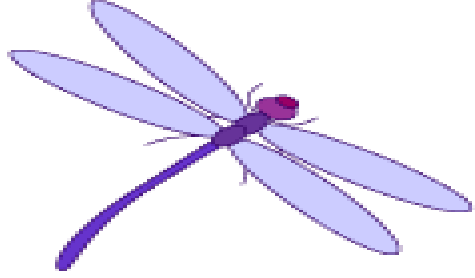




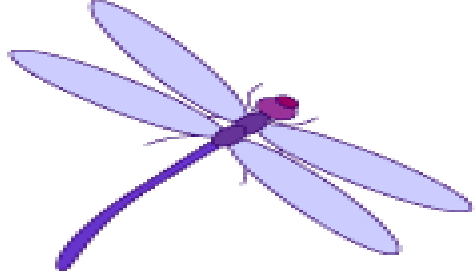
ARITH-ALGEBRA

SEQUENCE



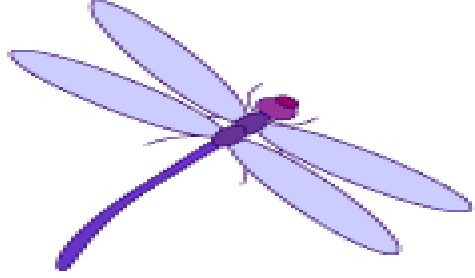
SEQUENCE

- **Sequence** is a → list or
→ set of numbers or
→ set of quantities **in a particular order.**
- Examples :
0,1,2,3... sequence of whole numbers
0,2,4,6... sequence of even numbers
1,3,5,7... sequence of odd numbers
- From the example we can see that all the sequence will have a “particular order” .



EXAMPLES

- 1) In a sequence “s”, third term is 12, second term is twice of the first term and the third term is three times the second term.
What is the first term of the sequence ?



SOLUTION

Given : third term = 12

second term = 2 x first term

third term = 3 x second term

To Find : first term of the sequence

Solution : Let first term be “x”

Second term = 2 (x) = 2x

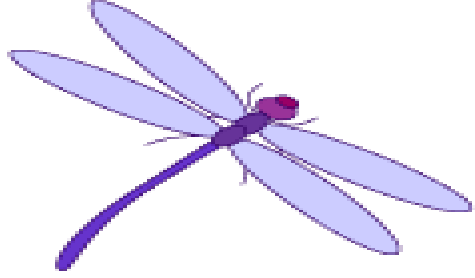
Third term = (3)(2)(x) = 6x

Which implies, $6x = 12$

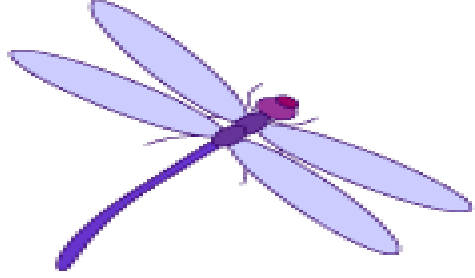
$$x = 12/6$$

$$x = 2$$

Therefore , first term of the sequence is 2



2) A, B, C, D, E are positive integers with an increasing order, such that the value of each successive integer is 3 more than the preceding integer and the value of E is 32, find the value of B ?



SOLUTION

2) Given : A,B,C,D,E are positive integers with increasing order.

