Noether to α v0.2

Draft for UBT Project

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1 Lagrangian in 5D

We start with the unified field $\Theta(x,\psi)$ and the gauge field $A_M(x,\psi)$ on $M^4 \times S^1_{\psi}$. The action in natural units $(c=\hbar=1)$ is

$$S = \int d^4x \int_0^{L_{\psi}} d\psi \left[(D_M \Theta)^{\dagger} (D^M \Theta) - m^2 \Theta^{\dagger} \Theta - \frac{1}{4} F_{MN} F^{MN} \right], \quad (1)$$

with $D_M = \partial_M + ig_5 A_M$.

2 Noether Current

Under global $U(1): \Theta \to e^{i\lambda}\Theta$, the Noether current is

$$J^{M} = i \left(\Theta^{\dagger} \partial^{M} \Theta - (\partial^{M} \Theta^{\dagger}) \Theta \right). \tag{2}$$

The charge is

$$Q = \int d^3x \, d\psi \, J^0, \tag{3}$$

and we normalize such that the fundamental excitation has $Q=\pm 1.$

3 Dimensional Reduction

Assuming $A_{\mu}(x)$ is independent of ψ , the gauge term reduces to

$$S_{\text{gauge}} = -\frac{L_{\psi}}{4} \int d^4x \, F_{\mu\nu} F^{\mu\nu}. \tag{4}$$

Canonical normalization requires rescaling $A_{\mu} \to A_{\mu}/\sqrt{L_{\psi}}$, yielding an effective coupling

 $g_4 = \frac{g_5}{\sqrt{L_\psi}}. (5)$

4 Wilson Loop Quantization

On the compact ψ -cycle, gauge invariance implies quantization of the Wilson loop:

$$\exp\left(ig_5 \oint A_\psi \, d\psi\right) = e^{2\pi i n}, \qquad n \in \mathbb{Z}. \tag{6}$$

This condition links g_5 , L_{ψ} , and the background $\langle A_{\psi} \rangle$. Thus, the geometry of the ψ -sector fixes the effective gauge coupling.

5 Generalized Factor Z

In full UBT, the integration over ψ gives not only L_{ψ} but also a correction factor $f(\tau, BC)$, depending on the modular parameter τ of complex time and boundary conditions:

$$Z = L_{\psi} \cdot f(\tau, BC). \tag{7}$$

The effective 4D action is then

$$S_{\text{eff}} = \int d^4x \left(-\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + g_4 J^{\mu} A_{\mu} \right). \tag{8}$$

6 Fine Structure Constant

After canonical normalization, the fine structure constant is

$$\alpha = \frac{g_4^2}{4\pi} = \frac{g_5^2}{4\pi Z}. (9)$$

7 Interpretation

• The relation $\alpha = g_5^2/(4\pi Z)$ follows rigorously from Noether symmetry and dimensional reduction.

- \bullet To obtain the numerical value, UBT must determine g_5 and Z from first principles:
 - g_5 fixed by Noether charge quantization $(Q = \pm 1)$.
 - Z fixed by ψ -geometry: $Z = L_{\psi} f(\tau, BC)$.
- Once these are determined ab-initio, α is no longer a free parameter but a derived constant.