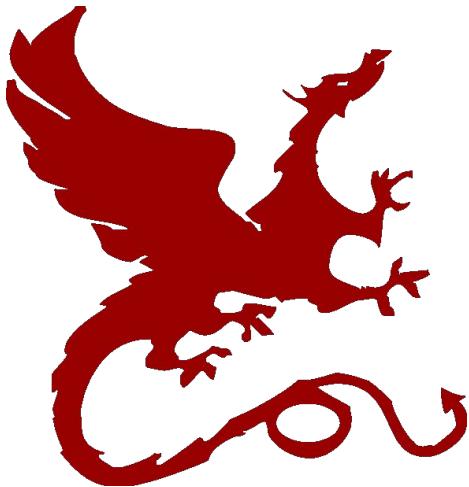


# Algorithms for NLP



## Parsing I

Yulia Tsvetkov – CMU

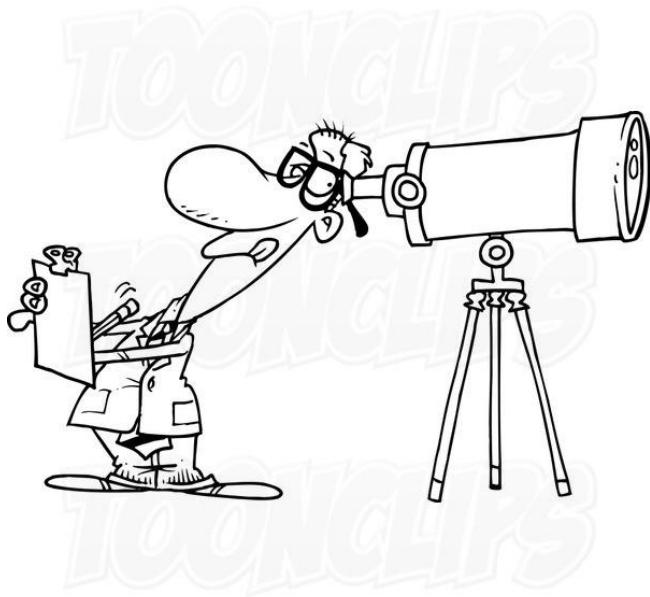
Slides: Ivan Titov – University of Edinburgh,  
Taylor Berg-Kirkpatrick – CMU/UCSD, Dan Klein – UC Berkeley



# Ambiguity

---

- I saw a girl with a telescope



Copyright © Ron Leishman \* <http://ToonClips.com/3005>



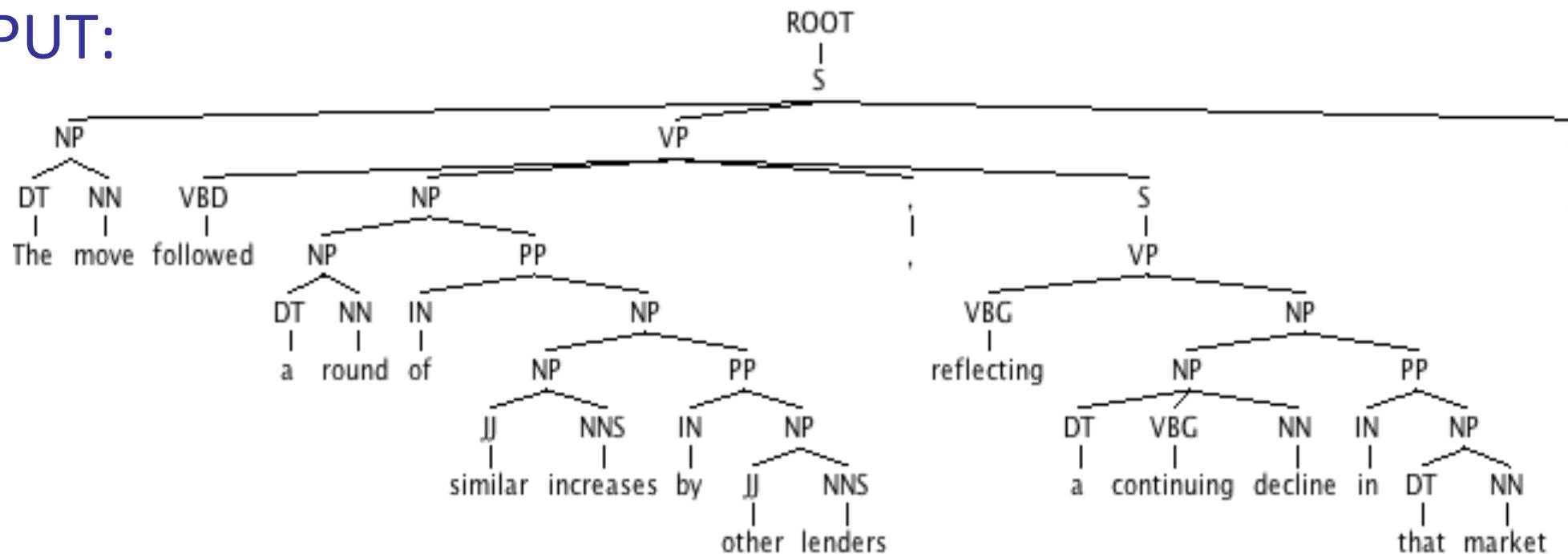


# Parsing

- INPUT:

- The move followed a round of similar increases by other lenders, reflecting a continuing decline in that market

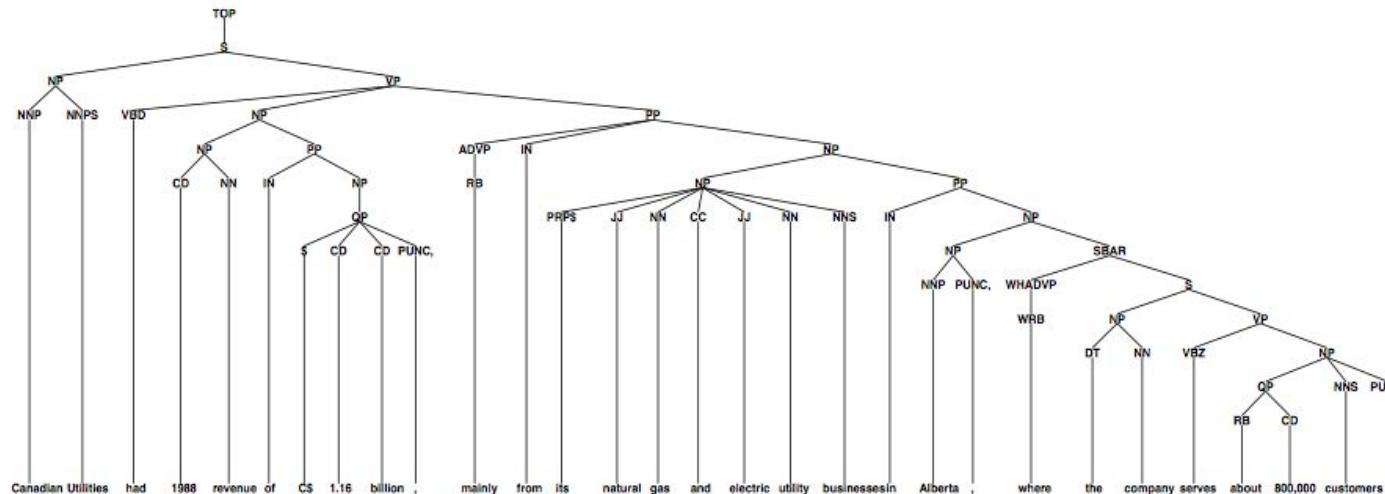
- OUTPUT:





# A Supervised ML Problem

- Data for parsing experiments:
  - Penn WSJ Treebank = 50,000 sentences with associated trees
  - Usual set-up: 40,000 training, 2,400 test



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers [from Michael Collins slides]



# Outline

---

- Syntax: intro, CFGs, PCFGs
- CFGs: Parsing
- PCFGs: Parsing
- Parsing evaluation

# Syntax



# Syntax

---

- The study of the patterns of formation of sentences and phrases from word

- my dog Pron N
- the dog Det N
- the cat Det N
  
- the large cat Det Adj N
- the black cat Det Adj N
  
- ate a sausage V Det N



# Syntax

---

- The study of the patterns of formation of sentences and phrases from word
  - Borders with **semantics** and **morphology** sometimes blurred

**Afyonkarahisarlılaştırabildiklerimizdenmişsinizcesinee**

in Turkish means

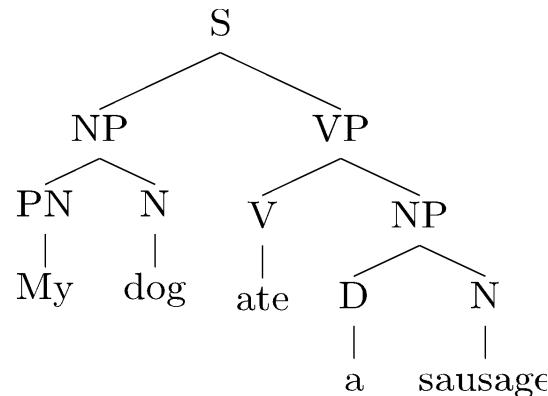
"as if you are one of the people that we thought to be originating from Afyonkarahisar" [wikipedia]



# Parsing

---

- The process of predicting syntactic representations
- Syntactic Representations
  - Different types of syntactic representations are possible, for example:

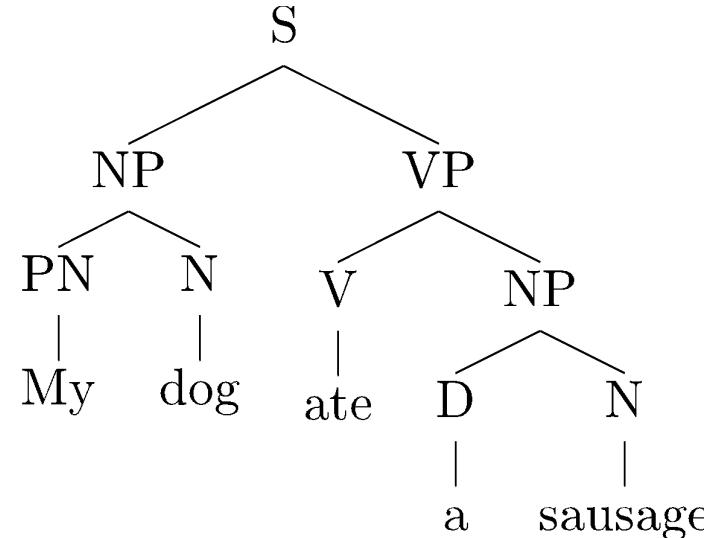


Constuent (a.k.a. phrase-structure) tree



# Constituent trees

---

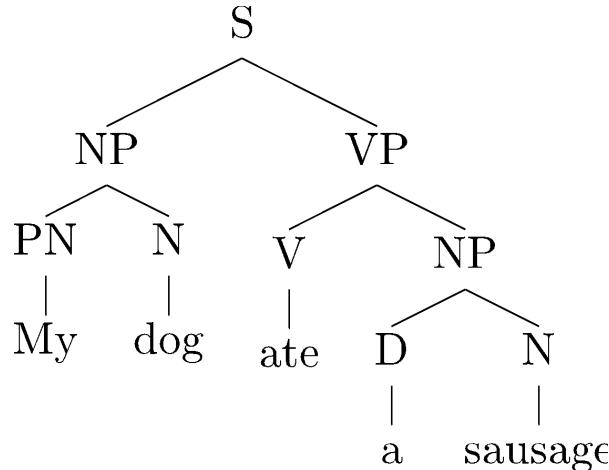


- Internal nodes correspond to phrases
  - S – a sentence
  - NP (Noun Phrase): My dog, a sandwich, lakes,..
  - VP (Verb Phrase): ate a sausage, barked, ...
  - PP (Prepositional phrases): with a friend, in a car, ...
- Nodes immediately above words are PoS tags (aka preterminals)
  - PN – pronoun
  - D – determiner
  - V – verb
  - N – noun
  - P – preposition



# Bracketing notation

---



- It is often convenient to represent a tree as a bracketed sequence

(S

  (NP (PN My) (N Dog) )

  (VP (V ate)

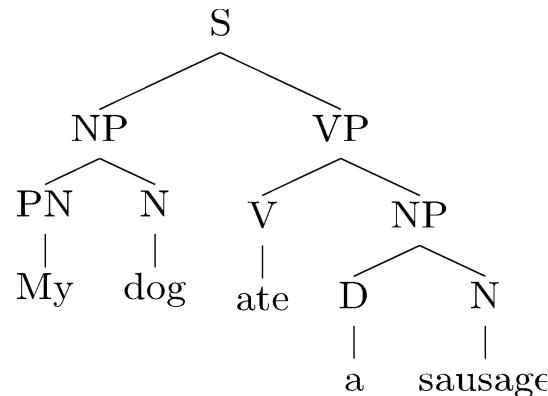
    (NP (D a ) (N sausage) )

)

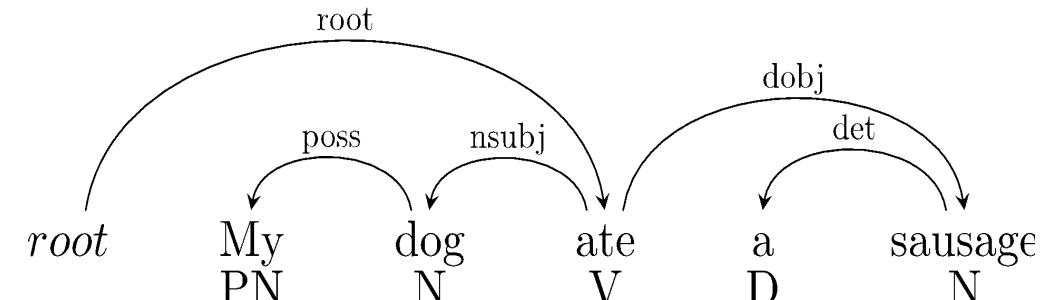


# Parsing

- The process of predicting syntactic representations
- Syntactic Representations
  - Different types of syntactic representations are possible, for example:



Constuent (a.k.a. phrase-structure) tree

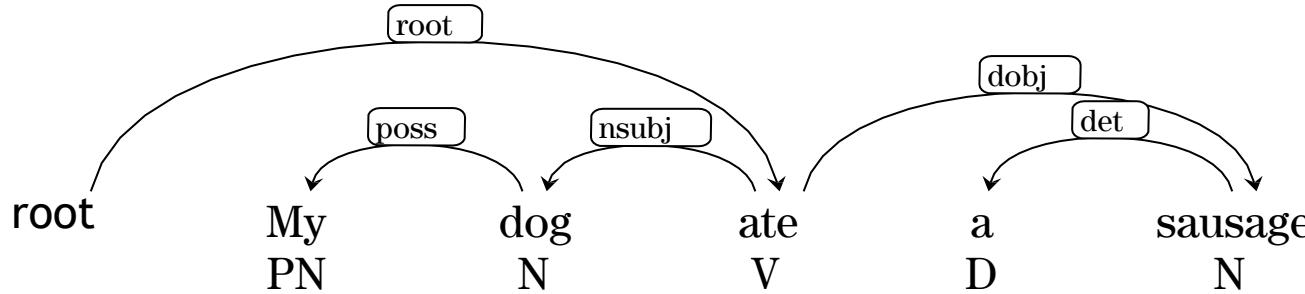


Dependency tree



# Dependency trees

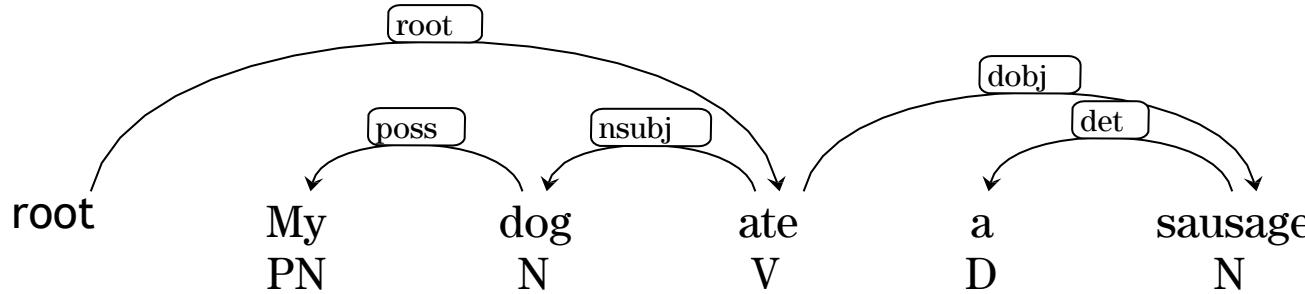
---



- Nodes are words (along with PoS tags)
- Directed arcs encode syntactic dependencies between them
- Labels are types of relations between the words
  - poss – possessive
  - dobj – direct object
  - nsubj - subject
  - det - determiner



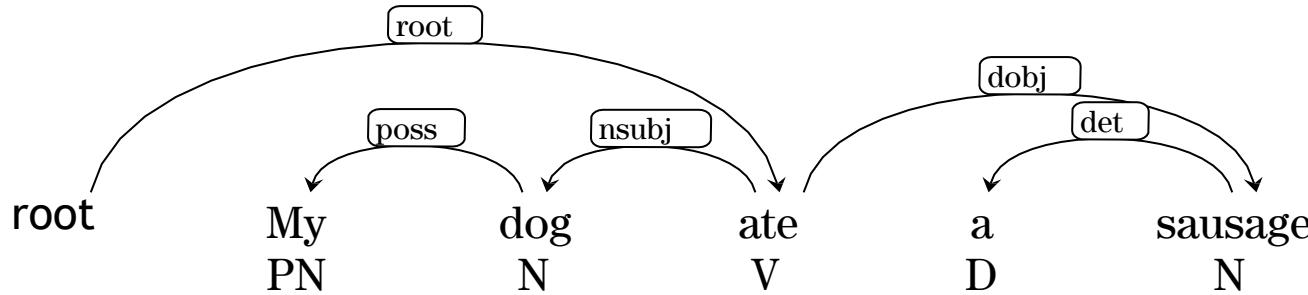
# Recovering shallow semantics



- Some semantic information can be (approximately) derived from syntactic information
  - Subjects (nsubj) are (often) agents ("initiator / doers for an action")
  - Direct objects (dobj) are (often) patients ("affected entities")



# Recovering shallow semantics

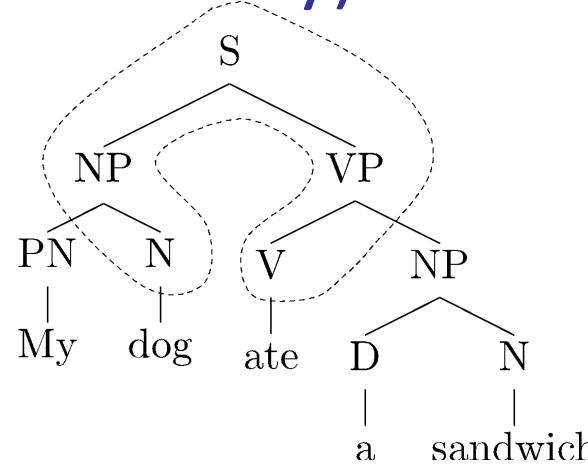


- Some semantic information can be (approximately) derived from syntactic information
  - Subjects (nsubj) are (often) agents ("initiator / doers for an action")
  - Direct objects (dobj) are (often) patients ("affected entities")
- But even for agents and patients consider:
  - Mary is baking a cake in the oven
  - A cake is baking in the oven
- In general it is not trivial even for the most shallow forms of semantics
  - E.g., consider prepositions: *in* can encode direction, position, temporal information, ...

