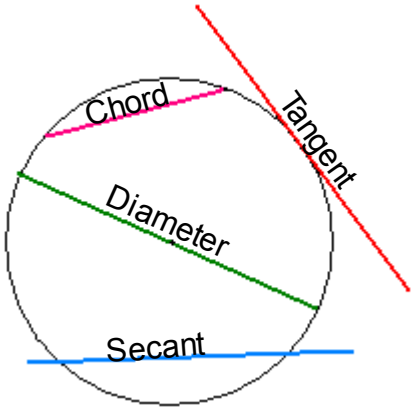
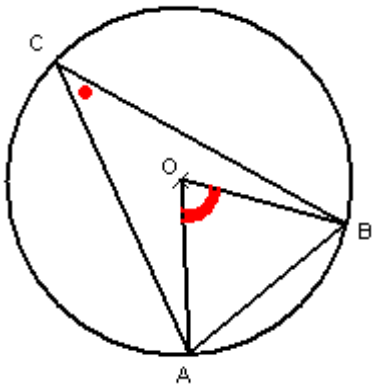
Chord, Tangent and Secant

Part2

Chord, Tangent and Secant

A chord of a circle is a line segment whose both endpoints lie on the circle.
A diameter is a chord passing through the center.
Tangent of the circle is the line or the segment which touches exactly one point of the circle whereas
secant is the line or the segment which
touches exactly two point of the circle





Central angle: Given a chord say AB in a circle and if O is the center of the circle then ∟AOB is known as the central angle subtended by the chord AB.

Inscribed angle: Given a chord say AB in a circle and if O is the center of the circle ∟ACB is known as the inscribed angle subtended by the chord AB

Chord properties

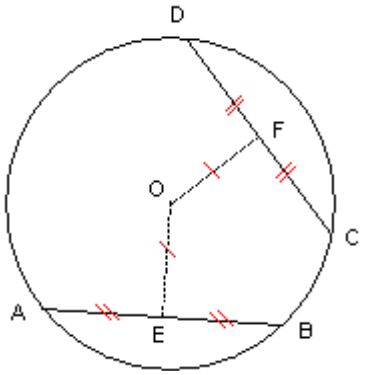
(These are important properties that will help you in problem solving in GRE. Ensure that you have understood these concepts well. Contact your SFA if you need further help in these concepts)

1) Chords are equidistant from the center of a circle if and only if they are equal (in length).

Let O be the center of the circle and AB and DC be two chords of the circle. OE be distance from the center to the chord AB and OF be the distance from the center to the chord CD.

If AB = CD then OE = OF

Conversely if OE = OF then AB = CD



2) The perpendicular bisector of a chord passes through the center of a circle

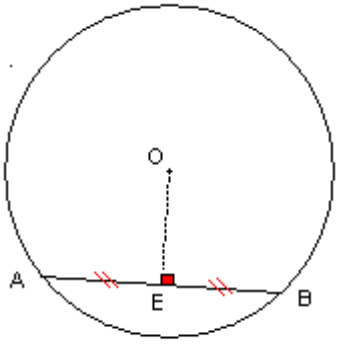
Equivalently

A perpendicular line from the center of a circle bisects the chord.

Equivalently

The line segment (Circular segment) through the center bisecting a chord is perpendicular to the chord.

is

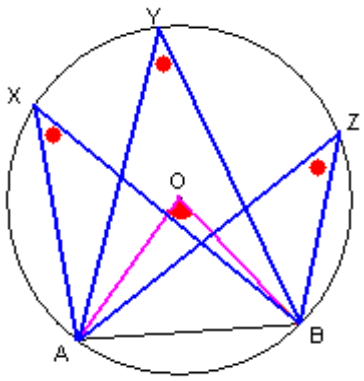


Let O be the center of the circle and AB is the chord of the circle. If E is the center of the chord AB then OE perpendicular to AB($OE \perp AB$), in other words $\angle OEB = 90^\circ$.

Or if $OE \perp AB$ then E is the midpoint of chord AB.

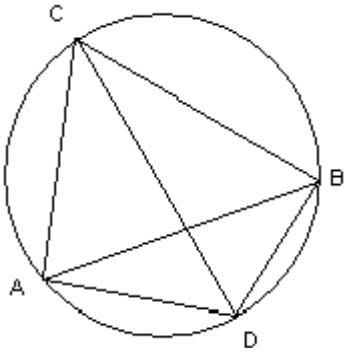
3) If two angles are inscribed on the same chord and on the same side of the chord, then they are equal. If a central angle and an inscribed angle of a circle are subtended by the same chord and on the same side of the chord, then the central angle is twice the inscribed angle.

In the figure O is the center and AB is the chord.
∟AOB is the central angle subtended by the chord AB.
∟AXB, ∟AYB and ∟AZB are the inscribed angle subtended by the same chord AB, then
∟AXB = ∟AYB = ∟AZB
and also
∟AOB = 2 ∟AXB = 2∟AYB = 2∟AZB



Guess: Find the measure of the central angle and inscribed angle subtended by the chord whose length is equal to the length of the radius of the circle .

4) If two angles are inscribed on the same chord and on opposite sides of the chord, then they are supplemental.



