1. Properties of Inequalities

The expression $a < b$ is read as

a is less than b

while the expression $a > b$ is read as

a is greater than b.

The < and > signs define what is known as the **sense** of the inequality (indicated by the direction of the sign).

Two inequalities are said to have

(a) the *same sense* if the signs of inequality point in the same direction; and

(b) the *opposite sense* if the signs of inequality point in the opposite direction.

Examples

The inequalities $x + 3 > 2$ and $x + 1 > 0$ have the same sense.

So do the inequalities $3x - 1 < 4$ and $x^2 - 1 < 3$.

The inequalities

$x - 4 < 0$ and $x > -4$

have the **opposite** sense as do the following 2 inequalities:

$2x + 4 > 1$ and $3x^2 - 7 < 1$.

The **solution** of an inequality consists of all the values of the variable that make the inequality a true statement.

Conditional inequalities are those which are true for some, but not all, values of the variable.

Absolute inequalities are those which are true for all values of the variable.

A solution of an inequality consists of only real numbers as the terms *"less than or greater than"* are not defined for complex numbers.

Examples

The inequality $x + 1 > 0$ is true for all values of x greater than -1.

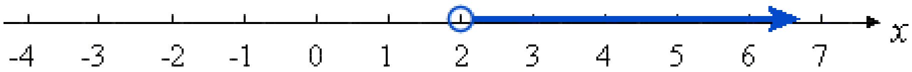
Hence the solution of the inequality is written as $x > -1$ and so this is a **conditional** inequality.

However, the inequality $x^2 + 1 > 0$ is true for all values of x and hence is an **absolute** inequality.

Graphical Representation of Inequalities

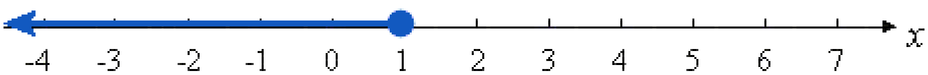
Examples

(a) To show $x > 2$ graphically, we use an open circle at 2 on the number line and a line to the right of this point, with an arrow pointing to the right:



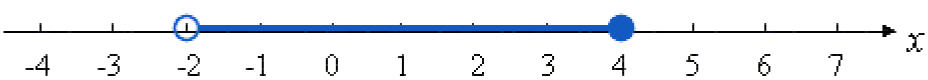
The open circle shows that the point is not part of the indicated solution.

(b) To show $x \leq 1$ graphically, we use a solid circle at 1 on the number line and a line to the left of this point, with an arrow pointing to the left:



The solid circle shows that the point is part of the indicated solution.

(c) To indicate $-2 < x \leq 4$ graphically, we draw a bold line between the 2 values, an open circle at -2 (since it is not included) and a closed circle at 4 (since it is included).



Properties of Inequalities

Property 1 - Adding or Subtracting a Number

The **sense** of an inequality is not changed when the same number is added or subtracted from both sides of the inequality.

Example

Using the inequality:

$9 > 6$

adding 4 to both sides gives

$9 + 4 > 6 + 4$

i.e. $13 > 10$ which is still true

subtracting 12 from each side gives

$9 - 12 > 6 - 12$

i.e. $-3 > -6$ which is still true

Property 2 - Multiplying by a Positive Number

The **sense** of the inequality is not changed if both sides are multiplied or divided by the same positive number.

Example

Using the inequality:

$8 < 15$

Multiplying both sides by 2 gives

$8 \times 2 < 15 \times 2$

i.e. $16 < 30$ which is still true

Dividing both sides by 2 gives

$8/2 < 15/2$

i.e. $4 < 7.5$ which is still true

Property 3 - Multiplying by a Negative Number

The sense of the inequality is **reversed** if both sides are multiplied or divided by the same negative number.

Example

We start with the inequality $4 > -2$.

Multiplying both sides by -3 gives

$4 \times -3 > -2 \times -3$

$-12 > 6$ which is **not true**

Hence the correct solution should be

$4 > -2$

$4 \times -3 < -2 \times -3$

$-12 < 6$ (Note the change in the sign used)

Similarly dividing both sides by -2 gives

$4 > -2$

$4 \div -2 < -2 \div -2$

$-2 < 1$ (Note the change in the sign used)

Property 4 - *n*-th Power

If both sides of an inequality are positive and ***n*** is a positive integer, then the inequality formed by the ***n***-th power or ***n***-th root of both sides have the **same sense** as the given inequality.

Example

Using the inequality:

$9 > 6$

Squaring both sides gives

$9^2 > 6^2$

i.e. $81 > 36$ which is still true

Taking square root of each side gives

$$\sqrt{9} > \sqrt{6}$$

i.e. $3 > 2.45$ which is still true

[**Note:** √9 does not equal ±3. By convention, we take the positive square root only. See the discussion at √16 - how many answers?]

Exercise

Graph the given inequality on the number line:

$1 < x \le 4$

Answer