

Converting the given equations into mathematical equations

Left side are the words used in the sentences – Right side will show the mathematical operator for the words mentioned.

(i) “Has the same value” - both the quantities are equal.

Example : m/n has the same value as $2/3$

It means , $m/n = 2/3$

(ii) “of ” - Multiply

Example : Find the value of $3/2$ of $4/3$

It means, $3/2 * 4/3 = 2$

(iii) “times” (or) “as much as”- Multiply

Example : 2 times p

It means, $2 * p = 2p$

(iv) “is” - equal to

Example : m is $9/4$

It means , $m = 9/4$

(v) If 20% more than (or larger than) n

Before analyzing how to form an equation, we will relate this with an easy statement.

Case 1 :

If I am having 5 chocolates and you are having 1 chocolate more than me.

Then how many you have on the whole?

It is 6 chocolates , Right?

Which means, $5 + 1$ (My amount of chocolate + your extra amount of chocolate)

Case 2 :

If I am having 5 chocolates and you have 20 % more than me.

In this case, how many you will have?

Its $5 + 20\%$ of 5 , Right?

Likewise, follow the same logic for the above question: “If 20% more than n ”

Relate this to the above case, 20% is extra and n is my amount of chocolate.

So, it should be $n + 20\% n$

(vi) Lets complicate the above statement.

“If m is 20% more than n ”

“is” in the sentence , means “equal”, so $m = n + 20\% n$

(vii) If x is 100% more than y

It is nothing but, x is twice as y.
“ $x = 2y$ ”

Now consider the problem:

If the ratio of 2x to 5y is 3 to 4, what is the ratio of x to y ?

Solution : The ratio of 2x to 5y is 3 to 4Statements

→ the ratio of 2x to 5y is the ratio 3 to 4

→ (ratio of 2x to 5y) = (ratio 3 to 4)

→ $\frac{2x}{5y} = \frac{3}{4}$ Mathematical equation

What is to be found?

We have to find “the ratio of x to y”

→ $\frac{x}{y} = ?$

Now $\frac{2x}{5y} = \frac{3}{4}$

→ $\frac{x}{5y} = \frac{3}{8}$

→ $\frac{x}{y} = \frac{15}{8}$

Hence the answer.

Now how to select the variables in the worded questions:

For a given two-digit positive integer, the tens digit is 5 greater than the units digit. The sum of the digits is 11. Find the integer.

a) Always remember : In the worded problems variables are assigned to the parameters whose values are unknown.

In the question, the two digits of positive integer is unknown.

Let us assign the variables. Let 'x' be the digit in the unit place and let 'y' be the digit in the tens digit. Hence the integer will be $10y + x$.

b) What are the conditions given here?

i) the tens digit is 5 greater than the units digit

→ y is 5 greater than x

→ $y = x + 5$

ii) the sum of the digits is 11

→ $x + y = 11$

we got equations with two variables:

Solving both the equations we get, $x = 3$ and $y = 8$

Hence the integer will be $10y + x = 10 \times 8 + 3 = 83$

If 3 times Jane's age, in years, is equal to 8 times Beth's age, in years, and the difference between their ages is 15 years, how old are Jane and Beth?

a) What is unknown here?

Unknown is Jane's age and Beth's age.

Now assign the variables

Let 'x' denotes Jane's age in years and 'y' denotes Beth's age in years

b) What are the conditions ?

i) 3 times Jane's age, in years, is equal to 8 times Beth's age, in years

→ (3 times Jane's age, in years) is equal to (8 times Beth's age, in years)

→ $3x = 8y$

One question arises here . Whether Jane is elder or Beth is elder ? How to find it?

We have one equation $3x = 8y$

$3x = 8y$

$x = 8 \text{ over } 3 y$

Note that $8 \text{ over } 3 > 1 \rightarrow 8 \text{ over } 3 y > y$

Hence $x > y$

that means Jane is elder than Beth.

ii) the difference between their ages is 15 years

the difference between Jane's age and Beth's age
= (Jane's age - Beth's age) or (Beth's age - Jane's age)
= $(x - y)$ or $(y - x)$

Since Jane is elder than Beth implies the difference between Jane's age and Beth's age
= (Jane's age - Beth's age)
= $(x - y)$
= 15

Hence $x - y = 15$

The two equations are

- i) $3x = 8y$
- ii) $x - y = 15$

Now solve both the equations
 $x = 24$ and $y = 9$

I will give you a few tips to solve an inequality problem to try out with other such problems.

(I) Suppose the question requires you to define the value of an integer x , if it is given that $2 < x < 5$ and $3 < 2x < 4$

If you are able to define the value of x as an integer, x can take values such as $\{....., -2, -1, 0, 1, 2,\}$

Then we can say that both the above ranges for x can define it.

When we take the above example

Given $2 < x < 5$:

Here x can be 3 and 4

It means that both are integers

Therefore, this defines x as an integer.

Also, given $3 < 2x < 4$

x cannot be 1, because $2x = 2(1) = 2$, which is not greater than 3.

x cannot assume values greater than or equal to 2, because if we substitute them, the second part of the condition (less than 4) will not be satisfied. Hence, in this case, x can assume any value from 1.6 to 1.9, which are not integers.

