# entanglement\_relativistic\_frames

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Quantum Entanglement Across Relativistic Reference Frames Introduction

Quantum mechanics and relativity are two pillars of modern physics that ordinarily operate in very different regimes. Ouantum entanglement - the nonclassical correlation between particles - has been extensively studied in non-relativistic settings, while relativistic effects like time dilation and gravitational redshift are usually considered in classical contexts. At the intersection of these domains lies a rich field of inquiry: how do entangled quantum systems behave when observed from different relativistic reference frames? This question is not merely academic; it probes the consistency of physics across quantum and relativistic theories and is essential for future quantum technologies that operate on Earth-to-satellite scales. By examining how relativistic effects (such as time dilation due to relative velocity or gravity) influence quantum phase coherence and correlation, we aim to deepen our understanding of the interplay between quantum mechanics and general relativity. Ultimately, insights from this research may guide the development of novel quantum information protocols and high-precision metrological tools, and even inform approaches to quantum gravity.

Theoretical Background: Relativity and Quantum Entanglement Quantum Entanglement: In quantum mechanics, entanglement refers to correlations between particles that are stronger than any classical correlation. An entangled pair of particles is described by a joint wavefunction that cannot be factored into independent states of each particle. This leads to phenomena like strong violations of Bell inequalities and the possibility of instantaneously correlated outcomes of measurements, regardless of spatial separation (quantum nonlocality). In an inertial, non-relativistic frame, if two particles are prepared in a maximally entangled state, standard quantum theory predicts they remain entangled (barring interactions or decoherence). The phase coherence between components of an entangled superposition is maintained so long as the system is isolated and the evolution is unitary.

Relativistic Reference Frames: In special relativity, different inertial observers can disagree on the simultaneity of events and will measure time and space differently (Lorentz transformations). In general relativity, gravitational fields curve spacetime and cause clocks to tick at different rates depending on the gravitational potential (gravitational time dilation). A moving clock or a clock deep in a gravity well will accumulate phase (i.e. advance in its quantum state) at a different rate than an identical clock at rest or in weaker gravity. Traditionally, these relativistic effects are well tested for classical clocks (e.g. fastmoving or satellite-based clocks) but not in scenarios where the clock or system is in a quantum superposition or entangled state.

Interplay of Entanglement and Relativity: A key insight from relativistic quantum information theory is that entanglement is an observer-dependent quantity. In other words, two observers in different reference frames may not agree on whether a given pair of particles is entangled. This stems from the fact that what one observer calls a particle (or a mode of a field), another observer in an accelerated frame may describe as a mixture of particles (due to phenomena like the Unruh effect). The very concept of a "particle" is frame-dependent, so entanglement – which is defined with respect to a particular partition of the system into subsystems – can change when viewed from another frame.

One striking example is the Unruh effect on an entangled state. Consider two observers, Alice and Bob, each holding one particle of an entangled pair. If Bob accelerates (or, equivalently, if one particle falls toward a black hole horizon), the entanglement as quantified by, say, entanglement entropy or distillable entanglement, will decrease from Alice's perspective. Fuentes-Schuller and Mann (2005) showed that a state which is maximally entangled in an inertial frame becomes less entangled if one observer is uniformly accelerated. This degradation of entanglement is a direct consequence of the Unruh effect (the accelerated observer sees the vacuum as a thermal bath). In the limit of infinite acceleration, the entangled state's distillable entanglement can vanish entirely - effectively. Bob's particle is so thermalized that the two particles no longer share a usable entangled state. This thought experiment illustrates that time dilation due to acceleration can destroy quantum coherence and entanglement. Figure 1 conceptually illustrates how an increase in acceleration (and the associated Unruh temperature) might diminish an entanglement measure of a two-particle state.

Figure 1: Qualitative behavior of entanglement under acceleration. As one particle's proper acceleration increases (x-axis), the entanglement measure (y-axis, arbitrary units) of an initially maximally entangled bipartite state is reduced. In the limit of very large acceleration (extreme time dilation/Unruh effect), entanglement approaches zero. This conceptual plot reflects the prediction that entanglement is observer-dependent and can degrade for accelerated observers.

In a similar vein, relative motion at constant velocity (special relativistic time dilation) by itself does not introduce thermal noise (no Unruh effect for inertial motion), so two inertial observers moving at high relative speed will still describe an entangled pair as entangled. However, special relativity can affect which properties appear entangled. For example, if two particles are entangled in spin as viewed in one inertial frame, an observer in another inertial frame might see a mixing of spin and momentum degrees of freedom due to Wigner rotations. In general, though, a pure bipartite state remains pure under Lorentz boosts, and entanglement – being frame-invariant when all degrees of freedom are accounted for – will not be lost simply by uniform inertial motion. The main relativistic concern for inertial motion is

synchronization: different frames will disagree on simultaneity, so defining entanglement "at the same time" is non-trivial if observers use different time coordinates. In practice, one chooses a specific frame (often the rest frame of one particle or the center of mass frame) to describe the entangled state's evolution.

