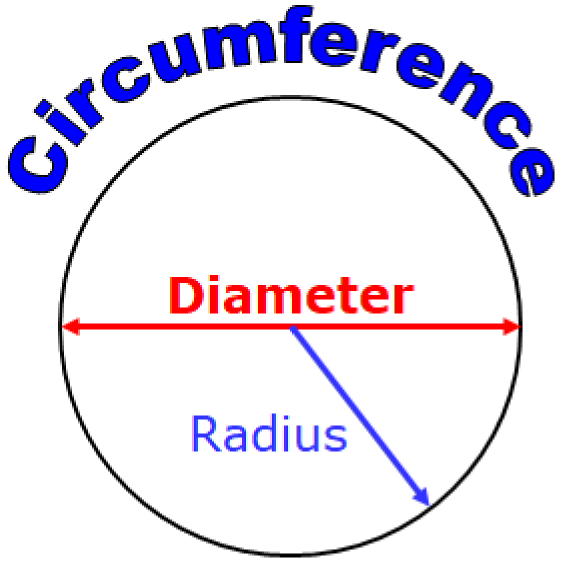
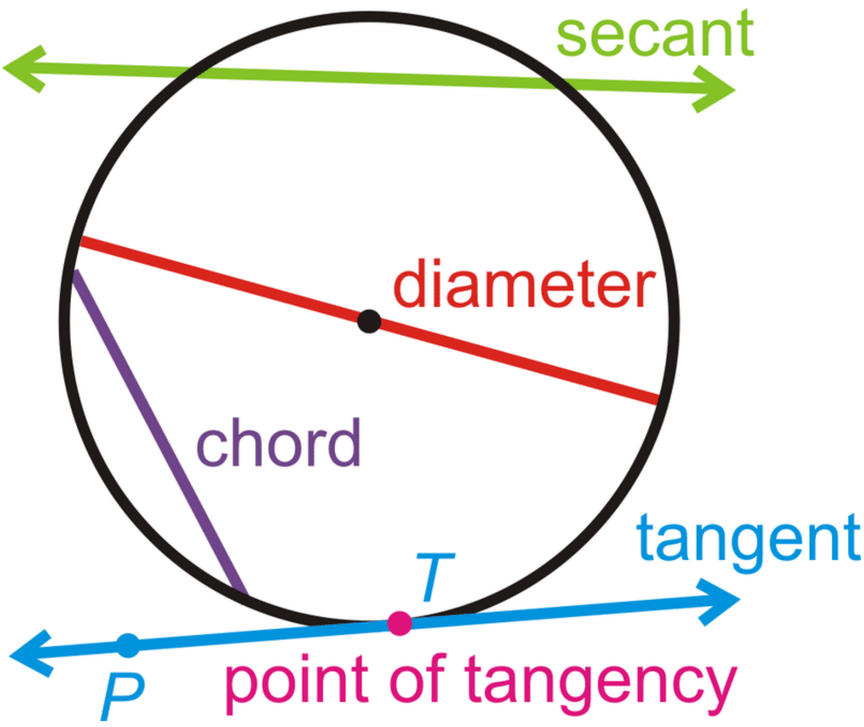
Circle



Circumference = $2\pi R$
Area = πR^2

A circle is set of all those points that are at a constant distance from a fixed point.
The fixed point is its **Centre** and fixed distance is its **radius.**



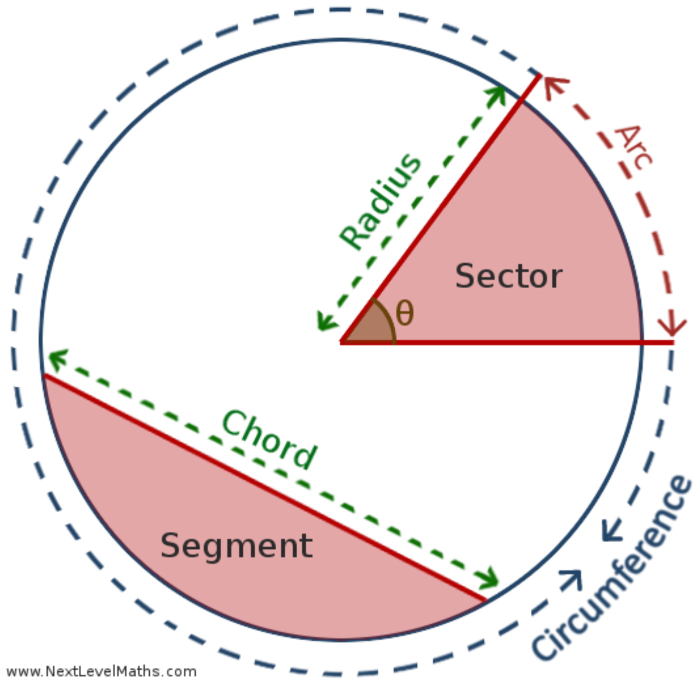
Chord: A chord is a line segment whose endpoints lie on the circle.

