LSE Statistics Department Practitioner's Challenge Technical Report

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Team 6

Huiyang Zhang, Yupeng Wu,
Caiyi Pan, Jiawei Huang, Le Zhi
Chiang

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1 Introduction

In the global financial landscape, the analysis of credit risk and its correlation is significant for the financial management and strategic investment. This is particularly relevant in the Latin American and Caribbean region, with its varied economies. Default correlation, which assesses the credit linkage between two parties, is critical for financial institutions to ensure adequate capital reserves against unexpected losses. Our report presents a methodology to calculate key correlation coefficients, employing theoretical and practical models to evaluate credit risk in different sectors within the region. We focus on global, country-specific, and sector-specific correlation coefficients, using First-Passage-Time (FPT) models to enrich our understanding of default correlations and their impact on the region's financial health.

The report is structured as follows: We begin by detailing the process of data collection and the subsequent exploratory data analysis conducted to prepare the dataset for in-depth investigation. We then define default correlation and establish two fundamental assumptions necessary for the FPT model. Moreover, we adopt the methodology based on model of [Zhou, 2001], as well as an alternative model proposed by [Li and Krehbiel, 2016], to calculate the default correlation. The results obtained from these models are then discussed, providing insights into the default correlation within the specified industry and region. Alongside our findings, we recognize the limitations inherent in our modeling approach. All our codes and datasets can be found in our Github repository.

2 Data Collection

We attempted to approximate a firm's credit quality based on three variables: the firm's quarterly total assets, the firm's quarterly total liabilities, and the information related to the firm's stocks, including the quarterly open price, quarterly close price, quarterly high price and quarterly low price. To make sure that we can access the publicly available stock price information, we decided to concentrate only on the publicly listed companies in Latin American&Caribbean Region.

We started with the S&P Capital IQ database, which provided us with a comprehensive list of the International Securities Identification Number (ISIN) and tickers of the listed companies in Latin America&Caribbean Region. Using the ISIN, we download the quarterly assets/liability data from the Compustat database in Wharton Research Data Service (WRDS). With the tickers available, we retrieved the stock prices of those companies from Yahoo Finance using the yfinance package in Python. As all the stock prices are denoted in the local currencies of the stock exchanges, we applied the annual exchange rate data from the Global Economic Monitor (GEM) database from the World Bank.

The resulting dataset comprises 199 listed companies, of which 38 are in the financial industry. The dataset contains their quarterly information of total assets, total liability and stock prices over 2006-2024 (72 quarters in total); we decided to research this particular period in order to cover the 2008 financial crisis, which resulted in a wave of defaults globally.