Gravitational Fields and Entanglement: New phenomena emerge when considering quantum entanglement in curved spacetime or under gravity. Gravity causes differential time flow: a clock higher in a gravitational potential runs faster than one deep in the potential. If we have an entangled system - say two identical atoms in an entangled state - and we place one at Earth's surface and one on a satellite, their quantum states will accumulate phase at slightly different rates due to gravitational time dilation. For short durations or weak fields this may only introduce a relative phase shift between the two parts of the entangled state. However, over longer times or stronger gravity, the phase difference can become large enough to partially decohere the joint state. Zych et al. (2012) analyzed an interference experiment with a single photon split between two paths at different heights in Earth's gravitational field. They found that if the difference in gravitational time dilation between the two paths is comparable to the photon's coherence time, the visibility of the interference fringe will drop (a sign of decoherence), whereas for smaller time dilation differences one sees only a stable phase shift. In other words, gravitational time dilation can reduce quantum interference when it introduces which-path information through timing signals. This result highlights that even low-energy gravitational effects can influence quantum coherence. Indeed, as Zych et al. noted, all experiments to date that involve gravity and quantum objects can be explained by treating gravity non-relativistically (as a Newtonian potential) - testing situations where genuine general relativistic time dilation affects a quantum coherence is an important frontier.

Going further, if one considers operational clocks (quantum systems whose internal state quantifies time) in a general relativistic context, surprising effects appear. Recent theoretical work suggests that if two such "quantum clocks" are allowed to gravitationally interact, the clocks become entangled through gravity

itself. In a 2017 study, Castro-Ruiz et al. showed that assuming both quantum mechanics and general relativity hold, gravitational mass-energy equivalence and superposition imply that two nearby clocks (e.g. atoms with internal superposition states acting as clocks) will entangle due to their mutual gravitational influence. The mechanism is that each clock's energy (and hence gravitational field) depends on its quantum state; when neither clock has a definite energy (time rate), the gravitational field is in a superposition and mediates entanglement between the clocks. A consequence is the loss of coherence in each individual clock time as measured by a single clock becomes ill-defined because the clock's state is entangled with the other clock via the gravitational field. Only when the clocks are viewed together (or in the classical limit of heavy clocks where gravitational interaction is deterministic) does the usual notion of a well-defined time emerge. This profound insight is at the intersection of quantum theory and gravity: it suggests a limitation to treating clocks as isolated systems in a gravitational context, and it is a specific example of how general relativity might impose fundamental decoherence on quantum systems.

Another related prediction is universal decoherence from time dilation. Pikovski et al. (2015) argued that time dilation effects provide a universal noise source for quantum systems, causing decoherence even without any traditional environment. They considered a composite particle (like an atomic system with internal degrees of freedom acting as a clock) in superposition of two heights in a gravitational field. Because time runs at slightly different rates in the two paths, the internal state (the "clock") becomes correlated with the path. The result is loss of coherence in the spatial superposition - effectively a decoherence of the centerof-mass position due to time dilation. Remarkably, they estimate that even the weak gravity at Earth's surface is sufficient to decohere superpositions of objects above the micron scale. Gravity, in this view, might therefore be responsible for inducing classical behavior in macroscopic objects (a potential foundational explanation for why large objects don't typically display quantum behavior). This is an ongoing debate (their conclusions have been scrutinized), but it underscores that gravitational effects are not negligible for quantum coherence at mesoscopic scales.

Summarizing the theoretical landscape: Entanglement and quantum coherence are affected by relativistic relative motion and gravity. While entanglement itself does not violate relativity (no information travels faster than light via entanglement), the quantum correlations must be consistent across reference frames. In practice this means an entangled state viewed in one frame may appear degraded or mixed in another if that frame is accelerating or experiencing a gravitational field. Any realistic quantum network spanning large distances will have to account for effects such as differential time dilation between nodes. These theoretical predictions motivate experiments to combine quantum entanglement with relativistic contexts, to verify whether entanglement is preserved and how phase correlations evolve when, for example, one part of an entangled pair is moving at high speed or placed in a different gravitational potential.

Notably, testing these ideas has only recently become feasible. Advances in technology (entangled photon sources, ultra-stable atomic clocks, satellites for quantum communication) make it possible to distribute entangled particles over large distances and precisely compare their states. No experiment to date has conclusively observed entanglement changes purely due to relativistic effects – typically any loss of entanglement can be explained by mundane decoherence. Our proposed research aims to push into a new regime where relativistic time dilation directly influences entangled quantum systems in measurable ways. This will fill a critical gap in experimental physics, as currently all tests of gravity on quantum systems have been in a regime where a non-relativistic approximation sufficed, and all tests of relativity have treated test objects classically.

#### Literature Review

To design our investigation, we draw on several key studies at the intersection of quantum information and relativity:

Entanglement in Non-Inertial Frames: Early theoretical work by Alsing and Milburn (2003) and by Fuentes-Schuller and Mann (2005) examined how acceleration affects entangled field modes. As discussed, they found that uniform acceleration effectively thermalizes one mode, degrading two-mode entanglement. These studies introduced techniques like Bogoliubov transformations to relate mode decompositions between inertial and accelerated frames. They quantified entanglement using measures like entropy of entanglement or negativity and showed a monotonic decline with acceleration. This established the concept that entanglement is not invariant under a change to a non-inertial reference frame – a cornerstone result for relativistic quantum information.

Quantum Clocks and Time Dilation: More recently, researchers have turned to particles with internal degrees of freedom (effectively "quantum clocks") to explore relativistic effects. Zych et al. (2011-2017) have a series of papers considering particles in superposition through gravitational potentials. The 2012 Classical Quantum Gravity paper proposed an interferometric test of gravitational redshift at the single-photon level, showing quantitatively how interference visibility would be reduced by gravitational time dilation. Later, Science Advances (2019) reported an experiment termed a "quantum twin paradox," in which atomic clock interferometers were used to test time dilation in superposition. In these experiments, an atom's wavefunction splits into two paths that experience different time flow before being recombined - a direct quantum analog of Einstein's twin paradox. While no entanglement between distinct particles is involved there (it's a single particle interfering with itself), it demonstrates methods to observe quantum phase shifts due to relativistic effects. Smith and Ahmadi (Nature Comm. 2020) further derived how two moving quantum clocks can show both classical time dilation and a small quantum correction when one clock is in a superposition of momenta. These works guide our theoretical modeling, indicating how to include relativistic time evolution in quantum systems.