Diameter: The diameter is the chord passing through the centre of the circle.

Secant: It is a line that intersects the circle in two distinct points.

Tangent: It is a line in the plane of the circle, which has one and only one point common with the circle.

Radius is always perpendicular to the tangent.



Arc length = $\frac{\theta}{360}\times 2\pi r$

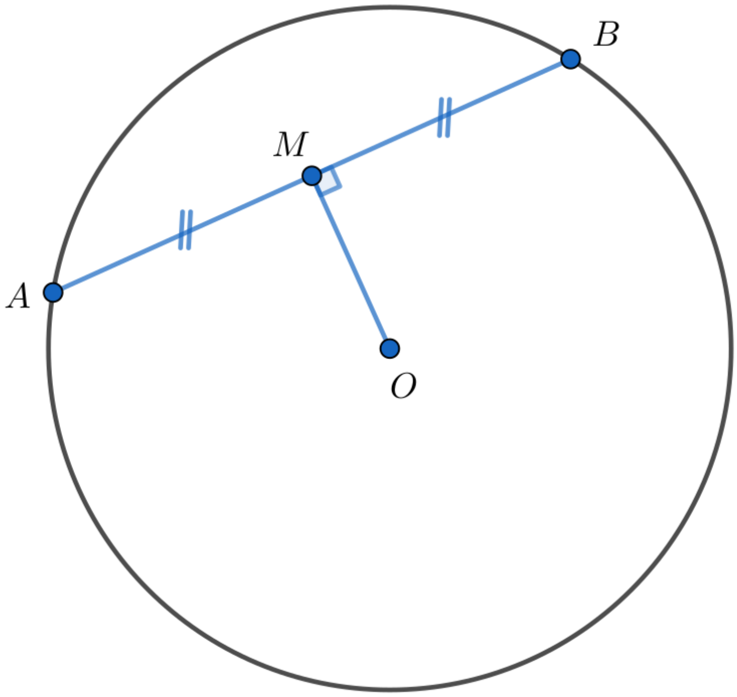
Area of sector = $\frac{\theta}{360}\times \pi r^2$

Perimeter of sector = 2r+ Arc length = $2r+\frac{\theta}{360}\times 2\pi r$

Area of Segment = Area of sector- Area of triangle

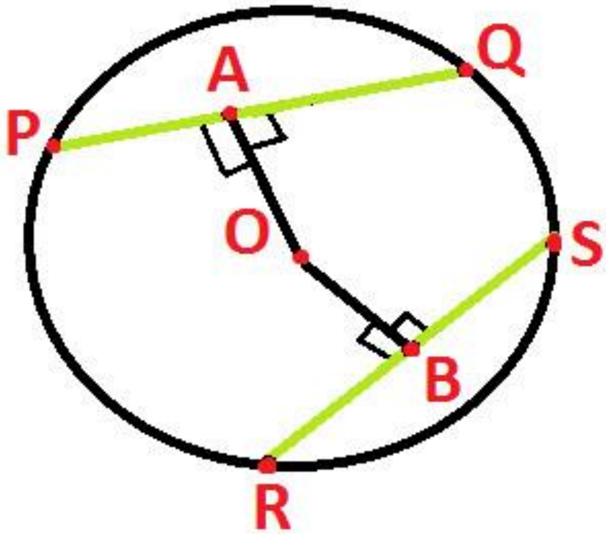
$$= \frac{\theta}{360}\times \pi r^2 - \frac{1}{2} r^2 \mathrm{Sin}\theta$$

Where θ is angle at the center



OM ⊥ AB then AM= BM

Reverse: ⊥ bisector of a chord passes through the center of the circle.



Equal chords are made by equal arcs and reverse is also true.

