# Appendix M – Dark Energy in the Unified Biquaternion Theory (UBT)

## M.1 Motivation and Scope

This appendix consolidates the UBT description of *dark energy* based on the complex-time framework $\tau = t + i\psi$ and the biquaternionic master field $\Theta(q, \tau)$. We derive an *effective cosmological sector* sourced by the slow phase $\psi$ and show how $\Lambda$CDM is recovered for $\psi \to 0$. Links to: Appendix F (psychons & $\psi$-sector dynamics), Appendix J (metric deformations), Appendix K (field propagation in curved backgrounds).

## M.2 UBT Action and Emergent Vacuum Sector

Consider the effective gravitational action (signature $-, +, +, +$)

$$S_{\text{UBT}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2\Lambda_0 \right) + S_\Theta[\Theta, g, \psi] + S_\psi[\psi, g] \,, \tag{1}$$

where $\Theta$ carries internal (biquaternionic/spinor) structure and the slow phase $\psi$ is the imaginary part of the complex time $\tau = t + i\psi$. At long wavelengths, integrating out fast $\Theta$-modes yields an *effective vacuum energy density*

$$\rho_{\text{vac}}^{(\text{UBT})}(\psi) = \rho_{\Lambda 0} \left( 1 + \kappa_\Lambda \psi \right) + \frac{1}{2} M_\psi^2 \psi^2 + \frac{\alpha_\psi}{2} (\nabla \psi)^2 + \cdots \,, \tag{2}$$

so that the *effective* cosmological term becomes

$$\Lambda_{\text{eff}}(\psi) = \frac{8\pi G}{c^4} \rho_{\text{vac}}^{(\text{UBT})}(\psi) \,. \tag{3}$$

The coefficients $(\kappa_\Lambda, M_\psi^2, \alpha_\psi)$ are UBT couplings; $\kappa_\Lambda \to 0$ restores $\Lambda$CDM with $\Lambda_{\text{eff}} = \Lambda_0$.

## M.3 Homogeneous and Isotropic Cosmology

For a spatially flat FLRW metric,

$$ds^2 = -c^2 dt^2 + a(t)^2 \, d\mathbf{x}^2, \qquad H \equiv \dot{a}/a, \tag{4}$$

the Friedmann equations with UBT dark-energy sector read

$$H^2 = \frac{8\pi G}{3} \left( \rho_m + \rho_r + \rho_{\text{vac}}^{(\text{UBT})}(\psi) \right), \tag{5}$$

$$\dot{H} = -4\pi G \left( \rho_m + \frac{4}{3}\rho_r + \rho_{\text{vac}}^{(\text{UBT})}(\psi) + p_{\text{vac}}^{(\text{UBT})}(\psi) \right) / c^2, \tag{6}$$

with effective equation of state

$$w_{\text{UBT}}(\psi) \equiv \frac{p_{\text{vac}}^{(\text{UBT})}}{\rho_{\text{vac}}^{(\text{UBT})}} \approx -1 + \frac{\alpha_\psi (\nabla\psi)^2 - M_\psi^2 \psi^2}{2\,\rho_{\Lambda 0}} + \mathcal{O}(\psi^2). \tag{7}$$

For a homogeneous slow phase ($\nabla\psi = 0$) we obtain $w_{\text{UBT}} \gtrsim -1$ for $M_\psi^2 \psi^2 \ll \rho_{\Lambda 0}$; phantom-like $w_{\text{UBT}} < -1$ requires parity-odd or higher-derivative mixings (cf. Appendix F).

## M.4 Linear Perturbations (Sketch)

Writing $\psi = \bar{\psi}(t) + \delta\psi(t, \mathbf{x})$, the scalar sector gains an extra gauge-invariant mode coupled to metric potentials $\Phi, \Psi$. At sub-horizon scales the effective dark-energy sound speed is

$$c_{s,\text{UBT}}^2 \simeq \frac{\alpha_\psi}{\alpha_\psi + M_\psi^2/k^2} \in (0, 1], \tag{8}$$

limiting clustering of the vacuum sector; $\alpha_\psi \to 0$ recovers an unclustered $\Lambda$.

