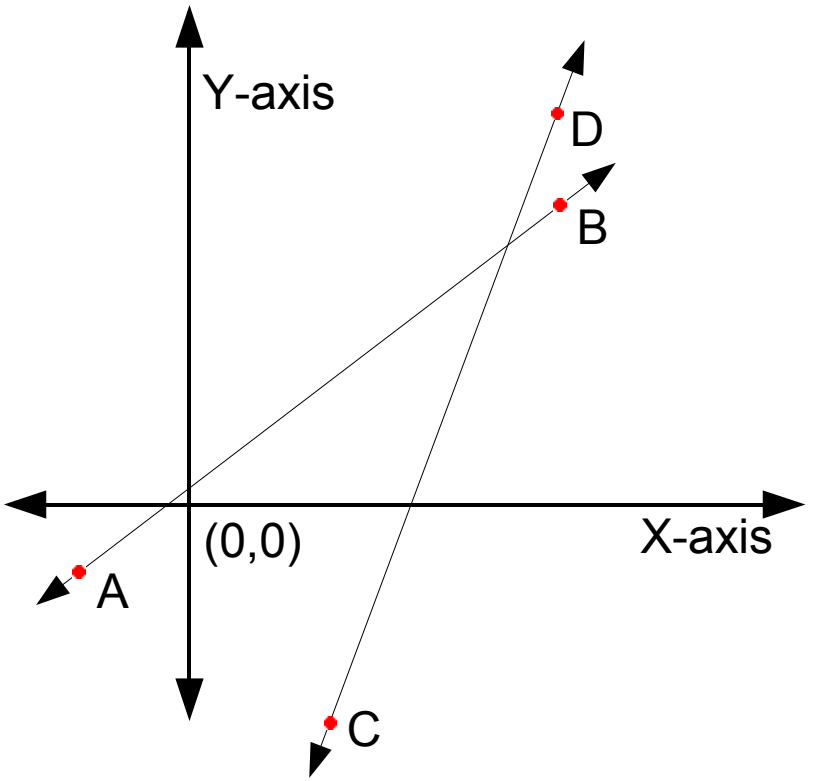
Coordinate Geometry

Part A : Slope of a Line

Slope

A straight line is characterized by a property called slope
Slope is measure of the inclination of the line with the X axis

The line which makes a larger angle with the X axis, has a greater slope ?

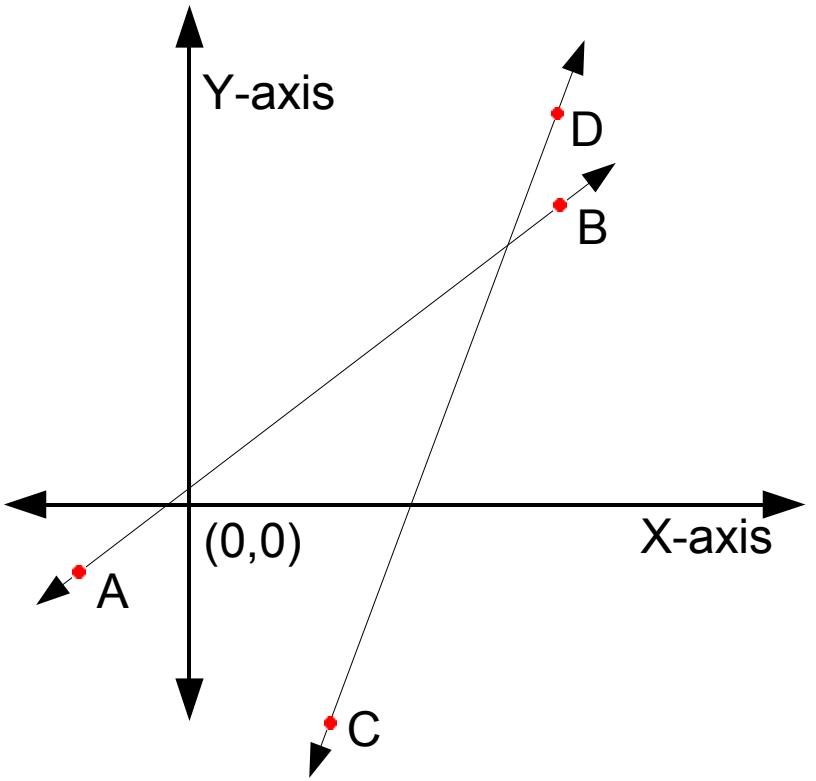


Guess: In the figure, which line has a higher slope, Line AB or Line CD ?

Slope

In the figure, line CD has a **higher** slope as compared to line AB

The angle made by CD with the positive X axis is higher than the angle made by line AB with the positive X axis.

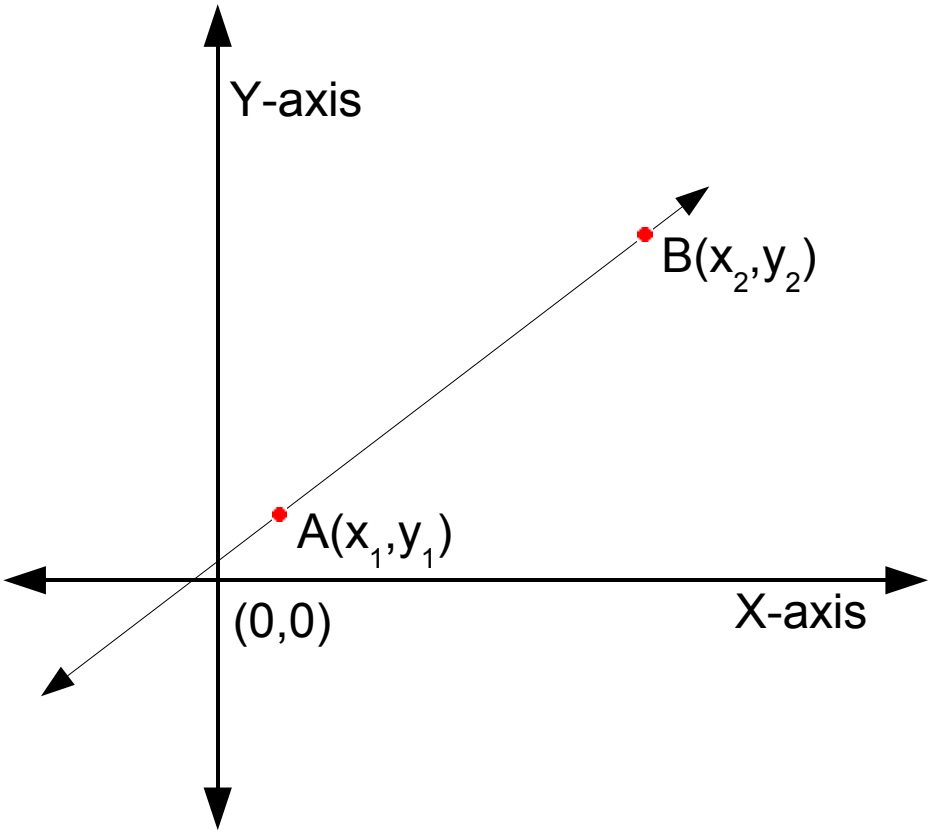


Formula for Slope of a line

In the figure, points $A(x_1,y_1)$ and $B(x_2,y_2)$ lie on line AB

Slope of line AB is defined as:

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$



Examples

Find the slope of the line passing through the points (1, 2) and (-4, -5) .

Solution: Let us apply the formula for slope

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{1 - (-4)} = 7 / 5$$

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 2}{-4 - 1} = 7 / 5$$

Guess ?

What is the slope of the X- axis ?

What is the slope of the Y axis ?

Note

Note that : The slope of the X- axis is 0,

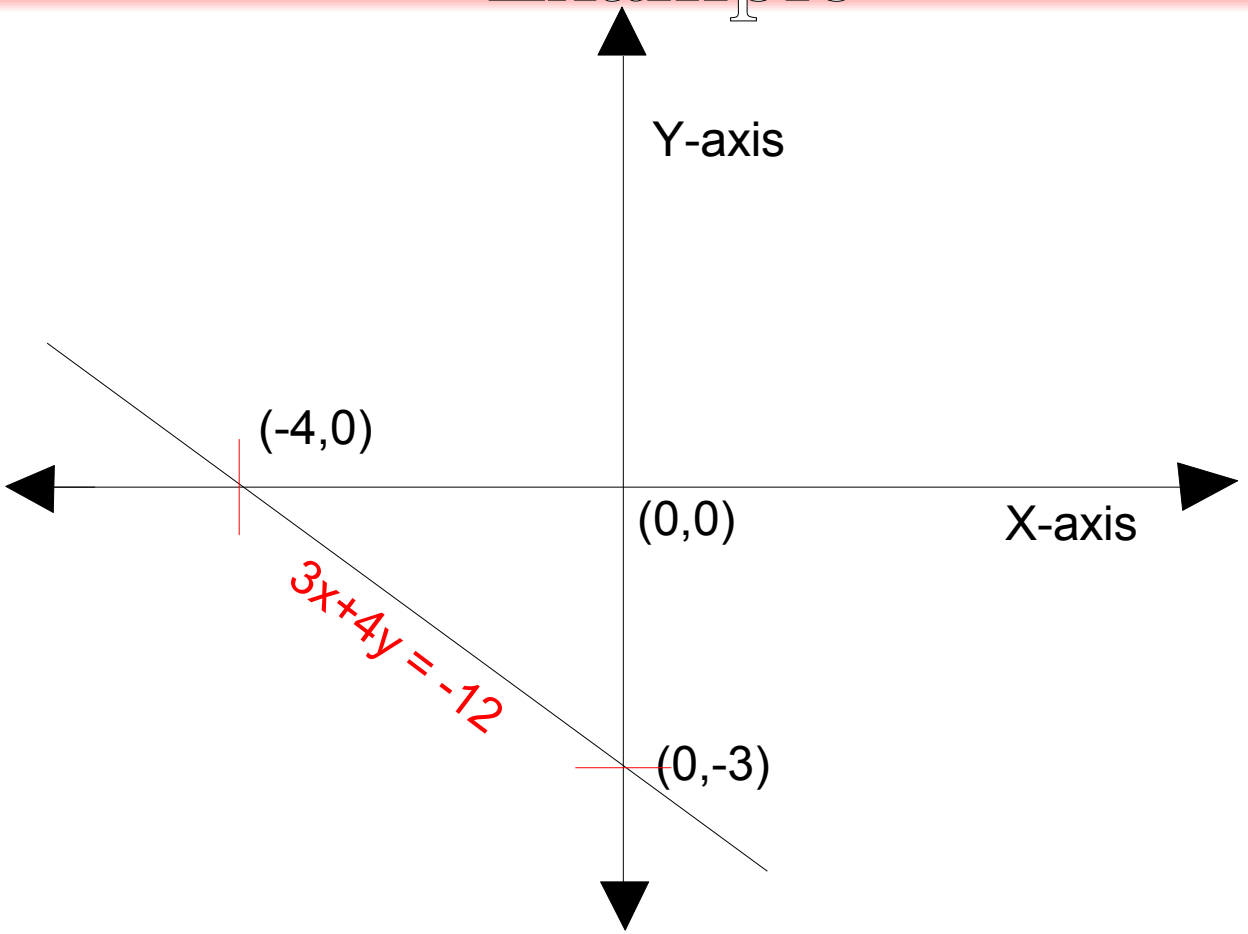
whereas the slope of the Y axis is infinite

