

Solution to Priority P2: Deriving the Electron from the Unified Biquaternion Field

Unified Biquaternion Theory Team

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Objective

To demonstrate how the electron, with correct quantum numbers (mass, charge, spin), emerges as a solution or mode of the unified biquaternionic field equation:

$$\square (q, \tau) + \mathcal{N}(\) = 0$$

1. Structure of the Unified Field

We define the total field:

$$(q, \tau) \in \mathbb{B}^{4 \times 4}$$

with components:

$$(q, \tau) = \psi_e(q, \tau) + \psi_g(q, \tau) + \dots$$

where ψ_e is the electron mode.

2. Ansatz for the Electron Mode

Let us define the electron excitation as:

$$\psi_e(q, \tau) = \psi(q) \otimes s$$

where $\psi(q)$ is a Dirac spinor and s is a fixed internal vector in \mathbb{B}^4 .

Assume time-dependence of the form:

$$\psi(q) = u(p)e^{-i\omega\tau}$$

This satisfies:

$$i\partial_\tau \psi = \omega \psi \quad \Rightarrow \quad m = \frac{\hbar\omega}{c^2}$$

3. Mass and Spin from the Unified Equation

The field ψ_e obeys a projected equation:

$$\square \psi_e + m^2 \psi_e = 0$$

and satisfies spin- $\frac{1}{2}$ algebra through commutators of its components:

$$[\psi_e^i, \psi_e^j] \sim i\epsilon^{ijk} \psi_e^k$$

implying intrinsic angular momentum (spin).

4. Charge Quantization

The coupling of ψ_e to the EM projection ψ_{em} yields:

$$j^\mu = \psi \gamma^\mu \psi$$

consistent with the standard QED current.

5. Geometric Embedding

The excitation ψ_e contributes to the stress-energy tensor:

$$T_{\mu\nu} = \frac{1}{2} \Re (\partial_\mu \psi_e^\dagger \partial_\nu \psi_e)$$

which sources the gravitational field in the Einstein equation.

Conclusion

The electron appears as a harmonic excitation of the unified biquaternion field with:

- Correct mass generation via internal time oscillation.
- Spin- $\frac{1}{2}$ behavior from algebraic structure.
- Electromagnetic coupling via projection.
- Gravitational interaction via stress-energy contribution.

This strongly supports the feasibility of UBT as a unification framework.