Set theory

A **Set** is defined as the collection of distinct and well-defined objects. The objects in a set are called the **members** of the set or the **elements** of the set. A set is usually designated by a **capital letter**.

Examples:

1. Let **A** denotes the multiples of 2 less than 10.

Then **A** = {2,4,6,8}

2. Let **C** denotes the employees of the Company XYZ who are absent today.

Then **C** = All the employees working in XYZ who were absent on that day.

Number of elements

Let **A** be the set. Then **number of elements** in a set is given by **n (A)**.

Example: Let **A** denote the letters of the word "difference".

Then **A** = {'D','I','F','F','E','R','E','N','C','E'}

Therefore **n (A) = 10**.

Universal set

A **universal set** is a set which contains all objects, including itself. More formally, a universal set, denoted by **U**, is a set that satisfying for all x, x ∈**U**.

For example, set of all real numbers R is a universal set.

Subset

If **A** and **B** are sets and every element of **A** is also an element of **B**, then **A is a subset of B**.

It is denoted by $A \subseteq B$.

From this definition, it is clear that

1) Every set is a subset of the universal set.

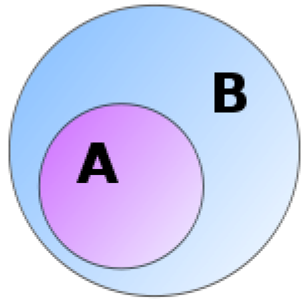
2) A set is a subset of itself.

Example: Let **A** = {5,7,3} and **B** = { 2,5,6,7,3,8}

In **B** we can able to find out the elements of **A**.

B = { 2,5,6,7,3,8}, the highlighted elements are the elements of **A**.

From which we can say that **A** is the subset of **B**.



Null Set

A set with no elements is an **Null set** and it is denoted by **{ }**. The empty set is a subset of every set, including the empty set itself.

Note: {0} is not a Null set.

Example: Find out a set **O** of squares with 5 sides.

We know that a square is of 4 sides.

Therefore **O** is an **Null set** and n (O) = 0.

Venn diagram

Venn diagram shows all possible logical relations between a finite collection of sets. Venn diagrams are pictorial ways of representing interactions among sets to give the information that can be read easily.

Properties:

- It is used for showing relationships between sets in math.
- It is used for Problem solving and Teamwork.
- It is used for sorting and classifying objects according to properties.

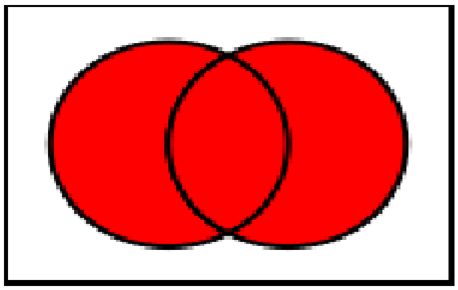
Fundamental operations on Sets

There are three fundamental operations performed on sets.
- Set Union,
- Set Intersection
- Set Complement.

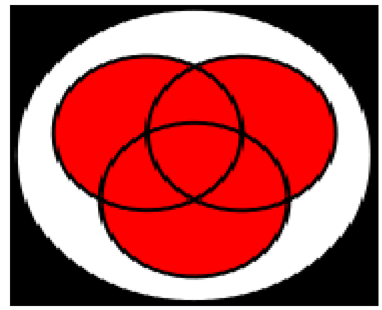
Set Union

The **union** of two sets **A** and **B** is the set which contains all of the elements in both **A** and **B**. It is usually denoted with the symbol **A ∪ B**.

The concept of Union is easily understood by the help of Venn diagram.



Union of two sets



Union of three sets

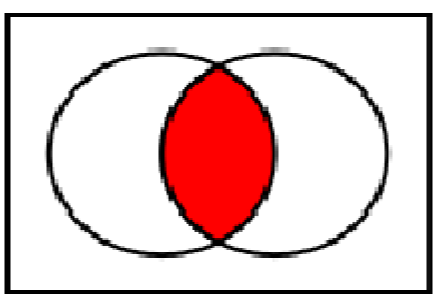
Example: Here is a simple example.