Explanation:

To determine slope of the X axis, we need 2 points on the X axis.
Lets take 2 points be (a, 0) and (b,0)
Slope of the X axis = (0 – 0) / (a-b) = 0

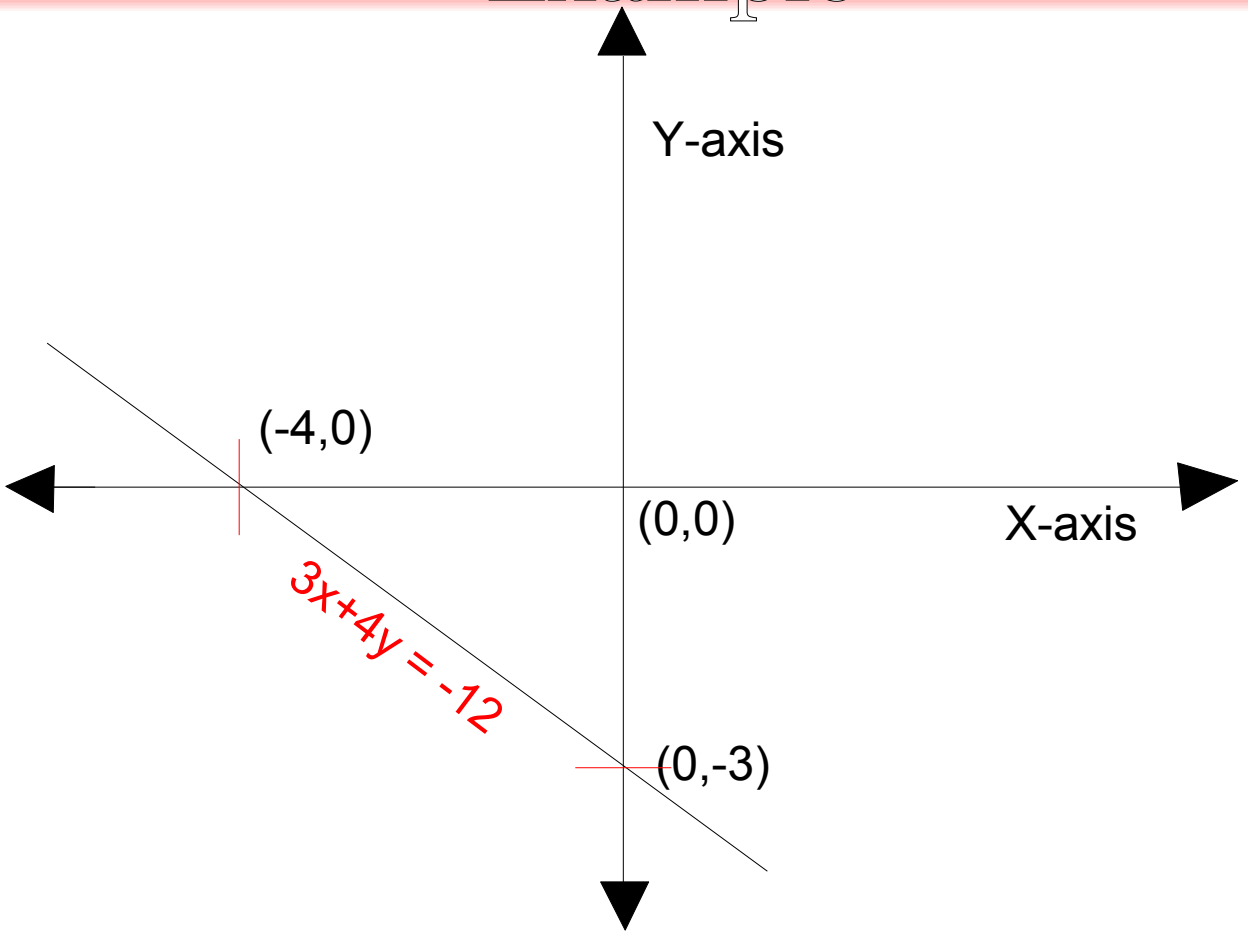
To determine slope of the Y axis, we need 2 points on the Y axis.
Lets take 2 points be (0, a) and (0,b)
Slope of the X axis = (a – b) / (0-0) = infinity

Example



Determine the slope of the line shown in the figure

Example



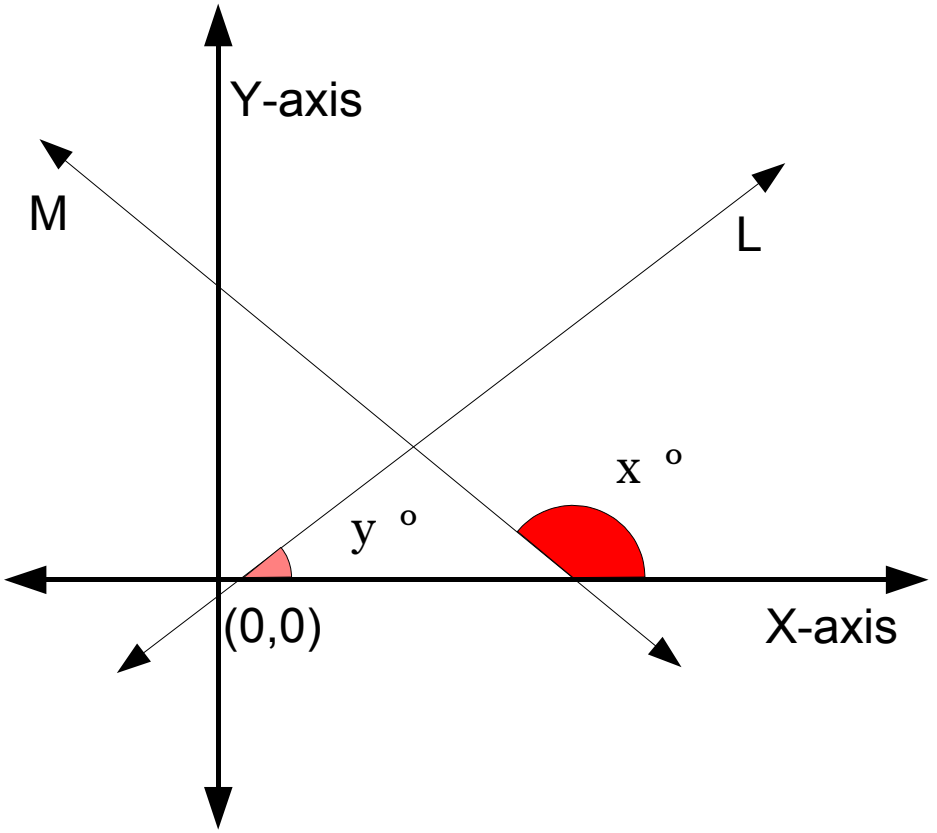
$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-3)}{-4 - 0} = -3/4$$

Properties of Slope

1) The slope can be either positive or negative.

- If the angle made by the line with the X-axis lies in between 0° to 90°, then slope is positive.

- Whereas if the angle made by the line with the X-axis lies in between 90° to 180°, then slope is negative .

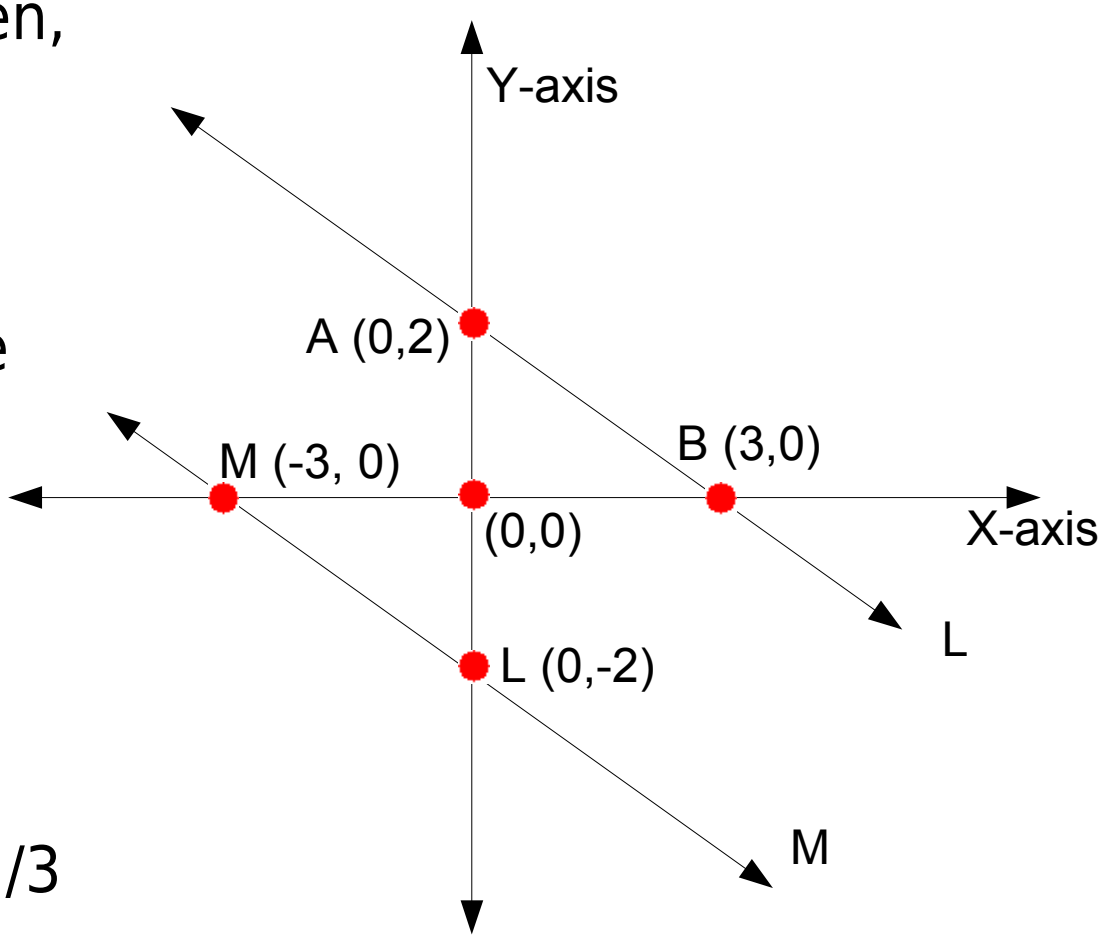


In the figure the slope of line L is positive whereas slope of line M is negative

Properties of Slope

2) If two lines are parallel then, their slopes are equal.

In the figure line L is parallel to line M
Now, let us calculate the slope lines L and M.



