

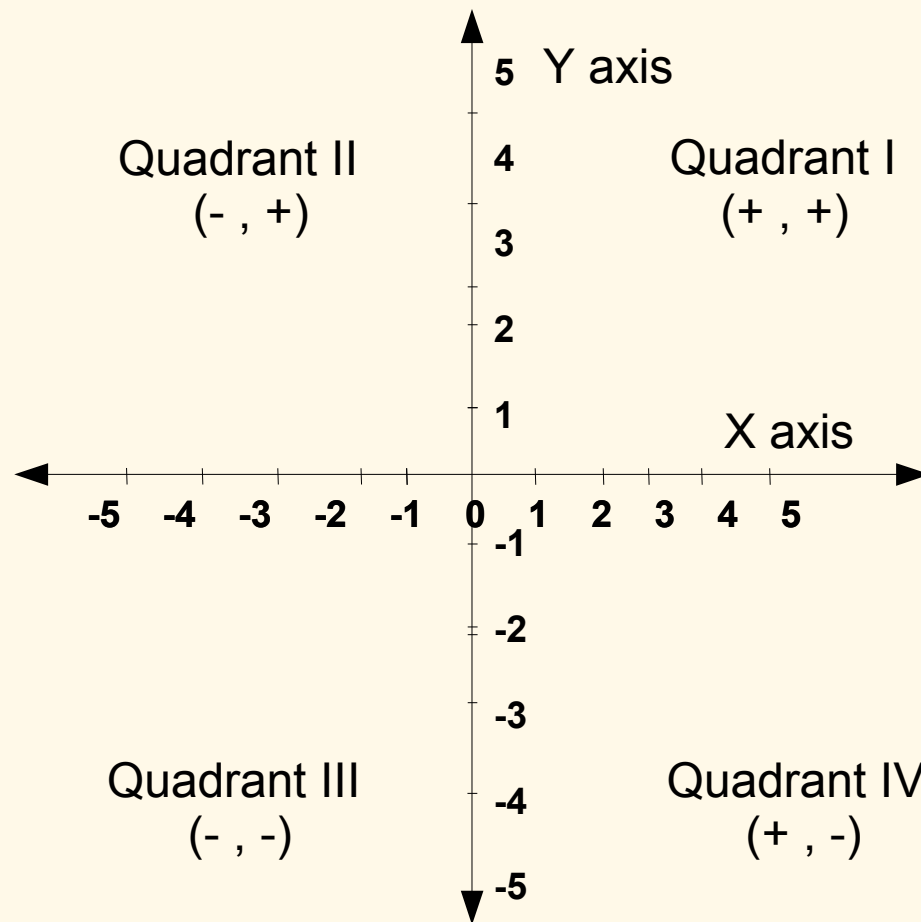


Coordinate Geometry I

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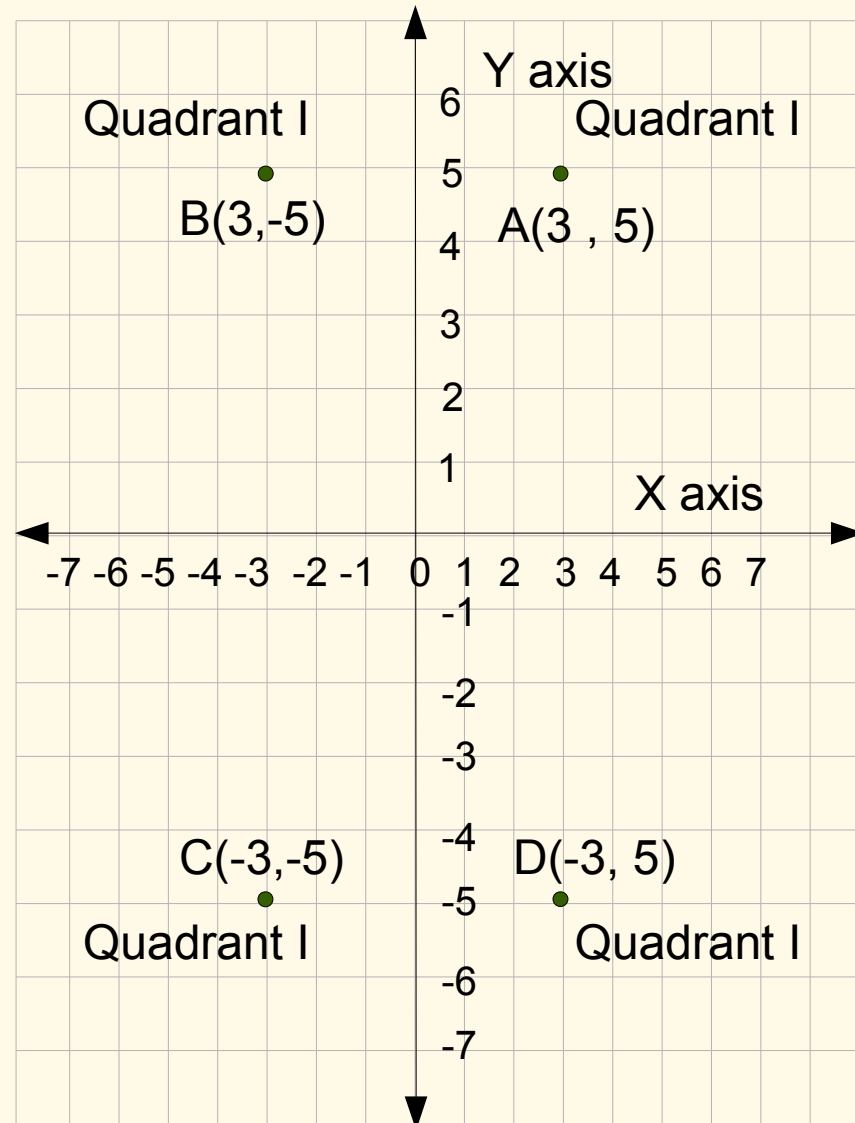
Basics In Coordinate geometry