1. Let **A** be the set of even numbers and **B** be the set of odd numbers.
   A = {x is an even number, x > 1}
   B = {x is an odd number, x > 1}
   Then **A ∪ B = {2,3,4,5,6,7,8,…}**
2. Let **A = {2,4,6,8}, B = {5,7} and C = { 3,9}.**
   Then **A ∪ B ∪ C = { 2,3,4,5,6,7,8,9}**.

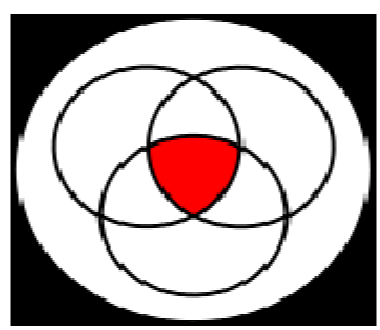
Set Intersection

The **intersection** of two sets **A** and **B** is the set which contains only those elements which are in both **A** and **B**. It is usually denoted by the symbol **A ∩ B**.

The concept of intersection is easily understood by the help of Venn diagram.



Intersection of two sets



Intersection of three sets

Example:

1. The intersection of the sets {1, 2, 3} and {2, 3, 4} is {2, 3}.
2. Let **A** be the set of prime numbers and **B** be the set of odd numbers.

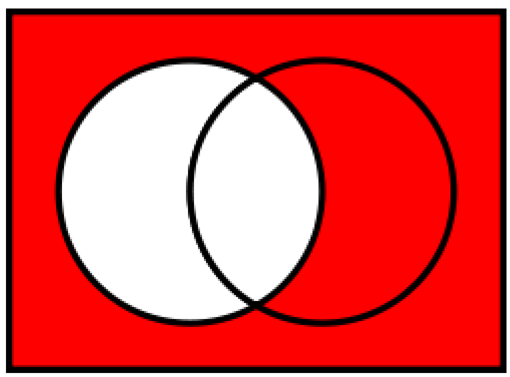
Then **A ∩ B = {The numbers which are prime and odd}**.

3. If A={2, 10, 12, 17, 21}, B={1, 8, 12, 16, 23} and C={2, 12, 18, 21, 27}
   Then find **A ∩ B ∩ C** and **A ∩ (B ∪ C)**.
   To find A ∩ (B ∪ C):
   B ∪ C = {1, 8, 12, 16, 23} ∪ {2, 12, 18, 21, 27} = {1, 2, 8, 12, 16, 18, 21, 23, 27}
   Hence, **A ∩ (B ∪ C)** = {2, 10, 12, 17, 21} ∩ {1, 2, 8, 12, 16, 18, 21, 23, 27} = {2, 12, 21}
   To find A ∩ B ∩ C
   A ∩ B ∩C means the common values of Sets A, B and C.
   Here 12 is the only common value.
   Therefore, A ∩ B ∩ C = {12}.

Set Complement

Complement of set A relative to set U, denoted A^C, is the set of all members of U that are not members of A. This terminology is most commonly employed when U is a universal set, as in the study of Venn diagrams. This operation is also called the set difference of U and A.

The concept of Complement is easily understood by the help of Venn diagram.



Complement of a set

Example: If the universal set U = {x: x integer; -6< x <7 } and

M = {y: y even number; 1< y <5}, what is the complement of M.

First, let's find all elements of sets U and M.

U = {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}; M = {2, 4}

Therefore $\mathbf{M^C}$ **= {-5, -4, -3, -2, -1, 0, 1, 3, 5, 6}**.

Note: Complement of an universal set is always a null set and vice versa.

Important formulas in Sets

Suppose A , B, C are the three sets then:

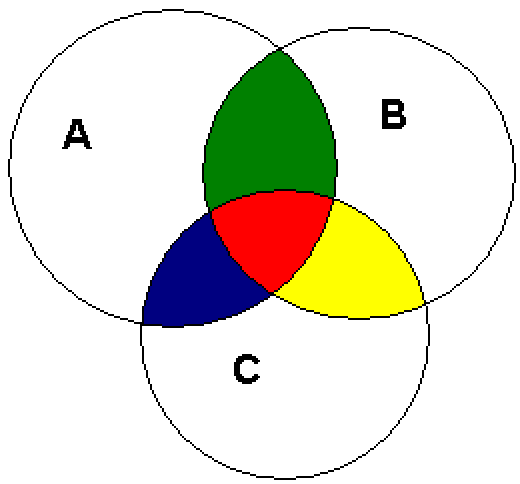
1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

3) If U is the universal set and A is any set then, $n(A^C) = n(U) - n(A)$

Properties:

The following Venn Diagram illustrates the elementary set operations.



