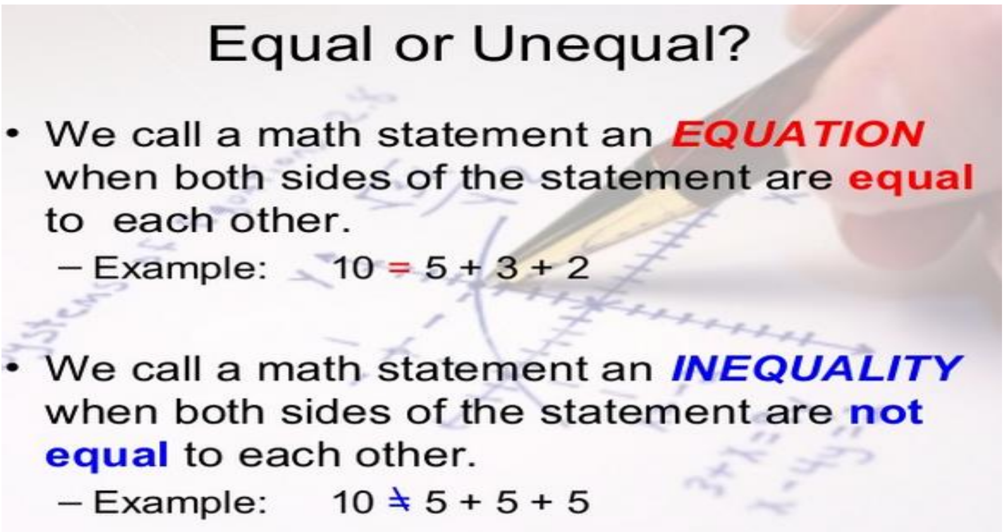
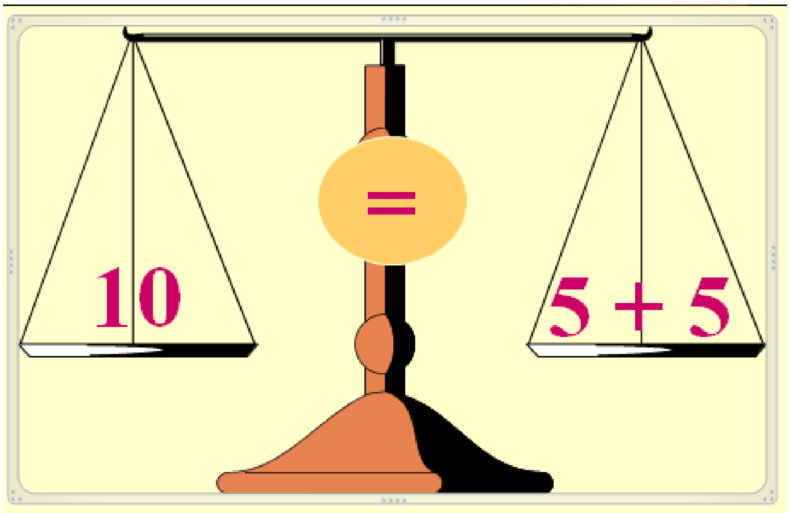
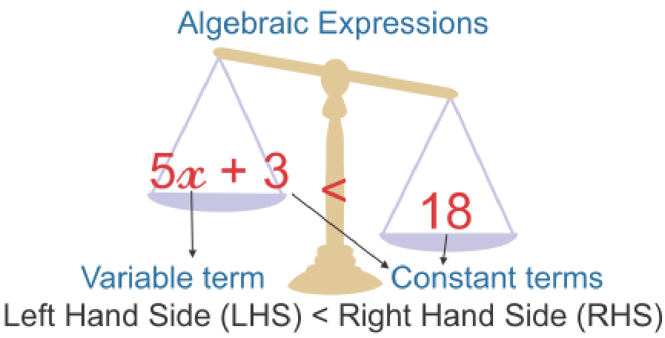
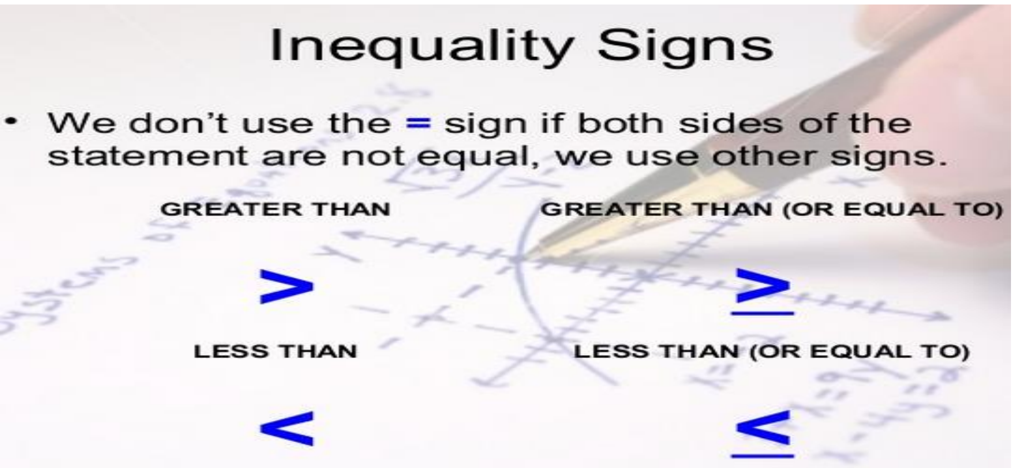
Algebra-2 (Inequality and Modulus)









