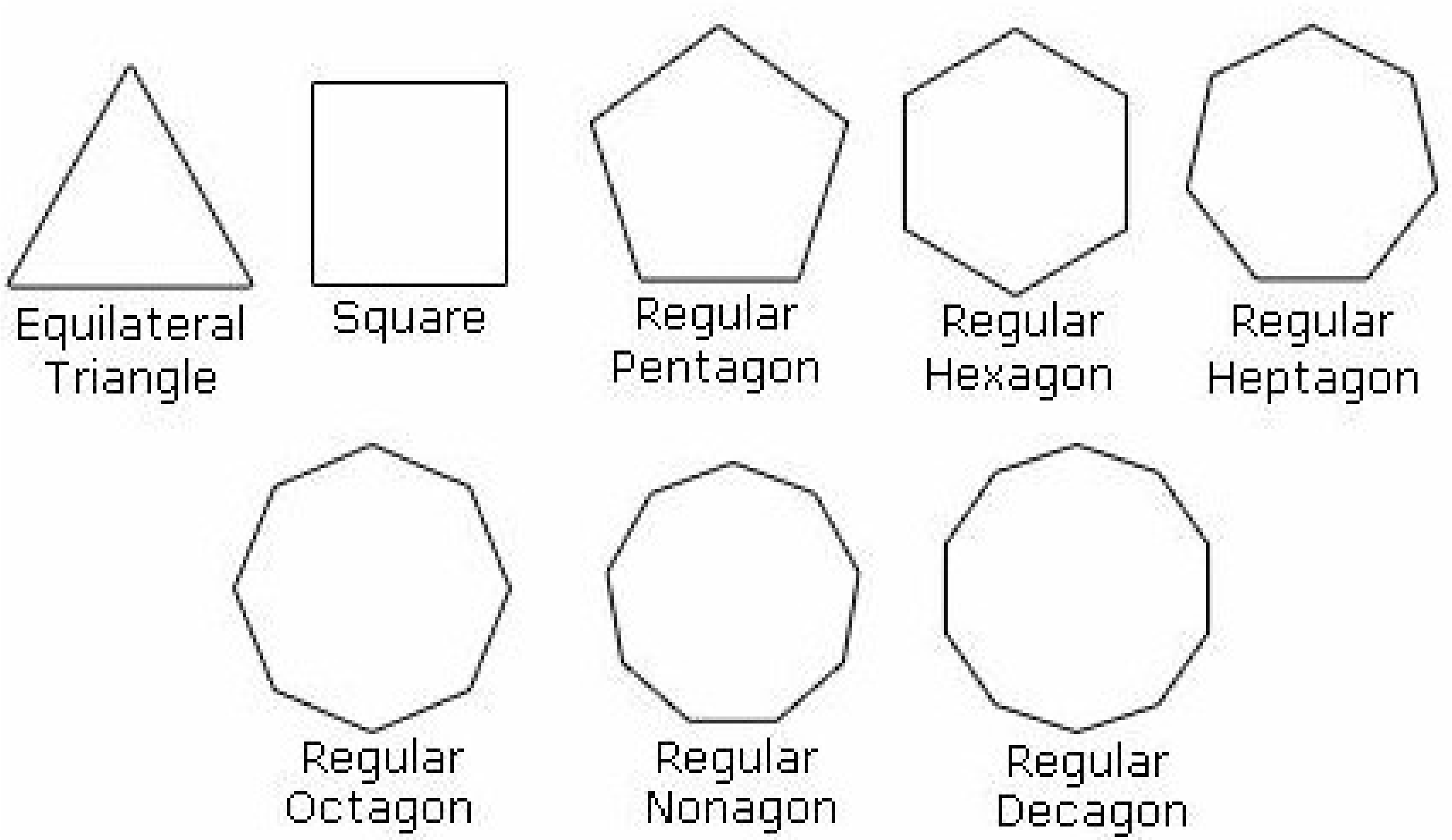
# POLYGONS


Equilateral Triangle
Square
Regular Pentagon
Regular Hexagon
Regular Heptagon
Regular Octagon
Regular Nonagon
Regular Decagon

