Fraction Tutorials

Fractions and Different Types

Concept : Fraction

Definition: **A fraction** is defined as a part of a whole. A fraction is always expressed as $\frac{p}{q}$ such that p and q are integers and $q \neq 0$. Here p is said to be **Numerator** and q is said to be **denominator**.

For example, 1) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{11}{45}$, $\frac{56}{18}$, $\frac{-16}{3}$, $\frac{29}{-4}$, $\frac{-15}{-6}$, $\frac{-9}{10}$ are the few examples of fractions.

2) Every integer can be expressed as fraction. As if a is any integer then, $a = \frac{a}{1}$.

Properties: Fraction can also be classified as positive fractions and negative fractions.

1) We say that given fraction as a **positive fraction** if the numerator and the denominator have same sign.
For example, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{-15}{-6} = \frac{15}{6}$ are the positive fractions.

2) Whereas the given fraction is said to **negative fraction** if the numerator and the denominator have different signs.
For example, $\frac{-16}{3}$, $\frac{29}{-4}$ are the negative fractions.

Additional information: Note that, in GRE we do not deal with fractions like $\frac{p}{0}$ or $\frac{0}{0}$. But if we have the fractions like $\frac{0}{q}$ then this type of fraction is equal to 0.

Now let us consider the different type of fractions.

Concept : Proper fraction.

Definition: A fraction $\frac{p}{q}$ is said to be **proper fraction** if the numeric value (or absolute value) of p (numerator) is less than the numeric value of q (denominator).

For example, $\frac{3}{8}$, $\frac{11}{45}$, $\frac{-9}{10}$ are the proper fraction.

Concept : Improper fraction.

Definition: A fraction $\frac{p}{q}$ is said to be **improper fraction** if the numeric value (or absolute value) of p is greater than the numeric value of q.

For example, $\frac{56}{18}$, $\frac{-16}{3}$, $\frac{29}{-4}$ are the improper fraction.

2) Every integer except -1, 0 1 are improper fractions.

Quiz time.

1) Is $-\{\frac{-(2-6)+3}{4-7}\}$ a positive fraction ?

i) True
ii) False
iii) Can't say.

2) Suppose p is a prime number and q is the positive integer multiple of p, what type of fraction will be $\frac{p}{q}$?

i) Proper Fraction
ii) Improper Fraction
iii) Any one of these.

3) If p is a non-zero integer then what type to fraction $\frac{p+3}{p}$ will be?

i) Proper Fraction
ii) Improper Fraction
iii) Any one of these.

Other types of fractions

Concept : Mixed fraction.

Definition: **Mixed fraction** is obtained by adding a whole number to a proper fraction. If a is s whole number and $\frac{p}{q}$ is a proper fraction then $a+\frac{p}{q}$ is a mixed fraction. Generally we represent $a+\frac{p}{q}$ as $a\frac{p}{q}$

$11\frac{1}{13}$, $8\frac{6}{9}$, $77\frac{9}{14}$ are few examples of mixed fraction.

Example: We can express given positive improper fraction into a mixed fraction

Consider the following fraction.

1) $\frac{56}{18}$

When we divide 56 by 18, the quotient will 3 and the remainder is 2

Hence, $\frac{56}{18} = \frac{54+2}{18} = \frac{54}{18}+\frac{2}{18} = 3+\frac{2}{18} = 3\frac{2}{18}$

2) $\frac{3}{2}$

When divide 3 by 2, the quotient is 1 and the remainder is 1.

Hence, $\frac{3}{2} = \frac{2+1}{2} = \frac{2}{2}+\frac{1}{2} = 1+\frac{1}{2} = 1\frac{1}{2}$

Example: We can express a given mixed fraction as a fraction.

For example, consider $11\frac{1}{13}$

$= \frac{11}{1}+\frac{1}{13} = \frac{11\times 13}{13}+\frac{1}{13}$

$= \frac{143+1}{13} = \frac{144}{13}$

2) $77\frac{-9}{14}$

$= \frac{77}{1}+\frac{-9}{14} = \frac{77\times 14}{14}+\frac{-9}{14}$

$= \frac{1078-9}{14} = \frac{1069}{14}$

Concept: Equivalent Fraction.