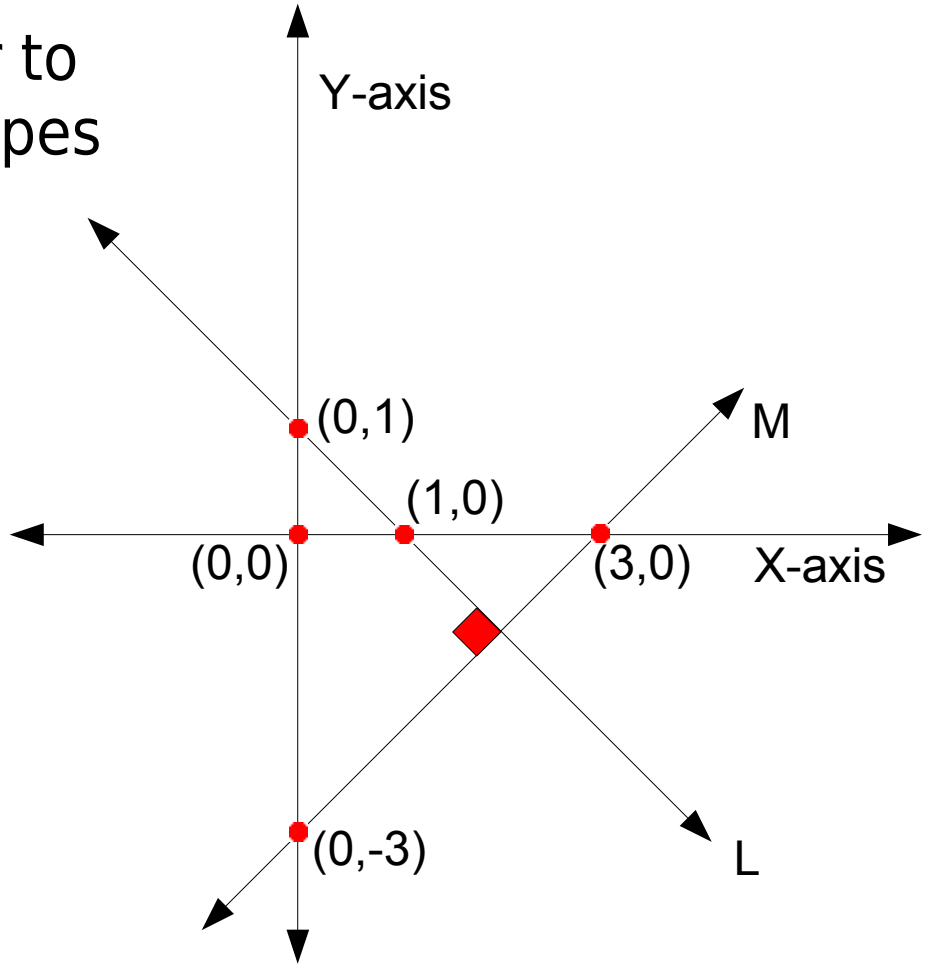
Slope of line L
= (2 – 0) / (0-3) = - 2 /3

Slope of line M
= (-2 – 0) / (0-(-3)) = - 2 /3

Here slope of both lines are equal.

Properties of Slope

3) When two lines are perpendicular to each other then product of the slopes of two lines is equal to -1

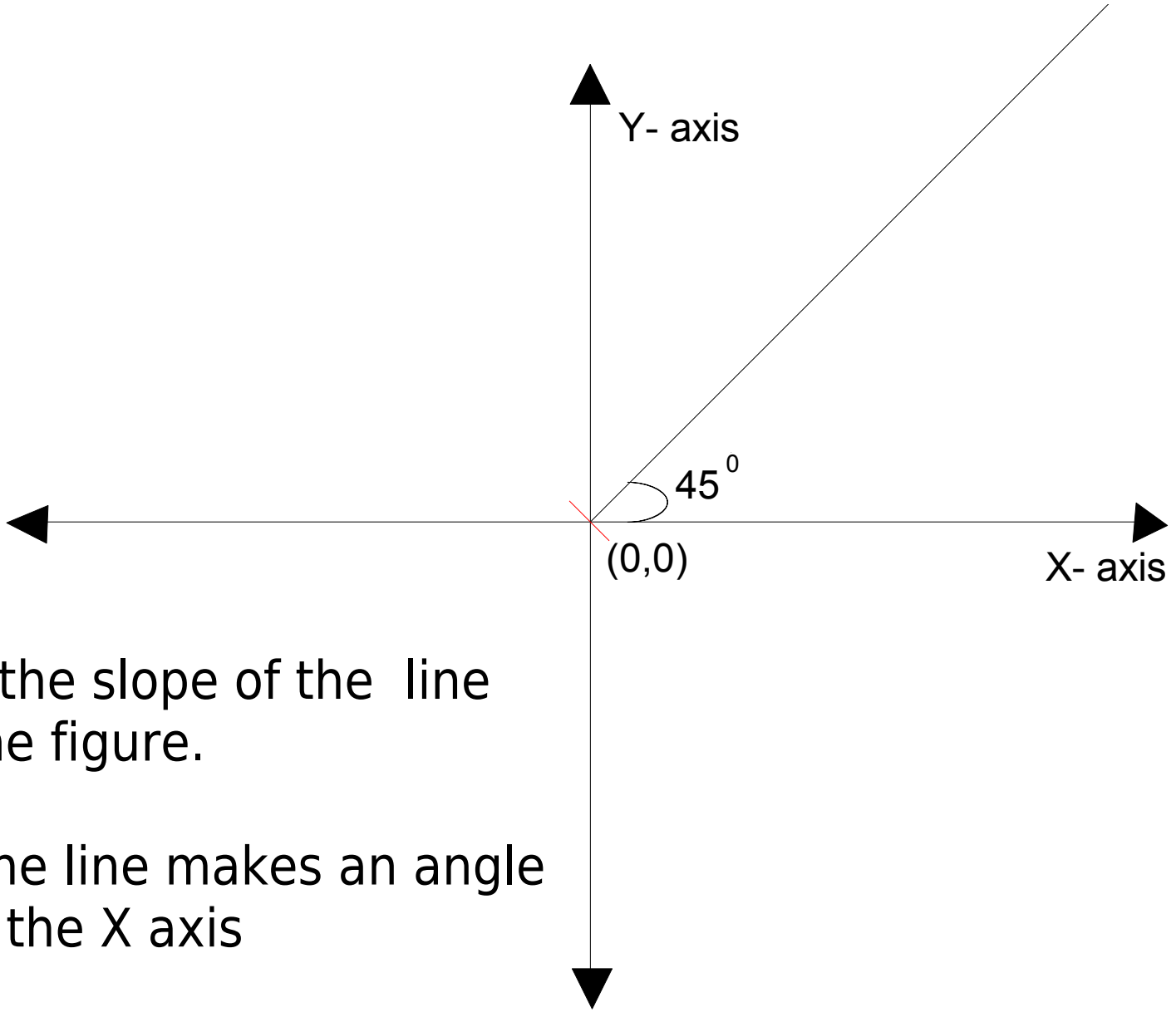


In the figure line M is perpendicular to line N
Now let us calculate their slopes.
Slope(line L) = (1-0) / (0-1) = -1

Slope (Line M) = (0-(-3)) / (3-0)
= 3/3
= 1

Slope(Line M) X Slope (Line N)
= -1 X 1
= -1

GUESS



Determine the slope of the line shown in the figure.

Note that the line makes an angle of 45° with the X axis

SOLUTION

In the diagram, the angle is given as 45 degree, which means the first quadrant is divided into two equal halves.

Therefore any point in the line will have same value for x and y. Hence, points on this line will be of the type (a, a) and (b,b)

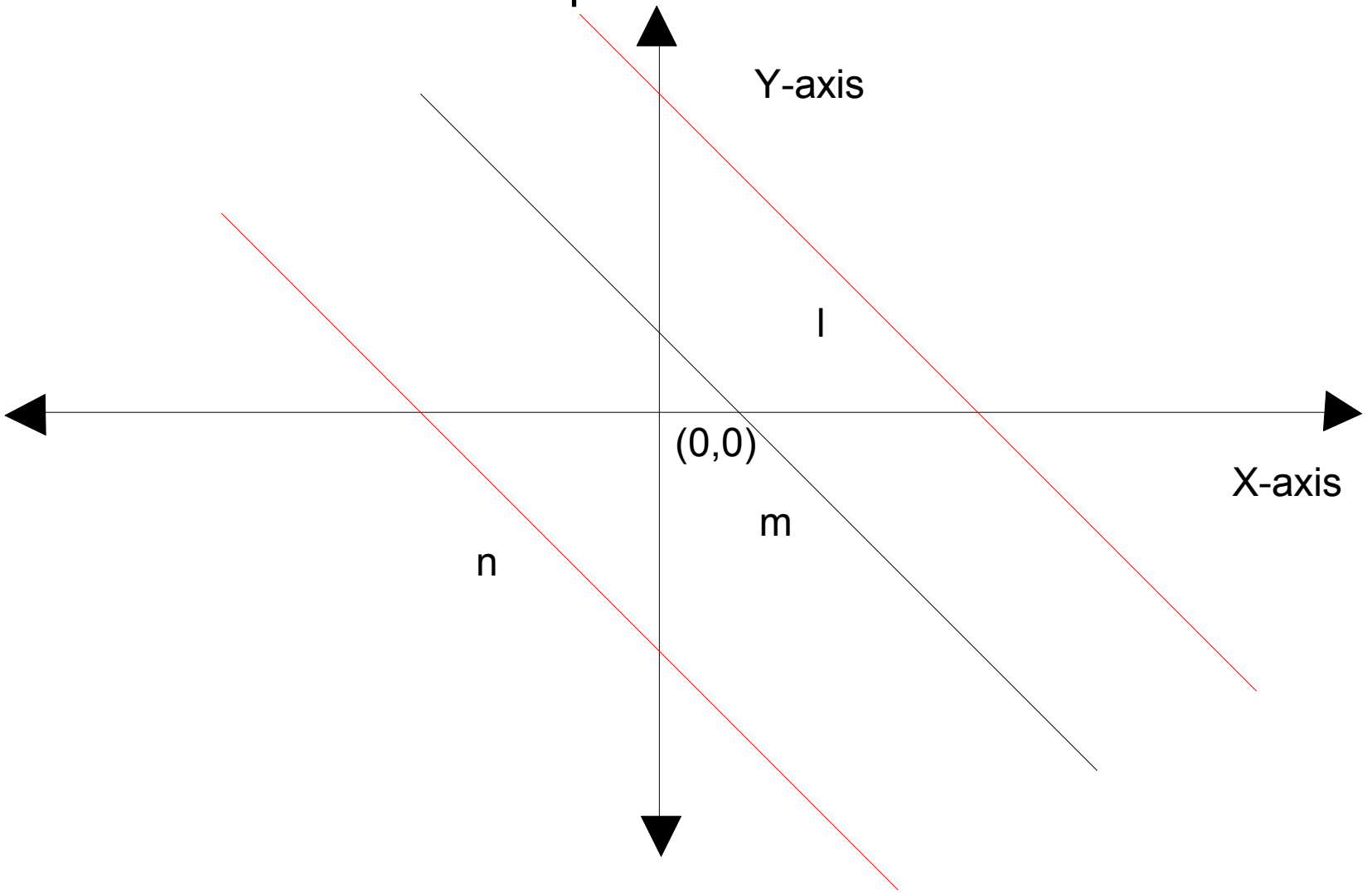
One point can be taken as (a,a) and the other point is the origin (0,0). By substituting these two points in the slope formula, we get,

m = (0 -a) / (0-a)
= -a / -a
= 1

Therefore, slope is equal to “1”.

