Digital Communications

« Linear » Carrier Modulations

« The complex envelop associated to the transmitted signal linearly depends on the message »

1) **One or two dimensional modulations**
2) **Complex envelop**
3) **Equivalent lowpass channel**
4) **Performance**

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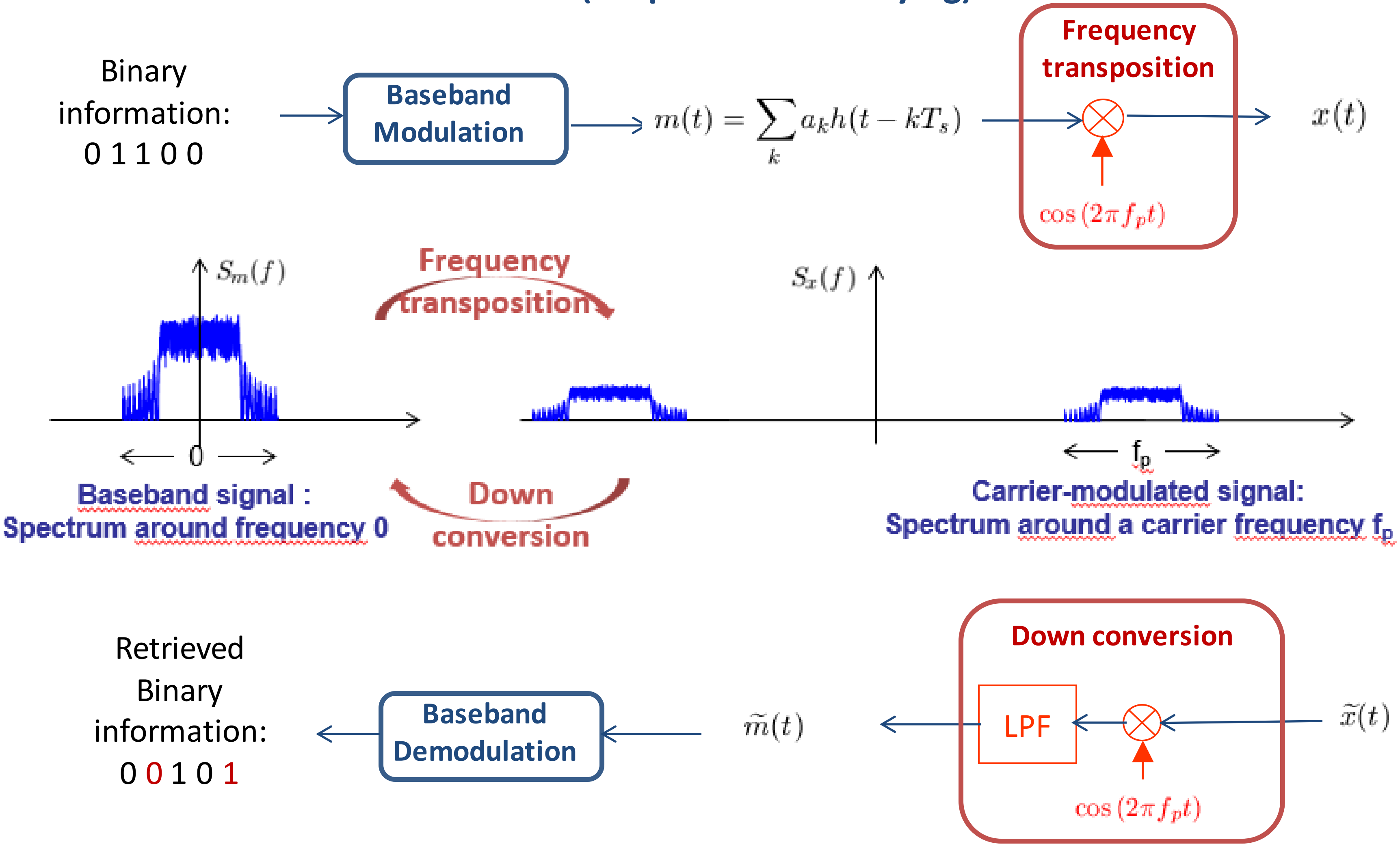
Linear Carrier Modulation
One-dimensionnal

M-ASK (Amplitude Shift Keying)



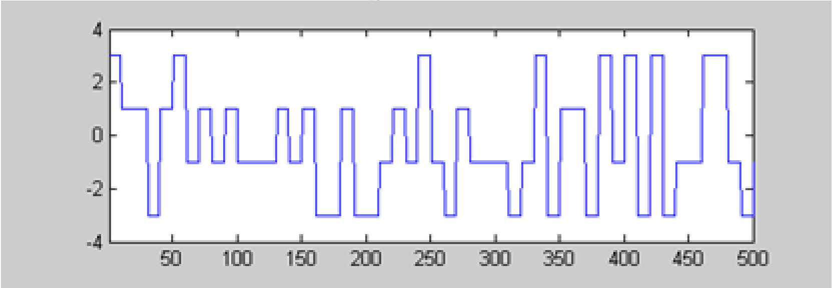
Coherent demodulation

Linear Carrier Modulation
One-dimensionnal

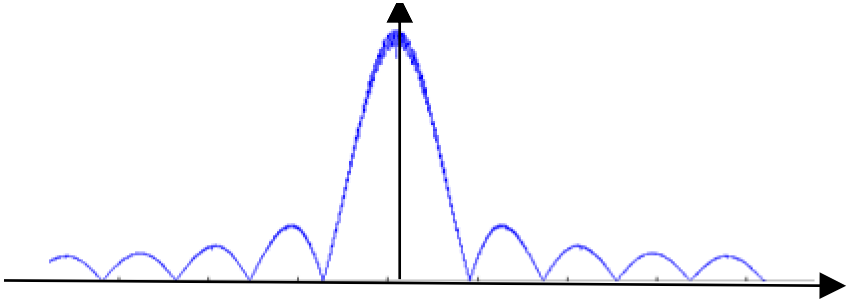
M-ASK (Amplitude Shift Keying)