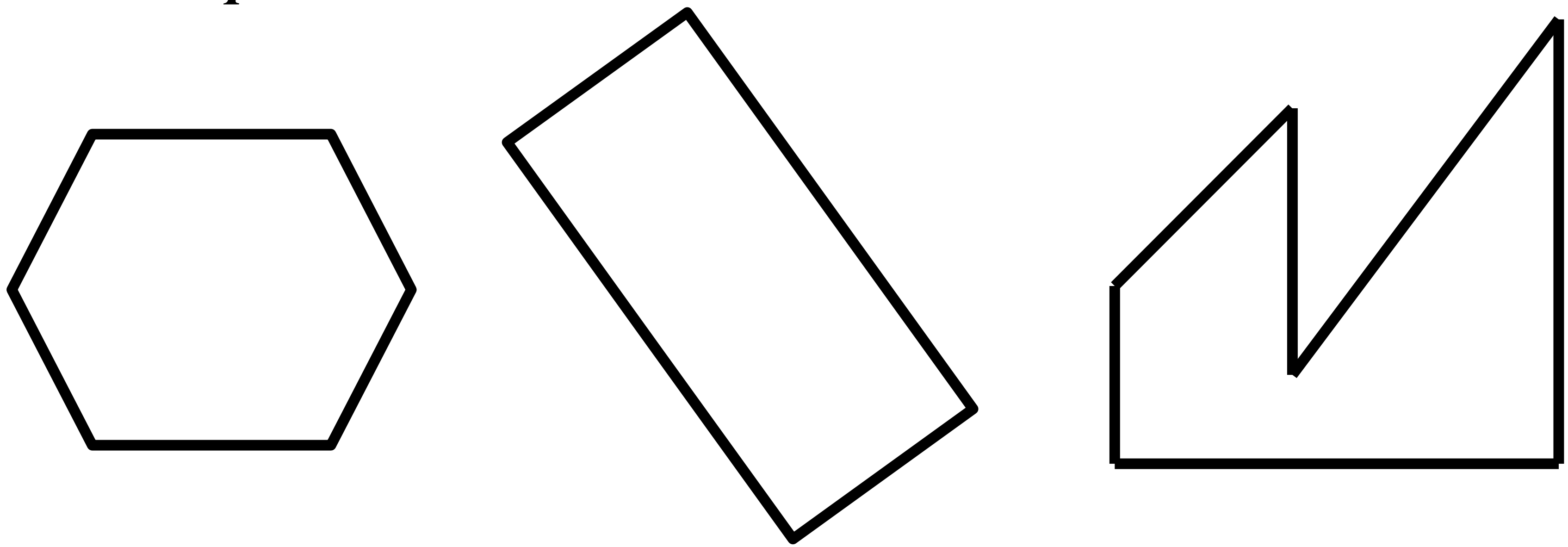
# Definition of a polygon

A **polygon** is a closed figure.

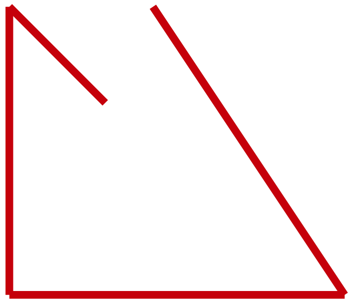
It is made up of joining the line segments.

Each line segment should intersect exactly at 2 points.

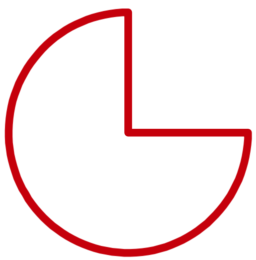
***Examples***



# Following are not polygons because



**It is not a closed figure.**



**It is not made up of line segments**

**A regular polygon will have all the sides equal and all the angles equal**

# Polygon names

**Polygons are differentiated with the help of its sides or edges .**

Important types of polygon are as follows:

N sided polygon → “n”-gon

3 sided polygon → Triangle (or) Trigon

4 sided polygon → Quadrilateral (or) Tetragon

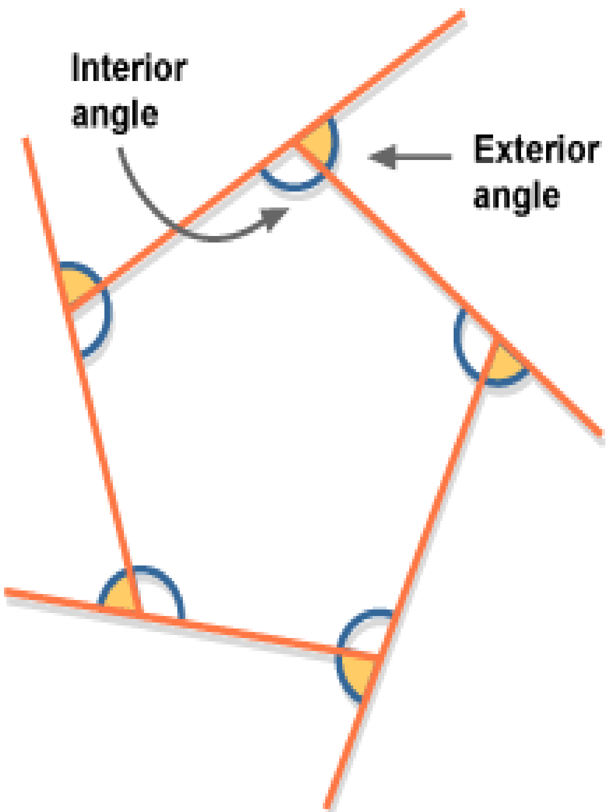
5 sided polygon → Pentagon

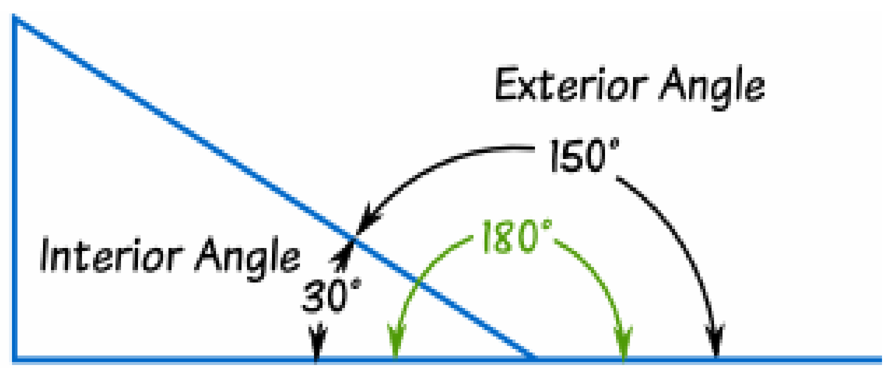
6 sided polygon → Hexagon

7 sided polygon → Heptagon

8 sided polygon → Octagon

# Formulas for interior and exterior angles of a polygon



## Sum of the Interior or internal angle of an n-gon

***(n-2) * 180***

Example:

sum of the interior angles of a triangle (3 sided polygon)

$= (3 - 2)*180 = 180$

sum of the interior angles of a hexagon( 6 sided polygon)

$= (6 - 2)*180 = 720$

## Measure of each interior or internal angle of a regular polygon

$$(n - 2) * 180^{o} \; / \; n$$

**<u>Example</u> :**

Find the measure of each interior angle of a Quadrilateral (4 sided polygon)?

