

# Topological Origin of Mass Hierarchy in Unified Biquaternion Theory

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August 31, 2025

## Abstract

We propose a novel explanation for the mass hierarchy of elementary particles based on the topological modes of the unified biquaternionic field  $\Theta(q, \tau)$ . This framework generalizes the concept of Hopfions to higher winding numbers, offering a natural mechanism for the existence of three generations of leptons and their sharply differing rest masses. Each particle generation corresponds to a stable topological mode indexed by its Hopf charge  $n$ , and its mass is derived from a universal topological energy function  $S(n)$ .

## 1 Introduction

The Standard Model of particle physics classifies leptons into three generations—electron, muon, and tau—with increasing rest masses. However, it does not provide a fundamental explanation for these mass ratios. We hypothesize that these generations correspond to quantized topological excitations of the  $\Theta$  field, each with a distinct Hopf charge  $n$ .

## 2 Topological Energy Function $S(n)$

The topological energy function  $S(n)$  approximates the rest energy of each stable excitation:

$$S(n) = an^p + b,$$

where  $n \in \mathbb{Z}_+$  is the Hopf index, and  $a, p, b$  are constants fitted to experimental mass values.

### 2.1 Fitting to Lepton Masses

Let  $m_e$ ,  $m_\mu$ , and  $m_\tau$  be the rest masses of electron, muon, and tau, respectively. We assign:

$$S(1) = m_e, \quad S(2) = m_\mu, \quad S(3) = m_\tau.$$

Assuming  $p = \frac{3}{2}$  and  $b = 0$ , solve for  $a$ :

$$a = \frac{m_\mu}{2^{3/2}} = \frac{m_\tau}{3^{3/2}}.$$

Using experimental values:

$$\begin{aligned}m_e &= 0.511 \text{ MeV}, \\m_\mu &= 105.66 \text{ MeV}, \\m_\tau &= 1776.86 \text{ MeV},\end{aligned}$$

we get:

$$a_\mu = \frac{105.66}{2.828} \approx 37.37, \quad a_\tau = \frac{1776.86}{5.196} \approx 341.96.$$

This suggests that a single power law may not fit all three values unless we include a correction term or consider different scaling regimes.

### 3 Discussion

We propose that each particle generation corresponds to a distinct topological structure. The sharp increase in mass between generations suggests a nonlinear scaling in topological complexity or self-interaction energy.

Possible future refinements:

- Introduce log-corrections to  $S(n)$ ,
- Use exact Hopfion energy functionals,
- Include interaction with curvature or field tension.

### 4 Conclusion

The mass hierarchy problem may be geometrically and topologically encoded in the  $\Theta$  field structure. The hypothesis is testable via the relationship between topological energy scaling and observed mass ratios, offering a unifying explanation within UBT.