# Analytical Derivation of Electron Mass from Electromagnetic Self-Energy

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October 30, 2025

## Overview

In this document, we analytically derive the electron mass from its electromagnetic self-energy, based on the hypothesis that the electron is a topological excitation of the $\Theta_1$ field.

## Assumptions and Ansatz

We assume that the charge distribution of the electron is spherically symmetric and approximated by a Gaussian:

$$\rho(r) = \frac{e}{\pi^{3/2} R^3} \exp\left(-\frac{r^2}{R^2}\right)$$

This allows analytical treatment and captures the finite localization scale of the electron.

## Electrostatic Potential

The electrostatic potential $\phi(r)$ is given by solving Poisson's equation:

$$\phi(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{|\vec{r} - \vec{r}'|} \, d^3r'$$

For the Gaussian source, this results in:

$$\phi(r) = \frac{e}{4\pi\epsilon_0 r} \operatorname{erf}\left(\frac{r}{R}\right)$$

## Self-Energy Integral

The total electromagnetic self-energy is:

$$\delta m_e c^2 = \frac{1}{2} \int \rho(r)\phi(r) \, d^3r$$

Evaluating the integral yields:

$$\delta m_e = \frac{e^2}{\sqrt{\pi}\epsilon_0 R c^2}$$

## Interpretation

This result links the electron mass to the scale $R$ of its internal structure, with no new parameters introduced. The remaining task is to derive $R$ from the stress-energy distribution of the $\Theta_1$ Hopfion solution.