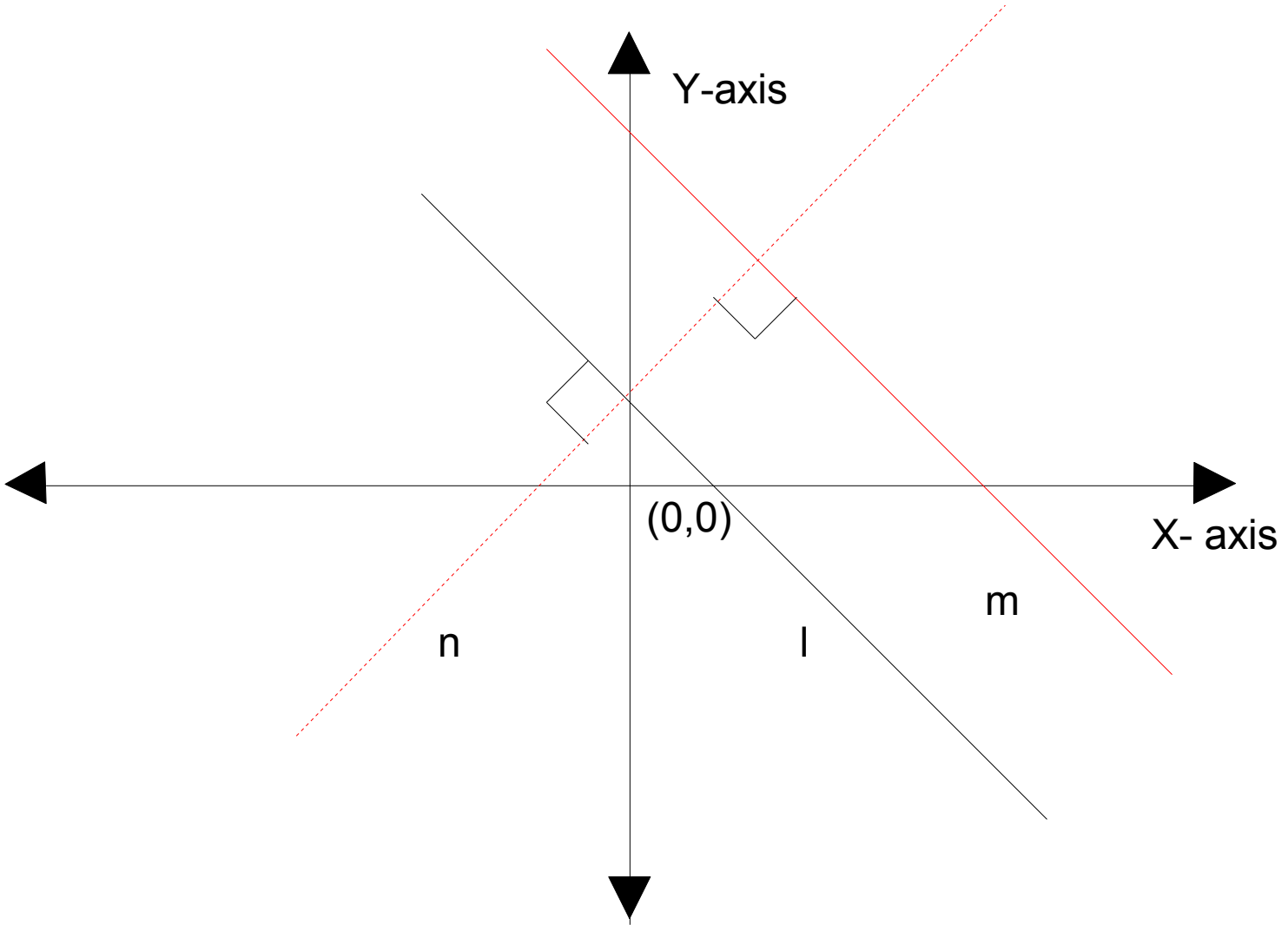
POINTS TO BE NOTED

(1) If a line “l” is parallel to line “m”, and line “m” is parallel to line “n”, then line “l” will be parallel to line “n”



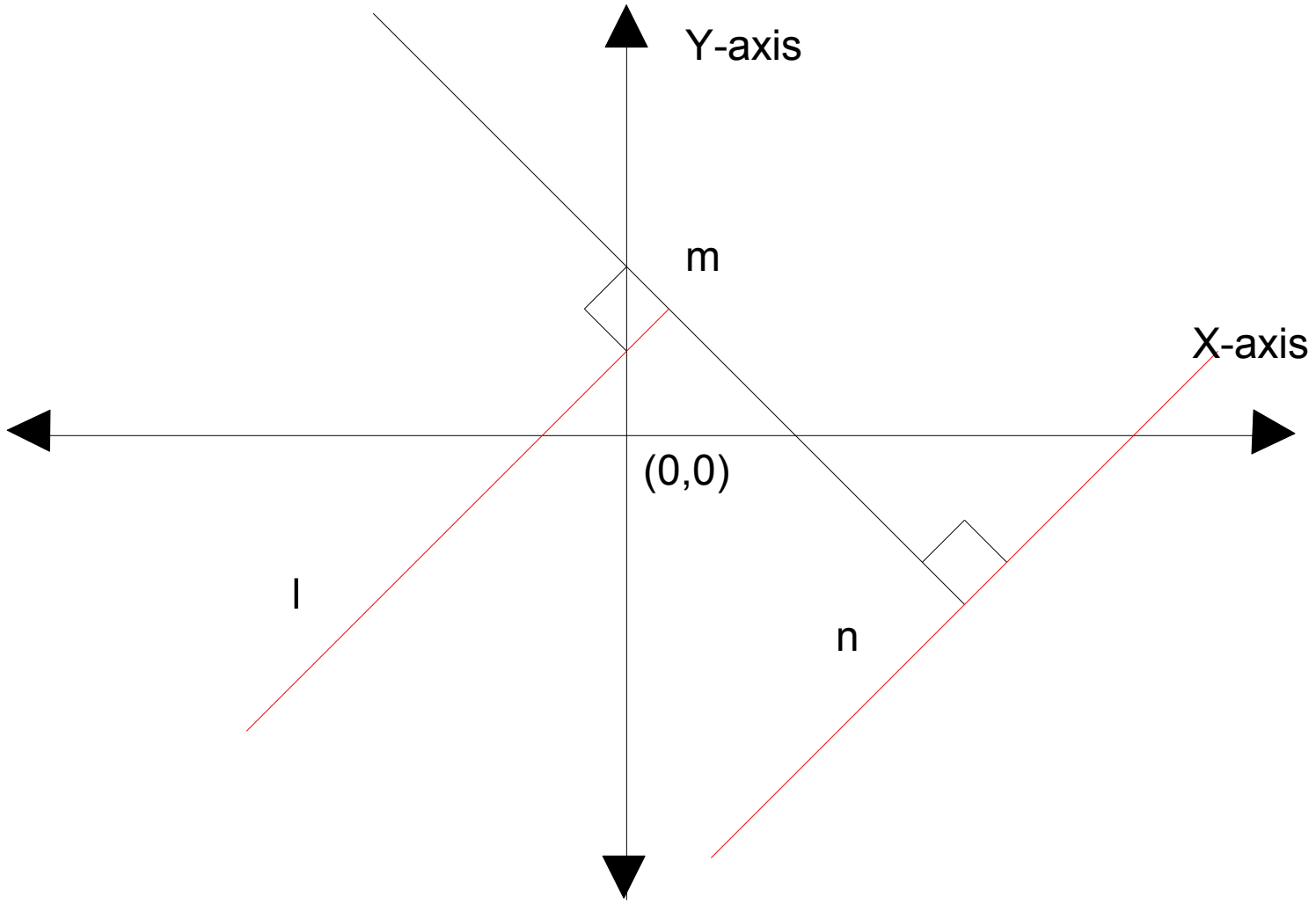
POINTS TO BE NOTED

(2) If line “l” is parallel to line “m” and
line “n” is perpendicular to line “l”, then
line “n” is also perpendicular to line “m”



POINTS TO BE NOTED

(3) If a line “l” is perpendicular to line “m” and line “m” is perpendicular to line “n”, then line “l” will be parallel to line “n”



QUIZ

(Send your solutions to support@greedge.com)

1) Find the slope of the line parallel to y-axis ?

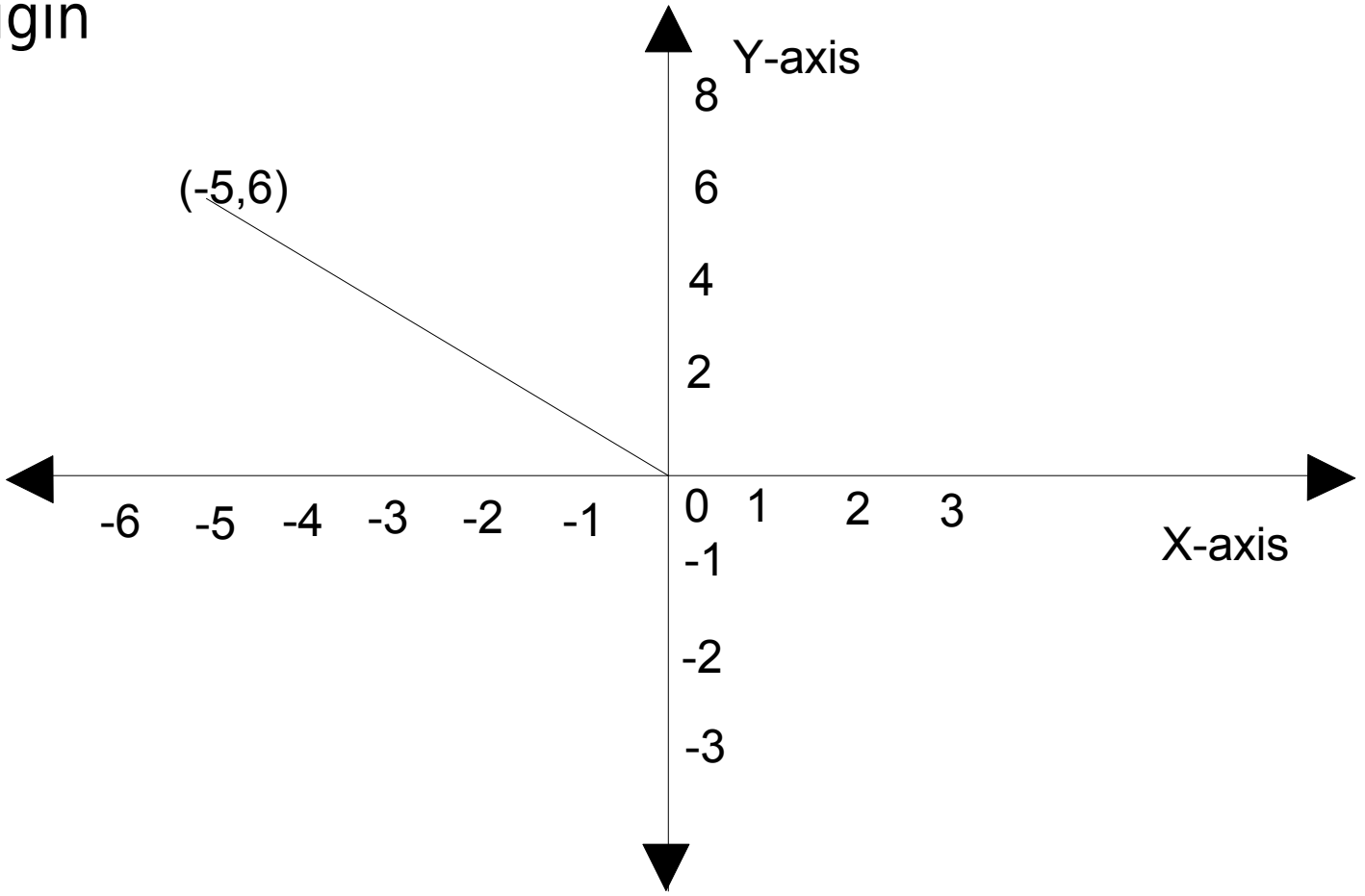
2) What is the slope of the line if point A is (4,-3) and point B is (1,2) ?