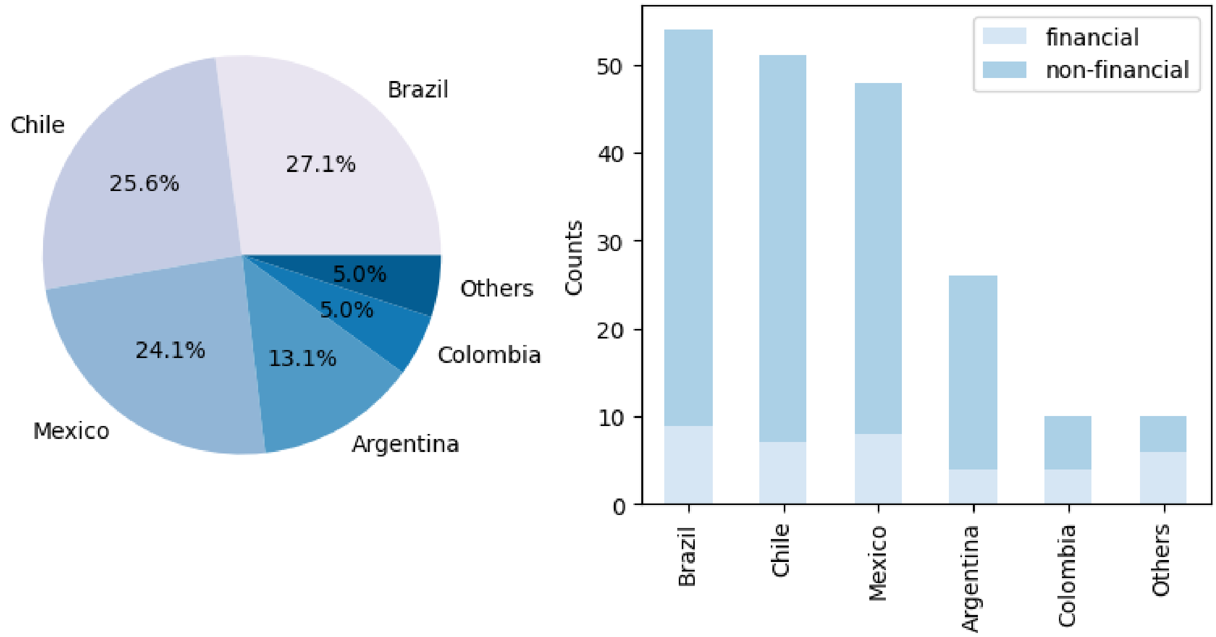

Figure 1: Distribution of countries

Country	**Financial**	**Non-financial**	**Total**
Brazil	9	45	54
Chile	7	44	51
Mexico	8	40	48
Argentina	4	22	26
Colombia	4	6	10
British Virgin Islands	1	2	3
Panama	1	1	2
Peru	1	0	1
Curacao	1	0	1
Jamaica	1	0	1
Cayman Islands	1	0	1
Trinidad and Tobago	0	1	1
Total	**38**	**161**	**199**

Table 1: Distribution of countries

3 Methodology

3.1 Default Correlation Overview

This section describes two random variables $D_1(t)$, $D_2(t)$, representing whether firms 1 and 2 respectively will default within a certain time period, t:

$$D_i(t) := \begin{cases} 1 & \text{if firm } i \text{ defaults by } t \\ 0 & \text{otherwise} \end{cases}$$

For simplicity, we denote $P(D_i(t))$ as P_i.
Then we can calculate the default correlation as the following equation:

$$corr(D_1(t), D_2(t)) = \frac{(P_1 = 1) + (P_2 = 1) - (P_{1or2} = 1) - (P_1 = 1) \cdot (P_2 = 1)}{\sqrt{(P_1 = 1) \cdot (1 - P_1 = 1) \cdot (P_2 = 1) \cdot (1 - P_2 = 1)}}$$

3.2 The First-Passage-Time Model of Default Correlation

3.2.1 The assumptions of First-Passage-Time Model

Here are two assumptions of the FPT Model:
Assumption 1. The total asset values of two firms, labeled as firm 1 and firm 2, with V_1 and V_2, represent these values respectively. It states that the changes in the logarithms of these asset values can be modeled by a certain type of stochastic process.

$$\left[\begin{array}{c} \frac{d\ln(V_1)}{dt} \\ \frac{d\ln(V_2)}{dt} \end{array} \right] = \left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array} \right] dt + \Omega \left[\begin{array}{c} dz_1 \\ dz_2 \end{array} \right], \tag{1}$$

where μ_1, μ_2 are constant drift terms, Z_1 and Z_2 are two independent standard Brownian Motions, and σ is a constant 2×2 matrix such that

$$\Omega \cdot \Omega' = \left[\begin{array}{cc} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{array} \right].$$

This correlation ρ, which is actually the correlation between the logarithmic changes in the asset values of the two firms, is a significant factor in evaluating the correlation of default risk between the firms.

$$\rho = \frac{\mathrm{cov}(\ln V_1, \ln V_2)}{\sqrt{\mathrm{var}(\ln V_1)\mathrm{var}(\ln V_2)}}.$$

Assumption 2. A company's default is initiated when the asset value drops. Each company, denoted by i, is associated with a variable threshold $C_i(t)$ that changes over time. The company remains operational and fulfills its financial commitments as long as its asset value $V_i(t)$ stays above this threshold. If the asset value dips below $C_i(t)$, the company is considered to default immediately, triggering restructuring. $C_i(t) = e^{\lambda_i t} K_i$, where K_i is a constant. In the following section, we assume that $\mu_i = \lambda_i$ for computation simplicity.