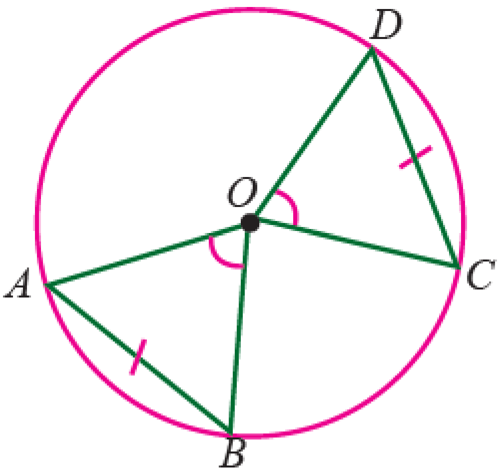
If OA = OB then PQ = RS

Or

If PQ = PS then OA = OB.

Chord closer to the center is longer.

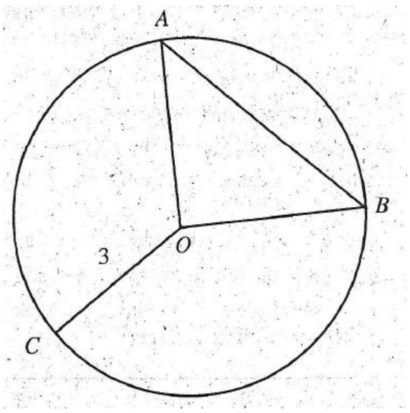
If OA> OB then RS > PQ and reverse is also true.

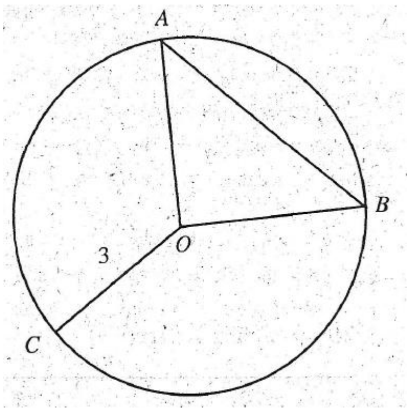


Equal equal arcs subtend equal angles at the centre.

If ∠AOB = ∠COD then AB =CD

Problem: In the circle shown in the figure the length of the arc ACB is 3 times the length of the arc AB. What is the length of the line segment AB?





Arc length AB = $\frac{1}{4}\times 2\pi R$

Arc length ACB = $\frac{3}{4}\times 2\pi R$

3×Angle subtended by arc AB at center = Angle subtended by arc ACB at center

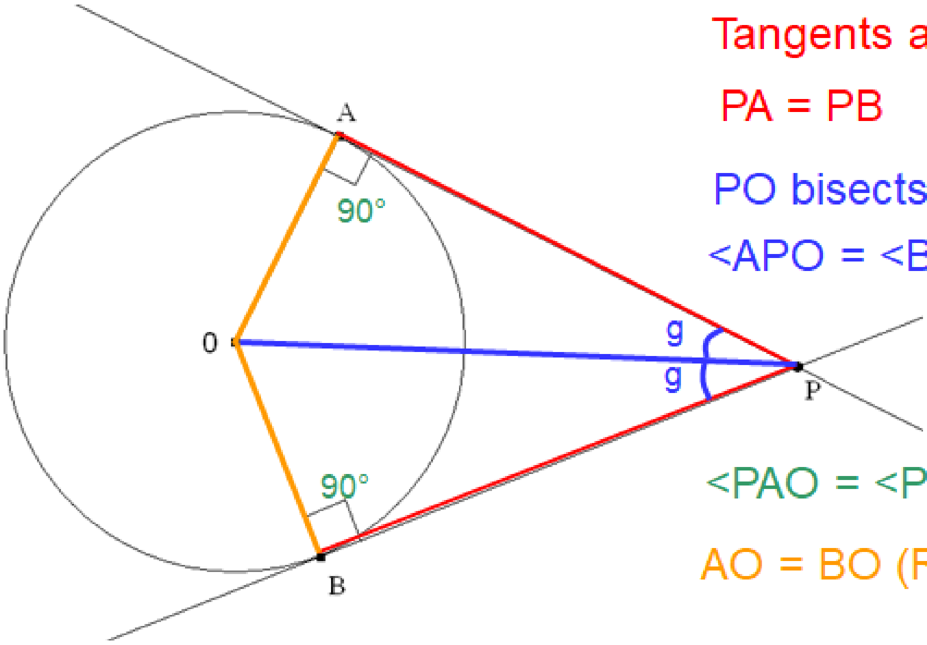
Therefore $\angle AOB=90$

Triangle AOB is 45, 45, 90.