3) Find the slope of the line CD which is parallel to the line AB. Where point A is (-6,-7) and B is (0, 5) ?

4) Find the slope of the line parallel to x-axis ?

QUIZ

5) From the diagram below, find the slope of the line from the origin



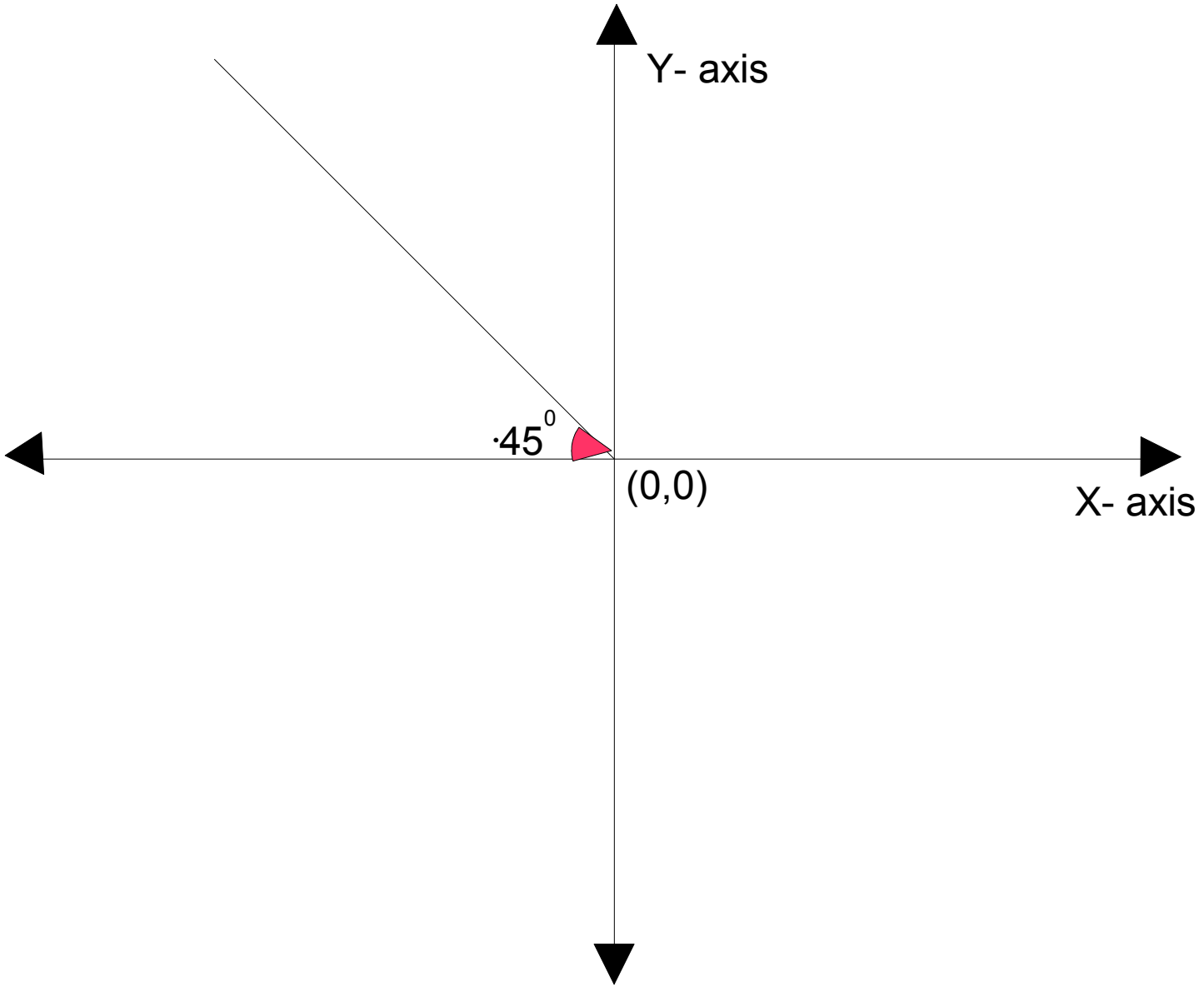
QUIZ

6) Find the slope of the line which is perpendicular to the line MN, where M is (0,-9) and N is (-5,6)?

7) Determine the slope of the line making 90 degree with x-axis?

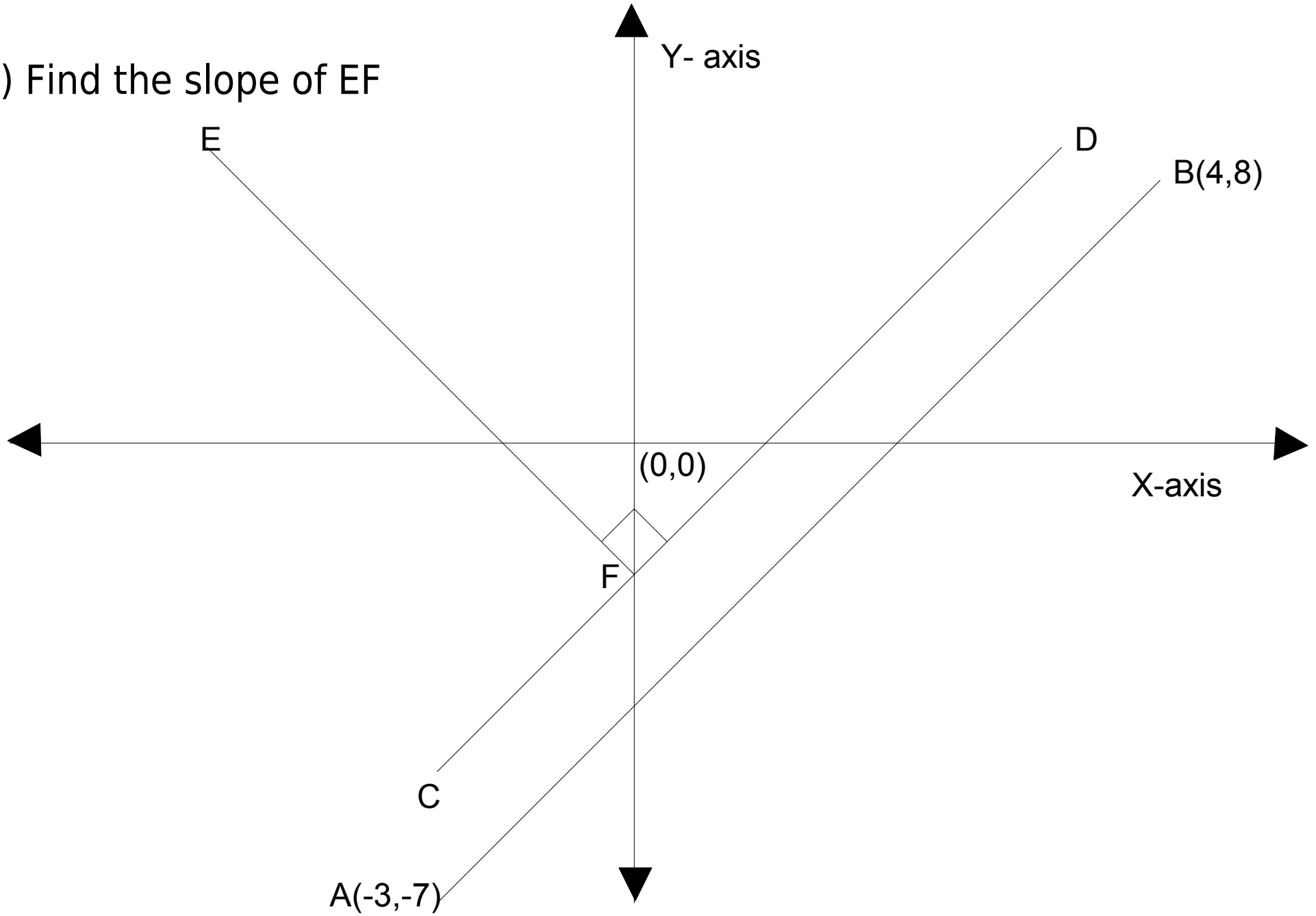
8) Find the slope of the line given below