<u>Example : 4-ASK, rectangular shaping</u>

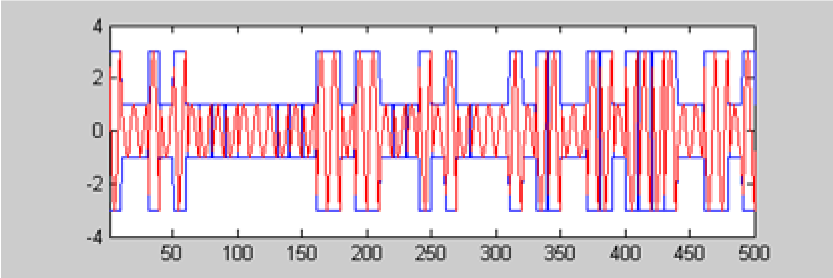
$$m(t) = \sum_k a_k h(t - kT_s)$$



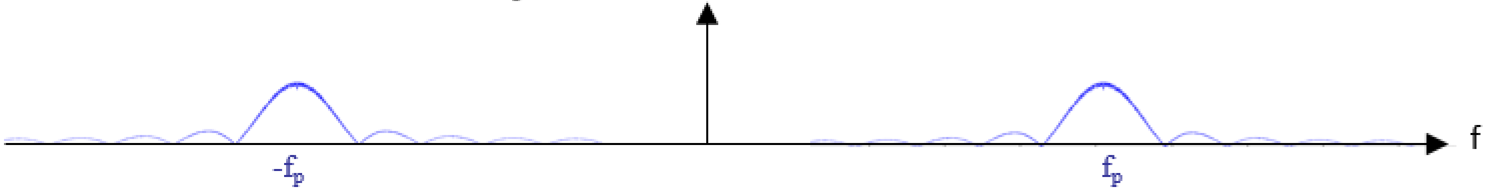
$$S_m(f) = 5T_s sinc^2(\pi f T_s)$$



Signal modulated on f_p : $x(t) = \sum_k a_k h(t - kT_s)\cos(2\pi f_p t)$

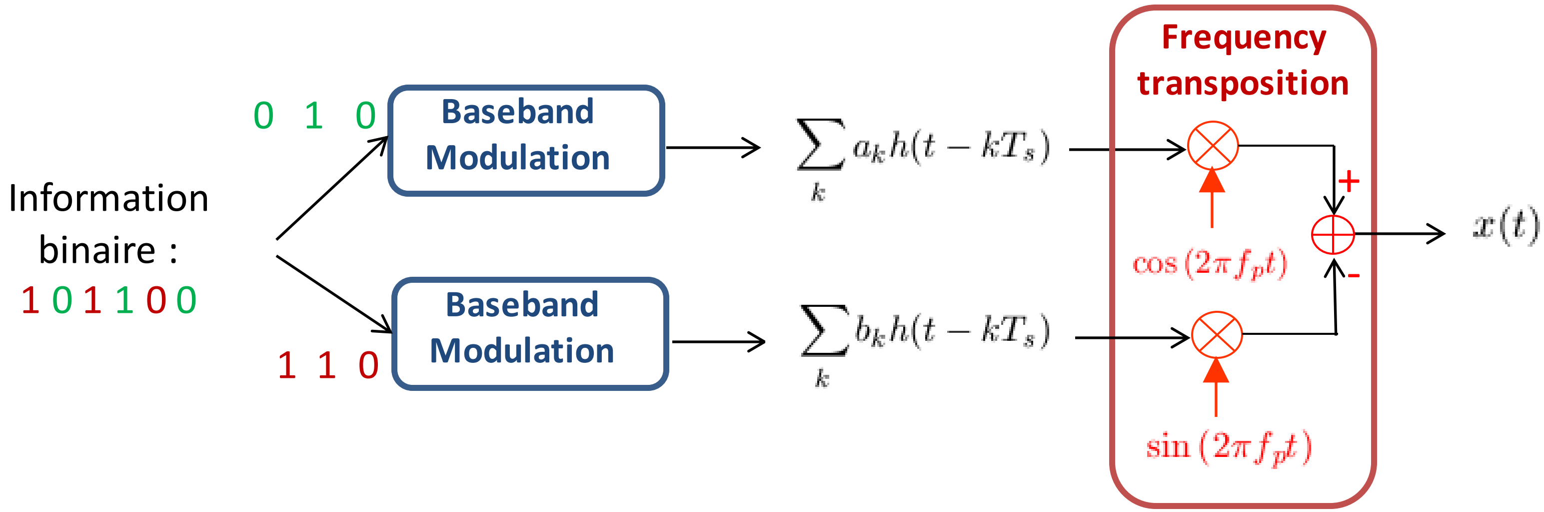


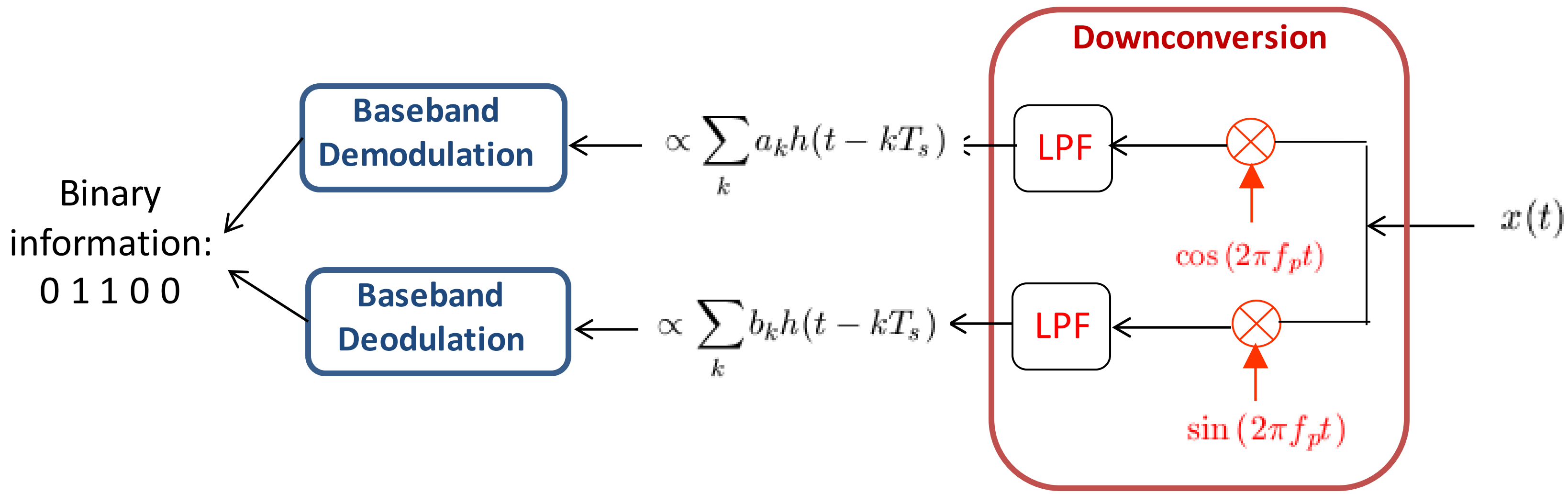
$$S_x(f) = \frac{5T_s}{4}\{sinc^2(\pi(f - f_p)T_s) + sinc^2(\pi(f + f_p)T_s)\}$$



