Appendix M – Dark Energy in the Unified Biquaternion Theory (UBT)

M.1 Motivation and Scope

This appendix consolidates the UBT description of *dark energy* based on the complex-time framework $\tau = t + i\psi$ and the biquaternionic master field $\Theta(q, \tau)$. We derive an *effective cosmological sector* sourced by the slow phase ψ and show how ΛCDM is recovered for $\psi \to 0$. Links to: Appendix F (psychons & ψ-sector dynamics), Appendix J (metric deformations), Appendix K (field propagation in curved backgrounds).

M.2 UBT Action and Emergent Vacuum Sector

Consider the effective gravitational action (signature $-, +, +, +$)

$$S_{\mathrm{UBT}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \left(R - 2\Lambda_0 \right) + S_\Theta[\Theta, g, \psi] + S_\psi[\psi, g] \,, \tag{1}$$

where Θ carries internal (biquaternionic/spinor) structure and the slow phase ψ is the imaginary part of the complex time $\tau = t + i\psi$. At long wavelengths, integrating out fast Θ-modes yields an *effective vacuum energy density*

$$\rho_{\mathrm{vac}}^{(\mathrm{UBT})}(\psi) \;=\; \rho_{\Lambda 0} \left(1 + \kappa_\Lambda \, \psi \right) + \frac{1}{2} M_\psi^2 \, \psi^2 + \frac{\alpha_\psi}{2} (\nabla \psi)^2 + \cdots \,, \tag{2}$$

so that the *effective* cosmological term becomes

$$\Lambda_{\mathrm{eff}}(\psi) \;=\; \frac{8\pi G}{c^4} \, \rho_{\mathrm{vac}}^{(\mathrm{UBT})}(\psi) \,. \tag{3}$$

The coefficients $(\kappa_\Lambda, M_\psi^2, \alpha_\psi)$ are UBT couplings; $\kappa_\Lambda \to 0$ restores ΛCDM with $\Lambda_{\mathrm{eff}} = \Lambda_0$.

M.3 Homogeneous and Isotropic Cosmology

For a spatially flat FLRW metric,

$$ds^2 = -c^2 dt^2 + a(t)^2 \, d\mathbf{x}^2, \qquad H \equiv \dot{a}/a, \tag{4}$$

the Friedmann equations with UBT dark-energy sector read

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r + \rho_{\rm vac}^{(\rm UBT)}(\psi)\right), \tag{5}$$

$$\dot{H} = -4\pi G\left(\rho_m + \frac{4}{3}\rho_r + \rho_{\rm vac}^{(\rm UBT)}(\psi) + p_{\rm vac}^{(\rm UBT)}(\psi)\right)/c^2, \tag{6}$$

with effective equation of state

$$w_{\rm UBT}(\psi) \equiv \frac{p_{\rm vac}^{(\rm UBT)}}{\rho_{\rm vac}^{(\rm UBT)}} \;\approx\; -1 + \frac{\alpha_\psi(\nabla\psi)^2 - M_\psi^2\psi^2}{2\,\rho_{\Lambda 0}} + \mathcal{O}(\psi^2). \tag{7}$$

For a homogeneous slow phase ($\nabla\psi = 0$) we obtain $w_{\rm UBT} \gtrsim -1$ for $M_\psi^2\psi^2 \ll \rho_{\Lambda 0}$; phantom-like $w_{\rm UBT} < -1$ requires parity-odd or higher-derivative mixings (cf. Appendix F).

M.4 Linear Perturbations (Sketch)

Writing $\psi = \bar{\psi}(t) + \delta\psi(t, \mathbf{x})$, the scalar sector gains an extra gauge-invariant mode coupled to metric potentials Φ, Ψ. At sub-horizon scales the effective dark-energy sound speed is

$$c_{s,\rm UBT}^2 \;\simeq\; \frac{\alpha_\psi}{\alpha_\psi + M_\psi^2/k^2} \in (0, 1], \tag{8}$$

limiting clustering of the vacuum sector; $\alpha_\psi \to 0$ recovers an unclustered Λ.