1. $A \cup B$ is the total of the white areas containing the letters A and B, together with the green, red, yellow and blue areas.

2. $A \cap B$ is the red area plus the green area.

3. $A \cup B \cup C$ is everything except the magenta colored area.

4. $A \cap B \cap C$ is the red area.

5. B^C is the total of the white areas containing the letters B and C, together with the blue and the Magenta area.

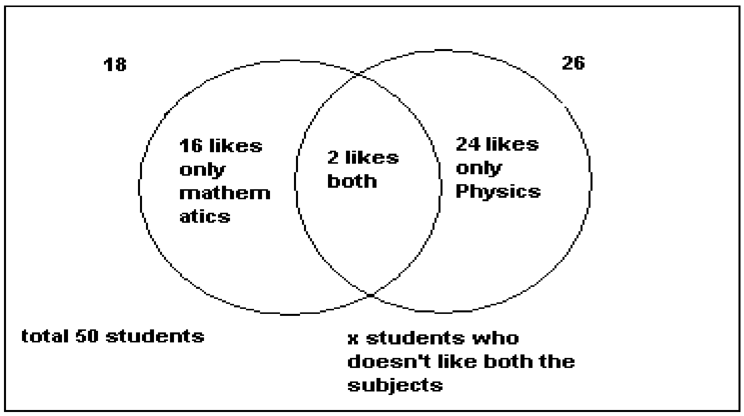
6. $(B \cup C)^C$ is the total of the white area containing the letter A and the magenta area.

7. $(A \cup B \cup C)^C$ is the magenta area.
8. $(A \cap C)^C$ is everything except the red area and blue area.
9. $(A \cap B \cap C)^C$ is everything except the red area.

Solved examples

1.In a class of 50 students, 18 likes Mathematics, 26 likes Physics, and 2 likes both. How many students in the class do not like both the subjects?

Solution:
It is given that 18 likes Mathematics and 26 likes physics.
It is also given that 2 likes both the subjects.
So 16 students likes only mathematics and 24 likes only Physics.

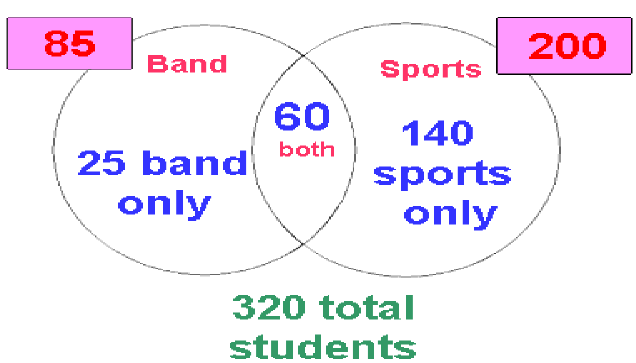


We know that total students are 50.

Therefore 16 + 2+ 24 +x = 50.
On simplification we get x = 8.
Which implies that **8 students who do not like both the subjects**.

2.In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are involved in either band or sports?

Solution:
It is given that 85 students are in band and 200 students are in sports.
It is also given that 60 students participate in both the activities.
So no.of students participate only in band is 25 and no.of students participate only in sports is 140.