Introduction



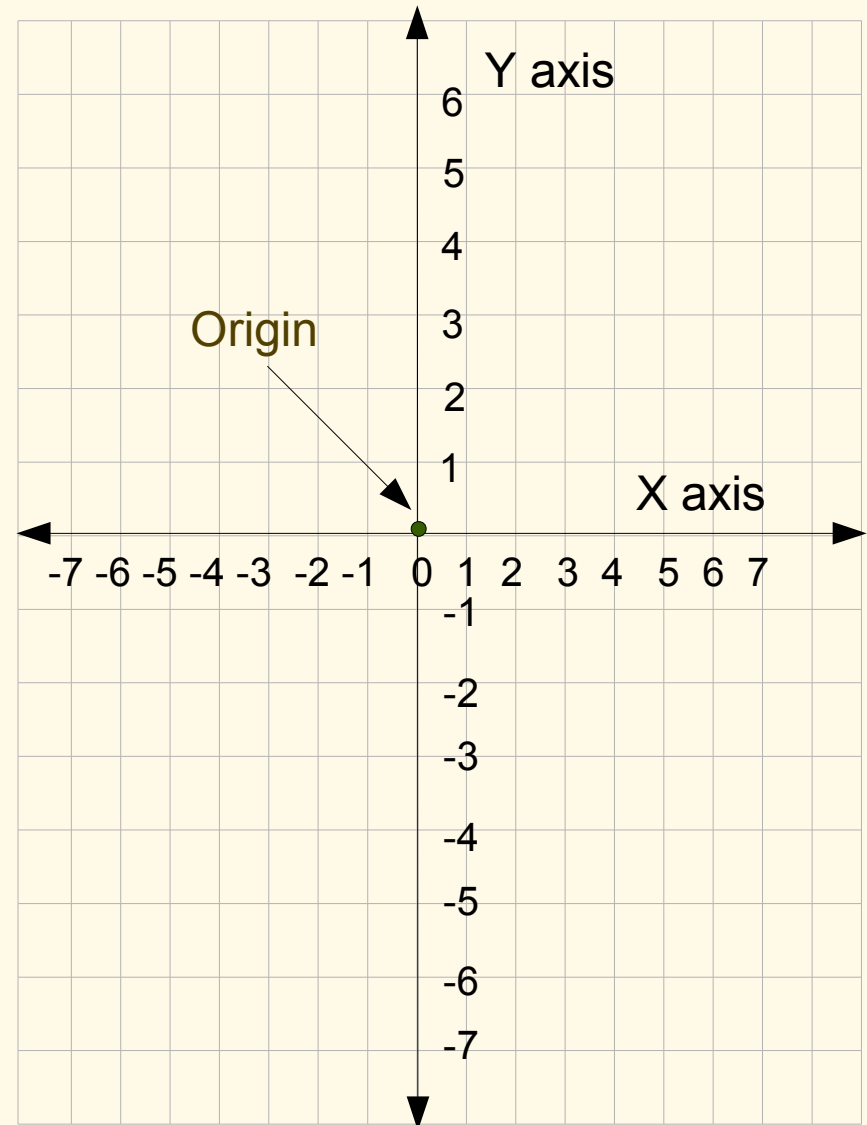
A system of geometry where the position of points on the plane is described using a pair of numbers.



Origin

Definition :

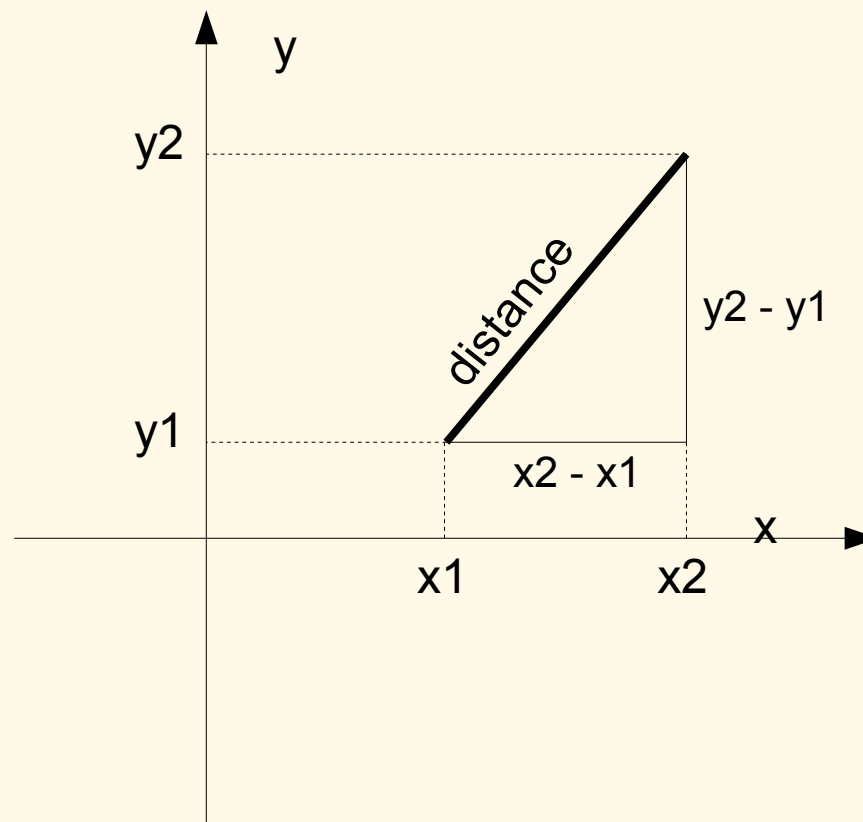
In a graph origin is the point $(0,0)$.
Where the x axis and y axis cross.