Given : $n = 4$

Solution : $( 4 - 2 ) * 180^{o} / 4 = 2 * 180^{o} / 4$

$= 90^{o}$

**Sum of the exterior angles(or external angle) of n-gon is 360º**

**Therefore,**

Sum of the exterior angles of a pentagon = 360º

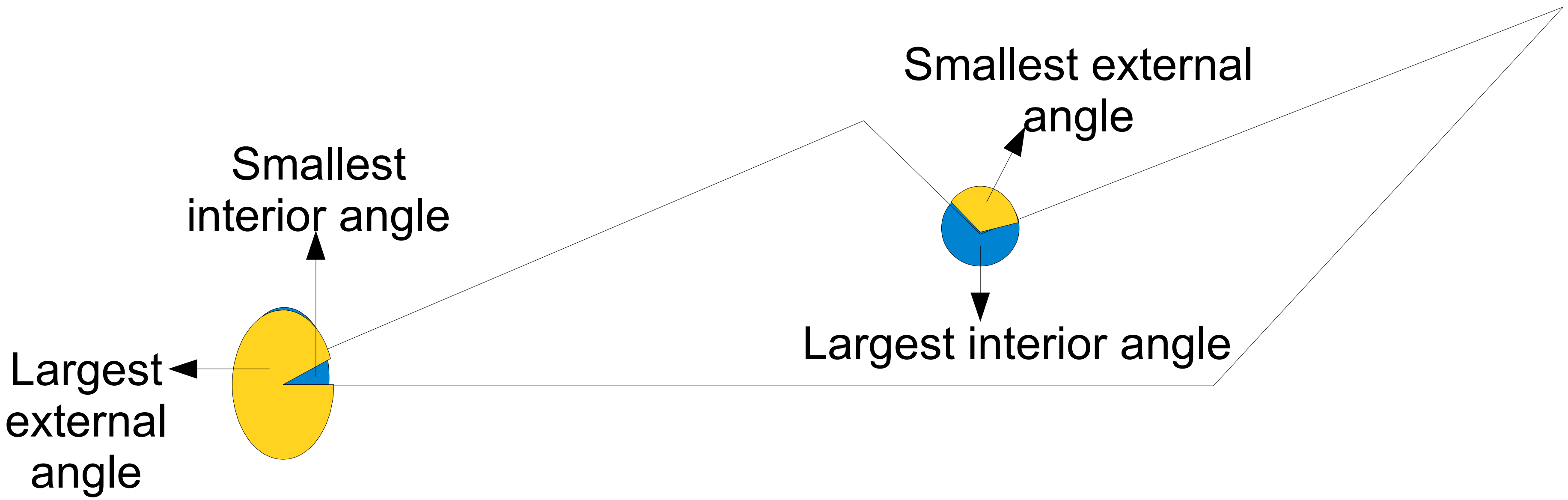
Sum of the exterior angles of a 15 sided polygon = 360º

**The measure of each exterior angle of a regular polygon is**

***360º/ n***

**Note : Interior angle + exterior angle = 180º**

**Smallest interior angle + Largest external angle = 360º**




**Largest interior angle + Smallest external angle = 360º**

# Example

**1. Each exterior angle of a regular polygon is  60 degrees. what is the sum of the interior angles of the polygon?**

The sum of the exterior angles of any polygon is 360 degrees.
Given, it is a regular polygon (one with equal length sides and angles).
~> 360/60 = 6

So,the regular polygon is eight-sided, i.e., an Hexagon.

WKT the interior and exterior angles are supplementary angles.
i.e their sum is 180 degrees.
Each interior angle is therefore 180 - 60 = 120 degrees.
Interior angle of a regular polygon = sum of interior angles ÷ number of sides.
The sum of the interior angles of a polygon with n sides is:
120* 6 = 720

**2. Find the sum of the measures of the interior angles of an Nonagon.**

It is an Nonagon (9 sides) which means n = 9. Therefore,
sum of the measures of the interior angles = 180 (9-2) = 180 (7) = 1260°

**3. If the sum of the measures of the interior angles of a polygon is 1800°, how many sides does the polygon have?**

Given the sum of the angle measures.
180(n-2) = 1800
180n – 360 = 1800
180n = 2160
n = 12, the shape has 12 sides (dodecagon).

