Nontrivial Solutions of the Scalar Constraint in the Unified Biquaternion Theory

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1 Introduction

We consider the scalar constraint from Priority 1:

$$\eta^{\mu\nu}(\partial_\mu \rho)(\partial_\nu \phi) = 0$$

where $\rho = |\Theta|$ is the amplitude and ϕ the phase of the field. This condition geometrically enforces the orthogonality of gradients of amplitude and phase in spacetime.

2 Nontrivial Minkowski Solutions

In Minkowski spacetime, one trivial solution is when either ρ or ϕ is constant. However, we can construct richer solutions.

Let:

$$\rho = f(t - x), \quad \phi = g(t + x)$$

Then:

$$\partial_\mu \rho = f'(t - x)(\delta^0_\mu - \delta^1_\mu), \quad \partial_\nu \phi = g'(t + x)(\delta^0_\nu + \delta^1_\nu)$$

so:

$$\eta^{\mu\nu}\partial_\mu \rho\, \partial_\nu \phi = f'(t - x)g'(t + x)(\eta^{00} - \eta^{01} + \eta^{10} - \eta^{11}) = 0$$

since the mixed terms cancel.

Thus, such left/right-moving wave combinations satisfy the scalar constraint.

3 Axially Symmetric Configurations

We explore solutions of the form:

$$\rho = \rho(r), \quad \phi = \phi(t - r)$$

with $r = \sqrt{x^2 + y^2 + z^2}$. Then:

$$\partial_t \rho = 0, \quad \nabla \rho = \rho'(r)\frac{\vec{r}}{r}$$

$$\partial_t \phi = \phi'(t - r), \quad \nabla \phi = -\phi'(t - r)\frac{\vec{r}}{r}$$

The scalar constraint becomes:

$$\eta^{\mu\nu}\partial_\mu \rho\, \partial_\nu \phi = -\nabla \rho \cdot \nabla \phi = -\rho'(r)\phi'(t - r)$$

This vanishes iff $\rho'(r)\phi'(t - r) = 0$, leading to conditional satisfaction.

4 Implications in FRW Spacetimes

In cosmology, consider an FRW metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

We look at configurations:

$$\rho = \rho(r), \quad \phi = \phi(t)$$

Then:

$$\partial_\mu \rho = \delta^i_\mu \partial_i \rho, \quad \partial_\nu \phi = \delta^0_\nu \dot{\phi}$$

so:

$$g^{\mu\nu} \partial_\mu \rho \partial_\nu \phi = g^{0i} \dot{\phi} \partial_i \rho = 0$$

since $g^{0i} = 0$, this is satisfied. But with $\phi = \phi(t, r)$, nontrivial structures emerge.

5 Conclusions and Future Work

We have demonstrated a family of exact solutions beyond trivial cases. These exhibit spatial-temporal interference patterns and allow for localized dynamics in scalar field evolution. This paves the way for:

- Numerical simulations of scalar-phase interaction.
- Exploring implications near strong gravitational fields.
- Deriving effective potentials from interactions.

Author's Note

This work was developed solely by Ing. David Jaroš. Large language models (ChatGPT-4o by OpenAI and Gemini 2.5 Pro by Google) were used strictly as assistive tools for calculations, LaTeX formatting, and critical review. All core ideas, equations, theoretical constructs and conclusions are the intellectual work of the author.