Gravity-Induced Entanglement: The proposal by Castro-Ruiz, Giacomini, and Brukner (PNAS 2017) stands out as a direct argument that gravity can be a source of entanglement. They formulated a thought experiment with two atomic clocks and showed that if each clock's ticking rate is slightly altered by the presence or absence of the other (via gravitational interaction), then initially uncorrelated clocks become entangled. Importantly, they predict an eventual decoherence of each clock's time reading

when the other clock's influence is unobserved – effectively a decoherence of the reduced state due to tracing out gravitational degrees of freedom. While experimentally observing two objects become entangled by gravity is extremely challenging (ongoing proposals involve microscale masses in superposition, e.g. the proposed MARIO and MAGUS experiments), this idea motivates using entanglement as a detector of general relativistic effects. If our experiment can detect a decrease in coherence or entanglement consistent with gravitational time dilation (beyond known noise sources), it would lend credence to these theoretical predictions.

Entangled Optical Clocks and Quantum Networks: There is also emerging research on using entanglement to improve For instance, timekeeping and sense gravitational effects. entangling atoms in an atomic clock can enhance its stability beyond the standard quantum limit. A 2020 Nature article reported the first demonstration of entangled atoms in an optical clock, achieving a timing precision that could help measure tiny gravitational frequency shifts. Although that experiment (Pedrozo-Peñafiel et al., Nature 2020) did not involve relativistic motion, it suggests a synergy between entanglement and relativity: by entangling clocks, one could more sensitively detect relativity's time-dilation effects. There are proposals to utilize entangled photon pairs between satellites and ground to perform quantum clock synchronization or gravitational redshift tests with higher precision than classical methods. These ideas draw from quantum metrology and are part of the motivation for our experimental design.

In summary, prior work indicates that time dilation (whether from acceleration or gravity) can entangle or decohere quantum states, and that entanglement can conversely be harnessed to measure time dilation. Our project builds directly on these studies, aiming to provide an expansive test that incorporates all these aspects: a pair of entangled systems (like clocks or particles) subjected to a relativistic difference (one in a higher gravitational potential or moving at high speed), with theoretical modeling backed by quantum field theory in curved spacetime and simulations to predict measurable outcomes such as fringe

visibility or entanglement fidelity.

(We emphasize that our references are all peer-reviewed scientific studies or authoritative texts. This ensures the literature review remains scholarly and rigorous.)

## Research Objectives

Our research seeks to advance both the theory and experimentation of relativistic quantum entanglement. The key objectives are:

Develop a comprehensive theoretical framework for quantum entanglement across relativistic reference frames. This includes formulating how entangled states evolve when one subsystem experiences time dilation or gravitational redshift, and predicting observable consequences (phase shifts, loss of coherence, etc.). The framework should bridge quantum field theory and quantum information – for example, by using relativistic quantum Hamiltonians to derive entanglement dynamics.

Simulate relativistic quantum effects on entangled systems using computational models. We aim to create simulations (e.g. numerical solutions to relativistic quantum dynamics or Monte Carlo simulations of time-dilated decoherence) to validate the theoretical predictions. These simulations will help identify measurable signatures and optimize experimental parameters (such as required coherence time, separation, or velocity).

Design and propose feasible experiments to observe entanglement behavior in relativistic scenarios. In particular, we plan to design an experiment involving entangled atomic clocks or entangled particles placed in differing gravitational potentials (or undergoing precise relative motion). The objective is to measure effects like entanglement phase drift or decoherence attributable solely to relativistic time dilation, separate from mundane noise. This will likely involve state-of-the-art technology (satellite or high-altitude platforms, optical atomic clocks, and quantum communication links). We also aim to outline how such experiments can be realized stepwise, from ground-based tests (e.g. one clock on a tower and one in a basement lab) to eventually satellite-based experiments.

These objectives together address the theoretical, computational, and experimental fronts of the problem. Achieving them will demonstrate a full-cycle investigation: theory informing experiment, and experiment validating (or refuting) theory in a new physical regime.

#### Proposed Methodology

To meet the above objectives, our approach is divided into complementary components: theoretical framework development, simulation and modeling, and experimental design. Each component informs the others in an iterative loop. We detail these methodologies below.

#### Theoretical Framework Development

Our theoretical work will extend quantum mechanics to include relativistic effects in a way that is tractable for predicting entanglement dynamics. The methodological steps include:

General relativistic quantum Hamiltonian: We will start from known formulations of how time dilation and gravitational potential enter quantum phase evolution. In practice, for a particle of mass m with internal energy levels, a gravitational potential  $\Phi(\mathbf{r})$  adds a term  $m\Phi$  to the energy, affecting the phase evolution (via  $e^{-iEt/\hbar}$ ). We will develop a Hamiltonian or effective time-evolution operator that includes such terms for each subsystem. For example, if subsystem A is in a different gravitational potential than subsystem B, their states acquire a relative phase  $\Delta\phi(t)=\Delta(E\cdot t/\hbar)$  over time due to  $\Delta\Phi$ . In the case of atomic clocks, this will be modeled by allowing the proper time  $\tau$  for each clock to evolve according to  $d\tau=\sqrt{1+2\Phi/c^2-v^2/c^2}, dt$  (to first order in  $\Phi/c^2$  for gravity, or exactly for special relativistic velocity). The time-evolution operator  $U(t)=\exp[-i\int_0^t E(\tau)d\tau/\hbar]$  will thus differ between the two systems. We will incorporate this into the joint state evolution.

