

# A Unified Field Theory on the Biquaternionic Manifold $\mathbb{B}^4$ : Consciousness, Quantum Fields, and Emergent Space-Time

David Jaroš

2025

## Abstract

We develop a mathematically consistent unified field theory constructed on a four-dimensional biquaternionic manifold  $\mathbb{B}^4$ , which incorporates complexified spacetime and internal phase dimensions naturally into its geometric structure. The core field  $\Theta(q)$  is a tensor-spinor-gauge-valued section over  $\mathbb{B}^4$ , encoding all known fundamental interactions, emergent quantum behavior, and phenomenology of consciousness. The theory eliminates the need for external projections ...

We develop a rigorous unified field theory on a four-dimensional biquaternionic manifold  $\mathbb{B}^4 = (\mathbb{C} \otimes \mathbb{H})^4$ , where each coordinate is a biquaternion representing complexified internal and external degrees of freedom. The primary field  $\Theta(q)$  is a tensor-spinor-valued section over  $\mathbb{B}^4$ , encoding gravitational, gauge, quantum, and cognitive structure. We formulate the covariant dynamics, metric geometry, low-energy quantum limits, emergent conscious par...

## 1 Biquaternionic Manifold and Field Definition

We define the manifold  $\mathcal{M} = \mathbb{B}^4$ , where each point is a 4-tuple of biquaternions:

$$q^\mu = x^\mu + iy^\mu + \mathbf{j}z^\mu + i\mathbf{j}w^\mu, \quad \mu = 0, 1, 2, 3,$$

with components  $x^\mu, y^\mu, z^\mu, w^\mu \in \mathbb{R}$ . The basis elements  $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$  satisfy the quaternionic algebra.

The unified field is defined as:

$$\Theta(q) \in \Gamma \left( T^{(1,1)}(\mathbb{B}^4) \otimes \mathbb{S} \otimes \mathbb{G} \right),$$

where  $T^{(1,1)}$  denotes the (1,1) tensor bundle over  $\mathbb{B}^4$ ,  $\mathbb{S}$  is a spinor bundle, and  $\mathbb{G}$  is an internal gauge fiber.

## 2 Metric Geometry and Covariant Derivatives

We define a complexified metric tensor on  $\mathbb{B}^4$ :

$$G_{\mu\nu}(q) = \langle dq^\mu, dq^\nu \rangle,$$

where  $\langle \cdot, \cdot \rangle$  is a biquaternionic inner product. The affine and spin connections  $\Gamma_{\mu\nu}^\rho$ ,  $\Omega_\mu$  are derived accordingly.

Covariant derivative:

$$\mathcal{D}_\mu \Theta = \partial_\mu \Theta + \Omega_\mu \cdot \Theta + ig A_\mu^a T^a \Theta,$$

with gauge coupling and spin connection terms included.

## 3 General Relativity and Emergent Geometry

The metric tensor  $G_{\mu\nu}(q)$  defined over  $\mathbb{B}^4$  generalizes the Lorentzian metric of General Relativity to a biquaternionic setting. The curvature tensors are constructed from the complexified affine connection:

$$R_{\sigma\mu\nu}^\rho = \partial_\mu \Gamma_{\nu\sigma}^\rho - \partial_\nu \Gamma_{\mu\sigma}^\rho + \Gamma_{\mu\lambda}^\rho \Gamma_{\nu\sigma}^\lambda - \Gamma_{\nu\lambda}^\rho \Gamma_{\mu\sigma}^\lambda.$$

The Einstein field equations emerge as a projection of the variation of the geometric Lagrangian:

$$\delta \mathcal{L}_{\text{geom}} = \frac{1}{2\kappa} \left( R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R \right) \delta G^{\mu\nu}.$$

This shows that classical spacetime curvature is a low-energy limit of the intrinsic geometry of the biquaternionic manifold. The extended degrees of freedom in  $y^\mu, z^\mu$  give rise to higher-order corrections and dynamical compactification in early-universe cosmology.