$$B = A + 3$$

$$C = B + 3$$

$$D = C + 3$$

$$E = D + 3 \quad \text{and} \quad E = 32$$

To Find : B

Solution: $D + 3 = 32$

$$D = 32 - 3 = 29$$

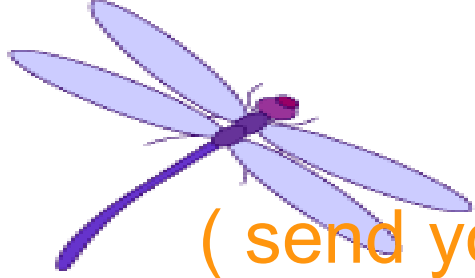
$$C + 3 = 29$$

$$C = 29 - 3 = 26$$

$$B + 3 = 26$$

$$B = 26 - 3 = 23$$

Therefore, value of B in the sequence is 23



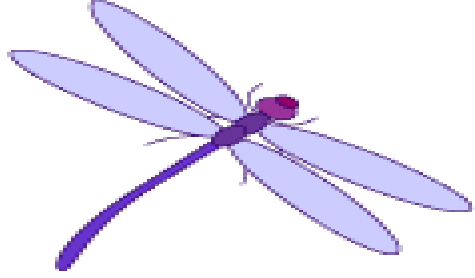
QUIZ

(send your solutions to support@greedge.com)

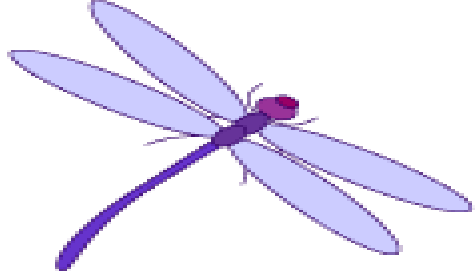
1) Except for the first two numbers, every number in the sequence -1,3,2,..... is the sum of the two immediately preceding numbers, find the 8th term of the sequence ?

2)	64	x	192	-256
	16	-32	y	-64
	4	-8	12	-16
	1	-2	3	-4

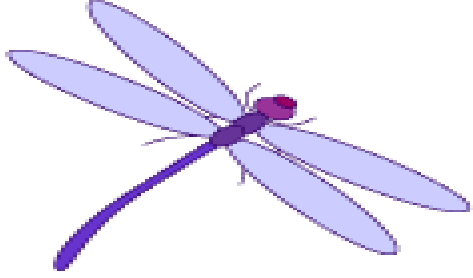
In the array of numbers above , each row above is a multiple of 4 of the row below, find $(x-y)$?



- Different types of sequences are
 - Arithmetic progression/ sequence.
 - Geometric progression/ sequence.

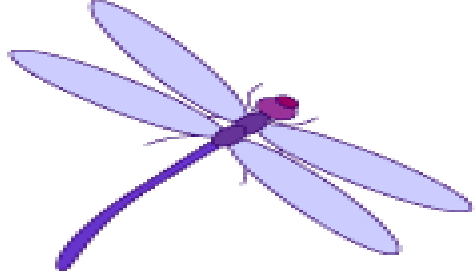


ARITHMETIC PROGRESSION



- An **arithmetic progression** is a sequence of numbers.
- where each new term is found by adding a fixed amount to the previous term in the sequence.
- Fixed amount is called the **common difference**.
- Example 1 : **1 , 3 , 5 , 7 ,**

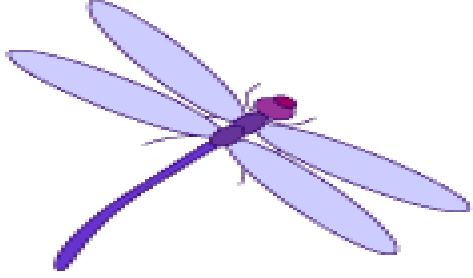
Here, the first term is 1 and the new term is found by adding 2 to the previous term. So, the **common difference is 2**.



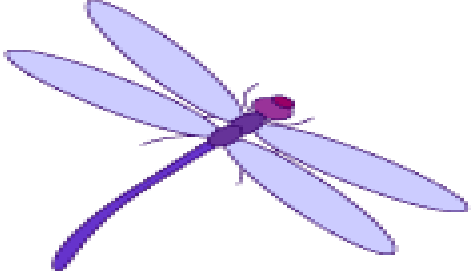
- Example 2: 4 , 1 , -2 , -5,

In this case, the first number is 4 and the new term is calculated by adding “-3” to the previous term, hence **common difference is “-3”** .