You must be 18 or older to vote.

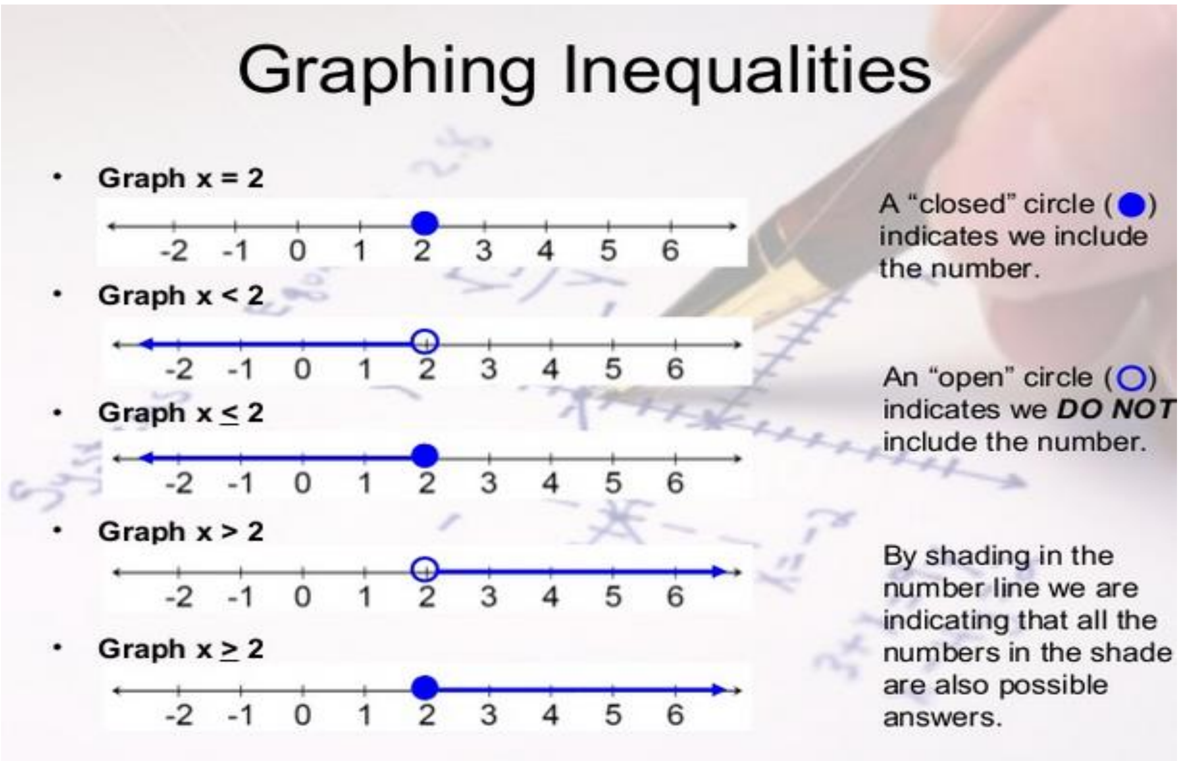
Your age must be "greater than **or** equal to 18", which is written:

$$\text{Age} \geq 18$$

ALWAYS EXPRESS THE ANSWER AS AN INTERVAL!

If $x>11$, the solution is $(11,\infty)$

If $x\leq 3$, the solution is $(-\infty,3]$



Note: Infinity is always excluded.

Addition property of inequalities:	If $A < B$ then, $A + c < B + c$
Subtraction property of inequalities:	If $A < B$, then $A - c < B - c$
Multiplication property of inequalities:	If $A < B$, then $cA < cB$
	If $A < B$, then $-cA > -cB$
Division property of inequalities:	If $A < B$, then $\frac{A}{c} < \frac{B}{c}$
	If $A < B$, then $\frac{A}{-c} > \frac{B}{-c}$

Even root is not allowed whereas odd root is allowed.

Example: If $x^2 > 36$ then x>6 or x<-6.

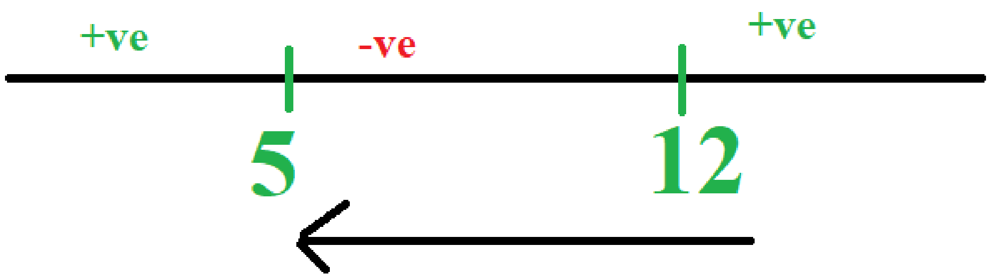
If $x^3 < 64$ then x< 4.

Solve the inequality for x:

(i) $x^2 - 17x + 60 \geq 0$

Soln: Roots are 5 &12.

Represent the roots on the number line.



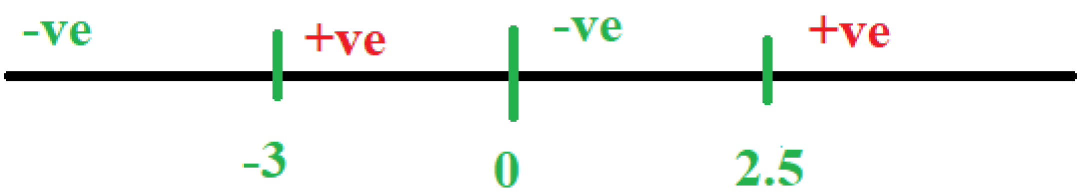
Here, we've to find the solution set for the greater than 0 i.e. +ve.

Considering +ve regions

$x \in (-\infty, 5] \cup [12, \infty)$

(ii) $(x+3)(2x-5)x \leq 0$

Soln: Roots are 0, -3, 2.5



Considering -ve regions

$x \in (-\infty,-3] \cup [0, 2.5]$

(iii) $(x^2+5)(x-7)(x+3) > 0$

Soln: $\min(x^2) = 0$ & $\min(x^2+5) = 5$

Means (x^2+5) is always +ve.