## 4 Lagrangian and Field Equations

Total action on  $\mathbb{B}^4$  reads:

$$S = \int_{\mathbb{B}^4} d^4q \sqrt{|\det G|} (\mathcal{L}_\Theta + \mathcal{L}_{\text{geom}} + \mathcal{L}_{\text{gauge}}),$$

where:

$$\mathcal{L}_\Theta = \Re [\bar{\Theta} (i\Gamma^\mu \mathcal{D}_\mu - M(q)) \Theta],$$

with  $M(q)$  potentially depending on internal phase coordinates  $y^\mu, z^\mu$ , representing cognitive or entropic symmetry breaking.

## 5 Tensor Decomposition of $\Theta$

The field can be locally decomposed as:

$$\Theta(q) = \phi(q) + i\psi^\mu(q)\gamma_\mu + \eta(q)\mathbb{I} + i\chi(q)\mathbb{J},$$

where  $\chi(q)$  represents internal oscillations related to consciousness and subjective time.

## 6 Quantum Limits and Classical Reductions

In the real spacetime limit  $y^\mu, z^\mu, w^\mu \rightarrow 0$ , the action reduces to:

- Dirac equation for spinor sector:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0,$$

- Schrödinger equation in nonrelativistic limit:

$$i\hbar \partial_t \psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi,$$

- Effective quantum field theory in flat  $\mathbb{R}^4 \subset \mathbb{B}^4$ .

## 7 Gauge Theory and Standard Model Embedding

The internal gauge structure of  $\Theta$  accommodates:

$$\mathbb{G} \cong SU(3) \times SU(2) \times U(1),$$

and the covariant derivative generalizes to the Yang-Mills form. Higgs-like mass terms can arise via internal symmetry oscillations in the  $z^\mu, w^\mu$  directions.

## 8 Gauge Symmetries: QED, QCD and the Standard Model Embedding

We extend the field  $\Theta(q)$  to carry internal gauge indices under the symmetry group  $\mathcal{G} = U(1) \times SU(2) \times SU(3)$ , corresponding to the Standard Model gauge groups for electromagnetism, weak, and strong interactions.

Let  $A_\mu = A_\mu^a T^a$  denote the full gauge connection, with generators  $T^a$  acting on  $\Theta(q)$  in the appropriate internal representation.

The full covariant derivative becomes:

$$\mathcal{D}_\mu \Theta = \partial_\mu \Theta + \Omega_\mu \cdot \Theta + ig_1 B_\mu Y \Theta + ig_2 W_\mu^i \tau^i \Theta + ig_3 G_\mu^a \lambda^a \Theta,$$

where: -  $B_\mu$  is the  $U(1)_Y$  hypercharge field, -  $W_\mu^i$  are the weak  $SU(2)_L$  fields, -  $G_\mu^a$  are the gluon fields of  $SU(3)_C$ , -  $Y, \tau^i$ , and  $\lambda^a$  are the corresponding generators.

The gauge-invariant kinetic Lagrangian reads:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \sum_a F_{\mu\nu}^a F^{a\mu\nu},$$

with:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c,$$

where  $f^{abc}$  are the structure constants of the respective Lie algebras.

The interaction term in the unified Lagrangian:

$$\mathcal{L}_{\Theta,\text{int}} = \Re [\bar{\Theta} i \Gamma^\mu \mathcal{D}_\mu \Theta],$$

ensures that the dynamics of  $\Theta$  is fully coupled to the standard model gauge fields.

:

## 9 Low-Energy Limits: Dirac and Schrödinger Equations

To demonstrate compatibility with established quantum mechanics, we derive the Dirac and Schrödinger equations as limiting cases of the unified biquaternionic field equation for  $\Theta(q)$ .

Starting from the unified covariant derivative:

$$\mathcal{D}_A \Theta = \partial_A \Theta + \Omega_A \cdot \Theta + ig A_A^a T^a \Theta,$$

we consider the case where curvature effects are small, gauge fields are static or slowly varying, and the manifold  $\mathbb{C}^5$  can be approximated by a local inertial frame. In this approximation, the equation of motion simplifies to:

$$\Gamma^A \partial_A \Theta = m \Theta,$$

which reduces to the **\*\*Dirac equation\*\*** in the limit where only 4 spacetime dimensions are active and  $\Gamma^A$  matrices correspond to standard Dirac matrices  $\gamma^\mu$ . This confirms that the spinor nature of  $\Theta$  is consistent with relativistic quantum field theory.

