Derivation of the Electron Mass from Electromagnetic Self-Energy

Unified Biquaternion Theory

Overview

In this document, we present a derivation of the electron mass as arising from its own electromagnetic self-energy, in line with the hypothesis of dual mass origin proposed in the Unified Biquaternion Theory (UBT).

1. Self-Energy of a Smeared Charge Distribution

We start from the classical expression for the electrostatic self-energy:

$$\delta m_e = \frac{1}{2} \int \rho(\vec{x}) \phi(\vec{x}) \, d^3x$$

Assuming a Gaussian charge distribution for the topological field configuration (Hopfion), we solve Poisson's equation and obtain the electrostatic potential. The resulting self-energy is:

$$\delta m_e = \frac{e^2}{\sqrt{\pi} R}$$

where R is the effective "size" of the charge distribution.

2. Total Energy of the Hopfion Field Θ_1

We consider the topological solution:

$$\Theta_1(\vec{x}) = \frac{1}{R} \cdot \frac{1}{(1 + r^2)^2}$$

Computing the total energy density of this configuration:

$$T_{00}(\vec{x}) = |\nabla \Theta_1|^2$$

we obtain the total energy:

$$E = \frac{\pi^2}{2R^3}$$

3. Effective Radius from Energy Density

From the normalized energy density we compute the effective spatial variance:

$$R_{\text{eff}}^2 = \frac{\int r^2 T_{00}(\vec{x}) \, d^3x}{\int T_{00}(\vec{x}) \, d^3x} = 5R^2 \quad \Rightarrow \quad R = \frac{R_{\text{eff}}}{\sqrt{5}}$$

Conclusion

The parameter R used in the self-energy calculation is not a free constant. It is uniquely determined by the topological field solution Θ_1, completing the prediction of the electron mass from first principles.