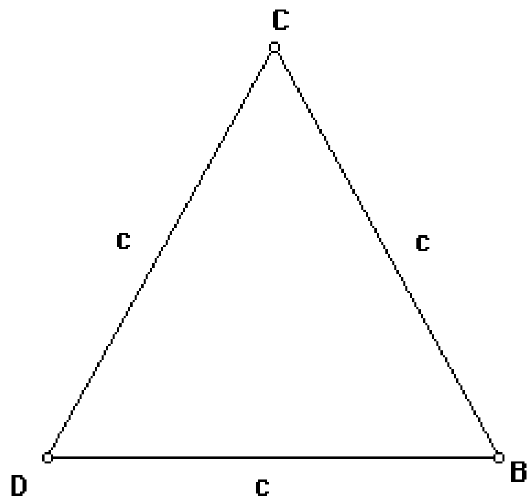
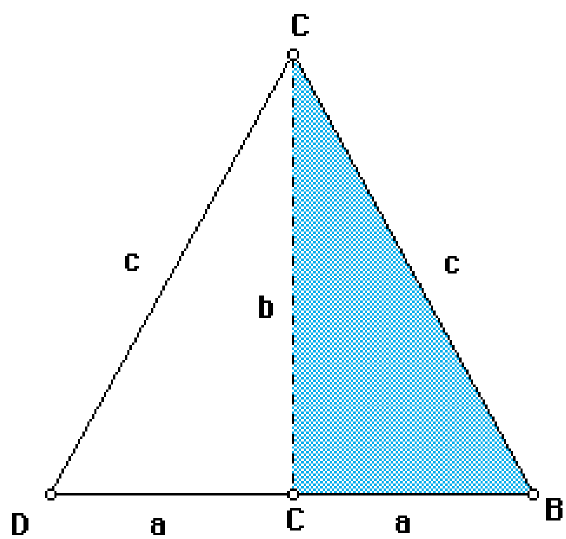
Special Right Triangles: Proof





Draw equilateral ΔABD with three 60° angles (Figure 1). Draw an altitude AC which creates a 30° - 60° - 90° right triangle ΔACB with right $\angle ACB$. Label the side across from the 60° angle as side b, the side across from the 30° angle as side a, and the hypotenuse as side c. Since ΔABD is equilateral, we know all the sides have length c. Therefore, DB = c. Since the altitude cuts side DB into two congruent segments, we know that DB = 2a. Using algebra, we know that c = 2a. Therefore, we know that the hypotenuse = 2 * short leg.

We can use this fact and the Pythagorean Theorem to calculate the value of b.

$$a^2+b^2=c^2$$
$$a^2+b^2=(2a)^2$$
$$a^2+b^2=4a^2$$
$$b^2=3a^2$$
$$b=\pm\sqrt{3a^2}$$
$$b=\pm|a|\sqrt{3}$$
$b = a\sqrt{3}$ (since length of segments are positive in Euclidean geometry)

Therefore, we have shown that the long leg b is $\sqrt{3}$ times the short leg.