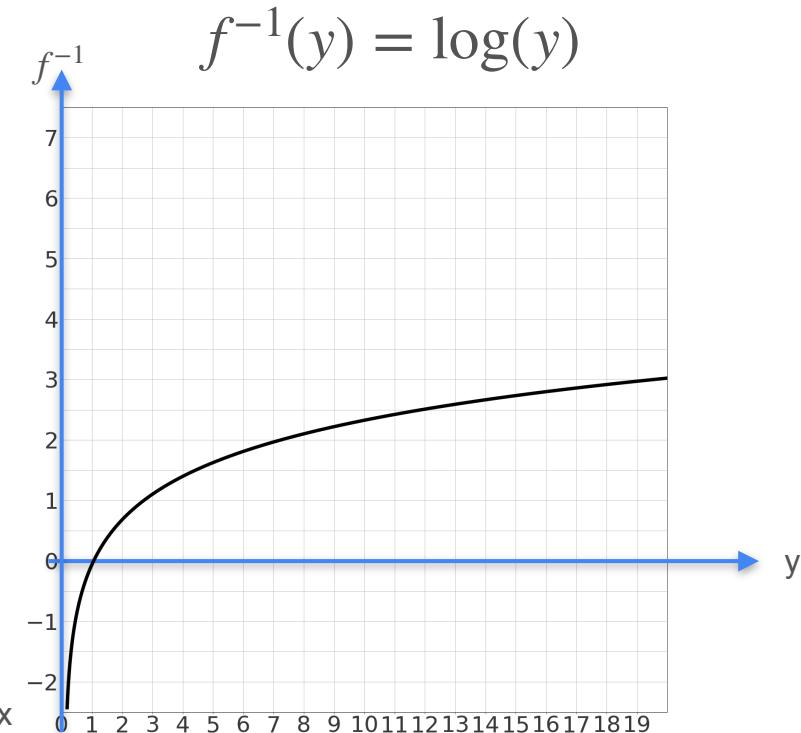
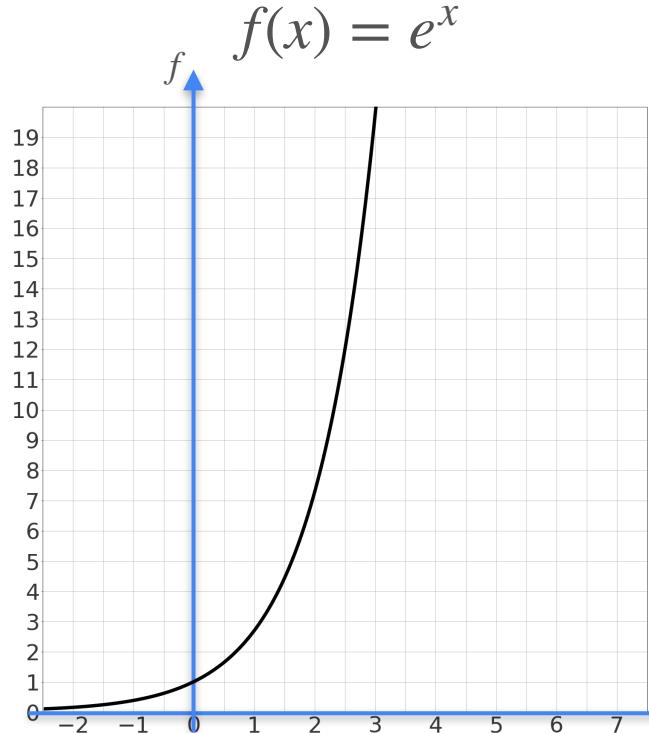
$$e^{\log(x)} = x \quad \log(e^y) = y$$

# Logarithm

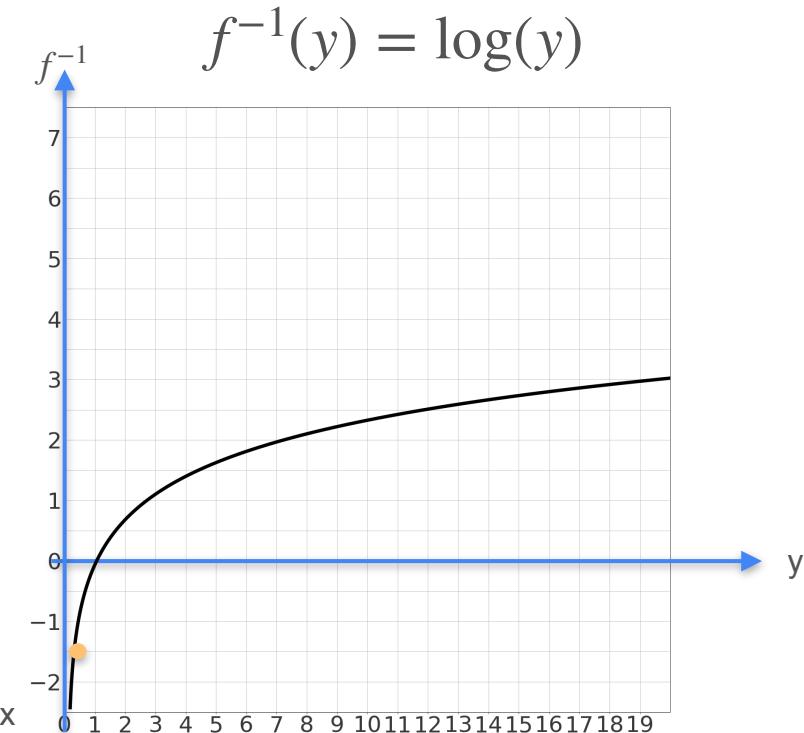
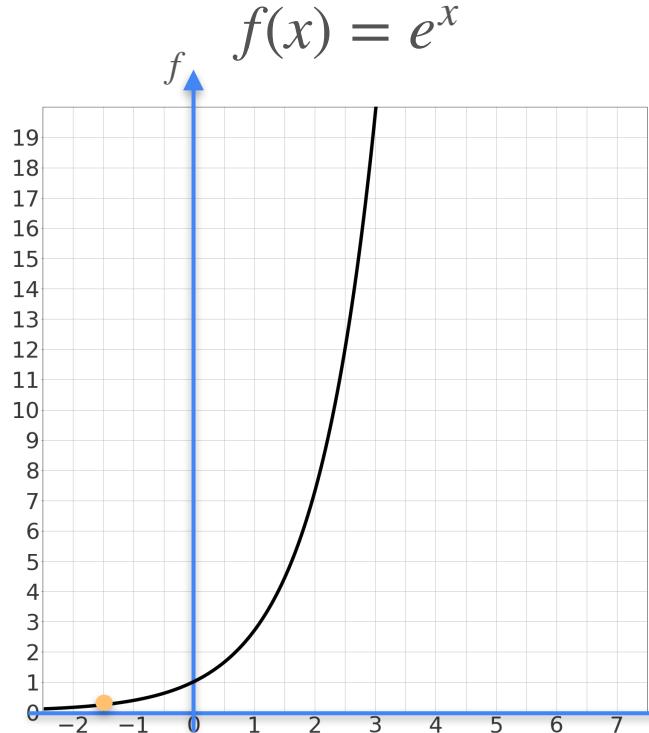
$$f(x) = e^x$$

$$f^{-1}(y) = \log(y)$$

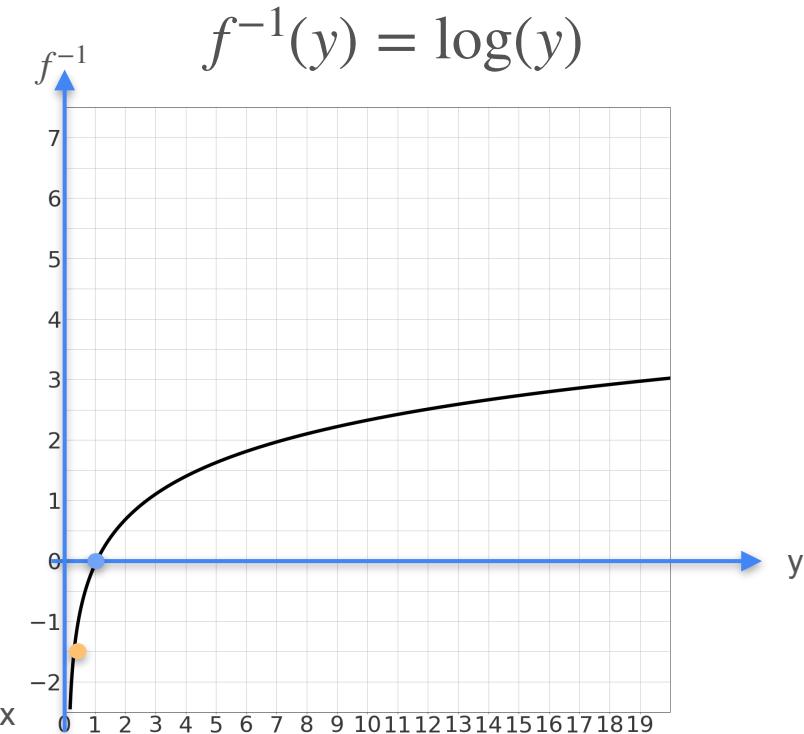
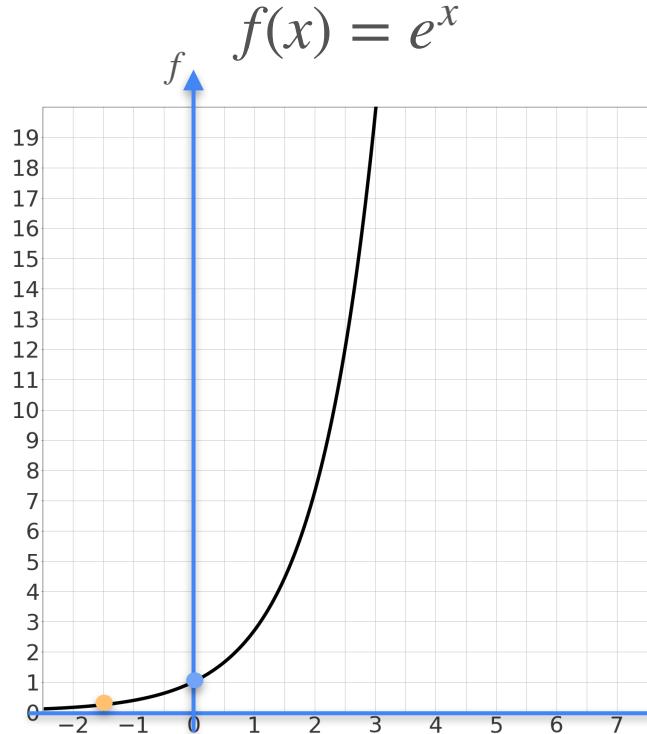
# Logarithm



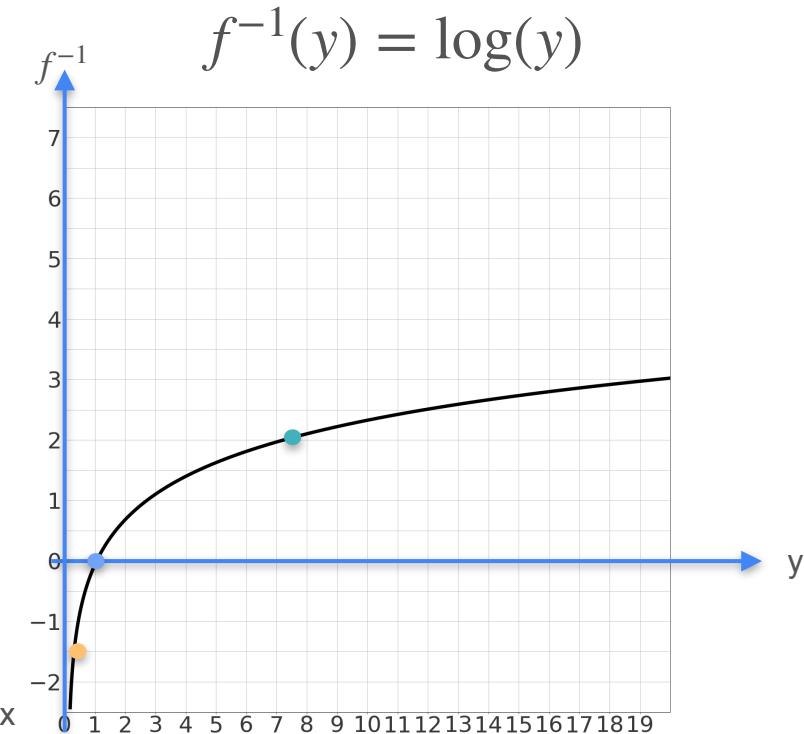
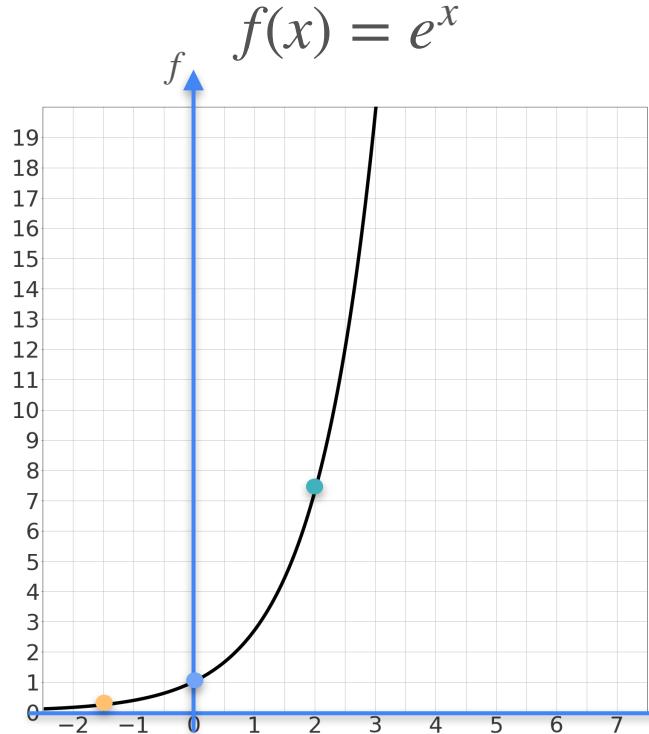
# Logarithm



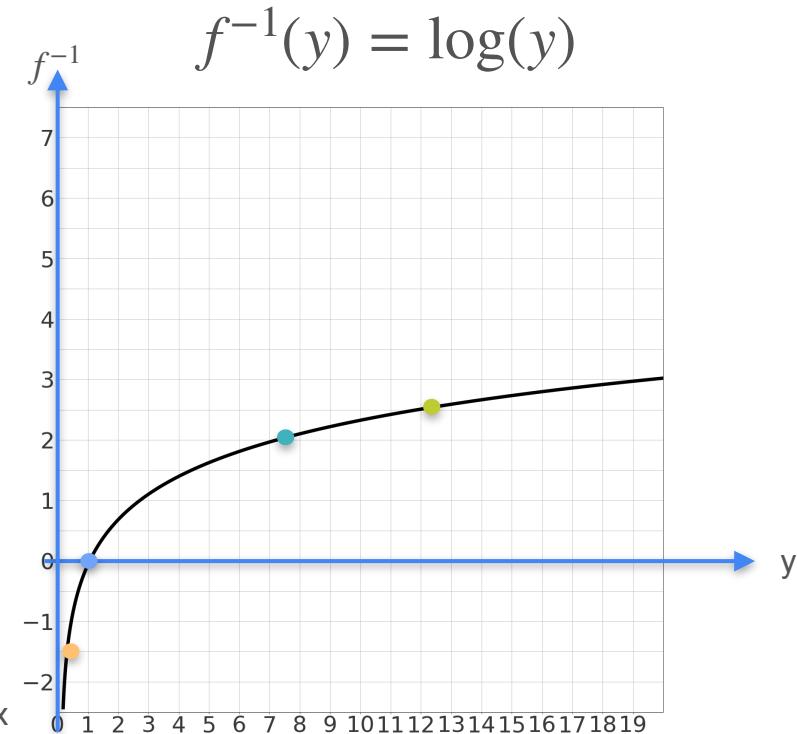
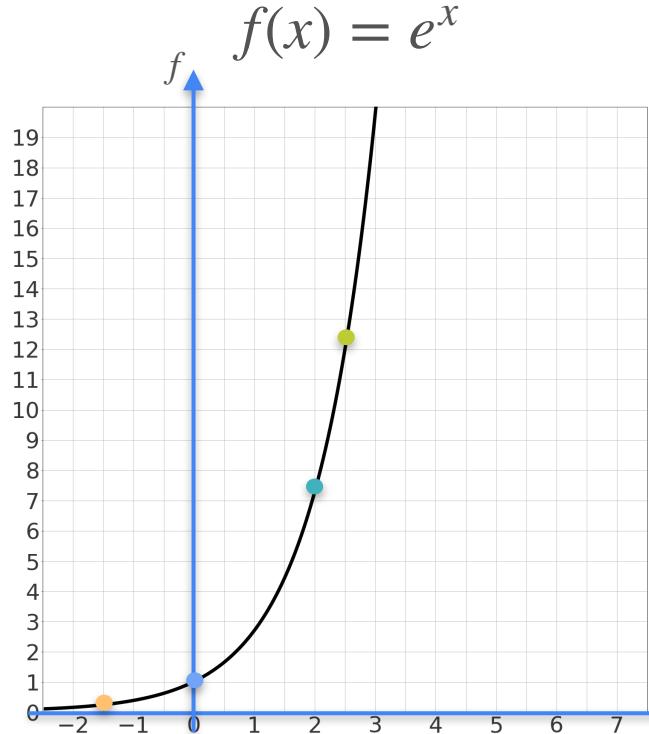
# Logarithm



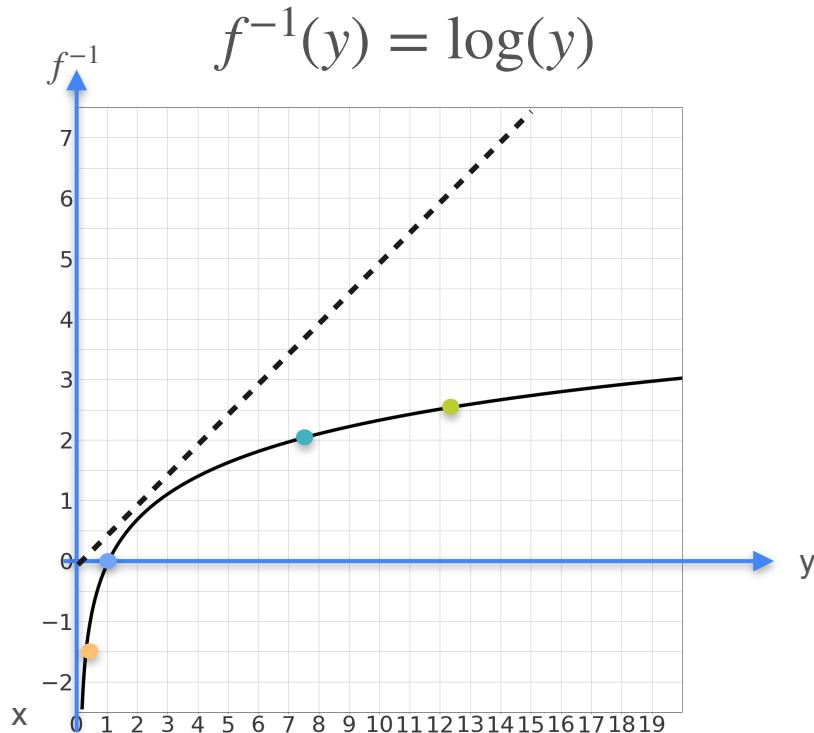
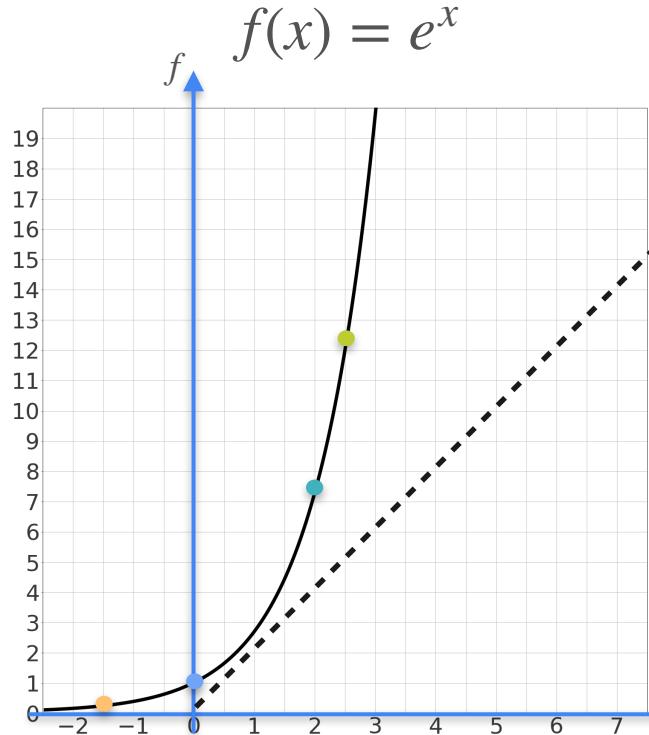
# Logarithm



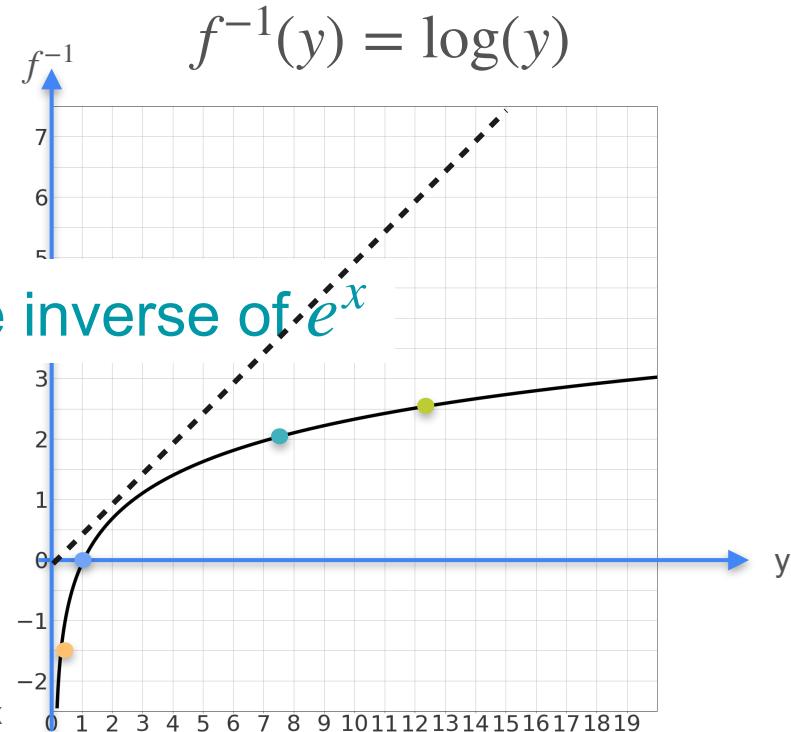
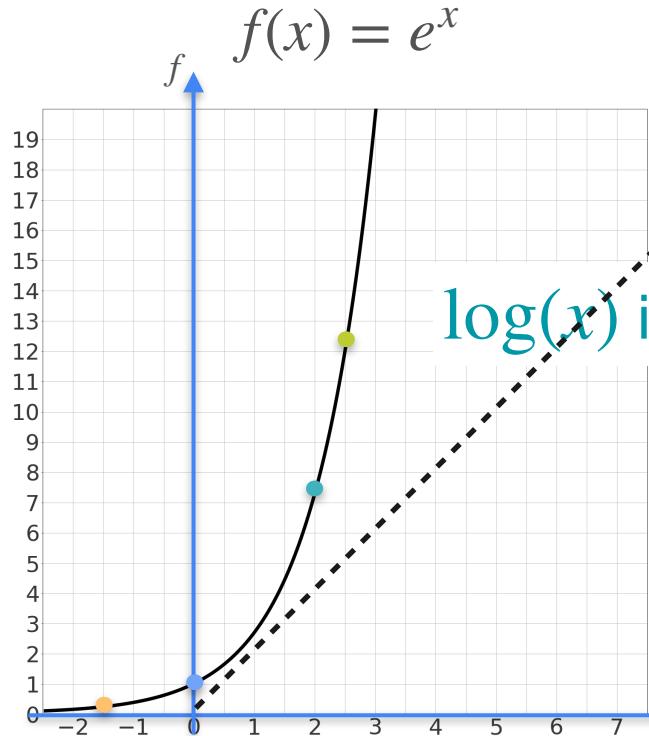
# Logarithm



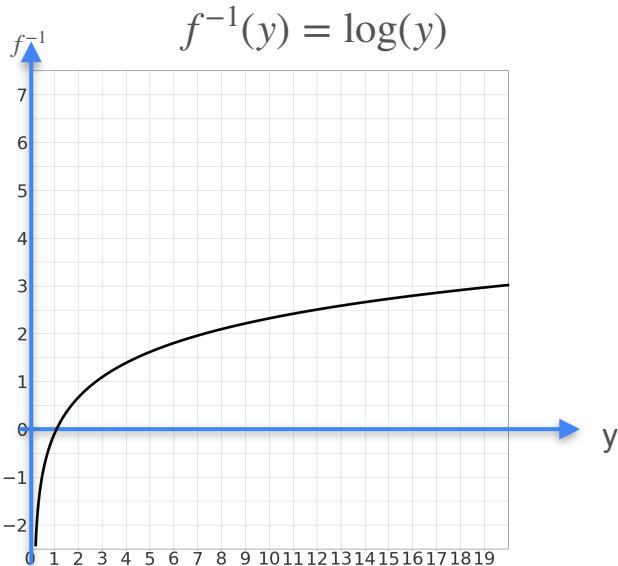
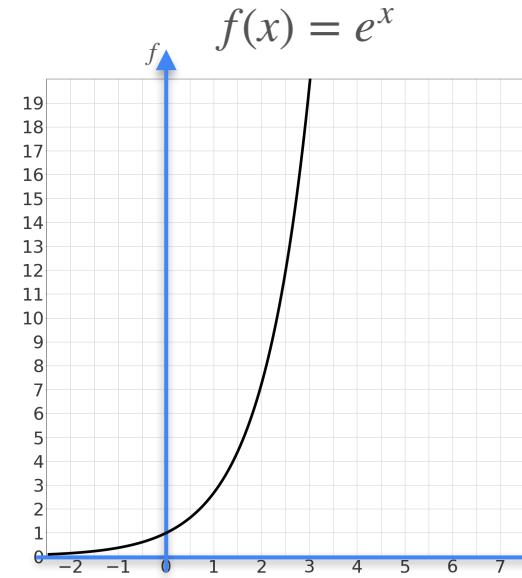
# Logarithm



# Logarithm

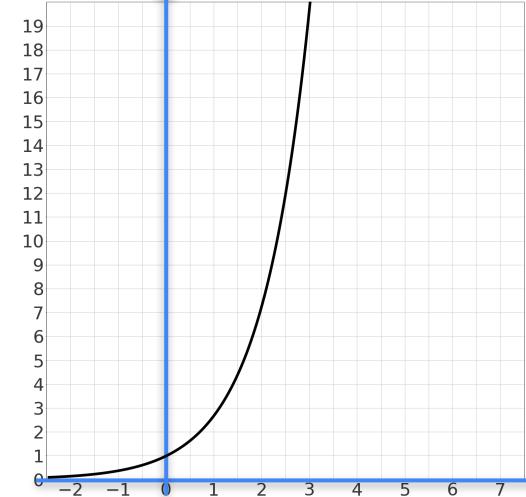


# Logarithm

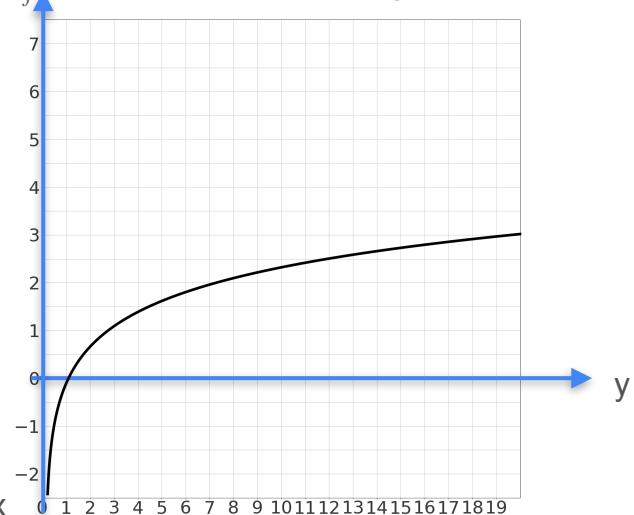


# Logarithm

$$f(x) = e^x$$



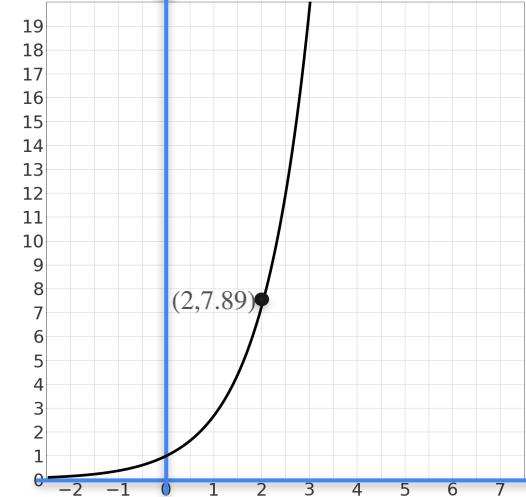
$$f^{-1}(y) = \log(y)$$



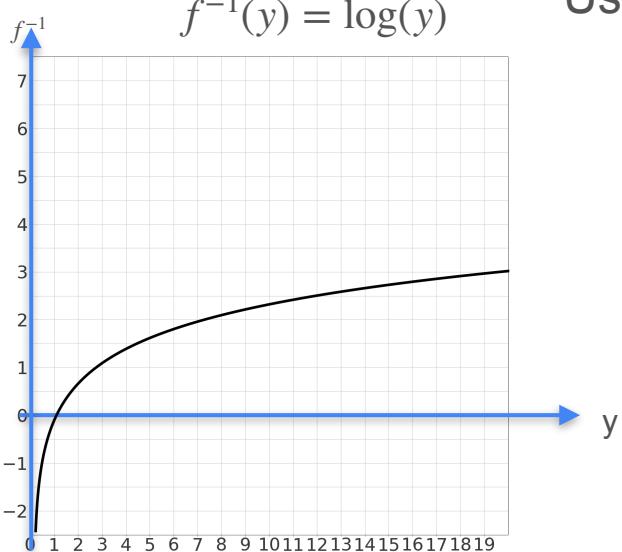
Using the result for inverses

# Logarithm

$$f(x) = e^x$$



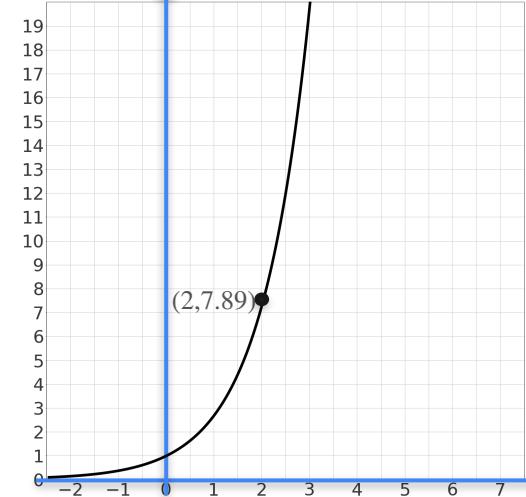
$$f^{-1}(y) = \log(y)$$



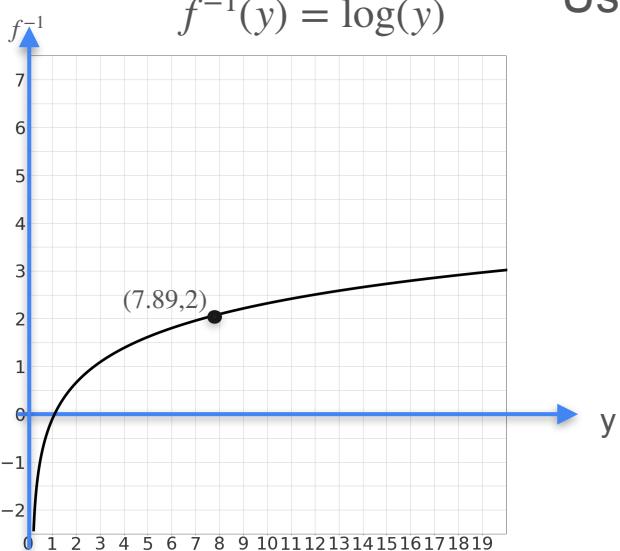
Using the result for inverses

# Logarithm

$$f(x) = e^x$$



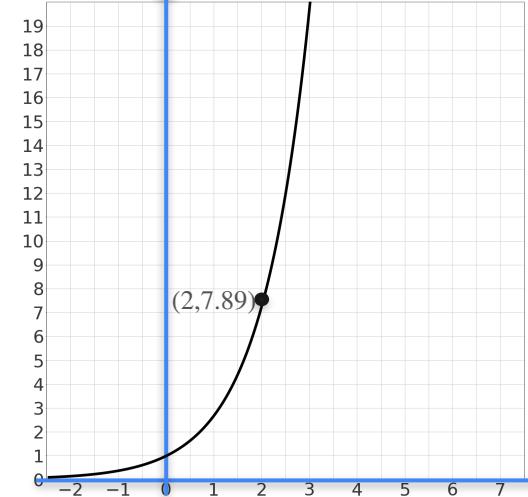
$$f^{-1}(y) = \log(y)$$



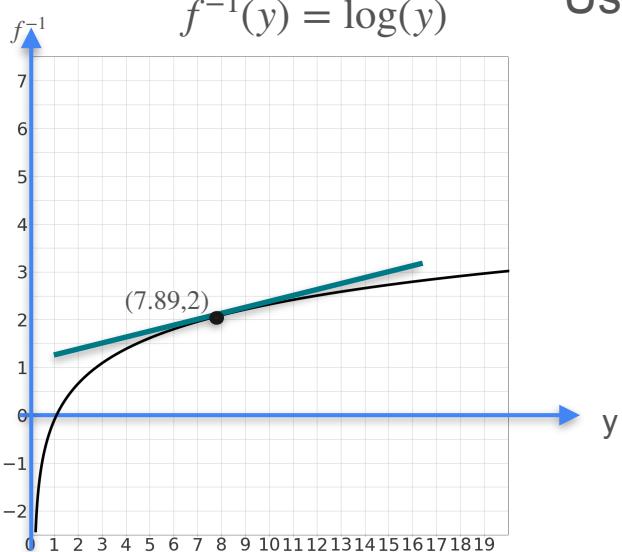
Using the result for inverses

# Logarithm

$$f(x) = e^x$$



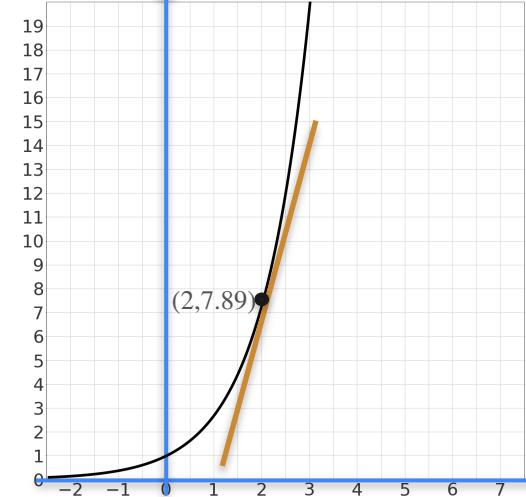
$$f^{-1}(y) = \log(y)$$



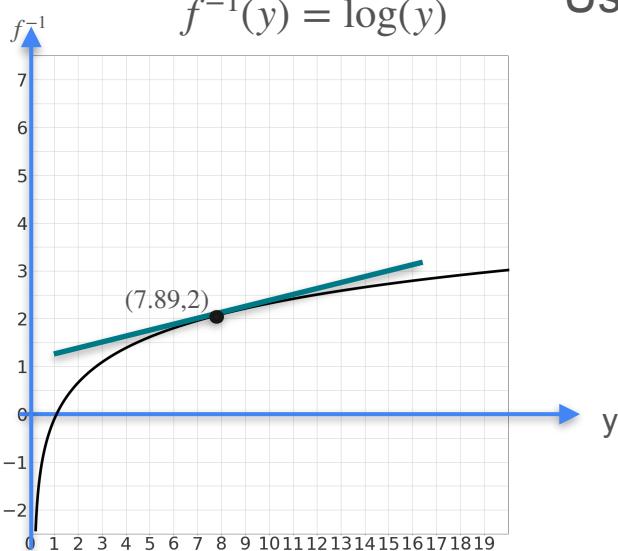
Using the result for inverses

# Logarithm

$$f(x) = e^x$$



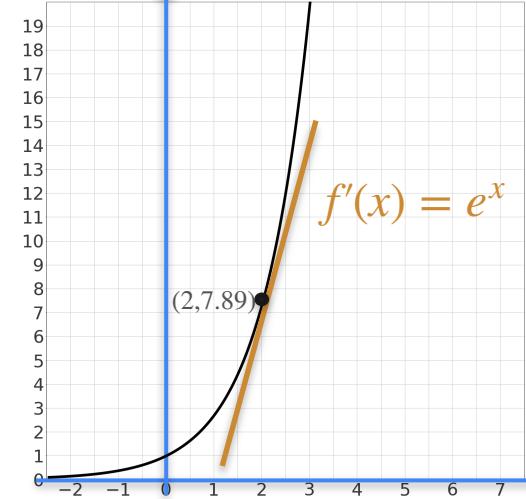
$$f^{-1}(y) = \log(y)$$



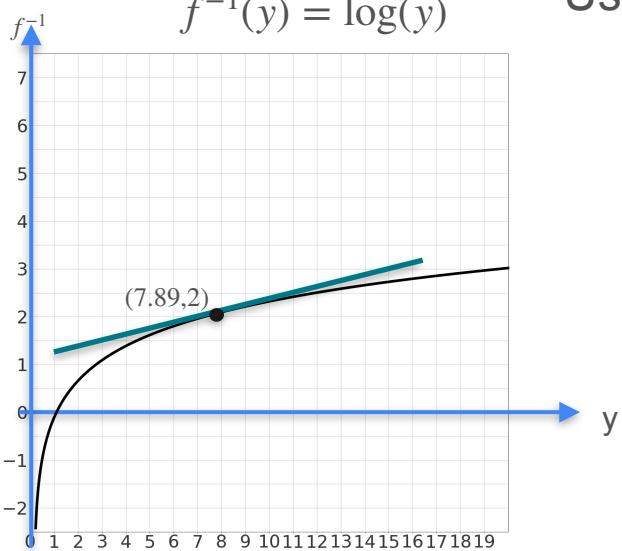
Using the result for inverses

# Logarithm

$$f(x) = e^x$$

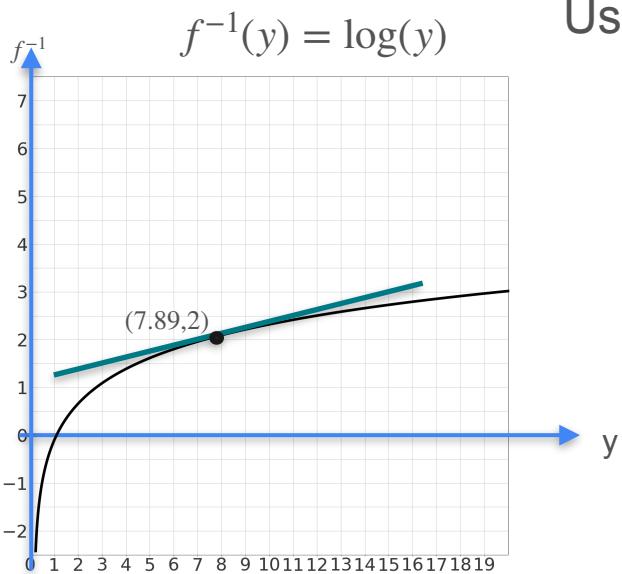
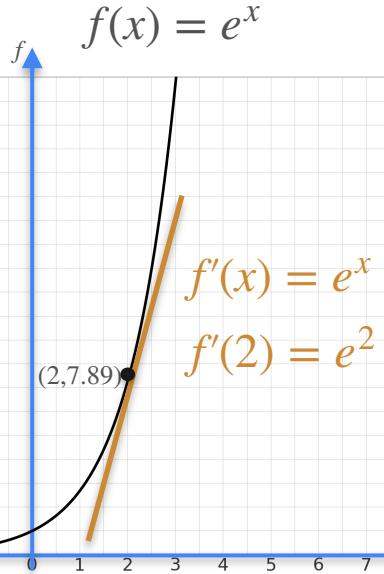


$$f^{-1}(y) = \log(y)$$



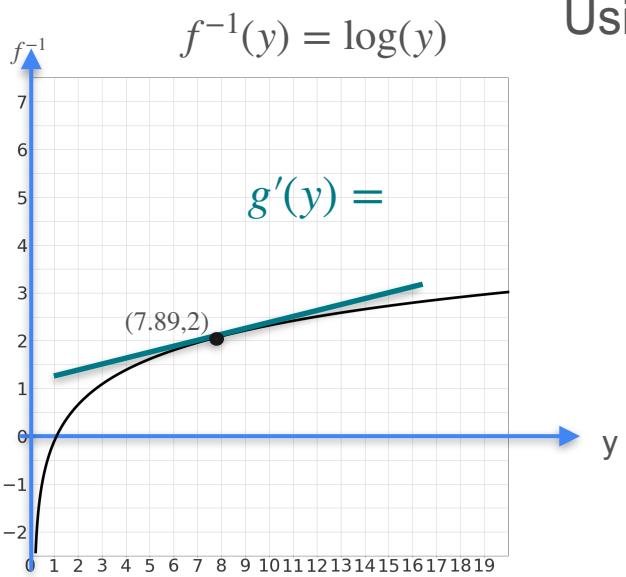
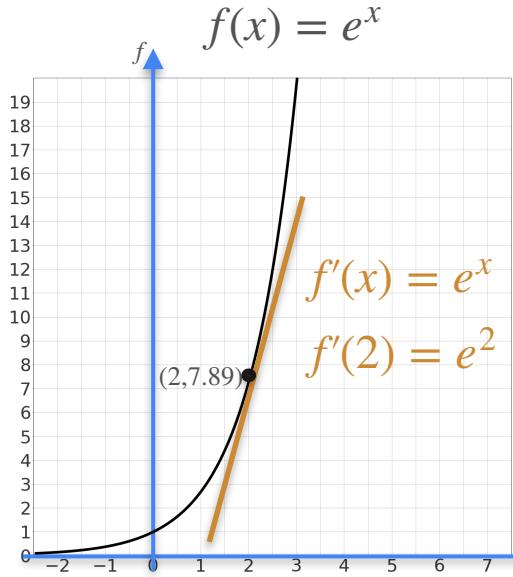
Using the result for inverses

# Logarithm



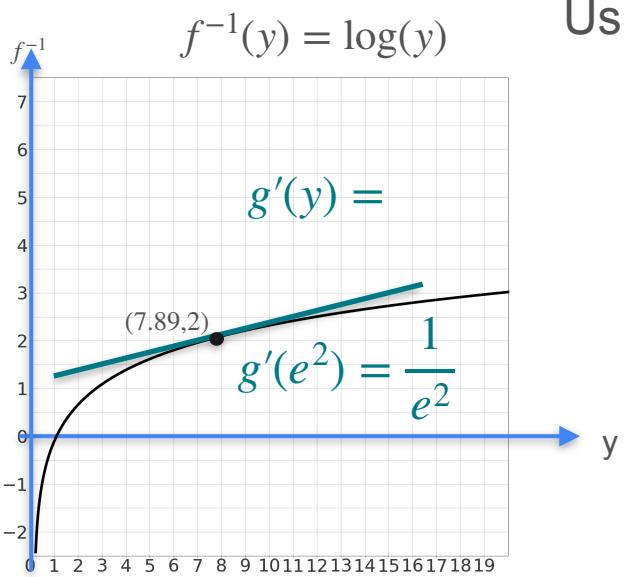
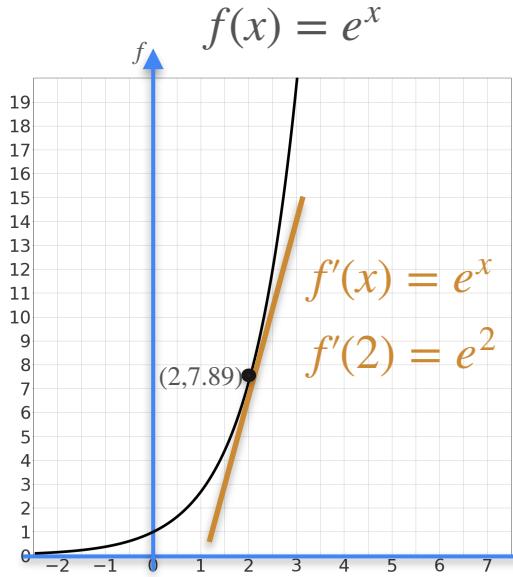
Using the result for inverses

# Logarithm



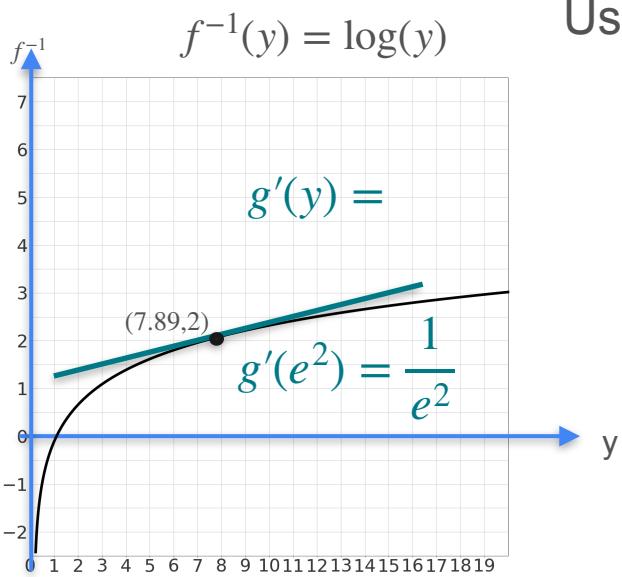
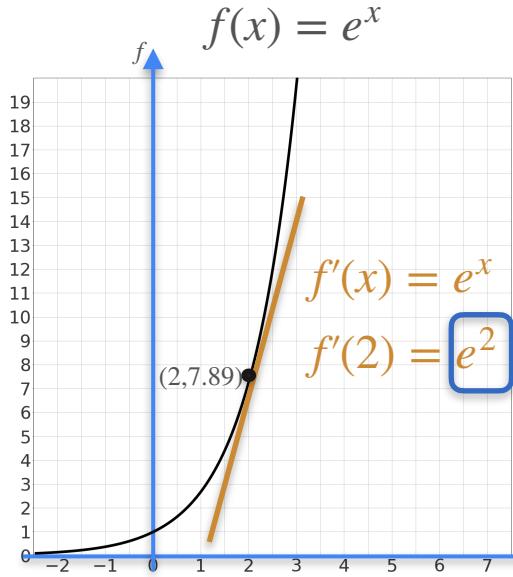
Using the result for inverses

# Logarithm



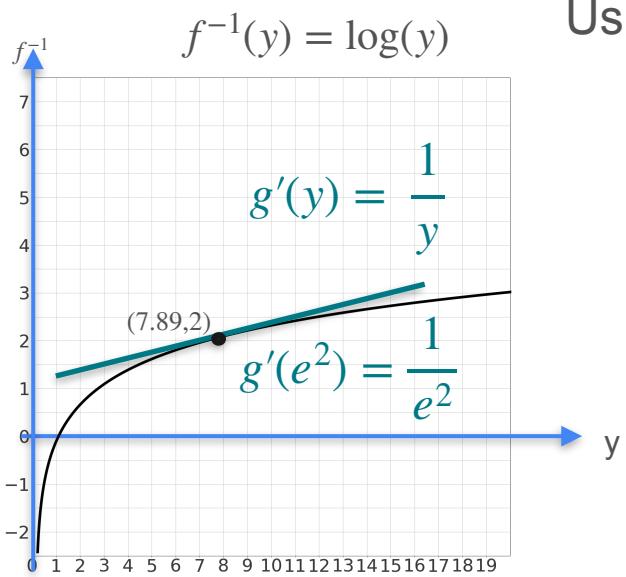
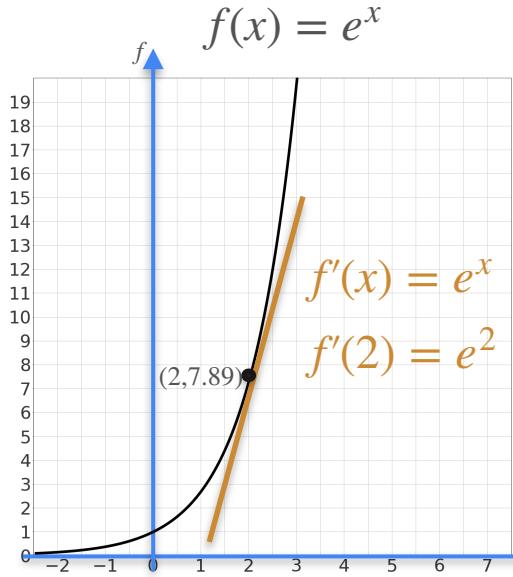
Using the result for inverses

# Logarithm



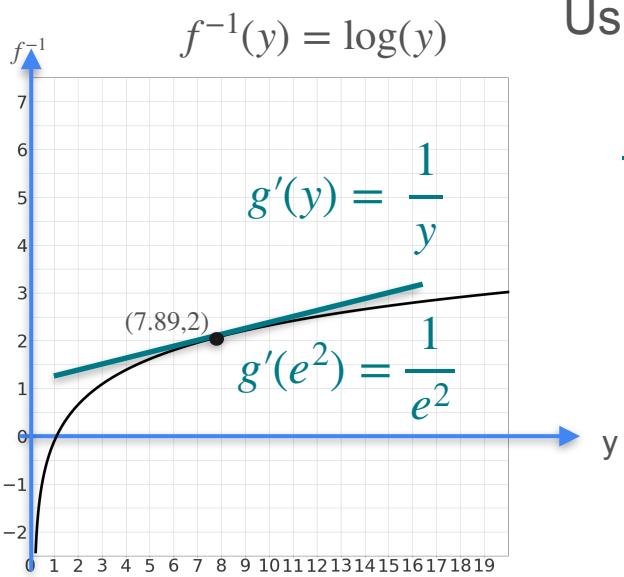
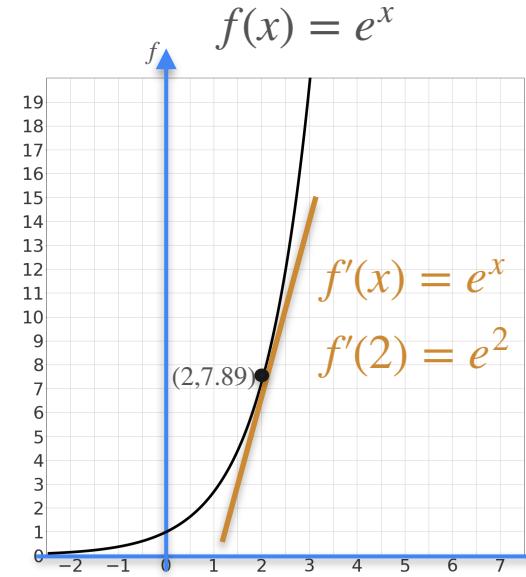
Using the result for inverses

# Logarithm



Using the result for inverses

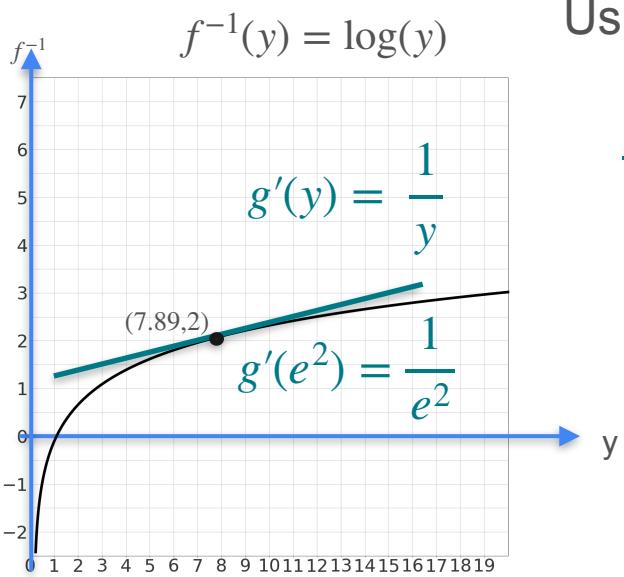
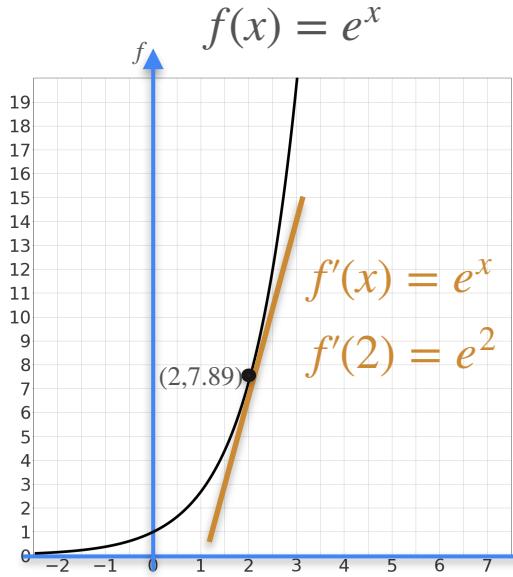
# Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(x)}$$

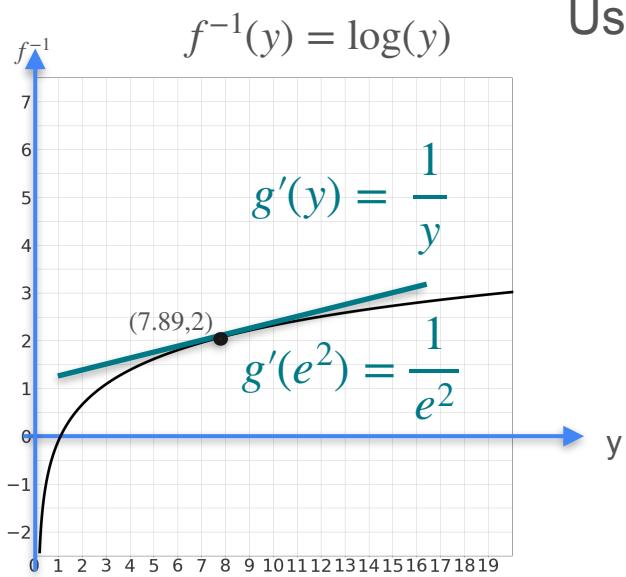
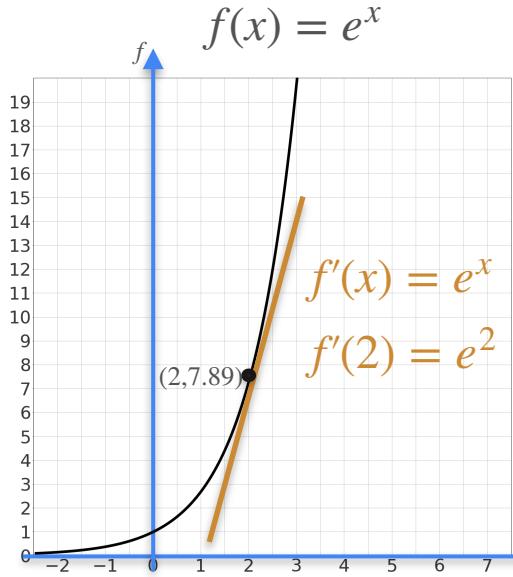
# Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

# Logarithm

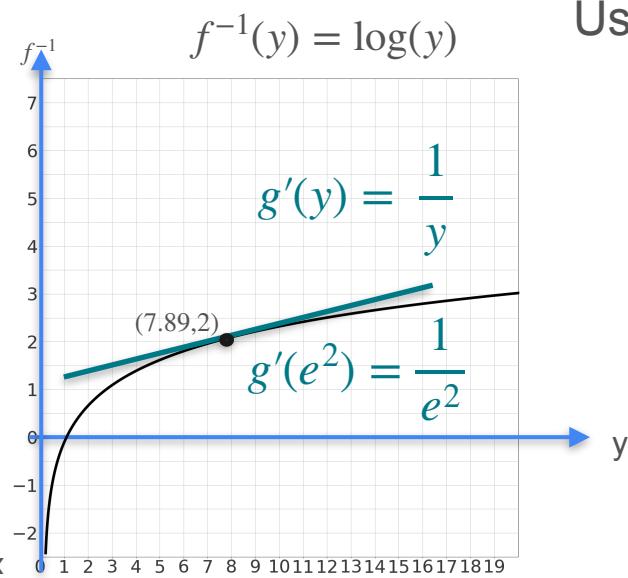
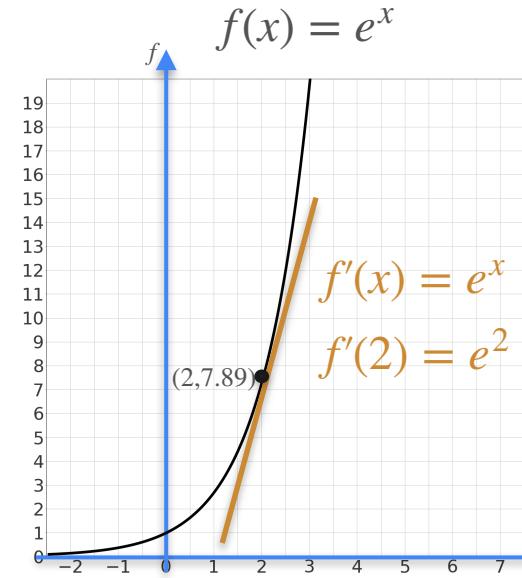


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$\log(y)$

# Logarithm

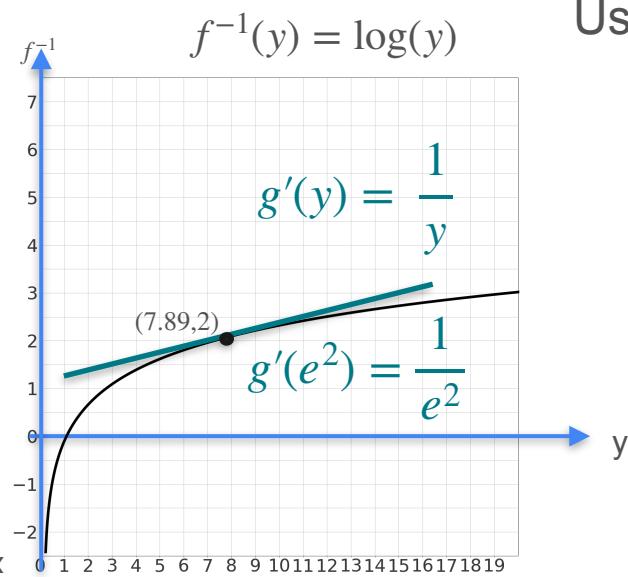
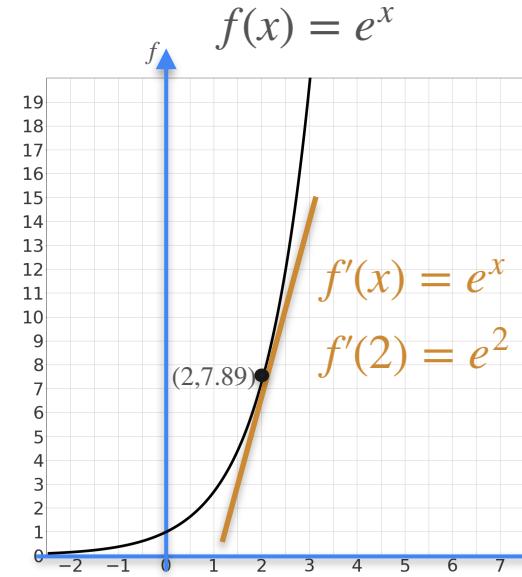


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$$\frac{d}{dy} \log(y)$$

# Logarithm

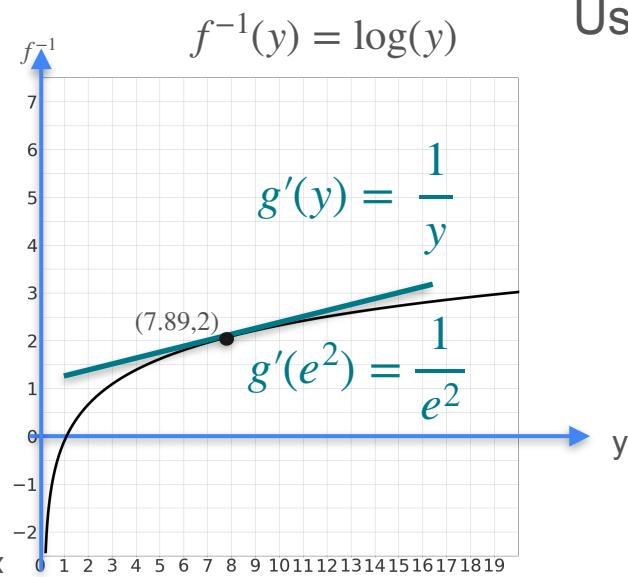
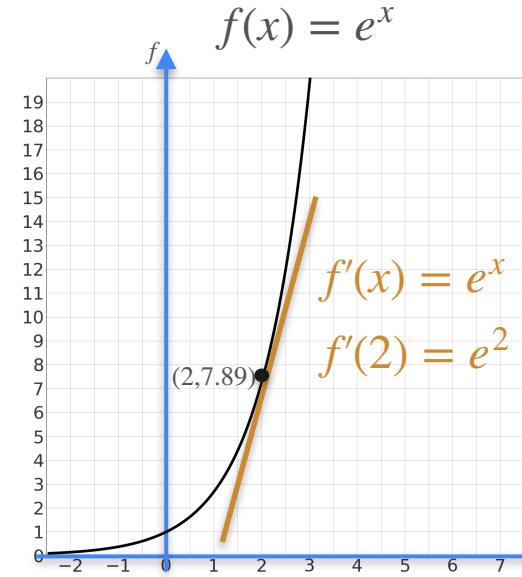


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y) = \frac{1}{y}$$

# Logarithm

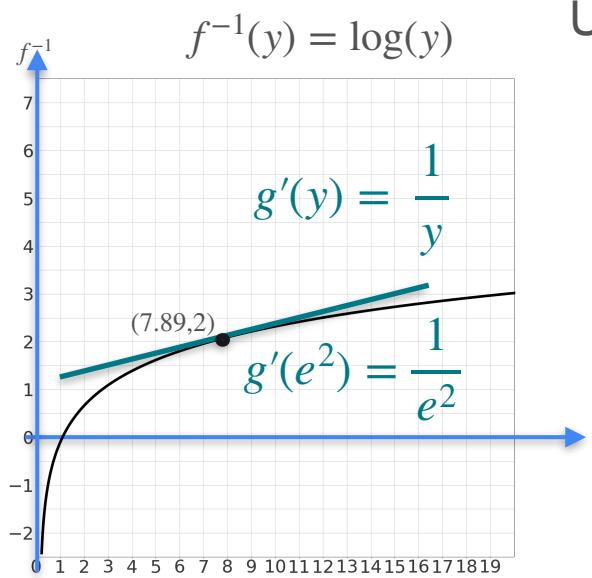
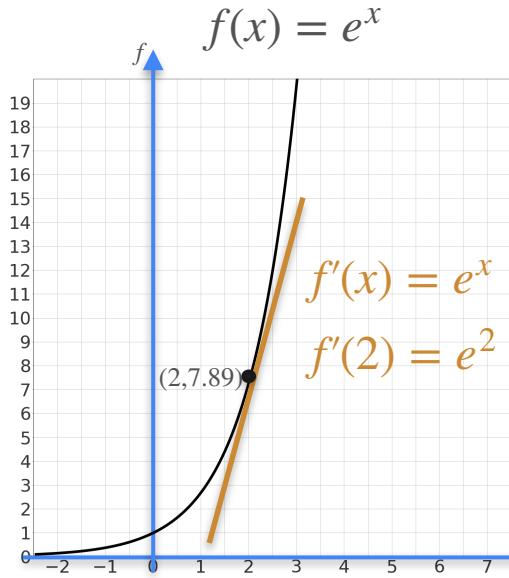


Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y) = \frac{1}{e^{\log(y)}}$$

# Logarithm

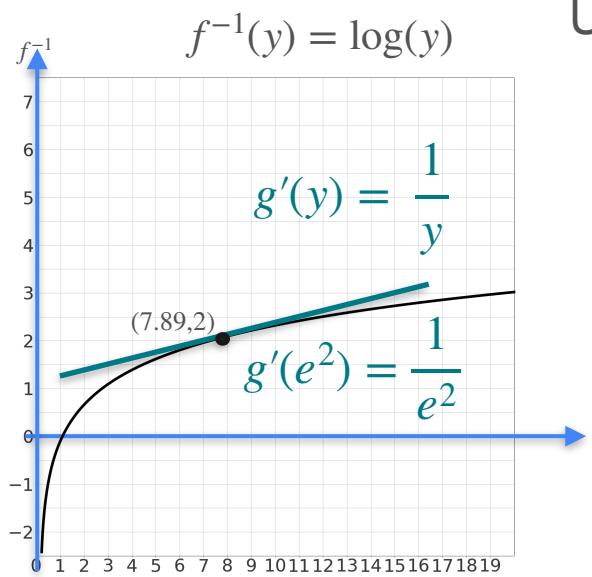
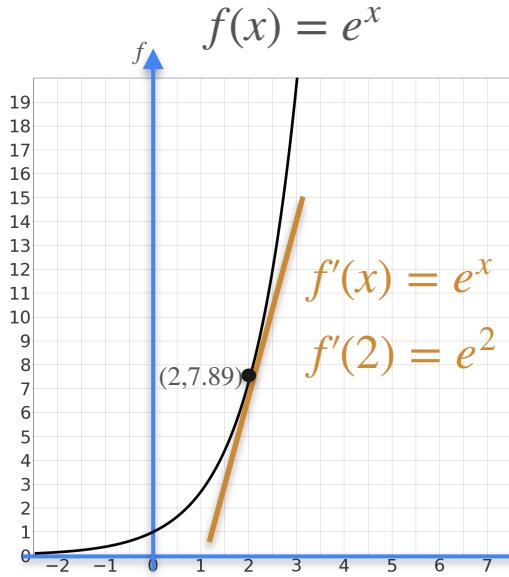


Using the result for inverses

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# Logarithm



Using the result for inverses

$$\frac{d}{dy} f^{-1}(y) = \frac{1}{f'(f^{-1}(y))}$$

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$$\frac{d}{dy} \log(y) = \frac{1}{y}$$



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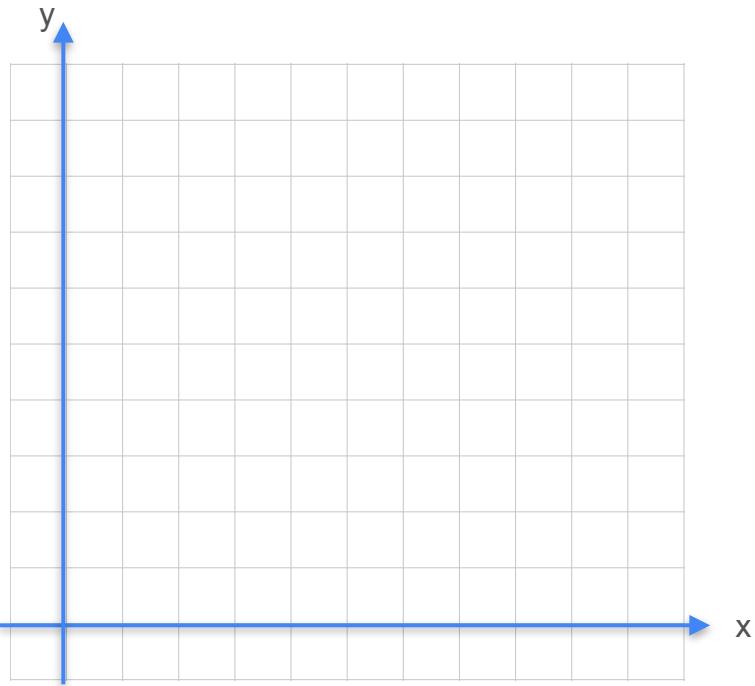
# Derivatives and Optimization

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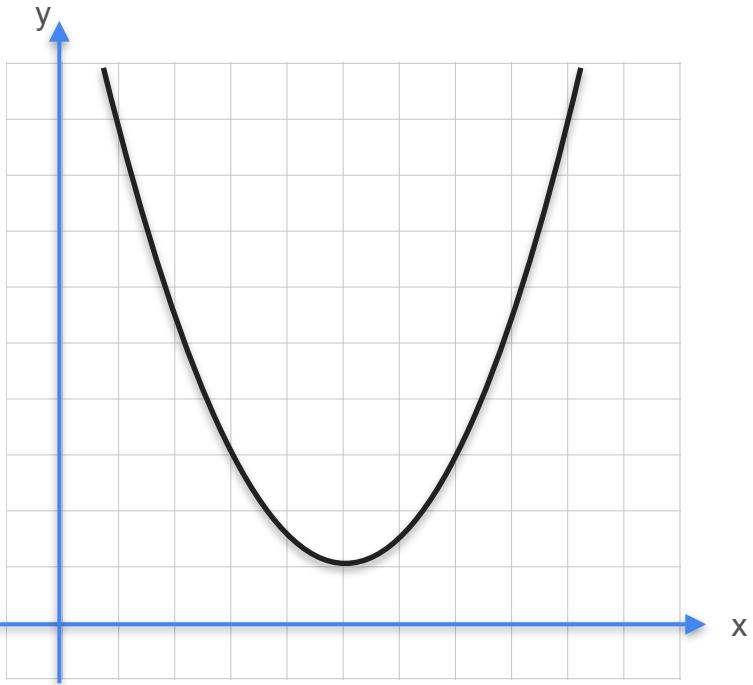
## Existence of the derivative

# Differentiable Function

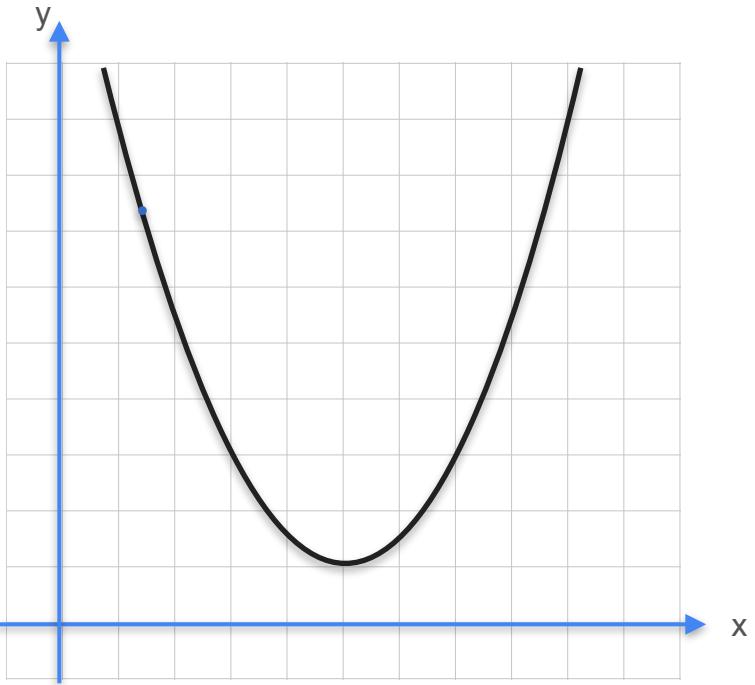
# Differentiable Function



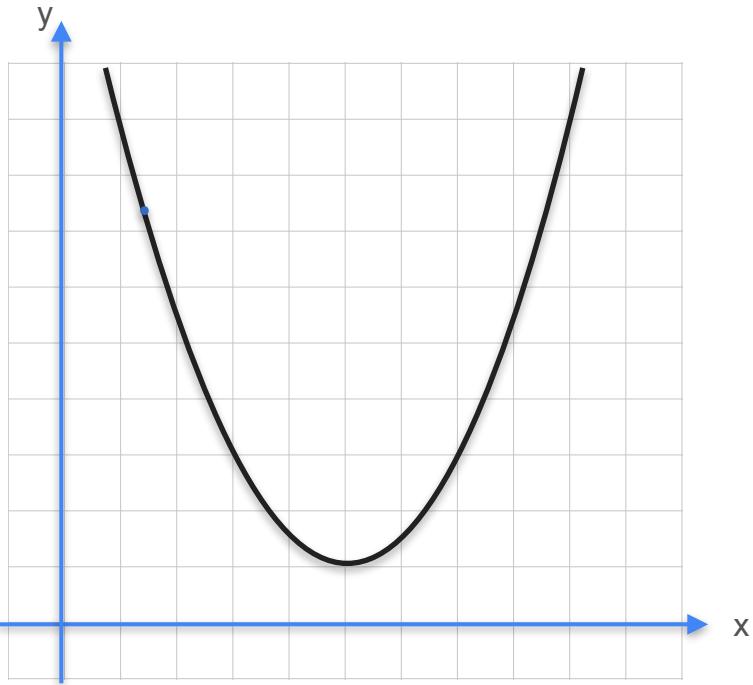
# Differentiable Function



# Differentiable Function



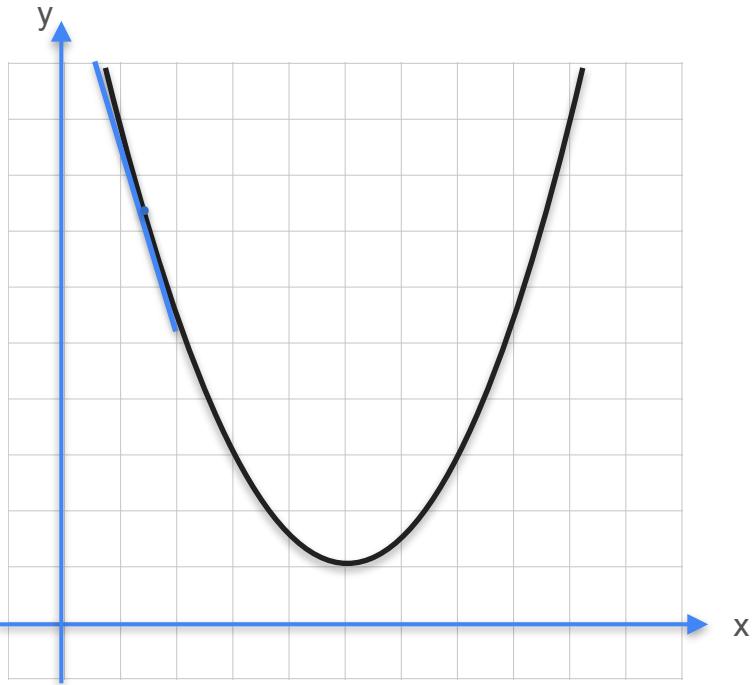
# Differentiable Function



**For a function to be differentiable at a point:**

The derivative has to exist for that point

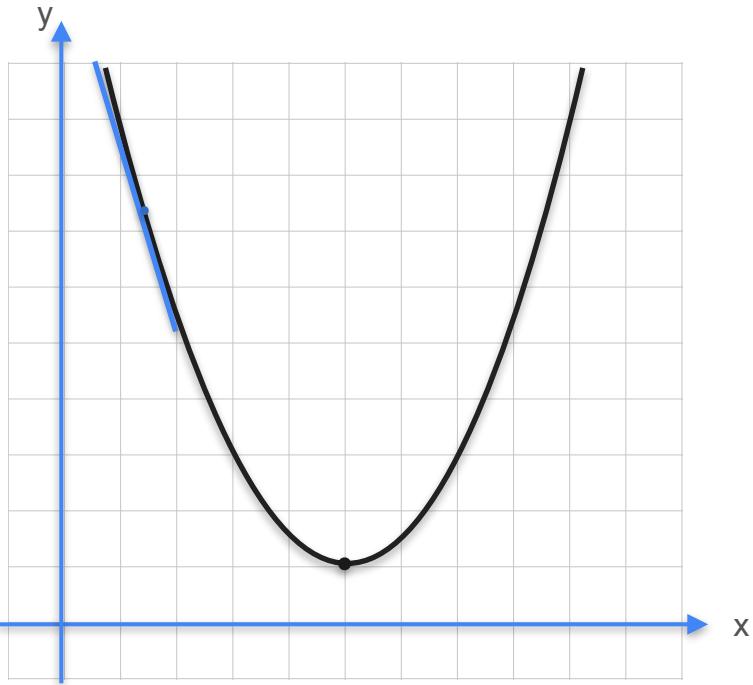
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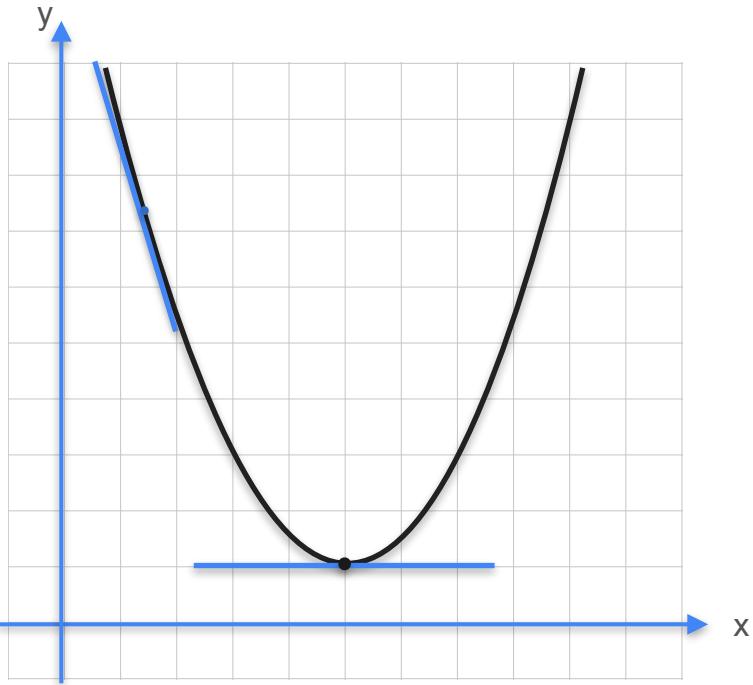
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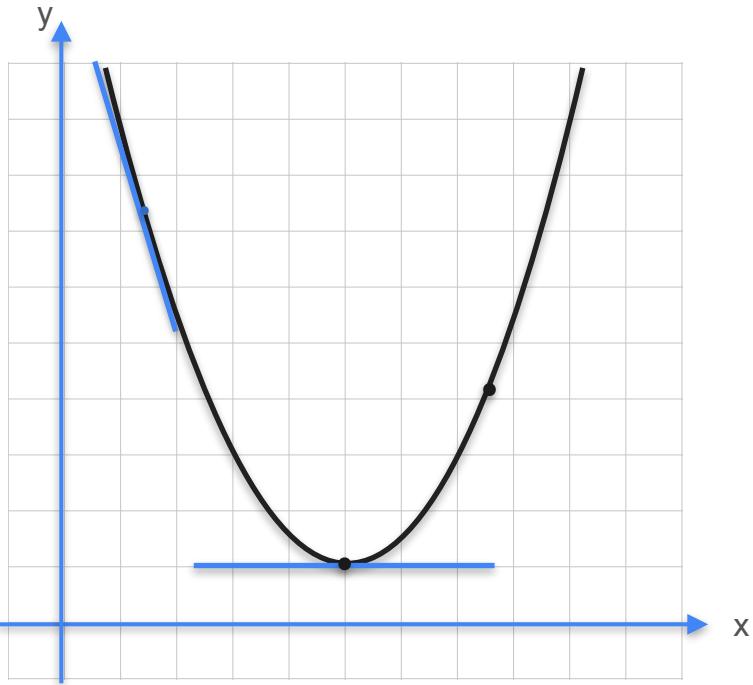
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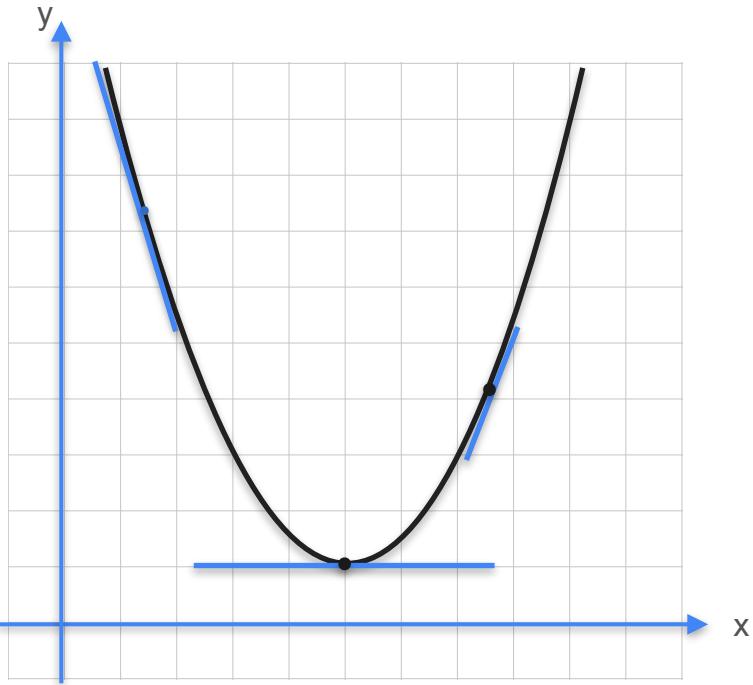
# Differentiable Function



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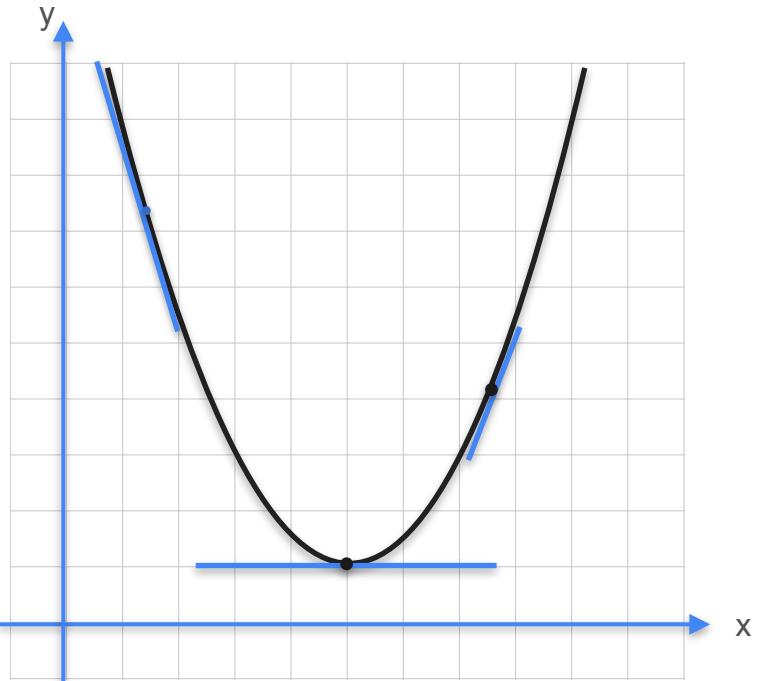
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# Differentiable Function



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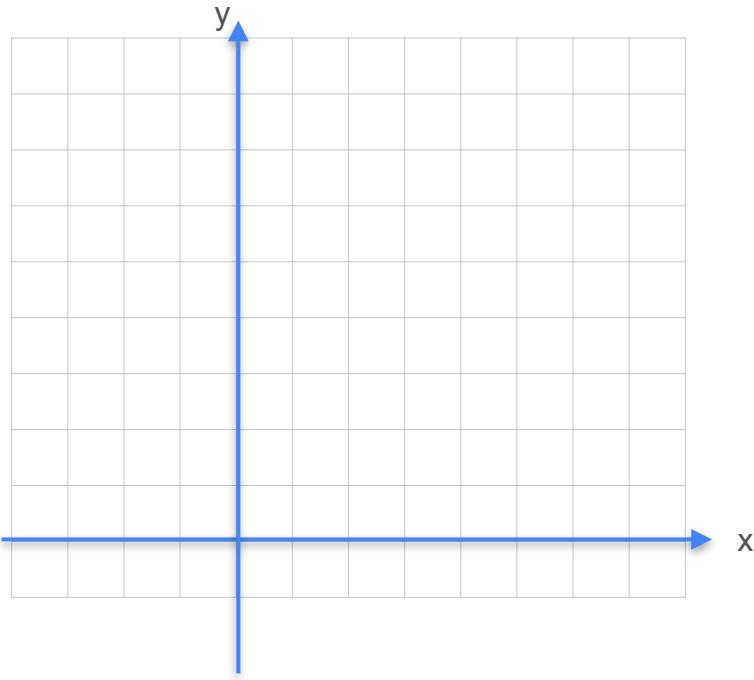
The derivative has to exist for that point

**For a function to be differentiable at an interval:**

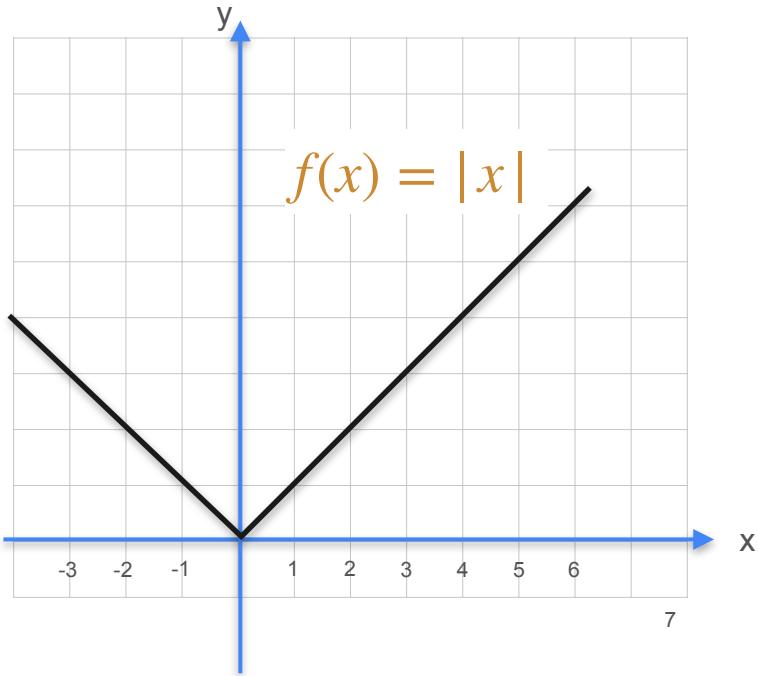
The derivative has to exist for *every* point in the interval

# Non Differentiable Functions

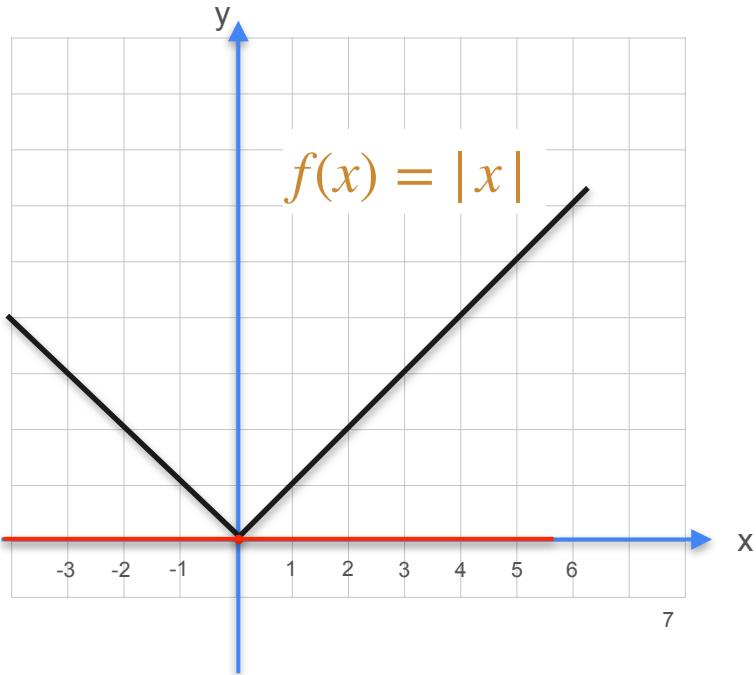
# Non Differentiable Functions



# Non Differentiable Functions



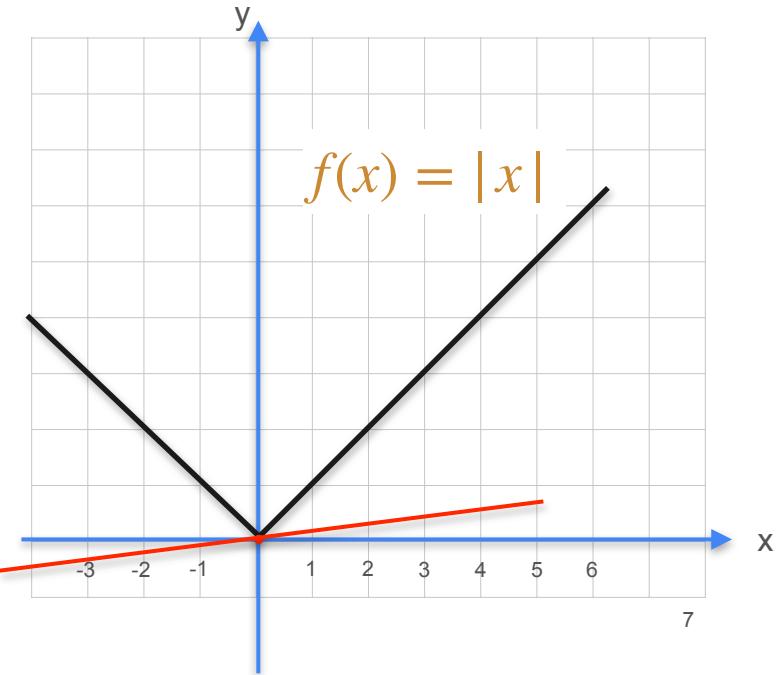
# Non Differentiable Functions



The absolute value function.

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

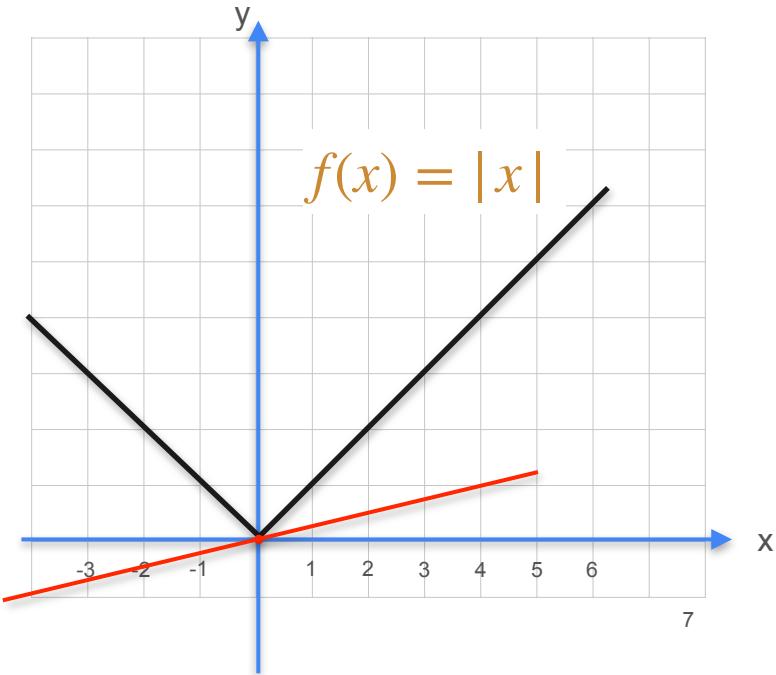
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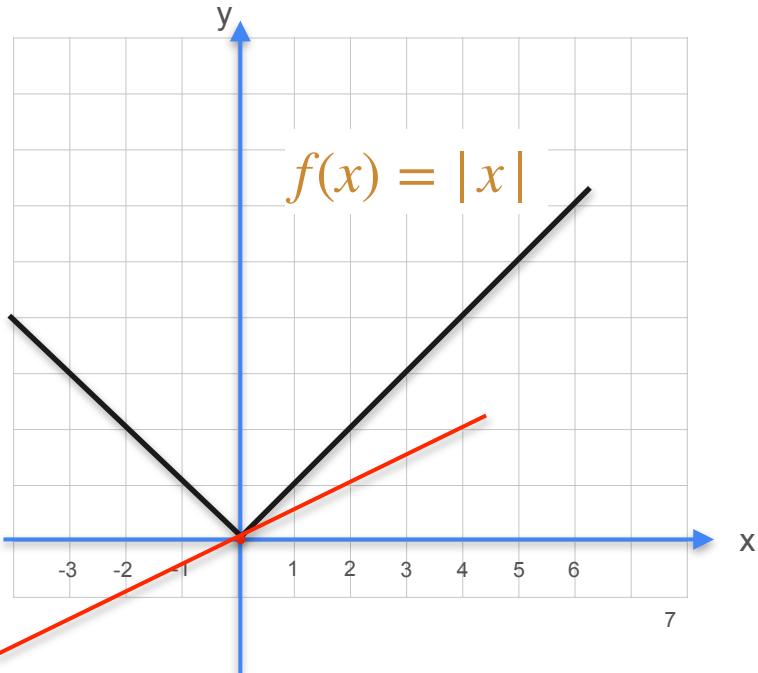
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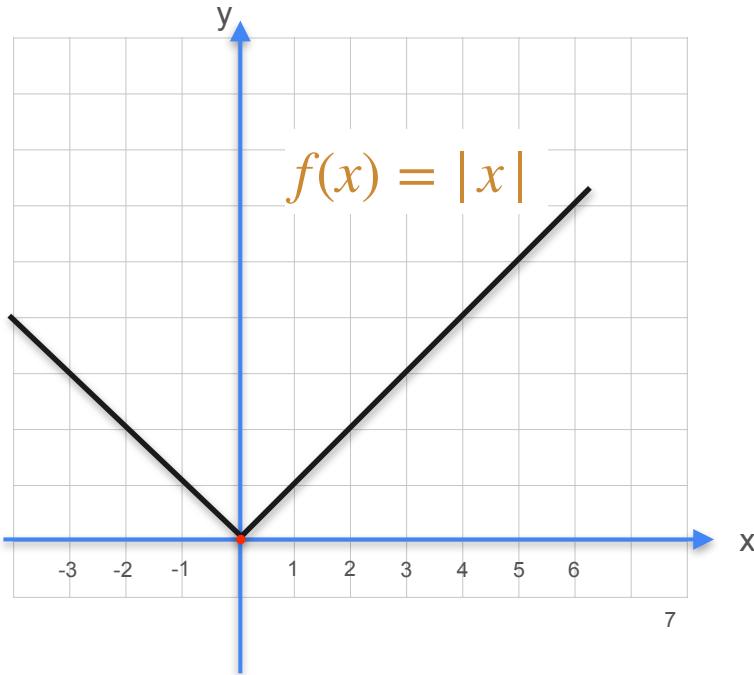
# Non Differentiable Functions



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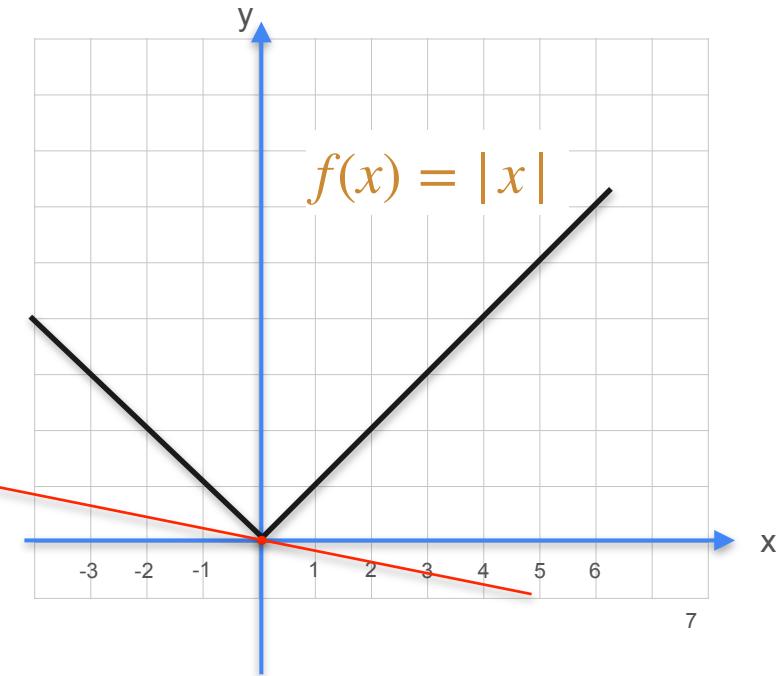
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# Non Differentiable Functions



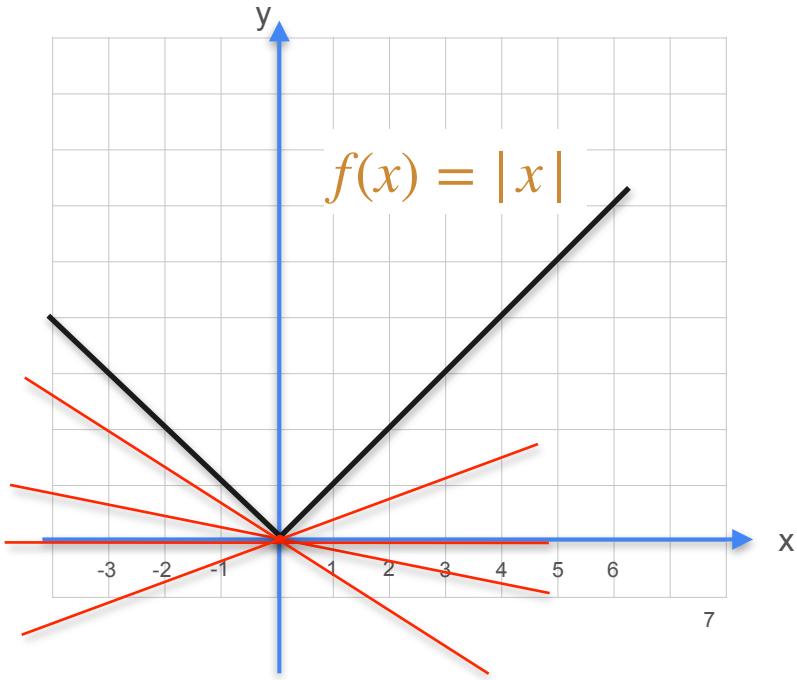
The absolute value function.

