Dealing with Coins

- There are only 2 outcomes when you throw a coin – Head or Tail. Set of all outcomes {H, T}
- When you throw 2 coins, you have 4 outcomes

  { **HT**, **HH**
  TT, **TH** }

  You can view this in multiple ways:
  - The first coin has 2 outcomes and the second coin has 2.

    So, total number of outcomes = 2 x 2 = $2^2 = 4$
- If you throw 3 coins, you have 8 outcomes: $8 = 2 \times 2 \times 2 = 2^3$

  You can visualize this as:

  { **H** { **HT**, **HH** , **TT**, **TH** }, **T** { **HT**, **HH** , **TT**, **TH** } }

  same as : { **HHT**, **HHH** , **HTT**, **HTH** , **THT**, **THH** , **TTT**, **TTH** }
- If you throw 4 coins, you have 16 outcomes: $16 = 2 \times 2 \times 2 \times 2 = 2^4$

Examples

1. One coin is thrown. Head is desired.
   Total outcomes = 2
   Favorable outcome = 1

2. Two coins are thrown. Head is desired on both
   Total outcomes = 4
   Favorable outcome = 1 { **HH** }

3. Two coins are thrown. At least one Head is desired
   Total outcomes = 4
   Favorable outcome = At least 1 head
   = (1 Head and 1 Tail) OR (2 heads)
   = { **HT**, **TH** } OR { **HH** } = 2 + 1 = 3

Note : OR implies +

4. Two coins are thrown one after the other. Head is desired on the first one and Tail on the second
   Total outcomes = 4
   Favorable outcomes : 1 { **HT** }

5. 3 coins are thrown. Head is desired on all 3.
   Total outcomes = 8
   Favorable outcomes = 1 {**HHH**}

6. 3 coins are thrown. At least 1 head is desired.
   Total outcomes = 8
   Favorable outcomes = At least 1 head
   = Total - (not a single Head)
   = Total - (all Tails)
   = 8 – 1 = 7

Points to Ponder

- What is the probability that the next 2 customers that walk into a store are both females ?
  - Can you consider this as a problem of tossing 2 coins ? Each person can be treated as coin that can assume 2 values {Male, Female} instead of {Heads, Tails}
  - Total outcomes : 4 - {MM, MF, FM, FF }
    Favorable : 1 {FF}
- A painter needs to paint 3 rooms of a house. He has only 2 colors: **Blue** and **Pink**. What are the total number of ways in which he can paint the house, given that he has to paint a room in a single color.
  - Can you treat this as a problem of tossing 3 coins ? Each room can be treated as coin that can assume 2 values {**Blue**, **Pink**} instead of {Heads, Tails}
  - Total ways : 8 = 2 x 2 x 2
  - Number of ways that the rooms can be painted such that at least one room is **Pink** ?

Is this question mathematically same as Qn 6 above involving 3 coin ?

Dealing with Dice

A Die can be treated as a Coin with 6 faces

- There are only 6 outcomes when you throw a Die – Set of all outcomes {1, 2, 3, 4, 5, 6}
- When you throw 2 dice, you have 36 outcomes

  { 1 1, 1 2, 1 3, 1 4, 1 5, 1 6,
  2 1, 2 2, 2 3, 2 4, 2 5, 2 6,
  3 1, ..., 3 6,
  4 1, ..., 4 6,
  5 1, ..., 5 6,
  6 1, ..., 6 6,
  }

  You can view this in multiple ways:
  - The first die has 6 outcomes and the second coin has 6.