- Therefore, common difference can be positive or negative.



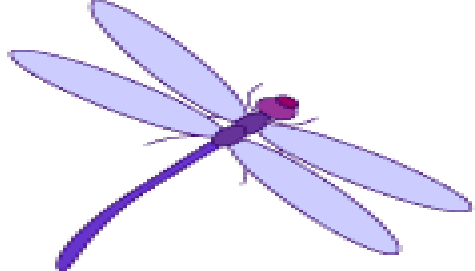
- In general arithmetic sequence can be written as
 $a+0d, a+1d, a+2d, a+3d, \dots, a+nd, \dots$
- Where,
 a is the first term of the sequence
 d is the common difference
 n is the number of terms
- In the general sequence, we can see that the common difference is multiplied by $(n-1)$ of that term.



- In the general sequence, we can see that the common difference is multiplied by $(n-1)$ of that term.
-
- For instance , first term $\rightarrow (n-1)d = (1-1)d = 0d$
second term $\rightarrow (n-1)d = (2-1)d = 1d$
third term $\rightarrow (n-1)d = (3-1)d = 2d$

- n th term of the arithmetic sequence is

$$n \text{ th term of A.P } (t_n) = a + (n-1)d$$



EXAMPLE

- Find the 8th term of the A.P sequence 0,3,6,9,.....

nth term of the A.P Sequence (t_n) = $a + (n-1)d$

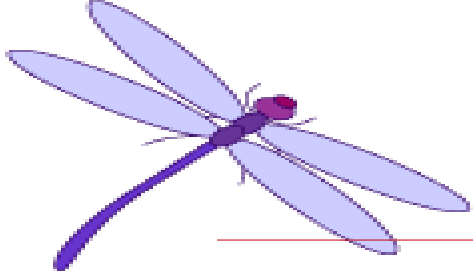
a = first term = 0

d = common difference = $3 - 0 = 3$

$(n-1) = (8 - 1) = 7$

Therefore, $t_8 = 0 + (7)(3) = 21$

Sum of first n terms of A.P



- Sum of first n terms (S_n) = n (average of first and last term)
- There are two ways to find the sum of n terms of a sequence.
- Method 1 : when the first term and the difference is given,

First term = a ,

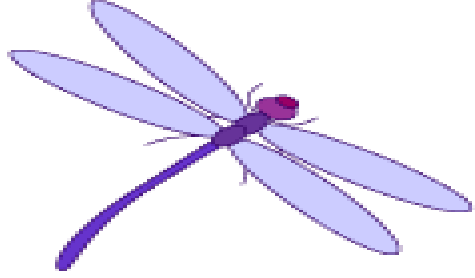
difference = d ,

last term (l) = $a + (n-1)d$

First term + last term = $a + a + (n-1)d$

= $2a + (n-1)d$

$$S_n = n/2 [2a + (n-1)d]$$

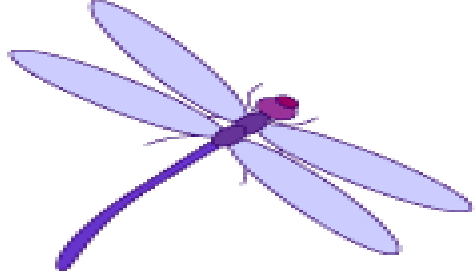


- Method 2 : when first term and last term are given

first term = a

last term = l

Therefore $S_n = n/2 (a + l)$



EXAMPLES

- Find the sum of the first five terms of the A.P where the first term is 3 and common difference is 5.