# Non Differentiable Functions



The absolute value function.

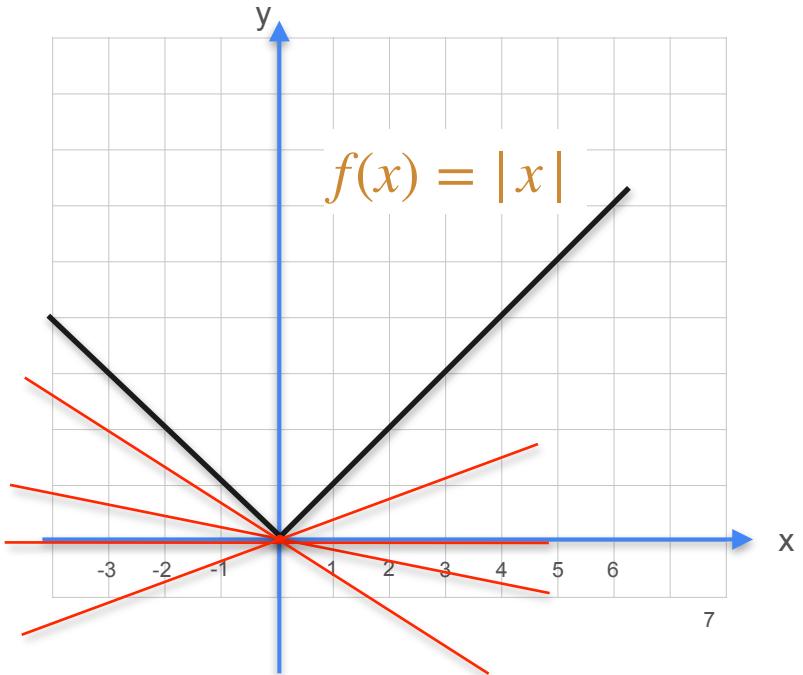
# Non Differentiable Functions



The absolute value function.

At  $x = 0$ , the derivative does not exist

# Non Differentiable Functions



The absolute value function.

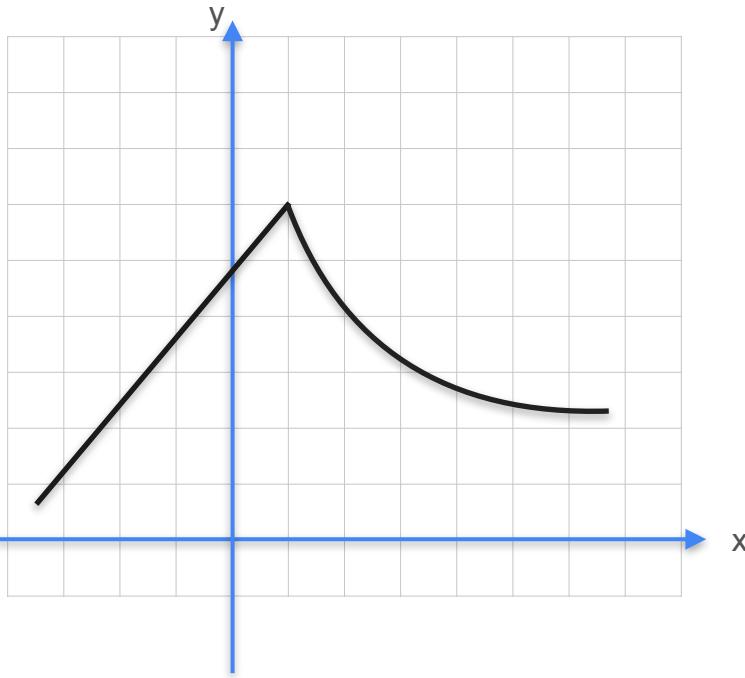
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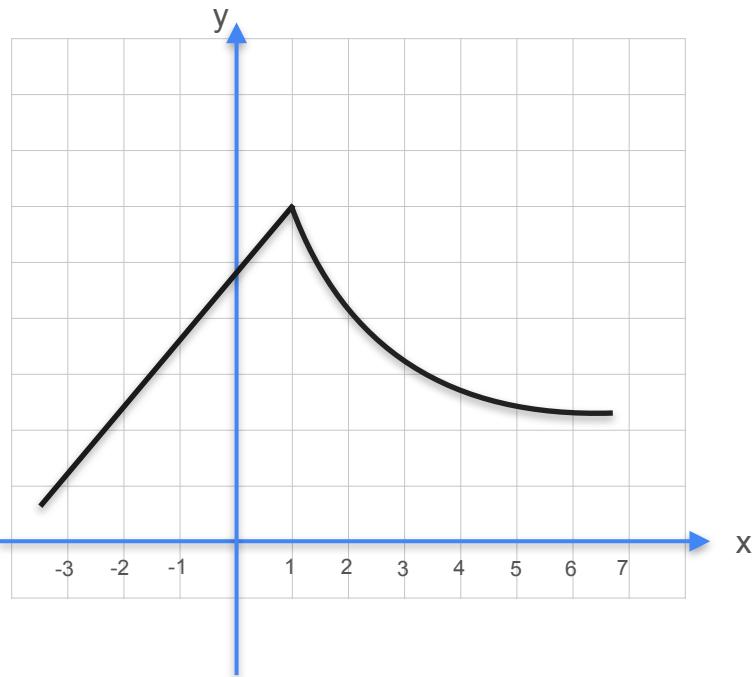
Generally, when a function has a corner or a cusp, the function is not differentiable at that point.

# Non Differentiable Functions - Quiz 1

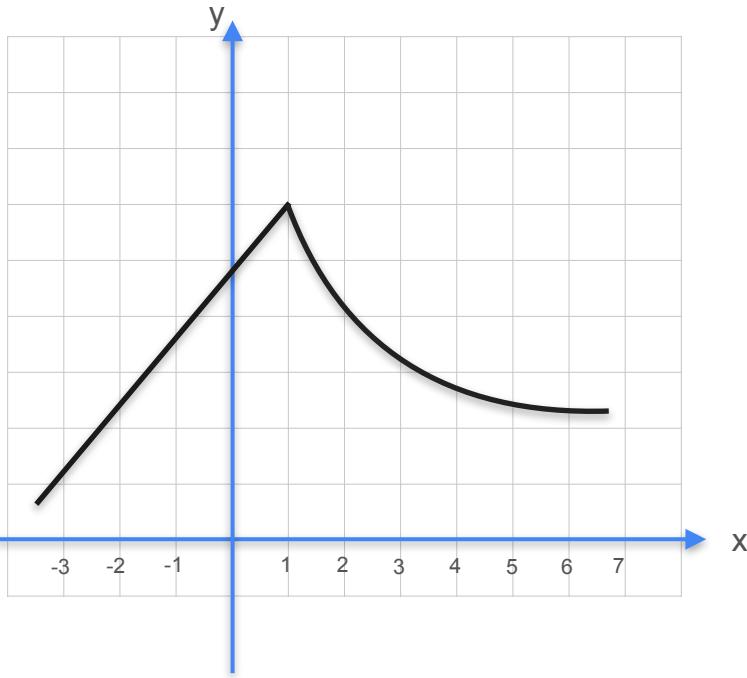
# Non Differentiable Functions - Quiz 1



# Non Differentiable Functions - Quiz 1

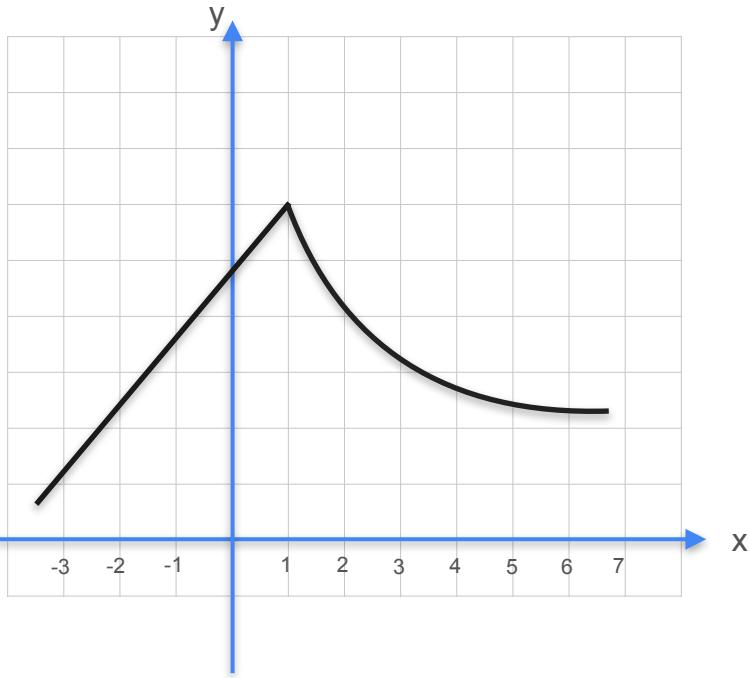


# Non Differentiable Functions - Quiz 1



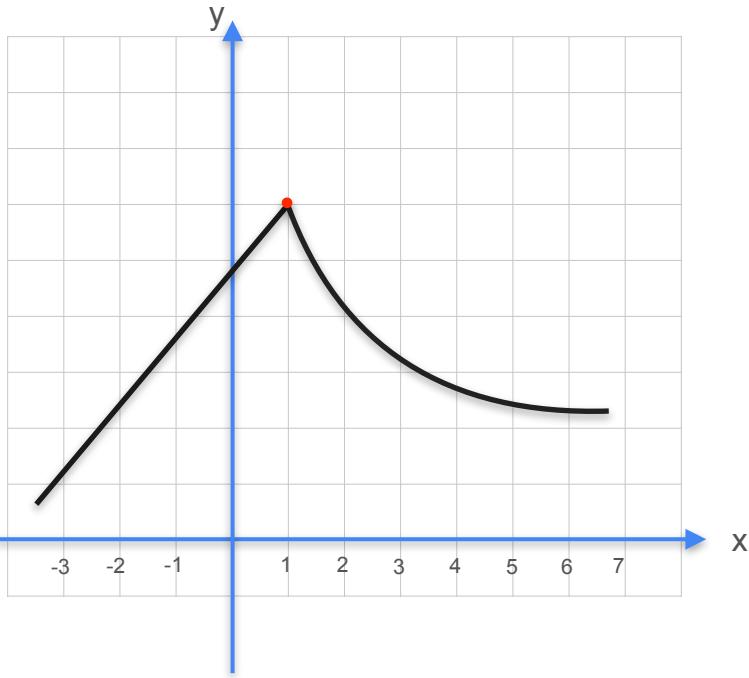
At which point in this function does the derivative not exist?

# Non Differentiable Functions - Quiz 1



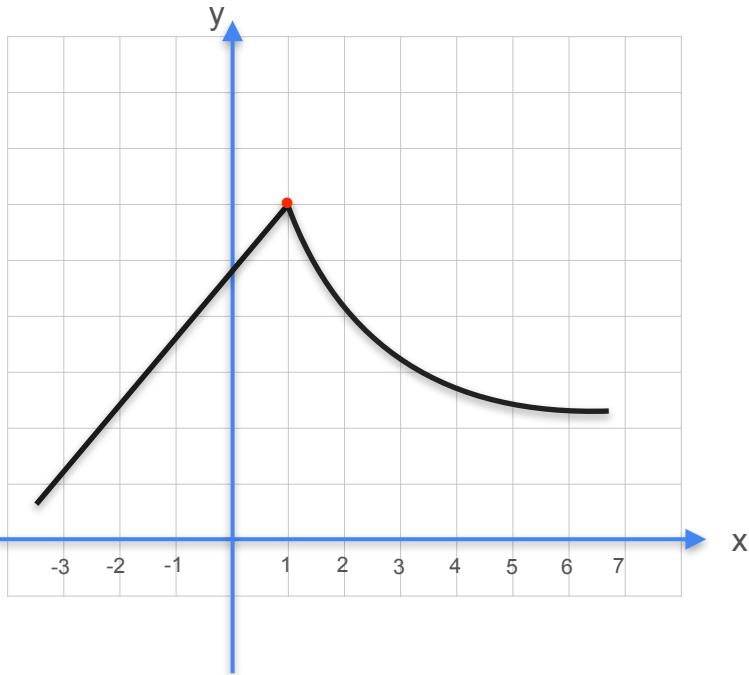
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# Non Differentiable Functions - Quiz 1



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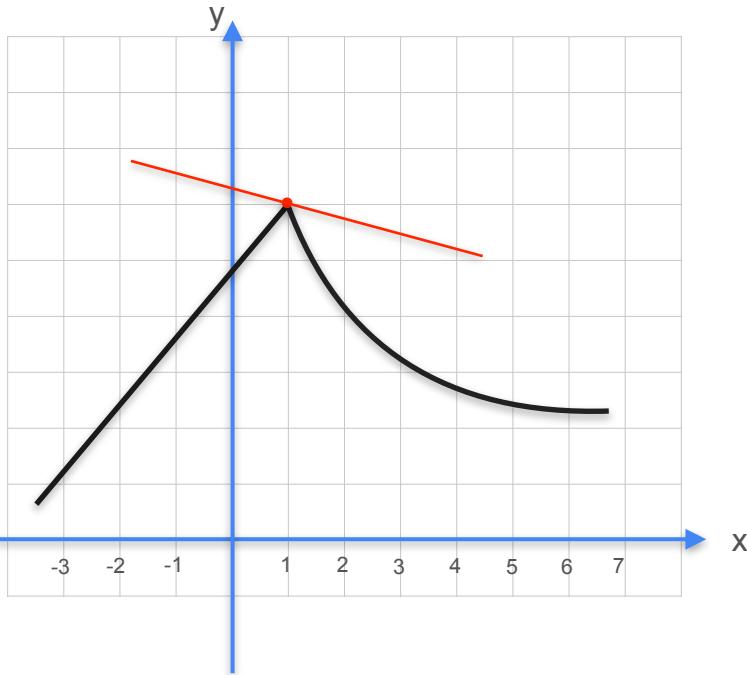
# Non Differentiable Functions - Quiz 1



At which point in this function does the derivative not exist?

The entire function is non-differentiable because a derivative does not exist for all points in the domain.

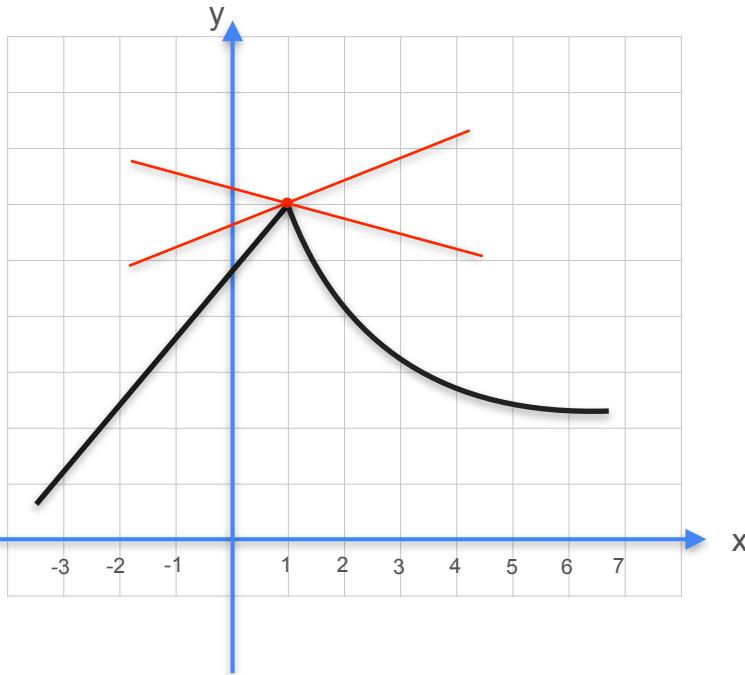
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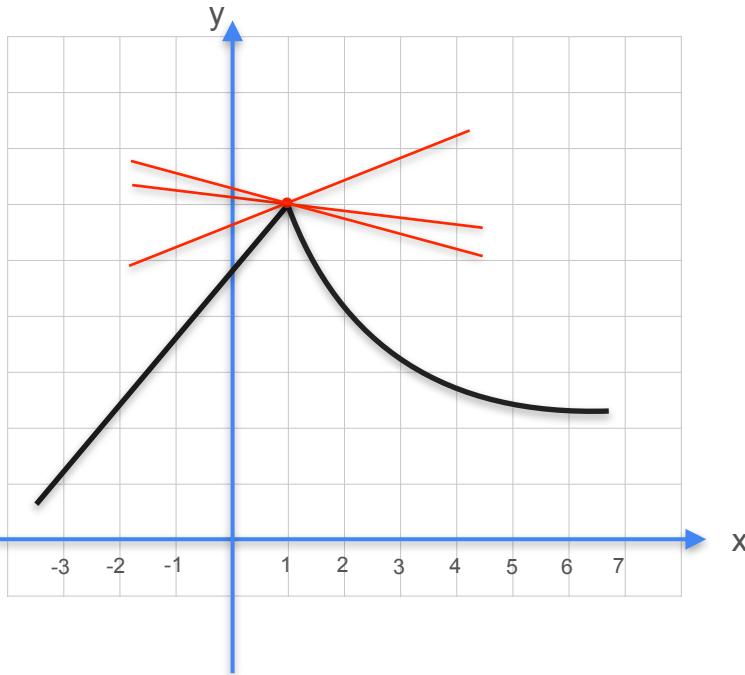
# Non Differentiable Functions - Quiz 1



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# Non Differentiable Functions - Quiz 1

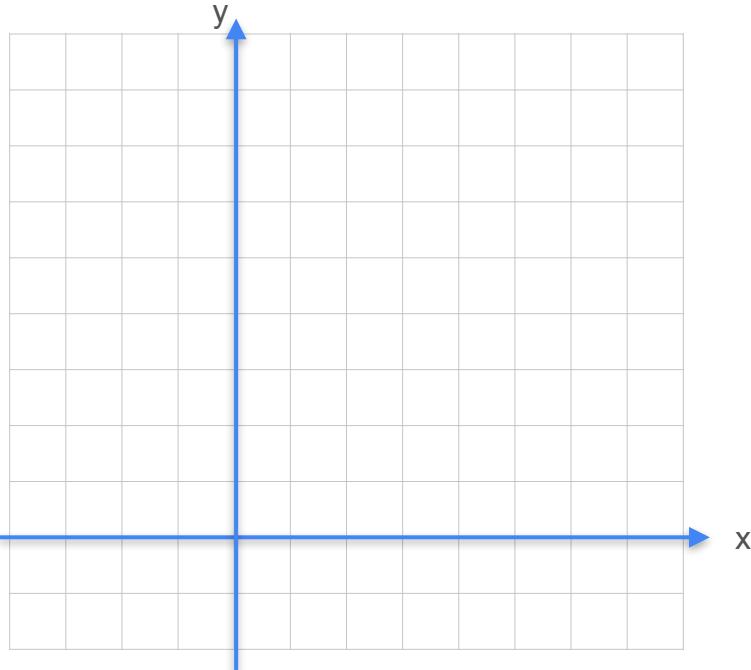


At which point in this function does the derivative not exist?

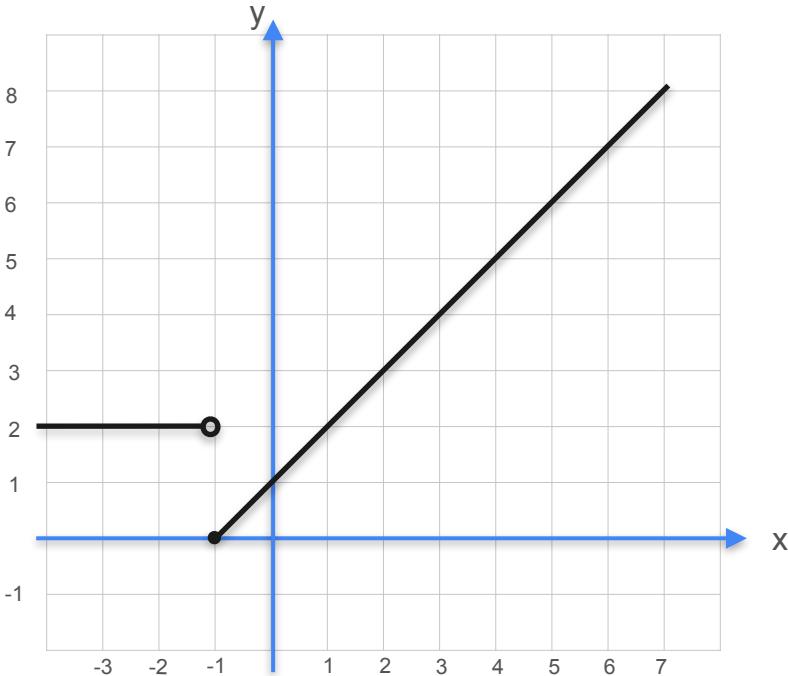
The entire function is non-differentiable because a derivative does not exist for all points in the domain.

# Non Differentiable Functions - Quiz 2

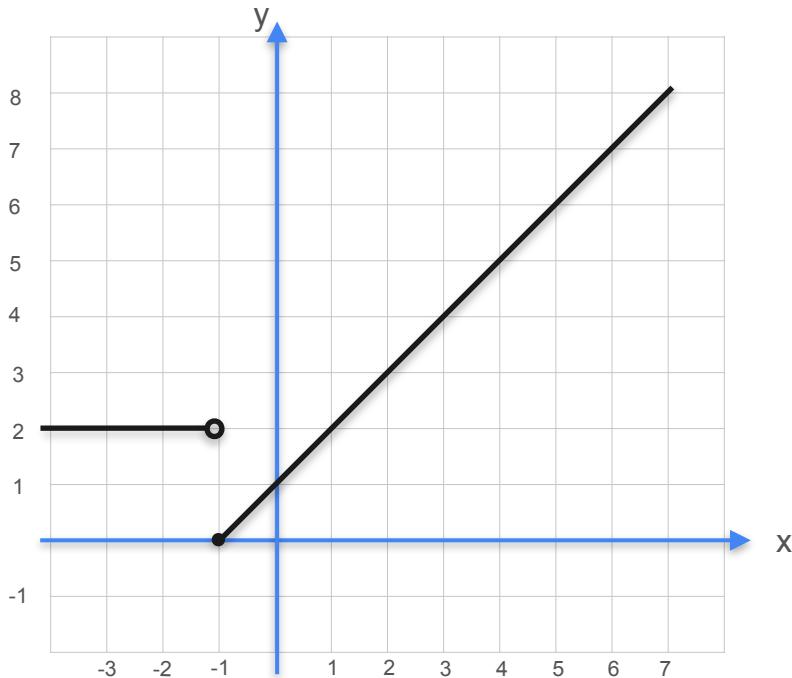
# Non Differentiable Functions - Quiz 2



# Non Differentiable Functions - Quiz 2

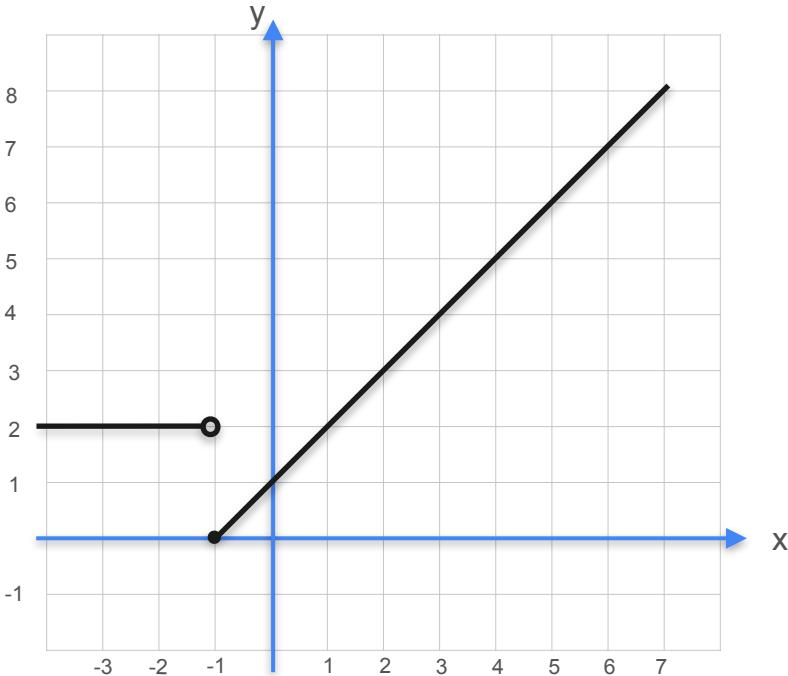


# Non Differentiable Functions - Quiz 2

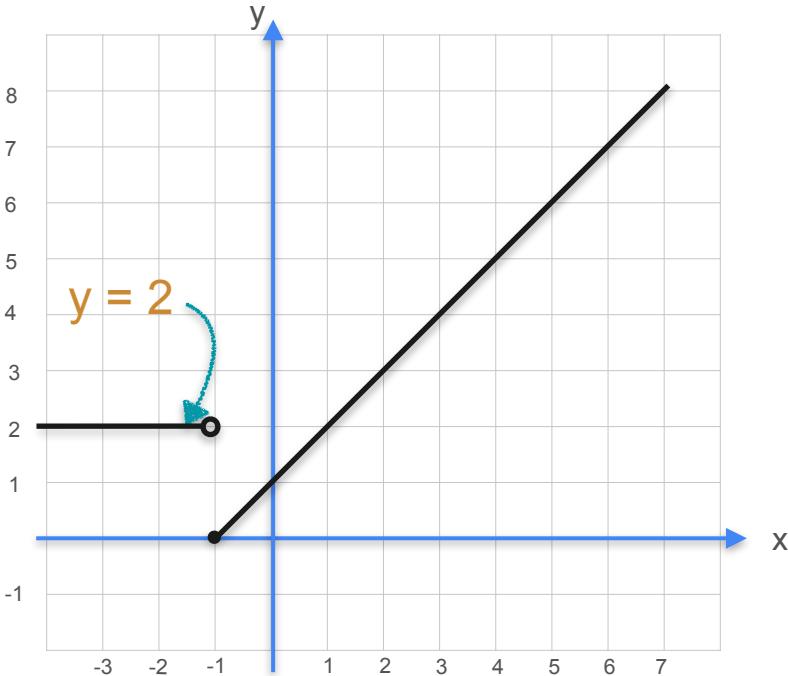


At which point in this function does the derivative not exist?

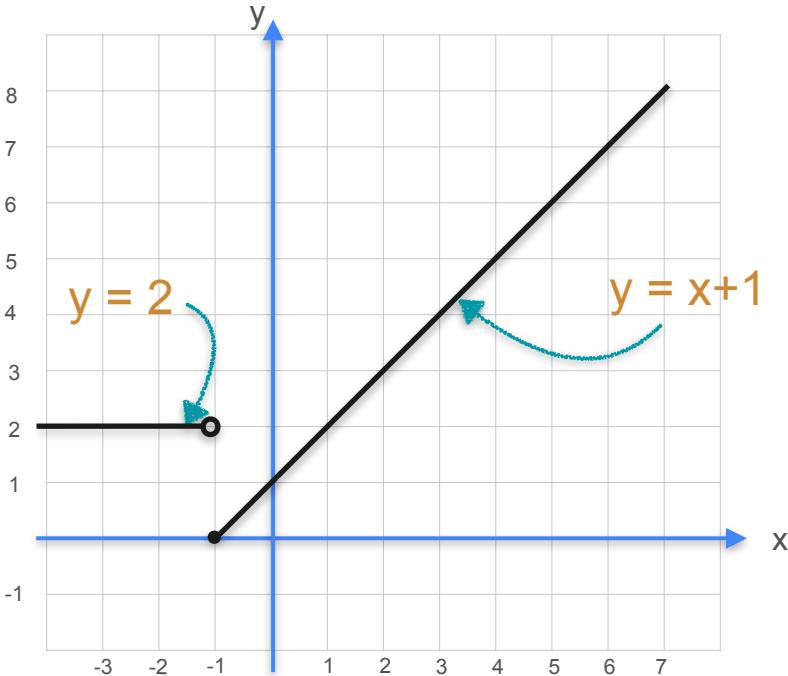
# Non Differentiable Functions



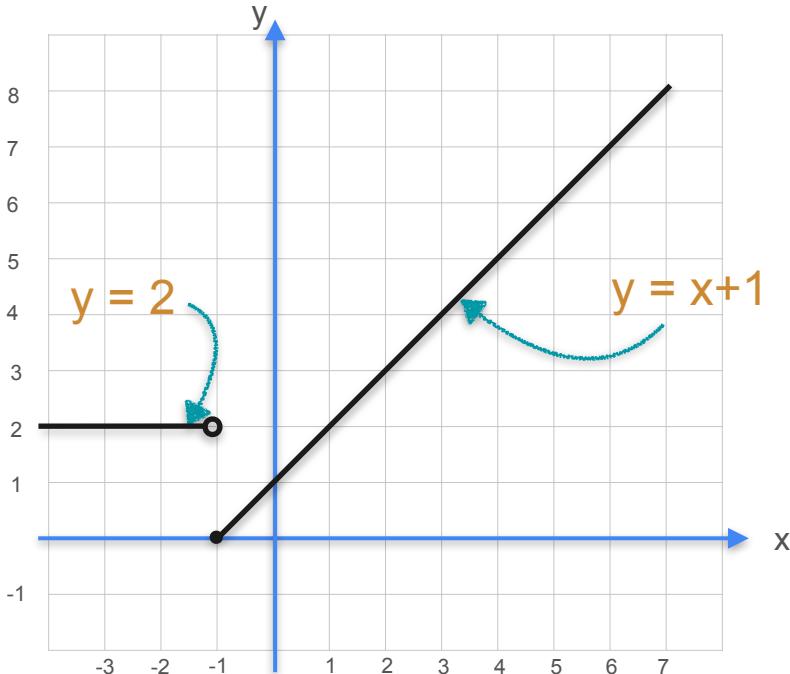
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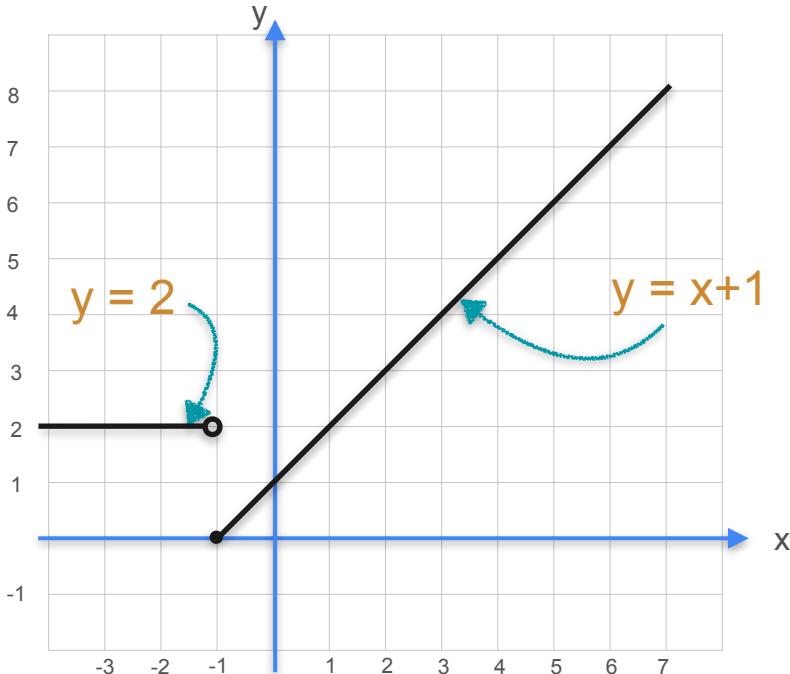
# Non Differentiable Functions



This is a piece-wise function.

$$f(x) = \begin{cases} 2, & \text{if } x < -1 \\ x + 1, & \text{if } x \geq -1 \end{cases}$$

# Non Differentiable Functions

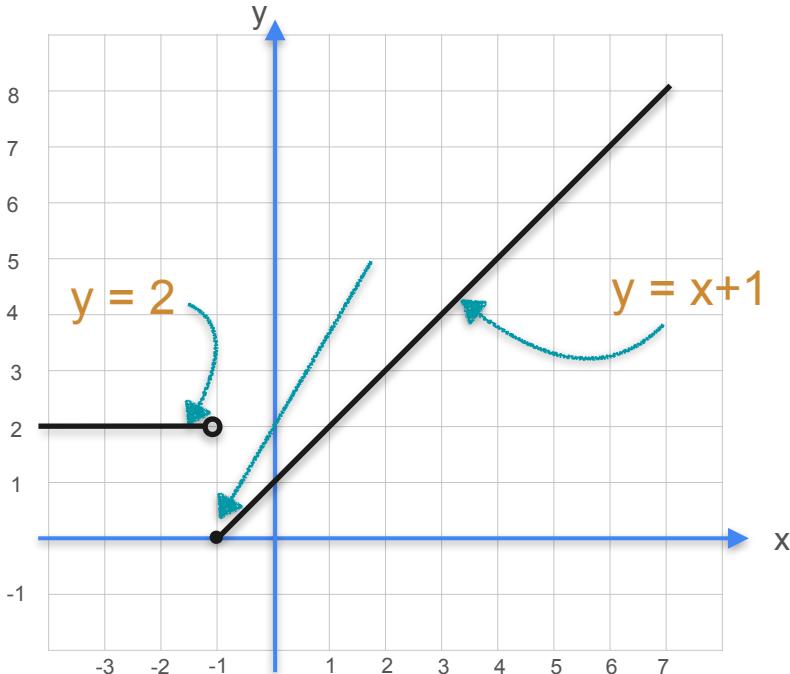


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Jump Discontinuity

# Non Differentiable Functions

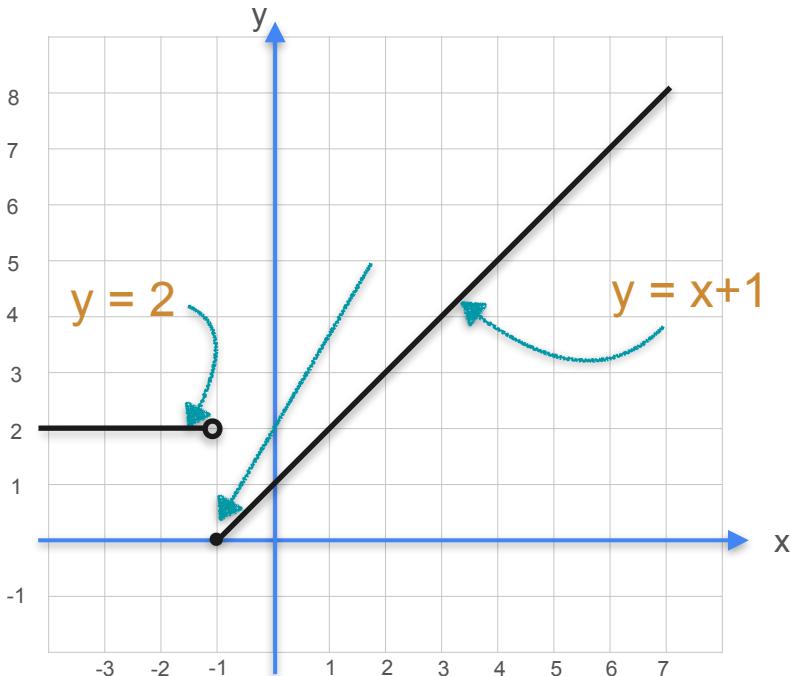


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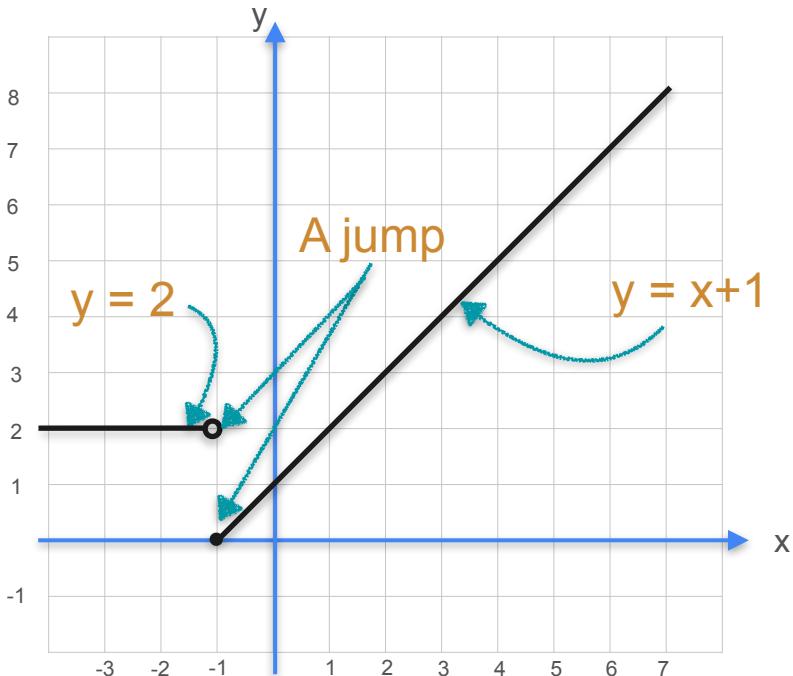
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Jump Discontinuity

The graph of the function does not appear to be continuous

# Non Differentiable Functions



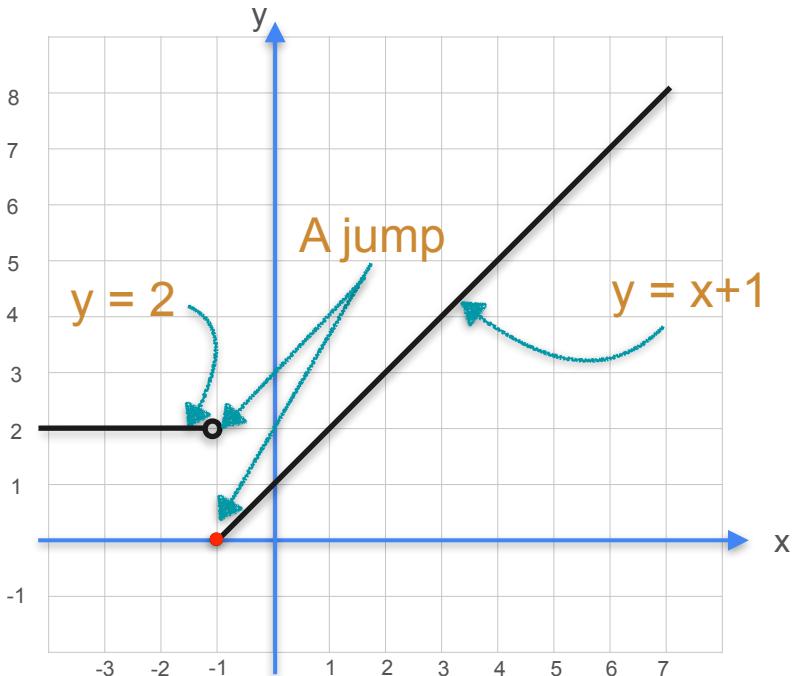
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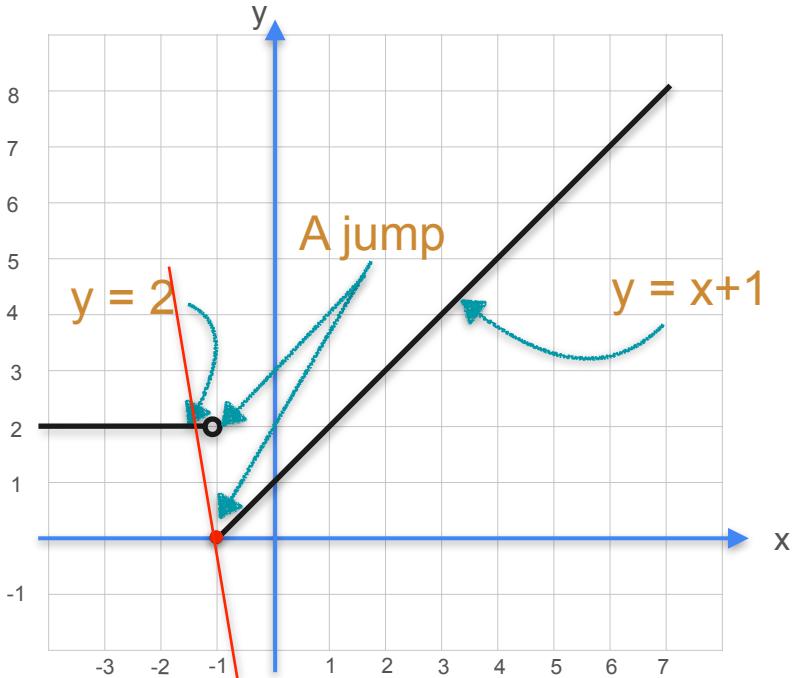
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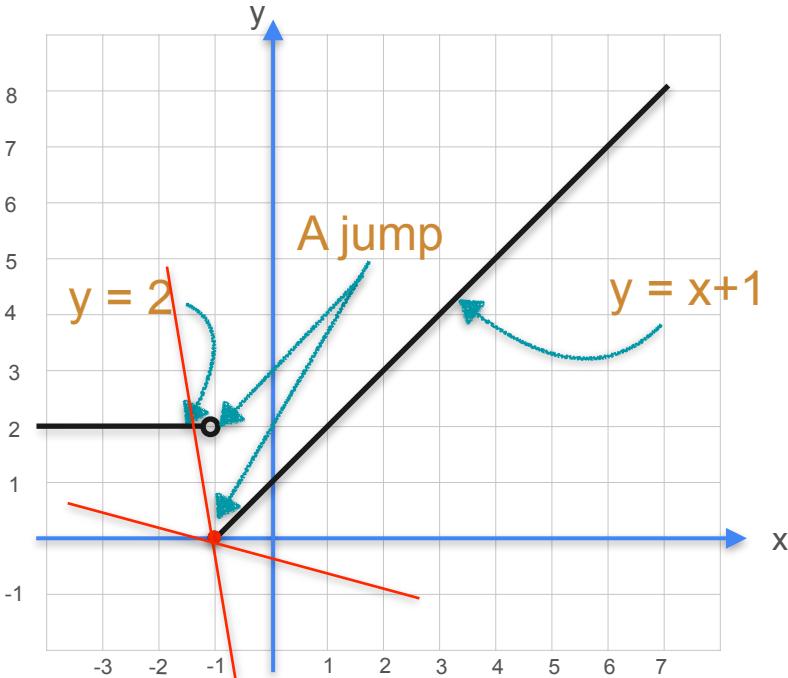
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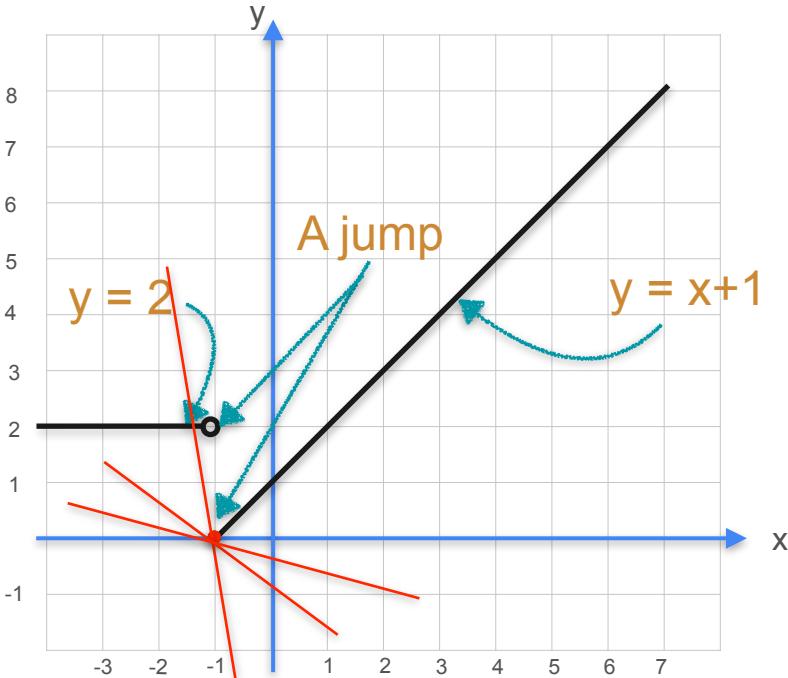
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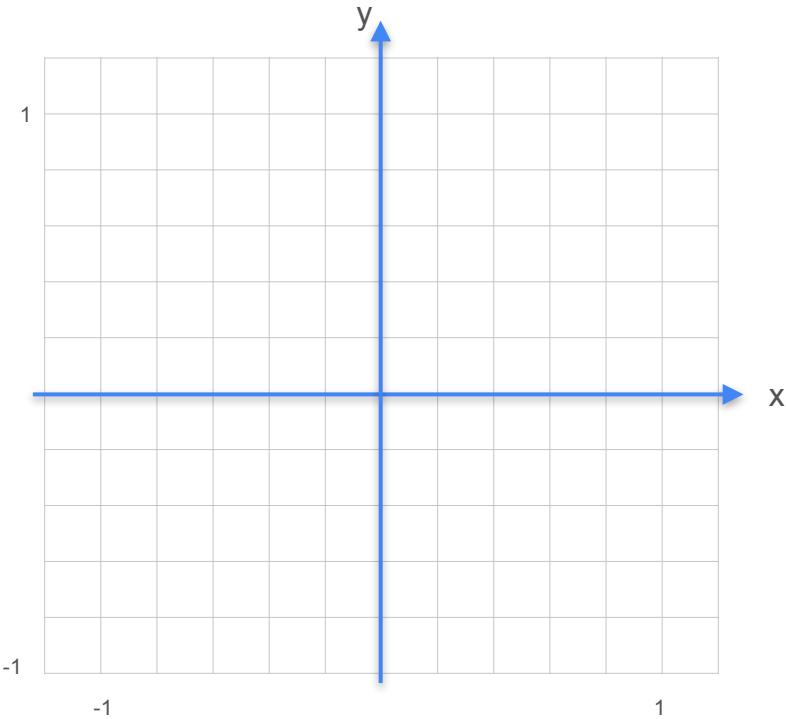
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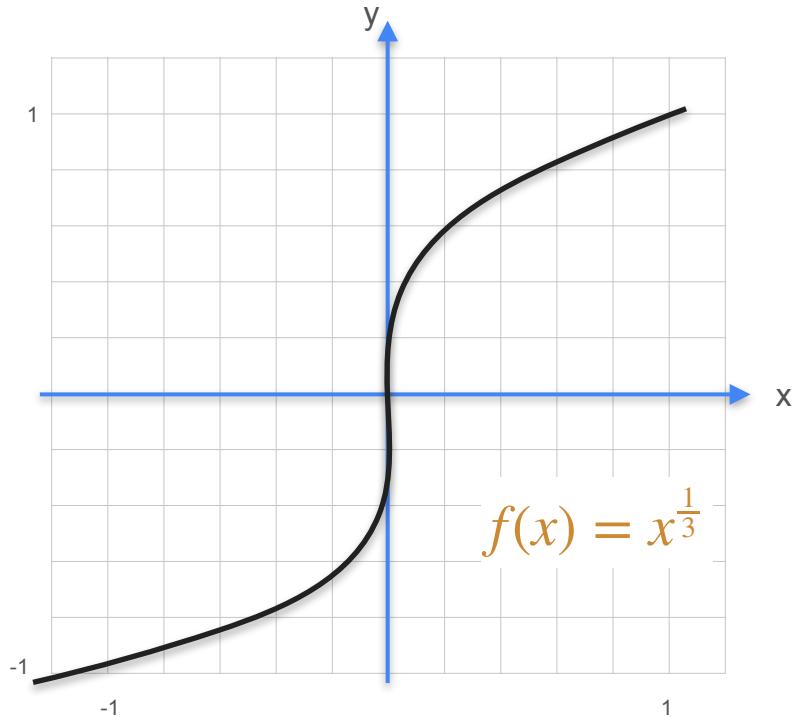
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# Non Differentiable Functions

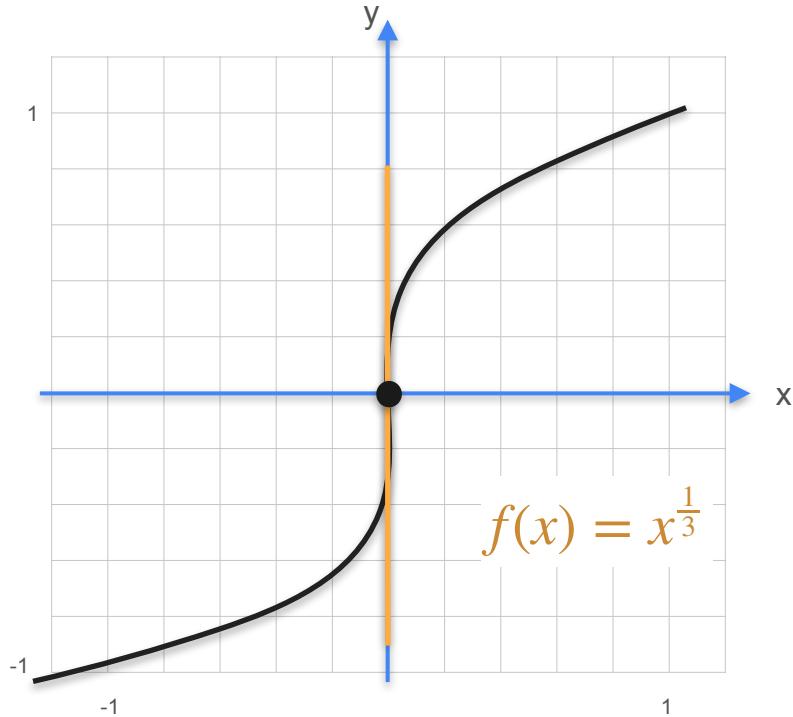
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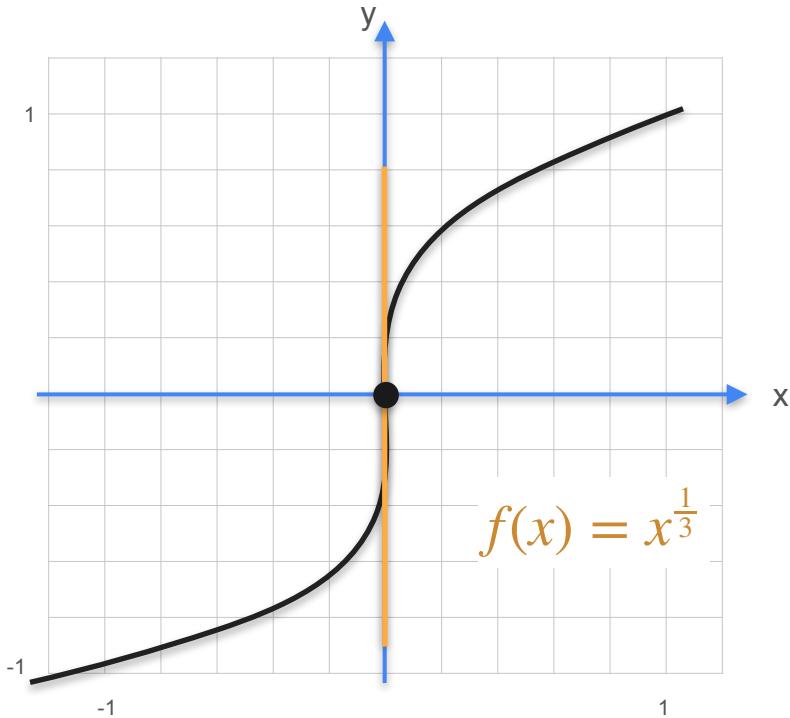
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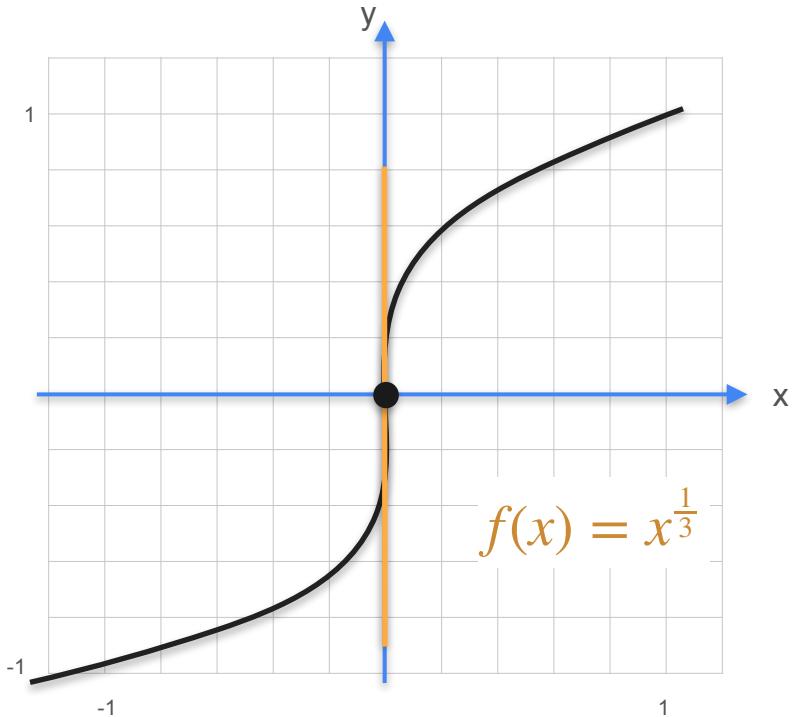


# Non Differentiable Functions



Vertical tangents

# Non Differentiable Functions

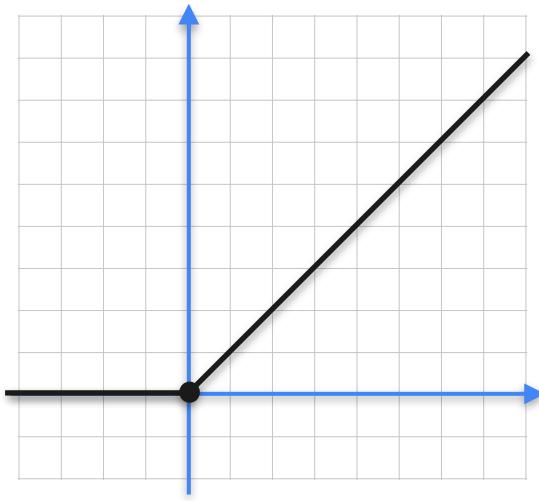


Vertical tangents

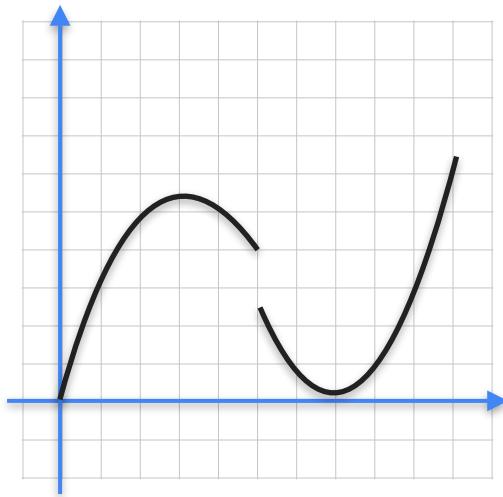
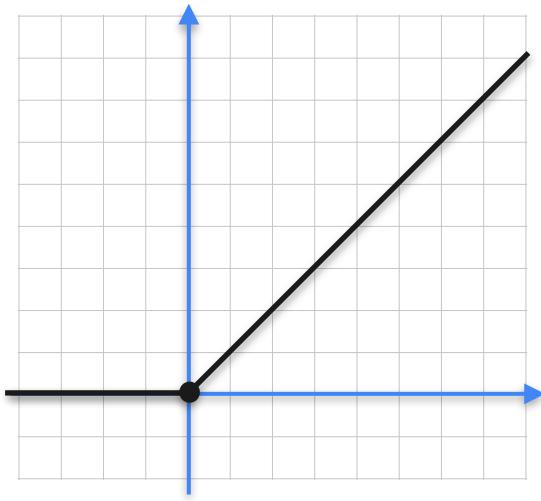
At  $x = 0$ , this graph has a tangent line that runs straight up parallel to the y-axis

# Recap: Non-Differentiable Functions

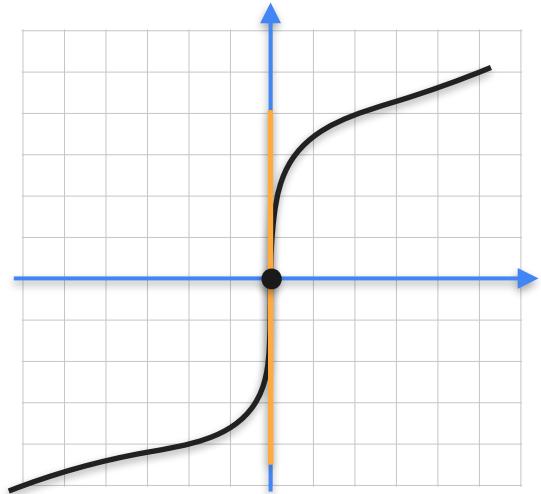
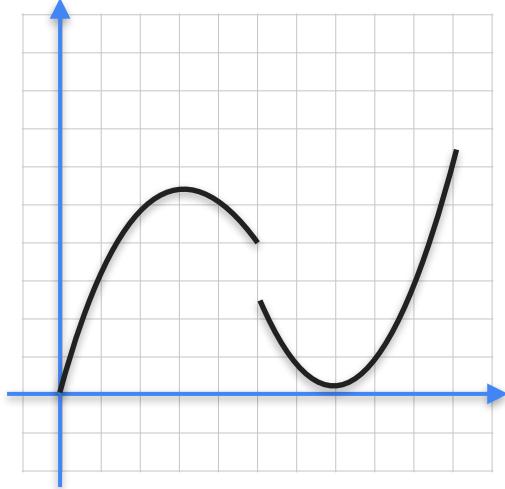
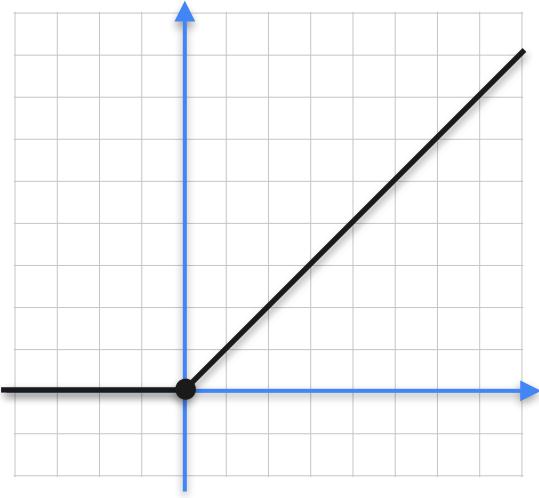
# Recap: Non-Differentiable Functions



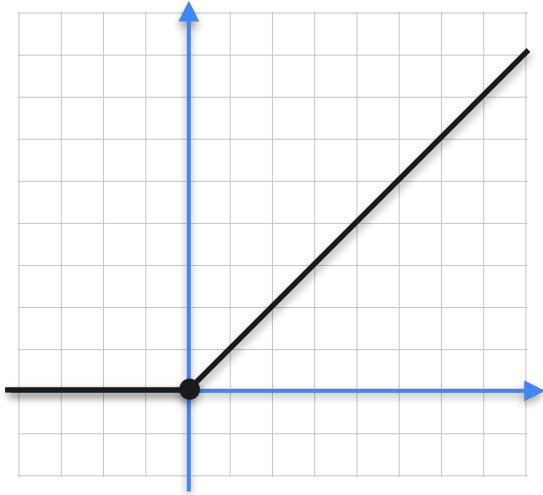
# Recap: Non-Differentiable Functions



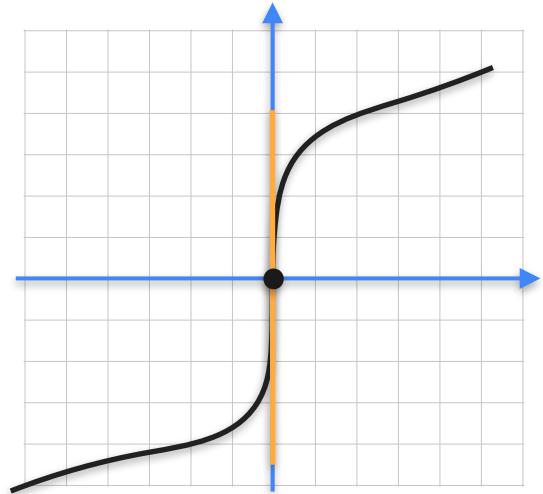
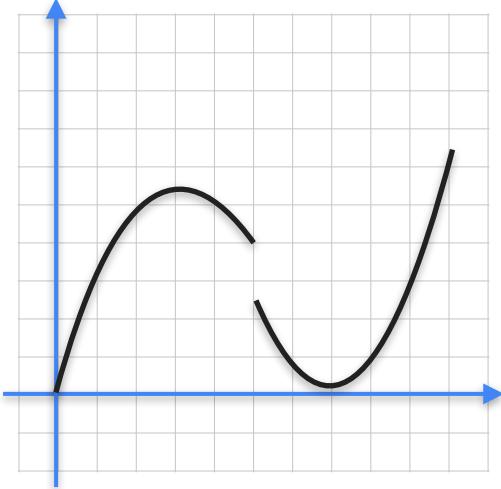
# Recap: Non-Differentiable Functions



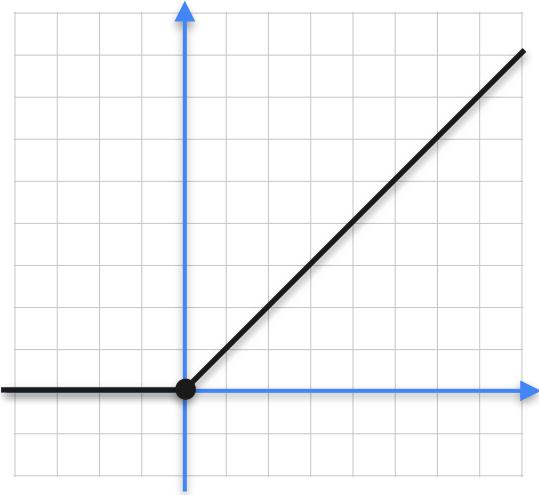
# Recap: Non-Differentiable Functions



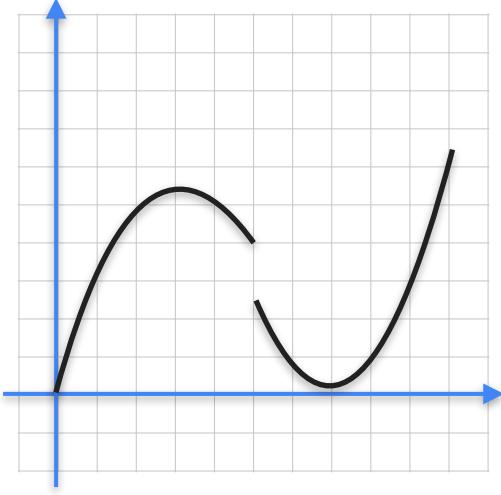
Corners/Cusps



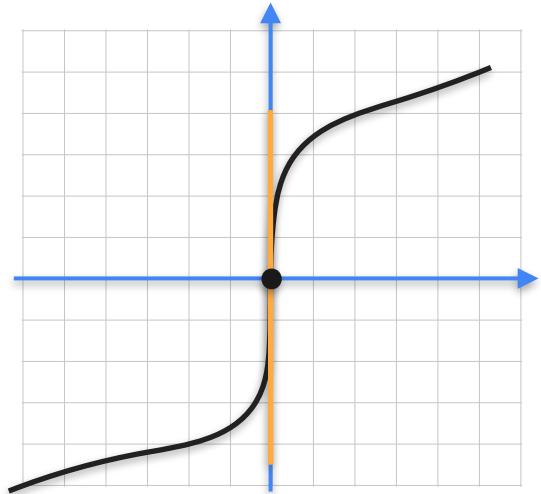
# Recap: Non-Differentiable Functions



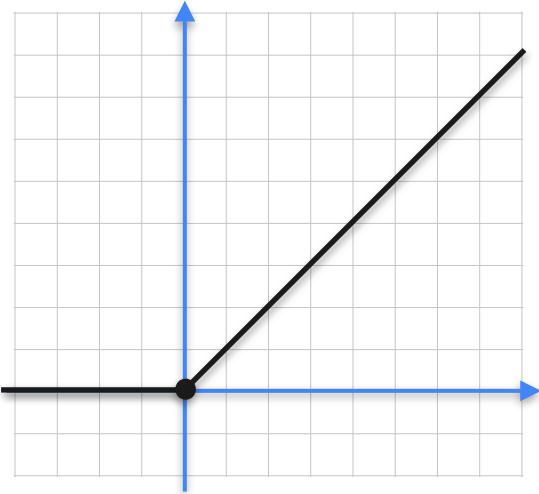
Corners/Cusps



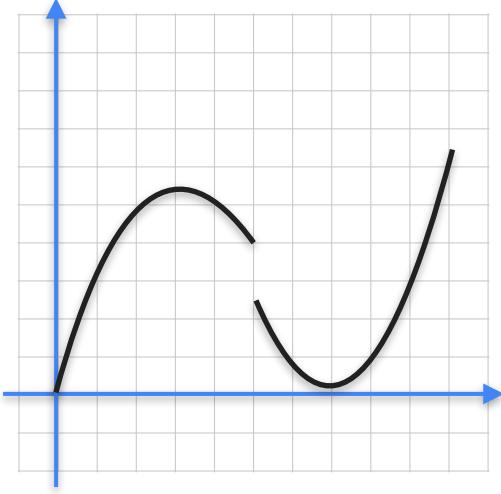
Jump Discontinuity



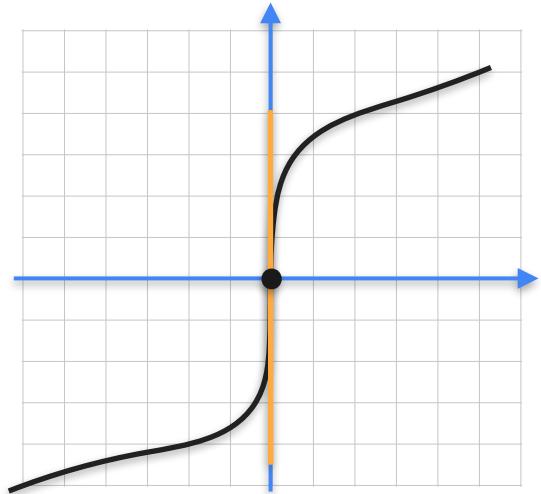
# Recap: Non-Differentiable Functions



Corners/Cusps



Jump Discontinuity



Vertical tangents



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# Derivatives and Optimization

---

**Properties of the derivative:  
Multiplication by scalars**

# The Sum Rule

$$f = 4g$$

# The Sum Rule

$$f' = 4g$$

# The Sum Rule

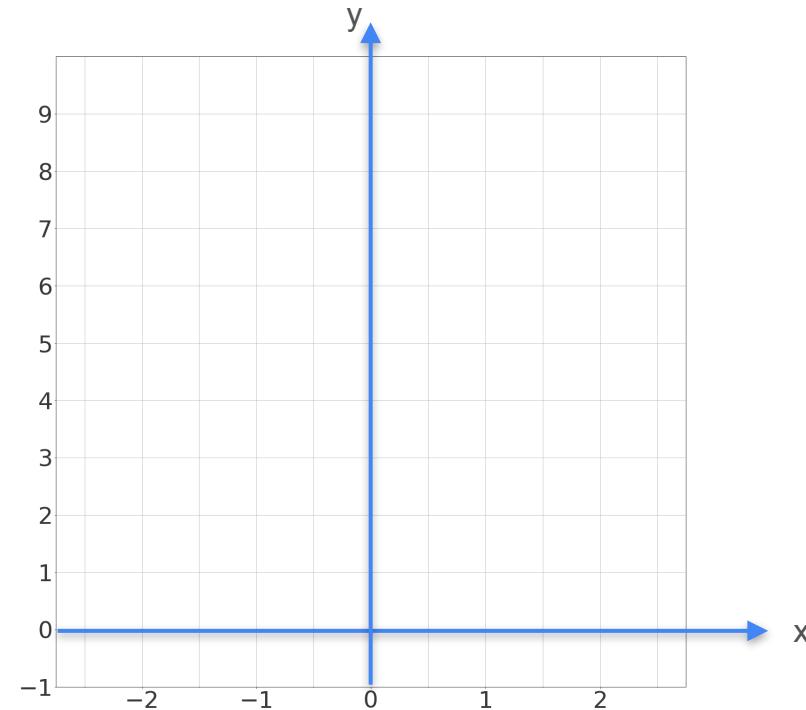
$$f' = 4g'$$

# The Sum Rule

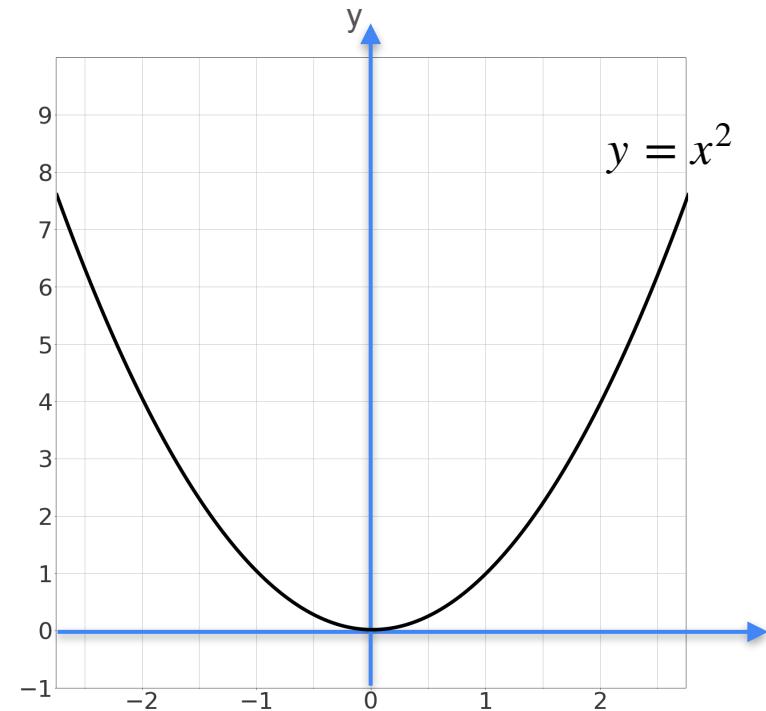
$$f' = c g'$$

# Multiplication by a Scalar

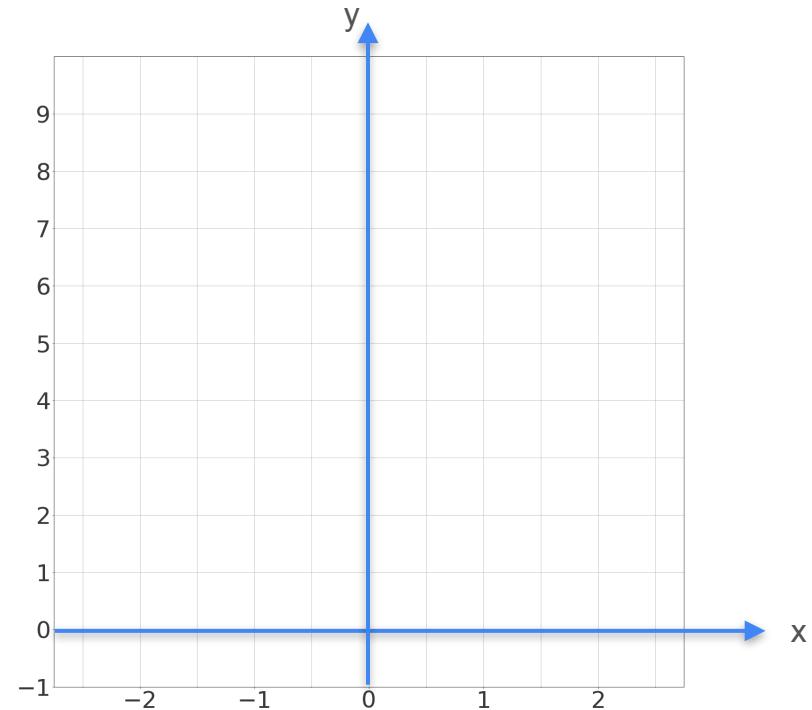
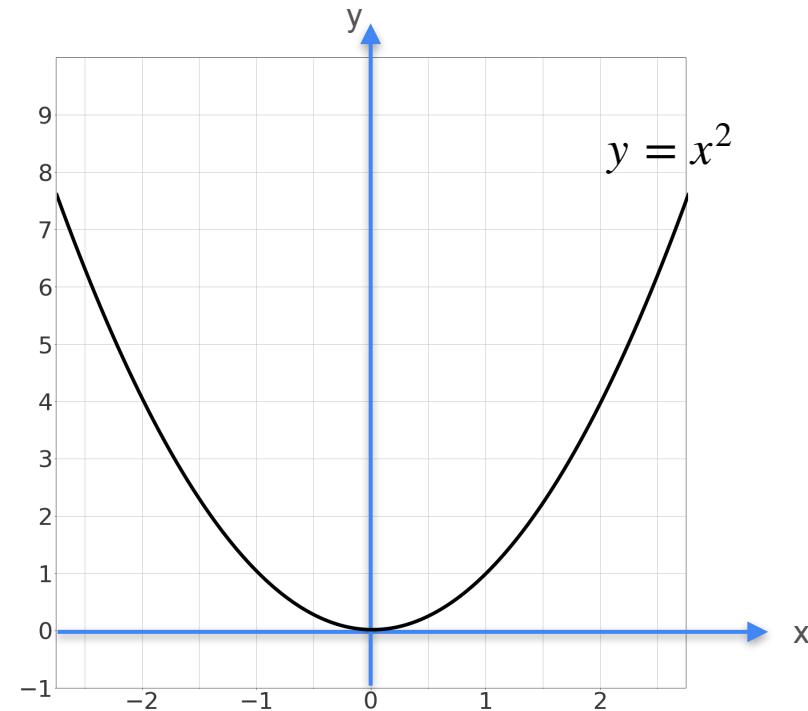
# Multiplication by a Scalar



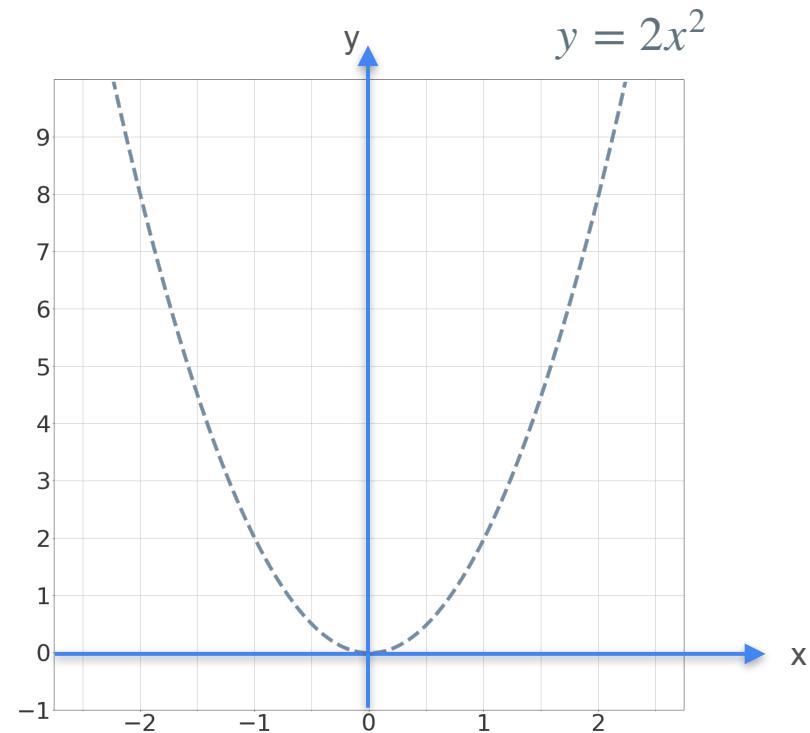
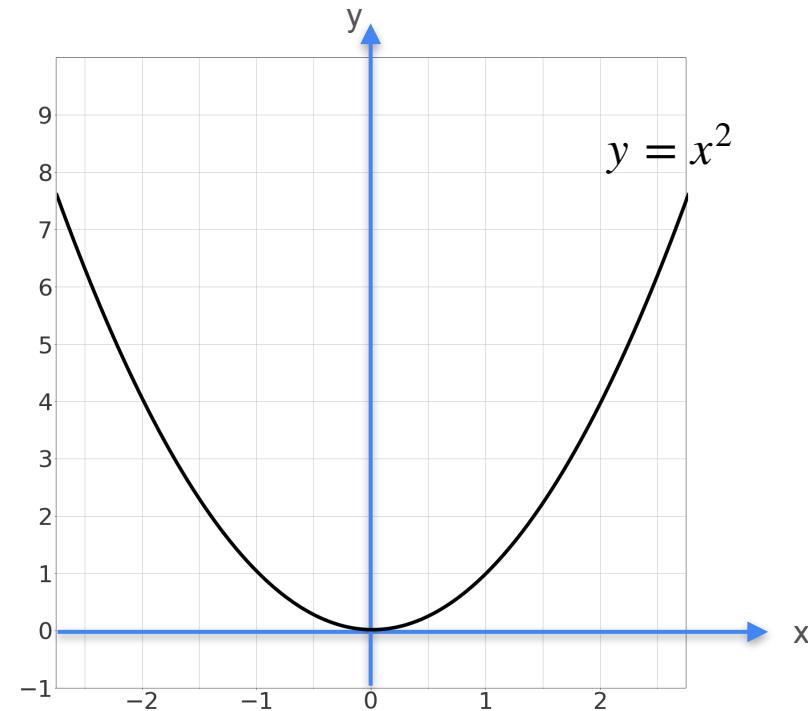
# Multiplication by a Scalar



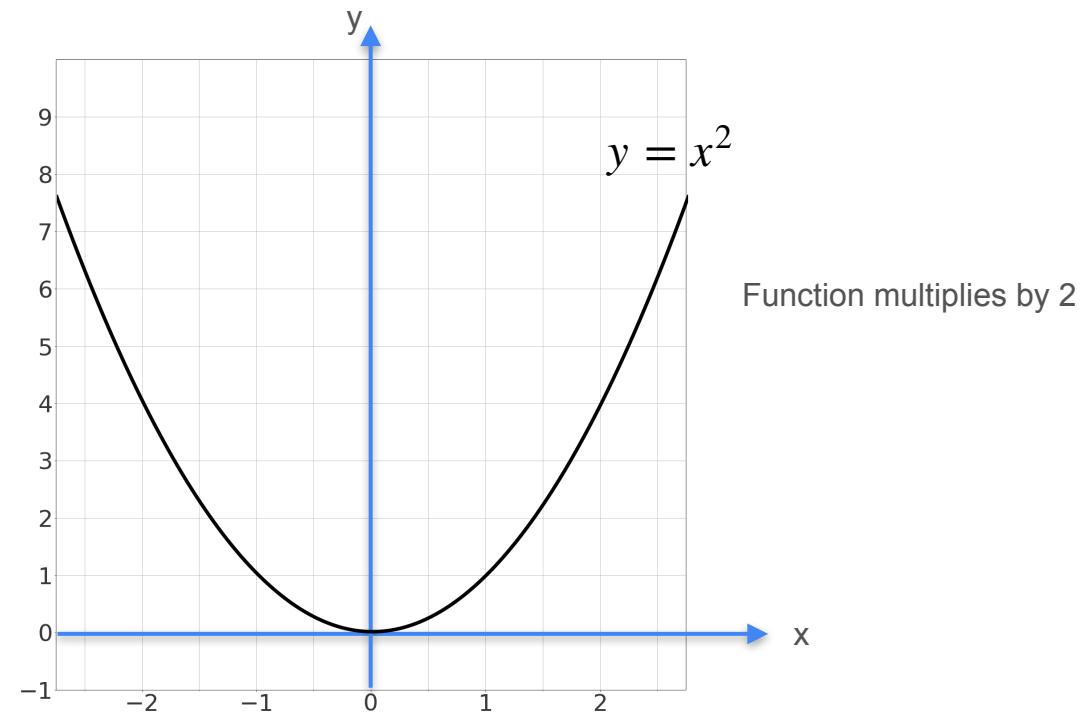
# Multiplication by a Scalar



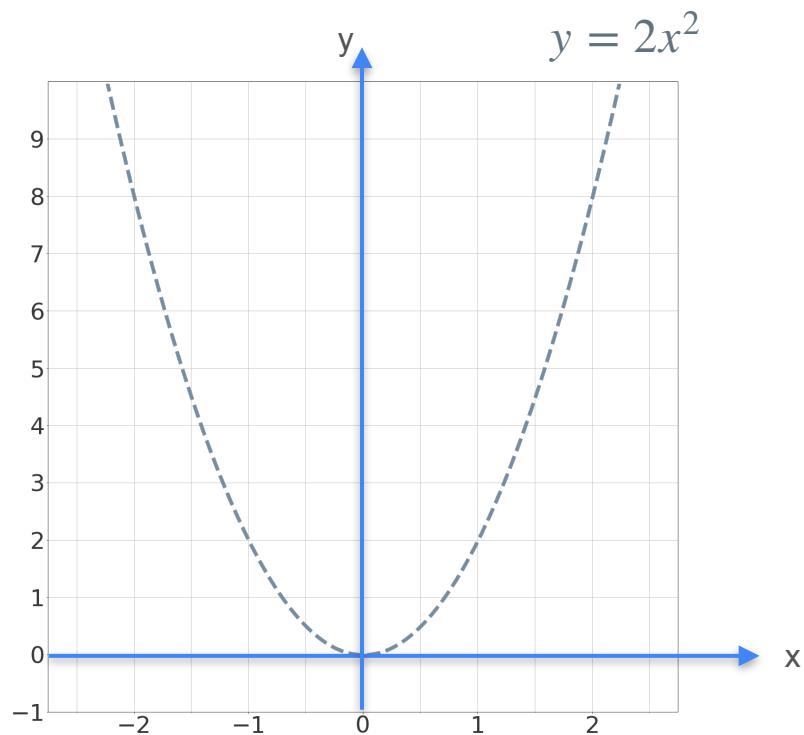
# Multiplication by a Scalar



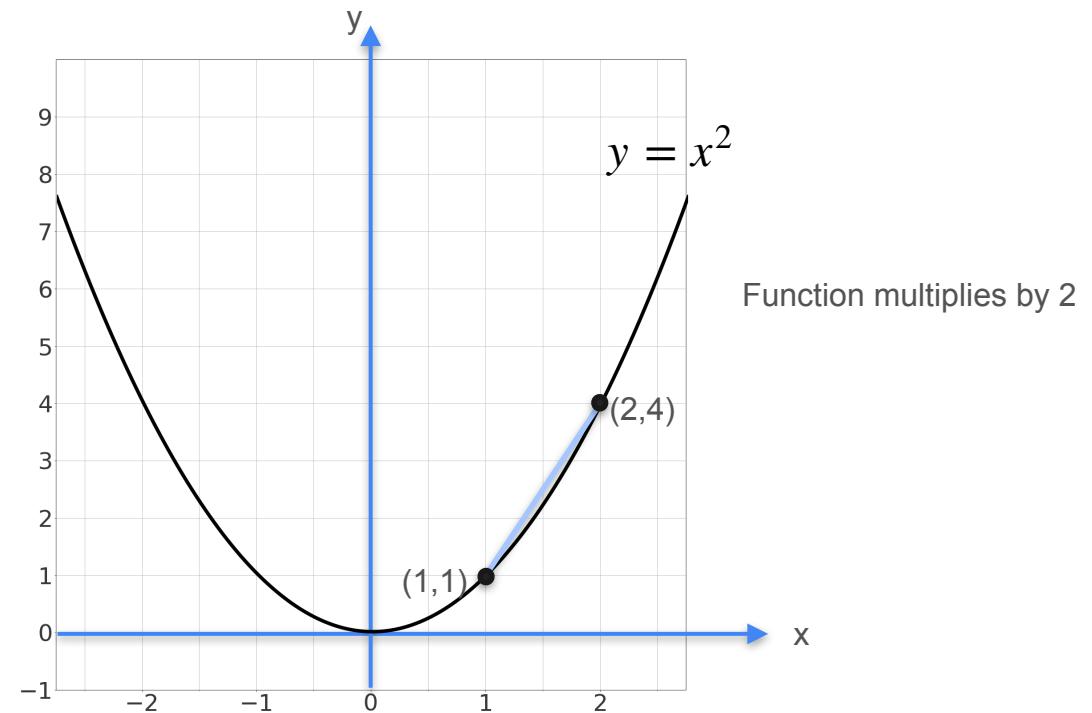
# Multiplication by a Scalar



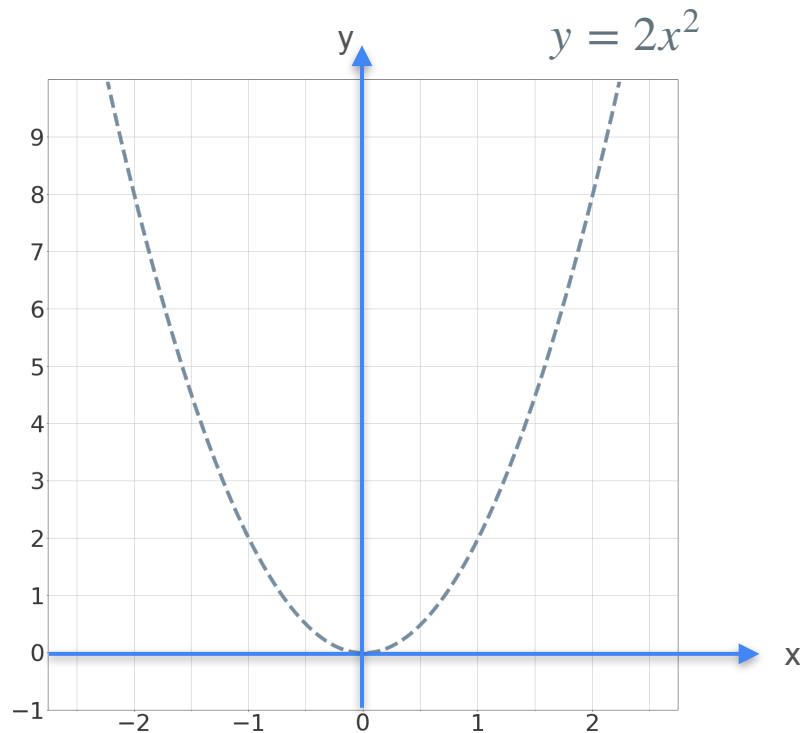
Function multiplies by 2



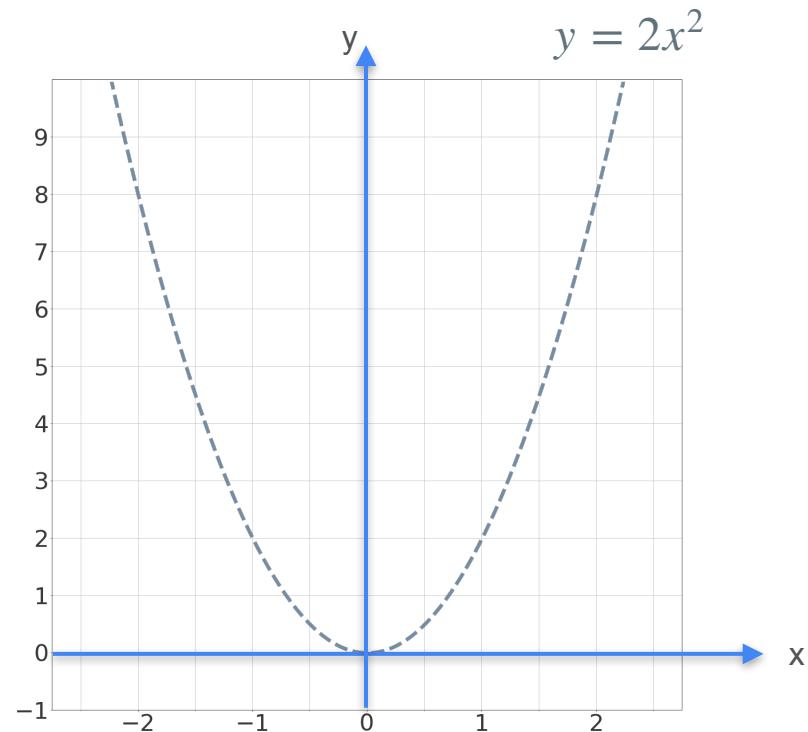
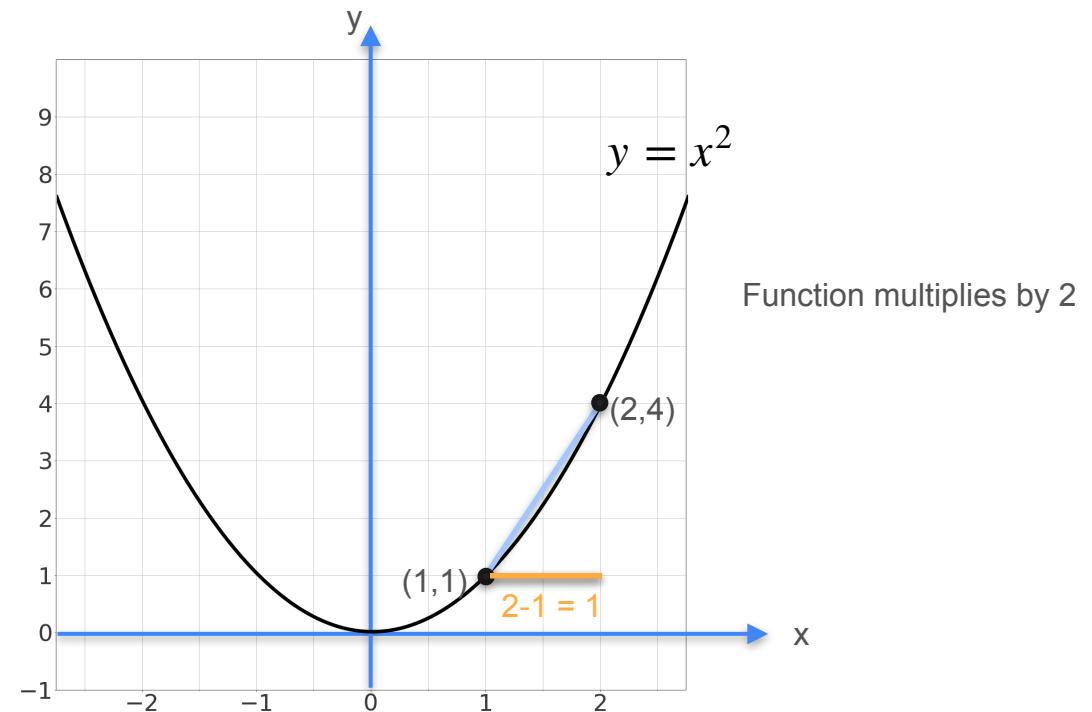
# Multiplication by a Scalar



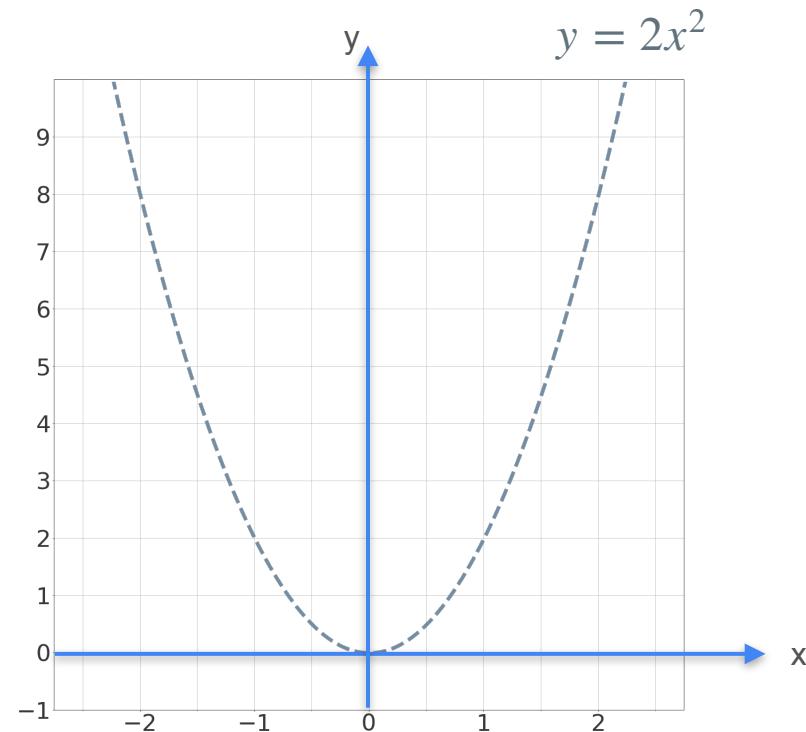
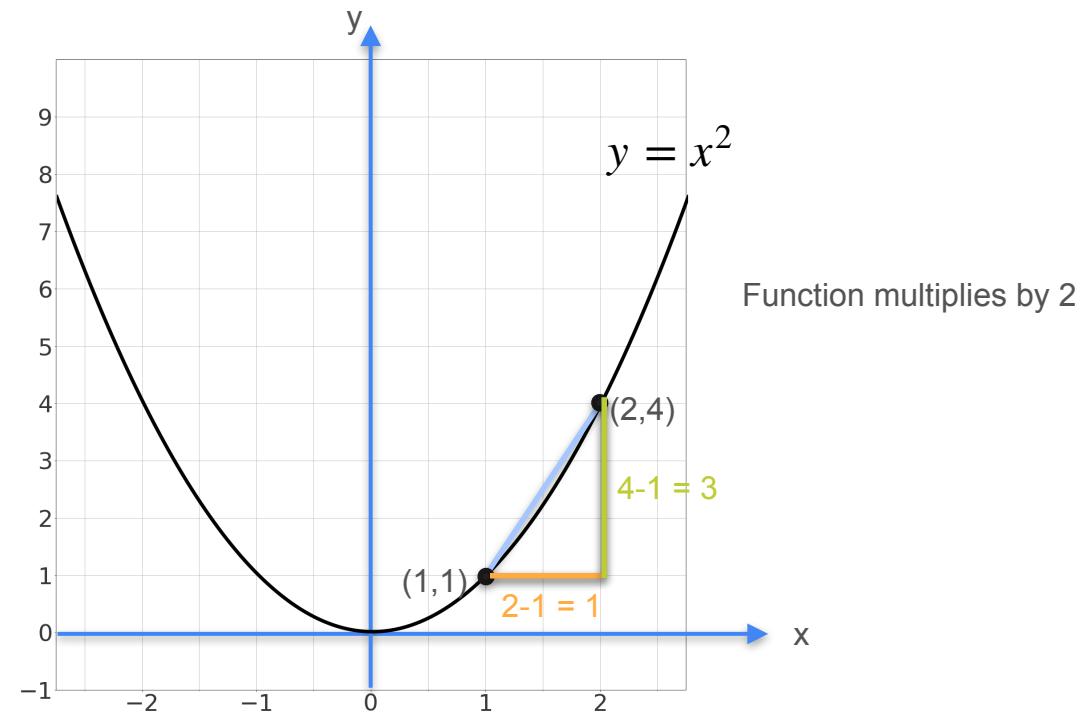
Function multiplies by 2