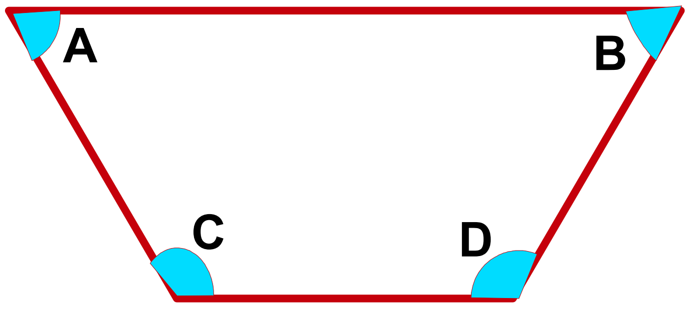
# Properties of polygons

# Properties of quadrilateral

**Quadrilateral means " four sided " .**

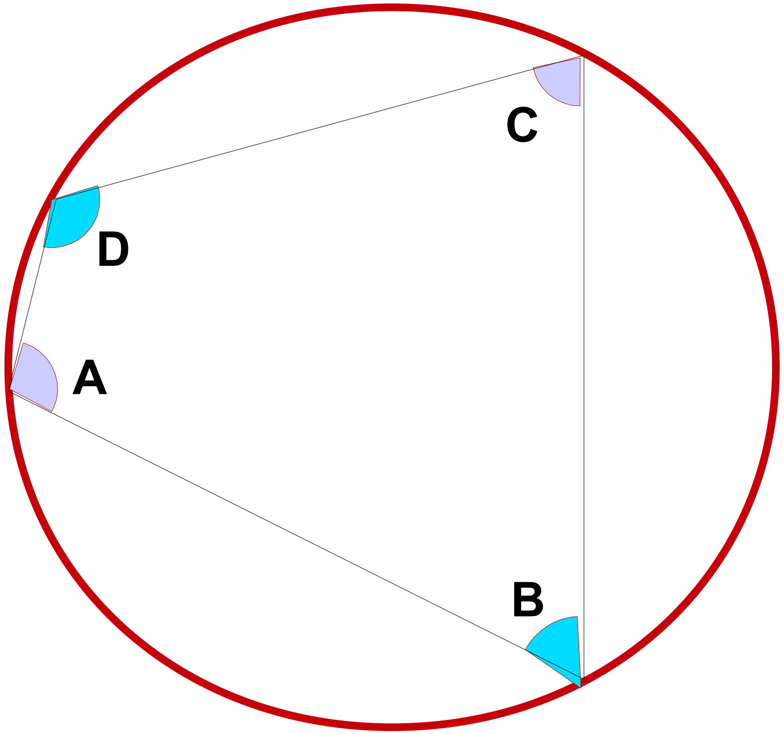
**Any four sided shape is a quadrilateral.**