Given : First term (a) = 3

common difference (d) = 5

number of terms(n) = 5

Solution : $S_n = n/2 [2a +(n-1)d]$

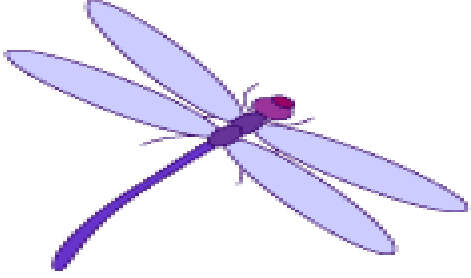
$$S_5 = 5/2 [(2)(3) + (5 - 1)(5)]$$

$$= 5/2 [6 + 20]$$

$$= 5/2 [26]$$

$$= (5)(13)$$

Therefore $S_5 = 65$



- Find the sum of first 7 terms of A.P, if the first term is 2 and the last term is 20.

Given : $a = 2$, $l = 20$, $n = 7$

solution : $S_n = n/2 (a + l)$

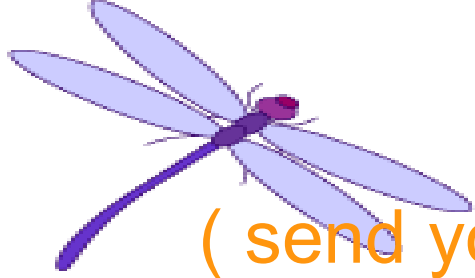
$$S_7 = 7/2 (2 + 20)$$

$$= 7/2 (22)$$

$$= (7)(11)$$

$$= 77$$

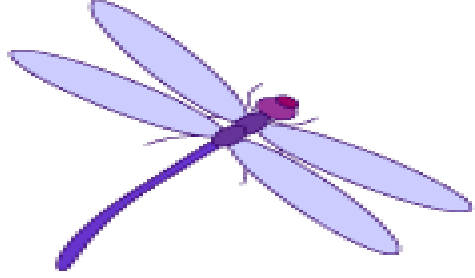
Therefore $S_7 = 77$



QUIZ

(send your solutions to support@greedge.com)

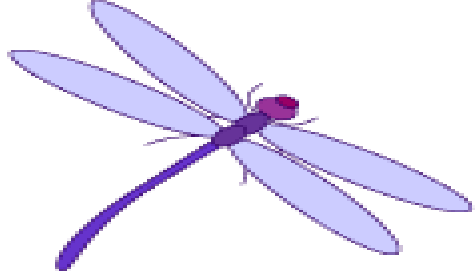
- 1) Write down the arithmetic sequence for 8 terms , if the first term is 34 and the common difference is 9 ?
- 2) Find the 59th term of the sequence, where the first term is 70 and the common difference is -2 ?
- 3) Find the sum of first 10 terms of the Arithmetic sequence 4 , 7 , 10 , 13 ,



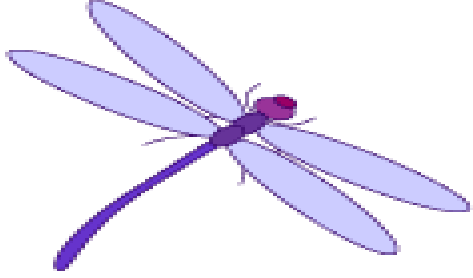
4) Find the Sum of first **20** terms of the sequence, where, $d = -45$ and $a = 12$?

5) Find the sum of first 1000 odd numbers ?

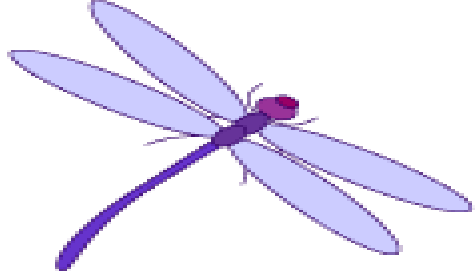
6) Find the **23rd** term of an A.P with first term **2** and common difference **7**.



GEOMETRIC PROGRESSION



- **Geometric progression** is the sequence of numbers.
- Where the new term is found by multiplying the fixed amount with the previous term in the sequence.
- **Fixed amount** is known as the **common ratio**.
- Common ratio can be positive or negative or a fraction.



