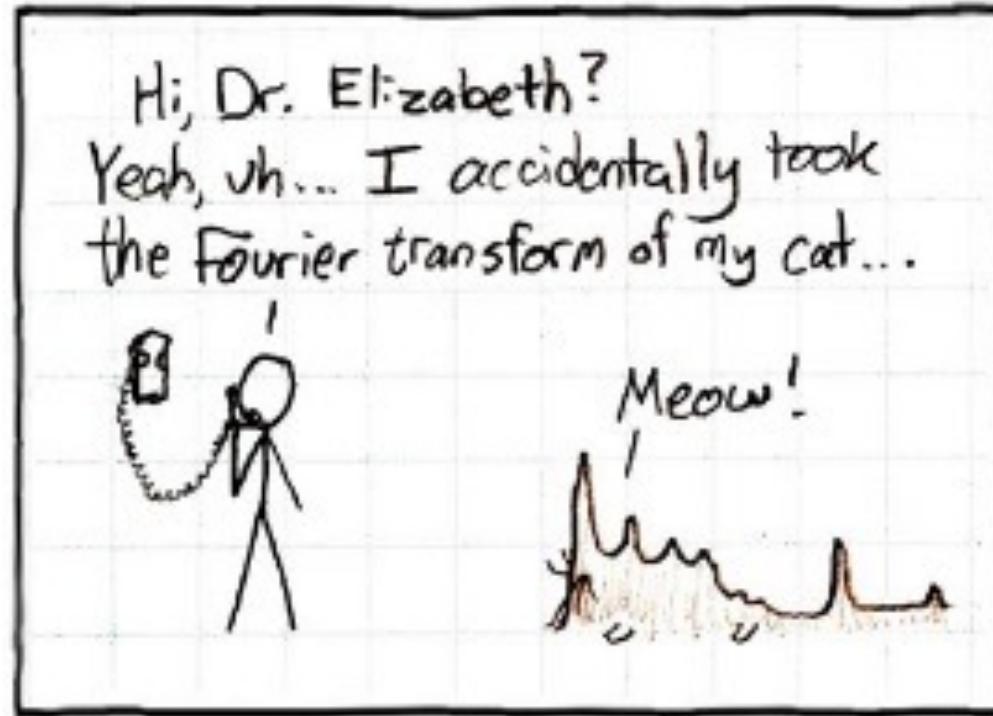


Image pyramids and frequency domain



Overview of today's lecture

- Image downsampling.
- Aliasing.
- Gaussian image pyramid.
- Laplacian image pyramid.
- Fourier series.
- Frequency domain.
- Fourier transform.
- Frequency-domain filtering.
- Revisiting sampling.

Slide credits

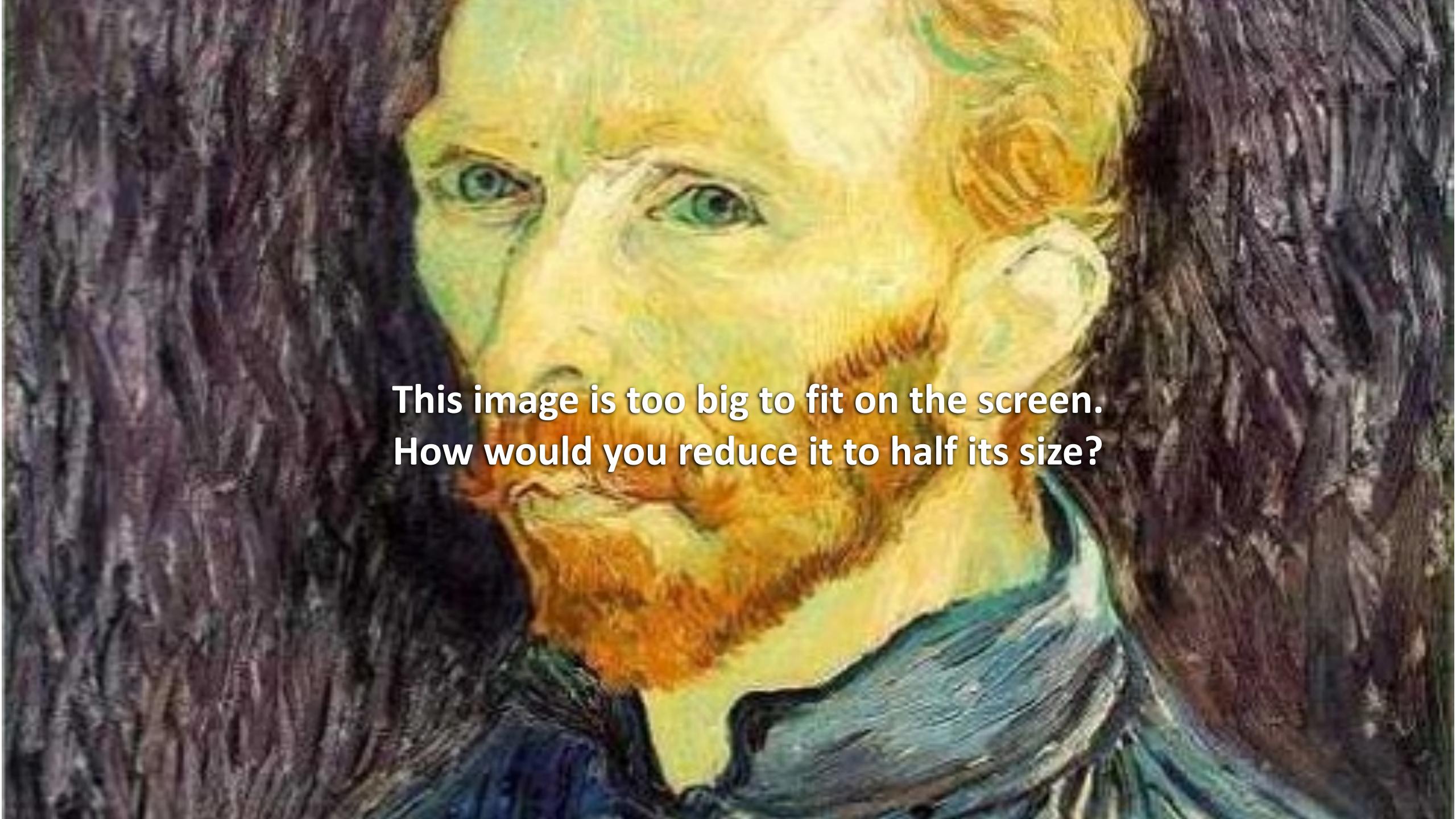
Most of these slides were adapted directly from:

- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

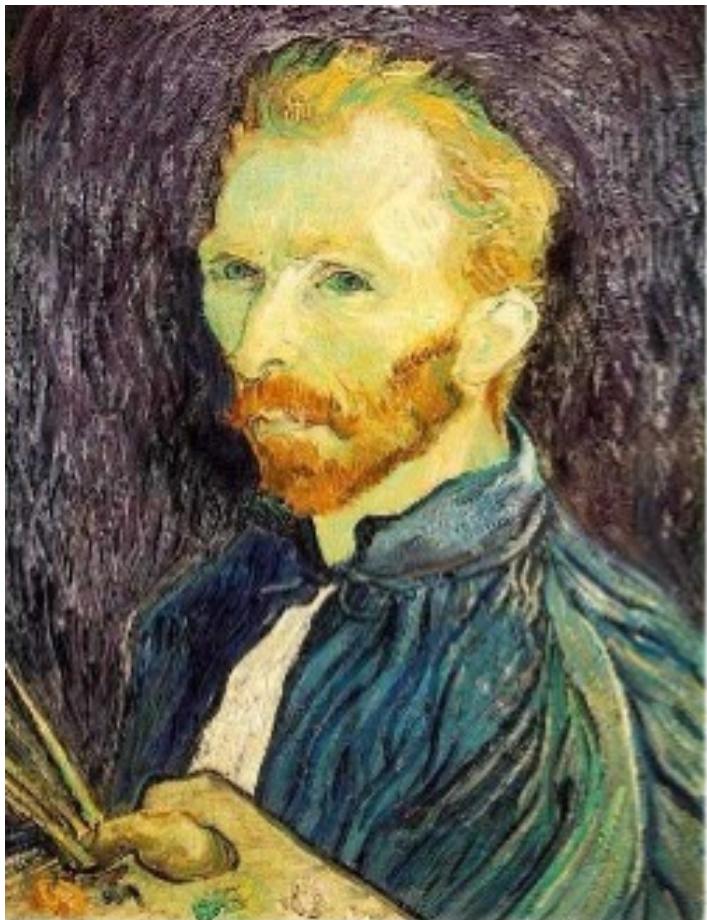
- Fredo Durand (MIT).
- Bernd Girod (Stanford University).
- James Hays (Georgia Tech).
- Steve Marschner (Cornell University).
- Steve Seitz (University of Washington).

Image downsampling



**This image is too big to fit on the screen.
How would you reduce it to half its size?**

Naïve image downsampling



1/2

Throw away half the rows and columns

delete even rows
delete even columns



1/4

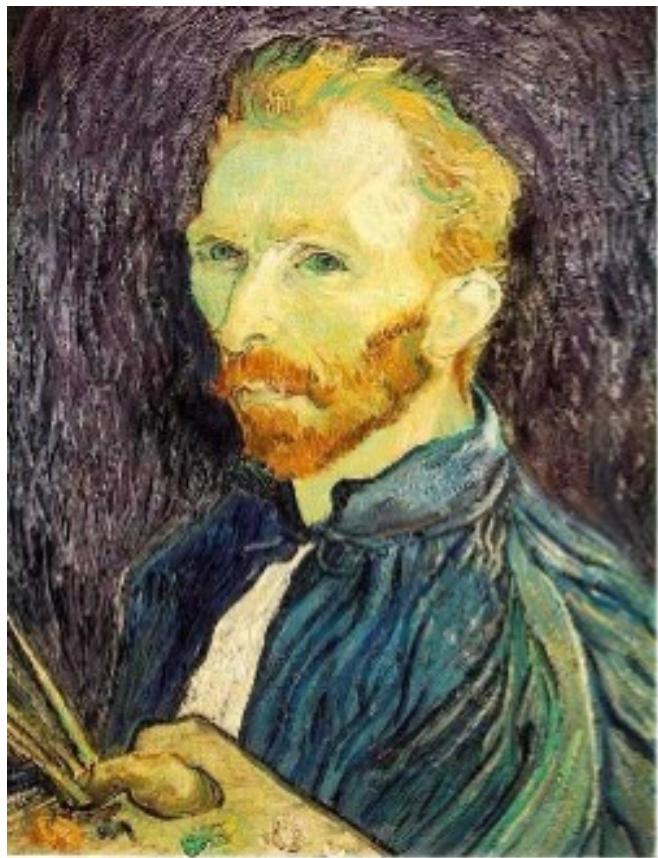
delete even rows
delete even columns



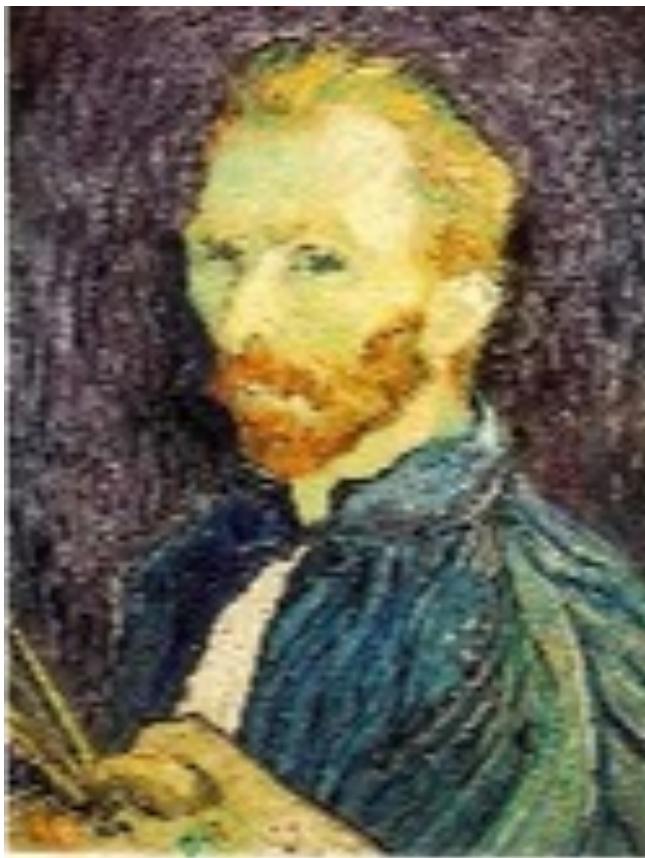
1/8

What is the problem with this approach?

Naïve image downsampling



1/2



1/4 (2x zoom)

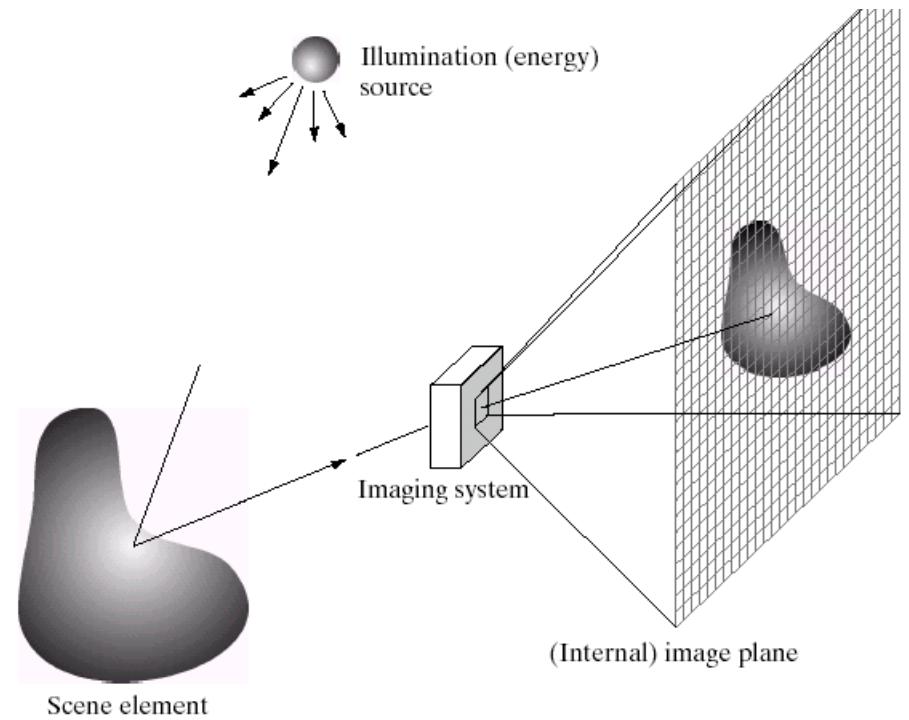


1/8 (4x zoom)

Why is the 1/8 image so pixelated (and do you know what this effect is called)?

Aliasing

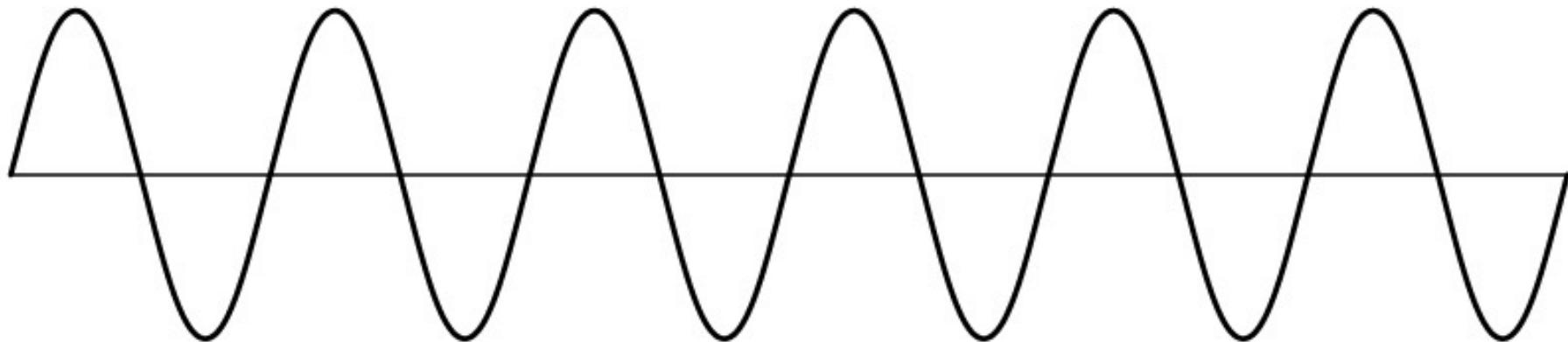
Reminder



Images are a *discrete*, or *sampled*, representation of a *continuous* world

Sampling

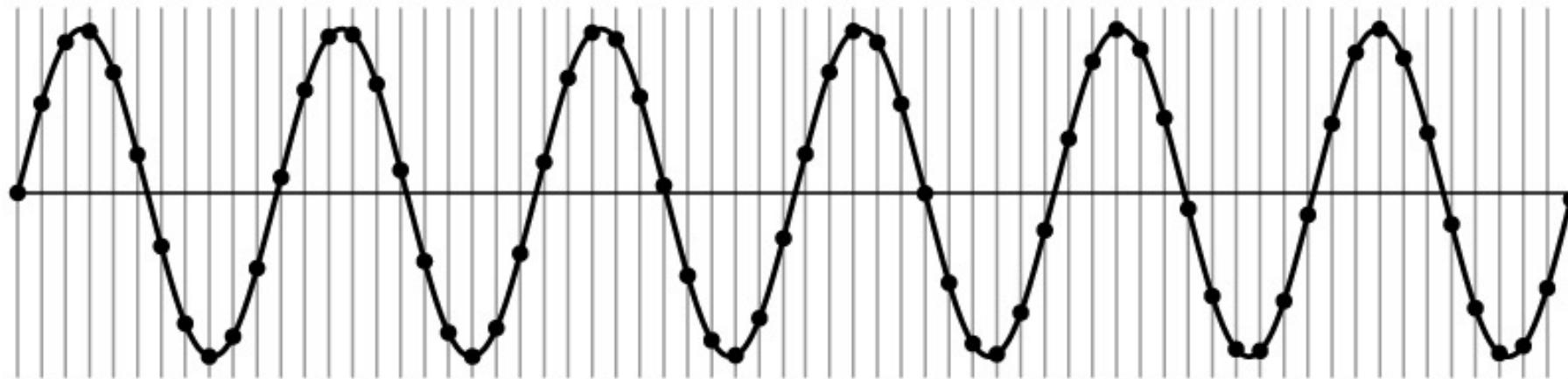
Very simple example: a sine wave



How would you discretize this signal?

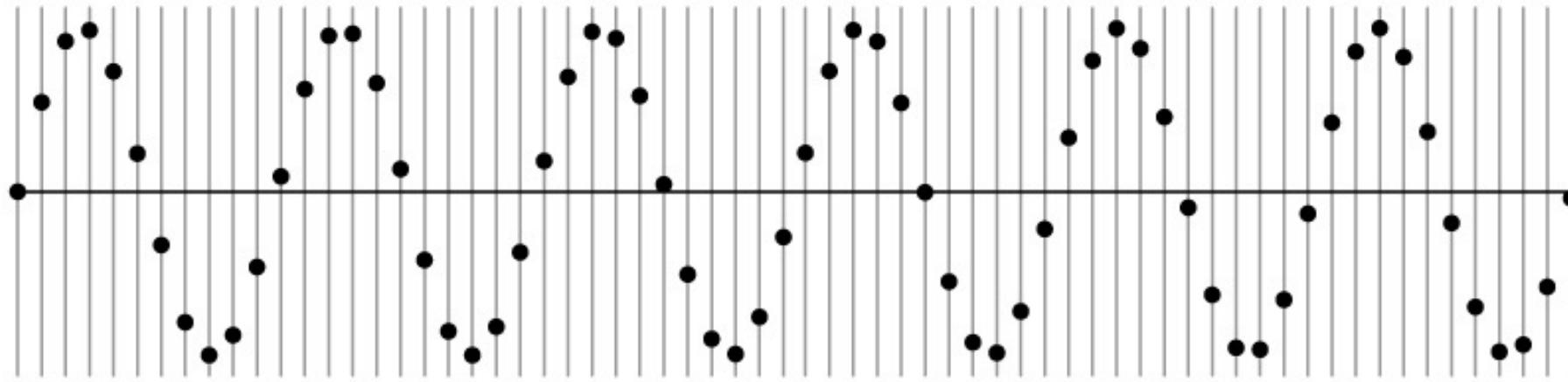
Sampling

Very simple example: a sine wave



Sampling

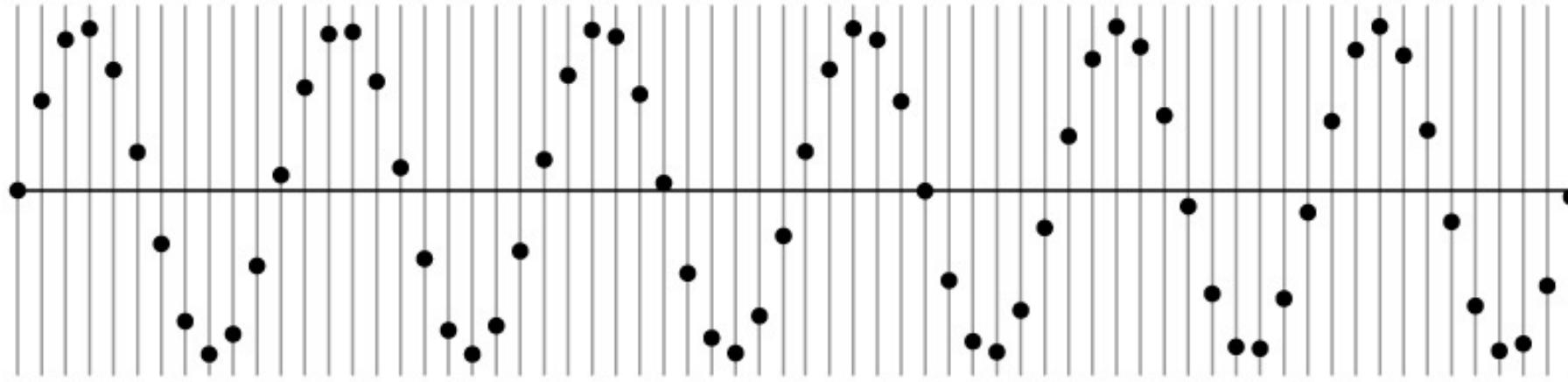
Very simple example: a sine wave



How many samples should I take?
Can I take as *many* samples as I want?

Sampling

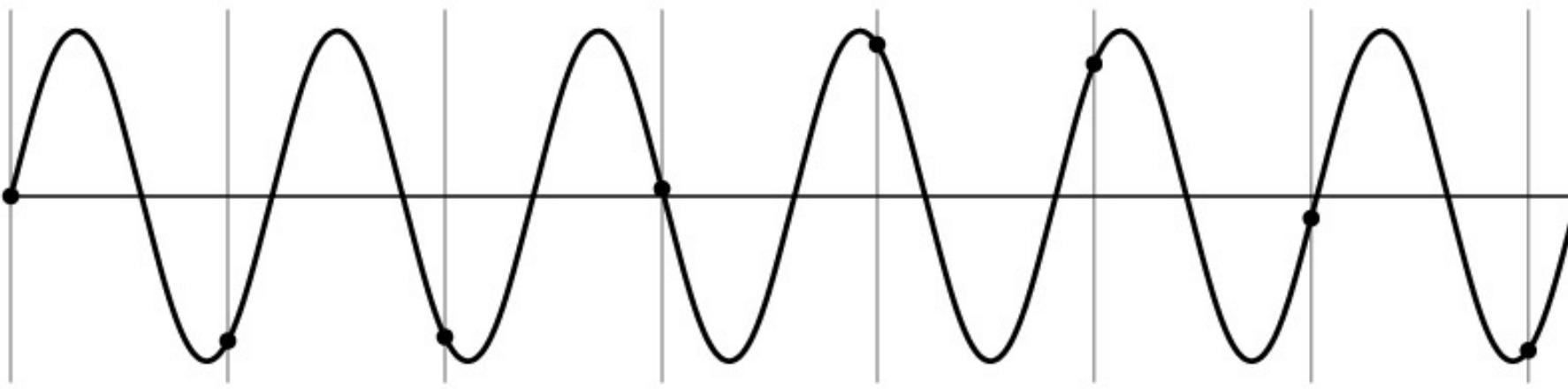
Very simple example: a sine wave



How many samples should I take?
Can I take as *few* samples as I want?

Undersampling

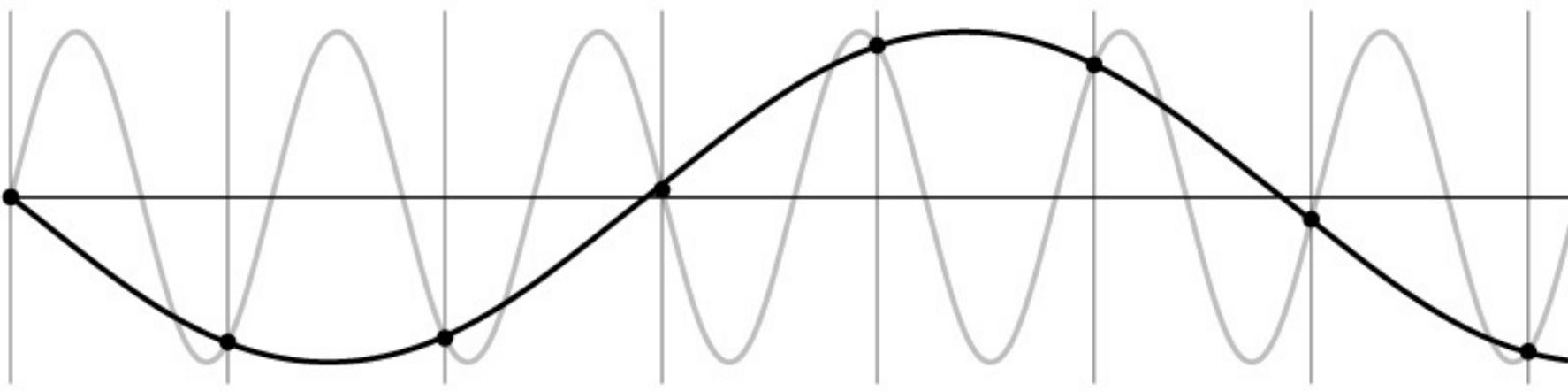
Very simple example: a sine wave



Unsurprising effect: information is lost.

Undersampling

Very simple example: a sine wave

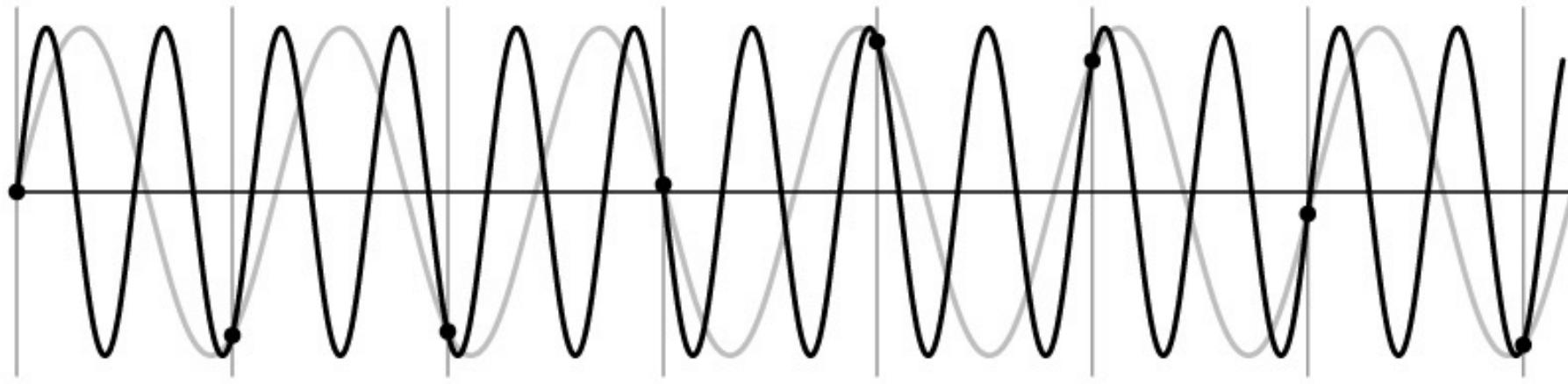


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Undersampling

Very simple example: a sine wave



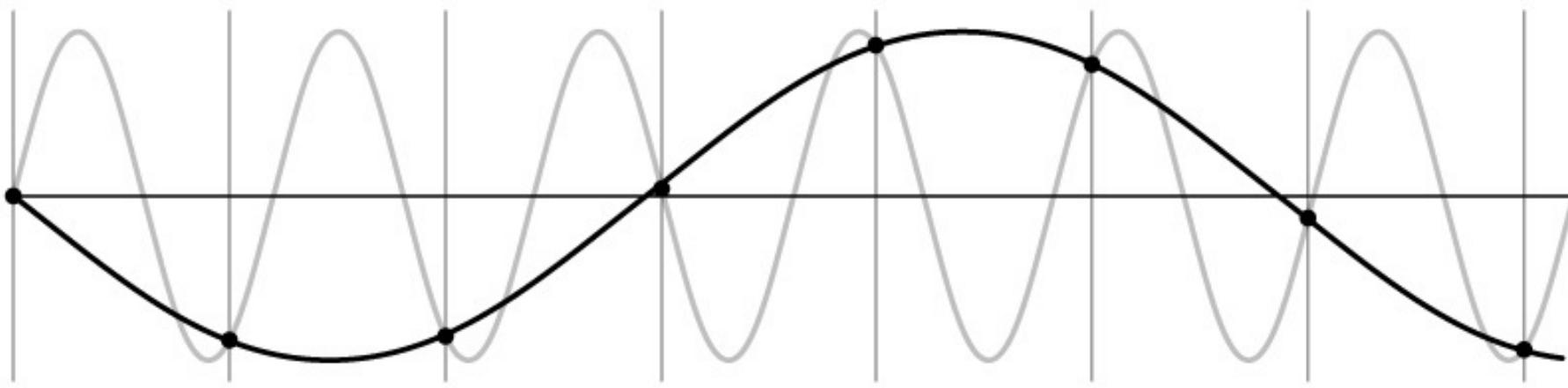
Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

Aliasing

Fancy term for: *Undersampling can disguise a signal as one of a lower frequency*

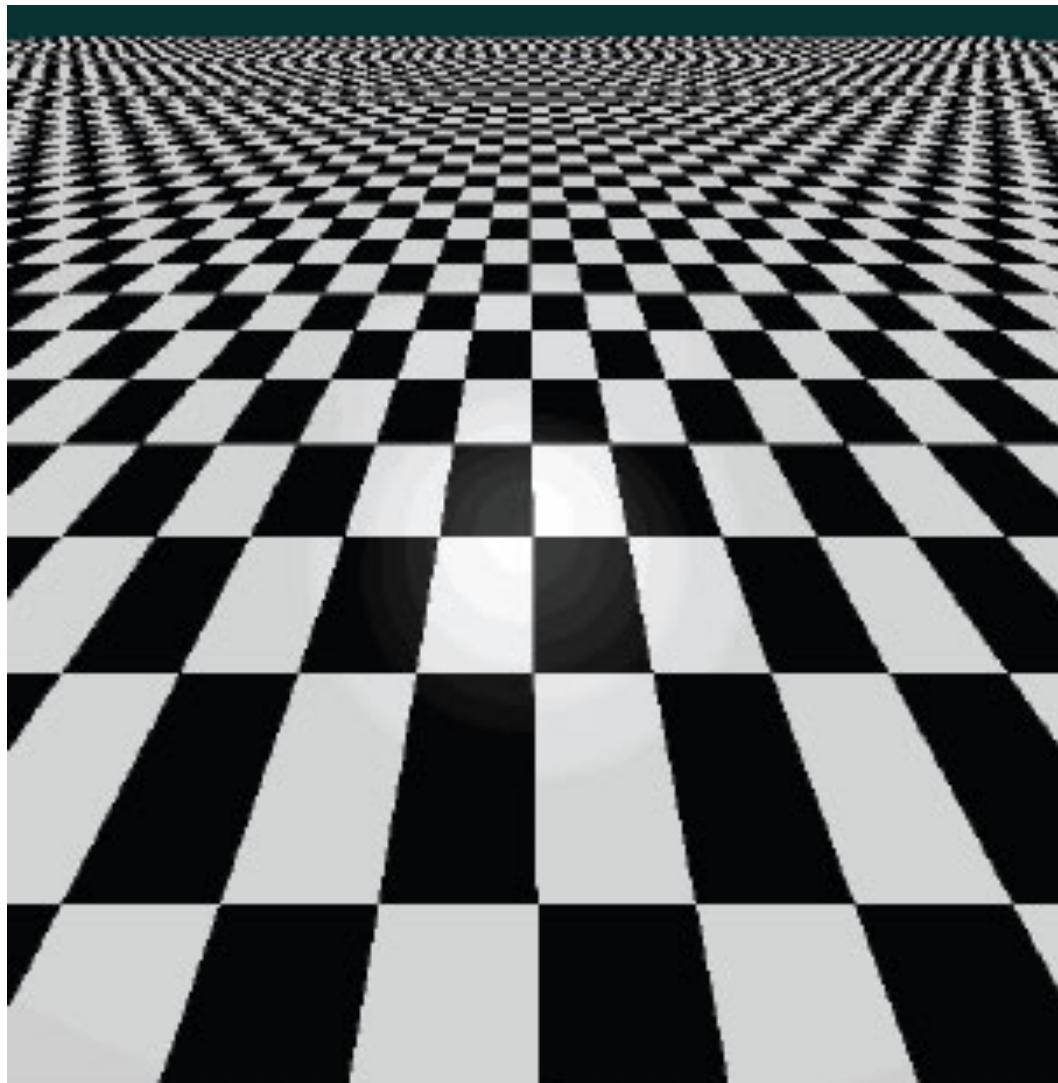


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

Aliasing in textures



Aliasing in photographs

This is also known as “moire”