3.2.2 Method of [Zhou, 2001]

To our knowledge, this is the first model which uses a closed formula to evaluate default correlation in the First-Passage time setting. Let's define (τ_i) as the earliest point in time when the value of firm i, denoted by $V_i(t)$, falls below a predetermined default threshold K_i. Accordingly, $P(D_i(t) = 1) = P(\tau_i \leq t)$. Based on [Harrison, 1986] findings, this probability can be expressed as

$$\begin{aligned} P(D_i(t) = 1) &= 2 \cdot N\left(\frac{\ln(V_{i,0}/K_i)}{\sigma_i\sqrt{t}}\right) && (2) \\ &= 2 \cdot N\left(\frac{-Z_i}{\sqrt{t}}\right). && (3) \end{aligned}$$

where

$$Z_i \equiv \frac{\ln(V_{i,0}/K_i)}{\sigma_i} \quad (4)$$

After knowing the marginal default probability for each firm, the only item unknown is the probability of either firm defaults. The main result when $\mu_i = \lambda_i$ was proved by Rebholz (1994), which could be expressed as:

$$\begin{aligned} P(D_1(t) = 1 \text{ or } D_2(t) = 1) = 1 - \frac{2r_0}{\sqrt{2\pi t}} \cdot e^{-\frac{r_0^2}{4t}} \cdot \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n} \cdot \sin\left(\frac{n\pi\theta_0}{\alpha}\right) && (5) \\ \cdot \left[I_{\frac{\nu}{2}}\left(\frac{n\pi r_0}{\alpha t}\right) + I_{\frac{\nu}{2}}\left(\frac{n\pi r_0}{\alpha t}\right)\right] && (6) \end{aligned}$$

where $I_\nu(z)$ is the modified Bessel function I with order ν and

$$\alpha = \begin{cases} \tan^{-1}\left(-\frac{\sqrt{1-\rho^2}}{\rho}\right) & \text{if } \rho < 0, \\ \pi + \tan^{-1}\left(-\frac{\sqrt{1-\rho^2}}{\rho}\right) & \text{otherwise.} \end{cases}$$

$$\theta_0 = \begin{cases} \tan^{-1}\left(\frac{Z_2\sqrt{1-\rho^2}}{Z_1-\rho Z_2}\right) & \text{if } \rho > 0, \\ \pi + \tan^{-1}\left(\frac{Z_2\sqrt{1-\rho^2}}{Z_1-\rho Z_2}\right) & \text{otherwise,} \end{cases}$$

$$r_0 = \frac{Z_2}{\sin(\theta_0)}.$$

For simplicity, we denote $P(D_i(t))$ as P_i.
Then we can calculate the default correlation as following equation:

$$corr(D_1(t), D_2(t)) = \frac{(P_1 = 1) + (P_2 = 1) - (P_{1or2} = 1) - (P_1 = 1) \cdot (P_2 = 1)}{\sqrt{(P_1 = 1) \cdot (1 - P_1 = 1) \cdot (P_2 = 1) \cdot (1 - P_2 = 1)}} \tag{7}$$

The main result can also derived by [Li and Krehbiel, 2016] from evaluating:

$$F(b_1, b_2, t) = \int_{-\infty}^{b_1}\int_{-\infty}^{b_2} f(x_1, x_2, t)dx_1dx_2 = 1 - P(D_1(t) = 1 \text{ or } D_2(t) = 1) \tag{8}$$

where $f(x_1, x_2, t)$ is the solution to the PDE, i.e. the transition probability density is the solution of the Kolmogorov forward equation:

$$\frac{1}{2}\left(\sigma_1^2\frac{\partial^2 f}{\partial x_1^2} + 2\rho\sigma_1\sigma_2\frac{\partial^2 f}{\partial x_1 \partial x_2} + \sigma_2^2\frac{\partial^2 f}{\partial x_2^2}\right) = \frac{\partial f}{\partial t}, \quad x_1 < b_1, \quad x_2 < b_2 \tag{9}$$

subject to the following boundary conditions:

$$f(-\infty, x_2, t) = f(x_1, -\infty, t) = 0,$$

$$f(x_1, x_2, 0) = \delta(x_1)\delta(x_2),$$
$$F(b_1, b_2, t) \leq 1, \quad t > 0,$$
$$f(b_1, x_2, t) = f(x_1, b_2, t) = 0.$$