QUIZ



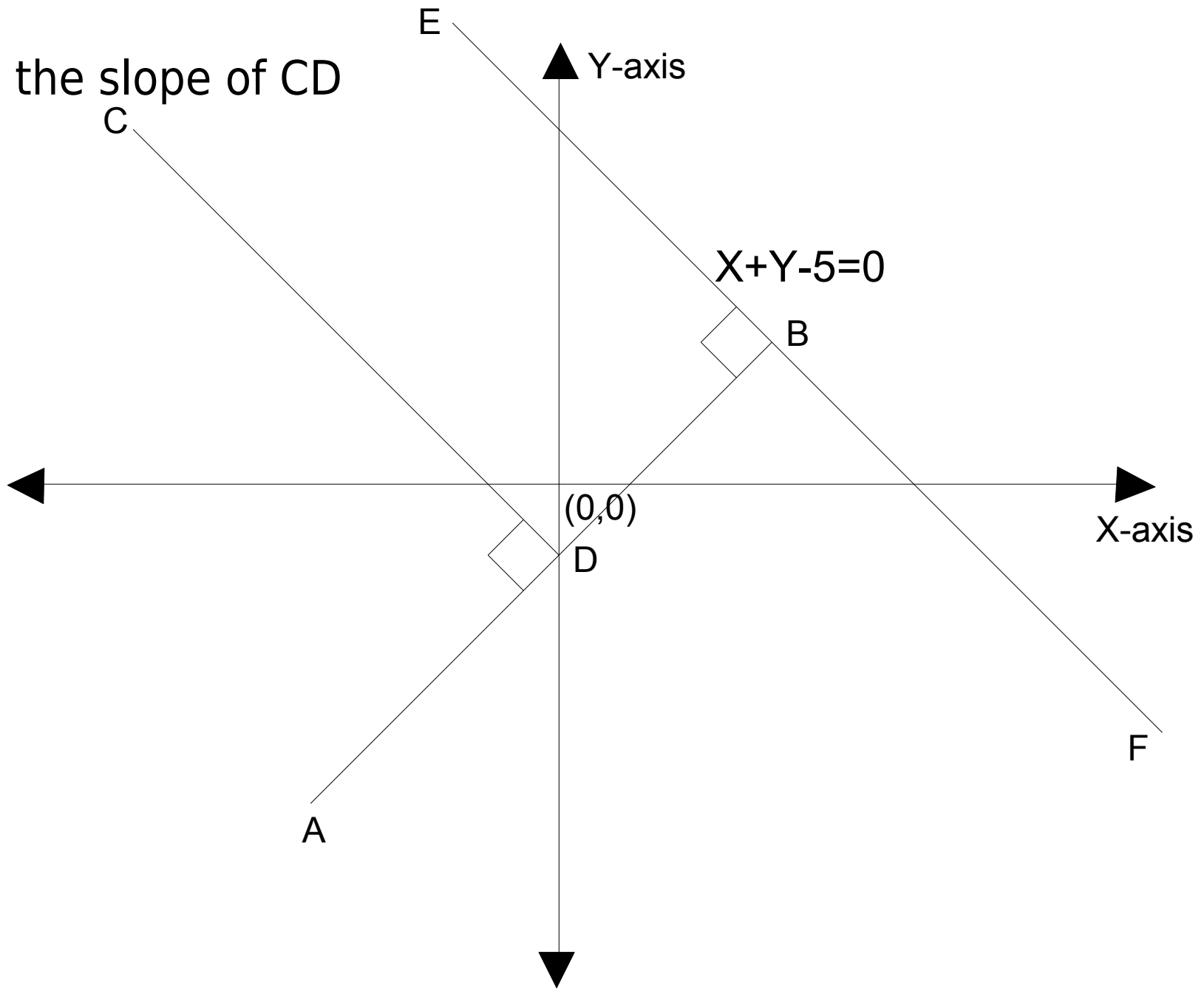
QUIZ

9) Find the slope of EF



QUIZ

10) Find the slope of CD



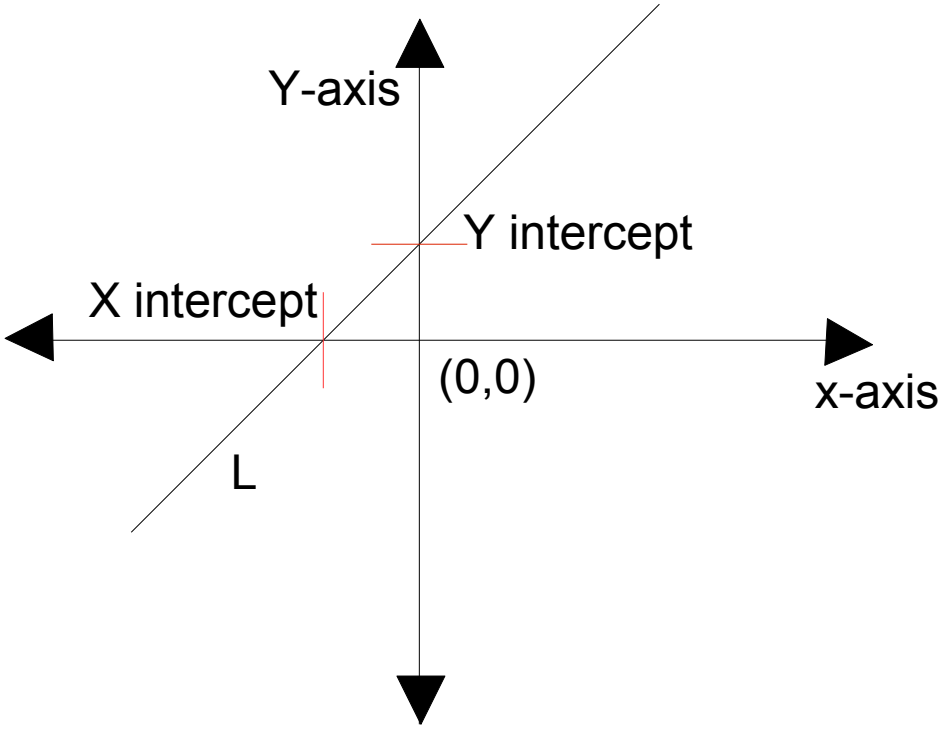
End of Part A

Part B: INTERCEPTS

Intercepts

Every line in the XY-plane touches X-axis and Y-axis at some point.

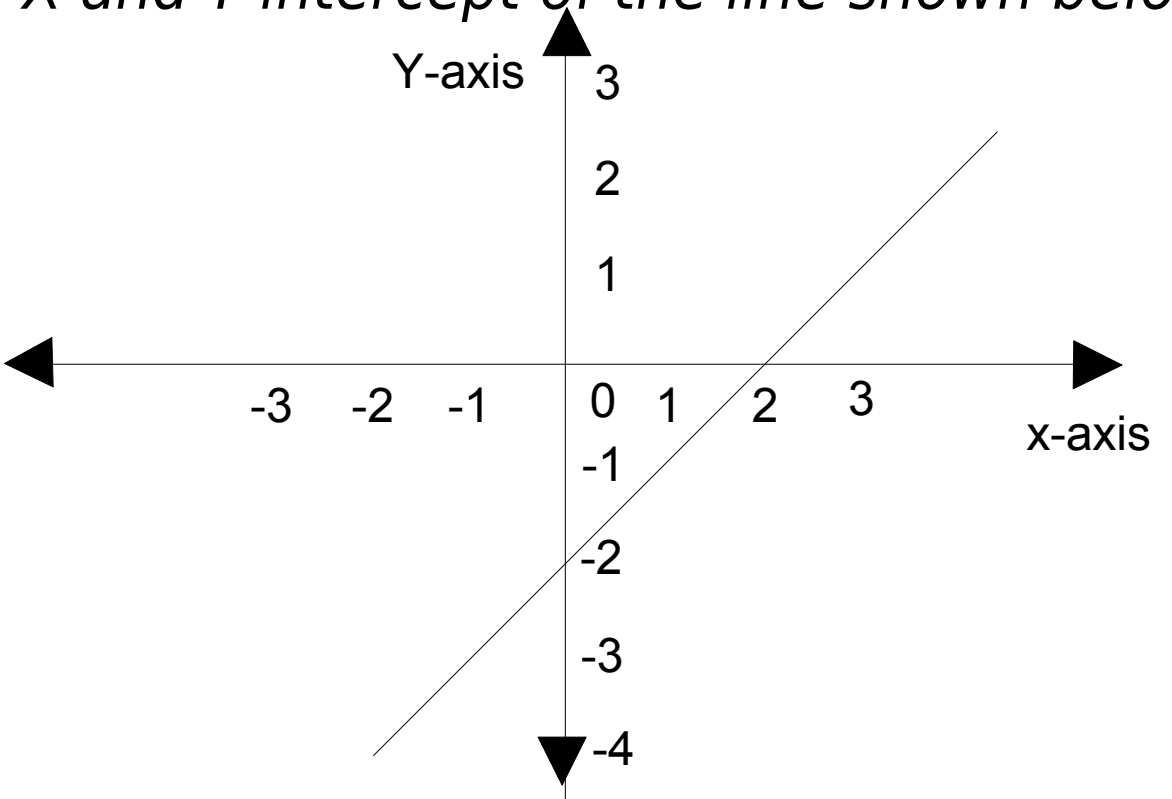
In the XY-plane, given a line L, the point at which line L intersects with X-axis is known as X-intercept of line L whereas the point at which line L intersects with Y-axis is known as Y-intercept of line L.



Intercept

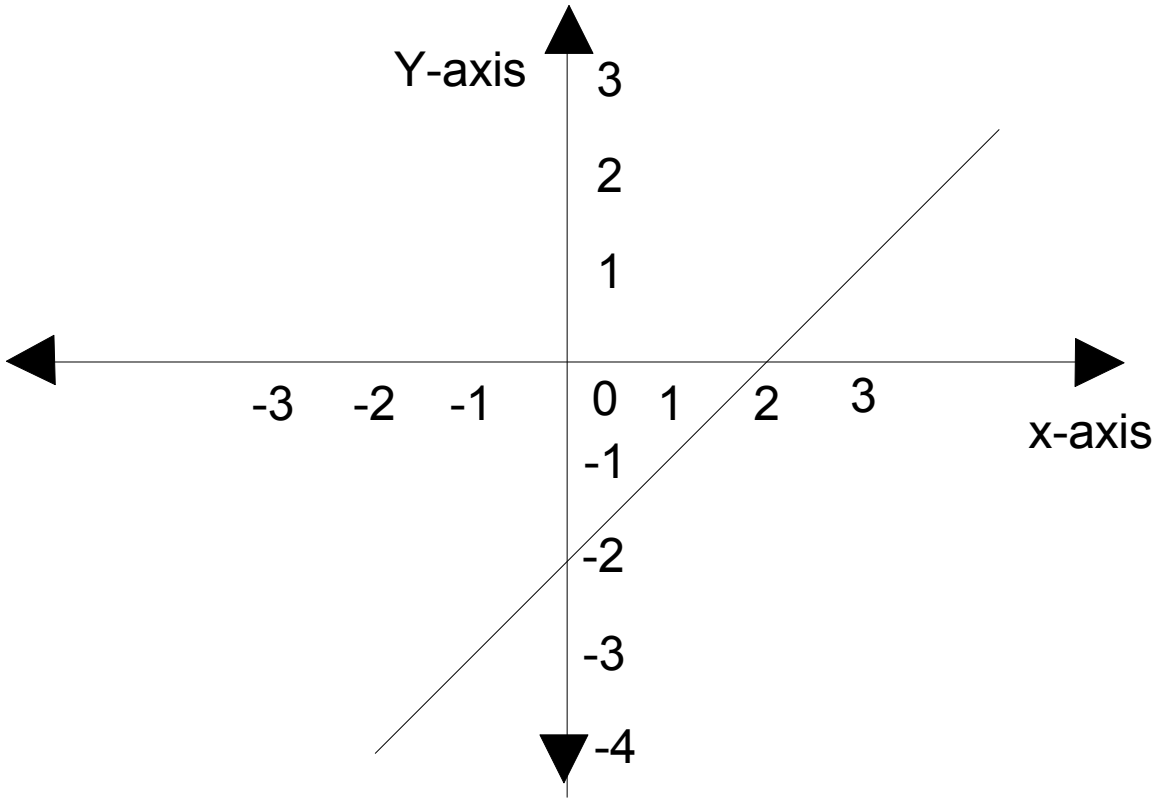
- X intercept of a line = x co-ordinate of the point where the line meets the X axis
- Y intercept of a line = y co-ordinate of the point where the line meets the Y axis

What is the X and Y intercept of the line shown below ?