EXAMPLES

1) $1, 3, 9, 27, \dots$

First term = 1

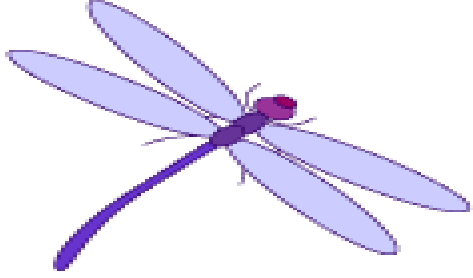
Common ratio = 3

(the new term is calculated by multiplying the previous term with common ratio, that is 3)

2) $2, -2/3, 2/9, -2/27, \dots$

Here, first term = 2

common ratio = $-1/3$



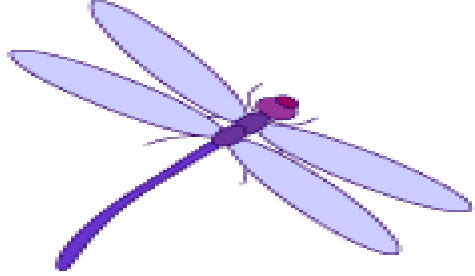
- In general, geometric sequence can be written as

$$a, ar, ar^2, ar^3, \dots, ar^n, \dots$$

- Where,
 $a \rightarrow$ first term
 $r \rightarrow$ common ratio
 $n \rightarrow$ number of terms

- Nth term of G.P is

$$t_n = a \cdot r^{(n-1)}$$



EXAMPLE

- Find the 7th term of G.P, if the first term is 2 and common ratio is 3 ?

Given : $a = 2$, $r = 3$, $n = 7$

Solution : $t_n = a \times r^{(n-1)}$

$$t_7 = (2) \times (3)^{(7-1)}$$

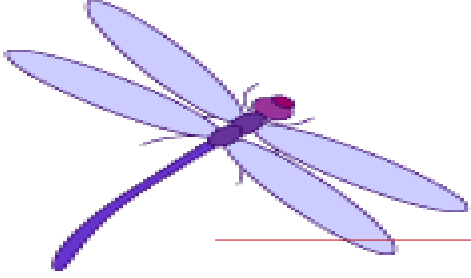
$$= (2) \times 3^6$$

$$= (2) \times (729)$$

$$= 1458$$

Therefore 7th term of G.P is 1458

Sum of the first n terms of G.P



- Method 1 : when common ratio (r) is greater than 1.

$$S_n = a \times (r^n - 1) / (r - 1)$$

- Method 2 : when common ratio (r) is less than 1.

$$S_n = a \times (1 - r^n) / (1 - r)$$

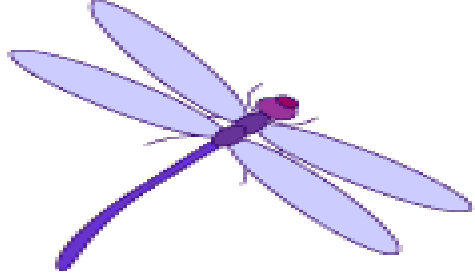
Where,

S_n = sum of first n terms

a = first term

r = common ratio

n = number of terms



EXAMPLES

1) Find the sum of the first five terms of the G.P, where first term is **3** and common ratio is **2**.