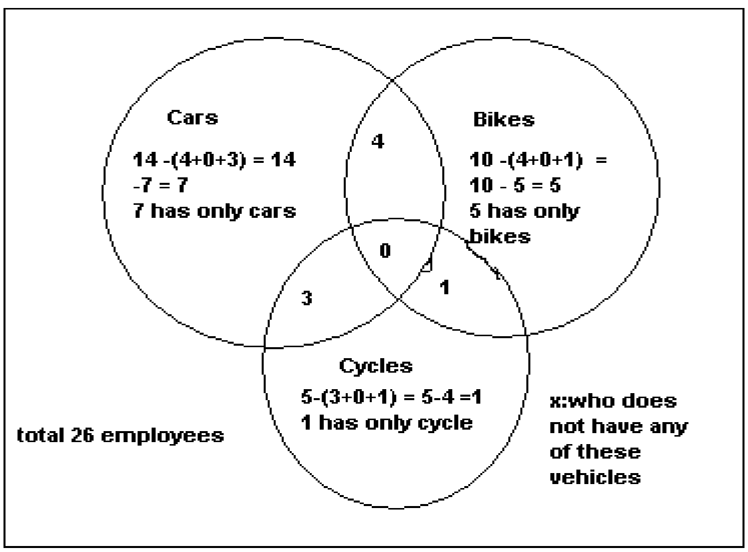
We have to find No.of students involved in either band or sports.
For which just add

No.of student participate in band only + No.of students participate in both + No.of student participate in sport only = **No.of students participate in either band or sports**

This is 25 + 60 +140 = 225.
Therefore **225 students participate in either band or sports**.

3.A manager surveys 26 of his employees for travel allowances. He discovers that 14 have cars, 10 have bikes, and 5 have Cycles. Four have cars and bike, 3 have cars and cycles, and one has bike and cycles. If no one has all three kinds of vehicles, how many have none of these?

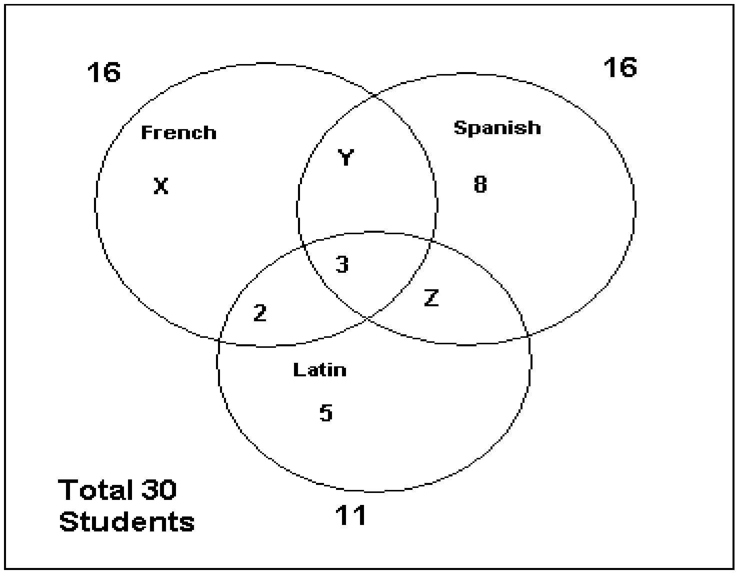
Solution:
Given that total number of employees is 26. In that 14 have cars, 10 have bikes and 5 have cycles.
It is also given that 4 have cars and bike, 3 have cars and cycles and one has bike and cycles.
Let us put this given information in a Venn diagram for better understanding.



From the Venn diagram we found that 7 employees have only cars, 5 have only bikes and 1 have only cycles.
We needs to find how many of them have do not have any of these vehicles.
We know that total number of employees is 26.
Therefore 7 + 5 + 1 + 4 + 3 + 1 + 0 + x = 26
On simplification we get x = 5.
Thus **5 employees do not have any these vehicles**.

4.A guidance counselor is planning schedules for 30 students. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. Five want only Latin, and 8 want only Spanish. How many students want French only?

Solution:
It is given that 16 students want to take French, 16 wants to take Spanish and 11 wants to take Latin.
It is also given that 5 wants only Latin and 8 wants only Spanish and 5 wants both French and Latin, but out of these 3 also wants to take Spanish. This implies that 3 students want to all the Subjects.
Let us import these in formations in a Venn diagram for better understanding.



