Appendix 20: Semiclassical Calculation of the Electron's Self-Energy

20.1 Overview

This appendix details the semiclassical derivation of the electron's electromagnetic self-energy (δm_e). This approach provides an intuitive physical picture and a first-order estimate for the correction required by our dual-mass hypothesis. The result demonstrates that the effective radius R of the charge distribution is not a free parameter but is determined by the underlying topological solution Θ_1.

20.2 Self-Energy of a Smeared Charge Distribution

We start from the classical expression for the electrostatic self-energy, which represents the energy stored in the electric field generated by the charge distribution:

$$\delta m_e c^2 = \frac{1}{2}\int \rho(\vec{x})\phi(\vec{x})\, d^3x \tag{1}$$

To make the problem analytically tractable, we assume the charge distribution $\rho(r)$ corresponding to the Θ_1 Hopfion can be approximated by a spherically symmetric Gaussian function:

$$\rho(r) = \frac{e}{\pi^{3/2}R^3}\exp\left(-\frac{r^2}{R^2}\right) \tag{2}$$

where R is the effective radius of the distribution.

20.3 Electrostatic Potential and Final Integration

The electrostatic potential $\phi(r)$ for this Gaussian source is found by solving the Poisson equation $\nabla^2\phi = -\rho/\epsilon_0$. The solution is:

$$\phi(r) = \frac{e}{4\pi\epsilon_0 r}\,\mathrm{erf}\left(\frac{r}{R}\right) \tag{3}$$

where erf is the error function. Substituting $\rho(r)$ and $\phi(r)$ back into the self-energy integral yields the result:

$$\delta m_e = \frac{e^2}{2\sqrt{\pi}(4\pi\epsilon_0)Rc^2} = \frac{\alpha\hbar}{\sqrt{\pi}Rc} \tag{4}$$

In natural units ($\hbar = c = 1$, $e^2 = 4\pi\alpha$), this simplifies to $\delta m_e = \frac{\sqrt{4\pi}\alpha}{R}$. *(Poznámka: Ve vašem souboru byl vzorec bez $4\pi\epsilon_0$, což odpovídá jinému systému jednotek. Zde jsem použil standardní SI-odvozenou formu, která je explicitnější.)*

20.4 The Nature of the Radius R

Crucially, the parameter R is not a free constant. As shown in the document 'electron$_{m}ass_{f}rom_{s}el f_{e}nerg$
For instance, by calculating the root-mean-square radius of the field's energy density distribution:

$$R^2 = \frac{\int r^2|\nabla\Theta_1|^2\, d^3x}{\int |\nabla\Theta_1|^2\, d^3x} \tag{5}$$

This calculation links the effective size R directly to the fundamental structure of the theory, thus completing the prediction of the electron mass from first principles.