Construction of a Dark Mode Hopfion Solution in Unified Biquaternion Theory

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Abstract

This document presents the analytical ansatz and geometric characteristics of a topologically nontrivial Hopfion solution Θ_D within the Unified Biquaternion Theory (UBT), proposed as a candidate configuration for dark matter.

1 Ansatz

We define the Hopfion-like solution in stereographic coordinates for a map $\Theta : \mathbb{R}^3 \to S^2 \subset \mathbb{C}^2$ via the rational map:

$$\Theta_D(x, y, z, t) = \frac{(2(x + iy))^p}{(2z + i(r^2 - 1))^q},$$

where $r^2 = x^2 + y^2 + z^2$, and $p, q \in \mathbb{Z}^+$ define the topological charge.

2 Properties

- **Topological Charge:** The Hopf invariant $H = pq$.
- **Energy Density:** Localized in a toroidal region around the core, where $|\Theta_D|$ varies rapidly.

- **Electromagnetic Neutrality:** Imposed via projection onto gauge-neutral components of Θ.

3 Stress-Energy Tensor

The energy-momentum tensor $T_{\mu\nu}$ is derived from the UBT Lagrangian:

$$T_{\mu\nu} = \mathrm{Re}\left[\partial_\mu \Theta^\dagger \cdot \partial_\nu \Theta - \frac{1}{2}\eta_{\mu\nu}\left(\partial^\alpha \Theta^\dagger \cdot \partial_\alpha \Theta\right)\right],$$

ensuring conserved gravitational energy.

4 Next Steps

To validate this configuration:

- Numerically simulate the stability of Θ_D,
- Compute the resulting gravitational potential from $T_{\mu\nu}$,
- Fit predicted rotation curves to observed galactic data.

Author's Note

This work was developed solely by Ing. David Jaroš. Large language models (ChatGPT-4o by OpenAI and Gemini 2.5 Pro by Google) were used strictly as assistive tools for calculations, LaTeX formatting, and critical review. All core ideas, equations, theoretical constructs and conclusions are the intellectual work of the author.