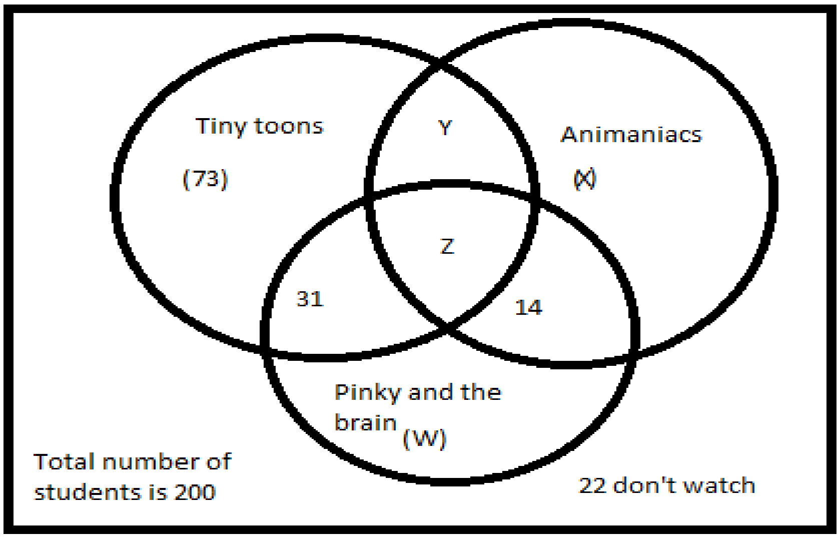
Here X is the number of students who wants to take French only.
Y is the number of students who wants to take French and Spanish and Z is the number of students who wants to take Latin and Spanish.

We know that 11 students want to take Latin.
Therefore 5 + 2 + 3 + Z = 11
On Simplification we get Z = 1… (1)
This implies that **1 student wants to take Latin and Spanish**.
We also know that 16 students want to take Spanish.
Therefore 8 + Y + Z + 3 = 16
From (1) we get Z =1.
This implies that 8 + Y + 1 + 3 = 16
On simplification we get Y = 4…. (2)
This implies that **4 students want to take French and Spanish**.
Similarly we know that 16 students want to take French.
Therefore X + Y + 3 + 2 = 16
From (2) we get Y =4.
This implies that X + 4 + 3 + 2 = 16
Again on simplification we get X = 7.
Therefore **7 students want to take French only**.

5.A study was made of 200 students to determine what TV shows they watch. 22 students don't watch these cartoons. 73 students watch only Tiny Toons. 136 students watch Tiny Toons. 14 students watch only Animaniacs and Pinky & the Brain. 31 students watch only Tiny Toons and Pinky & the Brain. 63

students watch Animaniacs. 135 students do not watch Pinky & the Brain (for some completely incomprehensible reason). How many of them watch only Animaniacs and only Pinky and the brain.

Solution:
It is given that 73 students watch only Tiny toons.
14 students watch Animaniacs and Pinky & the Brain and 31 students watch only Tiny Toons and Pinky & the Brain.
Let us draw the Venn diagram.



Let X is number of students who watch only Animaniacs and Y is number of students who watch both Tiny toons and Animaniacs.
Let W be number of students who watch only Pinky and the brain and Z be number of students who watch all the three shows.
It is given that 136 watch Tiny toons. This implies that **73 + Y + Z + 31 = 136**
On simplification we get **Y + Z = 32….. (1)**
It is also given that 63 students watch Animaniacs. This implies **X + Y + Z + 14 = 63**
On simplification we get **X + Y + Z = 49…. (2)**
It is also given that 135 students do not watch Pinky and the brain. This implies that **73 + Y + X + 22 = 135**
On simplification we get **X + Y = 40…. (3)**
We need to solve the three questions. By substitute (3) in (2) we get 40 + Z = 49 Which implies that **Z = 9**. Substitute the value of Z in (1) we get **Y = 23**. Again substituting Y = 23 in (3) we get **X = 17**.
Now we need to find the value of W.
We know that total no.of students = 73 + X + W + Y + 14 + 31 + Z + 22…. (4)
By substituting the values of X, Y, Z in (4) we get **73 + 17 + W + 23 + 14 + 31 + 9 + 22 = 200**.
We get W = 11.

Therefore **17 students watch only Animaniacs and 11 students watch only Pinky and the brains**.