Linear Carrier Modulation

Two-dimensionnal



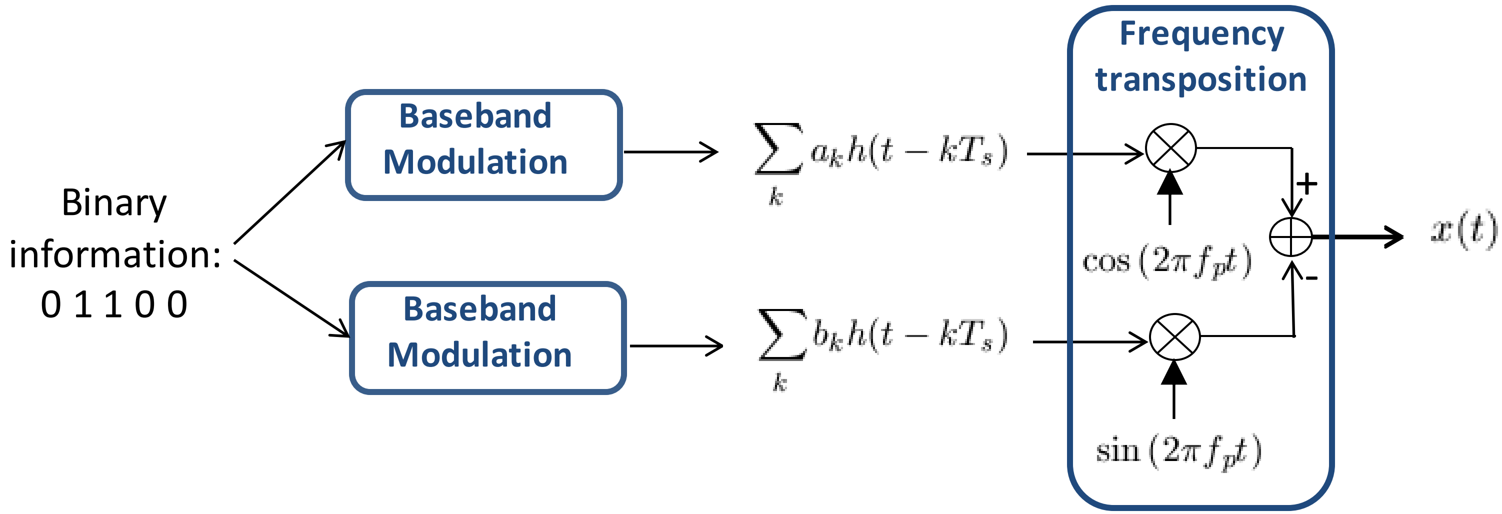


Coherent demodulation

Orthogonal signals

Linear Carrier Modulation

Complex envelop



$$x(t) = \underbrace{\sum_k a_k h(t - kT_s)}_{\substack{I(t) \\ \text{In Phase Component}}} \cos(2\pi f_p t) - \underbrace{\sum_k b_k h(t - kT_s)}_{\substack{Q(t) \\ \text{Quadrature Component}}} \sin(2\pi f_p t)$$

