Noether $\to \alpha$ v0.5: Current Matching and What Fixes g_5

Draft for UBT Project

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1 Goal

We derived

$$\alpha(\mu_0) \; = \; \frac{g_5^2}{4\pi \, Z^\star}, \qquad Z^\star = \int_0^{L_\psi^\star} d\psi \, e^{B-2A} \, |\xi_0(\psi)|^2 = L_\psi^\star f^\star(\tau, \mathrm{BC}). \tag{1}$$

This section shows how the Noether charge and canonical normalization fix the *field scale*, and how g_5 is or is not fixed, depending on the UV principle chosen in UBT.

2 Zero-Mode Decomposition and Canonical Normalization

Expand the unified field in ψ-modes (for simplicity: complex scalar prototype for the charged excitation)

$$\Theta(x, \psi) = \chi_0(\psi) \, \phi(x) + \cdots , \qquad \int_0^{L_\psi} d\psi \; e^{B+2A} \, |\chi_0(\psi)|^2 = 1. \tag{2}$$

Similarly, the photon zero-mode (canonically normalized in 4D) is

$$A_\mu(x, \psi) = \xi_0(\psi) \, A_\mu^{(0)}(x) + \cdots , \qquad \int_0^{L_\psi} d\psi \; e^{B-2A} \, |\xi_0(\psi)|^2 = Z. \tag{3}$$

With these conventions, the 4D kinetic term of ϕ is canonical,

$$\mathcal{L}_{4\mathrm{D}} \supset |\partial_\mu \phi|^2 - m_0^2 |\phi|^2, \tag{4}$$

and the 4D Noether current takes the standard form

$$j^\mu = i\left(\phi^\dagger \partial^\mu \phi - (\partial^\mu \phi^\dagger)\phi\right). \tag{5}$$

3 Current Matching: g_4 from the 5D Coupling

The 5D interaction is $\int d^5x \sqrt{|g_5|}\, g_5\, J^\mu A_\mu$, which reduces to

$$S_{\mathrm{int}}^{(0)} = \int d^4x\, g_5 \Big[\int_0^{L_\psi} d\psi\, e^B\, |\chi_0|^2\, \xi_0\Big]\, j^\mu A_\mu^{(0)} = \int d^4x\, \frac{g_5}{\sqrt{Z}}\, j^\mu A_\mu^{(0)}. \tag{6}$$

Hence

$$g_4 \equiv \frac{g_5}{\sqrt{Z}}, \qquad \alpha = \frac{g_4^2}{4\pi}. \tag{7}$$

The result is robust (spinor/vector generalizations give the same overlap structure).

4 Noether Charge and Field Normalization

The 5D Noether charge

$$Q = \int d^3x \int_0^{L_\psi} d\psi\, \sqrt{|g_5|}\, J^0 \tag{8}$$

reduces, for the normalized zero-mode, to the 4D number operator. Choosing the fundamental excitation to have $Q = \pm 1$ *fixes the normalization of* ϕ and removes any free rescaling of Θ. This does *not* fix the numerical value of g_5 by itself.

5 What Can Fix g_5 (and Thus α) Ab-Initio?

Noether symmetry and compactification give $\alpha = g_5^2/(4\pi Z^\star)$ but leave g_5 as a *UV parameter*. To arrive at a number, UBT needs one of the following principles (each consistent with the framework):

(A) Bare Gauge Term From UBT Lagrangian + Extra Constraint

Assume a tree-level gauge kinetic term $-\frac{1}{4}F^2$. Then g_5 is a fundamental 5D coupling of mass dimension $-1/2$ and is not fixed by Noether alone. It can be related to other UBT scales if UBT supplies a constraint, e.g.

- tying g_5 to the Θ sector (coupling unification in the biquaternionic algebra),
- matching to gravity via a universal scale (Sakharov-like, but determined by the CCT/UBT cutoff),
- or quantization coming from a topological term (e.g. a 5D abelian Chern–Simons coefficient that is quantized and links to minimal charge units).

(B) *Induced* Photon Kinetic Term (Emergent Gauge Field)

Set the bare coefficient to zero and generate F^2 entirely from loops of Θ-sector fields. Compactification to 4D yields

$$\frac{1}{g_4^2(\mu)} = \sum_{j,\,\mathrm{KK}} \frac{b_j}{8\pi^2} \ln\frac{\Lambda}{m_{j,\mathrm{KK}}} + \cdots , \tag{9}$$

with b_j the standard 4D beta-function weights for each KK mode and Λ the physical UV scale of UBT (finite in CCT/UBT due to the compact ψ and possible warping). In this scenario,

$$\alpha(\mu_0) \text{ is fixed by field content, BC, and } (L_\psi^\star, \theta_H^\star, A, B) \quad (\text{no } g_5 \text{ input}). \tag{10}$$

This produces a numerical α once the spectrum is specified; the calculation is technically involved but conceptually parameter-free.

(C) Holonomy Quantization With Dynamical A_ψ

If the vacuum fixes a nontrivial $\langle A_\psi \rangle$ through the Hosotani mechanism and UBT relates $\langle A_\psi \rangle$ to a geometric invariant, then

$$\theta_H^\star = g_5 \int_0^{L_\psi^\star} A_\psi \, d\psi = 2\pi n^\star \tag{11}$$

imposes a relation among g_5 and $L_\psi^\star$ that *can* remove the g_5 freedom. Whether this happens depends on the detailed UBT background equations; in many simple models $\theta_H^\star$ is 0 or π independently of g_5, leaving g_5 unfixed.

6 Recommendation

For a **fully ab-initio** numeric value of α within the present UBT framework, the cleanest route is (B): treat the photon as emergent and compute the induced $1/g_4^2$ from KK towers of the Θ-sector at the vacuum point $(L_\psi^\star, \theta_H^\star, A, B)$. This uses only symmetries, geometry and field content, with no free gauge coupling left.

7 Next Steps (Minimal Viable Calculation)

1. Choose a concrete Θ-sector (spins, charges q_j, masses m_j) and BC.
2. Determine $(L_\psi^\star, \theta_H^\star)$ by minimizing the one-loop V_{eff} (Sec. v0.4).
3. Compute the KK spectrum $m_{j,\text{KK}}(n)$ on that background.
4. Evaluate the induced gauge kinetic term in 4D, e.g. summing the standard one-loop polarizations over KK levels with a regulator consistent with UBT (compact ψ and warp imply a physical cutoff).
5. Read off $\alpha(\mu_0)$, then run to M_Z for comparison.

Consistency with m_e. The same background must yield the lightest charged eigenmode mass m_e; matching both α and m_e with a *single* background is the decisive non-numerological test.