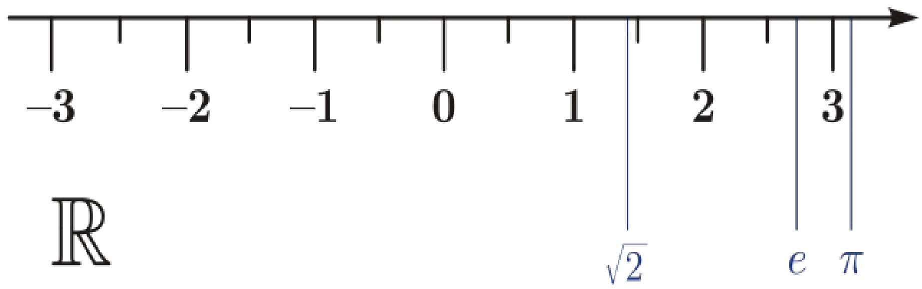
Inequalities

Real number line:
The real number system can be visualized as a horizontal line that extends from a special point called the Origin in both directions towards infinity. The origin corresponds to the number 0. All positive numbers lie towards the right side of 0 (infact all positive numbers are greater than 0) and all negative numbers lie towards the left side of 0 (all negative numbers are lesser than 0).



Inequality: An inequality is a statement about the relative size or order of two objects, *or* about whether they are the same or not. Let us look at them in details one by one.

1) *a > b*   *a* is greater than *b*

Statement I: We say that a is greater than b if a lies to the right of b on a real line.

For example: We know that **3 > 2**
Point them on a Real line
If we look at the Real line 3 lies to the right side of 2.
Hence any number 'a' which lies to the right side of any particlar number 'b' then we can say that 'a' is greater than 'b'.

Similarly which one is greater -2 or -3?
Is your answer is -3?? Then look at the following note.
NOTE:
People who say that -3 is greater than -2
Generally most of the students confuse with the magnitude of the number to compare which one is greater.
Let me explain you in details.
Everyone knows that 3 is greater than 2. But when it comes to -3 and -2 you people generally compare with the 3 , 2 and misjudge that -3 is greater than -2.
But in fact the pattern for negative number follow exactly the opposite way the positive numbers behave.
That is if 3 is greater than 2 then -2 is greater than -3.
Did you understand it now?
Let me summarize it breifly.
As we discussed earlier also any number 'a' which lies to the right side of a particular number 'b' then we can say that 'a' is greater than 'b'.

So, likewise if we look at the Real line -2 lies to the right of -3 hence we can conclude that -2 is greater than -3
So, whenever you are confused then try to point out these numbers on Real line and then compare according to the **Statement I.**

2) $p < q$ *p* is less than *q*

Statement II: We say that p is lesser than q if p lies to the left of q on a real line.

For example:
Consider the same example which we discussed earlier for 2, 3.
We know that 2 is lesser than 3.
If we look at the Real line 2 lies to the left side of 3
Hence $2 < 3$.

Now tell which one is greater among

(i) e and √2 (ii) e and π (iii) π and √2

(iv) -√2 and √2

(v) -1 and -√2

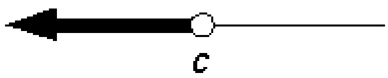
Where e = 2.71828183 , √2 = 1.414, π = 3.14 (Look at the figure 1)

Let me briefly introduce you to each and every inequality symbol which involve a single variable.

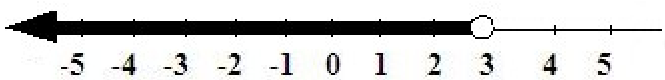
1) $x < c$

When **x** is less than a constant, you darken in the part of the number line that is to the left of the constant.

Also, because there is no equal line, we are not including where *x* is equal to the constant. That means we are not including the endpoint. One way to notate that is to use an open hole at that point.



Example:
Look at this $x < 3$

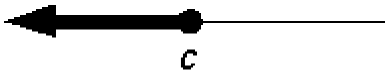


The dark line in the above figure corresponds to the region where x can lie. That is according to the **Statement II** x lies to the left side of 3 ($x < 3$) in the Real line and since x is a variable it can be any value to the left of 3. Hence the whole darkened line correspond to the region of x which is less than 3.

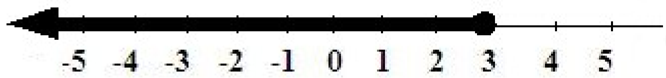
Note that we are not including where *x* is equal to 3 as there is no equal line between them.

2) $x \leq c$

This case is exactly same as that of previous case and the only difference is that because of an equal line, we are including where *x* is equal to the constant. That means we are including the endpoint. One way to notate that is to use an closed hole at that point.