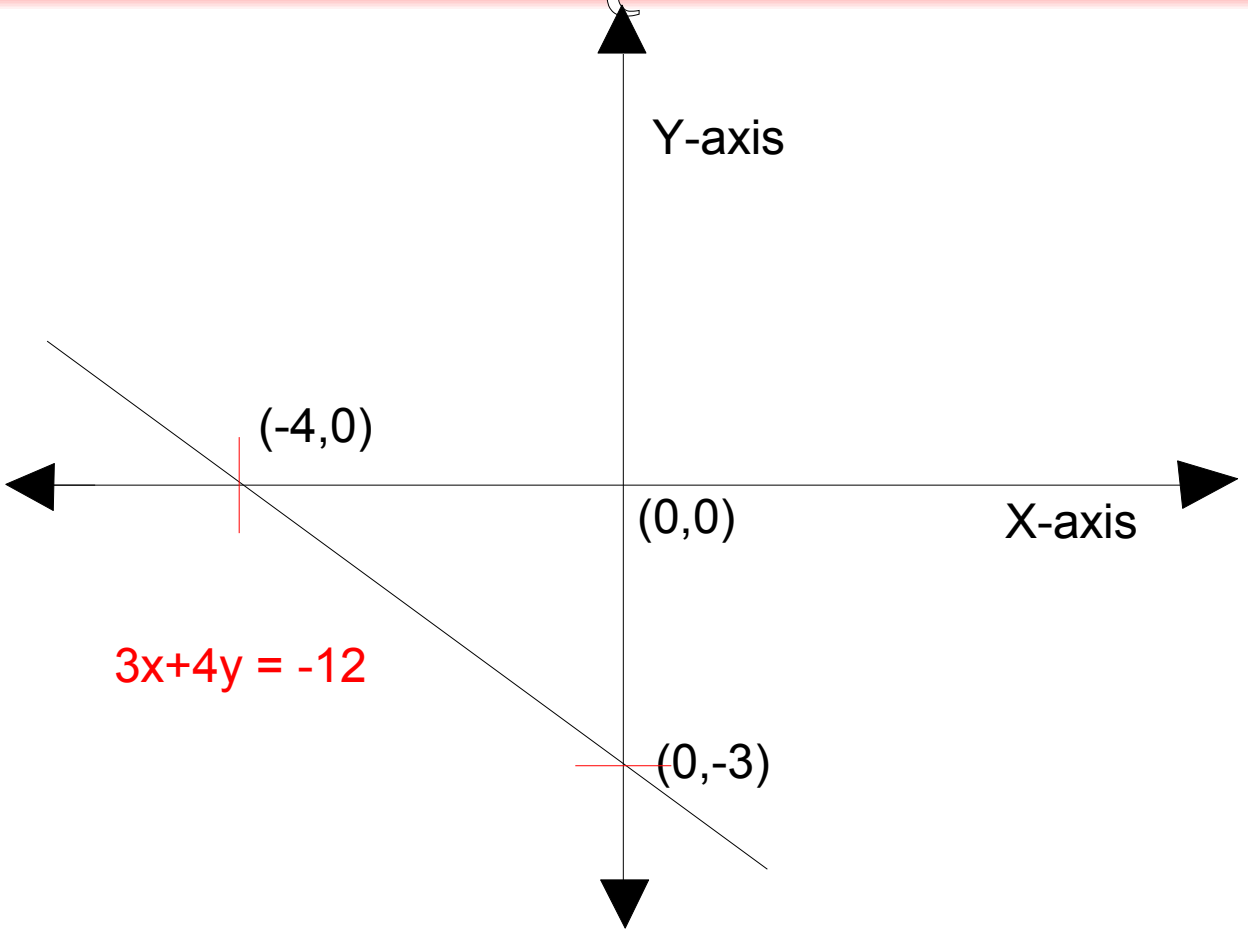
Intercept

- X intercept of the line = 2
- Y intercept of the line = -2



(Send your solutions to support@greedge.com)

Quiz



X intercept of the above line = _____
Y intercept of the above line = _____

X intercept of the Y axis = _____
Y intercept of the X axis = _____

End of Part B

Part C : EQUATION OF A LINE

Equation of a line

Equation of a line : Every line in XY-plane can be determined by an unique equation. This equation have two variables x and y. There are different methods in which equation of a line can be determined.

METHOD 1 :

If one point and slope is given, then the equation of the line is

$y = mx + c$, where m = the slope of the line and c = y intercept of the line

Note: Every point lying on this line satisfies the equation of this line.

In other words, if (a,b) are the co-ordinates of a point lying on this line then,

$$b = ma + c$$

EXAMPLE

The slope of a line is 4/5 and the line passes through the point (1,1) then find the equation of the line.

Solution: m = 4/5 and (x,y) = (1,1)

find the value of c use the equation:

y = mx + c ----------(eqn 1)

Now substitute the value of m = 4/5 and (1,1) in (eqn 1)

$1 = (4/5)\,1 + c$

$\Rightarrow c = 1/5$

Hence the equation of the line is

$y = (4/5)x + (1/5)$

$\Rightarrow \quad 4x - 5y = -1$

Equation of a line

METHOD 2 :

If two points of a line is given, say (x_1,y_1) and (x_2,y_2), then the equation will be,

$$(y-y_1) = m\,(x-x_1)\ ,$$

Where $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

substitute m in the equation, we get:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\,(x - x_1)$$

EXAMPLE

Consider the line passing through (-4,2) and (3,-3).

The equation of the line is given by

$(y - (-3)) / (x - 3) = (2 - (-3)) / (-4 - 3)$

$\Rightarrow (y + 3) / (x-3) = - (5 / 7)$(*Solve*)

$\Rightarrow 5x + 7y = -6$

Hence the equation of the line is given by $5x + 7y = -6$.

Equation of a line

METHOD 3:

Equation of a line can also be determined with the help of intercepts. If x intercept and y intercept are given, then the equation of the line is

$$\mathbf{x/a + y/b = 1}$$

Where, a is the x intercept and
b is the y intercept

(You have been exposed to X and Y intercept in part B of this tutorial)

Equation of a line

Note that : If x intercept of a line = a and y intercept = b, then slope of the line will be equal to -b/a

Or**, m = -b/a**

How to determine the Slope

If suppose equation of a line is given and we need to find the slope and the constant value.

First, change the given equation into the standard form, that is "y=mx+c".

We can pick up the values of m and c from the equation, which is nothing but the slope and the constant value respectively.

Example : Determine the slope and the constant of a line given by equation:

$$4y = -3x - 12$$

Let us write the equation in the form of y = mx + c

$$4y = -3x - 12$$
$$y = -(3/4)x - 3$$

Hence here the slope is (-3/4) and the constant is -3.

EXAMPLE

Find the equation of the line, if x intercept is 8 and y intercept is 4 ?

Since the X intercept and Y intercept are given, lets use this formula for the equation of the line: x/a + y/b = 1

x/8 + y/4 = 1

(x +2y) / 8 =1

x +2y = 8

Therefore equation of the line is x +2y -8 =0

Intercepts

Example:
Let us find the X-intercept and Y-intercept of the line whose equation is $x - 3y = 1$.

X-intercept of the line is the point at which the line and X-axis meet. At the X-axis the y coordinate is 0.

If y=0 then the equation will be $x - 0 = 1$
$\Rightarrow x = 1$

Similarly put x coordinate as 0 in the equation, we get $y = -1/3$

X-intercept of the line is (1,0) and Y-intercept of the line is (0,-1/3).

QUIZ

(Send your solutions to support@greedge.com)

1) Find the equation of the line. Where slope is 5 and one point is (2,-4) ?

2) What is the equation of the line parallel to x- axis?

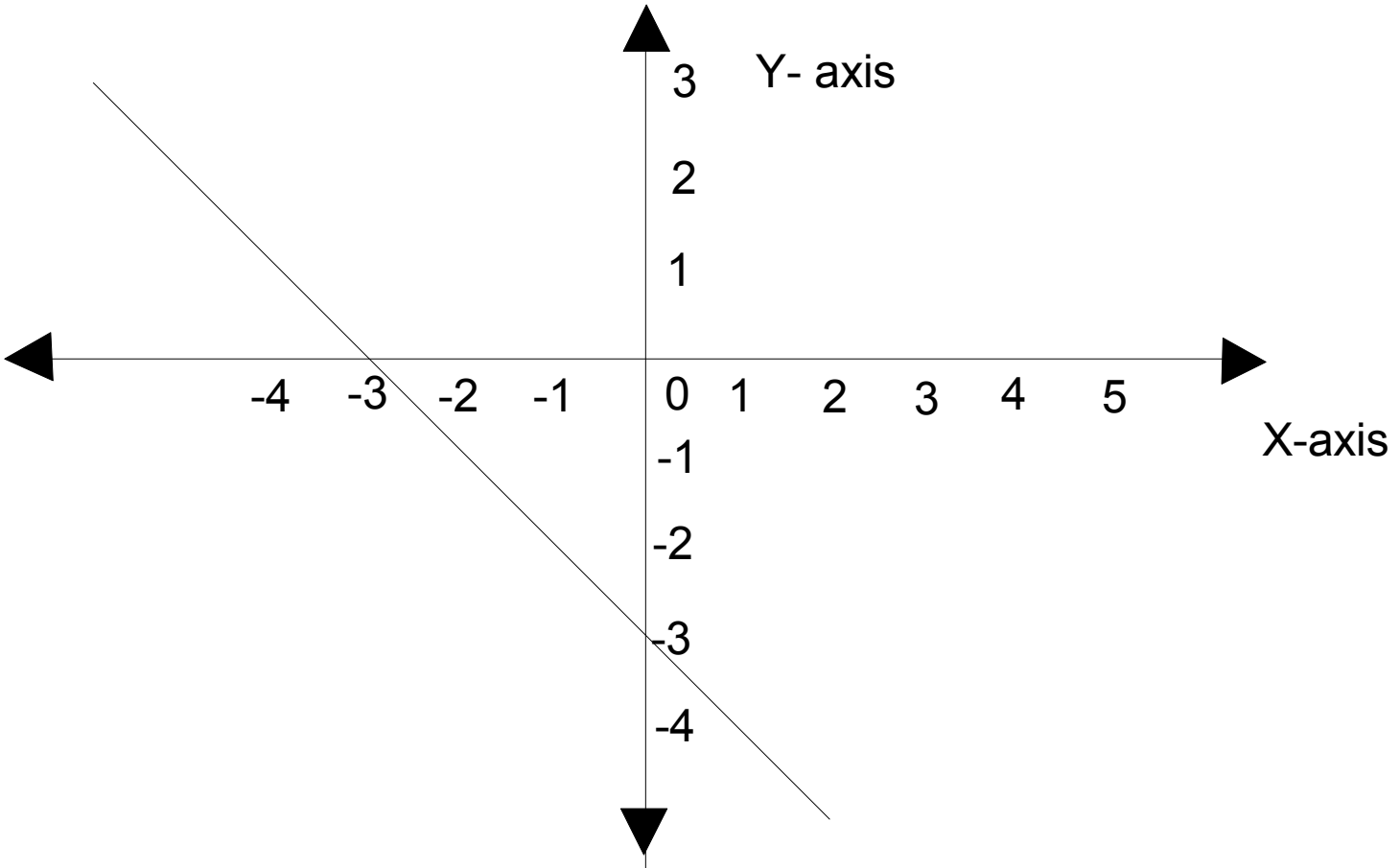
3) What is the equation of the line parallel to y-axis?

4) Find the equation of the line passing through the center from the point (6,-2) ?

QUIZ

5) If the x intercept is -1/5 and y intercept is 2/3, Find the equation of the line ?