M.5 Recovery of ΛCDM

Setting $(\kappa_\Lambda, \alpha_\psi, M_\psi) \to 0$ freezes ψ and yields $\rho_{\rm vac}^{(\rm UBT)} \to \rho_{\Lambda 0}$ with constant $w = -1$ and standard distances, growth, and CMB background. This ensures compatibility with precision cosmology when the UBT phase sector is inactive.

M.6 Illustrative Hubble Curves (No External Files)

Below we plot $E(z) \equiv H(z)/H_0$ for three small UBT deformations parameterized by $\kappa_\Lambda\psi \equiv \epsilon \in \{-0.05, 0, +0.05\}$, keeping $\Omega_{m0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$ at $z = 0$.

M.7 Observable Consequences (Qualitative)

- Slight shifts in distance–redshift relations $D_L(z), D_A(z)$ and in the derived H_0 if $\epsilon \neq 0$ today.
- Modified ISW effect and low-ℓ CMB for time-varying $\bar{\psi}(t)$.
- Growth rate changes $f(z)\sigma_8$ suppressed by $c_{s,\rm UBT}^2 \lesssim 1$; $\epsilon \to 0$ reproduces ΛCDM.

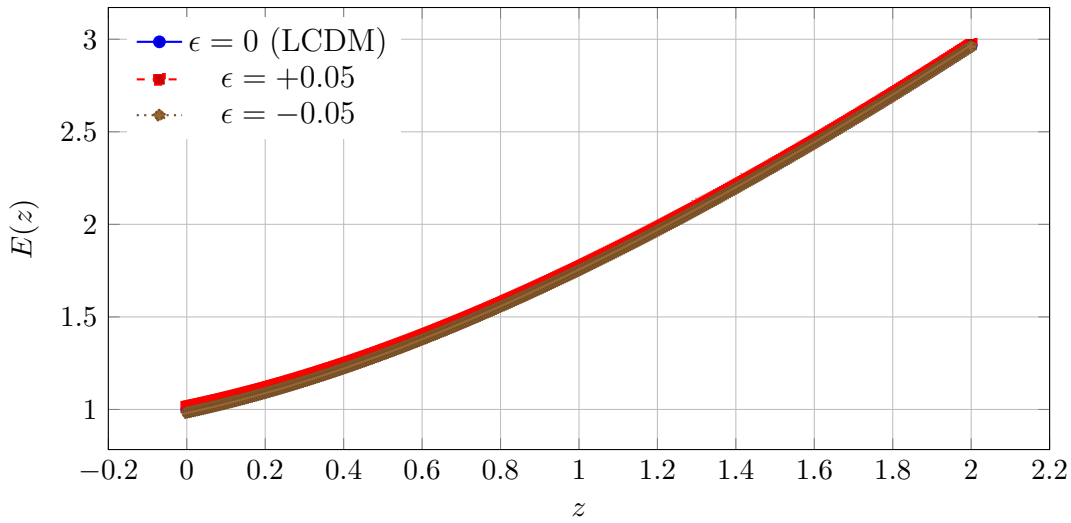

Figure 1: Illustrative expansion histories with a small UBT dark-energy deformation $\epsilon = \kappa_\Lambda \psi$. For $\epsilon \to 0$ we recover ΛCDM.

M.8 Relation to Psychon Sector and Local Tests

The same ψ that sources Λ_{eff} couples to local experiments (Appendix L/N). Constraints from cosmology (global $\bar{\psi}$) and laboratory (local ψ modulations) are complementary; combined fits determine $(\kappa_\Lambda, M_\psi, \alpha_\psi)$ or bound them.

M.9 Summary

UBT dark energy arises from a slow phase sector ψ that perturbs the vacuum energy density and hence the effective cosmological constant. The framework recovers ΛCDM when the phase sector is inactive and predicts small, testable deviations otherwise. This ties cosmic acceleration to the same ψ dynamics appearing in local UBT protocols.