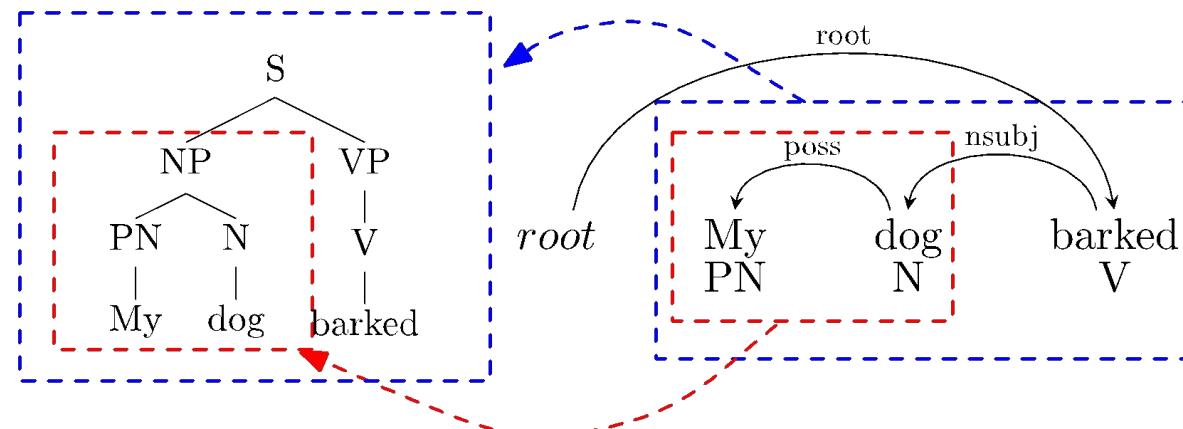


# Constituent and dependency representations

- Constituent trees can (potentially) be converted to dependency trees



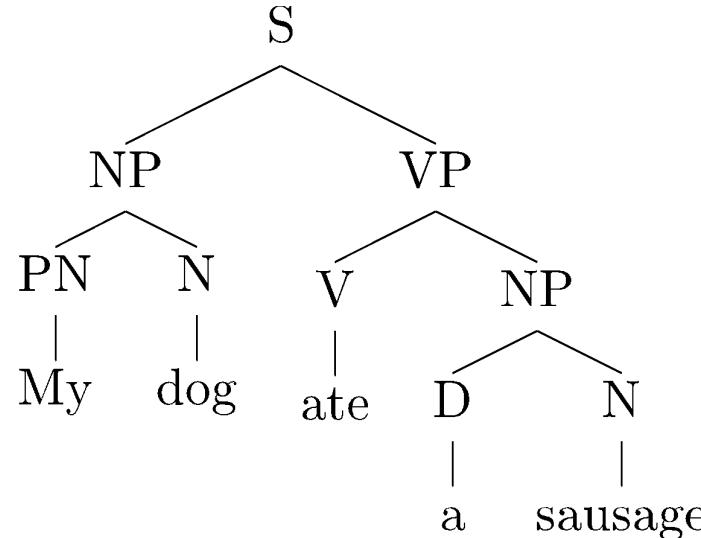
- Dependency trees can (potentially) be converted to constituent trees





# Constituent trees

---



- Internal nodes correspond to phrases
  - S – a sentence
  - NP (Noun Phrase): My dog, a sandwich, lakes,..
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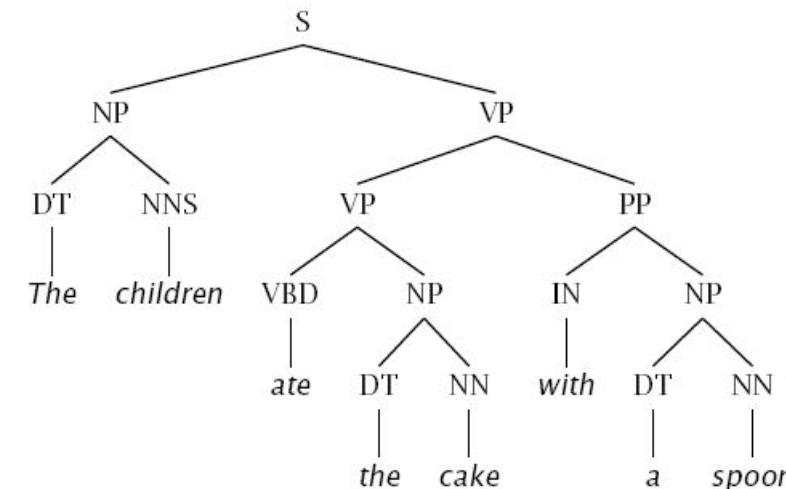
# Constituency Tests

---

- How do we know what nodes go in the tree?

- Classic constituency tests:

- Substitution by *proform*
- Movement
  - Clefting
  - Preposing
  - Passive
- Modification
- Coordination/Conjunction
- Ellipsis/Deletion





# Conflicting Tests

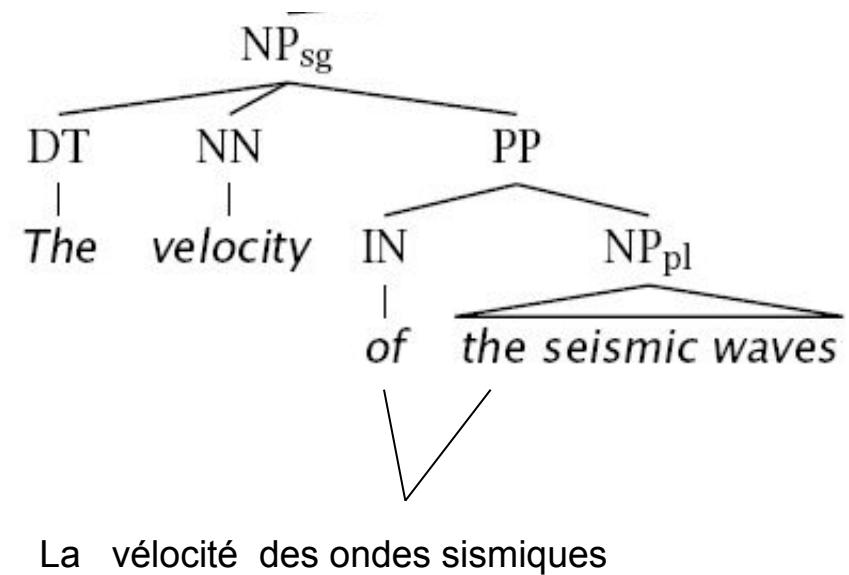
- Constituency isn't always clear

- Units of transfer:

- think about ~ penser à
    - talk about ~ hablar de

- Phonological reduction:

- I will go → I'll go
    - I want to go → I wanna go
    - a le centre → au centre



# CFGs



# Context Free Grammar (CFG)

---

## Grammar (CFG)

$\text{ROOT} \rightarrow S$	$NP \rightarrow NP\ PP$
$S \rightarrow NP\ VP$	$VP \rightarrow VBP\ NP$
$NP \rightarrow DT\ NN$	$VP \rightarrow VBP\ NP\ PP$
$NP \rightarrow NN\ NNS$	$PP \rightarrow IN\ NP$

## Lexicon

$NN \rightarrow \text{interest}$
$NNS \rightarrow \text{raises}$
$VBP \rightarrow \text{interest}$
$VBZ \rightarrow \text{raises}$

...

- Other grammar formalisms: LFG, HPSG, TAG, CCG...



# Treebank Sentences

---

```
( (S (NP-SBJ The move)
      (VP followed
        (NP (NP a round)
          (PP of
            (NP (NP similar increases)
              (PP by
                (NP other lenders)))
              (PP against
                (NP Arizona real estate loans))))))
    ,
    (S-ADV (NP-SBJ *)
      (VP reflecting
        (NP (NP a continuing decline)
          (PP-LOC in
            (NP that market))))))
  .))
```



# CFGs

S

$S \rightarrow NP \ VP$

$N \rightarrow girl$

$VP \rightarrow V$

$N \rightarrow telescope$

$VP \rightarrow V \ NP$

$N \rightarrow sandwich$

$VP \rightarrow VP \ PP$

$PN \rightarrow I$

$NP \rightarrow NP \ PP$

$V \rightarrow saw$

$NP \rightarrow D \ N$

$V \rightarrow ate$

$NP \rightarrow PN$

$P \rightarrow with$

$PP \rightarrow P \ NP$

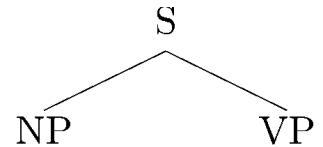
$P \rightarrow in$

$D \rightarrow a$

$D \rightarrow the$

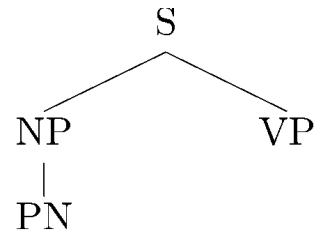


# CFGs

 $S \rightarrow NP \ VP$  $N \rightarrow girl$  $VP \rightarrow V$  $N \rightarrow telescope$  $VP \rightarrow V \ NP$  $N \rightarrow sandwich$  $VP \rightarrow VP \ PP$  $PN \rightarrow I$  $NP \rightarrow NP \ PP$  $V \rightarrow saw$  $NP \rightarrow D \ N$  $V \rightarrow ate$  $NP \rightarrow PN$  $P \rightarrow with$  $PP \rightarrow P \ NP$  $P \rightarrow in$  $D \rightarrow a$  $D \rightarrow the$

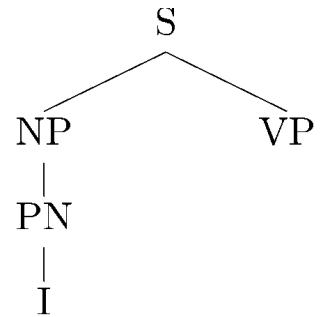


# CFGs


$$S \rightarrow NP \ VP$$
$$N \rightarrow girl$$
$$VP \rightarrow V$$
$$N \rightarrow telescope$$
$$VP \rightarrow V \ NP$$
$$N \rightarrow sandwich$$
$$VP \rightarrow VP \ PP$$
$$PN \rightarrow I$$
$$NP \rightarrow NP \ PP$$
$$V \rightarrow saw$$
$$NP \rightarrow D \ N$$
$$V \rightarrow ate$$
$$NP \rightarrow PN$$
$$P \rightarrow with$$
$$PP \rightarrow P \ NP$$
$$P \rightarrow in$$
$$D \rightarrow a$$
$$D \rightarrow the$$



# CFGs



$$S \rightarrow NP \ VP$$

$$N \rightarrow girl$$

$$\begin{array}{c} VP \rightarrow V \\ \boxed{VP \rightarrow V \ NP} \\ VP \rightarrow VP \ PP \end{array}$$

$$N \rightarrow telescope$$

$$\begin{array}{c} N \rightarrow sandwich \\ PN \rightarrow I \end{array}$$

$$PN \rightarrow I$$

$$V \rightarrow saw$$

$$NP \rightarrow NP \ PP$$

$$V \rightarrow ate$$

$$NP \rightarrow D \ N$$

$$P \rightarrow with$$

$$NP \rightarrow PN$$

$$P \rightarrow in$$

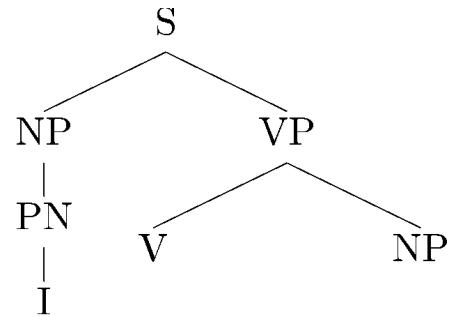
$$PP \rightarrow P \ NP$$

$$D \rightarrow a$$

$$D \rightarrow the$$



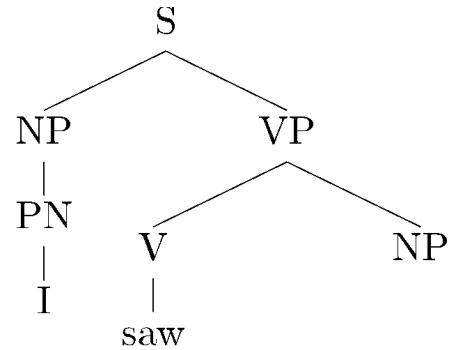
# CFGs


$$S \rightarrow NP \ VP$$
$$N \rightarrow girl$$
$$VP \rightarrow V$$
$$N \rightarrow telescope$$
$$VP \rightarrow V \ NP$$
$$N \rightarrow sandwich$$
$$VP \rightarrow VP \ PP$$
$$PN \rightarrow I$$
$$NP \rightarrow NP \ PP$$
$$V \rightarrow ate$$
$$NP \rightarrow D \ N$$
$$P \rightarrow with$$
$$NP \rightarrow PN$$
$$P \rightarrow in$$
$$PP \rightarrow P \ NP$$
$$D \rightarrow a$$
$$D \rightarrow the$$

V → saw



# CFGs



$$S \rightarrow NP \ VP$$

$$N \rightarrow girl$$

$$VP \rightarrow V$$

$$N \rightarrow telescope$$

$$VP \rightarrow V \ NP$$

$$N \rightarrow sandwich$$

$$VP \rightarrow VP \ PP$$

$$PN \rightarrow I$$

$$NP \rightarrow NP \ PP$$

$$V \rightarrow saw$$

$$NP \rightarrow D \ N$$

$$V \rightarrow ate$$

$$NP \rightarrow PN$$

$$P \rightarrow with$$

$$PP \rightarrow P \ NP$$

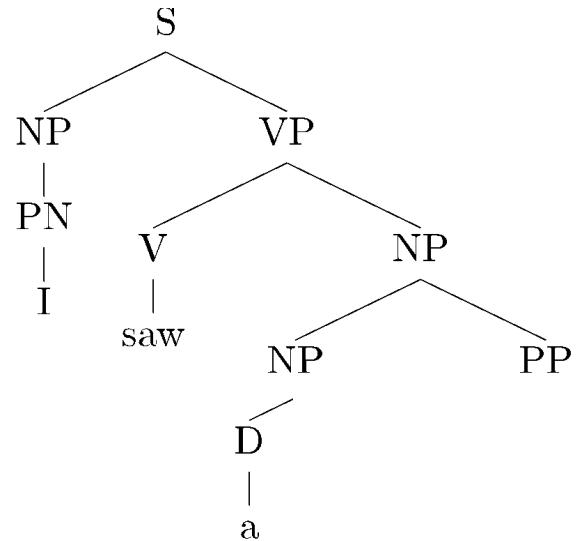
$$P \rightarrow in$$

$$D \rightarrow a$$

$$D \rightarrow the$$



# CFGs



$$S \rightarrow NP \ VP$$

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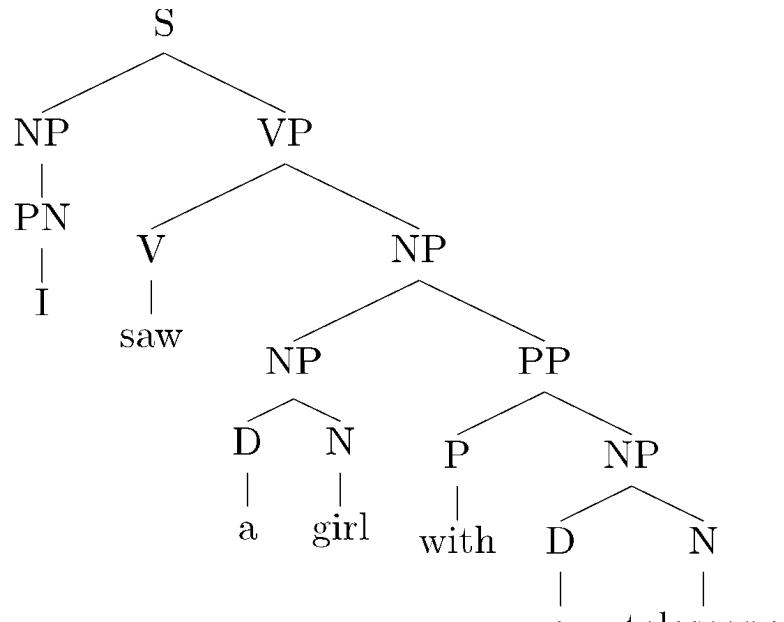
$$P \rightarrow in$$

$$D \rightarrow a$$

$$D \rightarrow the$$



# CFGs

 $S \rightarrow NP \ VP$  $N \rightarrow girl$  $VP \rightarrow V$  $N \rightarrow telescope$  $VP \rightarrow V \ NP$  $N \rightarrow sandwich$  $VP \rightarrow VP \ PP$  $PN \rightarrow I$  $NP \rightarrow NP \ PP$  $V \rightarrow saw$  $NP \rightarrow D \ N$  $V \rightarrow ate$  $NP \rightarrow PN$  $P \rightarrow with$  $PP \rightarrow P \ NP$  $P \rightarrow in$  $D \rightarrow a$  $D \rightarrow the$



# Context-Free Grammars

---

- A context-free grammar is a 4-tuple  $\langle N, T, S, R \rangle$ 
  - $N$  : the set of non-terminals
    - Phrasal categories: S, NP, VP, ADJP, etc.
    - Parts-of-speech (pre-terminals): NN, JJ, DT, VB
  - $T$  : the set of terminals (the words)
  - $S$  : the start symbol
    - Often written as ROOT or TOP
    - *Not* usually the sentence non-terminal S
  - $R$  : the set of rules
    - Of the form  $X \rightarrow Y_1 Y_2 \dots Y_k$ , with  $X, Y_i \in N$
    - Examples:  $S \rightarrow \text{NP VP}$ ,  $\text{VP} \rightarrow \text{VP CC VP}$
    - Also called rewrites, productions, or local trees



# An example grammar

$N = \{S, VP, NP, PP, N, V, PN, P\}$

$T = \{\text{girl}, \text{telescope}, \text{sandwich}, I, \text{saw}, \text{ate}, \text{with}, \text{in}, a, \text{the}\}$

$S = \{S\}$

$R :$

$S \rightarrow NP \ VP$

(NP A girl) (VP ate a sandwich)

$VP \rightarrow V$

$VP \rightarrow V \ NP$

(V ate) (NP a sandwich)

$VP \rightarrow VP \ PP$

(VP saw a girl) (PP with a telescope)

$NP \rightarrow NP \ PP$

(NP a girl) (PP with a sandwich)

$NP \rightarrow D \ N$

(D a) (N sandwich)

$NP \rightarrow PN$

$PP \rightarrow P \ NP$

(P with) (NP with a sandwich)

Called **Inner rules**

Preterminal rules

$N \rightarrow \text{girl}$

$N \rightarrow \text{telescope}$

$N \rightarrow \text{sandwich}$

$PN \rightarrow I$

$V \rightarrow \text{saw}$

$V \rightarrow \text{ate}$

$P \rightarrow \text{with}$

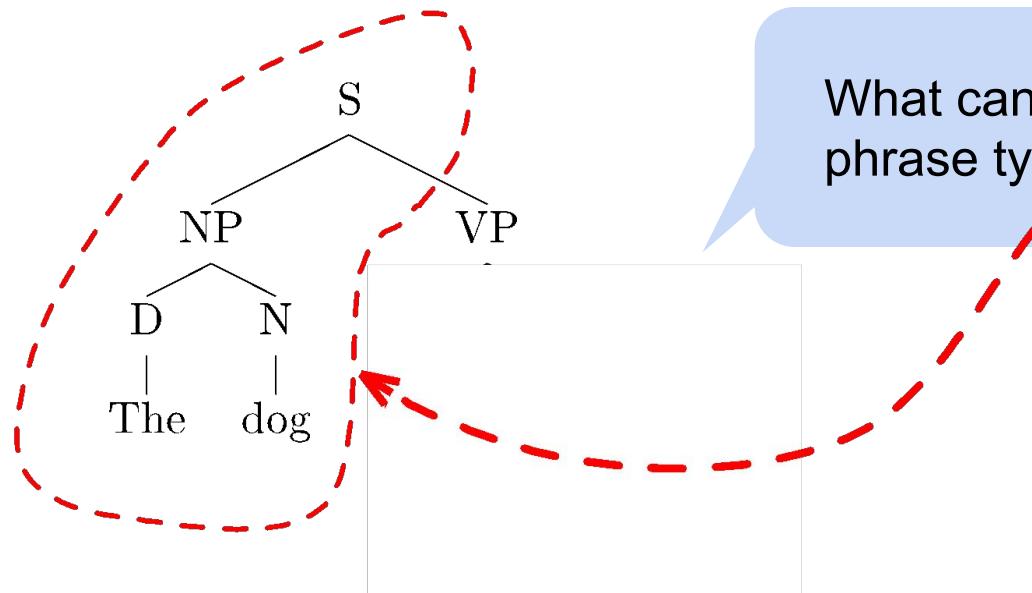
$P \rightarrow \text{in}$

$D \rightarrow a$

$D \rightarrow \text{the}$



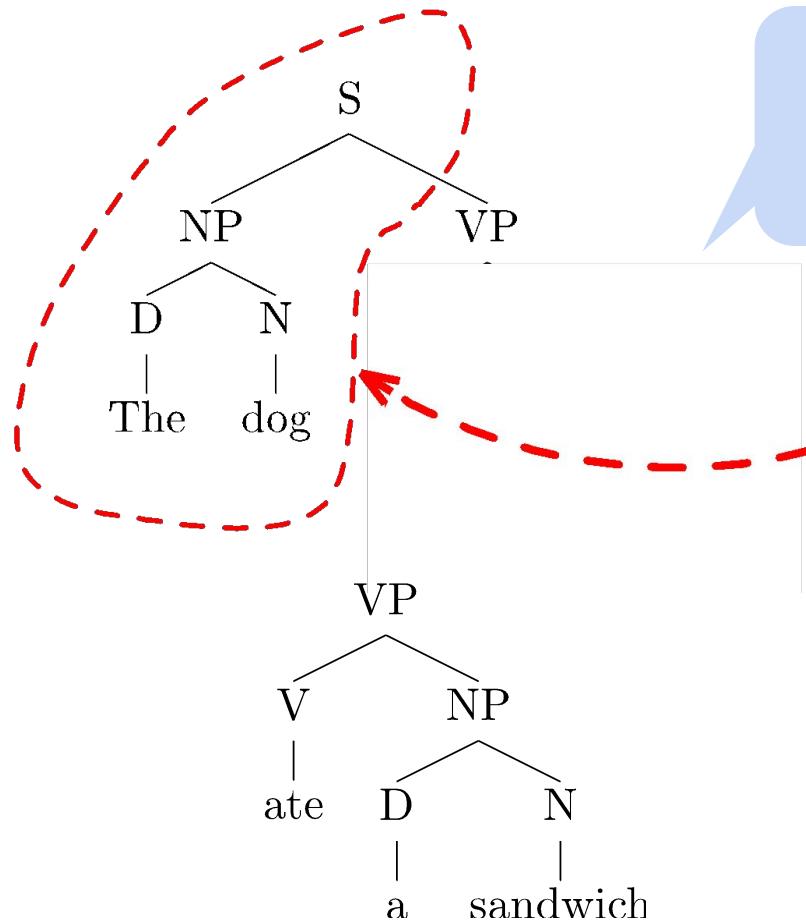
# Why context-free?