Sides are 3,3, $3\sqrt{2}$

AB = $3\sqrt{2}$

Two tangents from a point outside circle



Tangents are equal

PA = PB

PO bisects angle APB

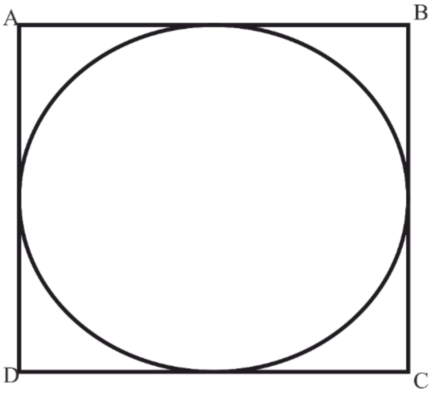
<APO = <BPO

<PAO = <PBO = 90°

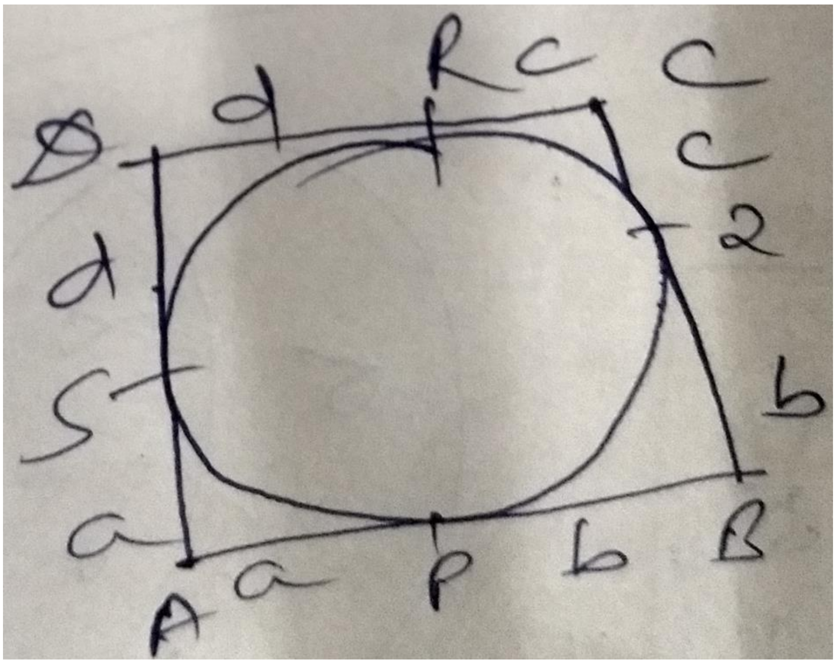
AO = BO (Radii)

The two Triangles APO and BPO are Congruent

Problem: AB = 20cm, CD = 15cm, BC = 17cm then find AD, if a circle is inscribed in a Quadrilateral ABCD.



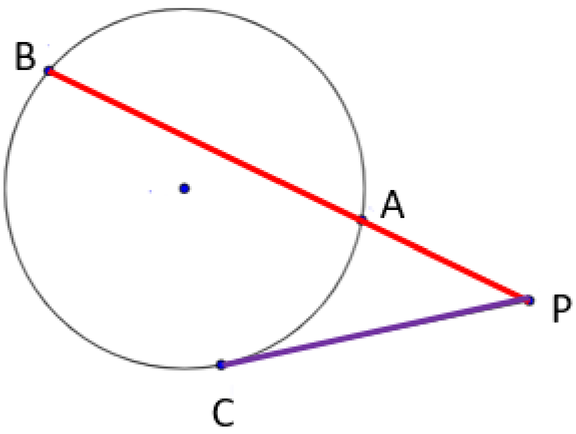
AB = 20cm, CD = 15cm, BC = 17cm then find AD, if a circle is inscribed in a Quadrilateral ABCD