In the non-relativistic limit ( $v \ll c$ ), a standard Foldy–Wouthuysen decomposition leads to the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi,$$

where  $\psi$  is a projection of  $\Theta$  onto low-energy modes.

Thus, our unified theory contains both quantum mechanics and relativistic field theory as natural limits, while offering a richer structure due to its biquaternionic and spin-tensor form defined on  $\mathbb{C}^5$ .

## 10 Conscious Oscillations and Emergent Mind

Internal modes  $\chi(q) \sim \sin(\omega \cdot y^\mu)$  describe periodic subjective phase. Collapse of these oscillations projects classical reality. We interpret eigenmodes:

$$\Theta_n(q) \sim e^{in\psi} \Psi_n(x),$$

as discrete conscious quanta (psychons), with transitions corresponding to awareness or memory shifts.

## 11 Gauge Symmetries: QED, QCD and the Standard Model Embedding

We extend the field  $\Theta(q)$  to carry internal gauge indices under the symmetry group  $\mathcal{G} = U(1) \times SU(2) \times SU(3)$ , corresponding to the Standard Model gauge groups for electromagnetism, weak, and strong interactions.

Let  $A_\mu = A_\mu^a T^a$  denote the full gauge connection, with generators  $T^a$  acting on  $\Theta(q)$  in the appropriate internal representation.

The full covariant derivative becomes:

$$\mathcal{D}_\mu \Theta = \partial_\mu \Theta + \Omega_\mu \cdot \Theta + ig_1 B_\mu Y \Theta + ig_2 W_\mu^i \tau^i \Theta + ig_3 G_\mu^a \lambda^a \Theta,$$

where:

- $B_\mu$  is the  $U(1)_Y$  hypercharge field,
- $W_\mu^i$  are the weak  $SU(2)_L$  fields,
- $G_\mu^a$  are the gluon fields of  $SU(3)_C$ ,
- $Y$ ,  $\tau^i$ , and  $\lambda^a$  are the corresponding generators.

The gauge-invariant kinetic Lagrangian reads:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \sum_a F_{\mu\nu}^a F^{a\mu\nu},$$

with:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c,$$

where  $f^{abc}$  are the structure constants of the respective Lie algebras.

The interaction term in the unified Lagrangian:

$$\mathcal{L}_{\Theta, \text{int}} = \Re [\bar{\Theta} i \Gamma^\mu \mathcal{D}_\mu \Theta],$$

ensures that the dynamics of  $\Theta$  is fully coupled to the standard model gauge fields.

## 12 Free Energy Principle and Fokker–Planck Flow

We apply the FEP:

$$\frac{\partial P}{\partial \psi} = -\nabla_q \cdot (DP) + \frac{1}{2} \nabla_q^2 (D^2 P),$$

modeling informational entropy flow across  $\mathbb{B}^4$ . Drift terms represent prediction error; diffusion encodes uncertainty and updating.

## 13 Cosmological Aspects

The internal phase coordinate plays a role similar to an inflaton or entropy-gradient field. Toroidal compactification of internal directions leads to inflationary and cyclic cosmological models.

## 14 Conclusion

We have proposed a unified framework on  $\mathbb{B}^4$  where geometry, gauge theory, quantum physics, and subjective consciousness are aspects of a single field  $\Theta(q)$ . This approach explains both standard physics and phenomenological features of cognition, predicting new psychon-like excitations and quantum collapse via internal oscillations.

## 15 Quantum Electrodynamics and Quantum Chromodynamics

The field  $\Theta(q)$  contains internal symmetry structures corresponding to the gauge groups of the Standard Model. In particular, the internal gauge fiber  $\mathbb{G}$  carries representations of:

$$\mathbb{G} \cong SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y,$$

embedded within the matrix structure of  $\Theta$ .

