

Noether to α v0.2

Draft for UBT Project

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1 Lagrangian in 5D

We start with the unified field $\Theta(x, \psi)$ and the gauge field $A_M(x, \psi)$ on $M^4 \times S^1_\psi$. The action in natural units ($c = \hbar = 1$) is

$$S = \int d^4x \int_0^{L_\psi} d\psi \left[(D_M \Theta)^\dagger (D^M \Theta) - m^2 \Theta^\dagger \Theta - \frac{1}{4} F_{MN} F^{MN} \right], \quad (1)$$

with $D_M = \partial_M + ig_5 A_M$.

2 Noether Current

Under global $U(1) : \Theta \rightarrow e^{i\lambda} \Theta$, the Noether current is

$$J^M = i(\Theta^\dagger \partial^M \Theta - (\partial^M \Theta^\dagger) \Theta). \quad (2)$$

The charge is

$$Q = \int d^3x d\psi J^0, \quad (3)$$

and we normalize such that the fundamental excitation has $Q = \pm 1$.

3 Dimensional Reduction

Assuming $A_\mu(x)$ is independent of ψ , the gauge term reduces to

$$S_{\text{gauge}} = -\frac{L_\psi}{4} \int d^4x F_{\mu\nu} F^{\mu\nu}. \quad (4)$$

Canonical normalization requires rescaling $A_\mu \rightarrow A_\mu/\sqrt{L_\psi}$, yielding an effective coupling

$$g_4 = \frac{g_5}{\sqrt{L_\psi}}. \quad (5)$$

4 Wilson Loop Quantization

On the compact ψ -cycle, gauge invariance implies quantization of the Wilson loop:

$$\exp\left(ig_5 \oint A_\psi d\psi\right) = e^{2\pi i n}, \quad n \in \mathbb{Z}. \quad (6)$$

This condition links g_5 , L_ψ , and the background $\langle A_\psi \rangle$. Thus, the geometry of the ψ -sector fixes the effective gauge coupling.

5 Generalized Factor Z

In full UBT, the integration over ψ gives not only L_ψ but also a correction factor $f(\tau, \text{BC})$, depending on the modular parameter τ of complex time and boundary conditions:

$$Z = L_\psi \cdot f(\tau, \text{BC}). \quad (7)$$

The effective 4D action is then

$$S_{\text{eff}} = \int d^4x \left(-\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + g_4 J^\mu A_\mu \right). \quad (8)$$

6 Fine Structure Constant

After canonical normalization, the fine structure constant is

$$\alpha = \frac{g_4^2}{4\pi} = \frac{g_5^2}{4\pi Z}. \quad (9)$$

7 Interpretation

- The relation $\alpha = g_5^2/(4\pi Z)$ follows rigorously from Noether symmetry and dimensional reduction.

- To obtain the numerical value, UBT must determine g_5 and Z from first principles:
 - g_5 fixed by Noether charge quantization ($Q = \pm 1$).
 - Z fixed by ψ -geometry: $Z = L_\psi f(\tau, \text{BC})$.
- Once these are determined ab-initio, α is no longer a free parameter but a derived constant.