ALGEBRA

Algebraic expressions

Tip #1

If the question asks for |x + y|, this means we have to find the absolute values of x+y (not the individual absolute values, i.e. |x| + |y|). The same goes for |x-y|, |xy| and |x/y|

Tip #2

Be careful when converting statements into mathematical expressions, pay attention to each and every word in the statement.

What is the difference between "A is sixty percent more than B" and "A is sixty percent of B"?

A is sixty percent more than B

A (A) is (=) sixty percent (+60%) more than B (B)

A = B + 60% of B

A = B + 0.6 B

A = 1.6B

A is sixty percent of B

A is (=) sixty percent (60%) of B

A = 60% B

A = 0.6B

P is forty percent less than Q

P (P) is (=) forty percent (-40%) less (-) than Q (Q)

$P = Q - 40\%Q$

$P = Q - 0.4Q$

$P = 0.6Q$

R is thirty percent less than three-fifth of T

R(R) is (=) thirty percent less (-30%) than three-fifth of T ($\frac{3T}{5}$)

$R = \frac{3T}{5} - 30\% \left(\frac{3T}{5}\right)$

$R = \frac{3T}{5} - 0.3\left(\frac{3T}{5}\right)$

$R = \frac{3T}{5} [1 - 0.3]$

$R = 0.7 \times \left(\frac{3T}{5}\right)$

Tip #3

Tangible items cannot be in decimal or fractional numbers.

For example: There cannot be 3.56 flowers in a garden or 55.6 cycles cannot be sold.

Tangible items as these should be in integer values.

Tip #4

When asked the percentage increase of something, which value comes in the denominator? New value or old value?

Use this formula

$$\left[\frac{\text{(New value - Old value)}}{\text{Old value}}\right] \times 100$$

Tip #5

What is the difference between 'third' and 'one - third'?

There is no such difference; they are one and the same

A is one-third of B is the same as

A is third of B

Both statements mean that $A = \frac{1}{3}\ B$

P is a fourth of half-Q

P is (=) a fourth $(\frac{1}{4})$ of half Q $(\frac{Q}{2})$

$$P = \frac{1}{4} \times \frac{Q}{2}$$

One-half = half = $\frac{1}{2}$

One-third = third = $\frac{1}{3}$

One-fourth = a fourth = quarter = $\frac{1}{4}$

Tip #6

Suppose you are asked to compare between x and y in the data comparison question.

For example:

Column A: x

Column B: y

The same values for 'x' and 'y' can be taken (Unless it is given that 'x' is not equal to 'y'). For example: $x = y = \frac{-3}{5}$

Also, 'x' and 'y' can take negative or fractional values.

Tip #7

What is the difference between "at least" and "at most"?

Let x = the number of dishes served at the wedding

If the question says "The buffet served at least forty five dishes"

Here the number of dishes served could be 45, 46, 47.....or any number greater than or equal to 45

$x \geq 45$

If the question says "The buffet at the wedding served at most forty five dishes"

Here the number of dishes can be 2, 4, 5.....45 any number less than or equal to forty five

$x \leq 45$

Tip #8

What is the difference between "more than" and "at least"

"The buffet at the wedding served **at least** forty five dishes"

Here, the number of dishes can be 45, 46.... or any number more than or equal to 45

$x \geq 45$

“The buffet at the wedding served **more than** forty five dishes”

The number of dishes can be 46, 47.... any number of dishes more than 45

$x > 45$

What is the difference between "at most" and "less than"

"The buffet at the wedding served **at most** forty five dishes"

Here, the number of dishes can be 45, 46.... or any number less than or equal to 45

$x \leq 45$

"The buffet at the wedding served **less than** forty five dishes"

Here, the number of dishes can be 1, 2, 3, 4.....44 or any number less than 45

$x < 45$

Tip #9

If the question asks you to find the maximum value of x-y, you have to take the maximum value of x and the minimum value of y (considering that both numbers are positive)

Now, if the question asks to find the minimum value of x-y, you have to take the minimum value of x and the maximum value of y (considering that both numbers are positive)

Linear and Quadratic Equations

Tip #1:

$(x + y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$x^2 - y^2 = (x + y)(x - y)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

In case you forget one of the formulas, or are not sure where the negative and positive signs appear in the formulas you can double check using the below method

For example:

If you forget the formula for $(x+y)^2$, then you should do the following

$(x + y)^2 = (x + y)(x + y)$

$= x^2 + xy + xy + y^2$

$= x^2 + 2xy + y^2$

You can do this method for any of the above listed formulas.