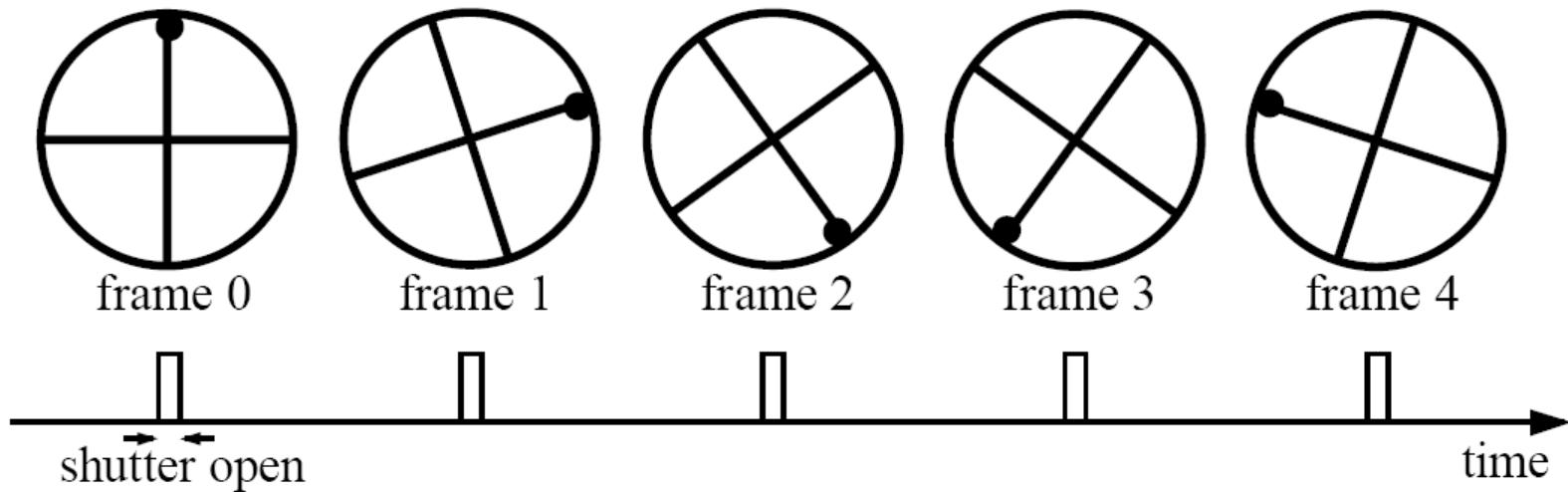


Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

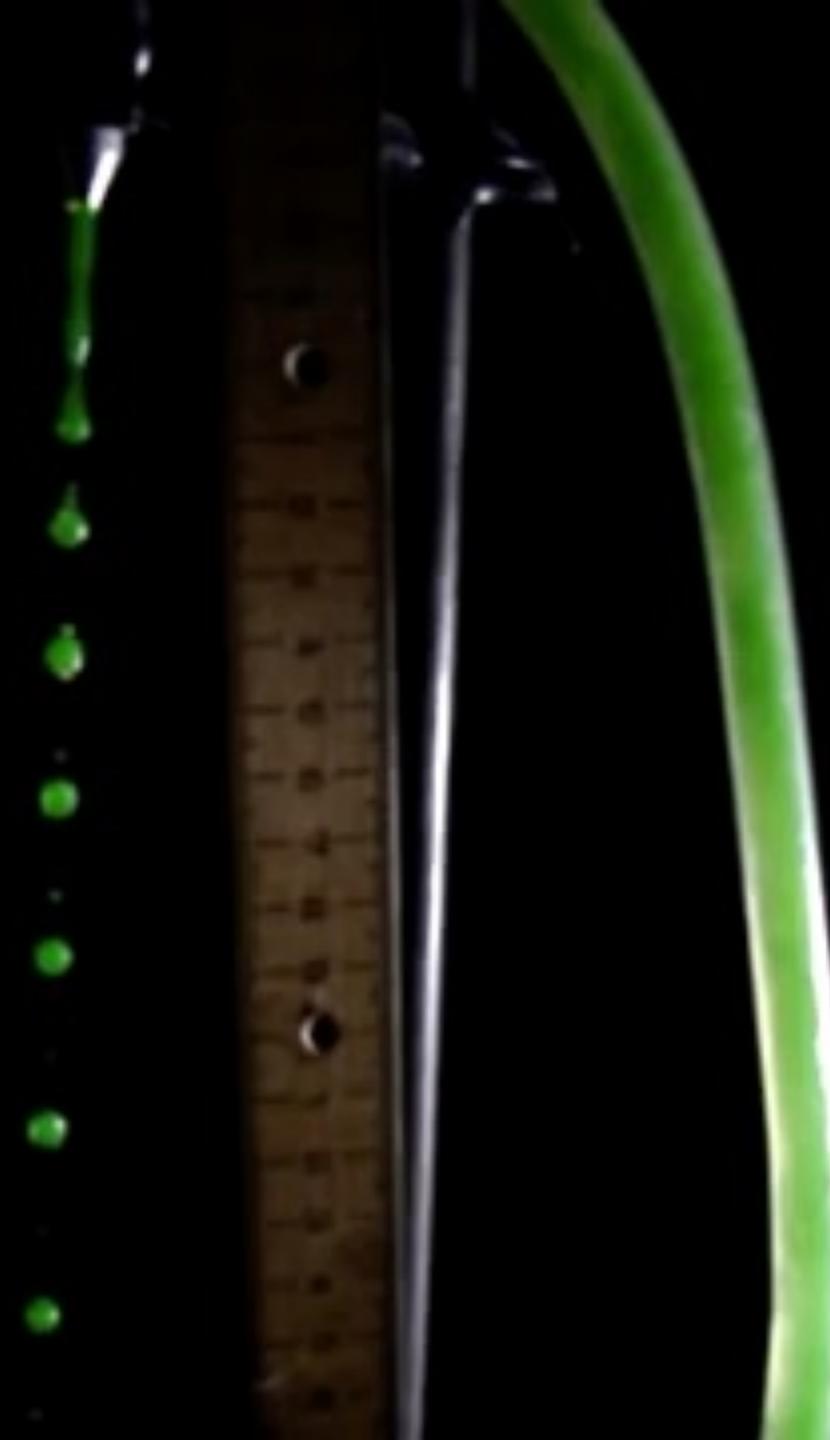
If camera shutter is only open for a fraction of a frame time (frame time = $1/30$ sec. for video, $1/24$ sec. for film):



Without dot, wheel appears to be rotating slowly backwards!
(counterclockwise)

Wagon wheel effect







Anti-aliasing

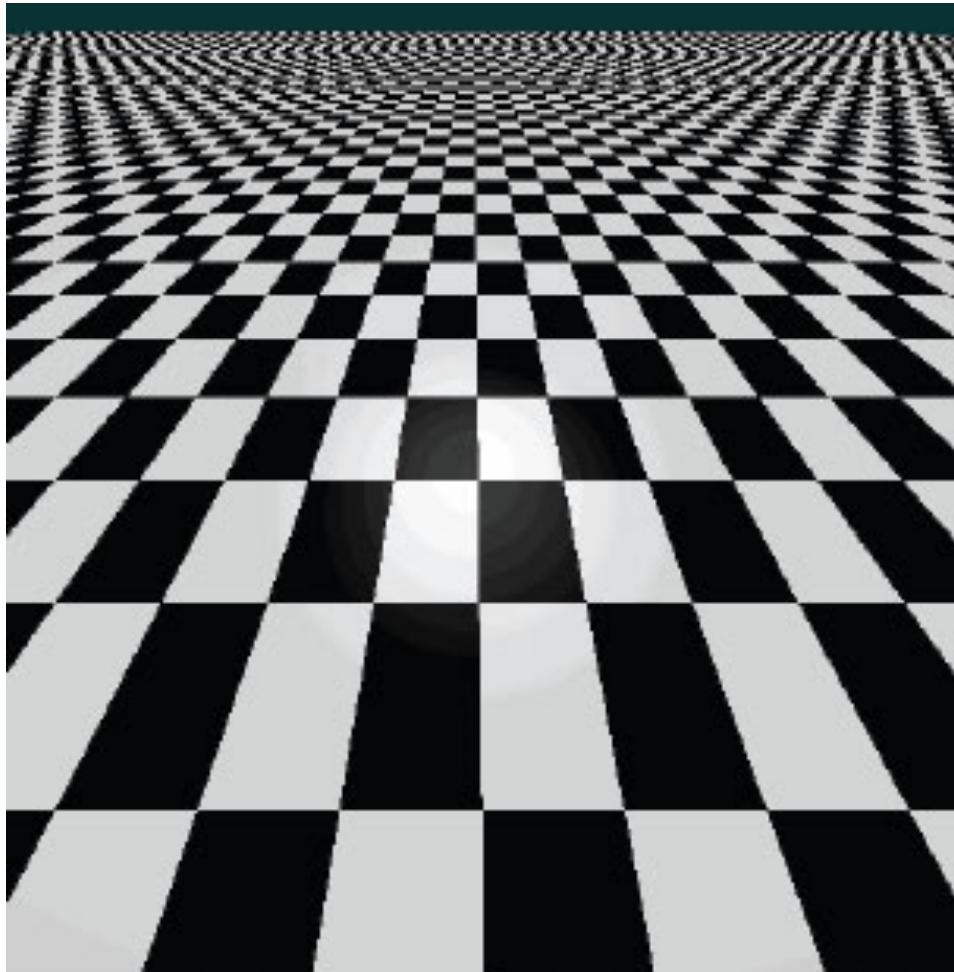
How would you deal with aliasing?

Anti-aliasing

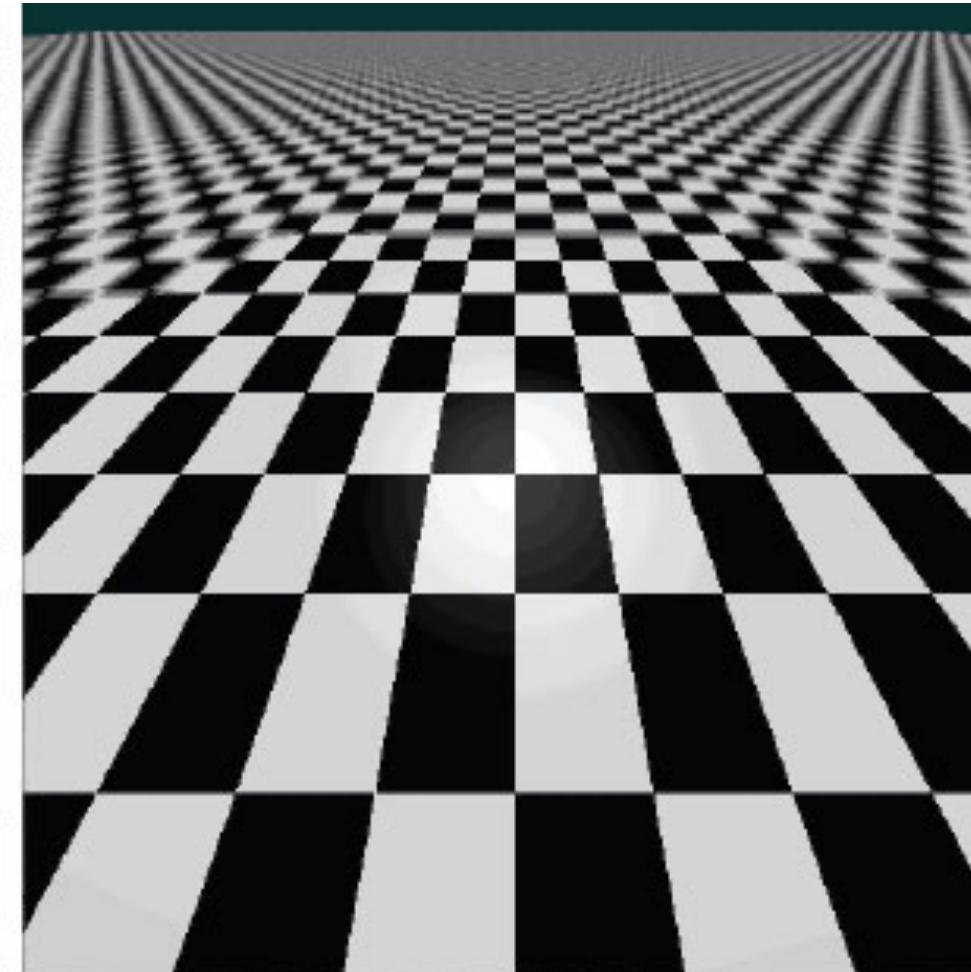
How would you deal with aliasing?

Approach 1: Oversample the signal

Anti-aliasing in textures



aliasing artifacts



anti-aliasing by oversampling

Anti-aliasing

How would you deal with aliasing?

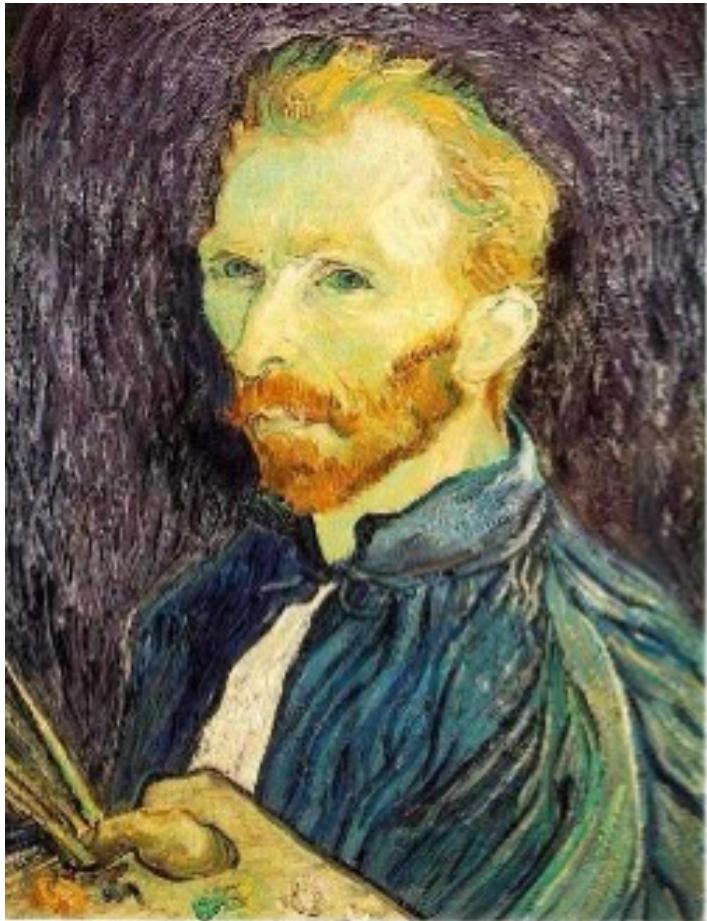
Approach 1: Oversample the signal

Approach 2: Smooth the signal

- Remove some of the detail effects that cause aliasing.
- Lose information, but better than aliasing artifacts.

How would you smooth a signal?

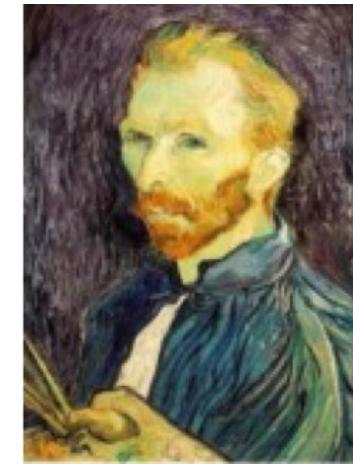
Better image downsampling



1/2

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter
delete even rows
delete even columns



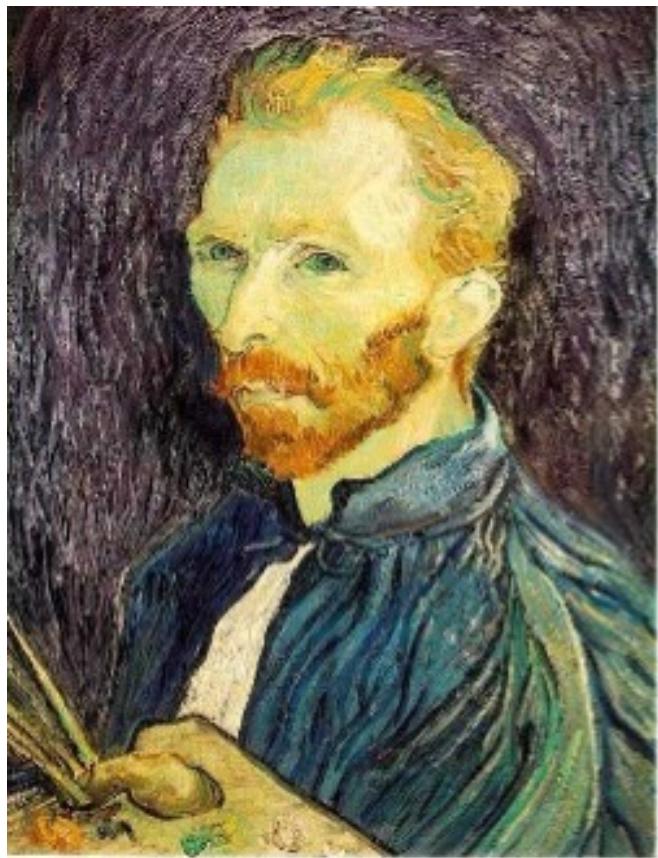
1/4

Gaussian filter
delete even rows
delete even columns

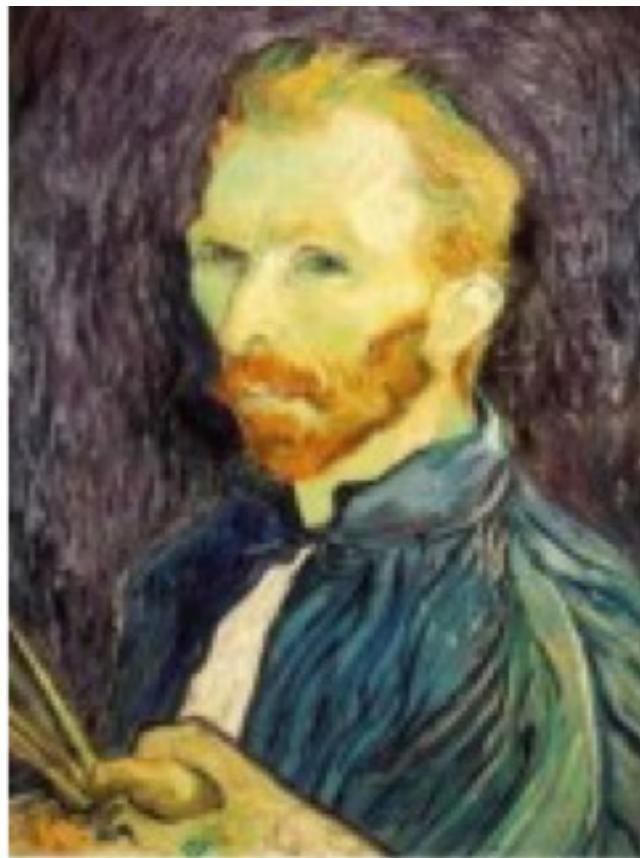


1/8

Better image downsampling



1/2

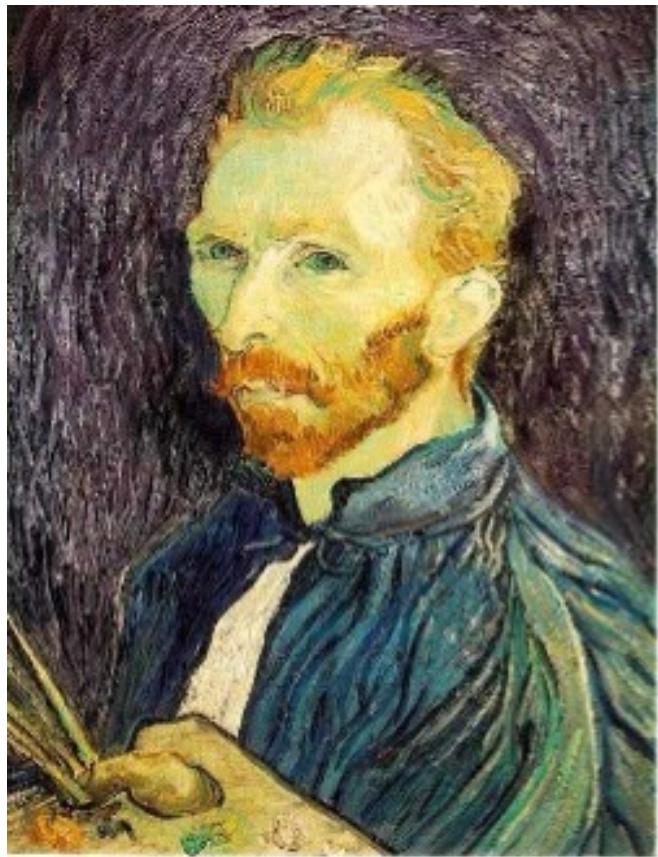


1/4 (2x zoom)

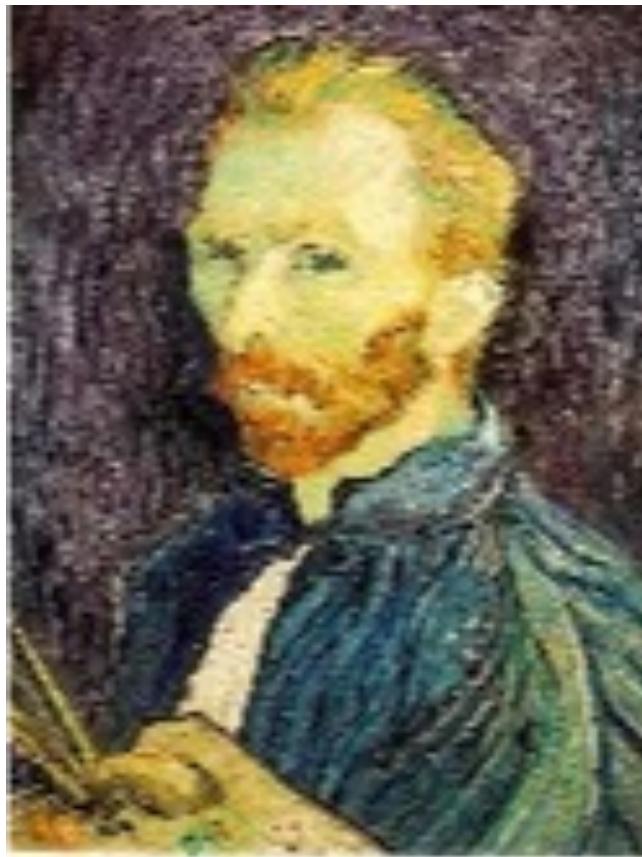


1/8 (4x zoom)

Naïve image downsampling



1/2



1/4 (2x zoom)



1/8 (4x zoom)

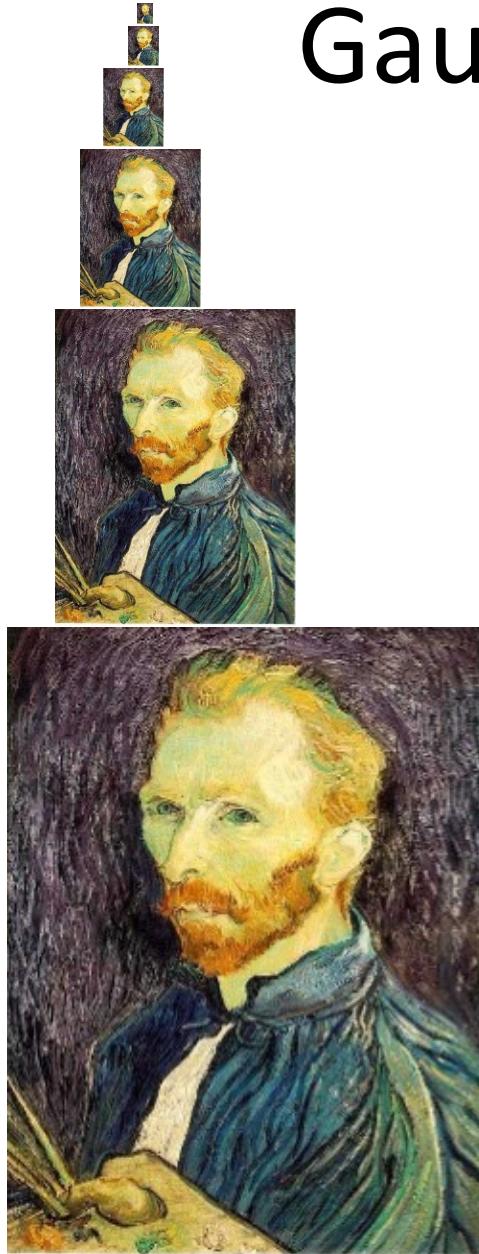
Anti-aliasing

Question 1: How much smoothing do I need to do to avoid aliasing?

Question 2: How many samples do I need to take to avoid aliasing?

Answer to both: Enough to reach the Nyquist limit. (We'll see what this means soon.)

