

Binomial Distribution Questions:

A coin is tossed 8 times and counting the number of tails. Find the probability of getting exactly 5 tails.

Solution:

We know that,

$$P(x) = ({}^nC_x) p^x (1-p)^{(n-x)}$$

Here, $x = 5$ and $n = \text{no of events} = 8$

$P = \text{probability of success} = \frac{1}{2} = 0.5$

$(1-p) = \text{not getting exactly 5 tail} = 1-0.5 = 0.5$

Substitute all the values in the formula we get,

$$\begin{aligned} P(x = 5) &= {}^8C_5 (0.5)^5 (0.5)^{(8-5)} \\ &= 56 * 0.125 * 0.03125 \\ &= 0.21875 \end{aligned}$$

A test contains 10 multiple choice questions. Each question has five choices for the correct answer. Only one of the choice is correct. What is the probability of getting 70% with random guessing ?

Solution:

$$P(x) = (nCx) p^x (1-p)^{(n-x)}$$

Here, $x = (70/100) * 10 = 7$ and $n = \text{no of events} = 10$

$P = \text{probability of success} = 1/5 = 0.2$

$$P(x = 7) = (10C7) (0.2)^7 (1-0.2)^{(10 - 7)}$$

$$= 120 * 0.0000128 * 0.512$$

$$= 0.0007864$$

**An experiment consist of free throw shot, probability of making it is 25%.
If 15 shots are taken, find the probability of making less than 3 shots.**

Solution:

The possible outcomes that will make this happen are 2 shots, 1 shot, and 0 shots.

Since these are mutually exclusive, we can add these probabilities.

$$P(x < 3) = P(x = 0) + P(x = 1) + P(X = 2)$$

$$= {}^{15}C_2 (0.25)^2 (0.75)^{13} + {}^{15}C_1 (0.25)^1 (0.75)^{14} + {}^{15}C_0 (0.25)^0 (0.75)^{15}$$

$$= 0.156 + 0.067 + 0.013 = 0.236$$

There is a 24 percent chance of sinking less than 3 shots.

A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let X denote the number of heads that come up.

Calculate:

- (i) $P(X = 2)$
- (ii) $P(X = 3)$
- (iii) $P(1 < X \leq 5)$.

Solution:

(i) If we call heads a success then this X has a binomial distribution with parameters $n = 6$ and $p = 0.3$.

$$P(X = 2) = 0.324135$$

$$(ii) P(X = 3) = 0.18522:$$

(iii) We need $P(1 < X \leq 5)$

$$\begin{aligned} &P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= 0.578 \end{aligned}$$

A Math quiz consists of **10 multiple-choice questions**. Each question has **five possible answers**, only one of which is correct. Suppose a student plans to **guess the answer** to each question.

(i) What is the probability that the student gets no answers correct?

(ii) What is the probability that the student gets two answers correct?

Solution:

This is a binomial experiment,

Where:

$n = \text{number of trials} = 10$

“success” of each trial is “correctly guessing the answer”.

Each answer is independent of the others.

Since each answer is guessed and there are five choices for each question, we have

$P(\text{success}) = 1/5 = 0.20$

So, $n=10$, and $P(\text{success}) = 0.20$

1) What is the probability that the student gets *no answers* correct?

We need to find,

$$P(X=0) = P(0) = 0.1074$$

Hence the student has 11% chance of getting no answer correct.

2) What is the probability that the student gets *two answers* correct?

Here we need to find,

$$P(X=2) = 0.3020$$

Hence the student has 30% chance of getting exactly two answers correct using the guessing strategy.

A product-quality researcher runs a study on a particular brand of Memory Chips. The Memory chip have a 1% probability of being defective. What is the probability that at most one chip is defective in a batch of 25 chips?

Solution:

Here it is a binomial distribution,
Given that, 25 chips being checked one-by-one. Each chip either works (success) or not (failure).

x = number of defective chips in a batch of 25.

$$P(x > 1) = 1 - P(x \leq 1) = 1 - 0.9742 = 2.58\%$$