Given : first term (a) = 3

common ratio (r) = 2

number of terms (n) = 5

Solution : $r > 1$, therefore $S_n = a \times (r^n - 1) / (r - 1)$

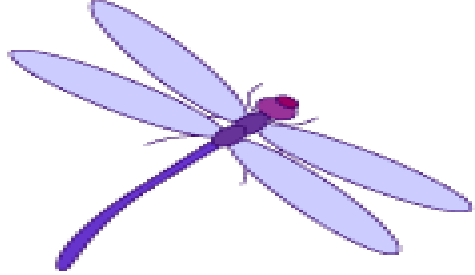
$$S_5 = 3 \times (2^5 - 1) / (2 - 1)$$

$$= 3 \times (32 - 1) / 1$$

$$= 3 \times 31$$

$$= 93$$

Therefore fifth term of G.P is 93



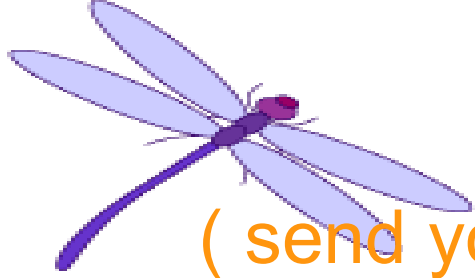
2) Find the sum of 6 terms of the G.P , Where a is 2 and r is $\frac{1}{2}$

Given : $a = 2$, $r = \frac{1}{2}$, $n = 6$

Solution : $r < 1$, therefore $S_n = a \times (1-r^n)/(1-r)$

$$\begin{aligned} S_6 &= 2 \times [1 - (\frac{1}{2})^6] / (1 - \frac{1}{2}) \\ &= 2 \times [1 - (1/64)] / (\frac{1}{2}) \\ &= 2 \times [63/64] \times 2 \\ &= 63/16 \end{aligned}$$

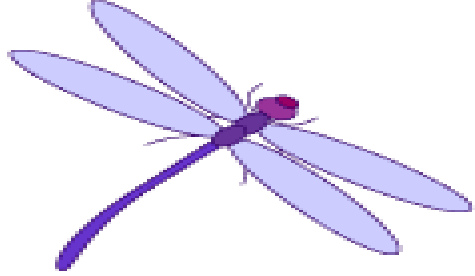
Therefore sum of 6 terms of G.P is 63/16



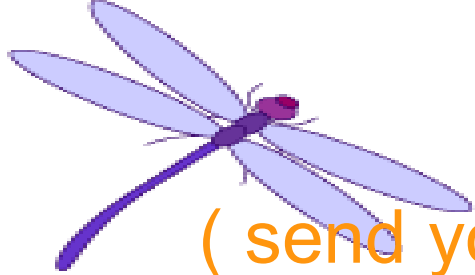
QUIZ

(send your solutions to support@greedge.com)

- 1) Find the common ratio for the geometric sequence $1, \frac{1}{2}, \frac{1}{4}, \dots$
- 2) Write down the geometric sequence for ten terms, if the first term is 3 and the common ratio is $-\frac{2}{3}$?
- 3) Find the 50^{th} term of the geometric progression $5, 10, 20, 40, 80, \dots$



- 4) Find the sum of first **5** terms of the geometric sequence **1** , **$2/3$** , **$4/9$** ,
- 5) Find the **20^{th}** term of G.P **$5/2$** , **$5/4$** , **$5/8$** ,
- 6) Find the sum of **9** terms of G.P, where **$a = 5$** and **$r = 4$** .
- 7) Given a geometric sequence with **$a = 729$** and the **7^{th} term is 64**,
find **S_7** ?



GUESS

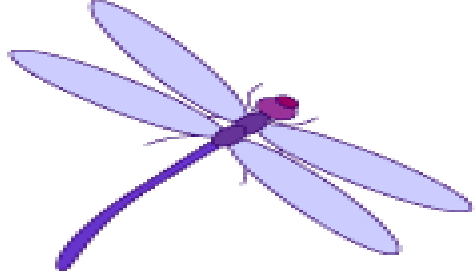
(send your solutions to support@greedge.com)

1) Write the first three terms for the sequence

$$t_n = (n+3)/4$$

2) If the 3rd term of G.P (t_3) is 24 and 6th term of G.P (t_6) is 192,
find 10th term (t_{10}) .