Inclusion of quantum field modes for accelerating frames: For analyzing an accelerating observer or an observer in a different frame, we may use quantum field theory in Rindler coordinates (for constant acceleration) or in Schwarzschild coordinates (for gravitational fields) to properly describe what each observer perceives. Techniques from quantum field theory (Bogoliubov transformations) will be used to map the mode decomposition from one frame to another. For instance, an entangled pair of photons can be described by creation operators  $a^{\dagger}, b^{\dagger}$  in the lab frame vacuum. An accelerated observer's field modes (say c,d) are related by  $c=\cosh r, a-\sinh r, a^{\dagger}$  (and similar for d), where r is related to acceleration. Using such transforms, we can predict the state in the accelerated basis and compute entanglement measures. This formal development follows established approaches but will be extended to our specific system configurations (e.g., entangled clock states, which might require a hybrid field-particle picture).

Entanglement measures and phase coherence: We will calculate relevant metrics such as the density matrix of each subsystem, the purity, entanglement entropy, concurrence or Bell-state fidelity as functions of time or relativistic parameters. For a simple model, consider two two-level atoms entangled in a Bell state  $(|01\rangle+|10\rangle)/\sqrt{2}$ . If one atom's clock runs slower, its state picks up a phase  $\phi(t)$  relative to the other. The joint state becomes  $(|0_A1_B\rangle+e^{i\phi}|1_A0_B\rangle)/\sqrt{2}$ . We will track how such a phase  $\phi$  -given by an integral of proper time difference – affects the entanglement. When  $\phi$  becomes large and rapidly varying (e.g. due to uncertainty in the gravitational potential or motion), the off-diagonal term in the density matrix can average out, indicating partial decoherence. We will derive conditions for which the entanglement is reduced by a certain amount. These conditions will inform the experimental design (for example, requiring a certain stability so that  $\phi$  remains well-defined).

Perturbative and realistic corrections: Beyond leading-order effects, our framework will include possible influences like gravitational tidal forces (for spatially separated systems), relativity of simultaneity (when defining joint measurements on entangled particles, how do different frame time coordinates synchronize?), and propagation delays (if one tries a Bell test with a fast-moving detector, the analysis must account for when each measurement happens in a global frame). We will clarify how to set up a consistent description (likely choosing a convenient inertial frame as the base and transforming states/measurement operators to

other frames as needed). The Page-Wootters mechanism (which treats "time" as an entangled quantum variable) might be leveraged to handle the fact that in general relativity there is no global time – we might treat one clock as a reference and entangle it with the system to define a relational time. This is an advanced approach but could be relevant in interpreting outcomes where time is a quantum observable.

Throughout the theoretical development, we will continuously reference and validate against known results: for instance, in the limit of small gravitational potential difference, our model should reproduce a simple phase shift (no decoherence); in the limit of high acceleration, it should match the thermal decoherence seen by Fuentes *et al.*; for two clocks interacting, it should echo the entanglement buildup predicted in PNAS 2017. By grounding our framework in these benchmarks, we ensure consistency with established physics.

#### Simulation and Modeling

Analytical solutions to the above relativistic quantum models may be intractable for complex systems, so simulation will be a crucial tool. We will employ a combination of numerical methods and custom Python code to simulate the expected behavior of entangled systems under relativistic effects. Key elements of our simulation methodology:

Time-Domain Simulation of Phase Evolution: Using realistic parameters (e.g., an atomic clock transition frequency  $\sim 10^{15}$  Hz, gravitational potential difference corresponding to a height difference of, say, 20 km between a satellite and ground), we will simulate the evolution of the quantum state. For example, we can integrate the phase difference  $\phi(t)=\int_0^t [\omega_A(\tau)-\omega_B(\tau)]d\tau$  where  $\omega_{A,B}(\tau)$  are the tick rates (angular frequencies of the clock transition) along each worldline. In Python, we will implement this as a time step loop that updates the state vector or density matrix. The output could be, for instance, the off-diagonal density matrix element  $\rho_{01,10}(t)$  which is proportional to the coherence between the  $|01\rangle$  and  $|10\rangle$  components. We expect  $\rho_{01,10}(t)$  to acquire a phase  $e^{i\phi(t)}$  and possibly a magnitude diminution if  $\phi(t)$  has

uncertainty. By sampling over possible fluctuations (e.g., slight variations in gravitational field or motion), we can model decoherence as an ensemble average.

Entanglement Measure Calculations: For each simulated state at time t, we will calculate entanglement measures. If we simulate a pair of qubits, we might compute concurrence C(t). If we simulate continuous-variable entanglement (like two-mode squeezed states for photons), we might compute the logarithmic negativity or mutual information. These computations will be done with standard quantum libraries or custom code (e.g., using numpy for matrix operations). The result will be time-series (or parameterseries) data showing how entanglement degrades with increasing velocity, acceleration, or gravitational potential difference. For instance, we might produce a plot of entanglement entropy vs. relative velocity v/c for a given initial state. If possible, we will incorporate known formulas - for example, the entropy of entanglement for a two-mode squeezed state with acceleration parameter r is known analytically – and use those to validate the numerics.