Definition: When we multiply the numerator and denominator of a given fraction by the same (non-zero) number, we say that the new fraction is said to be **equivalent** to the original fraction. If a is any non zero number and $\frac{p}{q}$ is any fraction then $\frac{a \times p}{a \times q}$ equivalent to $\frac{p}{q}$

Note that:

1) Given any fraction, the quotient obtained by dividing the numerator by the denominator of the fraction is known as the **value of the fraction**.

2) If $\frac{p}{q}$ and $\frac{a \times p}{a \times q}$ are the equivalent fraction, then the value of $\frac{p}{q}$ will be same as the value of $\frac{a \times p}{a \times q}$.

Hence we get $\frac{p}{q} = \frac{a \times p}{a \times q}$.

Example: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$ and $\frac{100}{300}$ are all equivalent fractions.

In fact, $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{100}{300}$ and the value of this fraction is 0.333333...

Concept: Irreducible Fraction

Definition: A fraction $\frac{p}{q}$ is said to be an **irreducible fraction**, if GCD (p, q) = 1.

$\frac{3}{8}$, $\frac{11}{45}$, $\frac{29}{-4}$ are the few examples of irreducible fractions.

Procedure:

How to convert the given fraction into irreducible fraction?

Let $\frac{p}{q}$ is not the irreducible fraction.

This means GCD (p, q) > 1.
Let r be an integer such that r = GCD (p, q) .

Now divide the numerator and denominator by r.

The obtained fraction will be irreducible.

Example: Convert $\frac{6}{15}$ into irreducible fraction.

GCD(6, 15) = 3.

$\frac{6/3}{15/3} = \frac{2}{5}$.

Hence $\frac{2}{5}$ is an irreducible fraction.

Quiz Time:

1. Which of the following improper fractions can be obtained from $7\frac{3}{4}, 33\frac{1}{3}$?

- $\frac{28}{4}$, $\frac{99}{3}$
- $\frac{21}{4,}$ $\frac{33}{4}$
- $\frac{31}{4,}$ $\frac{100}{3}$

2) Which of the following mixed fraction can be obtained from $\frac{11}{3}$, $\frac{15}{4}$

- $3\frac{2}{3}$, $3\frac{3}{4}$
- $11\frac{1}{3}$, $15\frac{1}{4}$
- $2\frac{5}{3}$, $2\frac{7}{4}$

3) Is the improper fraction which is obtained by an mixed fraction, always an irreducible fraction?

- True
- False
- Can't say.

4) Which of the following are the equivalent fraction?

- $\frac{3}{4}$, $\frac{9}{16}$
- $\frac{4}{5}$, $\frac{6}{7}$
- $\frac{7}{8}$, $\frac{-14}{16}$
- $\frac{6}{9}$, $\frac{4}{6}$

5) Find the equivalent irreducible fraction for $\frac{52}{60}$.

- $\frac{26}{30}$
- $\frac{13}{30}$
- $\frac{26}{15}$
- $\frac{13}{15}$

Addition and Subtraction of Fractions

Let us know how to add, subtract, multiply and divide the fractions.

Equalizing the Denominators .

We can add, subtract or compare two or more given fractions only if the denominators of the given fractions are same. Let us learn how to make the denominators of the given fractions equal without changing the values of the given fraction.

Procedure:

Consider two fractions $\frac{p}{q}$ and $\frac{l}{m}$ such that $q \neq m$, then

- Find the Least Common Multiple (LCM) of q and m.
  Suppose r = LCM(q, m) .
  Let s and t be the positive factors of r, such that $r = q \times s$ and $r = m \times t$.

- Now multiply the numerator and denominator of the fraction $\frac{p}{q}$ with s and multiply the numerator and denominator of $\frac{l}{m}$ with t.

Hence the new fractions are $\frac{p \times s}{q \times s} = \frac{p \times s}{r}$ and $\frac{l \times t}{m \times t} = \frac{l \times t}{r}$ where the denominators for both the fractions are same.