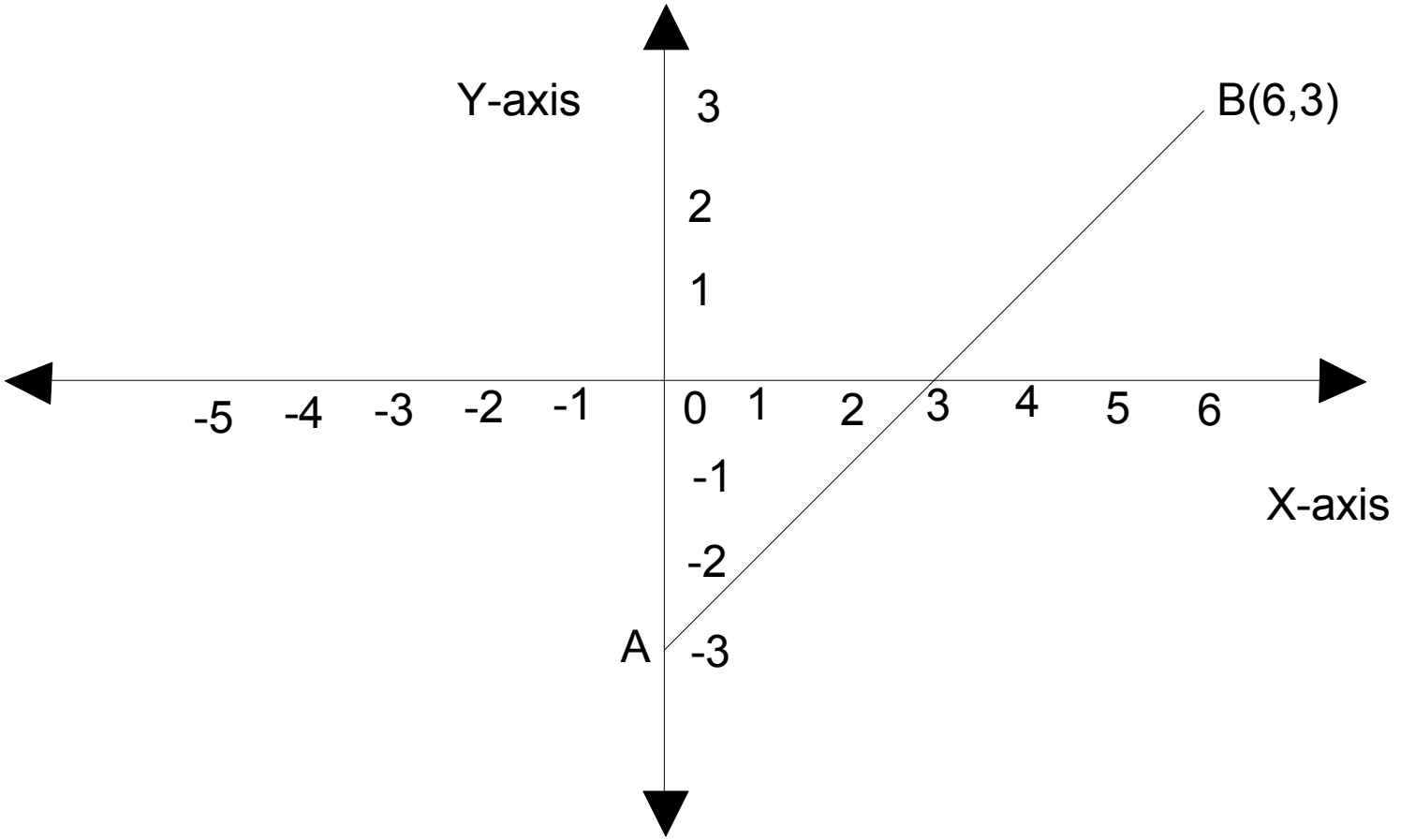
6) Find the equation of the line from the diagram below



QUIZ

7) Find the value of Y intercept, if slope is 2 and one point is (5,2) ?

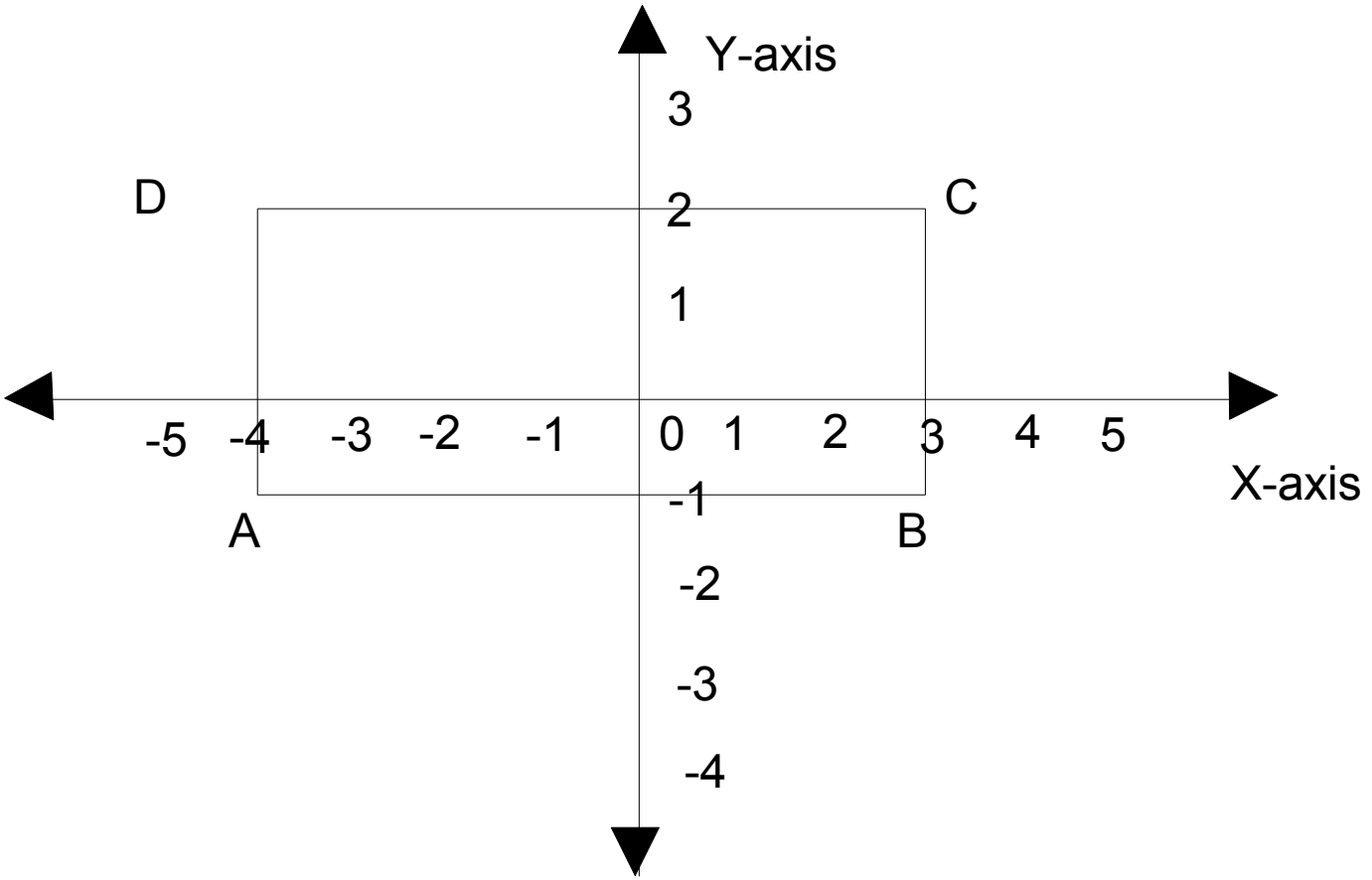
8) Find the equation of the line



QUIZ

9) Find the slope of the line if X-intercept is 5 and Y-intercept is -3 ?

10)



QUIZ

Answer the following with the help of the above diagram

(a)What is the distance between AB , BC , CD , DA ?

(b)Find the slopes of AB , BC , CD and DA ?

(c)Find the equation of the lines AB , BC, CD, DA?

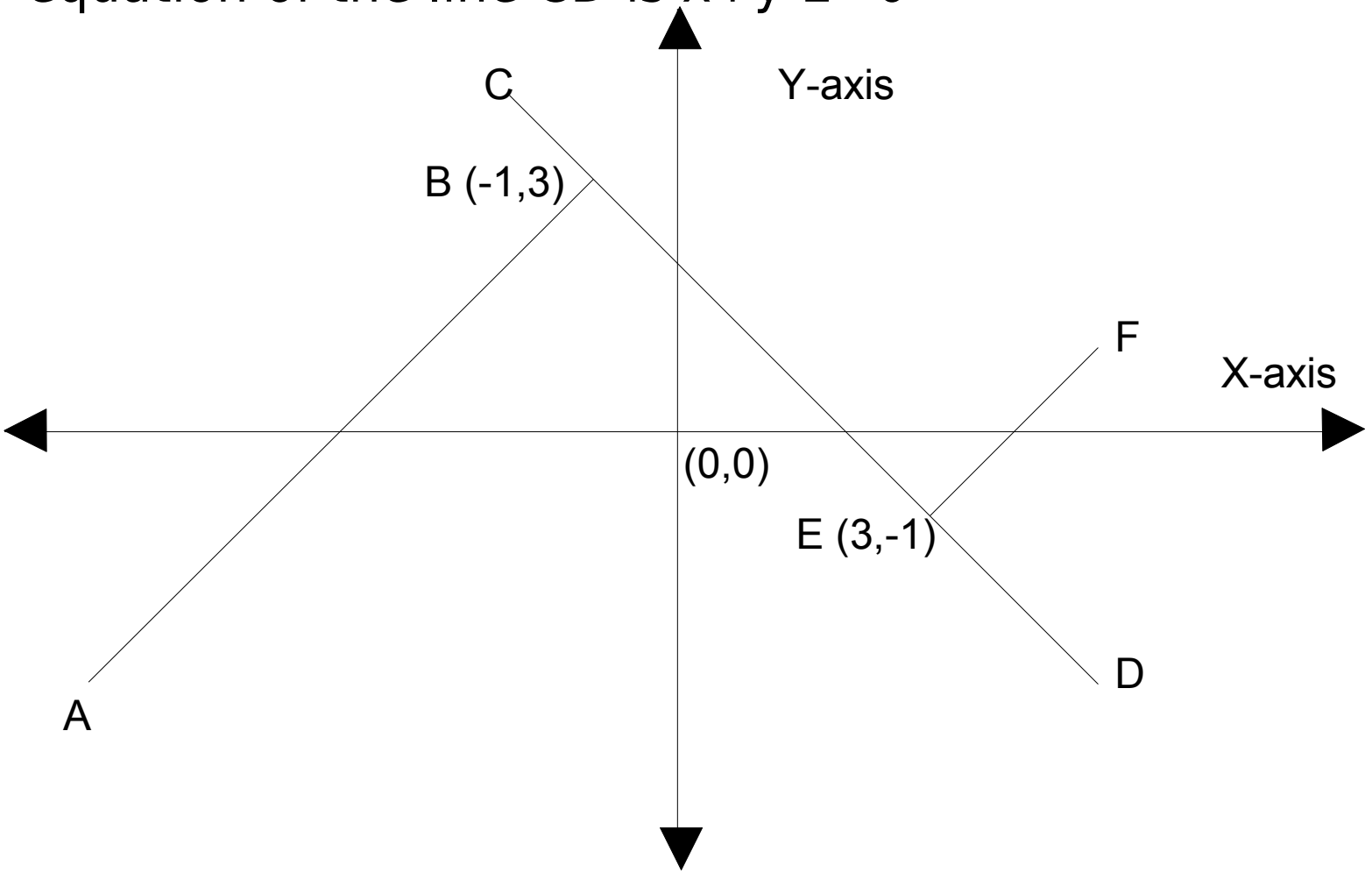
(d)Find the area of the figure?

(e)Find the coordinates of A, B, C and D .

(f)Find the perimeter of the figure ?

QUIZ

11) Find the equation of the line EF and AB, if the equation of the line CD is x+y-2=0



QUIZ

12) If the equation of the line is Y= 3x - 7, what is the Y- intercept ?

13) If the equation of the line is Y = 5x+2, what is the X- intercept ?