Gaussian image pyramid



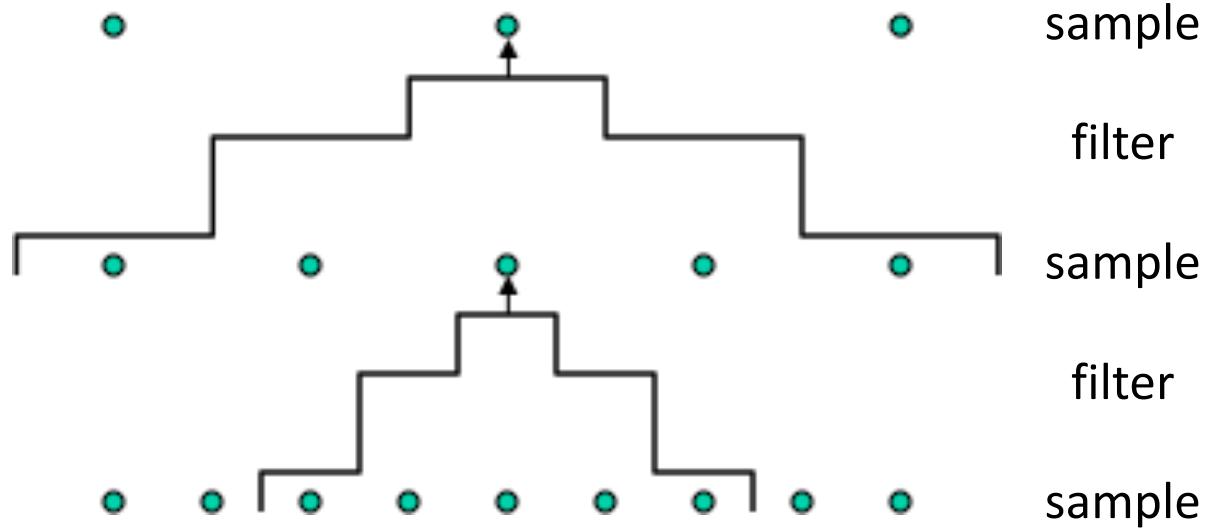
Gaussian image pyramid

The name of this sequence of subsampled images

Constructing a Gaussian pyramid

Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution reached
```

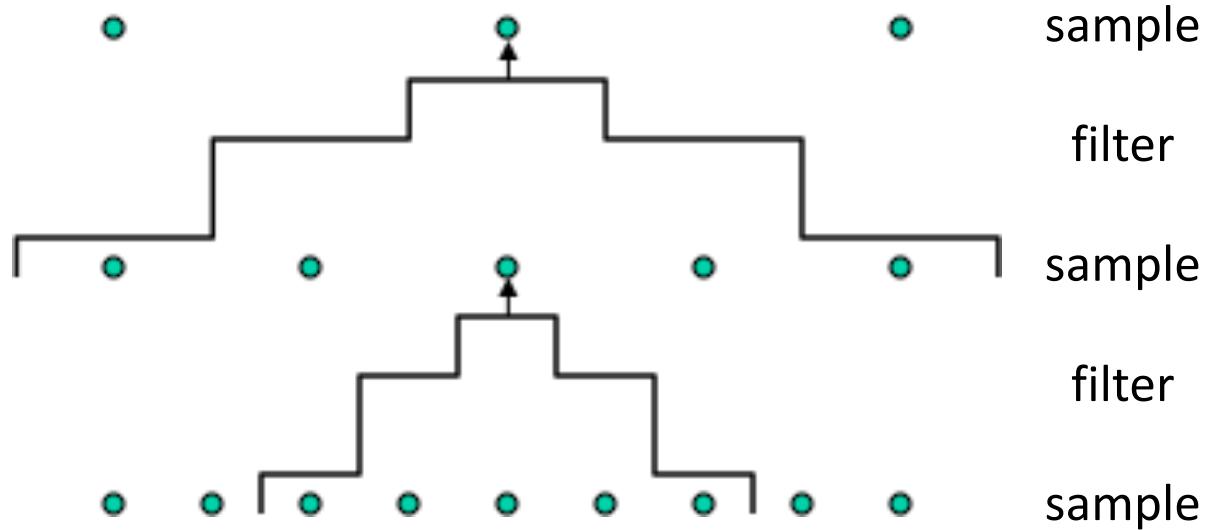


Question: How much bigger than the original image is the whole pyramid?

Constructing a Gaussian pyramid

Algorithm

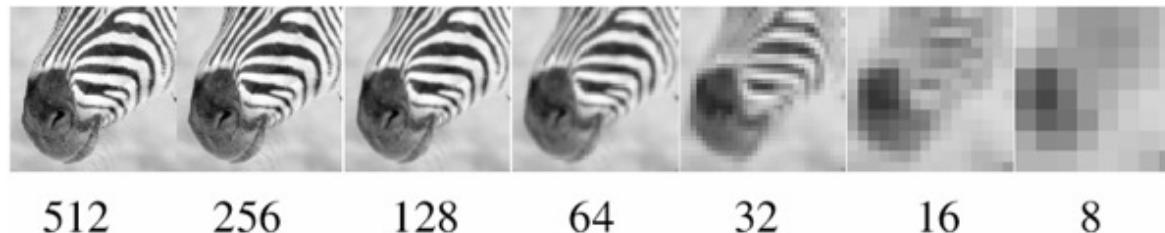
```
repeat:  
    filter  
    subsample  
until min resolution reached
```



Question: How much bigger than the original image is the whole pyramid?

Answer: Just $4/3$ times the size of the original image! (How did I come up with this number?)

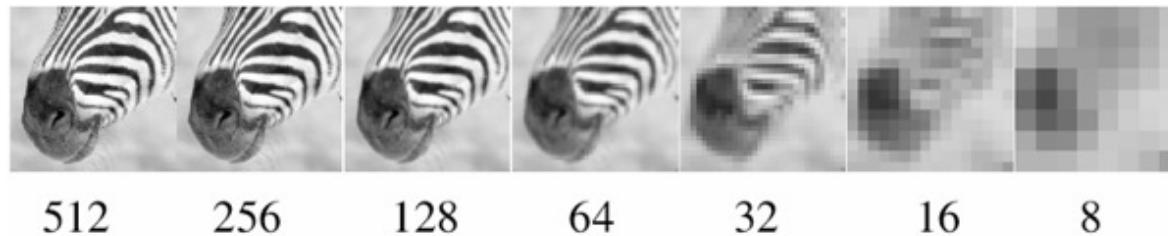
Some properties of the Gaussian pyramid



What happens to the details of the image?



Some properties of the Gaussian pyramid



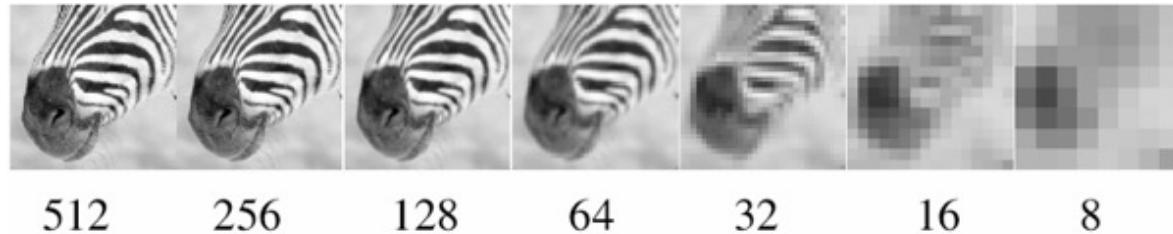
What happens to the details of the image?

- They get smoothed out as we move to higher levels.



What is preserved at the higher levels?

Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

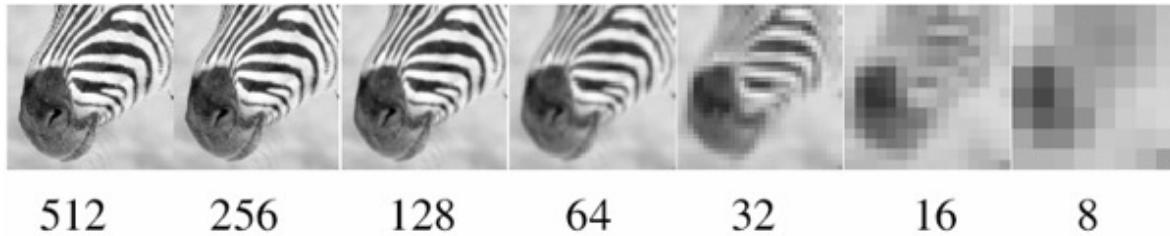


What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.



What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.

Blurring is lossy



level 0



level 1 (before downsampling)

What does the residual look like?

Blurring is lossy



level 0



level 1 (before downsampling)

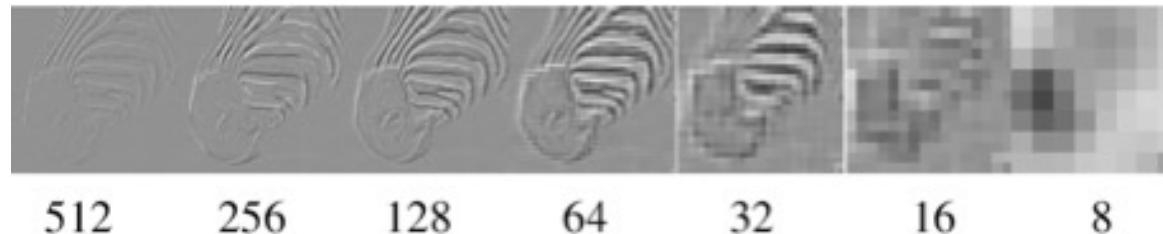


residual

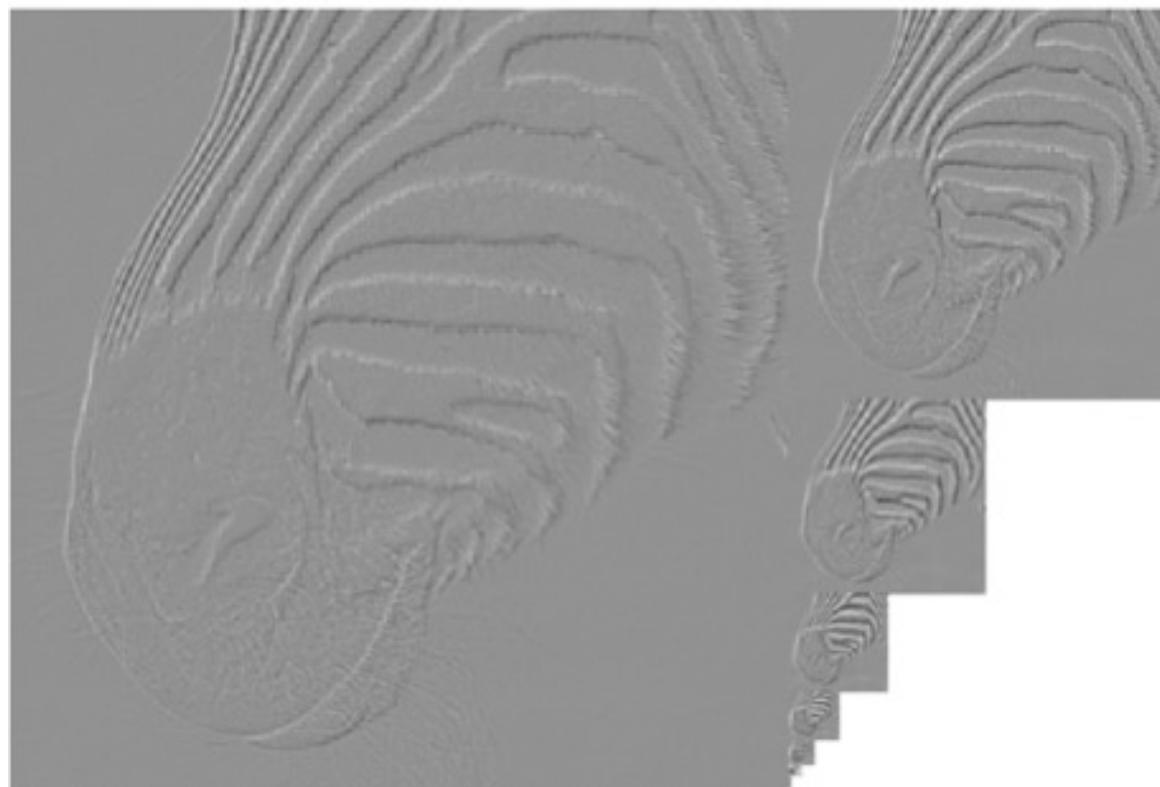
Can we make a pyramid that is lossless?

Laplacian image pyramid

Laplacian image pyramid

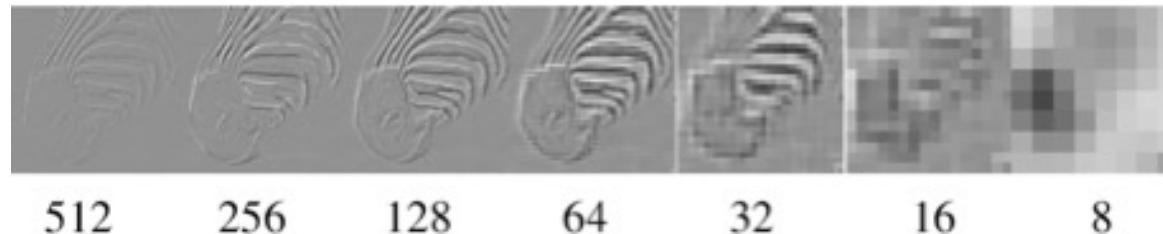


At each level, retain the residuals instead of the blurred images themselves.

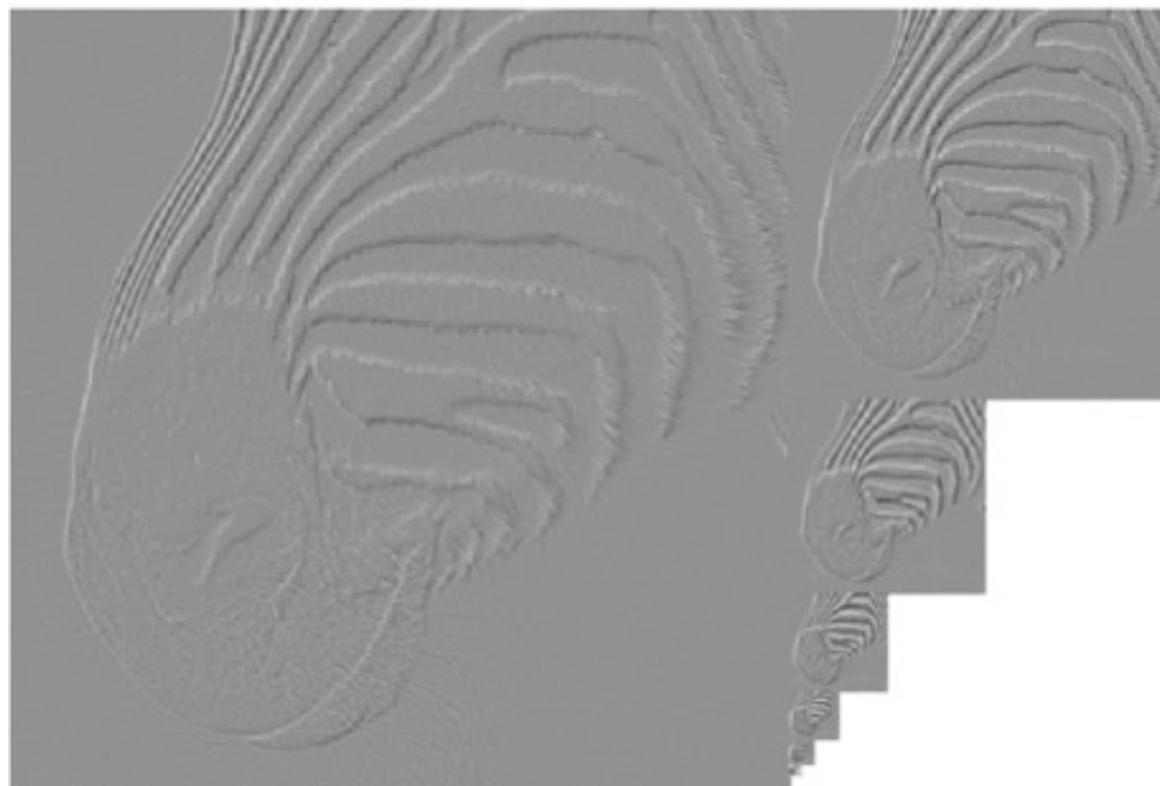


Can we reconstruct the original image using the pyramid?

Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.



Can we reconstruct the original image using the pyramid?

- Yes we can!

What do we need to store to be able to reconstruct the original image?

Let's start by looking at just one level



level 0



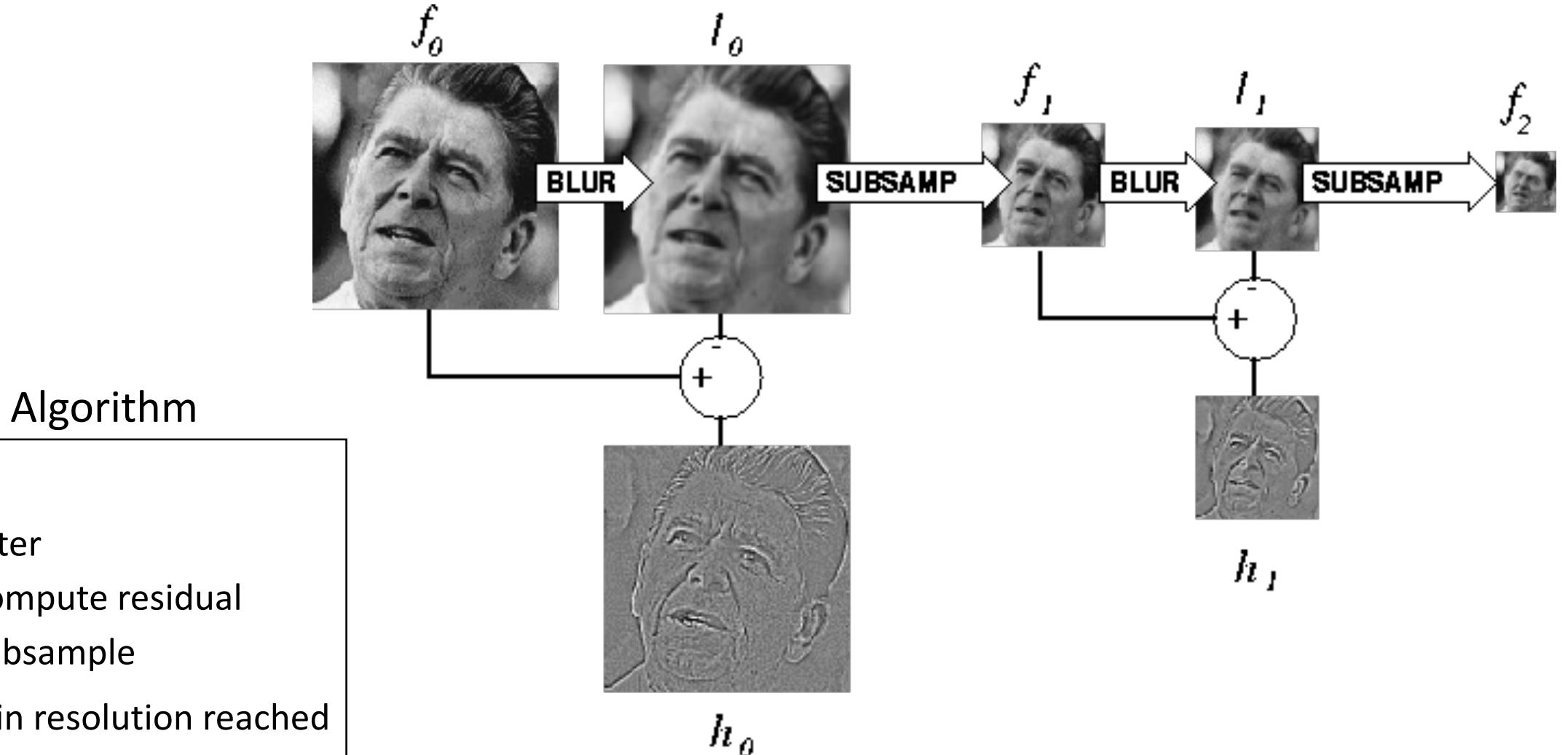
level 1 (upsampled)



residual

Does this mean we need to store both residuals and the blurred copies of the original?

Constructing a Laplacian pyramid



Constructing a Laplacian pyramid

What is this part?

Algorithm

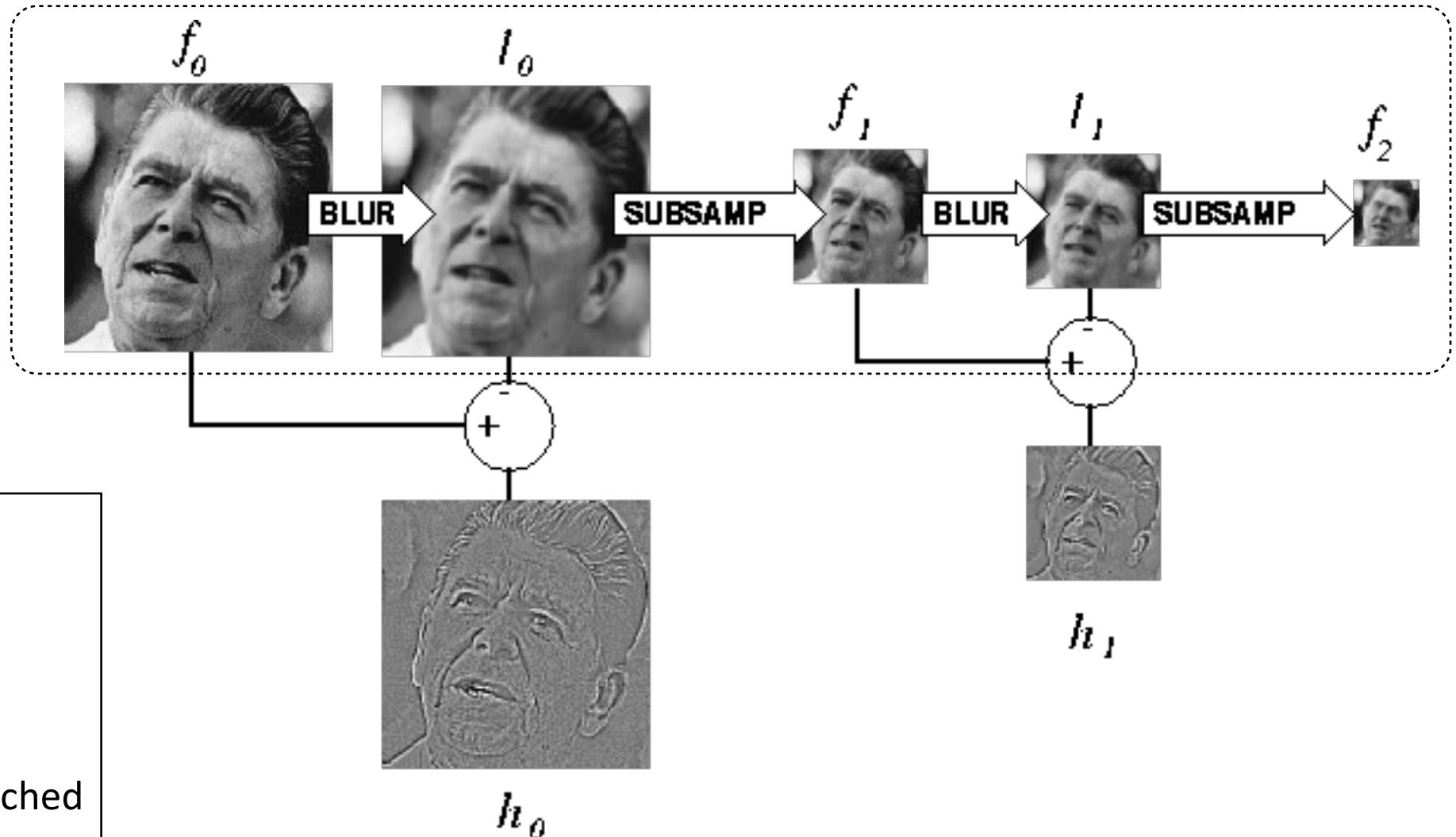
repeat:

filter

compute residual

subsample

until min resolution reached



Constructing a Laplacian pyramid

It's a Gaussian pyramid.

Algorithm

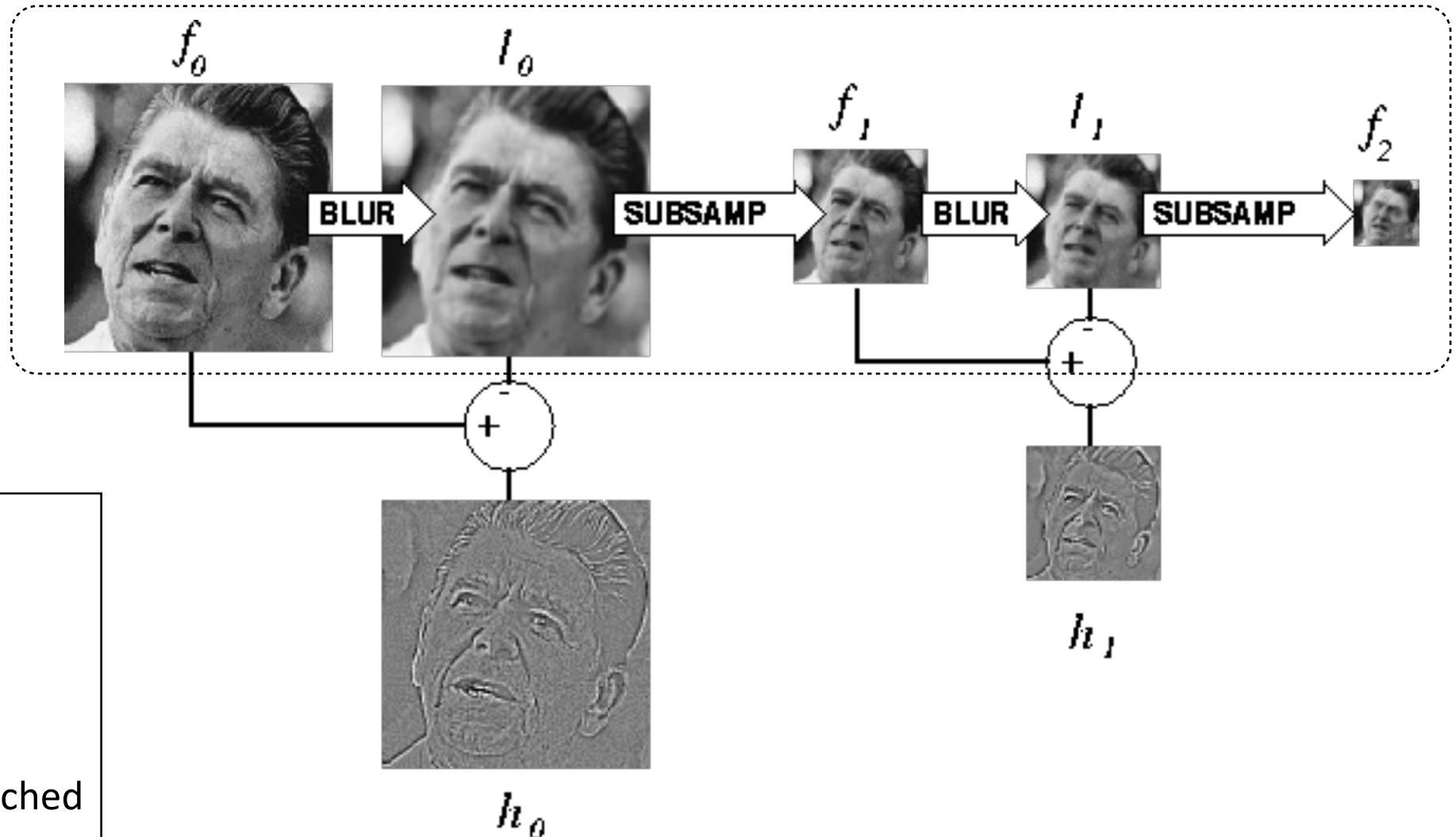
repeat:

filter

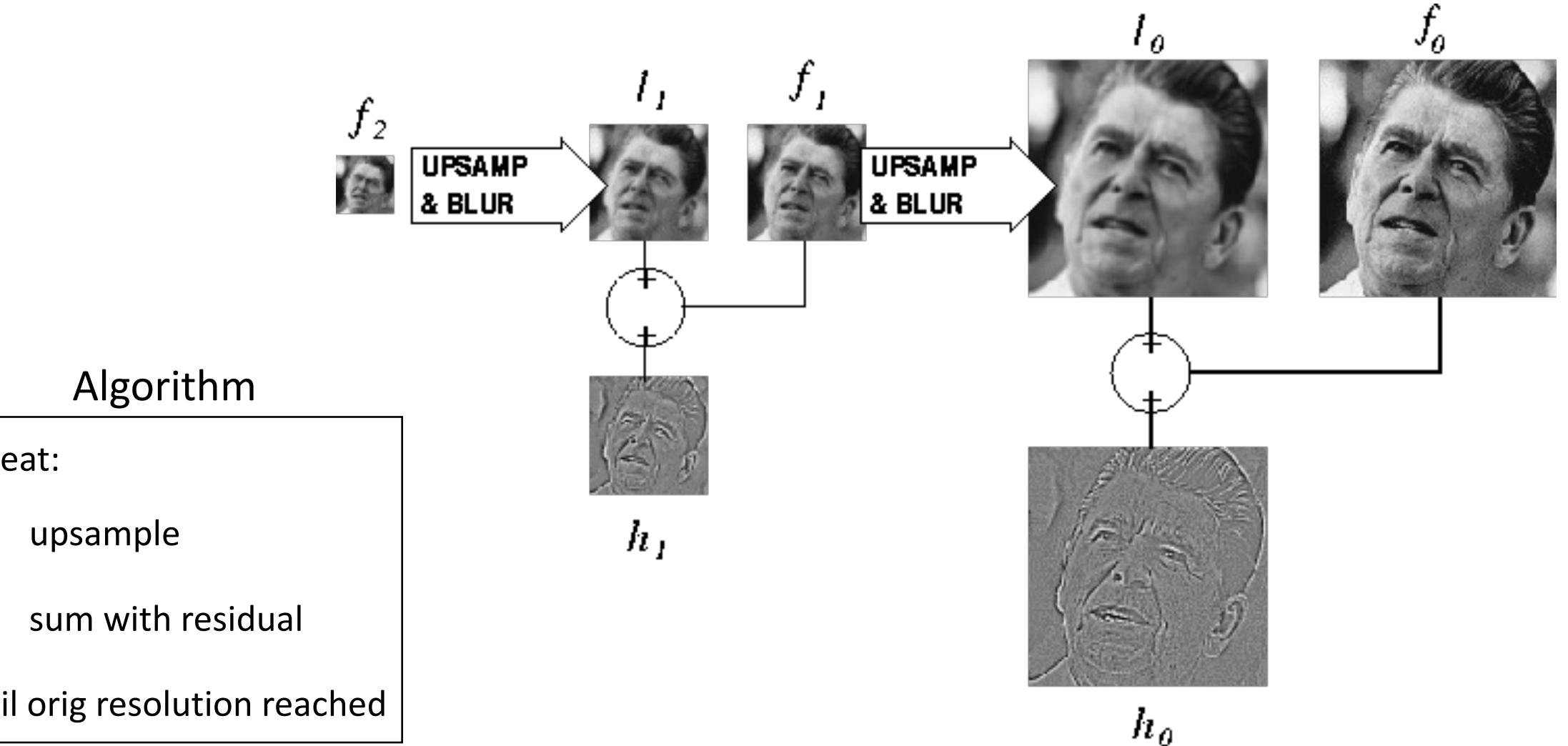
compute residual

subsample

until min resolution reached



Reconstructing the original image

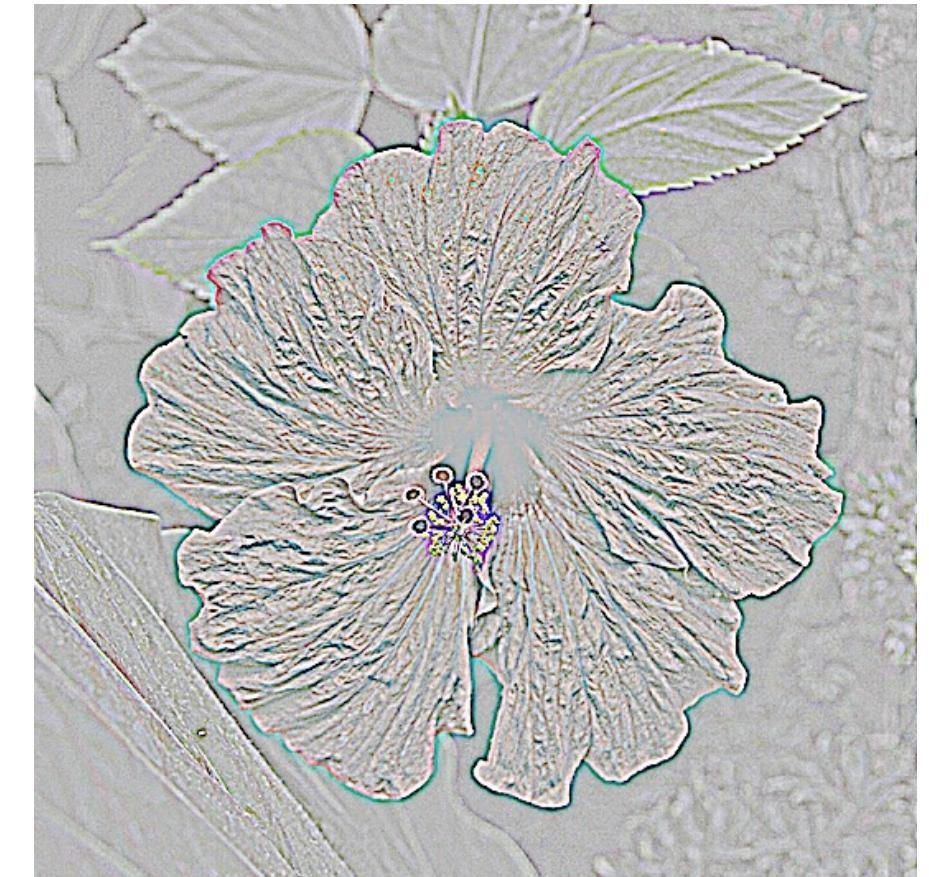
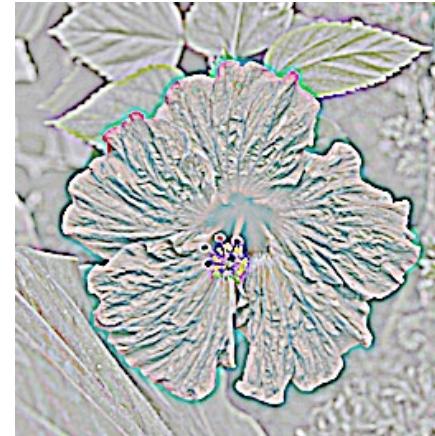
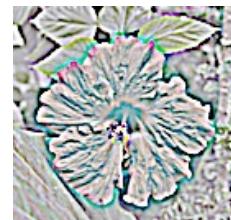


Gaussian vs Laplacian Pyramid



Shown in opposite
order for space.

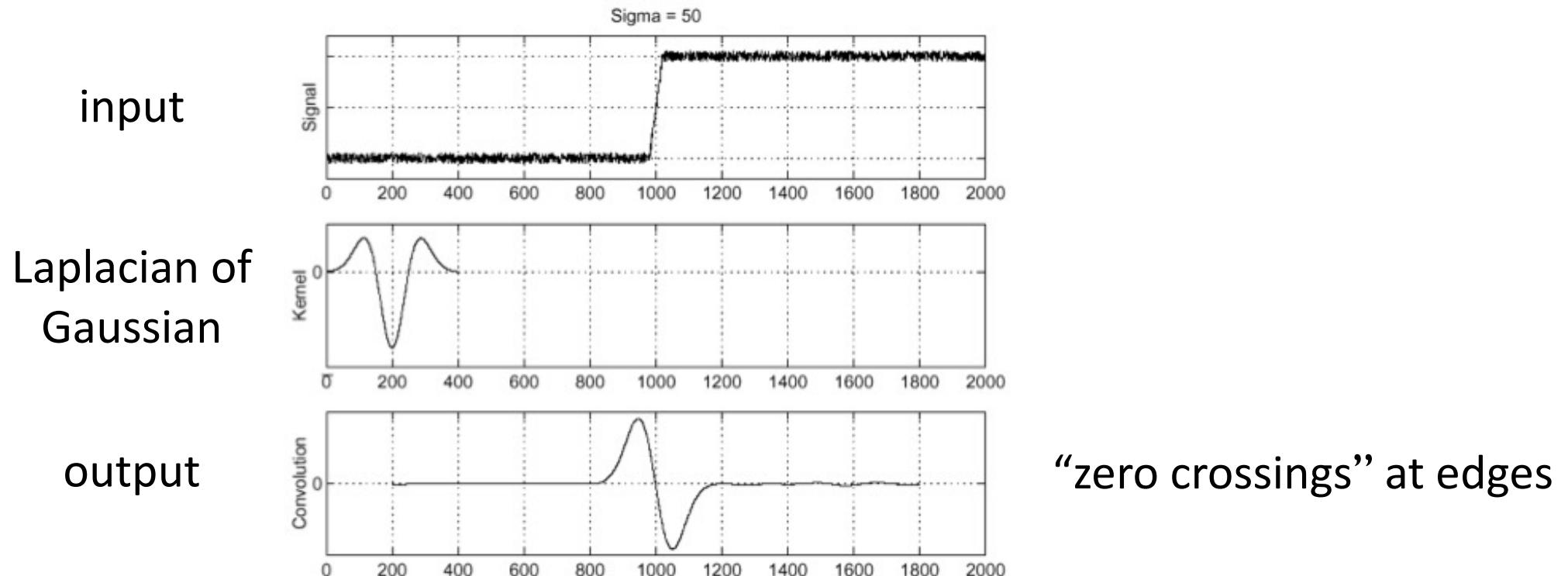
Which one takes
more space to store?



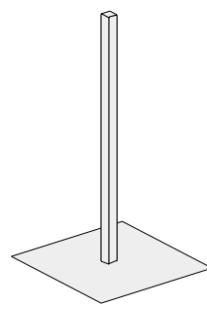
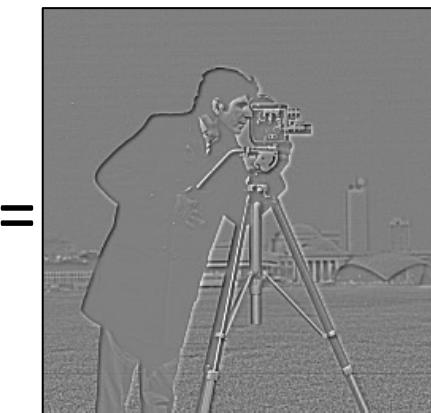
Why is it called a Laplacian pyramid?

Reminder: Laplacian of Gaussian (LoG) filter

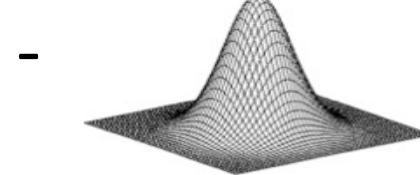
As with derivative, we can combine Laplace filtering with Gaussian filtering



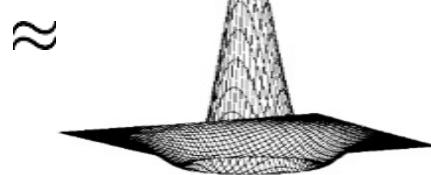
Why is it called a Laplacian pyramid?



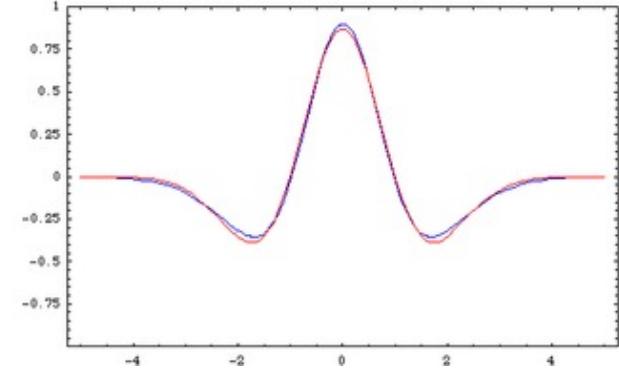
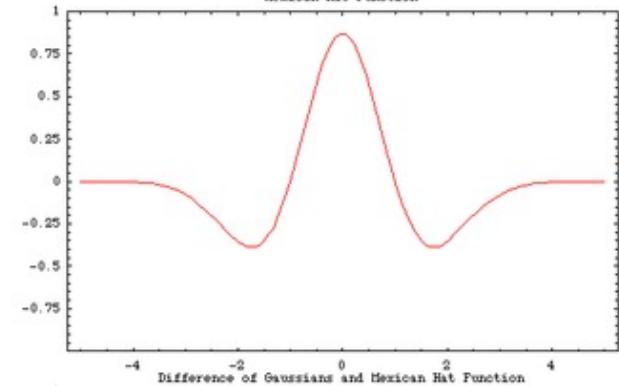
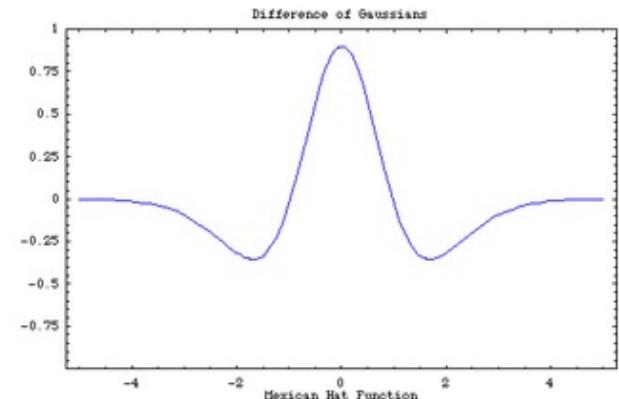
unit



Gaussian

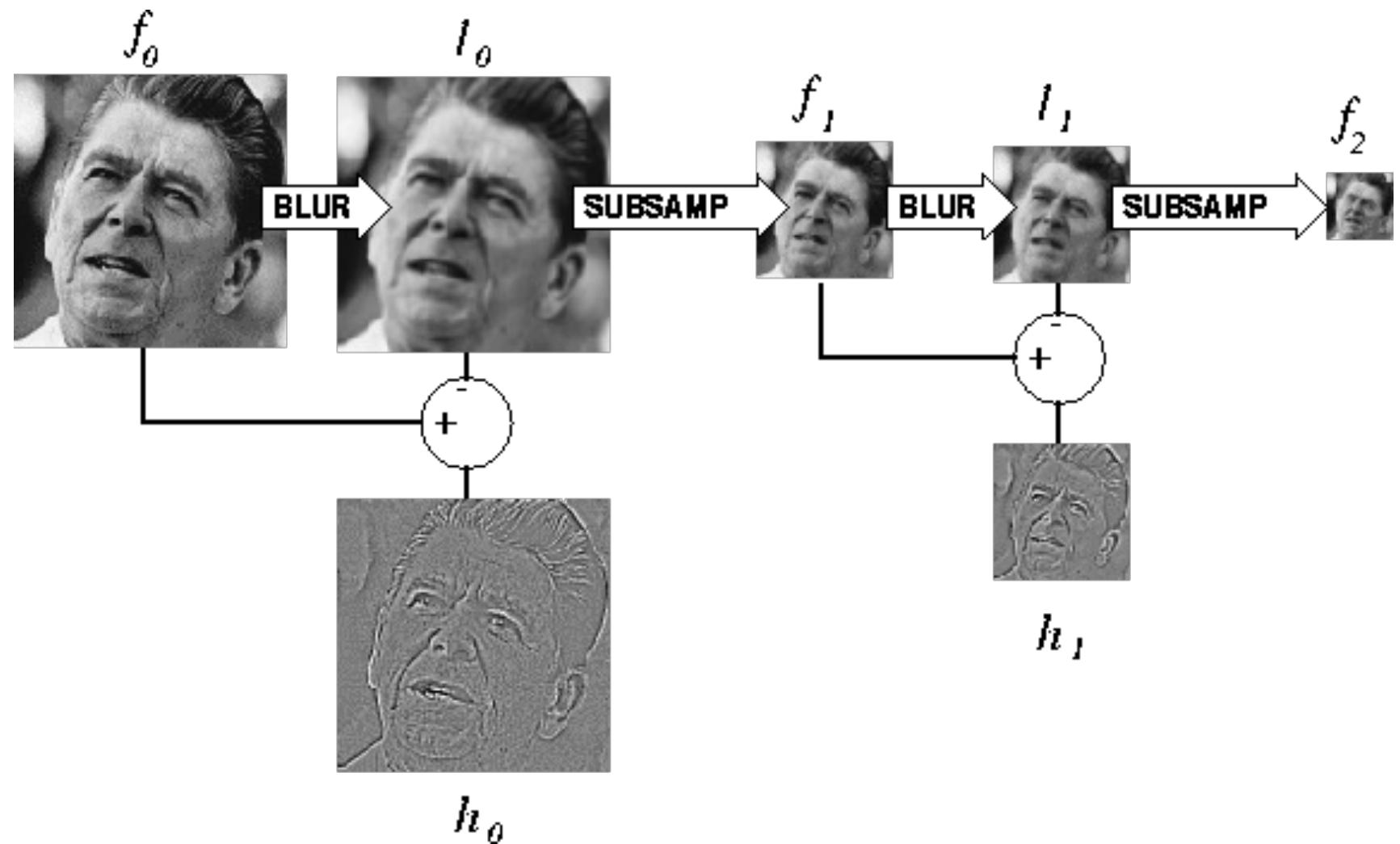


Laplacian



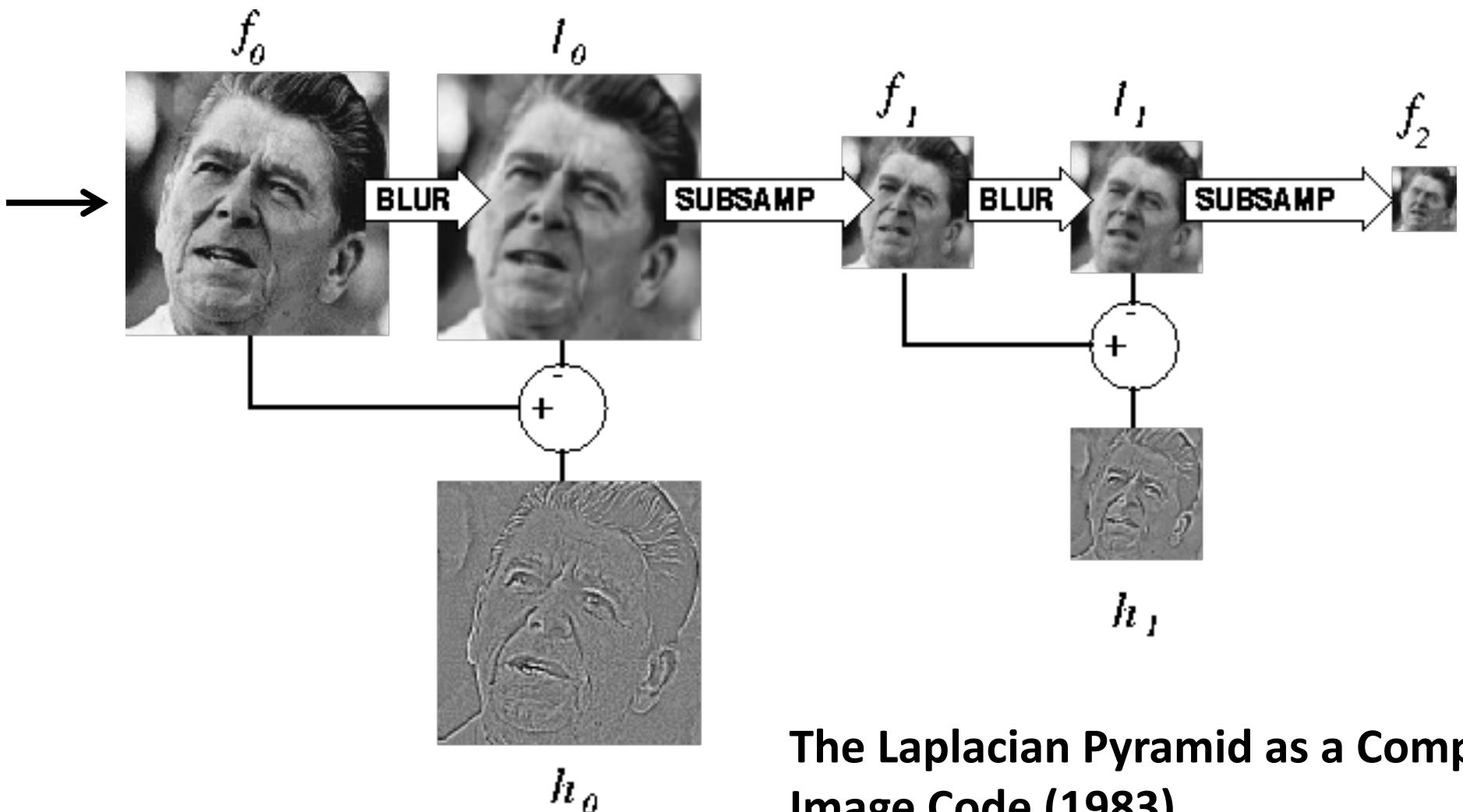
Difference of Gaussians approximates the Laplacian

Why Reagan?



Why Reagan?

Ronald Reagan was President when the Laplacian pyramid was invented



The Laplacian Pyramid as a Compact Image Code (1983)

Peter J. Burt , Edward H. Adelson

Still used extensively