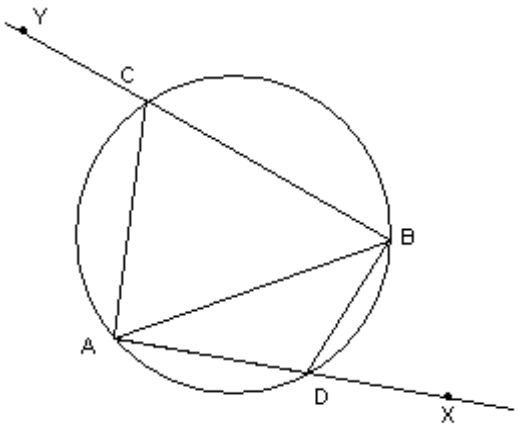
In the figure AB and CD are the two chords of the circle. ∟ACB and ∟ADB are the two inscribed angles by the chord AB. Hence ∟ACB + ∟ADB = 180°.
Similarly ∟CAD and ∟DBC are the two inscribed angles by the chord AB. Hence ∟CAD + ∟DBC = 180°.
ADBC is the known as cyclic quadrilateral.

NOTE:

- A quadrilateral is said to be cyclic if the opposite angle are supplementary.
- Sum of interior angles of a triangle is always180° and sum of the interior angles of a quadrilateral is always 360°.

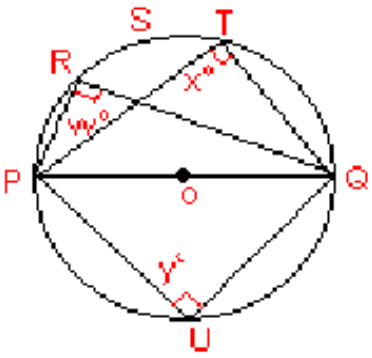
5) For a cyclic quadrilateral, the exterior angle is equal to the interior opposite angle.

Consider the figure,



we know that ∟ADB + ∟BDX = 180° ------(1)
(since AD and DX is the straight line and
the angle formed by straight lines is always 180°)
and also from above concept we know that
∟ADB + ∟ACB = 180°----------(2)
hence ACB = ∟BDX
Similarly prove that ∟ACY = ∟ ADB.

6) An inscribed angle subtended by a diameter is a right angle.



Consider the figure,
Suppose PQ is the diameter and every diameter is a chord. From the figure ∟PRQ, ∟PTQ and ∟PUQ are inscribed angles by the chord AB.
Hence ∟PRQ = ∟PTQ
But since PQ is the diameter which is a straight line hence, the angle made by PO and OQ is 180°.
=> ∟PRQ = ∟PTQ = 90°
and also we know that ∟PRQ + ∟PUQ
= ∟PRQ + ∟PUQ
= 180°
=> ∟PUQ = 90°.

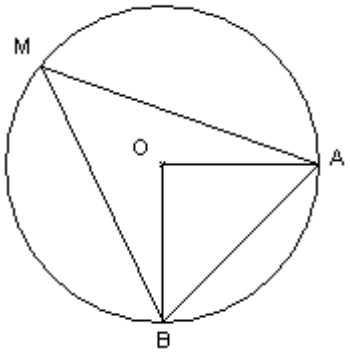
Quiz

(Send your solutions to support@greedge.com)

1) What is the length of the chord if the radius of the circle is 5cm and distance between chord and center is 3cm?

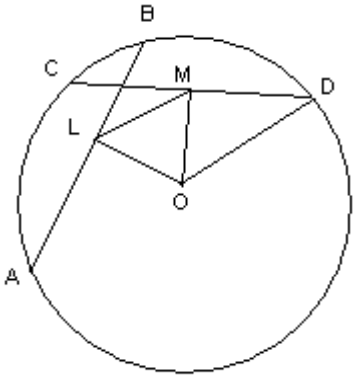
2) If the length of the chord is 12 cm and the distance between the center and chord is 8 cm, find the radius of the circle.

3) O is the center of the circle. AB is the chord. If the length AB is $\sqrt{2}$ times the length of its radius, find the following

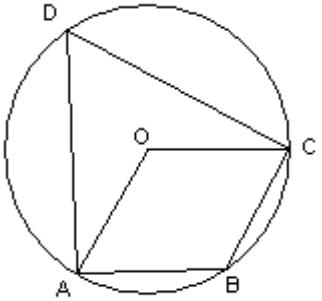


   a) measure of the angle $\angle AOB$.
   b) measure of the angle $\angle AMB$
   c) Compare $\angle AMB$ and $\angle OAB$.

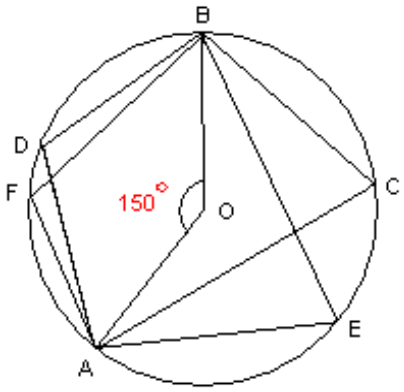
4) In the figure, O is the center of the circle. AB and CD are two chords. L and M are the midpoints of the AB and CD respectively. ΔLMO is an equilateral triangle(all three sides of an equilateral triangle have same length), compare the length of AB and CD.



5) In the figure, O is the center of the circle If $\angle AOC = 120^\circ$ find $\angle ABC$.
Hint: ADC is the inscribed angle subtended by chord AC and ABCD is a cyclic quadrilateral.



6) In the figure, O is the center of circle . If $\angle AOB = 150^\circ$ then find $\angle AEB$, $\angle ACB$, $\angle ADB$ and $\angle AFB$.



Which of the following is true?

a) $\angle DBC + \angle FAE = 180^\circ$

b) $\angle FBC + \angle DAC = 180^\circ$

c) $\angle AFB + \angle AOB = 180^\circ$

d) $\angle AFB + \frac{1}{2} \angle AOB = 180^\circ$

e) $\angle AEB + \frac{1}{2} \angle AOB = 180^\circ$