Construction of a Dark Mode Hopfion Solution in Unified Biquaternion Theory

UBT Project

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Abstract

This document presents the analytical ansatz and geometric characteristics of a topologically nontrivial Hopfion solution Θ_D within the Unified Biquaternion Theory (UBT), proposed as a candidate configuration for dark matter.

1 Ansatz

We define the Hopfion-like solution in stereographic coordinates for a map $\Theta : \mathbb{R}^3 \to S^2 \subset \mathbb{C}^2$ via the rational map:

$$\Theta_D(x, y, z, t) = \frac{(2(x + iy))^p}{(2z + i(r^2 - 1))^q},$$

where $r^2 = x^2 + y^2 + z^2$, and $p, q \in \mathbb{Z}^+$ define the topological charge.

2 Properties

- **Topological Charge:** The Hopf invariant $H = pq$.
- **Energy Density:** Localized in a toroidal region around the core, where $|\Theta_D|$ varies rapidly.
- **Electromagnetic Neutrality:** Imposed via projection onto gauge-neutral components of Θ.

3 Stress-Energy Tensor

The energy-momentum tensor $T_{\mu\nu}$ is derived from the UBT Lagrangian:

$$T_{\mu\nu} = \mathrm{Re}\left[\partial_\mu \Theta^\dagger \cdot \partial_\nu \Theta - \frac{1}{2}\eta_{\mu\nu}\left(\partial^\alpha \Theta^\dagger \cdot \partial_\alpha \Theta\right)\right],$$

ensuring conserved gravitational energy.

4 Next Steps

To validate this configuration:

- Numerically simulate the stability of Θ_D,
- Compute the resulting gravitational potential from $T_{\mu\nu}$,
- Fit predicted rotation curves to observed galactic data.