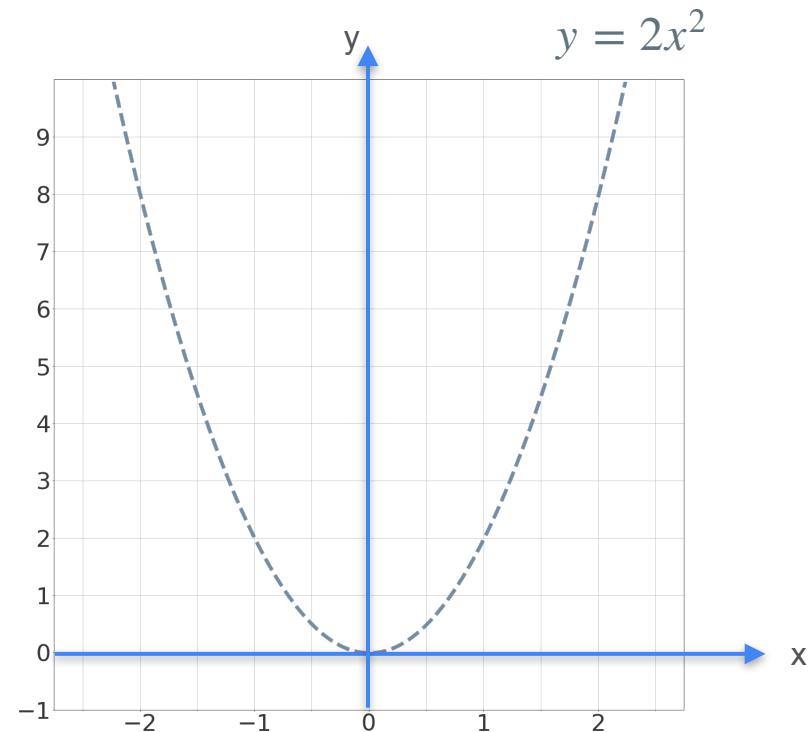
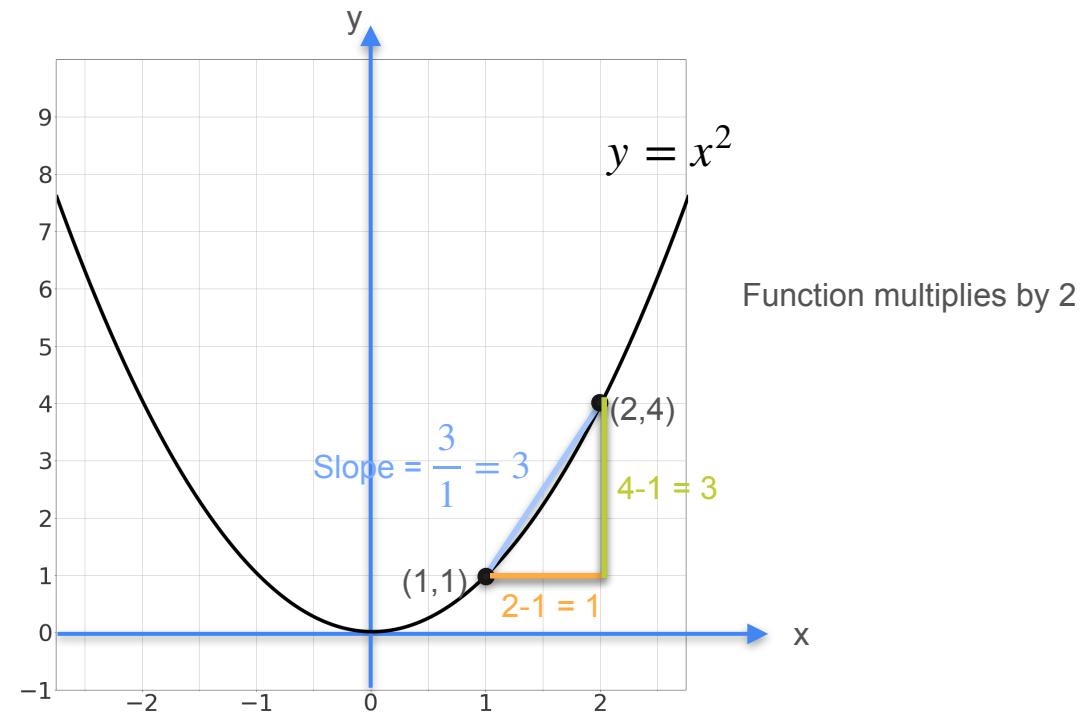
# Multiplication by a Scalar



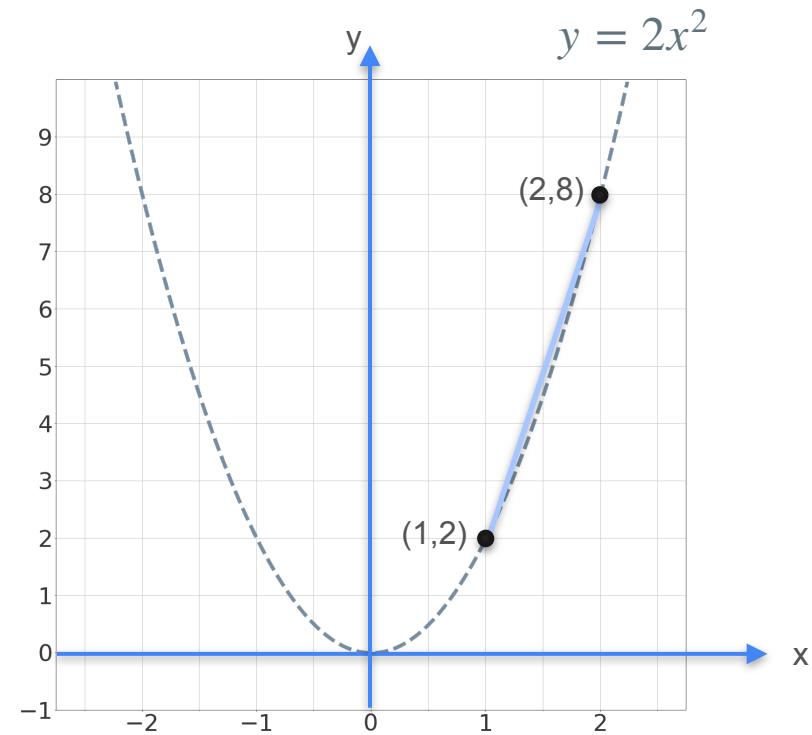
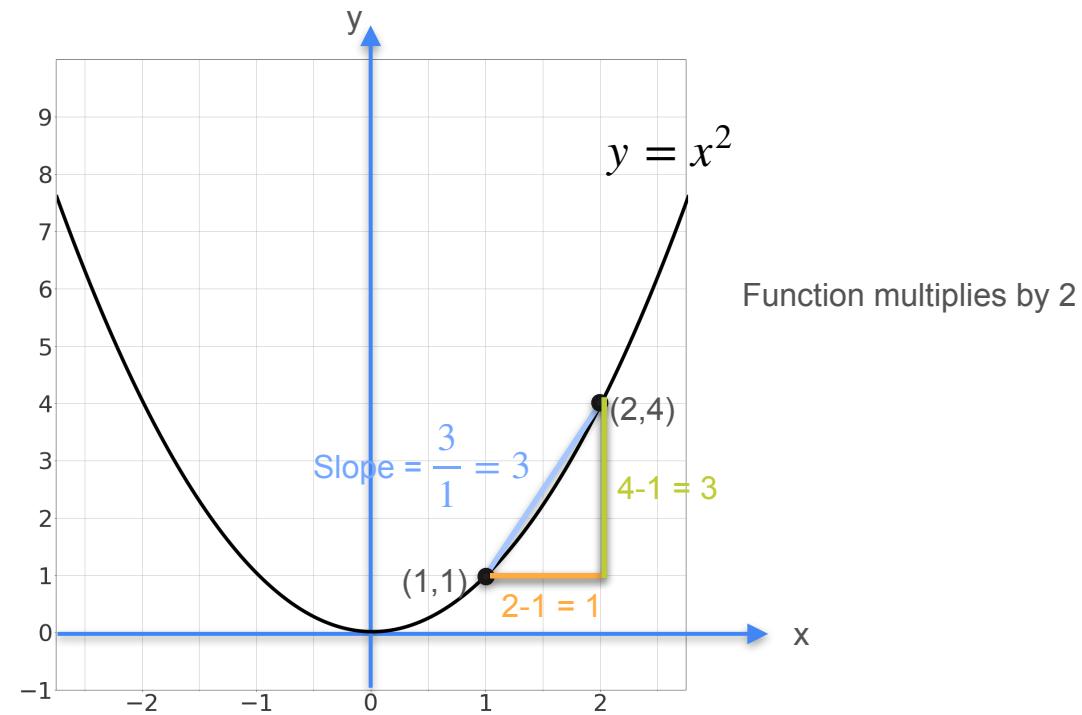
# Multiplication by a Scalar



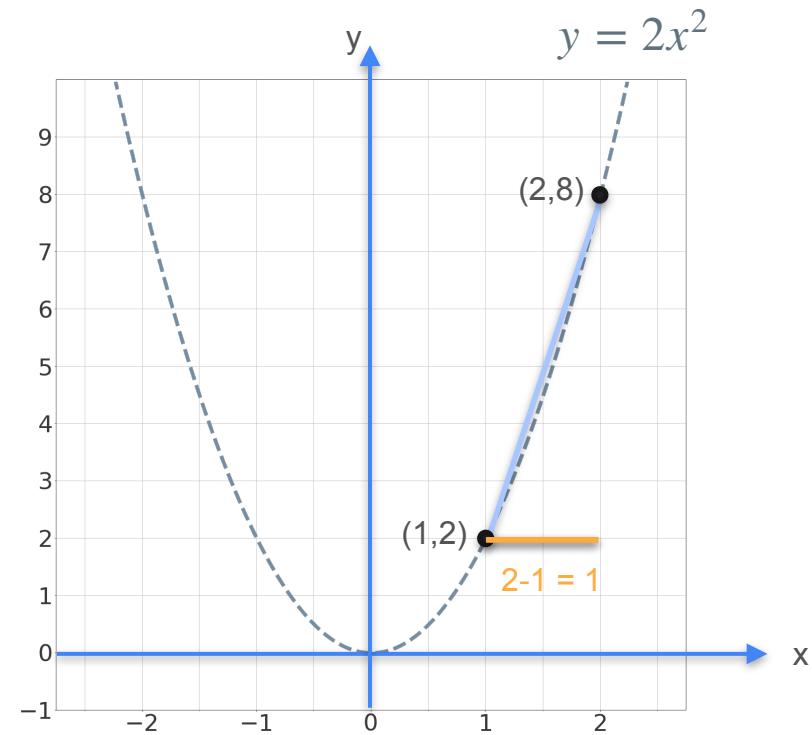
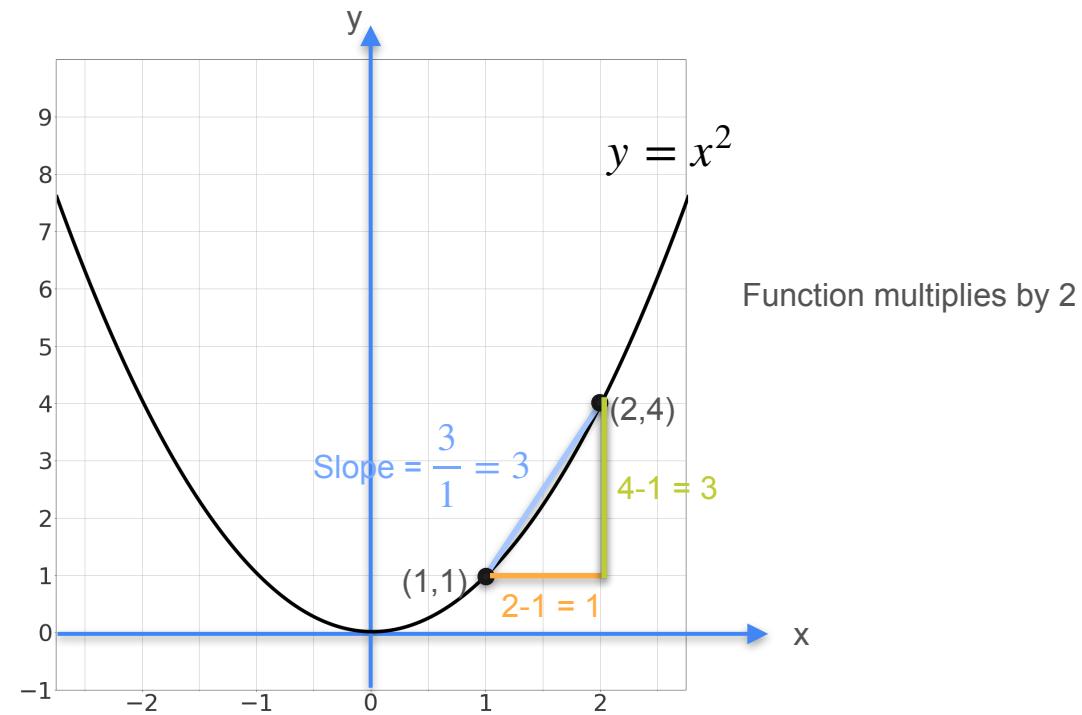
# Multiplication by a Scalar



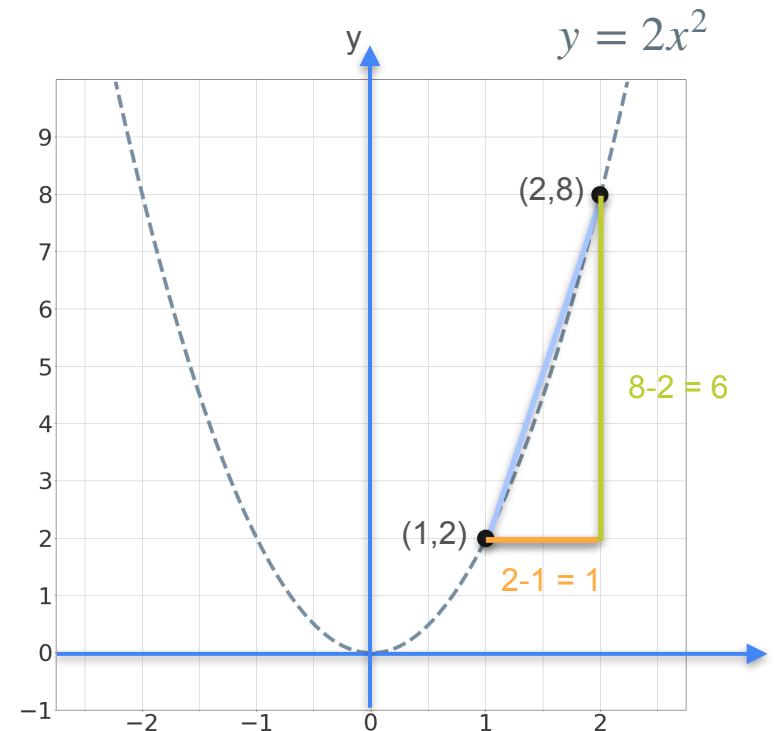
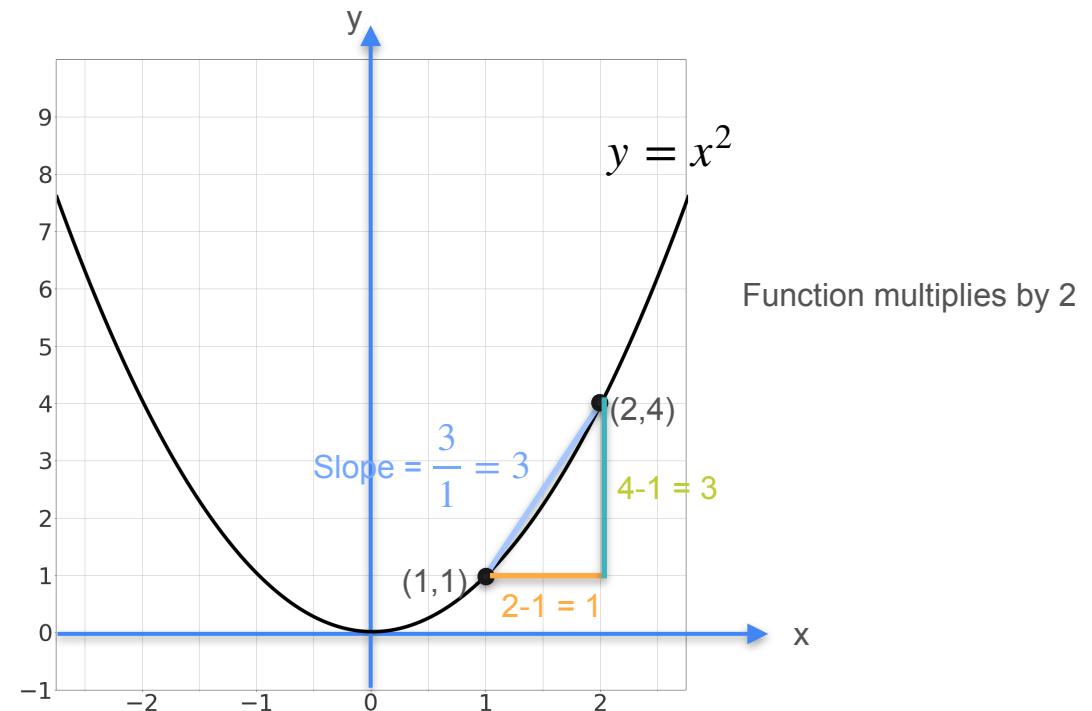
# Multiplication by a Scalar



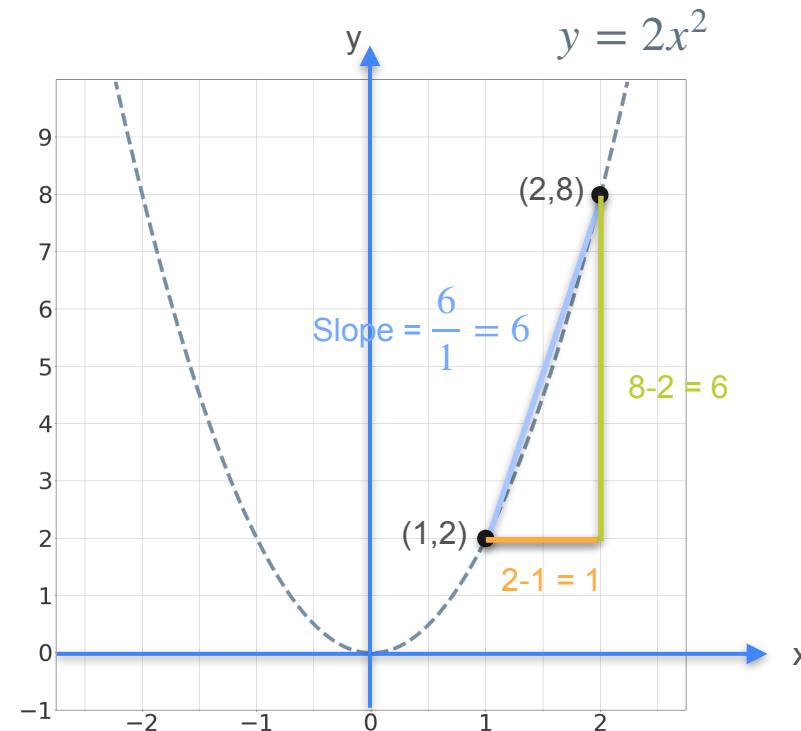
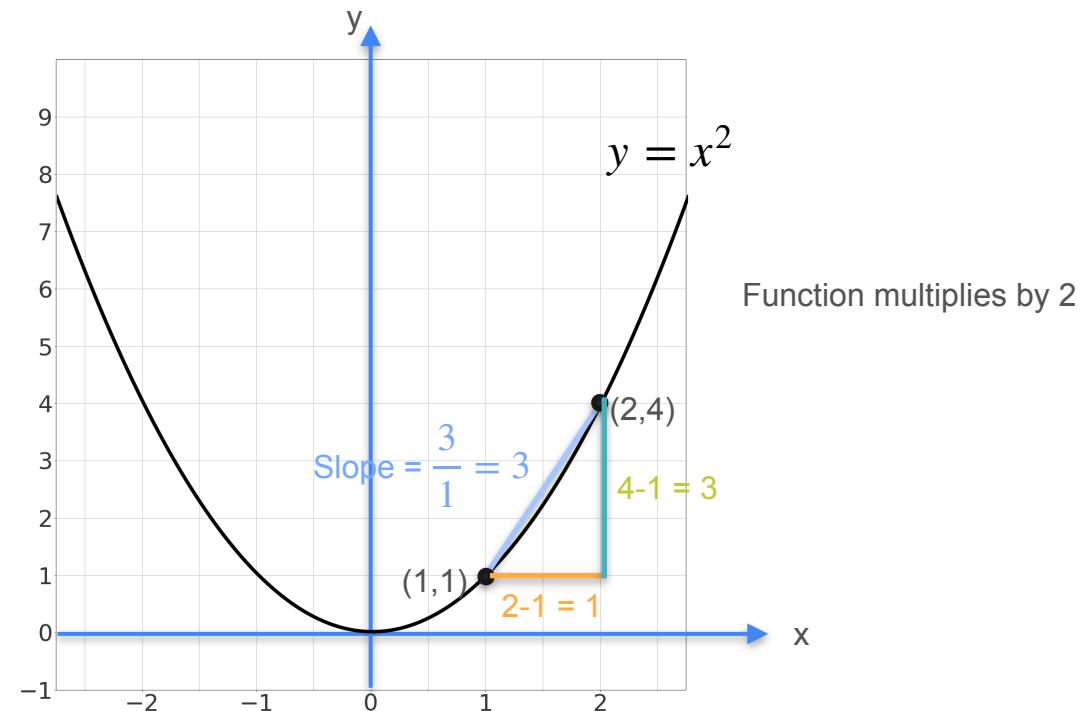
# Multiplication by a Scalar



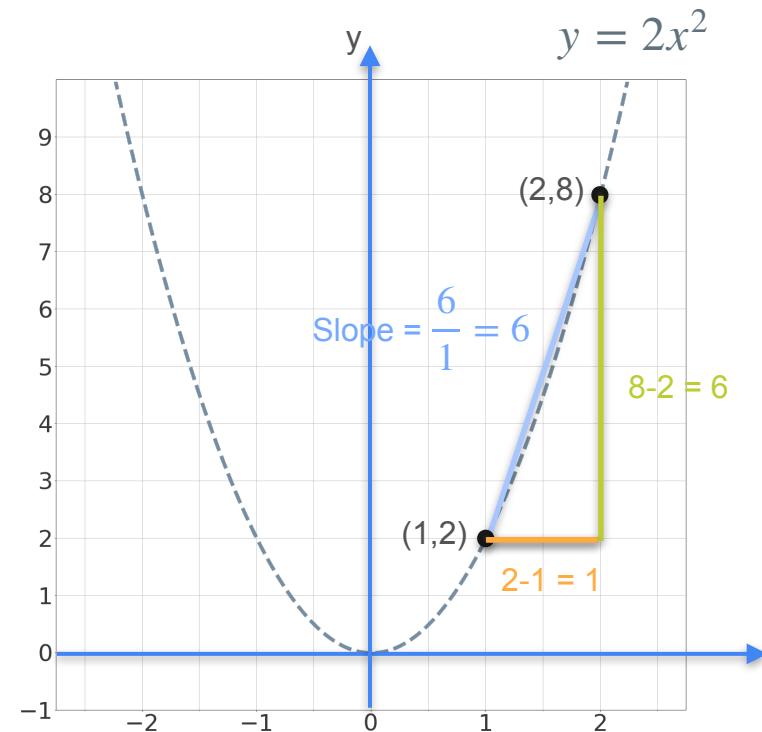
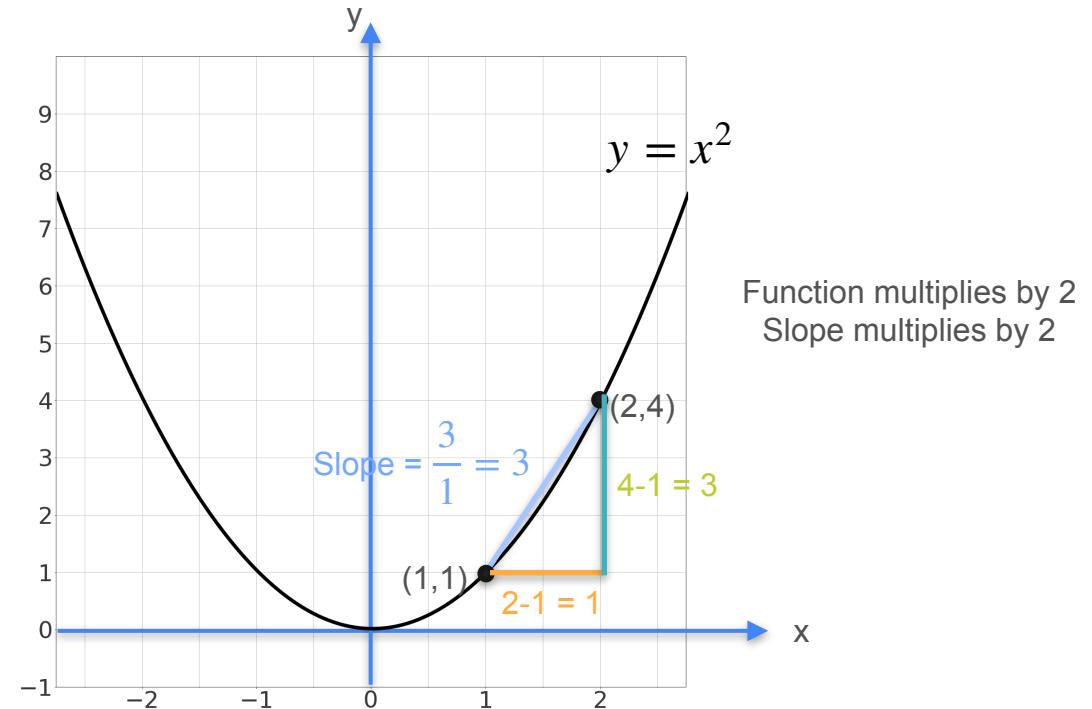
# Multiplication by a Scalar



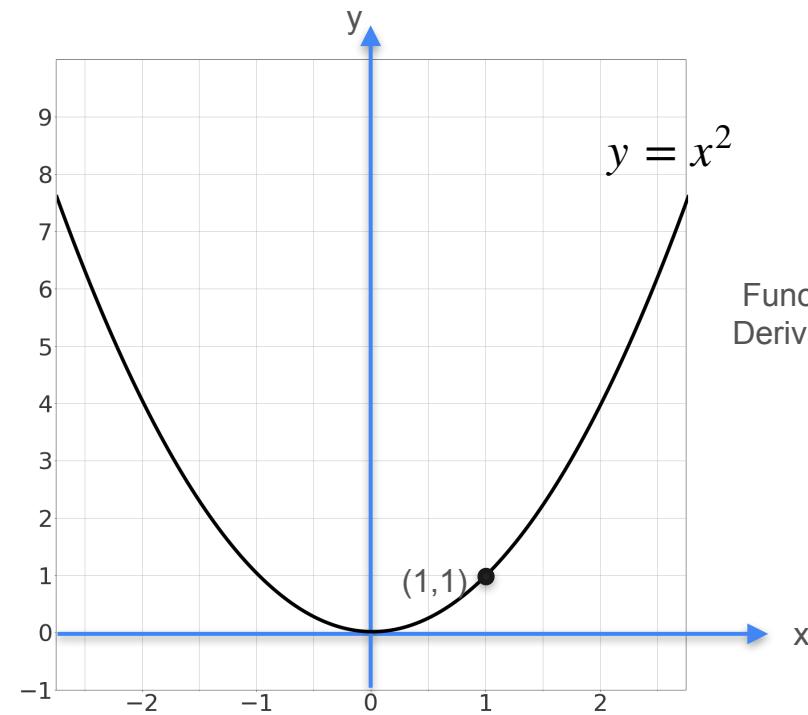
# Multiplication by a Scalar



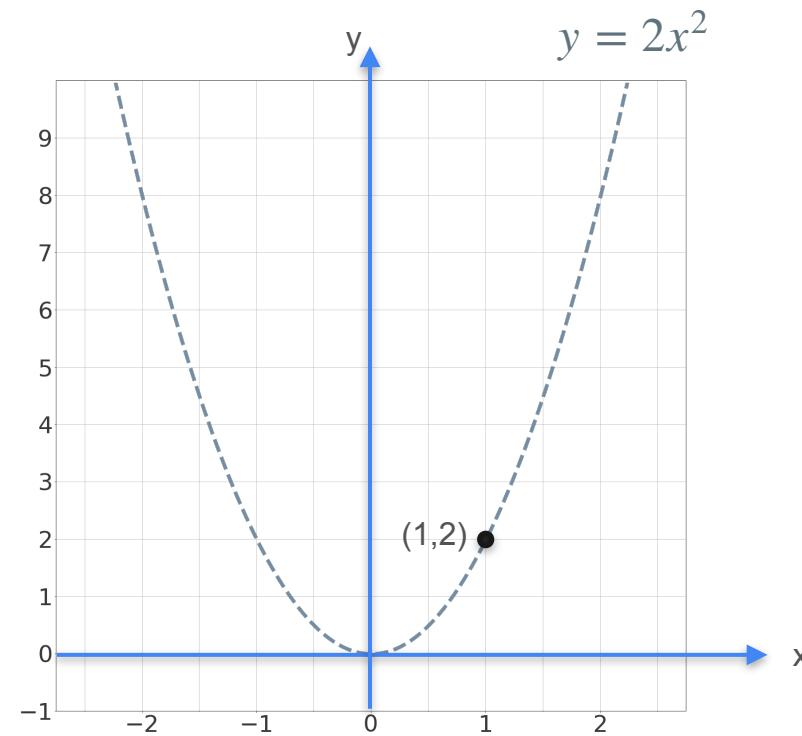
# Multiplication by a Scalar



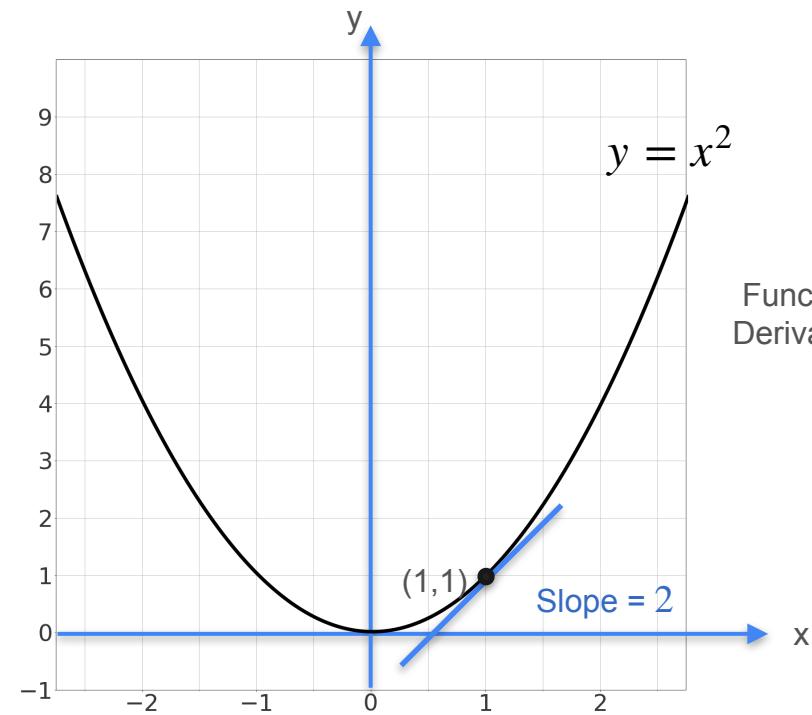
# Multiplication by a Scalar



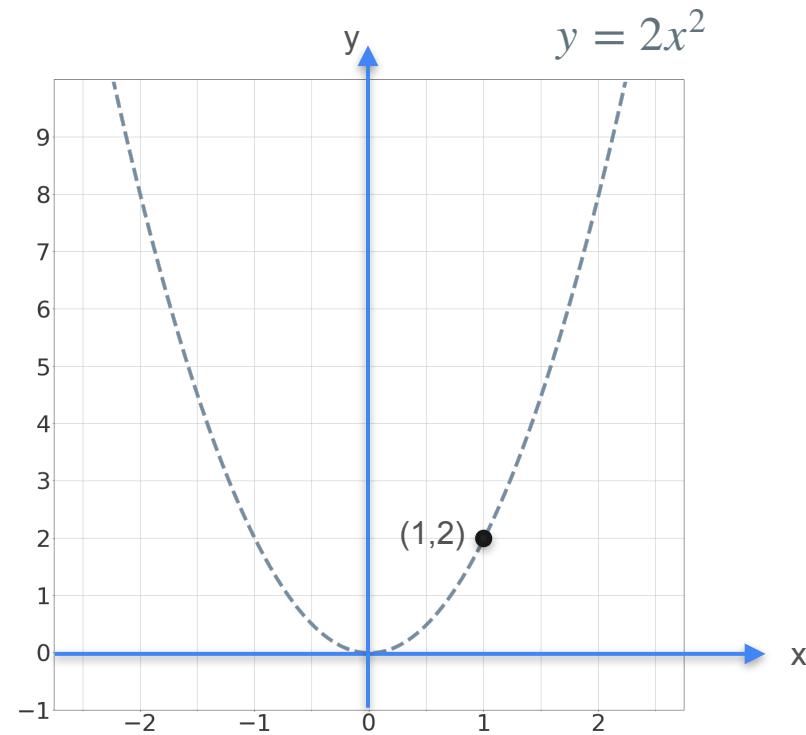
Function multiplies by 2  
Derivative multiplies by 2



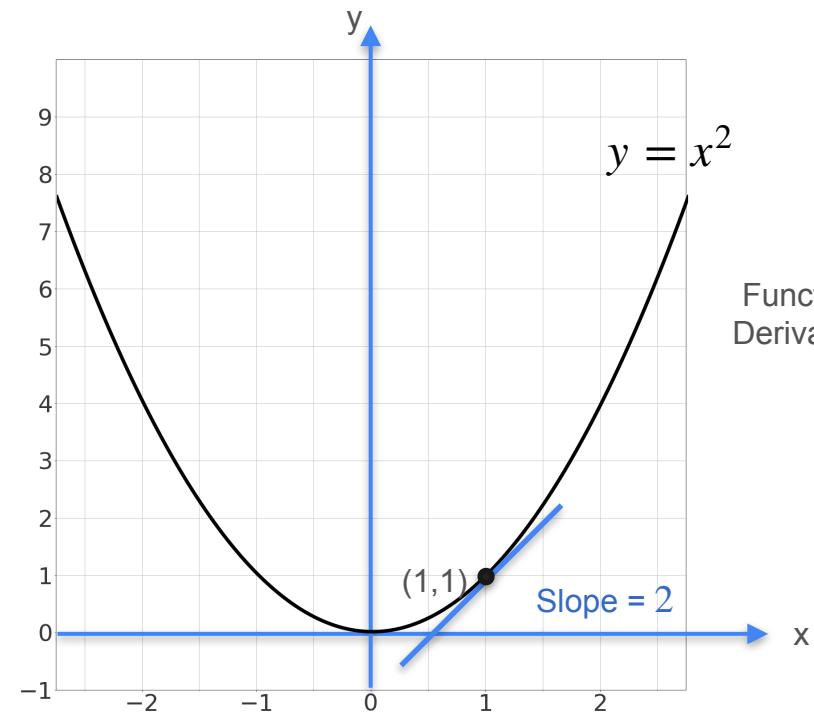
# Multiplication by a Scalar



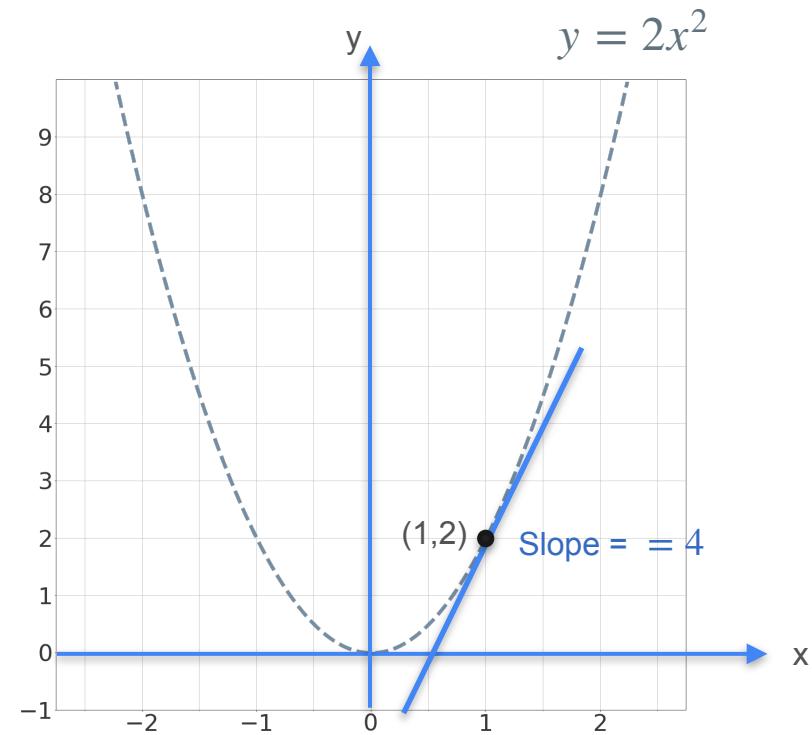
Function multiplies by 2  
Derivative multiplies by 2



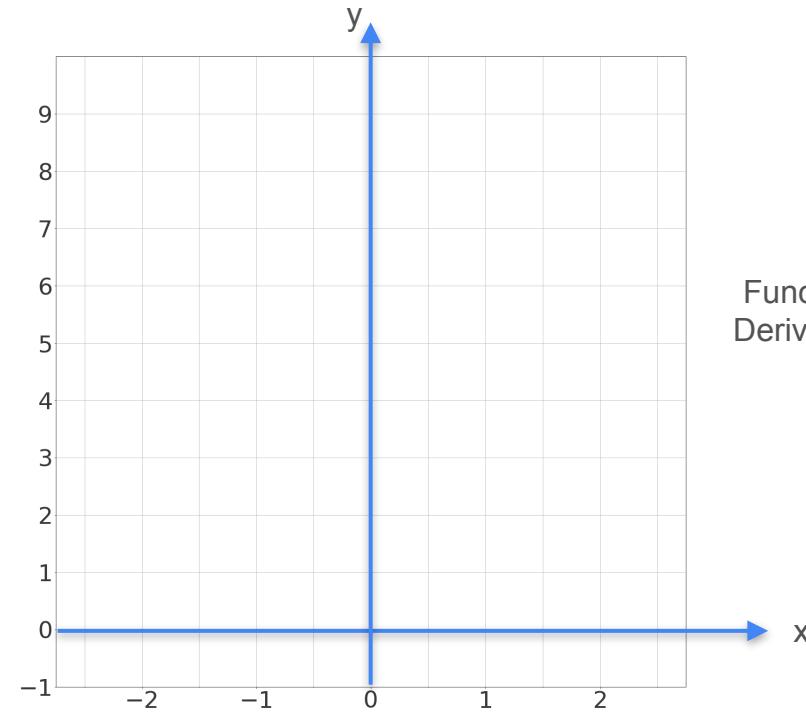
# Multiplication by a Scalar



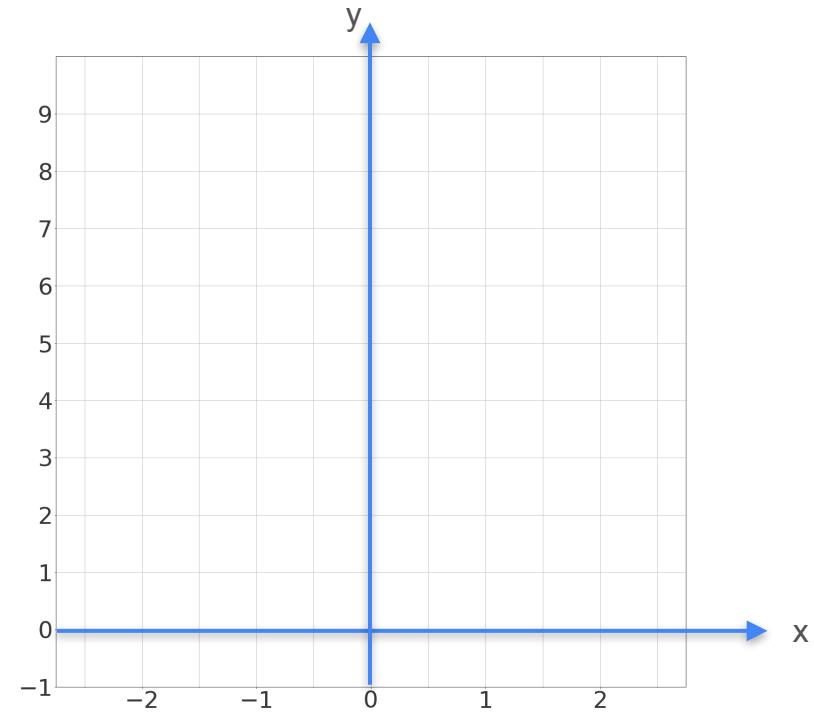
Function multiplies by 2  
Derivative multiplies by 2



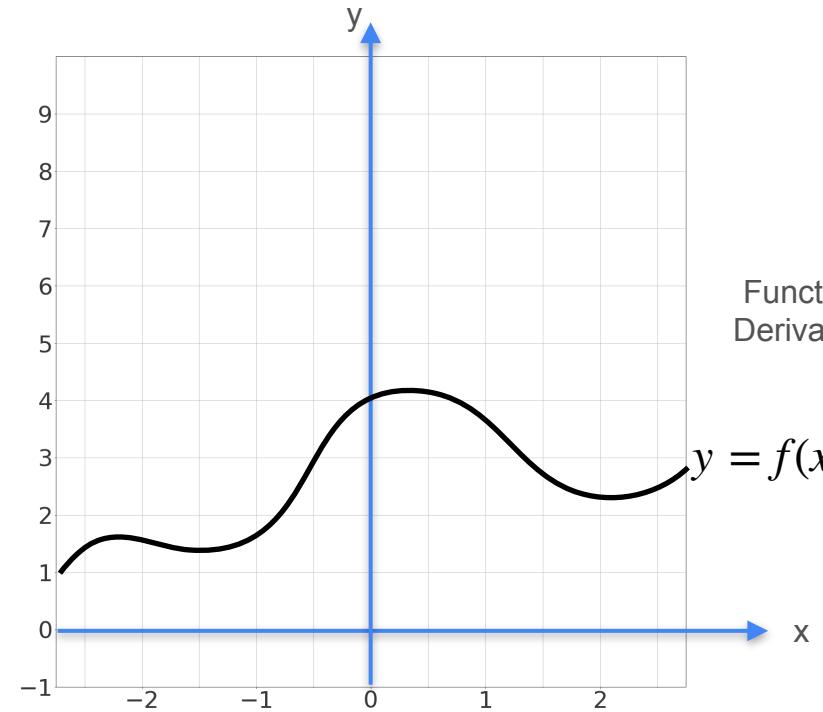
# Multiplication by a Scalar



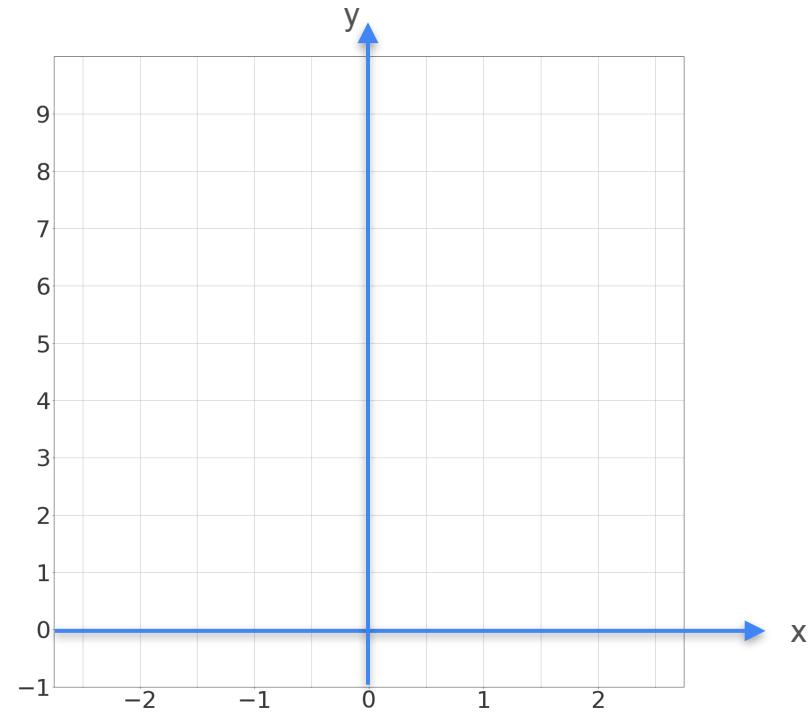
Function multiplies by  $c$   
Derivative multiplies by  $c$



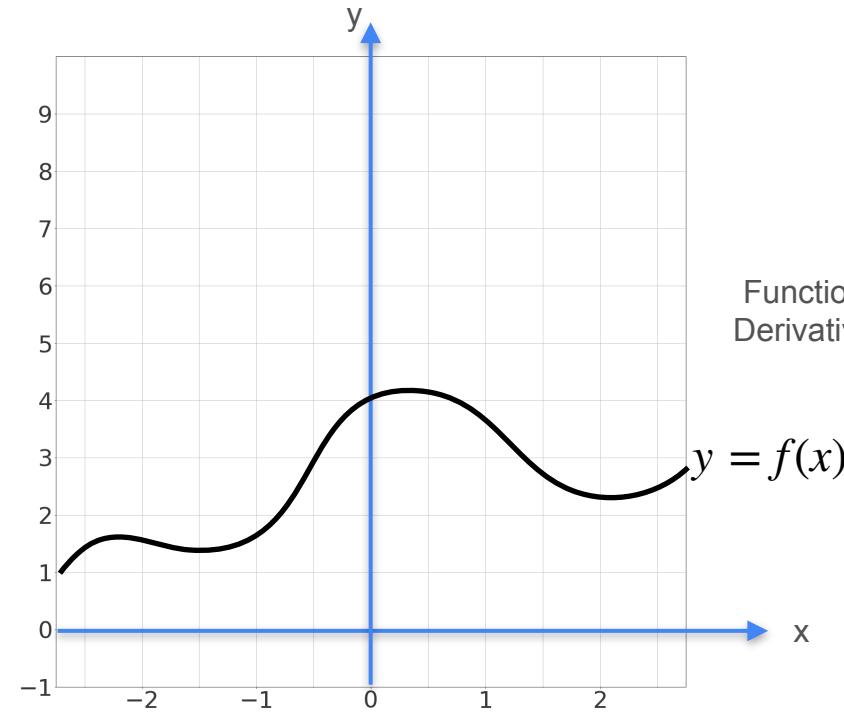
# Multiplication by a Scalar



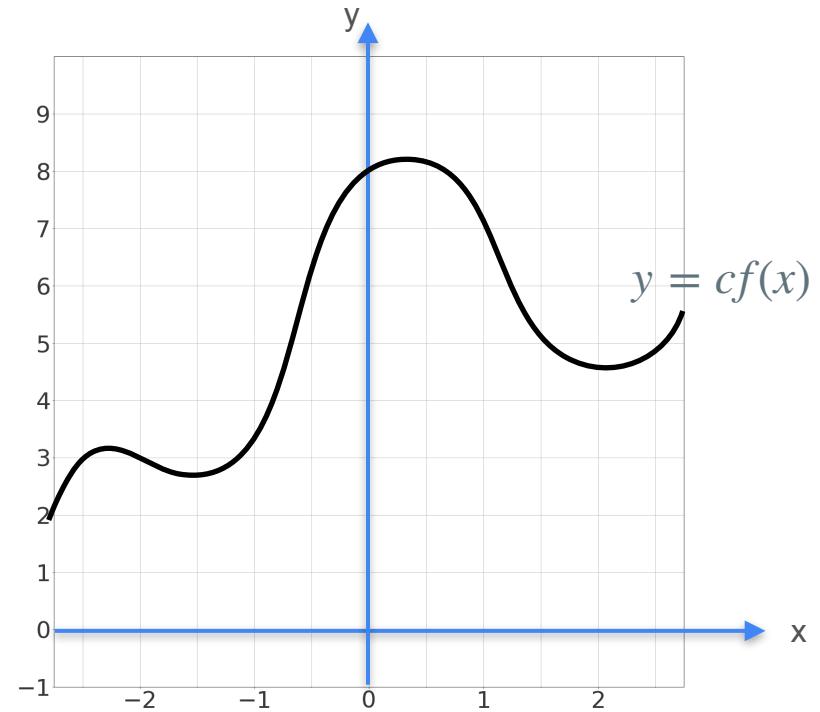
Function multiplies by  $c$   
Derivative multiplies by  $c$



# Multiplication by a Scalar

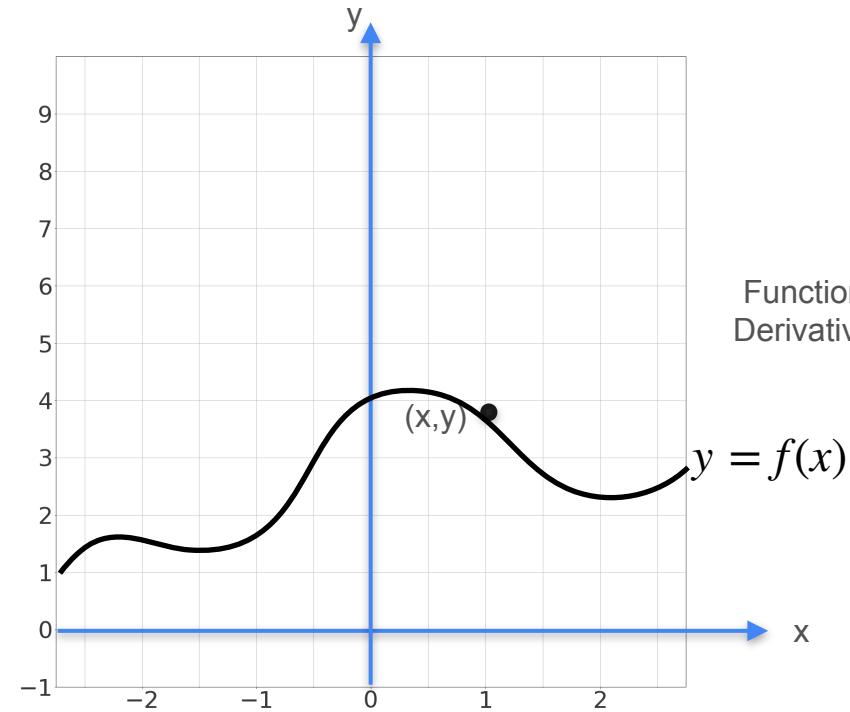


Function multiplies by  $c$   
Derivative multiplies by  $c$

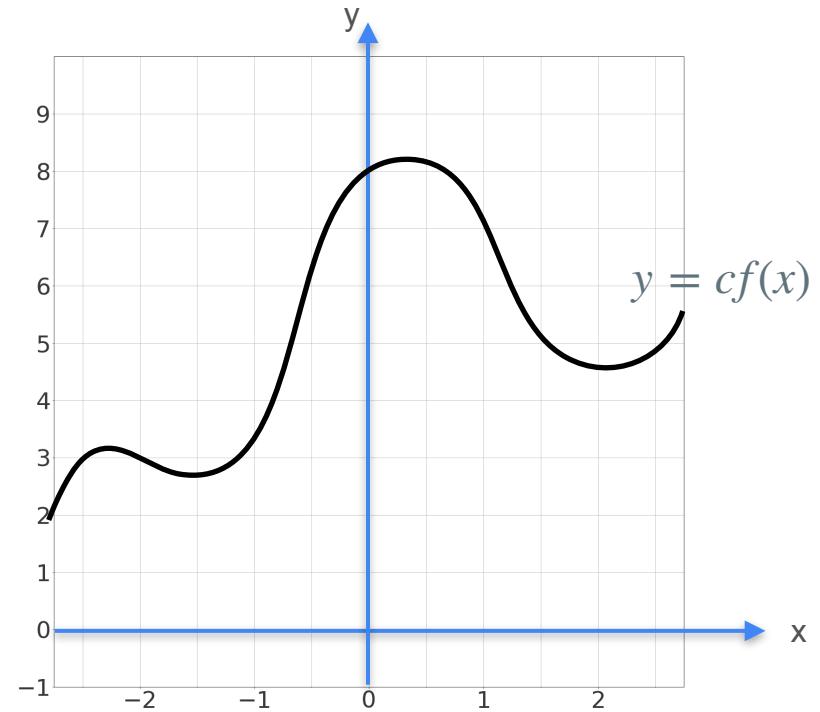


$$y = cf(x)$$

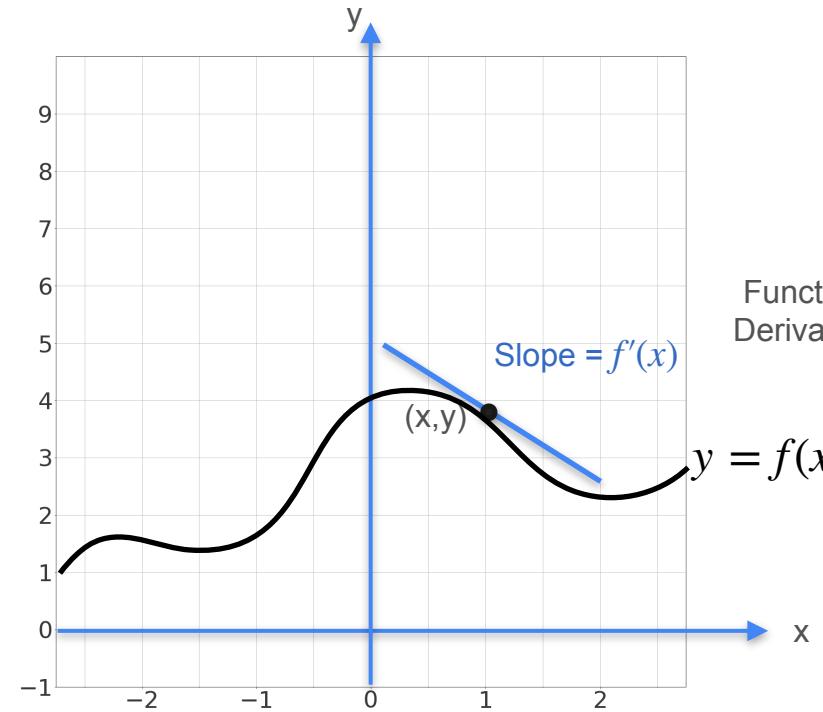
# Multiplication by a Scalar



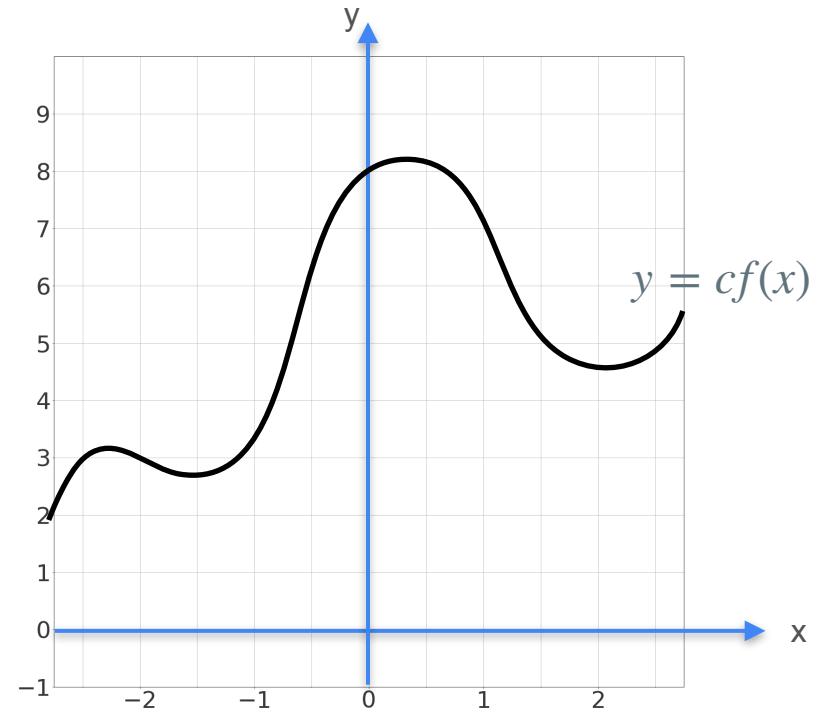
Function multiplies by  $c$   
Derivative multiplies by  $c$



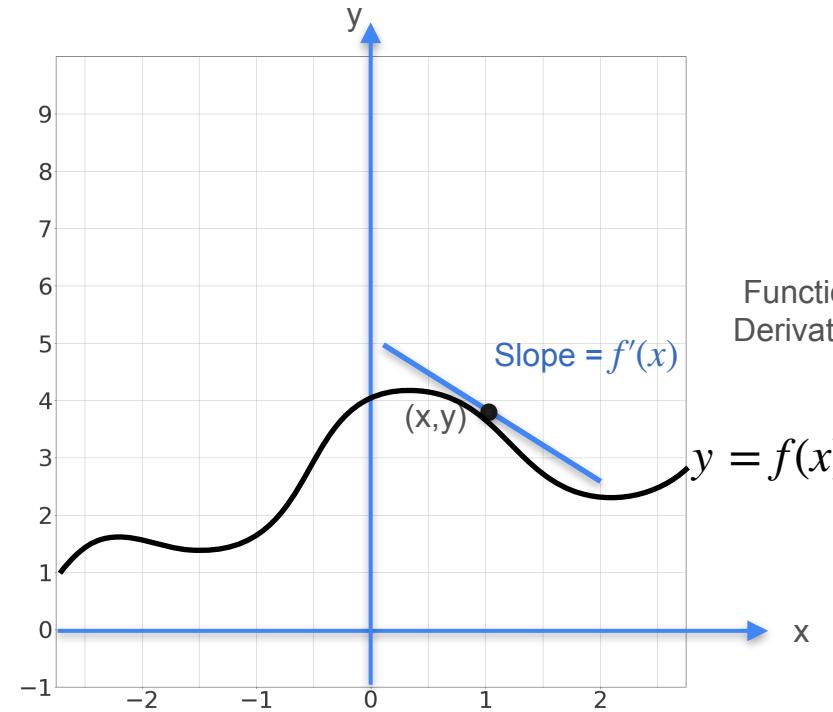
# Multiplication by a Scalar



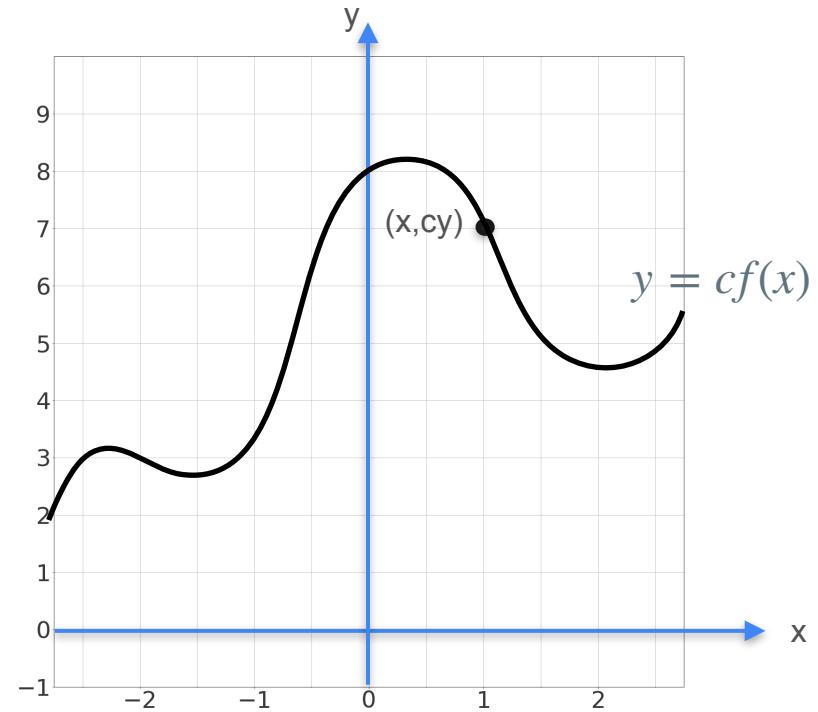
Function multiplies by  $c$   
Derivative multiplies by  $c$



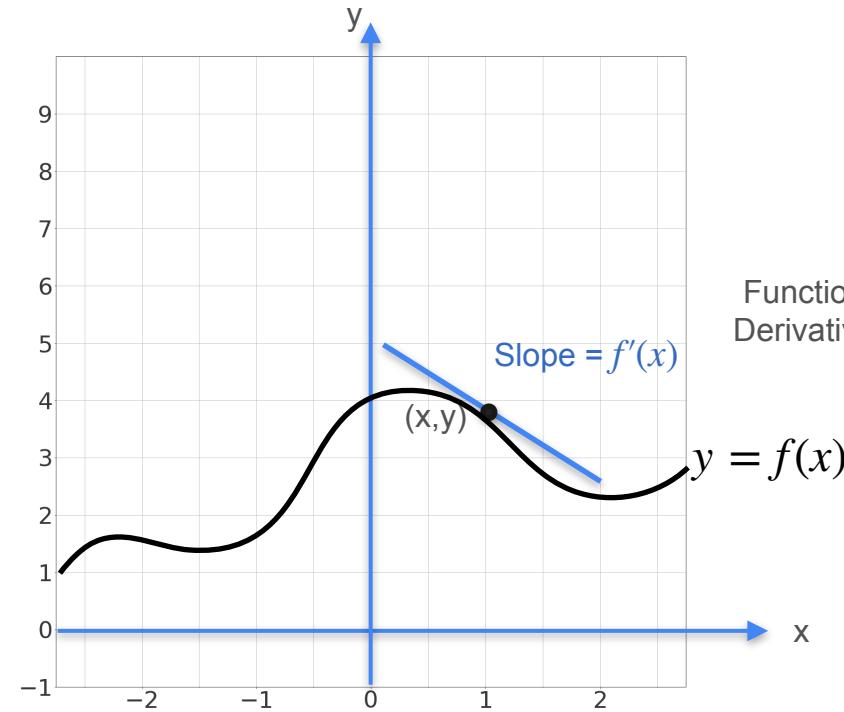
# Multiplication by a Scalar



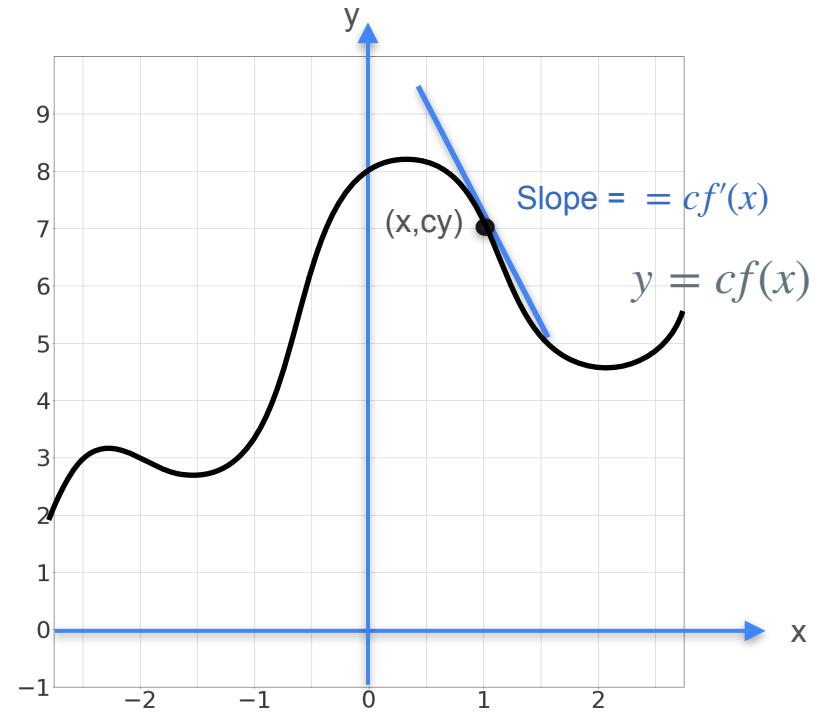
Function multiplies by  $c$   
Derivative multiplies by  $c$



# Multiplication by a Scalar



Function multiplies by c  
Derivative multiplies by c





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# Derivatives and Optimization

---

**Properties of the derivative:  
The sum rule**

# The Sum Rule

$$f = g + h$$

# The Sum Rule

$$f' = g + h$$

# The Sum Rule

$$f' = g' + h$$

# The Sum Rule

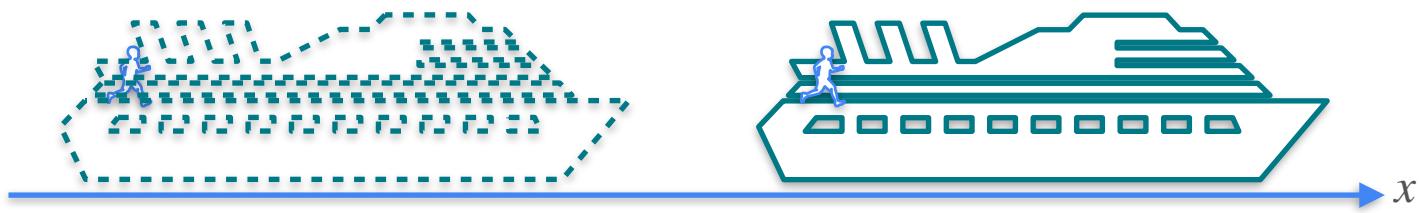
$$f' = g' + h'$$

# Sum Rule

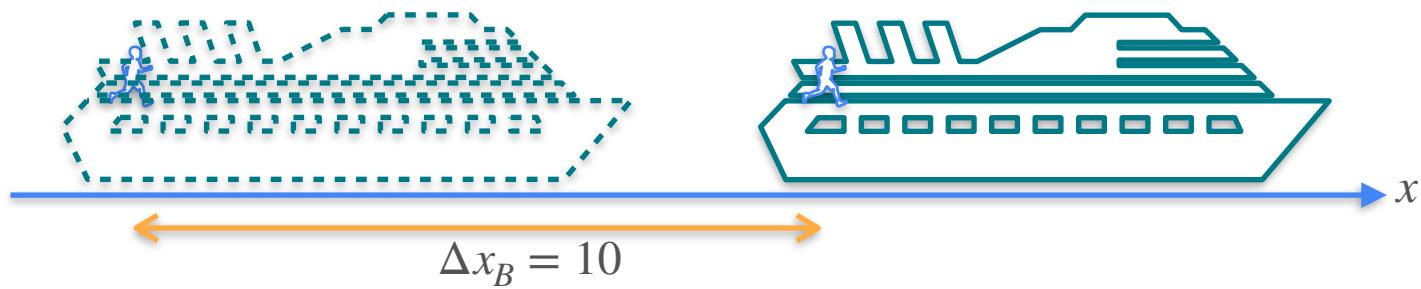
# Sum Rule



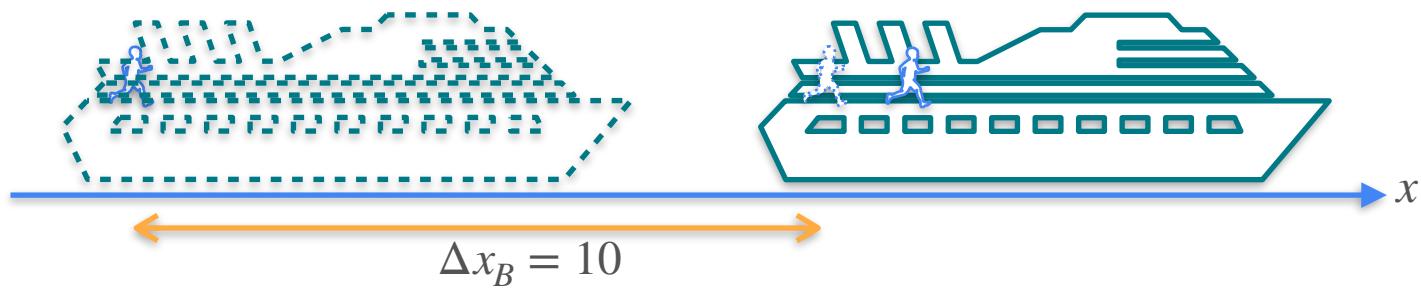
# Sum Rule



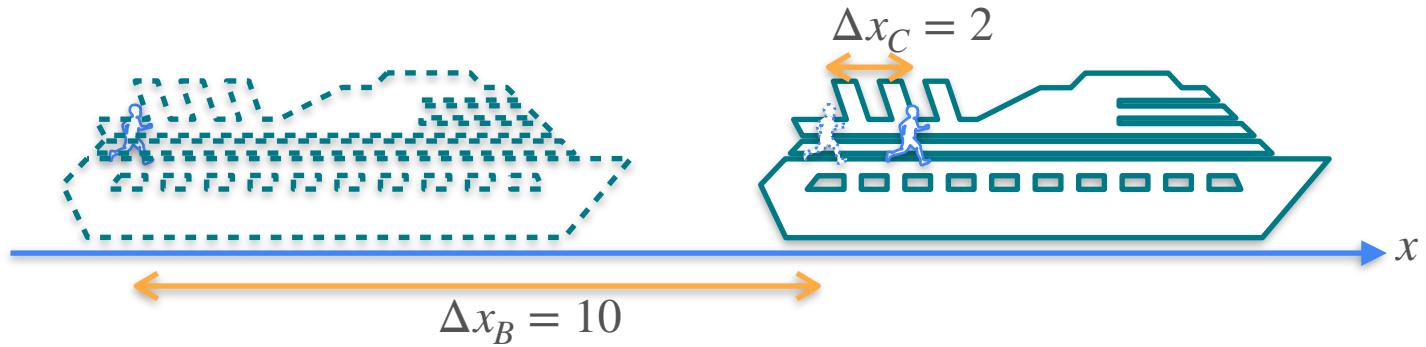
# Sum Rule



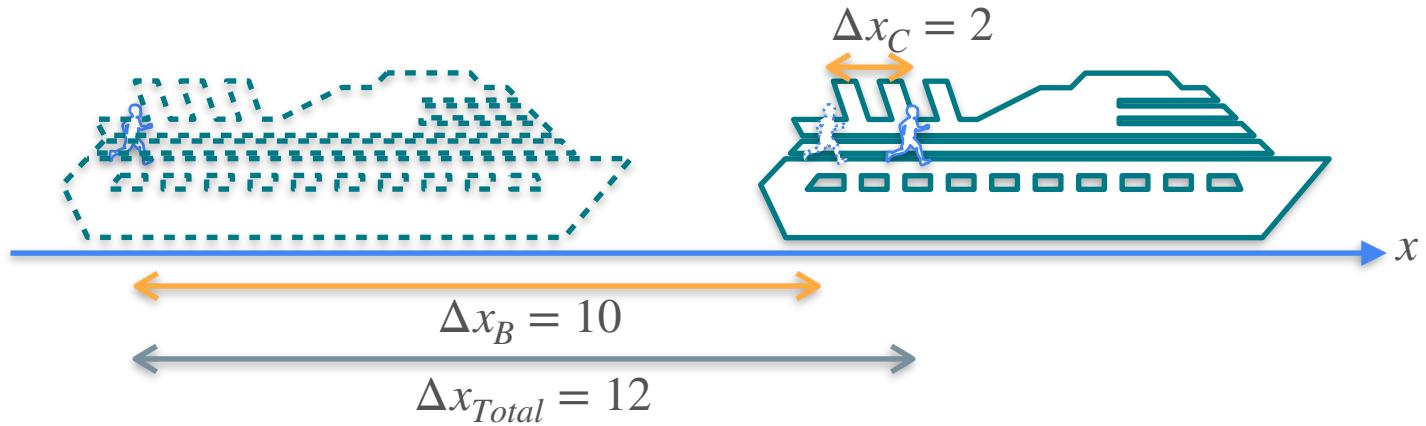
# Sum Rule



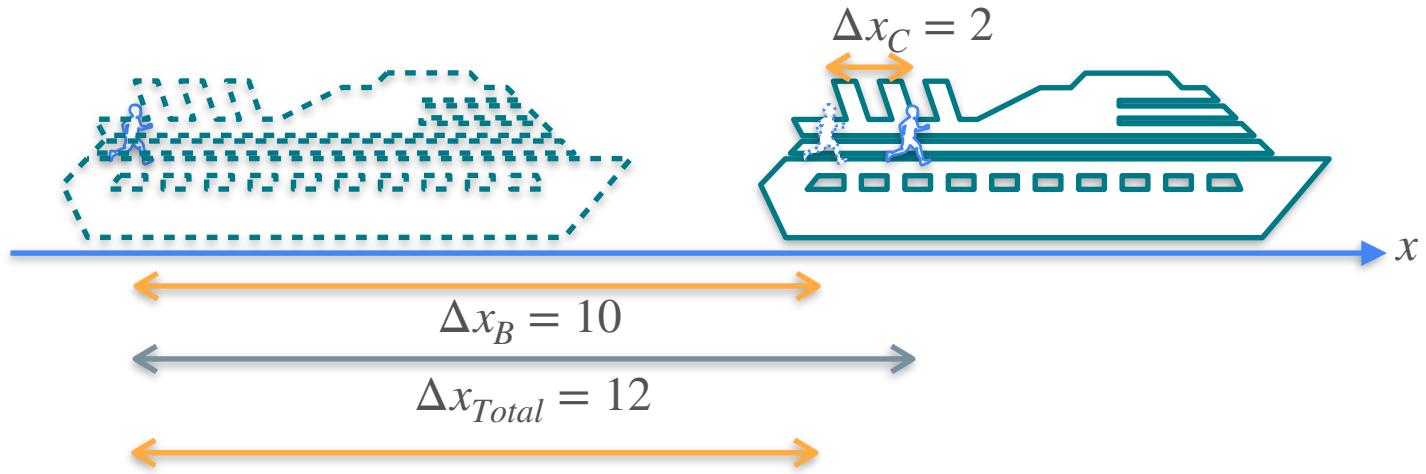
# Sum Rule



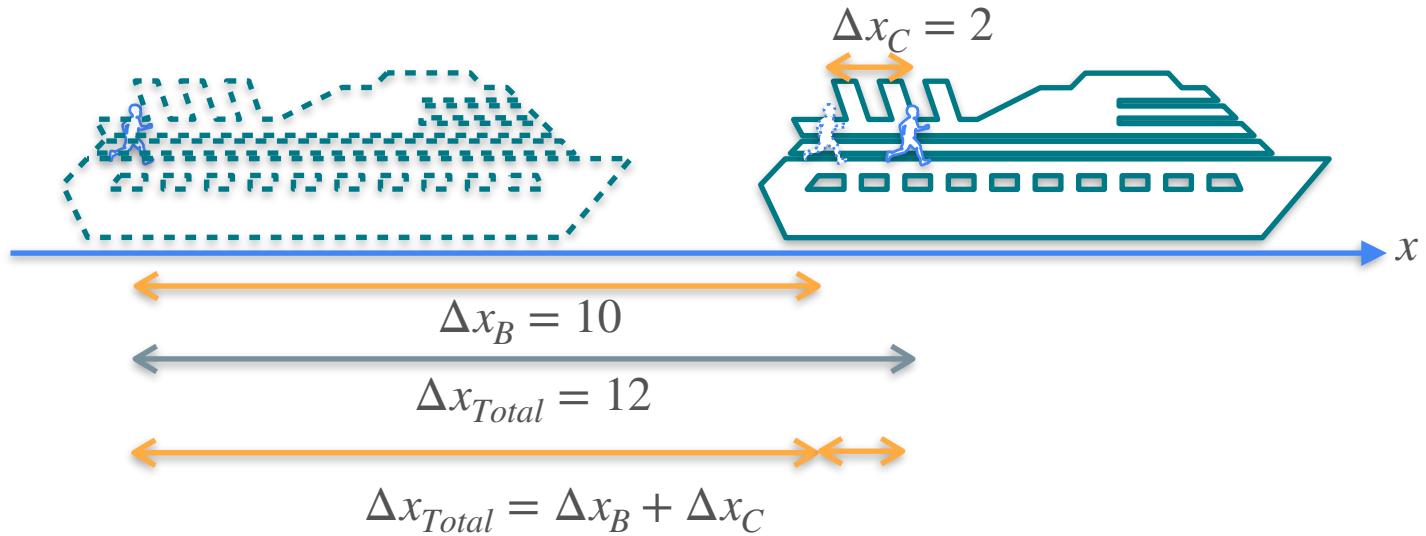
# Sum Rule



# Sum Rule



# Sum Rule



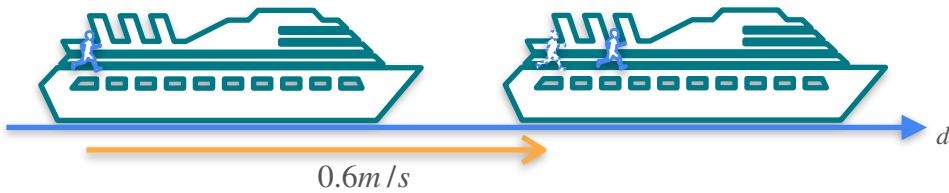
# Quiz:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

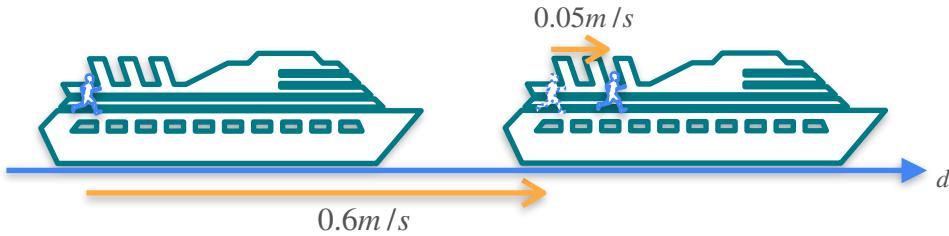
# Quiz:



- Speed of the boat:  $0.6 \text{ m/s}$
- Speed of the child inside the boat:  $0.05 \text{ m/s}$
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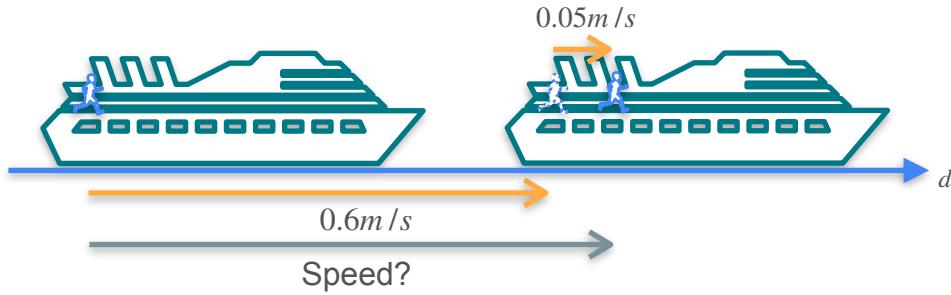
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- Speed of the boat:  $0.6$  m/s
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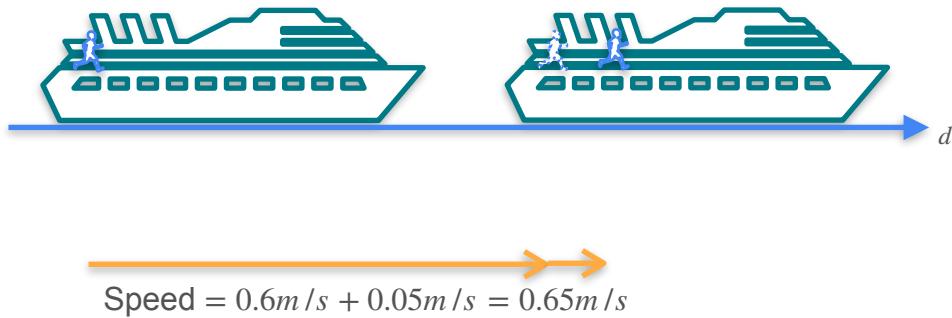
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- Speed of the boat:  $0.6 \text{ m/s}$
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What is the speed of the child with respect to the earth?

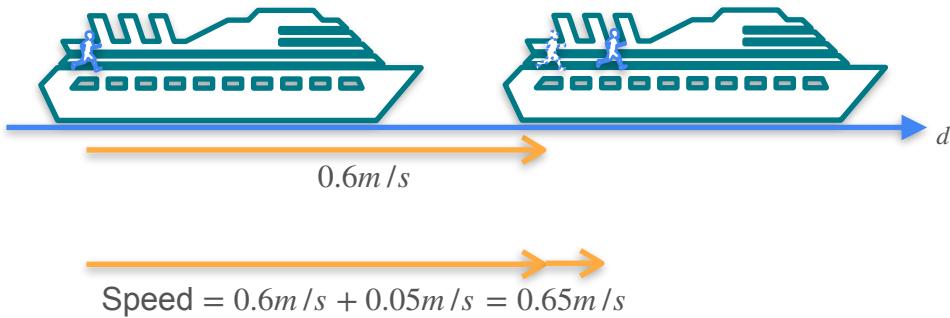
# Solution:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

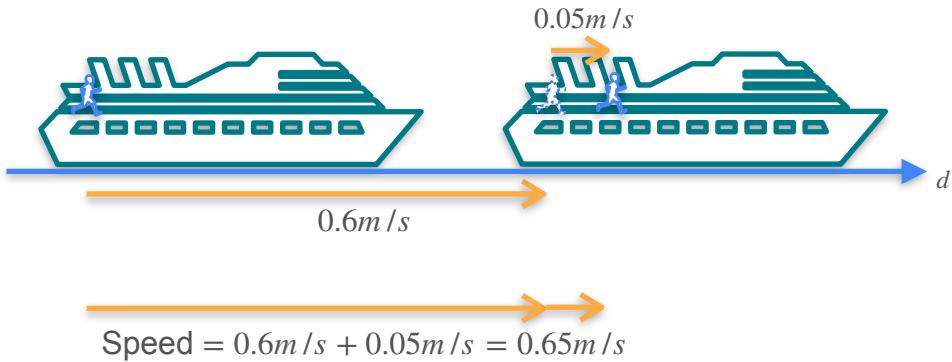
# Solution:



- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

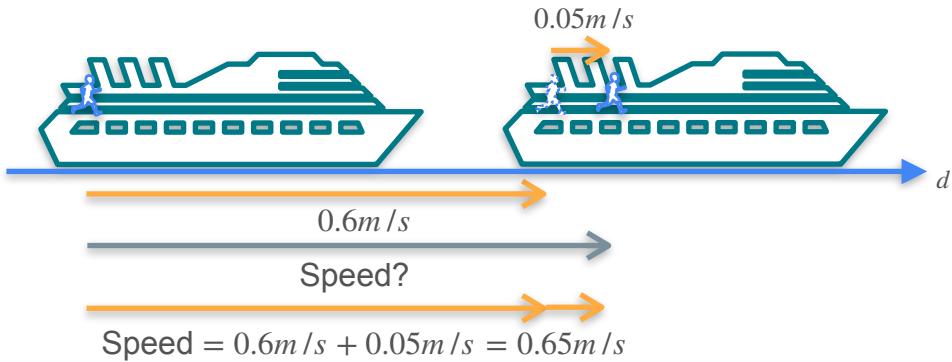
# Solution:



- Speed of the boat:  $0.6 \text{ m/s}$
- Speed of the child inside the boat:  $0.05 \text{ m/s}$
- Both travel in the same direction.

What is the speed of the child with respect to the earth?

# Solution:

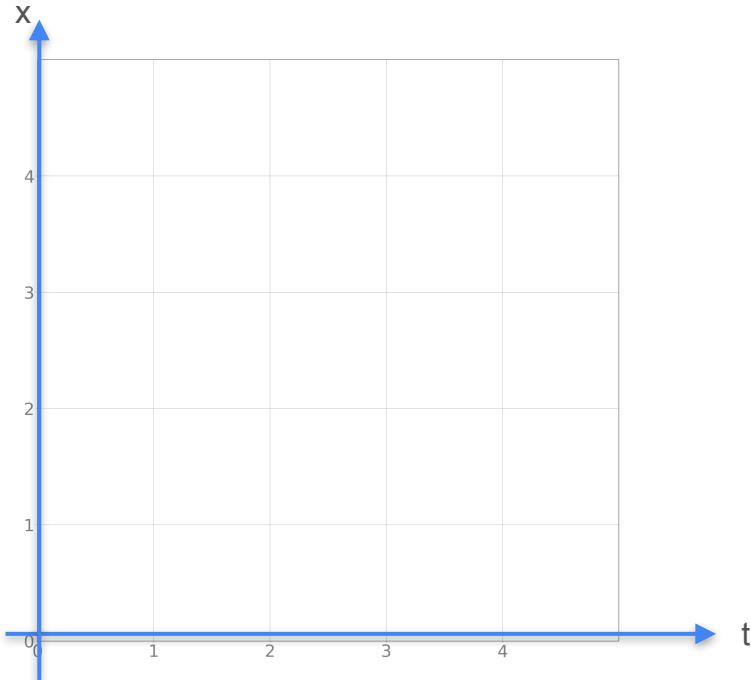


- Speed of the boat: 0.6 m/s
- Speed of the child inside the boat: 0.05 m/s
- Both travel in the same direction.

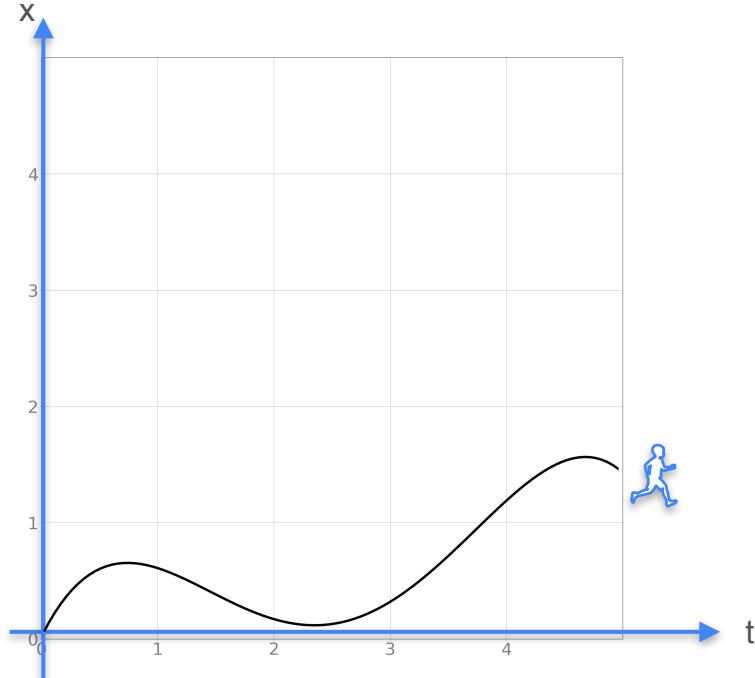
What is the speed of the child with respect to the earth?