Therefore, neglect (x^2+5).

Roots are 7 & (-3).

$x \in (-\infty,-3) \cup (7,\infty)$

(iv) $(x-5)^2(x+1)^3(x-10)^3 \geq 0$

Soln: Consider the roots with odd powers only.

Roots are (-1) & 10.

Note: All the EVEN powers are treated same and all the ODD powers are treated same.

$x \in (-\infty,-1] \cup [10,\infty) \cup \{5\}$

Q.

$$\frac{(x-7)}{(x+8)} \leq 0$$

Quantity A	Quantity B
Number of integral values x can take	15

Soln: Given question can be expressed as:

$(x-7)\times(x+8)^{-1} \leq 0$

Roots are 7 & -8

Considering –ve regions

$x \in (-8,7]$

Note: Here -8 is excluded because when x = -8 then denominator is 0.

Integers in the interval are {-7,-6,-5,…..5,6,7} i.e. total 15.

Answer is C.

Modulus (Absolute value)

"In life be like modulus so that the result is always positive or at least neutral." -HJ

In Mathematics if a number or quantity is –ve then it knocks the door of Modulus. Now I don't want be –ve anymore. Please make me +ve. Then modulus replies that you need to confine yourself into 2 walls(||) then only I can make you +ve.

$|x| = x$ if $x \geq 0$

$= -x$ if $x < 0$

$|x| = |-x|$

$|x|^2 = x^2$

Find the value of x?

$|x-7| = 5$

$|2x-7| = -7$

"God helps those who, helps themselves and modulus helps only negative."

$x^2+5|x|+6=0$, Find the number of real solutions?

Ans: 0

Q. Find the minimum value of y?

$y = 20+|2x-7|$

Ans: 20

a + minimum = minimum

a + maximum = maximum

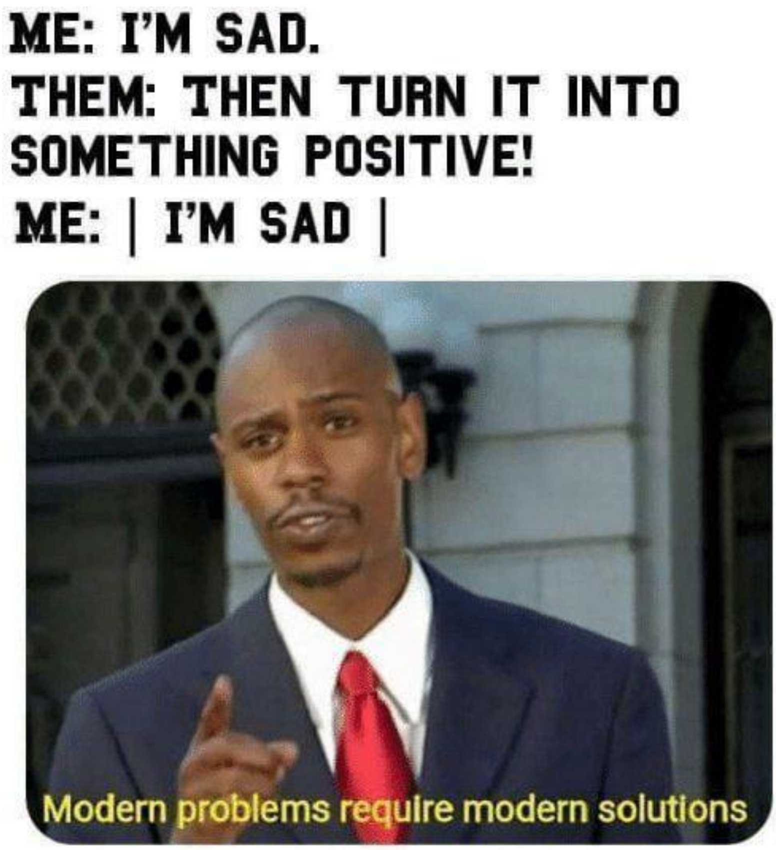
a - Minimum = Maximum

a – maximum = minimum

Q.