3.2.3 Method of [Li and Krehbiel, 2016]

The method of [Zhou, 2001] and [Li and Krehbiel, 2016] mainly differs in the way PDE (9) is solved. In [Zhou, 2001], the PDE is solved by the separable variable method, while in [Li and Krehbiel, 2016] it is solved through a technique called **reflection principle**. [Li and Krehbiel, 2016] successfully expressed the probability that either firm default as a function of the bivariate standard normal distribution function, which is way simpler than the analytical formula (5) of [Zhou, 2001]

$$\begin{aligned} P(D_1(t) = 1 \text{ or } D_2(t) = 1) = 1 - F(b_1, b_2, t) \\ &= 1 - N_{2,\rho}\left(\frac{b_1}{\sigma_1\sqrt{t}}, \frac{b_2}{\sigma_2\sqrt{t}}\right) + N_{2,-\rho}\left(-\frac{b_1}{\sigma_1\sqrt{t}}, \frac{b_2}{\sigma_2\sqrt{t}}\right) \\ &\quad - N_{2,\rho}\left(-\frac{b_1}{\sigma_1\sqrt{t}}, -\frac{b_2}{\sigma_2\sqrt{t}}\right) + N_{2,-\rho}\left(\frac{b_1}{\sigma_1\sqrt{t}}, -\frac{b_2}{\sigma_2\sqrt{t}}\right), \end{aligned} \tag{10}$$

where $N_{2,\rho}$ and $N_{2,-\rho}$ represents the CDF of the bivariate standard normal distribution with mean vector $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and covariance matrix $\Sigma_\rho = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ $\Sigma_{-\rho} = \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}$, $b_i = -\ln\left[\frac{K_i}{\sqrt{V_0}}\right]$.

[Li and Krehbiel, 2016] also points out that there is a consistency issue related to the individual default probability calculated in the model of [Zhou, 2001]. More specifically, he assumes that the asset correlation is 0 when calculating individual default probability, which contradicts the assumption of the FPT model. The consistent individual default probability should be calculated by

$$P(D_i(t) = 1) = 2N\left(-\frac{\ln(\frac{V_{i,0}}{K_i})}{\Sigma_i\sqrt{t}}\right) \tag{11}$$

where Σ_i is the ith eigenvalue of the covariance matrix $\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$

3.2.4 Advantages of these two models

- Both models give the closed-form formula to calculate default correlation, which makes both of them relatively easy to implement compared to other credit risk models (CreditRisk+, Moody KMV, CreditMetrics, etc)

- The models for first passage time account for the likelihood of a firm defaulting before the conclusion of each time horizon. This presents a substantial benefit over Merton-style models, which tend to underestimate default probabilities due to their oversight of early default risks.

- Compared to the reduced form model, the structural model utilizes the firm-specific information (Asset value, liability, stock price, etc)

- We did not manage to find the data required to implement the reduced form model (credit rating for each company, etc)

3.2.5 Comparison between these two models

- The model of [Li and Krehbiel, 2016] is way more computationally efficient than the model of [Zhou, 2001]. In fact, according to our experience, the running time of the former is 6 times faster than the latter.

- The consistency issue is addressed in [Li and Krehbiel, 2016] model mentioned in 3.2.3.