Still used extensively



input image



foreground details enhanced, background details reduced

user-provided mask

Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.



Wavelets: Huge area in image processing
(see 18-793).



What are image pyramids used for?

image compression



multi-scale texture mapping

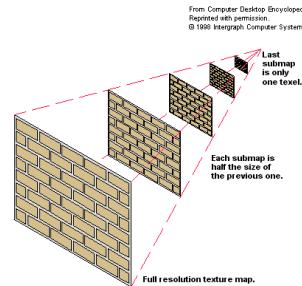
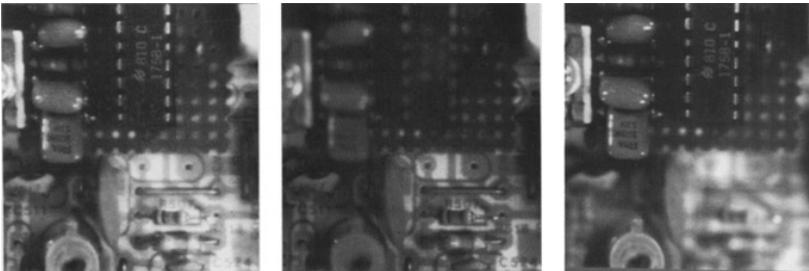


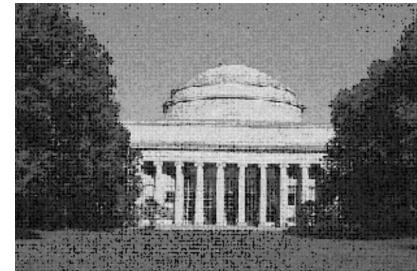
image blending



focal stack compositing



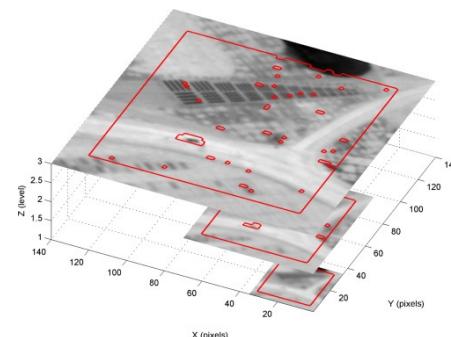
denoising



multi-scale detection



multi-scale registration



Some history

Who is this guy?



What is he famous for?



Jean Baptiste Joseph Fourier
(1768-1830)

What is he famous for?



Jean Baptiste Joseph Fourier
(1768-1830)

The Fourier series claim (1807):

'Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.'

... and apparently also for the discovery
of the greenhouse effect

Is this claim true?



Jean Baptiste Joseph Fourier
(1768-1830)

The Fourier series claim (1807):

'Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.'

Is this claim true?



Jean Baptiste Joseph Fourier
(1768-1830)

The Fourier series claim (1807):

'Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.'

Well, almost.

- The theorem requires additional conditions.
- Close enough to be named after him.
- Very surprising result at the time.

Is this claim true?



Jean Baptiste Joseph Fourier
(1768-1830)

The Fourier series claim (1807):

'Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.'

Well, almost.

- The theorem requires additional conditions.
- Close enough to be named after him.
- Very surprising result at the time.



Malus



Lagrange



Legendre



Laplace

The committee examining his paper had expressed skepticism, in part due to not so rigorous proofs

Fourier series

Basic building block

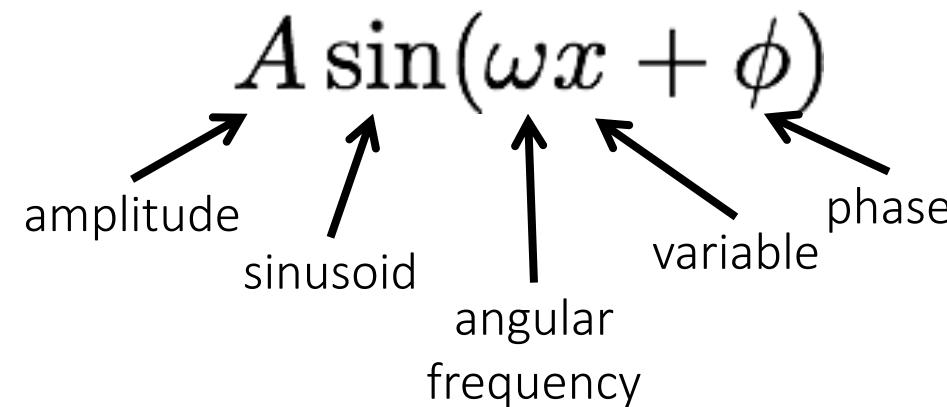
$$A \sin(\omega x + \phi)$$

Fourier's claim: Add enough of these to get any periodic signal you want!

Basic building block

$$A \sin(\omega x + \phi)$$

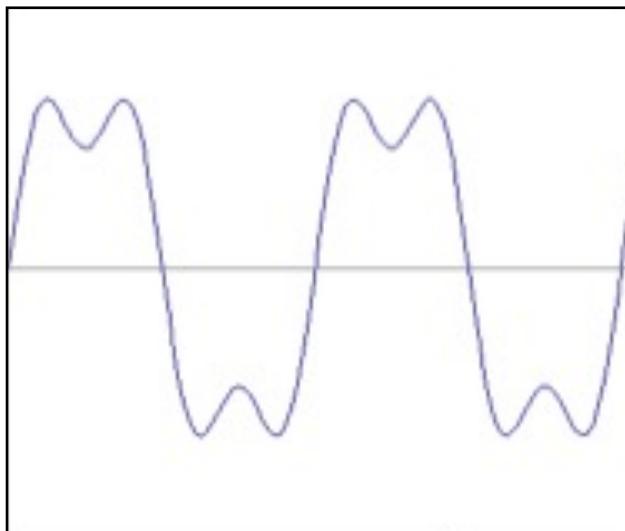
amplitude sinusoid angular frequency variable phase



Fourier's claim: Add enough of these to get any periodic signal you want!

Examples

How would you generate this function?



=

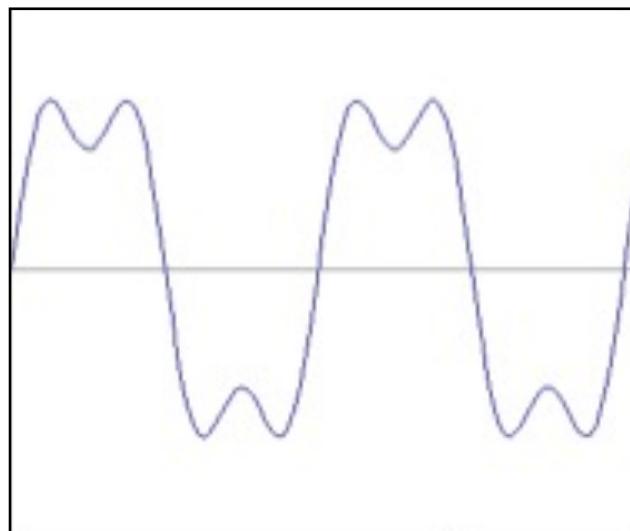
?

+

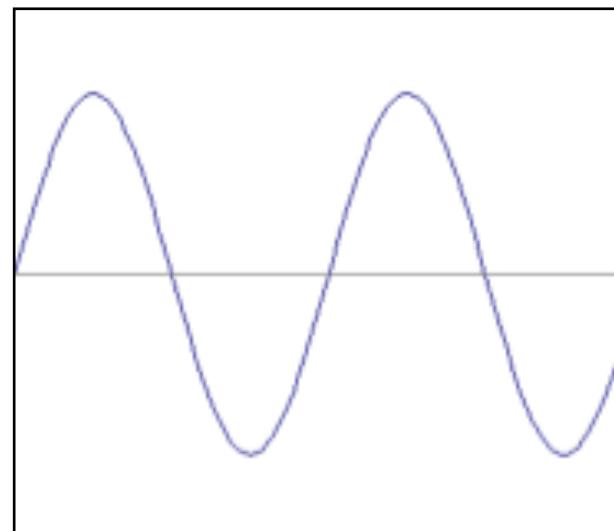
?

Examples

How would you generate this function?



=



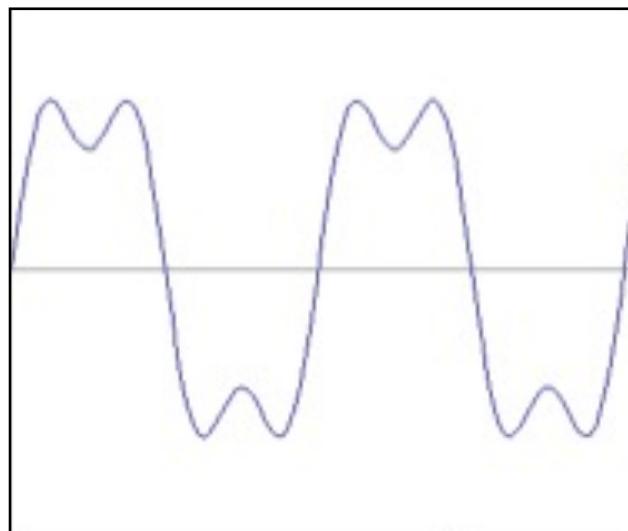
+

?

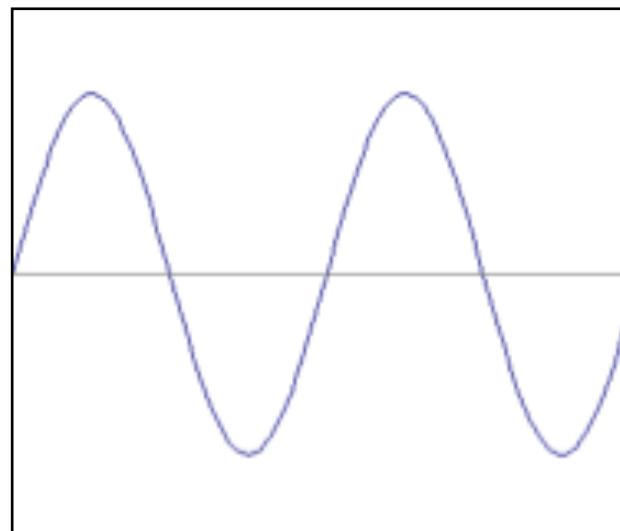
$$\sin(2\pi x)$$

Examples

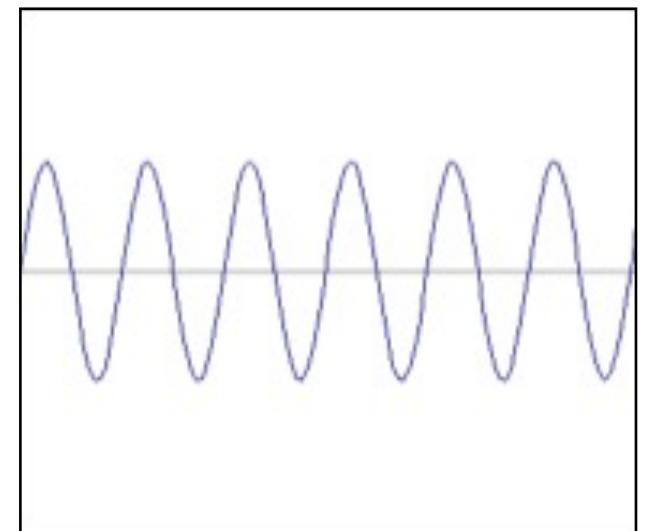
How would you generate this function?



=



+



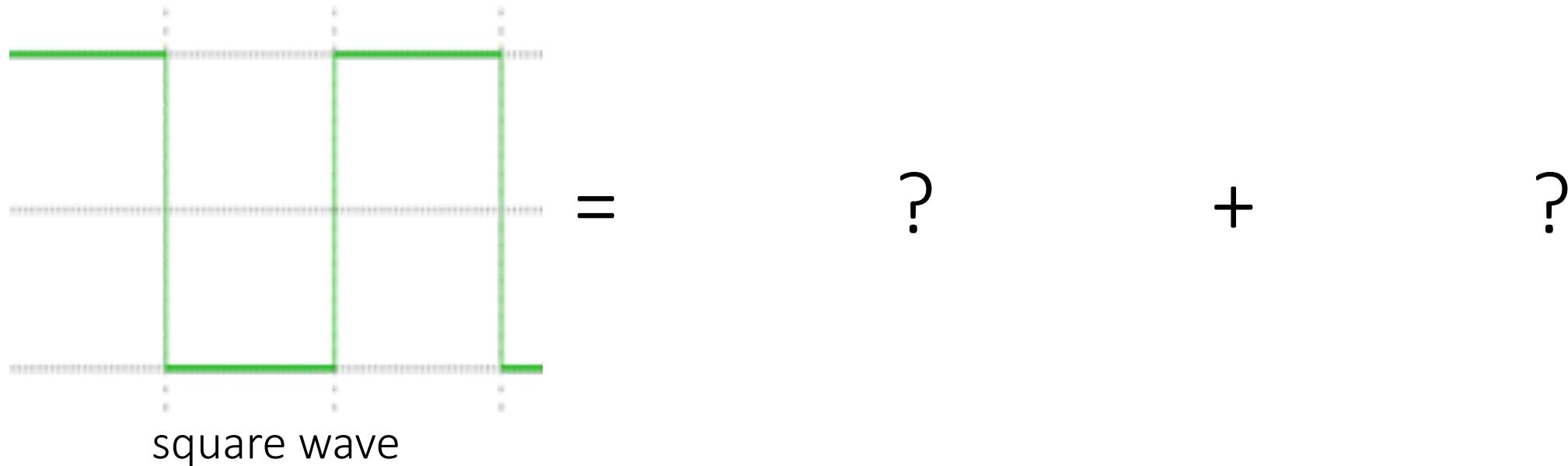
$$f(x) = \sin(2\pi x) + \frac{1}{3} \sin(2\pi 3x)$$

$$\sin(2\pi x)$$

$$\frac{1}{3} \sin(2\pi 3x)$$

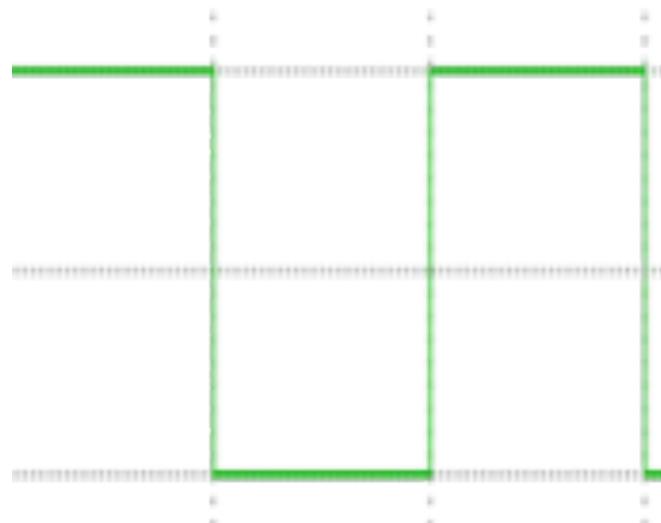
Examples

How would you generate this function?



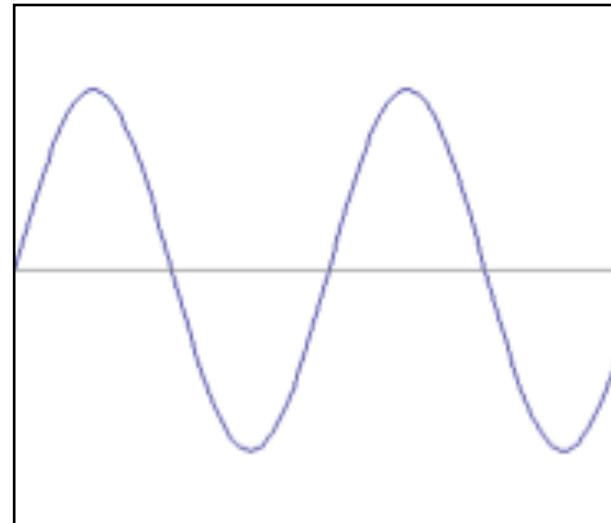
Examples

How would you generate this function?

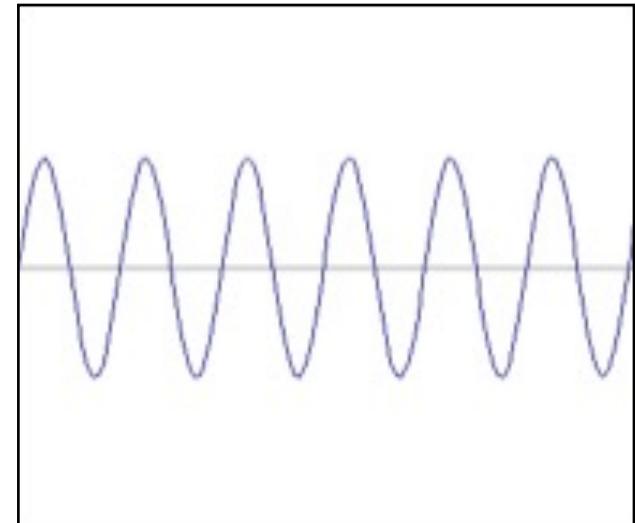


square wave

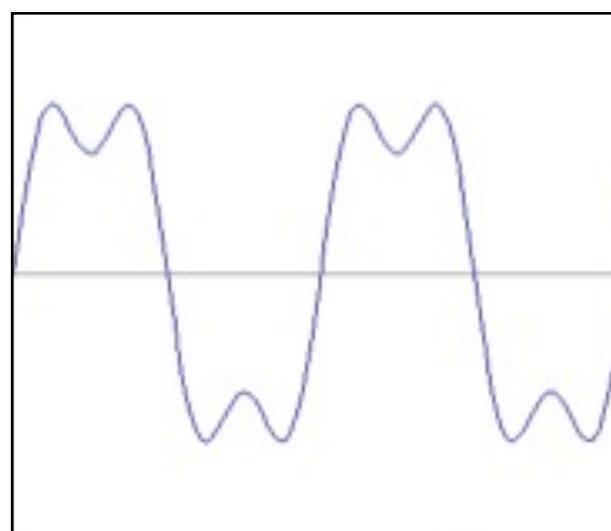
\approx



+

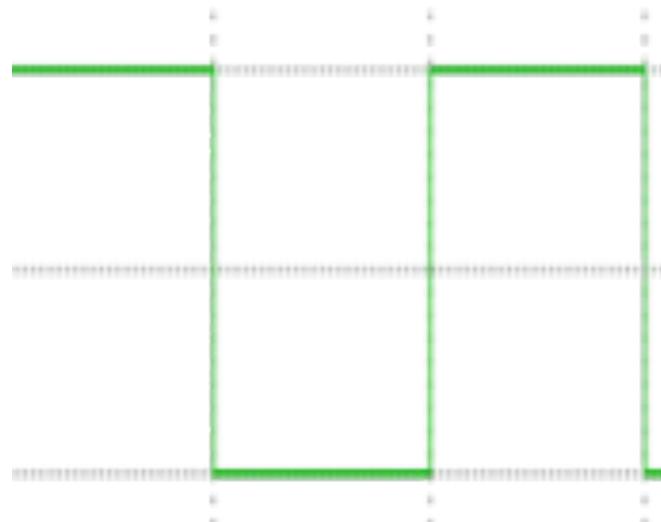


=



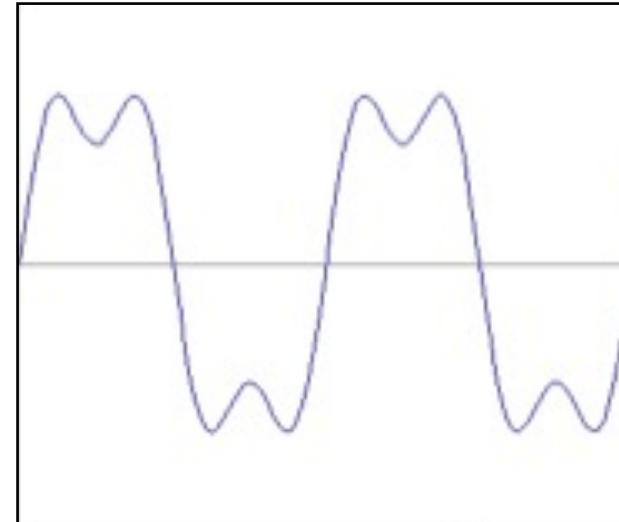
Examples

How would you generate this function?

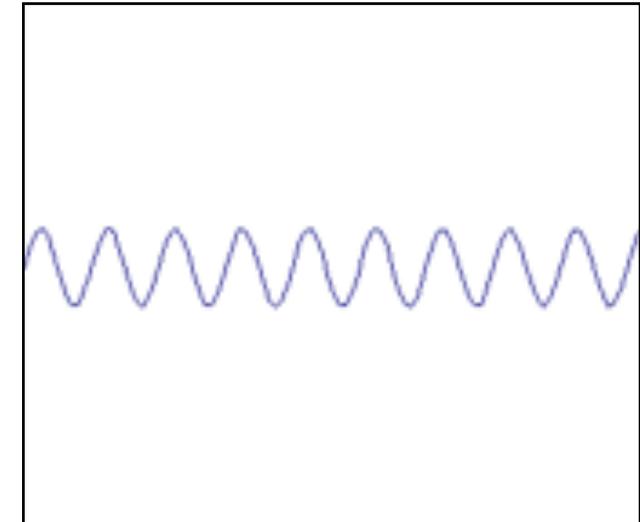


square wave

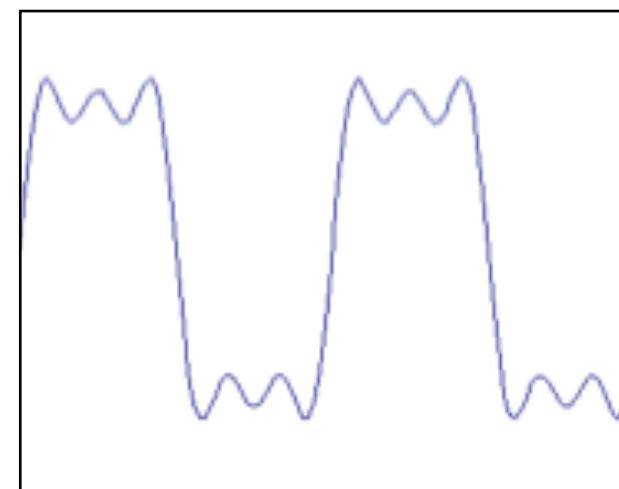
\approx



+

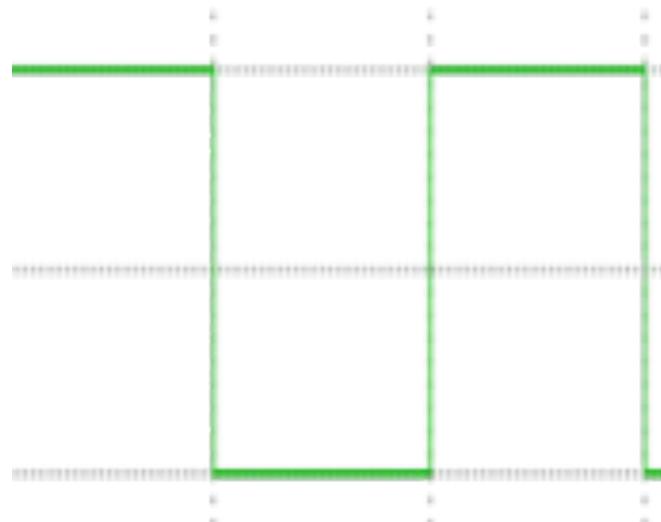


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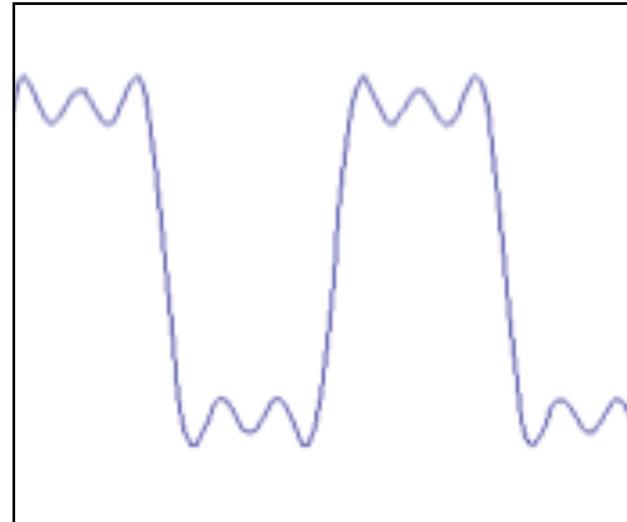
Examples

How would you generate this function?

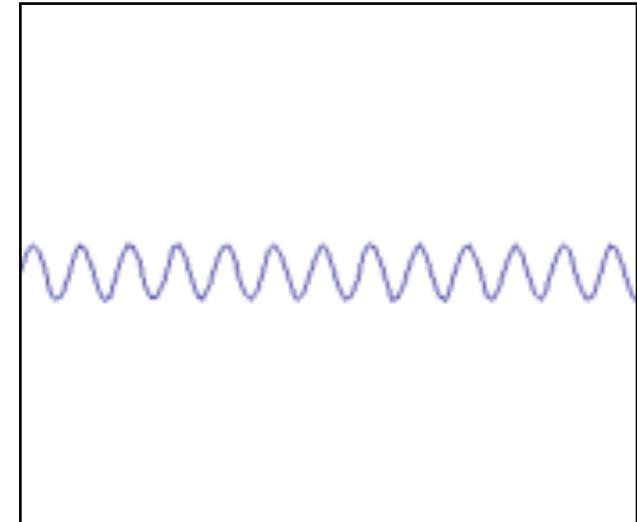


square wave

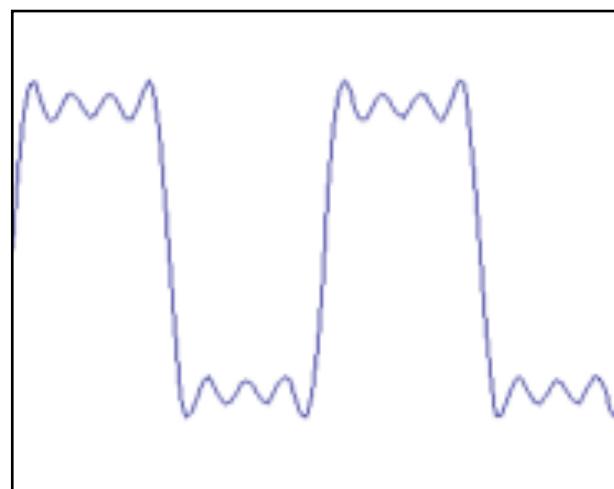
\approx



+

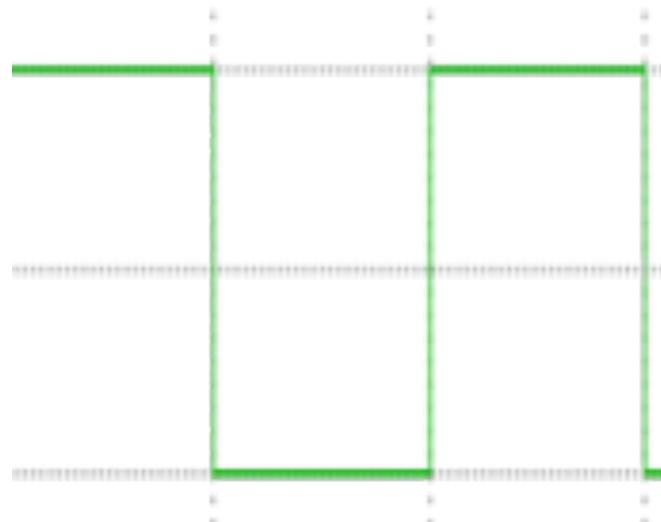


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Examples

How would you generate this function?

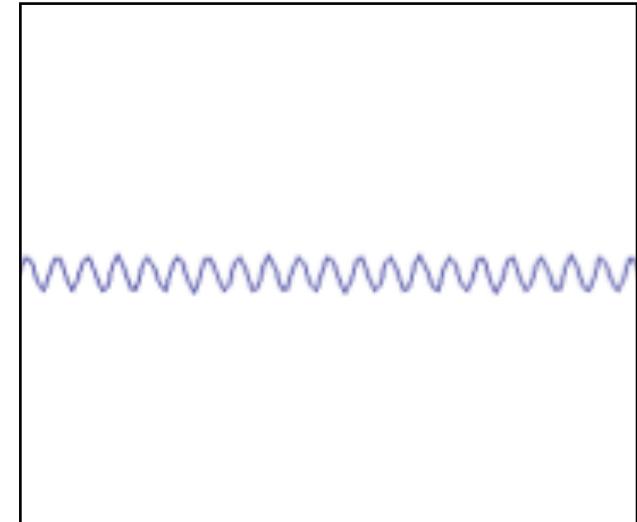


square wave

\approx



+

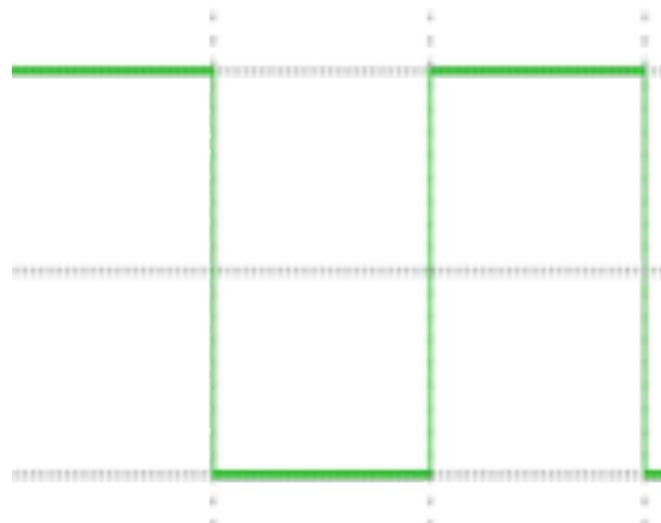