Distance between two points

Given the coordinates of two (x_1, y_1) and (x_2, y_2) , the distance d between the points is given by

$$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$$



Example

Find the distance between the points $(-1, -2)$ and $(-3, 5)$.

Can you find the answer?

Solution

Here, $(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (-3, 5)$.

Therefore, $x_1 = -1$, $y_1 = -2$, $x_2 = -3$, and $y_2 = 5$.

Using the distance formula we get,

$$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{((-3 - (-1))^2 + (5 - (-2))^2)}$$

$$d = \sqrt{((-3 + 1)^2 + (5 + 2)^2)} = \sqrt{((-2)^2 + 7^2)}$$

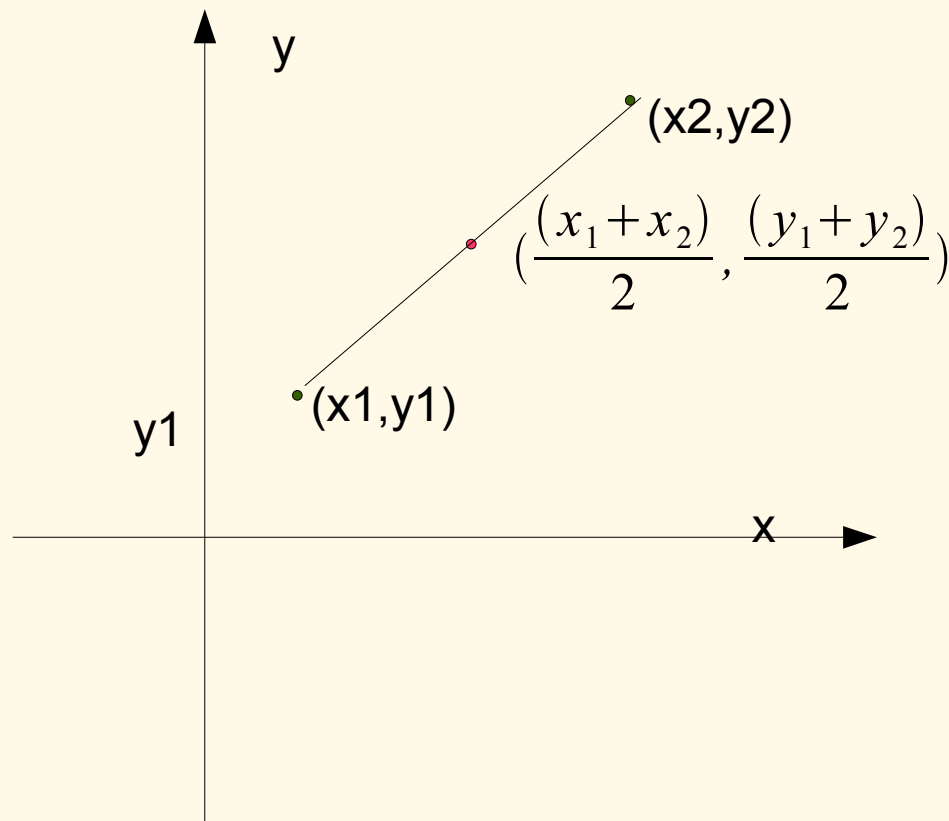
$$d = \sqrt{(4 + 49)} = \sqrt{53}$$

Midpoint formula



The **midpoint M** of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Example

Given the points $(-1, -2)$ and $(-3, 5)$, find the midpoint of the line segment joining them.

Try this question.

Solution

Using the midpoint formula, label the points as $x_1 = -1$, $y_1 = -2$, $x_2 = -3$, and $y_2 = 5$.

