NTS GAT General Past Papers Questions

Quantitative – Exam No. 18

Exponential Equations

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Formulas:

1. Let we have the polynomial $ax^b = 0$. Here, "a" is called co-efficient, "x" is called base (variable) and "b" is called power.
2. If power is same on the both sides of the equation, then we can equate the base. For example, if we have:

   $$(7x + 9)^4 = (3x - 4)^4$$

   It can be written as:

   $$7x + 9 = 3x - 4$$

3. If base is same on the both sides of the equation, then we can equate the power. For example, if we have:

   $$(4)^{4x-7} = (4)^{-4+x}$$

   It can be written as:

   $$4x - 7 = -4 + x$$

4. $\ln(a^x) = x \ln a$
5. $\ln(1) = 0$ (PP)
6. $\ln(0) = \infty$
7. $(a^x)^y = a^{x \times y}$ (PP)
8. $a^x \times a^y = a^{x+y}$ (PP)
9. $a^x \div a^y = a^{x-y}$
10. $x^a = {}^1/_{x^{-a}}$

Exercise:

1. If $4^{2a+3} = 4^{a-1}$, then find the value of a? (PP)

Solution:

$$4^{2a+3} = 4^{a-1}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$2a + 3 = a - 1$$
$$2a - a = -1 - 3$$
$$a = -4$$

2. If $64^{12} = 2^{a-3}$, what is the value of a? (PP)

Solution:

$$64^{12} = 2^{a-3}$$
$$(2^6)^{12} = 2^{a-3}$$
$$2^{6\times12} = 2^{a-3}$$
$$2^{72} = 2^{a-3}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$72 = a - 3$$
$$72 + 3 = a$$
$$a = 75$$

3. Find the value of x: (PP)

$$x = (8)^{\frac{4}{3}}$$

Solution:

$$x = (8)^{\frac{4}{3}}$$
$$x = ((2)^3)^{\frac{4}{3}}$$

$$x = (2)^{3\times\frac{4}{3}}$$

$$x = 2^4$$

$$x = 16$$

4. Find the value of x: (PP)

$$4^{x-2} = 16^{2-x}$$

Solution:

$$4^{x-2} = 16^{2-x}$$

$$4^{x-2} = (4^2)^{2-x}$$

$$4^{x-2} = (4)^{2(2-x)}$$

$$4^{x-2} = (4)^{4-2x}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$x - 2 = 4 - 2x$$

$$x + 2x = 4 + 2$$

$$3x = 6$$

$$x = 2$$

5. Find the value of x: (PP)

$$3^{-x} = \frac{1}{27^{(3+x)}}$$

Solution:

$$3^{-x} = 27^{-(3+x)}$$

$$3^{-x} = (3^3)^{-(3+x)}$$

$$3^{-x} = (3)^{-3(3+x)}$$

$$3^{-x} = (3)^{-9-3x}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$-x = -9 - 3x$$
$$-x + 3x = -9$$
$$2x = -9$$
$$x = -4.5$$

6. Find the value of x:

$$8^x = 13$$

Solution:

As neither the base nor the powers can be equated, so we will take natural log on both sides of the equation:

$$\ln(8^x) = \ln(13)$$
$$x \ln(8) = \ln(13)$$
$$x = \frac{\ln(13)}{\ln(8)}$$

7. Find the value of x:

$$1 = 2^x$$

Solution:

As neither the base nor the powers can be equated, so we will take natural log on both sides of the equation:

$$\ln(1) = \ln(2^x)$$
$$0 = x \ln(2)$$
$$x = \frac{0}{\ln(2)}$$
$$x = 0$$

8. Find the value of x: (PP)

$$x^2 = 5 \times 125$$

Solution:

$$x^2 = 625$$

$$x^2 = (25)^2$$

Since base is power on both sides of the equation, so we can equate the base as follows:

$$x = 25$$

9. Find the value of x: (PP)

$$4^{x+3} = 8^{x-1}$$

Solution:

$$((2)^2)^{x+3} = ((2)^3)^{x-1}$$

$$(2)^{2(x+3)} = (2)^{3(x-1)}$$

$$(2)^{2x+6} = (2)^{3x-3}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$2x + 6 = 3x - 3$$

$$3x - 2x = 6 + 3$$

$$x = 9$$

10. Find the value of x: (PP)

$$3^x = 81$$

Solution:

$$3^x = 3^4$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$x = 4$$

11. Find the value of 3^{2+x}: (PP)

$$2^{x+3} = 32$$

Solution:

$$2^{x+3} = 2^5$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$x + 3 = 5$$

$$x = 5 - 3$$

$$x = 2$$

We have to find the value of 3^{2+x}, so:

$$= 3^{2+x}$$

$$= 3^{2+2}$$

$$= 3^4 = 81$$

12. If $X^a . X^b = 1$, and $X = 1$, then find $a + b$? (PP)

Solution:

$$X^a . X^b = 1$$

Putting $X = 1$, we get:

$$(1)^a . (1)^b = (1)^1$$

$$(1)^{a+b} = (1)^1$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$a + b = 1$$

13. Find the value of x: (PP)

$$(27)^{11} = 3^{(x-6)}$$

Solution:

$$(27)^{11} = 3^{(x-6)}$$

$$((3)^3)^{11} = 3^{(x-6)}$$

$$(3)^{3\times 11} = 3^{(x-6)}$$

$$(3)^{33} = 3^{(x-6)}$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$33 = x - 6$$

$$33 + 6 = x$$

$$x = 39$$

14. Simplify: (PP)

$$\left(\frac{1}{8}\right)^{-\frac{1}{3}}$$

Solution:

$$= \left(\frac{1}{8}\right)^{-\frac{1}{3}} = (8)^{+\frac{1}{3}} = (2^3)^{\frac{1}{3}}$$

$$= (2)^{3\times\frac{1}{3}} = (2)^1 = 2$$

15. Simplify: (PP)

$$P^{-2} \times P^1 \times P^8$$

Solution:

We know that when several values with same base are multiplying, then their powers are added, so:

$$= P^{-2+1+8} = P^7$$

16. If $8 \times 8 = 4^x$, what is x? (PP)

Solution:

$$64 = 4^x$$

$$4^3 = 4^x$$

Since base is same on both sides of the equation, so we can equate the powers as follows:

$$x = 3$$