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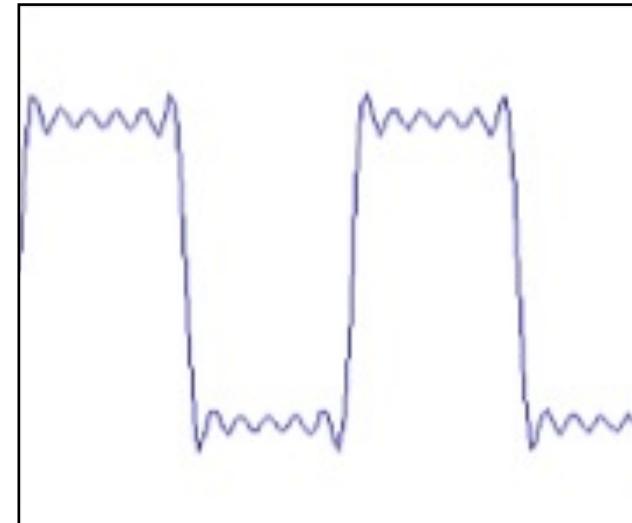
Examples

How would you generate this function?

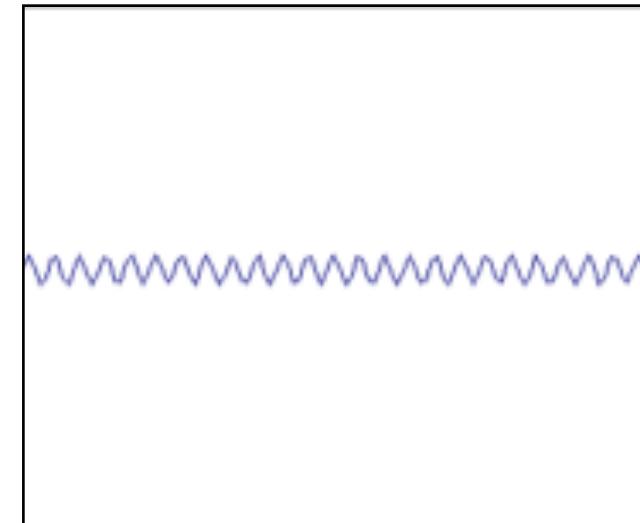


square wave

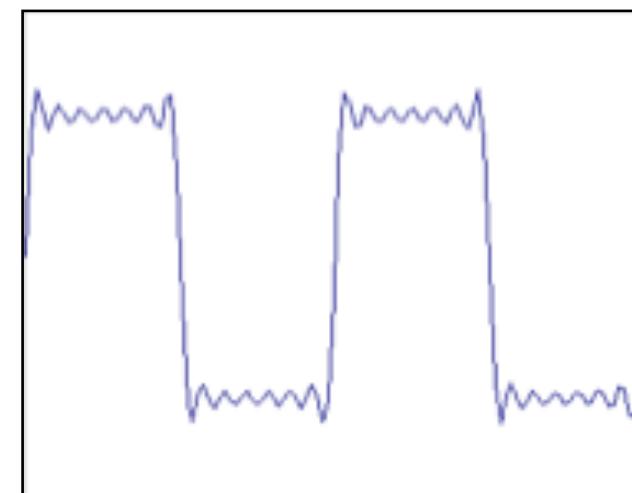
\approx



+

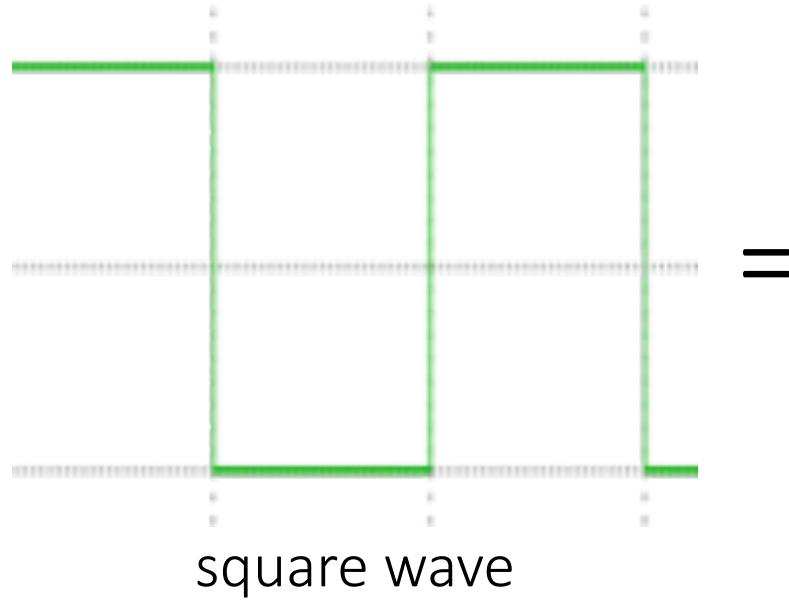


=



How would you express
this mathematically?

Examples



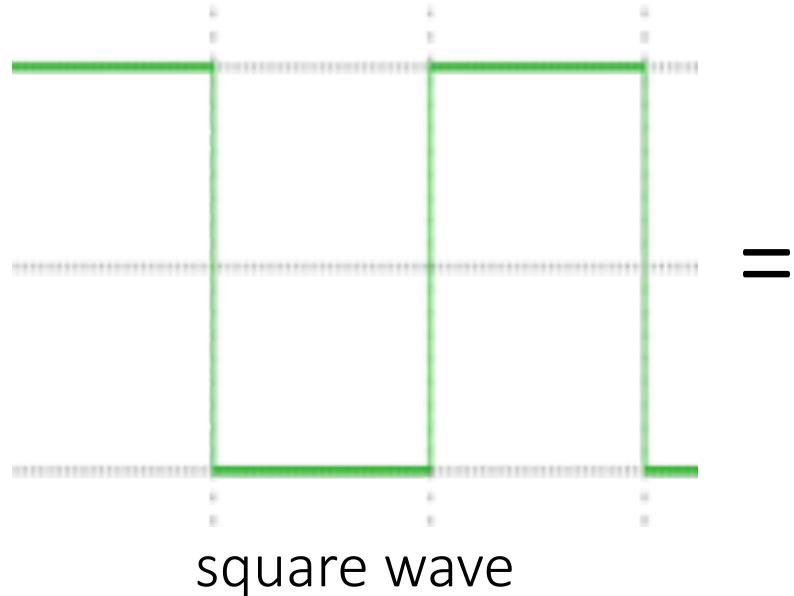
=

$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kx)$$

infinite sum of sine waves

How would you visualize this in the frequency domain?

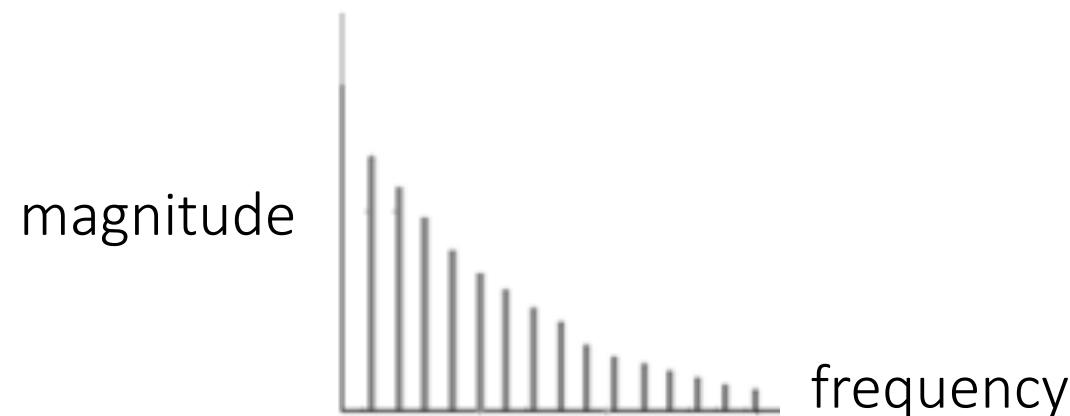
Examples



=

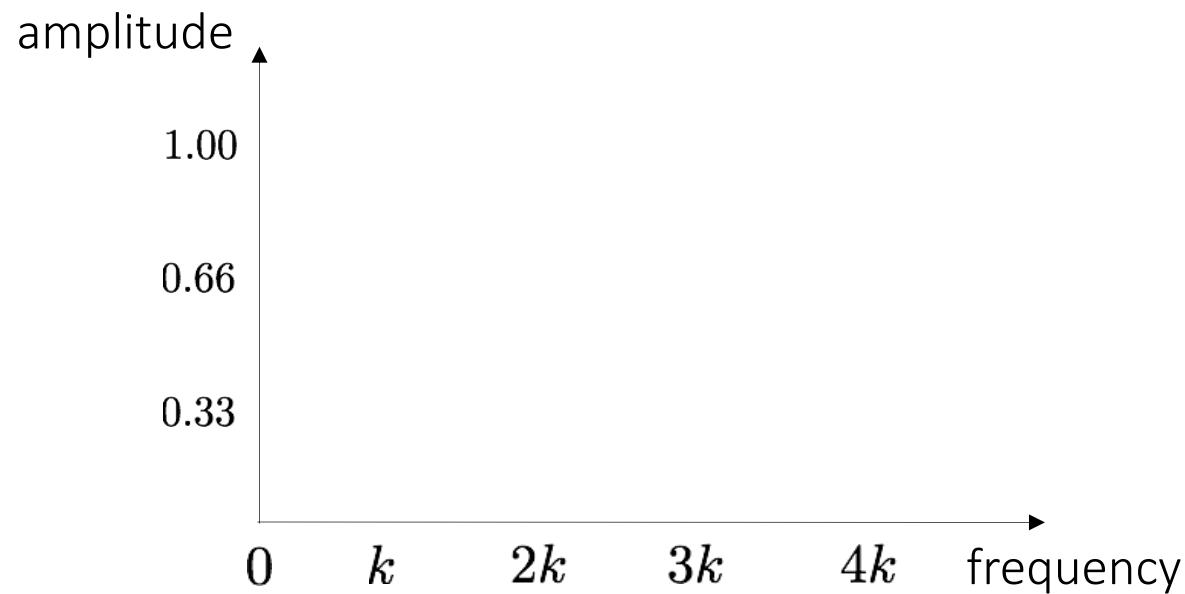
$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kx)$$

infinite sum of sine waves



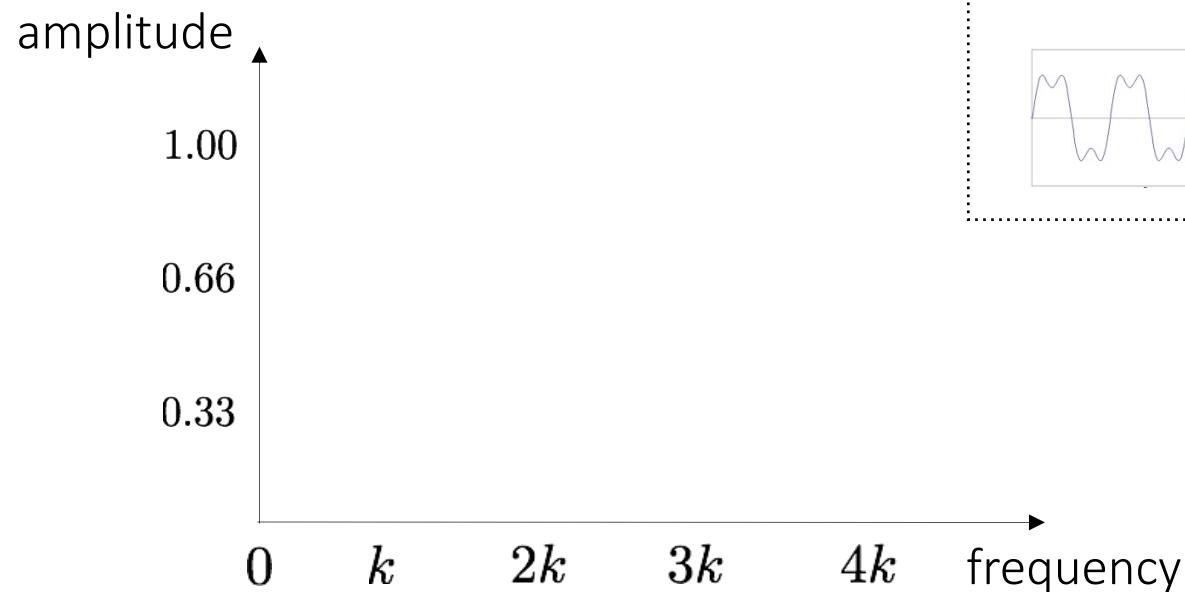
Frequency domain

Visualizing the frequency spectrum

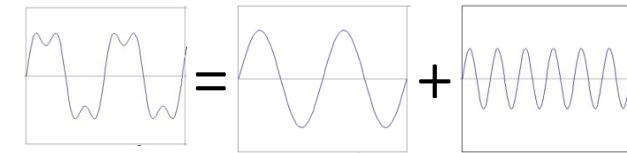


Visualizing the frequency spectrum

Recall the temporal domain visualization

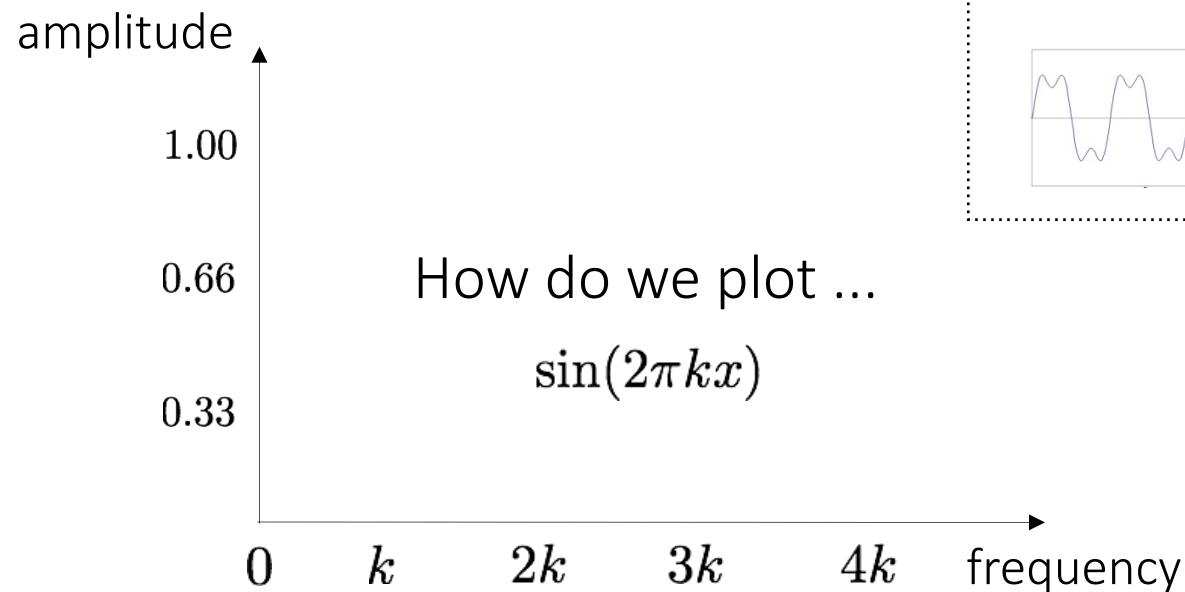


$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$

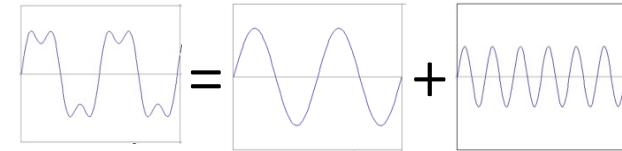


Visualizing the frequency spectrum

Recall the temporal domain visualization

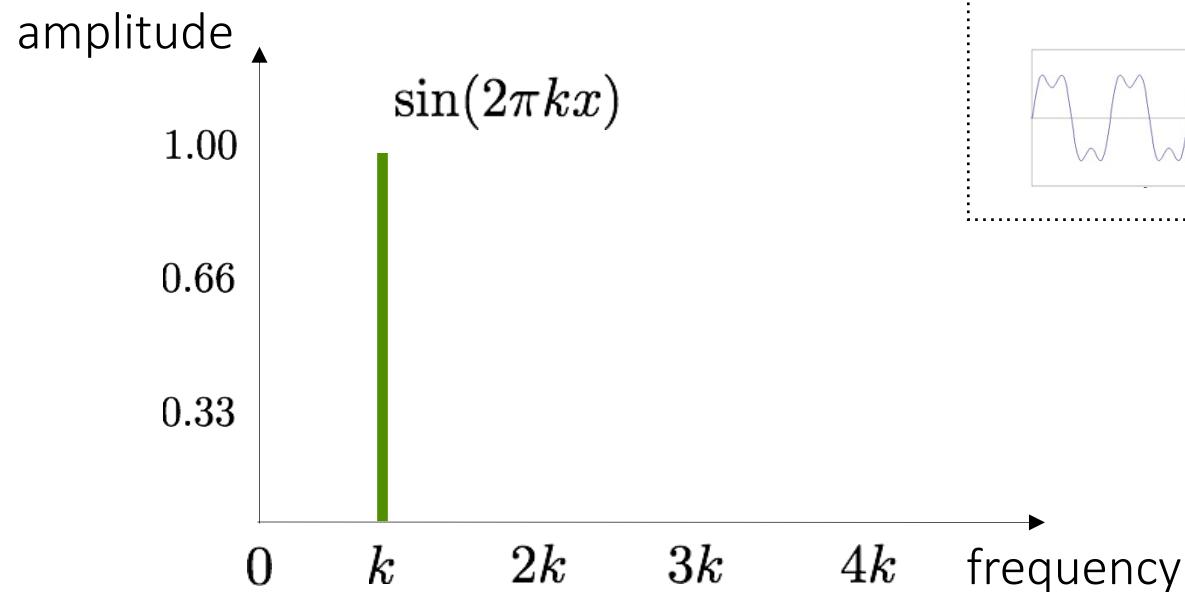


$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$



Visualizing the frequency spectrum

Recall the temporal domain visualization

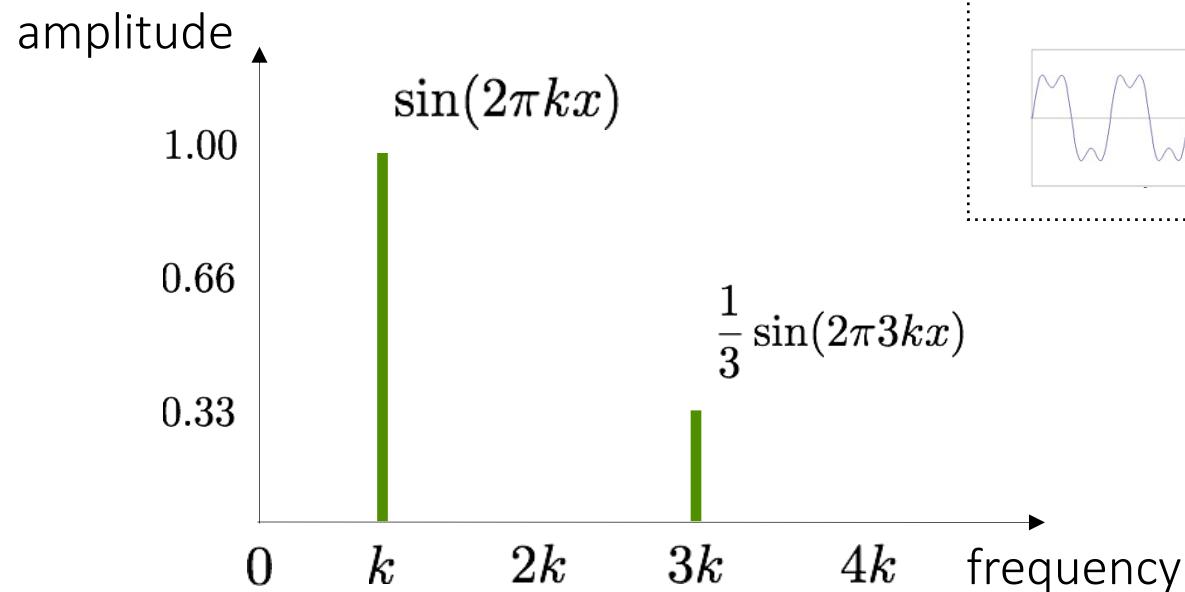


$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$

The equation $f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$ is shown within a dotted box. Below the equation is a diagram illustrating the decomposition of the function. It shows three separate plots: a blue square wave-like signal, a red sine wave, and a green sine wave. An equals sign between the first two plots indicates they sum to the third, which represents the original function $f(x)$.

Visualizing the frequency spectrum

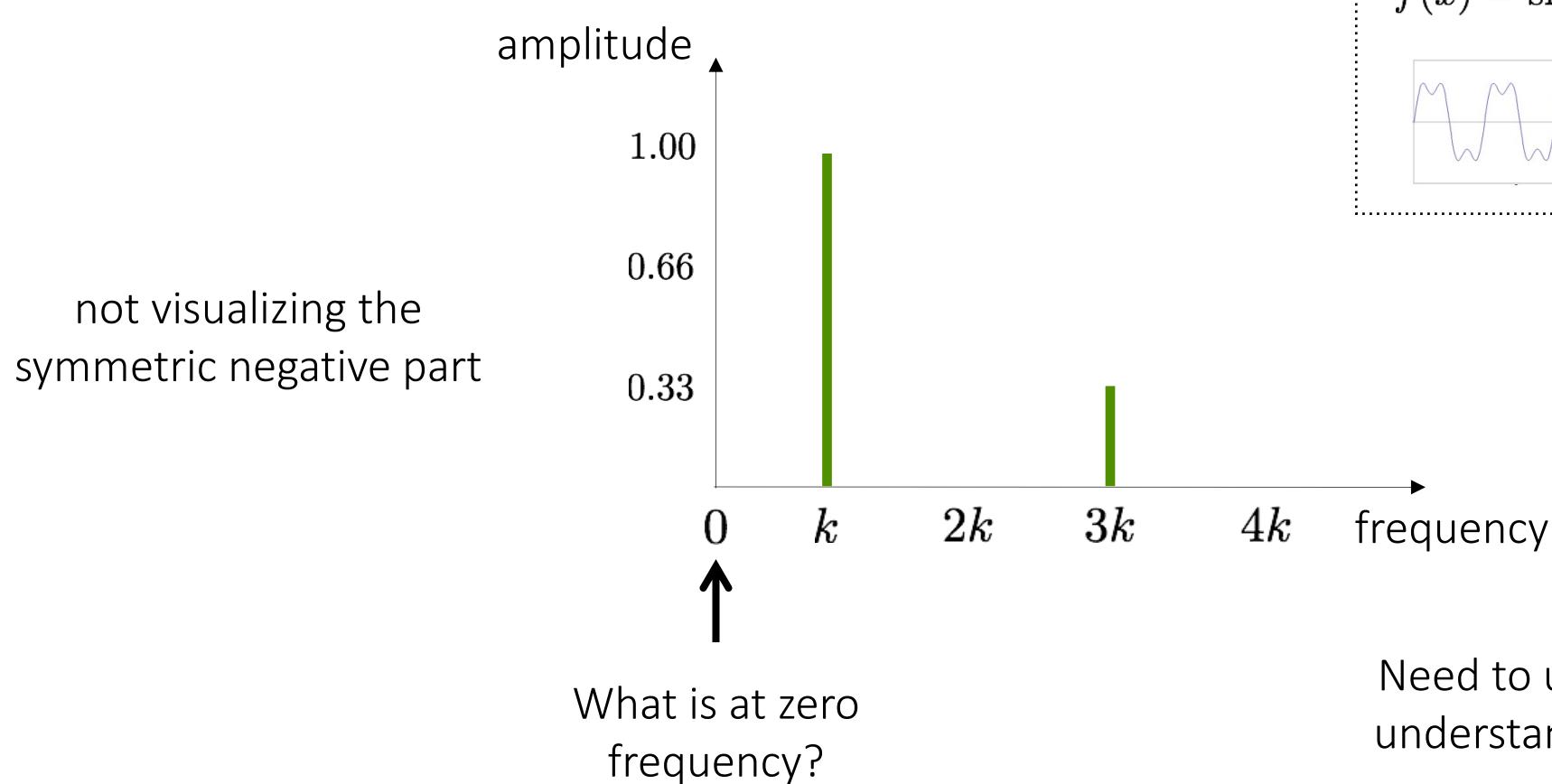
Recall the temporal domain visualization



$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$

The equation $f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$ is shown inside a dotted box. Below the box, three plots are shown: a blue square containing a periodic wave, followed by an equals sign, another blue square containing a periodic wave, and finally a plus sign followed by a third blue square containing a periodic wave.

Visualizing the frequency spectrum



Recall the temporal domain visualization

$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$
$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx) =$$

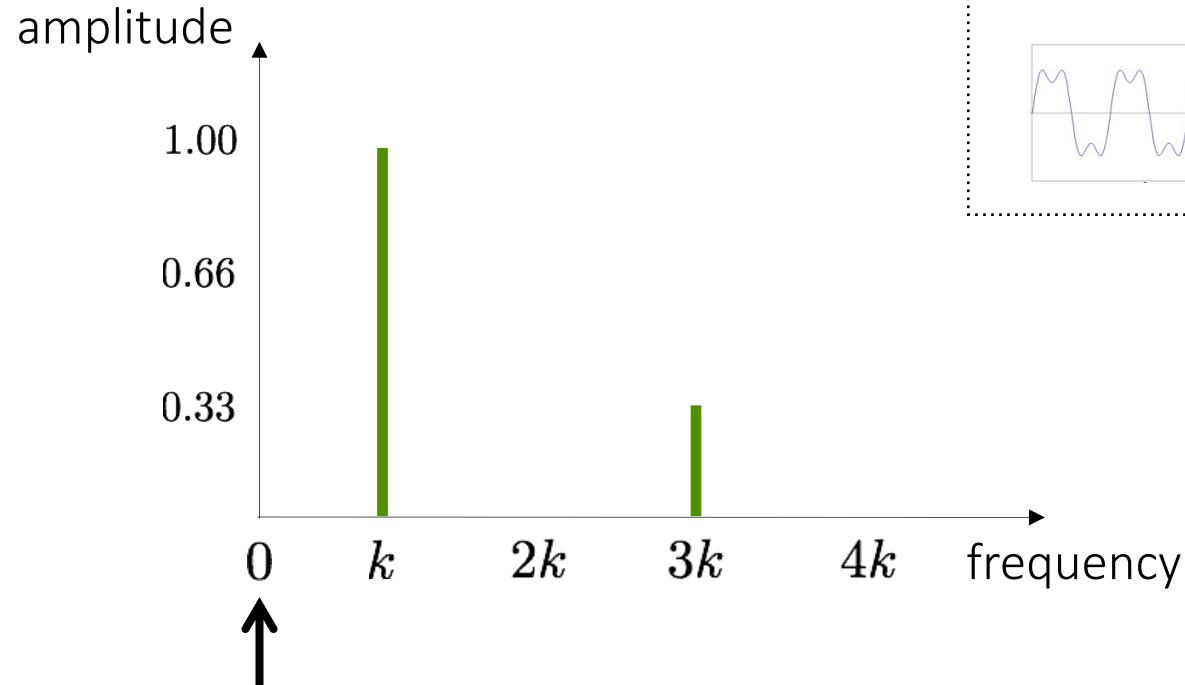
+

+

Need to understand this to understand the 2D version!

Visualizing the frequency spectrum

not visualizing the
symmetric negative part

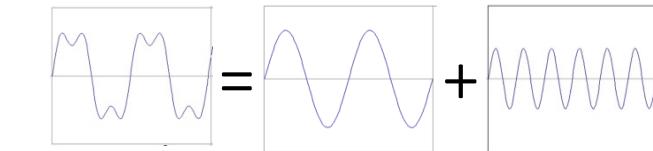


signal average (zero
for a sine wave with
no offset)

Recall the temporal domain visualization

$$f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$$

The equation $f(x) = \sin(2\pi kx) + \frac{1}{3} \sin(2\pi 3kx)$ is shown in a box. Below it, a diagram illustrates the decomposition of the function into its components. It shows three separate plots: a blue sine wave, a red sine wave, and their sum, which is a purple sine wave. The red sine wave has an amplitude of $\frac{1}{3}$.

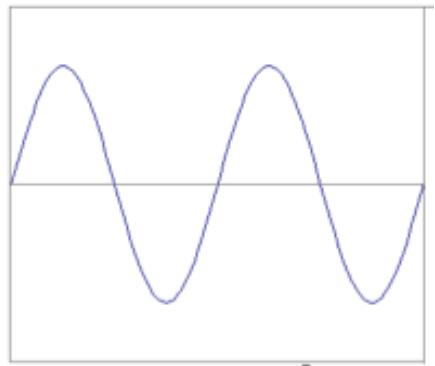


Need to understand this to
understand the 2D version!

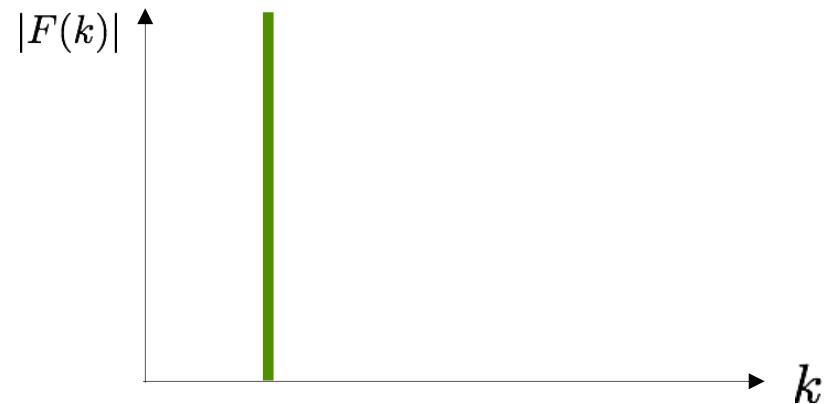
Examples

Spatial domain visualization

1D



Frequency domain visualization



2D

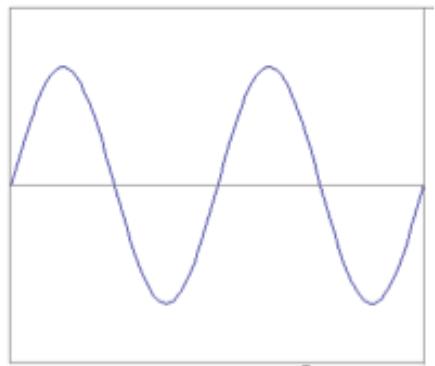


?

Examples

Spatial domain visualization

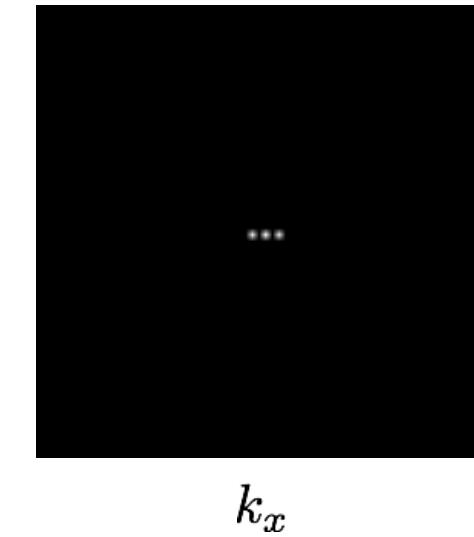
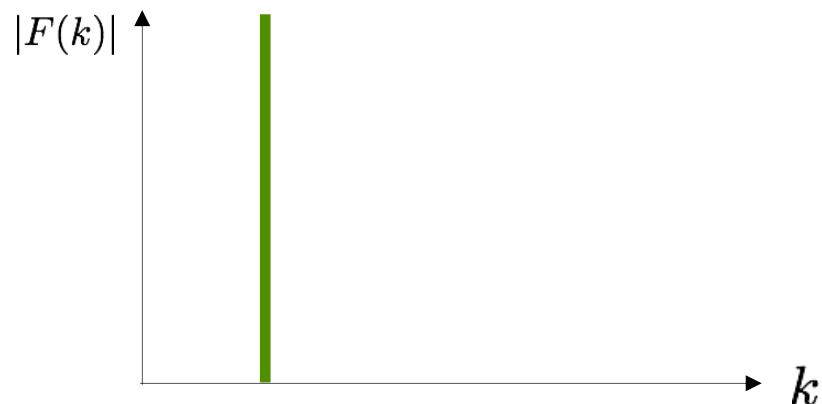
1D



2D



Frequency domain visualization



k_y

...

k_x

What do the three dots
correspond to?

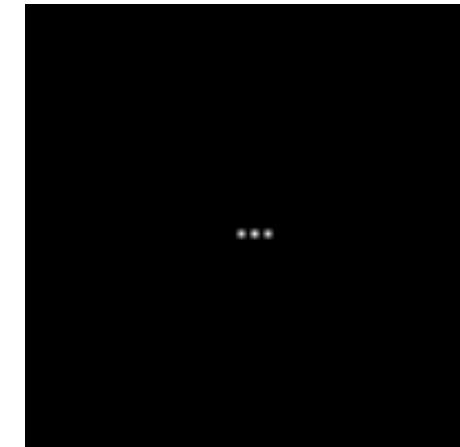
Examples

Spatial domain visualization



Frequency domain visualization

?