$$M = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right)$$

$$M = \left(\frac{(-1 - 3)}{2}, \frac{(5 - 2)}{2} \right)$$

$$M = \left(\frac{-4}{2}, \frac{3}{2} \right)$$

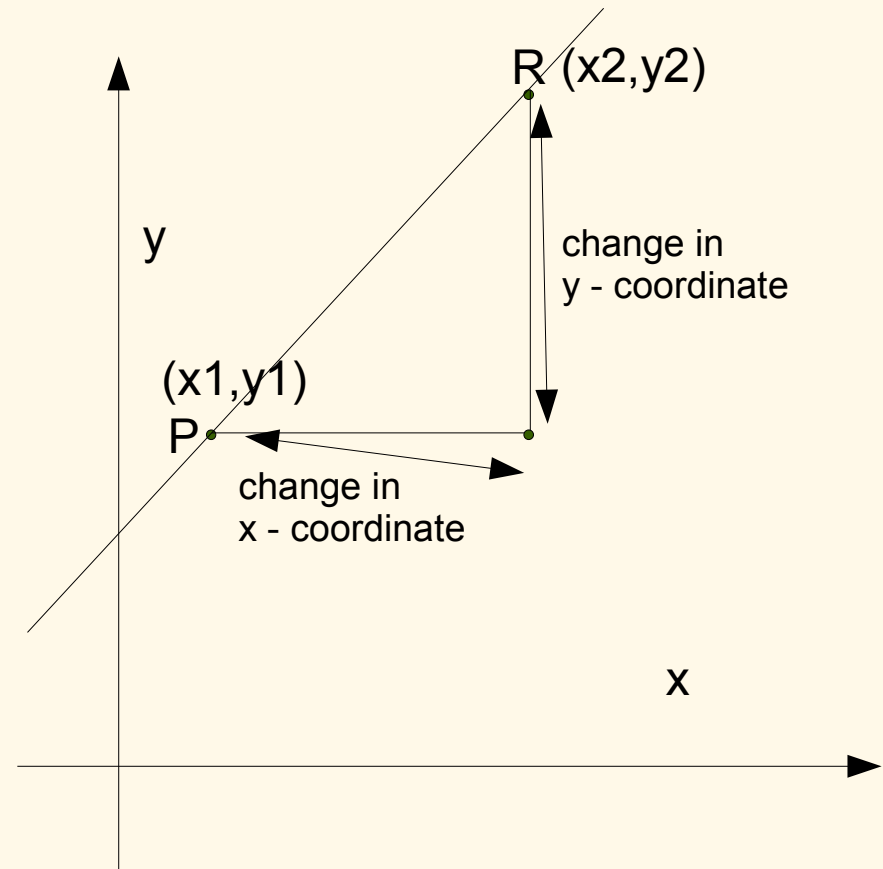
$$M = \left(-2, \frac{3}{2} \right)$$

Slope of a line

The **slope of a line** is a measurement of the steepness and direction of a non vertical line.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

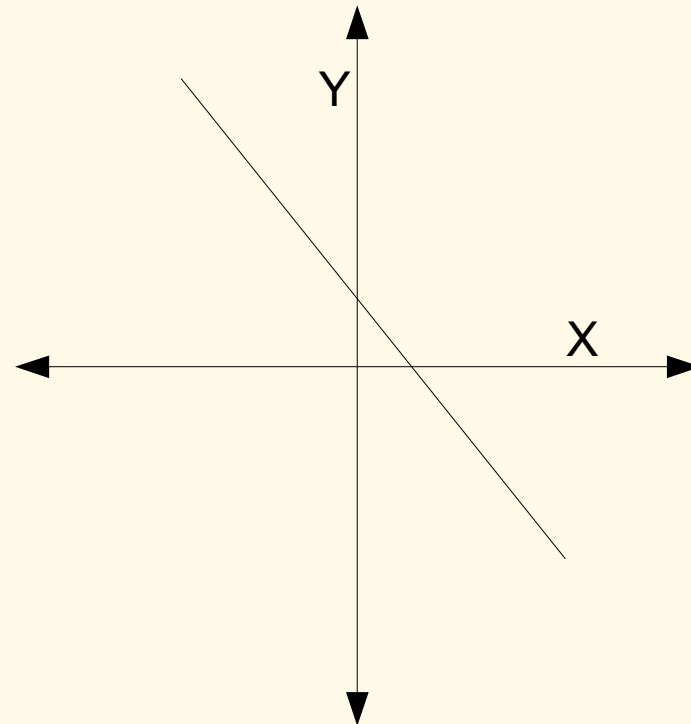
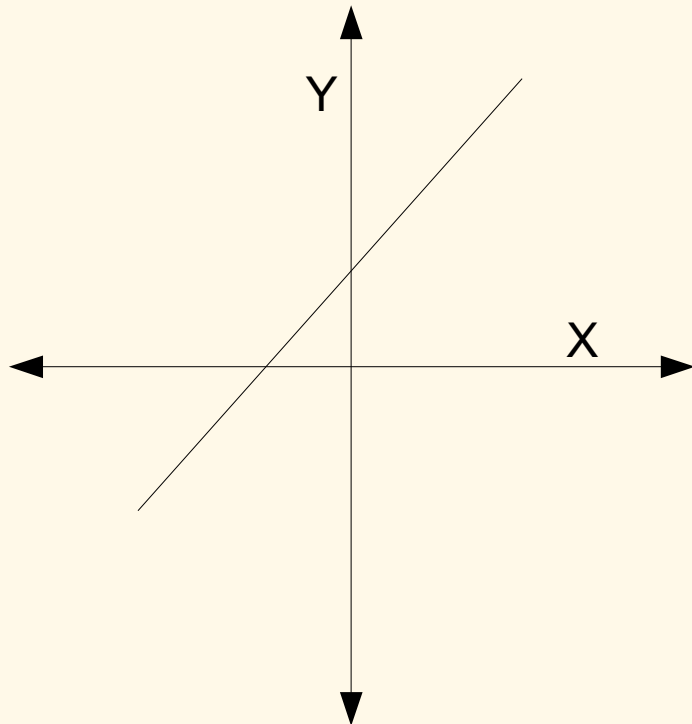
Note: The line which makes a larger angle with the X axis in the anti-clockwise direction has a greater slope.



Slope of a line

When a line rises from left to right, the slope is a positive number. Figure 1 (a) shows a line with a positive slope.