$|x+5|+|x-7| = 28$

Ans: 15,-13



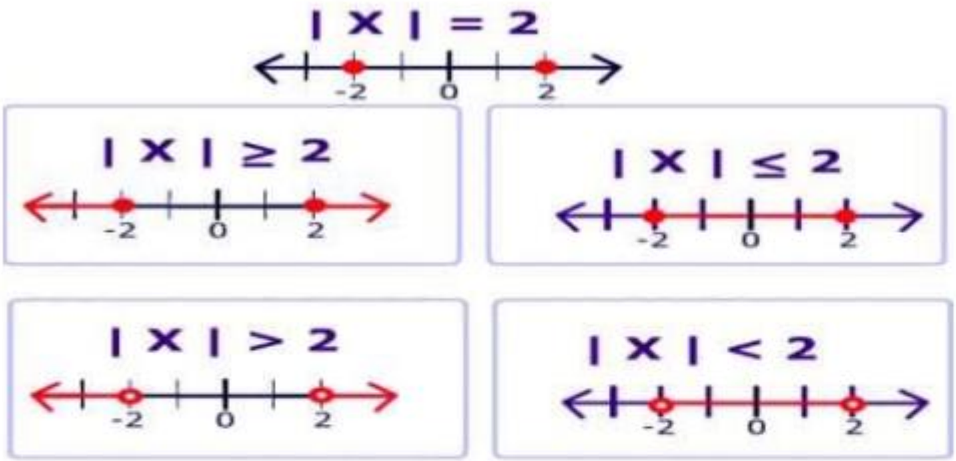
Find the minimum value of y as well as value of x for which y is minimum?

(i) $y = |x+4| + |x-7|$

Ans Min value = 11 for $-4 \leq x \leq 7$

(ii) $y = |2x-8| + |x+4|$

Ans : Minimum value = 8 for x = 4



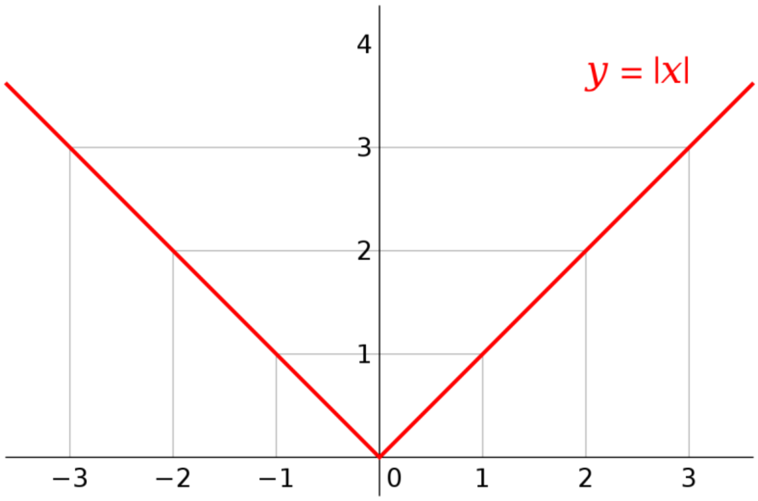
Q. $|x-5|> 7$, Solve for x?