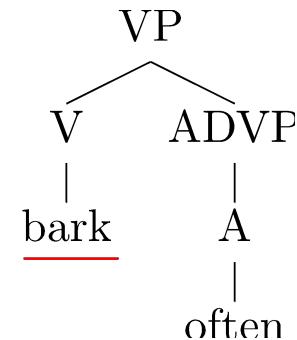
What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**



# Why context-free?



What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**

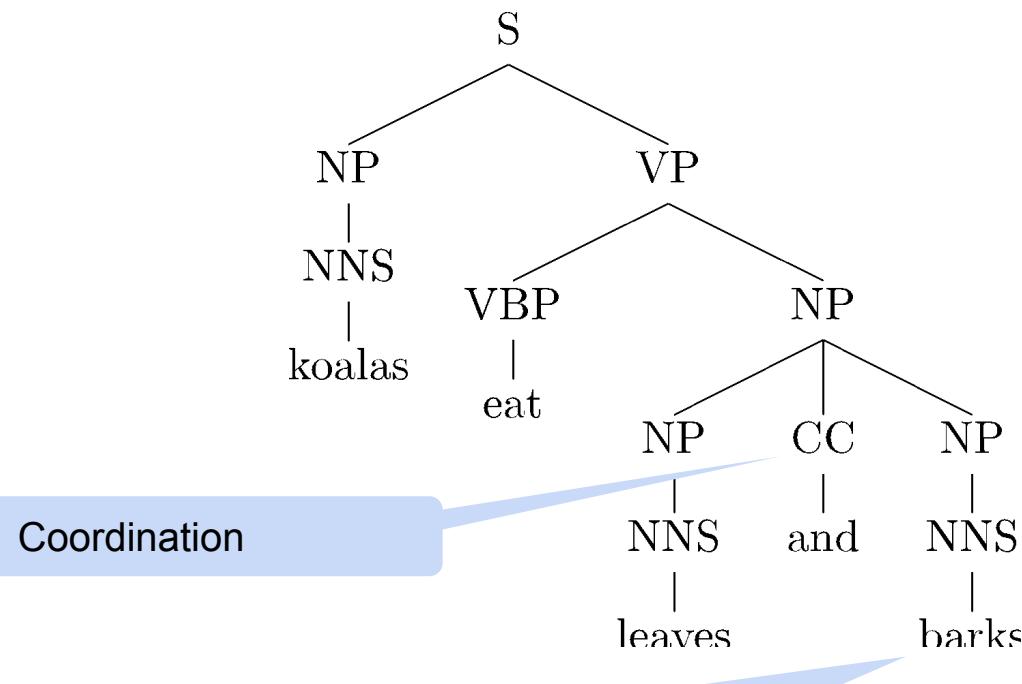


Not grammatical

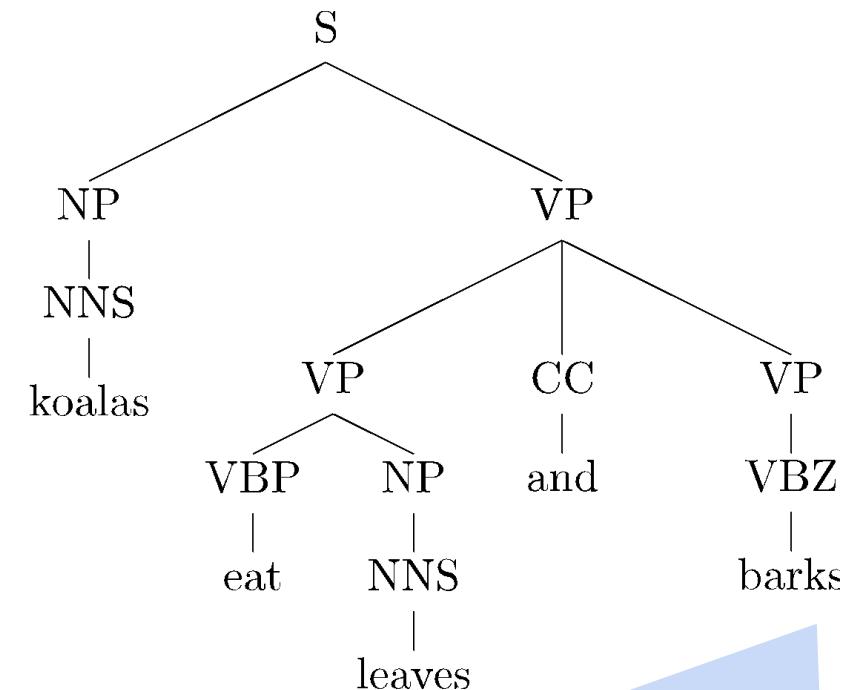


# Coordination ambiguity

- Here, the coarse VP and NP categories cannot enforce subject-verb agreement in number resulting in the coordination ambiguity



"Bark" can refer both to a noun or a verb



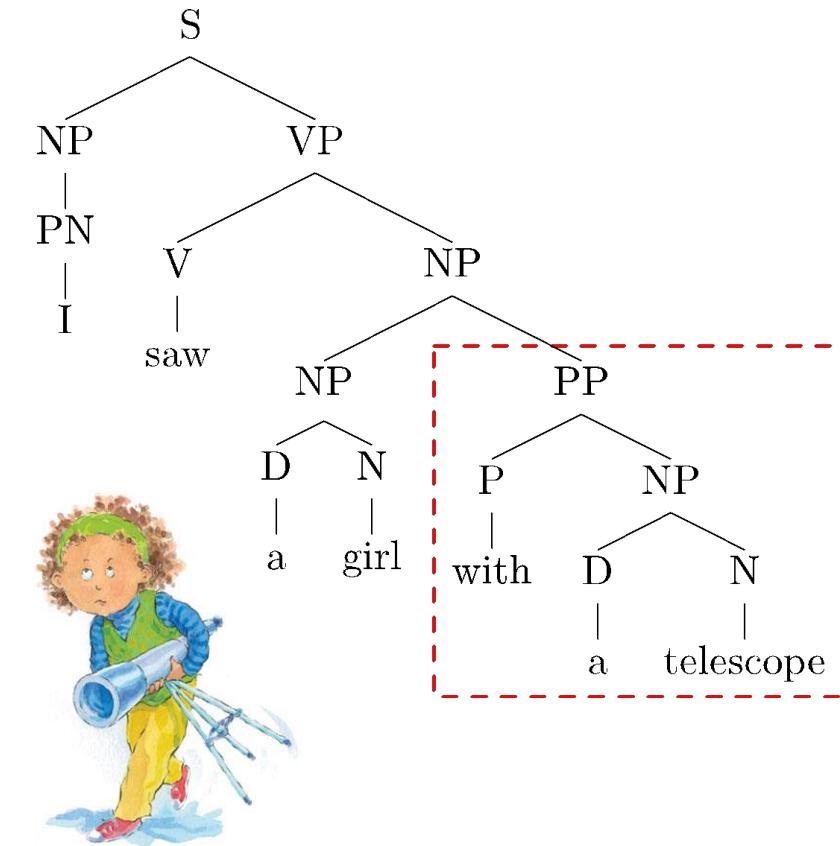
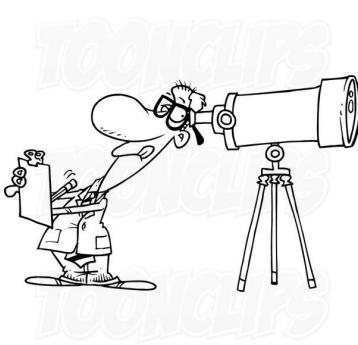
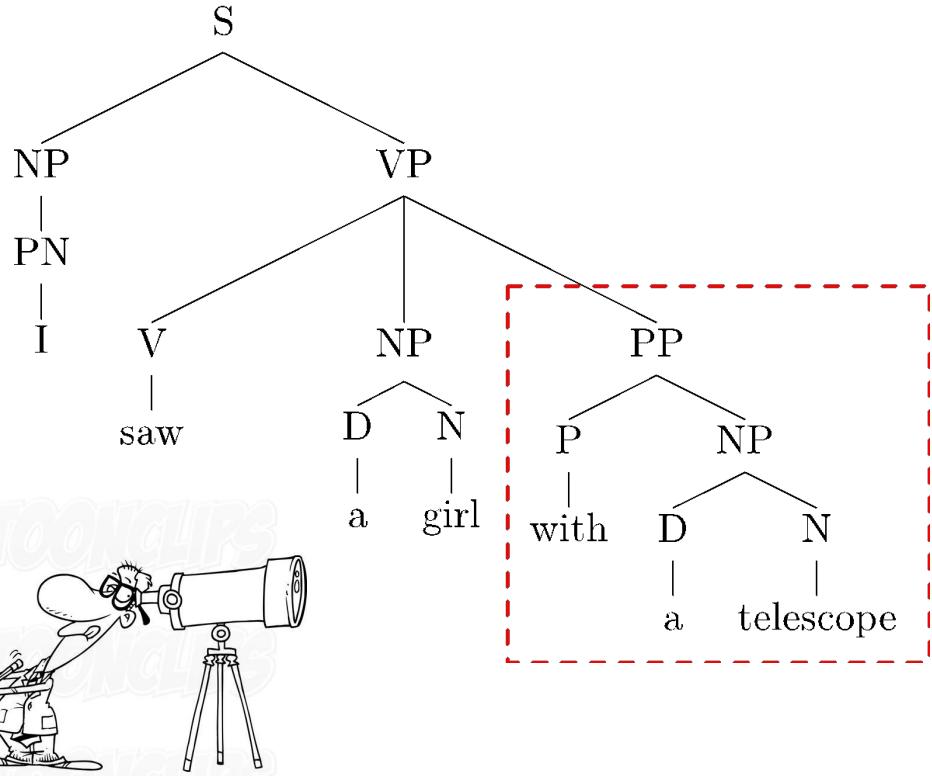
This tree would be ruled out if the context would be somehow captured (subject-verb agreement)

# Ambiguities



# Why parsing is hard? Ambiguity

- Prepositional phrase attachment ambiguity





# PP Ambiguity

---

***Put the block in the box on the table in the kitchen***

- 3 prepositional phrases, 5 interpretations:
  - Put the block (**(in the box on the table) in the kitchen**)
  - Put the block (in the box (on the table in the kitchen))
  - Put ((the block in the box) on the table) in the kitchen.
  - Put (the block (in the box on the table)) in the kitchen.
  - Put **(the block in the box) (on the table in the kitchen)**



# PP Ambiguity

***Put the block in the box on the table in the kitchen***

- 3 prepositional phrases, 5 interpretations:
  - Put the block ((in the box on the table) in the kitchen)
  - Put the block (in the box (on the table in the kitchen))
  - Put ((the block in the box) on the table) in the kitchen.
  - Put (the block (in the box on the table)) in the kitchen.
  - Put (the block in the box) (on the table in the kitchen)
- A general case:

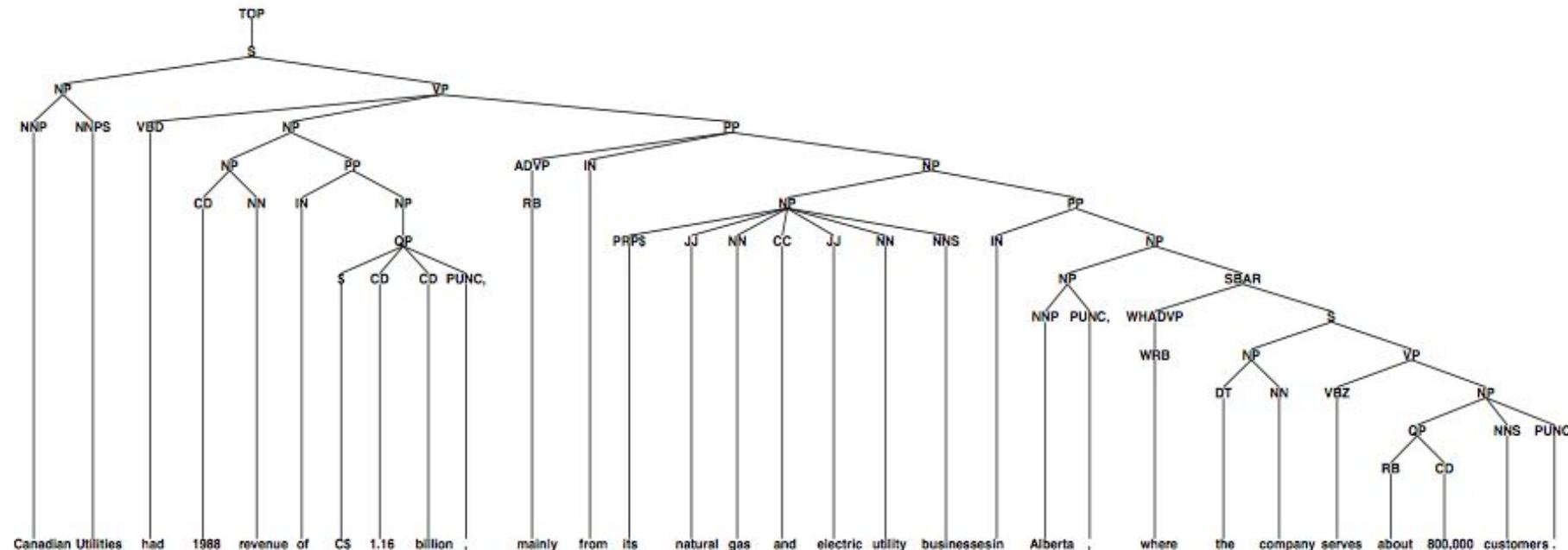
$$Cat_n = \binom{2n}{n} - \binom{2n}{n-1} \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$

Catalan numbers

1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...



## A typical tree from a standard dataset (Penn treebank WSJ)



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers .



# Syntactic Ambiguities I

---

- Prepositional phrases:

*They cooked the beans in the pot on the stove with handles.*

- Particle vs. preposition:

*The puppy tore up the staircase.*

- Complement structures

*The tourists objected to the guide that they couldn't hear.*

*She knows you like the back of her hand.*

- Gerund vs. participial adjective

*Visiting relatives can be boring.*

*Changing schedules frequently confused passengers.*



# Syntactic Ambiguities II

---

- Modifier scope within NPs

*impractical design requirements*  
*plastic cup holder*

- Multiple gap constructions

*The chicken is ready to eat.*  
*The contractors are rich enough to sue.*

- Coordination scope:

*Small rats and mice can squeeze into holes or cracks in the wall.*

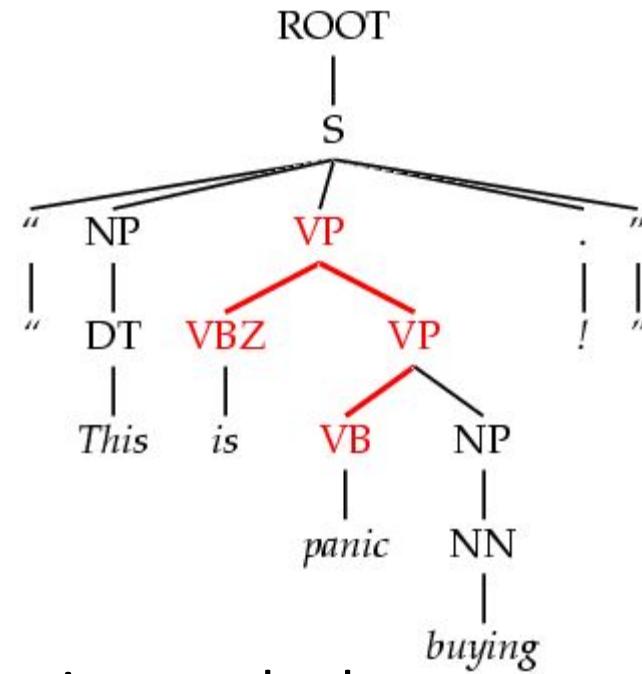


# Dark Ambiguities

- *Dark ambiguities*: most analyses are shockingly bad (meaning, they don't have an interpretation you can get your mind around)

This analysis corresponds to the correct  
parse of

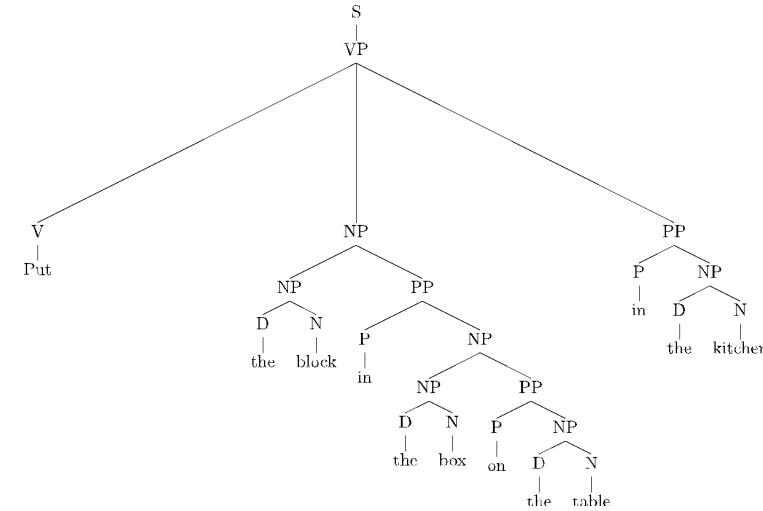
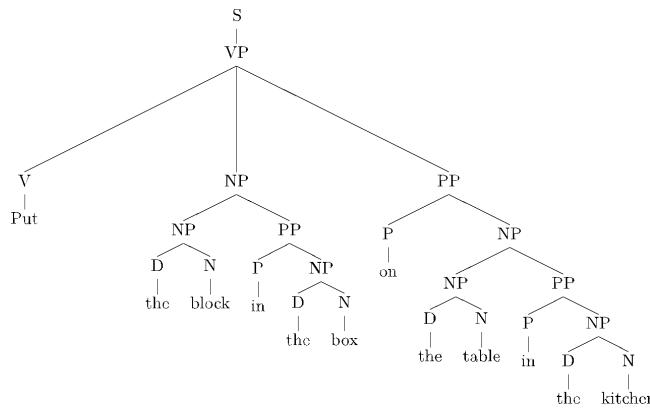
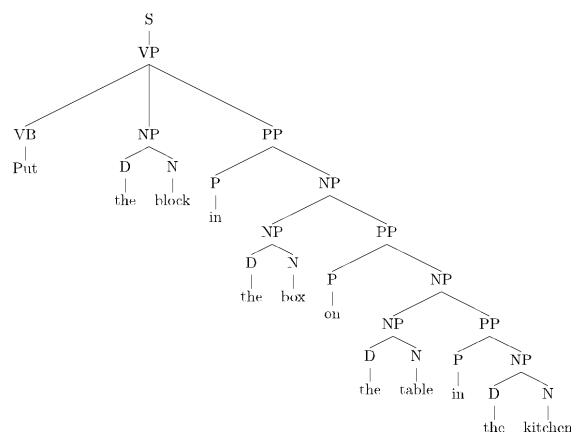
*“This is panic buying !”*



- Unknown words and new usages
- **Solution:** We need mechanisms to focus attention on the best ones, probabilistic techniques do this



# How to Deal with Ambiguity?



*Put the block in the box on the table in the kitchen*

- We want to score all the derivations to encode how plausible they are

# PCFGs



# Probabilistic Context-Free Grammars

---

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  - $N$  : the set of non-terminals
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    - Parts-of-speech (pre-terminals): NN, JJ, DT, VB
  - $T$  : the set of terminals (the words)
  - $S$  : the start symbol
    - Often written as ROOT or TOP
    - *Not* usually the sentence non-terminal S
  - $R$  : the set of rules
    - Of the form  $X \rightarrow Y_1 Y_2 \dots Y_k$ , with  $X, Y_i \in N$
    - Examples:  $S \rightarrow \text{NP VP}$ ,  $\text{VP} \rightarrow \text{VP CC VP}$
    - Also called rewrites, productions, or local trees
- A PCFG adds:
  - A top-down production probability per rule  $P(Y_1 Y_2 \dots Y_k | X)$



# PCFGs

Associate probabilities with the rules :  $p(X \rightarrow \alpha)$

$$\forall X \rightarrow \alpha \in R : 0 \leq p(X \rightarrow \alpha) \leq 1$$

$$\forall X \in N : \sum_{\alpha: X \rightarrow \alpha \in R} p(X \rightarrow \alpha) = 1$$

Now we can score a tree as a product of probabilities corresponding to the used rules

$S \rightarrow NP \ VP$	1.0	(NP A girl) (VP ate a sandwich)
-------------------------	-----	---------------------------------

$N \rightarrow girl$  0.2

$N \rightarrow telescope$  0.7

$N \rightarrow sandwich$  0.1

$PN \rightarrow I$  1.0

$V \rightarrow saw$  0.5

$V \rightarrow ate$  0.5

$P \rightarrow with$  0.6

$P \rightarrow in$  0.4

$D \rightarrow a$  0.3

$D \rightarrow the$  0.7

$VP \rightarrow V \ NP$	0.2	(VP ate) (NP a sandwich)
-------------------------	-----	--------------------------

$VP \rightarrow VP \ PP$	0.4	(VP saw a girl) (PP with ...)
--------------------------	-----	-------------------------------

$NP \rightarrow NP \ PP$	0.3	(NP a girl) (PP with ....)
--------------------------	-----	----------------------------

$NP \rightarrow D \ N$	0.5	(D a) (N sandwich)
------------------------	-----	--------------------

$NP \rightarrow PN$	0.2
---------------------	-----

$PP \rightarrow P \ NP$	1.0	(P with) (NP with a sandwich)
-------------------------	-----	-------------------------------