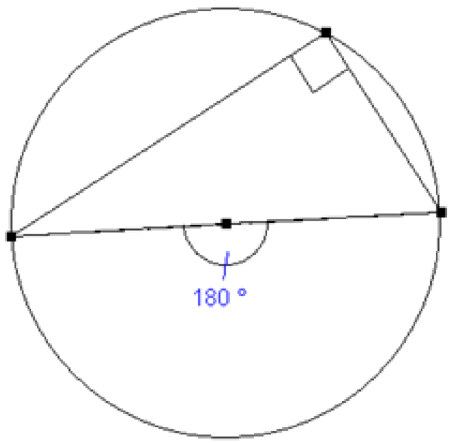
AP & AS are 2 tangents from the same point and therefore
AP =AS
Similarly
BP = BQ, CQ = CR & RD = DS
AB+CD = a+b+c+d
BC+AD= a+b+c+d
Sum of opposite sides is same.
AB+CD = BC+AD
20+15 = 17+AD

- ⇨ AD = 18 cm

If PBA is a secant intersecting the circle at A and B, and PC is a tangent, then $PA \times PB = PC^2$.



$PA \bullet PB = PC^2$



When the angle stands on the diameter, what is the size of angle a?

The diameter is a straight line so the angle at the centre is 180°

Angle a = 90°

"The angle in a semi-circle is a Right Angle"

Problem: Find the maximum area of a quadrilateral which is inscribed in a circle of radius 8cm?

Find the maximum area of a quadrilateral which is inscribed in a circle of radius 8cm

Note: When a polygon is inscribed in a circle its area is maximum when it is a regular polygon.

Therefore, this quadrilateral should be Square.

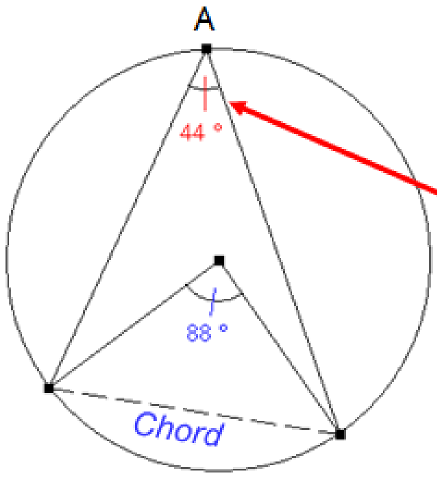
Angle in semi-circle is 90°.

Diagonal of Square = Diameter of the circle

$\sqrt{2} \times a = 2R$

- ⇨ $a = \dfrac{16}{\sqrt{2}} = 8\sqrt{2}$
- ⇨ Area of Square = $\left(8\sqrt{2}\right)^2 = 128$

Angle at the centre



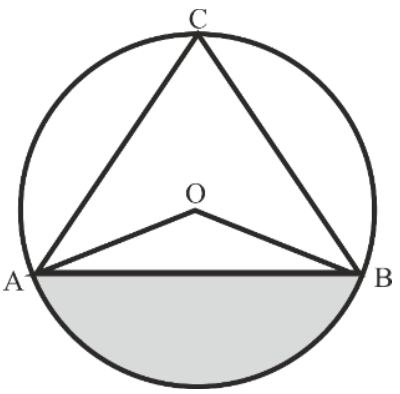
Consider the two angles which stand on this same chord

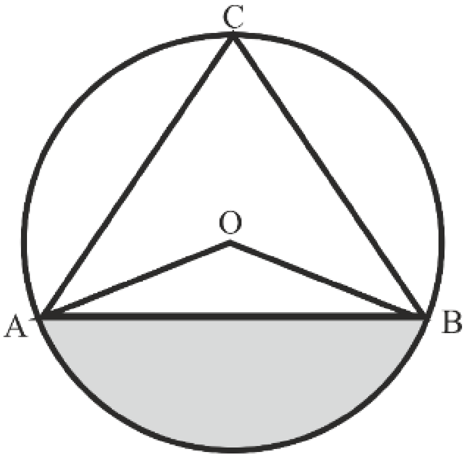
What do you notice about the angle at the circumference?

It is half the angle at the centre

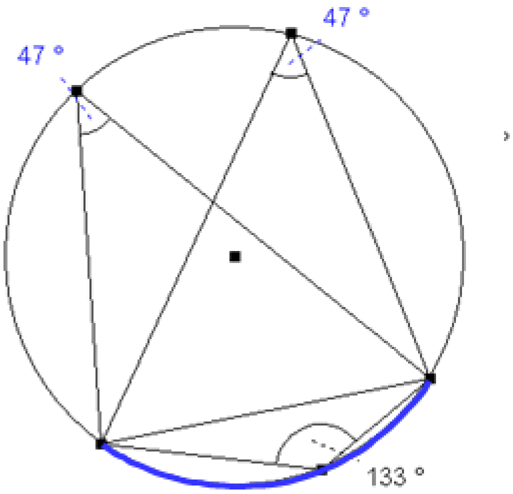
"If two angles stand on the same chord, then the angle at the centre is twice the angle at the circumference"

Problem: In ΔABC $\angle ACB = 15°$ then find the area of shaded region if radius of the circle is 42cm.



