Topological Origin of Mass Hierarchy in Unified Biquaternion Theory

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Abstract

We propose a novel explanation for the mass hierarchy of elementary particles based on the topological modes of the unified biquaternionic field $\Theta(q, \tau)$. This framework generalizes the concept of Hopfions to higher winding numbers, offering a natural mechanism for the existence of three generations of leptons and their sharply differing rest masses. Each particle generation corresponds to a stable topological mode indexed by its Hopf charge n, and its mass is derived from a universal topological energy function $S(n)$.

1 Introduction

The Standard Model of particle physics classifies leptons into three generations—electron, muon, and tau—with increasing rest masses. However, it does not provide a fundamental explanation for these mass ratios. We hypothesize that these generations correspond to quantized topological excitations of the Θ field, each with a distinct Hopf charge n.

2 Topological Energy Function $S(n)$

The topological energy function $S(n)$ approximates the rest energy of each stable excitation:

$$S(n) = an^p + b,$$

where $n \in \mathbb{Z}_+$ is the Hopf index, and a, p, b are constants fitted to experimental mass values.

2.1 Fitting to Lepton Masses

Let m_e, m_μ, and m_τ be the rest masses of electron, muon, and tau, respectively. We assign:

$$S(1) = m_e, \quad S(2) = m_\mu, \quad S(3) = m_\tau.$$

Assuming $p = \frac{3}{2}$ and $b = 0$, solve for a:

$$a = \frac{m_\mu}{2^{3/2}} = \frac{m_\tau}{3^{3/2}}.$$

Using experimental values:

$$m_e = 0.511 \text{ MeV},$$
$$m_\mu = 105.66 \text{ MeV},$$
$$m_\tau = 1776.86 \text{ MeV},$$

we get:

$$a_\mu = \frac{105.66}{2.828} \approx 37.37, \quad a_\tau = \frac{1776.86}{5.196} \approx 341.96.$$

This suggests that a single power law may not fit all three values unless we include a correction term or consider different scaling regimes.

3 Discussion

We propose that each particle generation corresponds to a distinct topological structure. The sharp increase in mass between generations suggests a nonlinear scaling in topological complexity or self-interaction energy.

Possible future refinements:

- Introduce log-corrections to $S(n)$,
- Use exact Hopfion energy functionals,
- Include interaction with curvature or field tension.

4 Conclusion

The mass hierarchy problem may be geometrically and topologically encoded in the Θ field structure. The hypothesis is testable via the relationship between topological energy scaling and observed mass ratios, offering a unifying explanation within UBT.