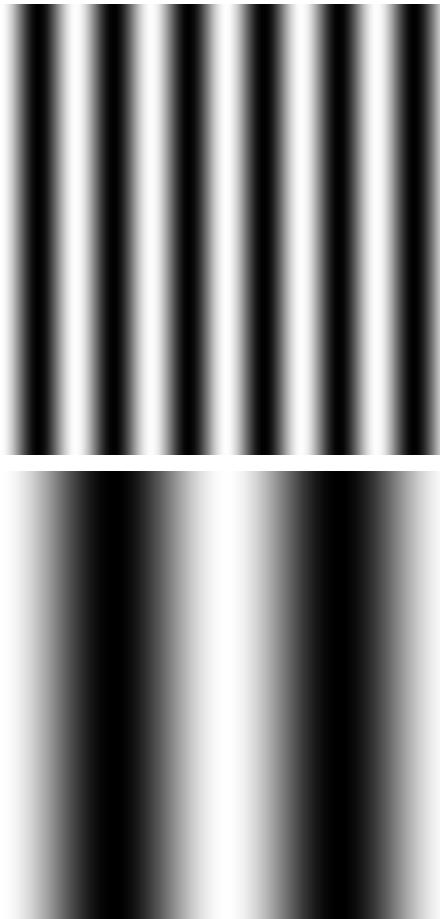
k_x

k_y

...

Examples

Spatial domain visualization

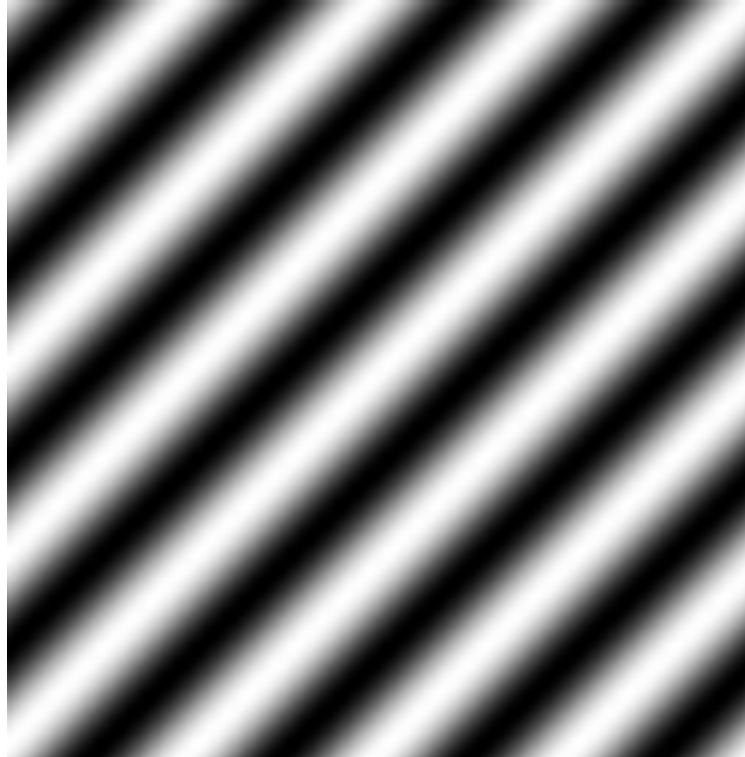


Frequency domain visualization



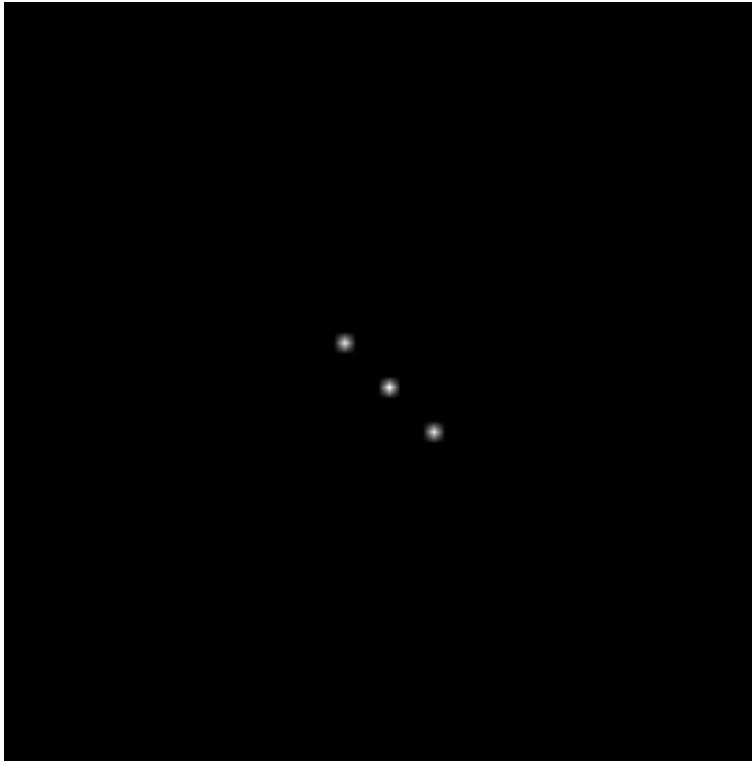
Examples

How would you generate this image with sine waves?



Examples

How would you generate this image with sine waves?

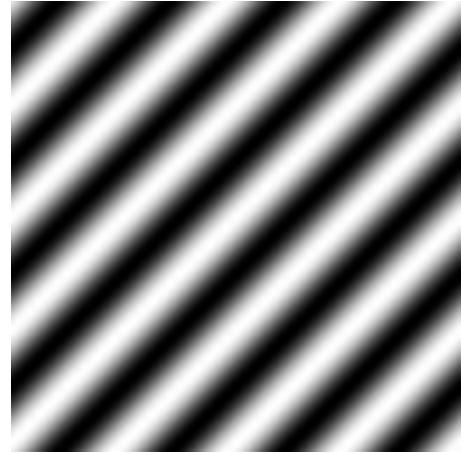


Has both an x and
y components

Examples



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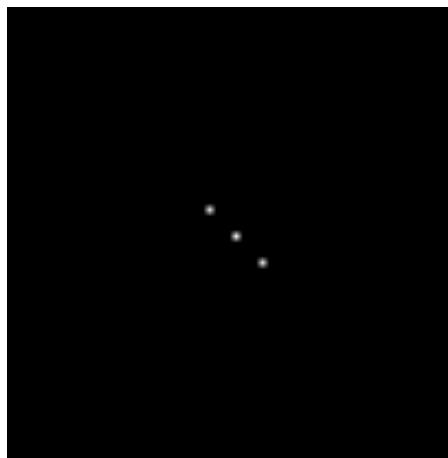
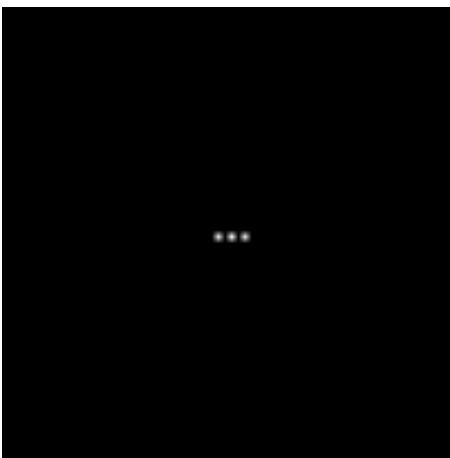


=

?

Examples

$$\begin{matrix} \text{Image 1} & + & \text{Image 2} & = & \text{Result} \end{matrix}$$

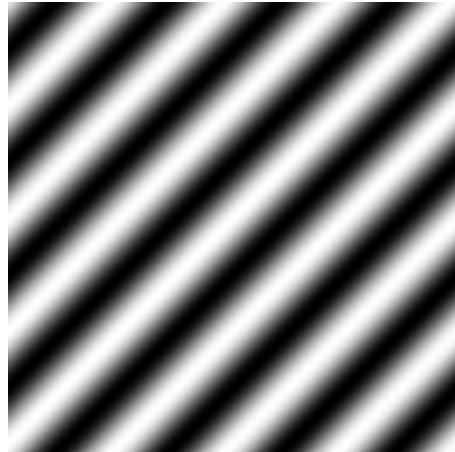


?

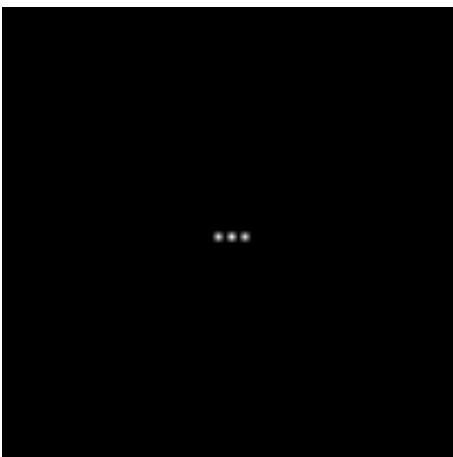
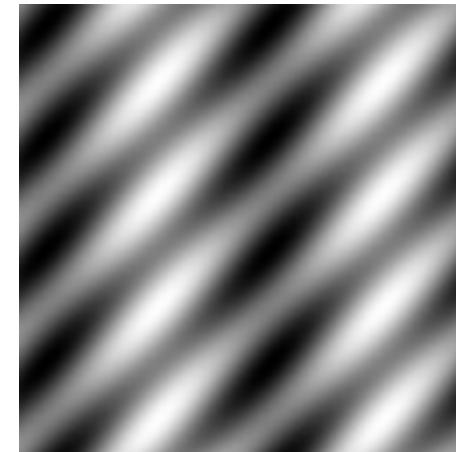
Examples



+



=



Basic building block

$$A \sin(\omega x + \phi)$$

amplitude sinusoid angular frequency variable phase

What about non-periodic signals?

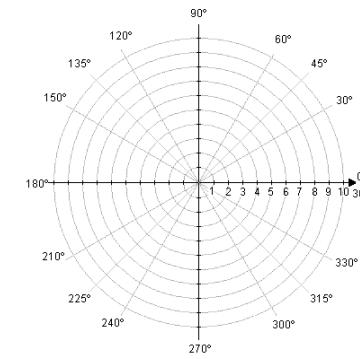
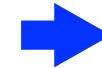
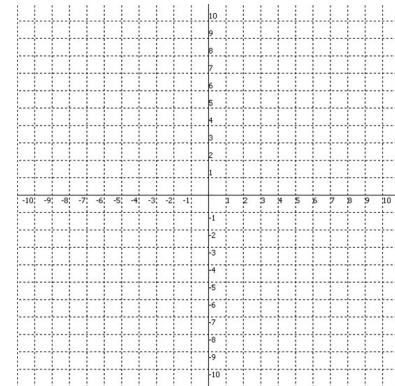
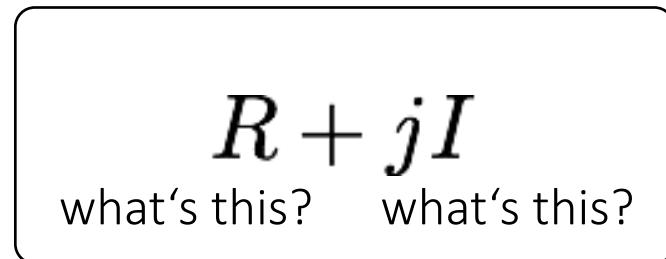
Fourier's claim: Add enough of these to get any periodic signal you want!

Fourier transform

Recalling some basics

Complex numbers have two parts:

rectangular
coordinates



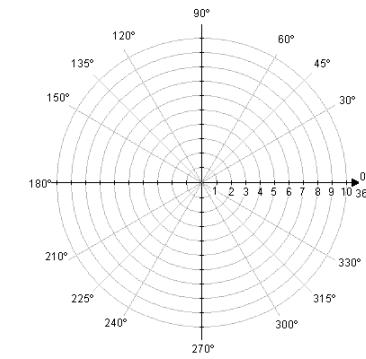
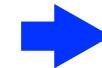
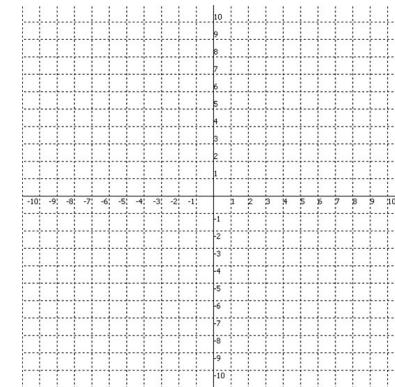
Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

real imaginary



Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

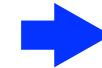
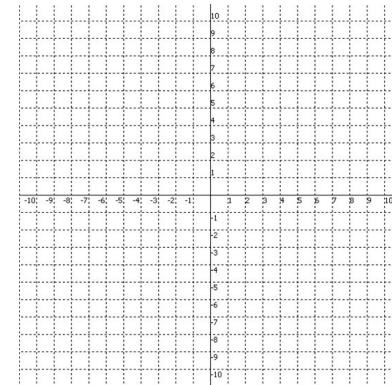
real imaginary

Alternative reparameterization:

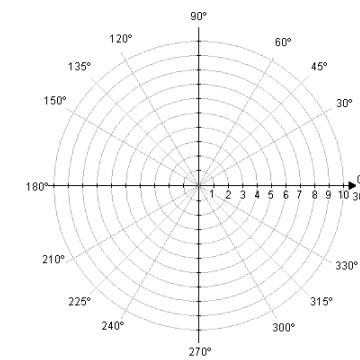
polar
coordinates

$$r(\cos \theta + j \sin \theta)$$

how do we compute these?



polar transform



Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

real imaginary

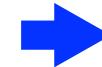
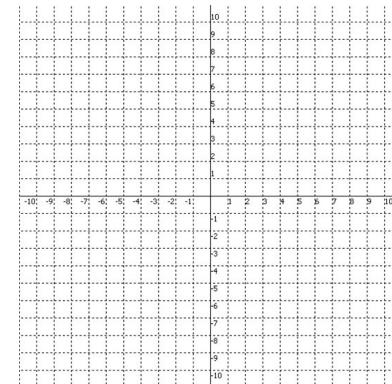
Alternative reparameterization:

polar
coordinates

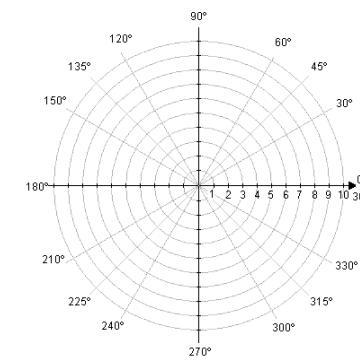
$$r(\cos \theta + j \sin \theta)$$

polar transform

$$\theta = \tan^{-1}\left(\frac{I}{R}\right) \quad r = \sqrt{R^2 + I^2}$$



polar transform



Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

real imaginary

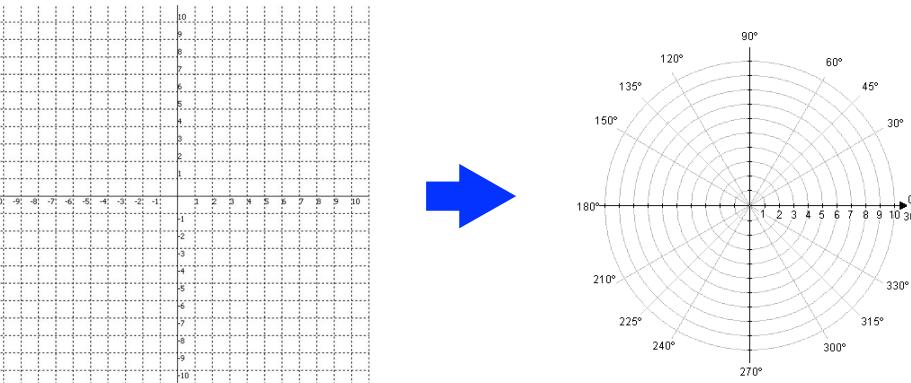
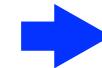
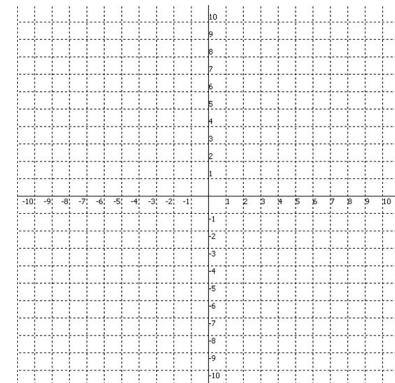
Alternative reparameterization:

polar
coordinates

$$r(\cos \theta + j \sin \theta)$$

polar transform

$$\theta = \tan^{-1}\left(\frac{I}{R}\right) \quad r = \sqrt{R^2 + I^2}$$



polar transform

How do you write
these in exponential
form?

Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

real imaginary

Alternative reparameterization:

polar
coordinates

$$r(\cos \theta + j \sin \theta)$$

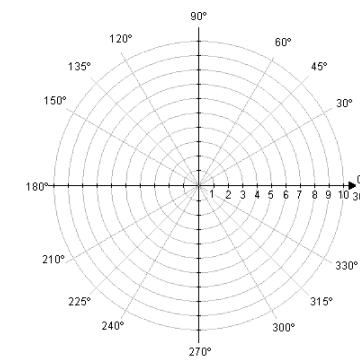
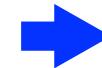
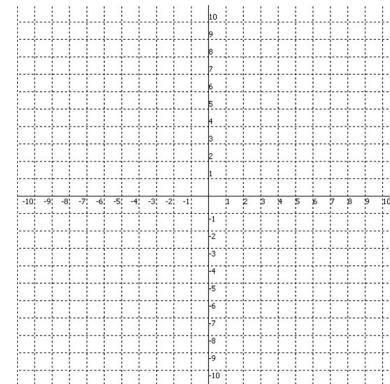
polar transform

$$\theta = \tan^{-1}\left(\frac{I}{R}\right) \quad r = \sqrt{R^2 + I^2}$$

or
equivalently

$$re^{j\theta}$$

how did we get this?



exponential
form

Recalling some basics

Complex numbers have two parts:

rectangular
coordinates

$$R + jI$$

real imaginary

Alternative reparameterization:

polar
coordinates

$$r(\cos \theta + j \sin \theta)$$

polar transform

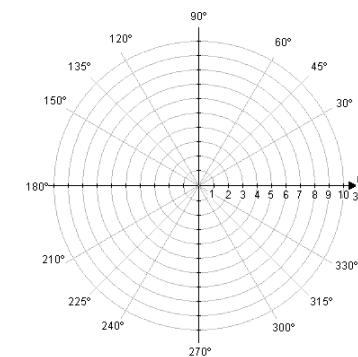
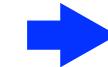
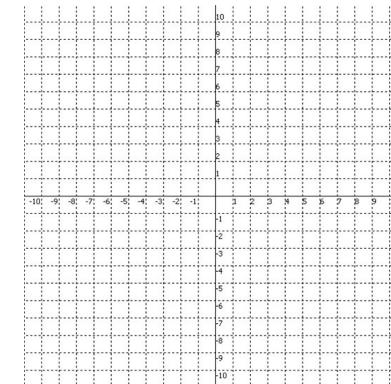
$$\theta = \tan^{-1}\left(\frac{I}{R}\right) \quad r = \sqrt{R^2 + I^2}$$

or
equivalently

$$re^{j\theta}$$

Euler's formula

$$e^{j\theta} = \cos \theta + j \sin \theta$$



exponential
form

This will help us understand the Fourier transform equations

Fourier transform

continuous

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi kx}dx$$

inverse Fourier transform

$$f(x) = \int_{-\infty}^{\infty} F(k)e^{j2\pi kx}dk$$

discrete

