Classification of Numbers

Numbers can be classified in various ways.

In this tutorial, we will look at some of the basic types, namely:

Natural Numbers,
Whole Numbers,
Integers.
Rational numbers,
Irrational numbers,
Real numbers.

- **Natural** numbers are: **{1, 2, 3, 4, 5, ...}**

- **Whole** numbers are: **{0, 1, 2, 3, 4, 5, ...}**

- **Integers** are the numbers: **{...-4, -3, -2, -1, 0, 1, 2, 3, 4,...}**
  Integers can be classified further.
  Positive Integers are : **{1, 2, 3, 4, ...}**
  Negative Integers are : **{..., -5, -4, -3, -2, -1}**
  Note: 0 is neither positive nor negative.

  Non positive Integers are : **{..., -5, -4, -3, -2, -1, 0}** **.....**
  Note: this includes Zero

  Non negative Integers: **{0, 1, 2, 3, 4, 5,...}**
  Note: this includes Zero

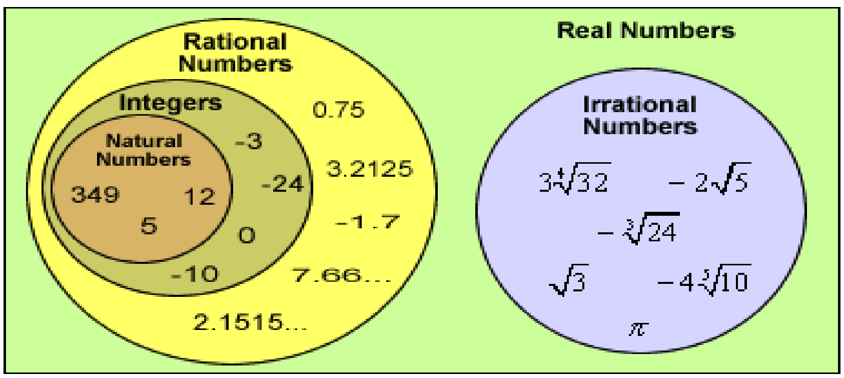
  So remember that: Integers is the set: {...-4, -3, -2, -1, 0, 1, 2, 3, 4,...} and that Integers can be positive, negative or Zero.

- **Rational number** is the number which is of the form $\frac{p}{q}$
  Where p and q are integers and $q \neq 0$.
  Example: $\frac{1}{2}$, $\frac{54}{79}$, 7, 8 ...
  Note: All the integers are rational numbers, because 7 can be written as $\frac{7}{1}$
  Similarly we can write any integer as a rational number.

- **Irrational number** is number that cannot be expressed as a ratio $\frac{p}{q}$, where p and q are integer and b is non-zero.

  Example: 2.71828182845904 …,√5 , 3.3166247..., etc

- **Real number** include all the **rational numbers** and all the **irrational numbers.**
  Note: Rational number include Integers, whole numbers and Natural numbers.



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Positive and Negative Numbers

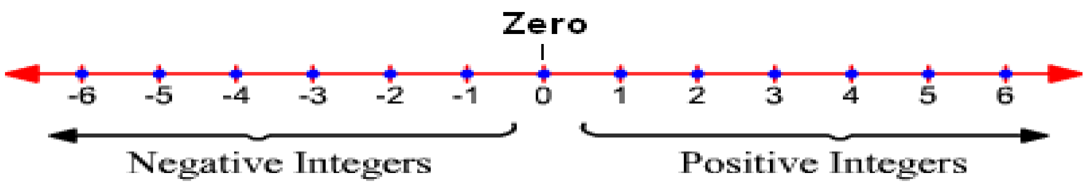
For a given number a, **only one** of the following is true:

1. **a is negative (a < 0)**
2. **a is zero (a = 0)**
3. **a is positive (a > 0)**