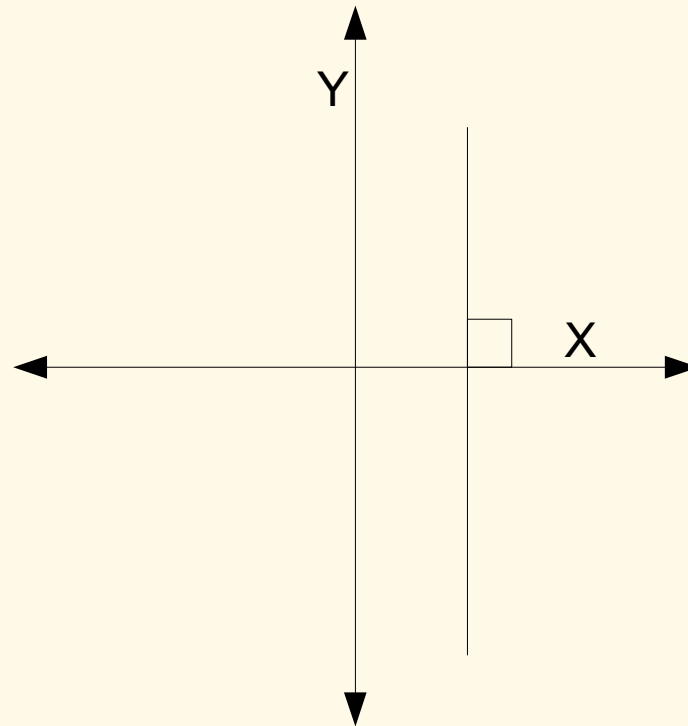
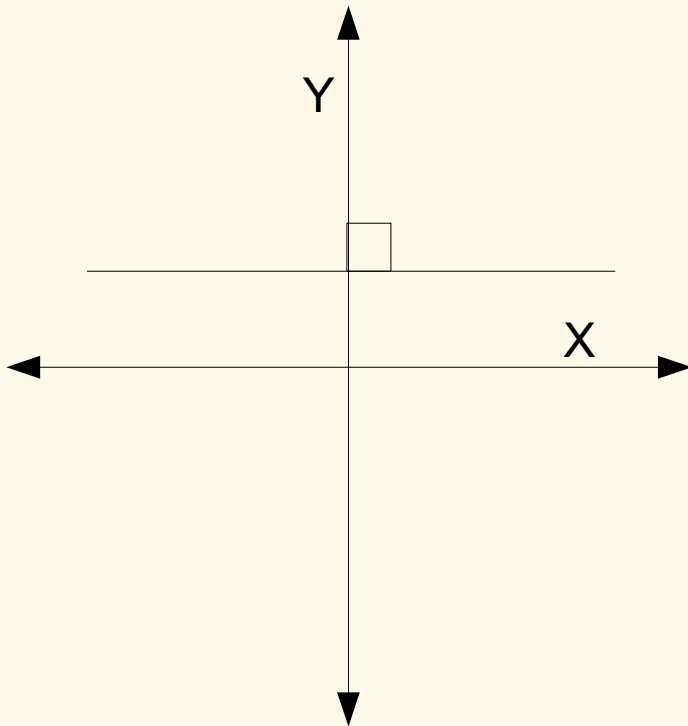
When a line falls from left to right, the slope is a negative number. Figure 1 (b) shows a line with a negative slope



Slope of a line

The x-axis or any line parallel to the x-axis has a slope of zero. Figure 1 (c) shows a line whose slope is zero.

The y-axis or any line parallel to the y-axis has no slope. Figure 1 (d) shows a line with an no slope.

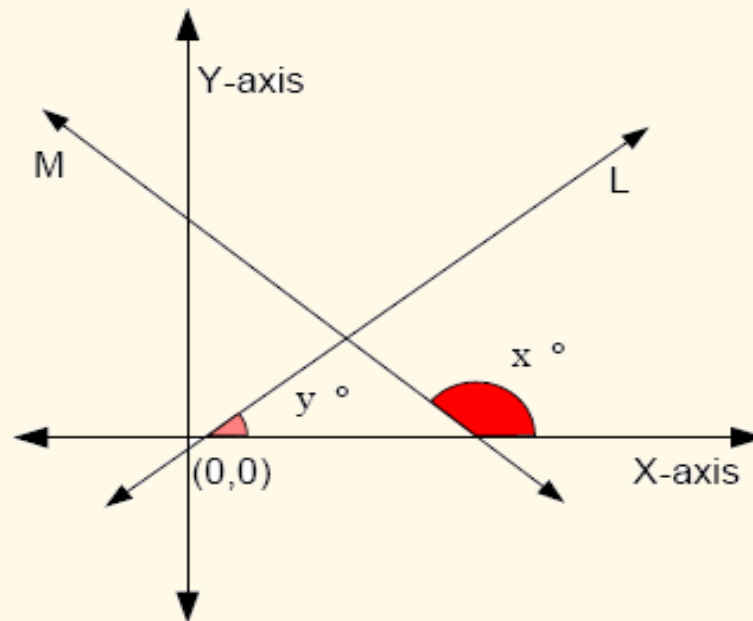


Properties of a Slope

If the angle made by the line with the X-axis lies in between 0° to 90° , then slope is positive.