$$F(k) = \frac{1}{N} \sum_{x=0}^{N-1} f(x)e^{-j2\pi kx/N}$$
$$k = 0, 1, 2, \dots, N-1$$

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k)e^{j2\pi kx/N}$$
$$x = 0, 1, 2, \dots, N-1$$

Where is the connection to the "summation of sine waves" idea?

Fourier transform

Where is the connection to the "*summation of sine waves*" idea?

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j2\pi kx/N}$$

sum over frequencies

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) \left\{ \cos\left(\frac{2\pi kx}{N}\right) + j \sin\left(\frac{2\pi kx}{N}\right) \right\}$$

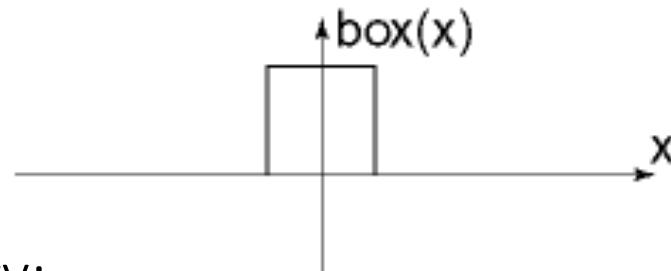
scaling parameter

wave components

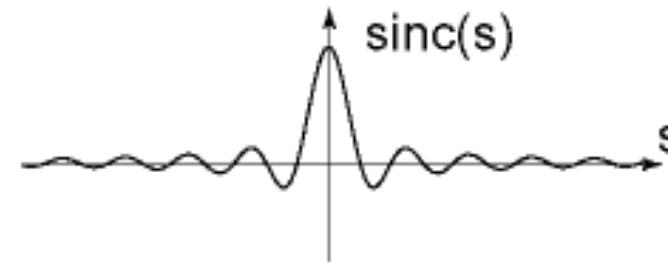
Euler's formula
 $e^{j\theta} = \cos(\theta) + j \sin(\theta)$

Fourier transform pairs

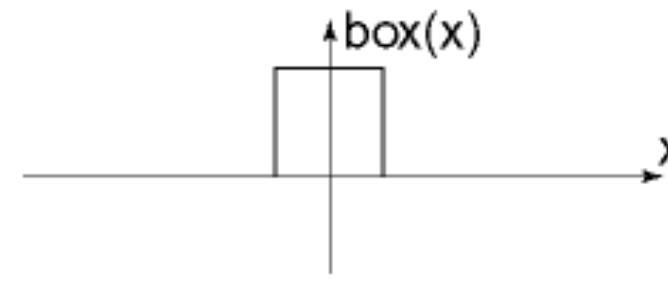
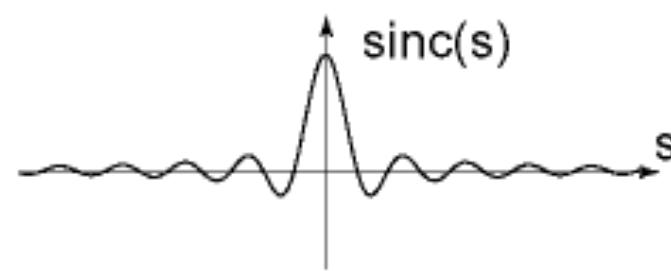
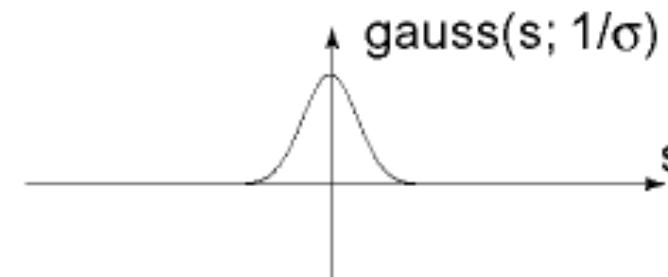
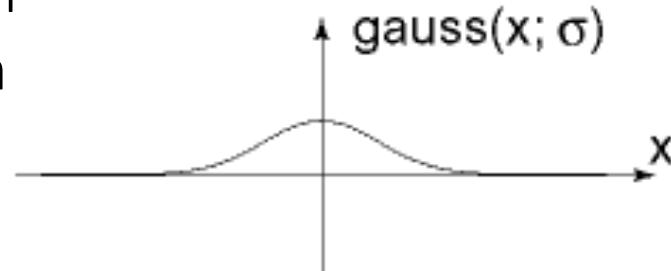
spatial domain



frequency domain



Note the symmetry:
duality property of
Fourier transform



Computing the discrete Fourier transform (DFT)

Computing the discrete Fourier transform (DFT)

$F(k) = \frac{1}{N} \sum_{x=0}^{N-1} f(x)e^{-j2\pi kx/N}$ is just a matrix multiplication:

$$\mathbf{F} = \mathbf{W}\mathbf{f}$$

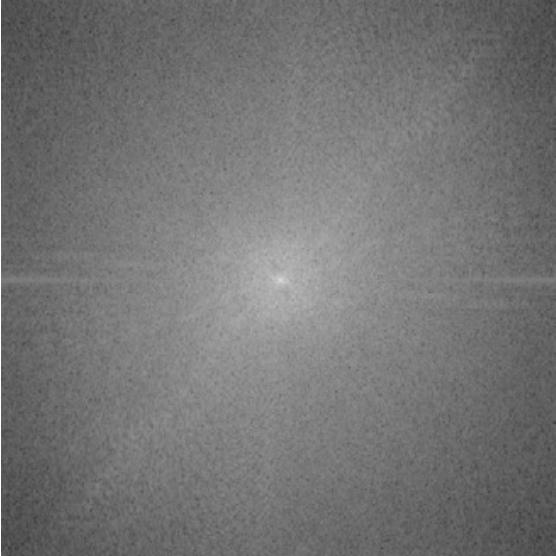
$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \\ \vdots \\ F(N-1) \end{bmatrix} = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 & \dots & W^0 \\ W^0 & W^1 & W^2 & W^3 & \dots & W^{N-1} \\ W^0 & W^2 & W^4 & W^6 & \dots & W^{N-2} \\ W^0 & W^3 & W^6 & W^9 & \dots & W^{N-3} \\ \vdots & \vdots & & & \ddots & \vdots \\ W^0 & W^{N-1} & W^{N-2} & W^{N-3} & \dots & W^1 \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \\ \vdots \\ f(N-1) \end{bmatrix} \quad W = e^{-j2\pi/N}$$

In practice this is implemented using the *fast Fourier transform* (FFT) algorithm.

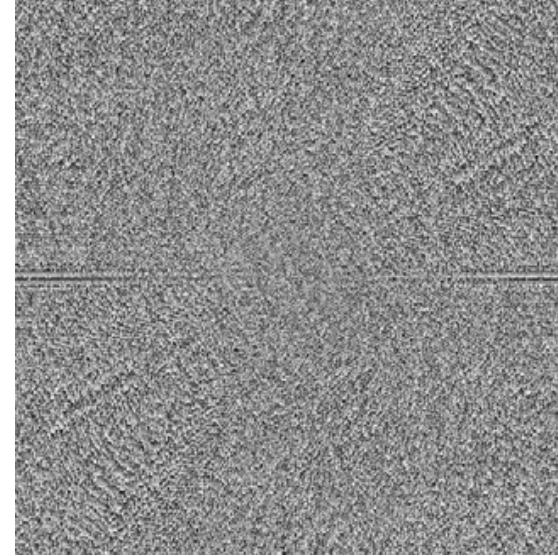
Fourier transforms of natural images



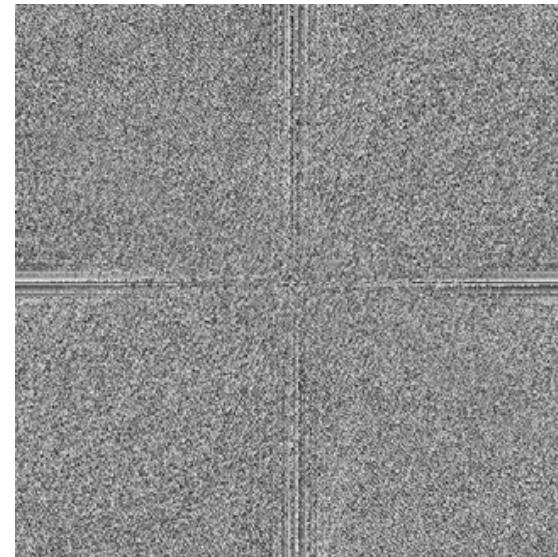
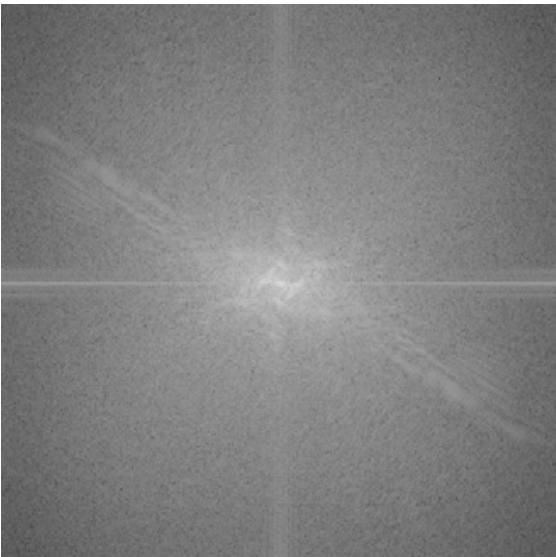
original



amplitude

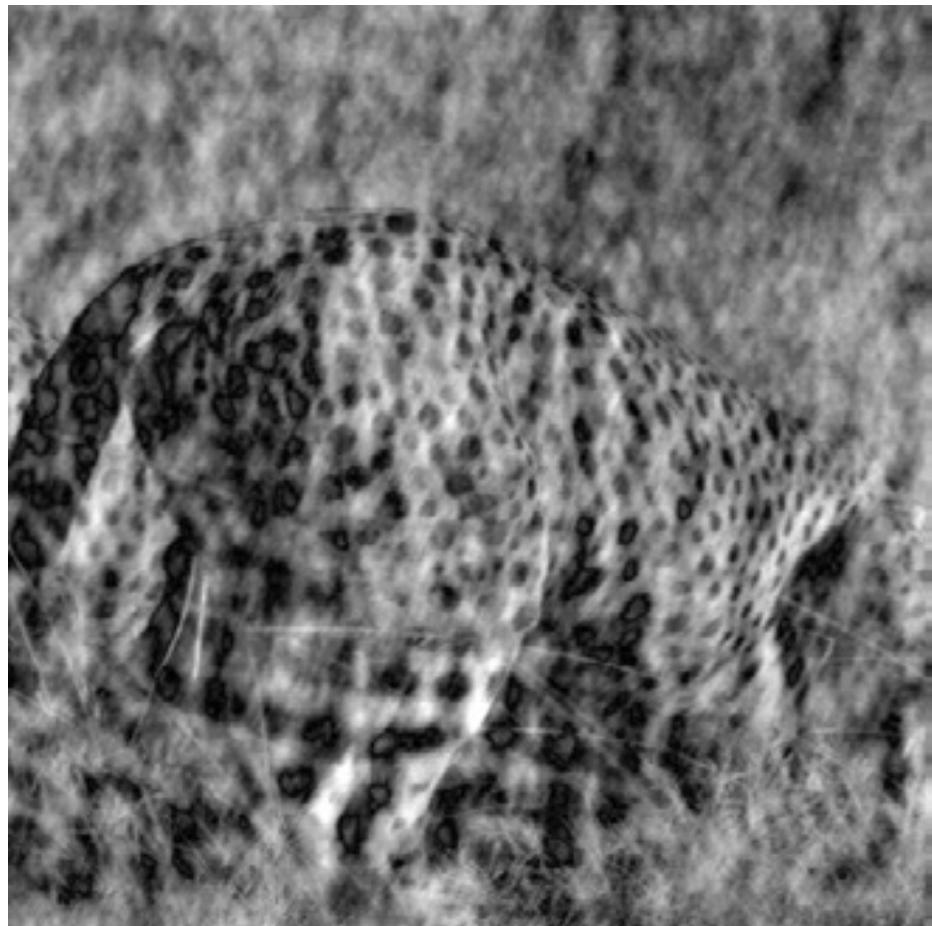


phase

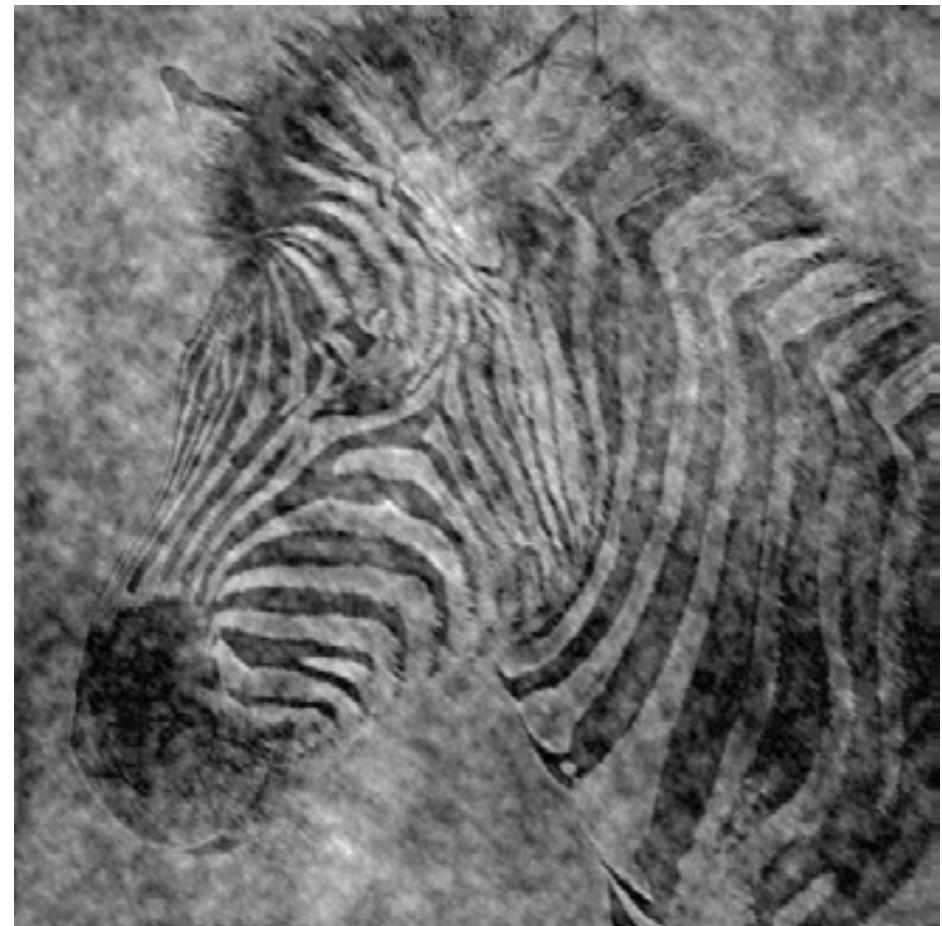


Fourier transforms of natural images

Image phase matters!



cheetah phase with zebra amplitude



zebra phase with cheetah amplitude

Frequency-domain filtering

The convolution theorem

The Fourier transform of the convolution of two functions is the product of their Fourier transforms:

$$\mathcal{F}\{g * h\} = \mathcal{F}\{g\}\mathcal{F}\{h\}$$

The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms:

$$\mathcal{F}^{-1}\{gh\} = \mathcal{F}^{-1}\{g\} * \mathcal{F}^{-1}\{h\}$$

Convolution in spatial domain is equivalent to multiplication in frequency domain!

What do we use convolution for?

Convolution for 1D continuous signals

Definition of linear shift-invariant filtering as convolution:

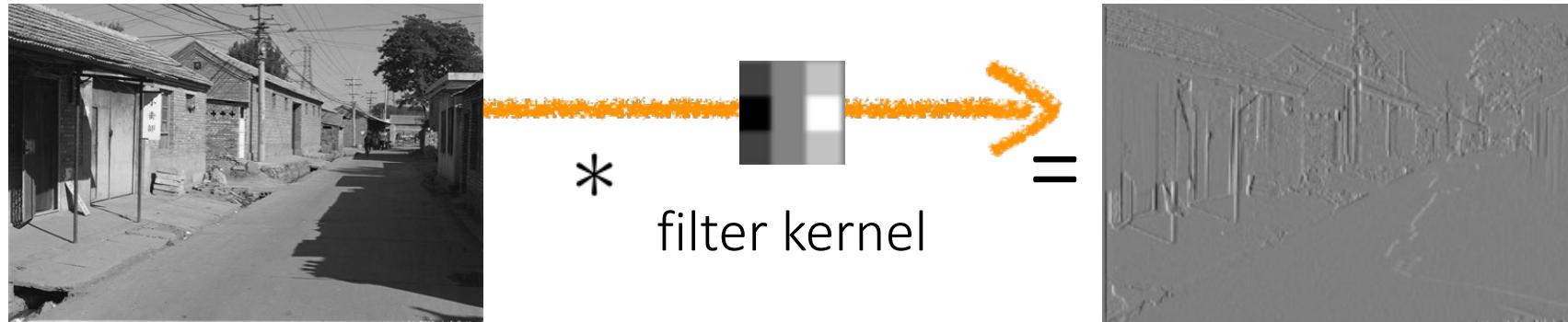
$$(f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x - y)dy$$

filtered signal filter input signal

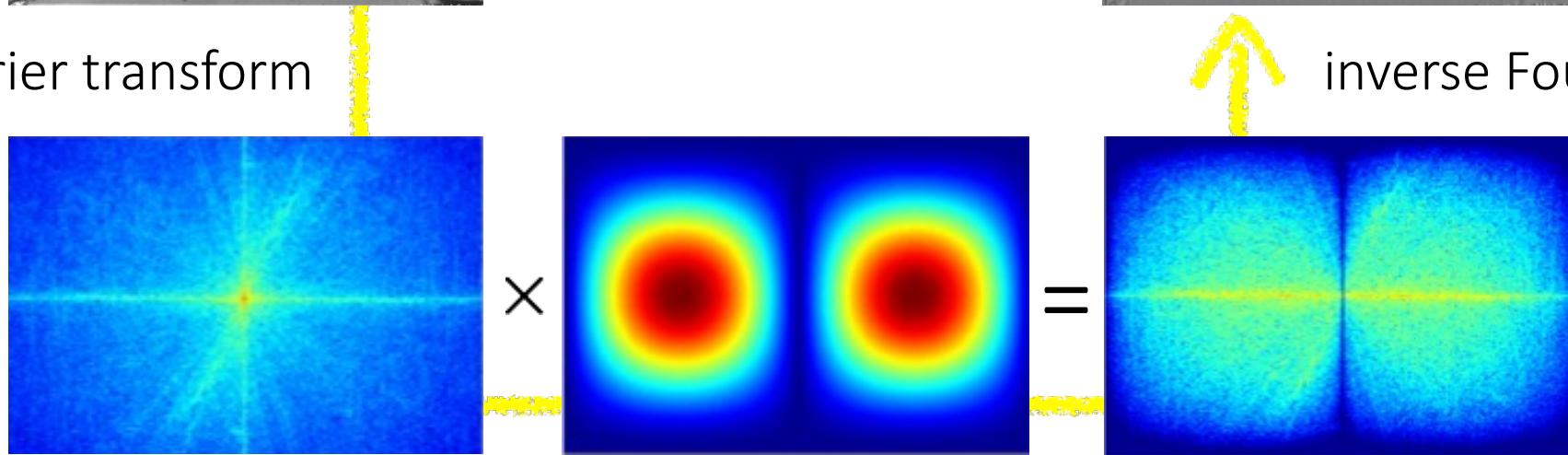
Using the convolution theorem, we can interpret and implement all types of linear shift-invariant filtering as multiplication in frequency domain.

Why implement convolution in frequency domain?

Spatial domain filtering



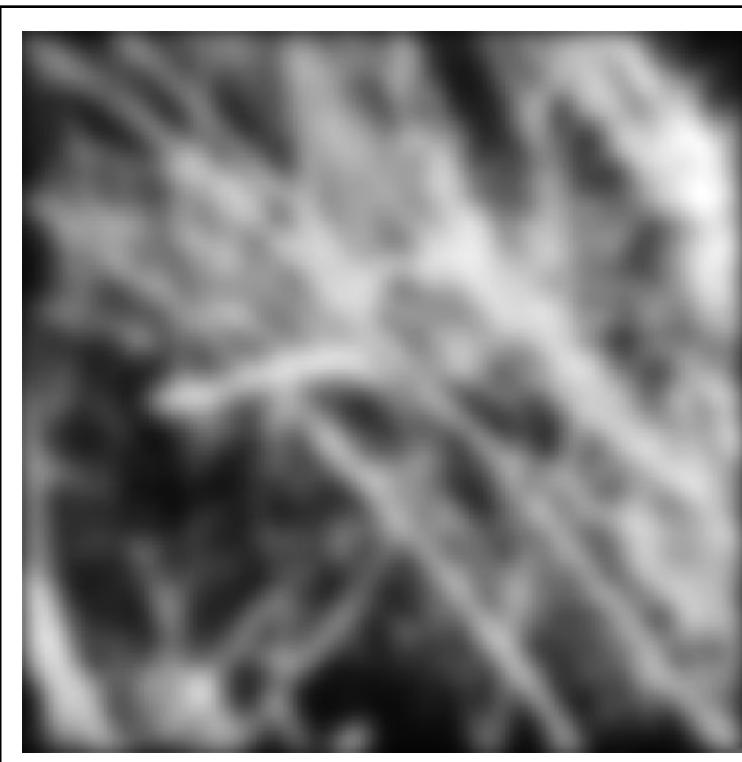
Fourier transform



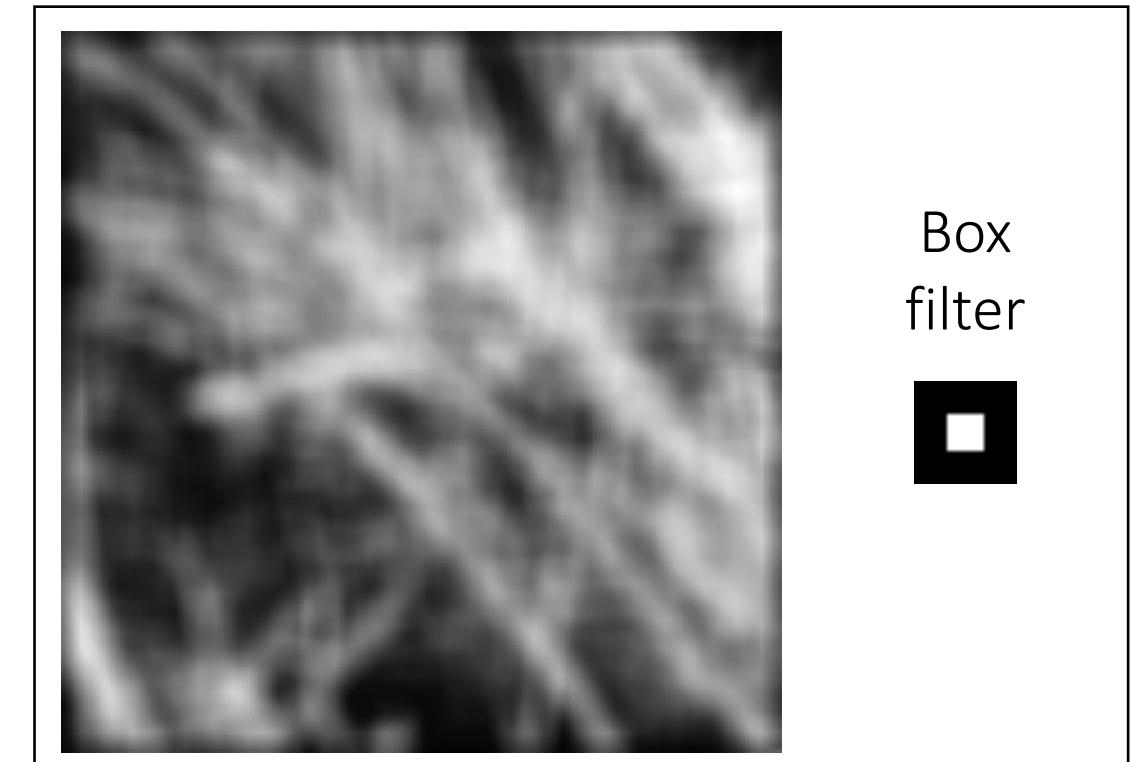
Frequency domain filtering

Revisiting blurring

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?



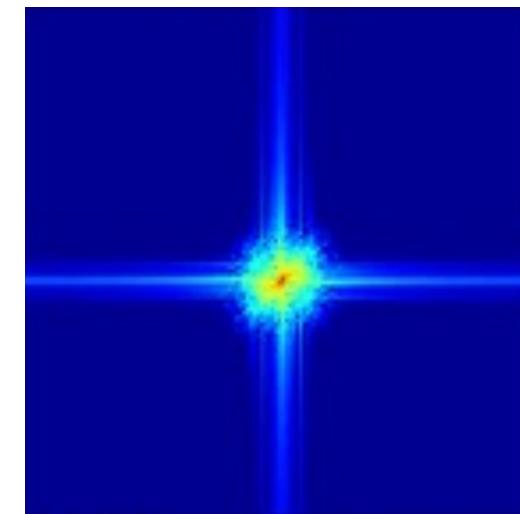
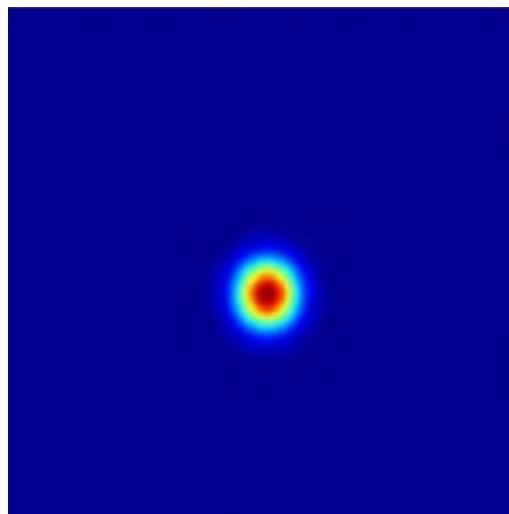
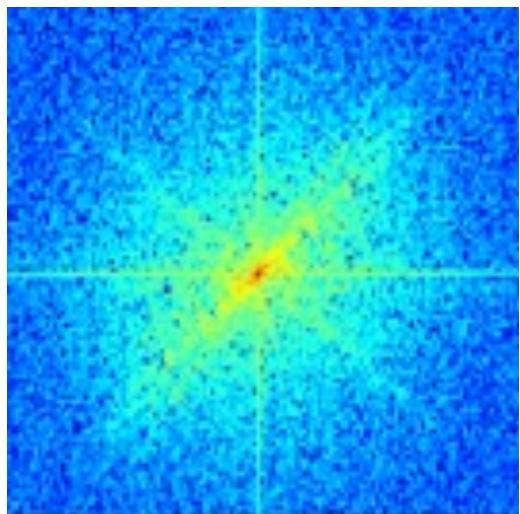
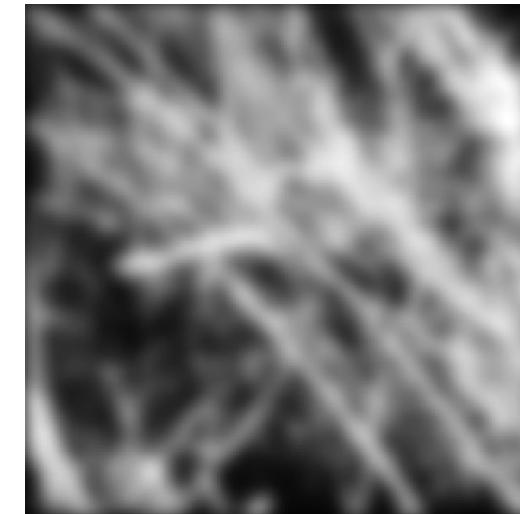
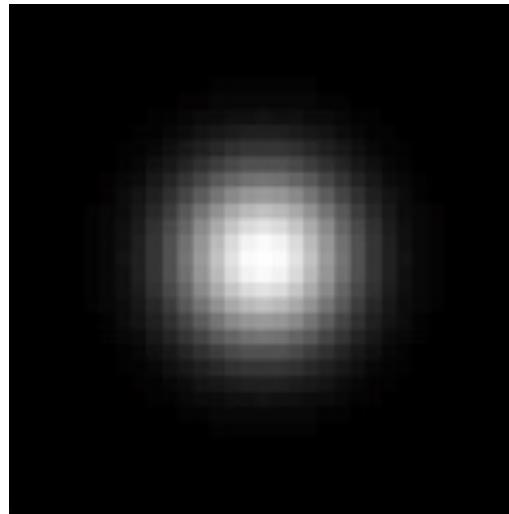
Gaussian
filter



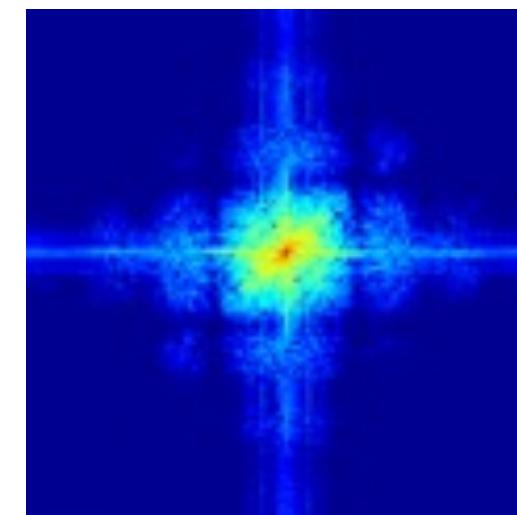
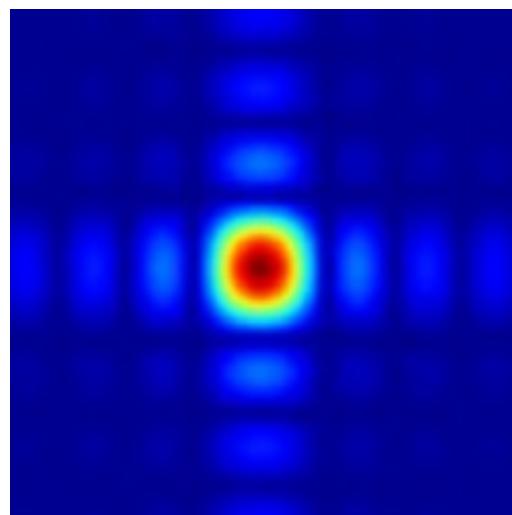
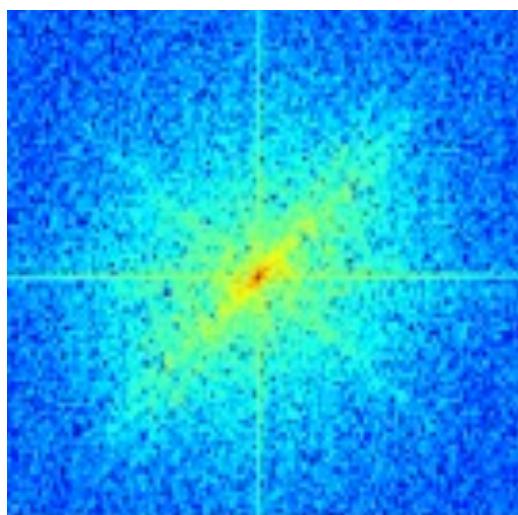
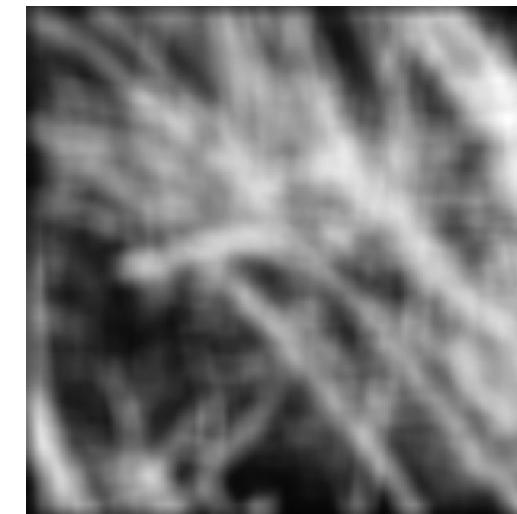
Box
filter