# Sum Rule

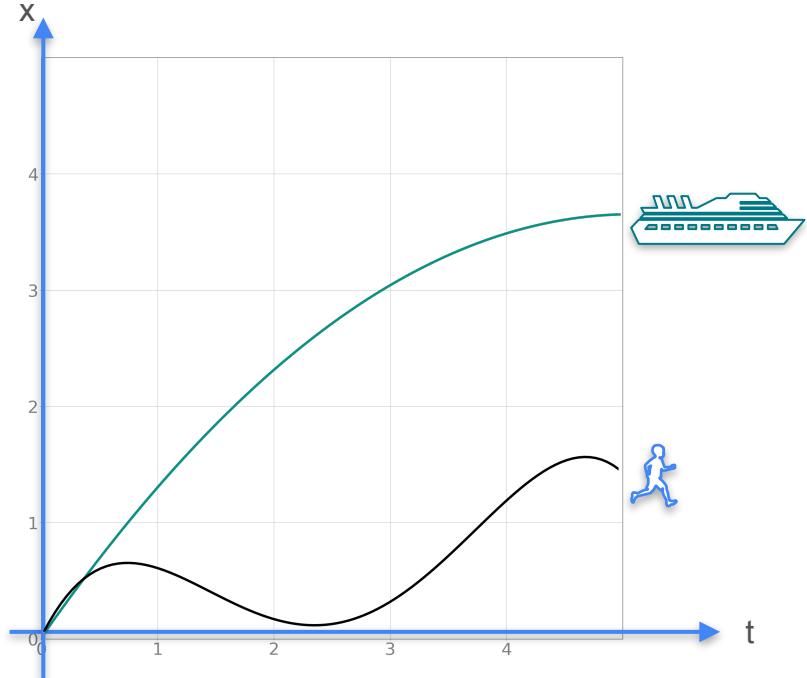
# Sum Rule



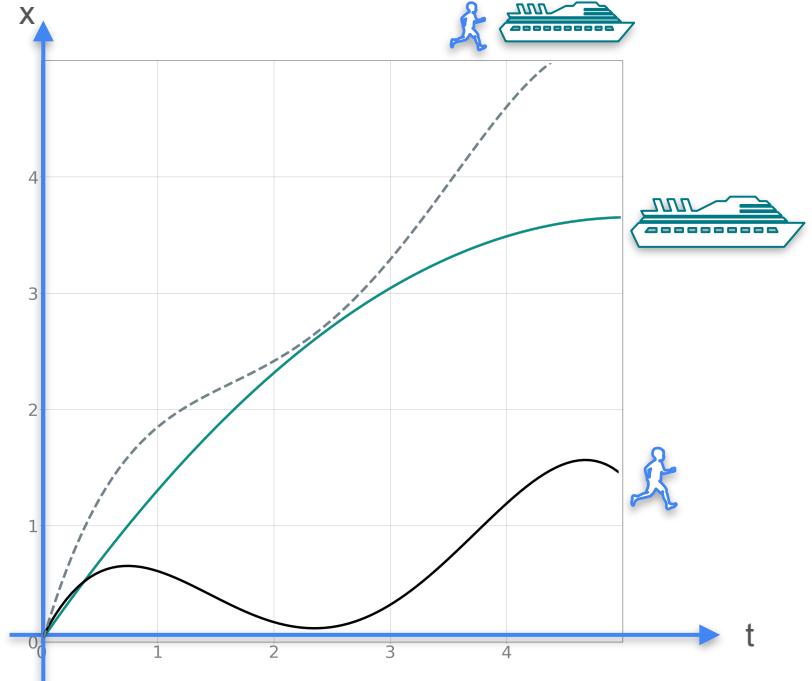
# Sum Rule



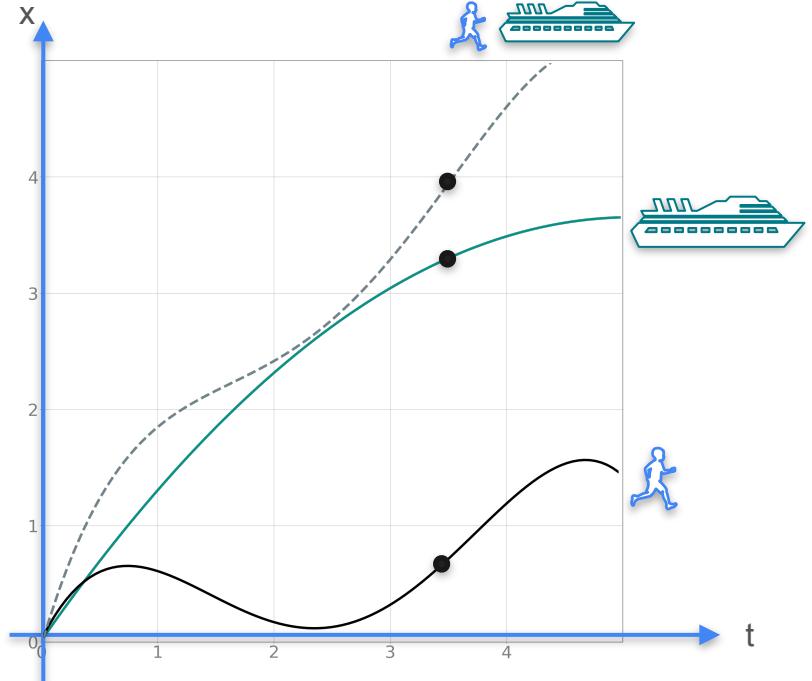
# Sum Rule



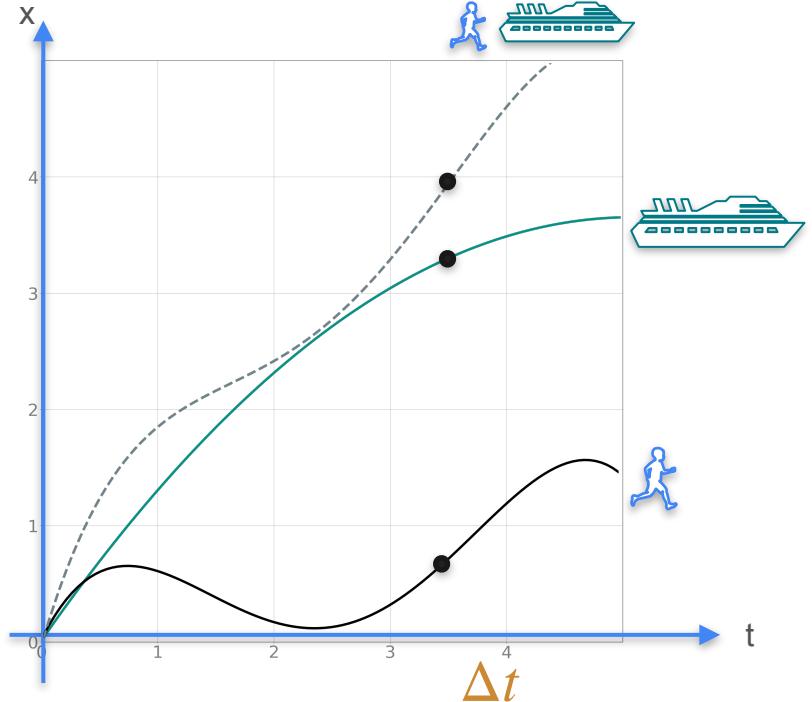
# Sum Rule



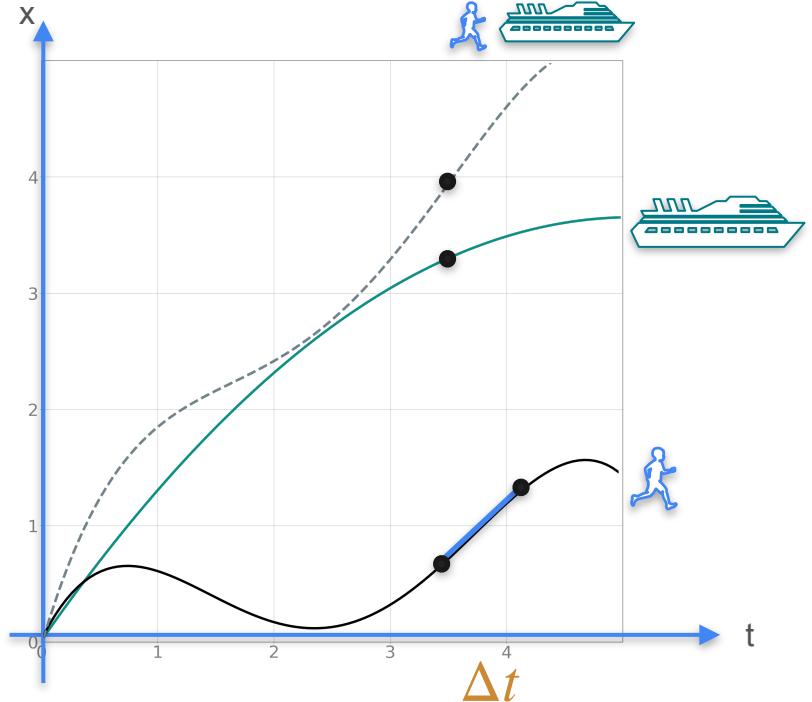
# Sum Rule



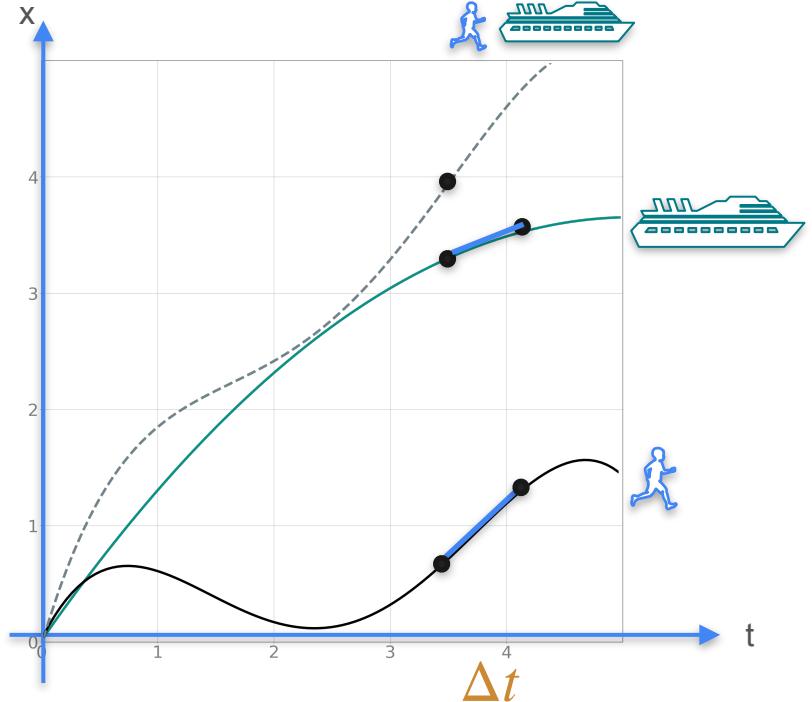
# Sum Rule



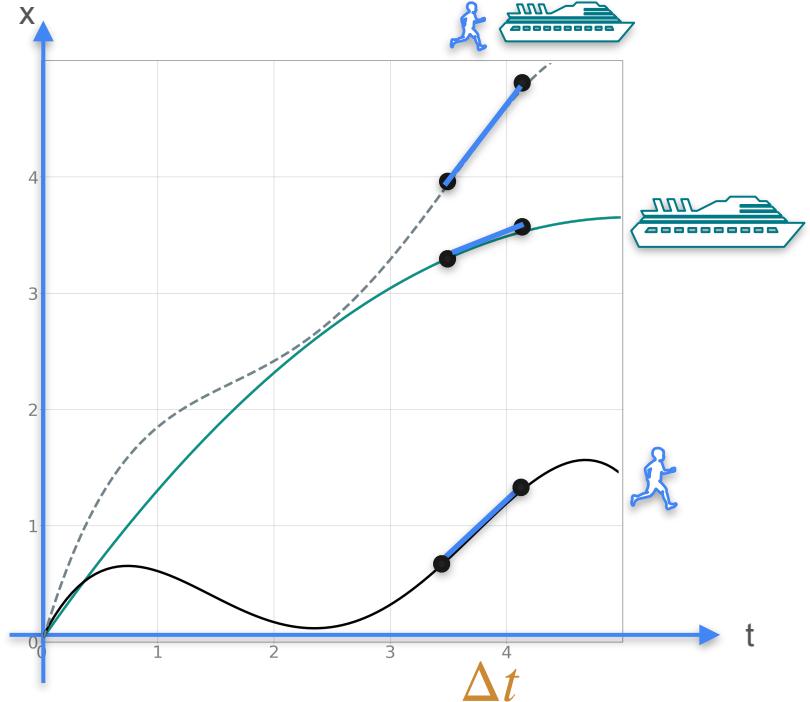
# Sum Rule



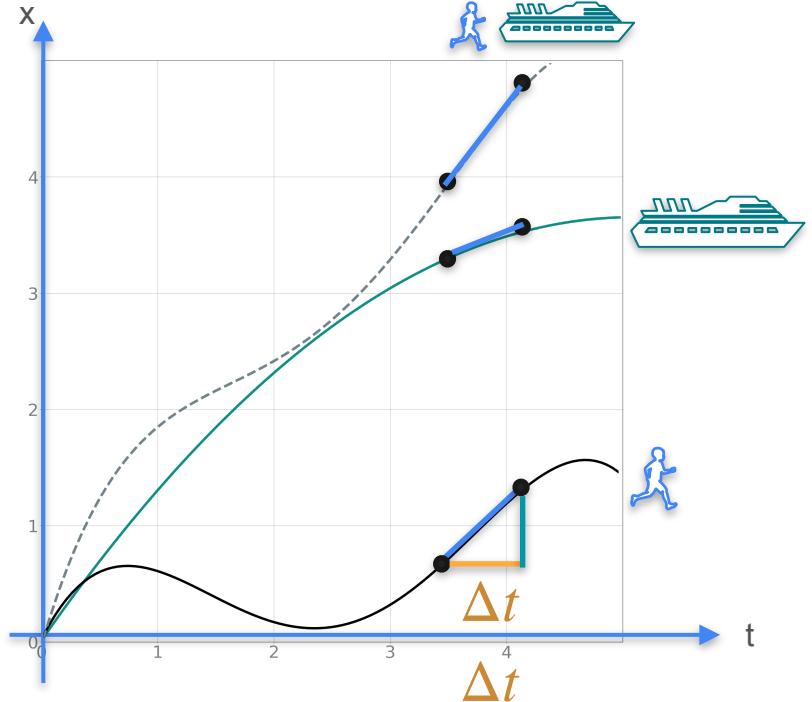
# Sum Rule



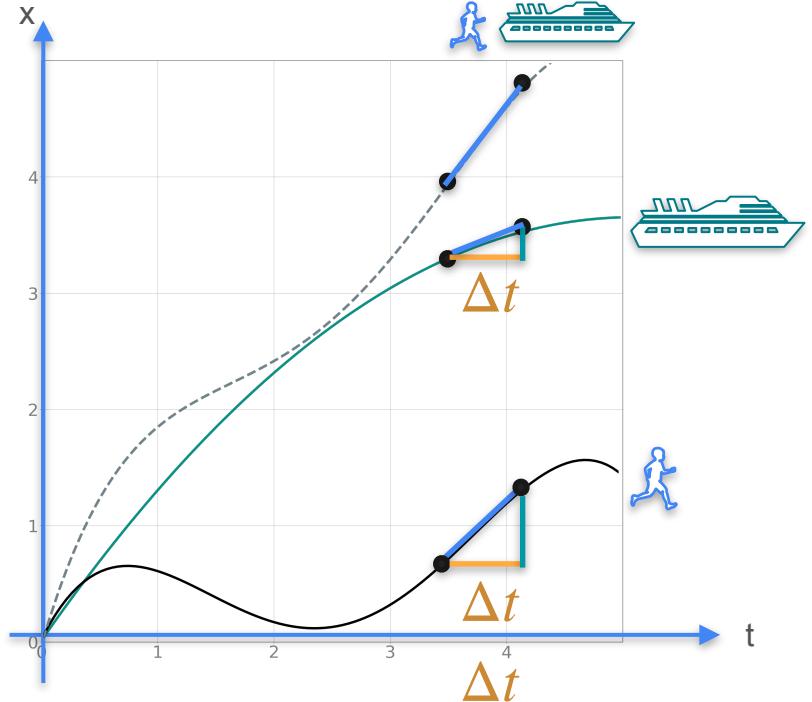
# Sum Rule



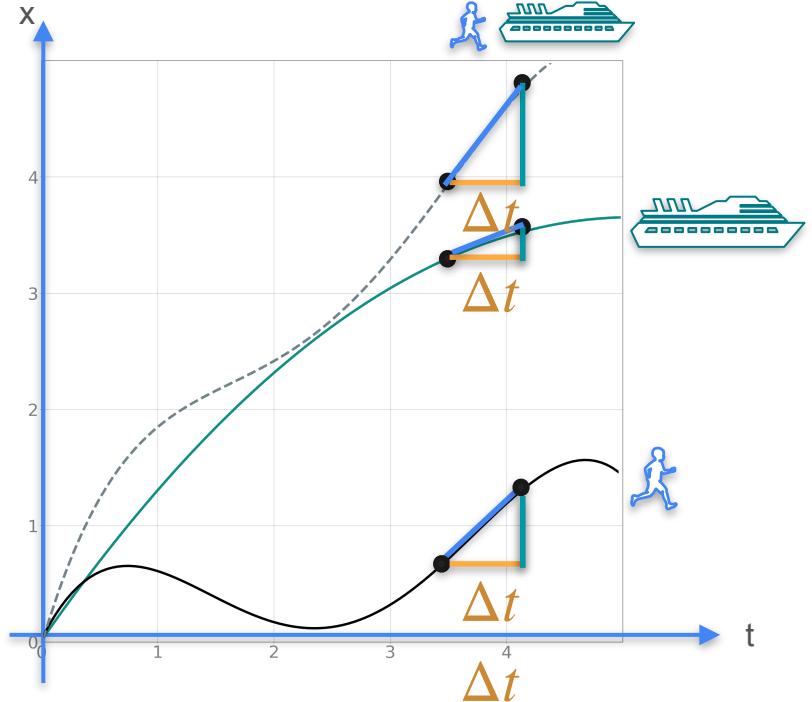
# Sum Rule



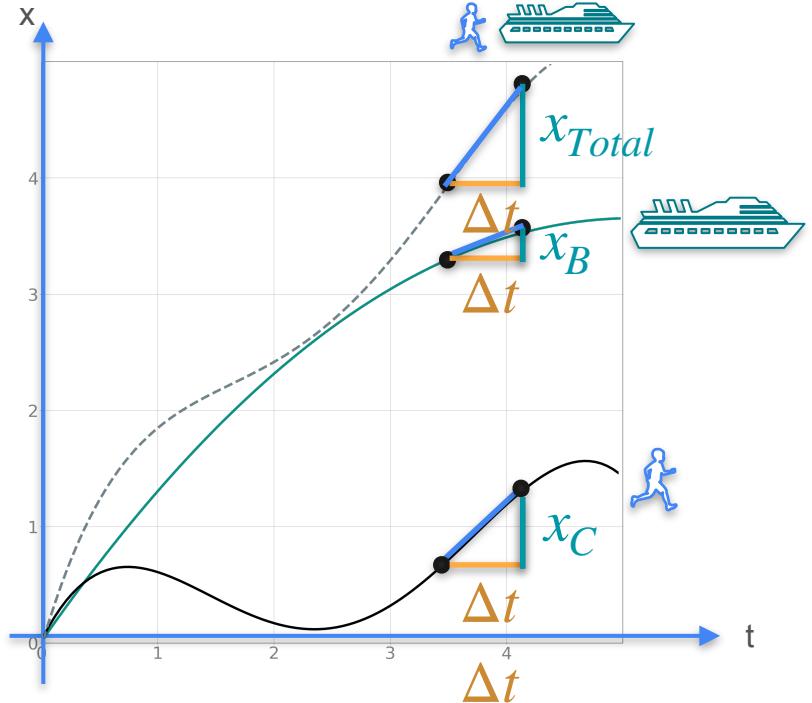
# Sum Rule



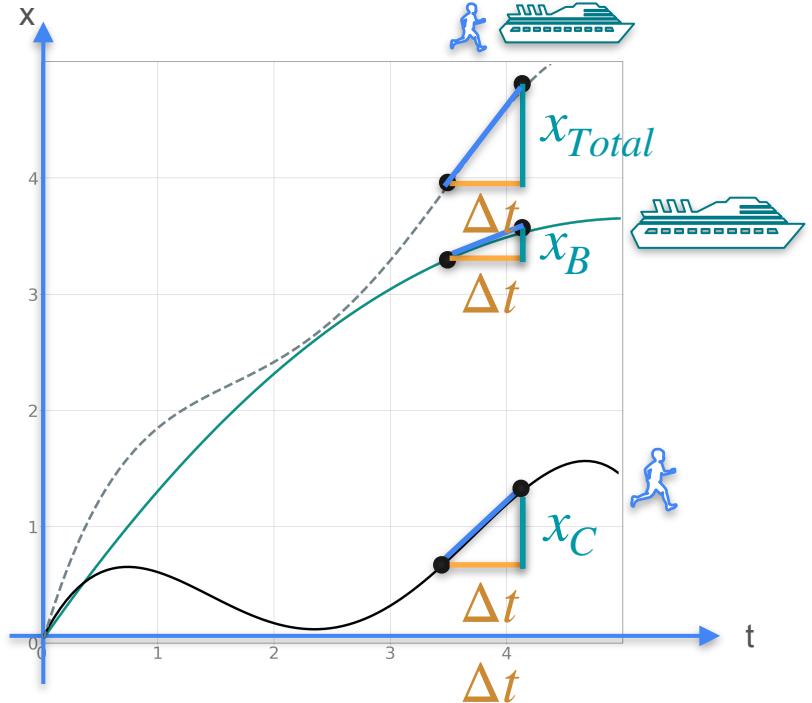
# Sum Rule



# Sum Rule

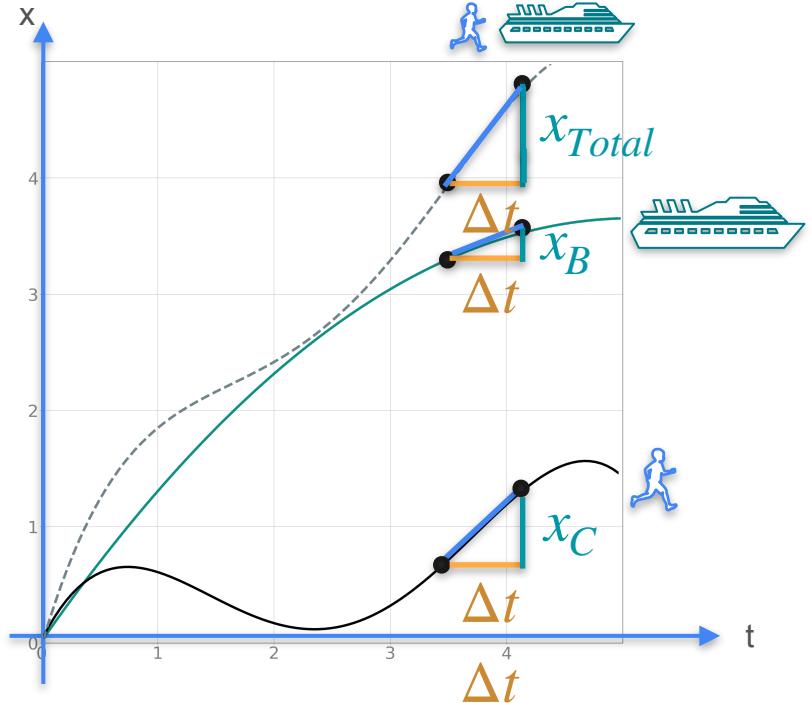


# Sum Rule



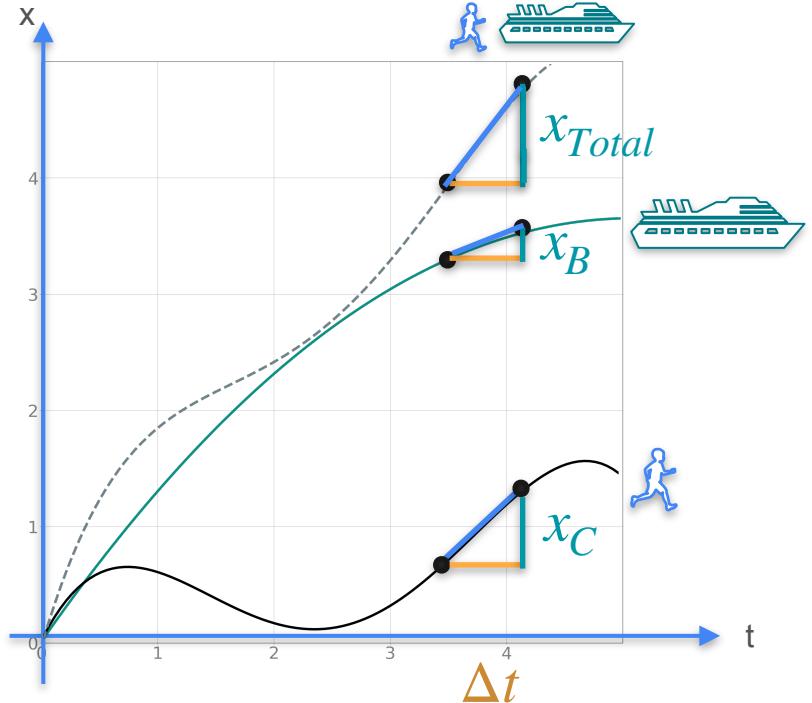
$$x_{Total} = x_B + x_C$$

# Sum Rule



$$x_{Total} = x_B + x_C$$

# Sum Rule

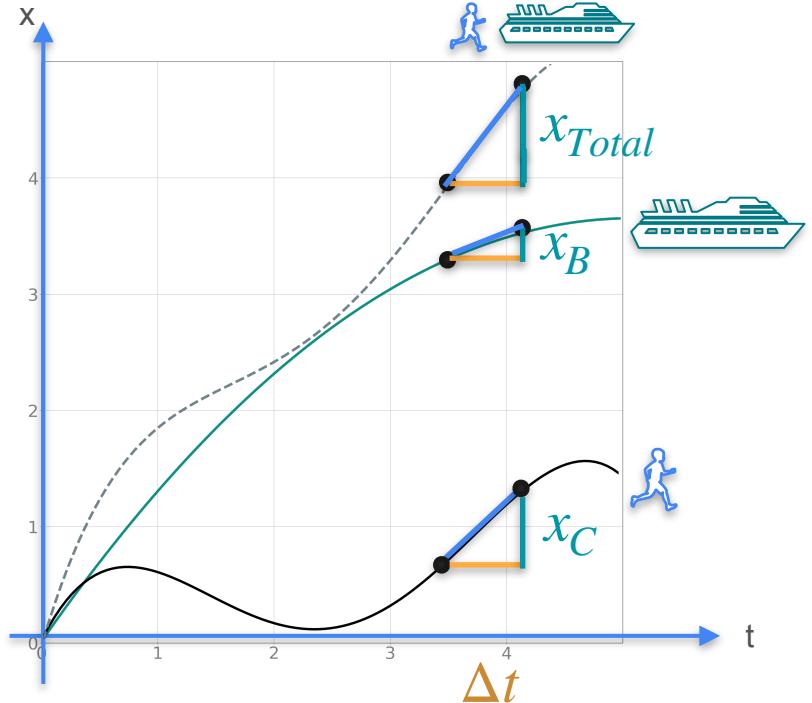


$$x_{Total} = x_B + x_C$$



$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$

# Sum Rule



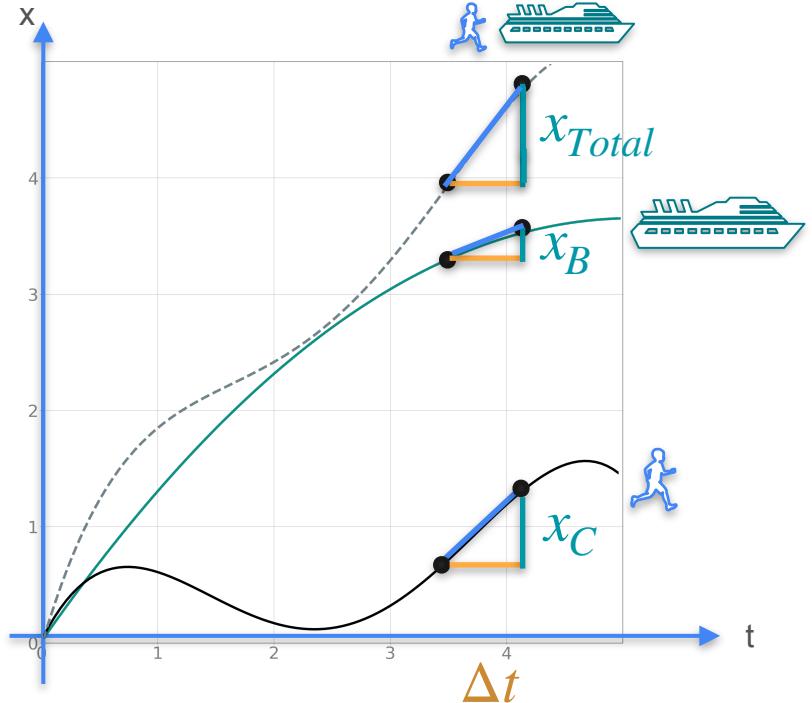
$$x_{Total} = x_B + x_C$$



$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$



# Sum Rule



$$x_{Total} = x_B + x_C$$

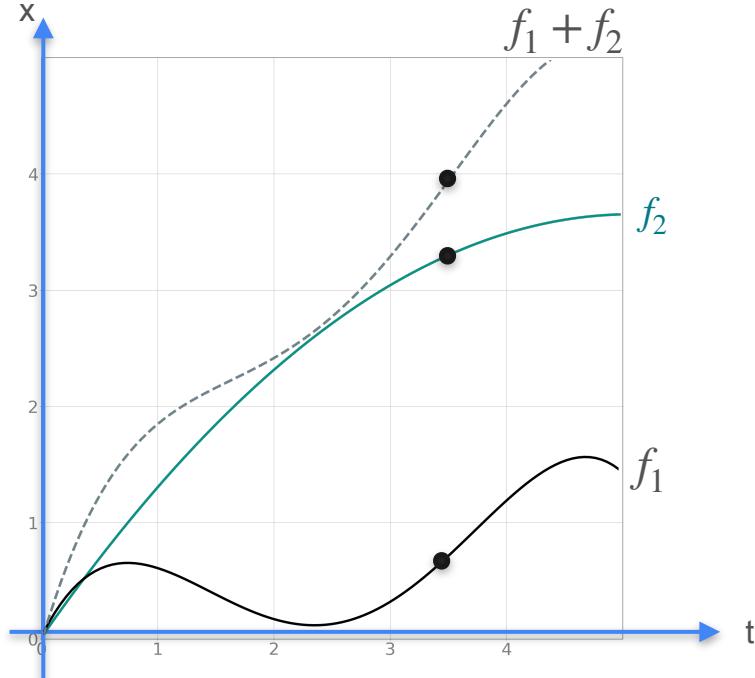


$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$

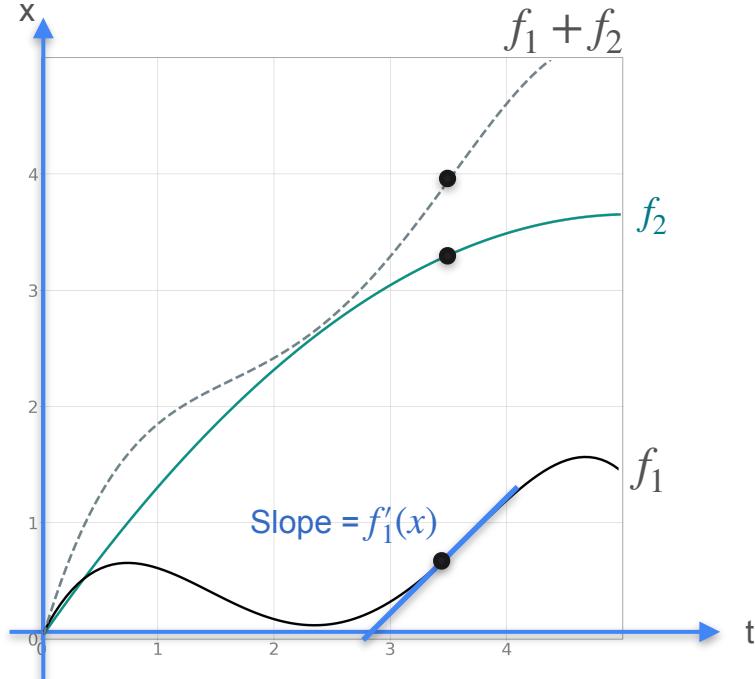


$$v_{Total} = v_B + v_C$$

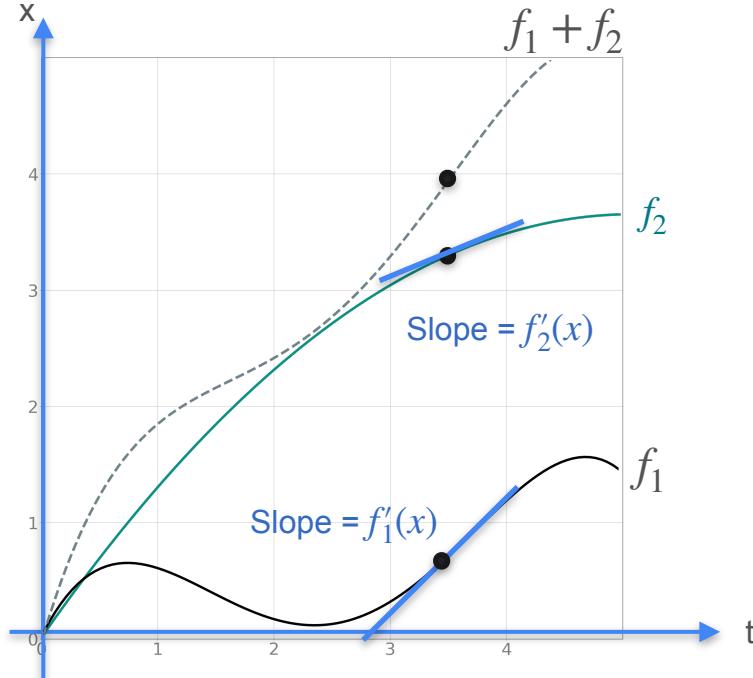
# Sum Rule



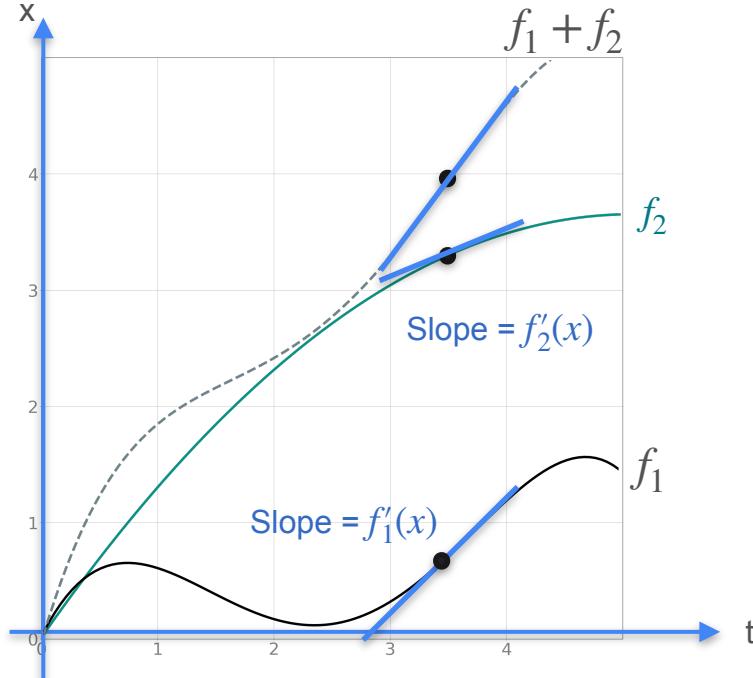
# Sum Rule



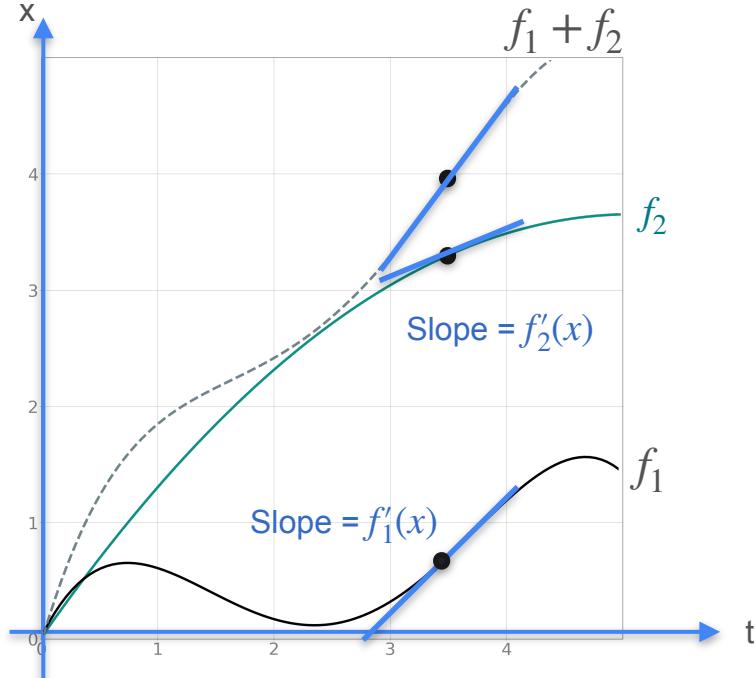
# Sum Rule



# Sum Rule

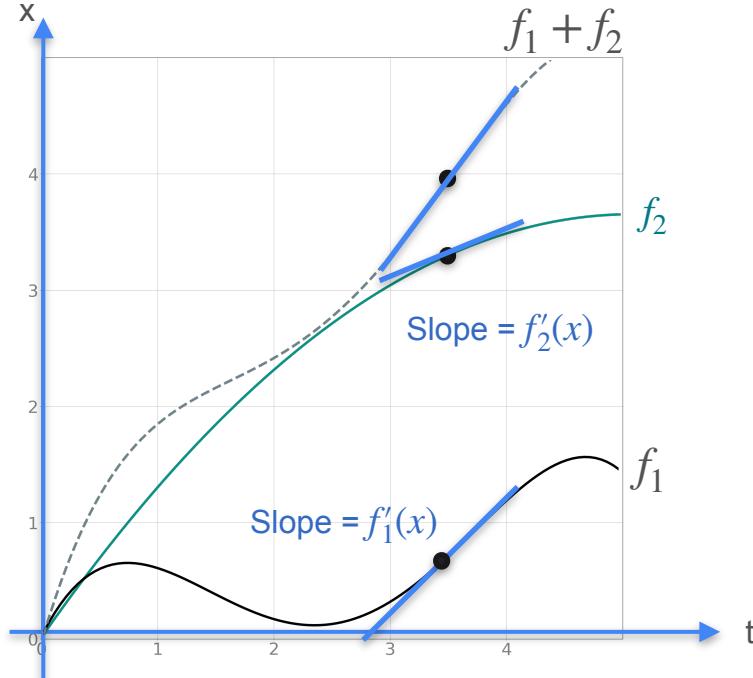


# Sum Rule



$$f = f_1 + f_2$$

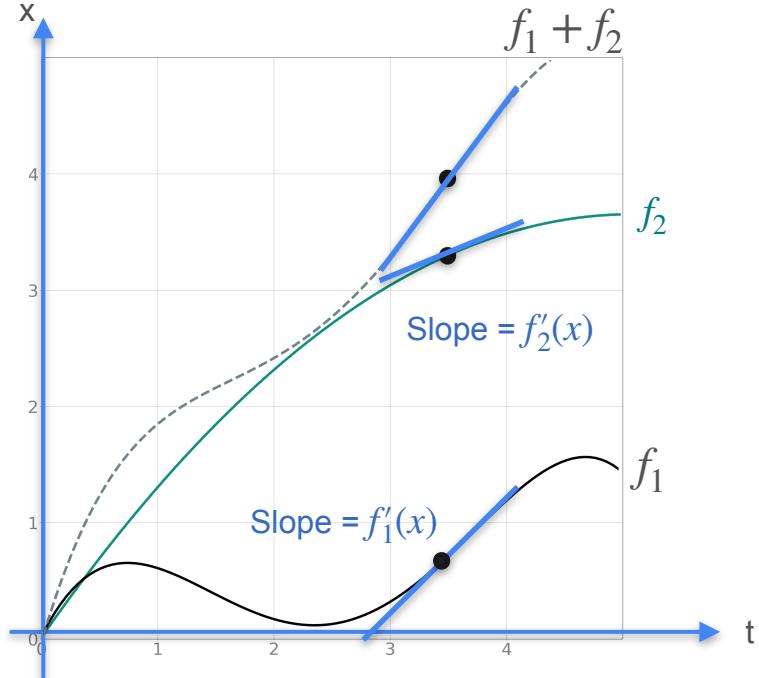
# Sum Rule



$$f = f_1 + f_2$$



# Sum Rule

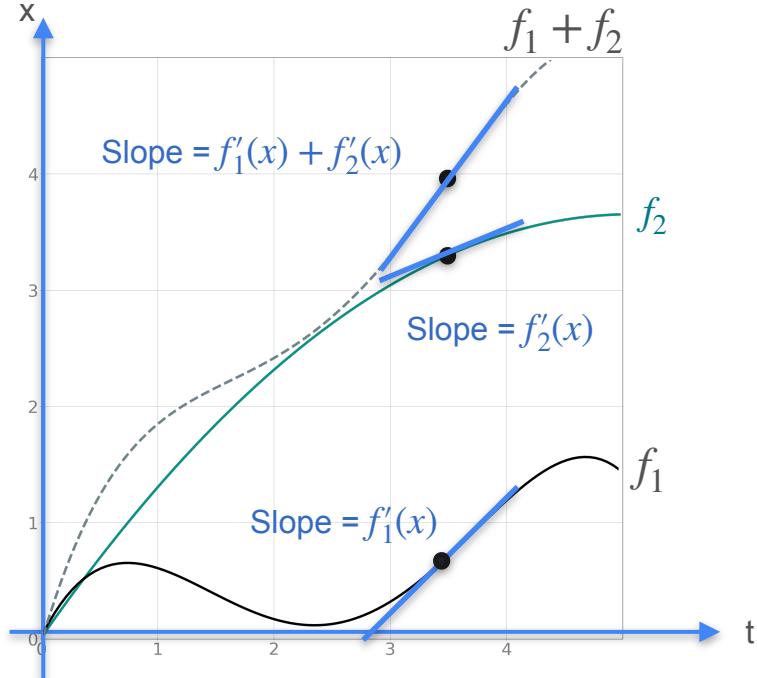


$$f = f_1 + f_2$$



$$f' = f'_1 + f'_2$$

# Sum Rule



$$f = f_1 + f_2$$



$$f' = f'_1 + f'_2$$



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# Derivatives and Optimization

---

**Properties of the derivative:  
The product rule**

# The Product Rule

$$f = gh$$



# The Product Rule

$$f = gh$$



$$f' = g'h$$

# The Product Rule

$$f = gh$$


$$f' = g'h + gh'$$

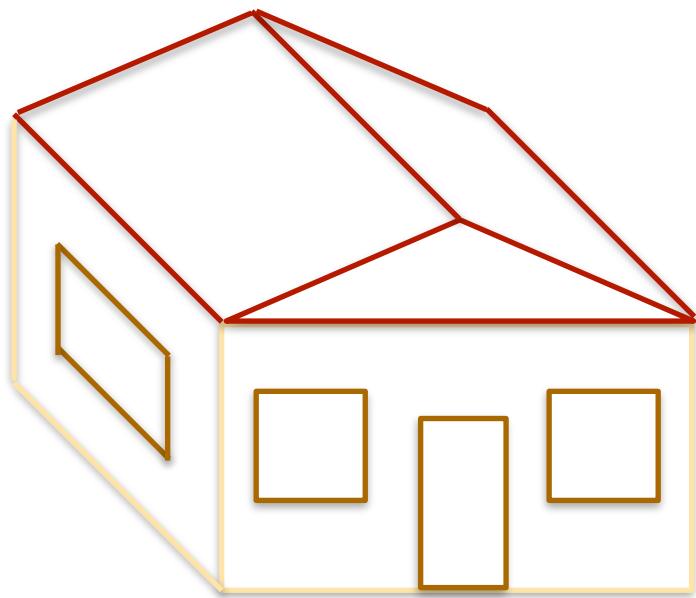
# The Product Rule

$$f = gh$$

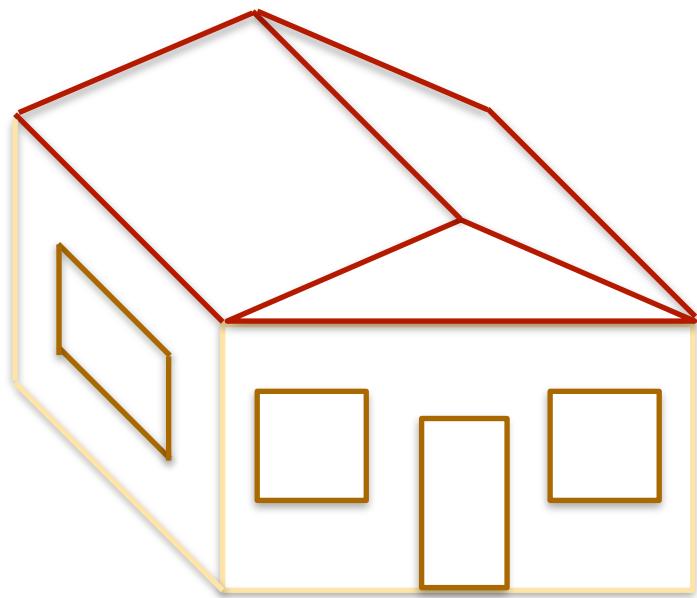
$$f' = g'h + gh'$$

# Product Rule

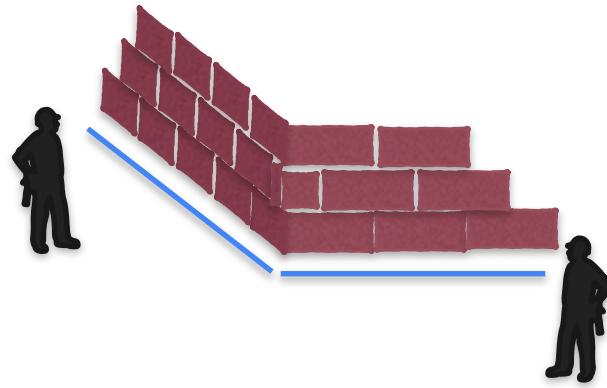
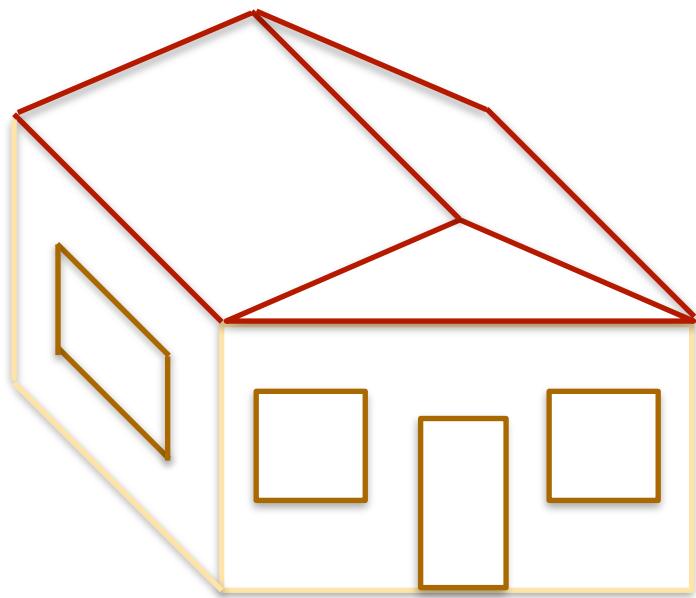
# Product Rule



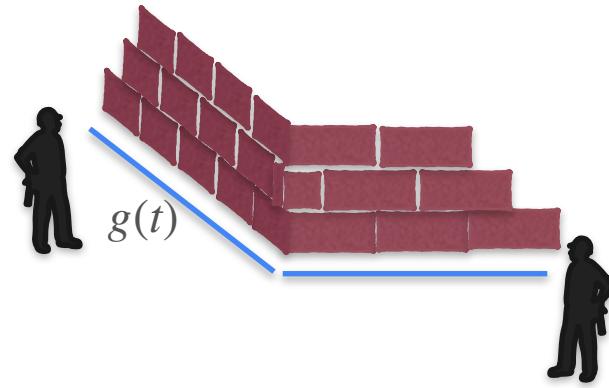
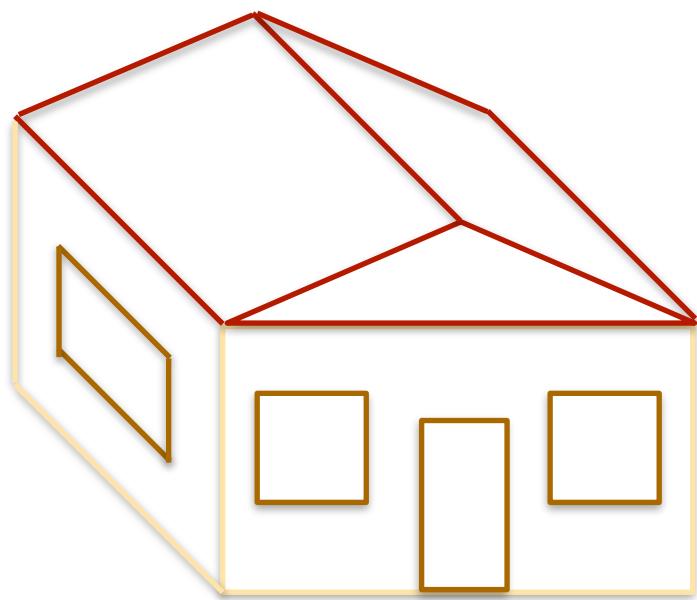
# Product Rule



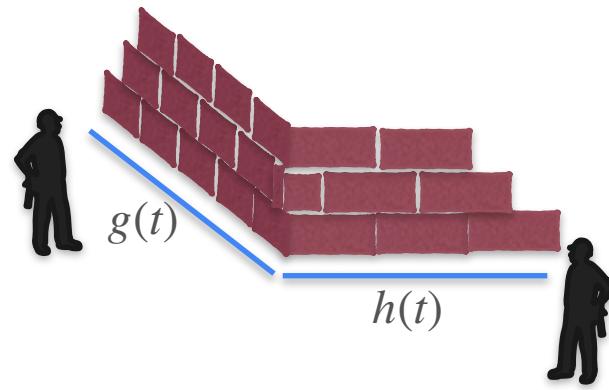
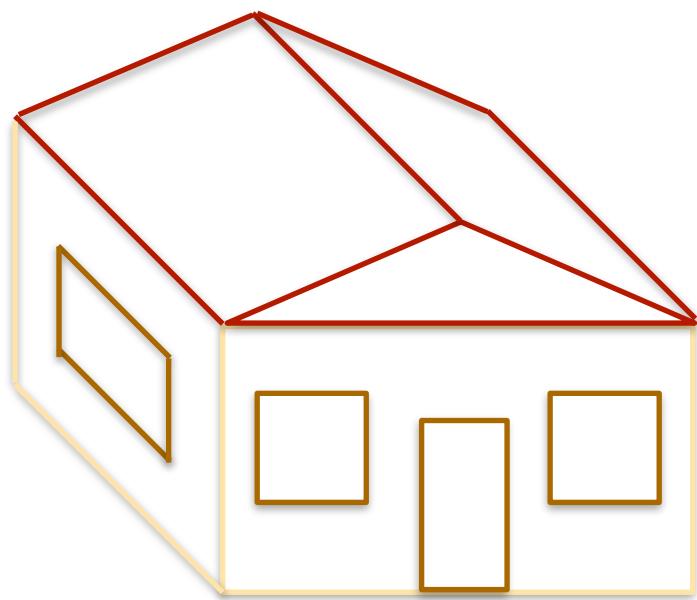
# Product Rule



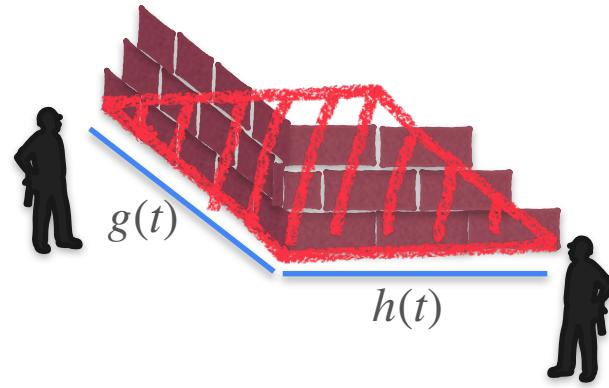
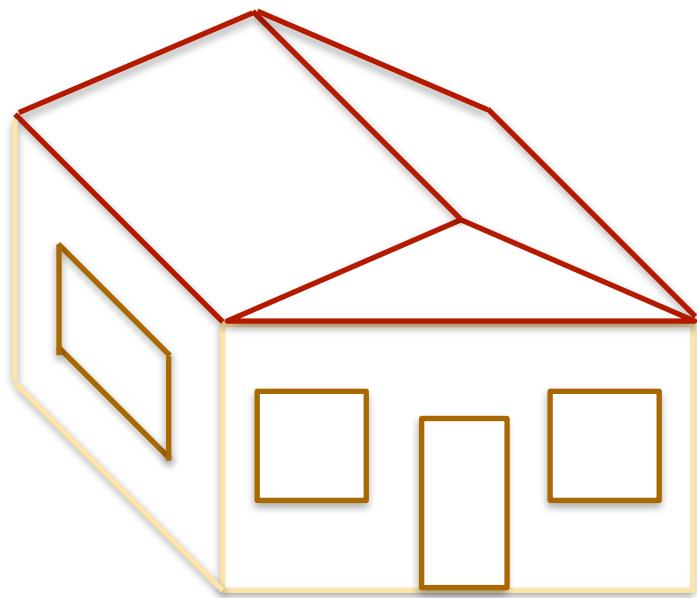
# Product Rule



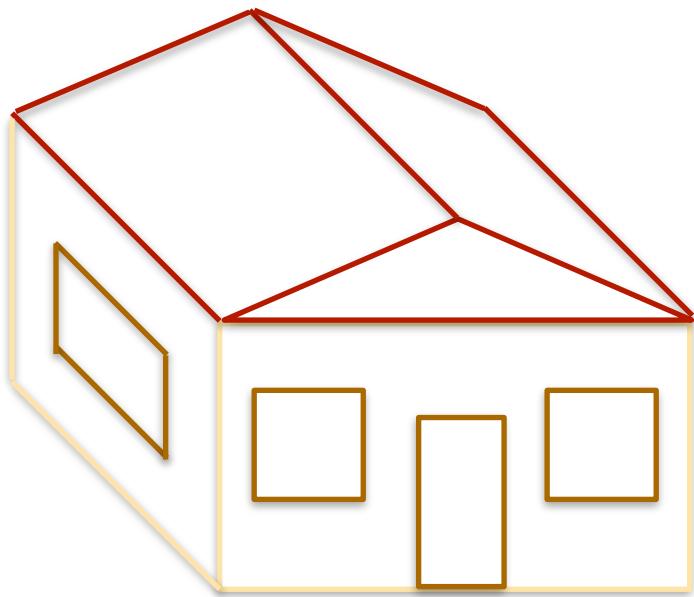
# Product Rule



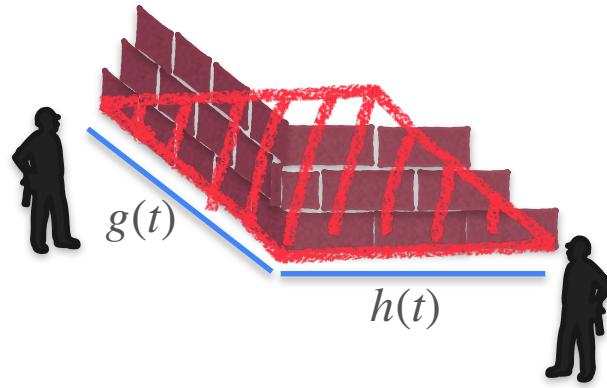
# Product Rule



# Product Rule



$$f(t) = \underline{g(t)h(t)}$$



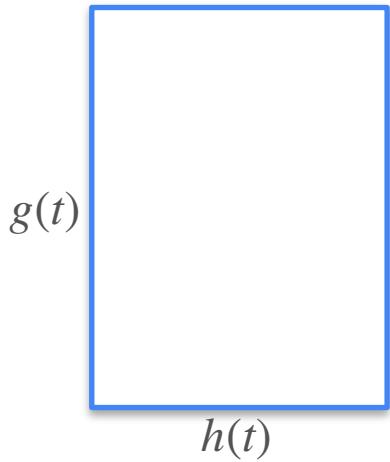
# Product Rule

# Product Rule

$$y = f(t) = g(t)h(t)$$

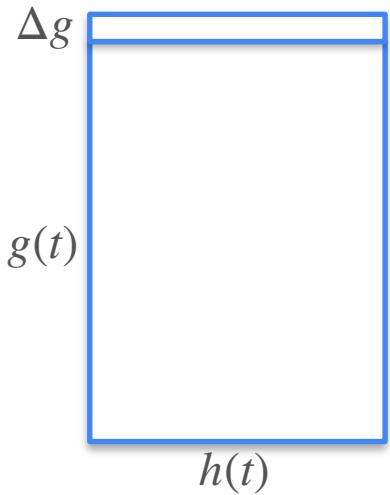
# Product Rule

$$y = f(t) = g(t)h(t)$$



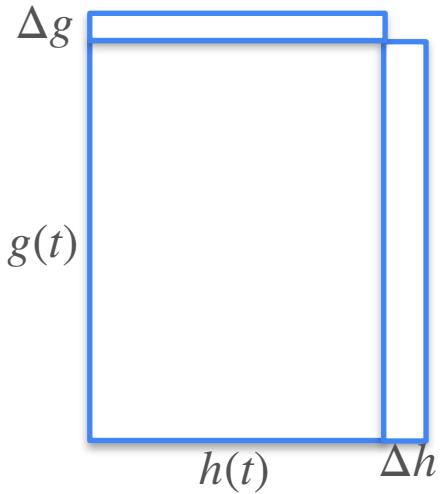
# Product Rule

$$y = f(t) = g(t)h(t)$$



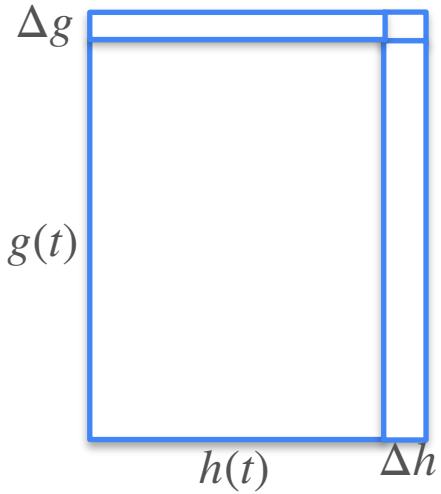
# Product Rule

$$y = f(t) = g(t)h(t)$$



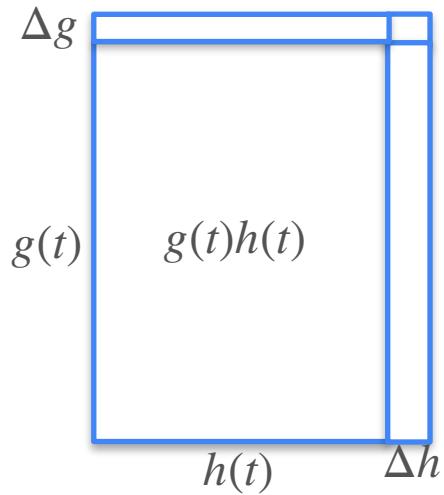
# Product Rule

$$y = f(t) = g(t)h(t)$$



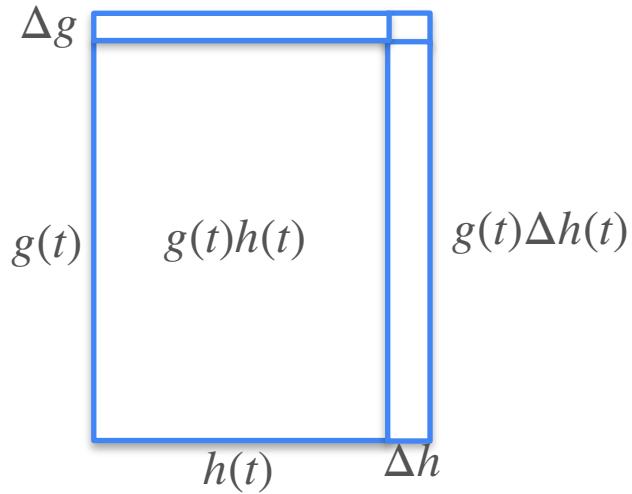
# Product Rule

$$y = f(t) = g(t)h(t)$$



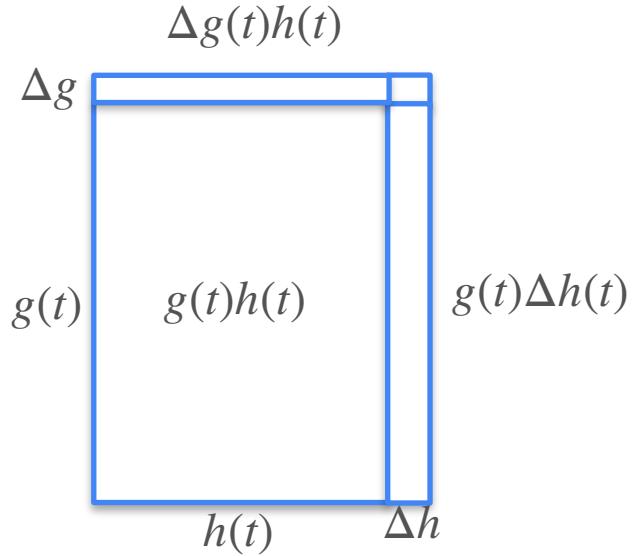
# Product Rule

$$y = f(t) = g(t)h(t)$$

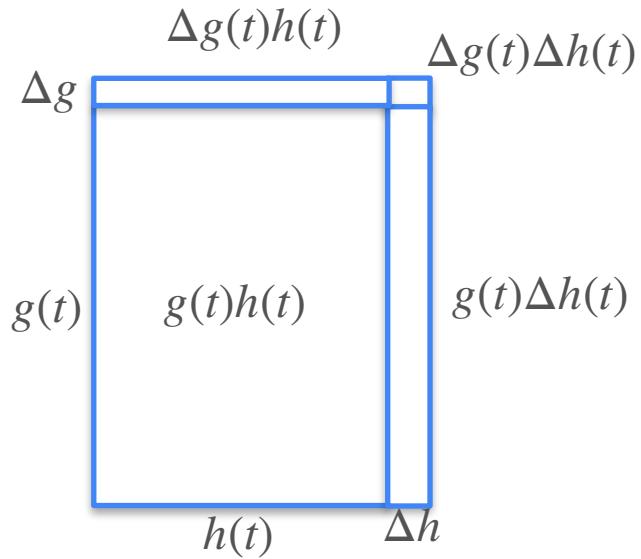


# Product Rule

$$y = f(t) = g(t)h(t)$$

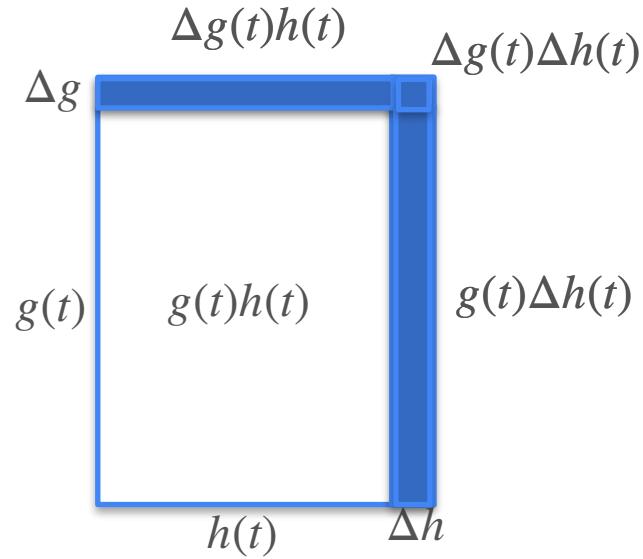


# Product Rule



$$y = f(t) = g(t)h(t)$$

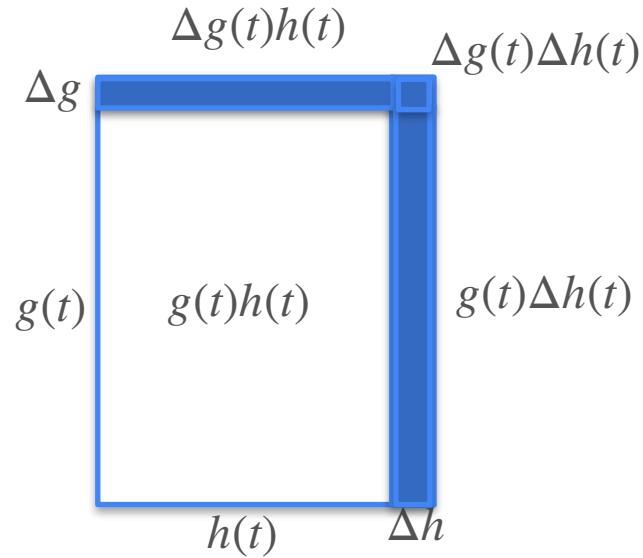
# Product Rule



$$y = f(t) = g(t)h(t)$$

$$\frac{\Delta f(t)}{\Delta t}$$

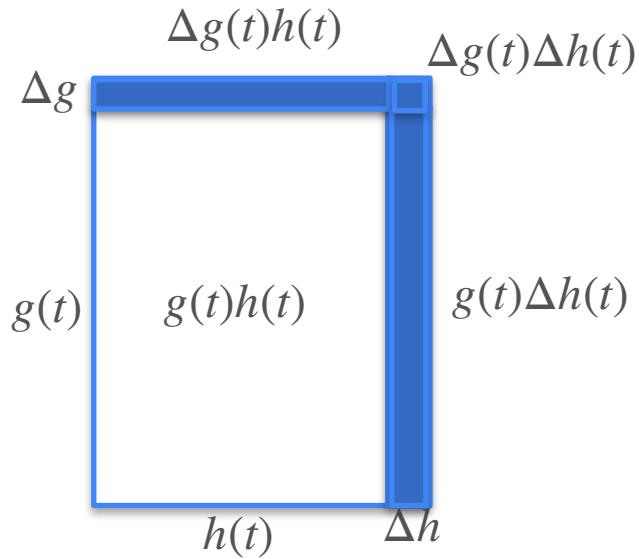
# Product Rule



$$y = f(t) = g(t)h(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

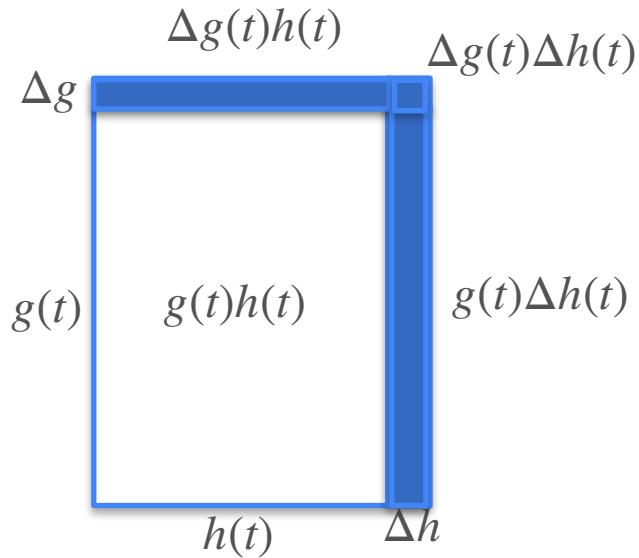
# Product Rule



$$y = f(t) = g(t)h(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

# Product Rule

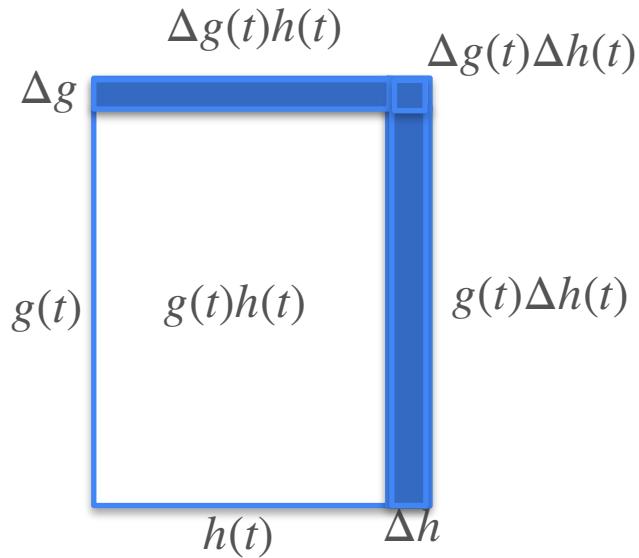


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as  $\Delta t \rightarrow 0$

# Product Rule



$$y = f(t) = g(t)h(t)$$

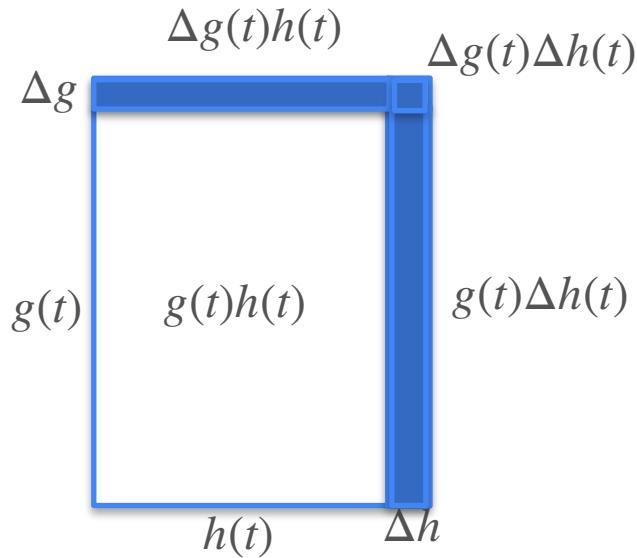
$$f'(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

$$= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}$$

as  $\Delta t \rightarrow 0$

# Product Rule



$$y = f(t) = g(t)h(t)$$

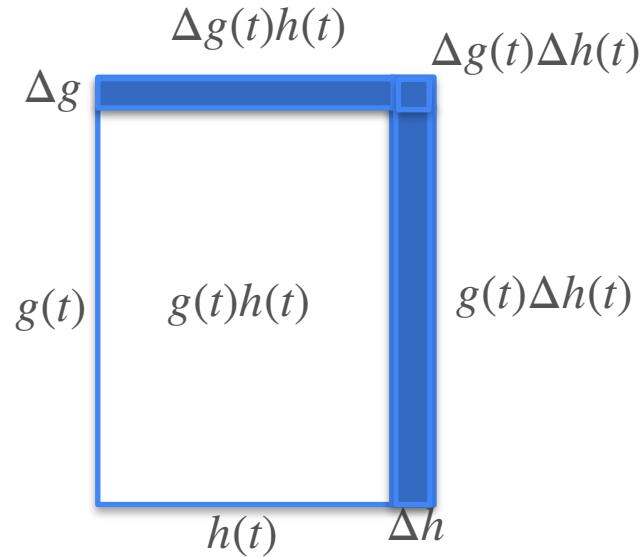
$$f'(t)$$

$$\frac{\Delta f(t)}{\Delta t} = \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t}$$

$$= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}$$

as  $\Delta t \rightarrow 0$

# Product Rule



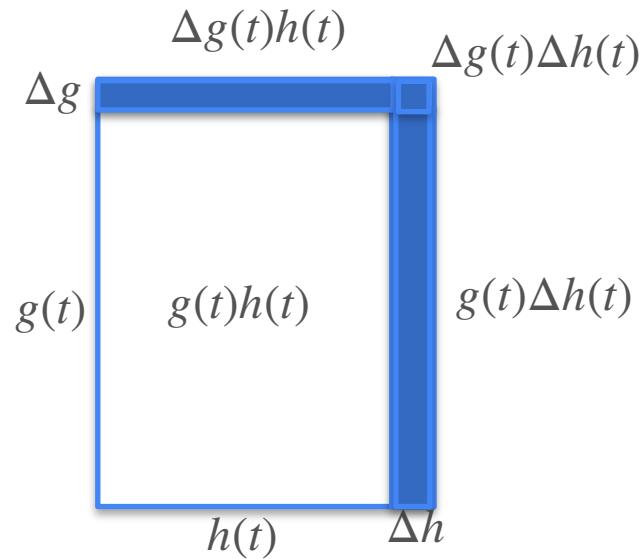
$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{\Delta g(t)}{\Delta t}h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

as  $\Delta t \rightarrow 0$

# Product Rule



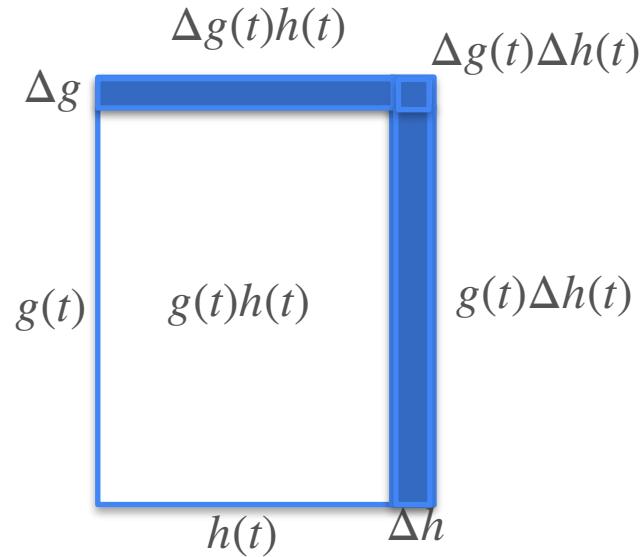
$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{g'(t)h(t)}{\Delta t} + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t}\end{aligned}$$

as  $\Delta t \rightarrow 0$

# Product Rule



$$y = f(t) = g(t)h(t)$$

$$f'(t)$$

$$\begin{aligned}\frac{\Delta f(t)}{\Delta t} &= \frac{\Delta g(t)h(t) + g(t)\Delta h(t) + \Delta g(t)\Delta h(t)}{\Delta t} \\ &= \frac{g'(t)h(t)}{\Delta t} + g(t)\frac{\Delta h(t)}{\Delta t} + \frac{\Delta g(t)\Delta h(t)}{\Delta t} \\ &\text{as } \Delta t \rightarrow 0\end{aligned}$$

$$f'(t) = g'(t)h(t) + g(t)h'(t)$$



DeepLearning.AI

# Derivatives and Optimization

---

**Properties of the derivative:  
The chain rule**

# The Chain Rule

# The Chain Rule

$$h(t)$$

# The Chain Rule

$$g(h(t))$$

# The Chain Rule

$$\frac{d}{dt} g(h(t))$$

# The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dh}{dt}$$

# The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \frac{dh}{dt}$$

# The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) \\ = \frac{dg}{dh} \cdot \frac{dh}{dt}\end{aligned}$$

# The Chain Rule

$$g(h(t))$$

$$\frac{dg}{dh} \frac{dh}{dt}$$

# The Chain Rule

$$f(g(h(t)))$$

$$\frac{dg}{dh} \quad \frac{dh}{dt}$$

# The Chain Rule

$$\frac{d}{dt} f(g(h(t)))$$

$$\frac{dg}{dh} \frac{dh}{dt}$$

# The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \frac{dg}{dh} \frac{dh}{dt}$$

# The Chain Rule

$$\begin{aligned} & \frac{d}{dt} f(g(h(t))) \\ &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \end{aligned}$$

# The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \cdot \frac{dh}{dt}$$

# The Chain Rule

$$\frac{d}{dt} g(h(t)) = \frac{dg}{dh} \cdot \frac{dh}{dt}$$

=

# The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) &= \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= h'(t)\end{aligned}$$

# The Chain Rule

$$\begin{aligned}\frac{d}{dt} g(h(t)) &= \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= g'(h(t)) \cdot h'(t)\end{aligned}$$

# The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

# The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

=

# The Chain Rule

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

=  $h'(t)$

# The Chain Rule

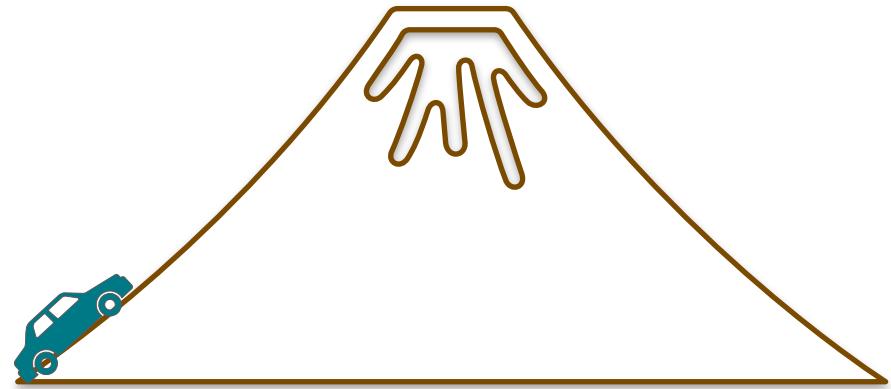
$$\begin{aligned}\frac{d}{dt} f(g(h(t))) &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= g'(h(t)) \cdot h'(t)\end{aligned}$$

# The Chain Rule

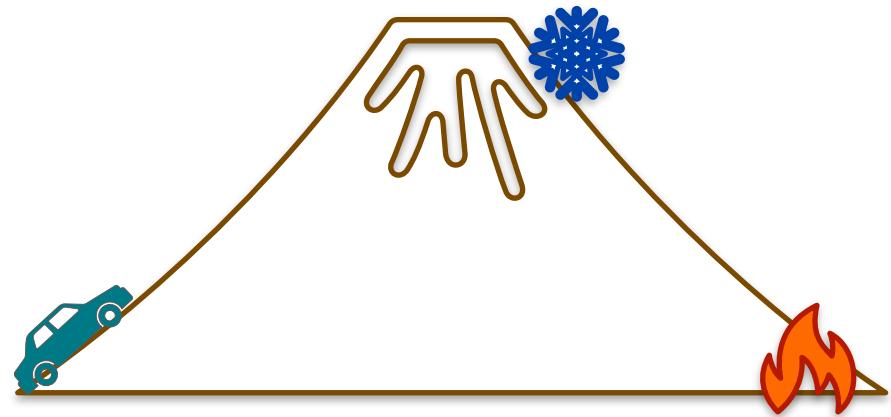
$$\begin{aligned}\frac{d}{dt} f(g(h(t))) &= \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt} \\ &= f'(g(h(t))) \cdot g'(h(t)) \cdot h'(t)\end{aligned}$$

# Idea of Chain Rule

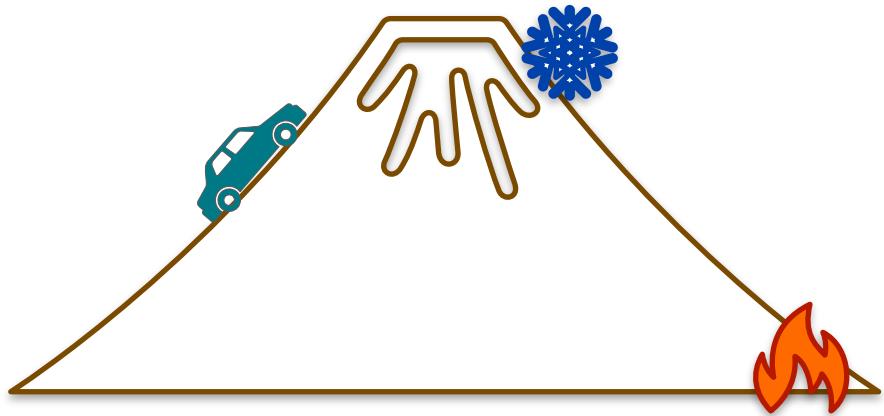
# Idea of Chain Rule



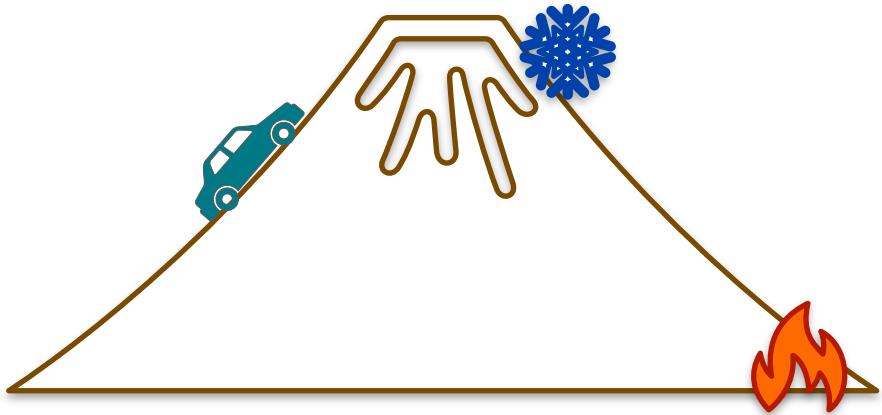
# Idea of Chain Rule



# Idea of Chain Rule



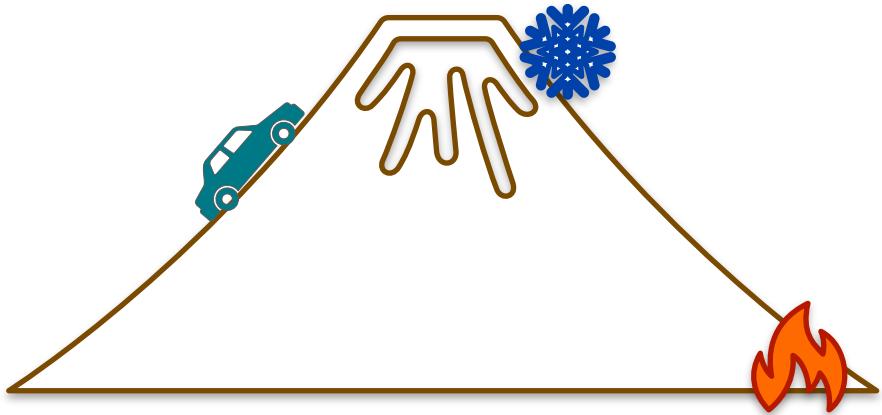
# Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

# Idea of Chain Rule



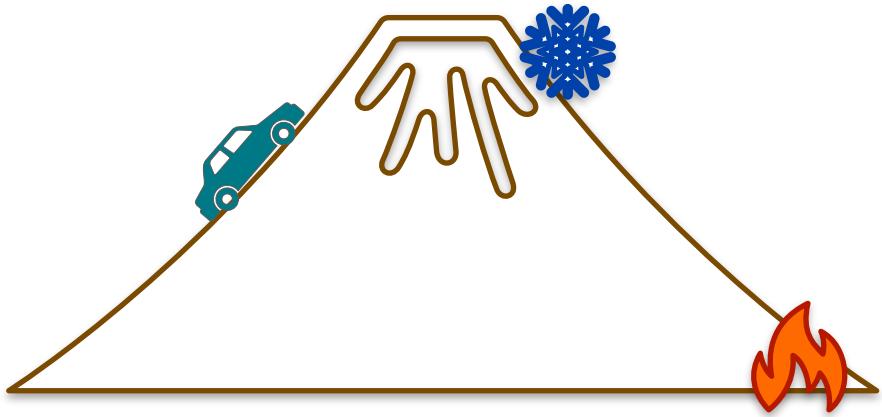
Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

$$\frac{dh}{dt}$$

# Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

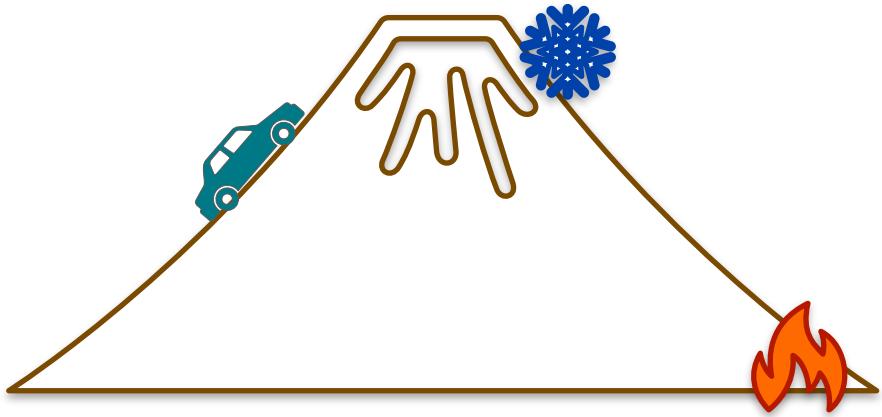
$$\frac{dh}{dt}$$



Temperature changes w.r.t. time

$$\frac{dT}{dt}$$

# Idea of Chain Rule



Temperature changes w.r.t. height

$$\frac{dT}{dh}$$

height changes w.r.t. time

$$\frac{dh}{dt}$$



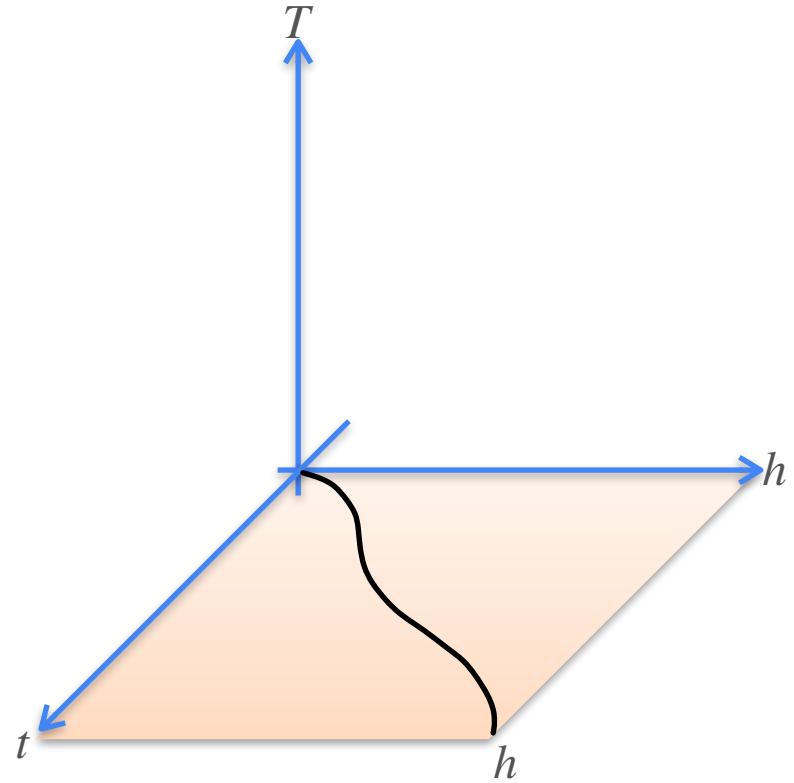
Temperature changes w.r.t. time

$$\frac{dT}{dt}$$

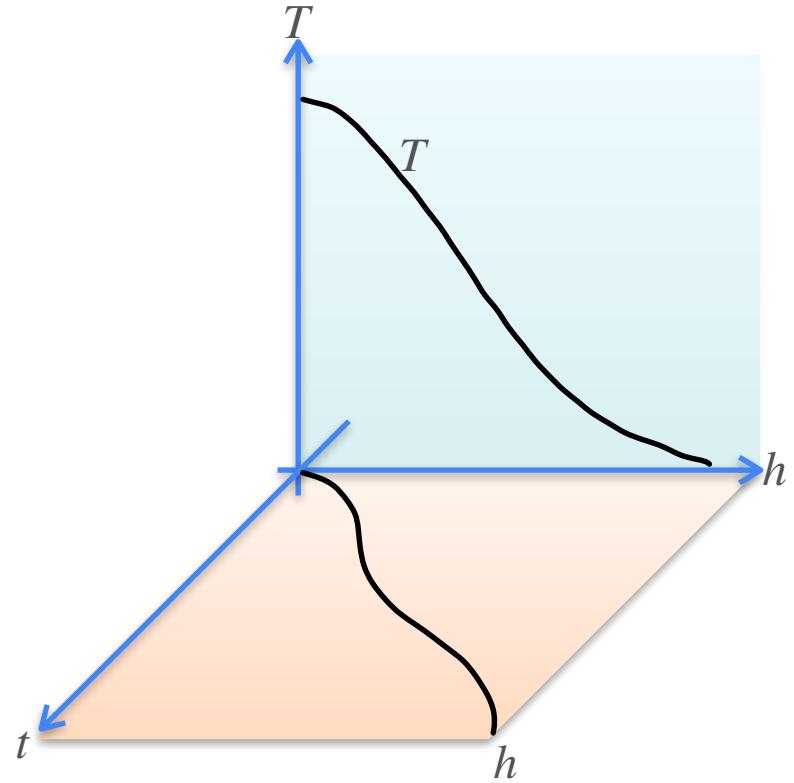
$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$

# Chain Rule

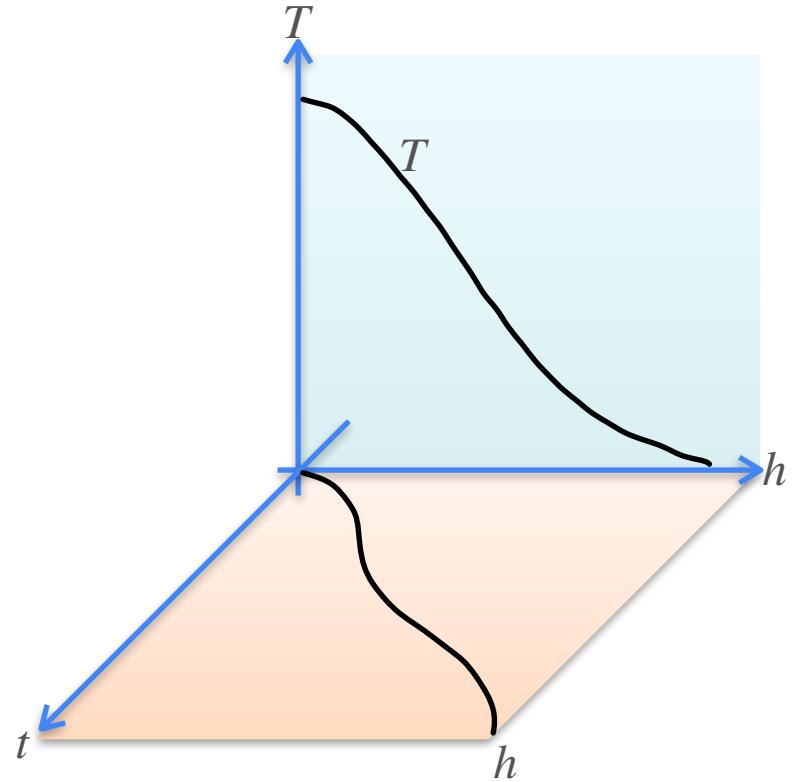
# Chain Rule



# Chain Rule

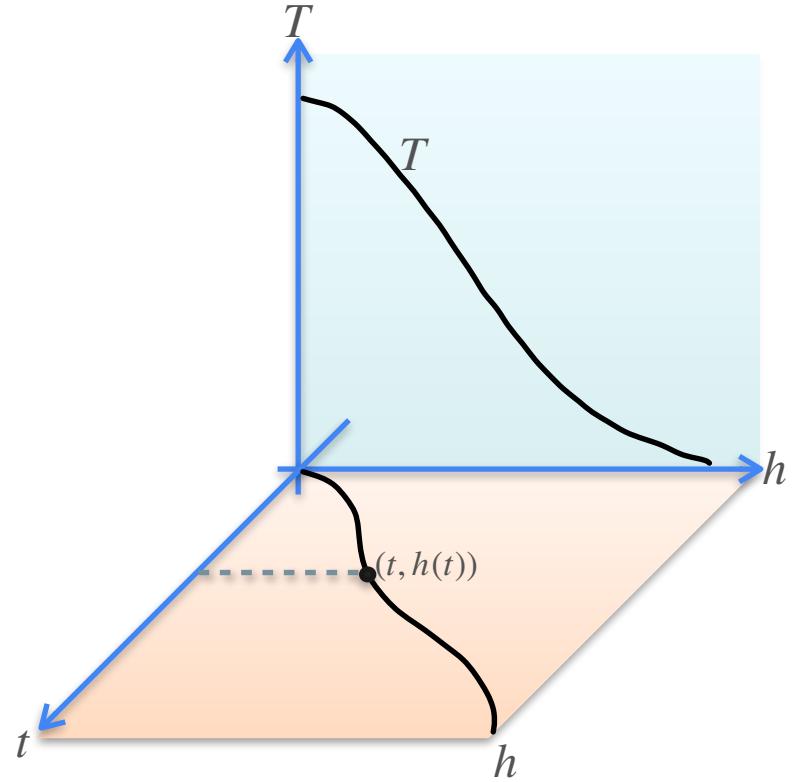


# Chain Rule



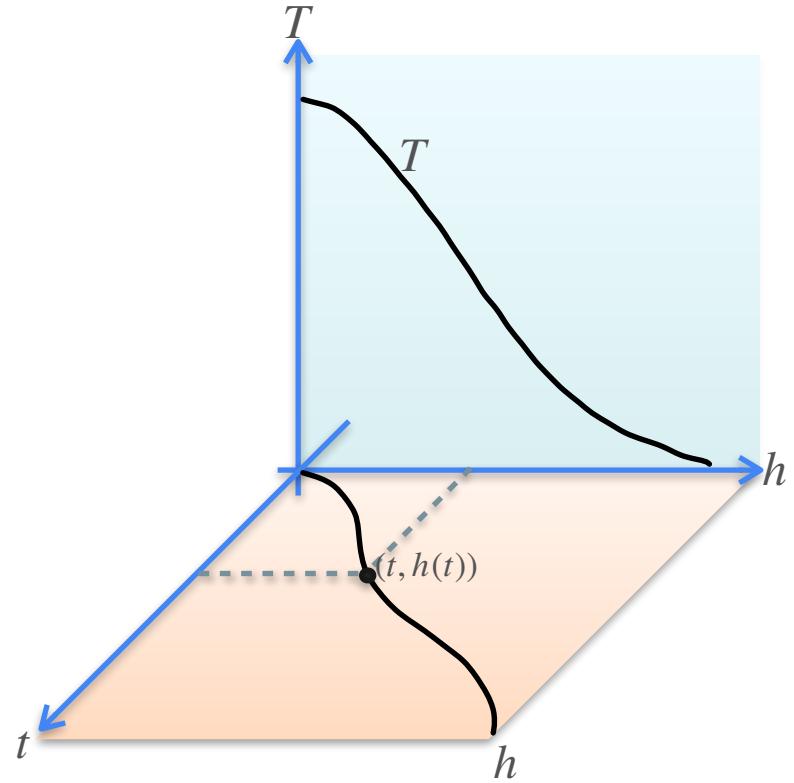
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



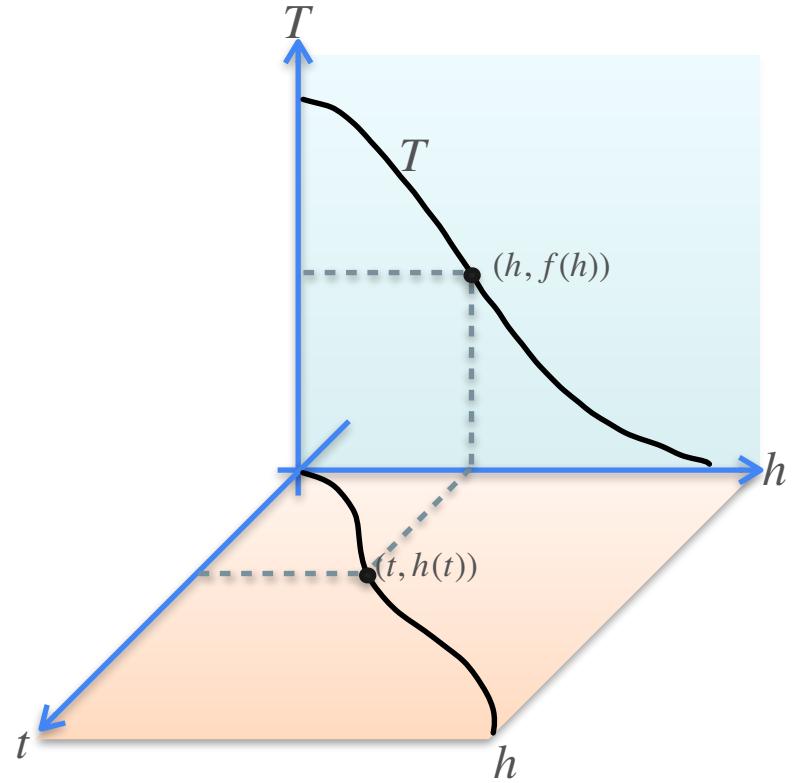
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



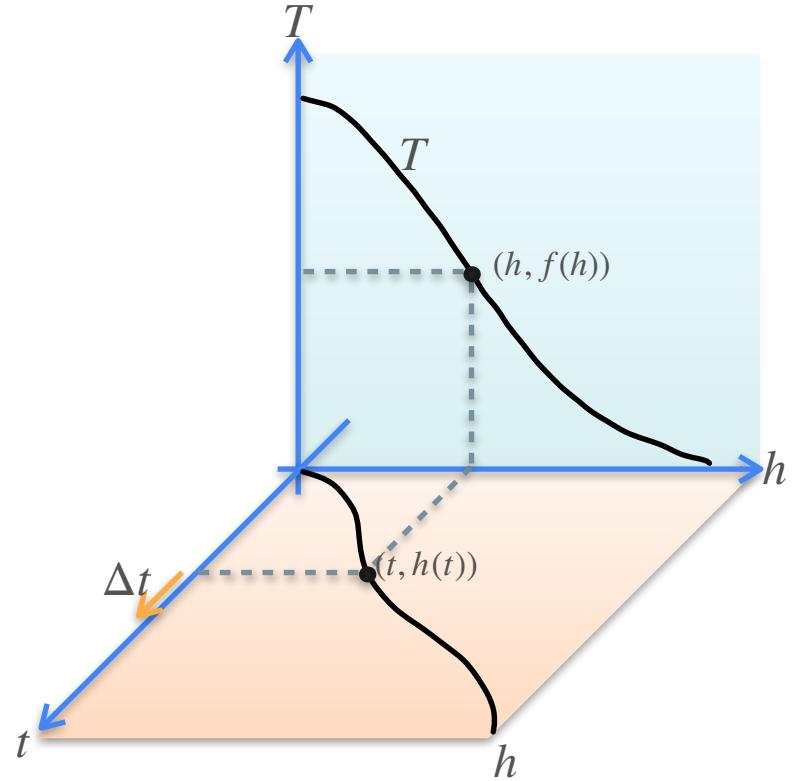
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



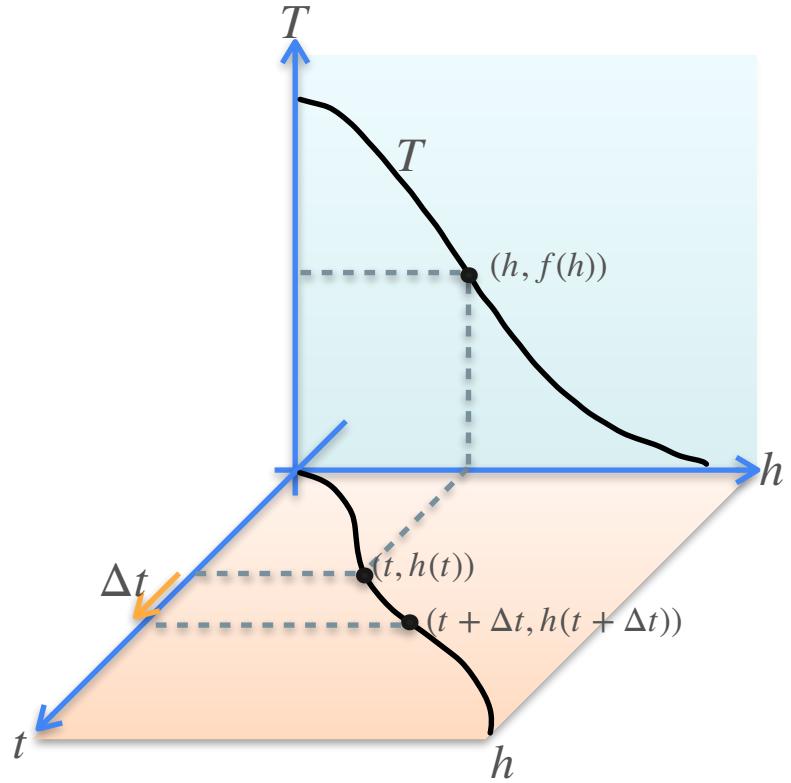
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



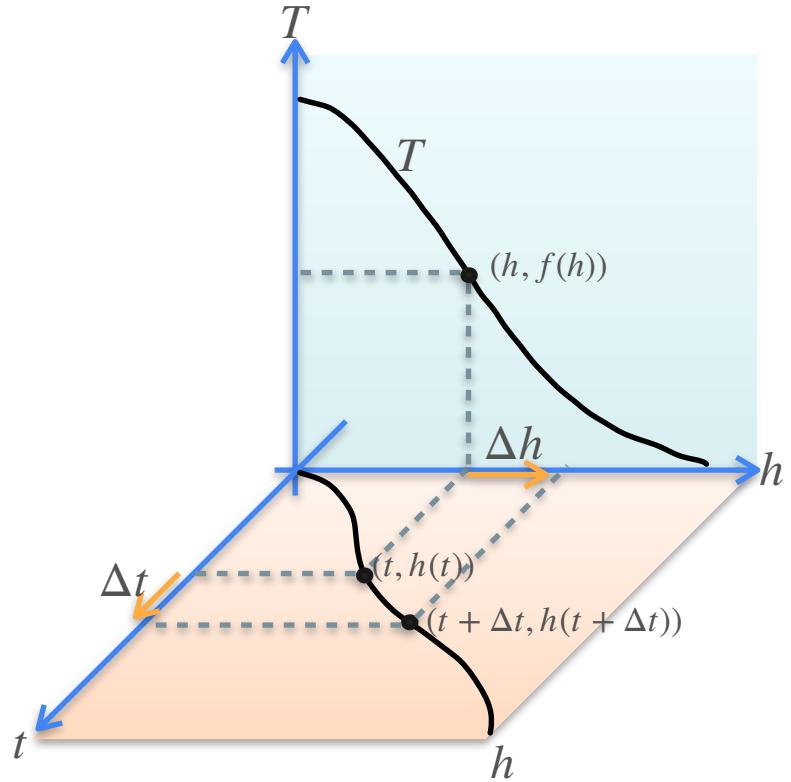
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



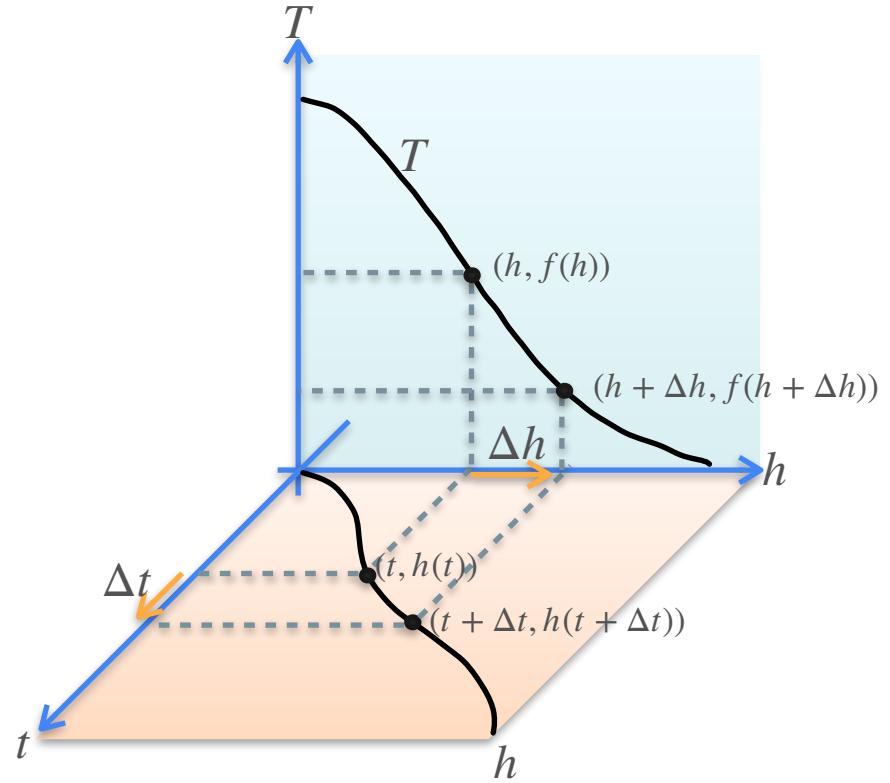
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



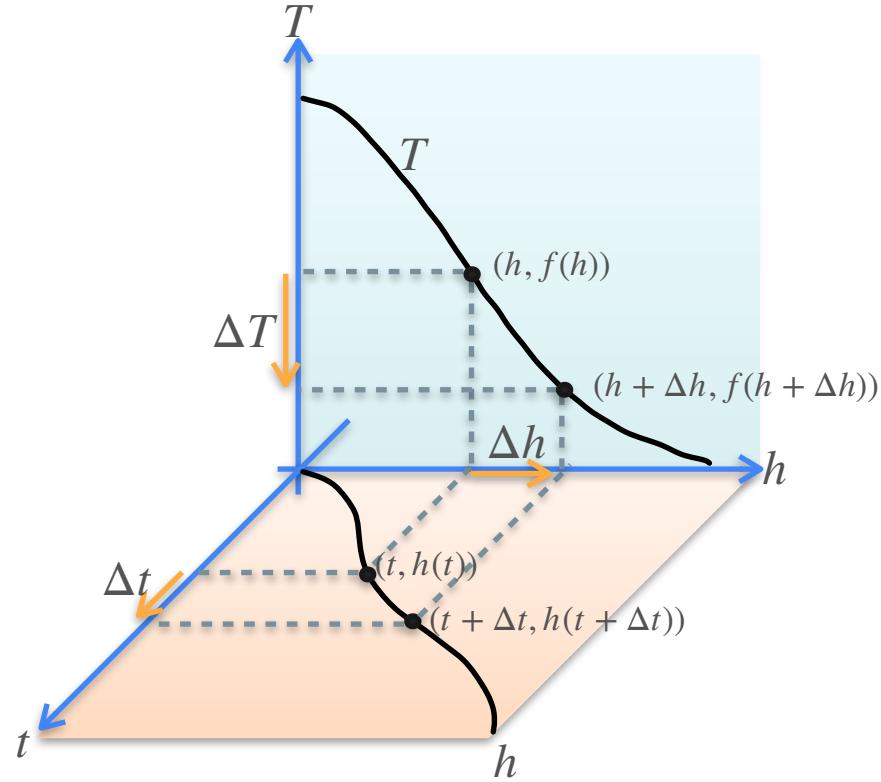
$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

