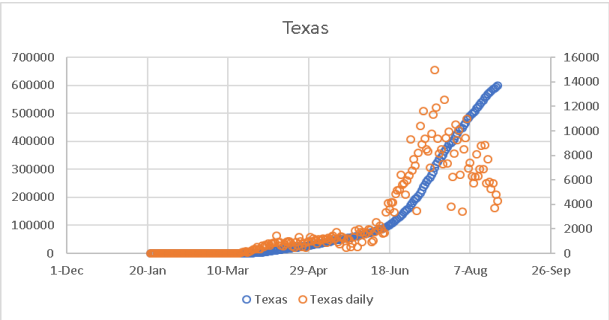
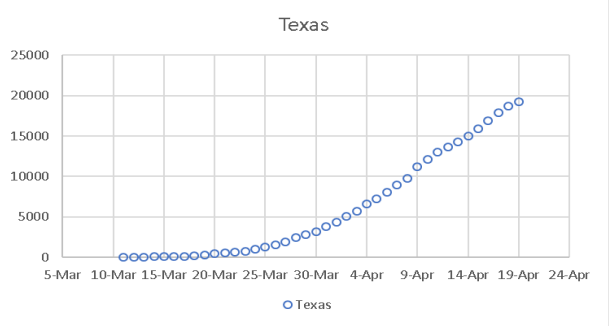
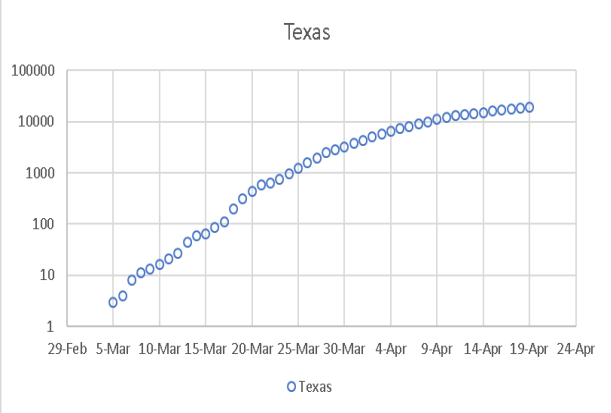
Explanation on R_o: If one can find the exponential part of the cumulative growth curve (a short section near the beginning of the curve that loses validity as N approaches zero), one finds the growth rate constant for an exponential, which is $K = d \ln(N) / dt$. [1]

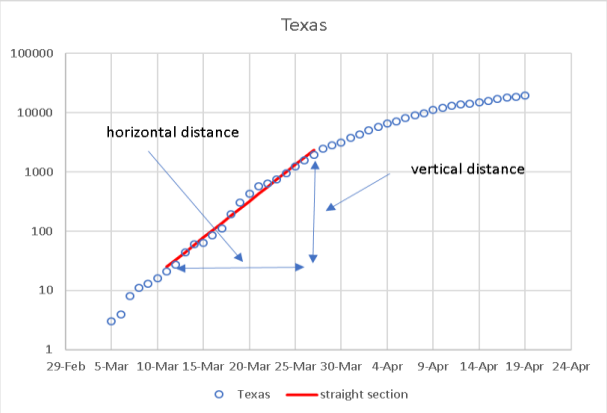
For example, here is some data on Texas' CV19 growth this year (the blue is cumulative), where the second graph zeros in on the very first days of this misery:





If we plot this semi-log (logarithmic vertical axis vs. arithmetic horizontal axis), this is what it looks like. The second graph highlights a straight line section, after which it begins to curve:





The straight section has a slope (ratio of rise over run, vertical over horizontal distance) that is 0.284 (done with a least squares regression). Afterwards, it is no longer exponential growth, since it curves off. This is called K, K = 0.284 for Texas. Notice this is a bit arbitrary, and avoids the values less than 40, at the extreme early stage.