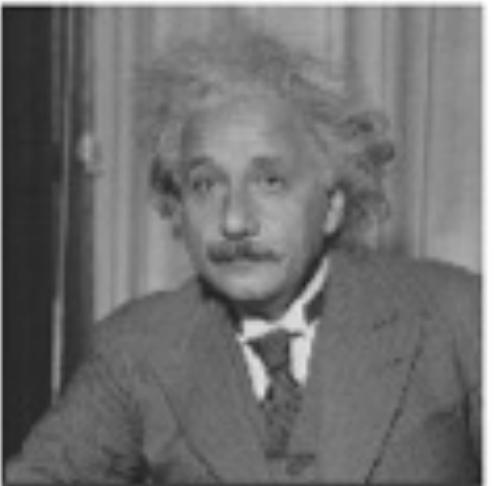
Gaussian blur



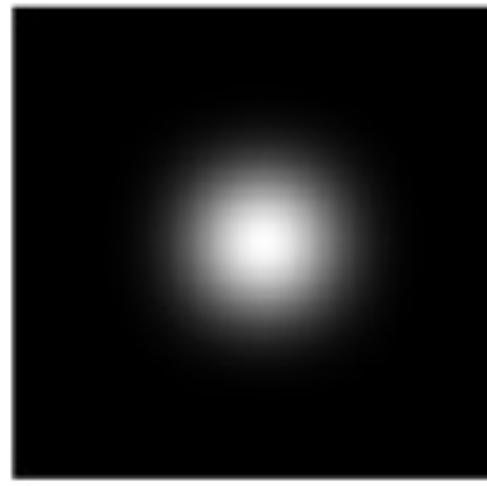
Box blur



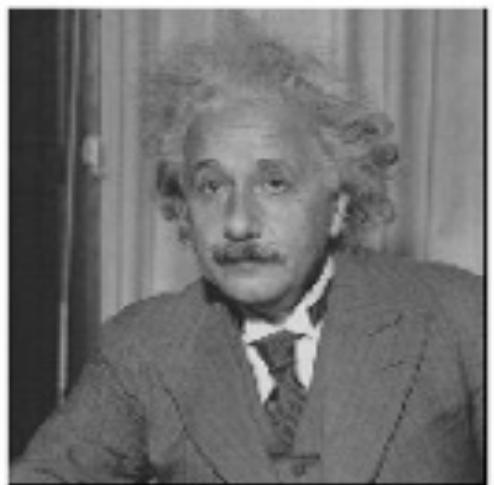
More filtering examples



?



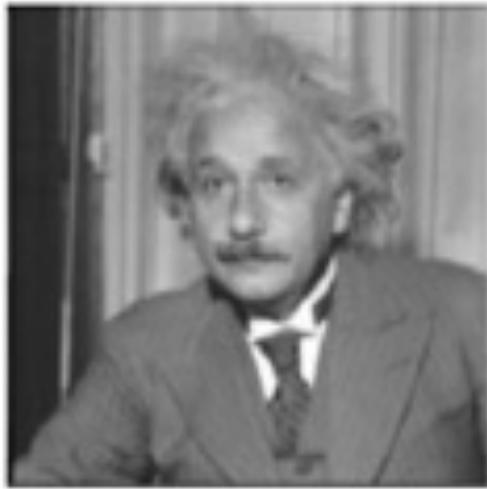
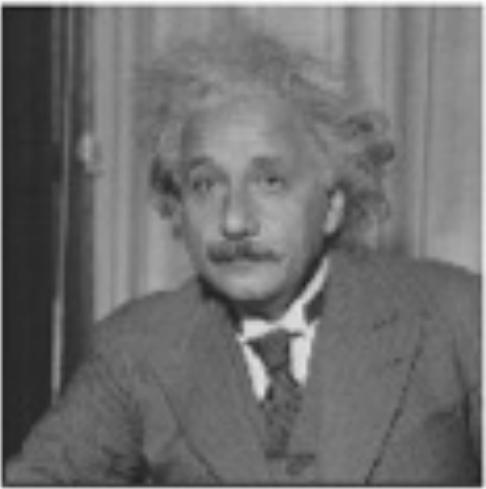
filters shown
in frequency-
domain



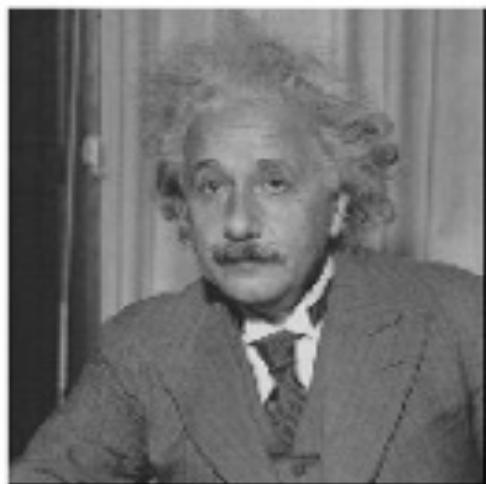
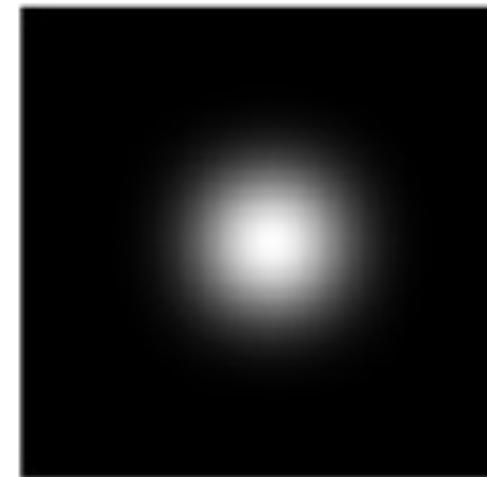
?



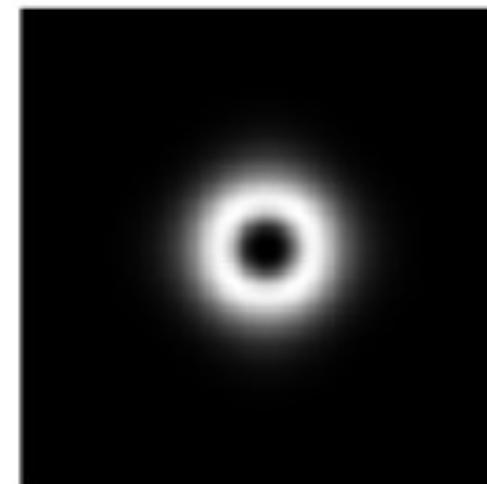
More filtering examples



low-pass



band-pass

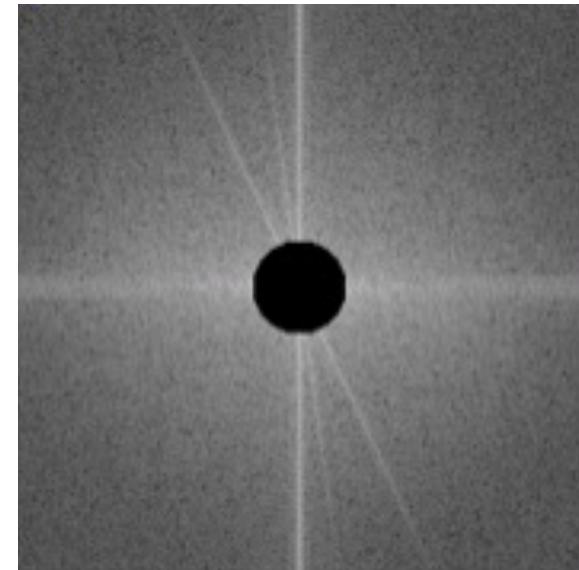


filters shown
in frequency-
domain

More filtering examples

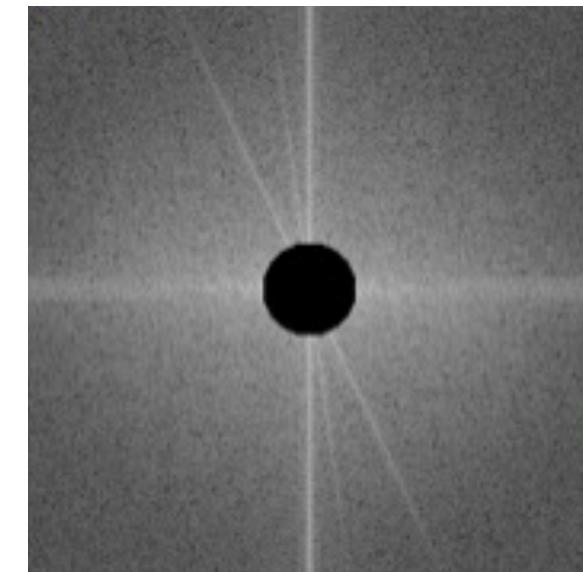


?



high-pass

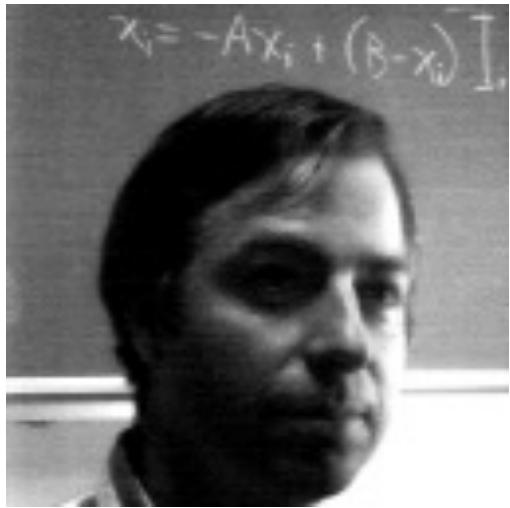
More filtering examples



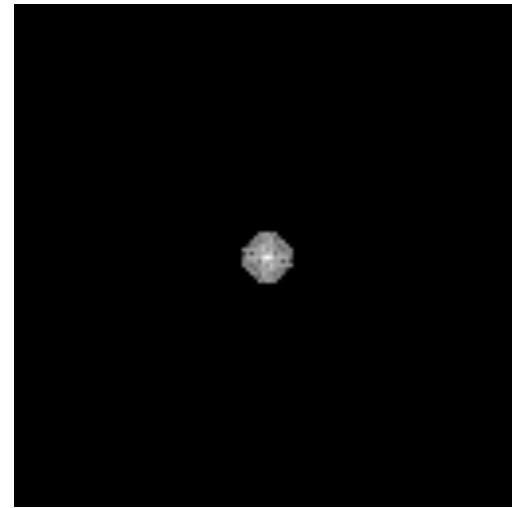
high-pass

More filtering examples

original image

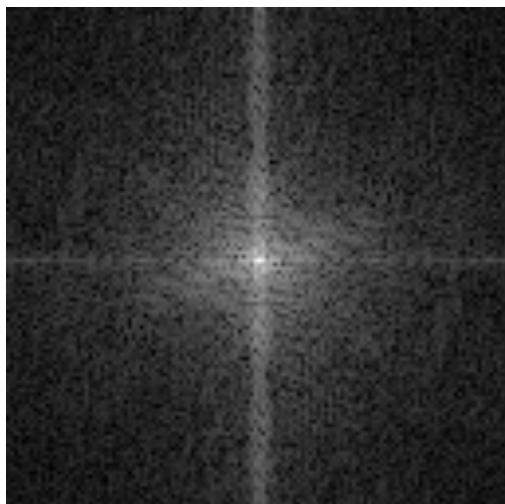


low-pass filter



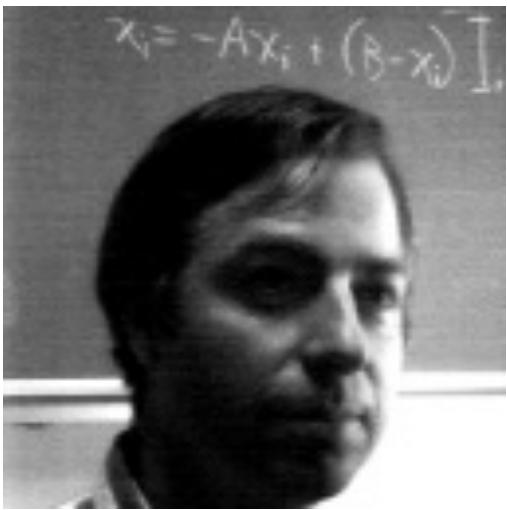
?

frequency magnitude

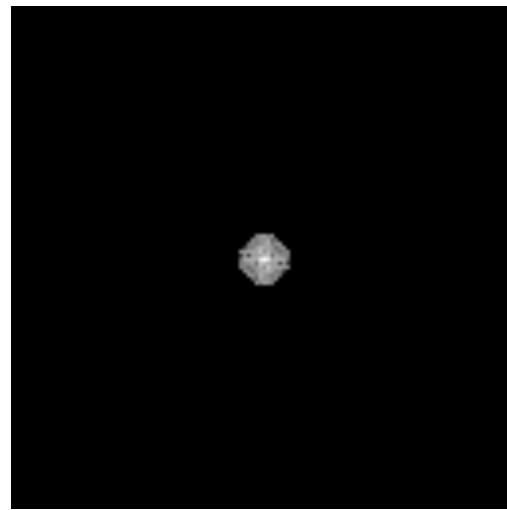


More filtering examples

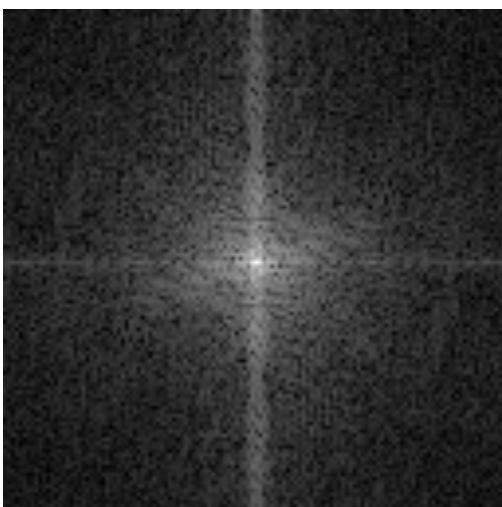
original image



low-pass filter

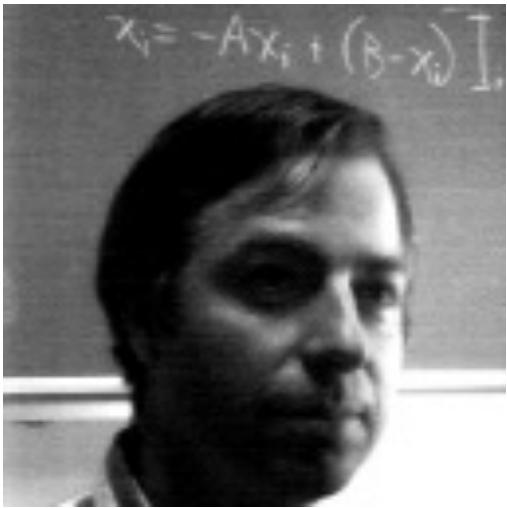


frequency magnitude

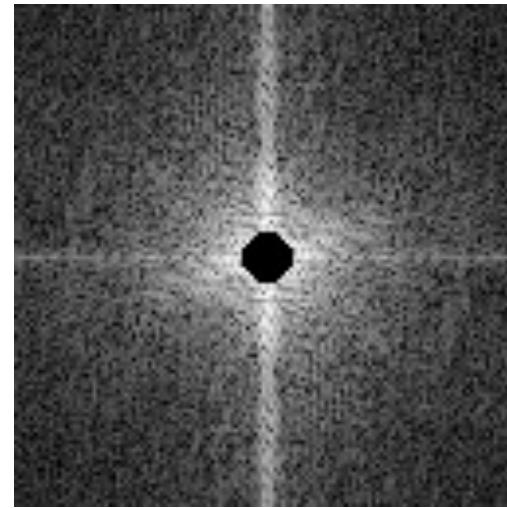


More filtering examples

original image

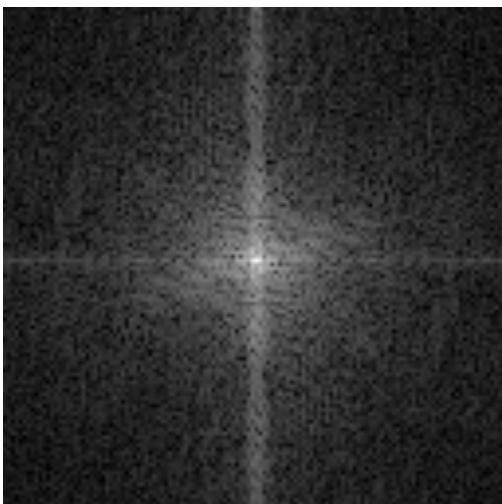


high-pass filter



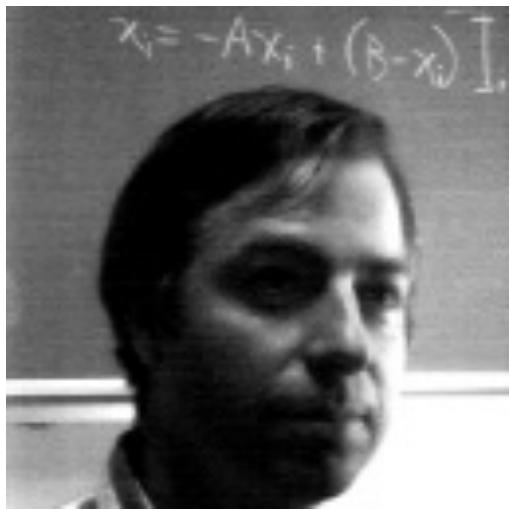
?

frequency magnitude

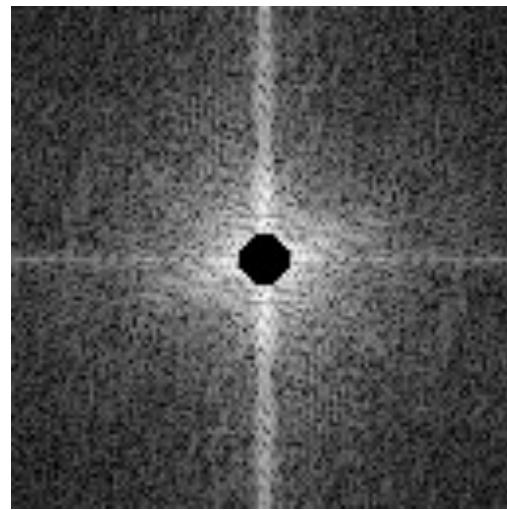


More filtering examples

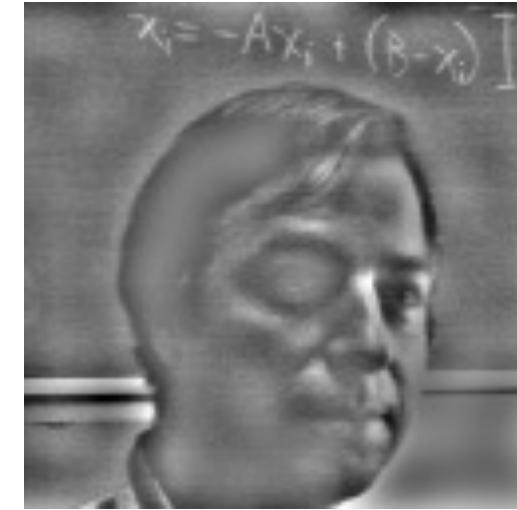
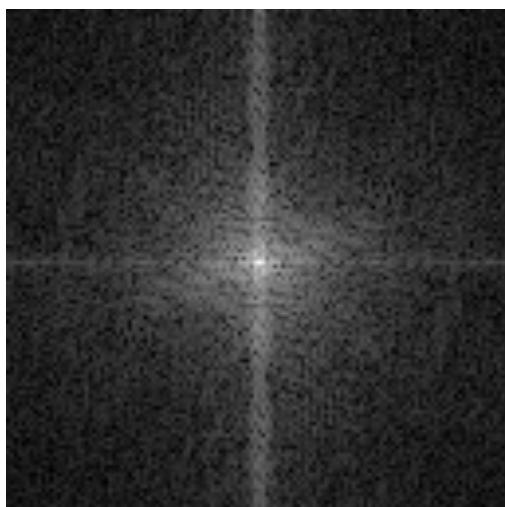
original image



high-pass filter

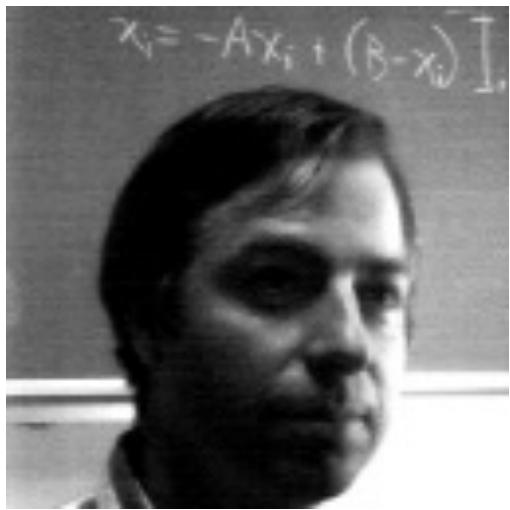


frequency magnitude

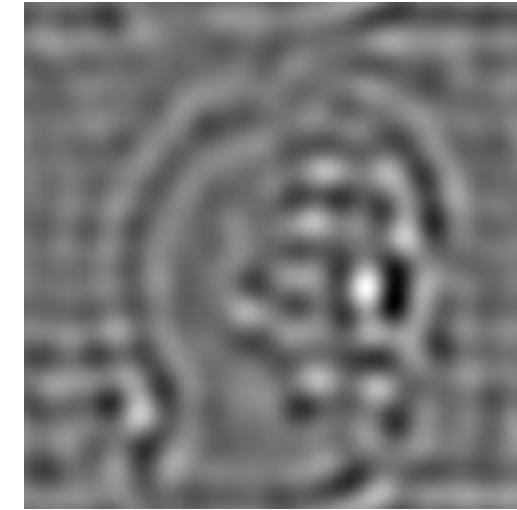
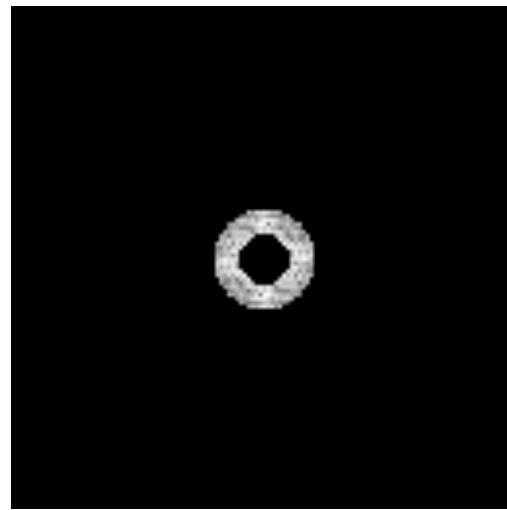


More filtering examples

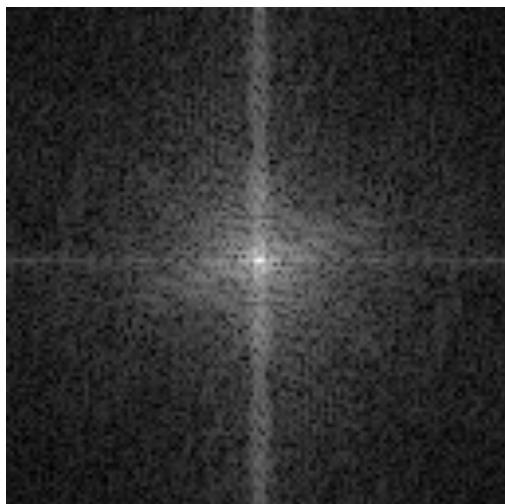
original image



band-pass filter

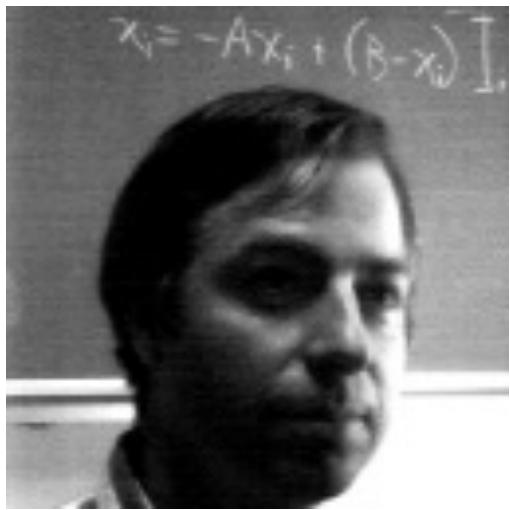


frequency magnitude

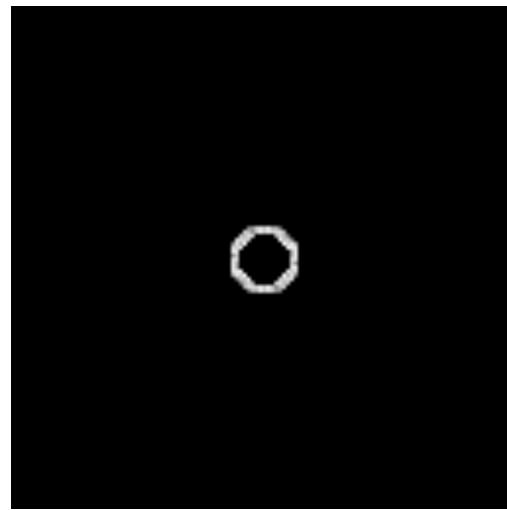


More filtering examples

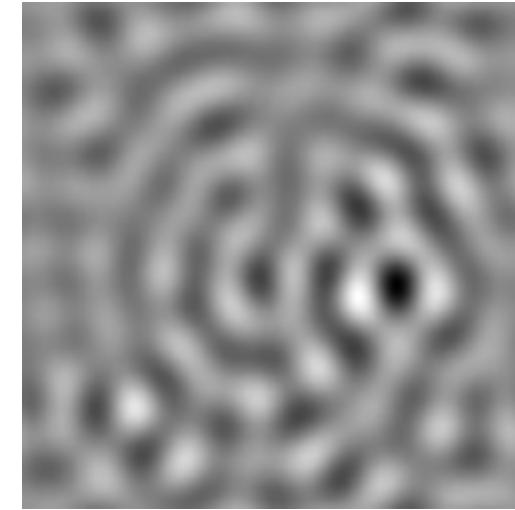
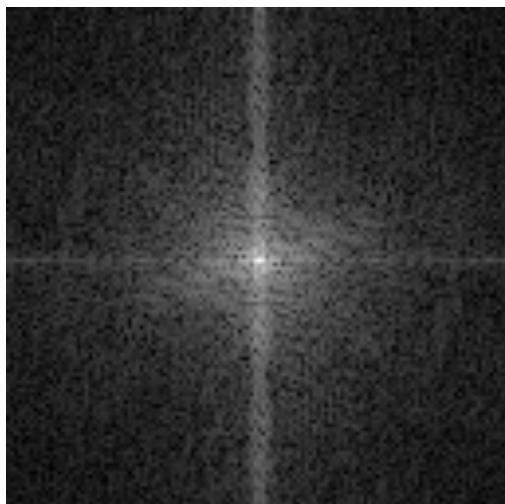
original image



band-pass filter

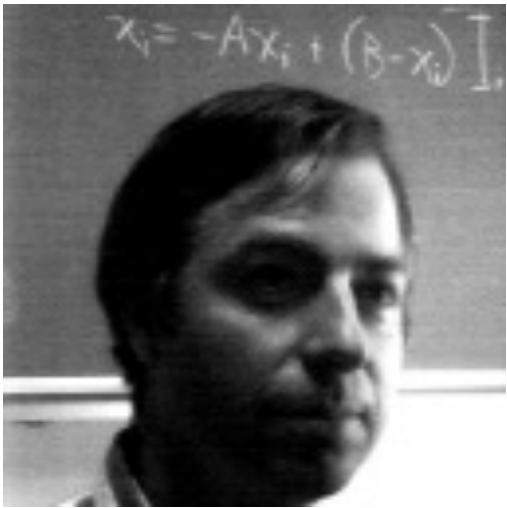


frequency magnitude

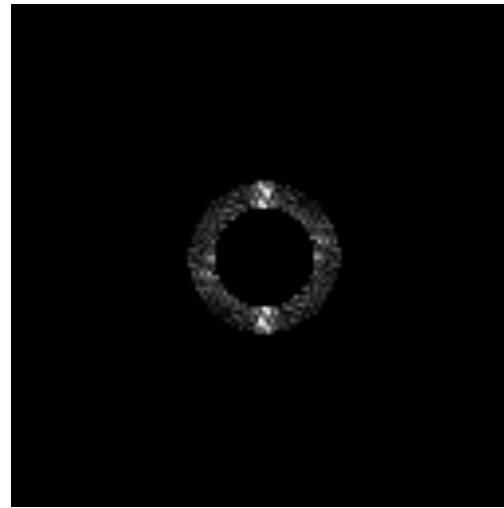


More filtering examples

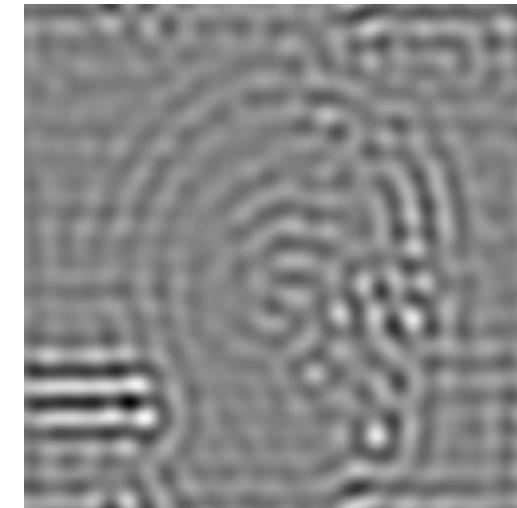
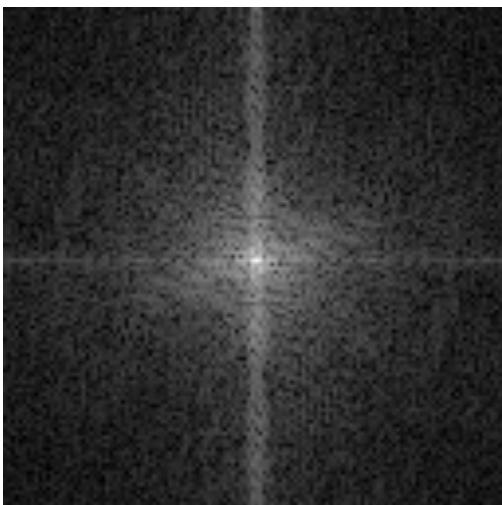
original image



band-pass filter

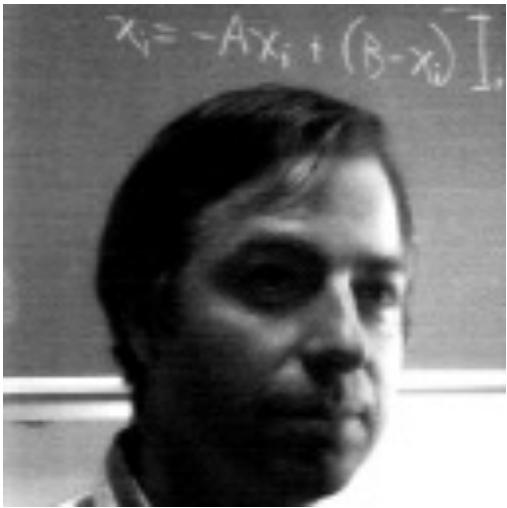


frequency magnitude

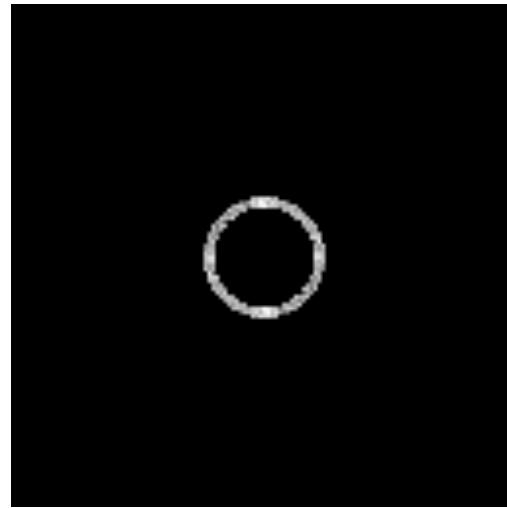


More filtering examples

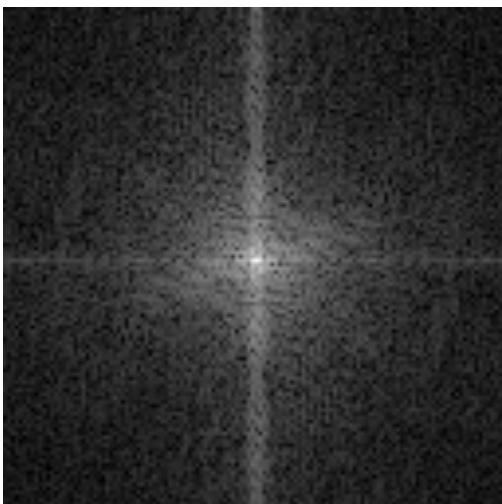
original image



band-pass filter



frequency magnitude



Revisiting sampling

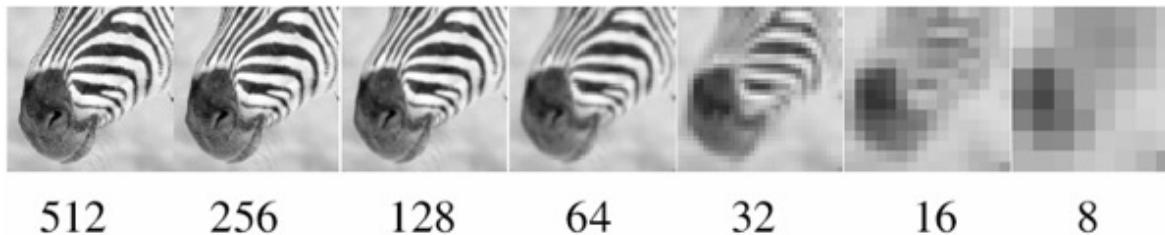
The Nyquist-Shannon sampling theorem

A continuous signal can be perfectly reconstructed from its discrete version using linear interpolation, if sampling occurred with frequency:

$$f_s \geq 2f_{\max} \quad \leftarrow \quad \text{This is called the Nyquist frequency}$$

Equivalent reformulation: When downsampling, aliasing does not occur if samples are taken at the Nyquist frequency or higher.

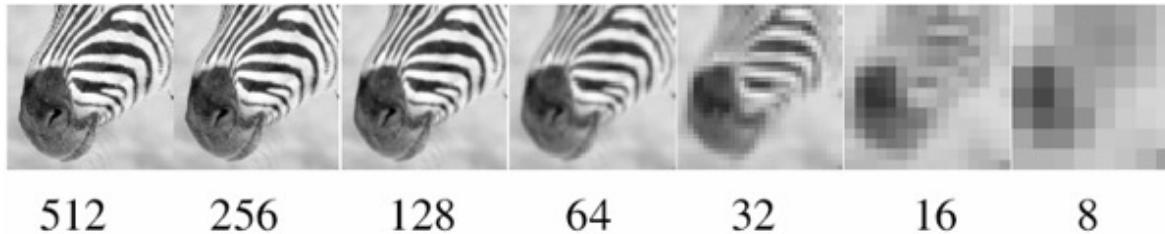
Gaussian pyramid



How does the Nyquist-Shannon theorem relate to the Gaussian pyramid?



Gaussian pyramid



How does the Nyquist-Shannon theorem relate to the Gaussian pyramid?

- Gaussian blurring is low-pass filtering.
- By blurring we try to sufficiently decrease the Nyquist frequency to avoid aliasing.

How large should the Gauss blur we use be?

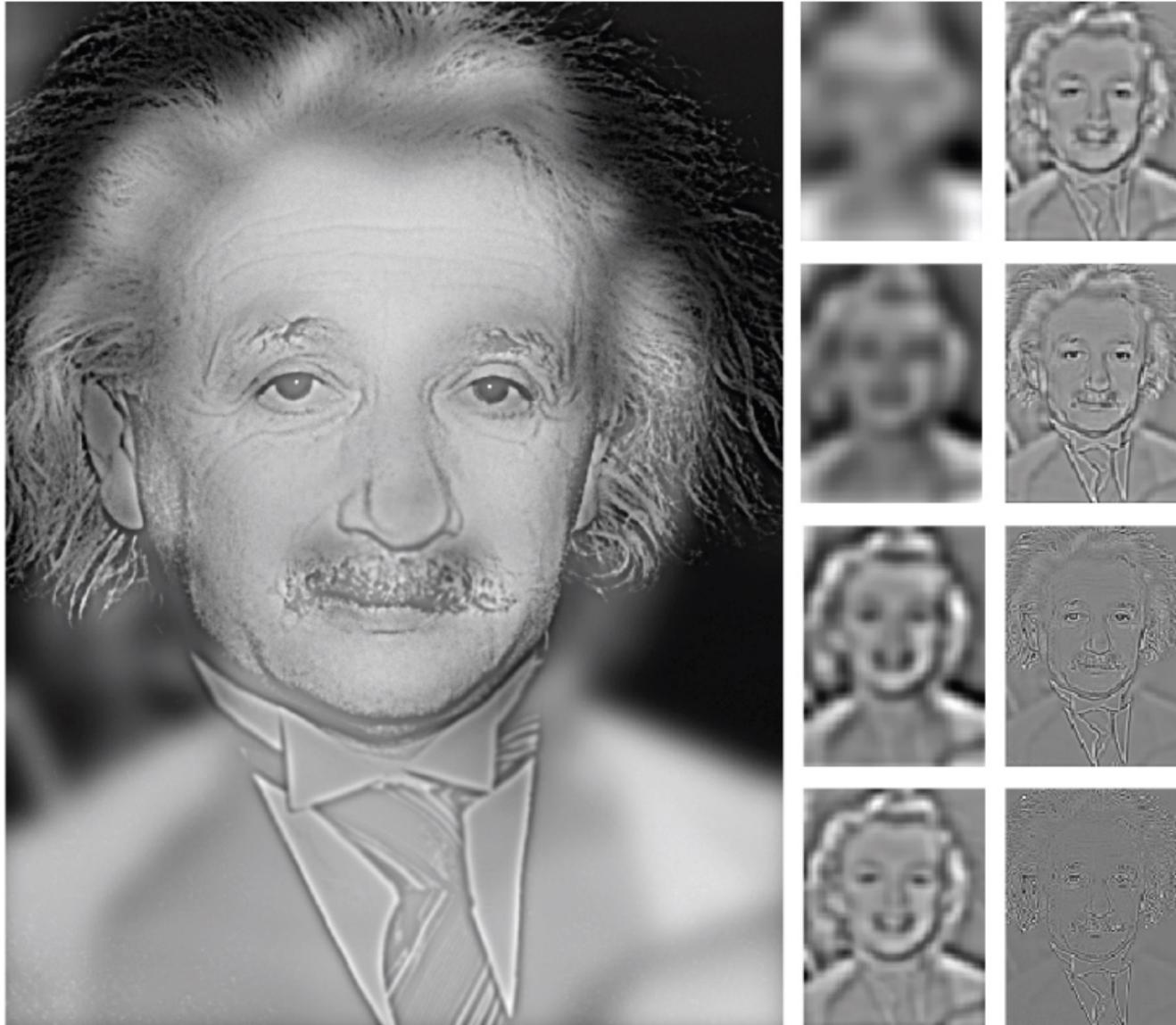
Frequency-domain filtering in human vision



“Hybrid image”

Aude Oliva and Philippe Schyns

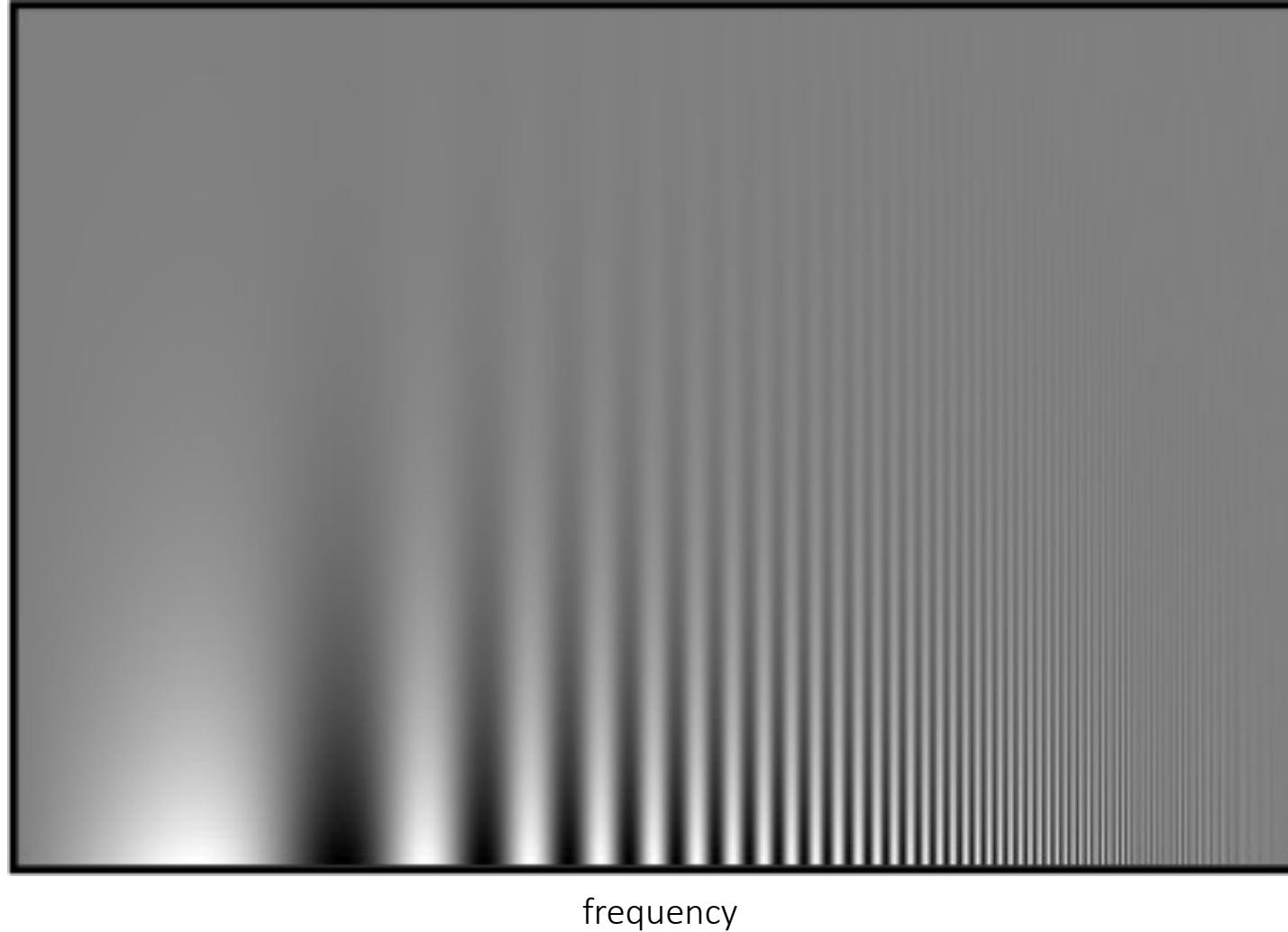
Frequency-domain filtering in human vision



Variable frequency sensitivity

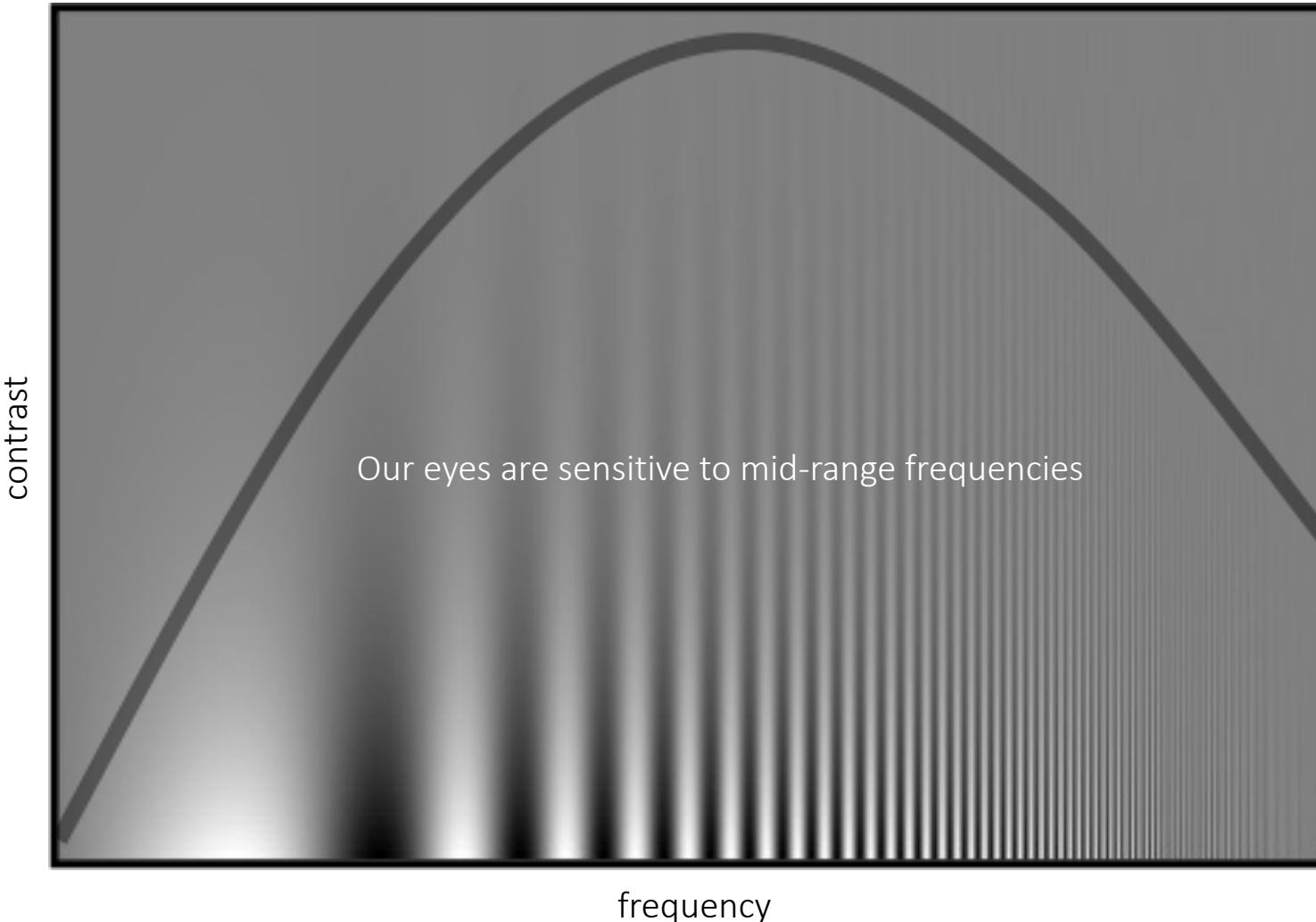
Experiment: Where do you see the stripes?

contrast



Variable frequency sensitivity

Campbell-Robson contrast sensitivity curve



- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid frequencies dominate perception