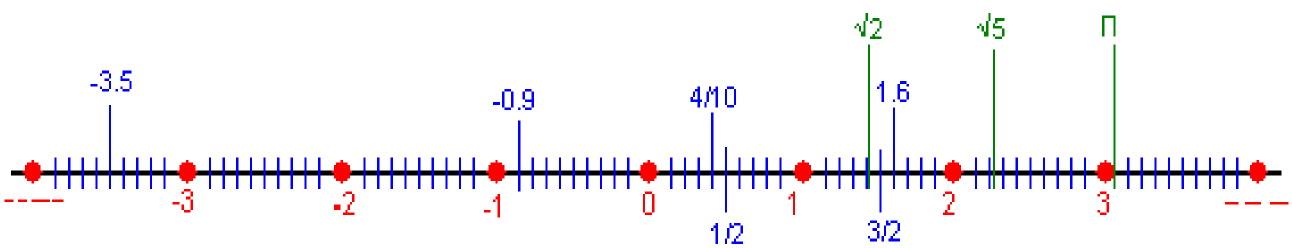
Thus, a number can be only one of the following:
Negative or Positive or Zero.

Number Line

Integers



Real numbers (Set of all numbers)



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Odd and Even Integers

If 2 divides a number perfectly, then the number is <u>even</u> number , else the number is said to be <u>odd</u> number.

{...-6, -4, -2, 0, 2, 4, 6, ...} are even integers.
{..., -5, -3, -1, 1, 3, 5, …} are odd integers.

<u>Note:</u>

1. **-6, -8, -10 ...... are Even, (though they are negative) and   -3, -7, -19 are odd (though negative).**

2. **Sum or difference of two even numbers is always even.**

   For example: 18 + 6 = 24,

$(-4) + 8 = 8 – 4 = 4$,
$12 + (-148) = - (148 – 12) = - 136$
$(-198) + (-126) = -198 – 126 = -(198 + 126) = - 324$
$(-198) - (-126) = -198 + 126 = -(198 - 126) = - 72$
$96 - (- 84) = 96 + 84 = 180$
$(-84) - (- 96) = -84 + 96 = 12$

3. **Sum or difference of two odd numbers is always even.**

For example: **$19 + 15 = 34$**
$(-5) + 81 = 81 – 5 = 76$
$(-25) + (-89) = -(25 + 89) = - 114$
$(-29) - (- 117) = -29 + 117 = 117 - 29 = 88$
$-7-(-57) = -7 + 57 = 57 – 7 = 50$

4. **Sum or difference of odd and even number is always odd**

For example: **$19 + 18 = 37$**
$(-19) + (-26) = - (19 + 26) = - 45$
(check)

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Absolute Values

The absolute value of a is denoted by $|a|$.

If $a < 0$, then $|a| = -(a)$.
For example: $|-3| = 3$, $|-6| = 6$, $|- 1.5| = 1.5$

If $a > 0$, then $|a| = a$.
For example: $|5| = 5$, $|8| = 8$, $|0.005| = 0.005$

If $a = 0$, then $|a| = |0| = | - 0| = 0$

Example:

Find the possible value of a .

i) $|a| = 4$
$a = -4$ or $a = 4$ (Since $|-4| = 4$)

ii) $|a - 2| = 4$

$a - 2 = -4$ or $a - 2 = 4$
case 1:
If $a - 2 = -4$
$a = -4 + 2 = -2$
$a = -2$
case 2:
If $a - 2 = 4$
$a = 4 + 2 = 6$
$a = 6$
So a can be either -2 or 6.

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Product

If $a * b = c$, and a, b , c are integers, then we say :

- c is the product of a and b.
- c is the multiple of a
- c is the multiple of b.

Example:
55 is a multiple of 11 because $55 = 11 \text{ X } 5$
36 is a multiple of 12, because $36 = 12 \text{ X } 3$

Points to remember

- If product of a and b is zero ($a \text{ X } b = 0$) than either $a = 0$ or $b = 0$.
  For example: $7 \text{ X } 0 = 0$, $0 \text{ X } 0 = 0$, $0 \text{ X } -9 = 0$.
- Multiplication of two number with different signs gives a negative number.
  For example: $7 \text{ X } (-8) = -56$, $(-7) \text{ X } 8 = -56$
- Multiplication of two number with same signs gives a positive number.