**Sum of the angles of any quadrilateral is equal to 360º.**




**∟A + ∟B + ∟C + ∟D = 360º**

# Sum of the opposite angle of a cyclic quadrilateral is 180º




$\angle A + \angle C = 180^{o}$

$\angle B + \angle D = 180^{o}$

# Types of quadrilaterals

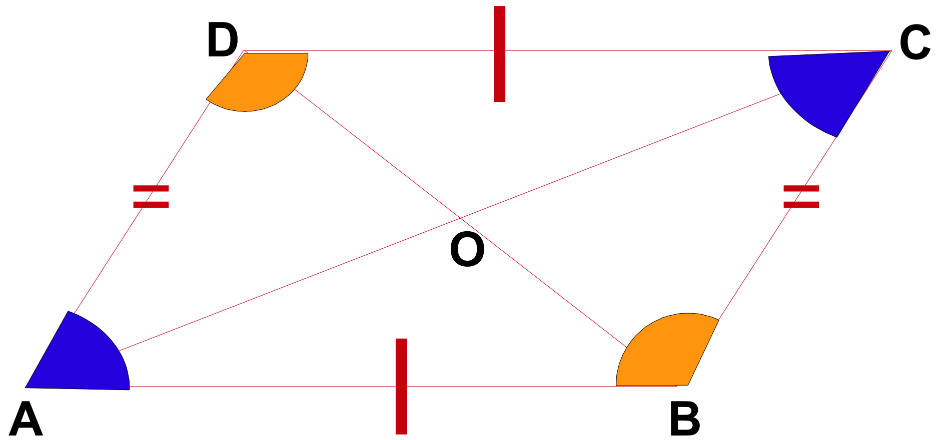
→ Parallelogram

→ Rectangle

→ Square

→ Trapezoid

# Properties of parallelogram



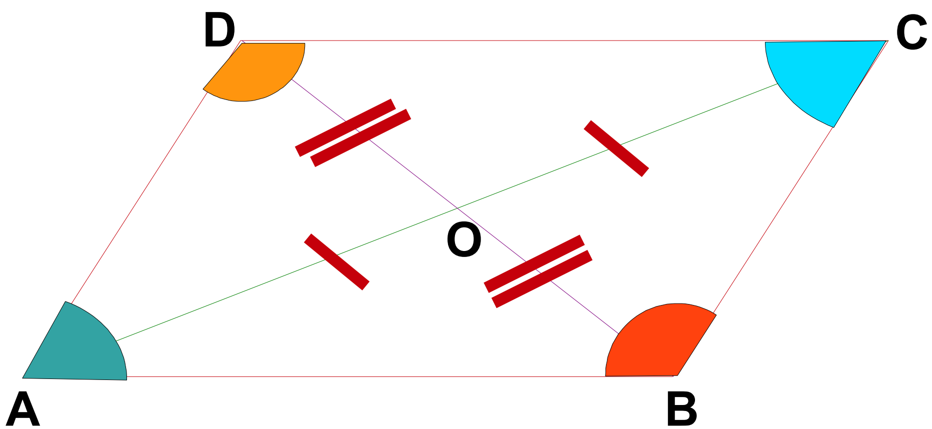

→ **opposite sides are parallel**

**AB ll CD**
**AD ll BC**

→ **opposite angles are equal**

**∟A = ∟C**
**∟B = ∟D**

# Properties of parallelogram

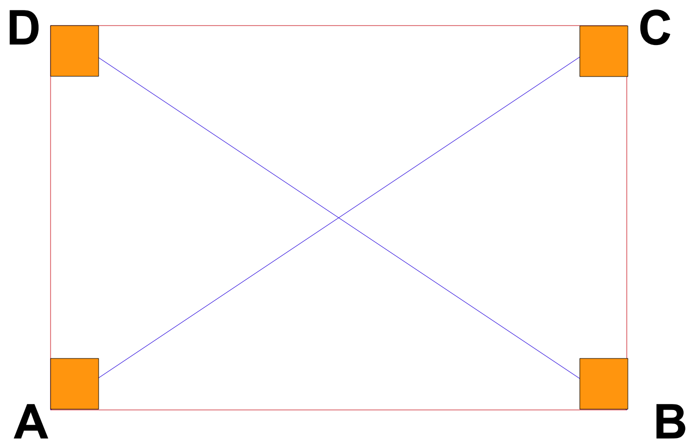



→ **Consecutive angles add up to 180º**

**∟A + ∟B = 180º ; ∟B + ∟C = 180º**
**∟C + ∟D = 180º ; ∟D + ∟A = 180º**

→ **Diagonals meet at o, therefore, AO = OC and BO = OD**

# Properties of rectangle




→ **All properties of parallelogram**

→ **All angles are equal to 90º**

**∟A = ∟B = ∟C = ∟D = 90º**

→ **Diagonals are of equal length**

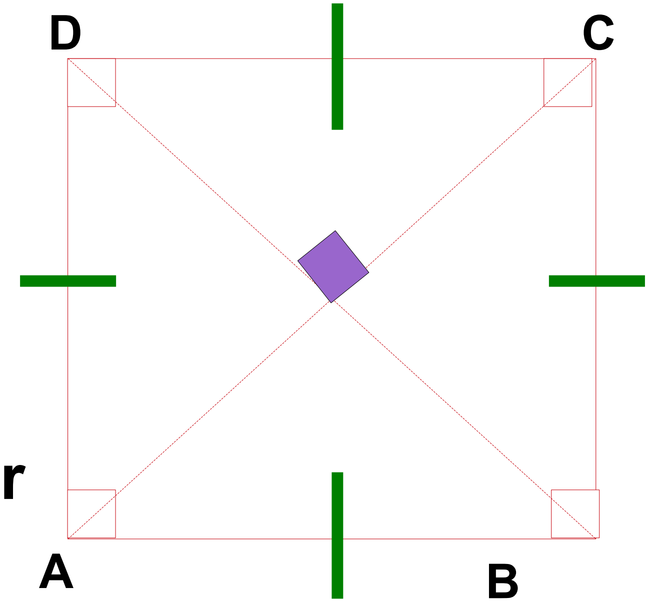
**AC = BD**

# Properties of square

- All properties of rectangle.
- All four sides are equal

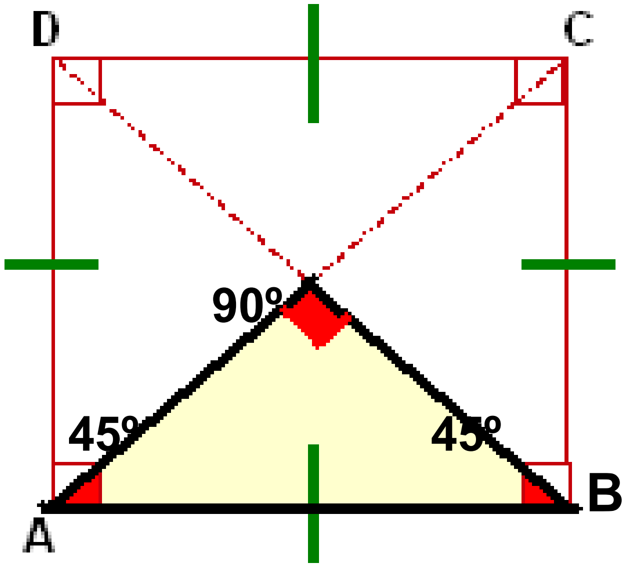
  **AB = BC = CD = DA**

- Diagonals are perpendicular to each other
- Diagonals divides the square into four equal parts.

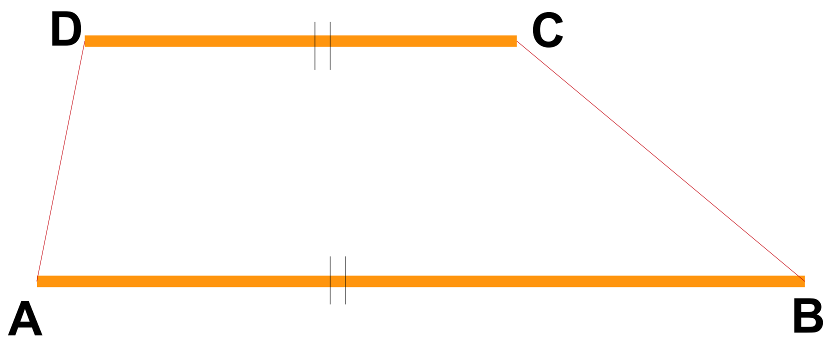


# Properties of square

- Each part forms a right angled triangle.
- Therefore, square has four right angle triangle.
- Angles formed in each right angle triangle are $45^{\circ}$, $90^{\circ}$, $45^{\circ}$.



