

UBT Appendix 5: Vector Part of the Biquaternionic Field Equation

1. Introduction

In this appendix, we examine the **vector part** of the unified field equation derived from the Biquaternion Gravity framework:

$$\mathbf{R}_a^\mu - \frac{1}{2}e_a^\mu \mathbf{R} = 0 \quad (1)$$

where all quantities are biquaternions.

Our goal is to extract and interpret the **vector part** of this equation, in analogy with how we previously analyzed the scalar and imaginary scalar components.

2. Decomposition into Vector Components

Recall that any biquaternion \mathbf{X} can be decomposed as:

$$\mathbf{X} = \text{Scal}(\mathbf{X}) + \vec{V}_R(\mathbf{X}) + i\vec{V}_I(\mathbf{X}) + i\text{PScal}(\mathbf{X})$$

We now apply the projection:

$$\text{Re}(\vec{V}(\mathbf{X})) = \text{Re}(\mathbf{X}) - \text{Scal}(\mathbf{X})$$

and similarly for the imaginary vector part.

Let us define:

$$\vec{V}_{Ra}^\mu = \text{ReVectorPart}(\mathbf{R}_a^\mu - \frac{1}{2}e_a^\mu \mathbf{R}) \quad (2)$$

$$\vec{V}_{Ia}^\mu = \text{ImVectorPart}(\mathbf{R}_a^\mu - \frac{1}{2}e_a^\mu \mathbf{R}) \quad (3)$$

Then the vector part equation is simply:

$$\vec{V}_{Ra}^\mu + i\vec{V}_{Ia}^\mu = 0 \quad (4)$$

which splits into two real vector equations:

$$\vec{V}_{Ra}^\mu = 0 \quad (5)$$

$$\vec{V}_{Ia}^\mu = 0 \quad (6)$$

3. Interpretation and Hypotheses

3.1 Real Vector Equation

The real vector equation $\vec{V}_{Ra}^\mu = 0$ may encode a constraint on torsion-free, metric-compatible geometries. It likely corresponds to Einstein-Cartan-like conditions or vectorial conservation laws.

3.2 Imaginary Vector Equation

The imaginary vector part $\vec{V}_I^\mu{}_a = 0$ is especially intriguing. It may encode:

- A generalized Maxwell-type field equation,
- A hidden vector field coupled to spacetime geometry,
- A remnant of conformal or chiral gauge symmetry.

In the simplified case $e_I = 0$, this equation may reduce to a divergence-type condition on ω_I :

$$\nabla_\mu \omega_I^{\mu ab} + (\text{nonlinear terms}) = 0$$

resembling Yang-Mills field dynamics.

4. Future Directions

- Attempt explicit calculation of $\vec{V}_I^\mu{}_a$ in terms of ω_R and ω_I .
- Test reductions in symmetric backgrounds (FLRW, Schwarzschild).
- Seek Lagrangian formulation that yields this vector equation as Euler-Lagrange equation.