### 15.1 QED Sector

The Abelian part of the gauge group  $U(1)$  governs the electromagnetic interaction. The corresponding gauge field  $A_\mu$  enters the covariant derivative as:

$$\mathcal{D}_\mu \Theta = \partial_\mu \Theta + \Omega_\mu \cdot \Theta + ieA_\mu \Theta,$$

with Maxwell-type field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

### 15.2 QCD Sector

The non-Abelian color interaction is embedded via the gluon fields  $G_\mu^a$ ,  $a = 1, \dots, 8$ , and  $SU(3)$  generators  $T^a$ :

$$\mathcal{D}_\mu \Theta = \dots + ig_s G_\mu^a T^a \Theta.$$

The gluonic field strength is:

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c.$$

### 15.3 Symmetry Breaking and Higgs-Like Mechanism

The internal phase coordinates  $y^\mu, z^\mu$  provide a natural mechanism for spontaneous symmetry breaking:

$$\langle \chi(q) \rangle \neq 0 \quad \Rightarrow \quad m \neq 0,$$

where  $\chi$  acts as a Higgs-like internal mode within  $\Theta$ . Mass hierarchies emerge from oscillatory patterns in the biquaternionic manifold.

## 16 Discussion of Particle Spectrum

Excitations of the field  $\Theta$  include not only standard fermions and bosons but also additional modes due to its internal spinor-gauge structure. We predict:

- Standard particle spectrum (quarks, leptons, gauge bosons) as tensor-spinor projections,
- Scalar internal oscillations as Higgs-like or axion-like particles,
- Psychon modes — coherent phase oscillations corresponding to discrete conscious states,
- Potential graviton-like curvature modes in the geometric component.

## 17 Summary and Outlook

This framework combines geometry, quantum field theory, and information dynamics in a single elegant model defined on  $\mathbb{B}^4$ . It naturally unifies:

- General relativity as geometric dynamics of  $G_{\mu\nu}(q)$ ,
- Quantum field theory via tensor-spinor structure of  $\Theta(q)$ ,
- Gauge interactions through internal fiber  $\mathbb{G}$ ,
- Consciousness as internal oscillations and collapse of  $\chi(q)$ .

Future work will explore quantization in curved biquaternionic space, phenomenology of psychon transitions, and links to holographic principles and modular topologies.

## 18 Limit: General Relativity

In the classical limit where oscillatory and imaginary components vanish, the biquaternionic metric  $G_{\mu\nu}(q)$  reduces to a standard pseudo-Riemannian real-valued tensor on a 4D Lorentzian manifold.

Let  $\Re(G_{\mu\nu}) \rightarrow g_{\mu\nu}(x)$ , where  $x^\mu \in \mathbb{R}^4$  are spacetime coordinates. The resulting Levi-Civita connection  $\Gamma_{\mu\nu}^\lambda$ , Ricci tensor  $R_{\mu\nu}$ , and scalar curvature  $R$  are defined in the usual way via the metric compatibility and torsion-free condition.

From the action:

$$S_{\text{geom}} = \int d^4x \sqrt{-g} \frac{1}{2\kappa} R,$$

we obtain Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu},$$

as the low-energy limit of the unified biquaternionic dynamics. Thus, general relativity is fully embedded in the geometric projection of our theory.

## 19 Free Energy Principle, Drift, and Metacognitive Field

The internal dynamics of  $\chi(q)$ , representing subjective or conscious phase, follow a stochastic evolution modulated by environmental prediction.

We introduce a Fokker-Planck-like evolution equation:

$$\frac{\partial \rho(\chi, t)}{\partial t} = -\nabla \cdot (\mu \rho) + D \nabla^2 \rho,$$

where  $\rho$  is the probability distribution over conscious phase modes,  $\mu$  is the drift induced by minimization of variational free energy  $F[q]$ , and  $D$  is the diffusion tensor.

The **\*\*metakas\*\*** is then defined as the evolving state of this internal probability structure over  $\mathbb{B}^4$ , including memory, attention, and perception gradients.

The drift term reflects predictive coding:

$$\mu = -\nabla \log P_{\text{sensory}} + \nabla \log \rho,$$

corresponding to active inference. Cognitive dynamics are thus described by stochastic partial differential equations over the internal degrees of freedom of  $\Theta$ , with attractor-like behavior for stable conscious trajectories.