# Properties of trapezoid

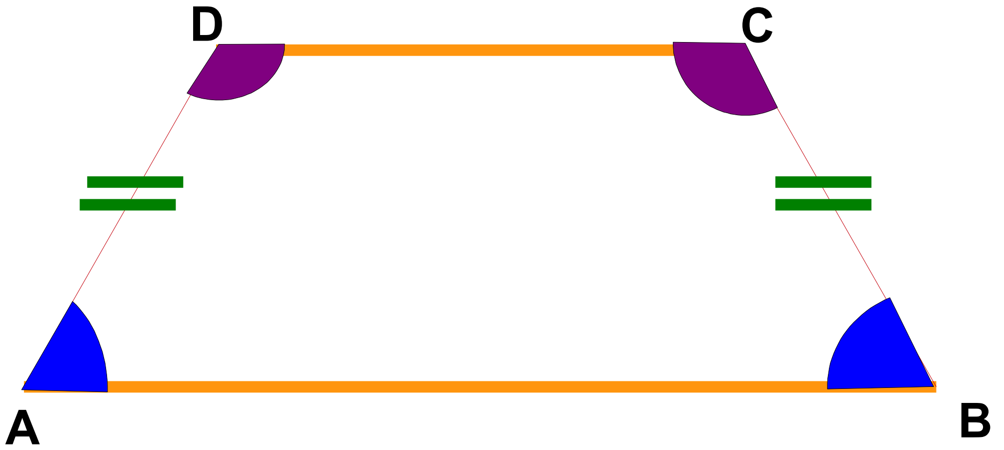



→ **Trapezoid will have one pair of opposite sides parallel.**

**AB II CD**

→ **Parallel sides are known as bases of the trapezoid.**

# Properties of trapezoid




Isosceles trapezoid

→ In an isosceles trapezoid, the two non-parallel sides will be of equal length,

AD = BC

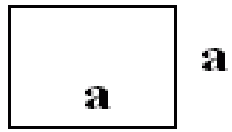
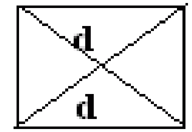
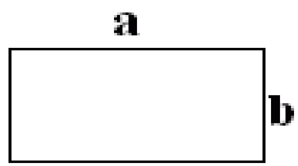
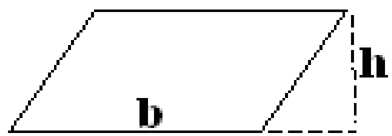
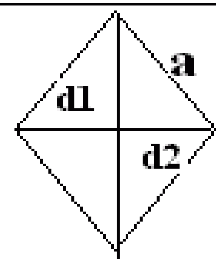
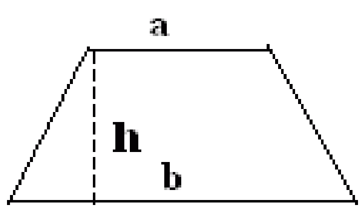
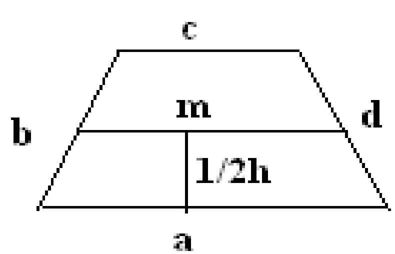
→ The base angles in isosceles trapezoid are equal

∟A = ∟B

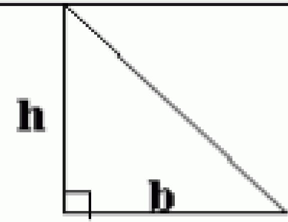
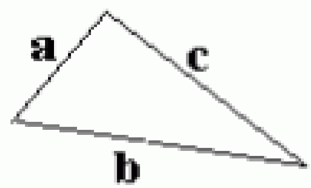
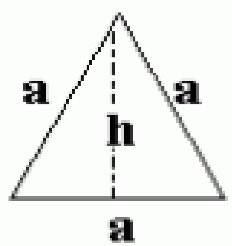
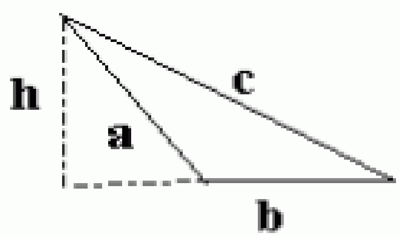
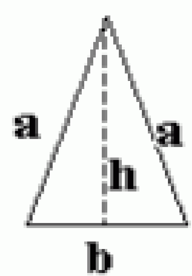
∟C = ∟D