# PCFGs

S

$p(T) =$

$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$

$VP \rightarrow V \ NP \ 0.4$

$VP \rightarrow VP \ PP \ 0.4$

$NP \rightarrow NP \ PP \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

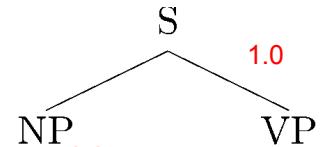
$P \rightarrow in \ 0.4$

$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$



# PCFGs



$$p(T) = 1.0 \times$$

$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$

$VP \rightarrow V \ NP \ 0.4$

$VP \rightarrow VP \ PP \ 0.4$

$NP \rightarrow NP \ PP \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

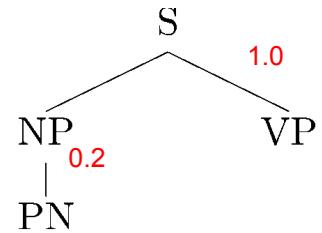
$P \rightarrow in \ 0.4$

$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$



# PCFGs



$$p(T) = 1.0 \times 0.2 \times$$

$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$

$VP \rightarrow V \ NP \ 0.4$

$VP \rightarrow VP \ PP \ 0.4$

$NP \rightarrow NP \ PP \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

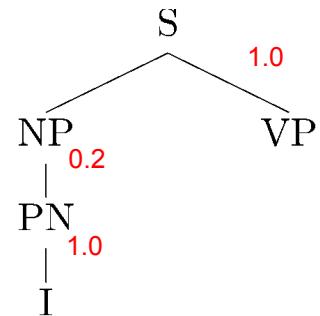
$P \rightarrow in \ 0.4$

$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$



# PCFGs



$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$

$VP \rightarrow V \ NP \ 0.4$

$VP \rightarrow VP \ PP \ 0.4$

$NP \rightarrow NP \ PP \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

$P \rightarrow in \ 0.4$

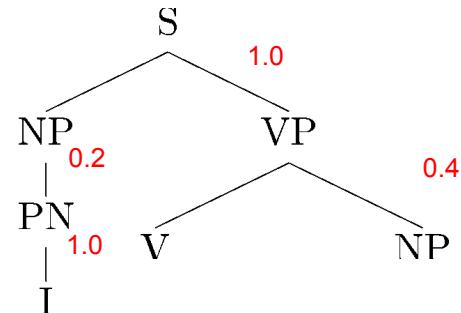
$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times$$



# PCFGs



$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$   
   $VP \rightarrow V \ NP \ 0.4$   
   $VP \rightarrow VP \ PP \ 0.4$

$NP \rightarrow NP \ PP \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

$P \rightarrow in \ 0.4$

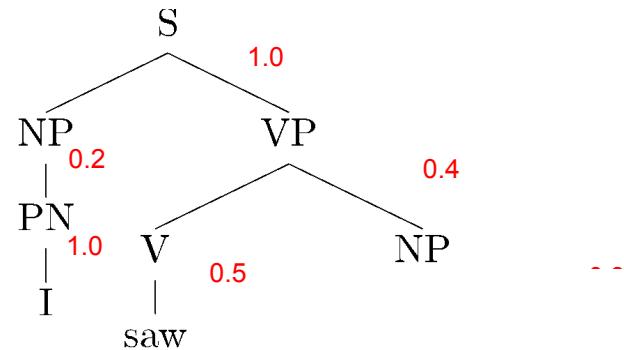
$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times$$



# PCFGs



$$S \rightarrow NP \ VP \ 1.0$$

$$VP \rightarrow V \ 0.2$$

$$VP \rightarrow V \ NP \ 0.4$$

$$VP \rightarrow VP \ PP \ 0.4$$

$$NP \rightarrow NP \ PP \ 0.3$$

$$NP \rightarrow D \ N \ 0.5$$

$$NP \rightarrow PN \ 0.2$$

$$PP \rightarrow P \ NP \ 1.0$$

$$N \rightarrow girl \ 0.2$$

$$N \rightarrow telescope \ 0.7$$

$$N \rightarrow sandwich \ 0.1$$

$$PN \rightarrow I \ 1.0$$

$$V \rightarrow saw \ 0.5$$

$$V \rightarrow ate \ 0.5$$

$$P \rightarrow with \ 0.6$$

$$P \rightarrow in \ 0.4$$

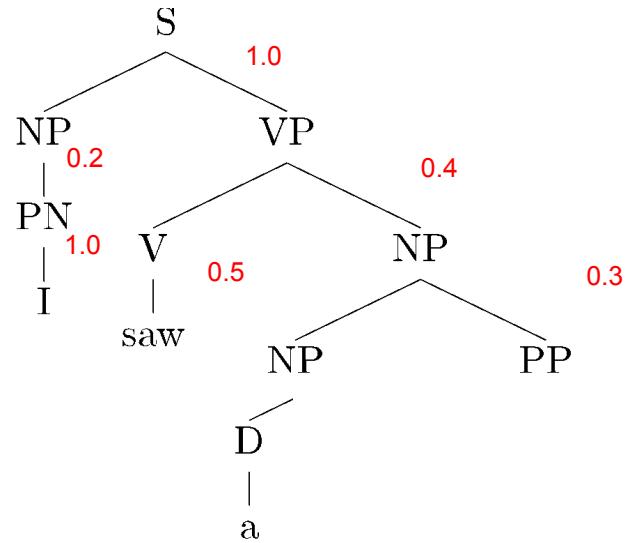
$$D \rightarrow a \ 0.3$$

$$D \rightarrow the \ 0.7$$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times$$



# PCFGs



$$S \rightarrow NP \ VP \ 1.0$$

$$VP \rightarrow V \ 0.2$$

$$VP \rightarrow V \ NP \ 0.4$$

$$VP \rightarrow VP \ PP \ 0.4$$

$$NP \rightarrow NP \ PP \ 0.3$$

$$NP \rightarrow D \ N \ 0.5$$

$$NP \rightarrow PN \ 0.2$$

$$PP \rightarrow P \ NP \ 1.0$$

$$N \rightarrow girl \ 0.2$$

$$N \rightarrow telescope \ 0.7$$

$$N \rightarrow sandwich \ 0.1$$

$$PN \rightarrow I \ 1.0$$

$$V \rightarrow saw \ 0.5$$

$$V \rightarrow ate \ 0.5$$

$$P \rightarrow with \ 0.6$$

$$P \rightarrow in \ 0.4$$

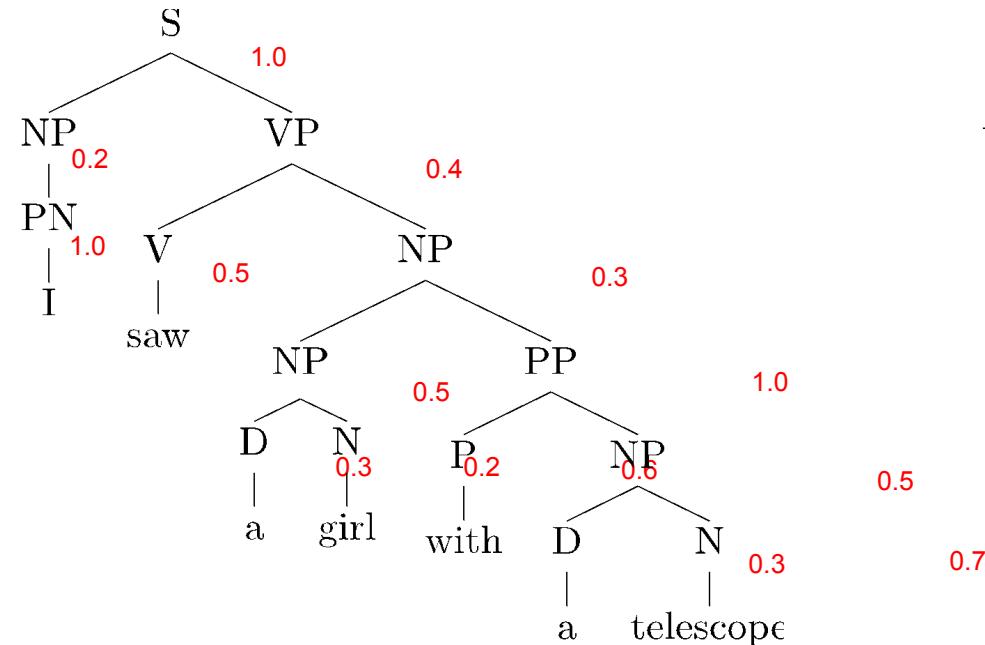
$$D \rightarrow a \ 0.3$$

$$D \rightarrow the \ 0.7$$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times$$



# PCFGs



$$S \rightarrow NP \ VP \ 1.0$$

$$VP \rightarrow V \ 0.2$$

$$VP \rightarrow V \ NP \ 0.4$$

$$VP \rightarrow VP \ PP \ 0.4$$

$$NP \rightarrow NP \ PF \ 0.3$$

$$NP \rightarrow D \ N \ 0.5$$

$$NP \rightarrow PN \ 0.2$$

$$PP \rightarrow P \ NP \ 1.0$$

$$N \rightarrow girl \ 0.2$$

$$N \rightarrow telescope \ 0.7$$

$$N \rightarrow sandwich \ 0.1$$

$$PN \rightarrow I \ 1.0$$

$$V \rightarrow saw \ 0.5$$

$$V \rightarrow ate \ 0.5$$

$$P \rightarrow with \ 0.6$$

$$P \rightarrow in \ 0.4$$

$$D \rightarrow a \ 0.3$$

$$D \rightarrow the \ 0.7$$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times \\ 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 = 2.26 \times 10^{-5}$$

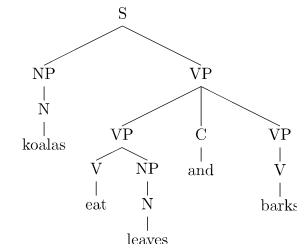
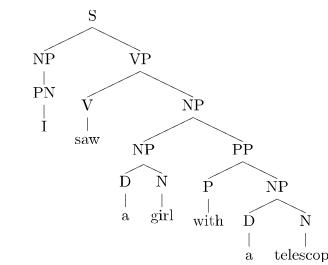
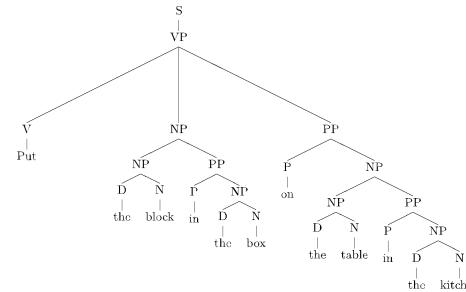
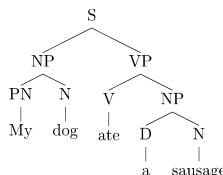


# PCFG Estimation



# ML estimation

- A treebank: a collection sentences annotated with constituent trees



- An estimated probability of a rule (maximum likelihood estimates)

$$p(X \rightarrow \alpha) = \frac{C(X \rightarrow \alpha)}{C(X)}$$

The number of times the rule used in the corpus

The number of times the nonterminal X appears in the treebank

- Smoothing is helpful
  - Especially important for preterminal rules



# Distribution over trees

---

- We defined a distribution over production rules for each nonterminal
- Our goal was to define a distribution over parse trees

Unfortunately, not all PCFGs give rise to a proper distribution over trees, i.e. the sum over probabilities of all trees the grammar can generate may be less than 1:  $\sum_T P(T) < 1$

- **Good news:** any PCFG estimated with the maximum likelihood procedure are always proper (Chi and Geman, 98)



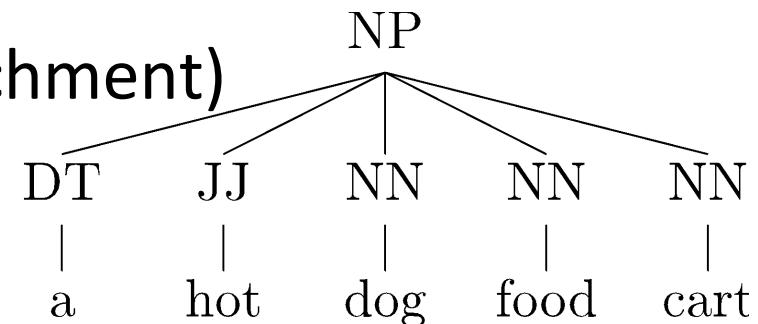
# Penn Treebank: peculiarities

---

- Wall street journal: around 40, 000 annotated sentences, 1,000,000 words
  - Fine-grained part of speech tags (45), e.g., for verbs

VBD	Verb, past tense
VBG	Verb, gerund or present participle
VBP	Verb, present (non-3 <sup>rd</sup> person singular)
VBZ	Verb, present (3 <sup>rd</sup> person singular)
MD	Modal

- Flat NPs (no attempt to disambiguate NP attachment)



# CKY Parsing



# Parsing

---

- Parsing is search through the space of all possible parses
  - e.g., we may want either any parse, all parses or the highest scoring parse (if PCFG):

$$\arg \max_{T \in G(x)} P(T)$$

- Bottom-up:
  - One starts from words and attempt to construct the full tree
- Top-down
  - Start from the start symbol and attempt to expand to get the sentence



# CKY algorithm (aka CYK)

---

- Cocke-Kasami-Younger algorithm
  - Independently discovered in late 60s / early 70s
- An efficient bottom up parsing algorithm for (P)CFGs
  - can be used both for the recognition and parsing problems
  - Very important in NLP (and beyond)
- We will start with the non-probabilistic version



# Constraints on the grammar

- The basic CKY algorithm supports only rules in the Chomsky Normal Form (CNF):

$$C \rightarrow x$$

Unary **preterminal** rules (generation of words given PoS tags)

$$N \rightarrow \text{telescope} \quad D \rightarrow \text{the}$$

$$C \rightarrow C_1 C_2$$

Binary **inner** rules  $S \rightarrow NP VP$      $NP \rightarrow D\ N$



# Constraints on the grammar

---

- The basic CKY algorithm supports only rules in the **Chomsky Normal Form (CNF)**:

$$C \rightarrow x$$

$$C \rightarrow C_1 C_2$$

- Any CFG can be converted to an equivalent CNF
  - Equivalent means that they define **the same language**
  - However (syntactic) **trees will look differently**
  - It is possible to address it by defining such transformations that **allows for easy reverse transformation**



# Transformation to CNF form

---

- What one need to do to convert to CNF form

- Get rid of unary rules:
- Get rid of N-ary rules:  $C \rightarrow C_1 C_2 \dots C_n \ (n > 2)$

$$C \rightarrow C_1$$

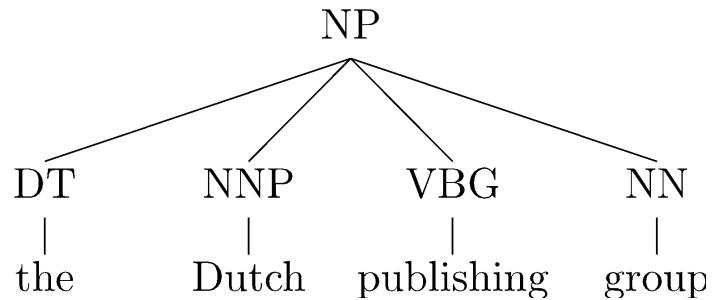
Not a problem, as our CKY algorithm will support unary rules

Crucial to process them, as required for efficient parsing



# Transformation to CNF form: binarization

- Consider  $NP \rightarrow DT \ NNP \ VBG \ NN$

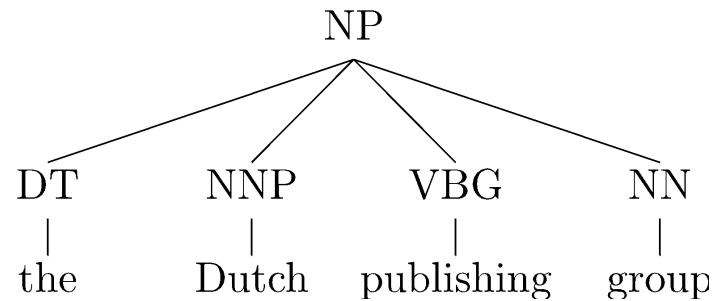


- How do we get a set of binary rules which are equivalent?



# Transformation to CNF form: binarization

- Consider  $NP \rightarrow DT \ NNP \ VBG \ NN$



- How do we get a set of binary rules which are equivalent?

$NP \rightarrow DT \ X$

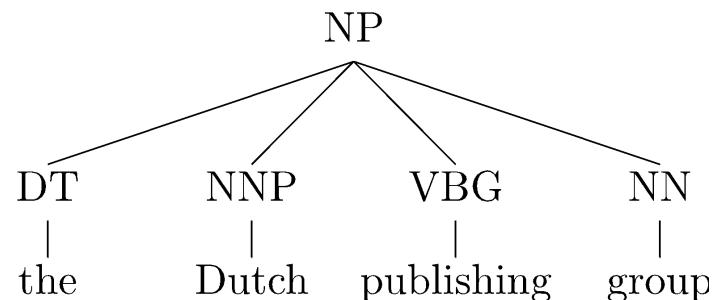
$X \rightarrow NNP \ Y$

$Y \rightarrow VBG \ NN$



# Transformation to CNF form: binarization

- Consider  $NP \rightarrow DT \ NNP \ VBG \ NN$



- How do we get a set of binary rules which are equivalent?

$$NP \rightarrow DT \ X$$
$$X \rightarrow NNP \ Y$$
$$Y \rightarrow VBG \ NN$$

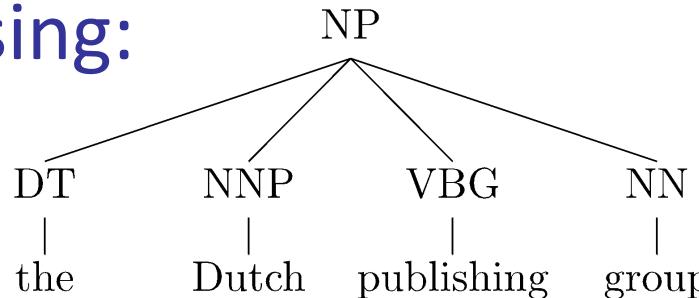
- A more systematic way to refer to new non-terminals

$$NP \rightarrow DT \ @NP|DT$$
$$@NP|DT \rightarrow NNP \ @NP|DT\_NNP$$
$$@NP|DT\_NNP \rightarrow VBG \ NN$$

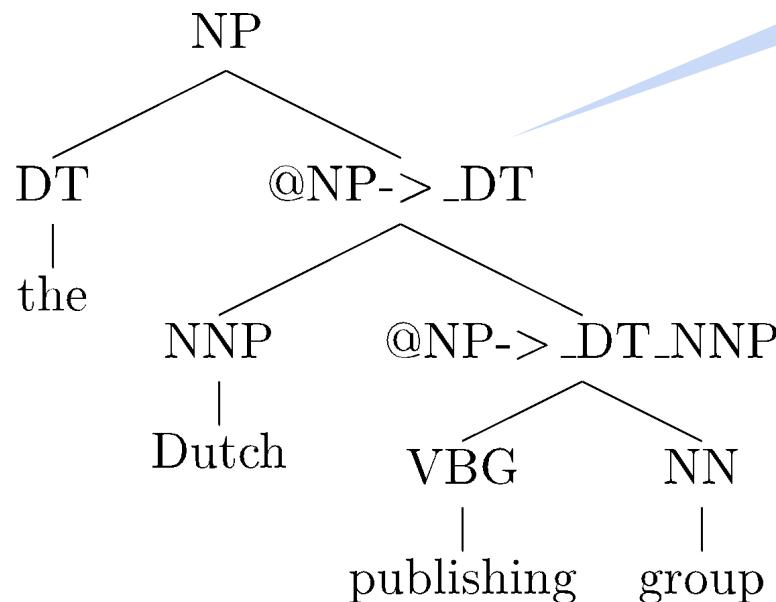


# Transformation to CNF form: binarization

- Instead of binarizing tuples we can binarize trees on preprocessing:



Also known as **lossless Markovization** in the context of PCFGs



Can be easily reversed on postprocessing



# CKY: Parsing task

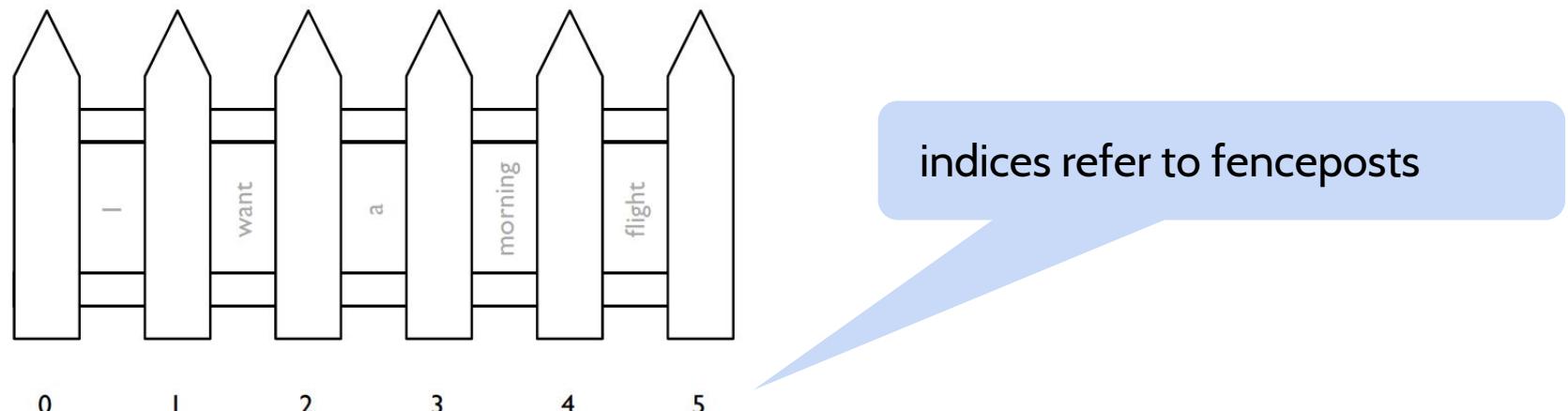
---

- We are given
  - a grammar  $\langle N, T, S, R \rangle$
  - a sequence of words  $w = (w_1, w_2, \dots, w_n)$
- Our goal is to produce a parse tree for  $w$



# CKY: Parsing task

- We are given
  - a grammar  $\langle N, T, S, R \rangle$
  - a sequence of words  $w = (w_1, w_2, \dots, w_n)$
- Our goal is to produce a parse tree for  $w$
- We need an easy way to refer to substrings of  $w$



