Noether to α v0.1

Draft for UBT Project

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1 Lagrangian and Normalization

We start with the unified field Θ and the electromagnetic field A_{μ} . The Lagrangian is written in natural units $(c = \hbar = 1)$:

$$\mathcal{L} = \partial_{\mu} \Theta^{\dagger} \partial^{\mu} \Theta - m^{2} \Theta^{\dagger} \Theta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{1}$$

Here $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. Normalization is chosen so that the Noether current has standard dimension.

2 Global U(1) Symmetry and Noether Current

Under global U(1):

$$\Theta \to e^{i\lambda}\Theta,$$
 (2)

the Noether current is

$$J^{\mu} = i \left(\Theta^{\dagger} \partial^{\mu} \Theta - (\partial^{\mu} \Theta^{\dagger}) \Theta \right). \tag{3}$$

The conserved charge is

$$Q = \int d^3x J^0. (4)$$

We identify the fundamental excitation with $Q = \pm 1$.

3 Gauging the Symmetry

Promote λ to a local function $\lambda(x)$ by introducing a gauge field A_{μ} :

$$\partial_{\mu} \to D_{\mu} = \partial_{\mu} + igA_{\mu}. \tag{5}$$

The interaction term becomes

$$\mathcal{L}_{\text{int}} = gJ^{\mu}A_{\mu}. \tag{6}$$

4 Integration over ψ -Cycle

In the complex-time/UBT framework, Θ also depends on the additional coordinate ψ . Integration over the compact ψ -cycle modifies the gauge kinetic term:

$$S_{\text{eff}} = \int d^4x \left(-\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + g J^{\mu} A_{\mu} \right), \tag{7}$$

where the factor Z encodes the geometry of the ψ sector.

5 Fine Structure Constant

By canonical normalization of the photon field,

$$\alpha = \frac{g^2}{4\pi Z}.\tag{8}$$

Thus, the fine structure constant arises from Noether charge quantization and the geometric factor Z from the ψ sector.

6 Next Steps

- Explicitly compute Z from the ψ -cycle geometry (period, modular parameter τ).
- Solve for eigenmodes of Θ in the ψ sector to determine particle masses, in particular m_e .
- Ensure no free fit parameters remain; all quantities must follow from geometry and normalization.