Average Speed

a) Suppose there are two trains X and Y. Each train starts at station A and reach station C passing through station B. Train X starts from station A moves at the speed of 150 miles/hr for 4 hrs and reach station B. The train X spends next 4 hours in the station and reach station C at the same speed. Train Y starts from station A moves at the speed of 100 miles/hr to station B. It spends 1 hour in the station and moves to station C at the same speed and reach station C in 4 hours.

Then find the following:

i) Distance between station A and station B
ii) Distance between station B and station C.
iii) Find the total distance covered by each train.
iv) If both the train starts from the station A at the same time then which train will reach the station C first.

Solution:

Given: Train X
Speed = 150 mph.
Time to reach AB = 4 hrs.
Rest time in B = 4 hrs

Train Y
Speed = 100 mph
Time to reach BC = 4 hrs
Rest time in B = 1 hr.

1) Distance between station A and station B.

Train X reaches B in 4 hrs with 150 mph.
So, **distance = speed x time**
= 150 x 4 = **600 miles.**
AB = 600 miles.

2) Distance between station B and C.

Train Y reaches C in 4 hrs with 100 mph.
So, BC = 100 x 4 = 400
BC = **400 miles**.

3) Total distance covered = 400 + 600 = **1000 miles**.

4) Which train will reach first?

Here we have compare the time taken by train X and Train Y .

Let us find the average speed of each train.
Average speed = total distance / total time.
Train X:

Time take to reach C from B = distance / speed = 400/ 150 = 8 / 3hrs
Average speed = 1000/ [4 + 4 + 8/3] = 3000/32 mph.
Train Y:
Time taken to reach B from A = 600 / 100 = 6 hrs.
Average speed = 1000/[6 + 1 + 4] = 3000/33
From these two average speed, the average speed of train X is greater.
So, **the train X will reach C from A first.**

b) A car covers a third of the total distance at the speed of 50 km/hr and remaining distance at the speed of 35 km/hr. Suppose the distance covered by the car is 60 km, then find the time taken by the car to cover the total distance.
Solution:
Given:
In $1/3^{rd}$ of the distance, speed of the car = 50 km/hr.
In the remaining distance ($2/3^{rd}$), speed of the car = 35 km/hr.
Total distance covered by the car = 60 km
$1/3^{rd}$ of 60 = 20 km.
$2/3^{rd}$ of 60 = 40 km.
Time = distance / speed
Time taken to cover 20 km in 50 km/hr speed = 20 / 50 hrs
Time taken to cover 40 km in 35 km/hr speed = 40 / 35 hrs
Total time time taken to cross 60 km = 20/50 + 40 / 35 = **54/35** hrs.

= 1.5426 hrs
= **1 hour 32 minutes and 34 seconds**.
The car takes 1 hour, 32 minutes, and 34 seconds to reach 60 km.

c) A car covers 80 miles in 2 hrs and 105 miles in 3 hrs. Find the average speed of the car.
Hint: Find the total distance and the total time taken.
Solution:
Given:
A car covers 80 miles and 105 miles in 2 hrs and 3 hrs respectively.
Total distance covered = 80 + 105 = 185 miles.
Total time taken = 2 + 3 = 5 hrs.
Average speed = total distance / total time taken.
= 185 / 5 = 37 miles / hr.
Average speed of the car is **37 miles / hr.**

Relative speed

d) Suppose Dick and Jerry are having running race. They have to cover 400 meters. Dick has covered first 100 meters with the speed of 10m/s and next 240 meters with the speed of 15 m/s. Finally Dick covered 400 meters in 32 seconds. Whereas Jerry has covered first 120 meters in 15 seconds, 180 meters in 10 seconds. Average speed of Jerry was 40/3 m/s. Answer the following.

i) By what speed Dick has covered last 60 meters?
ii) What was the speed of Jerry when he covered last 100meters ?
iii) What was the average speed of Dick?
iv) How much time is required for Jerry to cover 400 meters?
v) Who win the race?

Solution:
Given:
Total distance to cover 400 meters.
Dick has covers first 100 meters with the speed 10 m/s.
Next 240 meters with the speed 15 m/s.
Total distance in 32 seconds.

Jerry covered first 120 meters in 15 seconds.
Second 180 meters in 10 seconds.
Average speed of Jerry 40 / 3 m/s.

i) By what speed dick covered last 60 meters.

Total distance covered with the speeds 10 m/s and 15 m/s = 100 + 240
= 340 meters.
Total time taken to cover 340 meters = 100/10 + 240 / 15 = 26 seconds.
Dick covers the remaining 60 meters in 32 – 26 = 6 seconds.
So, speed to cover the last 60 seconds = 60 / 6 = **10 m/s**.

ii) Jerry's speed when he covers last 100 meters.

Let Jerry covers last 100 meters in x seconds.
So, average speed = 400/[15 + 10 + x] = 40 / 3 m/s.
=====> 30 = 25 + x
x = 5 seconds.
So, the speed of the Jerry in the last 100 meters = 100 / 5 = 20 m/s.
Speed of the Jerry in the last 100 meters = **20 m/s**.

iii) What was the average speed of Dick?

Total distance covered by Dick = 400 meters.
Total time take by Dick = 32 seconds.
Average speed = **12.5 m/s.**

iv) How much time is required for Jerry to cover 400 meters?