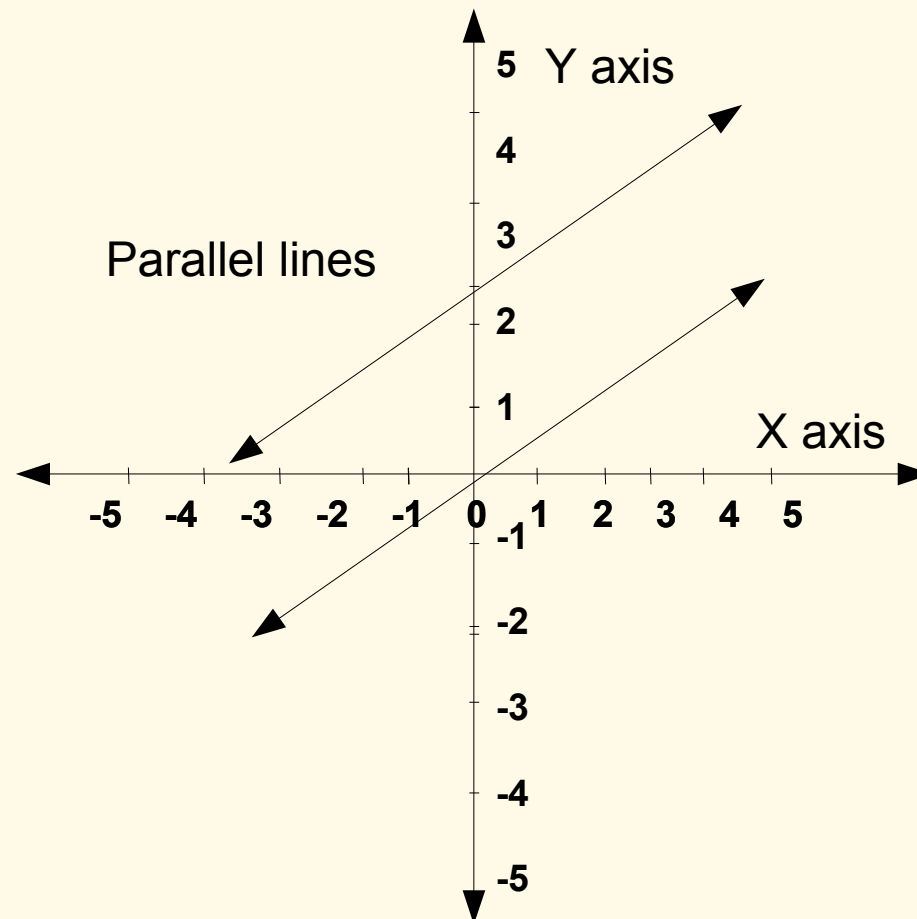
Whereas if the angle made by the line with the X-axis lies in between 90° to 180° , then slope is negative.

Hence in the above figure, slope of line L is positive (as $0^\circ < y < 90^\circ$) and the slope of line M is negative (as $90^\circ < x < 180^\circ$).



Parallel lines and their slopes

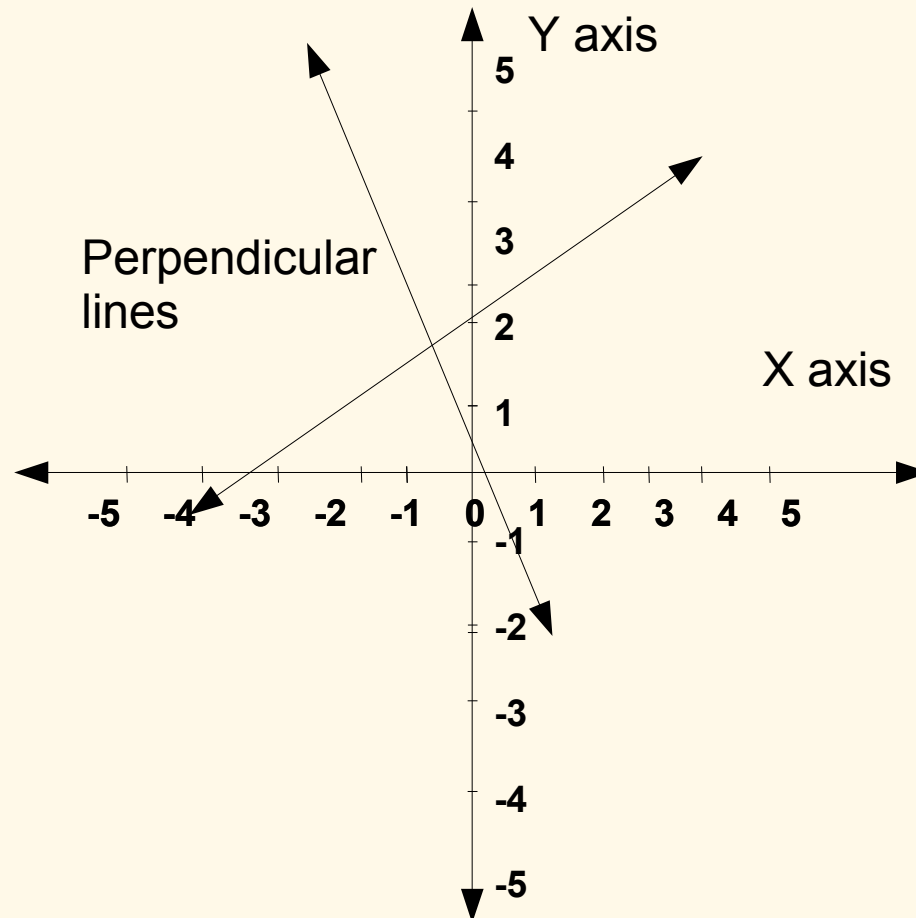
If two lines are parallel to each other then, their **slopes are equal**.



Perpendicular lines and their slopes



If two lines are perpendicular to each other then **product of the slopes of those two lines is equal to -1**.