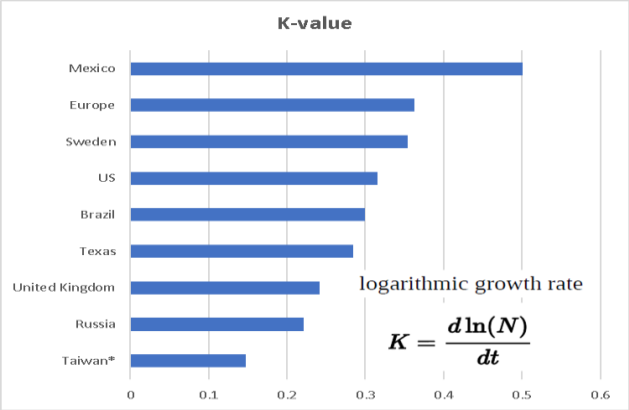
You can use this to find out infectious rate and contact rate, since R_o can be defined as the ratio of the contact rate to the infectious rate. β is often used for contact rate; τ is often used for infectious period, while γ (or κ) is used for infectious rate, its inverse. So,

$$R_o = \frac{\beta}{\gamma} = \beta\tau$$

R_o is also found graphically as $R_o = \exp(K\tau)$, where K is the slope of the exponential part of the cumulative curve just described.

[1] Basic Reproduction Rate. R_o is a measure of transmissibility: $R_o < 1$, disease disappears; $R_o = 1$, it's endemic; $R_o > 1$, epidemic. R_o is mentioned a lot during this epidemic, along with flattening of curves, with not a lot of understanding or relevance.

$R_o = e^{K\tau}$ where τ is the average infectious period for each person. Doubling time is also often mentioned. Reduce K by reducing contacts, reduce τ by isolation of infected individuals, for example. Here are some K values taken from Johns Hopkins data:



Example ($R_0 = 2.6$, $K_{TX} = 0.284$ / day)

$$R_0 = e^{K\cdot\tau}$$

$$\tau = \frac{ln(R_0)}{K} \qquad \frac{ln(2.6)}{0.284} = 3.4 \quad \text{days}$$

Example, doubling time (2/1) :

$$2 = 1\cdot e^{K\cdot\tau}$$

$$\tau = \frac{ln(2)}{K} \qquad \frac{ln(2)}{0.284} = 2.4 \quad \text{days}$$

Interesting comment in open letter from Belgian doctors (Sep 2020):

https://docs4opendebate.be/en/open-letter/

Confirmed cases are supposed to be proven through testing and symptoms, but it may not always be true. The actual number of infected people can be several times what is measured with confirmed cases, though. So there is a difference between case fatality rate (CFR) and infection fatality rate (IFR). CFR is the ratio of the number of deaths divided by the number of confirmed (preferably through testing) cases of disease. IFR is the ratio of deaths divided by the number of actual infections with SARS-CoV-2.

Modeling

Interestingly, one way these epidemics are visualized and modeled is based on susceptibles contacting infected people at some average rate. These susceptibles are quantified as a percentage or fraction of the entire population[2] (which implies that some fraction of the population *isn't* susceptible; think vaccine or herd immunity). So, over time, these susceptible people are catching the disease at some rate. You could put it in pseudo-algebraic terms like this:

rate of change, susceptibles = contact rate, times fraction of total that are susceptible, times number of infectious [3]

That needs to be refined, since the fraction of the population that are susceptibles will decrease, as they are converted to infectious. That means the rate is negative. So, add a negative sign, and abbreviated a bit:

susceptibles change = -contact rate X fraction susceptible X infectious

That's a bit cumbersome, so if you follow Leibniz' suggestion, and write a rate of change as the change divided by the corresponding change in time, where the symbol for change (difference) is a *d*, *S* for susceptibles, *t* for time:

$$\frac{dS}{dt} = -contact\ \ rate\ \ X\ \ fraction\ \ X\ \ infectiou\ \ s$$

[2] Another way to look at this is the ratio of susceptibles to the whole population is the basis for assigning a *probability* of infection of anyone in the whole population, not just susceptibles: contact rate X probability X infectious. The program of compartmentalizing by susceptibles, infectious, and recovered is known as the SIR model.

[3] Contact rate is average number of people a person has contact with, per unit time. Hence contact tracing, that not only tells how many people a certain infected person has had contact with, but hopefully who they were.

