Noether to α v0.2

Draft for UBT Project

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1 Lagrangian in 5D

We start with the unified field $\Theta(x, \psi)$ and the gauge field $A_M(x, \psi)$ on $M^4 \times S^1_\psi$. The action in natural units ($c = \hbar = 1$) is

$$S = \int d^4x \int_0^{L_\psi} d\psi \, \Big[(D_M\Theta)^\dagger(D^M\Theta) - m^2\Theta^\dagger\Theta - \tfrac{1}{4}F_{MN}F^{MN}\Big], \tag{1}$$

with $D_M = \partial_M + ig_5 A_M$.

2 Noether Current

Under global $U(1) : \Theta \to e^{i\lambda}\Theta$, the Noether current is

$$J^M = i\big(\Theta^\dagger\partial^M\Theta - (\partial^M\Theta^\dagger)\Theta\big). \tag{2}$$

The charge is

$$Q = \int d^3x \, d\psi \, J^0, \tag{3}$$

and we normalize such that the fundamental excitation has $Q = \pm 1$.

3 Dimensional Reduction

Assuming $A_\mu(x)$ is independent of ψ, the gauge term reduces to

$$S_{\text{gauge}} = -\frac{L_\psi}{4} \int d^4x \, F_{\mu\nu}F^{\mu\nu}. \tag{4}$$

Canonical normalization requires rescaling $A_\mu \to A_\mu/\sqrt{L_\psi}$, yielding an effective coupling

$$g_4 = \frac{g_5}{\sqrt{L_\psi}}. \tag{5}$$

4 Wilson Loop Quantization

On the compact ψ-cycle, gauge invariance implies quantization of the Wilson loop:

$$\exp\Big(ig_5 \oint A_\psi \, d\psi\Big) = e^{2\pi i n}, \qquad n \in \mathbb{Z}. \tag{6}$$

This condition links g_5, L_ψ, and the background $\langle A_\psi \rangle$. Thus, the geometry of the ψ-sector fixes the effective gauge coupling.

5 Generalized Factor Z

In full UBT, the integration over ψ gives not only L_ψ but also a correction factor $f(\tau, \mathrm{BC})$, depending on the modular parameter τ of complex time and boundary conditions:

$$Z = L_\psi \cdot f(\tau, \mathrm{BC}). \tag{7}$$

The effective 4D action is then

$$S_{\text{eff}} = \int d^4x \left(-\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + g_4 J^\mu A_\mu \right). \tag{8}$$

6 Fine Structure Constant

After canonical normalization, the fine structure constant is

$$\alpha = \frac{g_4^2}{4\pi} = \frac{g_5^2}{4\pi Z}. \tag{9}$$

7 Interpretation

- The relation $\alpha = g_5^2/(4\pi Z)$ follows rigorously from Noether symmetry and dimensional reduction.

- To obtain the numerical value, UBT must determine g_5 and Z from first principles:
  - g_5 fixed by Noether charge quantization ($Q = \pm 1$).
  - Z fixed by ψ-geometry: $Z = L_\psi f(\tau, \mathrm{BC})$.
- Once these are determined ab-initio, α is no longer a free parameter but a derived constant.