Equation of a Straight Line

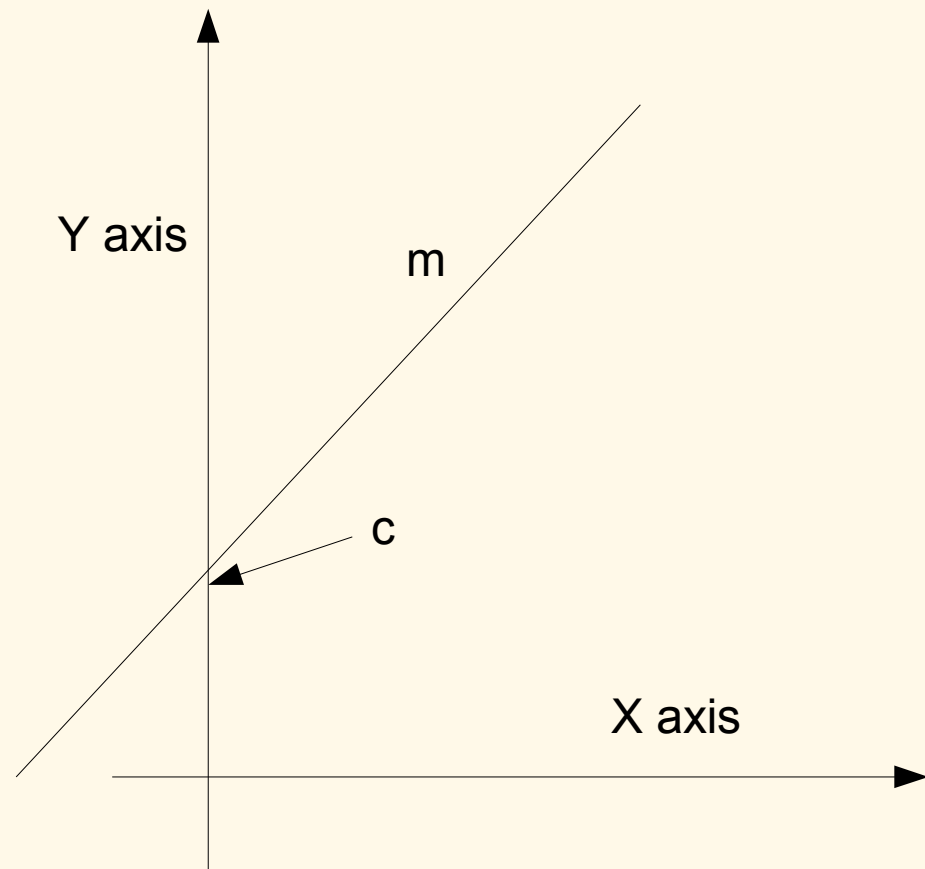
General Form of Equation of a Line

$$ax + by = c$$

Equation of a Straight Line

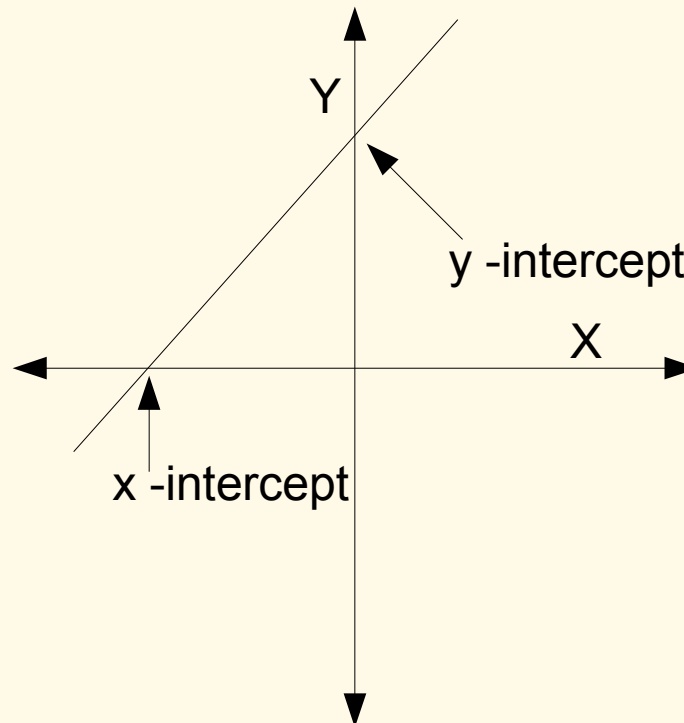
$$y = mx + c$$

↑ ↑
slope y intercept



X and Y intercepts

Every line in the XY plane which is neither parallel to X axis nor to Y axis intersects the X axis and Y axis at some point.



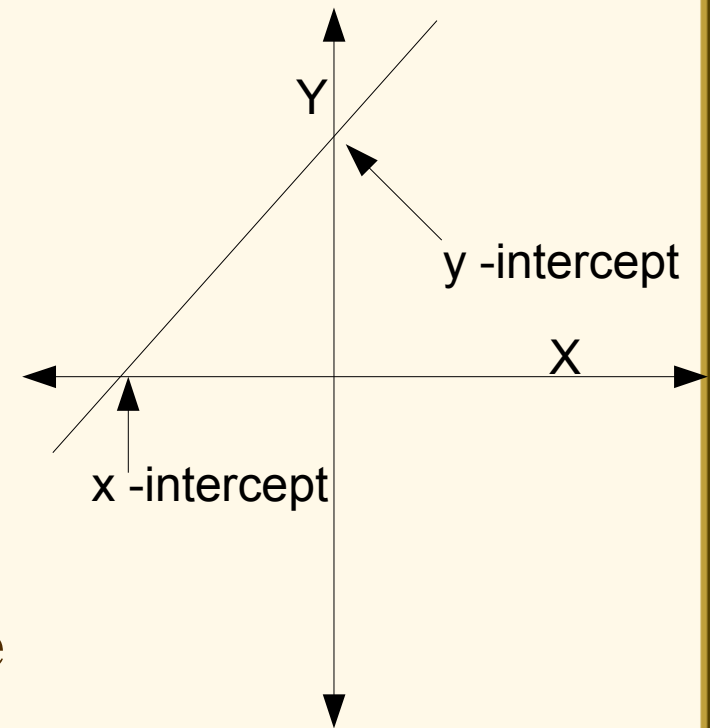
X and Y intercepts

The point at which line L intersects with X -axis is known as X -intercept of line L .

X intercept of a line = x -coordinate of the point where the line meets the X axis

The point at which line L intersects with Y axis is known as Y -intercept of line L .

Y intercept of a line = y co-ordinate of the point where the line meets the Y axis



Ways to Determine the Equation of a Straight Line



- Slope intercept form
- Point Slope Form
- Two-point form
- Intercept form



Slope Intercept Form

$$y = m x + c$$

Where,

m = Slope of the line

c = y- intercept

Example

Find the equation of the line that has a slope of 5 and a y-intercept of 2

Can you find the equation of the line?