Tip #2:

When asked to find the factors of the expression $x^3 - 6x^2 + 11x - 6$

You are given some options like the ones below, to choose an answer:

$(x+ 1) (x+ 2) (x+ 3)$

$(2x +3)(x +3)2$

$(x -1)(x -2)(x -3)$

$(2x -3)(x +3)2$

You can substitute the options to check for the answer:

Option A: $x = -1$

$x^3 - 6x^2 + 11x - 6$

$= (-1)^3 - 6(-1)^2 + 11(-1) - 6$

$= -1 - 6 - 11 - 6$

$= -24$(Not the answer)

Not zero

Option B: $x = -3$

$x^3 - 6x^2 + 11x - 6$

$= (-3)^3 - 6(-3)^2 + 11(-3) - 6$

$= -27 - 54 -33 -6$

Clearly not zero (Not the answer)

Option C: $x = 1$

$x^3 - 6x^2 + 11x - 6$

$= 1^3 - 6(1)^2 + 11(1) - 6$

$= 0$

This is the answer

Option D: $x = -3$

Same as Option B

This method gives you the answer in a short period of time.

Tip #3:

When asked to compare linear/quadratic equations in two columns (data comparison questions) you can add/subtract/divide/multiply positive terms or positive numbers from both columns to make the comparison much easier.

For example:

Column A

$x^2 + 2x + 2$

Column B

$x^2 + 2x$

Here you can cancel out x^2 from both sides, you cannot however multiply or divide with negative numbers or terms which could have negative values.

$+ 2x + 2$ and $+ 2x$

Divide both sides by 2

$x+1$ and x

Subtract 'x' from both sides

1 and 0

The comparison is much easier now, than when we started.

Linear Inequalities

Tip #1:

You can add, subtract, divide or multiply on both sides of the equations simultaneously.

For example:

$2x + 5 \geq 33$

Subtract 5 from both sides

$2x \geq 33-5$

Divide both sides by 2

$x \geq 14$

Doing this helps to arrive at the solution easily.

Tip #2:

Multiplication and division using negative numbers causes the sign to shift to the opposite direction

$\leq$ to become $\geq$ **or** $\geq$ to become $\leq$

For example:

$2x + 5 \geq -33$

Multiply both sides by -1

$-(2x + 5) \leq 33$

Similarly with division using negative numbers

$- 24x + 6 \geq 27$

Divide both sides by -3

$8x - 2 \leq -9$

$8x \leq -7$

$x \leq \frac{-7}{8}$

Exponents

Tip #1

If two numbers are equal then the exponents of these numbers would be also equal

$a^x = b^y$

if a = b, then x = y

For example: If $2^x = 2^9$ then x = 9

Tip #2

The exponents of two equal terms or numbers can be multiplied or divided with a number or a variable.

For example:

$4^2 = 4^{1/a}$

$4(^{2xa)} = 4(^{1/a \text{ x } a)}$ [Multiply both sides by 'a']

$4^{2a} = 4^1$

Subtraction or addition using numbers and variables on both sides is also possible

$4^2 = 4^{1/a}$

Divide both sides by 4

$4^{(2 -1)} = 4^{(1/a - 1)}$

$4^1 = 4^{(1-a)/a}$

And

$4^2 = 4^{1/a}$

Multiply both sides by 4

$4^{(2+1)} = 4^{1/a+1}$

Tip #3

The nth root of a number is actually the $\frac{1}{n}$ exponent of that number

$^{n}\sqrt{A} = A^{1/n}$

For example:

$^{3}\sqrt{8} = 8^{1/3}$

or

$^{10}\sqrt{12} = 12^{1/10}$

or

$\sqrt{9} = 9^{1/2}$

Tip #4

For a number to come outside of an nth root, it should have 'n' number of repetitions

Suppose

$^{5}\sqrt{4} = {}^{5}\sqrt{2x2}$

Since, 2’s appear only twice we cannot take 2 out of the root.

$^{5}\sqrt{32} = {}^{5}\sqrt{(2x\ 2x2x2x2)} = 2$

As '2' appears five times, here it comes out

Another way of looking at it is like this

$\sqrt[5]{32} = 32^{1/5} = (2^5)^{1/5} = 2^{5/5} = 2^1$

Tip #5

Distinguishing between $(a^m)^n$ and a^m x a^n

$(a^m)^n = a^{m \times n}$

$a^m \times a^n = a^{m+n}$

For example:

$(3^2)^4 = (3^2)^4 = 3^8$

$[(3^2)^4 = (3^2 \times 3^2 \times 3^2 \times 3^2 = 3^8)]$

$3^2 \times 3^4 = (3)^{2+4} = 3^6$

$[3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^6]$