People have come up with less cumbersome algebraic symbols, like β for the contact rate, and a real fraction for the fraction, S/N, where N is the number of the total population. Likewise, the average number of infectious would be well served with the letter I, so I:

$$\frac{dS}{dt} = -\beta \frac{S}{N} I$$

That's easier on the eye. It's important to remember all these quantities change over time, so they can be shown as functions of time (e.g. S written as $S(t)$), which clutters things up a bit. For example, deaths from an epidemic (not shown here) would reduce the value of $N(t)$ over time. S/N is almost always less than 1.0, except perhaps at the very beginning where just about everyone was susceptible and where R_o was found, keep in mind.

$$\frac{dS(t)}{dt} = -\beta \frac{S(t)}{N(t)} I(t)$$

Furthermore, the number of infectious changes, too. So, another, new equation, for the compartment of infectious:

$$\frac{d\,I(t)}{dt} = \beta \frac{S(t)}{N(t)} I(t) - \gamma\, I(t)$$

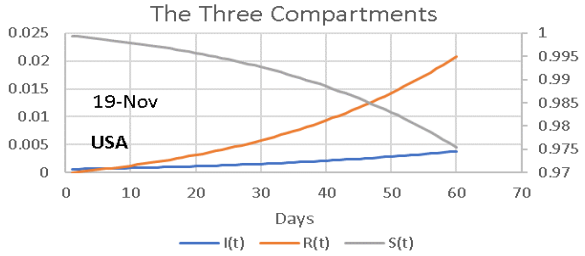
Notice the first term on the right side of the equation is just the negative of the change in susceptibles, since we're *adding* the same amount to infectious, that was *subtracted* from susceptibles before. So the change in infectious relates to the change in susceptibles, since members from one group are going over to the other. There is also the infectious period (not to be confused with contact rate). After a certain time, infectious either recover or die (this is kind of grim). So that has to be subtracted from the right side of the equation (the second term). γ is a common symbol for the frequency associated with the period of infection. Therefore γ^{-1} is the average period of infection. γ is the third letter in the Greek alphabet, gamma. $\gamma I(t)$ should be recognized as the rate of infectious by death or recovery, taken away from the overall rate.

The SIR model includes one more item in addition to the above, the recovered cases, $R(t)$. The three compartments should add up to a constant, throughout their evolution. Notice the following drops the N denominator in the ratio S/N, but S/N is easily recoverable, so:

$$\frac{dS}{dt} = -\beta\ S(t) I(t)$$

$$\frac{dI}{dt} = \beta\ S(t) I(t) - \gamma\ I(t)$$

$$\frac{dR}{dt} = \gamma\ I(t)$$



where $S(t) + I(t) + R(t)$ = constant. That's true, because they are all feeding off each other. An approximation to these expressions can be made by adding increments, using Euler's method:

$$f(t + \Delta t) = f(t) + \Delta t\, [\Delta f(t) / \Delta t] \text{ , because } \frac{df}{dt} \approx \frac{\Delta f}{\Delta t} .$$

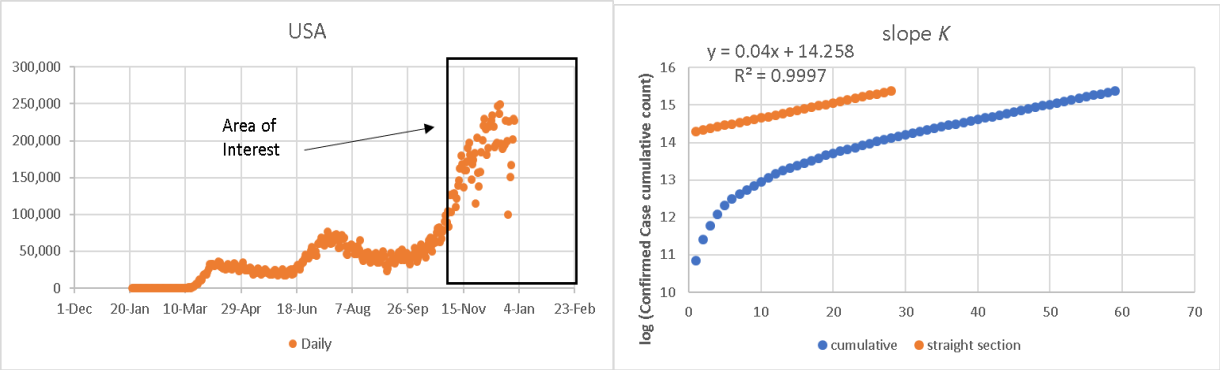
That way a table can be built for $I(t)$, for example.