# Chain Rule



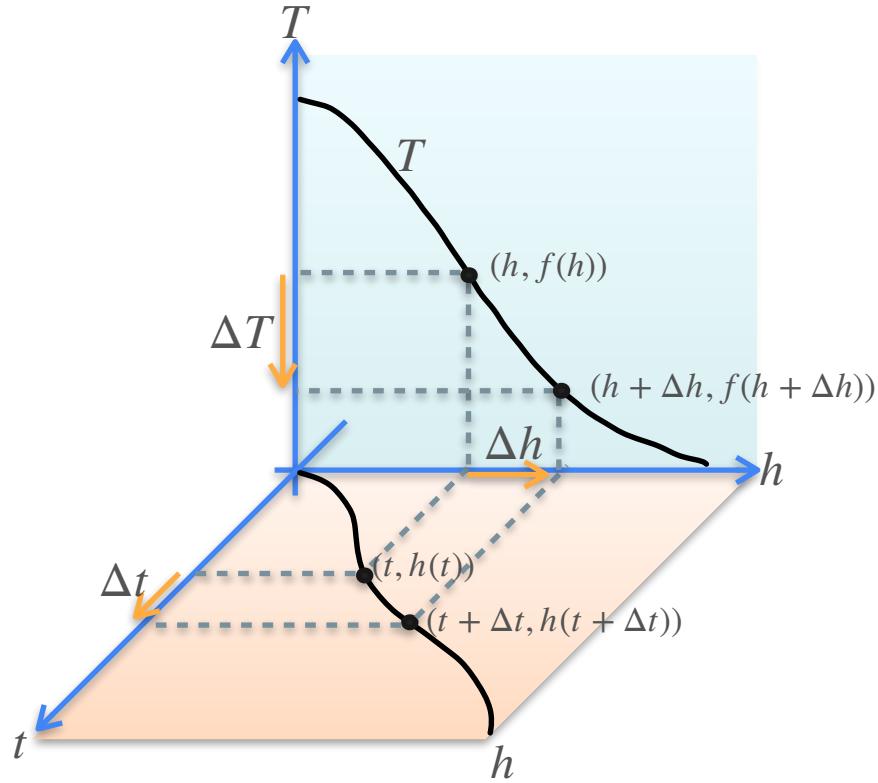
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# Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

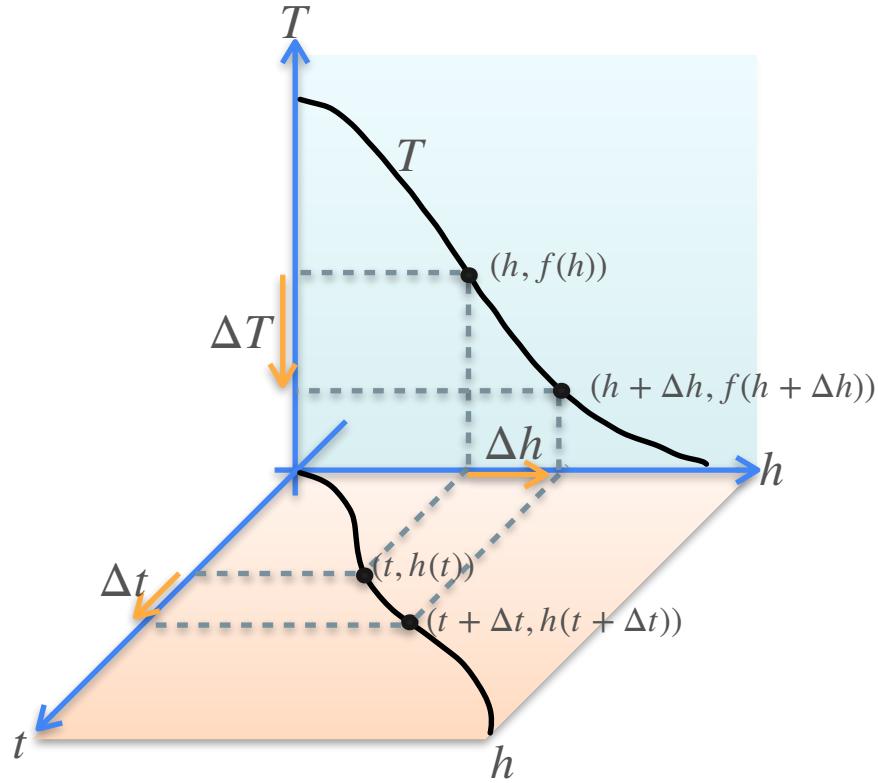
# Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

$$\frac{\Delta T}{\Delta t} =$$

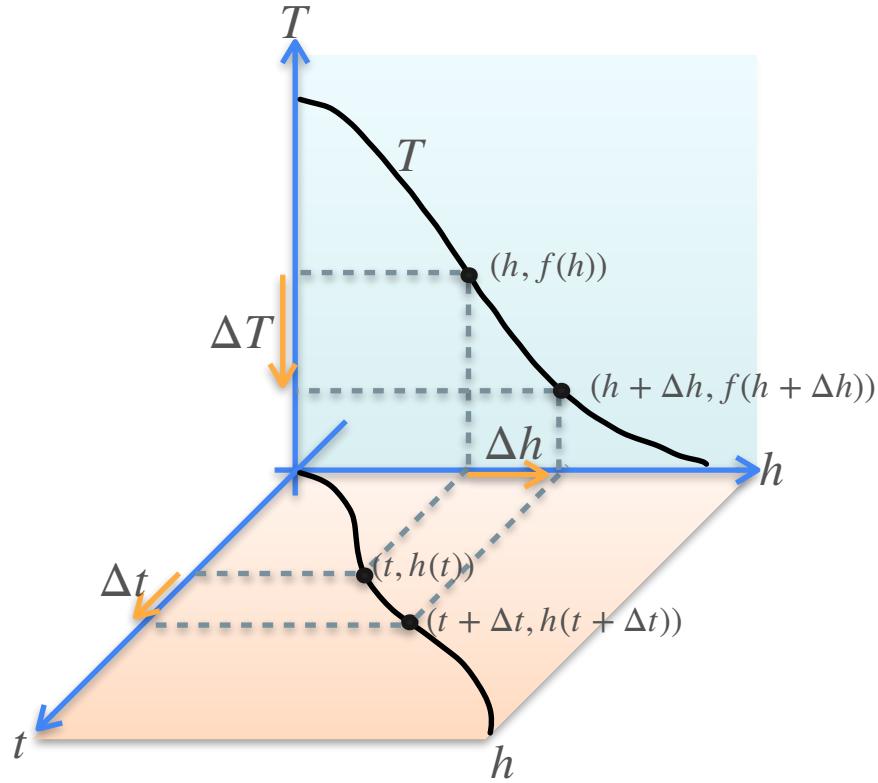
# Chain Rule



$$\Delta t \rightarrow \Delta h \rightarrow \Delta T$$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta h}{\Delta t}$$

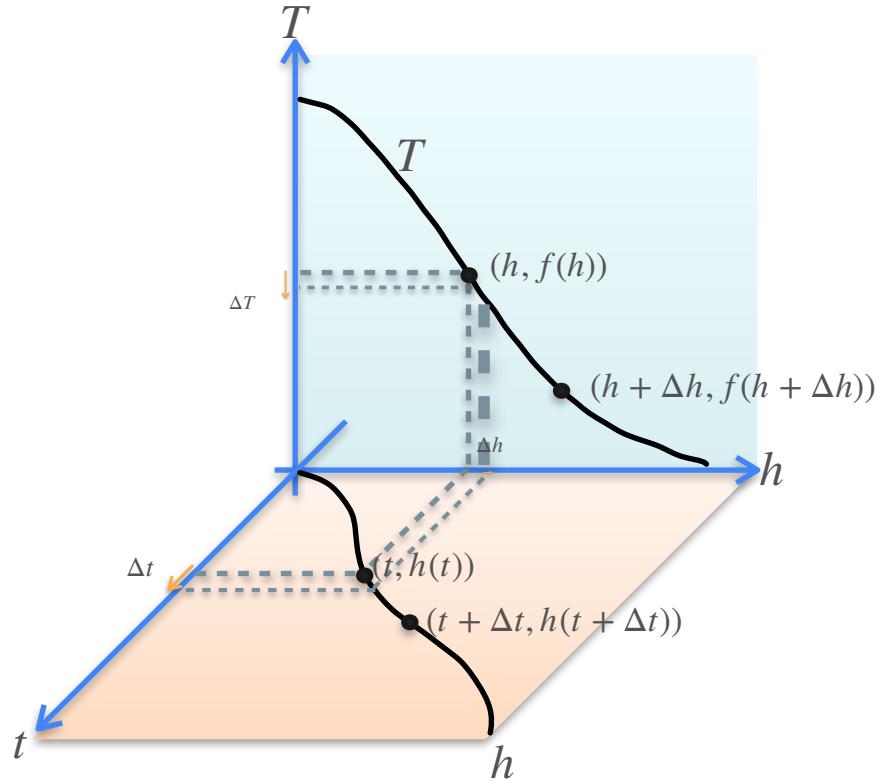
# Chain Rule



$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

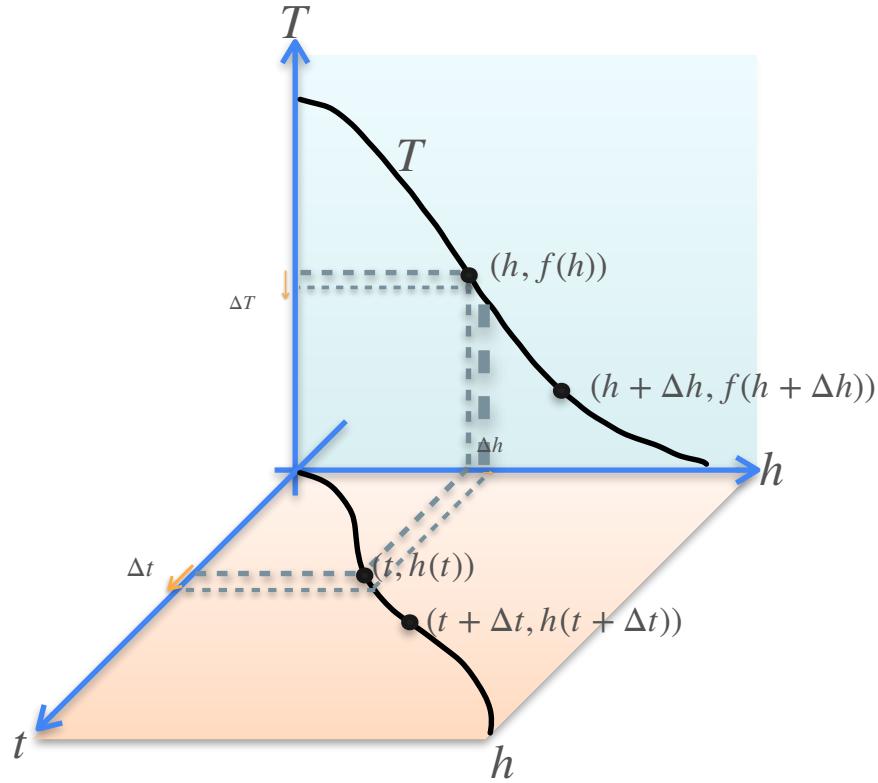
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$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

# Chain Rule

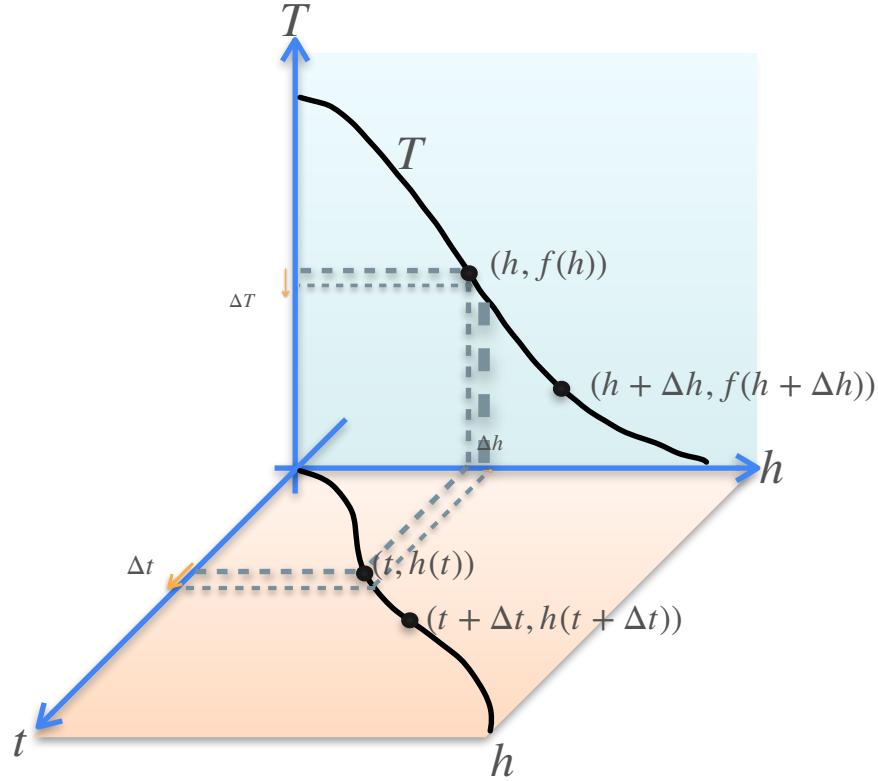


$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt}$$

# Chain Rule

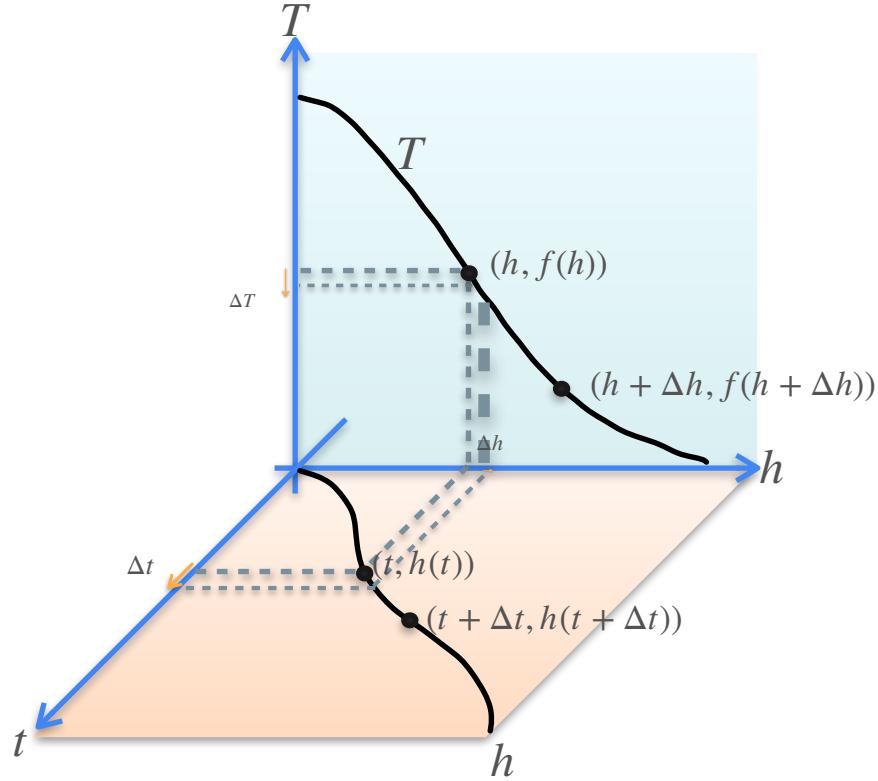


$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$

# Chain Rule



$\Delta t \rightarrow \Delta h \rightarrow \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt} = \frac{dT}{dh} \frac{dh}{dt}$$



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# Derivatives and Optimization

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## Introduction to optimization

# Motivation for Optimization

# Motivation for Optimization



# Motivation for Optimization



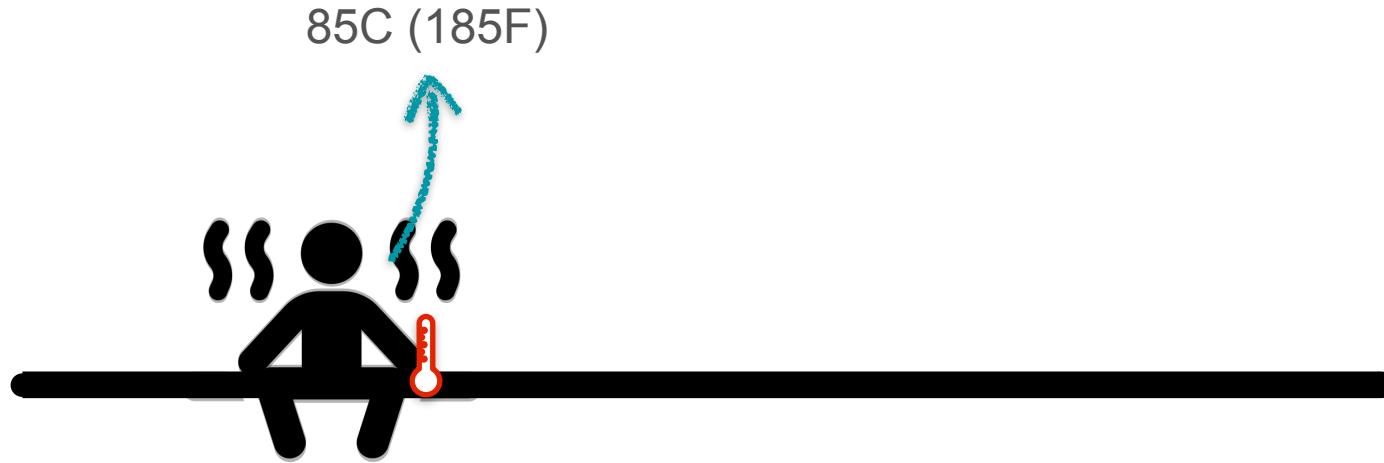
# Motivation for Optimization



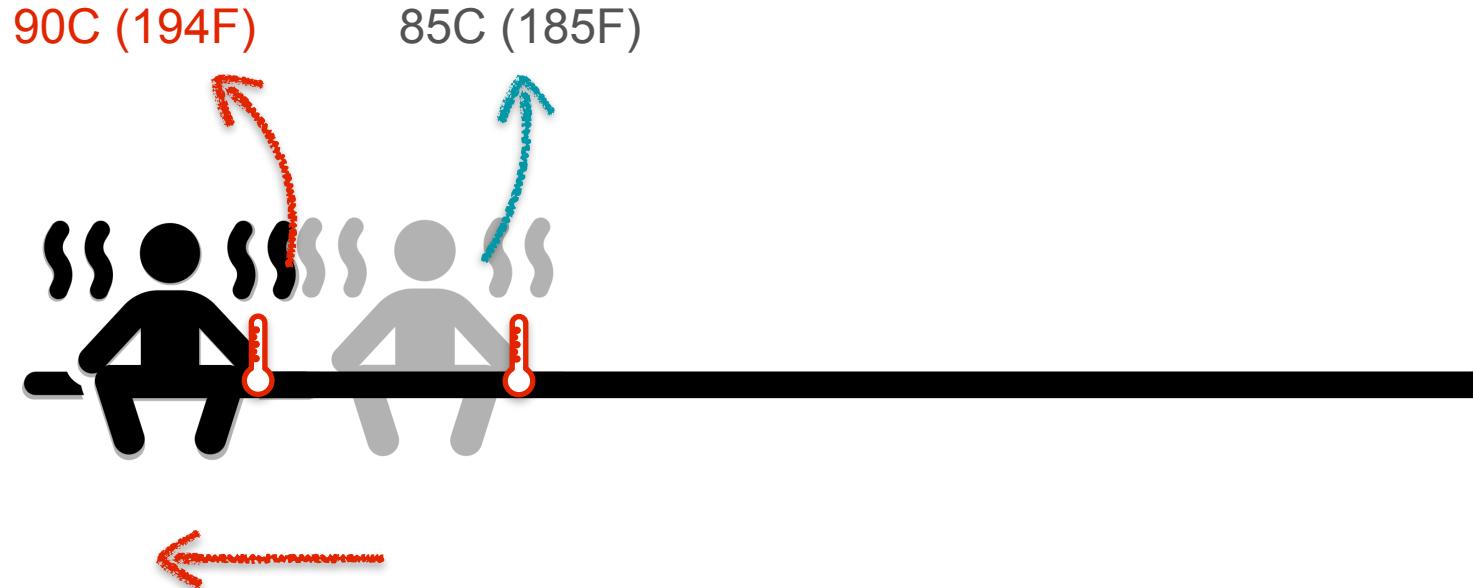
# Motivation for Optimization



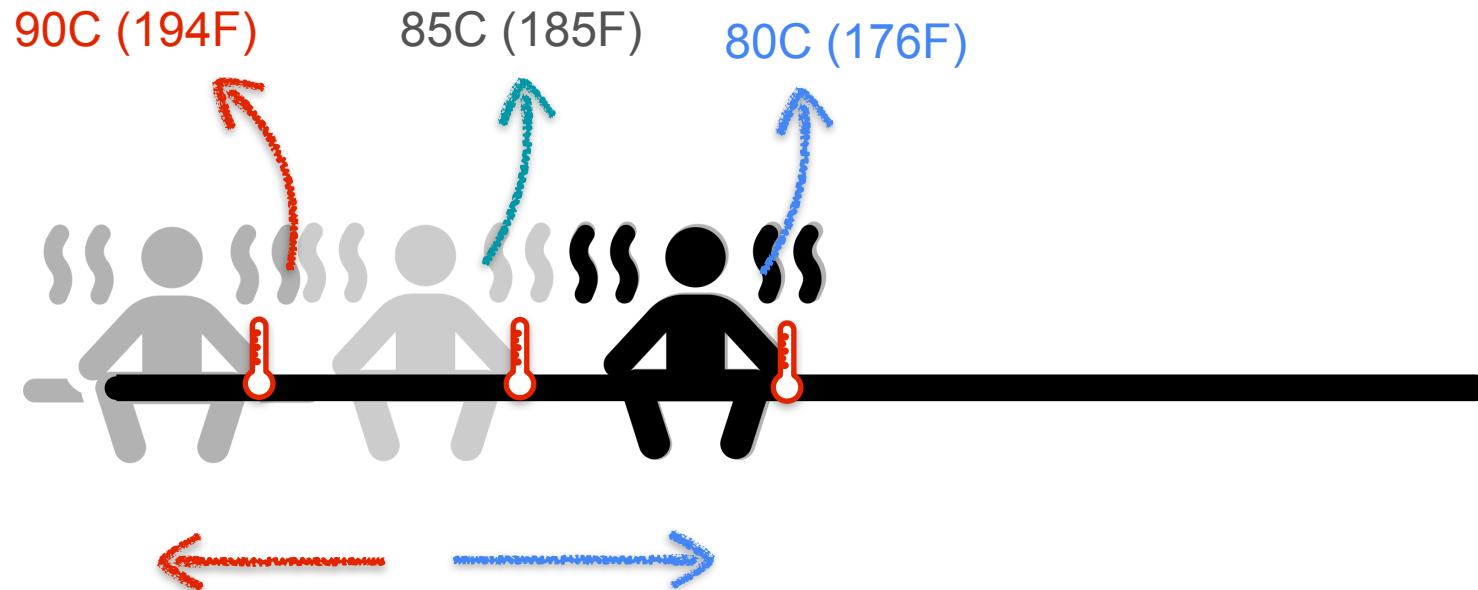
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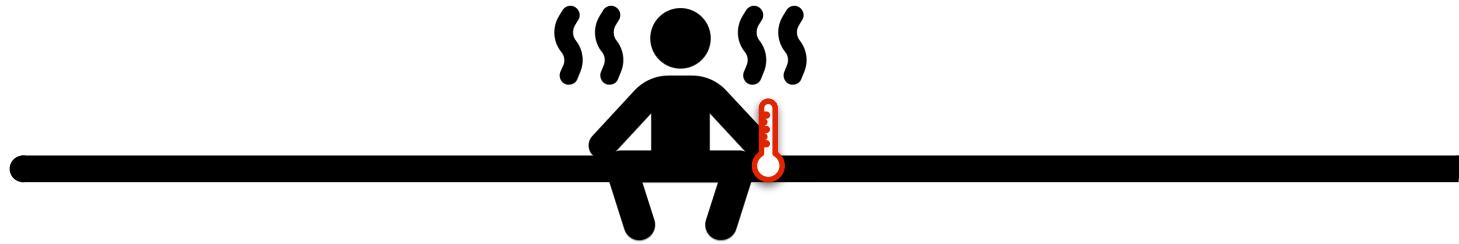
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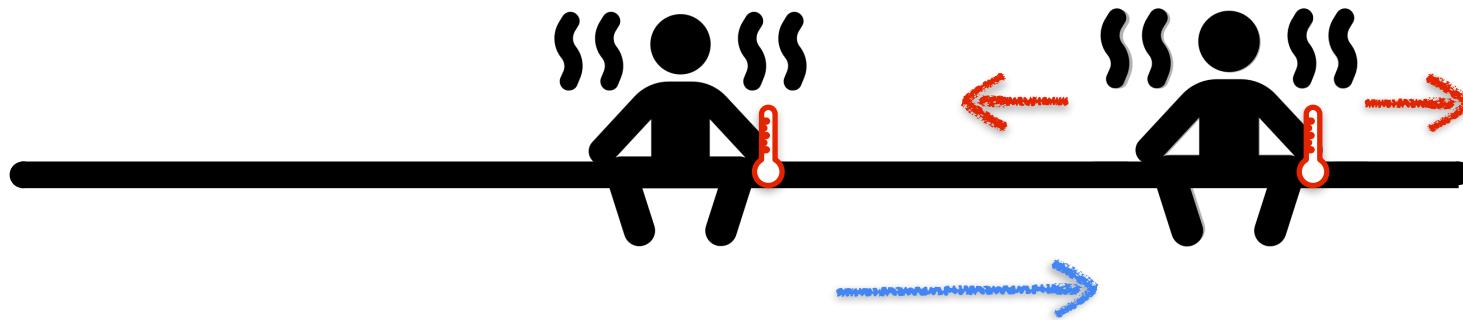
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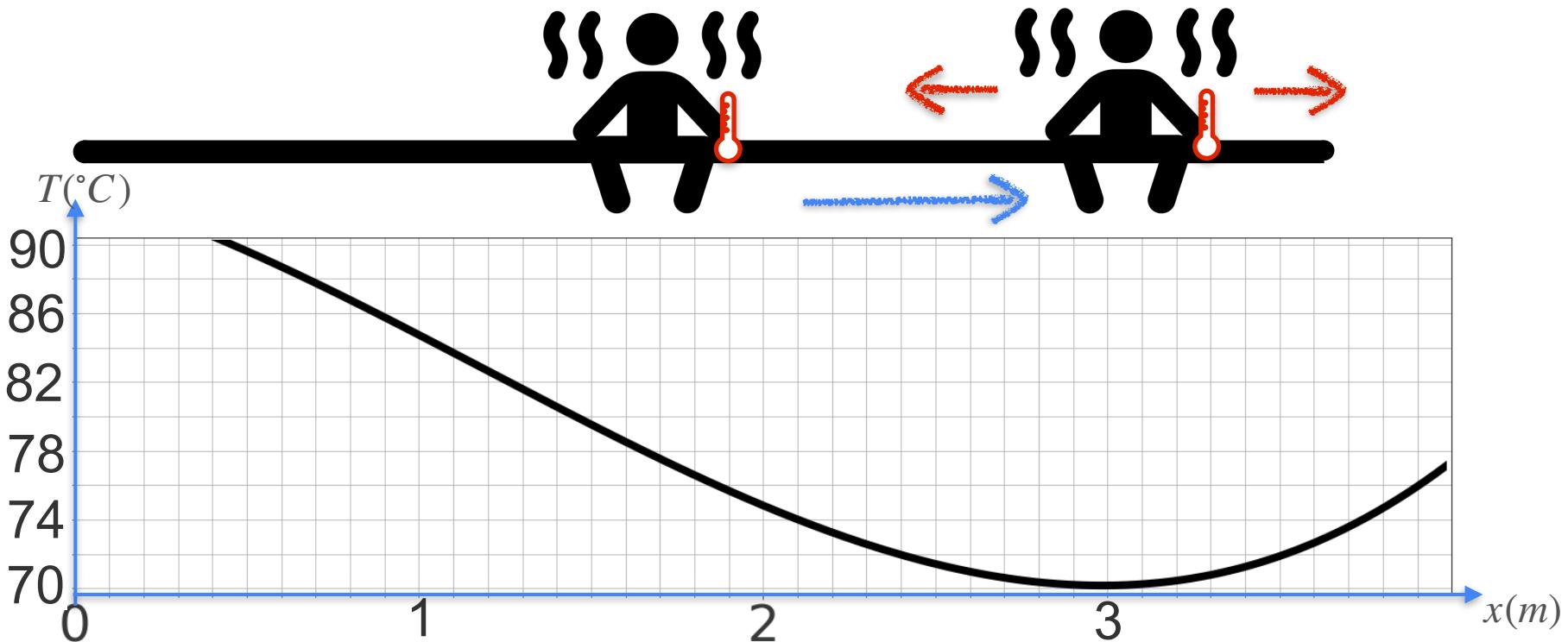
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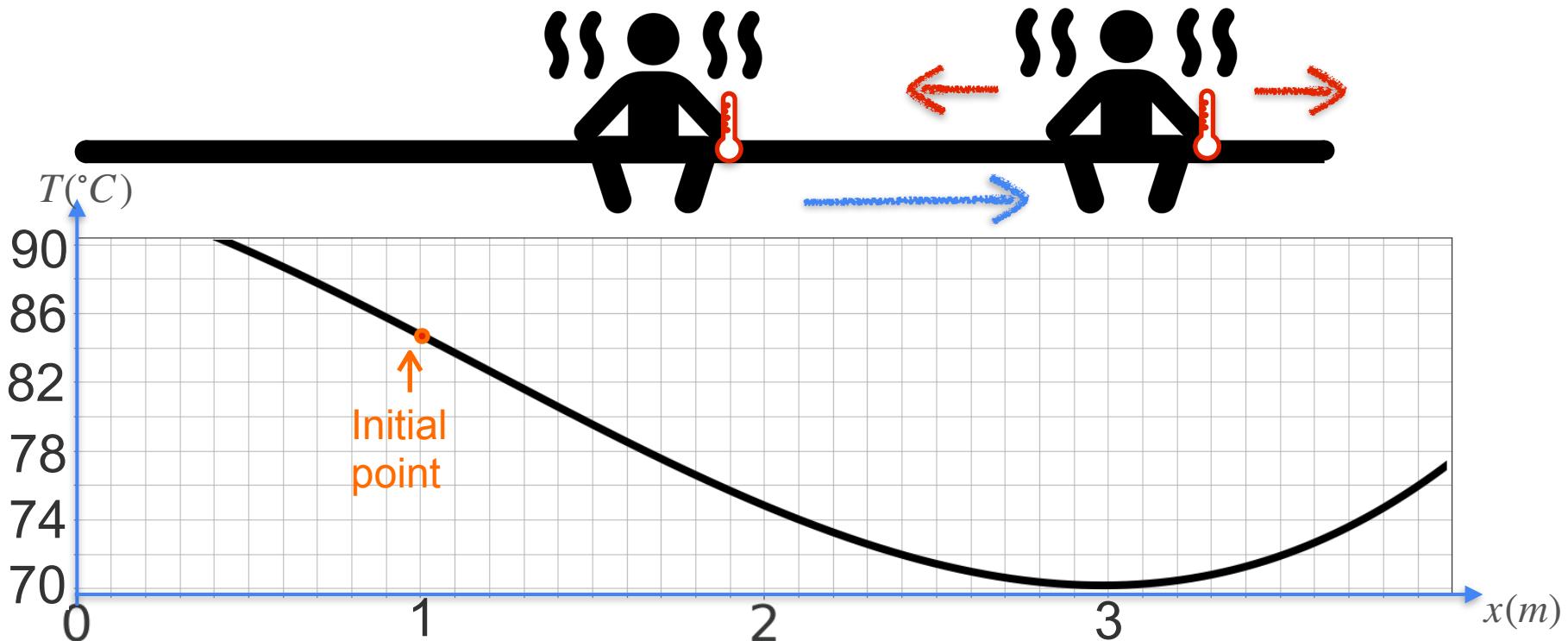
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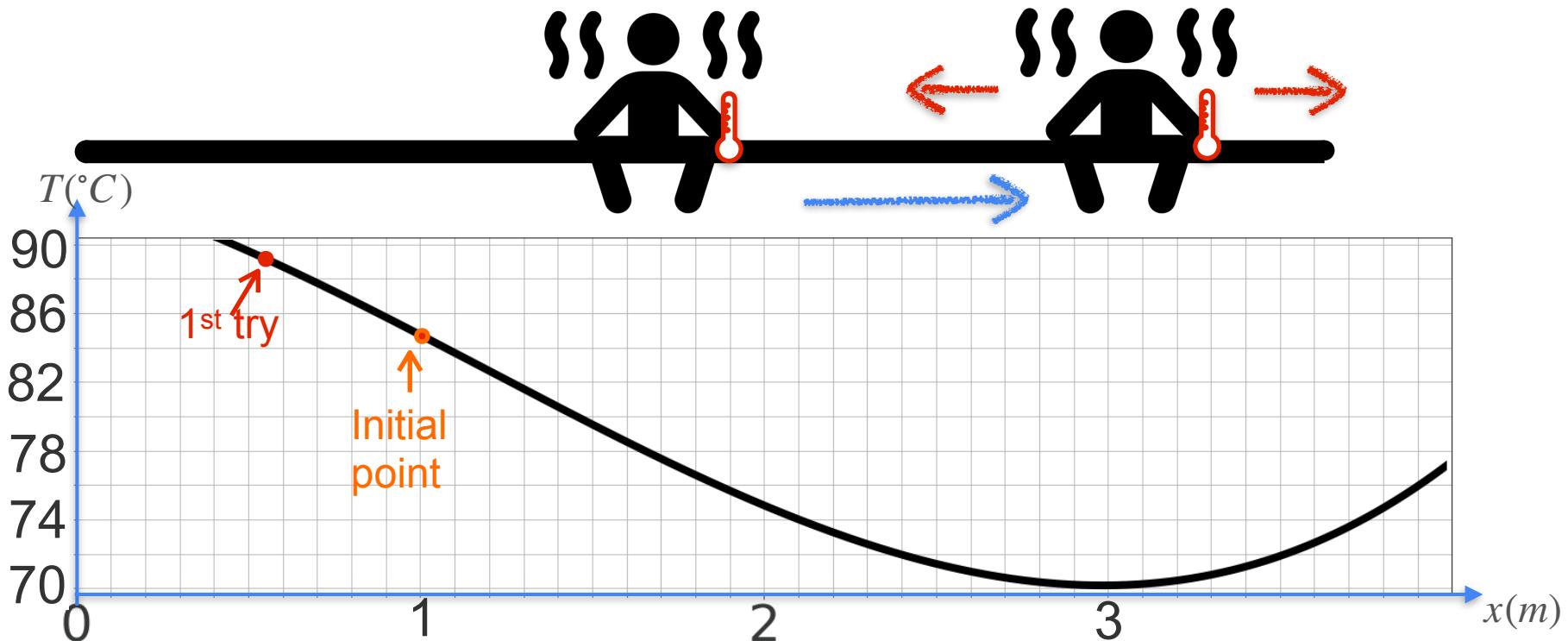
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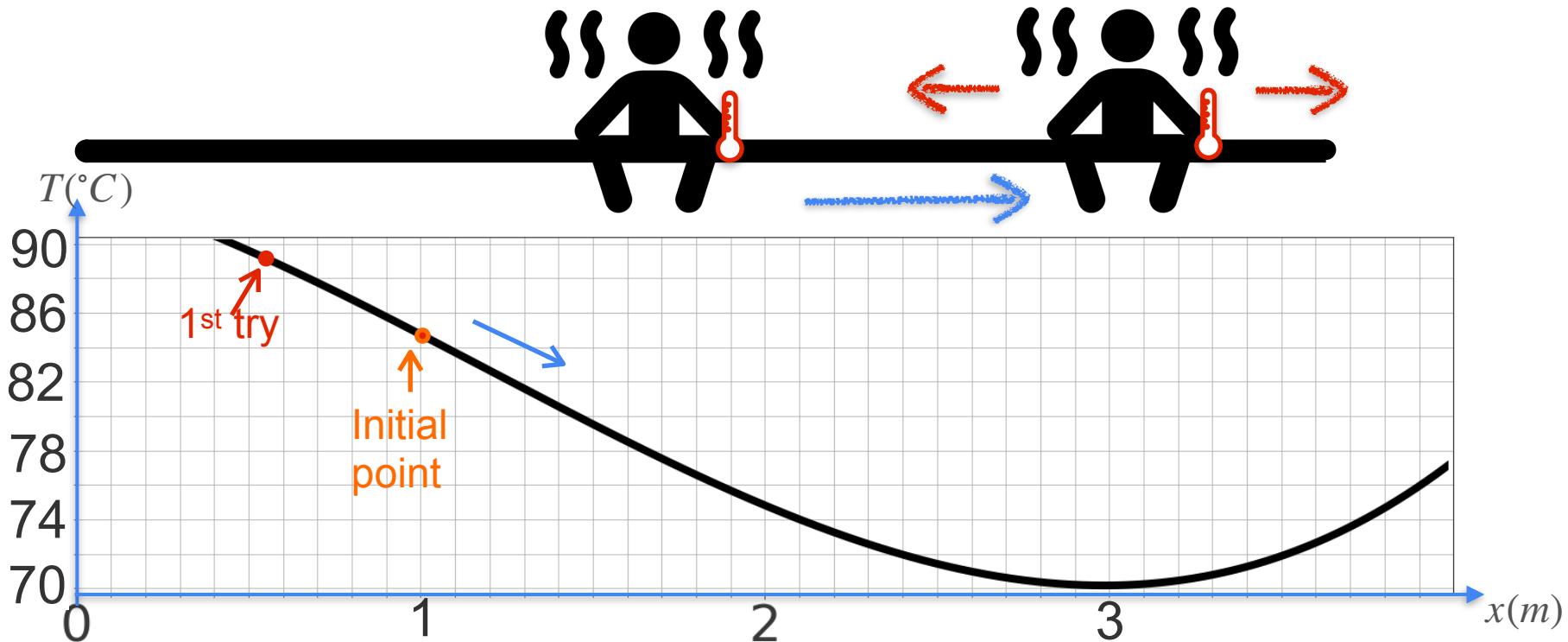
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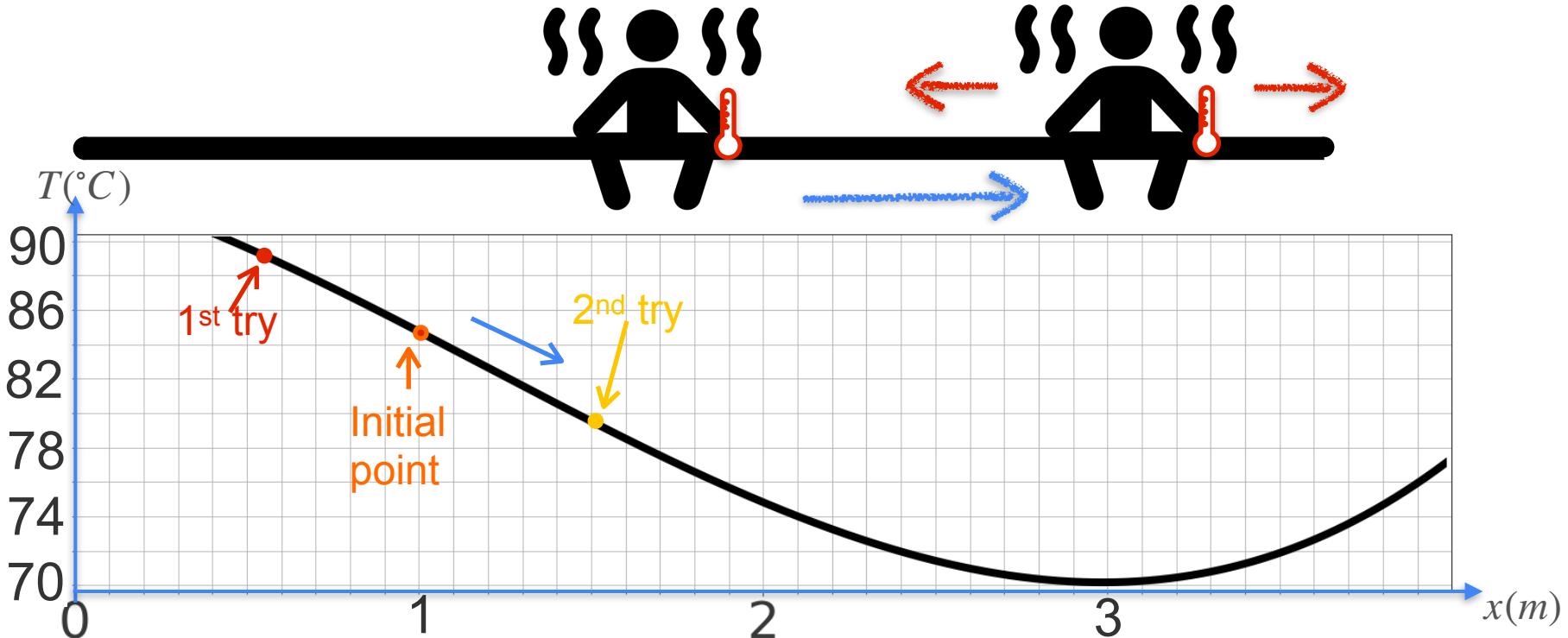
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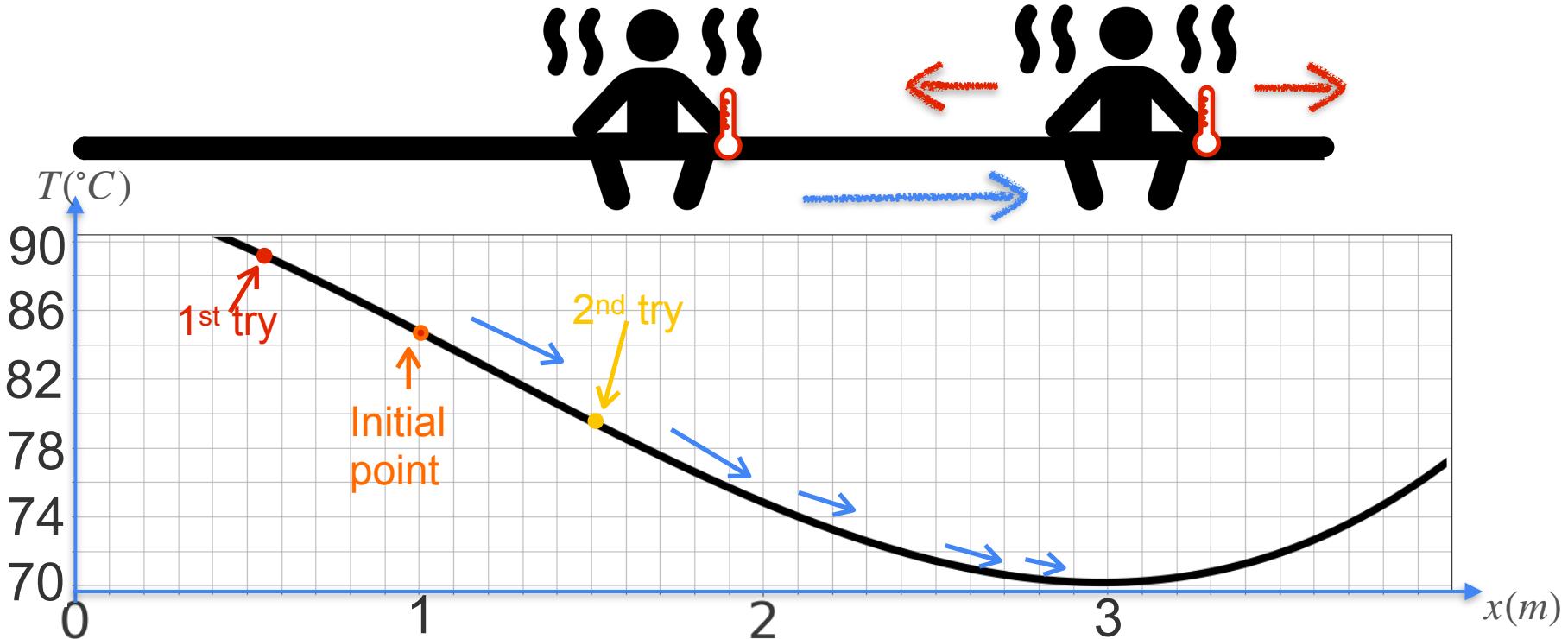
# Motivation for Optimization



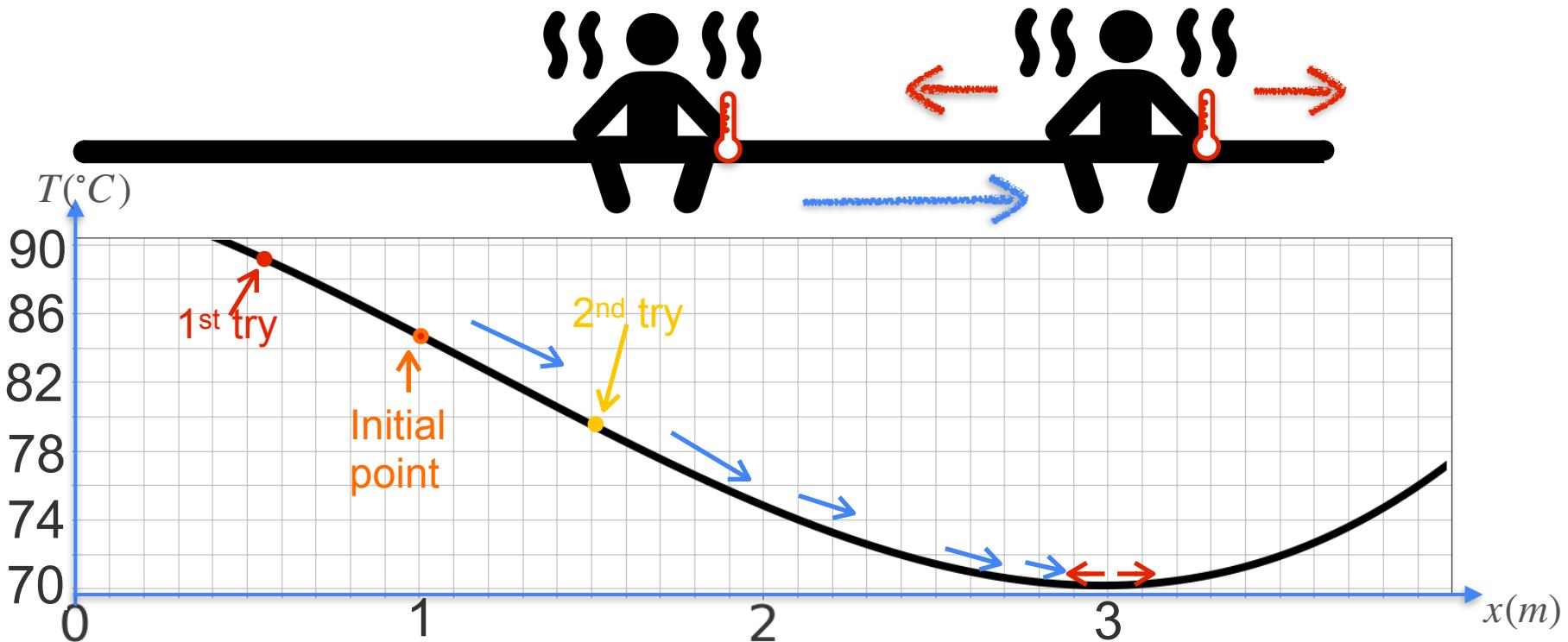
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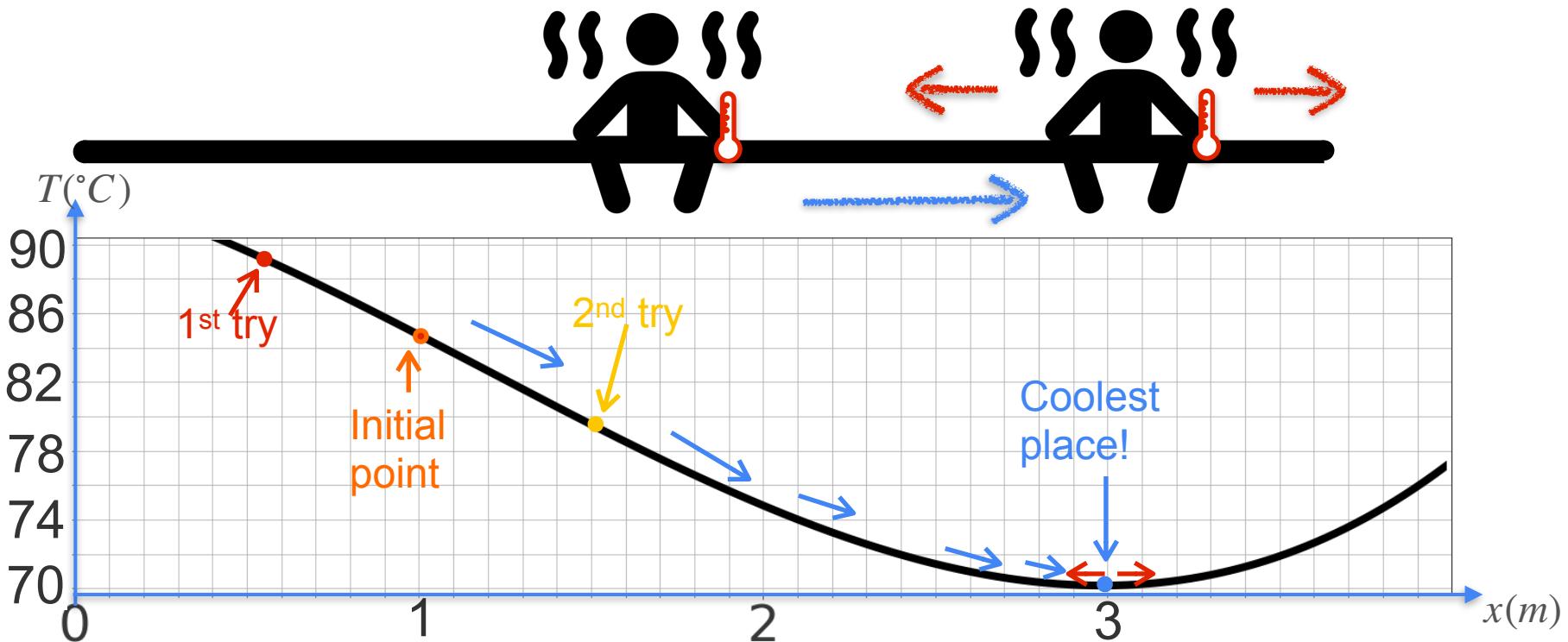
# Motivation for Optimization



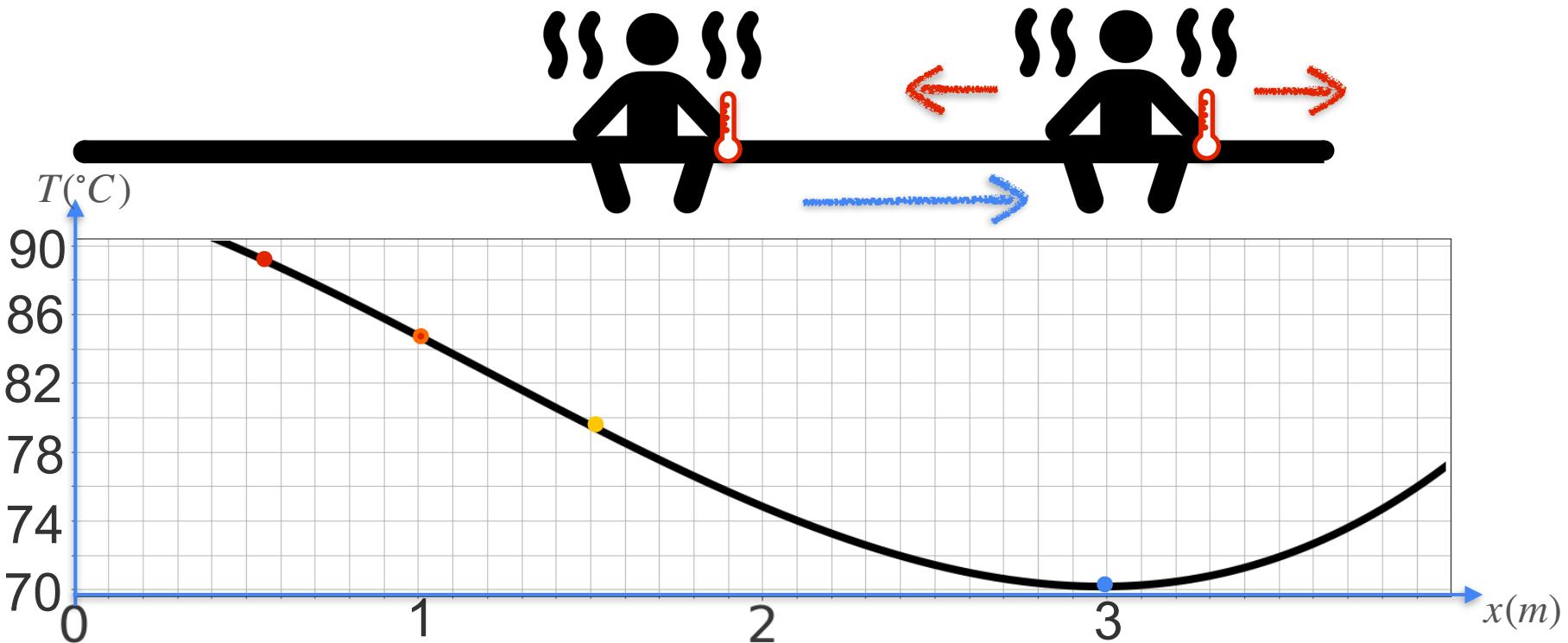
# Motivation for Optimization



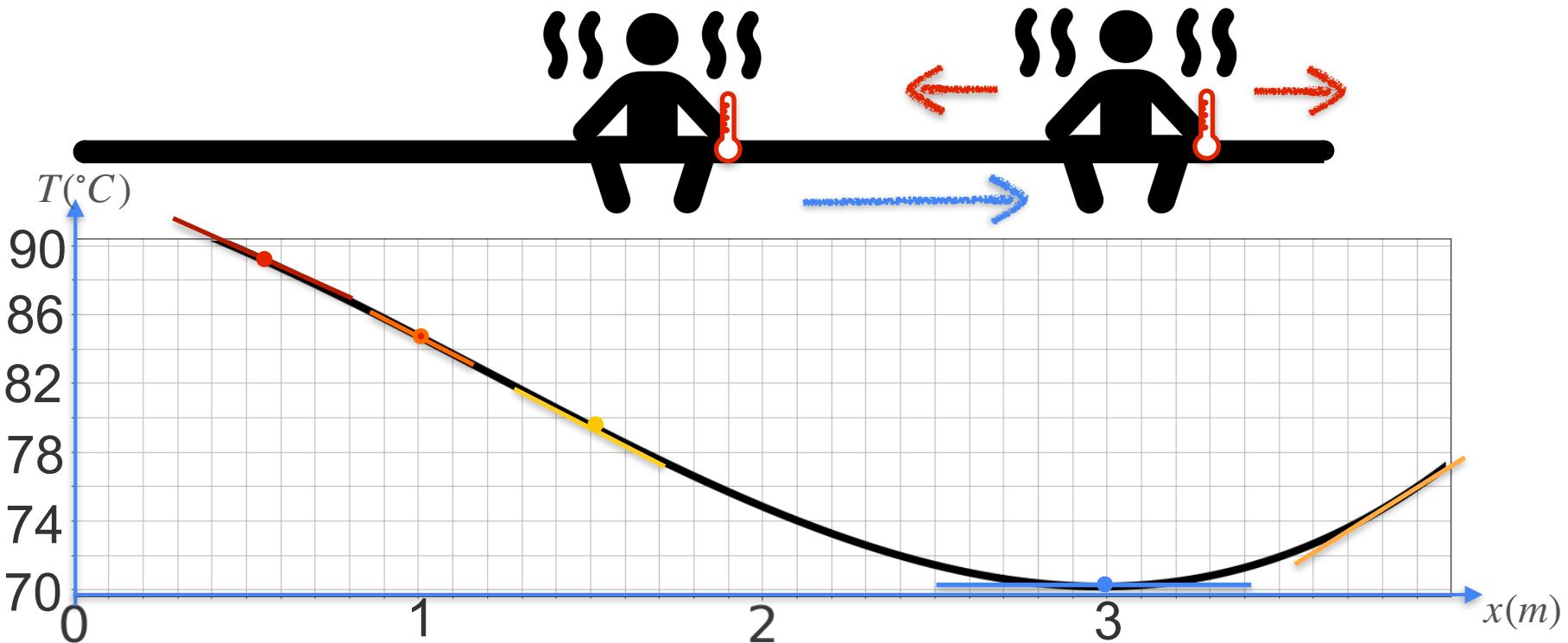
# Motivation for Optimization



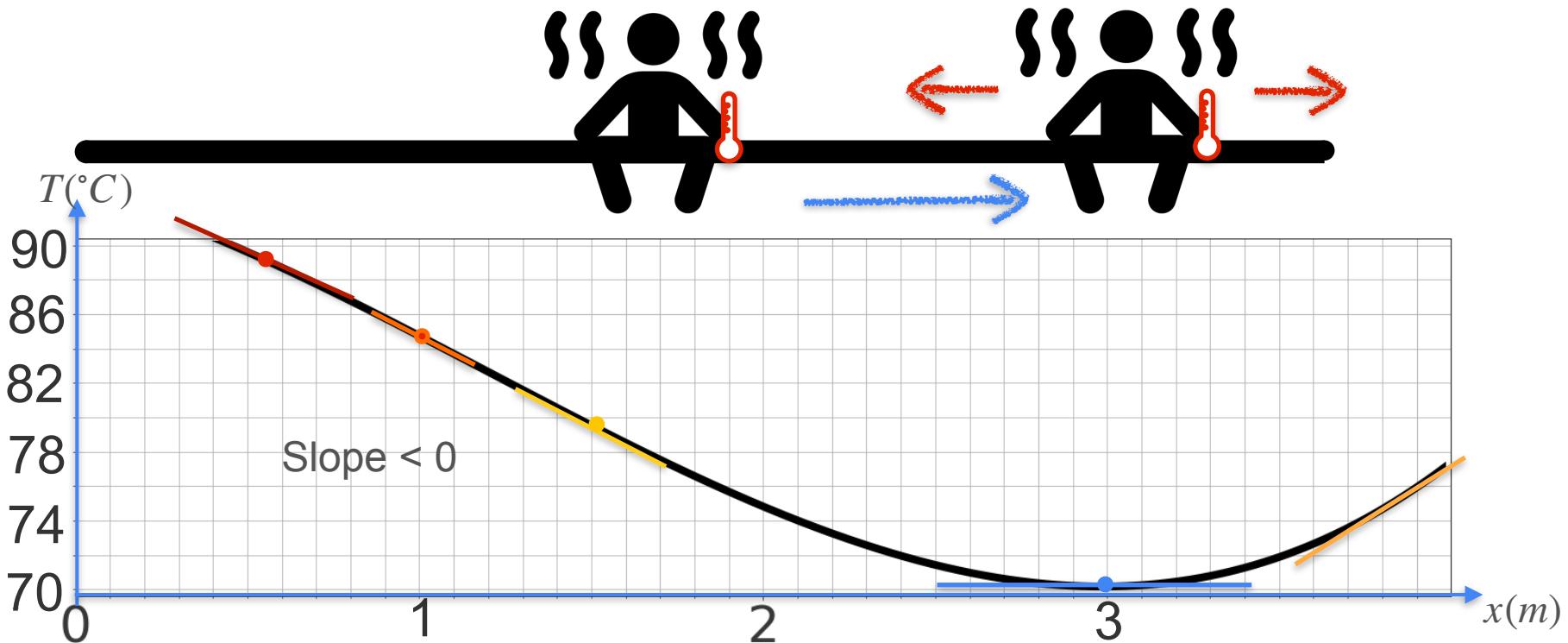
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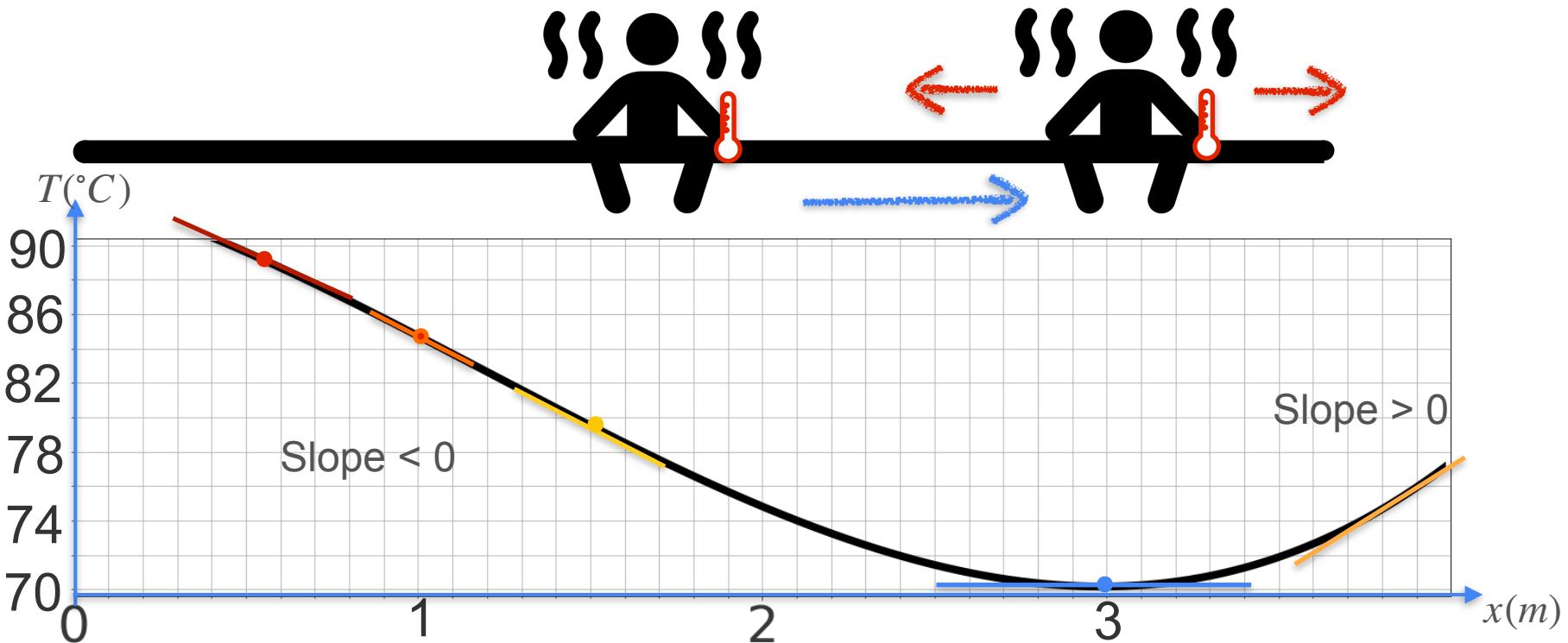
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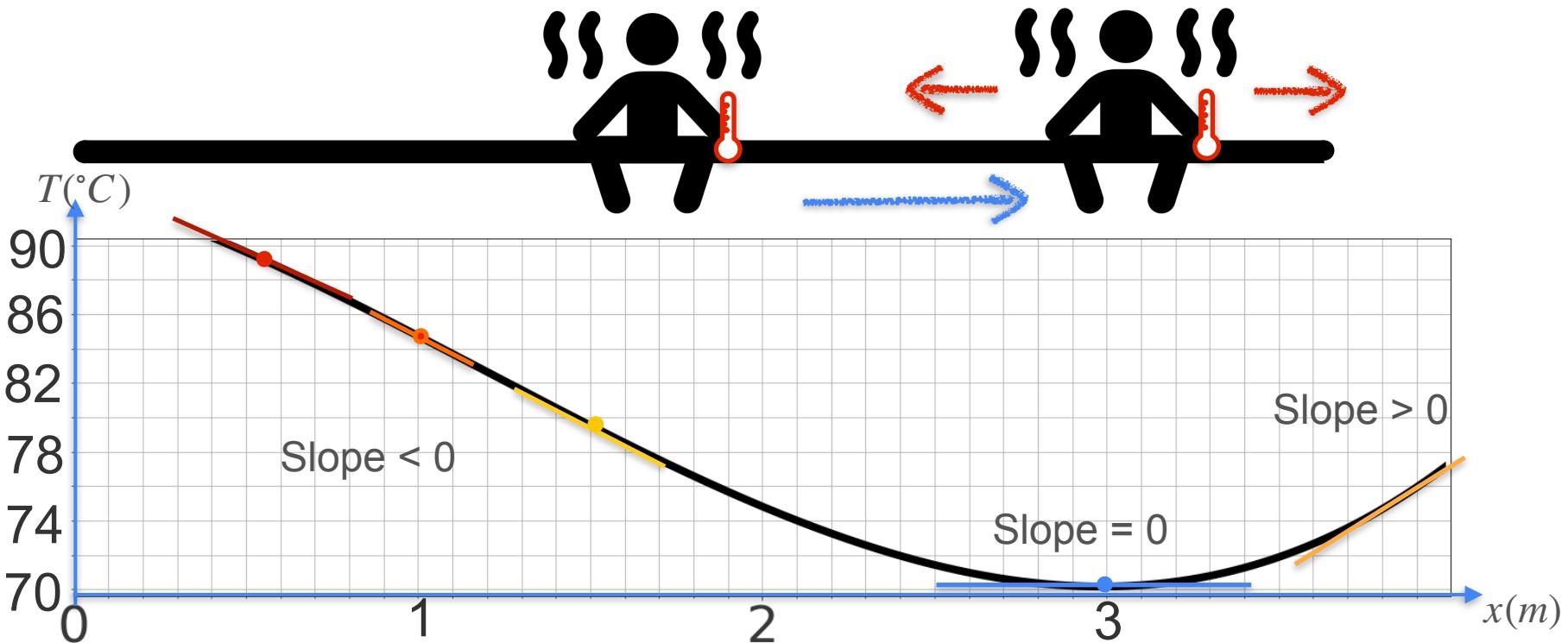
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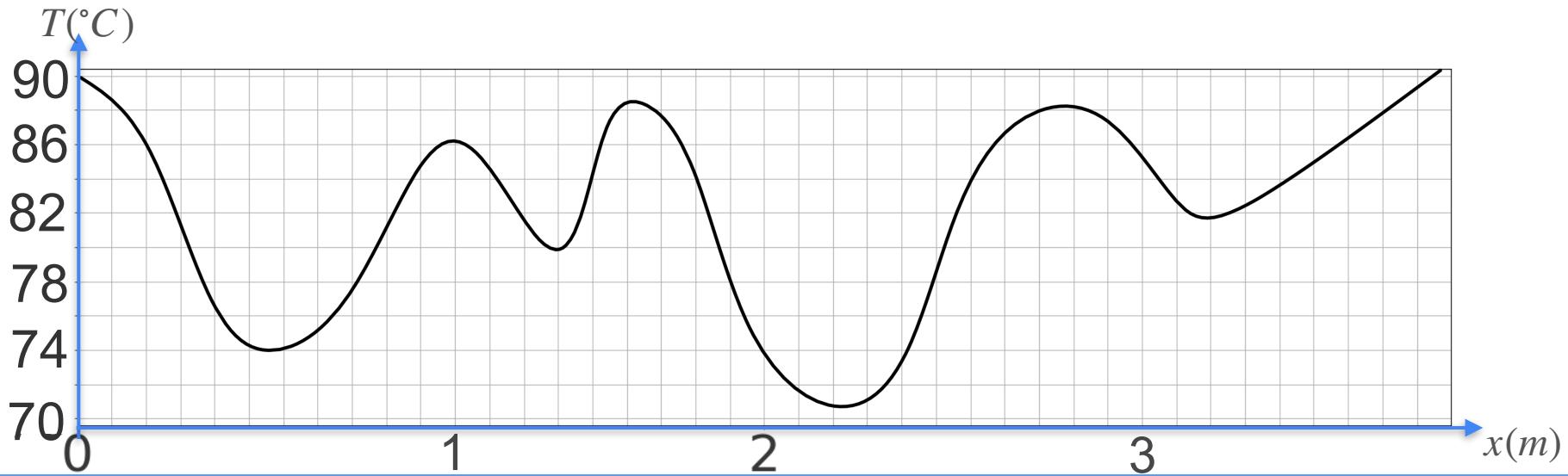


# Motivation for Optimization

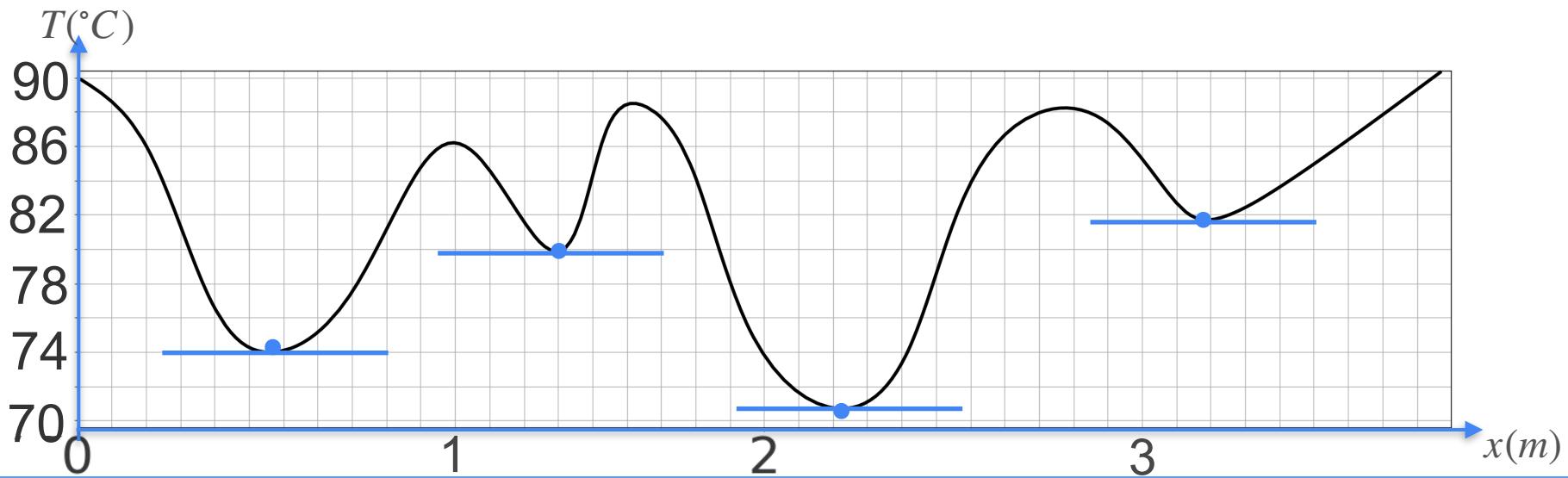


# Multiple Minima

# Multiple Minima

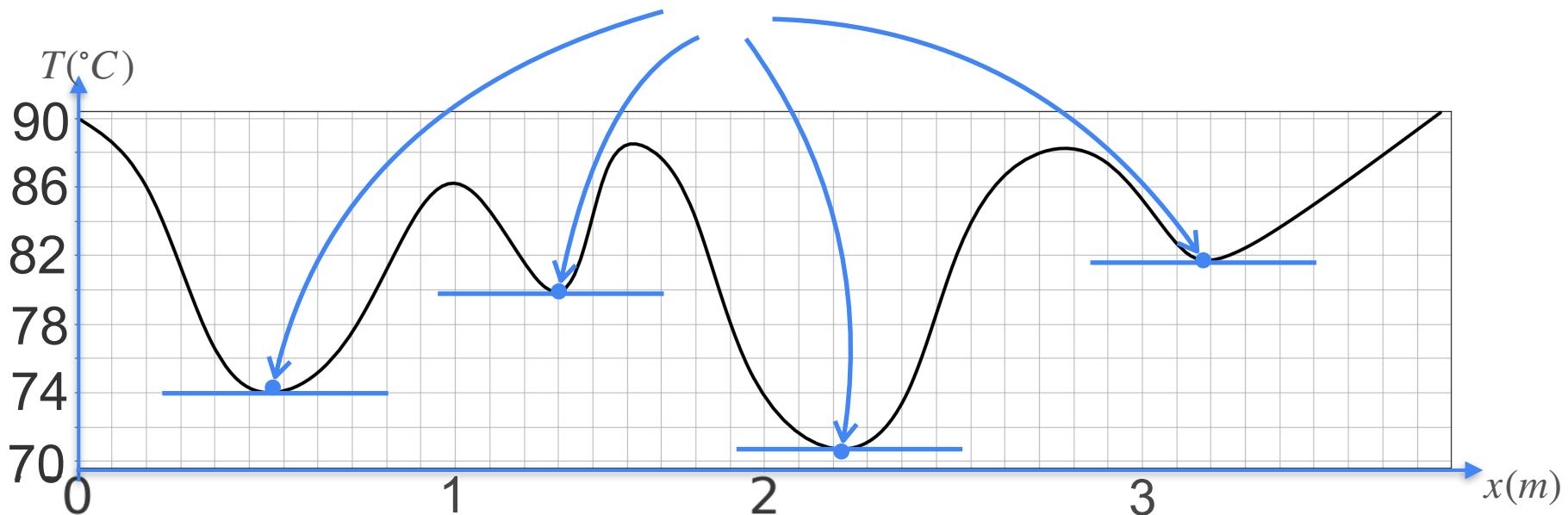


# Multiple Minima



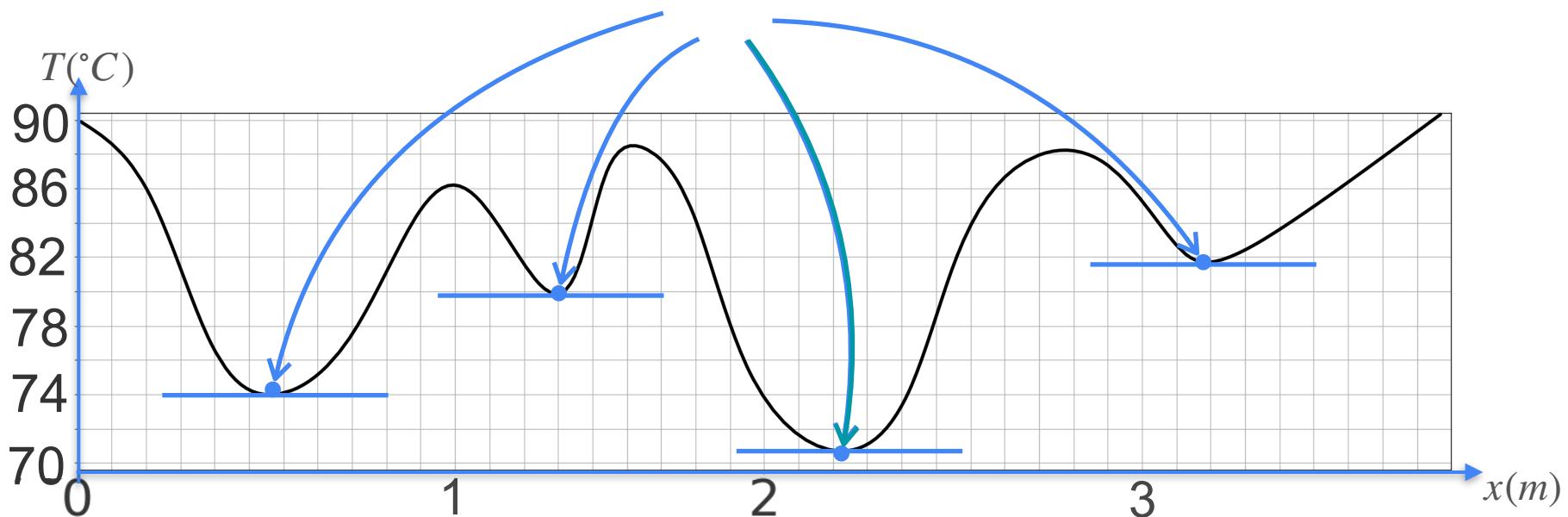
# Multiple Minima

Candidates for the minimum are at the points of zero slope



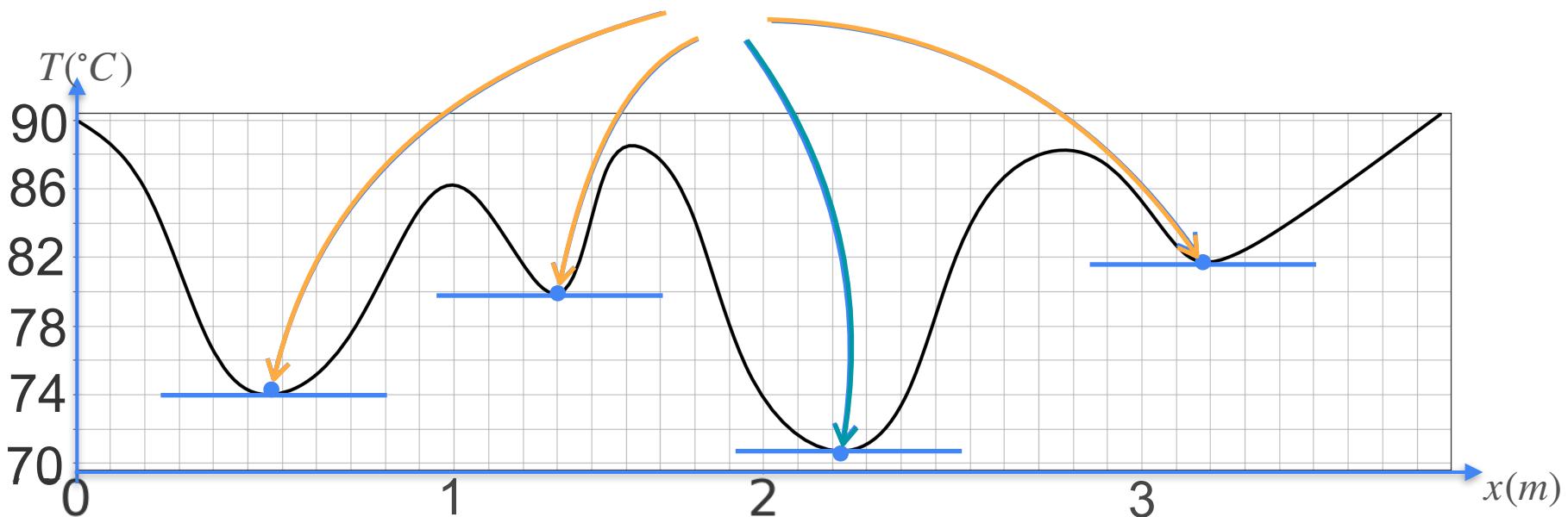
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Candidates for the minimum are at the points of zero slope



# Multiple Minima

Candidates for the minimum are at the points of zero slope





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# Derivatives and Optimization

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**Optimization of squared loss:  
The one powerline problem**

# Cost Function Motivation

# Cost Function Motivation

# Cost Function Motivation



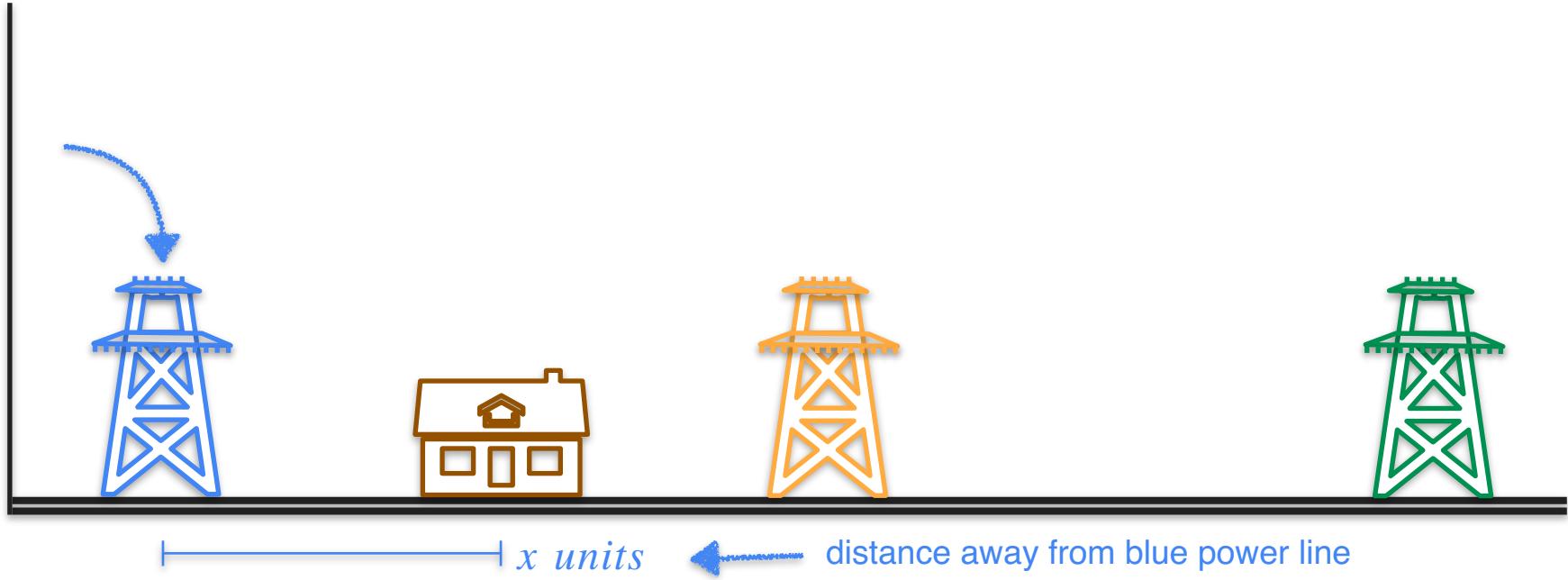
# Cost Function Motivation



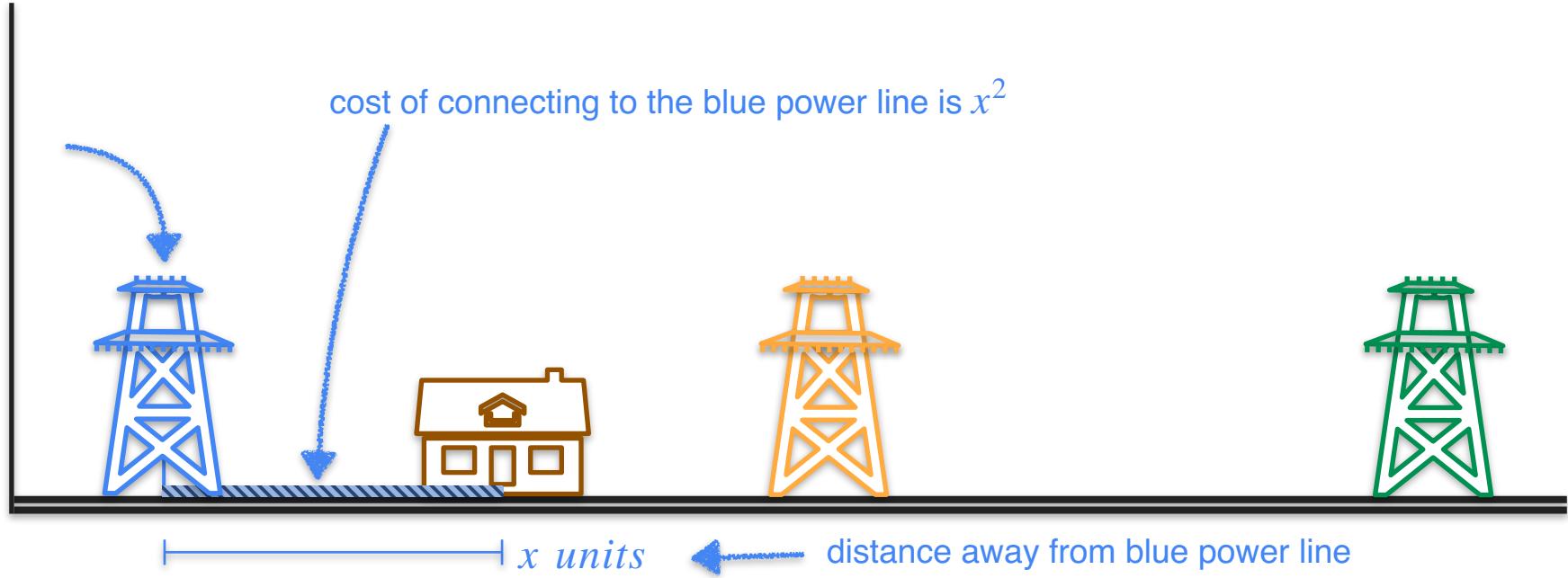
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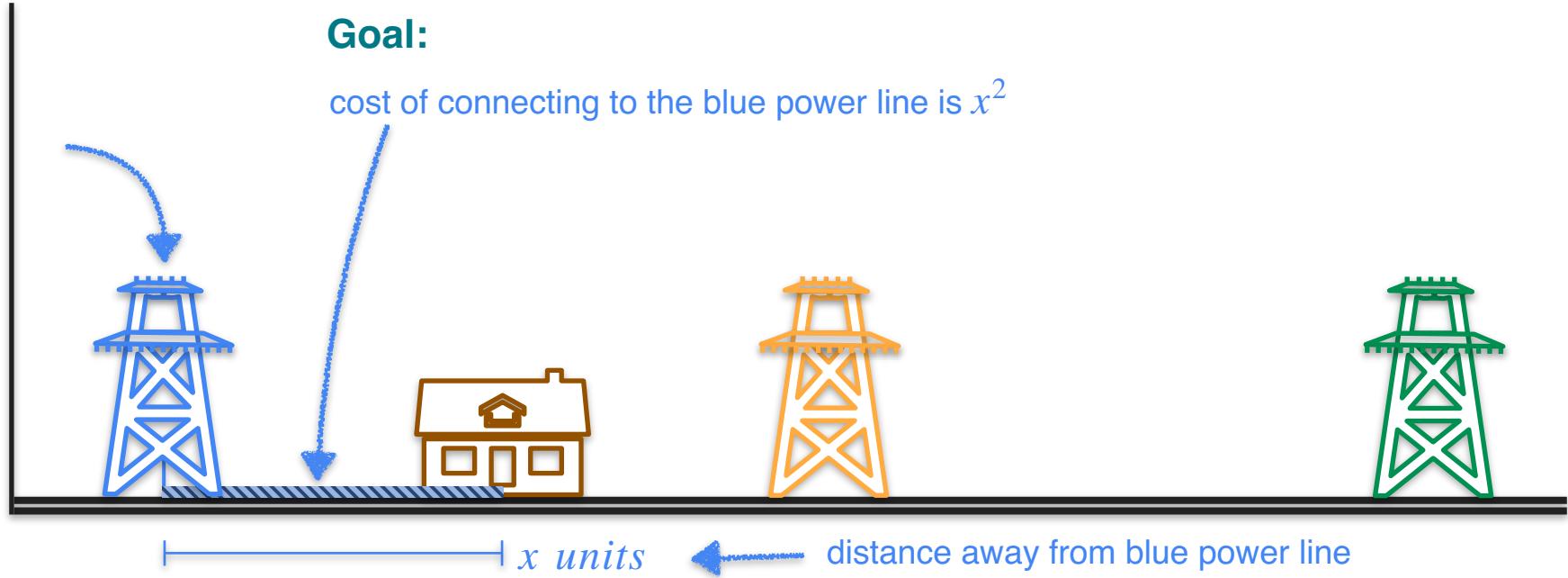
# Cost Function Motivation



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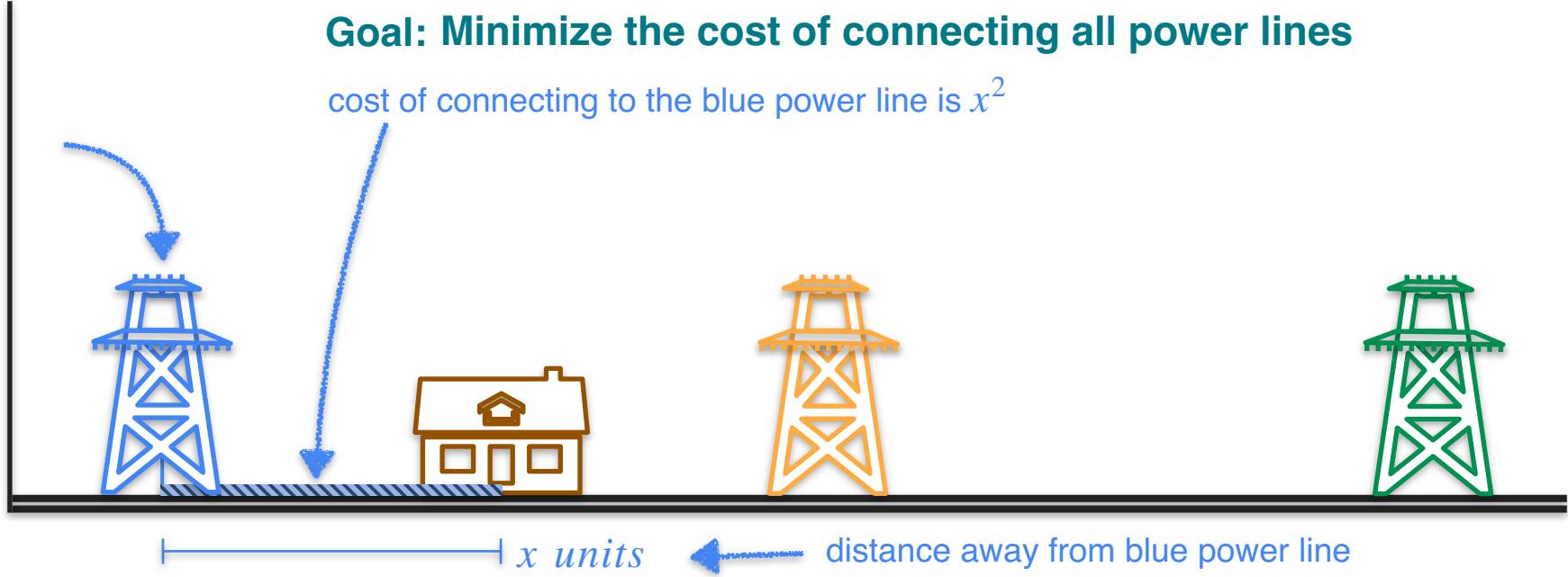
# Cost Function Motivation



# Cost Function Motivation

**Goal: Minimize the cost of connecting all power lines**

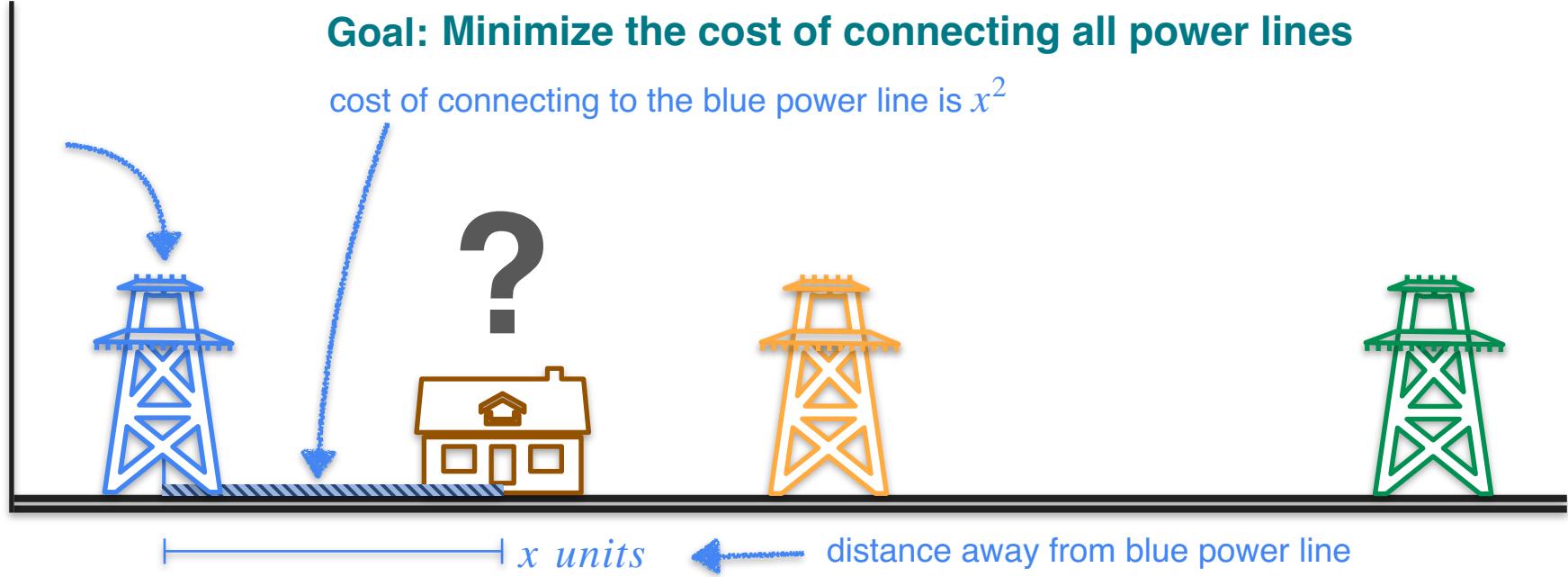
cost of connecting to the blue power line is  $x^2$



# Cost Function Motivation

**Goal: Minimize the cost of connecting all power lines**

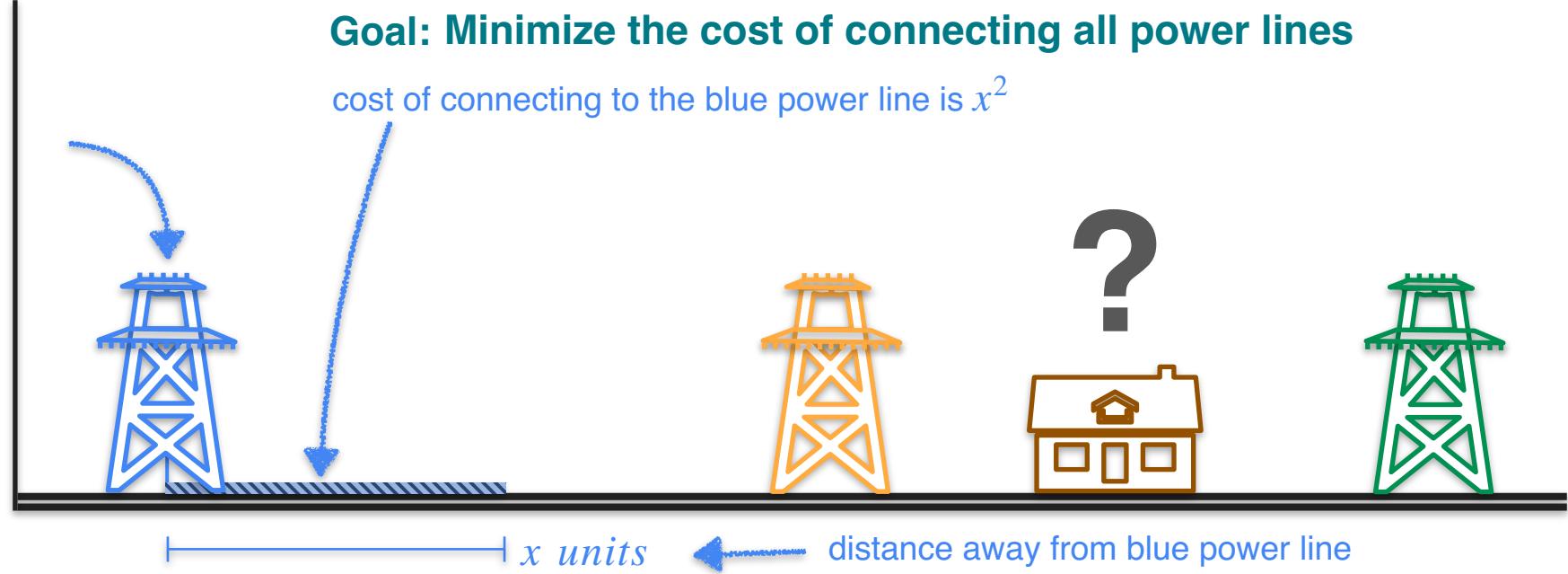
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# Cost Function Motivation