# Formula

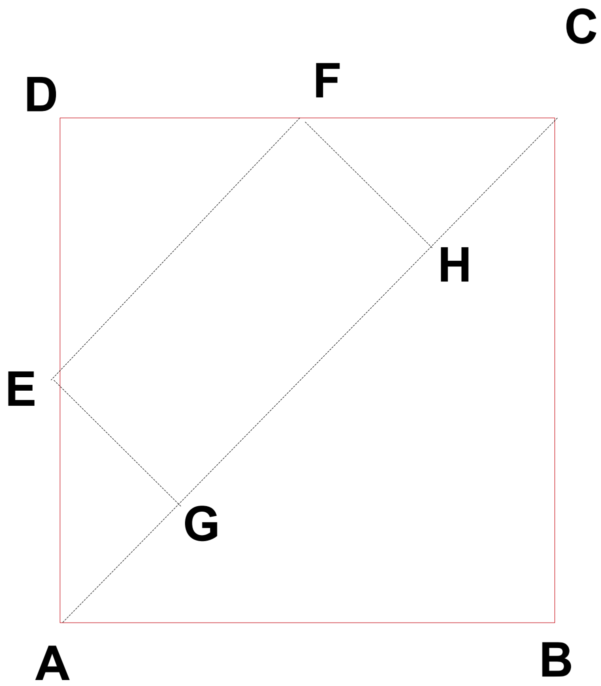
# Formula

| | | | |
|---|---|---|---|
| **Square** |  | a = side<br>Area=a x a =$a^2$<br>Perimeter = 4a | <br>d = diagonal<br>Area= (1/2) x $d^2$ |
| **Rectangle** |  | Perimeter=2a+2b<br>=2(a+b) | a=length, b=width<br>Area =a x b |
| **Parallelogram** |  | h = height<br>b = base | Area = b x h<br>Perimeter = a+b+a+b<br>=2a+2b |
| **Rhombus** |  | d1= diagonal<br>d2=diagonal | Area=1/2 x d1 x d2<br>Perimeter = a+a+a+a = 4a |
| **Trapezoid or Trapezium** |  | h = height or distance between the parallel sides<br>a and b = lengths of parallel sides | Area=1/2 x h (a x b)<br>Perimeter = a+b+c+d |
| |  | | Perimeter= a+b+c+d<br>= 2m+c+d |

# Formula

| | | | |
|---|---|---|---|
| **Right Angle Triangle** |  | $h$ = height<br>$b$ = base | Area $= 1/2\ (b \times h)$<br>= 1/2 (Product of the sides containing the right angle) |
| **Scalene Triangle: with length a, b, c** |  | $a$ = side<br>$b$ = side<br>$c$ = side | Area $= \sqrt{s(s-a)(s-b)(s-c)}$<br>Where $s = \dfrac{(a+b+c)}{2}$<br>Perimeter $= a + b + c$ |
| **Equilateral Triangle** |  | $a$ = three equal sides | Area $= \sqrt{3}/4 \times a^2$<br>Perimeter $= a + a + a$ |
| **Obtuse Angle Triangle** |  | $h$ = height<br>$b$ = base | Area $= 1/2\ (b \times h)$<br>Perimeter $= a + b + c$ |
| **Isosceles Triangle** |  | $a$ = two equal sides<br>$h$ = height<br>$b$ = base | Area $= 1/2\ (b \times h)$<br>$= 1/2 \times a \times b \sin c$<br>Perimeter $= a + a + c$ |

# Guess




(1) Find the value of EF and AC.
If E is the midpoint of AD and
F is the midpoints of DC and
one side of the square is 4cm.

(2) Find the area of the rectangle
EFHG ?

# Hints

**(1) Given : side of the square is 4cm.**

**E and F are mid points of the sides AD and CD**

**To Find : length EF and length AC**

**Solution : AC is the diagonal of the square**

**AC = √2*s, find the value of AC**

**EF is a line from the mid points, therefore, DE = DF = 2**

**DEF forms a right angle triangle, right angled at D.**

**Therefore find the value of hypotenuse EF, using the formula** $(EF)^2 = (DE)^2 + (DF)^2$

(2) Given : AE = 2, EF = (take the answer from previous solution)

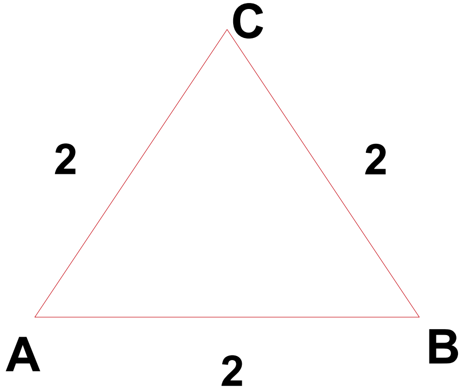
To Find : Area of rectangle

Solution : AGE forms the right angle triangle, right angled at G With the help of Pythagoras theorem ( square of the hypotenuse = sum of the square of the other two sides) solve and find the value of EG.