Therefore, the second part does not define the value of x as an integer.

Hence, for this problem, we should mark the answer as “only 1 and not 2”.

(II) Suppose the question requires you to check for some multiples, such as “x should be a multiple of 2”, we should check for that condition.

(III) If the data given is $2 < x < 5$ and $3 > y > 7$ and you are asked to find the maximum value of x-y. First think logically—they have asked for maximum value; so we need to find when x-y will be maximum.

**x-y will be maximum, when x is maximum and y is minimum.
So, find the values accordingly.**

When you are required to find the minimum value of x-y, it is just the opposite—x should be minimum and y should be maximum.

Now let us concentrate on how to calculate Percentages:

The formula for percentage increase and decrease are formed with the help of what is referred to as what.

Lets see few examples:

(i) By what percentage A is greater than B

Here, A is compared with B and B is the "Reference"

Numerator should be the "difference between them" and Denominator should be "Reference"

Therefore, it is $\frac{A-B}{B} \times 100$

(ii) By What percentage A is less than B.

B is the Reference.

Therefore, $\frac{|A-B|}{B} \times 100$.

If Modulus is removed, then it should be $\frac{B-A}{B} \times 100$

(iii) By what percentage B is greater than A

Here, A is the Reference.

Therefore, $\frac{B-A}{A} \times 100$

(iv) By what percentage B is less than A.

Here, A is the Reference

Therefore, $\frac{A-B}{A} \times 100$

When you face a worded problem, first break it into phrases and for each phrase form an equation. Only if you form an equation, will you be able to solve the problem. To begin with, you may find it difficult to form equations. However, this can be overcome by practice.

For example, let us consider one of the questions you got wrong in this session:

“A mall sells a muffler at a loss of 6.75% . A hike of \$2.50 in its S.P converts the deal into 3.25% profit. How much will the muffler cost the mall?”

Now let us try breaking the question into phrases and forming equations.

A mall sells a muffler at a loss of 6.75% → S.P = C.P – 6.75% C.P -----(1)
A hike of \$2.50 in its S.P converts the deal into 3.25% profit → S.P + \$2.50 = C.P + 3.25% C.P----- (2)

How much will the muffler cost the mall → Find the cost price (C.P)

C.P can be found by solving both the equations.

Questions for practice:

1. Seven years ago, Scott was 3 times as old as Kathy was at that time. If Scott is now 5 years older than Kathy, how old is Scott?
2. The age of B is half the sum of the ages of A and C. If B is 2 years younger than A and C is 32 years old, then what is the age of B?
3. The ages of three people are such that the age of one person is twice the age of the second person and three times the age of the third person. If the sum of the ages of the three people is 33, then the age of the youngest person is ?
4. At a fruit stand, apples can be purchased for \$0.15 each and pears for \$0.20 each. At these rates, a bag of apples and pears was purchased for \$3.80. If the bag contained exactly 21 pieces of fruit, how many were pears?
5. A mixture of 12 ounces of vinegar and oil is 40 percent vinegar (by weight). How many ounces of oil must be added to the mixture to produce a new mixture that is only 25 percent vinegar?
6. How many ounces of a solution that is 30 percent salt must be added to a 50-ounce solution that is 10 percent salt so that the resulting solution is 20 percent salt?
7. Kathleen's weekly salary was increased 8 percent to \$237.60. What was her weekly salary before the increase?
8. A particular stock is valued at \$40 per share. If the value increases 20 percent and then decreases 25 percent, what is the value of the stock per share after the decrease?
9. Ellen has received the following scores on 3 exams: 82, 74, and 90. What score will Ellen need to attain on the next exam so that the average (arithmetic mean) for the 4 exams will be 85 ?
10. In a driving competition, Jeff and Dennis drove the same course at average speeds of 51 miles per hour and 54 miles per hour, respectively. If it took Jeff 40 minutes to drive the course, how long did it take Dennis?
11. If it takes 3 hours for machine A to produce N identical computer parts, and it takes machine B only 2 hours to do the same job, how long would it take to do the job if both machines worked simultaneously?
12. It costs a manufacturer \$30 each to produce a particular radio model, and it is assumed that if 500 radios are produced, all will be sold. What must be the selling price per radio to ensure that the profit (revenue from sales minus total cost to produce) on the 500 radios is greater than \$8,200 ?
13. A theater sells children's tickets for half the adult ticket price. If 5 adult tickets and 8 children's tickets cost a total of \$27, what is the cost of an adult ticket?

14. Pat invested a total of \$3,000. Part of the money yields 10 percent interest per year, and the rest yields 8 percent interest per year. If the total yearly interest from this investment is \$256, how much did Pat invest at 10 percent and how much at 8 percent?
15. Two cars started from the same point and traveled on a straight course in opposite directions for exactly 2 hours, at which time they were 208 miles apart. If one car traveled, on average, 8 miles per hour faster than the other car, what was the average speed for each car for the 2-hour trip?
16. A group can charter a particular aircraft at a fixed total cost. If 36 people charter the aircraft rather than 40 people, the cost per person is greater by \$12. What is the cost per person if 40 people charter the aircraft?