Solution to the Scalar Constraint Equation (Priority P1)

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1 Introduction

In the Unified Biquaternion Theory (UBT), the imaginary scalar part of the main field equation leads to a novel constraint:

$$\Im\left[\partial_\mu \Theta^\dagger \, \partial^\mu \Theta\right] = 0,$$

where $\Theta(q, \tau)$ is a biquaternion-valued field over complexified spacetime.

This constraint is not a wave equation but rather an algebraic or geometric condition that relates the amplitude and phase of the field.

2 Field Decomposition and Reformulation

Assume the field Θ can be decomposed into amplitude and phase components:

$$\Theta = \rho(q)\, e^{i\phi(q)},$$

where $\rho \in \mathbb{R}$ and $\phi \in \mathbb{R}$ (local phase).

We compute:

$$\partial_\mu \Theta = (\partial_\mu \rho + i\rho\partial_\mu \phi)e^{i\phi},$$
$$\partial^\mu \Theta^\dagger = (\partial^\mu \rho - i\rho\partial^\mu \phi)e^{-i\phi},$$

Then:

$$\begin{aligned}\partial^\mu \Theta^\dagger \, \partial_\mu \Theta &= (\partial^\mu \rho - i\rho\partial^\mu \phi)(\partial_\mu \rho + i\rho\partial_\mu \phi) \\ &= \partial^\mu \rho \partial_\mu \rho + \rho^2 \partial^\mu \phi \partial_\mu \phi + i\left(\rho\partial^\mu \rho \partial_\mu \phi - \rho\partial_\mu \rho \partial^\mu \phi\right)\end{aligned}$$

The imaginary part yields:

$$\Im\left[\partial^\mu \Theta^\dagger \, \partial_\mu \Theta\right] = 2\rho\, \eta^{\mu\nu}\, \partial_\mu \rho\, \partial_\nu \phi = 0$$

This gives the scalar constraint:

$$\eta^{\mu\nu}\, \partial_\mu \rho\, \partial_\nu \phi = 0$$

which requires orthogonality between gradients of amplitude and phase.

3 Example: Spherical Symmetry

Consider $\rho = \rho(r)$ and $\phi = \phi(t)$. Then:

$$\partial_\mu \rho = \frac{d\rho}{dr}\delta^r_\mu, \quad \partial_\nu \phi = \frac{d\phi}{dt}\delta^t_\nu$$

and the constraint becomes:

$$\eta^{\mu\nu}\partial_\mu \rho \partial_\nu \phi = \eta^{rt}\frac{d\rho}{dr}\frac{d\phi}{dt} = 0$$

so it's satisfied trivially, as $\eta^{rt} = 0$.

4 Interpretation

This constraint may act as a filter on allowed configurations of the field, excluding those where amplitude and phase gradients align. It may also relate to topological or informational conditions of spacetime geometry.

5 Outlook

Future work will explore:

- General solutions in FRW and Schwarzschild backgrounds
- Topological classification of field configurations satisfying the constraint
- Role in quantum corrections and effective action

Author's Note

This work was developed solely by Ing. David Jaroš. Large language models (ChatGPT-4o by OpenAI and Gemini 2.5 Pro by Google) were used strictly as assistive tools for calculations, LaTeX formatting, and critical review. All core ideas, equations, theoretical constructs and conclusions are the intellectual work of the author.