**Goal: Minimize the cost of connecting all power lines**

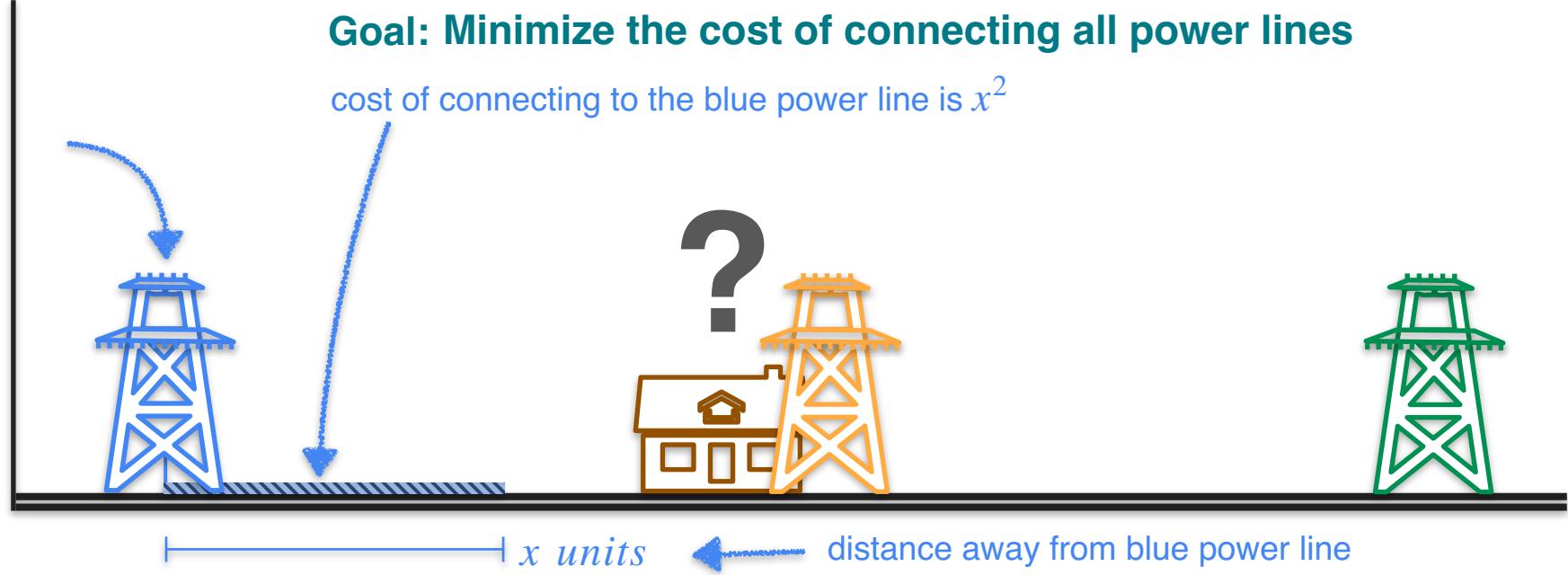
cost of connecting to the blue power line is  $x^2$



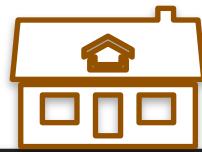
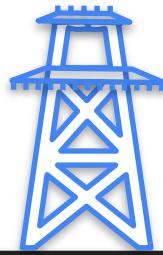
# Cost Function Motivation

**Goal: Minimize the cost of connecting all power lines**

cost of connecting to the blue power line is  $x^2$



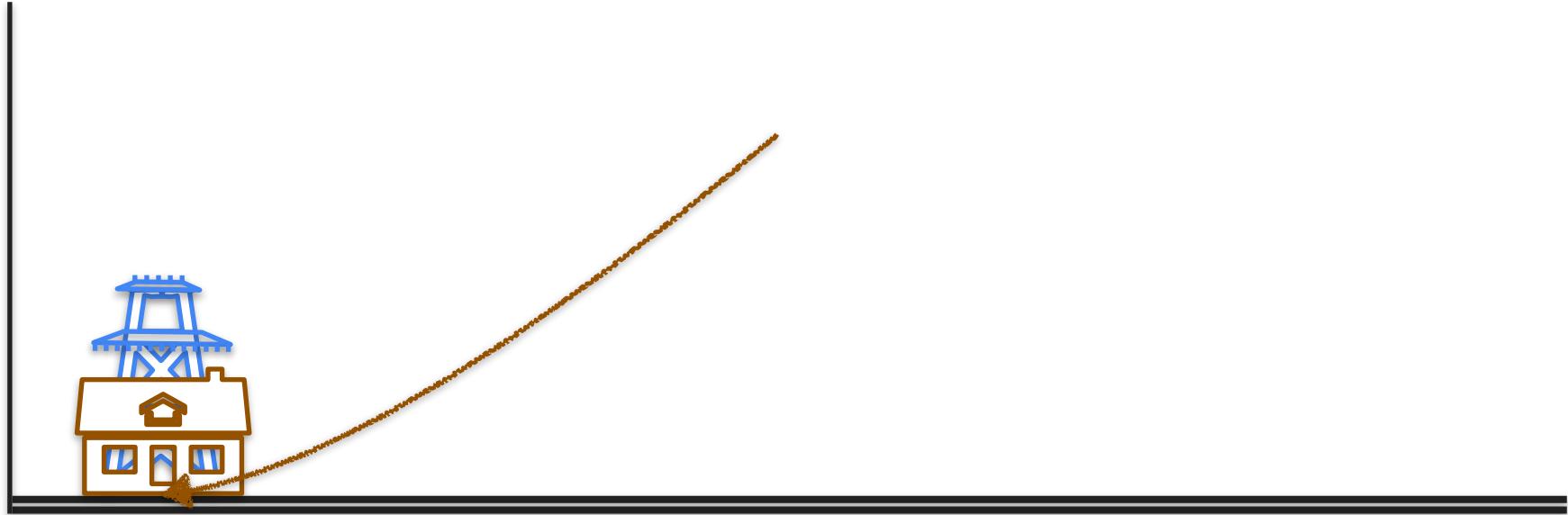
# One Power Line Problem



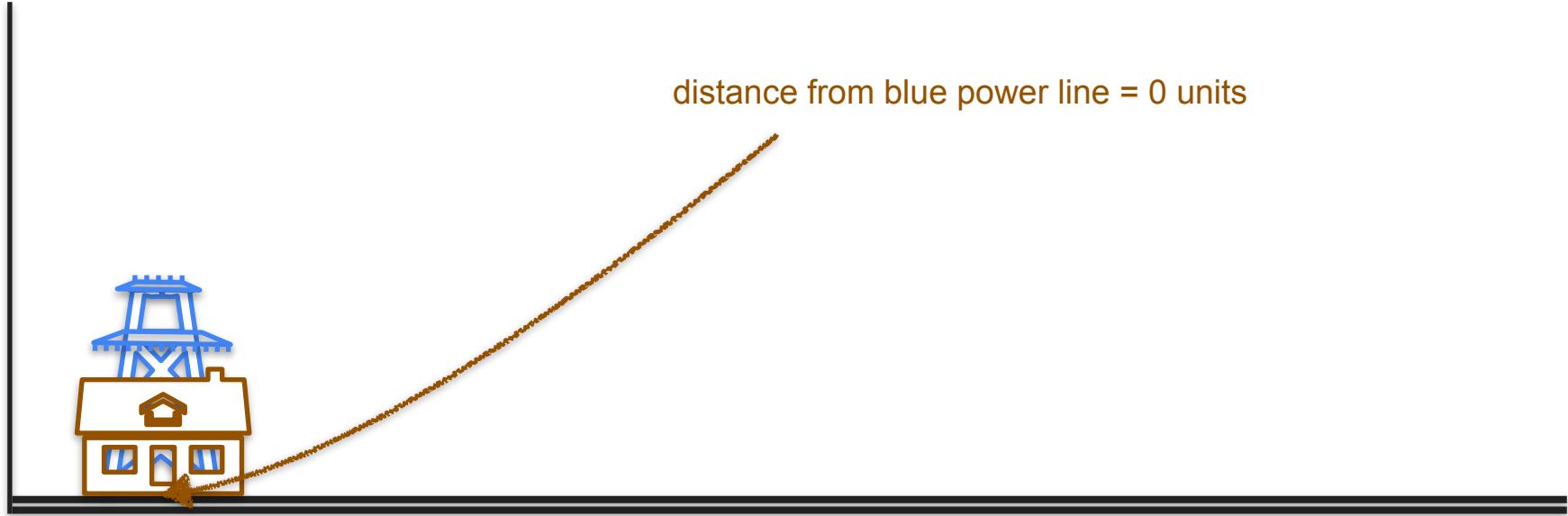
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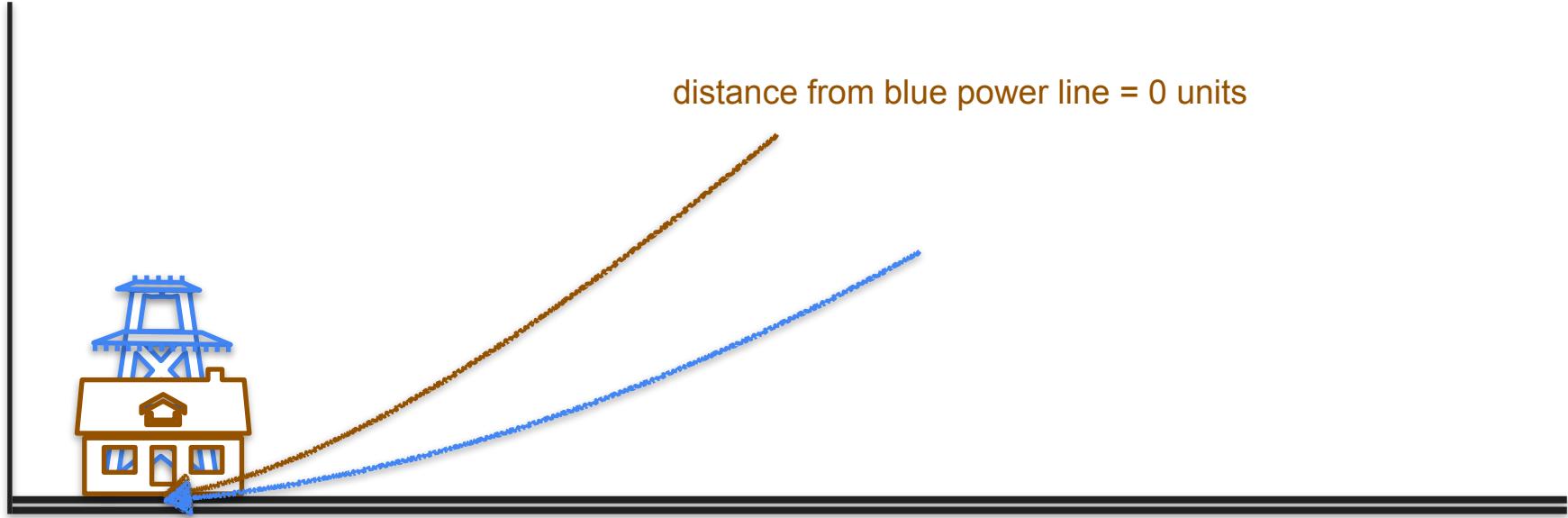
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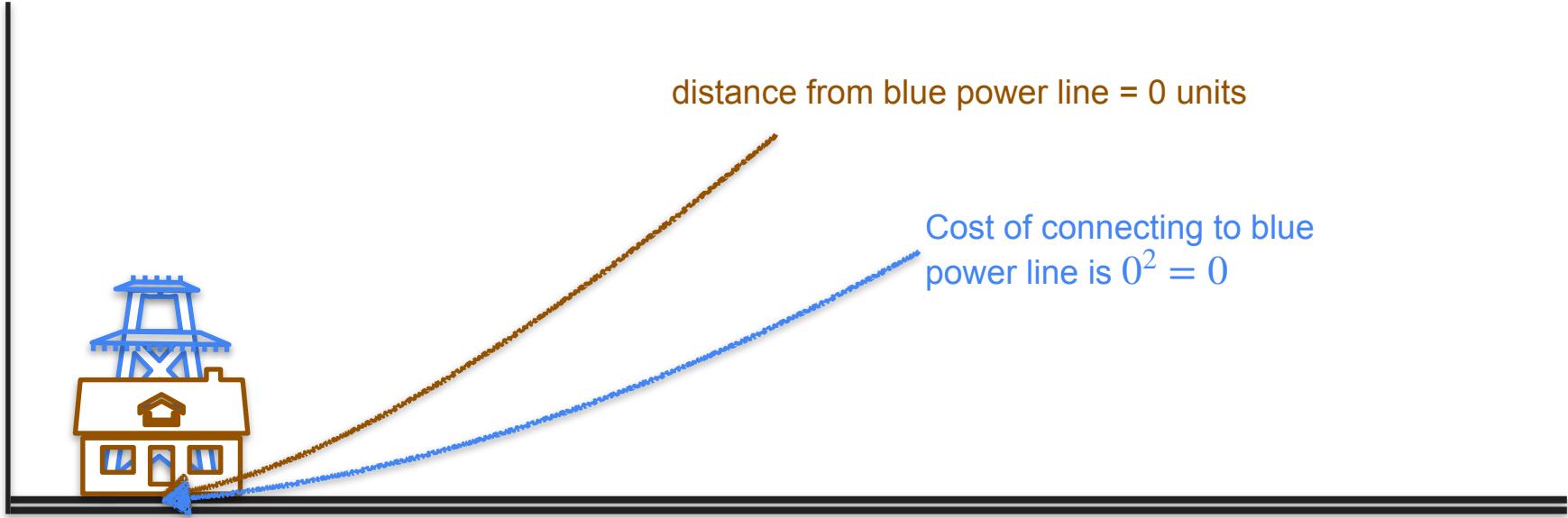
# One Power Line Problem



# One Power Line Problem



# One Power Line Problem





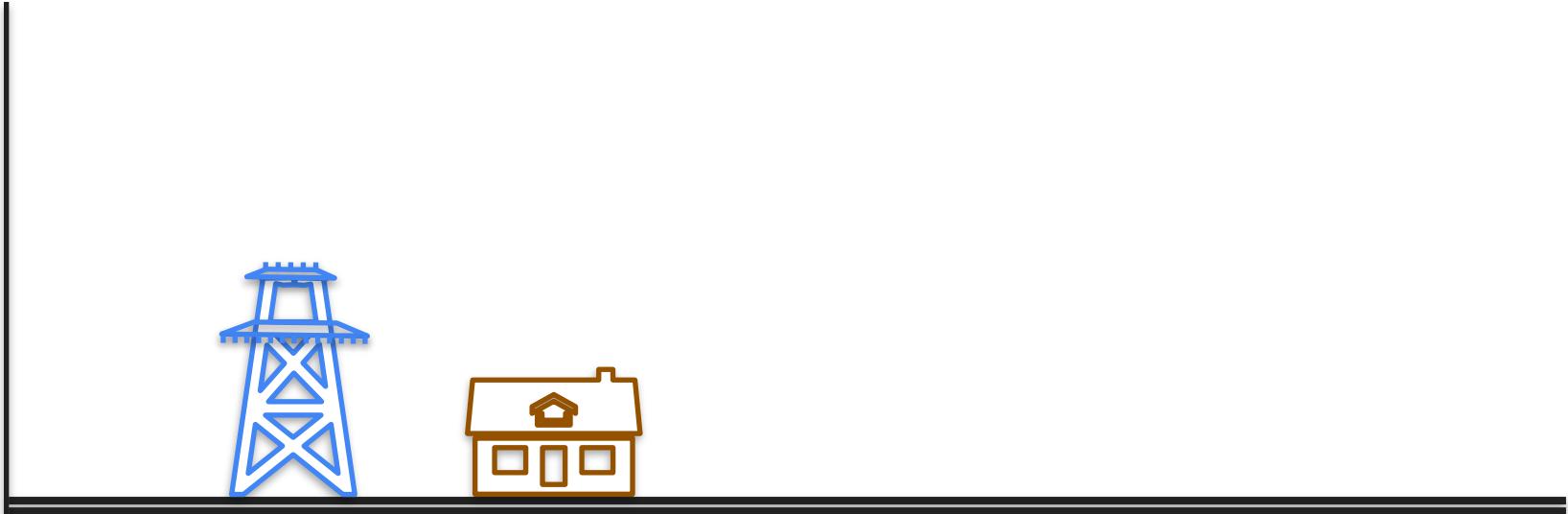
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# Derivatives and Optimization

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**Optimization of squared loss:  
The two powerline problem**

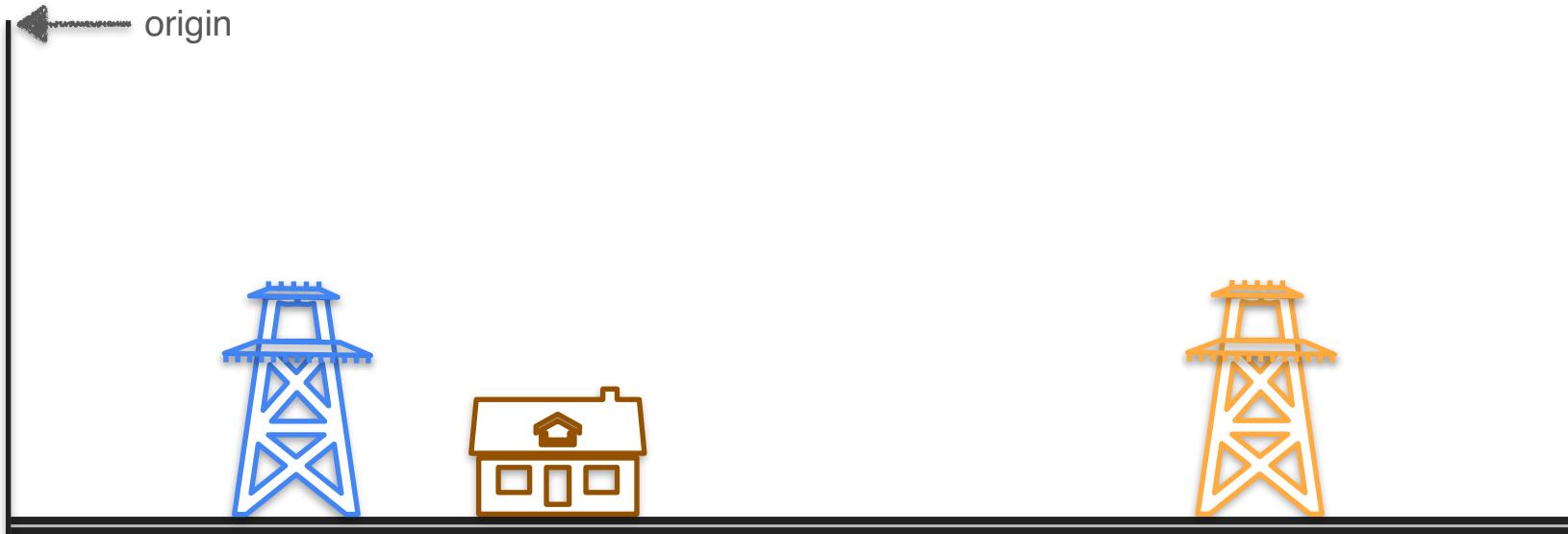
# Two Power Line Problem



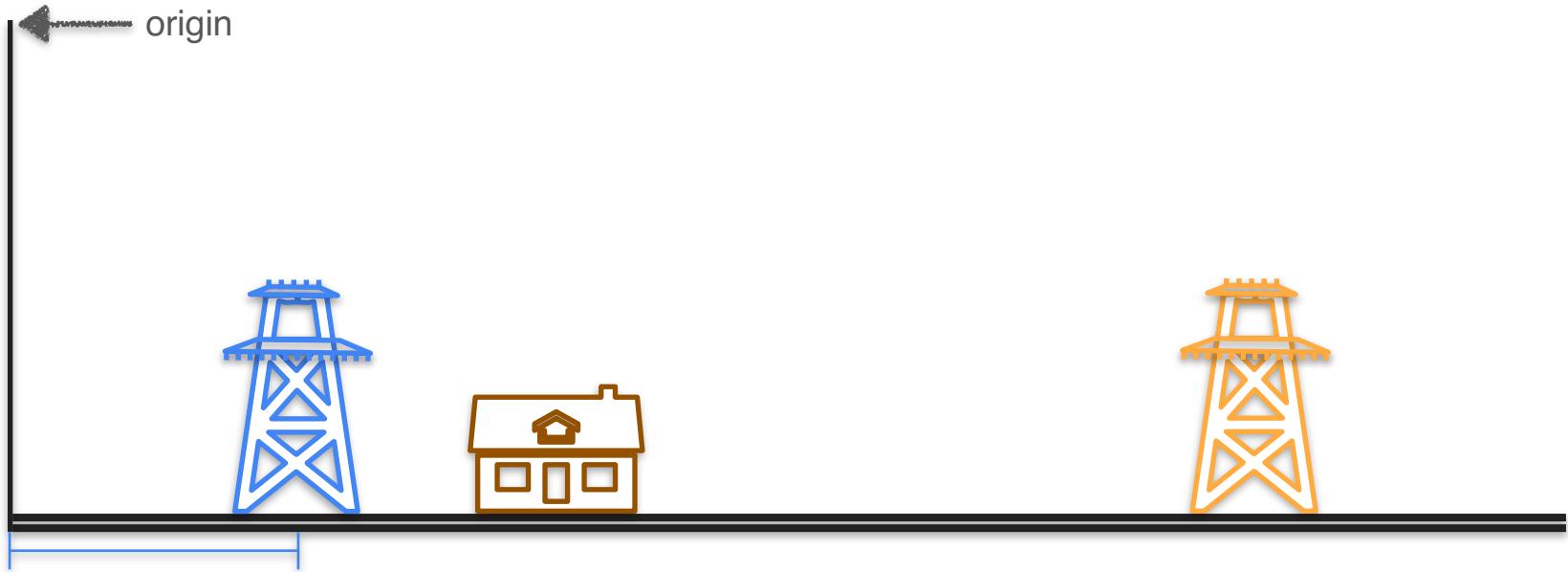
# Two Power Line Problem



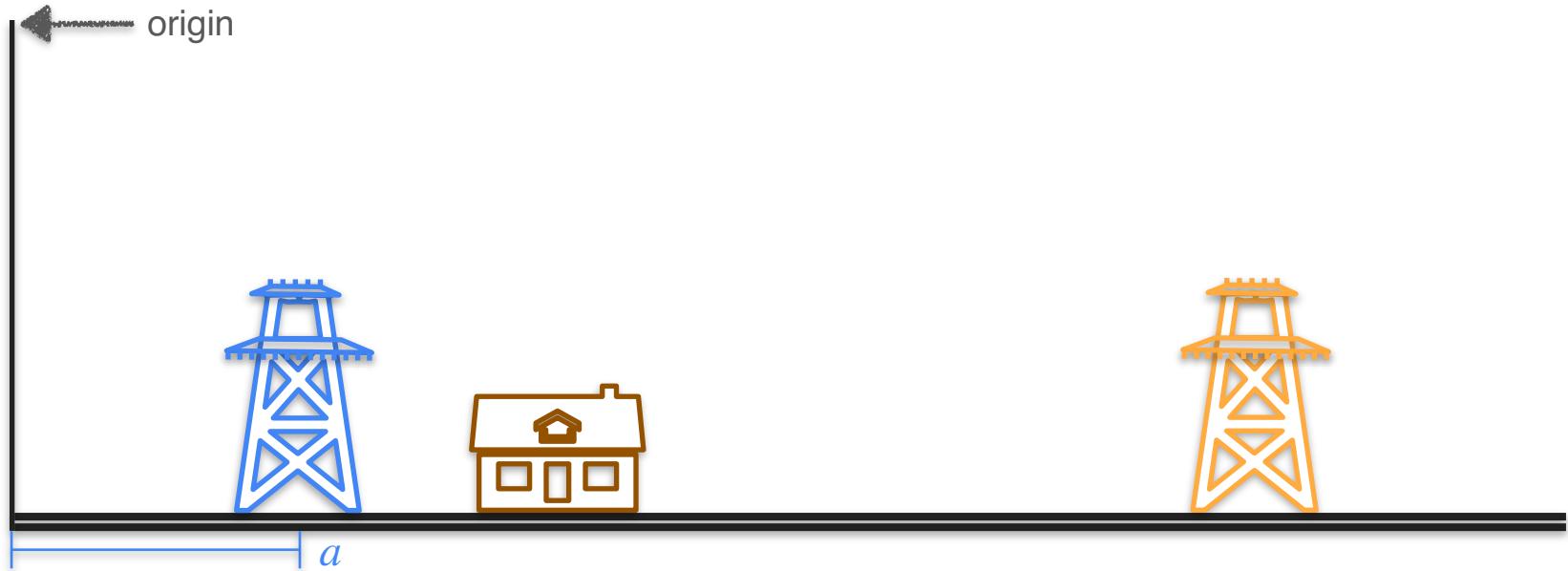
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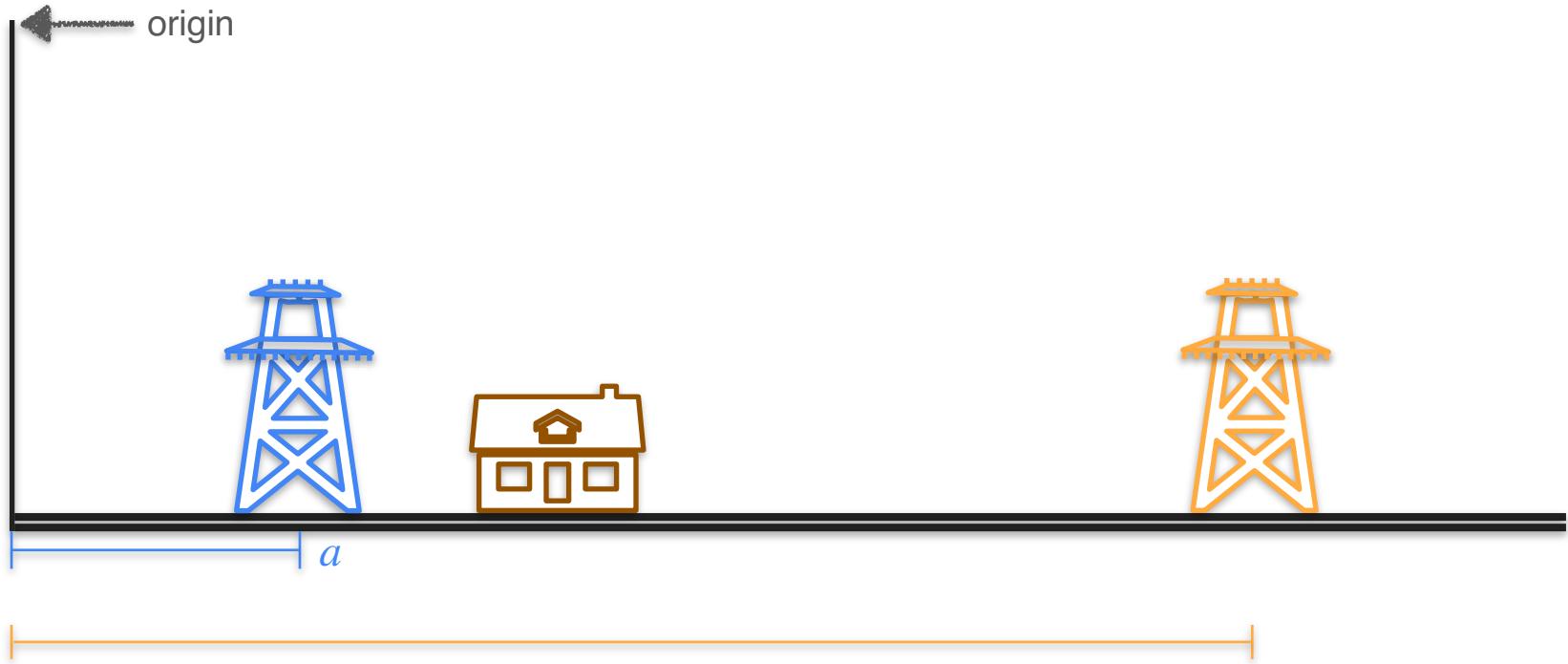
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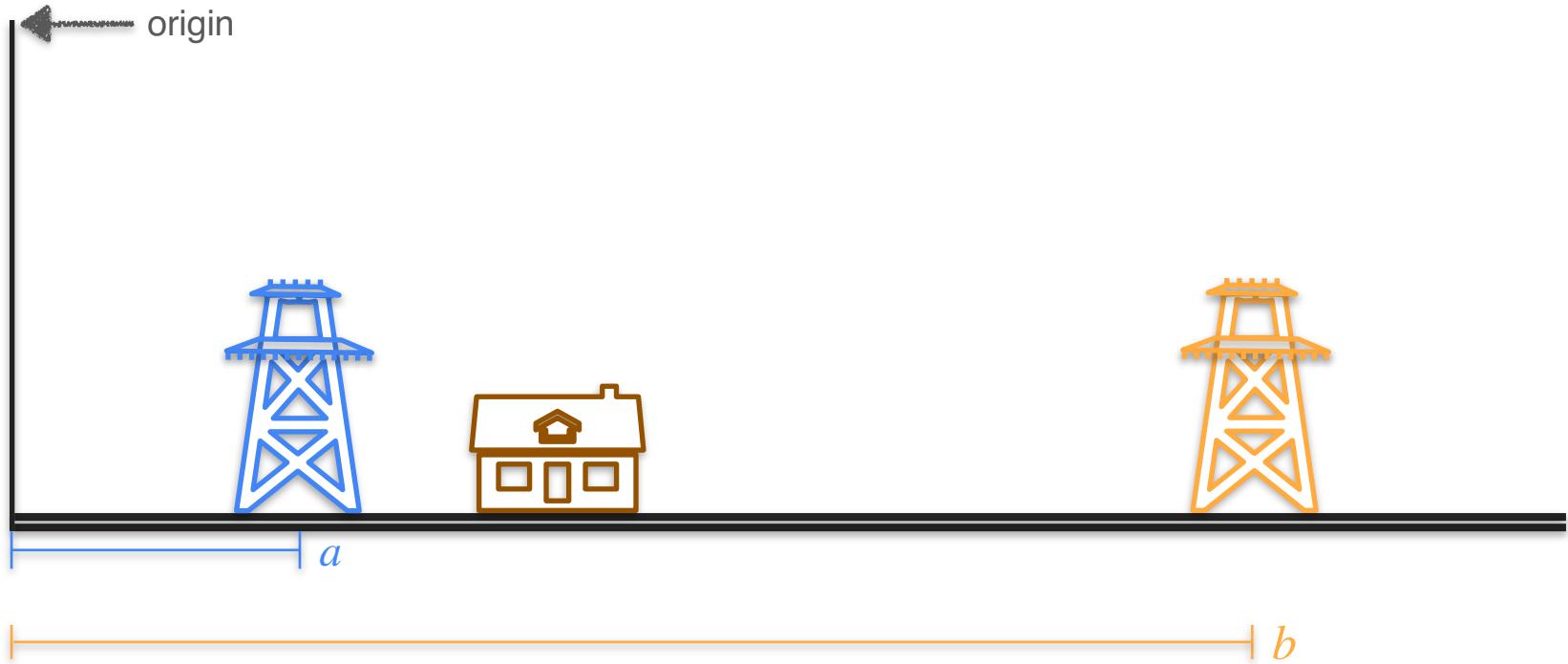
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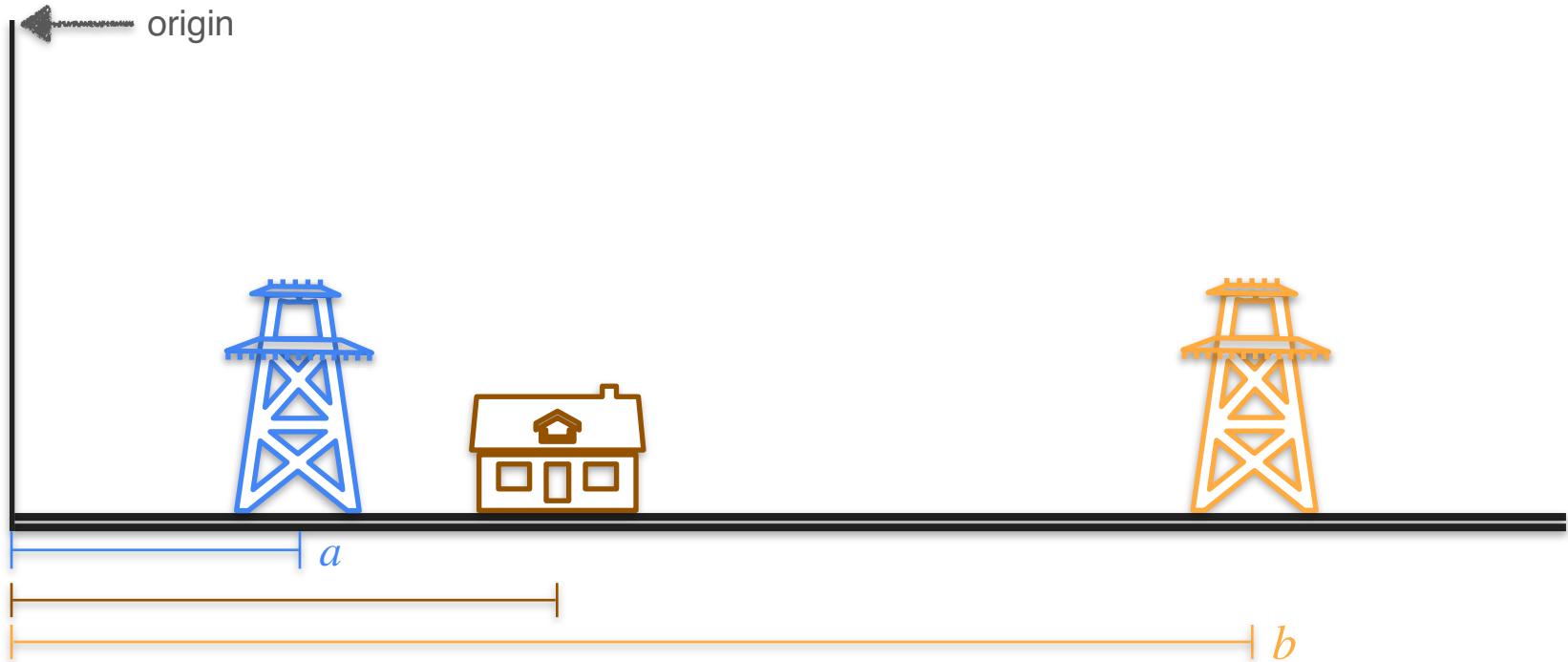
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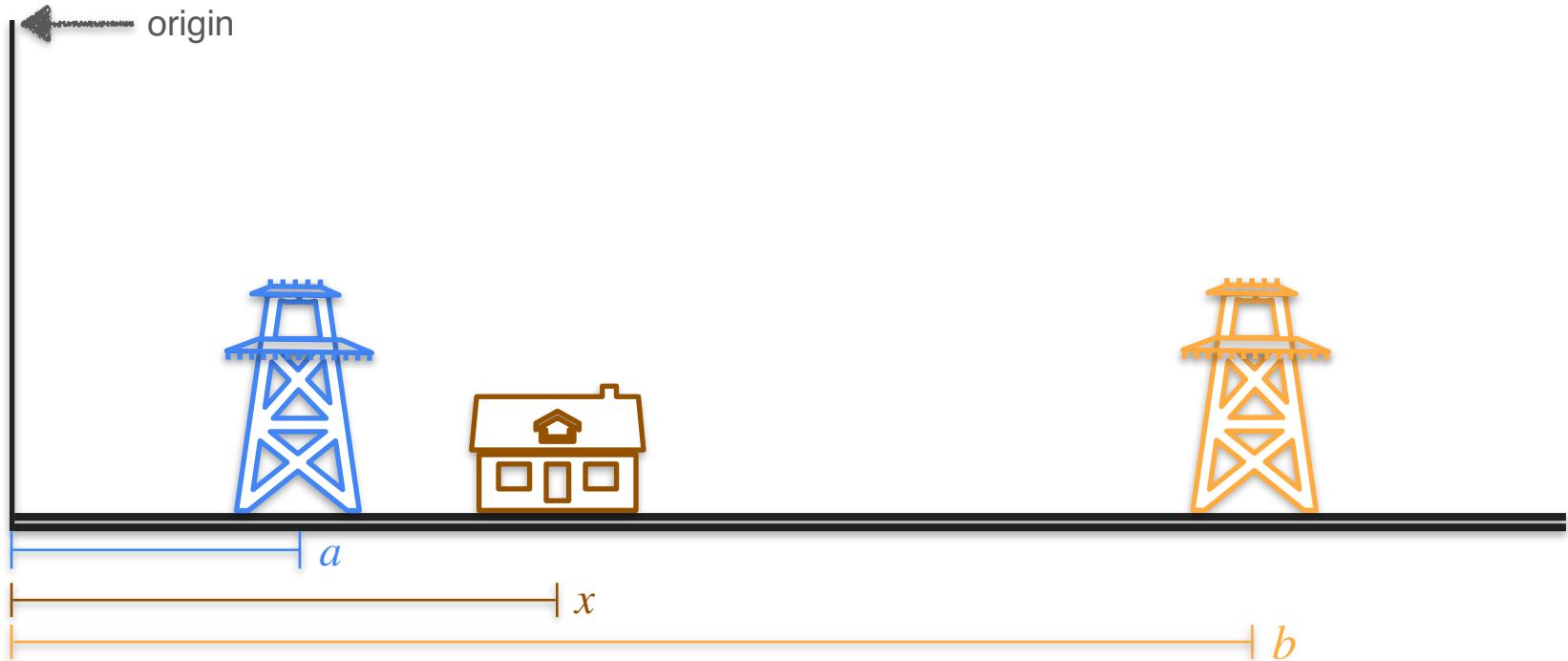
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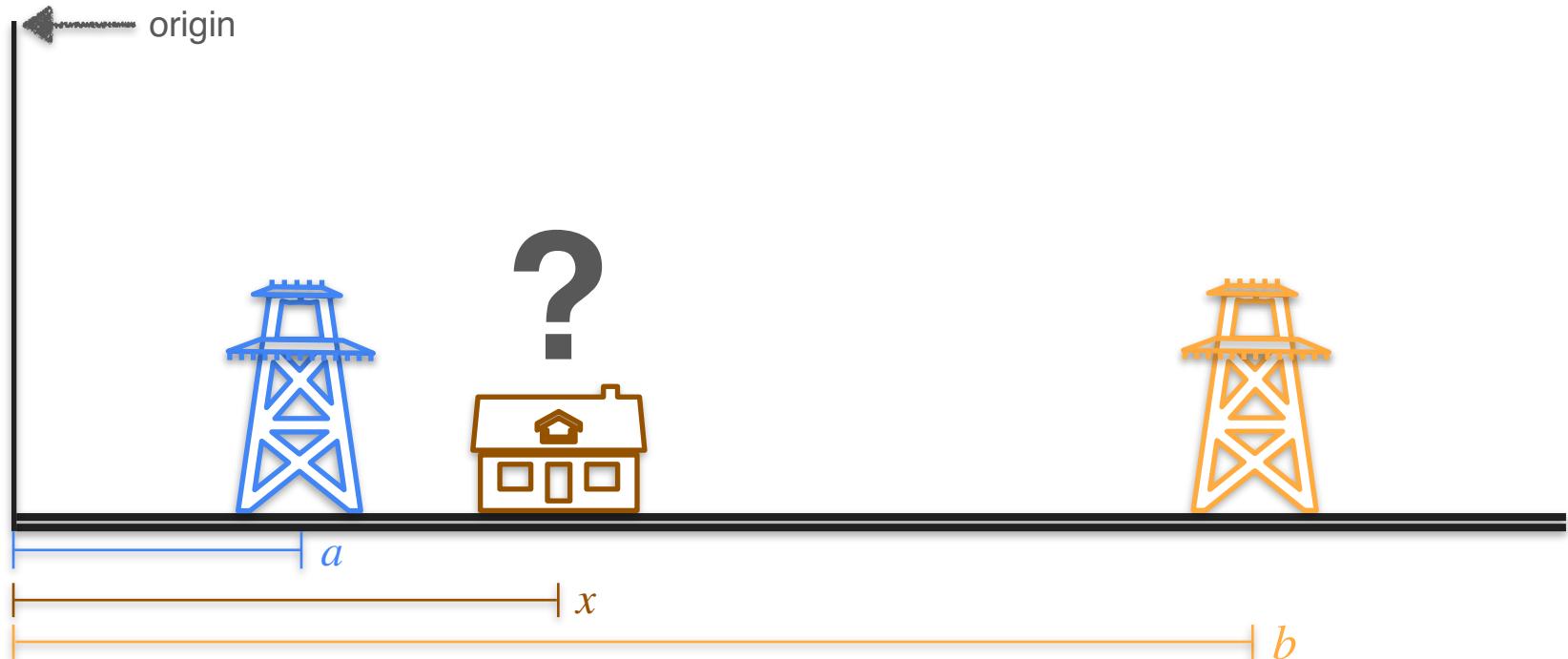
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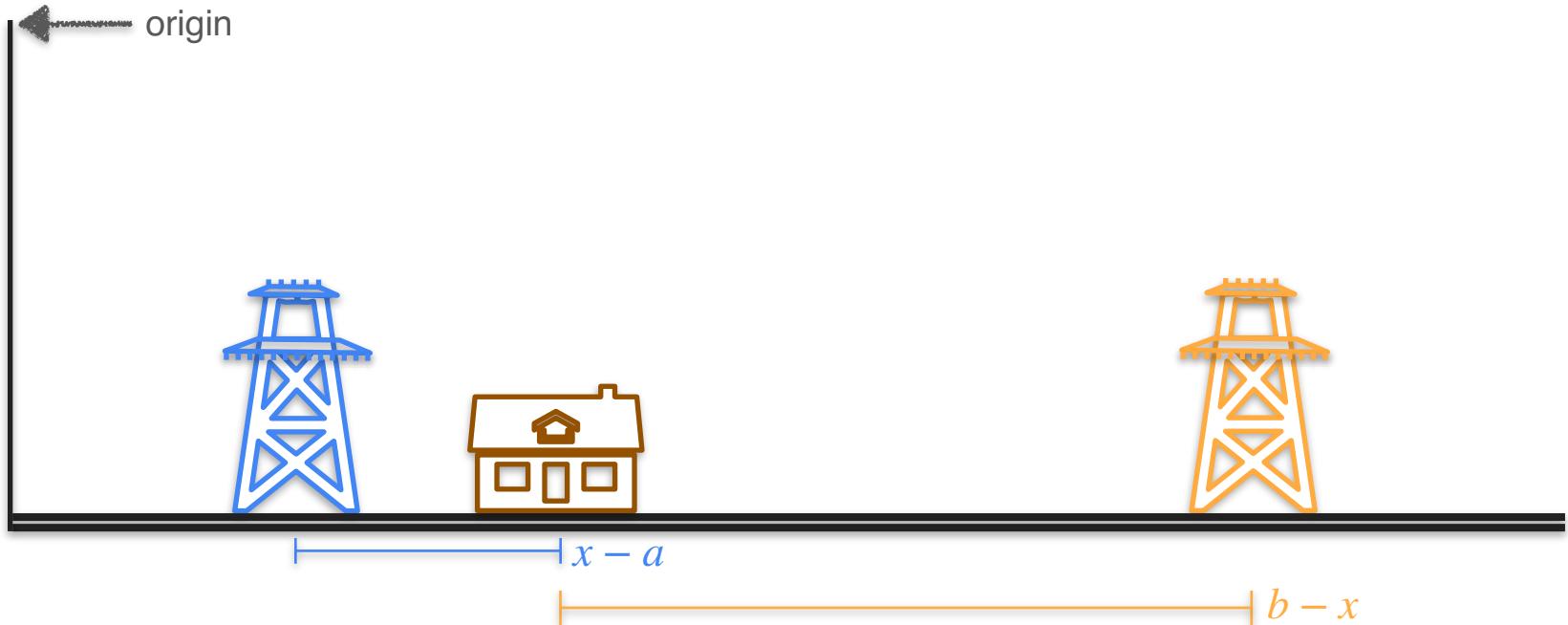
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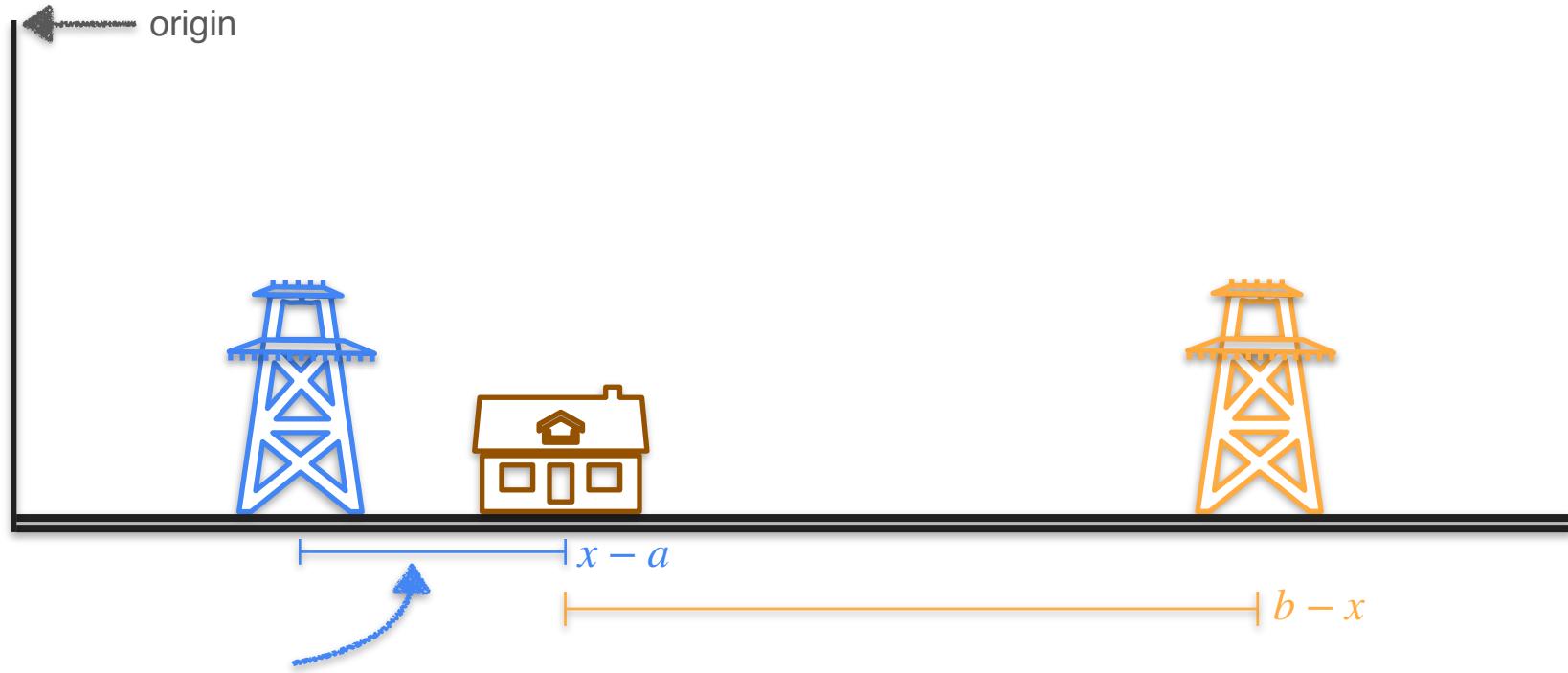
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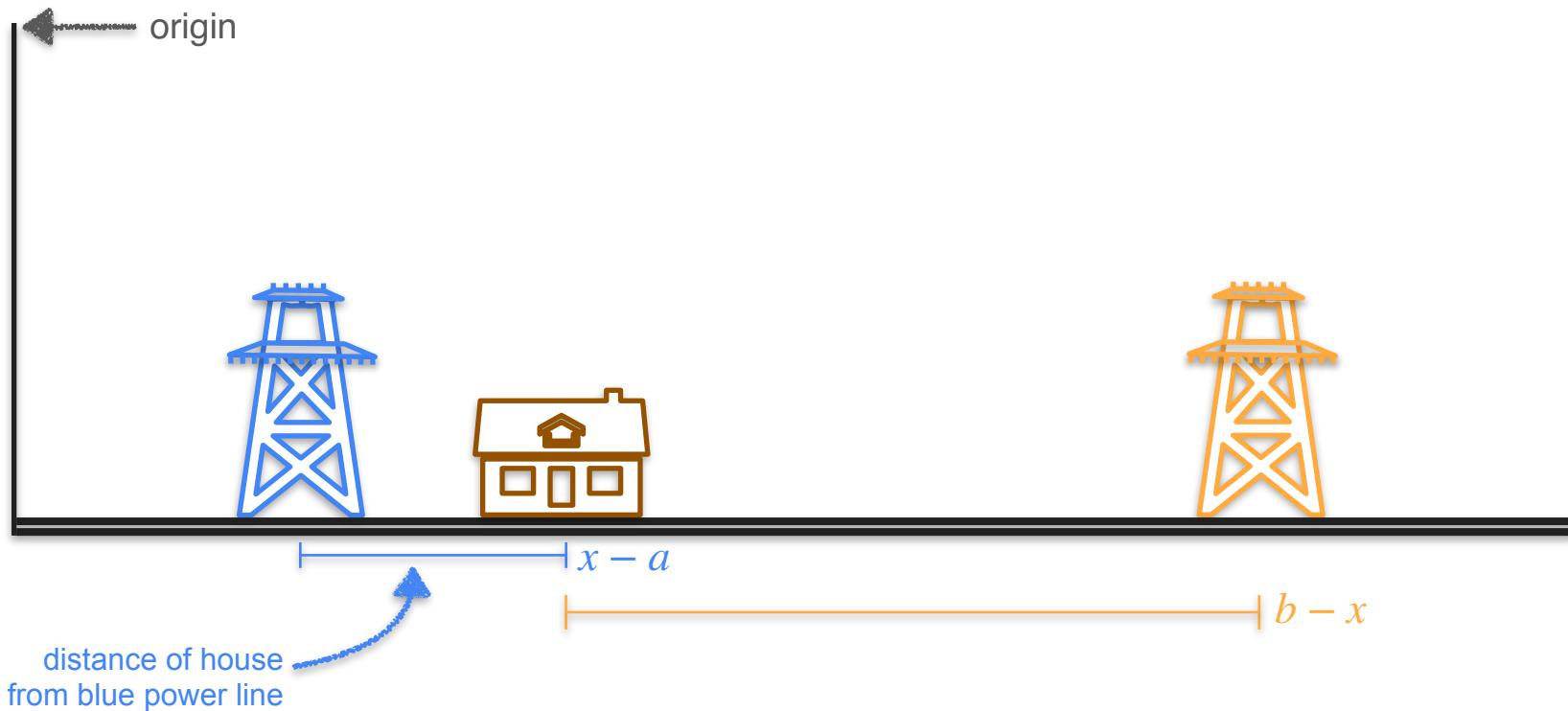
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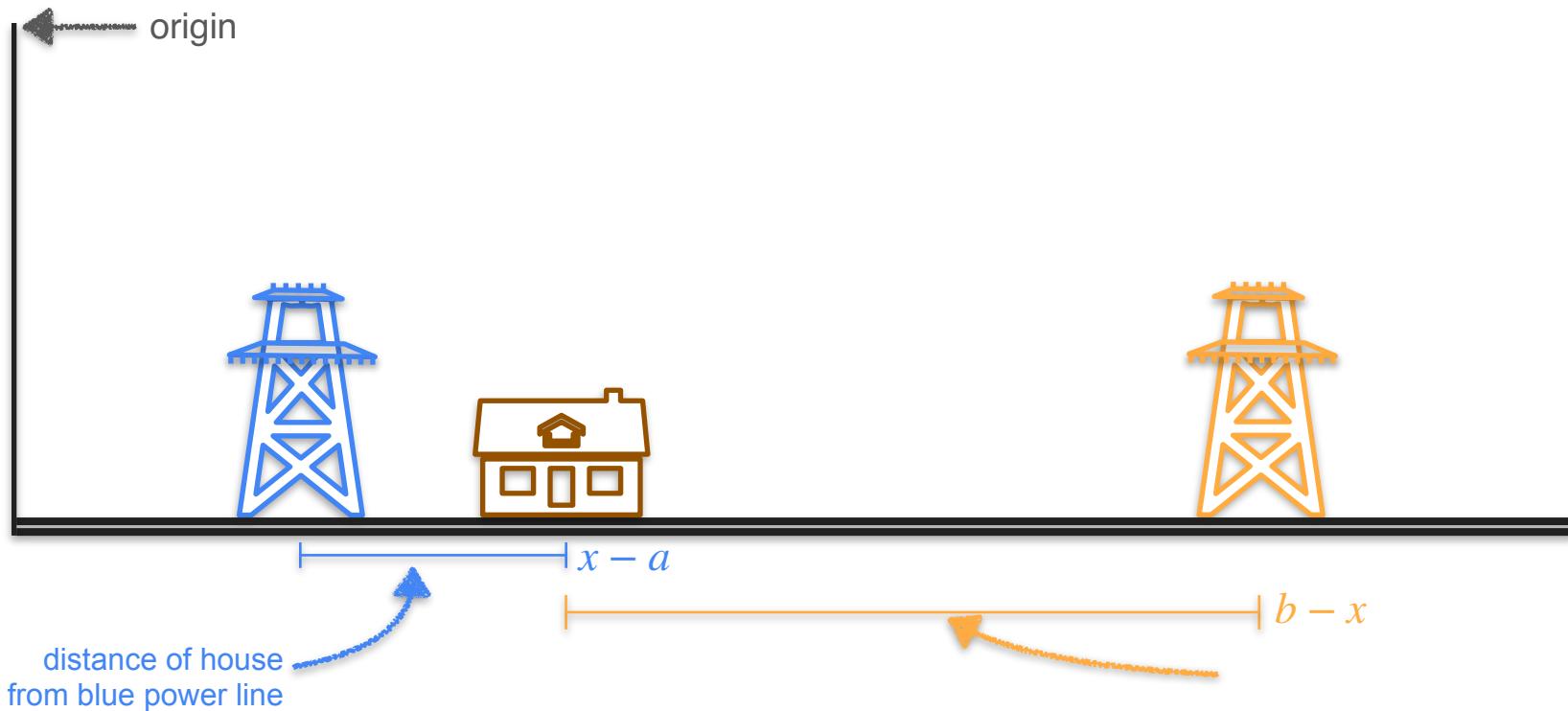
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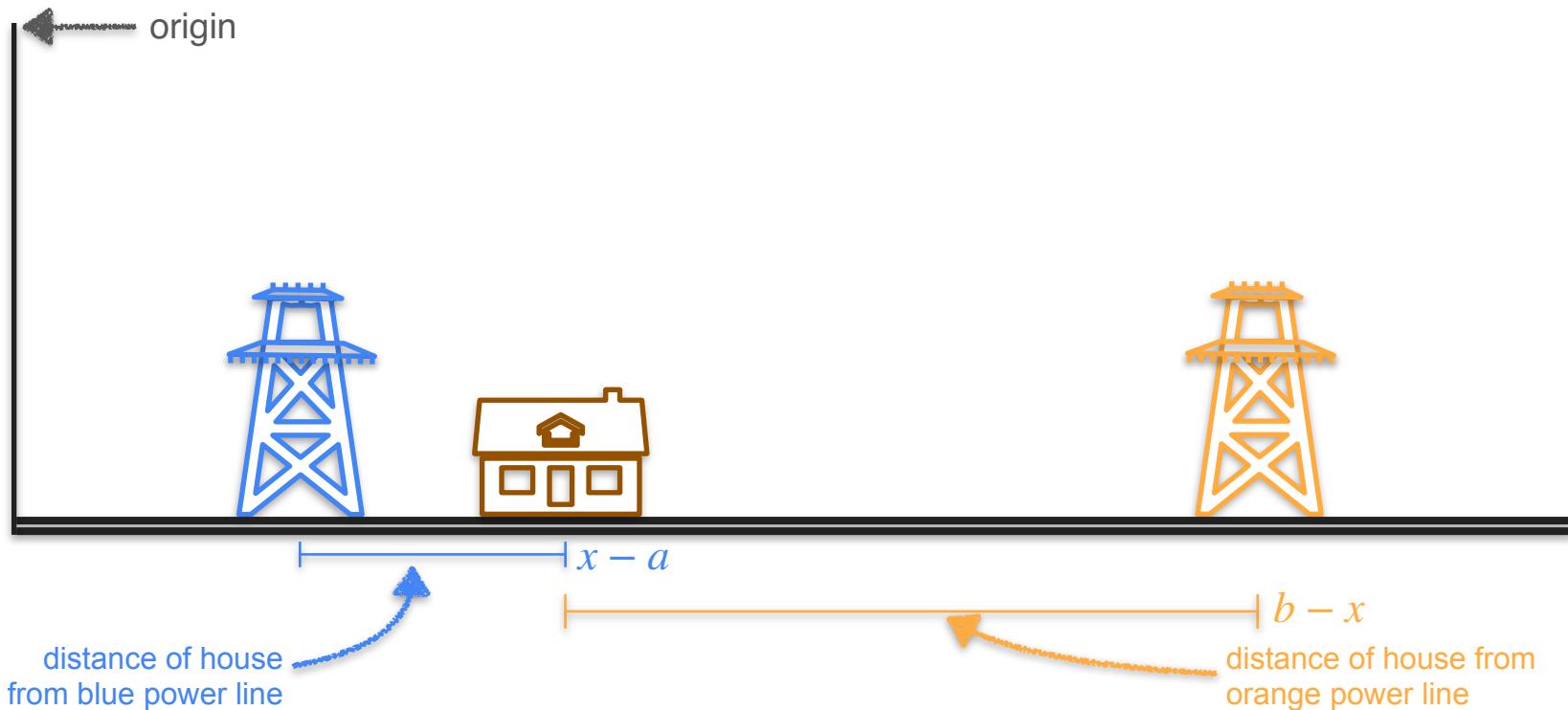
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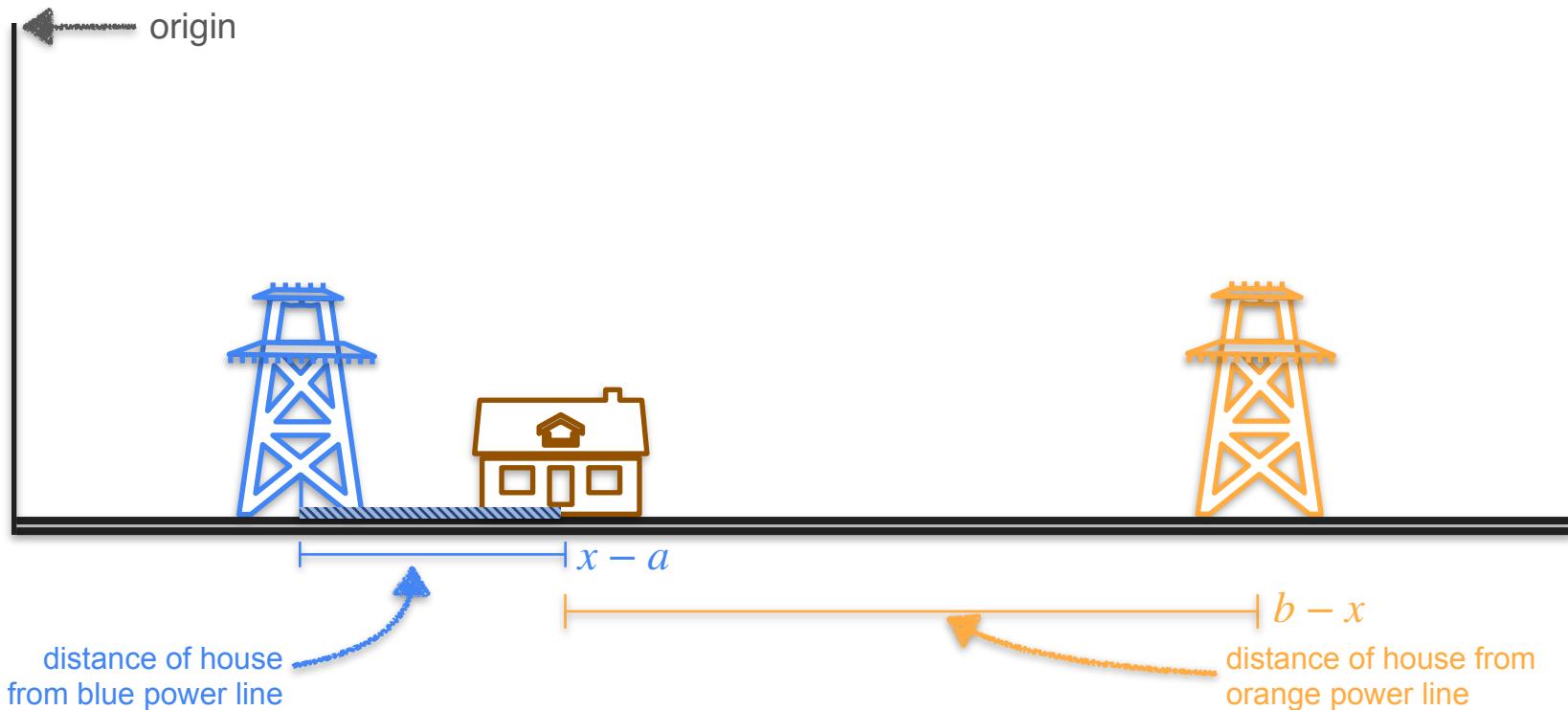
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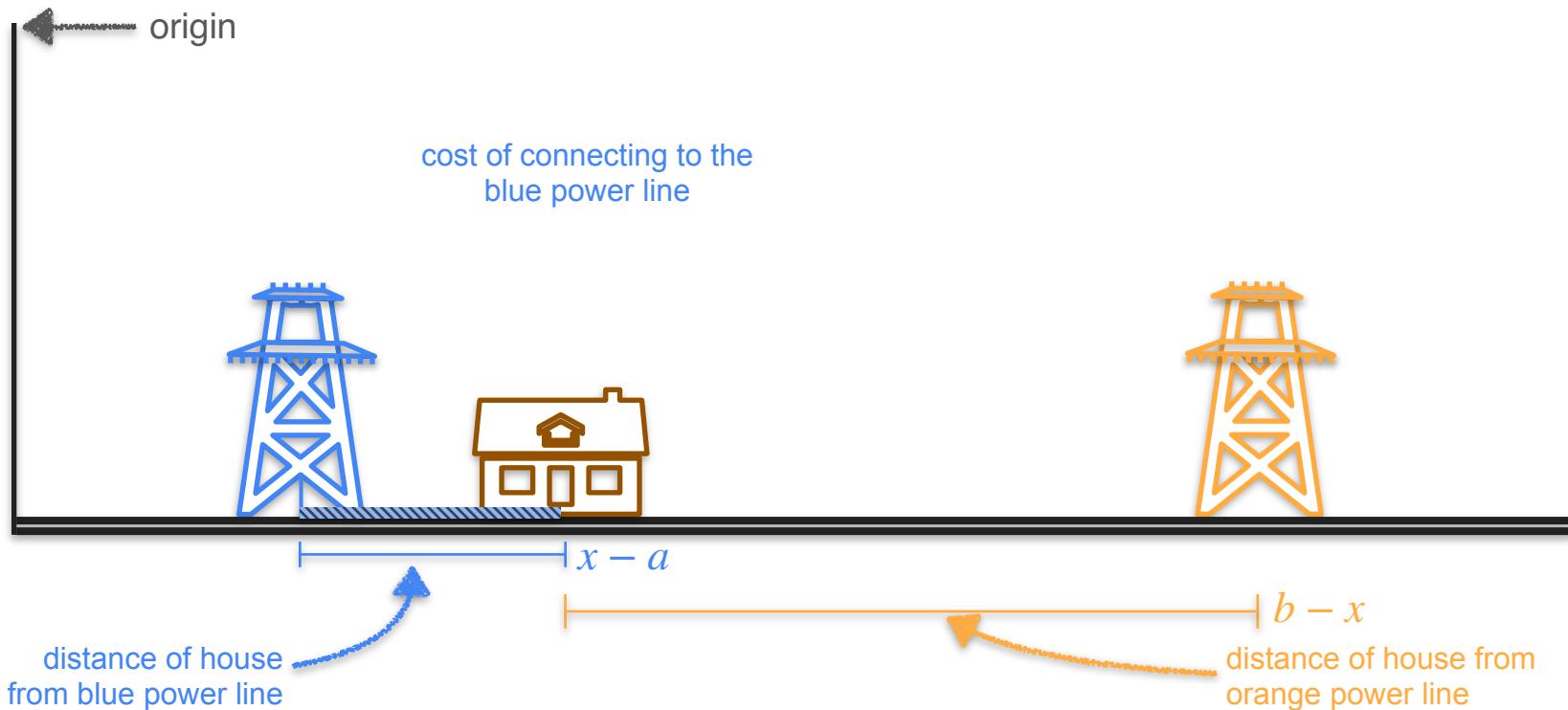
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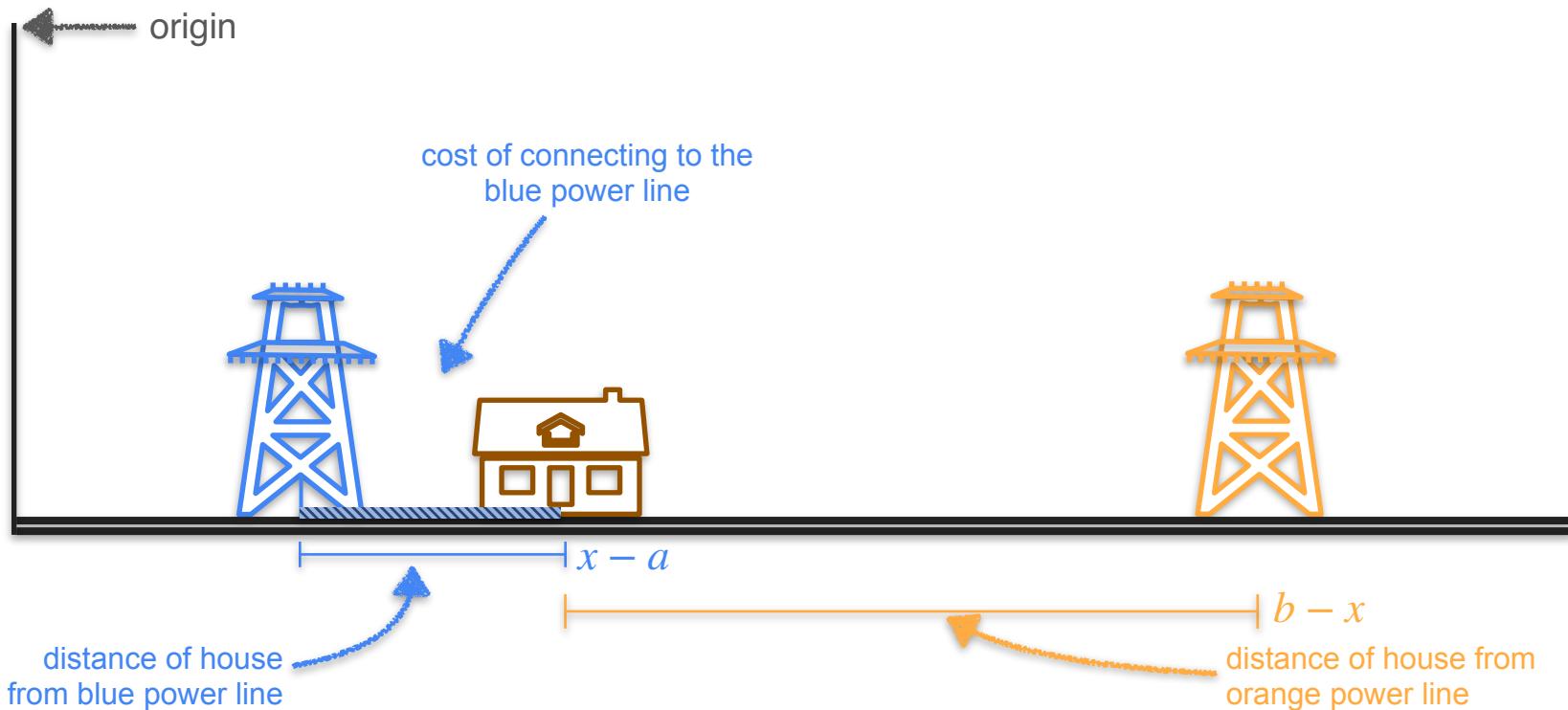
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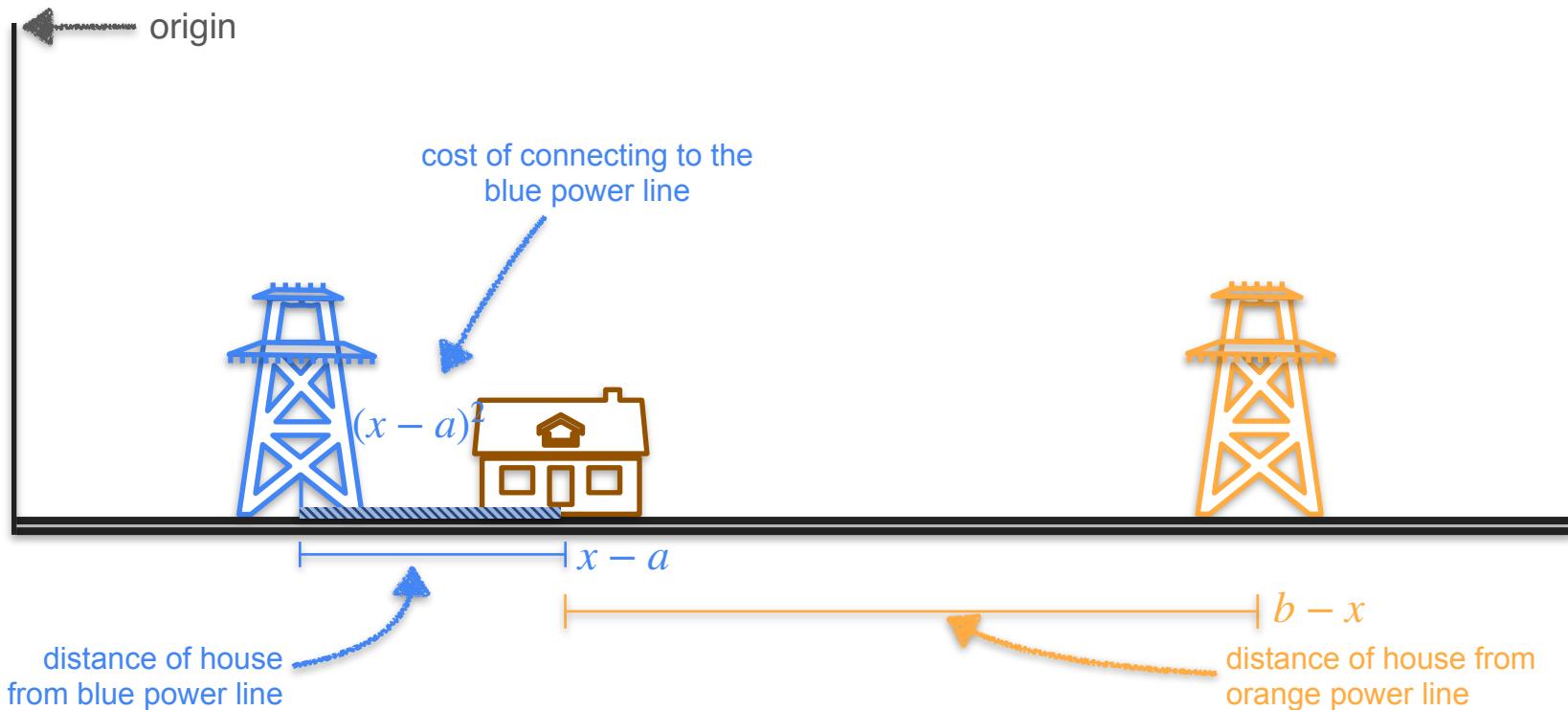
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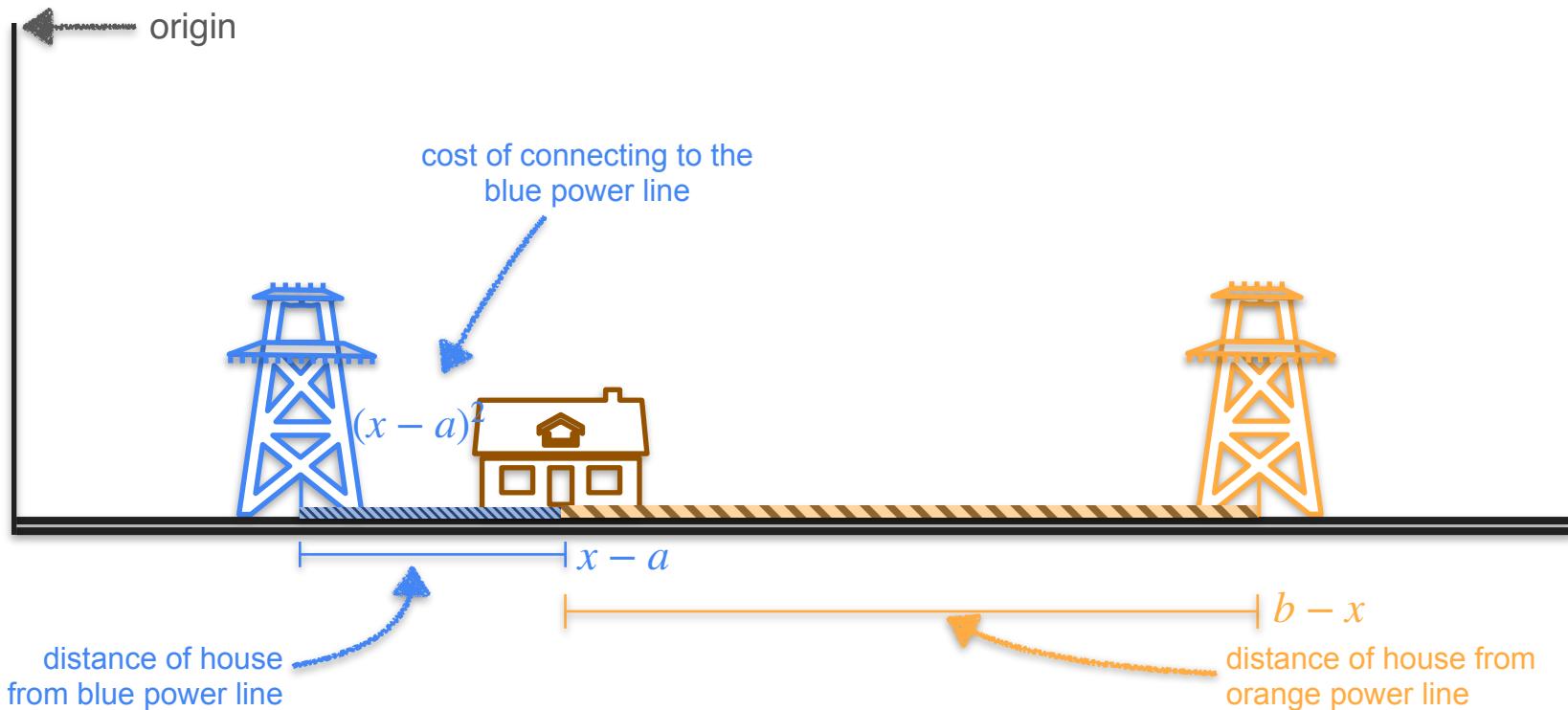
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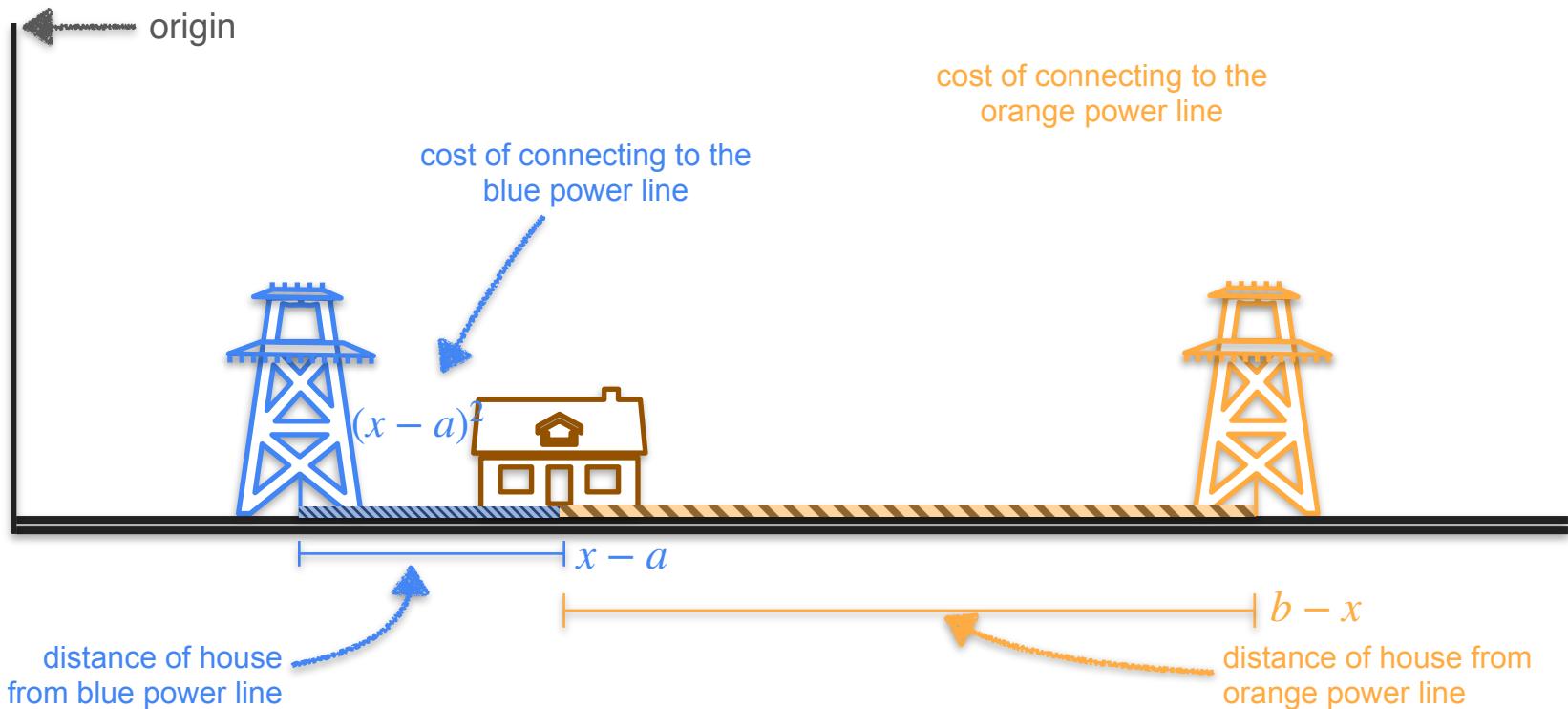
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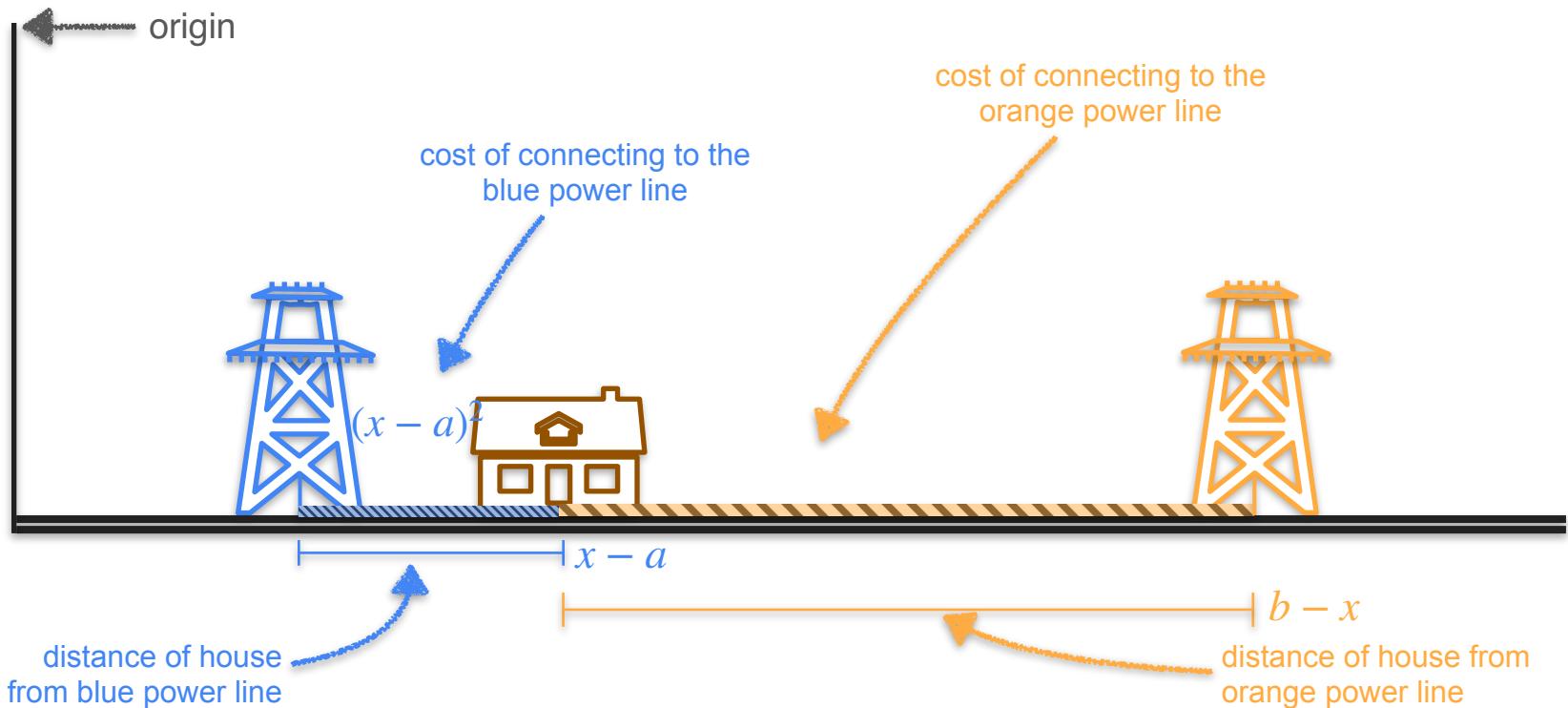
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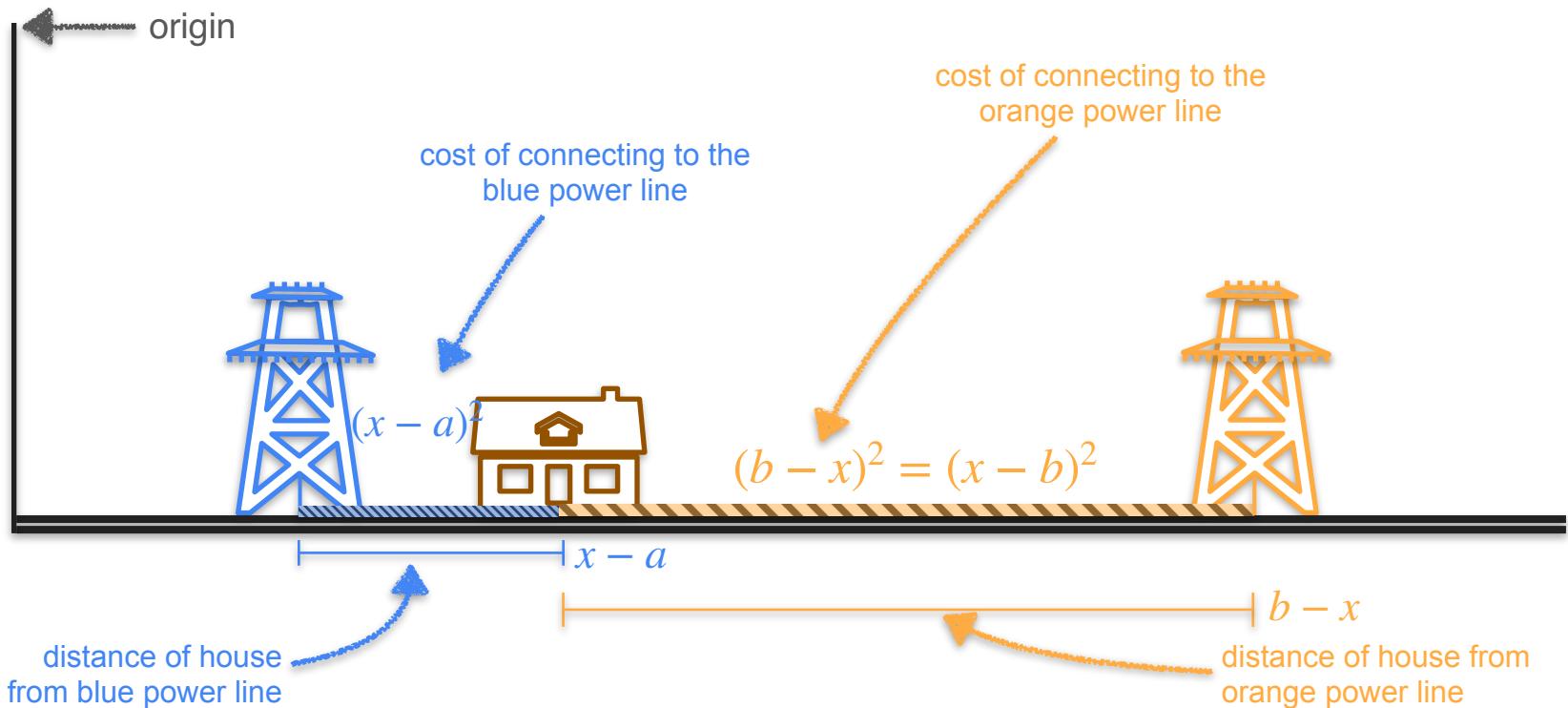
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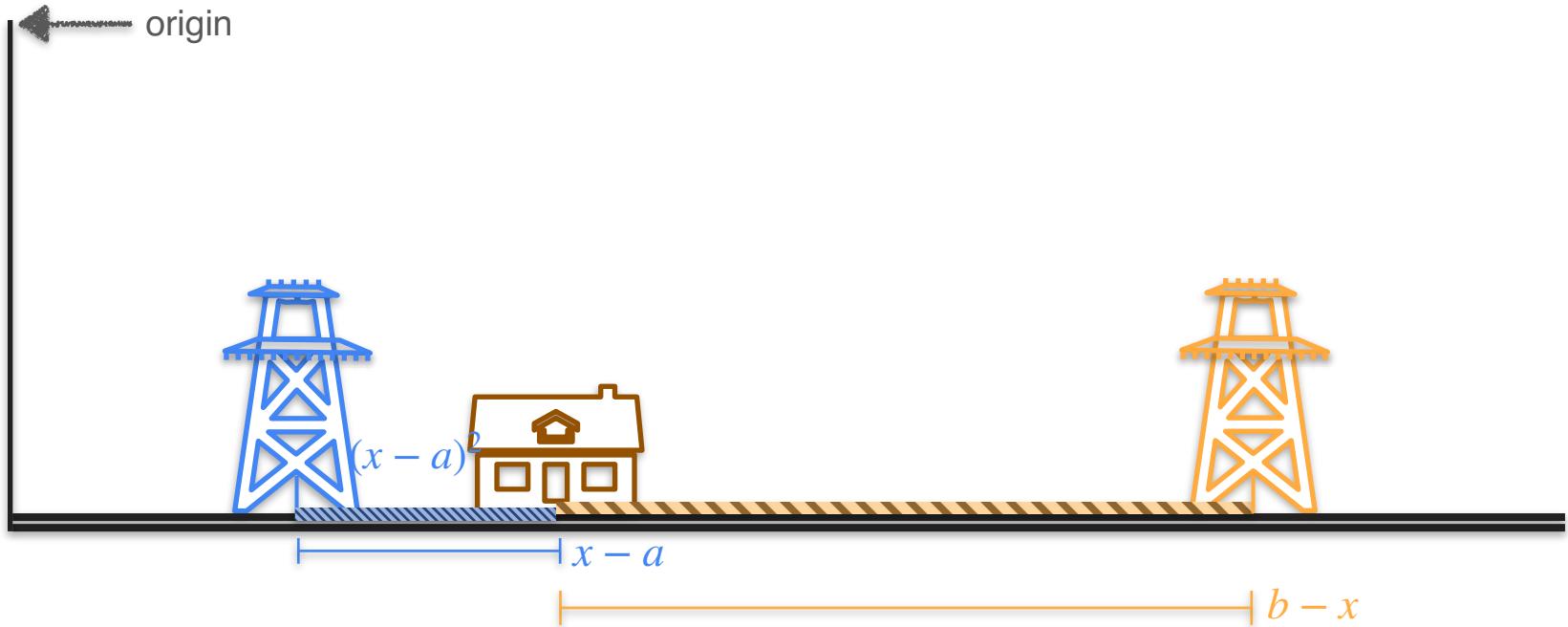
# Two Power Line Problem



# Two Power Line Problem



# Two Power Line Problem



# Two Power Line Problem

origin

Total cost of connecting to both power lines:

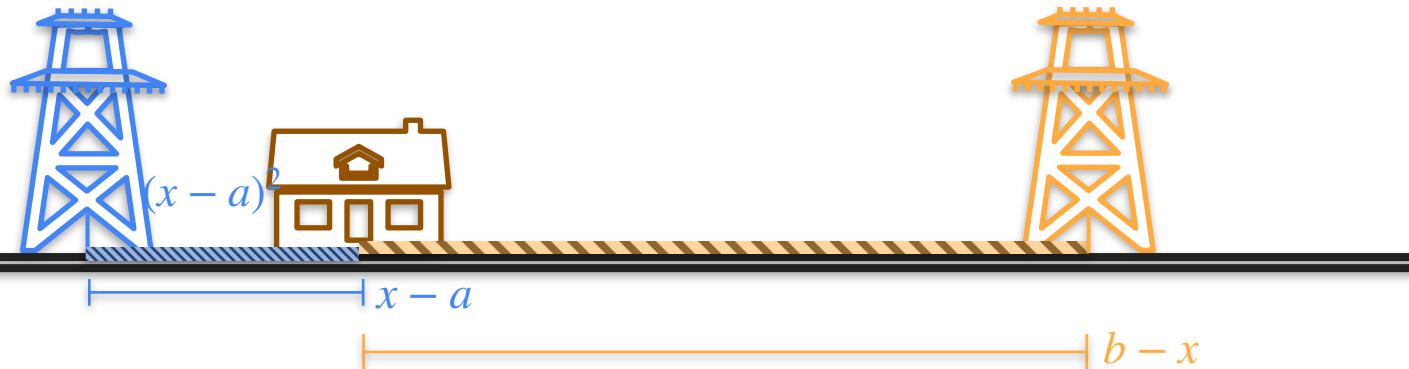


# Two Power Line Problem

origin

Total cost of connecting to both power lines:

$$(x - a)^2 + (x - b)^2$$

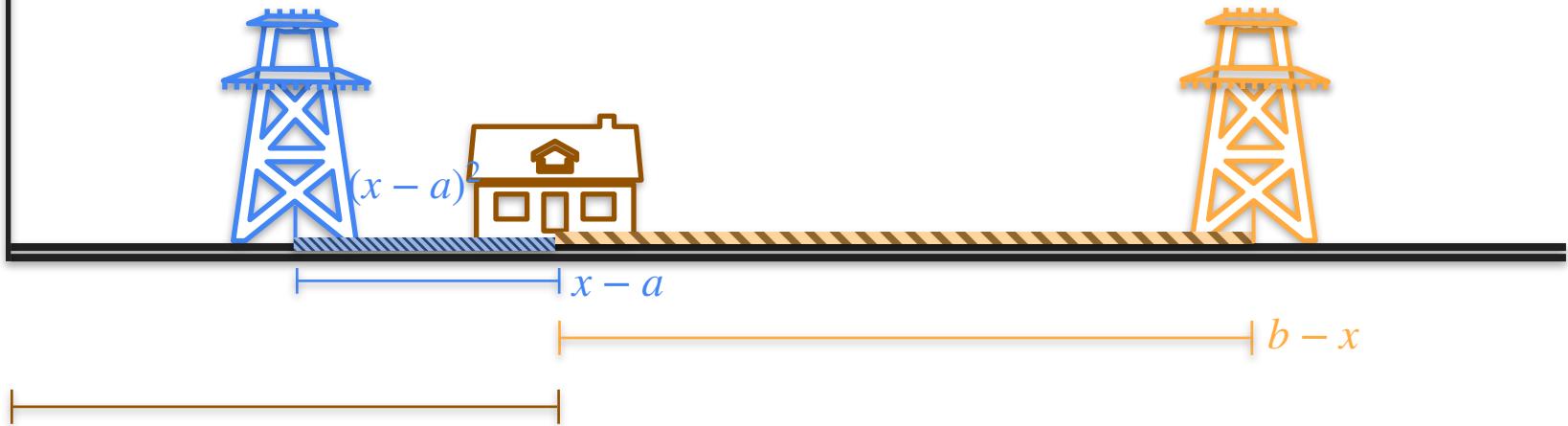


# Two Power Line Problem

origin

Total cost of connecting to both power lines:

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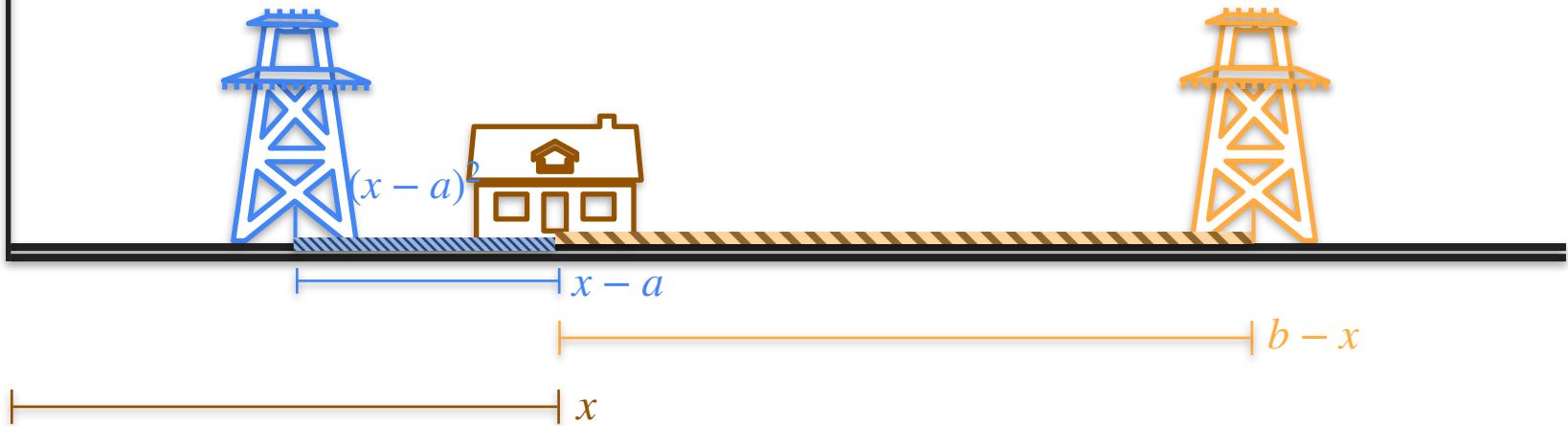


# Two Power Line Problem

origin

Total cost of connecting to both power lines:

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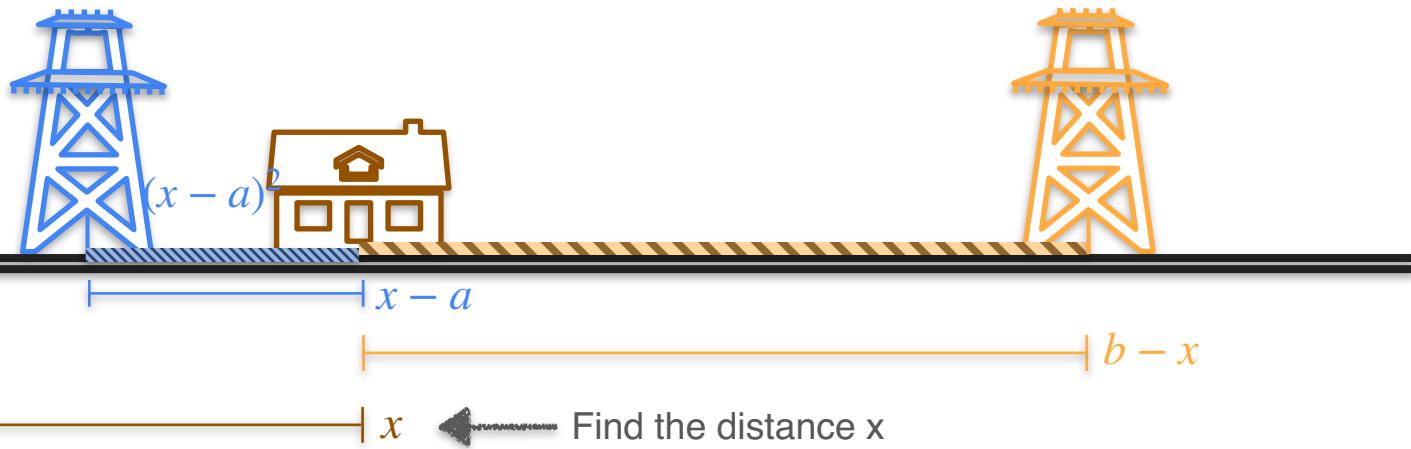


