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TC (complexity)

In theoretical computer science, and specifically computational complexity theory and circuit complexity, **TC** is a complexity class of decision problems that can be recognized by threshold circuits, which are Boolean circuits with AND, OR, and Majority gates. For each fixed i, the complexity class $\mathbf{TC^i}$ consists of all languages that can be recognized by a family of threshold circuits of depth $O(\log^i n)$, polynomial size, and unbounded fan-in. The class **TC** is defined via

$$\mathbf{TC} = \bigcup_{i \geq 0} \mathbf{TC}^i.$$

Relation to NC and AC

The relationship between the TC, NC and the AC hierarchy can be summarized as follows:

$$\mathbf{NC}^i \subseteq \mathbf{AC}^i \subseteq \mathbf{TC}^i \subseteq \mathbf{NC}^{i+1}.$$

In particular, we know that

$$\mathbf{NC}^0 \subsetneq \mathbf{AC}^0 \subsetneq \mathbf{TC}^0 \subseteq \mathbf{NC}^1.$$

The first strict containment follows from the fact that $\mathbf{NC^0}$ cannot compute any function that depends on all the input bits. Thus choosing a problem that is trivially in $\mathbf{AC^0}$ and depends on all bits separates the two classes. (For example, consider the OR function.) The strict containment $\mathbf{AC^0} \subsetneq \mathbf{TC^0}$ follows because parity and majority (which are both in $\mathbf{TC^0}$) were shown to be not in $\mathbf{AC^0}$.[1][2]

As an immediate consequence of the above containments, we have that NC = AC = TC.

References

1. Furst, Merrick; Saxe, James B.; Sipser, Michael (1984), "Parity, circuits, and the polynomial-time hierarchy", *Mathematical Systems Theory*, **17** (1): 13–27, doi:10.1007/BF01744431 (https://doi.org/10.1007%2FBF01744431), MR 0738749 (https://mathscinet.ams.org/mathscinet-getitem?mr=0738749).
2. Håstad, Johan (1989), "Almost Optimal Lower Bounds for Small Depth Circuits", in Micali, Silvio (ed.), *Randomness and Computation* (https://web.archive.org/web/20120222163102/http://reference.kfupm.edu.sa/content/a/l/almost_optimal_lower_bounds_for_small_de_134215.pdf) (PDF), Advances in Computing Research, vol. 5, JAI Press, pp. 6–20, ISBN 0-89232-896-7, archived from the original (http://reference.kfupm.edu.sa/content/a/l/almost_optimal_lower_bounds_for_small_de_134215.pdf) (PDF) on 2012-02-22

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