

# Noether to $\alpha$ v0.1

Draft for UBT Project

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## 1 Lagrangian and Normalization

We start with the unified field  $\Theta$  and the electromagnetic field  $A_\mu$ . The Lagrangian is written in natural units ( $c = \hbar = 1$ ):

$$\mathcal{L} = \partial_\mu \Theta^\dagger \partial^\mu \Theta - m^2 \Theta^\dagger \Theta - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (1)$$

Here  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . Normalization is chosen so that the Noether current has standard dimension.

## 2 Global $U(1)$ Symmetry and Noether Current

Under global  $U(1)$ :

$$\Theta \rightarrow e^{i\lambda} \Theta, \quad (2)$$

the Noether current is

$$J^\mu = i (\Theta^\dagger \partial^\mu \Theta - (\partial^\mu \Theta^\dagger) \Theta). \quad (3)$$

The conserved charge is

$$Q = \int d^3x J^0. \quad (4)$$

We identify the fundamental excitation with  $Q = \pm 1$ .

### 3 Gauging the Symmetry

Promote  $\lambda$  to a local function  $\lambda(x)$  by introducing a gauge field  $A_\mu$ :

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + igA_\mu. \quad (5)$$

The interaction term becomes

$$\mathcal{L}_{\text{int}} = gJ^\mu A_\mu. \quad (6)$$

### 4 Integration over $\psi$ -Cycle

In the complex-time/UBT framework,  $\Theta$  also depends on the additional coordinate  $\psi$ . Integration over the compact  $\psi$ -cycle modifies the gauge kinetic term:

$$S_{\text{eff}} = \int d^4x \left( -\frac{Z}{4} F_{\mu\nu} F^{\mu\nu} + gJ^\mu A_\mu \right), \quad (7)$$

where the factor  $Z$  encodes the geometry of the  $\psi$  sector.

### 5 Fine Structure Constant

By canonical normalization of the photon field,

$$\alpha = \frac{g^2}{4\pi Z}. \quad (8)$$

Thus, the fine structure constant arises from Noether charge quantization and the geometric factor  $Z$  from the  $\psi$  sector.

### 6 Next Steps

- Explicitly compute  $Z$  from the  $\psi$ -cycle geometry (period, modular parameter  $\tau$ ).
- Solve for eigenmodes of  $\Theta$  in the  $\psi$  sector to determine particle masses, in particular  $m_e$ .
- Ensure no free fit parameters remain; all quantities must follow from geometry and normalization.