    So, total number of outcomes = 6 x 6 = $6^2 = 36$
- If you throw 3 dies, you have 216 outcomes: $216 = 6 \times 6 \times 6 = 6^3$

  You can visualize this as:

  { 1 { 11, ... 66 }, 2 { 11, ... 66 }, 3 { 11, ... 66 }, 4 { 11, ... 66 }, 5 { 11, ... 66 }, 6 { 11, ... 66 } }

where { 11, ... 66 } is same as the set:

{ 1 1, 1 2, 1 3, 1 4, 1 5, 1 6,
2 1, 2 2, 2 3, 2 4, 2 5, 2 6,
3 1, ..., 3 6,
4 1, ..., 4 6,
5 1, ..., 5 6,
6 1, ..., 6 6,
}

Examples

(A die can be treated like a 6 sided coin. Check each Die question below with its counterpart in the coin section)

1. A die is thrown. Even number is desired

Total outcomes = 6
Favorable 3 {2, 4, 6 }

2. Two dies are thrown. 6 is desired on both
   Total outcomes = 36
   Favorable outcome = 1 { **66** }
3. Two dies are thrown. At least one 6 is desired
   Total outcomes = 36
   Favorable outcome = At least one 6
   = (1 six and any other non 6 number) OR (2 six)
   = { **61, 62, 63, 64, 65, 16, 26, 36, 46, 56**} OR { **66** } = 10 + 1 = 11

4. Two dies are thrown one after the other. 1 is desired on the first one and 2 on the second.
Total outcomes = 36
   Favorable outcomes : 1 { **12** }

5. 3 dies are thrown. 4 is desired on all 3.
   Total outcomes = 216
   Favorable outcomes = 1 {**444**}

6. 3 coins are thrown. At least one 5 is desired.
   Total outcomes = 216
   Favorable outcomes = At least one 5
   = Total - (not a single 5)
   Not a single 5, implies: Die 1 can assume any of {1,2,3,4,6} **AND** Die 2 can assume any of { 1,2,3,4,6} **AND** Die 3 can assume any of { 1,2,3,4,6}
   = 5 x 5 x 5 = 125
   Total - (not a single 5) = 216 – 125 = 91

Points to Ponder

- An ATM pin consists of 4 digits. Each digit can assume any of the 10 values {0, 1, 2, 3 ...8, 9}. What are the number of Pins where not a single digit is 0 ?
  - Can you consider this as a problem of tossing 4 dice where each die has 10 faces bearing numbers from 0 to 9 ? Each ATM digit can be treated as die that can assume 10 values {0, 1, 2, ... 9} instead of {1, 2, 3, 4, 5, 6 } like a die.
  - Total outcomes : 10 x 10 x 10 x 10 = 10^4

  Favorable : 9 x 9 x 9 x 9 = 9^4

Quiz

(send your solutions to support@greedge.com)

1. Two coins are thrown. Head is desired on exactly one
   Total outcomes = 4
   Favorable outcome = _____

2. Two coins are thrown. At least one Tail is desired
   Total outcomes = 4
   Favorable outcome = _____

3. Two coins are thrown one after the other. Head is desired on the first as well as on the second
   Total outcomes = 4
   Favorable outcomes : _____

4. 3 coins are thrown. Head is desired on exactly two.
   Total outcomes = 8
   Favorable outcomes = _____

5. 3 coins are thrown. At least 2 heads are desired.
   Total outcomes = 8
   Favorable outcomes = _____

6. A die is thrown. Prime number is desired
   Total outcomes = 6
   Favorable outcomes = _____

7. Two dies are thrown. It is desired that the sum of the numbers on the two faces should be 9
   Total outcomes = 36
   Favorable outcomes = _____

8. Two dies are thrown. It is desired that the sum of the numbers on the two faces should be at least 11
   Total outcomes = 36
   Favorable outcomes = _____

9. Two dies are thrown one after the other. Even number is desired on the first one and odd number on the second.
   Total outcomes = 36
   Favorable outcomes : _____

10. 3 dies are thrown. Even number is desired on all 3.

Total outcomes = 216
Favorable outcomes = _____

11. Three dies are thrown. It is desired that at the most two two faces should bear a 5
Total outcomes = 216
Favorable outcomes = _____