For example: (-7) X (-8) = 56 , 7 X 8 = 56

- Multiplication of two even numbers is always even. (Check)
- Multiplication of two odd numbers is always odd.(Check)
- Multiplication of any number with an even number is always even.(Check)

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Division

1. 55 / 11 = 5
   where 55 is called dividend
   11 is called divisor
   5 is called quotient
   here remainder is 0

Note: dividend = (divisor X quotient) + Remainder

(Note: Remainder < divisor)

Here 55 = (11 X 5) + 0

2. 47 / 5 = 9, (2 is a remainder)
   47 = (5 X 9) +2

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Multiple

If a x b = c, and a, b , c are integers, then:

- c is the multiple of a
- c is the multiple of b.

Example: 55 is a multiple of 11 because 55 = 11 x 5

36 is a multiple of 12, because 36 = 12 x 3

54 is not a multiple of 7 because 54 cannot be expressed as 7 x A, where A is some integer.

Note that, when B is a multiple of A, then B/A results in remainder that is 0

In the above examples : 55/11 = 5 (remainder = 0)

36/12 = 3 (remainder = 0)

But, 54/7 = 7 (and remainder = 5).

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FACTOR

A **factor** of a given number is every number that **divides exactly** into that number.

NOTE: Number 1 and the number itself are always factors of any number.

If b divides A perfectly and the remainder is 0, then b is a factor of A

Example:

1) 55 / 11 = 5 remainder = 0;
   55/5 = 11 remainder = 0;
   55 / 1 = 55 remainder = 0;

   So, 11, 5 and 1 are factors of 55

Note: 55 is a multiple of 11 and also we can say 11 is a factor of 55.

The complete set of factors (or divisors) of 55 are {1, 5, 11 , 55 }

2) 81 = 1 X 81
   = 1 X 3 X 27
   = 1 X 3 X 3 X 9
   = 1 X 3 X 3 X 3 X 3

   So the factors (or divisors) of 81 are 1, 3, 9, 27 and 81.

3) 29 = 1 X 29

   So the factors (or divisors) of 29 are 1 and 29.

<u>Here is another method to determine factors of a number:</u>

Consider the number 84.

Step 1: Determine the approximate square root of the number.
We know that 9 x 9 = 81 and 10x10 = 100.
Since 84 lies between 81 and 100, the square-root of 84 should lie between 9 and 10.

Step 2: Determine the factors (**divisors**) of 84 from 1 to 10
The factors of 84 between 1 to 10 are 1, 2, 3, 4, 6, 7
Write the number 84 as a product of the above factors.
That is
84 = 1 x 84 = 2 x 42 = 3 x 28 = 4 x 21 = 6 x 14 = 7 x 12
Step 3: Create the complete set.
As seem from above, if 2 is a factor of 84, then 42 is also a factor. Similarly for 3 and 28.

So the complete set of factors of 84 are : {1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84 }

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PRIME, NON PRIME AND COMPOSITE NUMBERS

A **prime number** has exactly 2 factors, the number **itself and 1.**
In other words, the prime number can be divided only by 1 and the number itself.
NOTE: 0 and 1 are not prime numbers.

Example

5 is a prime number, because the only factors (divisors) are 1 and 5.

The prime numbers less than 20 are 2,3,5,7,11,13,17,19

Example

Find all prime factors of 30.

Here you can use the **square root** method to find the factors**.**

All the factors of 30 are 30, 15, 10, 6, 5, 3, 2, 1

But only 5, 3 and 2 are prime numbers.

Therefore, the prime factors of 30 are 2, 3 and 5.

The positive integers which are not prime (except 1) are known as non prime numbers.
For example: 4, 6, 8, 9, 10, 12, ...