*span* ( $i, j$ ) refers to words between fenceposts  $i$  and  $j$



# Parsing one word

---

$C \rightarrow w_i$

$w_i$



# Parsing one word

---

$C \rightarrow w_i$

C



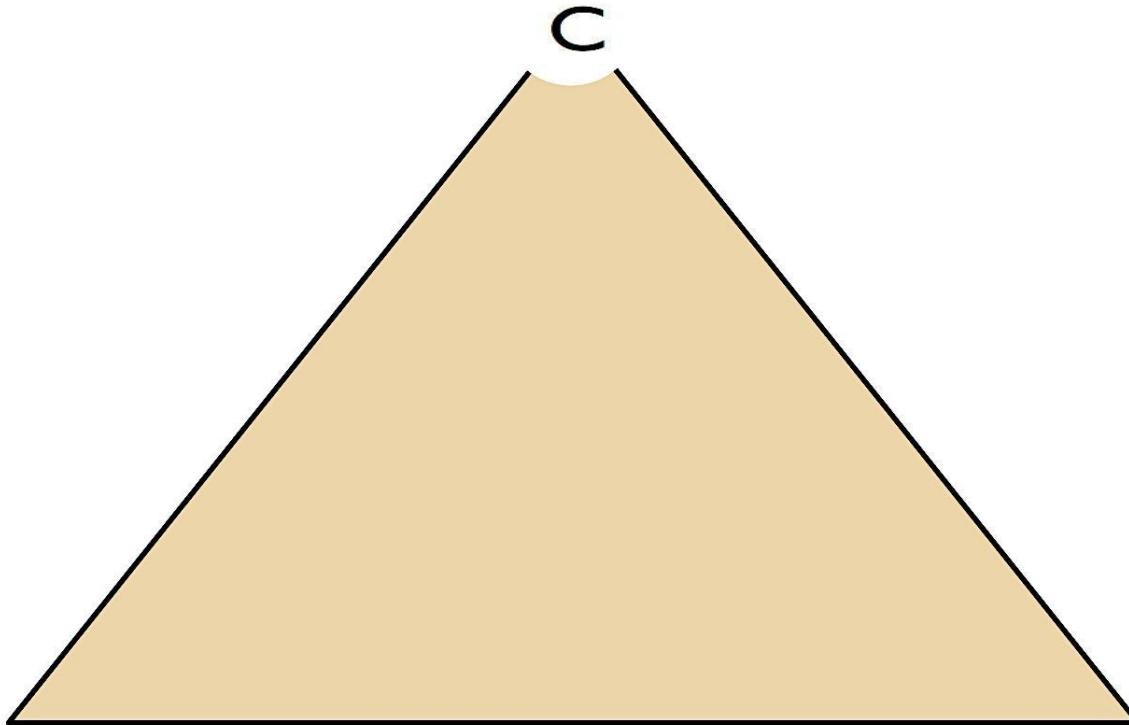
$w_i$



# Parsing one word

---

$C \rightarrow w_i$

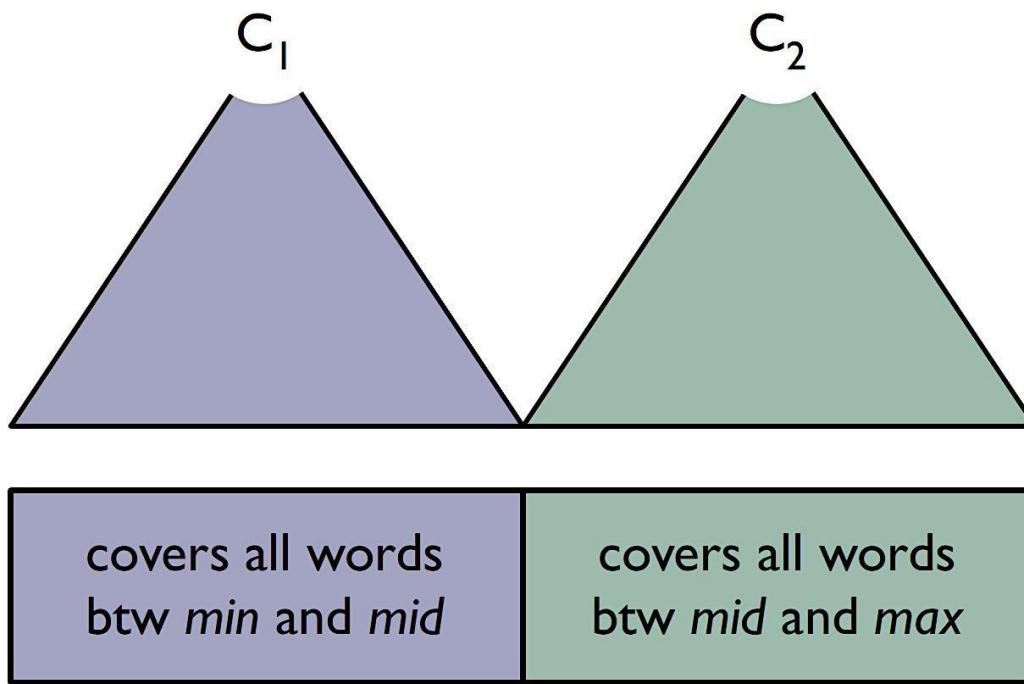


covers all words  
between  $i - l$  and  $i$



# Parsing longer spans

$C \rightarrow C_1 \ C_2$

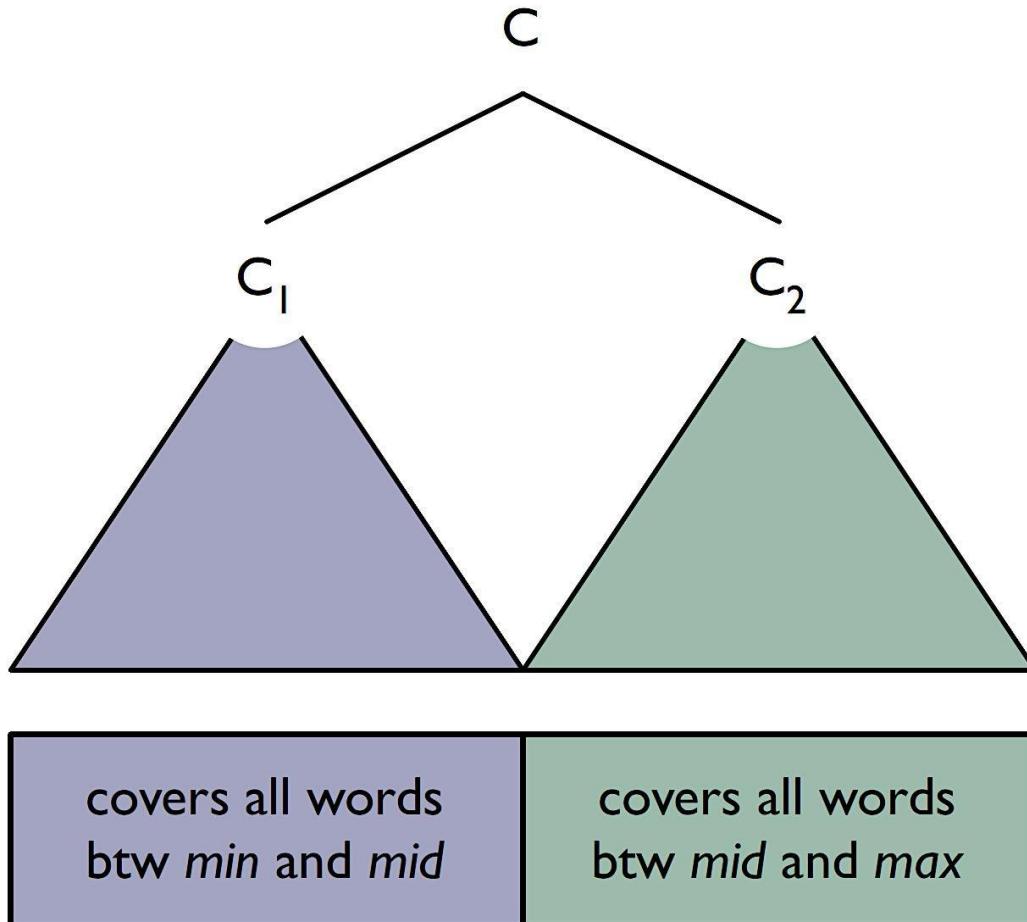


Check through all  
 $C_1, C_2, \text{mid}$



# Parsing longer spans

$C \rightarrow C_1 \ C_2$

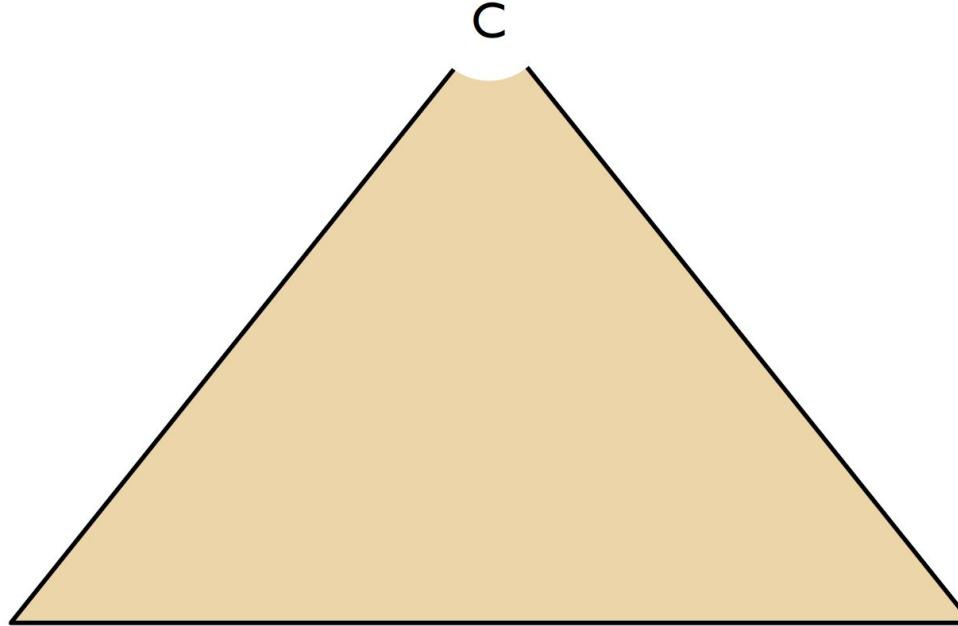


Check through all  
C<sub>1</sub>, C<sub>2</sub>, mid



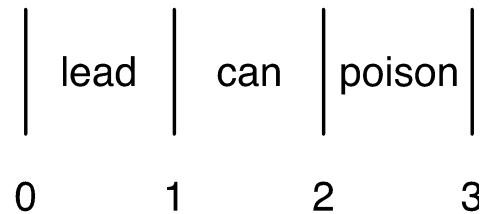
# Parsing longer spans

---



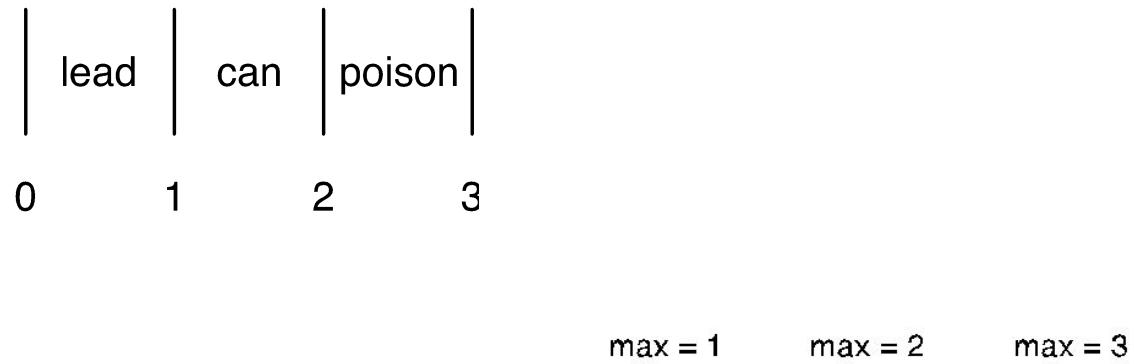
covers all words  
between *min* and *max*

# CKY in action


$$S \rightarrow NP \ VP$$
$$VP \rightarrow M \ V$$
$$VP \rightarrow V$$
$$NP \rightarrow N$$
$$NP \rightarrow N \ NP$$
$$N \rightarrow can$$
$$N \rightarrow lead$$
$$N \rightarrow poison$$
$$M \rightarrow can$$
$$M \rightarrow must$$
$$V \rightarrow poison$$
$$V \rightarrow lead$$

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

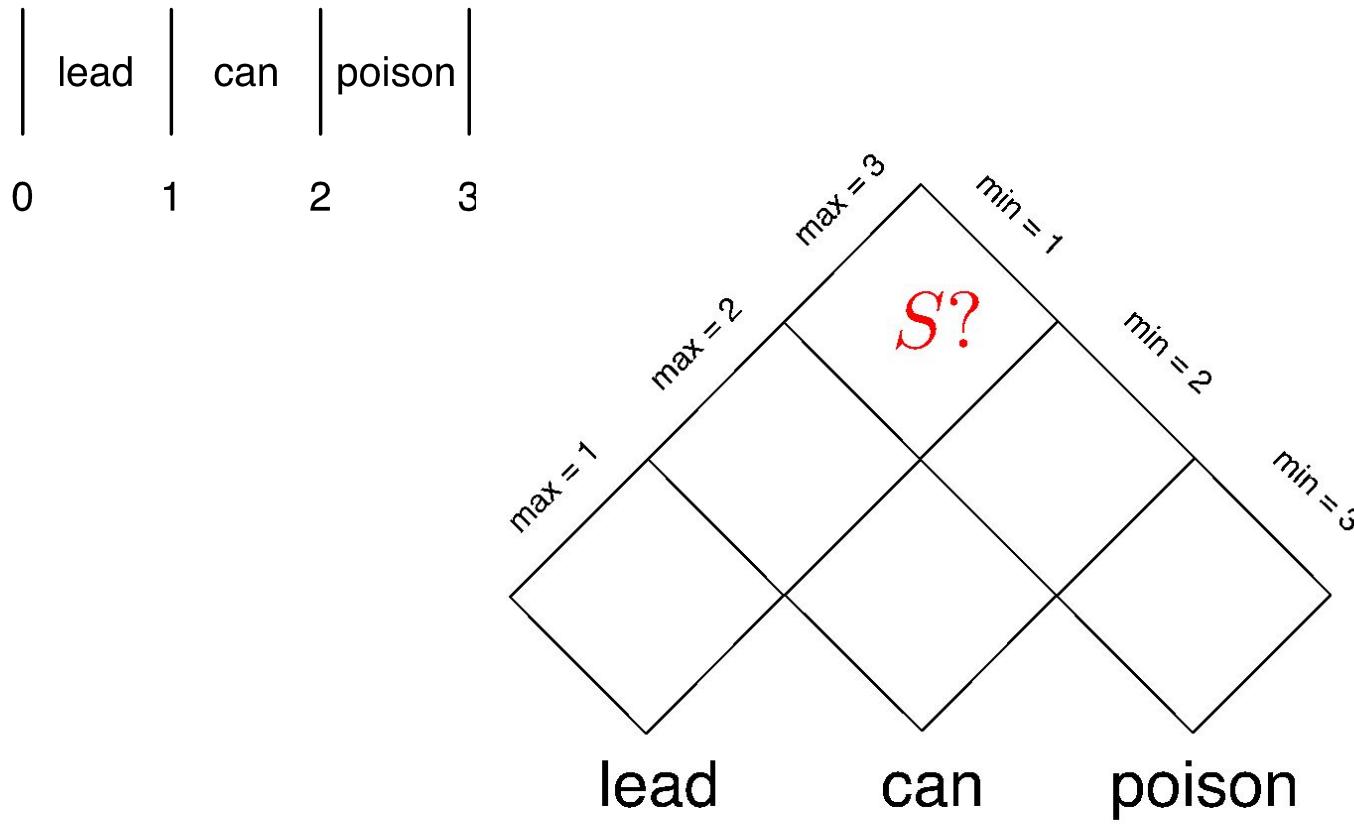
min = 0			<i>S?</i>
min = 1			
min = 2			

Chart (aka  
parsing  
triangle)

 $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$  $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

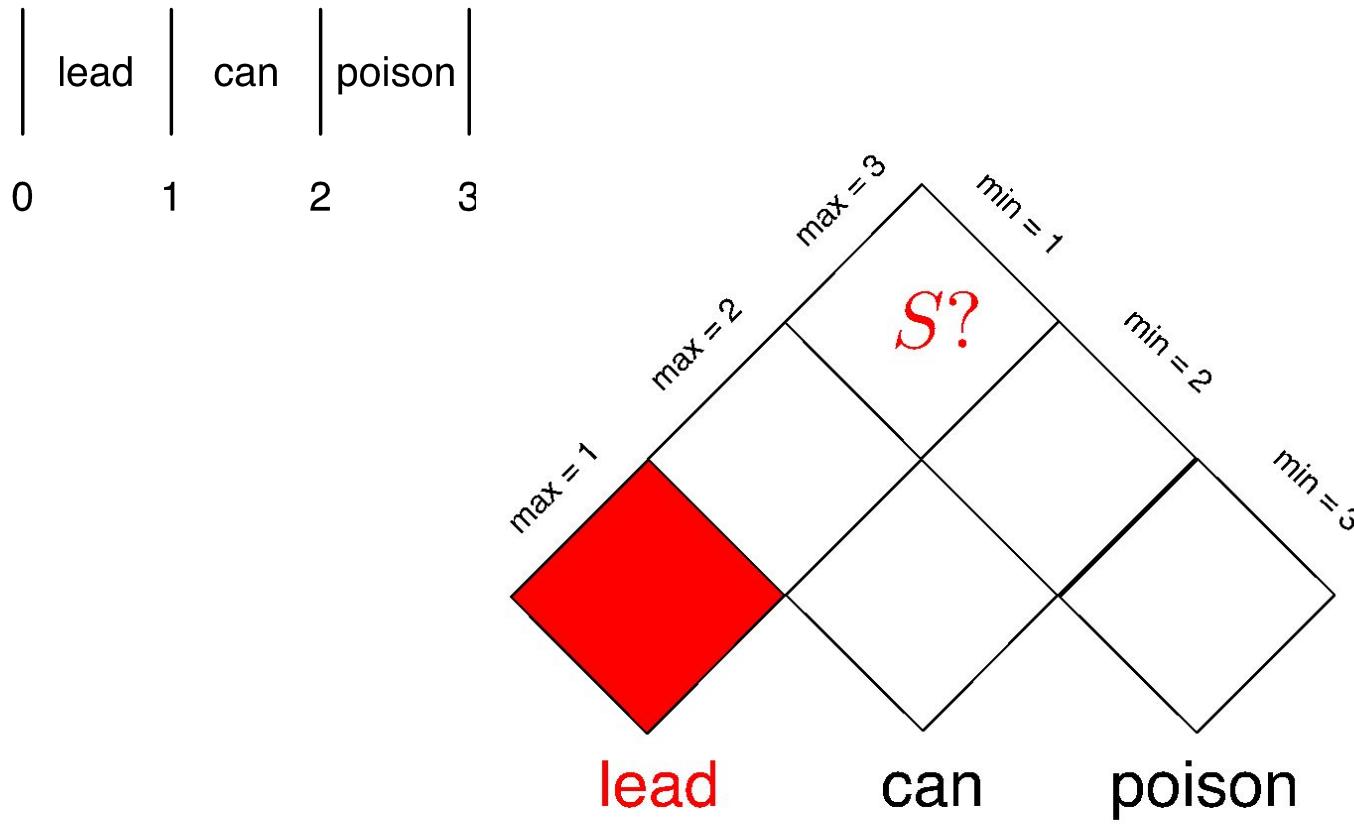
$S \rightarrow NP VP$  $VP \rightarrow M V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

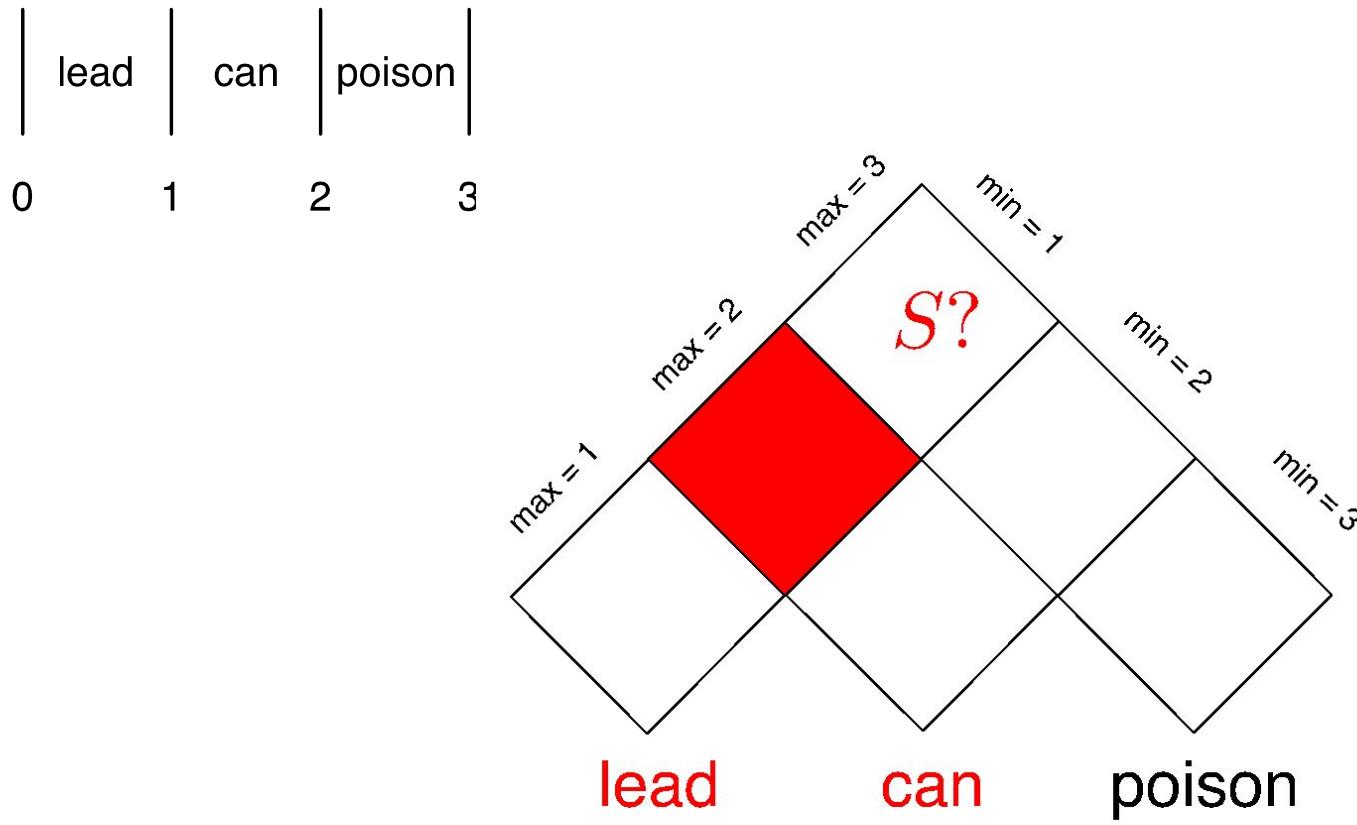
$S \rightarrow NP VP$  $VP \rightarrow M V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

