NTS GAT General Past Papers Questions

Quantitative – Exam No. 24

Probability

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Formulas:

1. **Probability of single event:**

$$P(A) = \frac{n(A)}{n(S)}$$

Where:

$$P(A) = Probability\ of\ event\ A$$

$$n(A) = Number\ of\ times\ event\ A\ occurred$$

$$n(S) = Sample\ space$$

2. **Probability of two independent events:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where:

$$P(A \cup B) = Probability\ of\ event\ A\ union\ B$$

$$P(A) = Probability\ of\ event\ A$$

$$P(B) = Probability\ of\ event\ B$$

$$P(A \cap B) = Probability\ of\ event\ A\ intersection\ B$$

3. **Probability of two dependent events:**

$$P(A.B) = P(A) \times P(B)$$

Where:

$$P(A.B) = Probability\ of\ event\ A\ and\ B$$

4. **Sample space of a coin:**

$$S = (2)^{Number\ of\ coins} = (2)^1 = 2$$

$$S = \{H, T\}$$

5. **Sample space of two coins:**

$$S = (2)^{Number\ of\ coins} = (2)^2 = 4$$

$$S = \{HH, HT, TH, TT\}$$

6. **Sample space of three coins:**

$$S = (2)^{Number\ of\ coins} = (2)^3 = 8$$

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Coin 1	H	H	H	H	T	T	T	T
Coin 2	H	H	T	T	H	H	T	T
Coin 3	H	T	H	T	H	T	H	T
Sample	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT

7. **Sample space of a dice:**

$$S = (6)^{Number\ of\ dices} = (6)^1 = 6$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

8. **Sample space of two dices:**

$$S = (6)^{Number\ of\ dices} = (6)^2 = 36$$

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$$
$$, (2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$$
$$, (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$$
$$, (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$$
$$, (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$
$$, (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

9. Probability limits: $0 \leq P \leq 1$ (PP)

10. Probability can be expressed in fraction form as well as in decimal form.

11. An event having probability equal to zero is called null event. (PP)

12. An event having probability equal to unity is called certain event.

Exercise

1. Two coins are tossed. Find the probability of getting all heads? (PP)

Solution:

$$S = \{HH, HT, TH, TT\}$$

$$P(all\ heads) = \frac{n(all\ heads)}{n(S)}$$

$$P(all\ heads) = \frac{1}{4}$$

2. Two coins are tossed. Find the probability of getting at least one tail? (PP)

Solution:

$$S = \{HH, HT, TH, TT\}$$

$$P(at\ least\ one\ tail) = \frac{n(at\ least\ one\ tail)}{n(S)}$$

$$P(at\ least\ one\ tail) = \frac{3}{4}$$

3. A coin is tosses twice. Find the probability of getting tail all the times? (PP)

Solution:

$$S = \{H, T\}$$

$$P(tail) = \frac{n(tail)}{n(S)}$$

$$P(tail) = \frac{1}{2}$$

Since coin is tossed twice, so:

$$P(tail) = \frac{1}{2} \times \frac{1}{2}$$

$$P(tail) = \frac{1}{4}$$

4. Three coins are tossed. Find the probability of getting two heads and one tail?

Solution:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$P(two\ heads\ and\ one\ tail) = \frac{n(two\ heads\ and\ one\ tail)}{n(S)}$$

$$P(two\ heads\ and\ one\ tail) = \frac{3}{8}$$

5. A coin is tossed three times. Find the probability of getting heads all the times?

Solution:

$$S = \{H, T\}$$

$$P(head) = \frac{n(head)}{n(S)}$$

$$P(head) = \frac{1}{2}$$

Since coin is tossed three times, so:

$$P(head) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$P(head) = \frac{1}{8}$$

6. A dice is rolled. Find the probability of getting multiple of 5? (PP)

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(multiple\ of\ 5) = \frac{n(multiple\ of\ 5)}{n(S)}$$

$$P(multiple\ of\ 5) = \frac{1}{6}$$

7. Find the probability of multiples of 4 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}? (PP)

Solution:

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$P(multiple\ of\ 4) = \frac{n(multiple\ of\ 4)}{n(S)}$$

$$P(multiple\ of\ 4) = \frac{2}{10} = \frac{1}{5}$$

8. A dice is rolled. Find the probability of getting an odd number or a number greater than 4? (PP)

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(O \cup > 4) = P(O) + P(> 4) - P(O \cap > 4)$$

$$P(O \cup > 4) = \frac{n(O)}{n(S)} + \frac{n(> 4)}{n(S)} - \frac{n(O \cap > 4)}{n(S)}$$

$$P(O \cup > 4) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

$$P(O \cup > 4) = \frac{3 + 2 - 1}{6}$$

$$P(O \cup > 4) = \frac{4}{6}$$

$$P(O \cup > 4) = \frac{2}{3}$$

9. A dice is rolled. Find the probability of getting an even number or 6?

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(E \cup 6) = P(E) + P(6) - P(E \cap 6)$$

$$P(E \cup 6) = \frac{n(E)}{n(S)} + \frac{n(6)}{n(S)} - \frac{n(E \cap 6)}{n(S)}$$

$$P(E \cup 6) = \frac{3}{6} + \frac{1}{6} - \frac{1}{6}$$

$$P(E \cup 6) = \frac{3+1-1}{6}$$

$$P(E \cup 6) = \frac{3}{6}$$

$$P(E \cup 6) = \frac{1}{2}$$

10. Two dices are rolled. Find the probability of getting a sum of 9? (PP)

Solution:

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$,(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$,(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$,(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$,(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$,(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$P(Sum\ of\ 9) = \frac{n(Sum\ of\ 9)}{n(S)}$$

$$P(Sum\ of\ 9) = \frac{4}{36}$$

$$P(Sum\ of\ 9) = \frac{1}{9}$$

11. Two dices are rolled. Find the probability of getting a sum of 7 or sum of11? (PP)

Solution:

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$,(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$,(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$,(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$,(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$

$$,(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(sum\ of\ 7 \cup sum\ of\ 11)$$

$$= P(sum\ of\ 7) + P(sum\ of\ 11) - P(sum\ of\ 7 \cap sum\ of\ 11)$$

$$P(sum\ of\ 7 \cup sum\ of\ 11)$$

$$= \frac{n(sum\ of\ 7)}{n(S)} + \frac{n(sum\ of\ 11)}{n(S)} - \frac{n(sum\ of\ 7 \cap sum\ of\ 11)}{n(S)}$$

$$P(sum\ of\ 7 \cup sum\ of\ 11) = \frac{6}{36} + \frac{2}{36} - \frac{0}{36}$$

$$P(sum\ of\ 7 \cup sum\ of\ 11) = \frac{6+2-0}{36}$$

$$P(sum\ of\ 7 \cup sum\ of\ 11) = \frac{8}{36}$$

$$P(sum\ of\ 7 \cup sum\ of\ 11) = \frac{2}{9}$$

12. There are 7 red, 8 white, 1 mauve, 3 blue and 5 pink lipsticks in a box. One lipstick is chosen at random. Find the probability that it is blue?

Solution:

$$n(S) = 7 + 8 + 1 + 3 + 5 = 24$$

$$P(blue) = \frac{n(blue)}{n(S)}$$

$$P(blue) = \frac{3}{24}$$

$$P(blue) = \frac{1}{8}$$

13. There are 3 red, 9 white, 11 mauve, 8 blue and 5 pink balls in a basket. One ball is chosen at random. Find the probability that it is either red or white?

Solution:

$$n(S) = 3 + 9 + 11 + 8 + 5 = 36$$

$$P(red \cup white) = P(red) + P(white) - P(red \cap white)$$

$$P(red \cup white) = \frac{n(red)}{n(S)} + \frac{n(white)}{n(S)} - \frac{n(red \cap white)}{n(S)}$$

$$P(red \cup white) = \frac{3}{36} + \frac{9}{36} - \frac{0}{36}$$

$$P(red \cup white) = \frac{3 + 9 - 0}{36}$$

$$P(red \cup white) = \frac{12}{36}$$

$$P(red \cup white) = \frac{1}{3}$$

14. There are 7 red, 5 blue and 3 green balls in a basket. One ball is chosen at random. Find the probability that it is white? (PP)

Solution:

As there is no white ball, so:

$$P(white) = 0$$

It is null event.

15. There are 100 people on a line. Asim is the 37th person and Faisal is the 67th person. If a person on line is chosen at random, what is the probability that the person is standing between Asim and Faisal? (PP)

Solution:

It is given that Asim is standing at 37th position and Faisal is standing at 67th position. So, if we count the people standing between Asim and Faisal, we got a value of 29. We know that:

$$P(Between\ Asim\ and\ Faisal) = \frac{n(Between\ Asim\ and\ Faisal)}{n(S)}$$

$$P(Between\ Asim\ and\ Faisal) = \frac{29}{100}$$

16. A jar has 5 marbles, 1 of each of the colors red, white, blue, green and yellow. If 4 marbles are removed from the jar, what is the probability that the yellow one was removed? (PP)

Solution:

We know that:

$$P(Yellow) = \frac{n(Yellow)}{n(S)}$$

$$P(Yellow) = \frac{1}{5}$$

As 4 marbles are removed so:

$$P(Yellow) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

$$P(Yellow) = \frac{4}{5}$$

17. There are 27 students on the college debate team. What is the probability that at least three of them have their birthdays in the same month? (PP)

Solution:

When we calculate the probability in any event, the data is uniformly distributed. For example, if we have a jar has 5 marbles, 1 of each of the colors red, white, blue, green and yellow; and one marble is removed from the jar, then each color has equal probability of appearing i.e., 1/5 or 0.2. It is said to be uniformly distributed. In the present question, there are 27 students; and if we distribute their birthdays uniformly in 12 months, then 24 students are done. We are left with three students. These three students can have their birthday in any of the 12 months. So, there must be three months which will contain the birthday of at least three students in the same month. So, it is certain event. Its probability will be unity.