Note:

- **1 is not a prime number.**
- **2 is a Prime number (even though it is even).**
- **Other than 2, no other even numbers are Prime**
- **The least positive even prime number is 2.**
- **The least positive odd prime number is 3.**

A **composite number** has **at least one more factor** that the number itself or 1.
In fact, **all whole numbers that are not prime are composite except for 1 and 0,**

1 and **0** are neither prime and nor composite.

For example:

4 is a composite number.

The factors of 4 are 1,2, and 4.

6 is a composite number.

The factors of 6 are 1,2,3,6.

The composite numbers l**ess than 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 .**

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DIVISIBILITY RULES

The simple divisibility rules will help you to find factors of a number.

The number is divisible by:

- **2** if the last digit is 0, 2, 4, 6, or 8
  (example: 12346, 9994,7958);
- **3** if the sum of digits in the number are divisible by 3
  (example: 1236, because 1+2+3+6 = 12 = 3 x 4 which is divisible by 3);

- **4** if the last 2 digits are divisible by 4
  (example: 897544, because 44 = 4 x 11);
- **5** if the last digit is 0 or 5
  (example: 178965 or 40980);
- **6** if it is divisible by 2 and 3;
- **7** sorry, no rule (you have to divide);
- **8** if the last 3 digits are divisible by 8
  (example: 124987080, because 080 = 8 x 10;
- **9** if the sum of digits is divisible by 9
  (example: 234612, because 2+3+4+6+1+2 = 18 = 9 x 2);
- **10** if the last digit is 0
  (example: 99990);
- **100** if the last 2 digits are 0
  (example 987600);

NOTE: If a number is divisible by 2 factors, it is also divisible by the **product** of these factors.

Example 1: Number 18 is divisible by 2 and 3, so it must be divisible by 6.

Example 2: Number 945 is divisible by 9 (why?) and by 5 (why?), so it must be divisible by 9 x 5 = 45. (Can you check it?)

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COMMON FACTORS

When two (or more) numbers have **the same factor**, that factor is called a **common factor.**

Example

Find all the common factors of 12 and 18.

Factors of 12 are 1, 2, 3, 4, 6, 12.
Factors of 18 are 1, 2, 3, 6, 18.
The common factors of **12 and 18 are 1, 2, 3 and 6.**

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HIGHEST COMMON FACTOR (H.C.F or G.C.D).

The **Highest Common Factor (H.C.F)** of two (or more) numbers is the **largest number that divides evenly** into both numbers.

In other words the H.C.F is the largest of all the common factors.

Method 1 :

GCD of 18 and 6:
The factors of 18 are : {1, 2, 3, 6, 9, 18}
The factors of 6 are : { 1, 2, 3, 6}

The highest factor that is common to both 6 and 18 is 6. So GCD = 6

Method 2 :
GCD of 12 and 18

12 = <u>1</u> X <u>2</u> X 2 X <u>3</u>
18 = <u>1</u> X <u>2</u> X 3 X <u>3</u>

common factors are 1 X 2 X 3 = 6

GCD of -49 , 35 and 14

\- 49 = -1 X <u>1</u> X <u>7</u> X 7
35 = <u>1</u> X 5 X <u>7</u>
14 = <u>1</u> X 2 X <u>7</u>

common factors are 1 X 7 = 7

GCD of 36 and 24

36 = 1 x 2 x 2x 3 x 3
24 = 1 x 2 x 2 x 2 x 3

Common factors are 1 x 2 x 2 x 3 = 12

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FINDING THE H.C.F. OF BIG NUMBERS

For larger numbers you can use the following method:

1. Find all prime factors of both numbers.
2. Write both numbers as a multiplication of prime numbers.
3. Find which factors are **repeating** in both numbers and multiply them to get H.C.F .

Example

Find the Highest Common Factor (H.C.F.) of 240 and 924.

Finding all prime factors of 240.

We will start with the smallest prime number and we will divide 240 into it if we can (the divisibility rules come handy).