Solution

Using the slope-intercept form,

$$y = mx + c$$

Where,

m = slope

c = y-intercept

Substituting the values $m=5$ and $c=2$

$$y = (5)x + 2$$

$y = 5x + 2$ is the Equation of the line.

Point Slope Form



$$y - y_1 = m(x - x_1)$$

Where,

m = Slope of the line

$P_1 = (x_1, y_1)$ is the point through which the line passes.

Example

Find the equation of the straight line that has slope $m = 4$ and passes through the point $(-1, -6)$.

Crack this question

Solution

Using Point slope form,

Here $x_1 = -1$ and $y_1 = -6$,
we get

$$y - (-6) = 4(x - (-1))$$

$$y + 6 = 4(x + 1)$$

$$y + 6 = 4x + 4$$

$y = 4x - 2$ is the Equation of the line

Try this by Slope intercept form.

Using the slope-intercept form

$$y = mx + c$$

$$(-6) = (4)(-1) + c$$

$$-6 = -4 + c$$

$$-2 = c$$

Then the line equation must be " $y = 4x - 2$ ".

Two point Form



$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Where,
(x₁ , y₁) and (x₂,y₂) are the points through which the line passes.

Example

Given that the line passes through the points $(-2, 4)$ and $(1, 2)$. Find the equation of the line.

Can you Solve this

Solution

Slope of the line is given by

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Here the points given are, $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (1, 2)$.
Substituting we get,

$$y - 4 = \left(\frac{2 - 4}{1 - (-2)} \right) (x - (-2))$$

$$y - 4 = \frac{(-2)}{3} (x + 2)$$

$$y - 4 = (-2x/3) - (2/3) * 2$$

$$y = (-2x/3) - 4/3 + 4$$

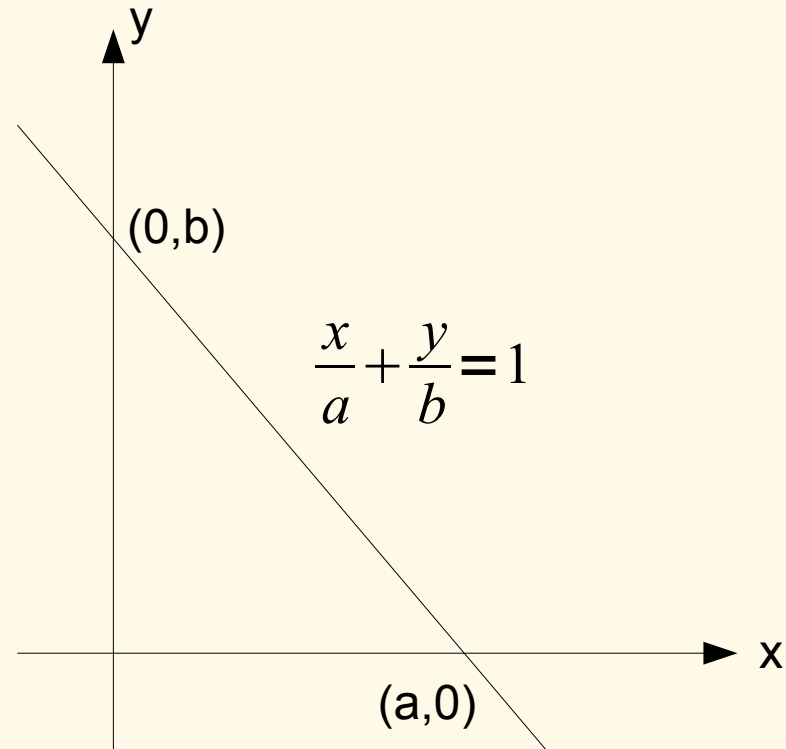
$$y = (-2/3)x - 8/3$$

Hence the equation of the line is, $y = (-2/3)x + 8/3$.

Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where,
a = x- intercept
b = y- intercept



Note:

x-intercept is a point on the graph where y is zero

y-intercept is a point on the graph where x is zero.

Example

Find the x and y intercepts of the graph of the equations given below.

$$2x - y = 2$$

Try this

Solution:

Write the given equation of the line in form of Intercept form.

$$\frac{x}{a} + \frac{y}{b} = 1$$

Given equation is $2x - y = 2$

Dividing 2 in the above equation

$$2x/2 - y/2 = 2/2$$

$$x/1 - y/2 = 1$$

$$\frac{x}{1} + \frac{y}{(-2)} = 1$$

Here $a = 1$ and $b = -2$

Hence the x intercept is 1 and the y intercept is -2.

Slope of a line when X and Y intercepts are given



If X intercept of a line L is 'a' and Y intercept is 'b' then the slope of the line is given by $(-b/a)$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$(bx + ay)/ab = 1$$

$$(bx + ay) = ab$$

$$ay = ab - bx$$

$$y = (-bx + ab)/a$$

$$y = -(b/a)x + b$$

Equation of the line is $y = mx + b$.

Where the slope, $m = (-b/a)$

Example

Find the slope of the line, if x intercept is 5 and y intercept is 8.

Solution:

Since X intercept and Y intercept are given.
The slope of line is given by $= (-b/a) = (-8/5)$

Hence the slope is $(-8/5)$.

All in one



Two points $A(3, -2)$ and $B(6, 4)$ are given. Answer the following questions.

- (a) mid point of A and B*
- (b) distance between mid point of A & B and B*
- (c) slope of line AB*
- (d) equation of line AB*
- (e) x-intercept and y-intercept of line AB*

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