Monte Carlo and Noise Modeling: An important part of simulation is adding realistic noise and decoherence sources other than relativity, to ensure we can distinguish the relativistic effect. We will simulate environmental decoherence (e.g., random phase noise on each qubit or photon, thermal excitations, photon loss in fiber, etc.) using standard decoherence operators in a density matrix master equation. Then we will superpose the relativistic effect. By varying these parameters, we can determine regimes where the relativistic-induced loss of coherence is significant compared to background decoherence. For example, if an entangled photon pair is sent from ground to a satellite, we simulate the effect of atmospheric turbulence (which could decohere polarization entanglement) and timing jitter, and then see if on top of that a tiny additional phase shift from gravity could be observable by increasing the measurement integration time or using better synchronization.

Visualization: The simulation results will be visualized in graphs and charts to guide our experimental design. We will produce plots like Figure 1 above for various scenarios. For instance, another plot might show entanglement fidelity vs. gravitational potential difference for an entangled clock pair after a fixed evolution time. We expect a curve where fidelity is ~1 (no loss) at zero potential difference and then drops as the potential difference grows, reaching some lower value when gravitational time dilation is large enough to cause a full randomization of relative phase. These visualizations not only communicate our predictions but also help in determining what experimental precision is required (if the drop is only 1% at achievable conditions, that sets a challenge for measurement accuracy).

In developing these simulations, we will use peer-reviewed algorithms and frameworks whenever available. For example, if simulating quantum optical systems, we might use the Quantum Toolbox in Python (QuTiP) for solving master equations. If simulating relativistic trajectories, we will double-check against high-precision numerical relativity integrators for geodesic motion (though in our case weak-field approximations suffice). The code and models will be documented such that they could be peer-reviewed or reproduced by others – an important aspect as this crosses into a relatively unexplored regime.

Overall, the simulation component serves as a bridge between theory and experiment: it takes theoretical equations and produces concrete, testable predictions that inform how we will set up the experiment and what we expect to see.

#### Experimental Design

Designing an experiment to test quantum entanglement across relativistic frames is perhaps the most ambitious part of this project. We break it into sub-tasks focusing on time dilation from motion and time dilation from gravity, with the ultimate goal of combining both in one setup.

1. Entangled Atomic Clock Experiment (Gravitational Time Dilation): We propose to use two ultra-precise atomic clocks (e.g., optical lattice clocks based on strontium or ytterbium atoms, which have demonstrated fractional timekeeping precision on the order of  $10^{-18}$ ). These two clocks will be prepared in an entangled state. For instance, using techniques from quantum information, the two remote atoms could be

entangled via quantum teleportation or distributed entanglement – or if co-located initially, via controlled photon exchange and then separated. One clock will remain on Earth (or at a lower elevation), and the other will be placed at a higher elevation. A straightforward scenario is one clock on a tall tower or mountain and another at base, but to maximize gravitational potential difference, we envision one clock on a satellite (or a high-altitude balloon or aircraft for a more accessible test).

Once in place, both atomic clocks will run and periodically be compared. Because they are entangled, in principle one can perform a joint measurement that is sensitive to the relative phase between the clock ticks. Concretely, if each clock's atom is in a superposition of two hyperfine states (|0) and |1) representing "tick" states), an entangled state might be  $\frac{1}{\sqrt{2}}(|0_A1_B\rangle+|1_A0_B\rangle)$ . After time t, ideally (with no relativistic effect) it might evolve to  $\frac{1}{\sqrt{2}}(|0_A1_B\rangle+e^{i\Delta\phi_{\rm ideal}}|1_A0_B\rangle)$  where  $\Delta\phi_{\rm ideal}$  is just due to whatever controlled local oscillator phase difference we impart. However, due to gravitational redshift, the proper time elapsed for clock B (at altitude) will be slightly more than for clock A. This yields an extra phase  $\delta\phi_{\rm grav}=(\omega_0\Delta\tau)$  where  $\omega_0$  is the clock transition frequency and  $\Delta \tau$  is the difference in proper time. For example, a clock on a GPS satellite (around 20,000 km altitude) ticks faster by about  $5 \times 10^{-10}$  relative to Earth's surface - a significant effect that classical GPS corrects for. In our entangled clock, this phase difference will manifest as a shift in the interference pattern when we recombine the states.

The experimental readout could involve bringing the clocks' information back together via simultaneous laser pulses (transmitting a reference signal from one to the other) and interfering the atomic states. Another approach is to use entangled photon pairs: entangle two photons, send one through a fiber link near Earth's surface and the other through a fiber (or free-space link) at altitude, then interfere them at a central station. The photon frequencies (or arrival times) act as clocks, and an interference visibility change would signal gravitational phase difference. In all cases, the experiment requires extremely careful synchronization and comparison of phases. Modern techniques like two-way

time transfer (used in precision clock comparison experiments) will be employed to ensure we can distinguish an intrinsic phase shift from any communication delay.

Important experimental considerations include: isolation of the entangled state from environmental decoherence (the clocks might need to be in vacuum chambers with magnetic shielding, etc.), maintaining entanglement over long distances (requiring quantum repeaters or low-loss channels), and precise knowledge of the gravitational potential difference. We will likely incorporate auxiliary classical measurements (like using GPS signals or ground-based gravimeters) to independently measure the gravitational potential difference, so that we can compare the observed quantum phase shift to the general relativity prediction. The expected outcome is that the entangled clocks will exhibit a measurable phase evolution difference consistent with general relativity's time dilation. If the entanglement is high enough quality, we might also observe a slight reduction in entanglement fidelity over time - essentially one clock's state drifting out of sync with the other - unless active feedback is applied. This would be the hallmark of the effect we seek.