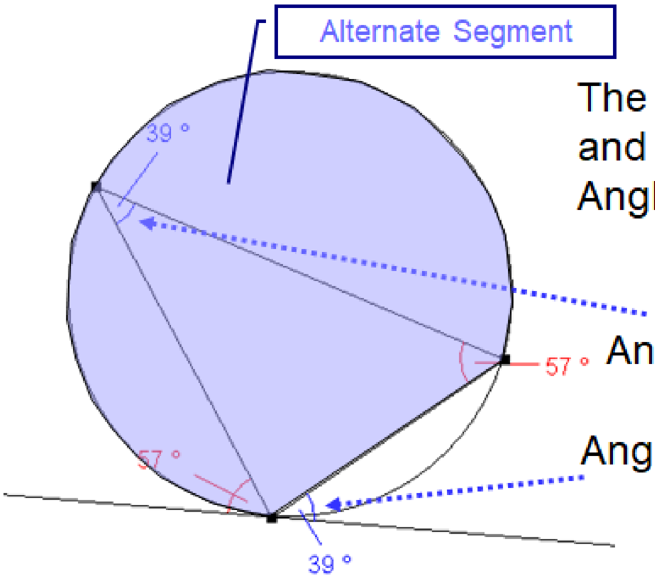


Area of sector = $\frac{30}{360} \times \frac{22}{7} \times 42 \times 42 = 462$



Angle made by same arc in the same segment are equal.

Alternate Segment Theorem



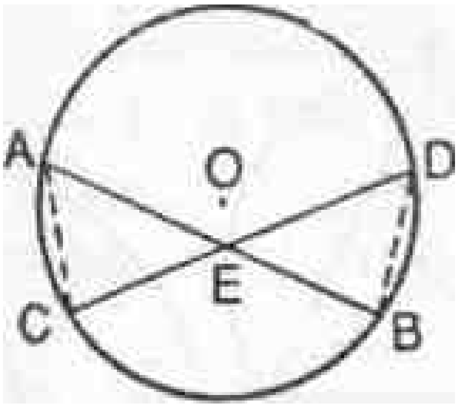
The angle between a tangent and a chord is equal to any Angle in the alternate segment

Angle in Alternate Segment

Angle between tangent & chord

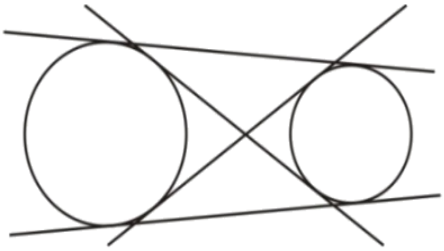
"The angle between a tangent and a chord is equal to any Angle in the alternate (opposite) segment"

If two chords AB and CD of a circle intersect at a point E, then in both the cases, AE x EB = DE x EC.

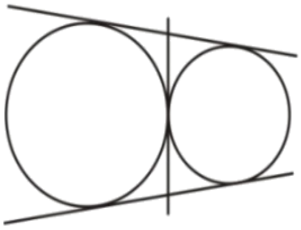


Common Tangents to Two Circles

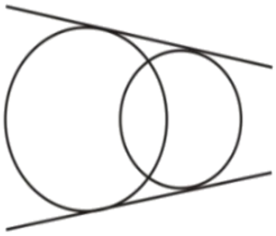
The number of common tangents to two circles could range from 4 to 0 depending on the relative placement of the circles.



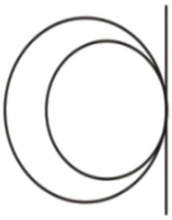
Case (i): Circles external to each other. 2 Direct and 2 Transverse Tangent



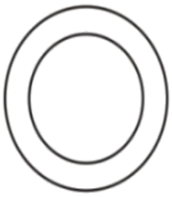
Case (ii): Circles touch externally. 2 Direct and 1 Transverse Tangent



Case (iii): Circles intersect. 2 Direct Tangent



Case (iv):Circles touch internally. 1 Transverse Tangent



Case (v): Circle within other. No common tangnet