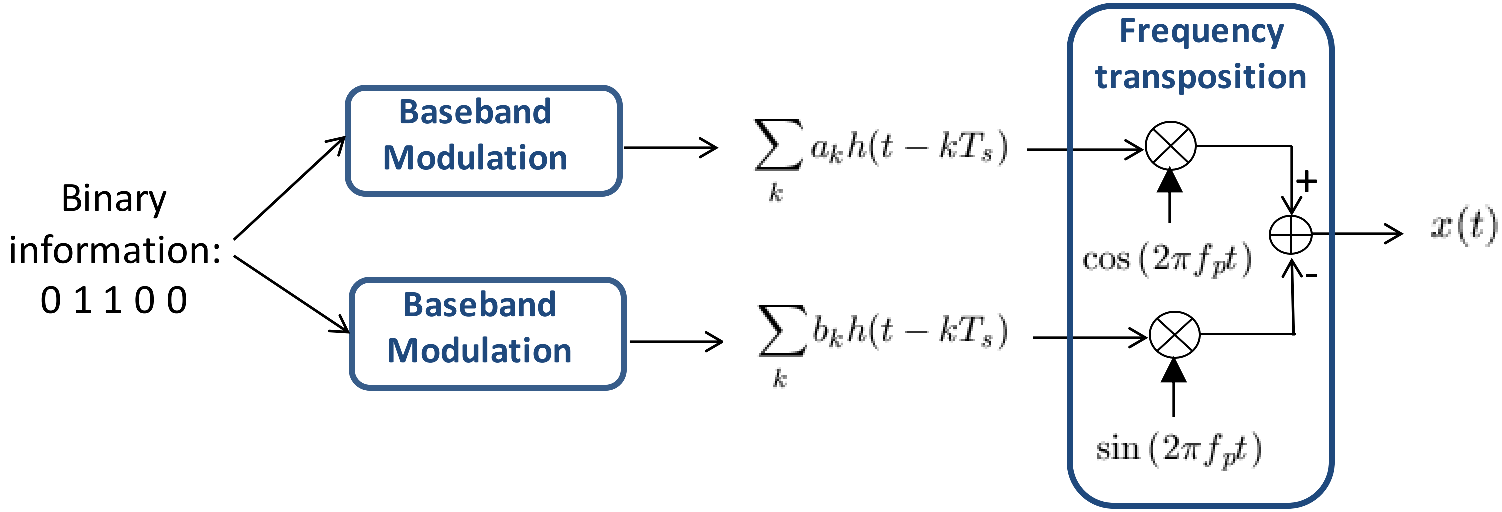
$$x(t) = \Re\left[\underbrace{(I(t) + jQ(t))}\, e^{j2\pi f_p t}\right]$$

$$\boxed{x_e(t) = I(t) + jQ(t) = \sum_k d_k h(t - kT_s)}$$

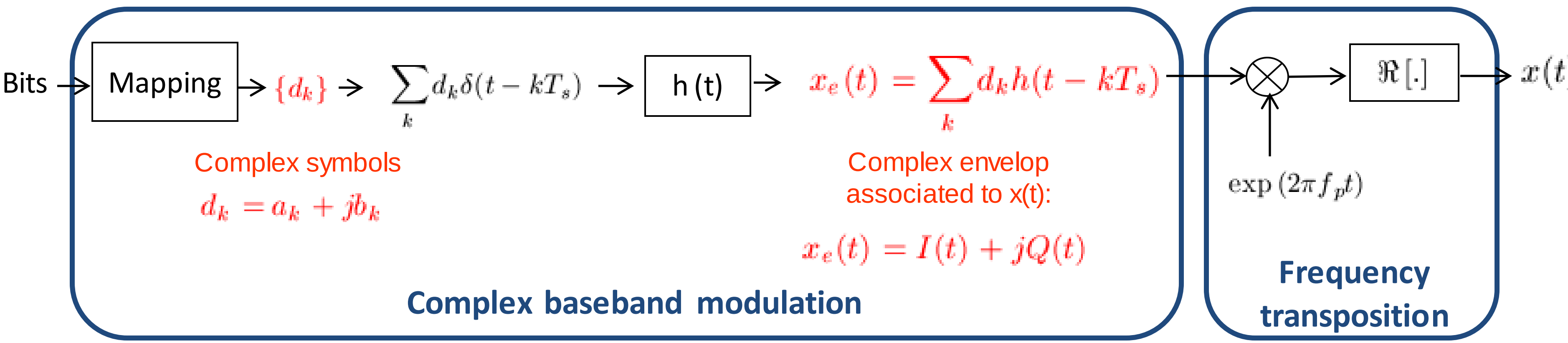
Complex envelop associated to x(t)