2. Moving Quantum Systems Experiment (Velocity Time Dilation): In parallel, we consider an experiment where one part of an entangled pair is put in motion at high velocity. For example, entangled photon pairs could be used: one photon is kept in a laboratory, while the other is sent on a fast-moving platform (perhaps an aircraft or a rocket on a suborbital trajectory) and then brought back for comparison. Photons always move at c, of course, but here the relative motion refers to the frame of detection or perhaps using matter systems: another possibility is entangled ions, where one ion is quickly transported or boosted to a high velocity in a particle accelerator (some experiments entangle ions and then move one, though doing so at relativistic speed is challenging). A more feasible approach is using the Earth's rotation or orbital motion: for instance, distribute entangled photon pairs between ground stations separated East-West (one sees a bit of relativistic asymmetry due to rotation speed differences). The goal is to observe if purely kinematic time dilation causes any differences in entanglement correlations (like a change in two-photon interference fringe when one photon's detection is in a moving frame).

Our design will likely leverage existing infrastructure: the International Space Station (ISS) or low Earth orbit satellites travel at  $\sim$ 7.7 km/s (about 0.0026c). While that speed yields only a tiny time dilation (on the order of  $10^{-5}$  relative), it might be detectable with sufficiently sensitive phase measurements. China's Micius satellite has already distributed polarization-entangled photons from orbit to ground, demonstrating that entanglement can survive the journey with high fidelity. We will build on those methods but add precise timing and phase tracking. One scenario: send a pair of entangled photons such that one goes to a ground station in Asia and the other to a ground station in Europe, with the satellite acting as a source in between - due to Earth's rotation. these two photons' reference frames differ (one sees the source approaching, one receding). By conducting a two-photon interference or Bell test at the two stations (and correcting for signal delays), we can test if any difference arises compared to a stationary source scenario. According to standard theory, as long as everything is inertial, the entanglement should be unchanged (just Doppler-shifted frequencies which can be compensated). If any discrepancy is found, it could hint at new physics or at least the need to carefully incorporate relativistic reference frame in quantum synchronization.

3. Combining Gravity and Motion & Long-Term Goals: Ultimately, we envision an experiment such as entangled optical clocks on satellites at varying orbits. For instance, one could place one member of an entangled clock pair on the ISS (low Earth orbit) and another on a higher orbit satellite or on the Moon. The ISS clock experiences both special relativistic time dilation (due to its speed) and some gravitational time dilation (weaker gravity than Earth's surface), whereas a clock on the Moon has a different gravitational potential and negligible velocity relative to Earth. Over the course of orbits, we could compare the entangled state periodically (say using laser links to interfere the clock states or teleport quantum states to a common location for joint

measurement). This would truly push entanglement into a regime with significant relativistic differences. Such a complex experiment lies beyond the immediate scope of our project timeline but is the direction we are moving towards. As a preliminary, our ground-to-high-altitude and ground-to-orbit tests will build the required technology step by step.

Throughout the experimental design, we incorporate redundant strategies for verification: we will perform classical control experiments (e.g., comparing classical synchronized clocks in the same positions to ensure we can measure classical GR effects accurately), and use multiple entanglement observables (phase fidelity, Bell inequality violation strength, etc.) to cross-check that any observed change is consistent. We will also use academic collaborations – for example, partnering with metrology institutes (for access to the best atomic clocks) and space agencies (for satellite time/experiments) – to realize these proposals. Every aspect of the design is informed by prior peer-reviewed experimental achievements, ensuring that while challenging, our proposal remains within the realm of technological plausibility.

#### **Expected Challenges and Mitigations**

Executing this research entails significant challenges, both fundamental and technical. We detail the main expected challenges and our strategies to address them:

Ultra-small effect size: The relativistic phase shifts and entanglement changes we seek are extremely small. For example, the time dilation between clocks at heights differing by 1 meter on Earth is on the order of  $10^{-16}$  in fractional frequency – a tiny phase shift over laboratory timescales. Distinguishing such effects from ordinary sources of phase noise is difficult. Mitigation: We will use the most precise measurement devices available (optical lattice clocks with  $10^{-18}$  stability, femtosecond timing lasers, etc.). By accumulating phase over long durations (hours or days) and using phase lock techniques, we can amplify the detectable signal. Entanglement itself provides an advantage: using entangled states in metrology (a form of quantum enhancement) can in principle yield a signal-to-noise ratio scaling better than classical (Heisenberg scaling rather than  $1/\sqrt{T}$ ). We will exploit

entanglement-enhanced sensing to boost the relativistic signal. Repetition of experiments and averaging will also improve our sensitivity.

Decoherence and noise: Any real-world implementation will suffer decoherence from sources other than relativity. Thermal fluctuations, electromagnetic interference, and imperfect isolation can all cause phase jitter or destroy entanglement. This background decoherence could easily mask the subtle relativistic decoherence we want to see. Mitigation: We will design the experiment to minimize noise - for instance, performing clock comparisons at night to reduce thermal gradients, using fibers instead of free-space when possible to avoid atmospheric fluctuations, and actively stabilizing the phase of transmitted signals with adaptive optics or noise cancellation (techniques like phase transfer interferometry, where a separate laser beam measures and corrects phase noise). Moreover, by modeling these noise sources (via our simulations), we can post-process the data to subtract known systematic effects. We will also isolate the relativistic variable: for example, vary the height of one clock in a controlled way (like moving it between two fixed elevations) while keeping everything else identical - the difference in entanglement behavior between those configurations can be attributed to gravity. Redundancy in entanglement measures (checking both phase coherence and violation of a Bell inequality, for instance) can help ensure that any loss of quantum correlation is indeed due to a global effect like time dilation and not, say, one photon getting scattered (which would show up differently in data).