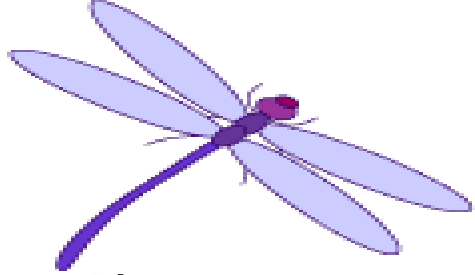
3) If the 4th term of A.P (t_4) is 30 and 8th term of A.P (t_8) is 42,
find 14th term of A.P(t_{14}).



4) Find the **12th term** of A.P, if the 8th term(t_8) is **66** and the common difference (**d**) is **12** ?

5) Find the **14th term** of a G.P. whose 9th term(t_9) is **192** and the common ratio (**r**) is **2**.

HINTS FOR THE GUESS SECTION



1) Substitute 1, 2, 3 in the place of “n” and find the values for the first three terms of the sequence.(t_1 , t_2 , t_3).

2) Given : $t_3 = 24$, $t_6 = 192$

To Find : t_{10}

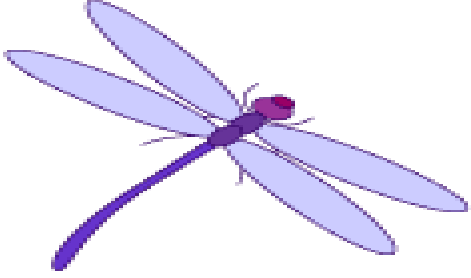
Solution : $t_n = a \times r^{(n-1)}$

Therefore, $t_3 \rightarrow a \times r^{(3-1)}=24 \rightarrow a \times r^2 = 24 \rightarrow (1)$

$t_6 \rightarrow a \times r^{(6-1)}=192 \rightarrow a \times r^5=192 \rightarrow (2)$

Divide equation (2) by equation (1) and find the value of of a and r.

With the value of a and r , find the value of t_{10}



3) Given : $t_4 = 30$, $t_8 = 42$

To find : t_{14}

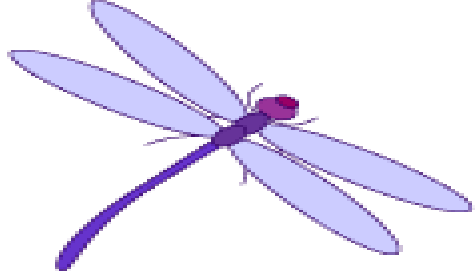
Solution : $t_n = a + (n-1)d$

Therefore, $t_4 \rightarrow a + (4-1)d = 30 \rightarrow a + 3d = 30 \rightarrow (1)$

$t_8 \rightarrow a + (8-1)d = 42 \rightarrow a + 7d = 42 \rightarrow (2)$

Solve equation (1) and (2) to find the values a and d .

Substitute a and d in the equation to find t_{14} .



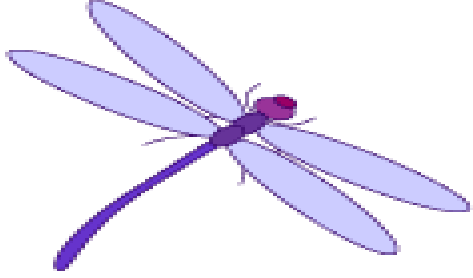
4) Given : $t_8 = 66$, $d = 12$

To Find : t_{12}

Solution : $t_n = a + (n-1)d$

$$t_8 \rightarrow a + (8 - 1)(12) = 66$$

solve the equation to find a , and substitute a and d in the general equation to find t_{12}



5) Given : $t_9 = 192$, $r = 2$

To find : t_{14}

Solution :

- check whether r is greater than or less than 2 and choose the appropriate formula.
- Form the equation for t_9 with the help of the formula and find the value of a .
- with the help of a and r , find t_{14}