Here $\frac{p \times s}{r} = \frac{p}{q}$ and $\frac{l \times t}{r} = \frac{l}{m}$

Example: Convert the given fractions into another fraction such that the denominators of the new fractions are same.

1) $\frac{4}{15}$, $\frac{6}{10}$

LCM (15, 10) = 30
We know that $30 = 5 \times 6$ and $30 = 10 \times 3$

Now, $\frac{4}{15} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$

and $\frac{6}{10} = \frac{6 \times 3}{10 \times 3} = \frac{18}{30}$

Hence the new fractions are $\frac{24}{30}$ and $\frac{18}{30}$.

2) $\frac{1}{2}$, $\frac{1}{3}$

Here, LCM(2, 3) = 6

Now, $\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$ and $\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6}$

Hence the new fractions are $\frac{3}{6}$ and $\frac{2}{6}$.

Addition between Fractions.

Definition: Suppose we have to add to fractions say $\frac{p}{q}$ and $\frac{l}{m}$, we have to follow certain rules.

1) Denominators of both the fractions should be equal.

Suppose, q ≠ m then, use **"Equalizing the Denominator"** method to have the common denominator.

2) When we are adding fractions having same denominators then, we need to only add the numerators whereas the denominator remains the same.

Hence $\frac{p}{q} + \frac{l}{m} = \frac{p\times s}{r} + \frac{l\times t}{r} = \frac{(p\times s)+(l\times t)}{r}$

Example: 1) $\frac{8}{5}+\frac{6}{5} = \frac{8+6}{5} = \frac{14}{5}$

2) $\frac{3}{2}+\frac{1}{2} = \frac{4}{2} = 2$

3) $\frac{5}{6}+\frac{11}{9}$

Now 6 ≠ 9

Hence LCM(6, 9) = 18. Now $18 = 6\times3$ and $18 = 9\times2$.

$$\frac{5}{6}+\frac{11}{9} = \frac{5\times3}{6\times3} + \frac{11\times2}{9\times2} = \frac{15}{18}+\frac{22}{18} = \frac{15+22}{18} = \frac{37}{18}$$

4) $\frac{2}{3}+\frac{6}{7}$

LCM (3, 7) = 21

Hence, $\frac{2}{3}+\frac{6}{7}=\frac{2\times7}{3\times7}+\frac{6\times3}{7\times3}=\frac{14}{21}+\frac{18}{21}=\frac{14+18}{21}=\frac{32}{21}$

Subtraction between Fractions.

Definitions: Suppose we have to find $\frac{p}{q}-\frac{l}{m}$, we have to follow certain rules.

1) Denominators of both the fractions should be equal.
Suppose, $q \neq m$ then, use **"Equalizing the Denominator"** method to have the common denominator.

2) Subtraction takes place only in the numerator not in the denominator.

Hence $\frac{p}{q}-\frac{l}{m}=\frac{p\times s}{r}-\frac{l\times t}{r}=\frac{(p\times s)-(l\times t)}{r}$. Here r = LCM (q, m) such that $r=q\times s$ and $r=m\times t$, where s and t are the integers.

Example: 1) $\frac{8}{5}-\frac{6}{5}=\frac{8-6}{5}=\frac{2}{5}$

2) $\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$

3) $\frac{5}{6}-\frac{11}{9}$

Now $6 \neq 9$
Hence LCM(6, 9) = 18. Now $18=6\times3$ and $18=9\times2$.

$$\frac{5}{6}-\frac{11}{9}=\frac{5\times3}{6\times3}-\frac{11\times2}{9\times2}=\frac{15}{18}-\frac{22}{18}=\frac{15-22}{18}=-\frac{7}{18}$$

4) $\frac{2}{3}-\frac{6}{7}$
LCM (3, 7) = 21

Hence, $\frac{2}{3}-\frac{6}{7}=\frac{2\times7}{3\times7}-\frac{6\times3}{7\times3}=\frac{14}{21}-\frac{18}{21}=\frac{14-18}{21}=-\frac{4}{21}$

Comparing Fractions:

Properties: If we want to compare two fractions, we have to compared the numerators of the fractions provided the denominators are equal.
Suppose the denominators of the fractions are not equal then use method of **"Equalizing the Denominators."**

Example: Compare the following.

