

# Construction of a Dark Mode Hopfion Solution in Unified Biquaternion Theory

UBT Project

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## Abstract

This document presents the analytical ansatz and geometric characteristics of a topologically nontrivial Hopfion solution  $\Theta_D$  within the Unified Biquaternion Theory (UBT), proposed as a candidate configuration for dark matter.

## 1 Ansatz

We define the Hopfion-like solution in stereographic coordinates for a map  $\Theta : \mathbb{R}^3 \rightarrow S^2 \subset \mathbb{C}^2$  via the rational map:

$$\Theta_D(x, y, z, t) = \frac{(2(x + iy))^p}{(2z + i(r^2 - 1))^q},$$

where  $r^2 = x^2 + y^2 + z^2$ , and  $p, q \in \mathbb{Z}^+$  define the topological charge.

## 2 Properties

- **Topological Charge:** The Hopf invariant  $H = pq$ .
- **Energy Density:** Localized in a toroidal region around the core, where  $|\Theta_D|$  varies rapidly.
- **Electromagnetic Neutrality:** Imposed via projection onto gauge-neutral components of  $\Theta$ .

### 3 Stress-Energy Tensor

The energy-momentum tensor  $T_{\mu\nu}$  is derived from the UBT Lagrangian:

$$T_{\mu\nu} = \text{Re} \left[ \partial_\mu \Theta^\dagger \cdot \partial_\nu \Theta - \frac{1}{2} \eta_{\mu\nu} (\partial^\alpha \Theta^\dagger \cdot \partial_\alpha \Theta) \right],$$

ensuring conserved gravitational energy.

### 4 Next Steps

To validate this configuration:

- Numerically simulate the stability of  $\Theta_D$ ,
- Compute the resulting gravitational potential from  $T_{\mu\nu}$ ,
- Fit predicted rotation curves to observed galactic data.