Noether to α v0.1

Draft for UBT Project

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1 Lagrangian and Normalization

We start with the unified field Θ and the electromagnetic field A_μ. The Lagrangian is written in natural units ($c = \hbar = 1$):

$$\mathcal{L} = \partial_\mu \Theta^\dagger \partial^\mu \Theta - m^2 \Theta^\dagger \Theta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{1}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Normalization is chosen so that the Noether current has standard dimension.

2 Global $U(1)$ Symmetry and Noether Current

Under global $U(1)$:

$$\Theta \to e^{i\lambda} \Theta, \tag{2}$$

the Noether current is

$$J^\mu = i \left(\Theta^\dagger \partial^\mu \Theta - (\partial^\mu \Theta^\dagger) \Theta \right). \tag{3}$$

The conserved charge is

$$Q = \int d^3x \, J^0. \tag{4}$$

We identify the fundamental excitation with $Q = \pm 1$.

3 Gauging the Symmetry

Promote λ to a local function $\lambda(x)$ by introducing a gauge field A_μ:

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu. \tag{5}$$

The interaction term becomes

$$\mathcal{L}_{\text{int}} = gJ^\mu A_\mu. \tag{6}$$

4 Integration over ψ-Cycle

In the complex-time/UBT framework, Θ also depends on the additional coordinate ψ. Integration over the compact ψ-cycle modifies the gauge kinetic term:

$$S_{\text{eff}} = \int d^4x \left(-\frac{Z}{4} F_{\mu\nu}F^{\mu\nu} + gJ^\mu A_\mu \right), \tag{7}$$

where the factor Z encodes the geometry of the ψ sector.

5 Fine Structure Constant

By canonical normalization of the photon field,

$$\alpha = \frac{g^2}{4\pi Z}. \tag{8}$$

Thus, the fine structure constant arises from Noether charge quantization and the geometric factor Z from the ψ sector.

6 Next Steps

- Explicitly compute Z from the ψ-cycle geometry (period, modular parameter τ).
- Solve for eigenmodes of Θ in the ψ sector to determine particle masses, in particular m_e.
- Ensure no free fit parameters remain; all quantities must follow from geometry and normalization.