Appendix 20: Semiclassical Calculation of the Electron's Self-Energy

20.1 Overview

This appendix details the semiclassical derivation of the electron's electromagnetic self-energy (δm_e). This approach provides an intuitive physical picture and a first-order estimate for the correction required by our dual-mass hypothesis. The result demonstrates that the effective radius R of the charge distribution is not a free parameter but is determined by the underlying topological solution Θ_1.

20.2 Self-Energy of a Smeared Charge Distribution

We start from the classical expression for the electrostatic self-energy, which represents the energy stored in the electric field generated by the charge distribution:

$$\delta m_e c^2 = \frac{1}{2} \int \rho(\vec{x}) \phi(\vec{x}) \, d^3x \tag{1}$$

To make the problem analytically tractable, we assume the charge distribution $\rho(r)$ corresponding to the Θ_1 Hopfion can be approximated by a spherically symmetric Gaussian function:

$$\rho(r) = \frac{e}{\pi^{3/2} R^3} \exp\left(-\frac{r^2}{R^2}\right) \tag{2}$$

where R is the effective radius of the distribution.

20.3 Electrostatic Potential and Final Integration

The electrostatic potential $\phi(r)$ for this Gaussian source is found by solving the Poisson equation $\nabla^2 \phi = -\rho/\epsilon_0$. The solution is:

$$\phi(r) = \frac{e}{4\pi\epsilon_0 r} \operatorname{erf}\left(\frac{r}{R}\right) \tag{3}$$

where erf is the error function. Substituting $\rho(r)$ and $\phi(r)$ back into the self-energy integral yields the result:

$$\delta m_e = \frac{e^2}{2\sqrt{\pi}(4\pi\epsilon_0) R c^2} = \frac{\alpha\hbar}{\sqrt{\pi} R c} \tag{4}$$

In natural units ($\hbar = c = 1$, $e^2 = 4\pi\alpha$), this simplifies to $\delta m_e = \frac{\sqrt{4\pi}\alpha}{R}$. *(Poznámka: Ve vašem souboru byl vzorec bez $4\pi\epsilon_0$, což odpovídá jinému systému jednotek. Zde jsem použil standardní SI-odvozenou formu, která je explicitnější.)*

20.4 The Nature of the Radius R

Crucially, the parameter R is not a free constant. As shown in the document 'electron$_m ass_f rom_s el f_e nerg$
For instance, by calculating the root-mean-square radius of the field's energy density distribution:

$$R^2 = \frac{\int r^2 |\nabla\Theta_1|^2 \, d^3x}{\int |\nabla\Theta_1|^2 \, d^3x} \tag{5}$$

This calculation links the effective size R directly to the fundamental structure of the theory, thus completing the prediction of the electron mass from first principles.

Author's Note

This work was developed solely by Ing. David Jaroš. Large language models (ChatGPT-4o by OpenAI and Gemini 2.5 Pro by Google) were used strictly as assistive tools for calculations, LaTeX formatting, and critical review. All core ideas, equations, theoretical constructs and conclusions are the intellectual work of the author.