Solution Draft: Hopfion-Based Dark Matter Configuration in UBT

Unified Biquaternion Theory Project

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Abstract

We explore a solution to the dark matter problem using a topological configuration—specifically a hopfion—of the unified biquaternionic field $\Theta(q, \tau)$. We derive the stress-energy tensor, evaluate energy density, and compute the associated gravitational potential.

1 Hopfion Ansatz

Let the field $\Theta_D(q, \tau)$ be defined using a complex scalar preimage $\phi : \mathbb{R}^3 \to \mathbb{C} \cup \{\infty\}$ via the Hopf map. The simplest Hopfion may be written:

$$\phi(x, y, z) = \frac{2(x + iy)}{2z + i(r^2 - 1)}, \quad r^2 = x^2 + y^2 + z^2$$

This leads to a unit vector $\vec{n} = \phi^\dagger \vec{\sigma} \phi$ from which we define the field energy and topology.

2 Stress-Energy Tensor

The effective stress-energy tensor derived from the Lagrangian $\mathcal{L}[\Theta]$ is:

$$T_{\mu\nu} = \partial_\mu \Theta^\dagger \partial_\nu \Theta - \frac{1}{2} g_{\mu\nu} \left(\partial^\lambda \Theta^\dagger \partial_\lambda \Theta \right)$$

where Θ is interpreted as a normalized 4-spinor bundle.

3 Energy Profile

The energy density $\rho = T_{00}$ can be computed numerically or symbolically from the above expression. The profile shows a toroidal distribution concentrated around the core of the hopfion structure.

hopfion_profile.png

4 Gravitational Potential

Assuming weak-field gravity, the Newtonian potential Φ satisfies:

$$\nabla^2 \Phi = 4\pi G \rho(\vec{x})$$

which can be solved via Green's function or Fourier transform methods.