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \beta\ S(t) I(t) - \gamma\ I(t) \text{ , so } I(t + \Delta t) \approx I(t)\left[1 + \Delta t\left(\beta\ S(t) - \gamma\right)\right].$$

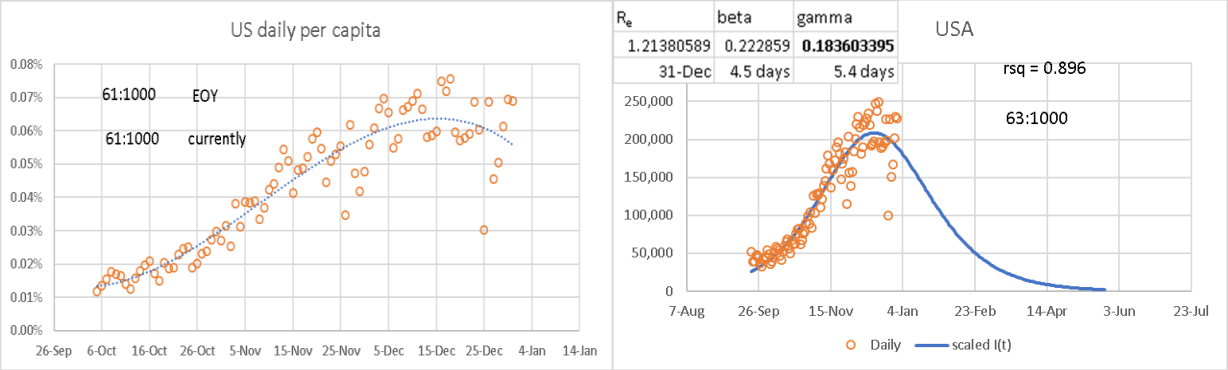
(Also, $S(t + \Delta t) \approx S(t)\left[1 - \Delta t\ \beta\ I(t)\right]$ and $R(t + \Delta t) \approx R(t) + \Delta t\ \gamma\ I(t)$.)

Taking the last few weeks of US numbers (just keep in mind, the US is comprised of thousands of similar sub-waves, at different magnitudes and starting points), and assuming that γ is (1/3)/day, taking daily increments (Δt = 1 day, in other words), a curve fit can be made, using R_o (use the definition from page 1) and a starting point as the two parameters to fit. The area of interest is shown below, with a logarithmic representation to the right, showing a

slope of 0.041. Since R_o can be approximated as $e^{K/\gamma}$, R_o is initially estimated, and then further modified through a least squares fit. R_o is the value used to estimate herd immunity. $R/N > 1 - 1/R_o$, R being Recovered.



One other thing interesting about the table so generated (it's called numerical integration, by the way) that you can see on the three compartments graph on the previous page is its arbitrary scale. That means the data to be fitted and the table of values have to be scaled to each other.



R_e	beta	gamma
1.21380589	0.222859	**0.183603395**
31-Dec	4.5 days	5.4 days

It's not a great fit, but gives some idea of how it works. This reproduction number is very different than the original back in March, so probably ought be called R_{eff} or R_e instead. It is almost not an epidemic, but keep in mind this is a composite value of a very simplified demonstration. (The original R_o number from the beginning of March in the US could be 5.02, based on $\gamma = 0.2$ and $K = 0.323$.) Using the scaling factors, the original scale can be recovered, and the table filled out towards the end, as in the chart on the right.

In actual models, effective values are used, which are dynamic, meaning they change over time. For example, $\gamma(t)$ or $R_e(t)$, effective infectious rate and effective reproduction number. $R_e(t)$ is roughly equivalent to R_o times $s(t)$, where $s(t)$ is a fraction between 1 and 0 of the fraction of susceptibles at time t.The above simplification described three compartments, but more sophisticated models will add another compartment of *exposed* to susceptible, infected, and recovered, for example. Most of these models will show R_o from 2.5 to 6 or even more, but it's not directly comparable to these results. In fact, the value of R_o depends strongly on the model used, so are not interchangeable.

It probably won't happen exactly as shown above, because of the uncertainty in any simplified projection and also the lockdowns and extreme methods that will likely occur (already occuring), which will change parameters like β and R_e. The actual infection count is probably 5 to 10 times the above based on confirmed cases. It would be useful to have a valid antigen test, so that excess vaccines don't have to be given (Pfizer probably wouldn't like that, though.) Given the bad building ventilation in the US, the lack of exercise and of eating good food, and just a lot of stubborn resistance to sensible measures, everyone will probably have to get jabbed, unfortunately. Another interesting thing to ponder is that once a country has achieved herd immunity the natural way, it won't need vaccines and won't need social distancing, masks, travel restrictions, etc. like the other countries that didn't will have to do for years. It will just need a lot of caskets.

P.S. See file "*Tri-County example*" for sensationlism example, too common with this epidemic.