## M.5 Recovery of $\Lambda$CDM

Setting $(\kappa_\Lambda, \alpha_\psi, M_\psi) \to 0$ freezes $\psi$ and yields $\rho_{\text{vac}}^{(\text{UBT})} \to \rho_{\Lambda 0}$ with constant $w = -1$ and standard distances, growth, and CMB background. This ensures compatibility with precision cosmology when the UBT phase sector is inactive.

## M.6 Illustrative Hubble Curves (No External Files)

Below we plot $E(z) \equiv H(z)/H_0$ for three small UBT deformations parameterized by $\kappa_\Lambda \psi \equiv \epsilon \in \{-0.05, 0, +0.05\}$, keeping $\Omega_{m0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$ at $z = 0$.

## M.7 Observable Consequences (Qualitative)

- Slight shifts in distance–redshift relations $D_L(z), D_A(z)$ and in the derived $H_0$ if $\epsilon \neq 0$ today.

- Modified ISW effect and low-$\ell$ CMB for time-varying $\bar{\psi}(t)$.

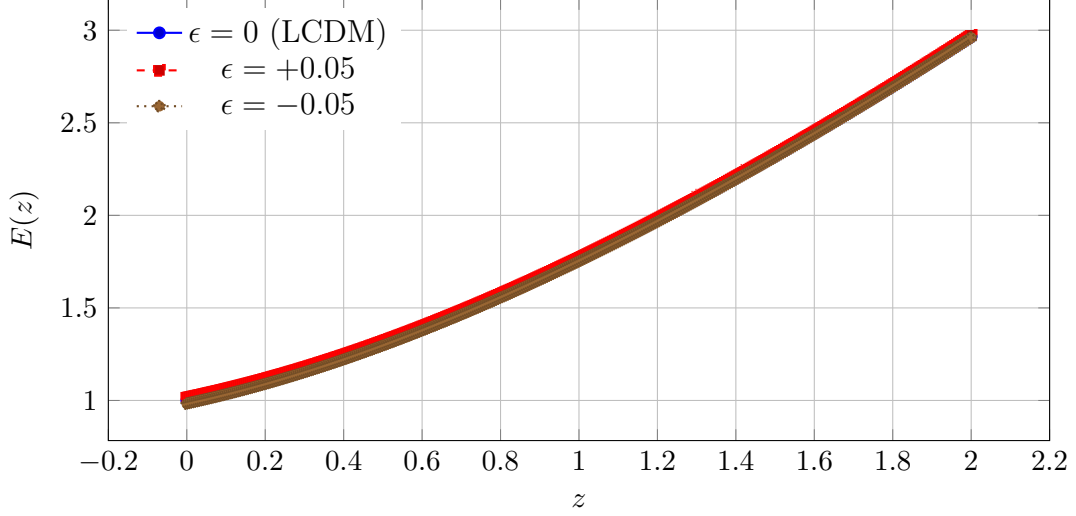- Growth rate changes $f(z)\sigma_8$ suppressed by $c_{s,\text{UBT}}^2 \lesssim 1$; $\epsilon \to 0$ reproduces $\Lambda$CDM.

Figure 1: Illustrative expansion histories with a small UBT dark-energy deformation $\epsilon = \kappa_\Lambda \psi$. For $\epsilon \to 0$ we recover $\Lambda$CDM.

# M.8 Relation to Psychon Sector and Local Tests

The same $\psi$ that sources $\Lambda_{\text{eff}}$ couples to local experiments (Appendix L/N). Constraints from cosmology (global $\bar{\psi}$) and laboratory (local $\psi$ modulations) are complementary; combined fits determine $(\kappa_\Lambda, M_\psi, \alpha_\psi)$ or bound them.

# M.9 Summary

UBT dark energy arises from a slow phase sector $\psi$ that perturbs the vacuum energy density and hence the effective cosmological constant. The framework recovers $\Lambda$CDM when the phase sector is inactive and predicts small, testable deviations otherwise. This ties cosmic acceleration to the same $\psi$ dynamics appearing in local UBT protocols.