

36700 – Probability and Mathematical Statistics

Spring 2019

Homework 1

Due Friday, January 25th at 12:40 PM

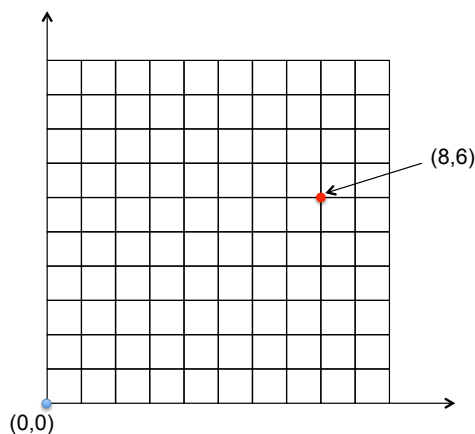
All homework assignments shall be uploaded using Gradescope through the Canvas portal). Late submissions are not allowed.

1. A box contains three coins with a head on each side, two coins with a tail on each side, and four fair coins.
 - (a) One of these nine coins is selected at random and tossed once, what is the probability of getting a tail?
 - (b) If we get a tail, what is the probability that the selected coin has a tail on both side? If we get a tail, what is the probability that it is a fair coin?
 - (c) If the first toss is tail, and another coin is selected at random from the remaining eight coins and tossed once, what is the probability of getting a tail again?
2. Three cards are drawn randomly without replacement from a deck of 52 poker cards (A deck of poker cards has four suits, each having 13 cards with numbers from 1, 2, 3, ..., 11, 12, 13).
 - (a) What is the probability that all three cards are in the same suit?
 - (b) What is the probability that all three cards have the same number?
 - (c) What is the probability that the three cards contain exactly one pair (a pair means the same number from two different suits)?
3. A civil engineer is studying a left-turn lane that is long enough to hold six cars. Let Y be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $Y = y$ is proportional to $(y + 1)(7 - y)$.
 - (a) Find the probability mass function of Y .
 - (b) Find the expectation and standard deviation of Y .
 - (c) Find the probability that Y will be at least 4.
4. Let the cumulative distribution function $F(y)$ of a continuous random variable Y be given by

$$F(y) = \begin{cases} 0, & y \leq 0 \\ \frac{2}{5}y, & 0 < y \leq 1 \\ ay - b, & 1 < y \leq 2 \\ 1, & y > 2 \end{cases}$$

where a and b are two numbers.

- (a) Find out the values of a and b .
 - (b) Write down the probability density function.
 - (c) Compute $\text{Var}[Y]$, the variance of Y . (You may leave your answer as a fraction!)
 - (d) What is the probability that an observed random variable Y is greater than 1.5, given that it is greater than 1?
5. There are 128 players in a tennis tournament. The tournament proceeds as follows. In the first round, the players are paired one against another at random. The loser in each pair is eliminated from the tournament. In the second round, all remaining players are paired at random and again, the loser in each pair is eliminated. The tournament continues in this way until only two players remain in the final round. The winner of the final round wins the tournament. Assume that all players have the same ability and all games are independent of each other. Find the probability that two specific players, A and B, will ever play against each other in the tournament.
6. A robot is placed at the origin (point $(0,0)$) on a 2-dimension integer grid (see the figure below). Denote the position of the robot by (x,y) . It can either move right to $(x+1,y)$, or move up to $(x,y+1)$.



- (a) Suppose each time the robot randomly moves right or up with equal chance. What is the probability that the robot will ever reach the point $(8,6)$?
- (b) Suppose another robot has a $2/3$ chance to move right and a $1/3$ chance to move up when $x+y$ is even, otherwise it has a $1/4$ chance to move right and a $3/4$ chance to move up. It stops whenever $|x-y| \geq 2$. Find the probability that $x-y=2$ when it stops.