- The model of [Zhou, 2001] produces a correlation coefficient with an absolute value greater than 1 when

  - The debt ratio $\frac{V}{K}$ is large
  - The asset correlation is high (say, $>= 0.8$)

  The model of [Li and Krehbiel, 2016], on the other hand, rarely produce bizarre results compared to the model of [Zhou, 2001]

3.2.6 Implementation

The parameters that need to be estimated are the following:

- ρ: Asset return correlation, approximated by stock price (equity) correlation ρ_E

- σ_i: Asset volatility of i_{th} firm, approximated by the average of classic annualised close-to-close price volatility. Note that this quantity is a fixed constant according to our assumption. More concretely, denote δ_j

$= \begin{bmatrix} c_{1j} \\ c_{2j} \\ c_{3j} \end{bmatrix}$ as the vector of percentage change of stock price in year j , the estimated annualized volatility $\sigma_{j_{annual}}$ for that year is then $\sqrt{4}\sigma_{j_{quarter}}$, where $\sigma_{j_{quarter}}$ is the standard deviation of vector δ_j. We then take the average of all the $\sigma_{j_{annual}}$ in the 18-year time horizon to be our estimate for σ_i

$$\hat{\sigma_i} = \frac{1}{18} \sum_{j=1}^{18} \sigma_{j_{annual}}$$

- V_i: Asset value of i_{th} firm at the beginning of the time horizon
- K_i: The Capital structure of i_{th} firm, approximated by 0.75 total liability

After getting the estimation to the above parameters, we can plug in the estimates to (5) and (10) to calculate the default correlation between any two firms in any time horizon we want.

4 Results

The following tables summarise our calculation of

- Global correlation coefficient
- Country Correlation coefficient
- Financial sector correlation coefficient
- Non-Financial sector correlation coefficient

over the horizon of 1 year (2022/12/31 - 2023/12/31), 2 years (2021/12/31 - 2023/12/31), 5 years (2018/12/31 - 2023/12/31) and 17 years (2006/12/31 - 2023/12/31).

We assign NaN to the correlation coefficients of the countries with fewer than 1 pair of companies to generate default correlation from.

We have used country codes for some of the tables: ARG - Argentina, PER - Peru, CUW - Curacao, BRA - Brazil, CHL - Chile, COL - Colombia, JAM

- Jamaica, CYM - Cayman Islands, MEX - Mexico, PAN - Panama, TTO - Trinidad and Tobago, VGB - British Virgin Islands.

1-year	2-year	5-year	17-year
0.022722	0.037581	0.111917	0.213928

Table 2: Global correlation (Li)

1-year	2-year	5-year	17-year
0.024484	0.020934	0.07645	0.082602

Table 3: Global correlation (Zhou)

Country	1-year	2-year	5-year	17-year
ARG	0.112114	0.373754	0.628496	0.659405
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.057081	0.099517	0.251666	0.478642
CHL	0.014388	0.008019	0.030594	0.098763
COL	0.014963	0.020901	0.070006	0.067606
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.017275	0.029649	0.080108	0.202627
PAN	0.316850	0.463421	0.788239	0.823506
TTO	NaN	NaN	NaN	NaN
VGB	NaN	NaN	NaN	0.008886

Table 4: Country correlation (Li)

Country	**1-year**	**2-year**	**5-year**	**17-year**
ARG	0.328526	0.395769	0.434135	0.322533
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.026413	0.042270	0.170974	0.184388
CHL	-0.058334	0.005728	0.036485	0.072069
COL	0.037521	0.017695	0.071388	0.039886
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.031872	0.023148	0.064817	0 123982
PAN	0.312606	0.394991	0.438377	0.434285
TTO	NaN	NaN	NaN	NaN
VGB	NaN	NaN	NaN	0.036688

Table 5: Country correlation (Zhou)

Country	**1-year**	**2-year**	**5-year**	**17-year**
ARG	0.674785	0.890560	0.950239	0.999897
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.194607	0.177068	0.251829	0.573294
CHL	0.001752	-0.027575	0.053190	0.102326
COL	0.002521	0.008312	0.076006	0.003563
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.031572	0.053253	0.154380	0.312132
PAN	NaN	NaN	NaN	NaN
TTO	NaN	NaN	NaN	NaN
VGB	NaN	NaN	NaN	NaN

Table 6: Financial correlation for each country (Li)