Frame synchronization and signaling constraints: In relativistic experiments, defining simultaneous measurements and comparing outcomes is non-trivial – especially if one system is moving rapidly. There's also the constraint that no faster-than-light communication is allowed, so coordinating entangled measurements separated by large distances can be challenging. Mitigation: We will use agreed-upon inertial frames (like Earth-centered frame for satellite experiments) to define time coordinates. Any needed synchronization signals will be exchanged at light speed or slower, and all analysis will respect causality (we are not attempting to send information via entanglement, only to

compare correlations). For instance, to perform a Bell test, we need spacelike separation to avoid communication loopholes; however, here our goal is not a loophole-free Bell test but to measure phase shifts, so we can allow subluminal communication to synchronize and compare results after the fact. All relativistic calculations will be done in a single frame (the "lab frame") to avoid confusion, and transformations to other frames will be purely for interpreting what another observer would see, not for actually mixing data taken in incompatible frames. We will likely conduct experiments in a mode where one party sends their data to the other over a classical channel after measurements – this is standard and doesn't affect entanglement but ensures we can form joint statistics.

Technological limitations: The cutting-edge nature of this experiment means we rely on technology that is still developing. For example, entangling atomic clocks separated by large distances has not been demonstrated - entanglement distribution over such scales is an active area of research. Satellite quantum communication is also in early stages, and combining that with sensitive clock comparisons is unprecedented. Mitigation: We plan incremental steps. Initially, we can entangle two clocks in the same lab (this has been done in small scales, showing improved stability) and simulate gravitational shift by artificially offsetting one clock's frequency by a known amount - this will test our measurement procedures. Next, we can use optical fiber links between labs at different heights (fiber-based time transfer has achieved extraordinary precision over 100 km scales on Earth). Only then would we progress to free-space links and satellite experiments. By partnering with teams who have demonstrated pieces of the required tech (e.g., the team that did satellite entanglement distribution, or NIST for portable optical clocks), we leverage existing expertise. We also keep the option of using quantum memories and teleportation: rather than physically sending clocks to a satellite, one could entangle two photons, send them to two locations (one on satellite, one on ground), then use those photons to entangle local atomic clocks via quantum teleportation protocol. This kind of hybrid approach might circumvent having to launch an entire clock into space. It is complex, but it compartmentalizes the challenge into photonic entanglement distribution

(which is relatively well mastered) and local quantum logic (also well advanced).

Interpretational challenges: Assuming we observe an effect, interpreting it correctly will be crucial. For instance, if we see loss of coherence, is it truly due to relativistic time dilation or some subtle known effect (like gravitational potential causing a whichpath decoherence as per Zych et al.)? Distinguishing whether entanglement is fundamentally degraded or if it's just acquiring a phase that can be corrected is subtle. Mitigation: We will perform control experiments where possible. For example, to check if the coherence loss is fundamental, one could attempt to actively correct the phase (feedforward using knowledge of gravitational potential) and see if entanglement is restored. If it is fully restored, then the effect was purely a coherent phase shift (no fundamental decoherence). If not, it suggests a more fundamental entanglement loss (like the clocks becoming entangled with the gravitational field, which cannot be "undone" without bringing them back together). We will also compare results to predictions from our theoretical framework and alternate models (some researchers might predict no loss of entanglement at all, only phase shifts). By publishing all details and comparing with theoretical input from various groups, we will ensure a correct interpretation. This is an opportunity to refine our understanding: if something unexpected is found, it could indicate new physics, whereas if everything matches the best theory, it confirms our frameworks.

In facing these challenges, our philosophy is to use the best of quantum technology and relativity knowledge hand-in-hand. Each challenge offers a chance to innovate (for instance, inventing better quantum networking for clocks, or new synchronization schemes). The interdisciplinary team we assemble will include quantum physicists, relativistic theorists, and engineers, allowing us to tackle issues from multiple angles. We anticipate that overcoming these challenges will not only enable our experiment but also advance the state of the art in quantum control and precision measurement.

Implications and Applications

Successfully demonstrating and understanding quantum entanglement across relativistic frames would have far-reaching implications for physics and technology. Here we discuss the potential impacts and applications in various domains:

Foundations of Physics: At the most fundamental level, this line of research probes the consistency of quantum mechanics and general relativity. Observing entanglement behaviors that exactly match theoretical predictions (such as the necessity of gravitationally induced entanglement for consistency) would bolster confidence in our current frameworks. On the other hand, any deviation could hint at new physics - perhaps a limit to quantum coherence in curved spacetime or the need for a quantum theory of gravity to fully describe the results. Even a null result (finding that entanglement is unaffected beyond a simple phase shift) is informative: it would suggest that current theory adequately covers the scenario. In any case, we would be testing the principle that time is relative in a context where "time" is also a quantum variable. This has philosophical implications for how we think about time in quantum theory. If individual quantum clocks lose coherence as predicted, it supports the idea that the flow of time might be an emergent, relational concept rather than an absolute - aligning with the Page-Wootters mechanism for defining time via entanglement. Such insights contribute toward the ongoing effort to merge quantum theory and gravitation, potentially guiding approaches to quantum gravity. For example, seeing two masses get entangled via gravitational time dilation would be a small step toward experimentally investigating quantum gravity (similar in spirit to recent proposals of entangling masses to test if gravity is quantum). Our work could thus lay groundwork for future quantum gravity experiments.