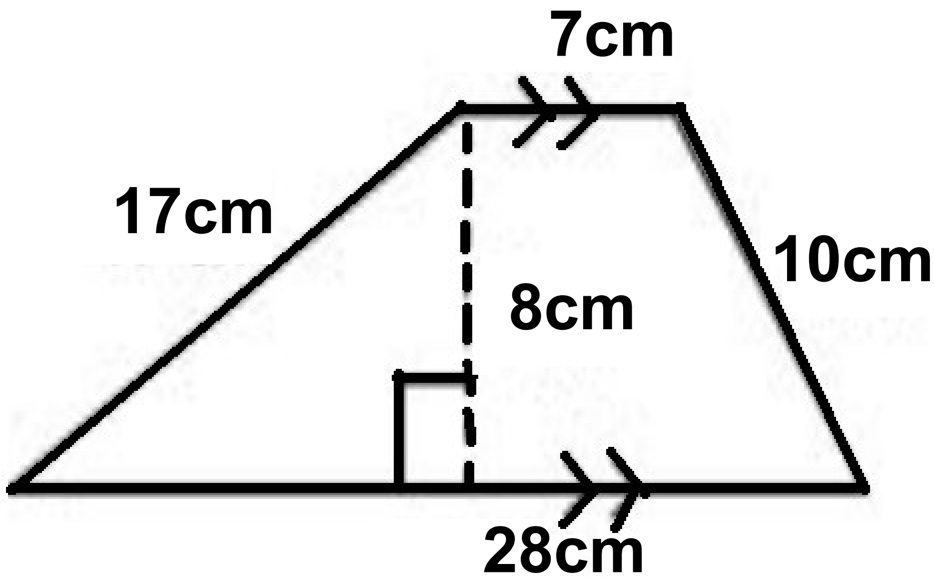
Therefore, area of rectangle = EF * EG

# Quiz

1. Find the area of the square if one of its side is 8cm ?
2. Find the area of the square if the diagonal is 14cm ?
3. Find the length of the diagonal of a rectangle if the length is 7cm and the width is 2 less than the length ?
4. If the perimeter of the square and perimeter of the rectangle are equal. Find the length of the rectangle , if the length is 4 more than the width and one side of the square is 9 ?
5. Find the area of the triangle given?

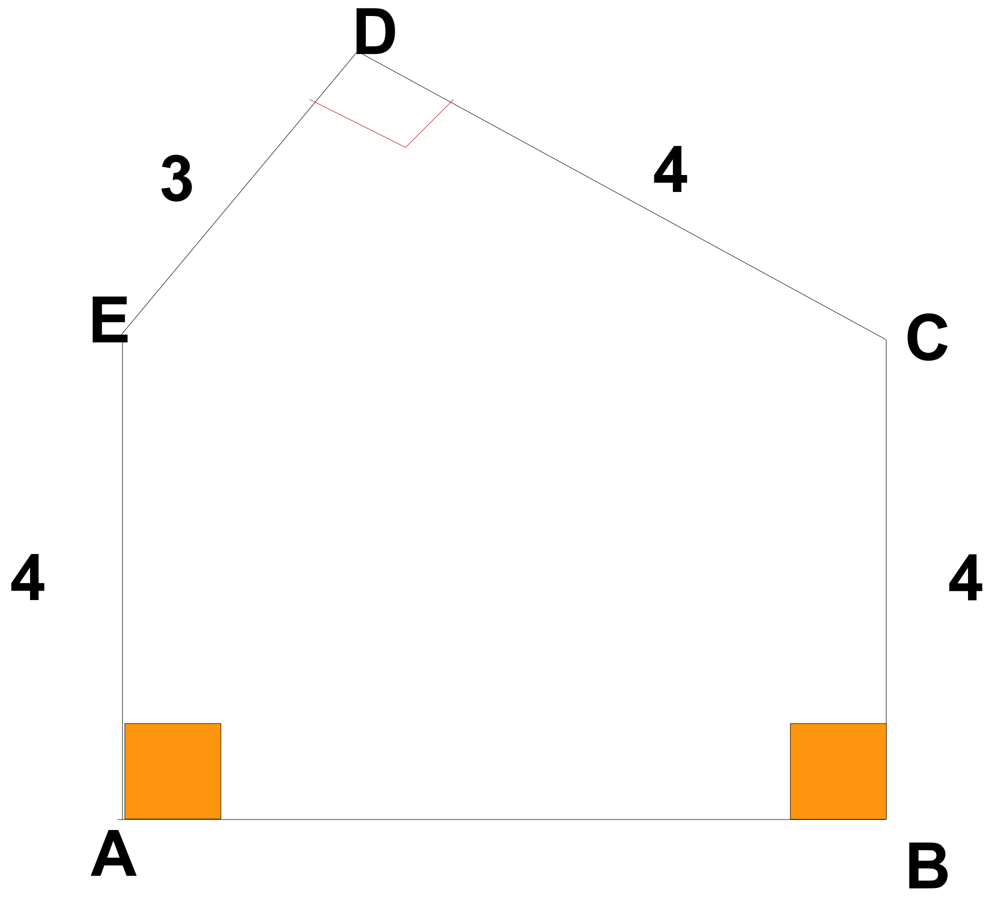


6)




**Find the perimeter and the area of the trapezoid ?**

# 7) Find the perimeter of the pentagon ?



# Solution

1} $s^2 = 8 * 8$
$= 64$ **sq.cm**

2} $14 = \sqrt{2}s$
$s = 7\sqrt{2}$ **cm**
**In the question it is asked to find the area.**
**Therefore, Area of the square =** $(7\sqrt{2})(7\sqrt{2}) = 98$ **sq cm**

3} **Length of the diagonal** $= \sqrt{(49 + 25)}$
$= \sqrt{74}$ **cm**

4} $4s = 2(l + b)$

$4 * 9 = 2 * [(x + 4) + x]$
**width** $= x$
$x = 7$ **cm**
**Length** $= 7 + 4 = 11$ **cm**

# Solution

5} $\sqrt{3}/4$ side sq = $\sqrt{3}/4 * 4$

$= \sqrt{3}$ sq.cm

6} $1/2 * (b1 + b2) * h = 1/2\ (28 + 7) * 8$

$= 140$ sq.cm

perimeter = 62 cm

7} $EC = \sqrt{(9 + 16)}$

$= 5$

ABCE is a rectangle <a = <b = 90

Therefore,

AB = EC

$= 5$

Perimeter = AB + BC + CD + DE + EA

$= 5 + 4 + 4 + 3 + 4$

$= 20$ cm