Appendix J – Extended Metric for Rotating Spacetime and CTC (UBT)

J.1 Overview

In the Unified Biquaternion Theory (UBT) the complex time $\tau = t + i\psi$ introduces a slow, phase-sector deformation of rotating spacetimes. We revisit the Kerr (and Kerr–Newman) geometry and use $g_{\phi\phi} < 0$ as a practical criterion for azimuthal closed timelike curves (CTC). Small ψ-dependent perturbations shift the ergosurface without creating exterior CTCs when energy conditions are respected.

J.2 Kerr Baseline and ψ-Perturbation

For Kerr with mass M and spin a, the outer ergosurface satisfies

$$r_E(\theta) = M + \sqrt{M^2 - a^2 \cos^2 \theta}\,. \tag{1}$$

In UBT we model a small phase-sector deformation by

$$r_E(\theta;\psi) \approx r_E(\theta)\,\big(1 + \varepsilon\,\psi\big), \qquad |\varepsilon\psi| \ll 1. \tag{2}$$

Below we plot $r_E(\theta;\psi)$ for $\psi \in \{-1, 0, +1\}$ and compare with the event horizon $r_+ = M + \sqrt{M^2 - a^2}$.

Parameters used in the figure: $M = 1$, $a = 0.8$, $\varepsilon = 0.10$. Angles are in degrees.

J.3 Interpretation and Relevance to UBT

What the plot shows. The curves are the outer ergosurface as a function of polar angle θ. At the poles ($\theta = 0^\circ, 180^\circ$) the ergosurface meets the horizon; at the equator ($\theta = 90^\circ$) it bulges outward. A small UBT phase ψ *perturbs* this shape: $\psi > 0$ inflates, $\psi < 0$ deflates.

Why it matters. The ergoregion controls frame dragging and the availability of Penrose-like processes. In the CTC analysis, the location of the $g_{\phi\phi} = 0$ boundary is the practical diagnostic; UBT predicts controlled shifts of this boundary via the ψ-sector, while keeping exterior causality intact when energy conditions are satisfied.

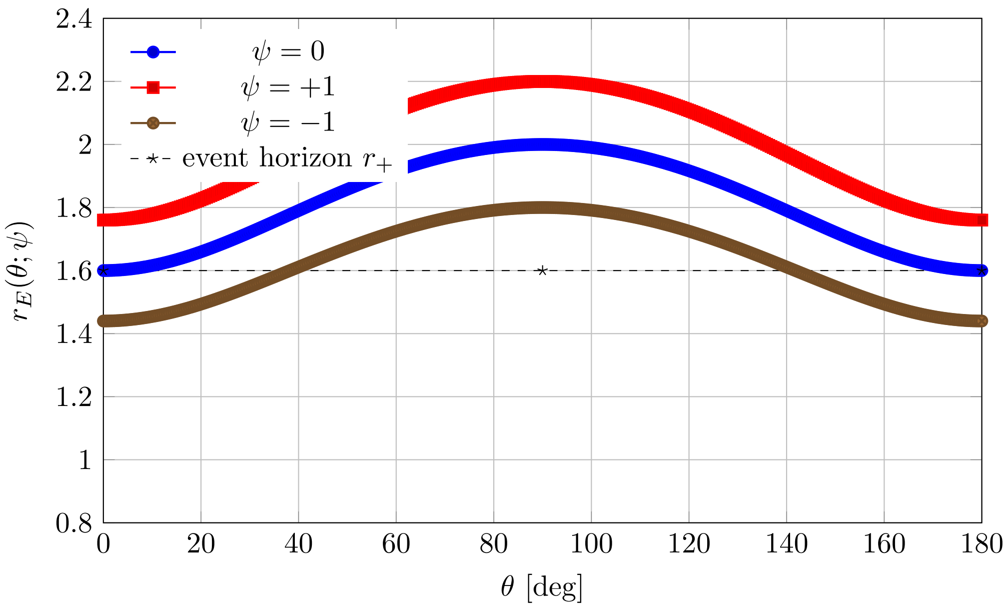

Figure 1: Ergosphere boundary $r_E(\theta;\psi)$ for a Kerr black hole with $M = 1$, $a = 0.8$ under a small UBT phase-sector deformation ($\varepsilon = 0.10$). Positive ψ expands, negative ψ contracts the ergosurface. The dashed line marks the event horizon r_+.