Homework:

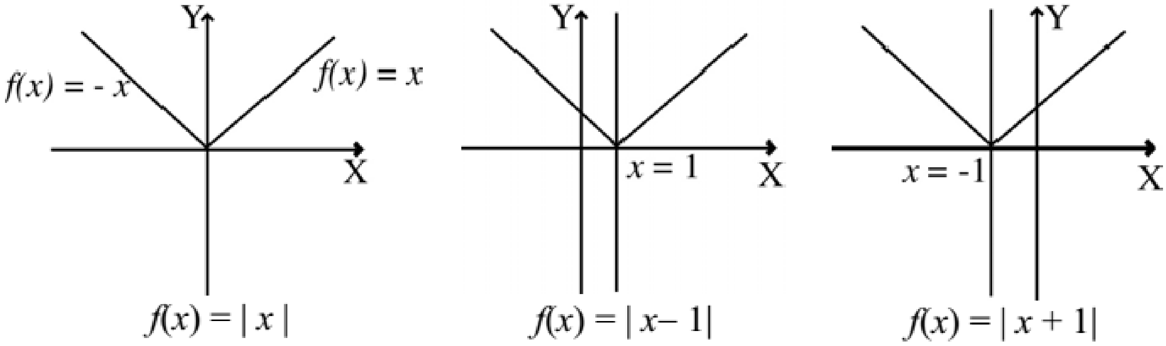
$|2x-4|+|x+7| = 30$

Q. $|x+2|\leq 10$

Quantity A	Quantity B
Number of integral values x can take	19



4
3
2
1
0
−3
−2
−1
1
2
3
y = |x|



Y
X
f(x) = - x
f(x) = x
f(x) = | x |
Y
X
x = 1
f(x) = | x– 1|
Y
X
x = -1
f(x) = | x + 1|

Q. $xy > 0$

Quantity A	Quantity B
$\frac{x}{y}$	0

Soln: A

Q. $a > 0$

Quantity A	Quantity B
a^b	0

Soln: A

Q. Quantity A | Quantity B

Quantity A	Quantity B
$\frac{2^{50}}{3^{50}}$	$\frac{2^{50}+7^{20}}{3^{50}+7^{20}}$

Soln: If $\frac{a}{b}<1$ and a,b,x are positive

Then $\frac{a}{b} < \frac{a+x}{b+x} < \frac{a+2x}{b+2x}$.....

Answer is B.

Q. x & y are positive

Quantity A	Quantity B
xy	$(xy)^2$

Soln: D

Q

Quantity A	Quantity B
$2\times3\times4\ldots..\times23$	$5\times6\times7\ldots..\times24$

Soln: 5 to 23 are common both the sides. Remove common part and now compare the remaining values.

$2\times3\times4 \quad = \quad 24$

Ans-C

Q. Quantity A	Quantity B
$\dfrac{\sqrt{65}-\sqrt[3]{63}}{\sqrt{15}}$	1

Soln: $\sqrt{65}>8$, $\sqrt[3]{63}<4$ & $\sqrt{15}<4$

$$\frac{(>8)-(<4)}{(<4)} = \frac{>4}{<4} > 1$$

Ans: A

Q. $\sqrt[3]{m^4} = \frac{7}{11}$

Quantity A	Quantity B
m	$\frac{7}{11}$

Soln: If 0<a<1 and 0<b<1, then $a < a^b < 1$

If 0<a<1 and b>1 then $a^b < a < 1$

If a>1 and 0<b<1 then $1 < a^b < a$

If a>1 and b>1 then $1 < a < a^b$

Ans: A

Q.Quantity A	Quantity B
$9\frac{3}{4}$	$9+\frac{3}{4}$

Ans C

Q. N = 113×133×239×169×209.

Quantity A	Quantity B
Increase in N when 113 is increased by 20	Increase in N when 169 is increased by 20

Soln: Answer is A

Q.x>y>0

Quantity A	Quantity B
$\left(\frac{x}{y}+\frac{y}{x}\right)$	2

Soln: For +ve numbers AM ≥ GM ≥ HM

Answer A

Q.Quantity A Quantity B

Quantity A	Quantity B
The tens digit of $(4^{100} \times 5^{99})$	The tens digit of $(4^{100} \times 5^{101})$

Soln: C

Q.n is an integer

Quantity A	Quantity B
$7.23 \times 10^{(n+1)}$	$723 \times 10^{(n-1)}$

Soln: C