# Two Power Line Problem

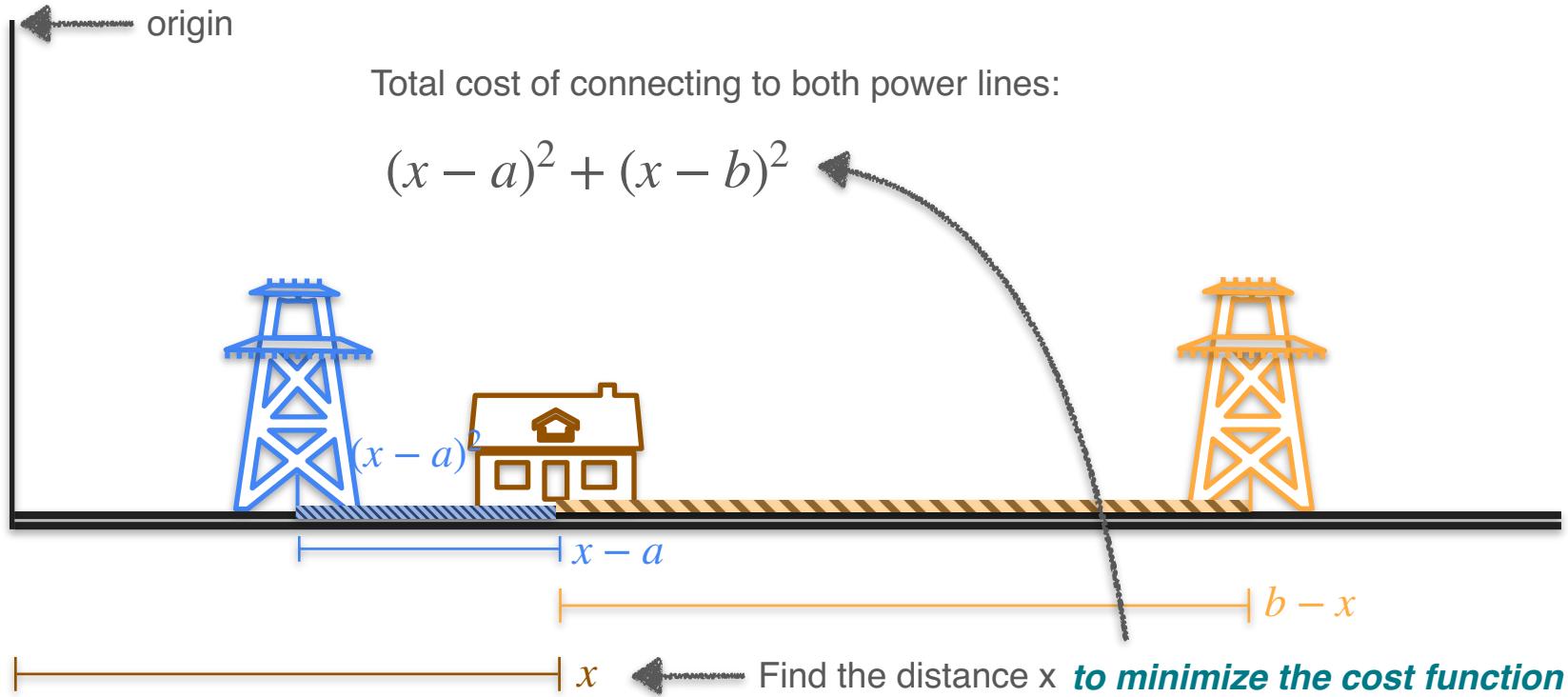
origin

Total cost of connecting to both power lines:

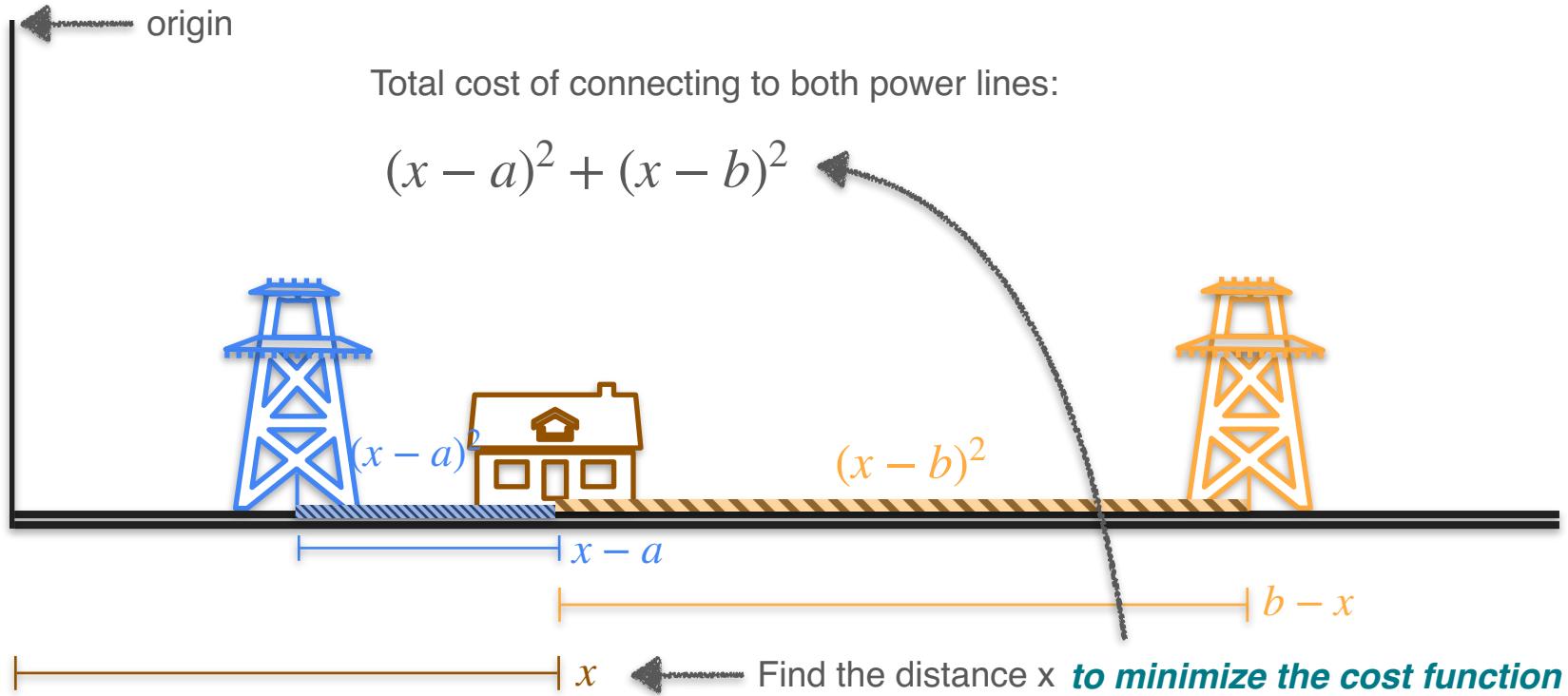
$$(x - a)^2 + (x - b)^2$$



# Two Power Line Problem



# Two Power Line Problem

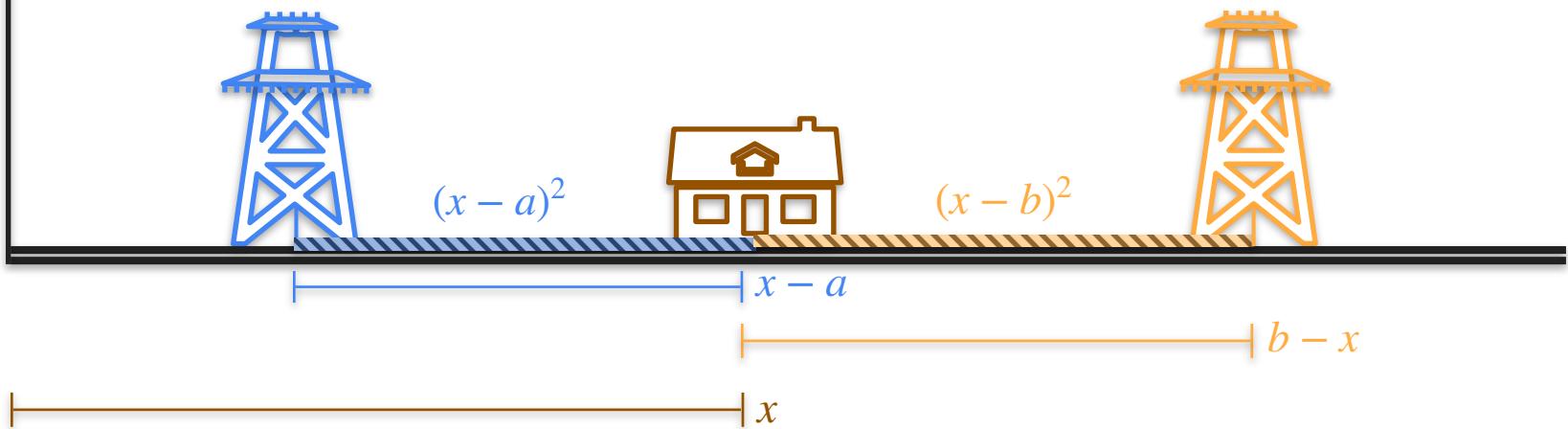


# Two Power Line Problem

origin

Total cost of connecting to both power lines:

$$(x - a)^2 + (x - b)^2$$

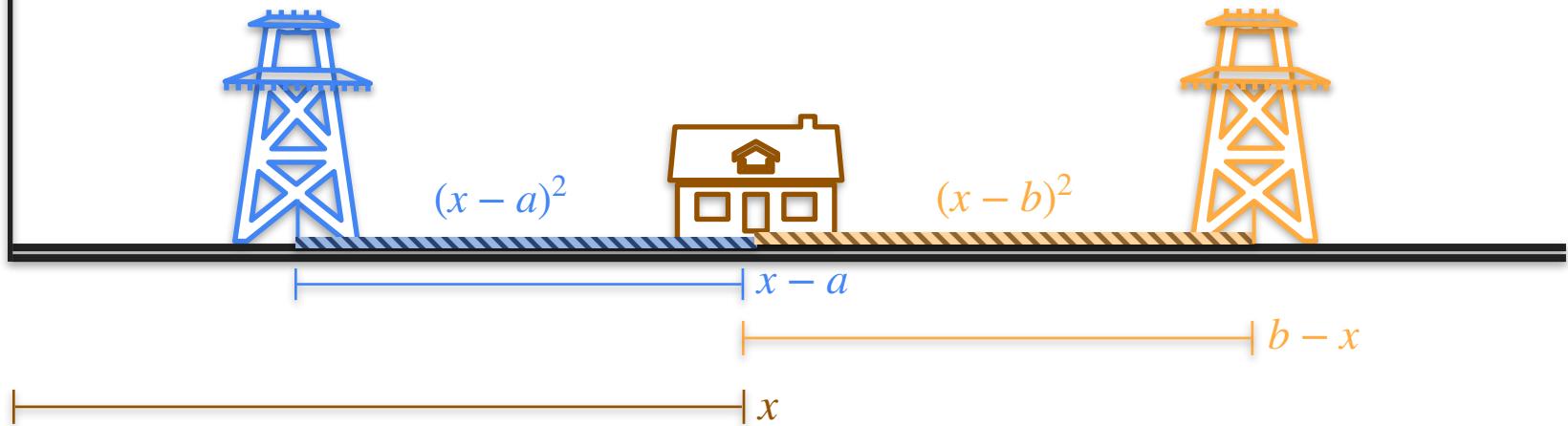


# Two Power Line Problem

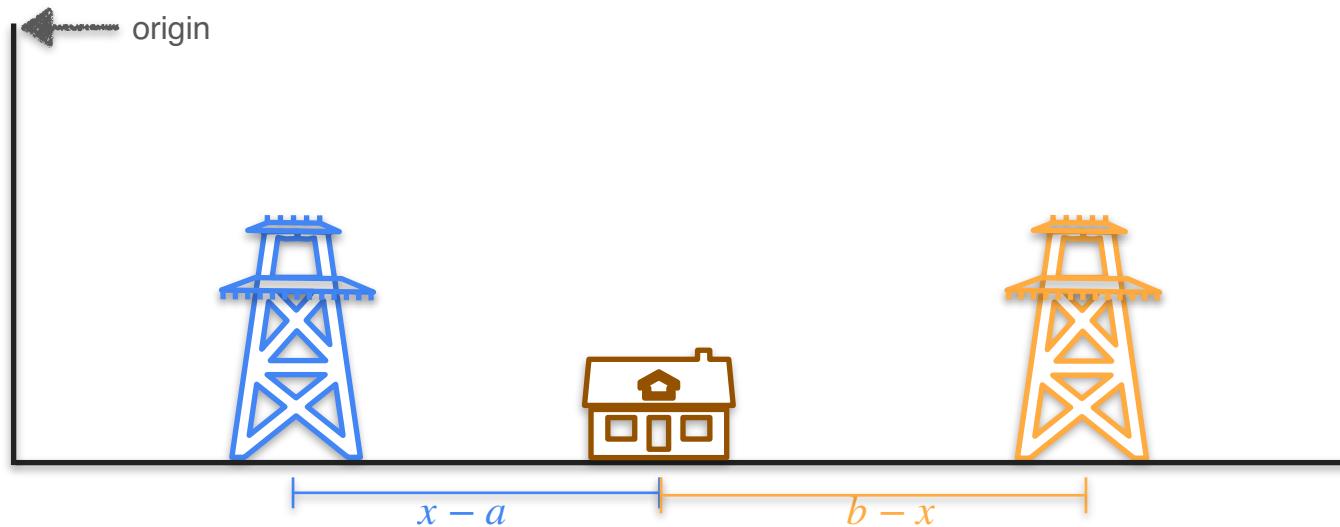
origin

Total cost of connecting to both power lines:

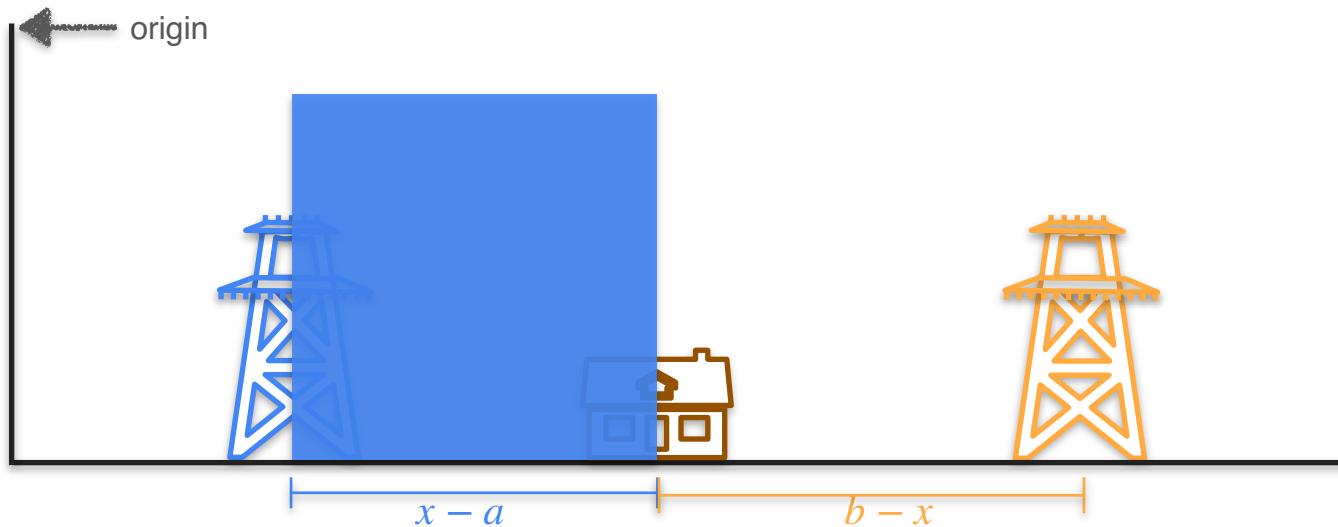
$$(x - a)^2 + (x - b)^2$$



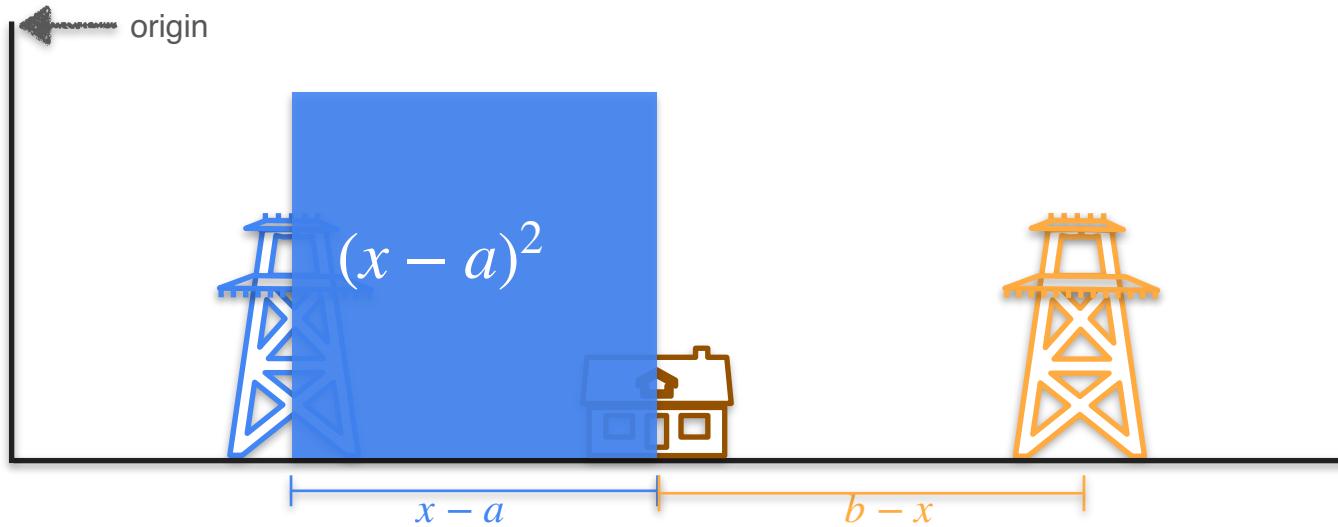
# Two Power Line Problem - Square Analogy



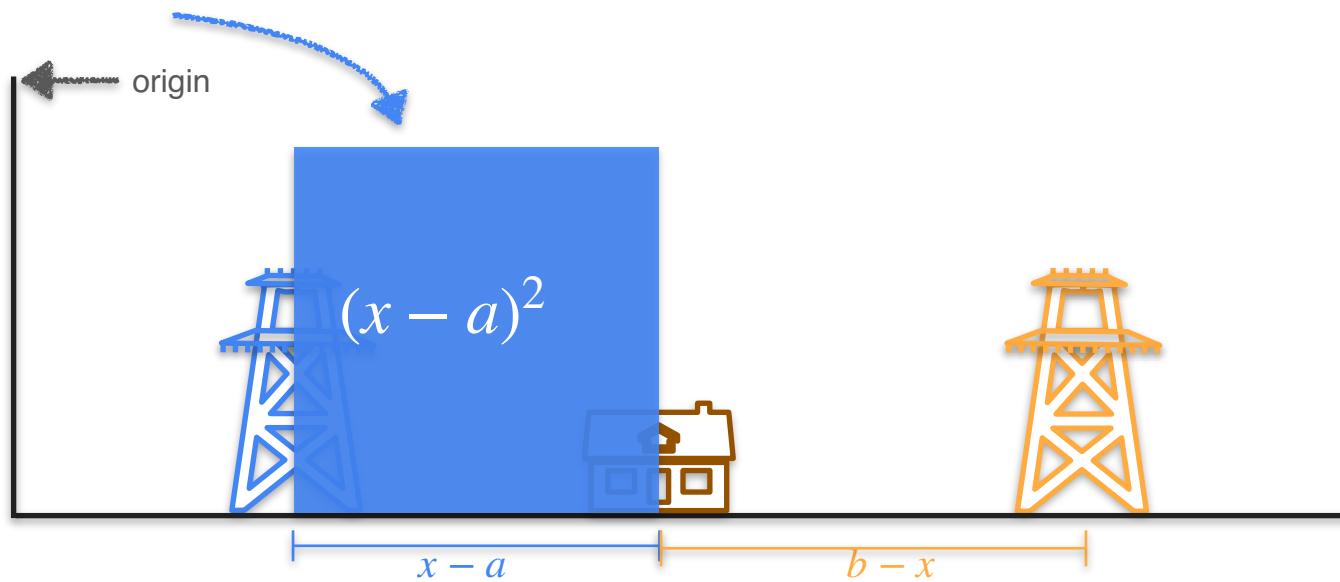
# Two Power Line Problem - Square Analogy



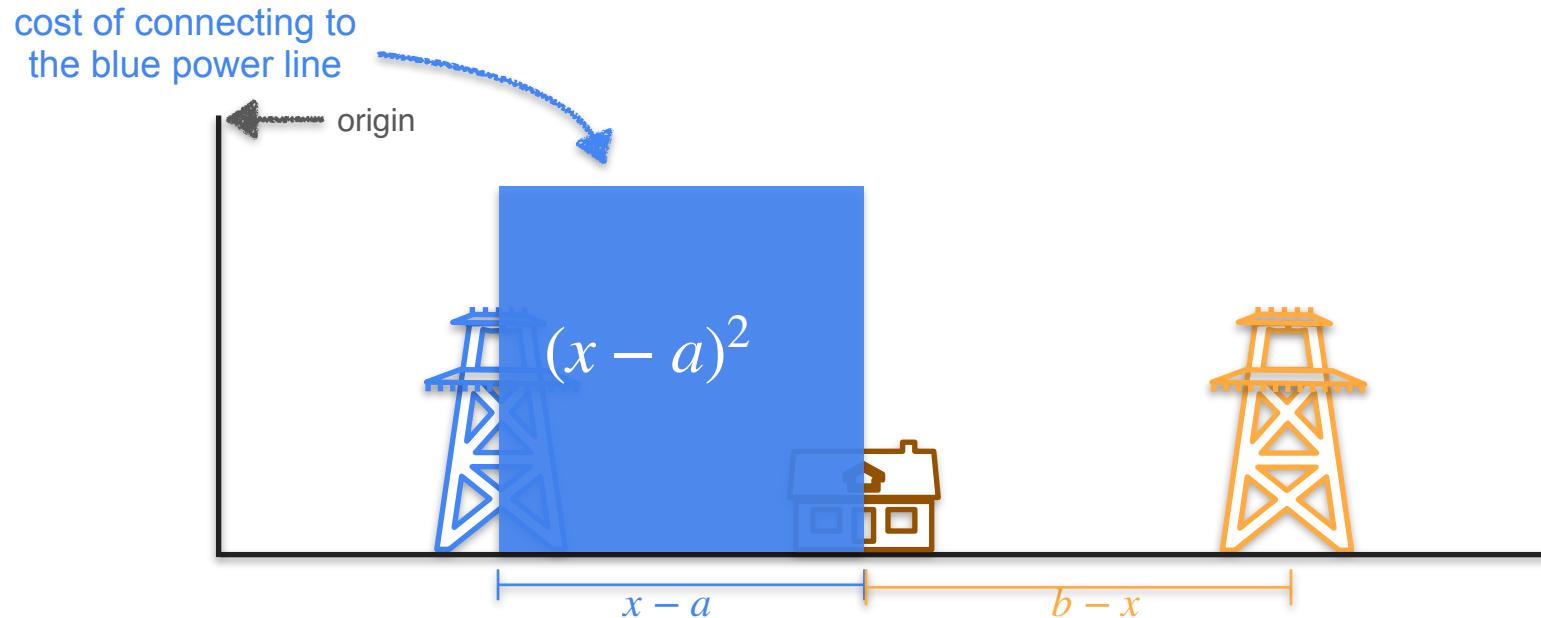
# Two Power Line Problem - Square Analogy



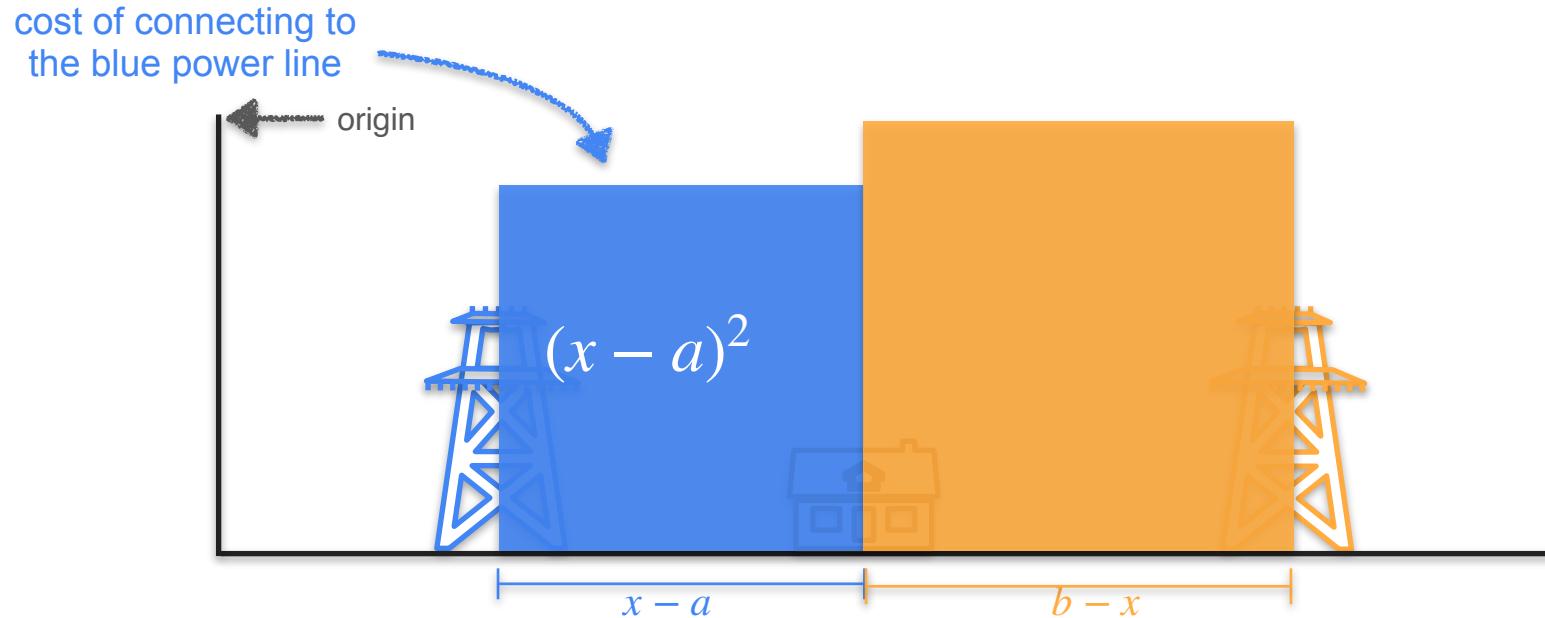
# Two Power Line Problem - Square Analogy



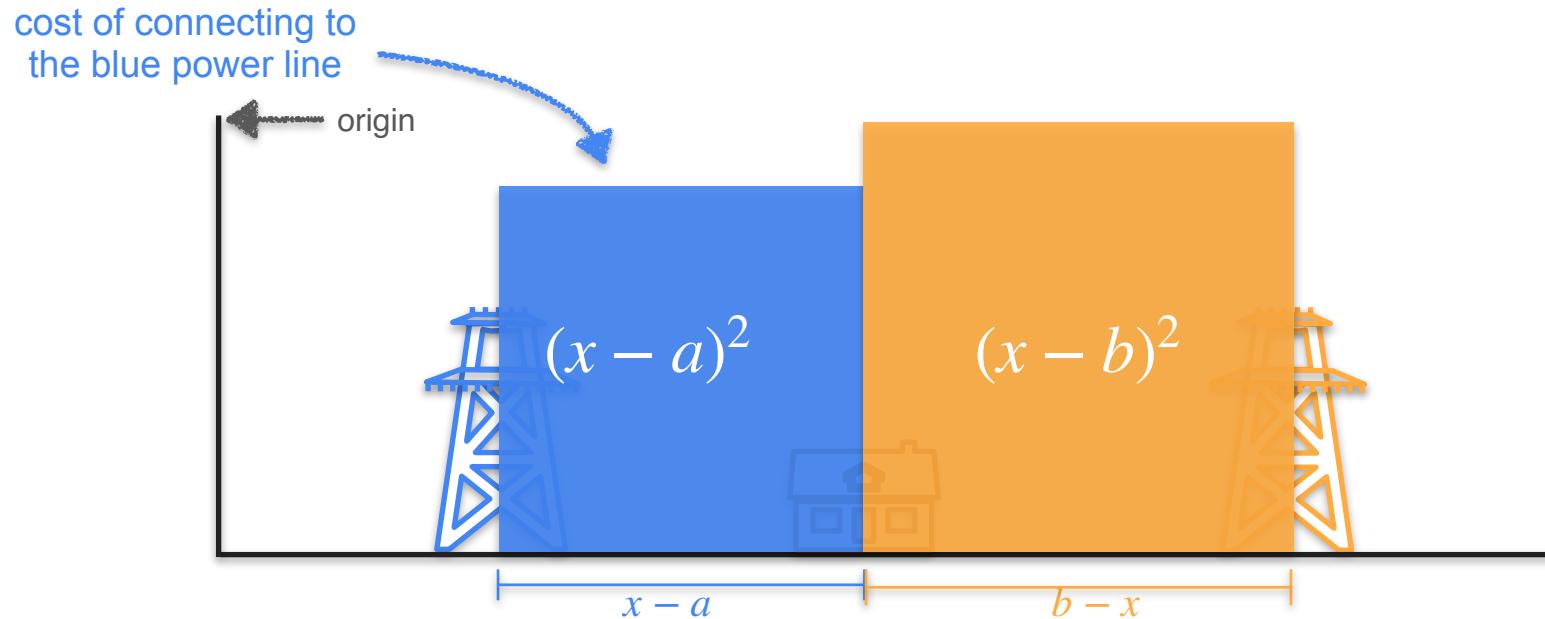
# Two Power Line Problem - Square Analogy



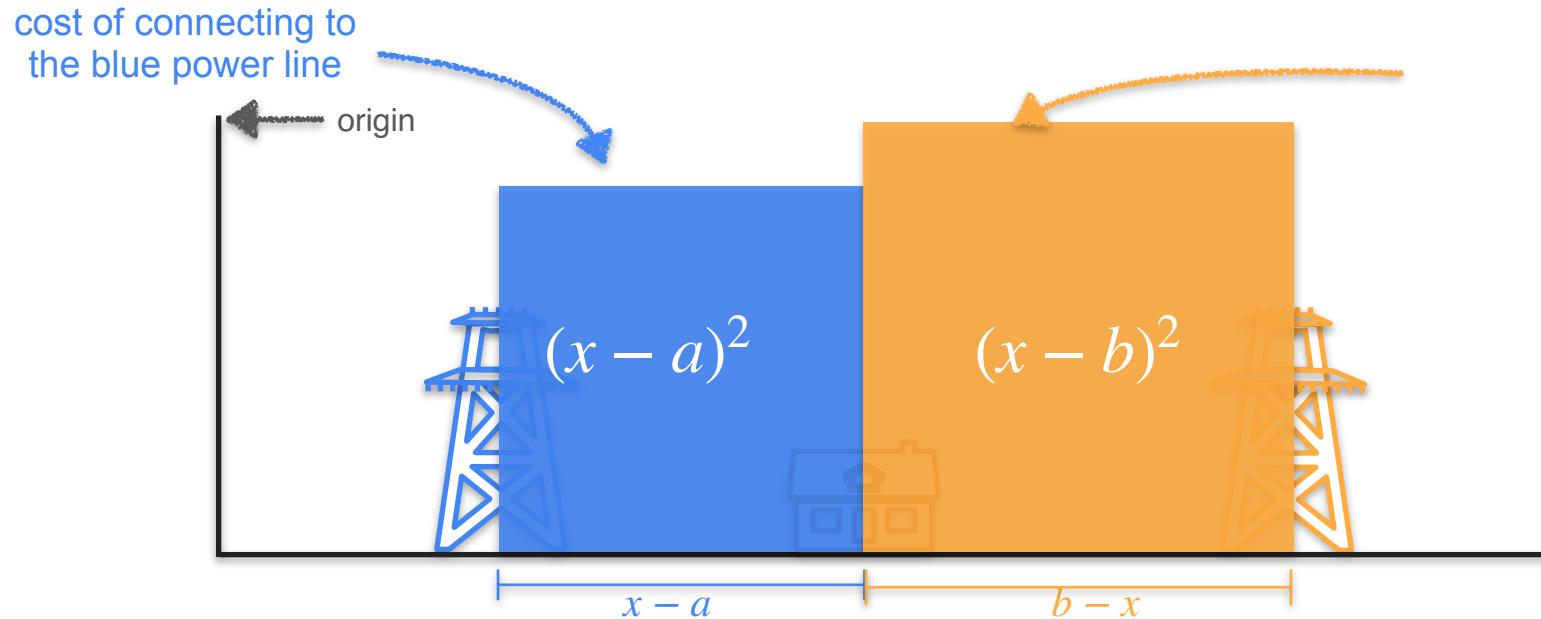
# Two Power Line Problem - Square Analogy



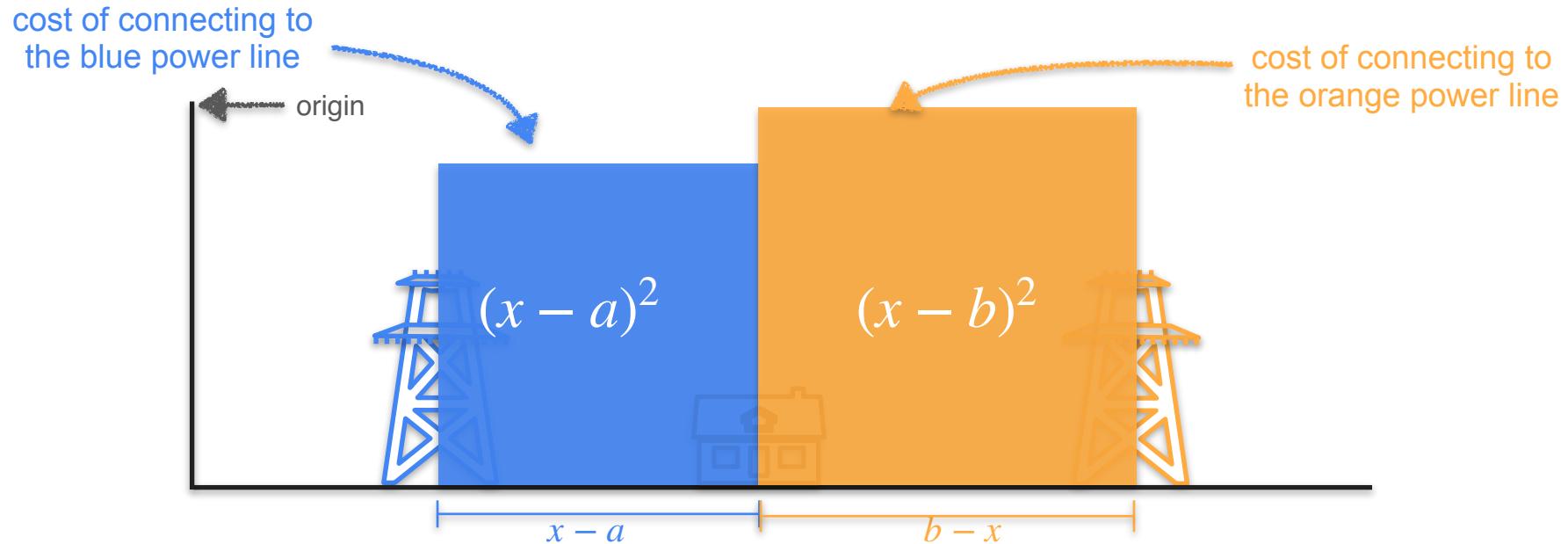
# Two Power Line Problem - Square Analogy



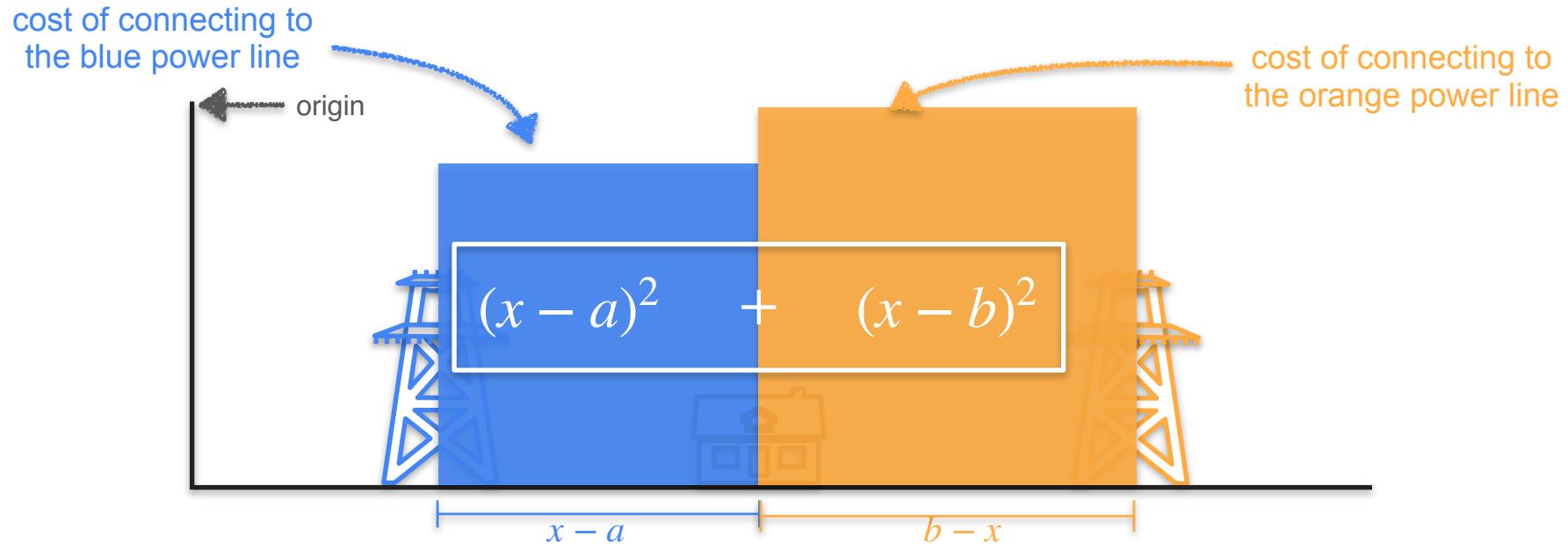
# Two Power Line Problem - Square Analogy



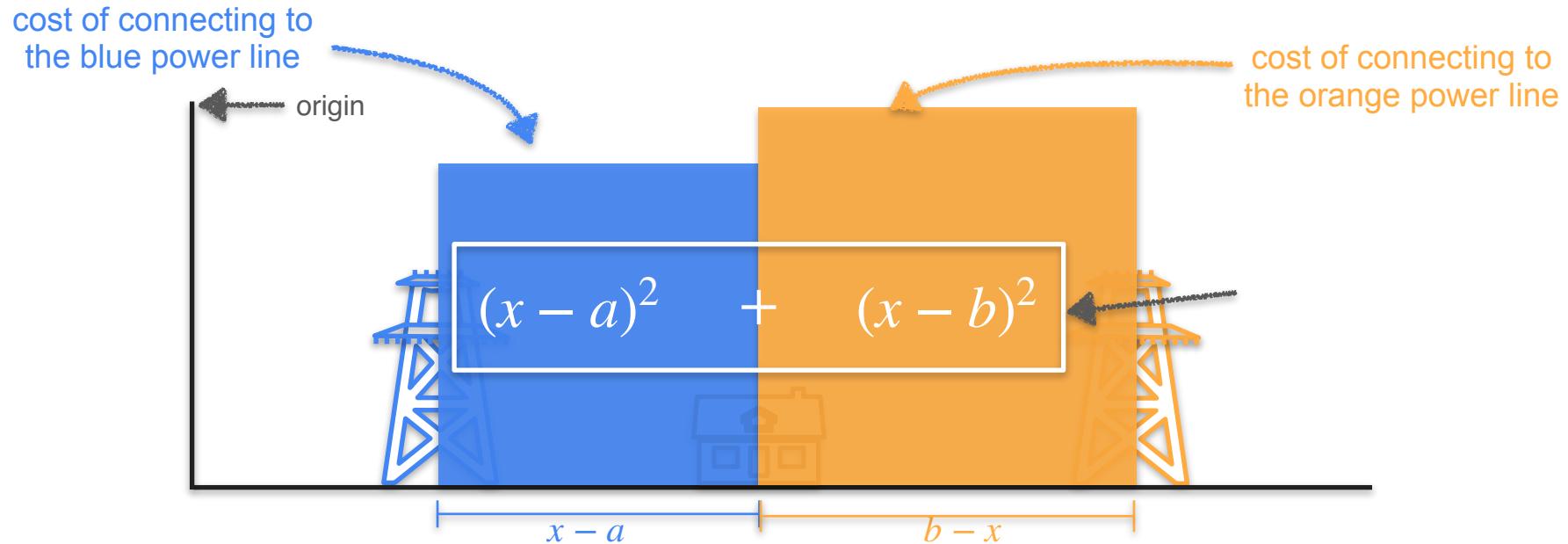
# Two Power Line Problem - Square Analogy



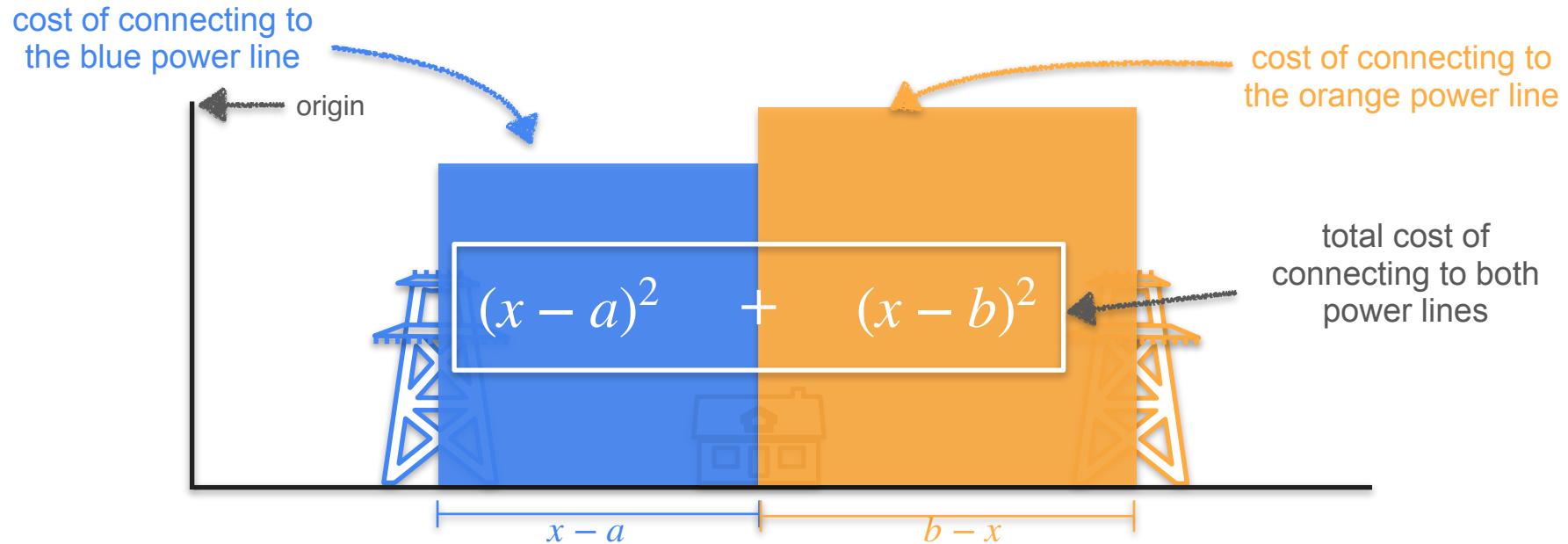
# Two Power Line Problem - Square Analogy



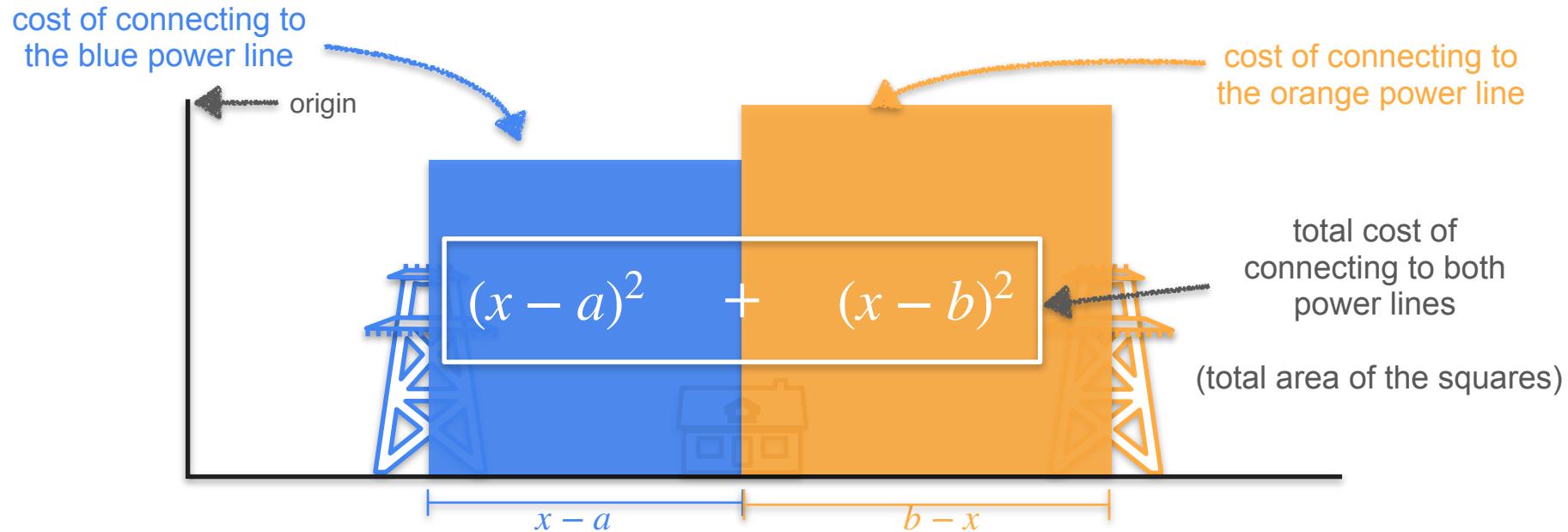
# Two Power Line Problem - Square Analogy



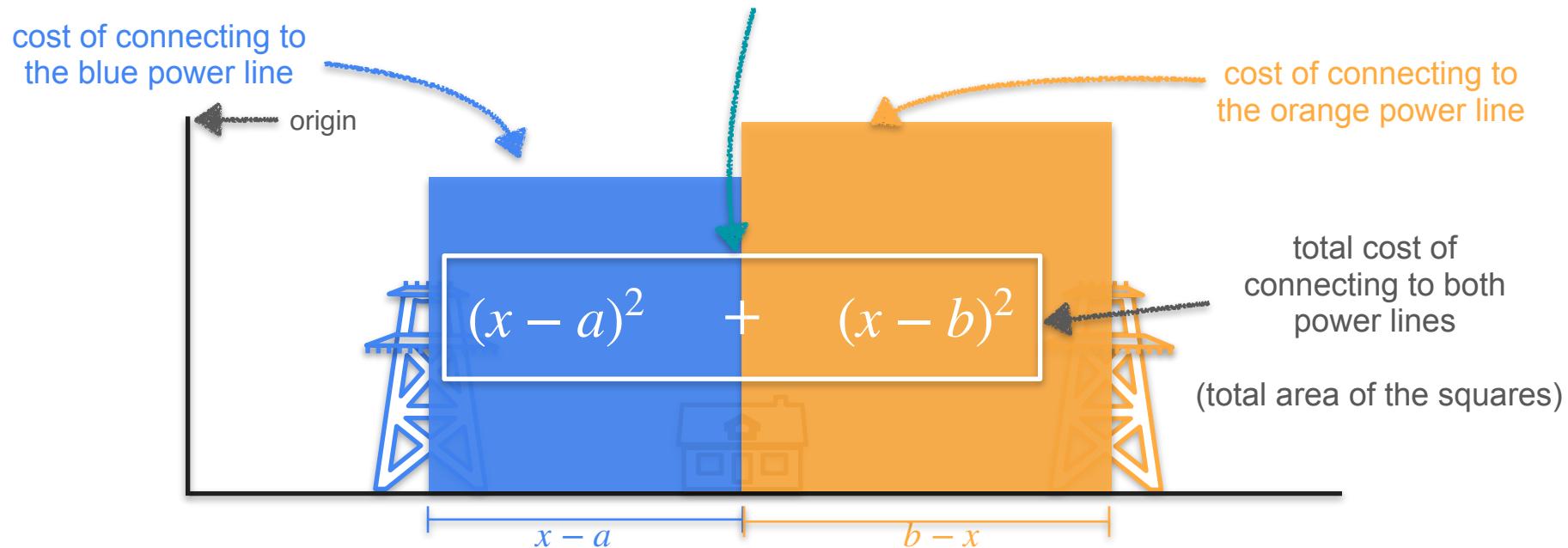
# Two Power Line Problem - Square Analogy



# Two Power Line Problem - Square Analogy

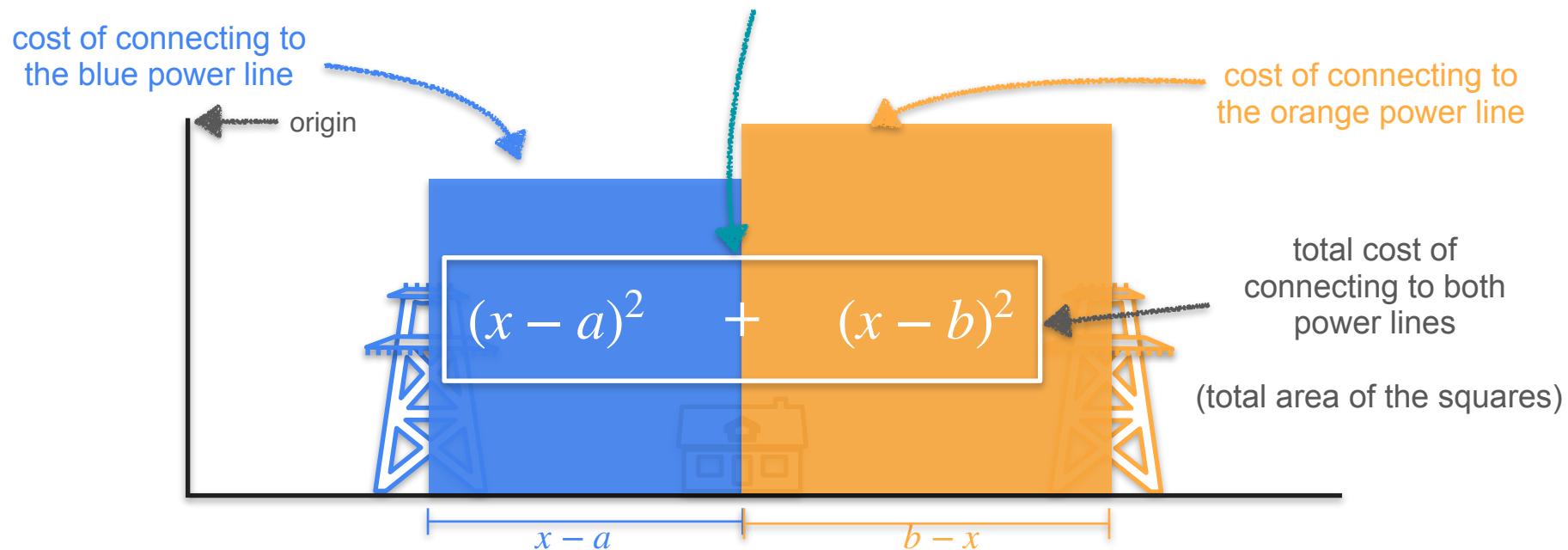


# Two Power Line Problem - Square Analogy

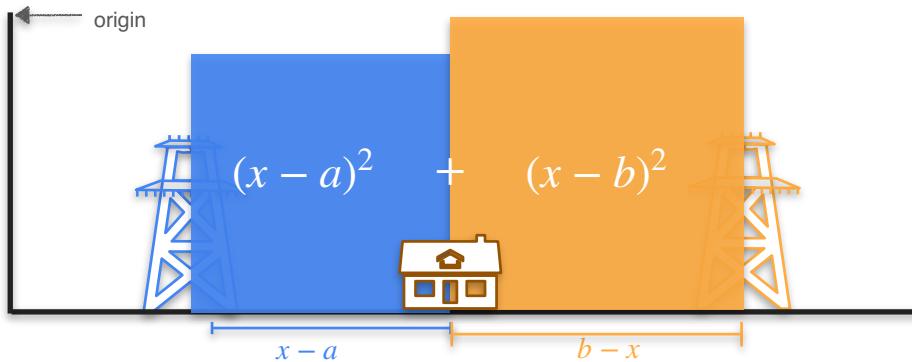


# Two Power Line Problem - Square Analogy

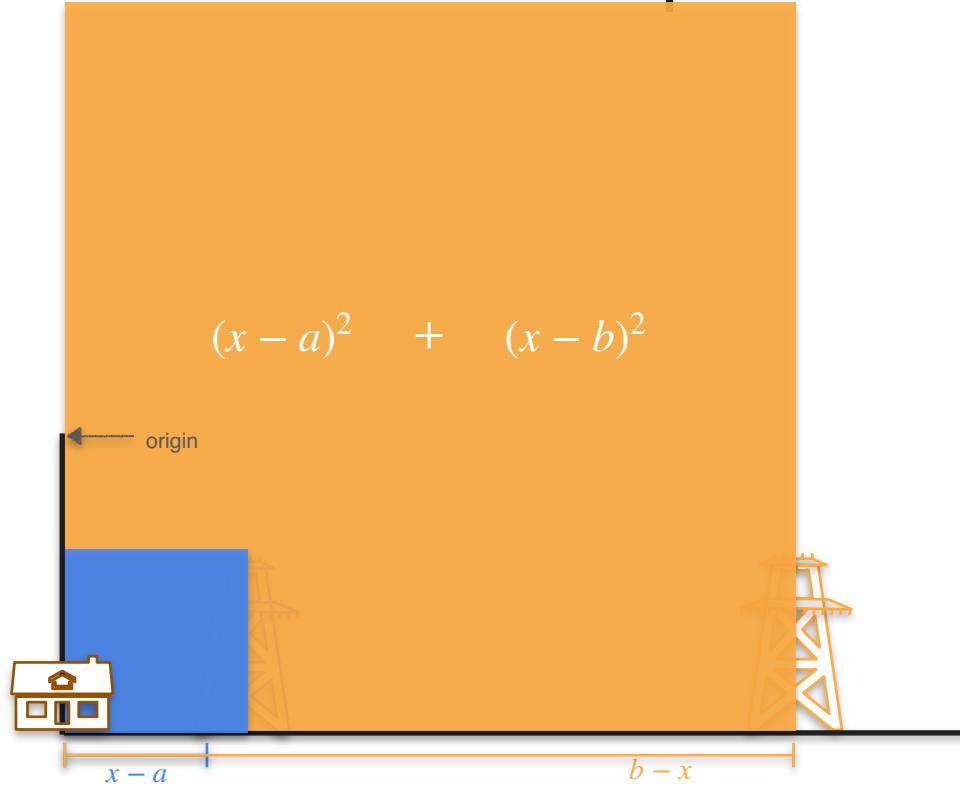
**Goal: Minimize the total area of the squares**



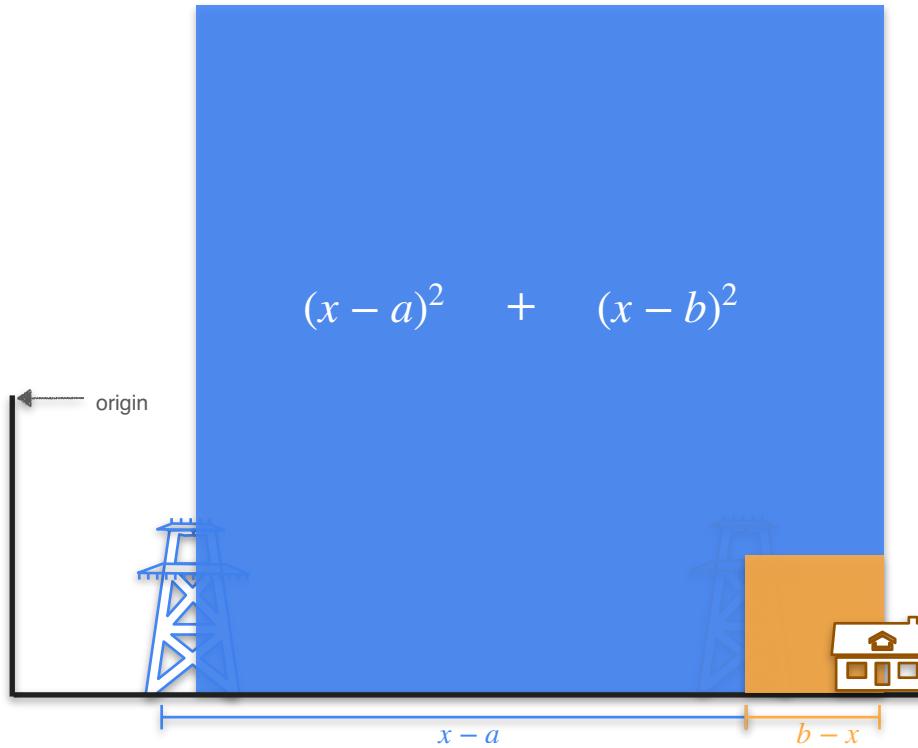
# Minimize the Total Area of Squares



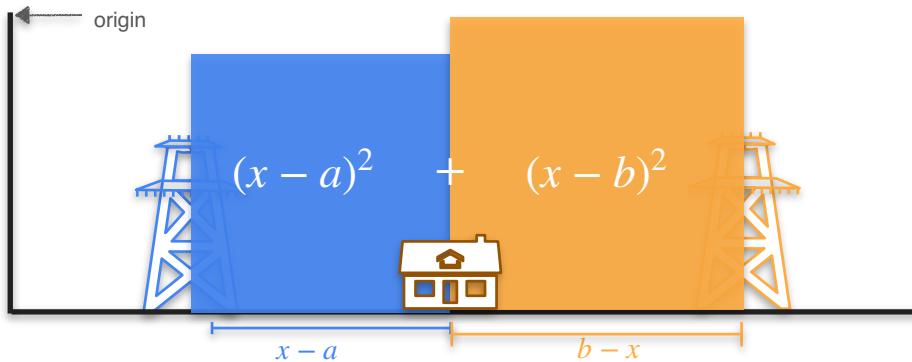
# Minimize the Total Area of Squares



# Minimize the Total Area of Squares



# Minimize the Total Area of Squares



# Two Power Line Problem

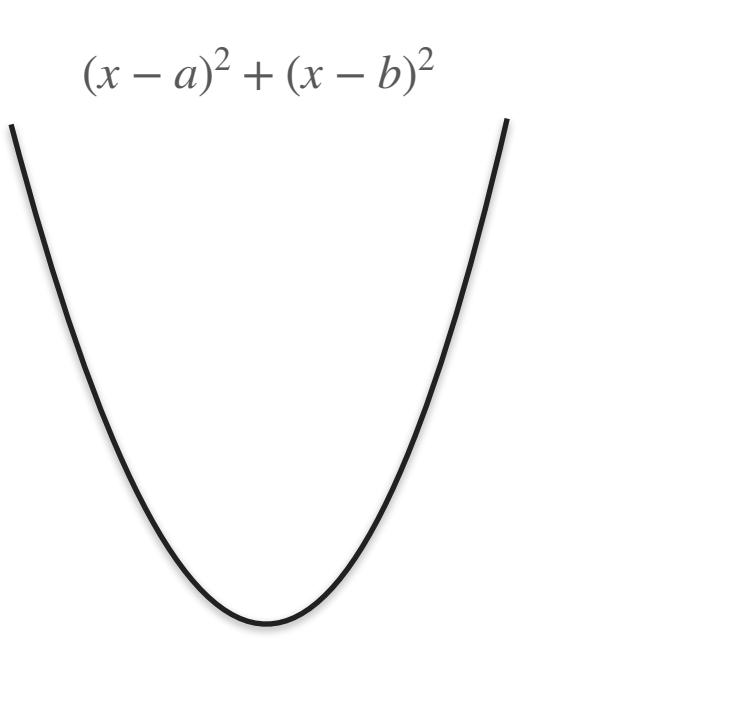


# Two Power Line Problem

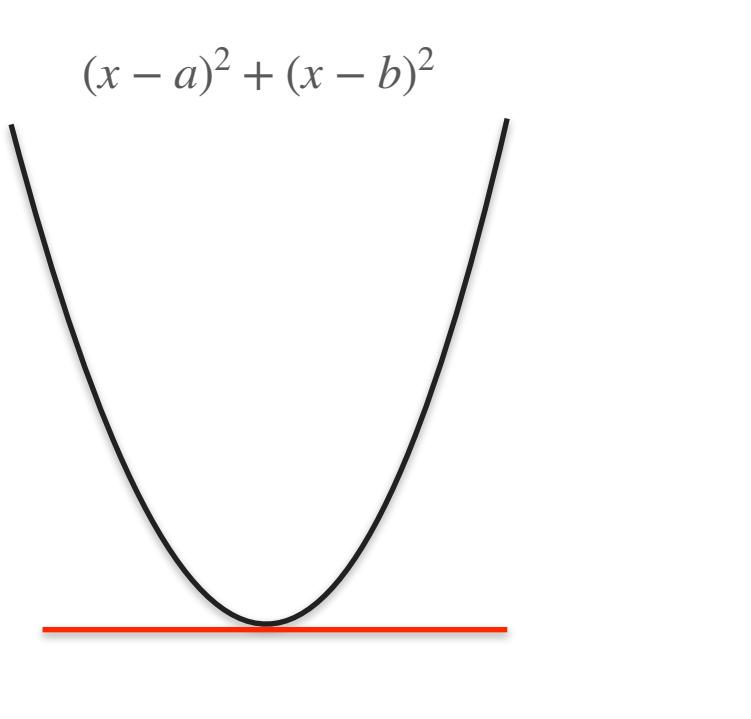
$$(x - a)^2 + (x - b)^2$$



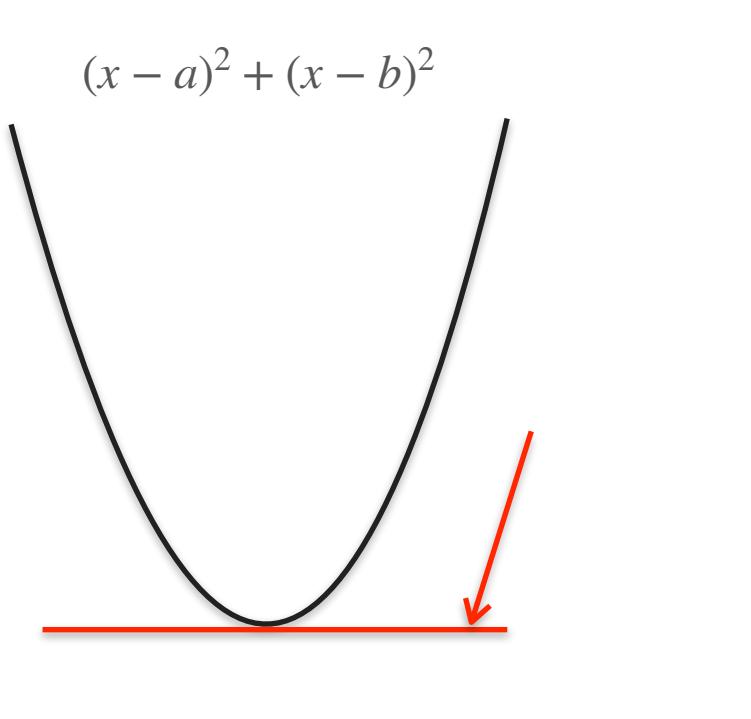
# Two Power Line Problem



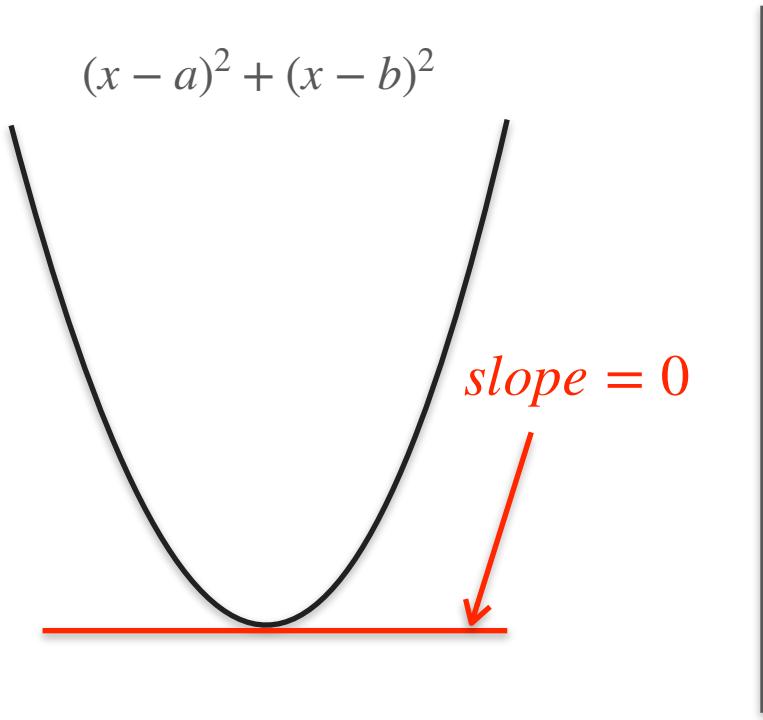
# Two Power Line Problem



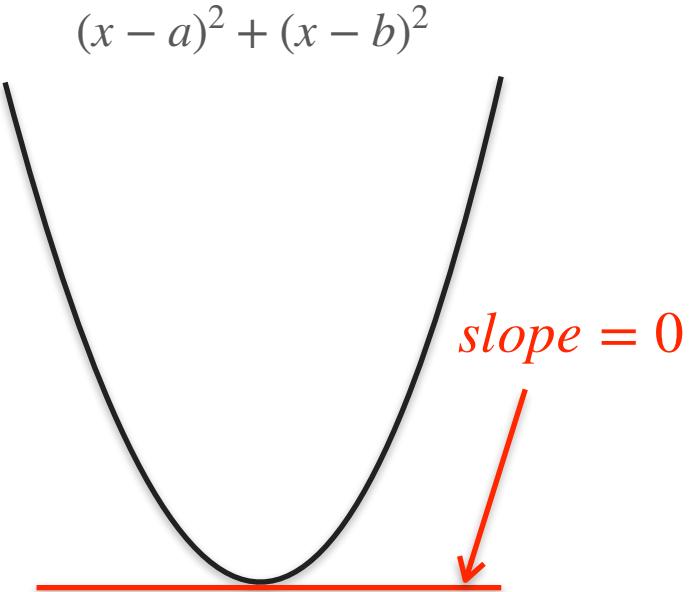
# Two Power Line Problem



# Two Power Line Problem

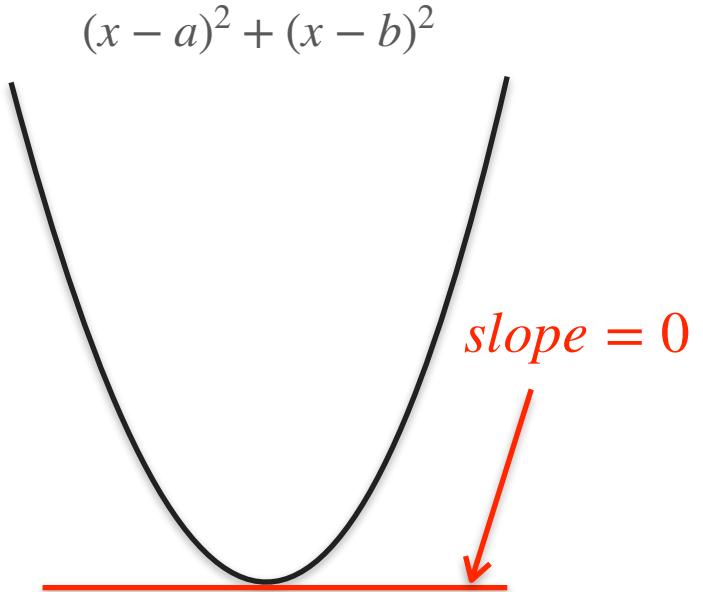


# Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

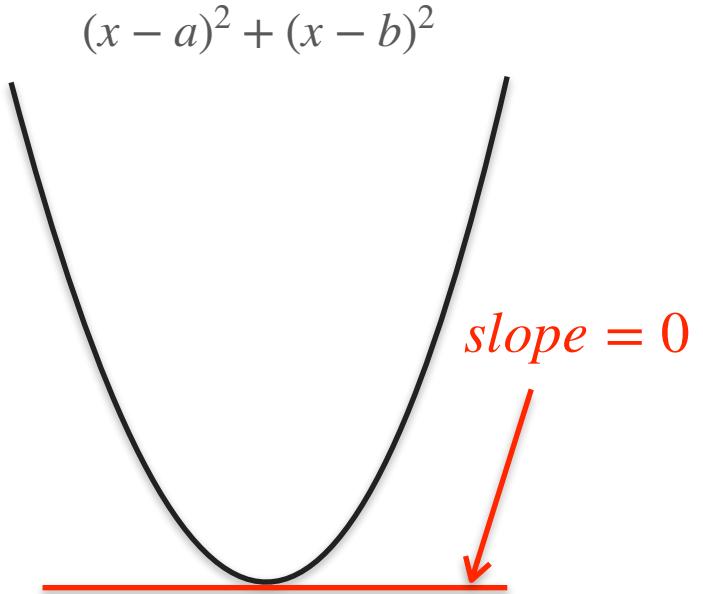
# Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

# Two Power Line Problem

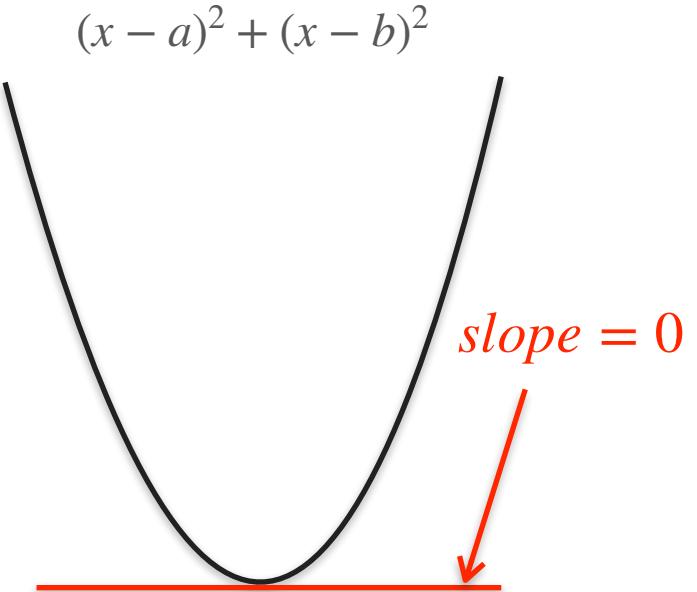


$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

# Two Power Line Problem



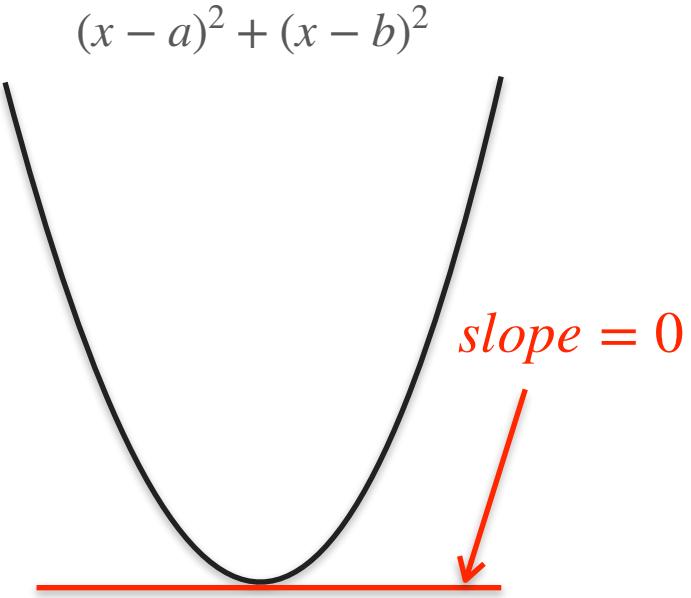
$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

# Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

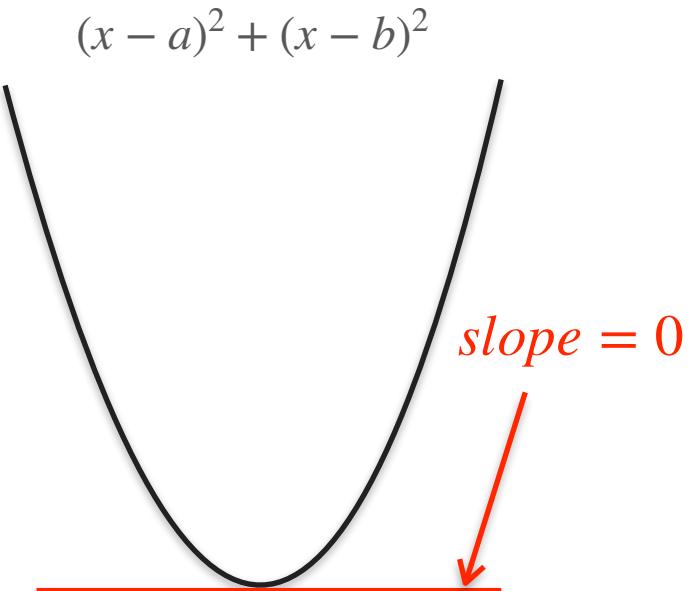
$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

$$2x = a + b$$

# Two Power Line Problem



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2] = 0$$

$$2(x - a) + 2(x - b) = 0$$

$$(x - a) + (x - b) = 0$$

$$2x - a - b = 0$$

$$2x = a + b$$

$$x = \frac{a + b}{2}$$



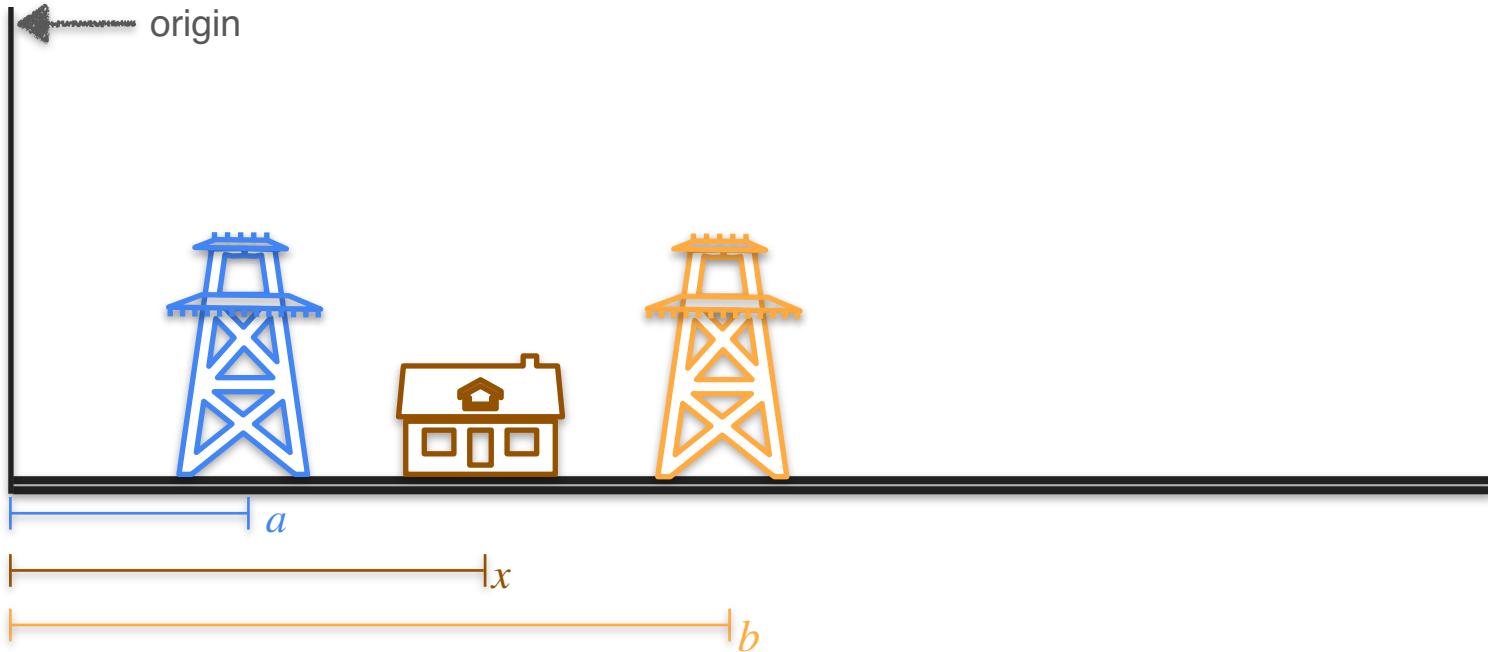
DeepLearning.AI

# Derivatives and Optimization

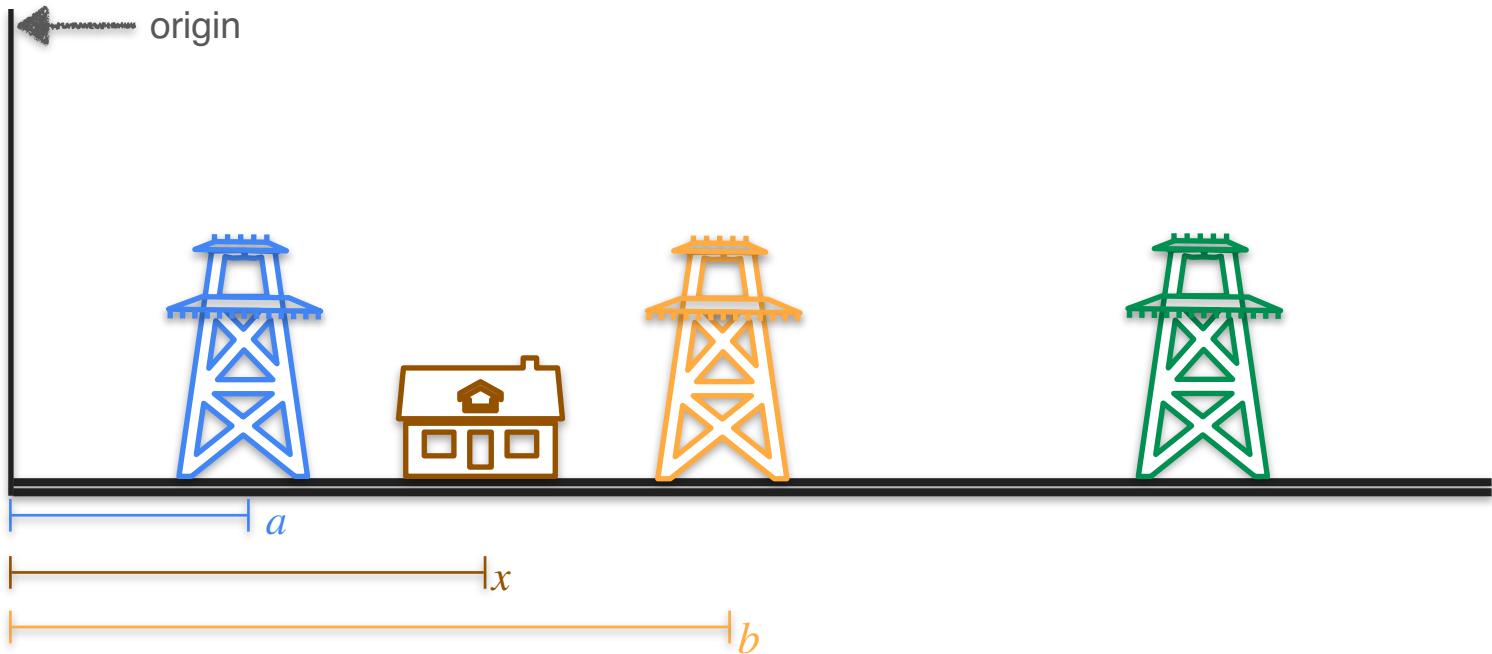
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**Optimization of squared loss:  
The three powerline problem**

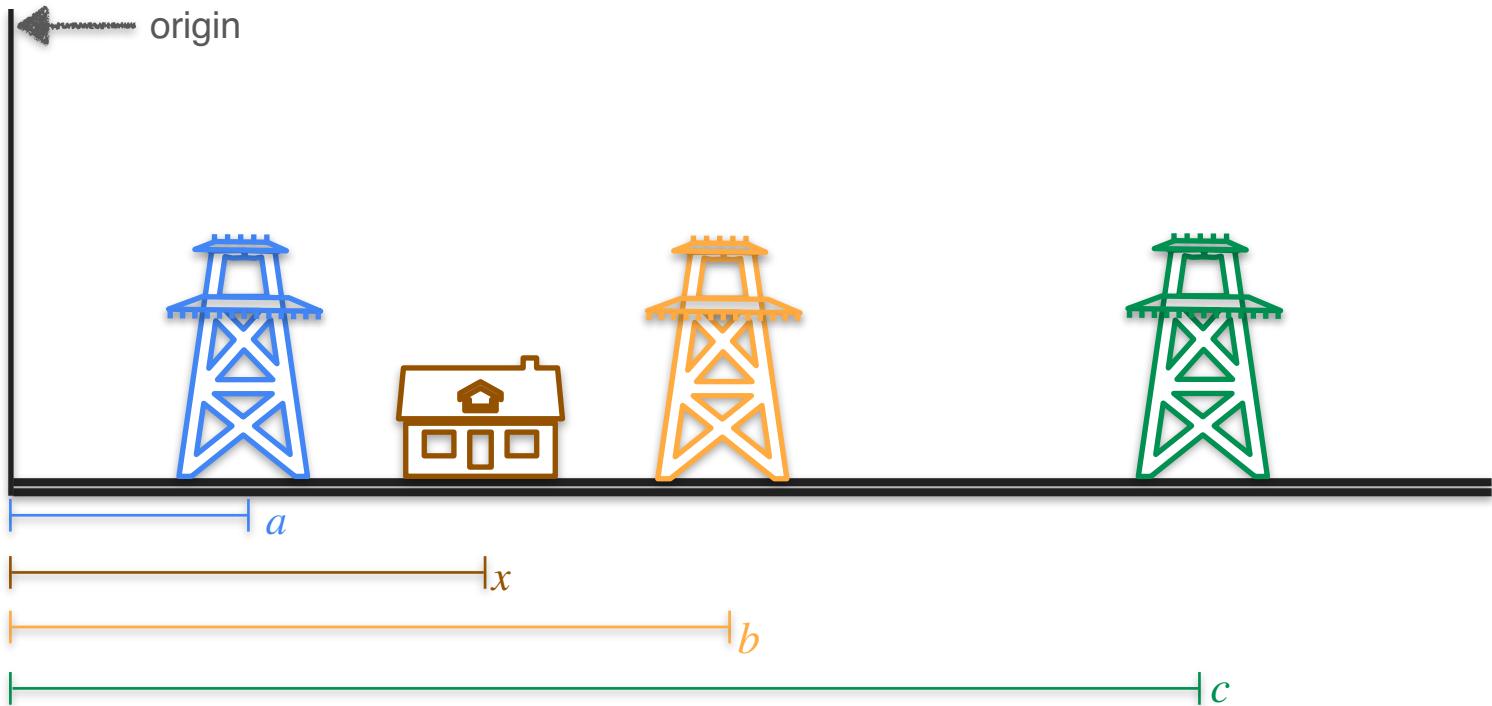
# Three Power Line Problem



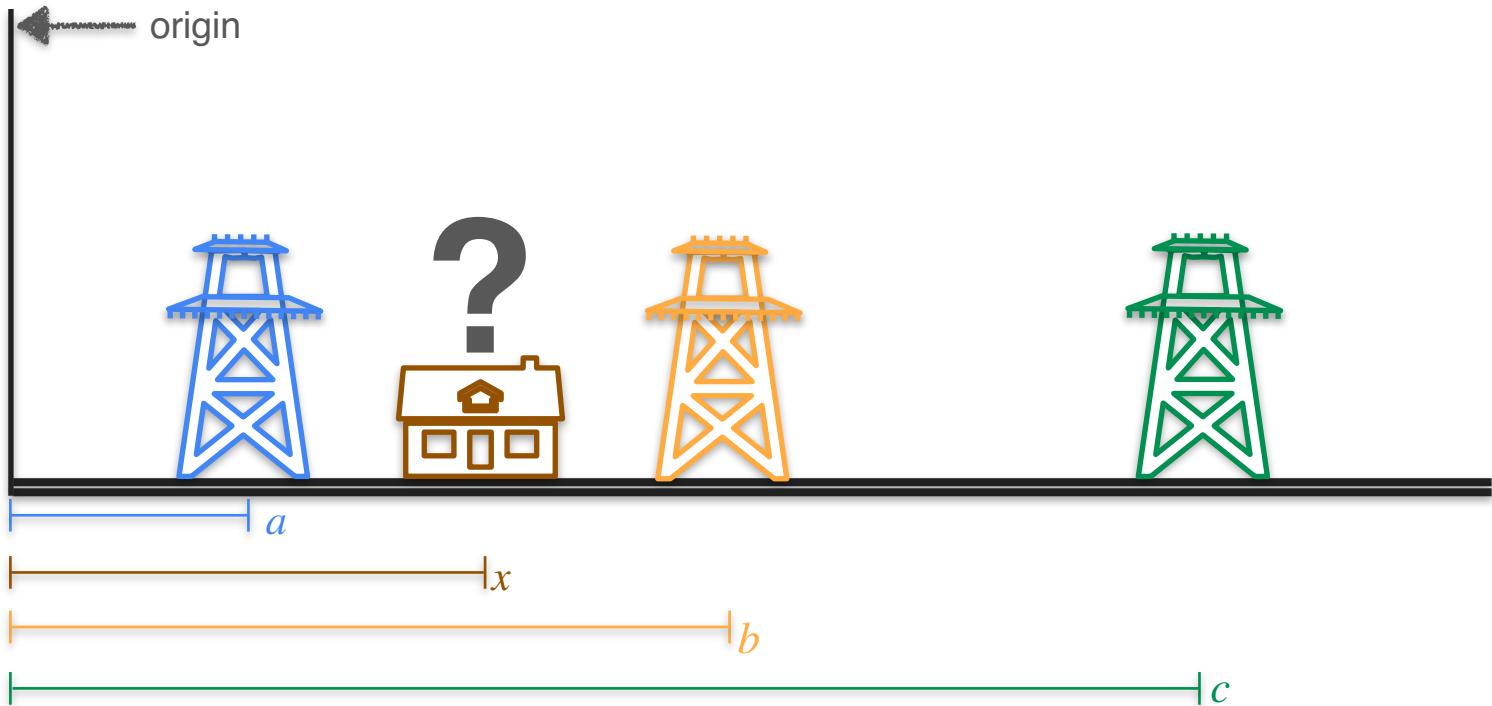
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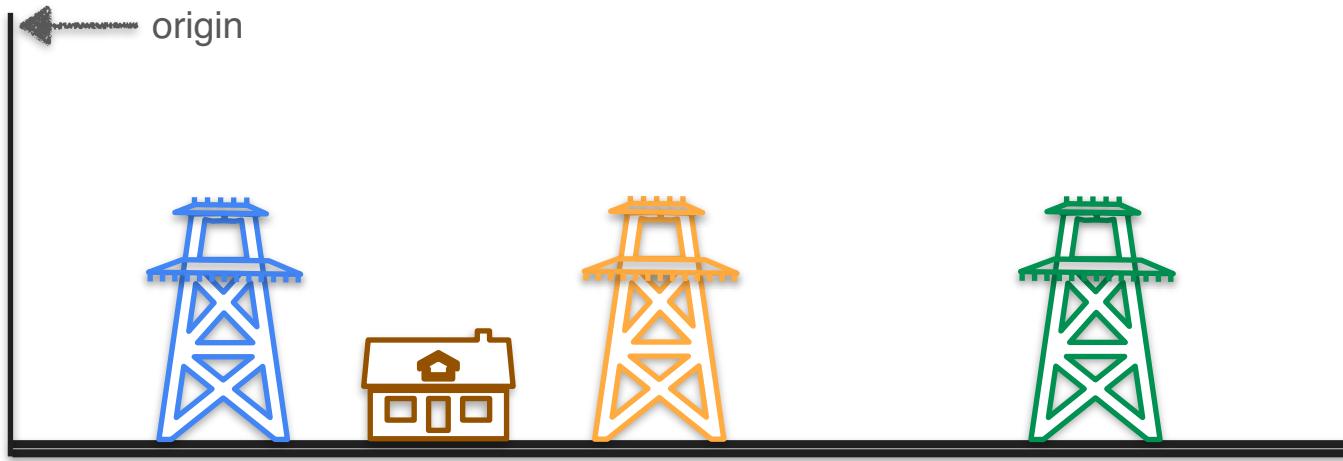
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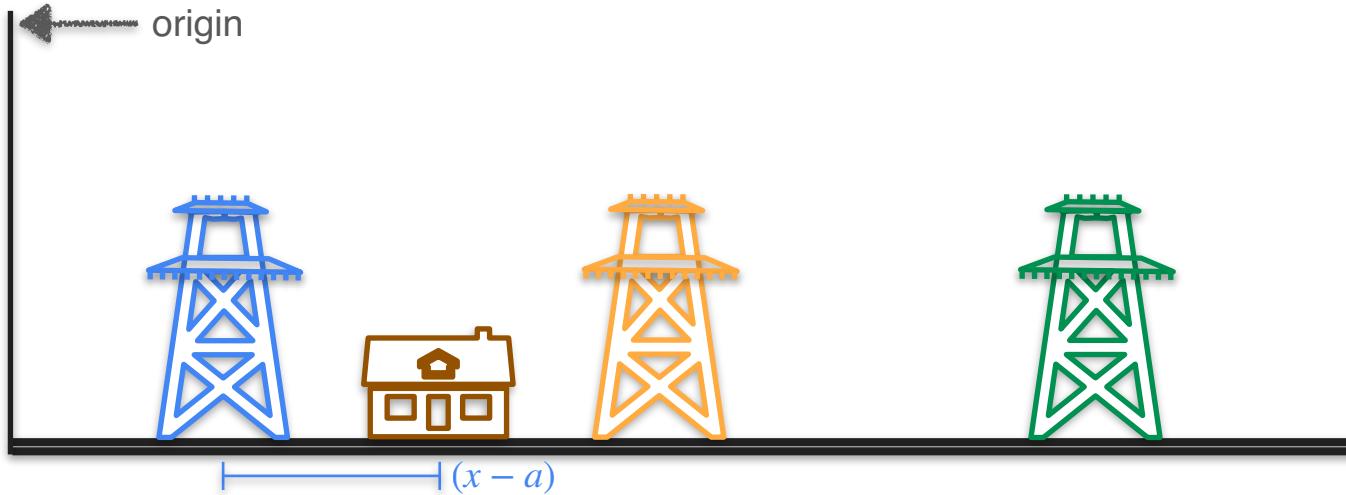
# Three Power Line Problem



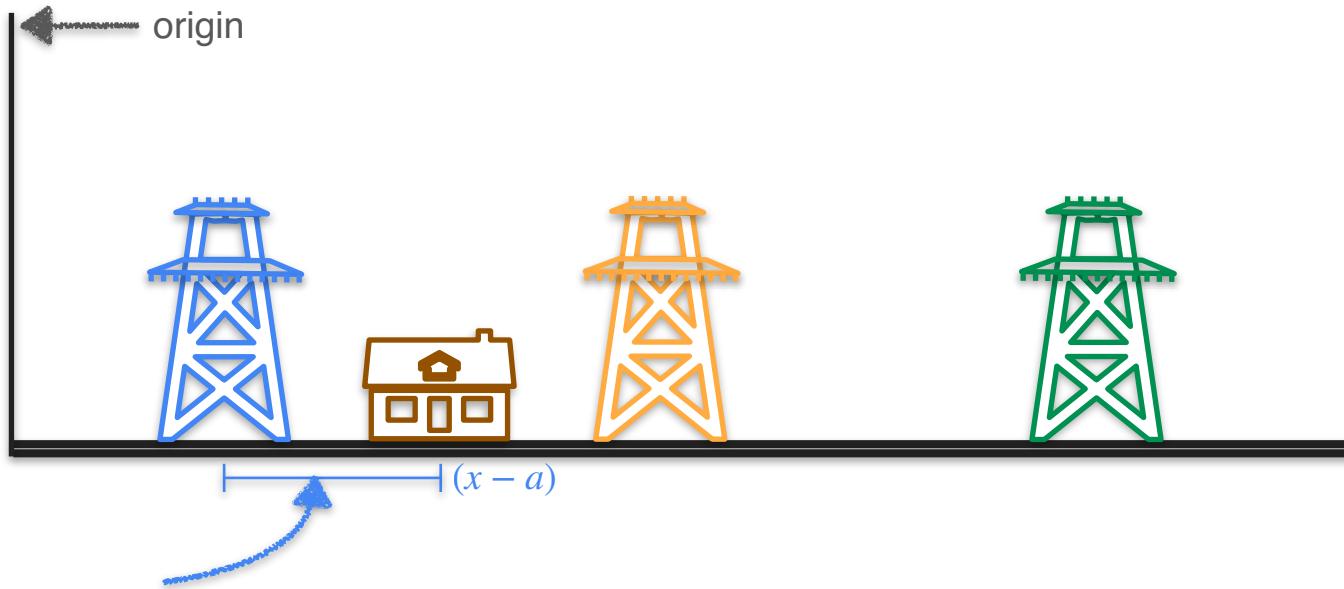
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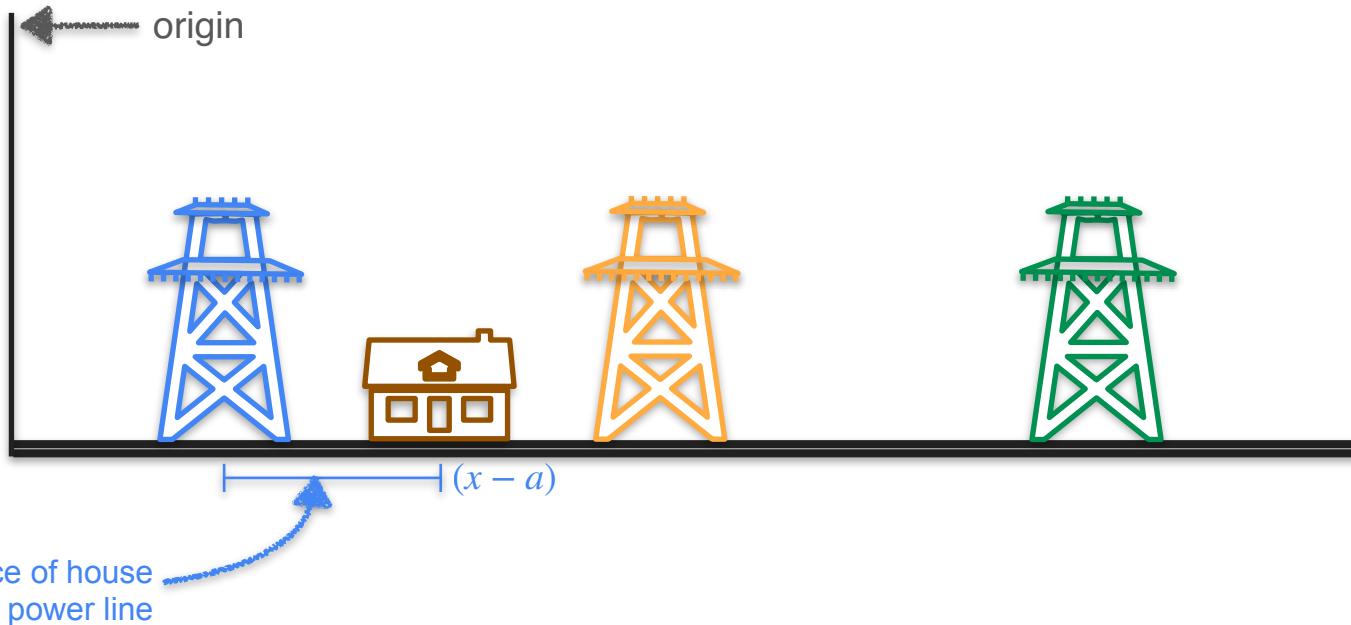
# Three Power Line Problem