Linear Carrier Modulation

Complex envelop



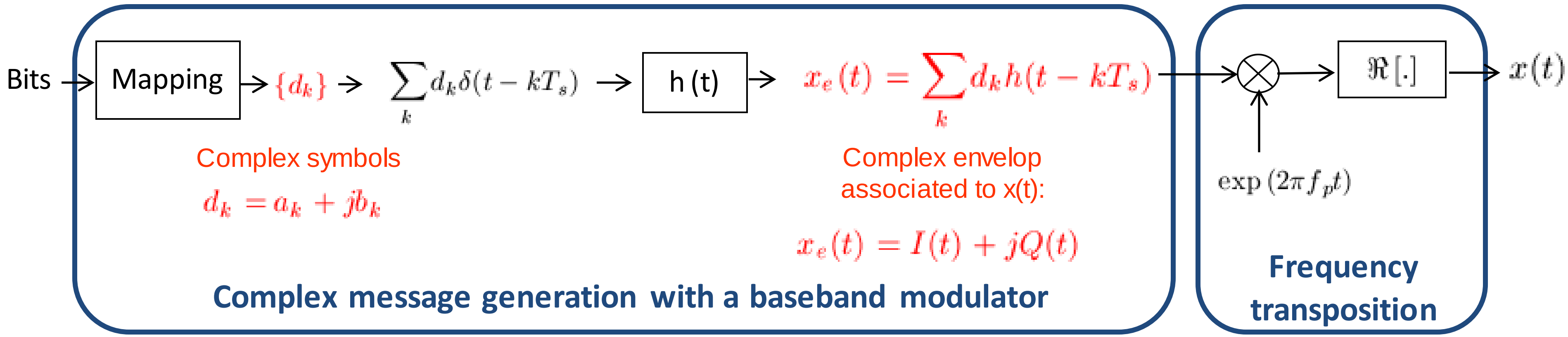
$$x(t) = \sum_k a_k h(t - kT_s) \cos(2\pi f_p t) - \sum_k b_k h(t - kT_s) \sin(2\pi f_p t)$$



$$x(t) = \Re\left[x_e(t) e^{j2\pi f_p t}\right]$$

Linear Carrier Modulation

Complex envelop



→ <u>The PSD of the carrier-modulated signal:</u>

$$x(t) = \Re\left[x_e(t)e^{j2\pi f_p t}\right] \longrightarrow R_x(\tau) = \frac{1}{2}\Re\left[R_{x_e}(\tau)e^{j2\pi f_p \tau}\right] \longrightarrow \boxed{S_x(f) = \frac{1}{4}\left(S_{x_e}(f - f_p) + S_{x_e}(-f - f_p)\right)}$$

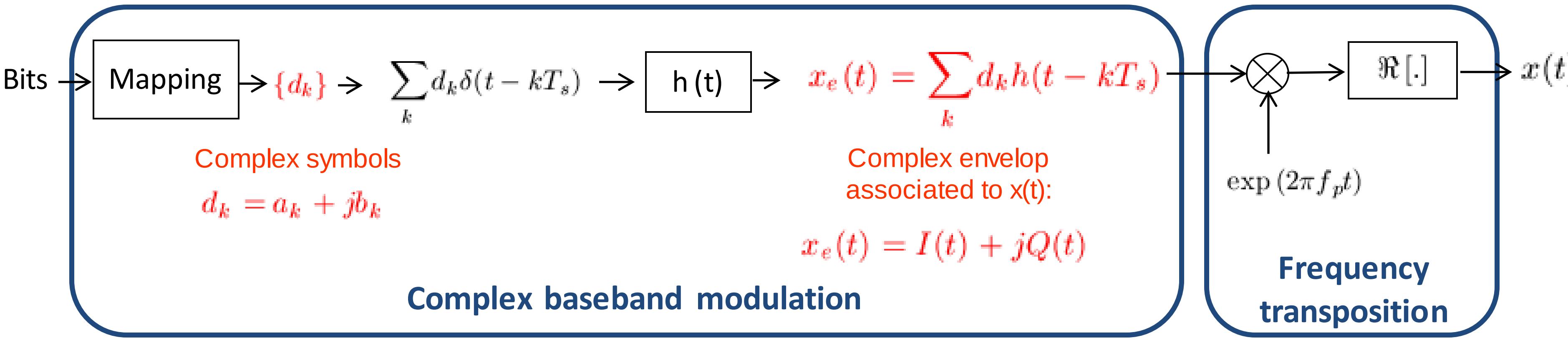
<u>is obtained from the PSD of its associated complex envelop (known baseband spectrum):</u>

$$S_{x_e}(f) = \frac{\sigma_d^2}{T_s}|H(f)|^2 + 2\frac{\sigma_d^2}{T_s}|H(f)|^2 \sum_{k=1}^{\infty} \Re\mathfrak{e}\left[R_d(k)e^{j2\pi f k T_s}\right] + \frac{|m_d|^2}{T_s^2}\sum_k \left|H\left(\frac{k}{T_s}\right)\right|^2 \delta\left(f - \frac{k}{T_s}\right)$$

Re-use the results obtained for baseband modulations

Linear Carrier Modulation

Two main classes of two-dimensionnal modulations



$\longrightarrow \{a_k\}$ and $\{b_k\}$ M-ary independent symbols $\in \{\pm 1, \pm 3, ..., \pm(\sqrt{M} - 1)\}$

square M-QAM (Quadrature Amplitude Modulation)

$$\longrightarrow \quad d_k \in \{e^{j\left(\frac{2\pi}{M}l + \frac{\pi}{M}\right)}\},\ l = 0, ..., M-1$$

M-PSK (Phase Shift Keying)

QUESTION

Let's assume that the symbols are independent, equaly likely and with a zero mean. For a given bit rate R_b, a transmission using a 8-PSK modulation and a rectangular shaping filter will be more spectrally efficient than:

A. a transmission using a QPSK modulation with the same rectangular shaping filter.

B. a transmission using a 8-PSK modulation with a square root raised cosine shaping filter.

C. a transmission using a 16-QAM modulation with the same rectangular shaping filter.

QUESTION

A QPSK modulation is:

A 4-state phase modulation:

A TRUE

B FALSE

A 4-state QAM modulation:

A TRUE

B FALSE

Less spectrally efficient than a BPSK modulation using the same shaping filter

A TRUE

B FALSE

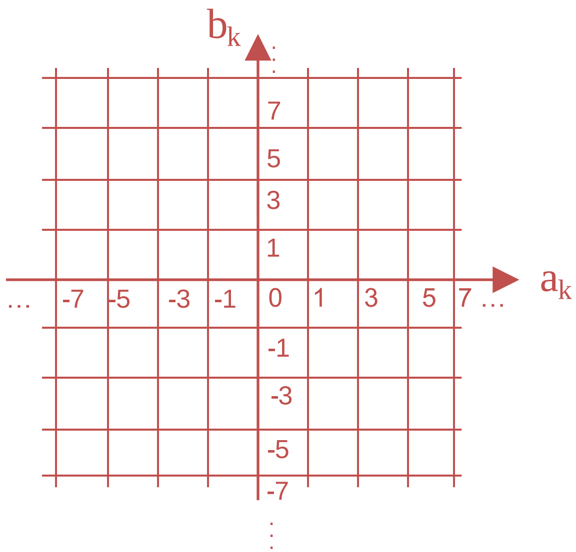
More robust, for the same level of noise and the same transmitted power, than a BPSK modulation:

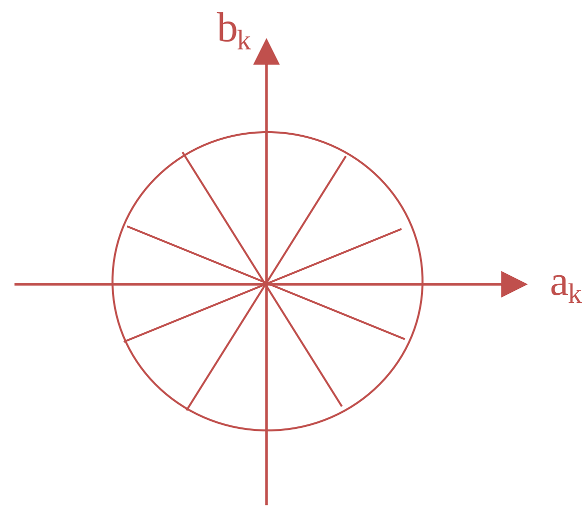
A TRUE

B FALSE

Linear Carrier Modulation
Constellation

Representation of possible d_ks in the (a_k, b_k) plane = « constellation » of the modulation





QAM Constellations

Power efficient

(DVB-C, DVB-T, xDSL)

PSK Constellations

Robust to non linearities

(DVB-S)

Hybrid modulations : APSK

(DVB-S2, DVB-S2X)

Linear Carrier Modulation

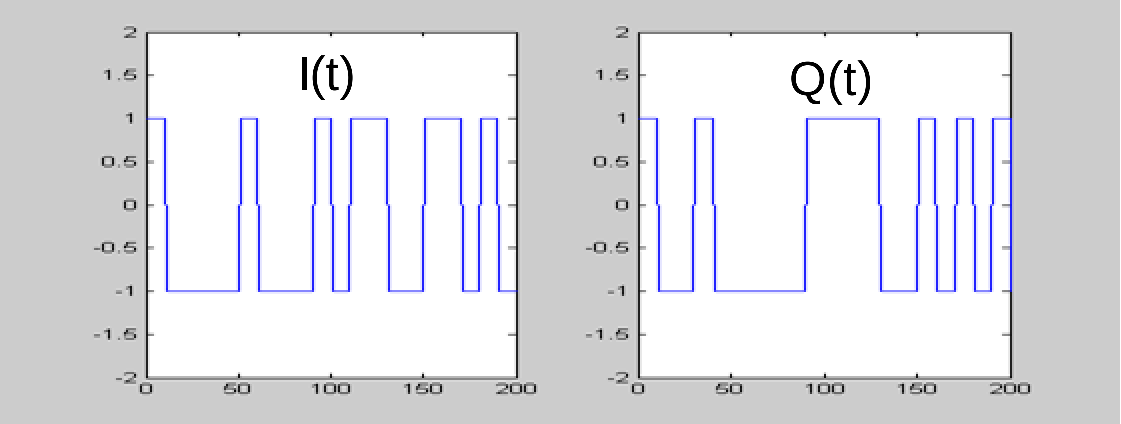
Examples

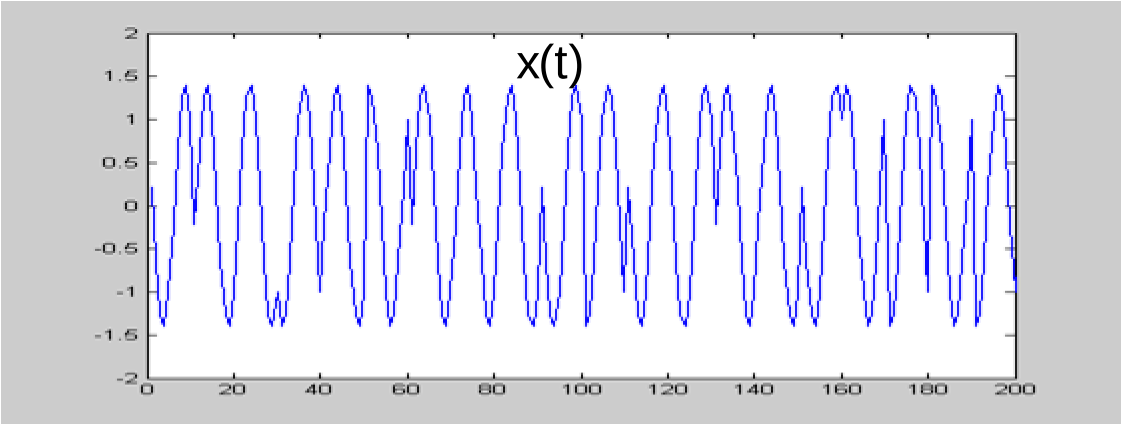
→ Two-dimensionnal linear modulations : M-QAM

