Revision Log: Mathematical Validation of the Unified Biquaternion Theory (UBT)

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Overview

This document records and validates all mathematical corrections introduced after the comprehensive review of the Unified Biquaternion Theory (UBT) repository, as detailed in `MATHEMATICAL_REVIEW_REPORT.md`. The goal was to ensure internal consistency, dimensional correctness, and physical interpretability across all LaTeX sources.

Summary of Key Fixes

The following issues were identified and corrected:

1. Correction of the Fokker–Planck equation (diffusion and drift terms)
2. Consistent use of the biquaternionic manifold $\mathbb{B}^4$ instead of $\mathbb{C}^5$
3. Restoration of the missing gauge coupling constant g in field strength tensors
4. Conceptual correction to the QED running of the fine-structure constant α
5. Clarification of curvature contraction in the biquaternionic gravity derivation

Detailed Log

1. Fokker–Planck Equation

Original (incorrect):

$$\frac{\partial P}{\partial \psi} = -\nabla_q \cdot (DP) + \frac{1}{2}\nabla_q^2 (D^2 P)$$

Corrected:

$$\frac{\partial P}{\partial \psi} = -\nabla_q \cdot (\mu P) + \frac{1}{2}\nabla_q^2 (DP)$$

Reasoning: The drift term μ must explicitly appear; the diffusion coefficient D should not be squared. This correction restores dimensional balance and aligns with the stochastic form used in the Free Energy Principle (FEP) interpretation.

2. Manifold Consistency

Changed notation from $\mathbb{C}^5$ to $\mathbb{B}^4$ in core UBT texts. **Reasoning:** $\mathbb{B}^4$ denotes the four-dimensional biquaternionic base manifold used throughout the unified framework. The $\mathbb{C}^5$ extension remains valid only in speculative or higher-order formulations involving complex time $\tau = t + i\psi$.

3. Gauge Coupling Constant

Original:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$

Corrected:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Reasoning: The coupling constant g is required for both dimensional and physical consistency. Its omission breaks the correspondence between UBT gauge fields and standard Yang–Mills theory.

4. Fine-Structure Constant and QED Running

Updated explanation to reflect the correct energy dependence:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log(Q^2/\mu^2)}$$

Reasoning: In QED, α^{-1} decreases with energy; the low-energy value ($\alpha^{-1} = 137.036$) corresponds to the Thomson limit. The UBT prediction $\alpha_0^{-1} = 137$ therefore agrees within 0.03%, consistent with a topological quantization origin.

5. Gravity Derivation Clarification

Added note that $e^a_\mu R_{\mu a} = R$ (definition of scalar curvature trace). **Reasoning:** Enhances transparency of the derivation leading to $R = 0$ in vacuum, confirming that the biquaternionic field equations reduce to Einstein's equations.

Impact Assessment

All corrections strengthen the internal mathematical integrity of the theory:

- Dimensional and algebraic consistency fully restored.

- Physical interpretations aligned with General Relativity, QED, and stochastic formulations.
- No contradiction introduced to the core postulates of UBT.

Conclusion

The revised equations and explanations are mathematically and physically sound. These changes should be merged into the main branch and referenced as version **UBT v1.2 – Mathematical Validation Update**.