1) $\frac{1}{2}$, $\frac{2}{3}$

LCM (2, 4) = 4

$\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$ and $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$

Hence the new fractions are $\frac{3}{6}$ and $\frac{4}{6}$. Now we can compare these two fractions.

Here numerator of $\frac{3}{6}$ is less than the numerator of $\frac{4}{6}$

Hence $\frac{3}{6} < \frac{4}{6}$

2) $\frac{-3}{4}$, $\frac{-1}{2}$

LCM (4, 2) = 4

Here the new fractions will be $\frac{-3}{4}$ and $\frac{-2}{4}$

Hence by comparing the numerators we get $\frac{-3}{4} < \frac{-2}{4}$.

Quiz Time:

Which of the following fractions are equivalent to the givens fraction such that the denominators of the new fractions are same.

1) $\frac{7}{9}$, $\frac{2}{6}$

2) $\frac{-4}{8}$, $1\frac{2}{3}$

3) $\frac{7}{8}$, $4\frac{1}{3}$

4) 1 , $\frac{1}{2}$, $\frac{1}{3}$

Solve the following and find the fraction.

1) $2 + \frac{4}{3}$

2) $\frac{3}{4} - \frac{1}{2}$

3) $\frac{-8}{11} - \frac{3}{11}$

4) $\frac{11}{-5} + 2\frac{1}{10}$

5) $9\frac{1}{7} - 2\frac{1}{7}$

Which of the columns are greater?

1) Column A: $\frac{4}{8} + \frac{2}{8}$

   Column B: $\frac{3}{4}$

2) Column A: $\frac{9}{5} - \frac{7}{4}$

   Column B : $-\{2\frac{1}{2}\}$

3) Column A: $\frac{7}{4} - \frac{3}{4} - 1$

   Column B: - 1

Multiplication and Division of Fractions

Multiplication of Fractions.

Definition: Multiplication of the fractions is simple than the addition or subtraction. When we multiply two fractions say $\frac{p}{q}$ and $\frac{l}{m}$, in the numerator of the resultant fraction will $p \times l$ and denominator will be $q \times m$.

Hence $\frac{p}{q} \times \frac{l}{m} = \frac{p \times l}{q \times m}$.

Example: 1) $\frac{2}{3} \times (\frac{4}{8}) = \frac{2 \times 4}{3 \times 8} = \frac{8}{24}$

2) $\frac{2}{5} \times \frac{1}{5} = \frac{2}{25}$

Inverse of Fraction.

Definition: If $\frac{p}{q}$ is the given fraction then the inverse of this fraction is given by $\frac{q}{p}$.

Example: 1) 2 is the inverse of $\frac{1}{2}$.

2) $\frac{1}{5}$ is the inverse of 5.

3) $\frac{2}{3}$ is the inverse of $\frac{3}{2}$.

4) 1 is the inverse of 1

5) Inverse of 0 does not exist

Division of Fractions.

Definition: Suppose we have to divide $\frac{p}{q}$ by $\frac{l}{m}$, then we will be multiplying $\frac{p}{q}$ by the inverse of $\frac{l}{m}$. Hence $\frac{p / q}{l / m} = \frac{p}{q} \times \frac{m}{l}$

Example: 1) $\frac{2}{4} / \frac{1}{2} = \frac{2}{4} \times \frac{2}{1} = \frac{2 \times 2}{4 \times 1} = \frac{4}{4} = 1$

2) $\frac{4}{5} / \frac{6}{7} = \frac{4}{5} \times \frac{7}{6} = \frac{4 \times 7}{5 \times 6} = \frac{28}{30}$

Quiz Time:

1) _____ is the inverse of $\frac{1}{1/5}$

2) Find the inverse of $\frac{7/8}{5/6}$ **?**

3) Find the irreducible fractions for $\frac{4}{-5}\times\frac{15}{26}$, $\frac{1}{7/14}$, $\frac{6/9}{9/6}$ **?**

4) Solve the following and find the irreducible fraction?