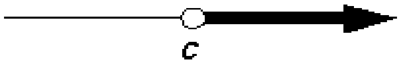
Example:
Look at this x ≤ 3



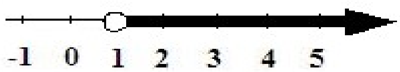
3) x > c

When *x* is greater than a constant, you darken in the part of the number line that is to the right of the constant.

Also, because there is no equal line, we are not including where *x* is equal to the constant. That means we are not including the endpoint. One way to notate that is to use an open hole at that point.



Example:
Look at this x > 1

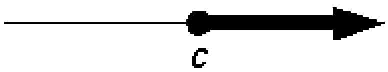


The dark line in the above figure corresponds to the region where x can lie. That is according to the **Statement I** x lies to the right side of 1 (if x > 1) in the Real line and since x is a variable it can be any value to the right side of 1. Hence the whole darkened line correspond to the region of x which is less than 1.

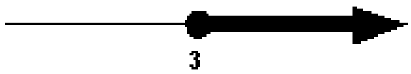
Note that we are not including where *x* is equal to 1 as there is no equal line between them.

4) x ≥ c

This case is exactly same as that of previous case and the only difference is that because of an equal line, we are including where *x* is equal to the constant. That means we are including the endpoint.



Example:
Look at this x ≥ 3

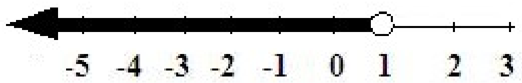


Addition/Subtraction Property for Inequalities

Adding or subtracting the same expression to both sides of an inequality does not change the inequality.

- If $a \leq b$, then $a + c \leq b + c$
- If $a < b$, then $a - c < b - c$
- If $a > b$ then $a + c > b + c$
- If $a \geq b$ then $a - c \geq b - c$

Example 1: Solve the inequality and graph the solution set $x - 5 < -4$
Solution: $x - 5 < -4$
In order to graph the solution it will be more convenient if we bring all the variables to one side and all constants to the other side.
From the addition property for inequality we can add 5 on both sides which will not affect the inequality.
=> $x - 5 + 5 < -4 + 5$
=> $x < 1$
Hence we need to plot $x < 1$



Note that the inequality (which is < in this example) remained the same throughout the problem. Adding or subtracting the same value to both sides does not change the inequality sign.
The answer '*x* is less than 1' means that if we put any number less than 1 back in $x - 5 < -4$, it would be a solution (the left side would be less than the right side).

Take $x = 0$ which is less than 1.

Sub this $x = 0$ in $x - 5 < -4$

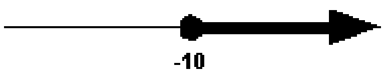
=> $0 - 5 < -4$

=> -5 is less than -4. Hence $x < 1$ satisfies the given inequality.

Therefore, by Adding/Subtracting the same expression on both sides of an inequality doesnot change the inequality.

Graph:
Since we needed to indicate all values less than 1, the part of the number line that was to the left of 1 was darkened. Since we are not including where it is equal to, an open hole was used.

Example 2: Solve the inequality and graph the solution set $x + 6 \geq -4$
Solution: Let us bring all the variables on one side and constants on the other side.
From the subraction property for inequality we can subtract 6 on both sides which will not affect the inequality.
=> $x + 6 - 6 \geq -4 - 6$
=> $x \geq -10$
Hence we need to plot $x \geq -10$



Multiplication/Division Properties for Inequalities with positive numbers

Multiplying or dividing the same **POSITIVE** number to both sides of an inequality does not change the inequality

- If $a < b$ and **c is positive**, then $ac < bc$
- If $a \geq b$ and **c is positive,** then $a/c \geq b/c$

Example 1: Solve the inequality and graph the solution set $3x < -9$

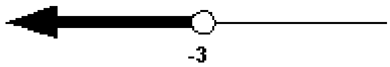
Solution: Given that $3x < -9$

It will be convenient to graph the solution for $3x < -9$ if we can bring x alone on one side and all constants on the other side.

So let us divide the given inequality with 3 (**a positive number**) on sides.

=> $3x/3 < -9/3$

=> $x < -3$



Note that the inequality sign stayed the same direction. Even though the right side was a -10, the number we were dividing both sides by, was a positive 5. **Multiplying or dividing both sides by the same positive value does not change the inequality.**

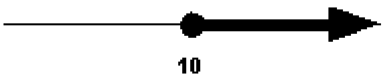
Example 2: Solve the inequality and graph the solution set. $x/5 \geq 2$

Solution: Given that $x/5 \geq 2$

Let us multiply with 5 (**a positive number**) on both sides to have an explicit x on one side and all constants on the other side.