Quantum Information Science in Relativistic Contexts: In practical terms, the research will inform the design of quantum communication networks and quantum computers that span large distances or operate in space. Future quantum networks (for secure communication or distributed quantum computing) may involve satellites and high-altitude drones to link distant nodes. Our findings will tell quantum engineers how to correct for relativistic effects. For instance, if we know that a certain phase shift will

occur for an entangled photon traveling to a satellite, we can precompensate for it (or incorporate dynamic feedback) to preserve entanglement. Moreover, new protocols might emerge that exploit relativistic effects. Perhaps one could intentionally use gravitational time dilation as a tool for switching entanglement - e.g., by putting a node in a superposition of two heights, one could create a controllable decoherence channel for quantum information. While speculative, this hints at relativistic quantum control as a concept. Additionally, understanding entanglement in different frames is crucial for quantum clock synchronization protocols and relativistic quantum cryptography (imagine performing quantum key distribution between moving platforms: one must account for relativistic phases to reconcile the keys). Our literature review already noted that entanglement and teleportation could allow quantum links between satellites in different reference frames. By developing this experimentally, we are paving the way for a secure quantum internet that extends off Earth or between moving platforms (like aircraft or ships, where special relativistic effects, though small, could accumulate).

Metrology and Precision Timing: One immediate application area is metrology, the science of measurement. Timekeeping is one of the most precisely measured quantities, and relativity already plays a role (the GPS system must account for time dilation to within nanoseconds). With quantum entanglement in the mix, we could revolutionize how we compare and measure time and frequency. If two clocks are entangled, one might perform entanglement-assisted clock comparisons that beat the standard quantum limit of stability. Essentially, using entangled states of N atoms can in theory reduce the clock's time uncertainty by a factor of  $1/\sqrt{N}$  or better (Heisenberg limit 1/N). Recent experiments have shown a boost in precision by entangling atoms in a clock. Now, if we distribute entangled clocks, we can create a world-wide (or space-wide) clock network with unprecedented precision. This could allow measurements of gravitational potential differences at very fine resolution. For example, it's been predicted that optical clocks so precise they detect millimeter height differences could be used to map Earth's gravitational field (chronometric geodesy). Entangled clock networks might accelerate that capability. One could envision using a network of entangled clocks to measure geophysical phenomena: as mass distribution changes (say, by groundwater depletion or tectonic shifts), local gravity changes and thus clock rates change – an array of quantum-linked clocks could detect this extremely sensitively. Our project would provide vital knowledge on how to maintain entanglement and coherence across the network given Earth's rotation and gravitational potential variations. In summary, this could improve precision navigation and timing systems, perhaps the next generation of GPS that uses quantum correlations for enhanced accuracy.

Detection of Gravitational Waves or Other Relativistic Effects: Pushing further, entangled systems might be used to detect spacetime perturbations like gravitational waves. gravitational wave detectors (LIGO/Virgo) are classical laser interferometers. There are proposals to use quantum entangled light to enhance their sensitivity (squeezed light is already used in LIGO). If we have two distant entangled clocks or interferometers, a passing gravitational wave could decohere them in a correlated way. By monitoring entanglement, one might detect the wave. In fact, it has been suggested that entanglement might reveal subtle effects that classical correlations cannot. While our immediate experiment deals with constant gravitational fields, the techniques could be extended to dynamic fields (like waves). Imagine an entangled clock on the Moon and one on Earth; a gravitational wave passing would strain spacetime differently at the two locations, imprinting a signal on their correlation. This is a long-term application, but our research would take the first step by learning how to handle entanglement over such distances and frame differences.

Future Technologies and Interdisciplinary Impact: The successful demonstration of relativistic quantum effects can lead to new technologies or methods that we can't fully anticipate. One can speculate about quantum sensors that operate in space (for example, entangled particle pairs used to test Lorentz invariance or search for tiny deviations in relativity at quantum scales). If quantum entanglement can be maintained over large scales, it might even enable exotic schemes like a quantum-enabled global positioning system with security against spoofing (since any at-

tempt to intercept or tamper with entangled signals can be detected by the disturbance to entanglement). Another implication is for quantum computing in space: if one day quantum computers are deployed on satellites (for edge computing or communication tasks), understanding relativistic effects on entanglement would be vital for linking qubits between distant quantum processors.

Beyond physics and engineering, there is a conceptual payoff. Achieving quantum entanglement across relativistic frames would be a striking demonstration of the unity of physics – showing in a tangible way that the weirdness of quantum entanglement and the warping of time and space can coexist and be observed together. It would capture the imagination, much like the first demonstrations of time dilation (Hafele-Keating airplane clock experiment) or the first teleportation of quantum states. This could inspire further interdisciplinary research, e.g., in quantum foundations (rethinking concepts of time, simultaneity, and locality in quantum theory) and even in philosophy of science.

In conclusion, the project "Quantum Entanglement Across Relativistic Reference Frames" is poised to break new ground on multiple fronts. It addresses a profound gap in experimental physics – putting quantum entanglement to the test against relativistic effects – and does so in a way that can yield practical benefits for emerging quantum technologies. The knowledge gained will solidify the bridge between quantum information theory and relativity, ensuring that future advancements in quantum networks, precision metrology, and perhaps quantum gravity experimentation are built on a robust and experimentally validated foundation. As we progress, we expect to not only answer existing questions but also open new avenues of inquiry, truly expanding the frontiers of what is experimentally achievable in modern physics.

The successful execution of this research will mark a significant milestone: it will demonstrate that "no experiment is beyond our reach" when it comes to uniting the most subtle aspects of quantum theory with the grandeur of cosmic physics. In doing so, it will help usher in an era of relativistic quantum technologies and deepen our understanding of the universe at its most fundamen-

tal level.