Scalar Constraint Equation in Biquaternionic Field

Starting from the decomposition of the field:

$$\Theta(q) = \rho(q)e^{i\phi(q)}$$

We derived the constraint:

$$\eta^{\mu\nu}\partial_\mu\rho\,\partial_\nu\phi = 0$$

This implies orthogonality between gradients of amplitude and phase in Minkowski space.

Example: Spherical Symmetry

Let:

$$\rho = \rho(r), \quad \phi = \phi(t)$$

Then:

$$\partial_\mu\rho\,\partial^\mu\phi = \left(\frac{d\rho}{dr}\right)^2 \cdot 0 + 0 \cdot \left(\frac{d\phi}{dt}\right)^2 = 0$$

So the constraint is trivially satisfied.