$\{5\frac{6}{13}-\frac{5}{4/3}\}\times\{\frac{7}{5}+\frac{3}{5}\}$

Solved Question.

Let us see solved examples to learn how to convert the statement into the mathematical equations:
Examples:

1) What is the two tenth of 175?

Sol: Two tenth = 2 / 10 = 1 / 5
Hence two tenth of 175 = 2 / 10 X 175 = 1 / 5 = 35

2) Find X if X is one fifth more than 25.

Sol: One fifth of 25 = 1/5 X 25 = 5
X is one fifth more than 25 means
X = 25 + one fifth of 25
= 25 + 1/5 X 25
= 25 + 5
= 30

3) Find Y if Y is one eighth less than 64.

Sol: One eighth of 64 = 1/8 X 64 = 8
Y is one eighth less than 64 means
Y = 64 - one eighth of 64
= 64 - 1/8 X 64
= 64 - 8
= 56

4) John has N number of tickets. He gives away half of them to his brother Jack. Jack distributes those among his four friends equally. Now, if each friend of Jack has 3 tickets, then how many tickets did John have with him initially ?

Solution: **Given that** John has N number of ticket .
The no. of ticket Jack got from the John = N/2

Jack distributed all his ticket equally into his four friends
So each friend of Jack will receive $\frac{N/2}{4} = \frac{N}{8}$ tickets

It is given that each friend of Jack has 3 ticket ,

$\rightarrow \frac{N}{8} = 3$
$\rightarrow$ N= 24

So John initially had 24 tickets.

5) Frank and Joe together had invested some amount in a company. Frank's investment was thrice the investment of Joe. At the end of the year, Frank and Joe got one fifth more than the amount that they had invested. If Joe alone had invested $25,000, then what is the total amount received by Frank and Joe?

Solution:

Let consider that the the Frank invested x amount and Joe invested y amount .

Given : Frank investment thrice as Joe investment

So $x = 3y$

The total investment $= x + y$

$= 3y + y = 4y$

At the end of the year Frank and Joe receiving one Fifth more than the total investment.

So the total amount received by the Frank and Joe $= 4y + \frac{1}{5}$ of $(4y)$

$= 4y + \frac{4y}{5} = \frac{24y}{5}$

Given that Joe alone had invested $ 25,000.

So here $y = 25{,}000$

So the total amount received by the Frank and Joe $= \frac{24y}{5} = \frac{24 \, X \, 25{,}000}{5}$ =$ 120,000

6) Jenny and Rose have the same birthday. The current age of Jenny is thrice the current age of Rose. If the sum of their current ages is 80, then what will be the age of Rose after 3 years ?

Solution:

Let Consider the current age of Jenny is x and Rose is y.

Given :

- The Current age of Jenny is thrice of Rose age
- And the sum of their age is 80.

Here $x = 3y$

And $x + y = 80$ --------(1)

put the value of the $x = 3y$ in the (1)

$3y + y = 80$

$4y = 80$

$y = 20$

So the current age of the Rose is 20 and jenny age is 3X20 = 60

The age of Rose after 3 year is given by

20 + 3 = 23

Review Test.

Solve the following question:

1) **What is one third of 36 / 12?**

2) **What is the irreducible fraction of inverse of** $4 \times \frac{3}{8} - \frac{7}{4} + 12$ **?**

3) **In a class room, three fifth of the students are boys. In a test that was conducted, only half of the total students have passed in history. The number of boys and girls who passed in history are 10 and 20 respectively. Find the fraction of the girls who passed in history.**

4) **Maddy had $240 in his account. He said that he will spend half the amount left in his account everyday. How much dollars will Maddy have after fours days, if no additional amount is added to his account?**

5) **Mary goes to a shop to purchase a dress. She liked the dress whose price was $490. But she found that she will need two fifth more than what she had. How much money did she have with her ?**