Topological Phase Defects in Scalar Field Θ

Consider a vortex configuration in 2D:

$$\phi(\theta) = n\theta, \quad \rho(r) \to 0 \text{ as } r \to 0$$

Then:

$$\nabla\phi = \frac{n}{r}\hat{\theta}, \quad \nabla\rho \sim \rho'(r)\hat{r}$$

Their scalar product is zero due to orthogonality:

$$\nabla\rho \cdot \nabla\phi = 0$$

So the constraint is preserved even with a topological defect at the origin.