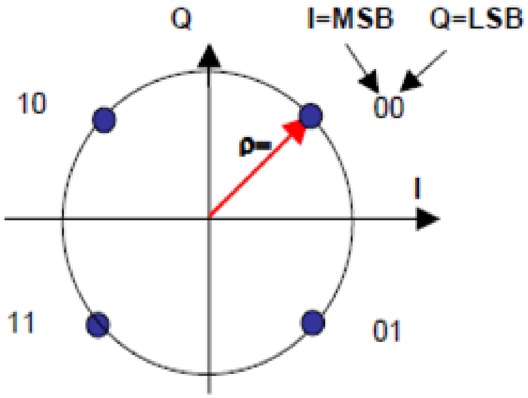
Independent $\{a_k\}$ and $\{b_k\}$

Example : 4-QAM or QPSK (DVB-S)

bits	a_k	b_k	d_k
00	-1	-1	-1-j
01	-1	+1	-1+j
10	+1	-1	1-j
11	+1	+1	1+j







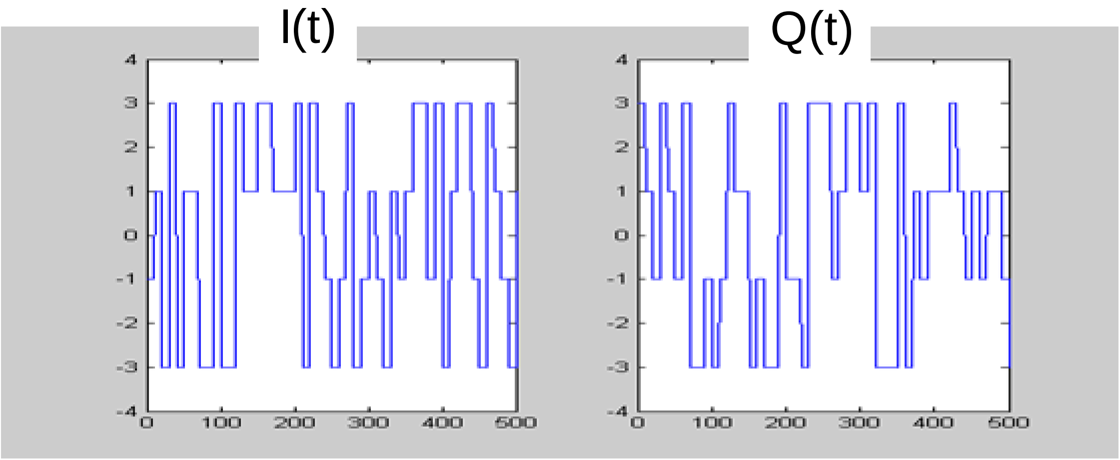
Linear Carrier Modulation
Examples

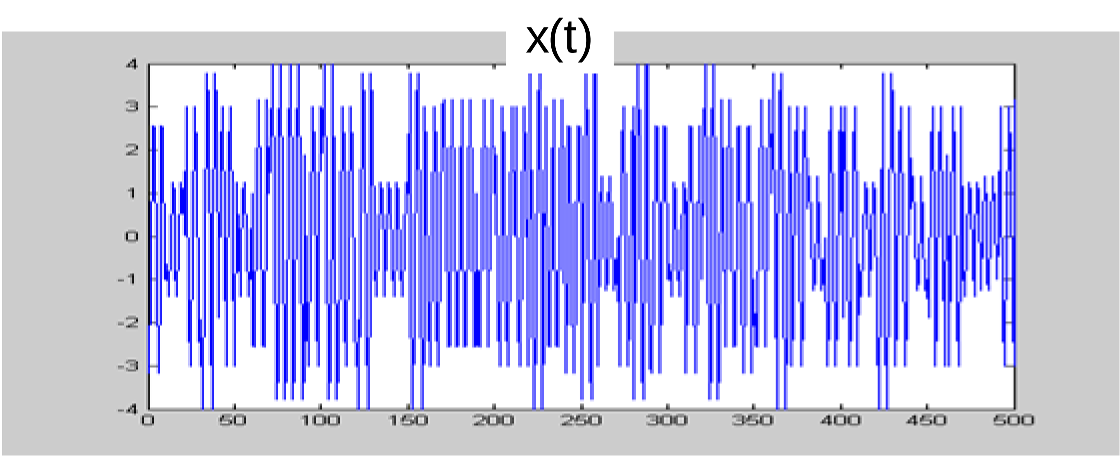
→ Two-dimensionnal linear modulations : M-QAM

