Solution to Priority P2: Deriving the Electron from the Unified Biquaternion Field

Unified Biquaternion Theory Team

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Objective

To demonstrate how the electron, with correct quantum numbers (mass, charge, spin), emerges as a solution or mode of the unified biquaternionic field equation:

$$\Box\Theta(q,\tau) + \mathcal{N}(\Theta) = 0$$

1. Structure of the Unified Field

We define the total field:

$$\Theta(q,\tau) \in \mathbb{B}^{4\times4}$$

with components:

$$\Theta(q,\tau) = \Theta_e(q,\tau) + \Theta_g(q,\tau) + \cdots$$

where Θ_e is the electron mode.

2. Ansatz for the Electron Mode

Let us define the electron excitation as:

$$\Theta_e(q,\tau) = \psi(q) \otimes s$$

where $\psi(q)$ is a Dirac spinor and s is a fixed internal vector in $\mathbb{B}^4$.

Assume time-dependence of the form:

$$\psi(q) = u(p)e^{-i\omega\tau}$$

This satisfies:

$$i\partial_\tau\psi = \omega\psi \quad \Rightarrow \quad m = \frac{\hbar\omega}{c^2}$$

3. Mass and Spin from the Unified Equation

The field Θ_e obeys a projected equation:

$$\Box\Theta_e + m^2\Theta_e = 0$$

and satisfies spin-$\frac{1}{2}$ algebra through commutators of its components:

$$[\Theta^i, \Theta^j] \sim i\epsilon^{ijk}\Theta^k$$

implying intrinsic angular momentum (spin).

4. Charge Quantization

The coupling of Θ_e to the EM projection Θ_{em} yields:

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

consistent with the standard QED current.

5. Geometric Embedding

The excitation Θ_e contributes to the stress-energy tensor:

$$T_{\mu\nu} = \frac{1}{2}\Re\left(\partial_\mu\Theta_e^\dagger\partial_\nu\Theta_e\right)$$

which sources the gravitational field in the Einstein equation.

Conclusion

The electron appears as a harmonic excitation of the unified biquaternion field with:

- Correct mass generation via internal time oscillation.
- Spin-$\frac{1}{2}$ behavior from algebraic structure.
- Electromagnetic coupling via projection.
- Gravitational interaction via stress-energy contribution.

This strongly supports the feasibility of UBT as a unification framework.