JUMLAH DAN SELISIH DUA SUDUT

- **Sinus**

$\sin(A+B) = sinA.cosB + cosA.sinB$

$\sin(A-B) = sinA.cosB - cosA.sinB$

- **Cosinus**

$\cos(A+B) = cosA.cosB - sinA.sinB$

$\cos(A-B) = cosA.cosB + sinA.sinB$

- **Tangen**

$$\tan(A+B) = \frac{tanA + tanB}{1 - tanA.tanB}$$

$$\tan(A-B) = \frac{tanA - tanB}{1 + tanA.tanB}$$

Contoh :

1. $\cos 75° = \sin(45 + 30)$

$= cos45°cos30° - sin45°sin30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

2. $sin15° = \sin(60° - 45°)$

$= cos60°cos45° + sin60°sin45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

3. Pada suatu segitiga ABC yang siku-siku di C, diketahui bahwa $\sin A.\sin B = \frac{2}{5}$ dan $\sin(A - B) = 5a$. Nilai a adalah…

$A + B = 90°$

$\cos(A + B) = \cos 90°$

$$cosAcosB - sinAsinB = 0$$

$$cosAcosB - \frac{2}{5} = 0$$

$$cosAcosB = \frac{2}{5}$$

$$\cos(A - B) = \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

$$\sin(A - B) = \frac{3}{5}$$

$$\frac{3}{5} = 5a$$

$$a = \frac{3}{25}$$

RUMUS SUDUT GANDA TRIGONOMETRI

- $sin2a$

  $sin2a = \sin(a + a)$

  $= sina \cdot cosa + cosa \cdot sina$

  $= 2sina \cdot cosa$

- $cos2a$

  $cos2a = \cos(a + a)$

  $= cosa \cdot cosa - sina \cdot sina$

  $= \cos^2 a - sin^2 a$

  $= 1 - 2sin^2 a$

  $= 2cos^2 a - 1$

- $tan2a$

$tan2a = \tan(a + a)$

$$= \frac{tana + tana}{1 - tana \cdot tana}$$

$$= \frac{2tana}{1 - tan^2 a}$$

Contoh:

1. Tentukanlah nilai $sin2a$, $cos2a$ dan $tan2a$ jika $\tan a = -\frac{1}{3}$, $(90° < a < 180°)$

Karena $(90° < a < 180°)$, $sina = \frac{1}{\sqrt{10}}$ dan $cosa = -\frac{3}{\sqrt{10}}$

$$sin2a = 2sina \cdot cosa = 2 \cdot \frac{1}{\sqrt{10}} \cdot \left(-\frac{3}{\sqrt{10}}\right) = -\frac{3}{5}$$

$$\cos^2 a = \cos^2 a - \sin^2 a = \frac{9}{10} - \frac{1}{10} = \frac{4}{5}$$

$$tan2a = \frac{2\tan a}{1 - \tan^2 a} = \frac{-\frac{2}{3}}{1 - \frac{1}{9}} = -\frac{3}{4}$$

2. Tentukanlah nilai $sin2a$ dan $cos2a$ jika $\tan a = \frac{4}{5}$, $(0° < a < 90°)$

Karena $(0° < a < 90°)$, $sina = \frac{4}{\sqrt{41}}$ dan $cosa = \frac{5}{\sqrt{41}}$

$$sin2a = 2sina \cdot cosa = 2 \cdot \frac{4}{\sqrt{41}} \cdot \frac{5}{\sqrt{41}} = \frac{40}{41}$$

$$cos2a = cos^2 a - \sin^2 a = \frac{25}{41} - \frac{16}{41} = \frac{9}{41}$$

RUMUS TRIGONOMETRI PERTENGAHAN SUDUT

- $\sin^2 \frac{a}{2}$

$\cos a = \cos\left(2 \cdot \frac{a}{2}\right) = 1 - 2sin^2 \frac{a}{2}$

$\sin^2 \frac{a}{2} = \frac{1 - \cos a}{2}$

- $\cos^2 \frac{a}{2}$

$\cos a = \cos\left(2 \cdot \frac{a}{2}\right) = 2\cos^2 \frac{a}{2} - 1$

$\cos^2 \frac{a}{2} = \frac{1 + \cos a}{2}$

- $\tan^2 \frac{a}{2}$

$= \frac{\sin^2 \frac{a}{2}}{\cos^2 \frac{a}{2}}$

$= \frac{\frac{1 - \cos^2 a}{2}}{\frac{1 + \cos a}{2}}$

$= \frac{1 - \cos a}{1 + \cos a}$

Contoh:

1. $\tan 22{,}5°$

$= \frac{1 - \cos 45°}{1 + \cos 45°} = \frac{2 - \sqrt{2}}{2 + \sqrt{2}} = 3 - 2\sqrt{2}$

Karena $tan22{,}5° > 0$

$tan22{,}5° = \sqrt{3 - 2\sqrt{2}} = \sqrt{2} - 1$

2. $\cos 22{,}5°$

Karena $\cos 22{,}5° > 0$

$$\cos 22{,}5° = \sqrt{\frac{1+\cos 45°}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

3. $\sin 11{,}25°$

Karena $sin 11{,}25 > 0$

$$sin 11{,}25° = \sqrt{\frac{1-\cos 22{,}5\,°}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{2+\sqrt{2}}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{2}$$