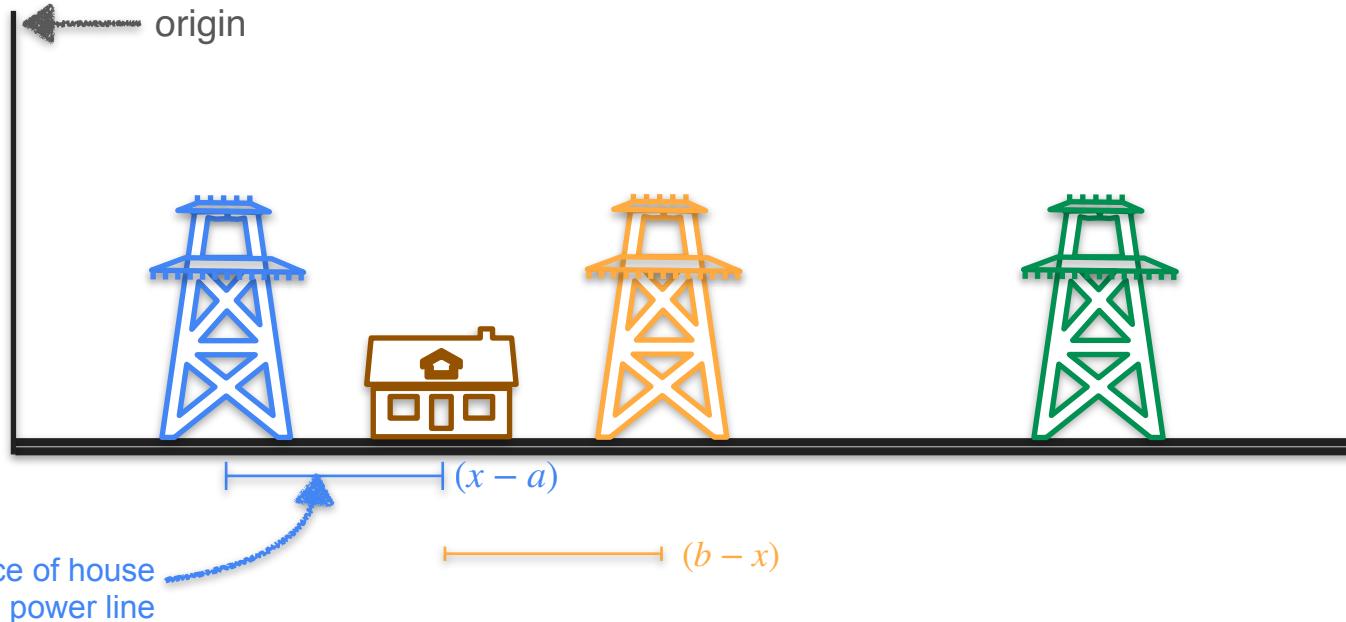
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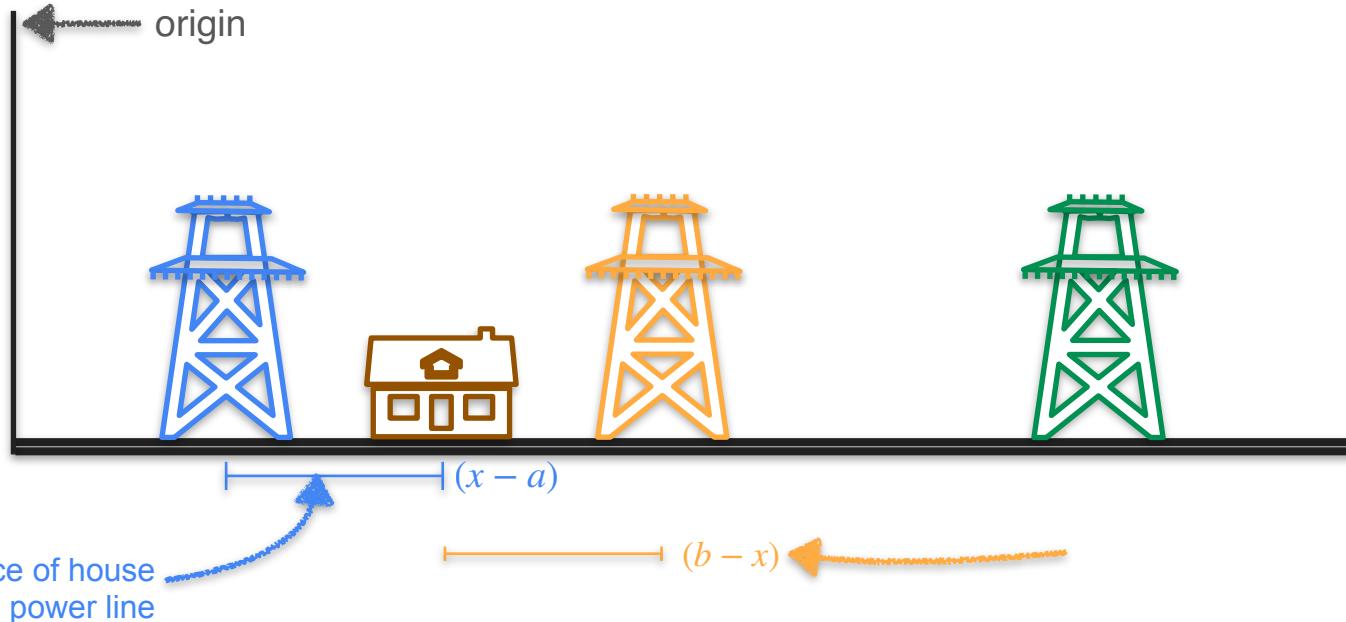
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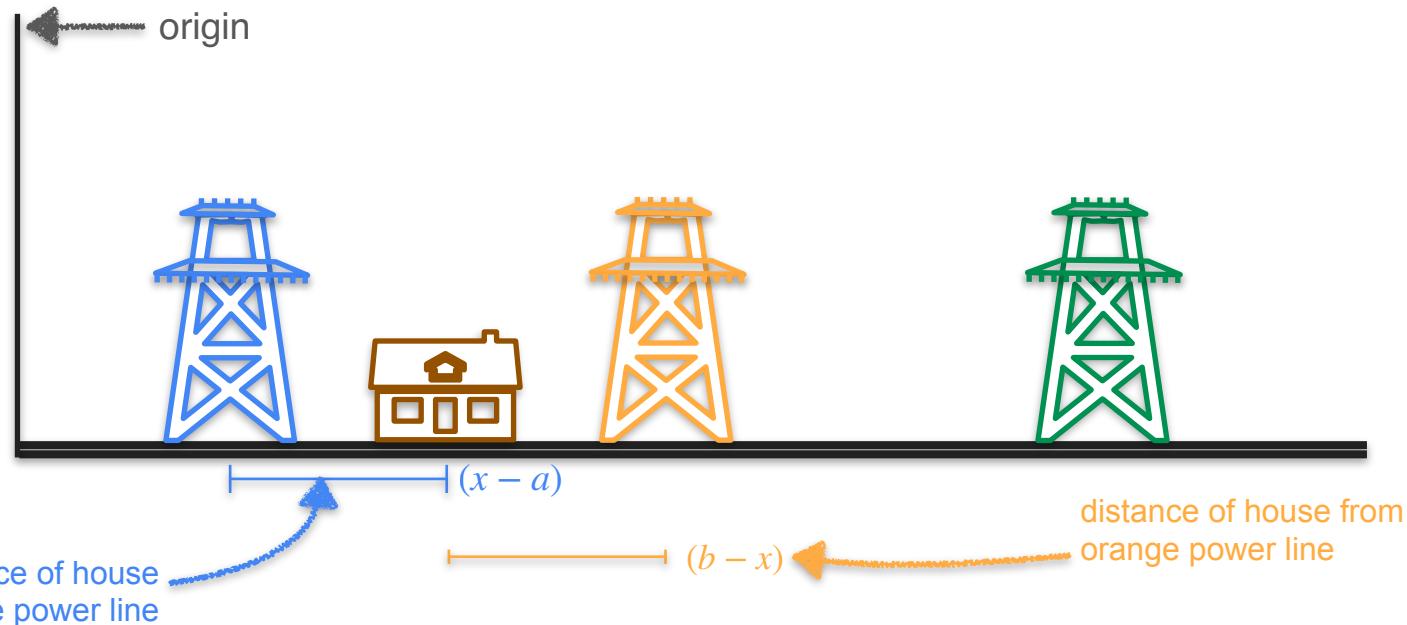
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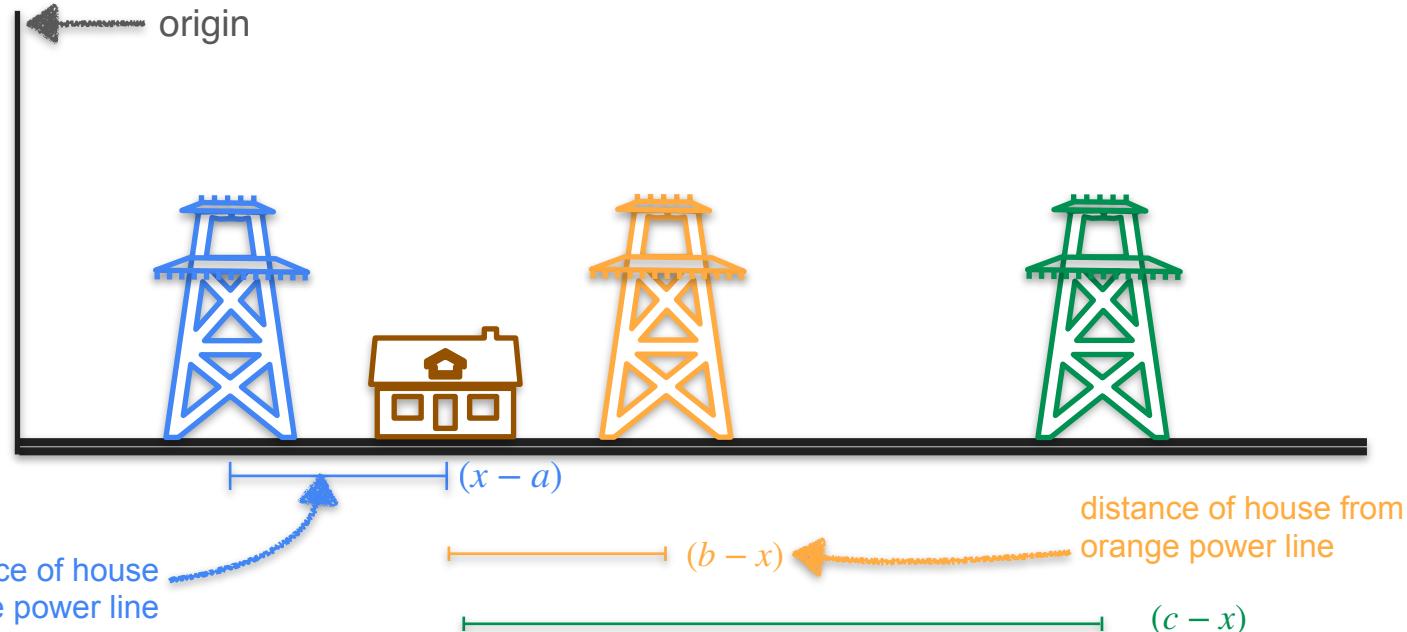
# Three Power Line Problem



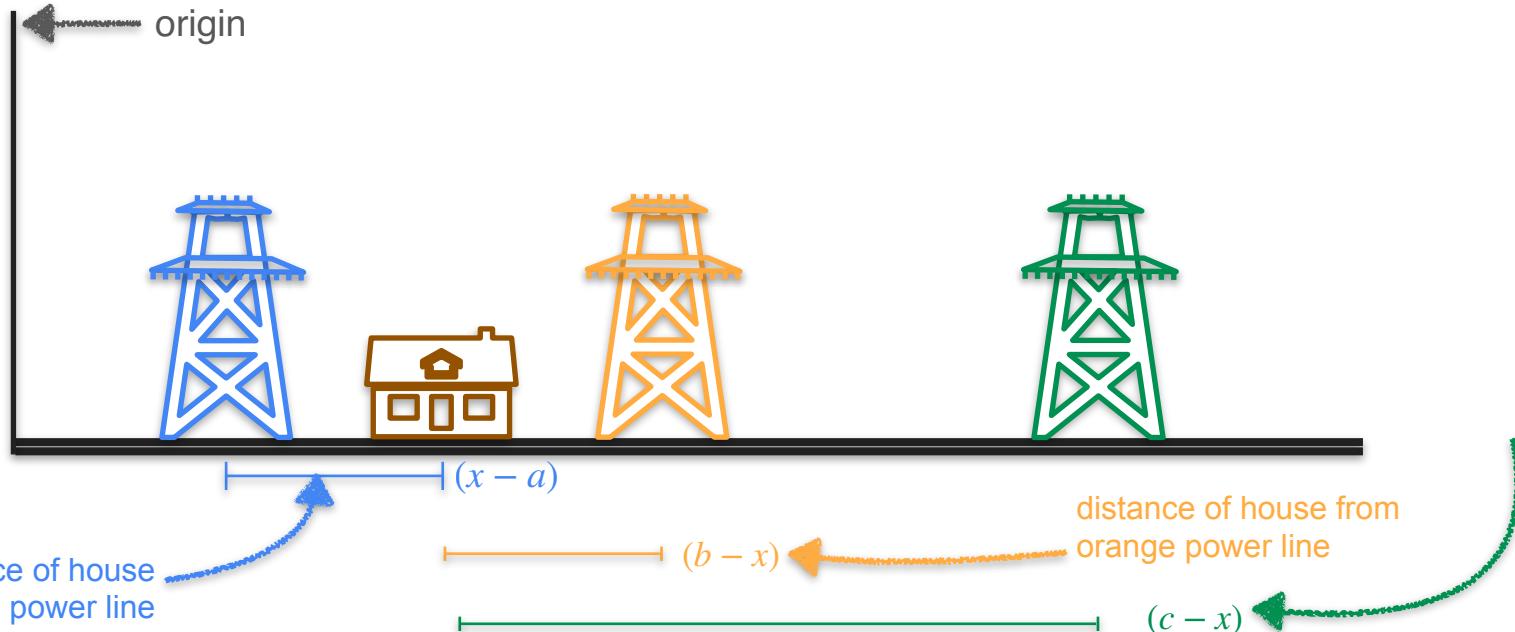
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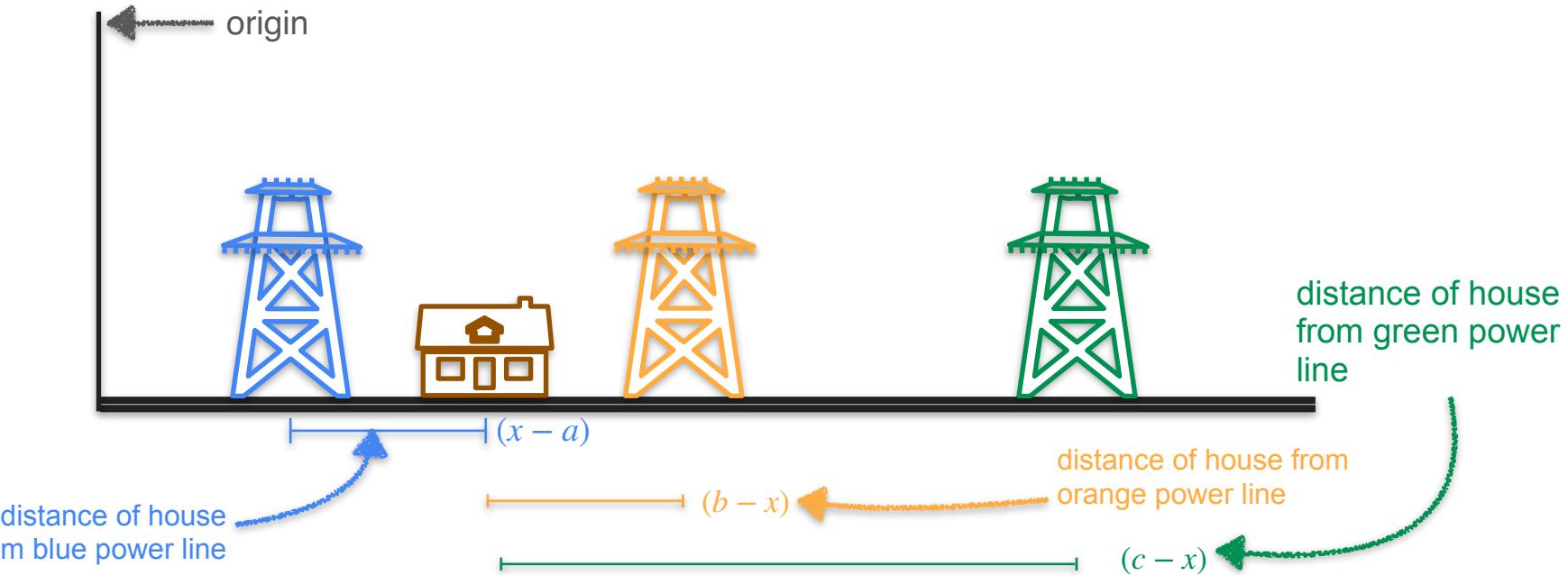
# Three Power Line Problem



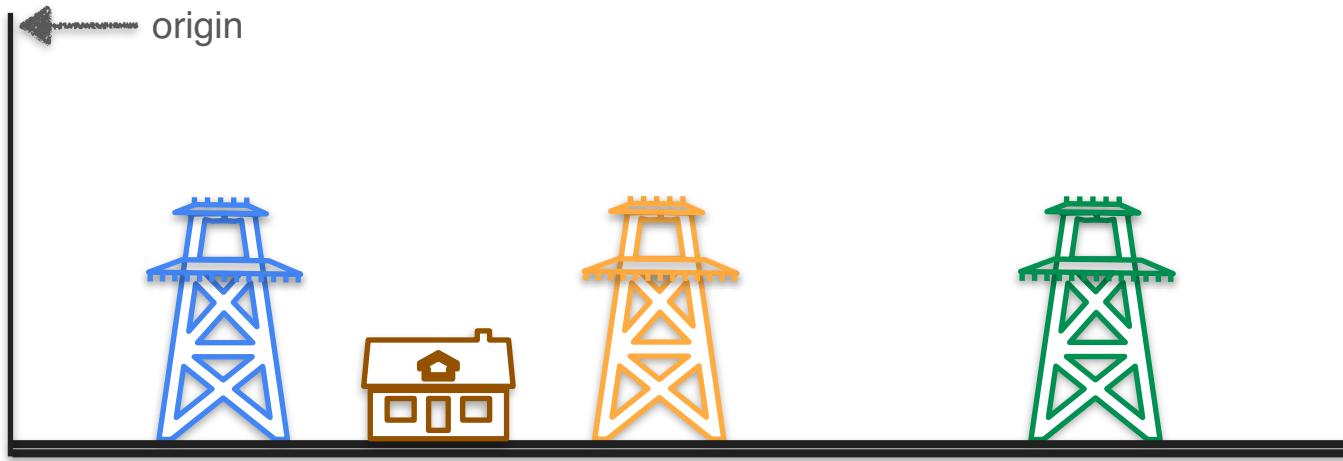
# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



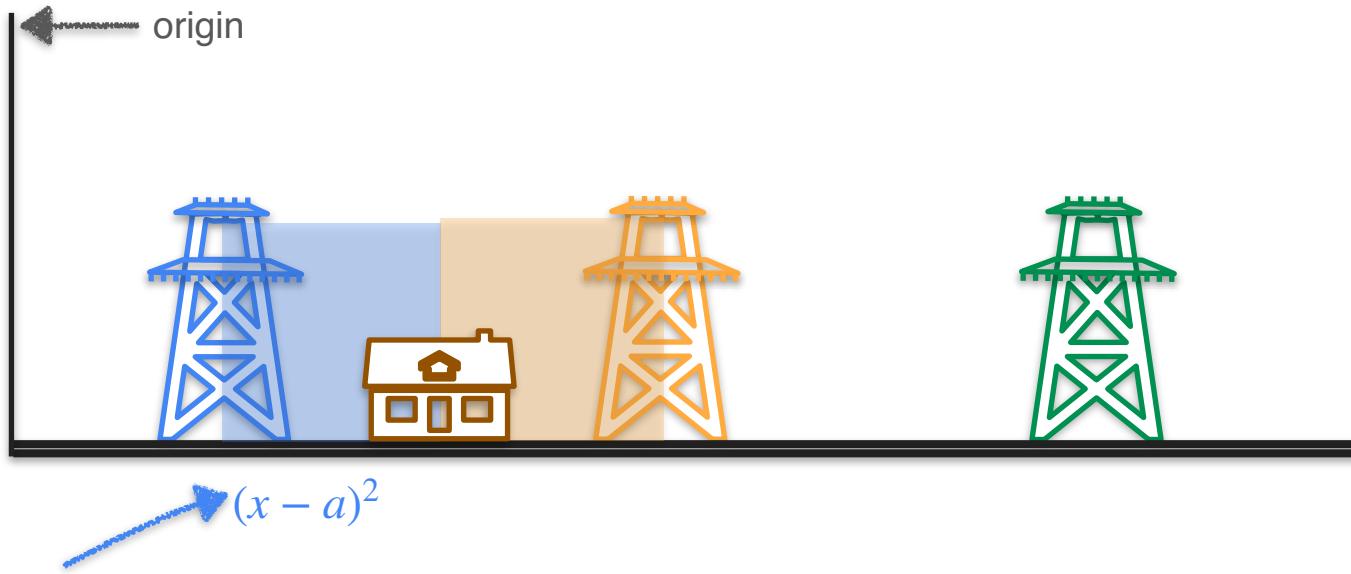
# Three Power Line Problem



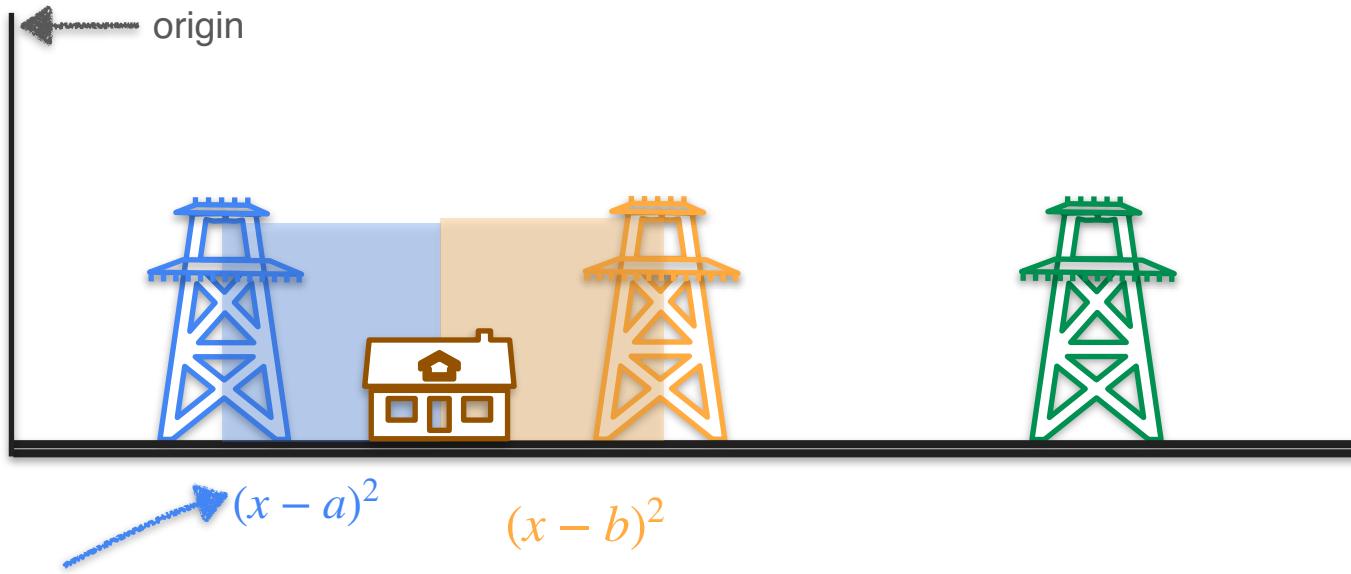
# Three Power Line Problem



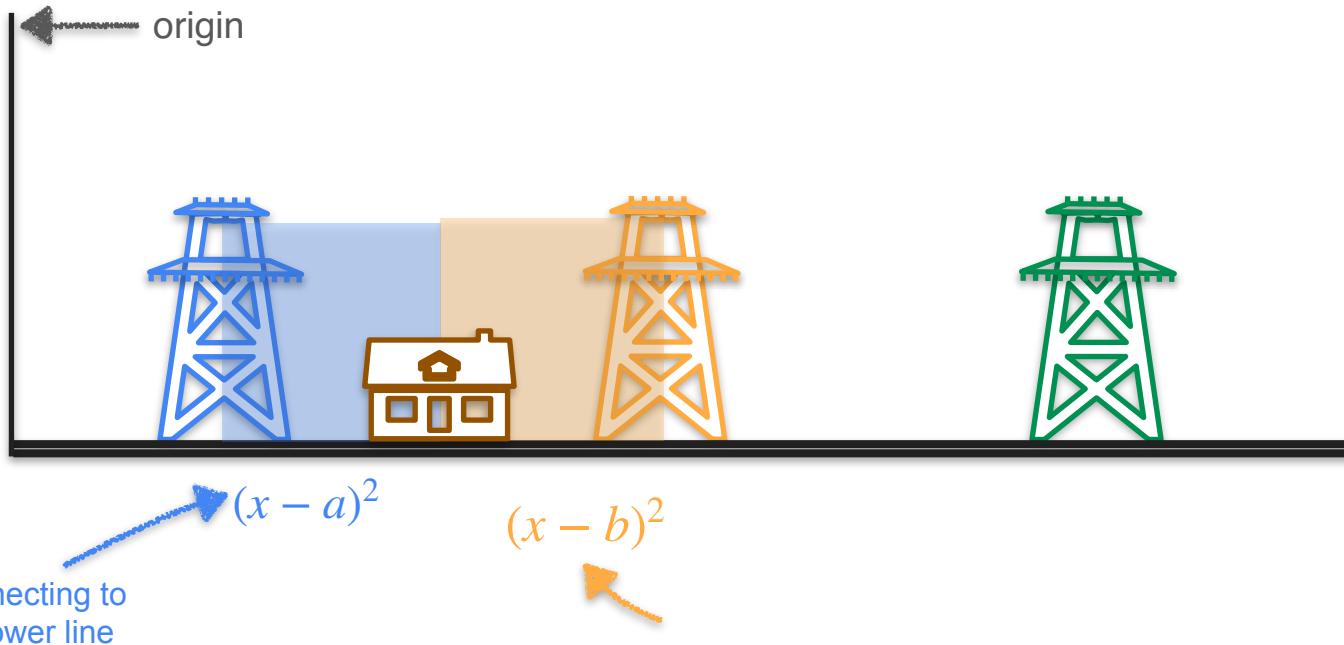
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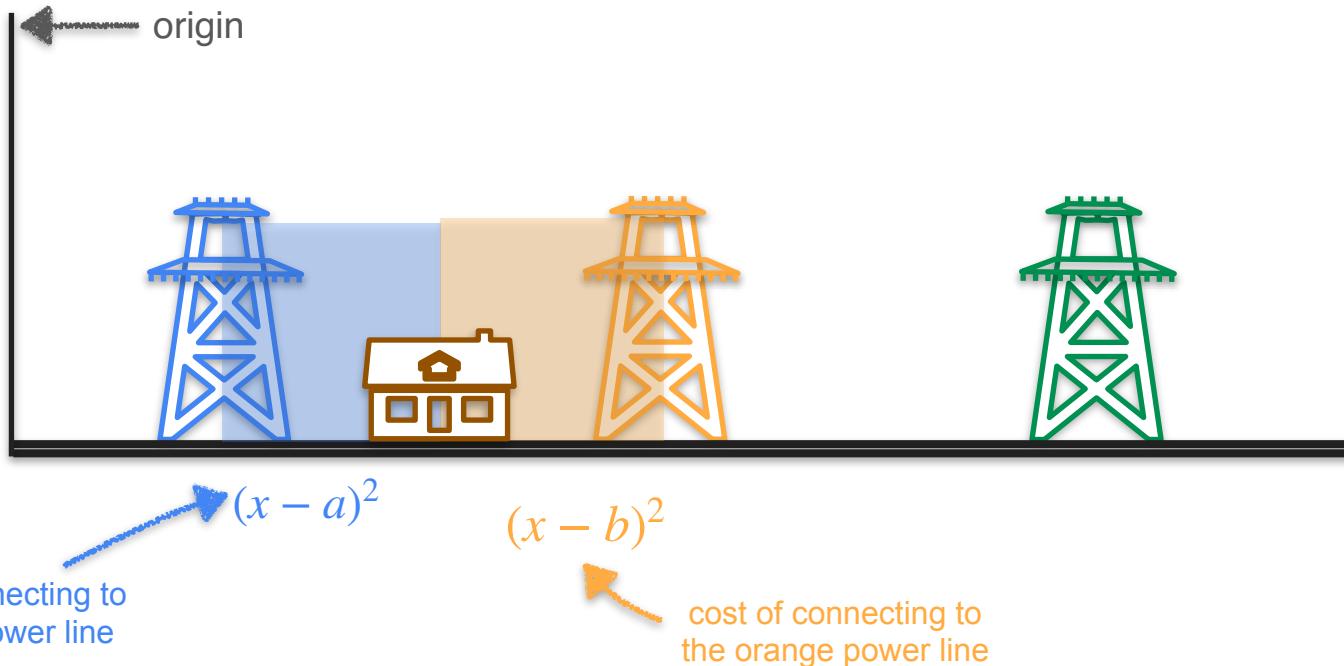
# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem

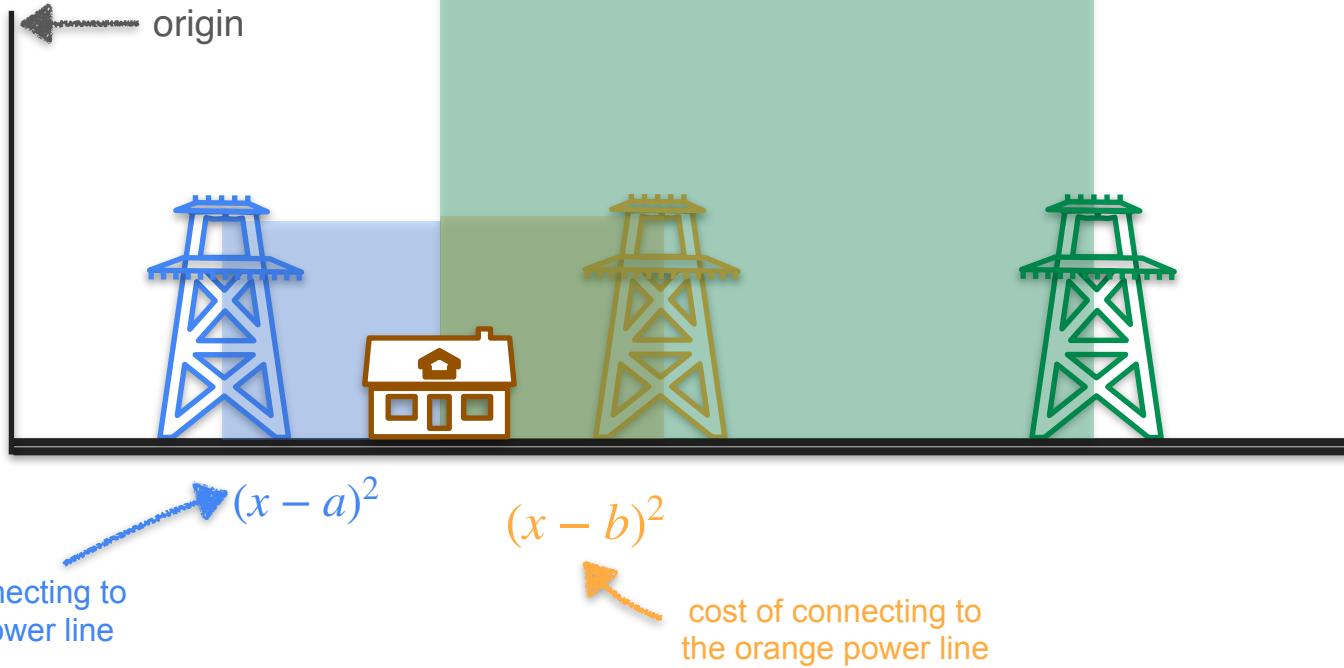


cost of connecting to  
the blue power line

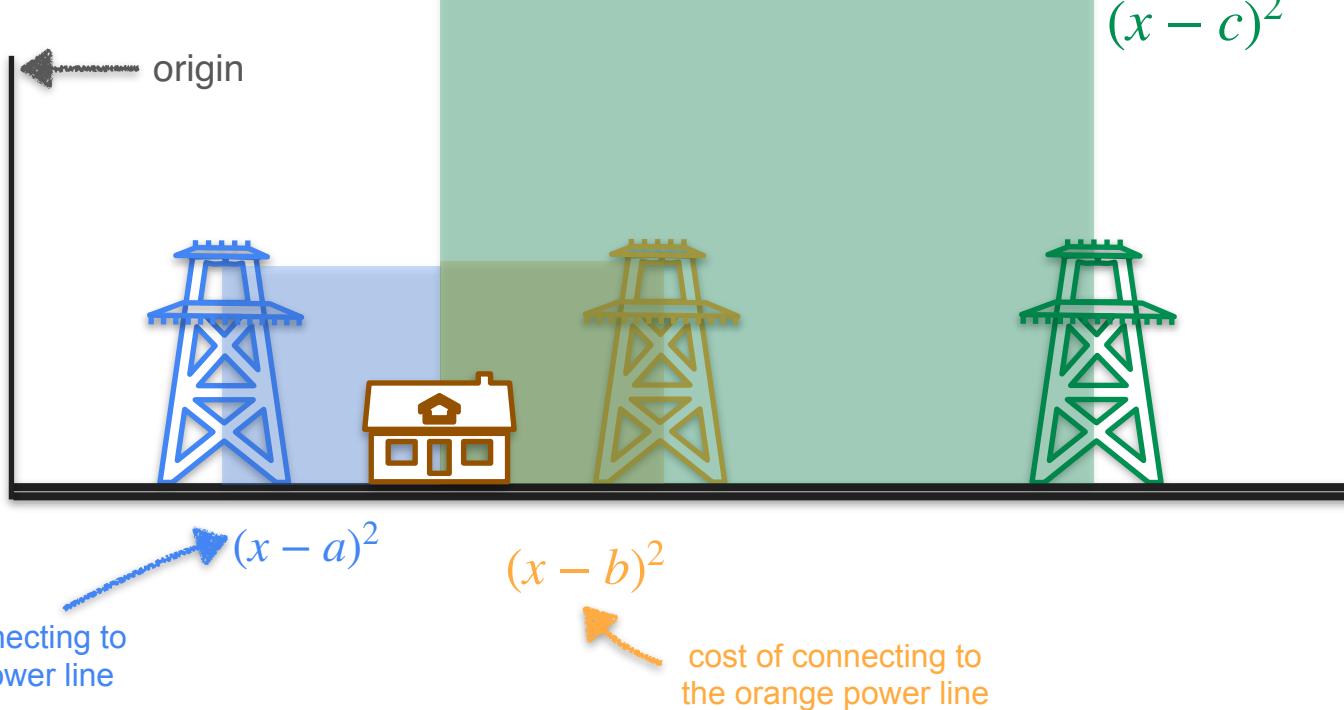
$(x - b)^2$

cost of connecting to  
the orange power line

# Three Power Line Problem



# Three Power Line Problem

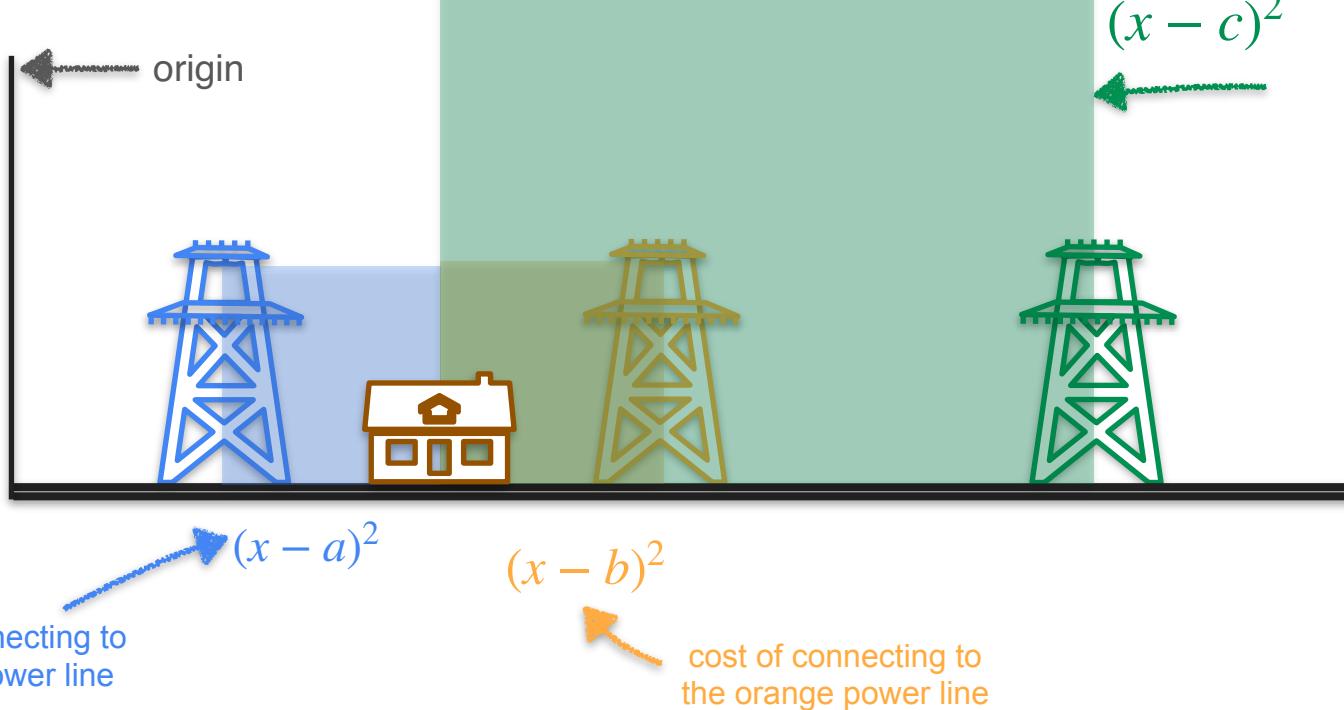


cost of connecting to  
the blue power line

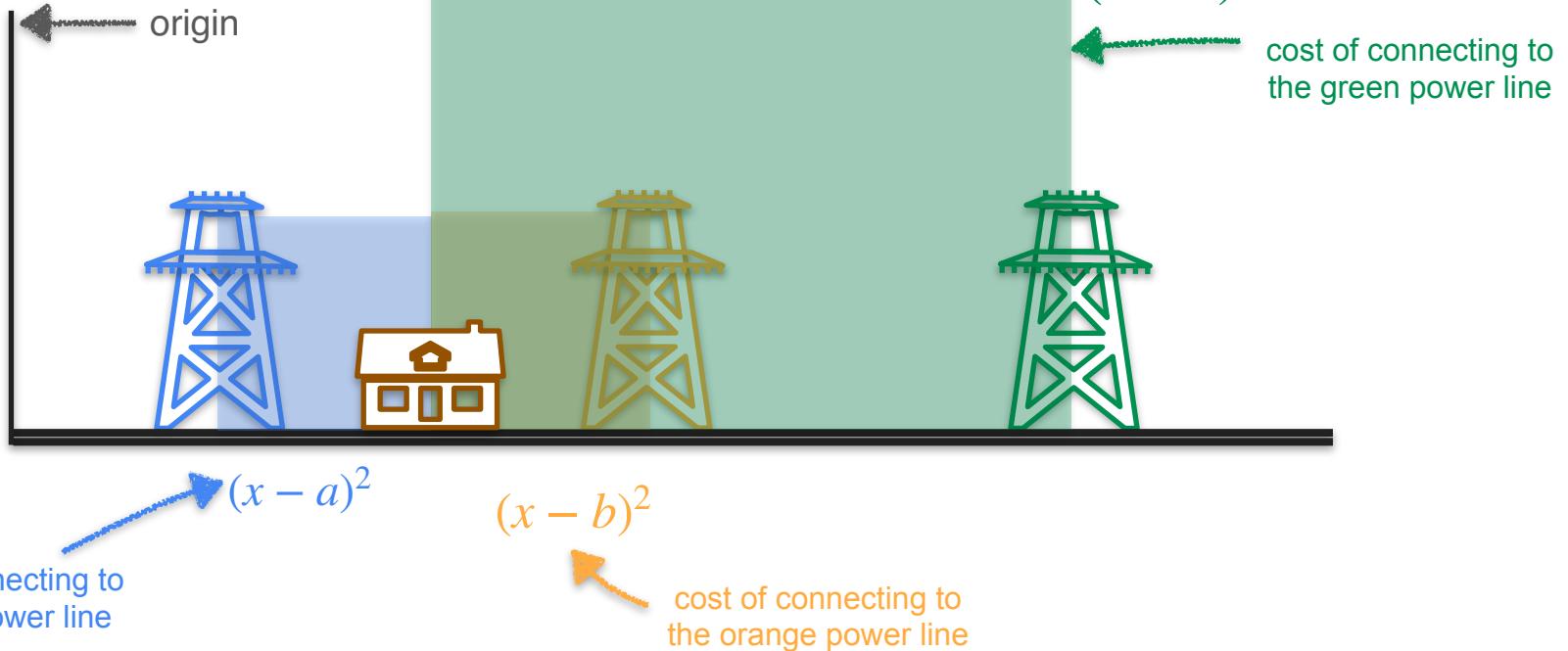
$(x - b)^2$

cost of connecting to  
the orange power line

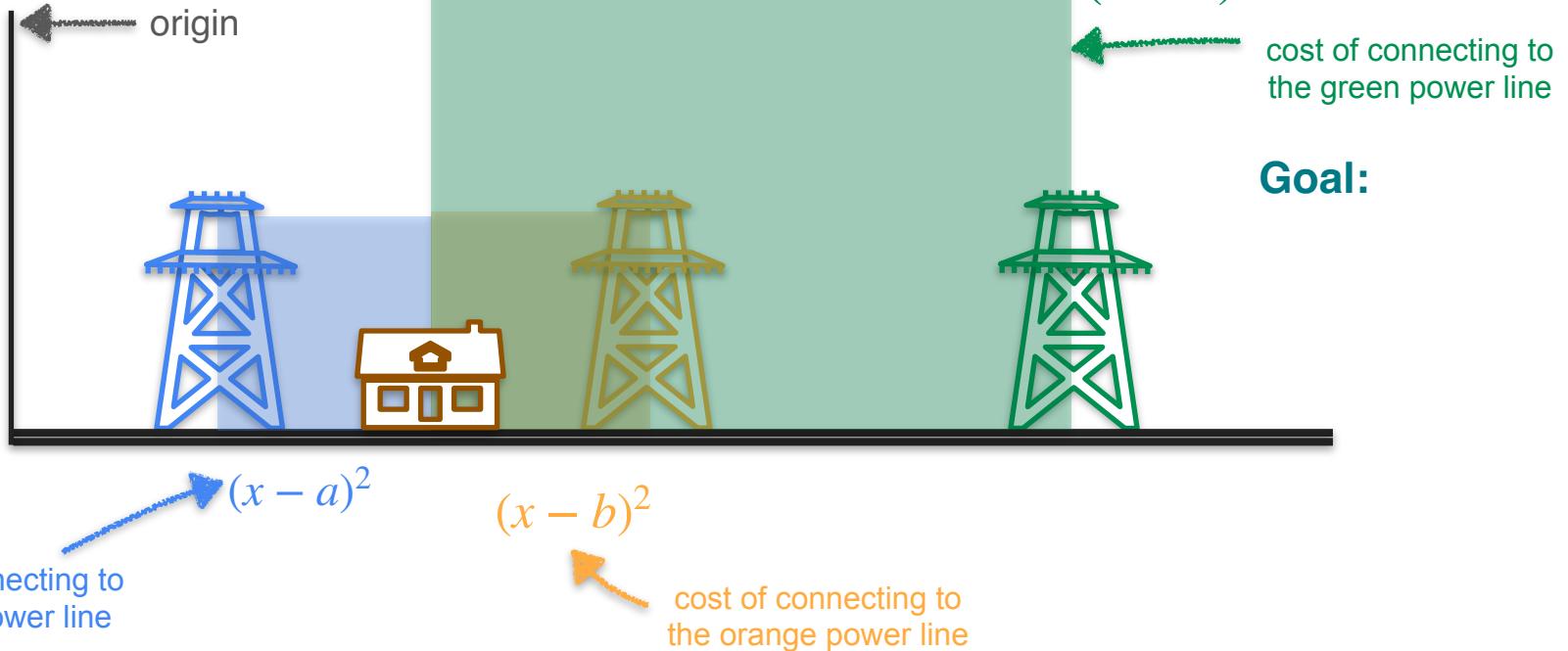
# Three Power Line Problem



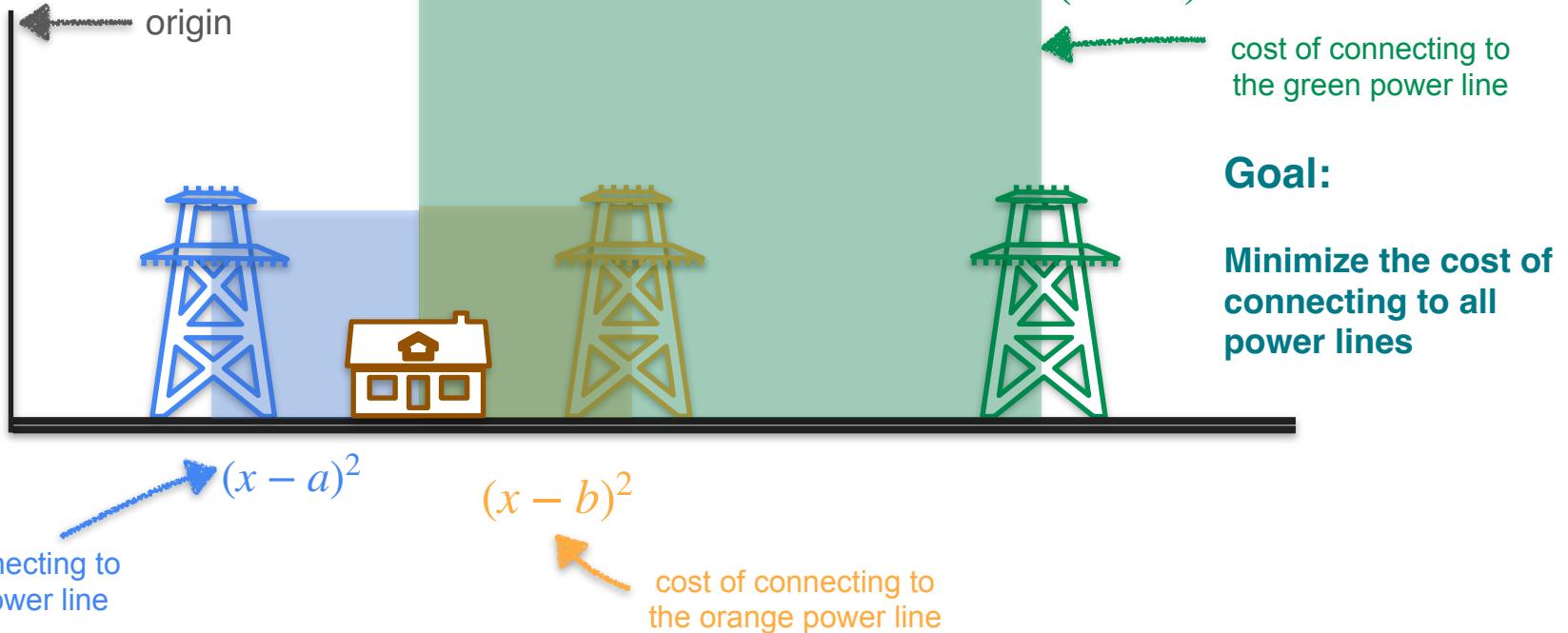
# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



cost of connecting to  
the blue power line

$(x - b)^2$

cost of connecting to  
the orange power line

What is the cost function of  
this problem?

$(x - c)^2$

cost of connecting to  
the green power line

**Goal:**

**Minimize the cost of  
connecting to all  
power lines**

# Three Power Line Problem



**Goal:**

**Minimize the cost of  
connecting all power  
lines**

$$(x - a)^2$$

$$(x - b)^2$$

$$(x - c)^2$$

# Three Power Line Problem



**Goal:**

Minimize the cost of  
connecting all power  
lines

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$

# Three Power Line Problem



**Goal:**

**Minimize the cost of  
connecting all power  
lines**

$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

# Three Power Line Problem



**Goal:**

**Minimize the cost of  
connecting all power  
lines**

$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

# Three Power Line Problem



**Goal:**

**Minimize the cost of  
connecting all power  
lines**

$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

# Three Power Line Problem



$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Total area of squares

Where should you put the house to minimize the cost?

# Three Power Line Problem



$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2$$

Minimize this cost function

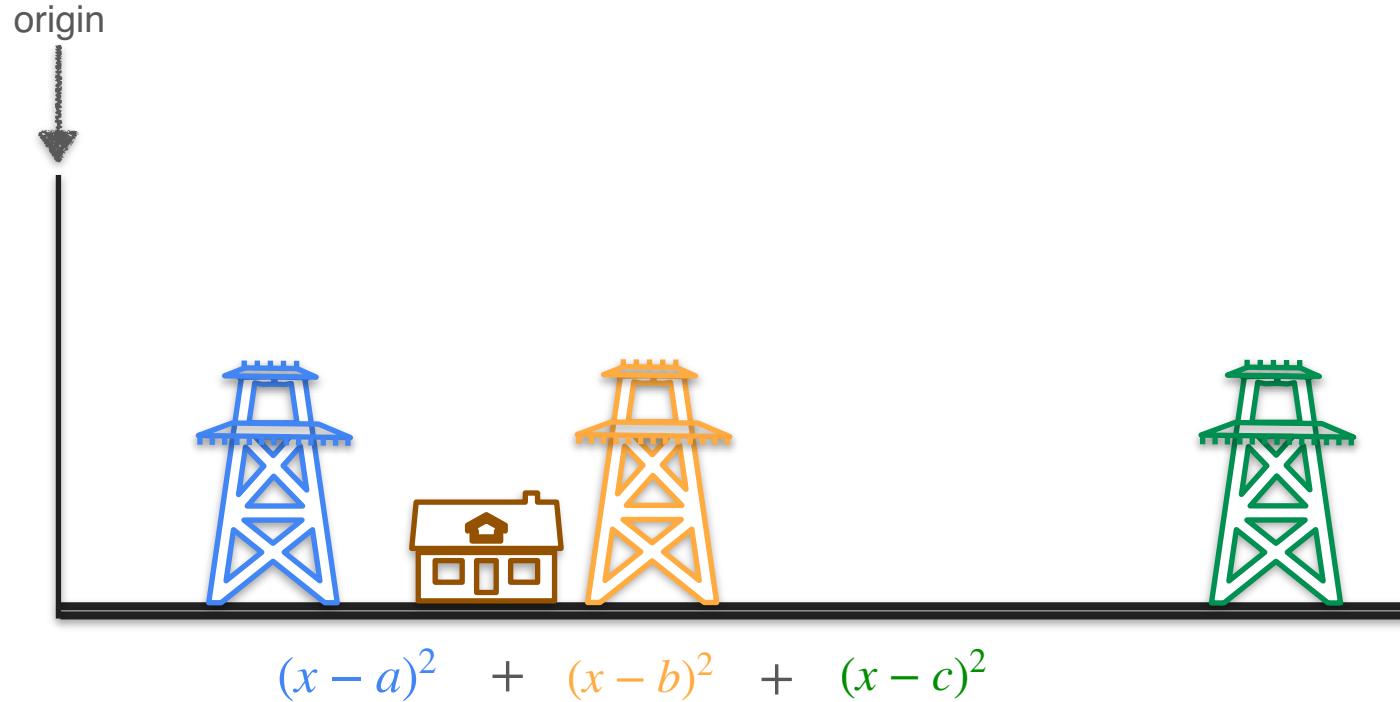
Solution:

# Three Power Line Problem

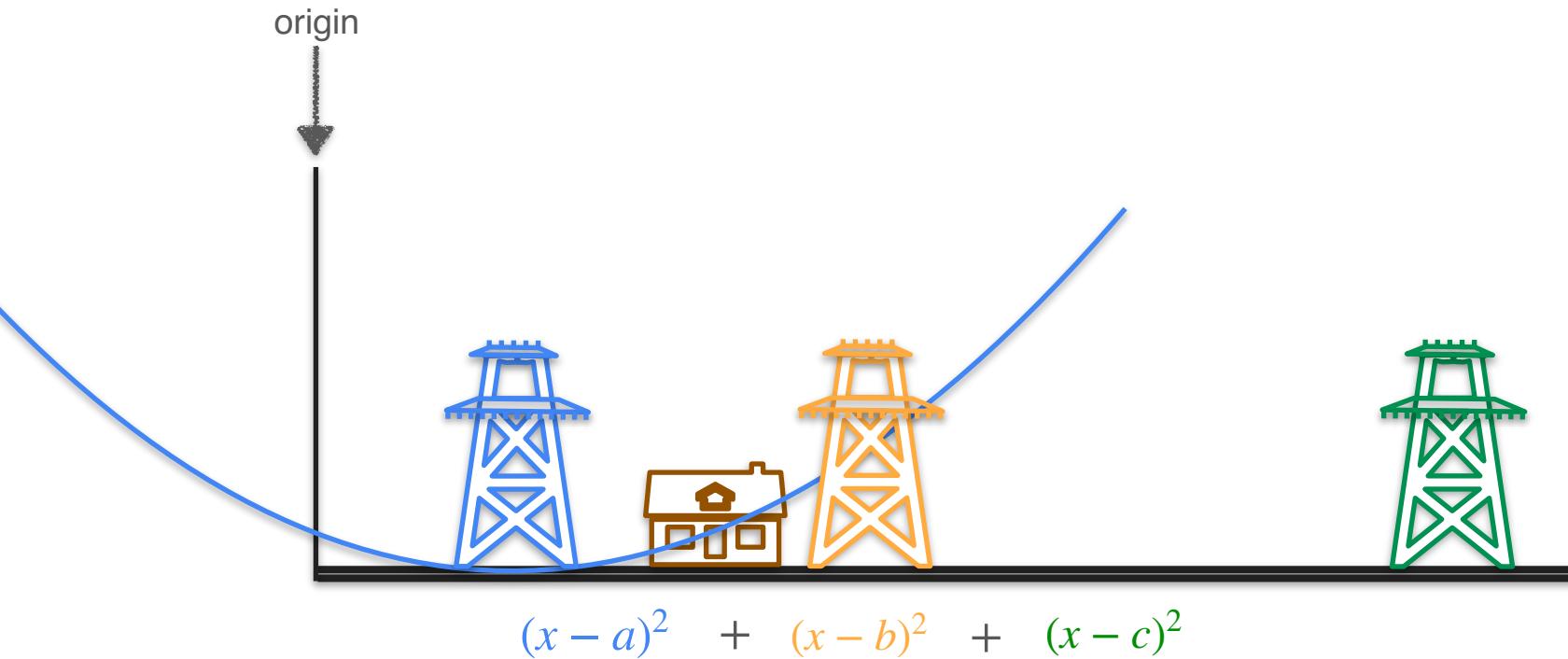


$$\text{Cost function} = (x - a)^2 + (x - b)^2 + (x - c)^2 \quad \leftarrow \text{Minimize this cost function}$$
$$\text{Solution: } x = \frac{a + b + c}{3}$$

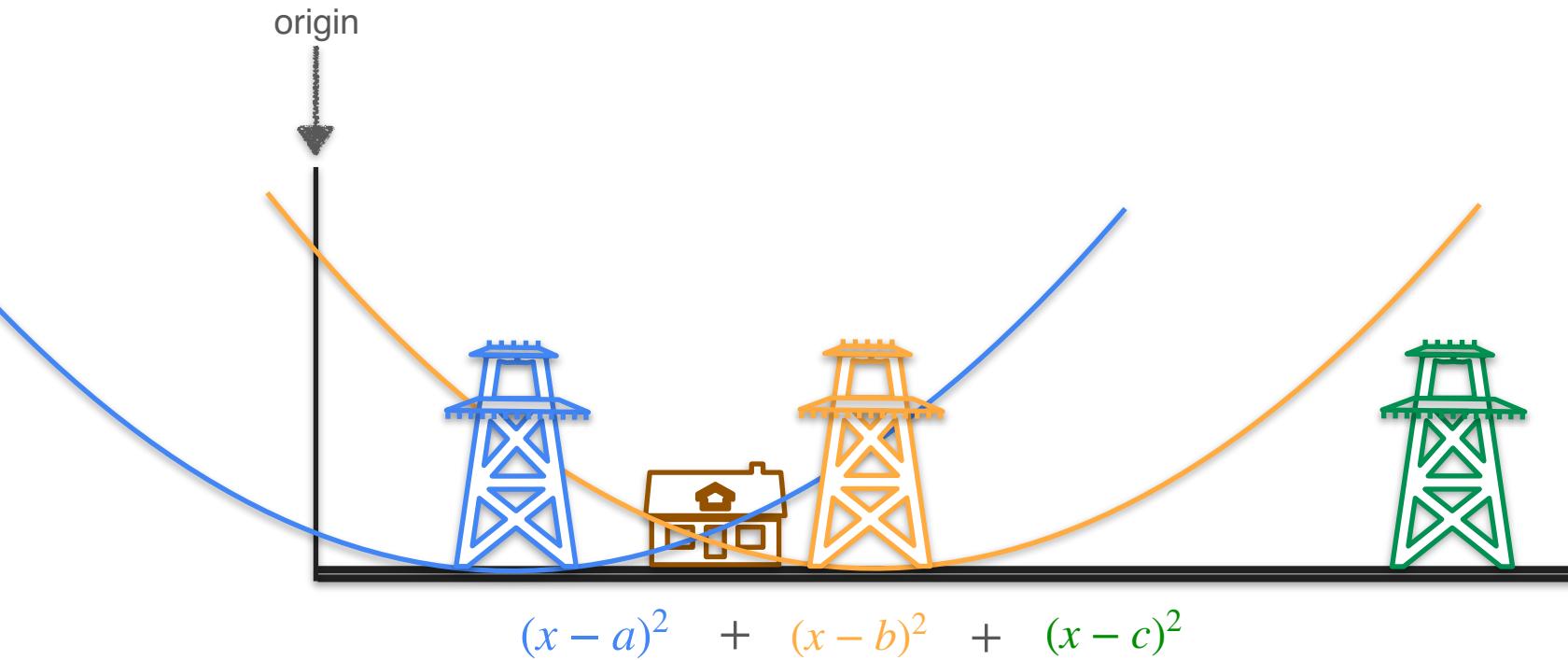
# Three Power Line Problem



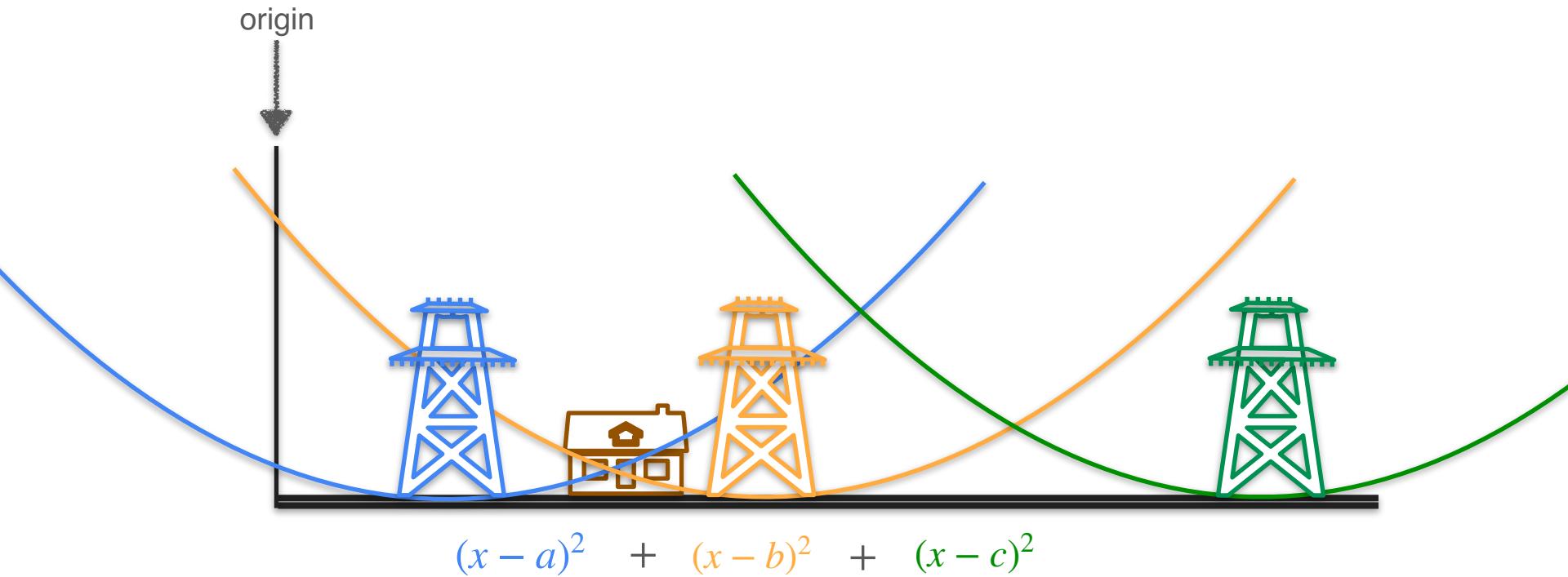
# Three Power Line Problem



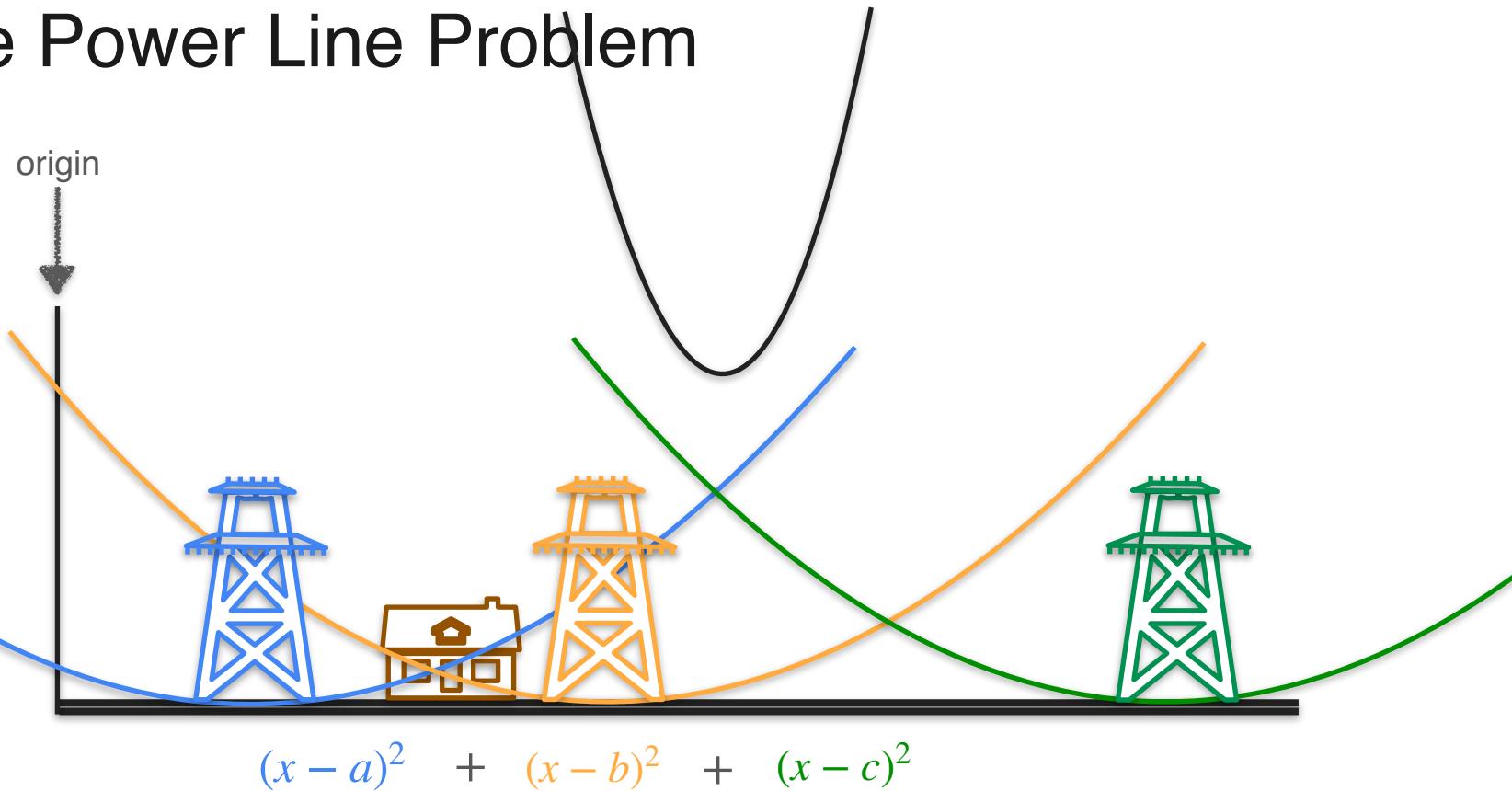
# Three Power Line Problem



# Three Power Line Problem



# Three Power Line Problem



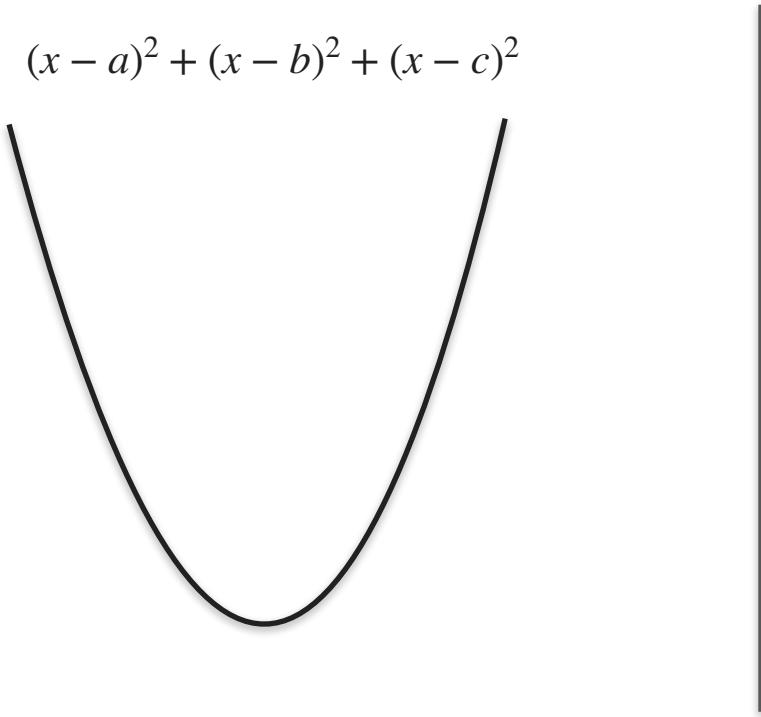
# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



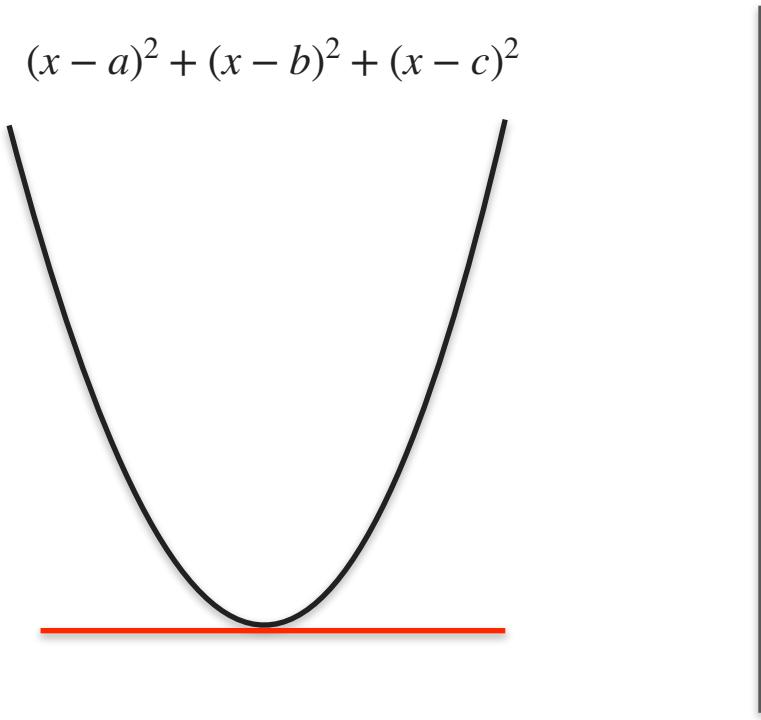
# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



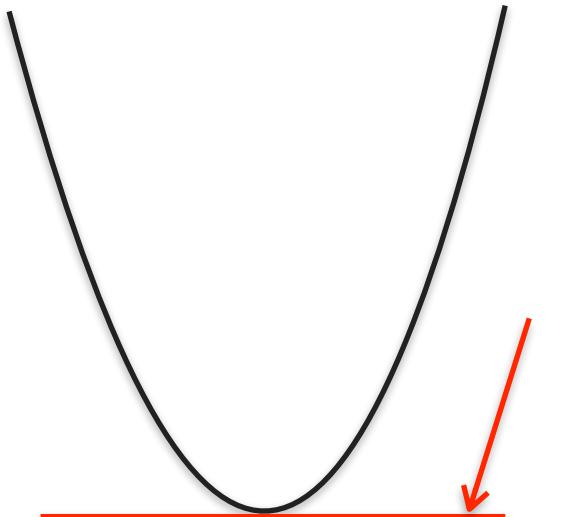
# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



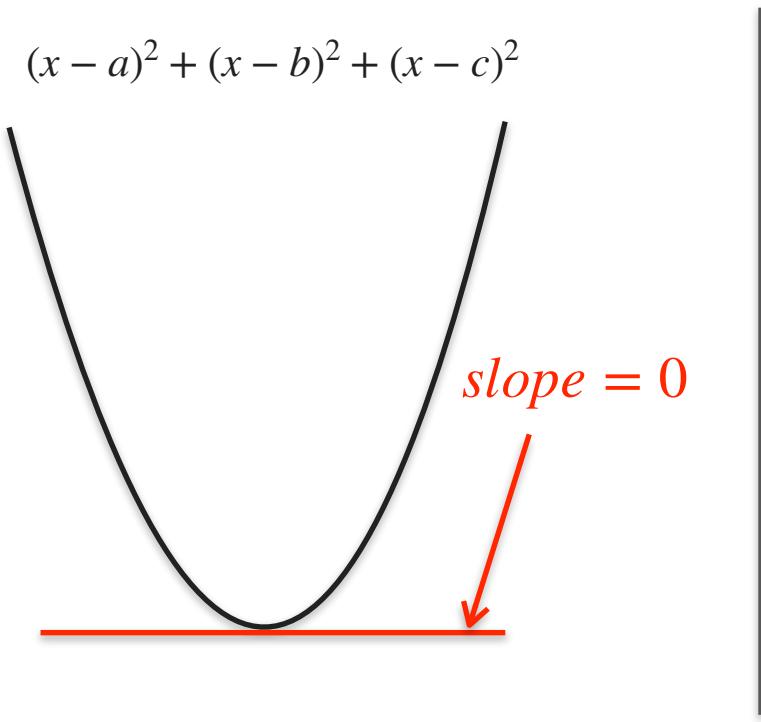
# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



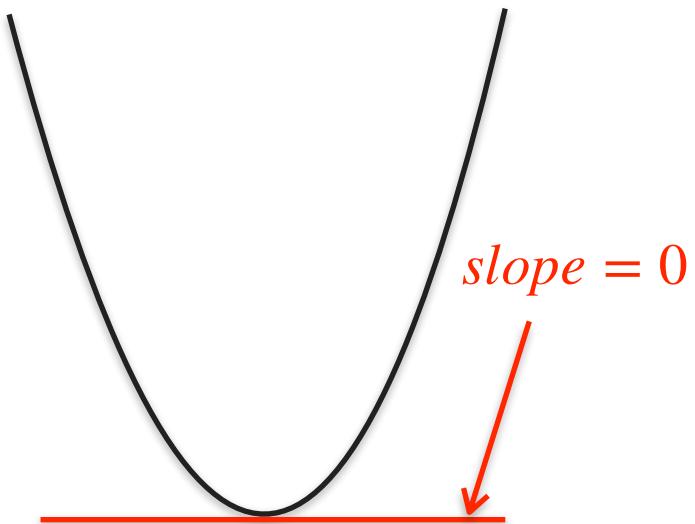
# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



# Three Power Line Problem

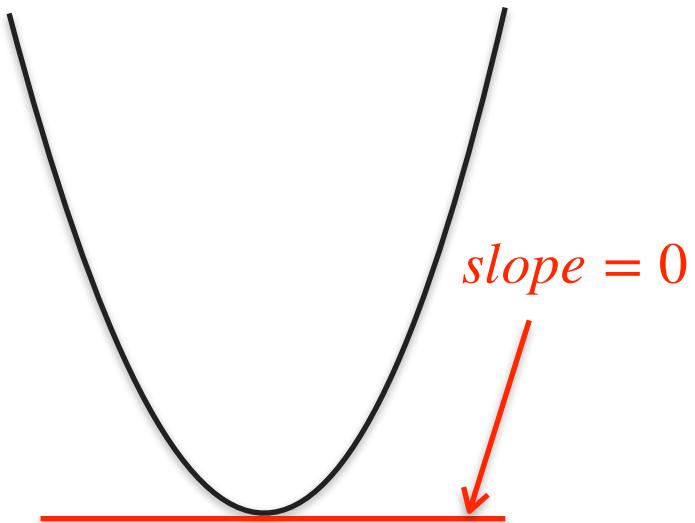
$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$

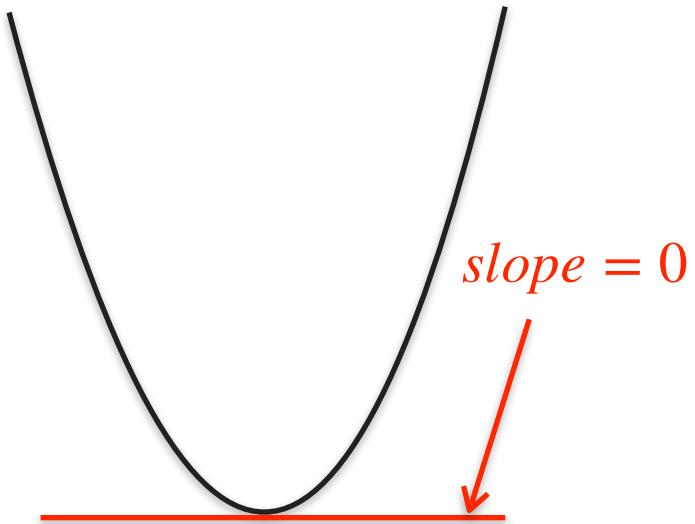


$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



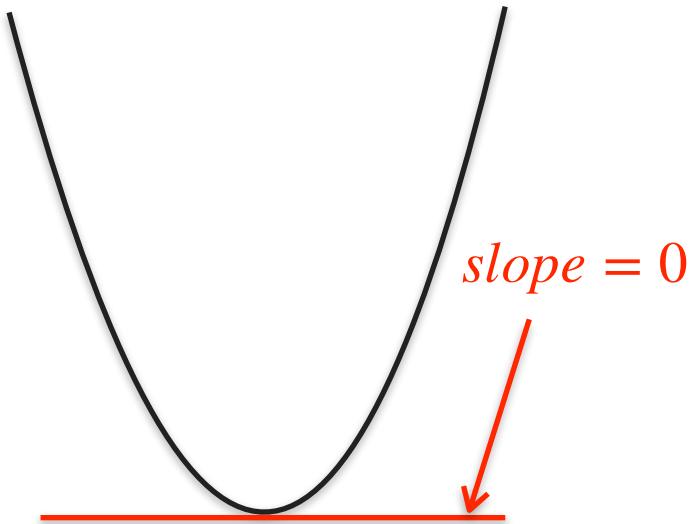
$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

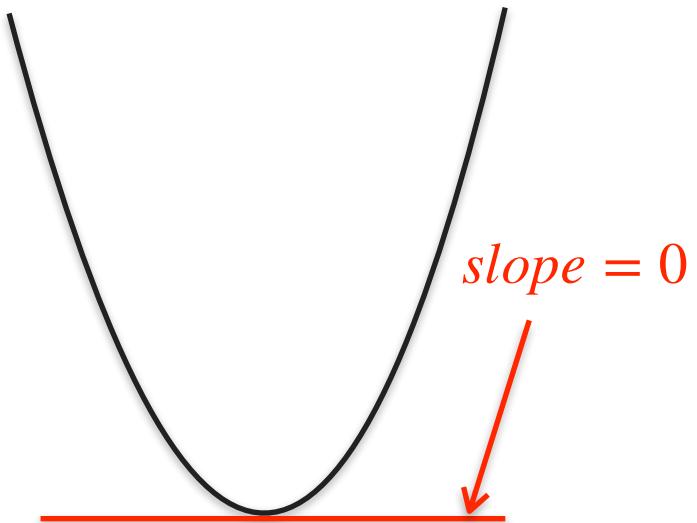
$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

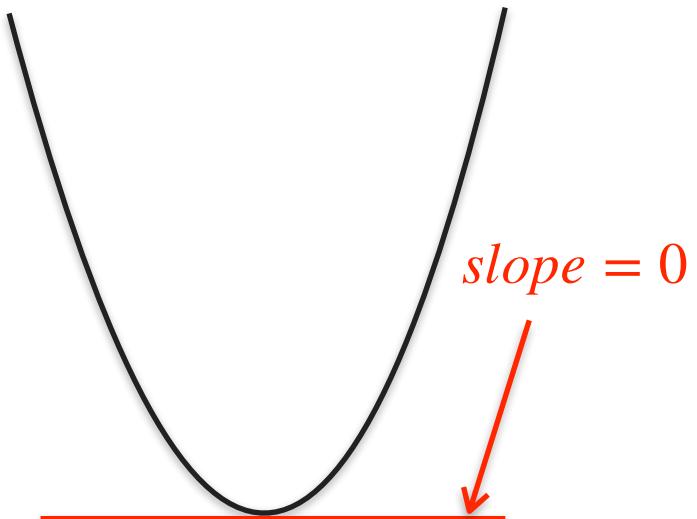
$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

$$3x = a + b + c$$

# Three Power Line Problem

$$(x - a)^2 + (x - b)^2 + (x - c)^2$$



$$\frac{d}{dx} [(x - a)^2 + (x - b)^2 + (x - c)^2] = 0$$

$$2(x - a) + 2(x - b) + 2(x - c) = 0$$

$$(x - a) + (x - b) + (x - c) = 0$$

$$3x - a - b - c = 0$$

$$3x = a + b + c$$

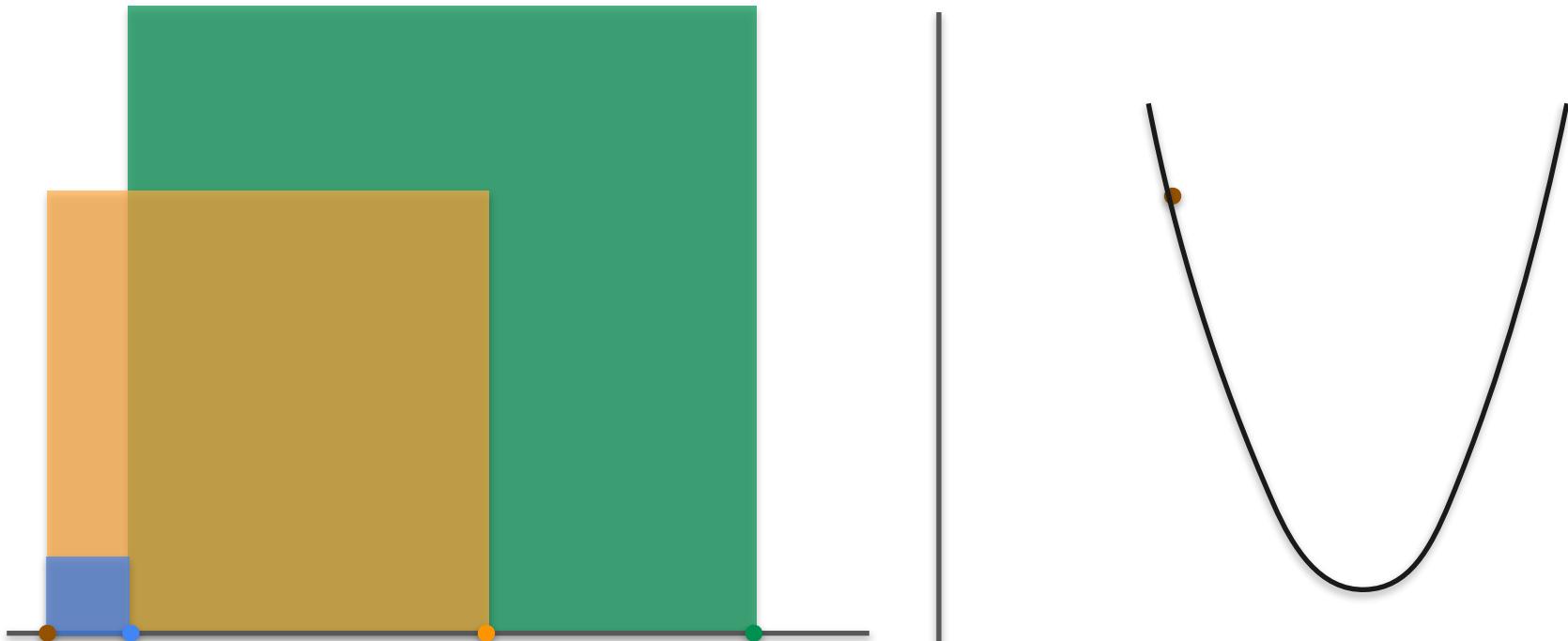
$$x = \frac{a + b + c}{3}$$

# Three Power Line Problem: Square Analogy



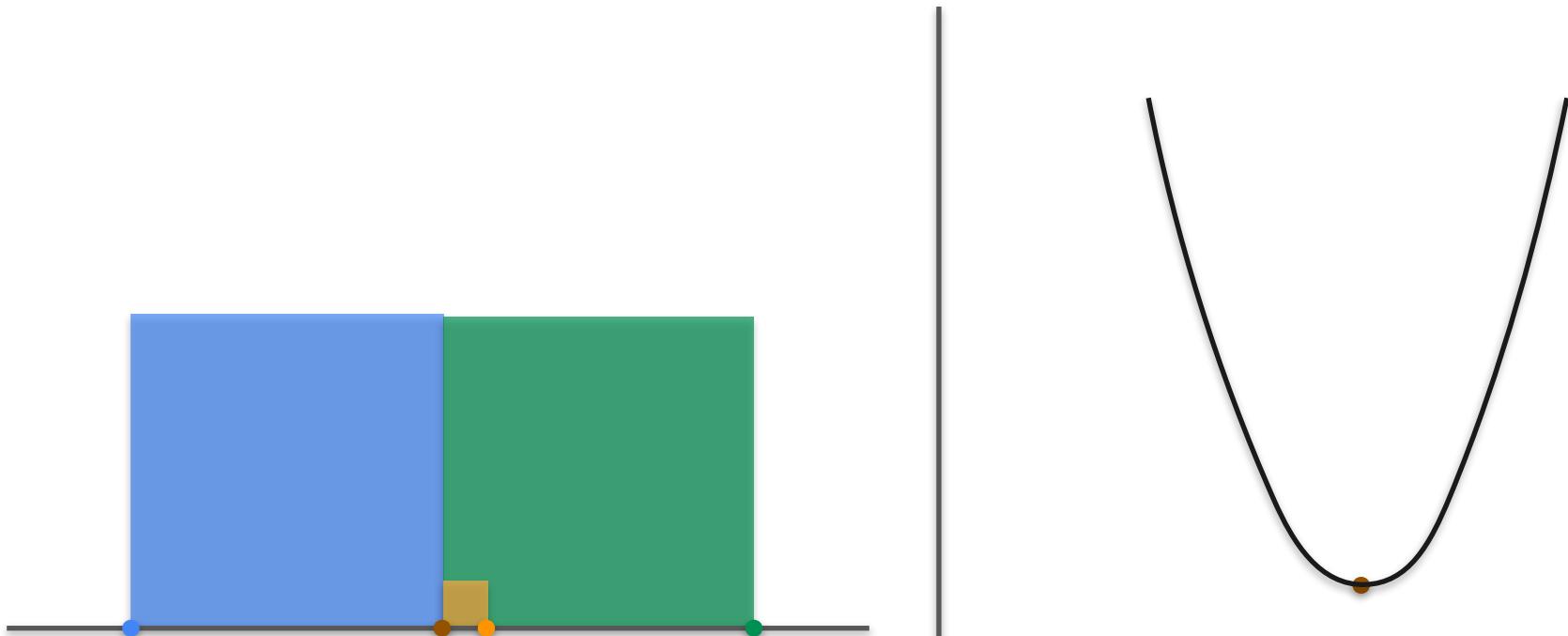
**Problem: Minimize total area of the squares**

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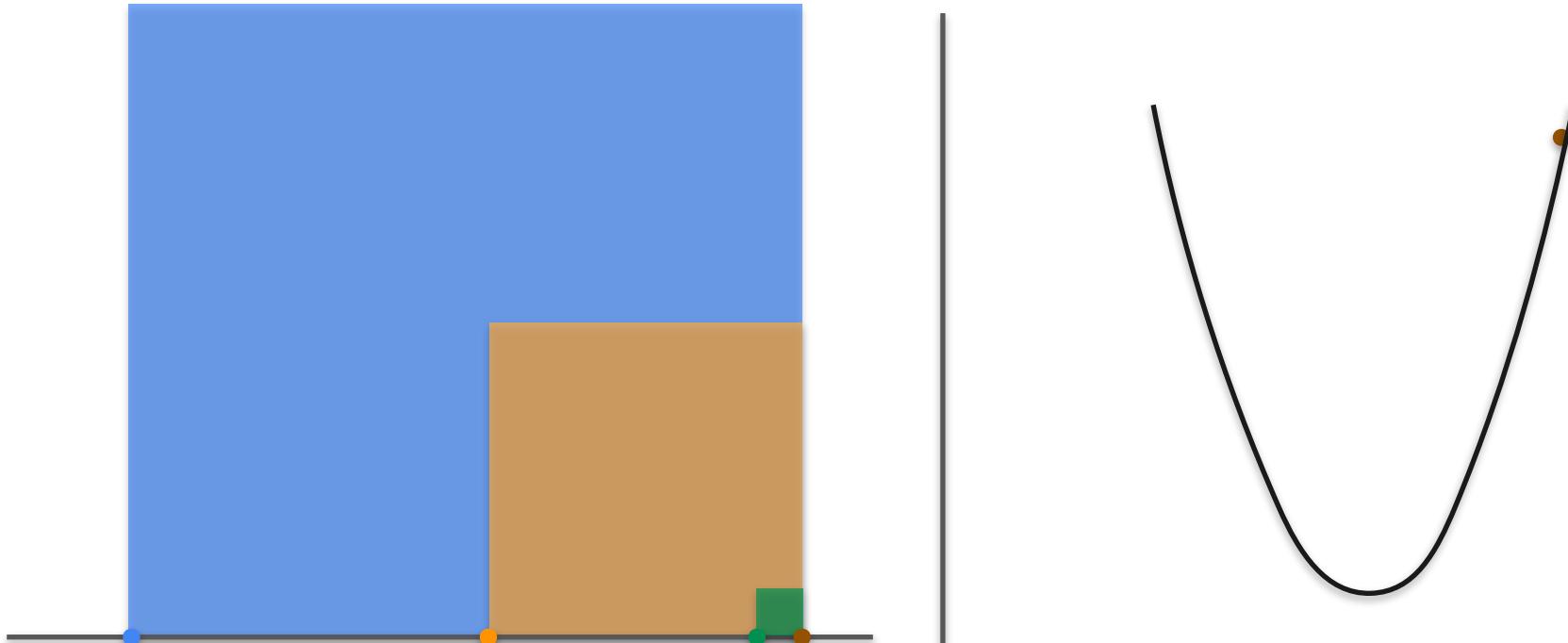
Problem: Minimize total area of the squares

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# The Square Loss

Minimize  $(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2$

# The Square Loss

Minimize  $(x - a_1)^2 + (x - a_2)^2 + \cdots + (x - a_n)^2$

Solution:  $x = \frac{a_1 + a_2 + \cdots + a_n}{n}$



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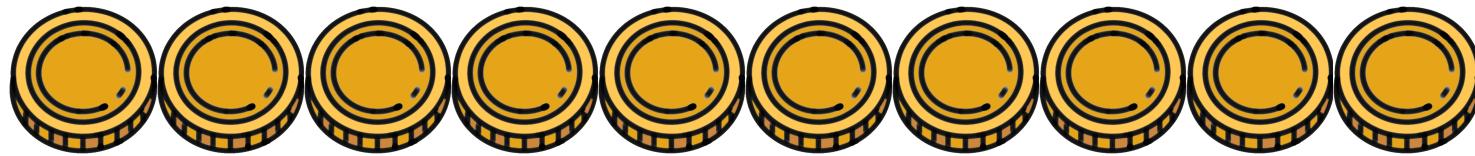
# Derivatives and Optimization

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## Optimization of log-loss Part 1

# Coin Toss

# Coin Toss



# Coin Toss



# Coin Toss



Coin 1



70%    30%

# Coin Toss



Coin 1



70%    30%

Coin 2



50%    50%

# Coin Toss



Coin 1



70%    30%

Coin 2



50%    50%

Coin 3



30%    70%

# Quiz

- Which of the three coins would you choose to maximize your chances of winning?

Coin 1



70%    30%

Coin 2



50%    50%

Coin 3



30%    70%

# Coins Toss



# Coins Toss



Coin 1  $0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3$   
 $= 0.00222$

# Coins Toss



$$\text{Coin 1} \quad 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3 \\ = 0.00222$$

$$\text{Coin 2} \quad 0.5 \times 0.5 = 0.5^7 0.5^3 \\ = 0.00097$$

# Coins Toss



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$$\text{Coin 3} \quad 0.3 \times 0.7 \times 0.7 \times 0.7 = 0.3^7 0.7^3 \\ = 0.00008$$

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# Coin Toss

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$p$

# Coin Toss



$p$        $(1 - p)$

# Coin Toss



$p$        $(1 - p)$

Chances of winning:  $p^7(1 - p)^3 = g(p)$

# Coin Toss



$p$        $(1 - p)$

Chances of winning:  $p^7(1 - p)^3 = g(p)$

Goal: maximize  $g(p)$

# Coin Toss

# Coin Toss

$$\frac{dg}{dp} = \frac{d}{dp}(p^7(1-p)^3)$$

# Coin Toss

Product rule

$$\frac{dg}{dp} = \frac{d}{dp}(p^7(1-p)^3) \stackrel{\text{Product rule}}{=} \frac{d(p^7)}{dp}(1-p)^3 + p^7 \frac{d((1-p)^3)}{dp}$$

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Chain rule

# Coin Toss

Product rule

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$\downarrow$

$$p = 0$$

$\rightarrow$

$$p = 1$$

$\rightarrow$

$$p = 0.7$$

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$p \neq 0$

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$\log(g(p))$

# Coin Toss

$$\log(g(p)) = \log(p^7(1-p)^3)$$

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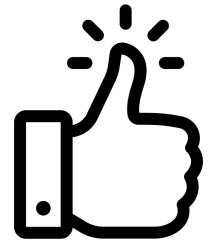
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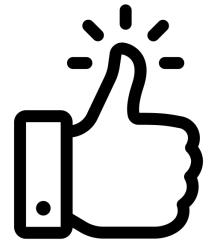
$$= 7\log(p) + 3\log(1-p) = G(p)$$

$-G(p)$  is the logloss

$$\frac{dG(p)}{dp} = \frac{d}{dp}(7\log(p) + 3\log(1-p)) = 7\frac{1}{p} + 3\frac{1}{1-p}(-1)$$

$$= \frac{7(1-p) - 3p}{p(1-p)} = 0$$

$$7(1-p) - 3p = 0 \quad p = 0.7$$





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# Derivatives and Optimization

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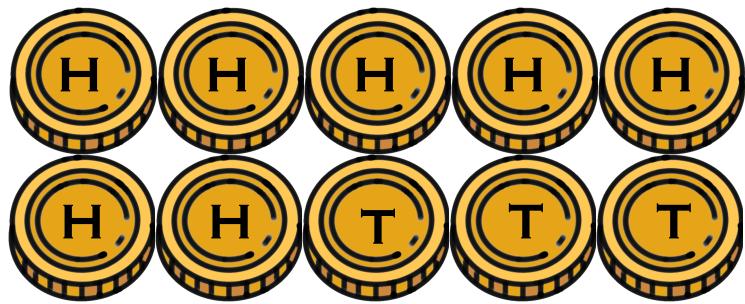
**Optimization of log-loss  
Part 2**

# Relationship With ML

# Relationship With ML



# Relationship With ML



Data

# Relationship With ML



Data

# Relationship With ML

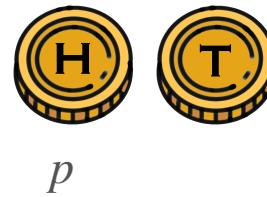


Data

# Relationship With ML



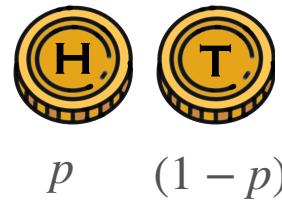
Data



# Relationship With ML



Data



# Relationship With ML



Data



Model

# Relationship With ML



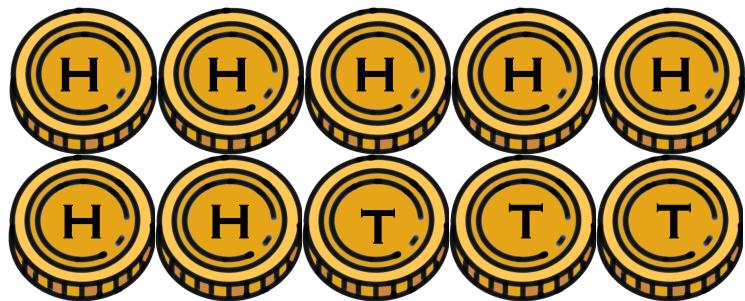
Data



Model

Minimized log-loss

# Relationship With ML



Data

Minimized log-loss



Model

$$p = 0.7$$

# Why the Logarithm?

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1. Derivative of products is hard, derivative of sums is easy

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$$f(p) = p^6(1 - p)^2(3 - p)^9(p - 4)^{13}(10 - p)^{500}$$

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$$f(p) = p^6(1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500}$$

$$\frac{df}{dp}$$



$$[6p^5](1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500} +$$

$$p^6 [2(1-p)](3-p)^9(p-4)^{13}(10-p)^{500}(-1) +$$

$$p^6(1-p)^2[9(3-p)^8](p-4)^{13}(10-p)^{500}(-1) +$$

$$p^6(1-p)^2(3-p)^9[13(p-4)^{12}](10-p)^{500} +$$

$$p^6(1-p)^2(3-p)^9(p-4)^{13}[500(10-p)^{499}](-1)$$

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1. Derivative of products is hard, derivative of sums is easy

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$$\frac{df}{dp}$$



$$[6p^5](1-p)^2(3-p)^9(p-4)^{13}(10-p)^{500} +$$

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$$p^6(1-p)^2(3-p)^9(p-4)^{13}[500(10-p)^{499}](-1)$$



$$\frac{d}{dp} \log(f)$$



$$\frac{6}{p} + \frac{2}{1-p}(-1) + \frac{9}{3-p}(-1) +$$

$$\frac{13}{p-4} + \frac{500}{10-p}(-1)$$

# Why the Logarithm?

1. Derivative of products is hard, derivative if sums is easy

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1. Derivative of products is hard, derivative if sums is easy

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$$\frac{df}{dp}$$



$$\frac{d}{dp} \log(f)$$



2. Product of lots of tiny things is tiny!



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# Derivatives and Optimization

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## Conclusion