Non-Trivial Solutions: Plane Wave Ansatz

Let:

$$\rho = \rho(\xi), \quad \phi = \phi(\xi), \quad \text{with } \xi = t - x$$

Then:

$$\partial_\mu \rho = \rho'(\xi)\partial_\mu \xi = \rho'(\xi)(1, -1, 0, 0)$$

$$\partial_\nu \phi = \phi'(\xi)(1, -1, 0, 0)$$

$$\eta^{\mu\nu}\partial_\mu \rho\, \partial_\nu \phi = \rho'(\xi)\phi'(\xi) \cdot (1^2 - (-1)^2) = 0$$

So the constraint is satisfied for arbitrary functions of $\xi = t - x$.