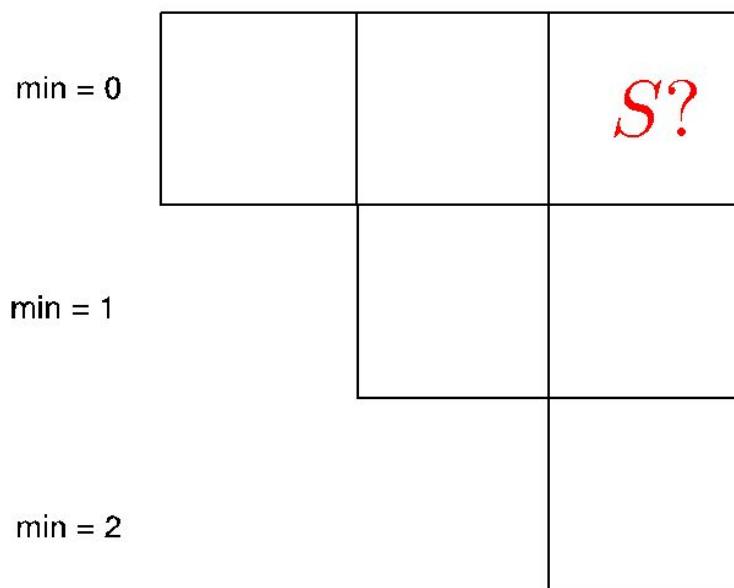
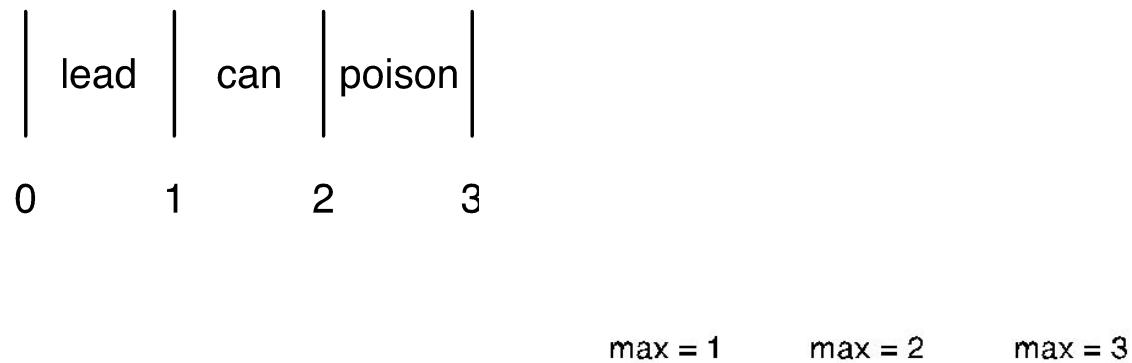
$S \rightarrow NP VP$  $VP \rightarrow M V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

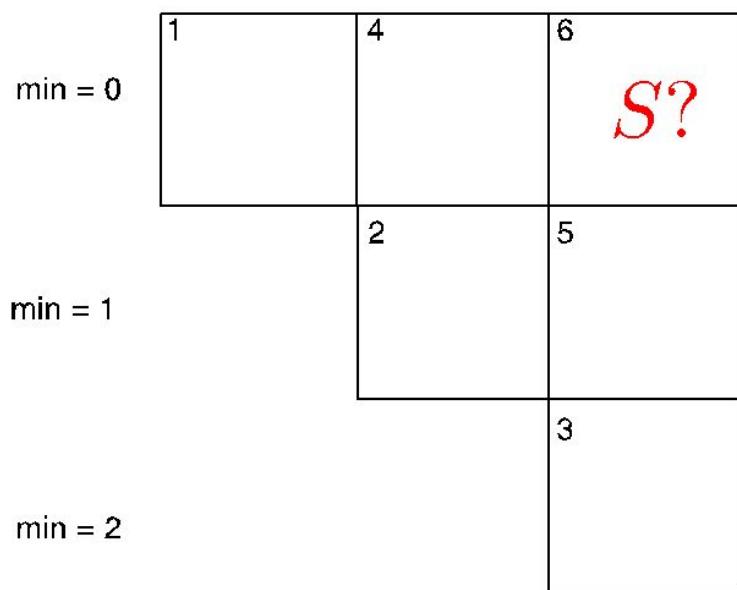
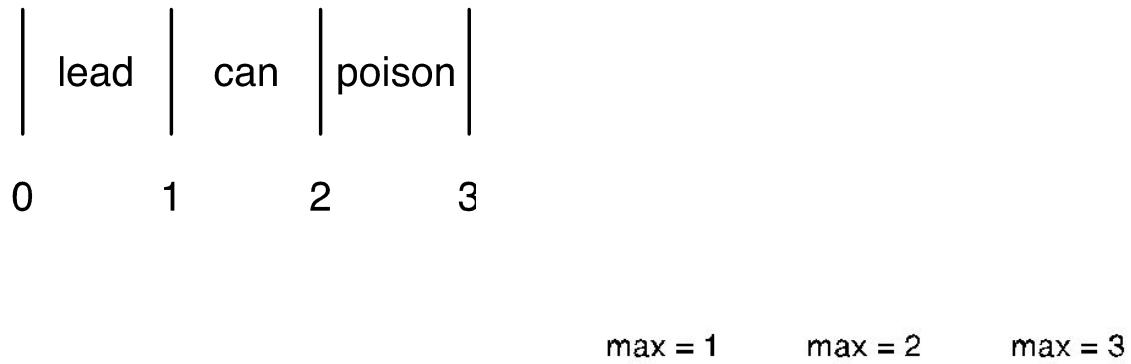
$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

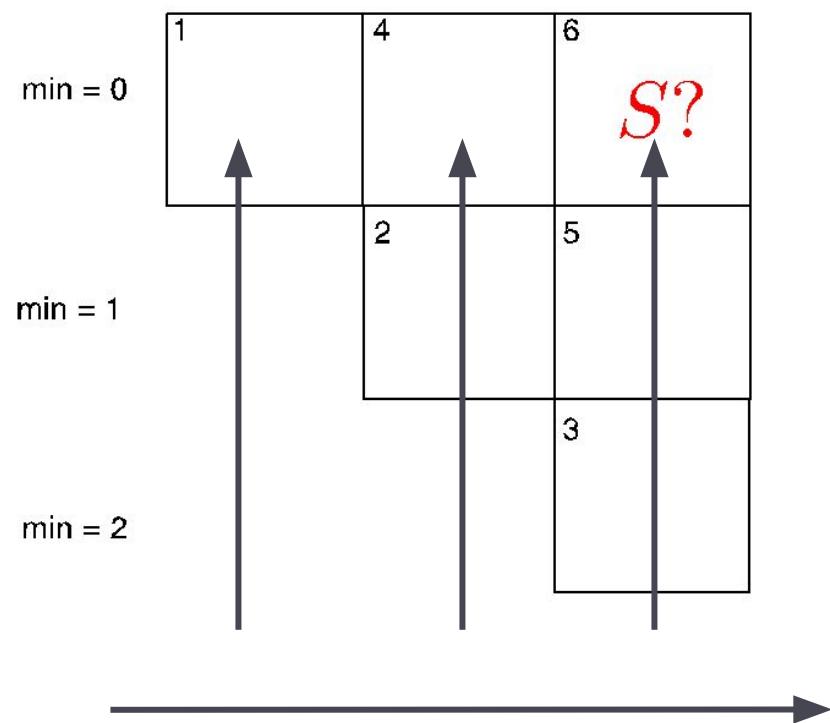
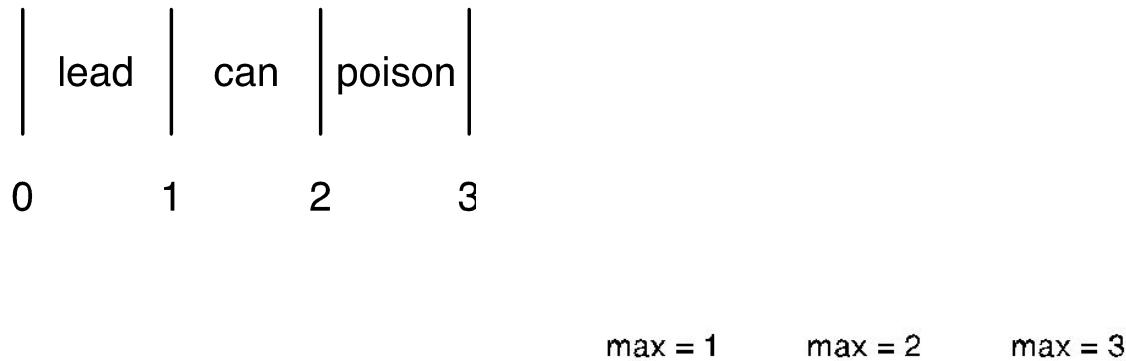
$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$  $NP \rightarrow N \ NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

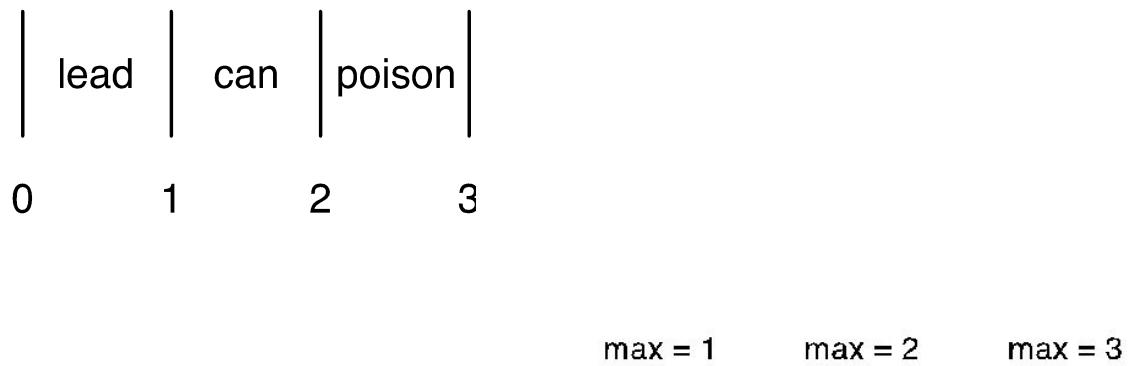
$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

1 min = 0	?		
2 min = 1	?		
3 min = 2		?	

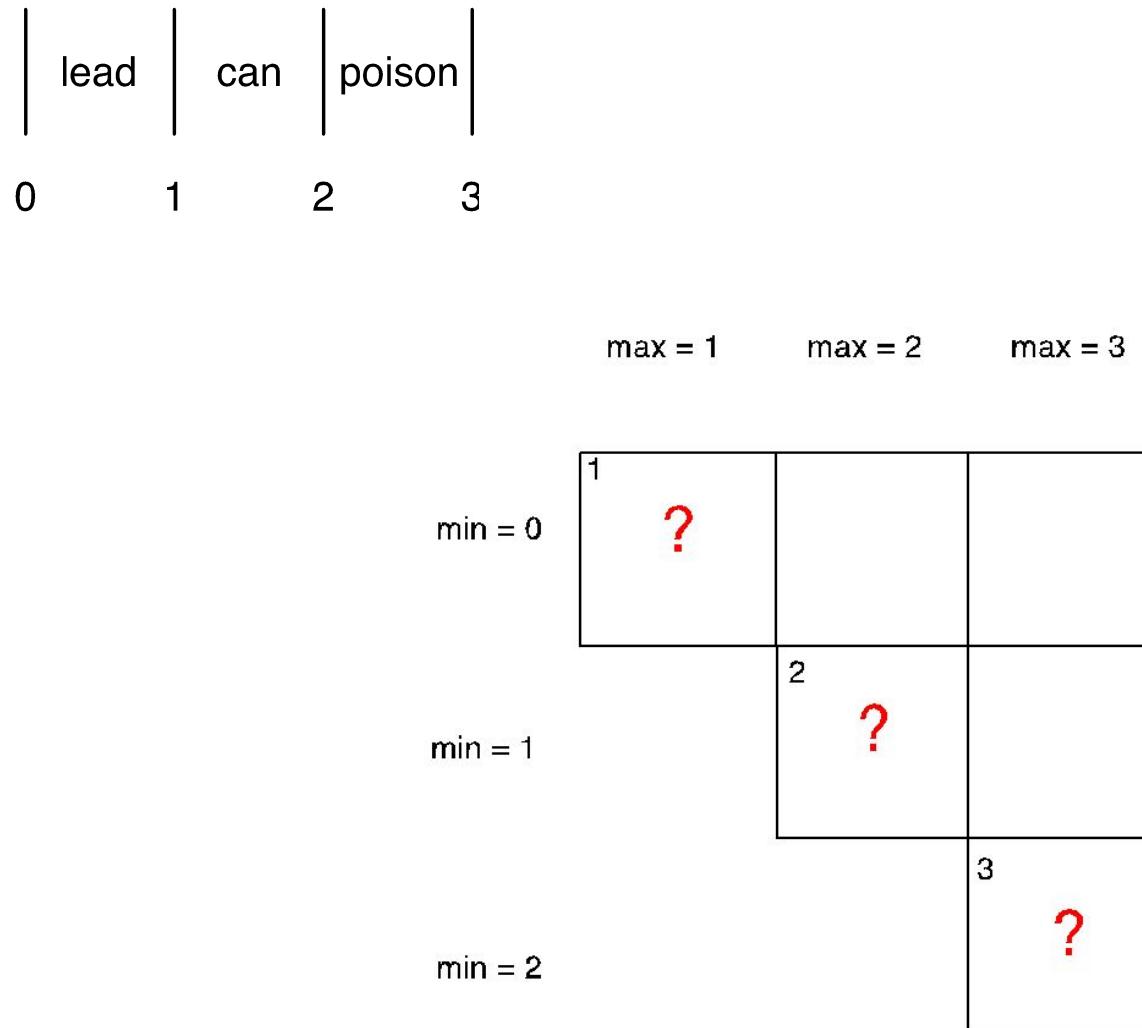
 $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$  $NP \rightarrow N \ NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

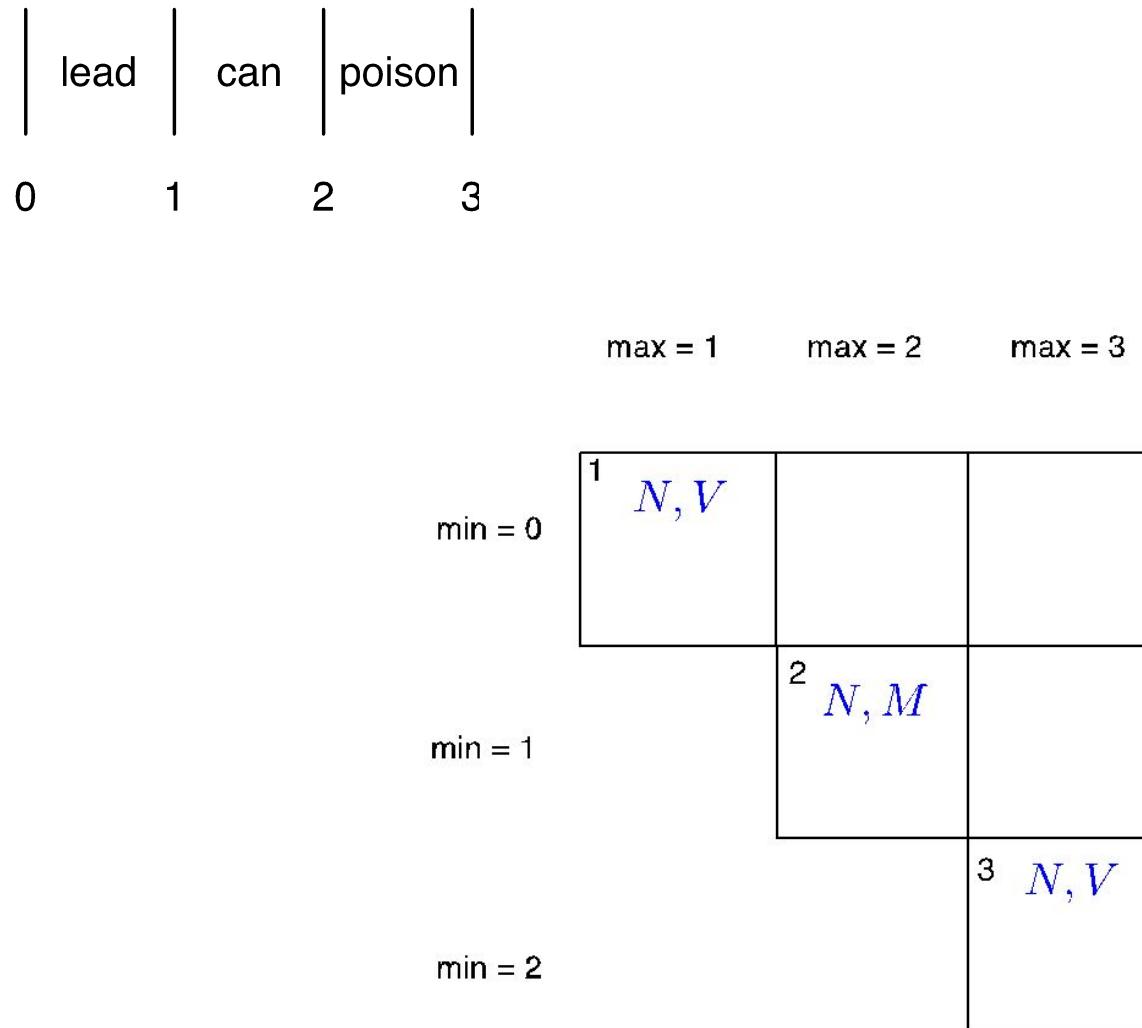
Inner rules

Preterminal rules

$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$  $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$  $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

	<sup>1</sup> $N, V$ $NP, VP$	
<sup>min = 0</sup>		
<sup>min = 1</sup>	<sup>2</sup> $N, M$ $NP$	
<sup>min = 2</sup>		<sup>3</sup> $N, V$ $NP, VP$

$VP \rightarrow M \ V$   
 $VP \rightarrow V$   
 $NP \rightarrow N$   
 $NP \rightarrow N \ NP$

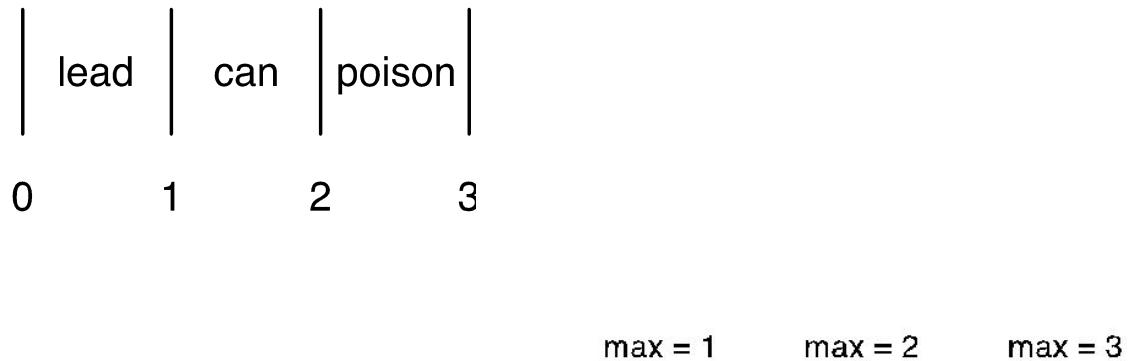
$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

	$1 \ N, V$ $NP, VP$	4 ?	
min = 0			
min = 1	$2 \ N, M$ $NP$		
min = 2		$3 \ N, V$ $NP, VP$	

 $VP \rightarrow M \ V$   
 $VP \rightarrow V$ 
 $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 
 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$ 
 $M \rightarrow can$   
 $M \rightarrow must$ 
 $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

