UBT Appendix 5: Vector Part of the Biquaternionic Field Equation

1. Introduction

In this appendix, we examine the **vector part** of the unified field equation derived from the Biquaternion Gravity framework:

$$\mathbf{R}_a^\mu - \frac{1}{2}e_a^\mu \mathbf{R} = 0 \tag{1}$$

where all quantities are biquaternions.

Our goal is to extract and interpret the **vector part** of this equation, in analogy with how we previously analyzed the scalar and imaginary scalar components.

2. Decomposition into Vector Components

Recall that any biquaternion $\mathbf{X}$ can be decomposed as:

$$\mathbf{X} = \text{Scal}(\mathbf{X}) + \vec{V}_R(\mathbf{X}) + i\,\vec{V}_I(\mathbf{X}) + i\,\text{PScal}(\mathbf{X})$$

We now apply the projection:

$$\text{Re}(\vec{V}(\mathbf{X})) = \text{Re}(\mathbf{X}) - \text{Scal}(\mathbf{X})$$

and similarly for the imaginary vector part.

Let us define:

$$\vec{V}_{R\,a}^\mu = \text{ReVectorPart}(\mathbf{R}_a^\mu - \tfrac{1}{2}e_a^\mu \mathbf{R}) \tag{2}$$

$$\vec{V}_{I\,a}^\mu = \text{ImVectorPart}(\mathbf{R}_a^\mu - \tfrac{1}{2}e_a^\mu \mathbf{R}) \tag{3}$$

Then the vector part equation is simply:

$$\vec{V}_{R\,a}^\mu + i\vec{V}_{I\,a}^\mu = 0 \tag{4}$$

which splits into two real vector equations:

$$\vec{V}_{R\,a}^\mu = 0 \tag{5}$$

$$\vec{V}_{I\,a}^\mu = 0 \tag{6}$$

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3. Interpretation and Hypotheses

3.1 Real Vector Equation

The real vector equation $\vec{V}^{\mu}_{R\,a} = 0$ may encode a constraint on torsion-free, metric-compatible geometries. It likely corresponds to Einstein-Cartan-like conditions or vectorial conservation laws.

3.2 Imaginary Vector Equation

The imaginary vector part $\vec{V}^{\mu}_{I\,a} = 0$ is especially intriguing. It may encode:

- A generalized Maxwell-type field equation,

- A hidden vector field coupled to spacetime geometry,

- A remnant of conformal or chiral gauge symmetry.

In the simplified case $e_I = 0$, this equation may reduce to a divergence-type condition on ω_I:

$$\nabla_{\mu}\omega_I^{\mu ab} + (\text{nonlinear terms}) = 0$$

resembling Yang-Mills field dynamics.

4. Future Directions

- Attempt explicit calculation of $\vec{V}^{\mu}_{I\,a}$ in terms of ω_R and ω_I.

- Test reductions in symmetric backgrounds (FLRW, Schwarzschild).

- Seek Lagrangian formulation that yields this vector equation as Euler-Lagrange equation.