Non-Trivial Solutions: Plane Wave Ansatz

Let:

$$\rho = \rho(\xi), \quad \phi = \phi(\xi), \quad \text{with } \xi = t - x$$

Then:

$$\partial_\mu \rho = \rho'(\xi)\partial_\mu \xi = \rho'(\xi)(1, -1, 0, 0)$$

$$\partial_\nu \phi = \phi'(\xi)(1, -1, 0, 0)$$

$$\eta^{\mu\nu}\partial_\mu \rho\, \partial_\nu \phi = \rho'(\xi)\phi'(\xi) \cdot (1^2 - (-1)^2) = 0$$

So the constraint is satisfied for arbitrary functions of $\xi = t - x$.

Author's Note

This work was developed solely by Ing. David Jaroš. Large language models (ChatGPT-4o by OpenAI and Gemini 2.5 Pro by Google) were used strictly as assistive tools for calculations, LaTeX formatting, and critical review. All core ideas, equations, theoretical constructs and conclusions are the intellectual work of the author.