=> $(x/5)*5 \geq 2*5$

=> $x \geq 10$



Inequality remained the same throughout the calculation.

<u>Multiplication/Division Properties for Inequalities with negative numbers</u>

Multiplying or dividing the same **NEGATIVE** number to both sides of an inequality reverses the sign of the inequality.

- If $a < b$ and **c is negative**, then $ac > bc$
- If $a \geq b$ and **c is negative,** then $a/c \leq b/c$

The reason for this is, when you multiply or divide an expression by a negative number, it changes the sign of that expression. On the number line, the positive values go in a reverse or opposite direction than the negative numbers go, so when we take the opposite of an expression, we need to reverse our inequality to indicate this.

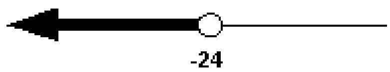
<u>Example 1</u>: Solve the inequality and graph the solution set $-x/4 > 6$

<u>Solution</u>: Given that $-x/4 > 6$

It will be convenient to graph the solution if we remove any values that are in front of the variable So let us multiply with -4 (**negative number**) on both sides.

=> $(-4)*(-x/4) < (-4)*6$

=> $x < -24$



Note that the inequality sign changed after multiplying with a negative number on both sides which is more clear from the second step.

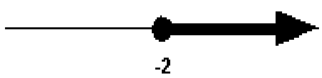
<u>Example 2</u>: Solve the inequality and graph the solution set $-4x \leq 8$

<u>Solution</u>: Given that $-4x \leq 8$

Let us divide with -4 (**a negative number**) on both sides to have an explicit x on one side and all constants on the other side.

=> $(-4x)/(-4) \geq 8/(-4)$

=> $x \geq -2$



Note in the step 2 after dividing with a negative number -4 the inequality sign changed.

Strategies for solving these inequalities

These problem solving strategies for inequalities is of same basic concepts that we generally use for solving equations. While solving any problem inequality follow all the basic properties that we have discussed earlier. Let us look at these problem solving strategies in detailed step wise procedure.

Step 1: Simplify each side, if needed.
This would involve things like removing ( ), removing fractions, adding like terms, etc.

Step 2: Use Add./Sub. Properties to move the variable term on one side and all other terms to the other side.

Step 3: Use Multiplication/Division Properties to remove any values that are in front of the variable.

Step 4: Follow these operations until we observe exclusive variables on one side and the other parameters on the other sides

Example 1: Solve the inequality $-4x - 8 \leq 16$ and find the solution set for x.
Sol:
Step 1: No simplications needed
Step 2: We need to add 8 on both sides to remove -8 term from LHS
=> $-4x - 8 + 8 \leq 16 + 8$
=> $-4x \leq 24$
Step 3: Divide with -4 on both sides (since -4 is a negative number inequality sign changes) to remove the values that are in front of the variable.
=> $-4x/(-4) \geq 24/(-4)$
=> $x \geq -6$

Example 2: Solve the inequality $3(x+2) \leq x + 4$ and find the solution set for x.
Sol:
Step 1: Simplify the inequality $3(x+2) \leq x + 4$
=> $3x + 6 \leq x + 4$
Step 2: We need to move all variables to one side and other terms to the other side.
We can do this by adding -x – 6 on both sides which removes 6 (constant from LHS) and x (variable from RHS)
=> $3x + 6 + (-x - 6) \leq x + 4 + (-x - 6)$
=> $2x \leq -2$
Step 3: Divide with 2 on both sides (since 2 is a positive number the inequality remains same) to remove the values that are in front of the variable.
=> $2x/2 \leq -2/2$
=> $x \leq -1$

<u>**Exercise problems**</u>

1) Solve the inequality and graph the solution set for

i) $6 < 5 - 3x$

ii) $4(x - 2) - 2 \leq 2x - 1$

iii) $\dfrac{7x+1}{4} \geq 2x - 1$

iv) $\dfrac{5-2x}{3} \leq \dfrac{x}{6} - 5$

v) $\dfrac{3x-7}{2} \geq \dfrac{x+1}{5} - 2$

2) The marks obtained by a student of Class X in first and second terminal examination are 62 and 48, respectively. Find the number of minimum marks in the third terminal so that he should get in the annual examination to have an average of at least 60 marks. (Each exam is conducted for 100 marks)

3) To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Samuel's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Samuel must obtain in fifth examination to get grade 'A' in the course.