

Statistics is the study of the patterns and relationships of numbers and data.

There are four main concepts that may appear on the test:

Median

When a set of numbers is arranged in order of size, the *median* is the middle number.

For example,

the **median** of the set {1, 3, 8, 9, **10**, 11, 12, 34, 42} is 10 because it is the middle number (4 numbers on either side of 10)

In this case, the median is also the mean (average). But this is usually not the case.

For example, the median of the set {1, 8, 9, **10**, 11, 17, 25} is 10 because it is the middle number.

But the mean is: $(1+8 + 9 +10 +11+17+25) / 7 = \mathbf{11.57}$

If a set contains an even number of elements, then the median is the average of the two middle elements.

For example, the median of the set {1, 1, 2, 3, 3, **4, 15**, 20, 23, 45, 56, 72} is : $(4 + 15)/2 = 9.5$

Example: What is the median of 0, -2, 200 , 18, 2, 157, 1 ?

First arrange the numbers from smallest to largest. We get

-2, 0, 1, **2** , 18, 157, 200.

The median is the middle number, 2

What is the median of 0, -2, 200 , 18, 2, 157, 1 , 70 ?

First arrange the numbers from smallest to largest. We get

-2, 0, 1, **2** , **18**, 70, 157, 200.

The median is the average of the numbers 2 and 18 = $(2+18)/2 = 10$

Mode

The *mode* is the number or numbers that appear most frequently in a set.

(Note that this definition allows a set of numbers to have more than one mode)

Example: What is the mode of 3, -1, 3, 7, 5, 7.5 ?

The number 3 is the mode because it is the only number that is listed more than once.

Example: What is the mode of 3, π , 3, -9, π , 5, -3 ?

Both 3 and π are modes because each occurs twice, which is the greatest number of occurrences for any number in the list.

Range

The *range* is the distance between the smallest and largest numbers in a set.

To calculate the range, merely subtract the smallest number from the largest number.

Example: What is the range of 2, 8, 1, -6, π , $1/2$?

The largest number in this set is 8, and the smallest number is -6. Hence, the range is $8 - (-6) = 8 + 6 = 14$.

Variance and Standard Deviation

If a, b, c, d, e, f, g be 7 numbers

$$\text{mean} = m = (a+b+c+d+e+f+g) / 7$$

Variance is defined as:

$$\frac{(a-m)^2 + (b-m)^2 + (c-m)^2 + (d-m)^2 + (e-m)^2 + (f-m)^2 + (g-m)^2}{7}$$

whereas standard deviation of the numbers is defined as square root of Variance

$$= \sqrt{\frac{(a-m)^2 + (b-m)^2 + (c-m)^2 + (d-m)^2 + (e-m)^2 + (f-m)^2 + (g-m)^2}{7}}$$

Interpretation of Standard Deviation:

Consider two sequence of seven numbers:

A = 29, 33, 36, 39, 45, 48, 50

B = 29, 34, 37, 39, 43, 48, 50

If we observe the mean, median and the range of the sequence A and B are same. (check!)

But both the sequence are different. So we need another tool to analyze the sequence. One of such tool is Standard deviation

Standard deviation measures how far the numbers in a set are scattered away from the set's mean.

Let us calculate standard deviation for the two sequence A and B.

Sequence A				Sequence B			
	x_i	$(x_i - m)$	$(x_i - m)^2$		y_i	$(y_i - m)$	$(y_i - m)^2$
	29	-11	121		29	-11	121
	33	-7	49		34	-6	36
	36	-4	16		37	-3	9
	39	-1	1		39	-1	1
	45	5	25		43	3	9
	48	8	64		48	8	64
	50	10	100		50	10	100
total	280		376	total	280		340
Mean	40	Variance	53.71	Mean	40	Variance	48.57

Standard deviation	7.33	Standard deviation	6.97
--------------------	-------------	--------------------	-------------

Here we see the standard deviation for two sequence varies.

You can observe that:

- if most of the numbers are closer to the mean, then the SD is smaller
- If the numbers are farther and more distributed around the mean, then the SD is higher.
- Two sets may have more or less same mean but the SD can be large.

Example: Which of the following sets has the larger standard deviation?

A = {1, 2, 3, 4, 5}

B = {1, 4, 15, 21, 27}

Mean of A = 3. Mean of B is 13.6

All the numbers in Set A are within 2 units of the mean, 3.

All the numbers in Set B are greater than 5 units from the mean, 13.6. Hence, the standard deviation of Set B is greater.

(you could have also reached the same conclusion by calculating)

Qn 1: Bob is a supervisor in a automobile factory. Bob requires sheets of thickness 3 mm with a tolerance of 0.1mm . Two manufacturers have supplied him samples of metal sheets. Bob finds that the sheets from manufacturer A have a mean thickness of 2.97 mm and a standard deviation of 1.37. The sheets from manufacturer B have a mean thickness of 3.03 mm and a standard deviation of 21.37.

If you were in Bob's position, whom would you place orders with, Manufacturer A or B ?

Qn 2:

Col A = the Standard Deviation of 5 consecutive positive numbers with mean X.

Col B = the Standard Deviation of 5 consecutive even positive numbers with mean Y.

Compare column A and column B

Qn 3:

10 students in a class have obtained the following GRE scores (out of 1600)

{940 , 1150, 1450, 1400, 1250, 1450, 940, 1120, 1510, 1020 }

What is the median score ?

What is the mode ?

Qn 4: Table shows the results of survey of GRE scores in an engineering college in an academic year. It shows the percentage of students who scored in 6 score ranges.

GRE score Range	Percentage
1400 to 1500	10.00%
1300 to1400	20.00%
1200 to 1300	10.00%
1100 to 1200	30.00%
1000 to 1100	10.00%
<1000	20.00%

What is the median score range in that college ?

(email your response and your doubts to your SFA at support@greedge.com)