

# Noether to $\alpha$ v0.2

Draft for UBT Project

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## 1 Lagrangian in 5D

We start with the unified field  $\Theta(x, \psi)$  and the gauge field  $A_M(x, \psi)$  on  $M^4 \times S^1_\psi$ . The action in natural units ( $c = \hbar = 1$ ) is

$$S = \int d^4x \int_0^{L_\psi} d\psi \left[ (D_M \Theta)^\dagger (D^M \Theta) - m^2 \Theta^\dagger \Theta - \frac{1}{4} F_{MN} F^{MN} \right], \quad (1)$$

with  $D_M = \partial_M + ig_5 A_M$ .

## 2 Noether Current

Under global  $U(1) : \Theta \rightarrow e^{i\lambda} \Theta$ , the Noether current is

$$J^M = i(\Theta^\dagger \partial^M \Theta - (\partial^M \Theta^\dagger) \Theta). \quad (2)$$

The charge is

$$Q = \int d^3x d\psi J^0, \quad (3)$$

and we normalize such that the fundamental excitation has  $Q = \pm 1$ .

## 3 Dimensional Reduction

Assuming  $A_\mu(x)$  is independent of  $\psi$ , the gauge term reduces to

$$S_{\text{gauge}} = -\frac{L_\psi}{4} \int d^4x F_{\mu\nu} F^{\mu\nu}. \quad (4)$$

Canonical normalization requires rescaling  $A_\mu \rightarrow A_\mu/\sqrt{L_\psi}$ , yielding an effective coupling

$$g_4 = \frac{g_5}{\sqrt{L_\psi}}. \quad (5)$$

## 4 Wilson Loop Quantization

On the compact  $\psi$ -cycle, gauge invariance implies quantization of the Wilson loop:

$$\exp\left(ig_5 \oint A_\psi d\psi\right) = e^{2\pi i n}, \quad n \in \mathbb{Z}. \quad (6)$$

This condition links  $g_5$ ,  $L_\psi$ , and the background  $\langle A_\psi \rangle$ . Thus, the geometry of the  $\psi$ -sector fixes the effective gauge coupling.

## 5 Generalized Factor $Z$

In full UBT, the integration over  $\psi$  gives not only  $L_\psi$  but also a correction factor  $f(\tau, \text{BC})$ , depending on the modular parameter  $\tau$  of complex time and boundary conditions:

$$Z = L_\psi \cdot f(\tau, \text{BC}). \quad (7)$$

The effective 4D action is then

$$S_{\text{eff}} = \int d^4x \left( -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + g_4 J^\mu A_\mu \right). \quad (8)$$

## 6 Fine Structure Constant

After canonical normalization, the fine structure constant is

$$\alpha = \frac{g_4^2}{4\pi} = \frac{g_5^2}{4\pi Z}. \quad (9)$$

## 7 Interpretation

- The relation  $\alpha = g_5^2/(4\pi Z)$  follows rigorously from Noether symmetry and dimensional reduction.

- To obtain the numerical value, UBT must determine  $g_5$  and  $Z$  from first principles:
  - $g_5$  fixed by Noether charge quantization ( $Q = \pm 1$ ).
  - $Z$  fixed by  $\psi$ -geometry:  $Z = L_\psi f(\tau, \text{BC})$ .
- Once these are determined ab-initio,  $\alpha$  is no longer a free parameter but a derived constant.