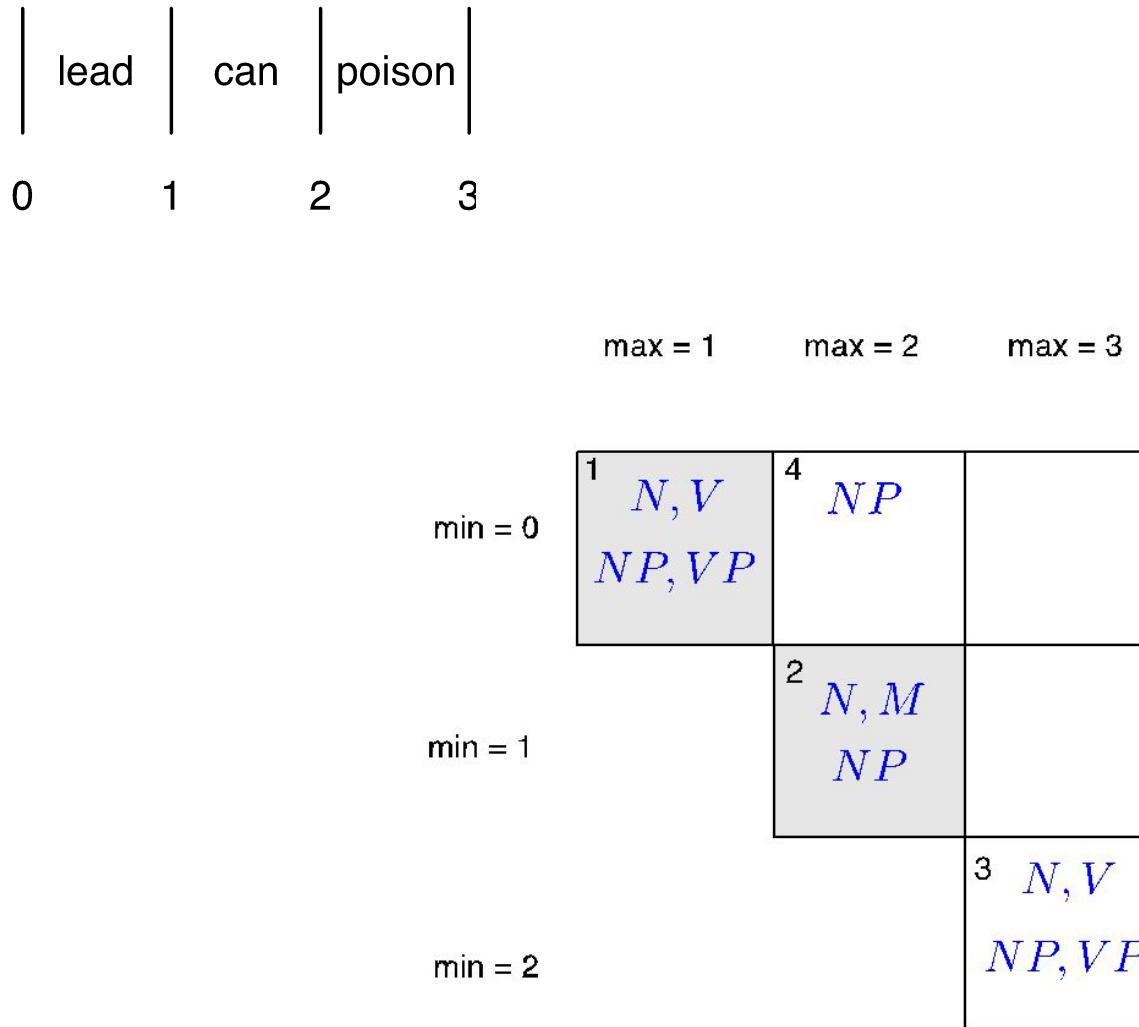
$S \rightarrow NP \ VP$ 

	lead	can	poison	
0	1	2	3	
	max = 1	max = 2	max = 3	
min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 ?		
min = 1		2 <i>N, M</i> <i>NP</i>		
min = 2			3 <i>N, V</i> <i>NP, VP</i>	

 $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N \ NP$  $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$  $VP \rightarrow M \ V$  $VP \rightarrow V$  $NP \rightarrow N$  $NP \rightarrow N \ NP$  $N \rightarrow can$  $N \rightarrow lead$  $N \rightarrow poison$  $M \rightarrow can$  $M \rightarrow must$  $V \rightarrow poison$  $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

lead	can	poison	
0	1	2	3

	max = 1	max = 2	max = 3
min = 0	1 $N, V$ $NP, VP$	4 $NP$	
min = 1		2 $N, M$ $NP$	
min = 2			3 $N, V$ $NP, VP$

Check about  
unary rules: no  
unary rules  
here

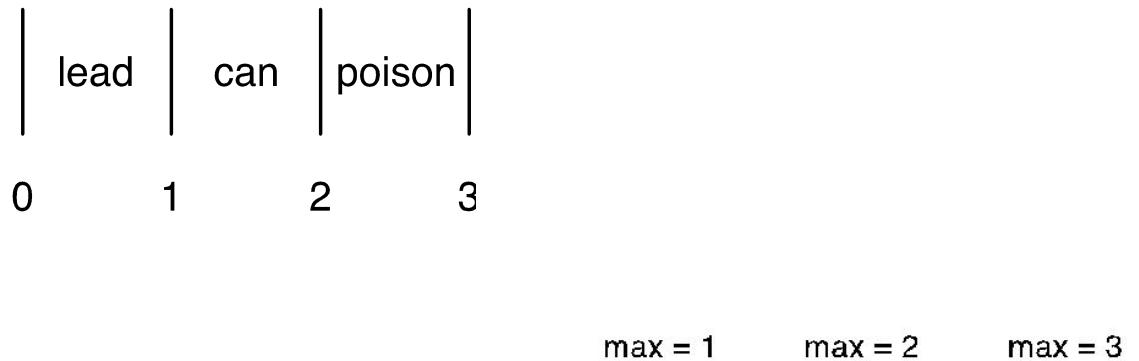
 $VP \rightarrow M \ V$   
 $VP \rightarrow V$  $NP \rightarrow N$  $NP \rightarrow N \ NP$ 

---

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 

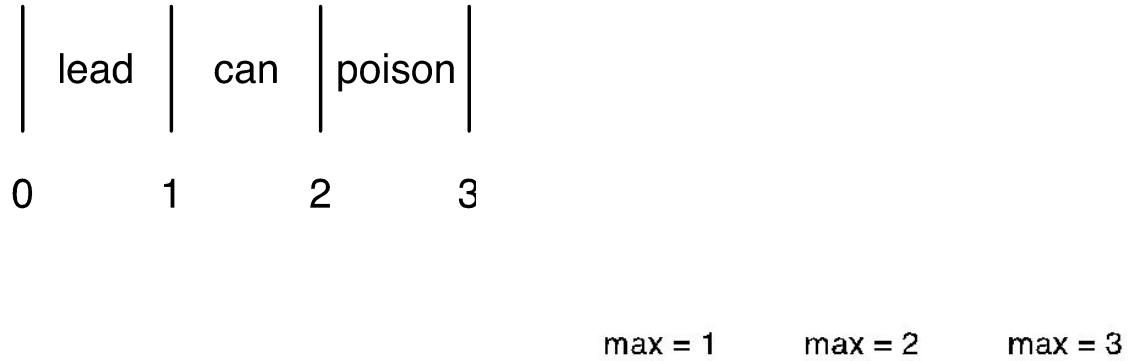
	$1 \ N, V$ $NP, VP$	$4 \ NP$	
min = 0			
min = 1	$2 \ N, M$ $NP$	$5 \ ?$	
min = 2		$3 \ N, V$ $NP, VP$	

 $VP \rightarrow M \ V$   
 $VP \rightarrow V$ 
 $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 
 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$ 
 $M \rightarrow can$   
 $M \rightarrow must$ 
 $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

Inner rules



	1 $N, V$ $NP, VP$	4 $NP$	
min = 0			
min = 1	2 $N, M$ $NP$	5 $S, VP,$ $NP$	
min = 2		3 $N, V$ $NP, VP$	

$S \rightarrow NP \ VP$

$VP \rightarrow M \ V$   
 $VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N \ NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules

# CKY in action

	lead	can	poison	
0	1	2	3	
	max = 1	max = 2	max = 3	
min = 0	1 N, V NP, VP	4 NP		
min = 1	2 N, M NP	5 S, VP, NP		
min = 2		3 N, V NP, VP		

Check about  
unary rules: no  
unary rules  
here

$$S \rightarrow NP \ VP$$

$$VP \rightarrow M \ V$$

$$VP \rightarrow V$$

$$NP \rightarrow N$$

$$NP \rightarrow N \ NP$$

$$N \rightarrow can$$

$$N \rightarrow lead$$

$$N \rightarrow poison$$

$$M \rightarrow can$$

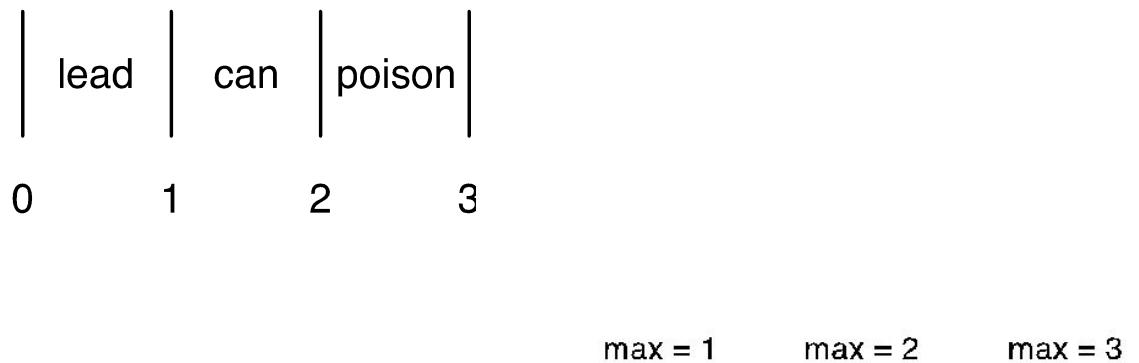
$$M \rightarrow must$$

$$V \rightarrow poison$$

$$V \rightarrow lead$$

Inner rules

Preterminal rules

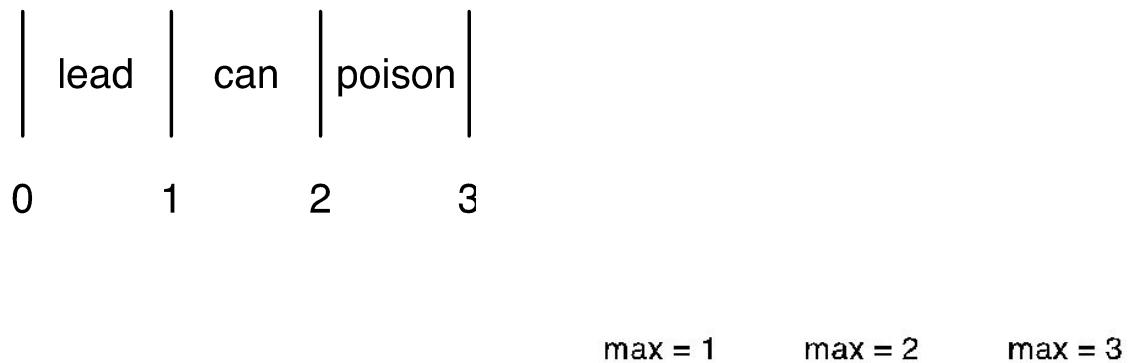
$S \rightarrow NP \ VP$ 

	$1 \ N, V$ $NP, VP$	$4 \ NP$	$6 \ ?$
min = 0			
min = 1	$2 \ N, M$ $NP$	$5 \ S, VP,$ $NP$	
min = 2		$3 \ N, V$ $NP, VP$	

 $VP \rightarrow M \ V$   
 $VP \rightarrow V$ 
 $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 
 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$ 
 $M \rightarrow can$   
 $M \rightarrow must$ 
 $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$S \rightarrow NP \ VP$ 
 $VP \rightarrow M \ V$   
 $VP \rightarrow V$ 
 $NP \rightarrow N$   
 $NP \rightarrow N \ NP$ 

	1 $N, V$ $NP, VP$	4 $NP$	6 ?
min = 0			
min = 1	2 $N, M$ $NP$	5 $S, VP,$ $NP$	
min = 2		3 $N, V$ $NP, VP$	

 $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$ 
 $M \rightarrow can$   
 $M \rightarrow must$ 
 $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$$S \rightarrow NP \ VP$$

lead	can	poison	
0	1	2	3

max = 1      max = 2      max = 3

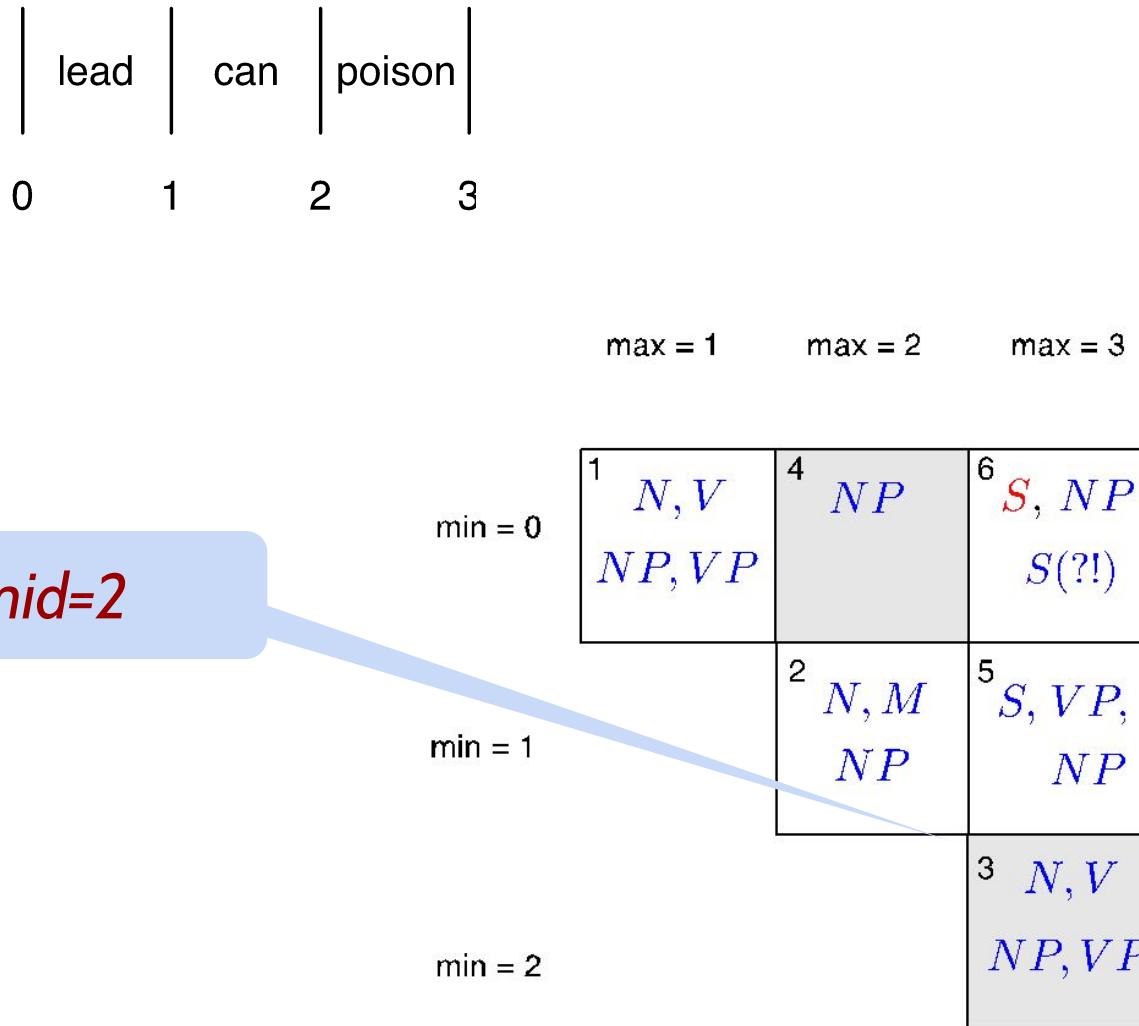
*mid=1*

$N, V$ $NP, VP$	$NP$	$S, NP$
$N, M$ $NP$	$S, VP,$ $NP$	
	$N, V$ $NP, VP$	

$$VP \rightarrow M \ V$$
$$VP \rightarrow V$$
$$NP \rightarrow N$$
$$NP \rightarrow N \ NP$$
$$N \rightarrow can$$
$$N \rightarrow lead$$
$$N \rightarrow poison$$
$$M \rightarrow can$$
$$M \rightarrow must$$
$$V \rightarrow poison$$
$$V \rightarrow lead$$

Inner rules

Preterminal rules

$S \rightarrow NP VP$  $VP \rightarrow M V$   
 $VP \rightarrow V$  $NP \rightarrow N$   
 $NP \rightarrow N NP$  $N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$  $M \rightarrow can$   
 $M \rightarrow must$  $V \rightarrow poison$   
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

$$S \rightarrow NP \ VP$$

	lead	can	poison	
0	1	2	3	
	max = 1	max = 2	max = 3	
min = 0	$^1 N, V$ $NP, VP$	$^4 NP$	$^6 S, NP$ $S(?)$	
min = 1		$^2 N, M$ $NP$	$^5 S, VP,$ $NP$	
min = 2			$^3 N, V$ $NP, VP$	

$$VP \rightarrow M \ V$$
$$VP \rightarrow V$$
$$NP \rightarrow N$$
$$NP \rightarrow N \ NP$$
$$N \rightarrow can$$
$$N \rightarrow lead$$
$$N \rightarrow poison$$
$$M \rightarrow can$$
$$M \rightarrow must$$
$$V \rightarrow poison$$
$$V \rightarrow lead$$

Apparently the sentence is ambiguous for the grammar: (as the grammar overgenerates)

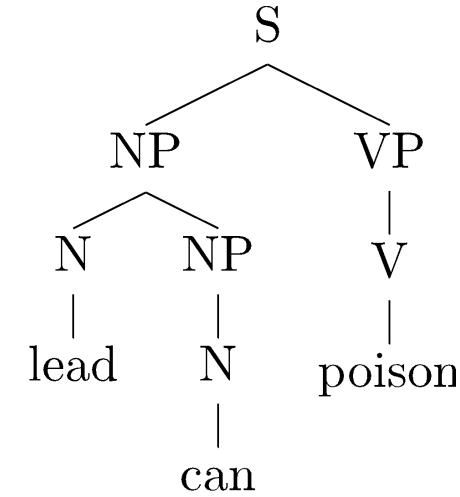
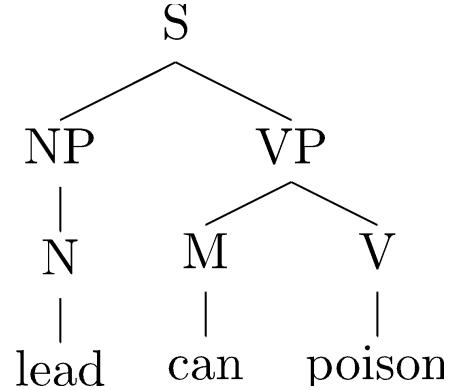
Inner rules

Preterminal rules



# Ambiguity

- 



No subject-verb agreement, and  
*poison* used as an intransitive verb



# CKY more formally

Here we assume that labels (C) are integer indices

Chart can be represented by a Boolean 3D array chart [min] [max] [C]

- Relevant entries have  $0 < \text{min} < \text{max} \leq n$

chart [min] [max] [C] = true if the signature (min, max, C) is already added to the chart;  
false otherwise.

		max = 1	max = 2	max = 3
min = 0		1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>	6 <i>S, VP,</i> <i>NP</i>
min = 1			2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i>
min = 2				3 <i>N, V</i> <i>NP, VP</i>



# Implementation: preterminal rules

---

```
for each  $w_i$  from left to right  
  for each preterminal rule  $C \rightarrow w_i$   
    chart[i - 1][i][C] = true
```



# Implementation: binary rules

```
for each max from 2 to n
```

```
    for each min from max - 2 down to 0
```

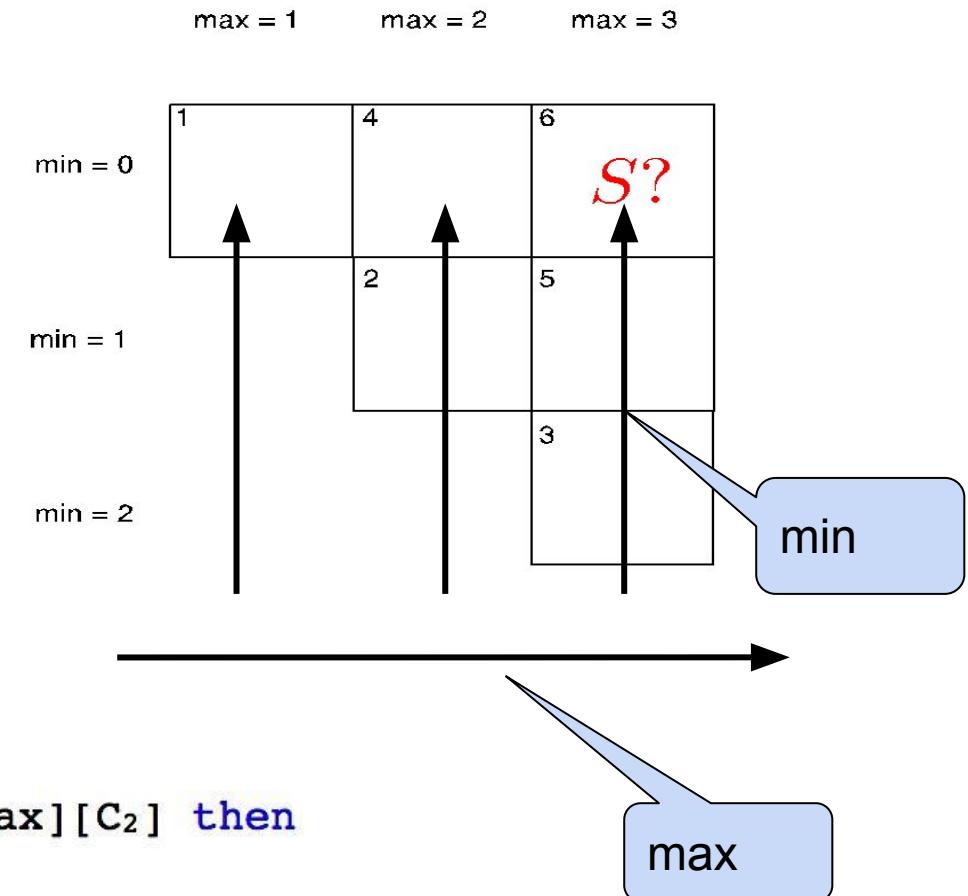
```
        for each syntactic category C
```