We will do the same with the result (or quotient), and we will keep dividing by prime numbers until we have 1 as a quotient.

Each time we write the prime factor to the right and the quotient below:

240 | 2	**2 is a factor of 240;**	**240 divided by 2 is 120**
120 | 2	**2 is a factor of 120;**	**120 divided by 2 is 60**
60 | 2	**2 is a factor of 60;**	**60 divided by 2 is 30**
30 | 2	**2 is a factor of 30;**	**30 divided by 2 is 15**
15 | 3	**3 is a factor of 15;**	**15 divided by 3 is 5**
5 | 5	**5 is a factor of 5;**	**5 divided by 5 is 1**
1		

240 = 2 x 2 x 2 x 2 x 3 x 5

NOTE: There are other methods for finding all prime factors of a number, for example a factor tree.

Finding all prime factors of 924.

924 | 2	**2 is a factor of 924;**	**924 divided by 2 is 462**
462 | 2	**2 is a factor of 462;**	**462 divided by 2 is 231**
231 | 3	**3 is a factor of 231;**	**213 divided by 3 is 77**
77 | 7	**7 is a factor of 77;**	**77 divided by 7 is 11**
11 | 11	**11 is a factor of 11;**	**11 divided by 11 is 1**
1		

924 = 2 x 2 x 3 x 7 x 11

240 = 2 x 2 x 2 x 2 x 3 x 5

Multiply the factors which repeat in both numbers to get the H.C.F.

The Highest Common Factor is 2 x 2 x 3 = 12

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MULTIPLES. COMMON MULTIPLES.

When you **multiply a given whole number by any other whole number,** the result is a **multiple** of that number.
For example, 5 is the first multiple of 5 (because 5 x 1 = 5), 10 is the second multiple of 5 (because 5 x 2 = 10), and so on.

Example 1:

Write down the first 3 multiples of 8.
Solution: 8 x 1 = 8, 8 x 2 = 16, 8 x 3 = 24, so the first 3 multiples of 8 are 8,16,24.

Example 2:

Write down all multiples of 3 greater than 10 but smaller than 20.
Solution: Multiples of 3 are 3, 6,9,**12,15,18,**21,24,...
Among these we have to choose the numbers which are greater than 10 but smaller than 20 .

The desired ones are 12,15,18.

The **common multiples** of two numbers are **multiples of both numbers.**

Example 3:

Find common multiples of 3 and 5.

Solution: Multiples of 3 are **3,6,9,12,15,18,21,24,27,30,33,...**
Multiples of 5 are **5,10,15,20,25,30,35,...**

Common multiples of **3 and 5 are 15, 30, 45...**

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LOWEST COMMON MULTIPLE (L.C.M.).

The **Lowest Common Multiple (L.C.M)** is the **smallest number that is a common multiple** of two or more numbers.

Method 1 :

For example, the L.C.M of 3 and 5 .

Multiples of 3 are **3,6,9,12,15,18,21,24,27,30,33,...**

Multiples of 5 are **5,10,15,20,25,30,35,...**

Common multiples of **3 and 5 are 15, 30,45 ...**

Here we have to find the least common multiple (least value) .

The least common multiple of 3 and 5 is 15**.**

Note : Other numbers are greater than 15.

Method 2 :

1) LCM of 15 and 25:

15 = 1 X 3 X 5
25 = 1 X 5 X 5

we see 1 X 5 is the common factors.