Time = total distance / average speed.
= 400 / [40/3] = 30 seconds.
Total time to cover 400 meters = **30 seconds.**

v) Who win the race?

Since Jerry covers 400 meters in 30 seconds when Dick covers 400 meters in 32 seconds, **Jerry wins the race.**

e) If the train of length is 80 meters crosses signal at 2 seconds, then find the speed of the train in meters per second and also in km per hr.

Solution:
Given:
length of the train = 80 meters = 80 / 1000
Time taken to cross the signal = 2 seconds = 2/[60 x 60] hours.
While crossing the signal, train covers it's own distance.
Speed of the train = Distance/time.
= 80 / 1000 / 2/[60 x 60] = 80 / 1000 x 60 x 60 / 2
= 144 km/hr.
Speed of the train = 144 km/hr.

f) Suppose the distance between the car and the bike is 3 km and both started to move towards each other in the same road. Speed of car is 10 m/s and the speed of the bike is 18 km/hr. There is a tea shop which is 1 km away from the bike. Answer the following .

i) What is the relative speed of the car and the bike?

ii) Who will reach the tea shop first?

Solution:

Given:

Distance between the car and bike = 3 km

Speed of the car = 10 m/s = 10 x 60 x 60 /1000 = 36 km/hr.

Speed of the bike = 18 km/hr.

Distance of the tea shop from the bike = 1 km.

So, distance of the tea shop from the car = 2 km.

i) What is the relative speed of the car and the bike?

Since both of them travels in opposite side, relative speed = 18 + 36 = 54 km/hr.

ii) Who will reach the tea shop first?

Car is 2 km away from the teashop and the bike is 1 km away from the teashop.

Time taken to cover 1 km by bike = 1 / 18 hrs.

Time taken to cover 2 km by car = 2 / 36 = 1 / 18 hrs

Since time taken to reach teashop by bike and car is 1/18 hrs, the both reach at the same time.

g) If train A and train B are moving towards each other at the distance of 81 km. The speed of both the trains is 108 km/hr. Length of train A is 150 meters. If the distance covered by train A to cross train B is 40.626 km then find the length of train B

Length of A = 150 m = 0.15 km

Let the length of train B = X m

Distance that has to be covered will be $L_1 + L_2 + D = 0.15 + 81 + X$.

Now the time taken to cover this distance by train A and train B is given by

$$T = \frac{0.15 + 81 + X}{108 + 108} = \frac{81.15 + X}{2 \times 108}$$ ----------(1)

This time is same as the time taken by Train A to cross Train B

Hence $$T = \frac{distance\ covered\ by\ train\ A}{speed\ of\ train\ A} = \frac{40.626}{108}$$ ----------(2)

Note that: *The distance 40.626 also included the length of train A. Hence no need to add 0.15 to 40.626*

Since in the equation (1) and (2) the time taken is equal.

Hence equating both the equations we get,

$$\frac{81.15 + X}{2 \times 108} = \frac{40.626}{108}$$

→ X = 0.102 km = 102m

Boats and Streams

1) The speed of a motor boat itself is 20 km/h and the rate of flow of the river is 4 km/h. Moving with the stream the boat went 120 km. What distance will the boat cover during the same time going against the stream?

Solution:

Given:

Speed of the boat = 20 km/hr.

Speed of the current = 4 km/hr.

Distance covered by the boat in down stream = 120 km.

Speed of the boat in down stream = 20 + 4 = 24 km/hr.

Time taken by the boat to cover 120 km = 120 / 24 = 5 hrs. [time = distance / speed]

Speed of the boat in up stream = 20 – 4 = 16 km/hr.

Distance which could be covered by the boat in upstream = 16 x 5 = 80 km. [distance = speed x time]

At the same time, the boat will cover 80 km in upstream.

2) A boat can travel 20 mph in still water. A downstream trip takes 1 hour, and the return trip takes 3 hours.

(a) What is the speed of the river's current?

(b) What is the upstream speed of the boat relative to the river bank?

(c) What is the downstream speed of the boat relative to the river bank?

(d) Find the round trip distance.

Solution:

Given:

Speed of the boat = 20 mph.

Time taken for a trip in downstream = 1 hr

Time taken for a trip in upstream = 3 hrs.

a) What is the speed of the river's current?

Let the speed of the current be 'x'.

In the trip, distance in down stream = distance in upstream.

Distance = speed x time.

Downstream distance = (20 +x) *1 = 20 + x miles.

Upstream distance = (20 – x) *3 = 60 – 3x miles.

Both the distance are same.

20 + x = 60 – 3x

4x = 40 ==========> x = 10.

The speed of the current = 10 mph.

(b) What is the upstream speed of the boat relative to the river bank?

Speed of the boat in upstream = speed of the boat – speed of the current

= 20 – 10 = 10 mph.

Speed of the boat in upstream = 10 mph.

(c) What is the downstream speed of the boat relative to the river bank?

Speed of the boat in downstream = speed of the boat + speed of the current

= 20 + 10 = 30 mph.

Speed of the boat in downstream = 30 mph.

(d) Find the round trip distance.
Distance = speed * time
Distance in one way trip = 30 * 1 = 30 miles. [boat takes 1 hr in downstream at 30 mph speed]
The round trip distance = 2 * 30 = 60 miles.