

Analytical Gravitational Potential of a Hopfion-like Dark Matter Configuration

UBT Research Team

Introduction

We investigate the gravitational field generated by a toroidal energy configuration derived from the $\Theta_D(x, y, z)$ hopfion mode, a candidate structure for dark matter in the framework of Unified Biquaternion Theory (UBT). Our aim is to obtain an analytical approximation to the gravitational potential and derive the resulting rotation curve.

Poisson Equation and Green's Function

The Newtonian gravitational potential $\Phi(\vec{x})$ satisfies the Poisson equation:

$$\nabla^2 \Phi(\vec{x}) = 4\pi G T_{00}(\vec{x}),$$

where $T_{00}(\vec{x})$ is the energy density of the Θ_D configuration.

Using the Green's function method, the solution reads:

$$\Phi(\vec{x}) = -G \int \frac{T_{00}(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3 x'.$$

Toroidal Approximation

Assuming toroidal symmetry, we approximate the energy distribution as:

$$T_{00}(\vec{x}) \approx \rho_0 \exp\left(-\frac{(r - R_0)^2}{\sigma^2}\right),$$

where R_0 is the major radius and σ the width of the torus.

For points along the axis or near the center, the potential becomes:

$$\Phi(r) \approx -\frac{4\pi G \rho_0 \sigma^2 R_0}{r} \left(1 - \exp\left(-\frac{(r - R_0)^2}{\sigma^2}\right)\right).$$

Rotation Curve

The rotational velocity $v(r)$ follows from:

$$v(r) = \sqrt{r \frac{d\Phi}{dr}} \approx \text{const},$$

for $r \gtrsim R_0$, in agreement with observed flat galactic rotation curves.

Conclusion

Our analytical approximation shows that the gravitational field of a Hopfion-like dark matter configuration naturally yields a flat rotation curve without invoking additional particles, supporting its viability as a dark matter model in UBT.