Country	1-year	2-year	5-year	17-year
ARG	0.640718	0.648866	0.735160	0.654970
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.025693	0.051492	0.178756	0.198349
CHL	0.000101	0.007554	0.050115	0.068338
COL	0.111406	0.027422	0.108902	0.010344
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.018942	0.024912	0.072021	0.140254
PAN	NaN	NaN	NaN	NaN
TTO	NaN	NaN	NaN	NaN
VGB	NaN	NaN	NaN	NaN

Table 7: Financial correlation for each country (Zhou)

Country	1-year	2-year	5-year	17-year
ARG	0.100145	0.321379	0.560532	0.571387
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.023452	0.062596	0.239999	0.419904
CHL	0.015040	0.000119	0.021084	0.072540
COL	-0.037740	0.009517	0.060980	0.138164
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.006761	0.011269	0.049983	0.140935
PAN	NaN	NaN	NaN	NaN
TTO	NaN	NaN	NaN	NaN
VGB	NaN	NaN	0.000009	0.051626

Table 8: Non-financial correlation for each country (Li)

Country	1-year	2-year	5-year	17-year
ARG	0.375851	0.416909	0.479446	0.346960
PER	NaN	NaN	NaN	NaN
CUW	NaN	NaN	NaN	NaN
BRA	0.030141	0.040817	0.188330	0.204357
CHL	0.008568	0.008171	0.033309	0.066200
COL	0.006764	0.003694	0.045338	0.054551
JAM	NaN	NaN	NaN	NaN
CYM	NaN	NaN	NaN	NaN
MEX	0.025467	0.024279	0.054534	0.118345
PAN	NaN	NaN	NaN	NaN
TTO	NaN	NaN	NaN	NaN
VGB	NaN	-0.000003	0.000367	0.021874

Table 9: Non-financial correlation for each country (Zhou)

Our results generally align with the previous findings of [Lucas, 1995] and [Zhou, 2001], which suggest that default correlation increases with the time horizon. That is because default events are essentially rare, especially when we consider a short period of time.

Our calculations suggest that the global correlation coefficients are generally lower than the other types of coefficients, which can be explained by the fact that firms from different companies and industries are exposed to very different sectoral and economic risks and shocks. The financial coefficients are always the highest among the four types of coefficients, indicating the high degree of interdependency financial firms exhibit relative to the non-financial ones: this is the consequence of the extensive borrowing activities and other transactions between the financial firms, which link their credit quality closely to one another. Meanwhile, the country coefficients are consistently higher than the non-financial coefficients, which reflects the significantly adverse impact a failing financial system could have on the stability of the non-financial sector.

We also noticed that the coefficients in Argentina are always substantially higher than the other countries. Li's model even suggests a financial coefficient approaching 1. We interpret it as a result of the particular economic challenges Argentina has been facing in the past decades. A prolonged period of hyperinflation has weakened Argentine borrowers (both individual