LCM of 15 and 25 will be multiple of :

- **common factor 15 and 25 (which is 1 x 5)**
- **uncommon factor of 15 (which is 3) and**
- **uncommon factor of 25 (which is 5).**

LCM of 15 and 25 = 1 X 5 X 3 X 5 = 75

2) LCM of 72 and 64

72 = 1 X 2 X 2 X 2 X 3 X 3
64 = 1 X 2 X 2 X 2 X 2 X 2 X 2

- we see 1 X 2 X 2 X 2 = 8 is the common factors.
- Uncommon factors of 72 : 3 x 3
- Uncommon factors of 64 : 2 x 2 x 2

LCM of 72 and 64 will be multiple of the following:

(Common factor 72 and 64) X (uncommon factor of 72, which is 3 X 3 = 9) X (uncommon factor of 64 (which is 2 X 2 X 2 = 8))

LCM of 72 and 64 = 8 X 9 X 8 = 576

3) LCM of 14, 49 and 35

49 = <u>1</u> X <u>7</u> X 7
35 = <u>1</u> X 5 X <u>7</u>
14 = <u>1</u> X 2 X <u>7</u>

LCM of 14, 49 and 35 will be multiple of common factor 14, 49 and 35

Common factor of 14, 49 and 35 is **7 ,** uncommon factor of 14 is **2 ,** uncommon factor of 35 is **5** and uncommon factor of 49 is **7**
Now multiply the values,

LCM of 14, 49 and 35 = 7 X 2 X 5 X 7 = 490.

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FINDING L.C.M. OF BIG NUMBERS

1. Find all the prime factors of both numbers.
2. Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included.

Example:

Find the Lowest Common Multiple (L.C.M.) of 240 and 924.

From the example of finding the H.C.F. we know the prime factors of both numbers.

924 = 2 x 2 x 3 x 7 x 11
240 = 2 x 2 x 2 x 2 x 3 x 5

Common factors of 924 and 240 are 2, 2 and 3, Uncommon factor of 924 are 7 and 11 and Uncommon factors of 240 are 2,2,5 .

The L.C.M. is (2 x 2 x 3) x (7 x 11) x (2 x 2 x 5) = 12 x 77 x 20 = 18,480

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Square Root

Square root ALWAYS takes the **"positive value"**. Because, square root is a function "that maps the positive real numbers to the positive real numbers only".

Note : The mathematical definition for square root --> when we take square root for any value, the answer will always be Positive.

Only if we need to find the solution of A, when A^2 is given, then we will get the answer as positive or negative. If the question requires you to find the value of Sqrt A, then the solution will always be positive.

Examples:

(a) Find the value of X, if $X^2 = 36$
Given, $X^2 = 36$
X = +6 or -6

(b) Find the value of, Sqrt A , if A = 36.
if **Sqrt (A)= Sqrt(36) = +6** (It will not be negative, this is the mathematical rule).

Quiz:

1. Every even number is always a multiple of ___________.

2. What is the product of 7 X 15 X 16 X (-48) X 0 X (-18) X 45

3. What is the remainder when 13 divides 91 ?

4. What is the quotient when 120 is divided by 17 ?

5. Can the Divisor be smaller than the Dividend ? (try various examples)

6. Can the Quotient be larger than the Divisor ? (try various examples)

7. Can the Remainder be larger than the Divisor ?

8. Determine all the factors of 12

9. Determine all the factors of 35

10. Determine all the factors of 72

11. Determine all the factors of 96

12. Determine all the factors of 105

13. Determine all the factors of 256

14. What are the factors of 119 ?

15. What are the factors of 225 ?

16. List out some of the multiples of 12

17. List five multiples that are common to 12 and 18

18. List five multiples that are common to 12, 18 and 24

19. List out all the odd factors of 90

20. List out all the even factors of 125

21. List out all the prime factors of 210

22. Is 81 a multiple of -9 ?

23. Can an even number be a Prime Number ?

24. Is 323 a prime number ?

25. ________________ is the least prime (even) integer and __________________ is the least prime (odd) integer.

26. Find the GCD of:

i) 12 and 16
ii)91, 51, 13

iii)14 , 35 and 5
iv)45 and -9

27. Find the LCM of:

i)54 and 36
ii)8 and 24
iii)18 and 12
iv)20, 25, 15
v)72, 6, 18

28. Find the possible value of a.

i) $|a - 6| = 8$
ii) $|2a – 5| = 15$
iii) $|4a – 8| = 24|$

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