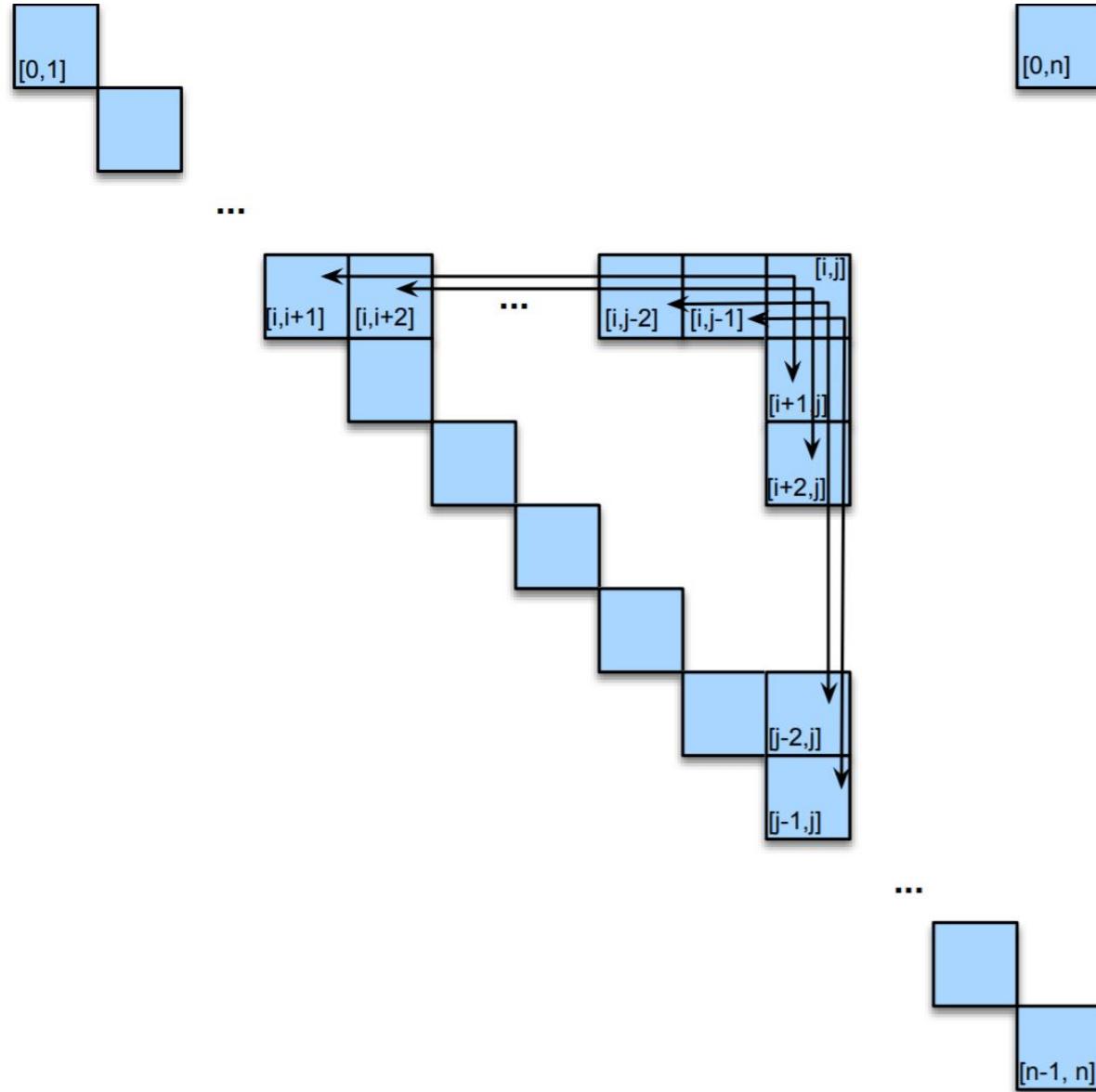
```
            for each binary rule C → C1 C2
```

```
                for each mid from min + 1 to max - 1
```

```
                    if chart[min][mid][C1] and chart[mid][max][C2] then
```

```
                        chart[min][max][C] = true
```







# Unary rules

---

- How to integrate unary rules  $C \rightarrow C_1$  ?



# Unary rules

- How to integrate unary rules  $C \rightarrow C_1$  ?

```
for each max from 1 to n ←
    for each min from max - 1 down to 0 → new bounds!
        // First, try all binary rules as before.

        ...
        // Then, try all unary rules.

        for each syntactic category C

            for each unary rule C -> C1

                if chart[min][max][C1] then

                    chart[min][max][C] = true
```



# Unary rules

- How to integrate unary rules  $C \rightarrow C_1$  ?

```
for each max from 1 to n ←  
      for each min from max - 1 down to 0 → new bounds!  
  
      // First, try all binary rules as before.  
  
      ...  
  
      // Then, try all unary rules.  
  
      for each syntactic category C  
  
          for each unary rule C -> C1  
  
              if chart[min][max][C1] then
```

But we forgot something!



# Unary closure

---

- What if the grammar contained 2 rules:

$$A \rightarrow B$$

$$B \rightarrow C$$

- But C can be derived from A by a chain of rules:

$$A \rightarrow B \rightarrow C$$

- One could support chains in the algorithm but it is easier to extend the grammar, to get the **transitive closure**

$$A \rightarrow B$$

$$B \rightarrow C$$

 $\Rightarrow$ 

$$A \rightarrow B$$

$$B \rightarrow C$$

$$A \rightarrow C$$



# Unary closure

---

- What if the grammar contained 2 rules:

$$A \rightarrow B$$

$$B \rightarrow C$$

- But C can be derived from A by a chain of rules:

$$A \rightarrow B \rightarrow C$$

- One could support chains in the algorithm but it is easier to extend the grammar, to get the **transitive closure**

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array}$$

 $\Rightarrow$ 

$$\begin{array}{lll} A \rightarrow B & A \rightarrow A & \\ B \rightarrow C & B \rightarrow B & \\ A \rightarrow C & C \rightarrow C & \end{array}$$

Convenient for  
programming  
reasons in the PCFG  
case



# Algorithm analysis

---

Time complexity?

for each max from 2 to n

    for each min from max - 2 down to 0

        for each syntactic category C

            for each binary rule  $C \rightarrow C_1 C_2$

                for each mid from min + 1 to max - 1



# Algorithm analysis

---

Time complexity?

for each max from 2 to n

    for each min from max - 2 down to 0

        for each syntactic category C

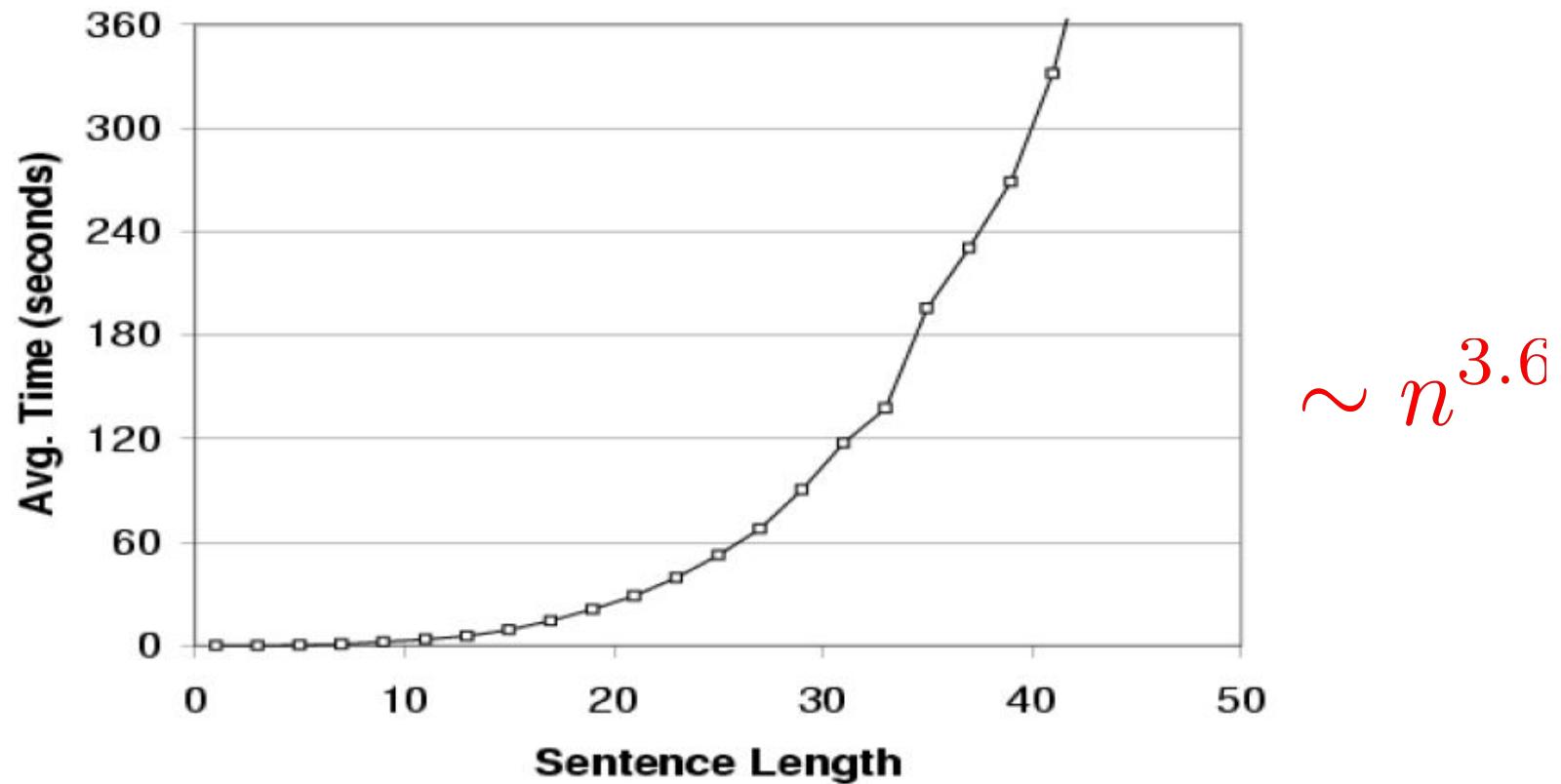
            for each binary rule C → C<sub>1</sub> C<sub>2</sub>

                for each mid from min + 1 to max - 1

$O(n^3|R|)$  where  $|R|$  is the number of rules in the grammar



# Practical time complexity

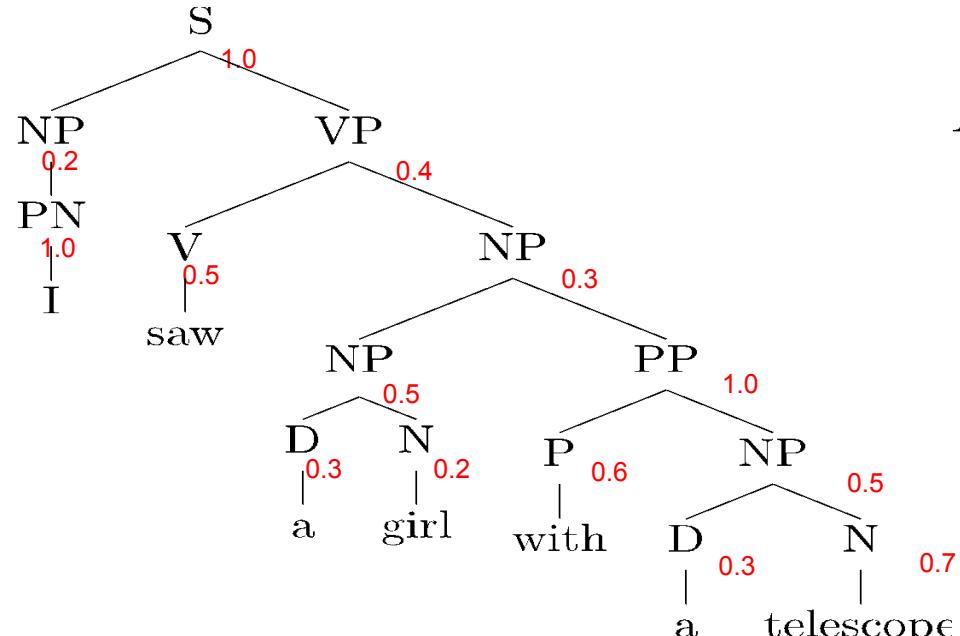




# Probabilistic CKY

---

# PCFGs



$$\begin{aligned}
 p(T) &= 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times \\
 &\quad 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 \\
 &= 2.26 \times 10^{-5}
 \end{aligned}$$

$S \rightarrow NP \ VP \ 1.0$

$VP \rightarrow V \ 0.2$

$VP \rightarrow V \ NP \ 0.4$

$VP \rightarrow VP \ PF \ 0.4$

$NP \rightarrow NP \ PF \ 0.3$

$NP \rightarrow D \ N \ 0.5$

$NP \rightarrow PN \ 0.2$

$PP \rightarrow P \ NP \ 1.0$

$N \rightarrow girl \ 0.2$

$N \rightarrow telescope \ 0.7$

$N \rightarrow sandwich \ 0.1$

$PN \rightarrow I \ 1.0$

$V \rightarrow saw \ 0.5$

$V \rightarrow ate \ 0.5$

$P \rightarrow with \ 0.6$

$P \rightarrow in \ 0.4$

$D \rightarrow a \ 0.3$

$D \rightarrow the \ 0.7$



# CKY with PCFGs

---

- Chart is represented by a 3d array of floats  
`chart [min] [max] [label]`
  - It stores probabilities for the most probable subtree with a given signature
- `chart [0] [n] [S]` will store the probability of the most probable full parse tree
-



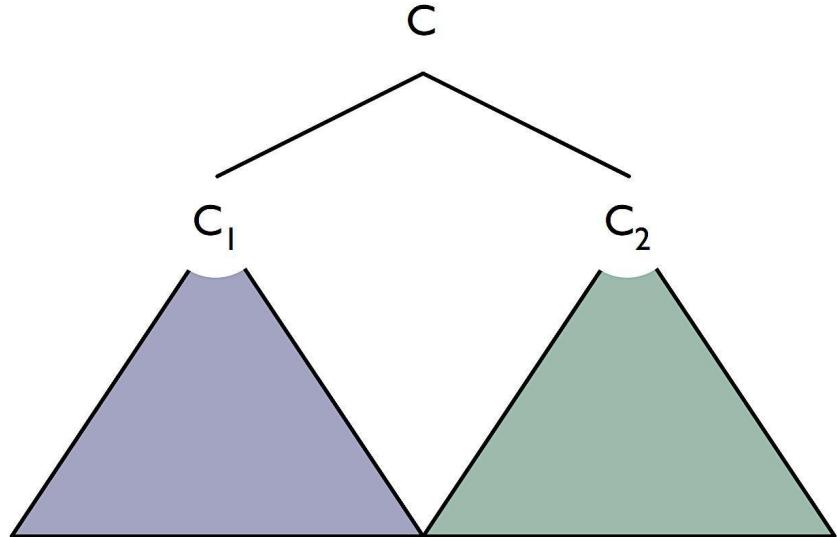
# Intuition

$$C \rightarrow C_1 \ C_2$$

For every  $C$  choose  $C_1, C_2$  and mid such that

$$P(T_1) \times P(T_2) \times P(C \rightarrow C_1 C_2)$$

is maximal, where  $T_1$  and  $T_2$  are left and right subtrees.



covers all words  
btw *min* and *mid*

covers all words  
btw *mid* and *max*



# Implementation: preterminal rules

---

```
for each  $w_i$  from left to right  
  for each preterminal rule  $C \rightarrow w_i$   
    chart[i - 1][i][C] = p( $C \rightarrow w_i$ )
```



# Implementation: binary rules

```
for each max from 2 to n

    for each min from max - 2 down to 0

        for each syntactic category C
            -----
            | double best = undefined
            |
            for each binary rule C → C1 C2

                for each mid from min + 1 to max - 1

                    double t1 = chart[min][mid][C1]

                    double t2 = chart[mid][max][C2]

                    double candidate = t1 * t2 * p(C → C1 C2)
                    -----
                    | if candidate > best then
                    |
                    |     best = candidate
                    |
                    -----
                    chart[min][max][C] = best
```



# Unary rules

---

- Similarly to CFGs: after producing scores for signatures (c, i, j), try to improve the scores by applying unary rules (and rule chains)
  - If improved, update the scores



# Unary (reflexive transitive) closure

$$\begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ \dots & \end{array} \Rightarrow \begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ A \rightarrow C & 0.2 \times 0.1 \\ \dots & \end{array} \begin{array}{ll} A \rightarrow A & 1 \\ B \rightarrow B & 1 \\ C \rightarrow C & 1 \end{array}$$

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent



# Unary (reflexive transition) closure

The fact that the rule is composite needs to be stored to recover the true tree

$$\begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ \dots & \end{array} \Rightarrow \begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ A \rightarrow C & 0.2 \times 0.1 \\ \dots & \end{array} \quad \begin{array}{ll} A \rightarrow A & 1 \\ B \rightarrow B & 1 \\ C \rightarrow C & 1 \\ \dots & \end{array}$$

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent



# Unary (reflexive transition) language

The fact that the rule is composite needs to be stored to recover the true tree

$$\begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ \dots & \end{array} \Rightarrow \begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ A \rightarrow C & 0.2 \times 0.1 \\ \dots & \end{array} \quad \begin{array}{ll} A \rightarrow A & 1 \\ B \rightarrow B & 1 \\ C \rightarrow C & 1 \\ \dots & \end{array}$$

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent

$$\begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.2 \\ A \rightarrow C & 1.e-5 \end{array} \Rightarrow \begin{array}{ll} A \rightarrow B & 0.1 \\ B \rightarrow C & 0.1 \\ A \rightarrow C & 0.02 \end{array} \quad \begin{array}{ll} A \rightarrow A & 1 \\ B \rightarrow B & 1 \\ C \rightarrow C & 1 \end{array}$$

What about loops, like:  $A \rightarrow B \rightarrow A \rightarrow C$  ?



# Recovery of the tree

---

- For each signature we store backpointers to the elements from which it was built (e.g., rule and, for binary rules, midpoint)
  - start recovering from  $[0, n, S]$
- Be careful with unary rules
  - Basically you can assume that you always used an unary rule from the closure (but it could be the trivial one  $C \rightarrow C$ )



# Speeding up the algorithm (approximate search)

Any ideas?



# Speeding up the algorithm

---

- Basic pruning (roughly):
  - For every span  $(i,j)$  store only labels which have the probability at most  $N$  times smaller than the probability of the most probable label for this span
  - Check not all rules but only rules yielding subtree labels having non-zero probability
- Coarse-to-fine pruning
  - Parse with a smaller (simpler) grammar, and precompute (posterior) probabilities for each spans, and use only the ones with non-negligible probability from the previous grammar



# Parsing evaluation

---

- **Intrinsic evaluation:**
  - **Automatic:** evaluate against annotation provided by human experts (gold standard) according to some predefined measure
  - **Manual:** ... according to human judgment
- **Extrinsic evaluation:** score syntactic representation by comparing how well a system using this representation performs on some task
  - E.g., use syntactic representation as input for a semantic analyzer and compare results of the analyzer using syntax predicted by different parsers.



# Standard evaluation setting in parsing

---

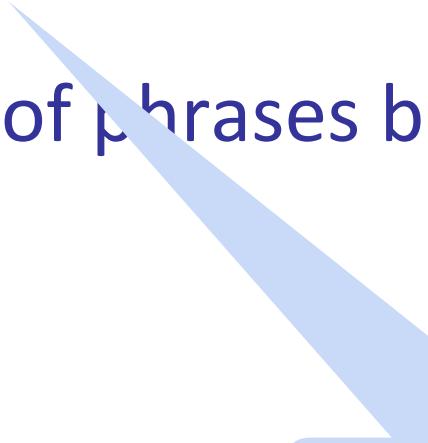
- Automatic intrinsic evaluation is used: parsers are evaluated against gold standard by provided by linguists
  - There is a standard split into the parts:
    - training set: used for estimation of model parameters
    - development set: used for tuning the model (initial experiments)
    - test set: final experiments to compare against previous work



# Automatic evaluation of constituent parsers

---

- **Exact match:** percentage of trees predicted correctly
- **Bracket score:** scores how well individual phrases (and their boundaries) are identified
- **Crossing brackets:** percentage of phrases boundaries crossing



The most standard measure;  
we will focus on it



# Brackets scores

Subtree signatures for  
CKY

- The most standard score is **bracket score**
- It regards a tree as a collection of brackets: $[min, max, C]$
- The set of brackets predicted by a parser is compared against the set of brackets in the tree annotated by a linguist
- Precision, recall and F1 are used as scores



# Preview: F1 bracket score

---

