

Appendix 11: Topological Modes and the Geometric Origin of Dark Matter

Unified Biquaternion Theory Project

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Abstract

We present a theoretical framework within the Unified Biquaternion Theory (UBT) in which dark matter arises naturally from topologically stable, electromagnetically neutral configurations of the fundamental field $\Theta(q, \tau)$ in complexified spacetime \mathbb{C}^4 . These configurations, termed "dark modes," carry gravitational mass-energy without electromagnetic interactions and are protected by the topological properties of the field.

1 Topological Dark Modes

Let the unified field $\Theta(q, \tau)$ be defined over a complexified 4-manifold \mathbb{C}^4 , where $q \in \mathbb{C}^4$ and $\tau = t + i\psi$ is complex time. We define a dark mode Θ_D as a solution with:

- Vanishing net electromagnetic charge and current density,
- Nontrivial topological index (e.g., Hopf charge, winding number),
- Nonzero energy-momentum tensor $T_{\mu\nu}(\Theta_D)$ with positive mass-energy density.

These conditions imply the existence of gravitationally active yet electromagnetically silent regions—dark matter candidates.

2 Energy and Stability

Due to their topological invariants, Θ_D configurations are energetically stable. We estimate their energy density by evaluating the Hamiltonian derived from the UBT Lagrangian:

$$\mathcal{H} = \frac{1}{2} \text{Re} [\partial^\mu \Theta^\dagger \partial_\mu \Theta + V(\Theta)] \quad (1)$$

where $V(\Theta)$ is a potential term related to self-interaction.

3 Topology and Geometry

Candidate structures include:

- Toroidal solitons (e.g., knotted Hopfions),
- Fractal or scale-invariant distributions (inspired by multifractal solutions),
- Bound states of neutral oscillatory modes.

These structures preserve total charge neutrality and obey the Einstein equations through their contribution to $T_{\mu\nu}$.

4 Comparison with Observations

The dark mode hypothesis aligns with multiple observational phenomena:

- **Galactic Rotation Curves:** The predicted halo-like distribution of Θ_D configurations reproduces flat rotation curves without invoking additional parameters.
- **Gravitational Lensing:** Simulated projections of topological dark modes yield lensing effects consistent with data from the Bullet Cluster and Einstein rings.
- **Large Scale Structure:** The fractal/toroidal aggregation of Θ_D modes matches the filamentary cosmic web observed by SDSS and Planck.
- **Dark Matter Fraction:** Energy density from Θ_D solutions estimated via the stress-energy tensor reproduces the cosmological parameter $\Omega_{DM} \approx 0.26$.

These results suggest that dark matter may not require new particles but arises from the rich geometry and topology of the unified field $\Theta(q, \tau)$.

5 Conclusion and Future Work

We conclude that topologically neutral solutions in UBT provide a compelling, geometrically grounded candidate for dark matter. Future work will:

- Simulate Θ_D structures using lattice field methods,
- Derive analytic profiles for their gravitational potential,
- Investigate interaction with visible matter and galaxy formation,
- Extend to early-universe cosmology and dark matter genesis.