$$P(A) = 1$$

18. How many different orders may 6 books be placed on a shelf? (PP)

Solution:

We know that orders can be found as follows:

$$Orders = (Number\ of\ books)!$$

$$Orders = (6)!$$

$$Orders = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$Orders = 720$$

19. How many different combinations can be made with "WORD"? (PP)

Solution:

We know that orders can be found as follows:

$$Combinations = (Number\ of\ alphabets)!$$

$$Combinations = (4)!$$

$$Combinations = 4 \times 3 \times 2 \times 1$$

$$Combinations = 24$$

20. In how many ways, 4 boys and 3 girls can be seated in a row so that they are alternate? (PP)

Solution:

Let the arrangement be as follows:

$$B \quad G \quad B \quad G \quad B \quad G \quad B$$

Hence, four boys can be arranged in 4! ways as follows:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

And, three girls can be arranged in 3! ways as follows:

$$3! = 3 \times 2 \times 1 = 6$$

So, total number arrangements will be:

$$Arrangements = 24 \times 6$$

$$Arrangements = 144$$

21. A bag contains 7 blue balls and 14 red balls. If one ball is drawn at a random from bag, what is the probability that ball is blue? (PP)

Solution:

$$n(S) = 7 + 14 = 21$$

$$P(blue) = \frac{n(blue)}{n(S)}$$

$$P(blue) = \frac{7}{21}$$

$$P(blue) = \frac{1}{3}$$

22. A number, x, is chosen at random from the set of positive integers less than 10. What is the probability that $\frac{9}{x} > x$? (PP)

Solution:

There are following possible values of x:

$$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Now, we will check the inequality but substituting each value of x one by one.

$$Let\ x = 1 \rightarrow \frac{9}{1} > 1 \rightarrow 9 > 1 \rightarrow True$$

$$Let\ x = 2 \rightarrow \frac{9}{2} > 2 \rightarrow 4.5 > 2 \rightarrow True$$

$$Let\ x = 3 \rightarrow \frac{9}{3} > 3 \rightarrow 3 > 3 \rightarrow False$$

$$Let\ x = 4 \rightarrow \frac{9}{4} > 4 \rightarrow 2.25 > 4 \rightarrow False$$

$$Let\ x = 5 \rightarrow \frac{9}{5} > 5 \rightarrow 1.8 > 5 \rightarrow False$$

$$Let\ x = 6 \rightarrow \frac{9}{6} > 6 \rightarrow 1.5 > 6 \rightarrow False$$

$$Let\ x = 7 \rightarrow \frac{9}{7} > 7 \rightarrow 1.28 > 7 \rightarrow False$$

$$Let\ x = 8 \ \rightarrow \ \frac{9}{8} > 8 \ \rightarrow \ 1.125 > 8 \ \rightarrow \ False$$

$$Let\ x = 9 \ \rightarrow \ \frac{9}{9} > 9 \ \rightarrow \ 1 > 9 \ \rightarrow \ False$$

Now, let's calculate the probability:

$$P(A) = \frac{Number\ of\ trues}{Sample\ space}$$

$$P(A) = \frac{2}{9}$$

23. Given that sample space is {2, 4, 5, 8, 10}. Find the probability of even number? (PP)

Solution:

$$n(S) = \{2, 4, 5, 8, 10\}$$

$$P(even) = \frac{n(even)}{n(S)}$$

$$P(even) = \frac{4}{5}$$

24. In a group of 900 students, 400 are girls. Find the probability of boys? (PP)

Solution:

$$P(Boys) = \frac{n(Boys)}{n(S)}$$

$$P(Boys) = \frac{900 - 400}{900}$$

$$P(Boys) = \frac{500}{900} = \frac{5}{9}$$

25. There are 5 red and 3 blue balls in a box. If two balls are taken out one by one without replacement, what is the probability that both balls are red? (PP)

Solution:

$$n(S) = 5 + 3 = 8$$

Probability of 1st ball to be red:

$$P(1^{st}\ ball\ is\ red) = \frac{n(red)}{n(S)}$$

$$P(1^{st}\ ball\ is\ red) = {}^{5}/_{8}$$

Probability of 2nd ball to be red:

$$P(2^{nd}\ ball\ is\ red) = \frac{n(red) - 1}{n(S) - 1}$$

$$P(2^{nd}\ ball\ is\ red) = \frac{5 - 1}{8 - 1}$$

$$P(2^{nd}\ ball\ is\ red) = {}^{4}/_{7}$$

Probability that both balls are red:

$$P(Both\ balls\ are\ red) = [\,P(1^{st}\ ball\ is\ red)] \times [P(2^{nd}\ ball\ is\ red)]$$

$$P(Both\ balls\ are\ red) = \frac{5}{8} \times \frac{4}{7}$$

$$P(Both\ balls\ are\ red) = \frac{5}{14}$$