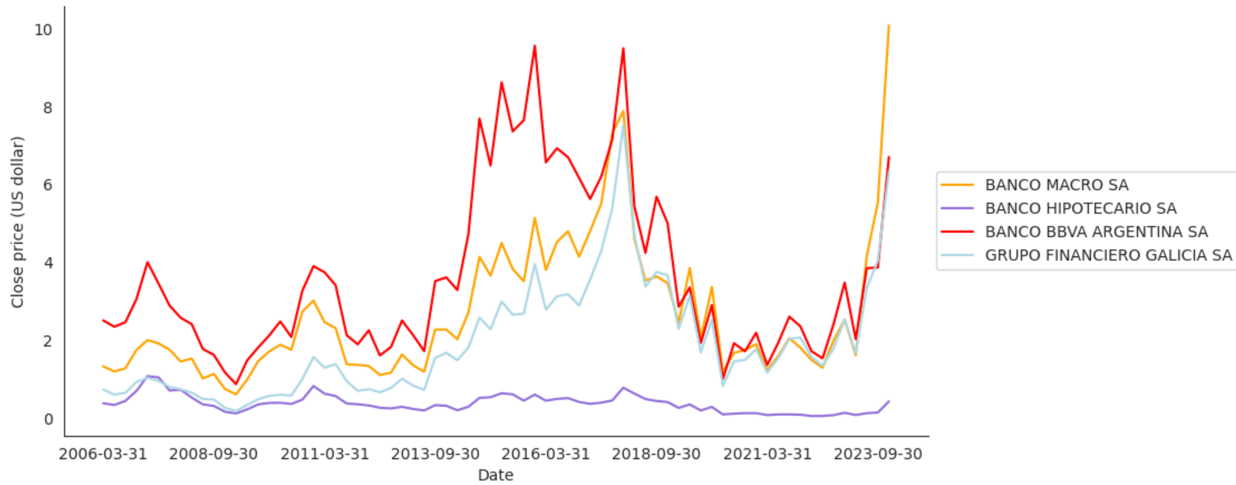

Figure 2: The close price of the four financial firms in Argentina

and corporate)' ability to repay debts, and the policy of capital control has restrained financial firms' access to foreign capital and opportunities, both cripple financial firms' ability to honour their debts. On top of that, many banks in Argentina have been increasing their holdings in central bank securities and sovereign-related bonds to combat inflation [Feliba and Gull, 2020], which makes them exposed to the associated risks - the government of Argentina is well-known for its longstanding deficits and its record of defaulting nine times in the past.

We could also rationalize Argentina's abnormal financial correlation coefficients using the model. Our model implies equity correlation might be highly correlated to default correlation. Figure 2 illustrates the behavior of the stock of the four Argentine financial firms in our dataset. They have demonstrated a significant correlation with each other over the period we are concerned.

5 Limitations and Possible Extensions

- Concerns about dataset quality and the absence of long-term liability data.
  - In the original dataset we downloaded there are about a quarter of rows with a null value in the long-time liability column. If we could have the complete long-term liability data we could have estimated the parameter K_i by short-term liability + 1.5 long-term liability,

as suggested in Section 3 by [Zhou, 2001]

  - some companies have bizarre debt ratios ($\frac{V}{K}$). Some of them are even greater than 100, or smaller than 0.2

- Assumption of constant volatility for each firm throughout the horizon ignores real-world fluctuations. Also, instead of the classic close-to-close estimator, we can use more advanced range-based estimators which take the high price, low price, and open price into account. A typical example of them is the Yang-Zhang estimator proposed in [Yang and Zhang, 2000]

- Using equity correlation to approximate the asset correlation might be problematic, especially for those companies with lower credit ratings, as suggested by [Qi et al., 2010]

- Lack of consideration for macroeconomic factors, which offers opportunities for further research.

- If we have access to the following data source:

  - Historical default data
  - Credit rating time series

  We could have done some reduced-form modeling. For example, by using the Generalised linear mixture model method proposed by [McNeil et al., 2015] to model the default correlation between different rating groups and countries by considering the economic factors and any other exogenous variables.

- Instead of using the Pearson correlation coefficient which only captures the linear relationship, other measures of dependence can also be used. For example, Kendall's tau can measure the concordance for bivariate random vectors, while Spearman's rho can be used to calculate the rank correlation.

6 Conclusion

In conclusion, our report introduces a robust methodology for calculating correlation coefficients to assess credit risk across various sectors in the Latin

American and Caribbean region. By using the analytical formula derived by [Zhou, 2001] and [Li and Krehbiel, 2016], we modeled the default correlation in the Latin American and Caribbean regions. We calculated the overall default correlation as well as financial, non-financial, and country' specific default correlation in this region. Our analysis, utilizing First-Passage-Time (FPT) models, sheds light on global, country-specific, and sector-specific correlations, offering valuable insights into the intricacies of default risk within different industries.

The methodology exhibits several limitations, encompassing concerns regarding the quality of the dataset, inconsistencies in equity correlation, constraints stemming from the assumption of constant volatility, and the omission of macroeconomic variables. Enhancing the accuracy and applicability of this methodology necessitates the utilization of historical default data, credit rating analysis, and thorough analysis of industry-specific analysis.

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