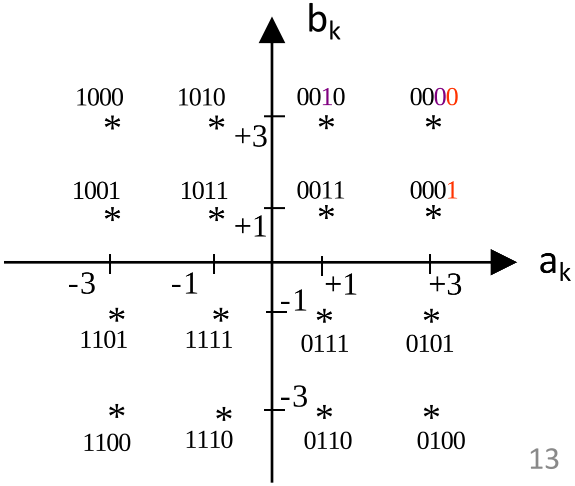
Independent $\{a_k\}$ and $\{b_k\}$

Example : 16-QAM (DVB-C)

Bits	0000	0001	…	1110	1111
a_k	+3	+3		-1	-1
b_k	+3	+1		-3	-1
d_k	3+3j	3+j		-1-3j	-1-j





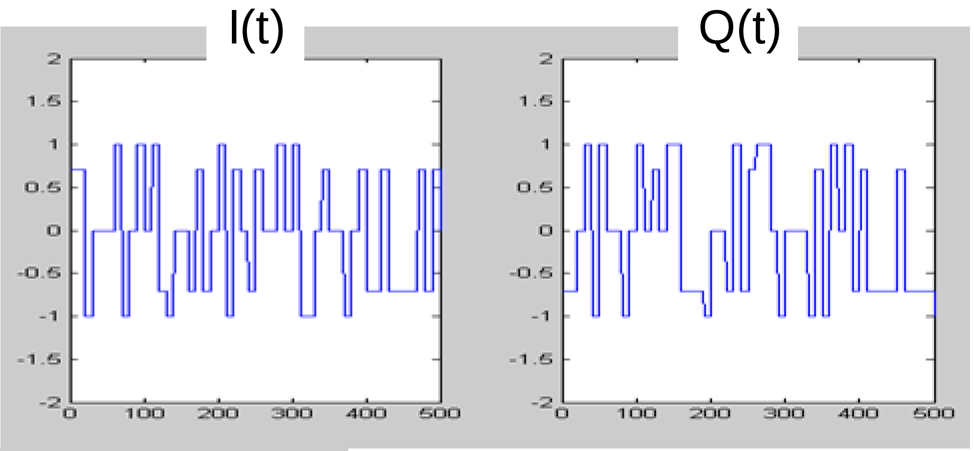


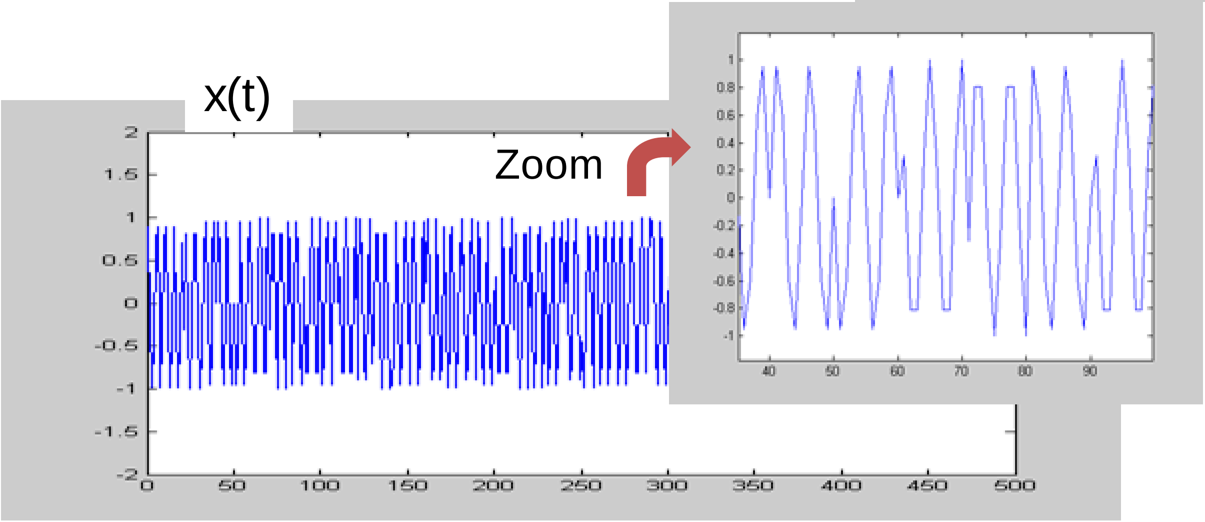
→ Two-dimensionnal linear modulations : M-PSK

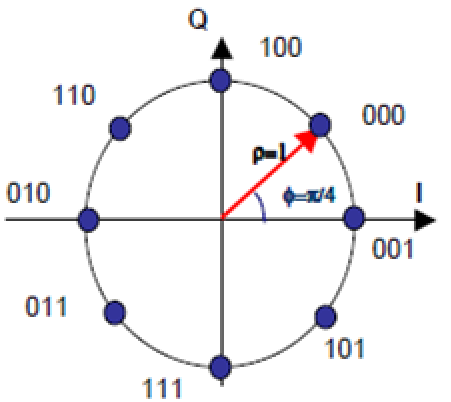
$\{a_k\}$ and $\{b_k\}$ are linked

Example : 8-PSK (DVB-S2)

$$d_k \in \{e^{j\left(\frac{2\pi}{8}l+\frac{\pi}{8}\right)}\},\ l = 0, \ldots, 7$$



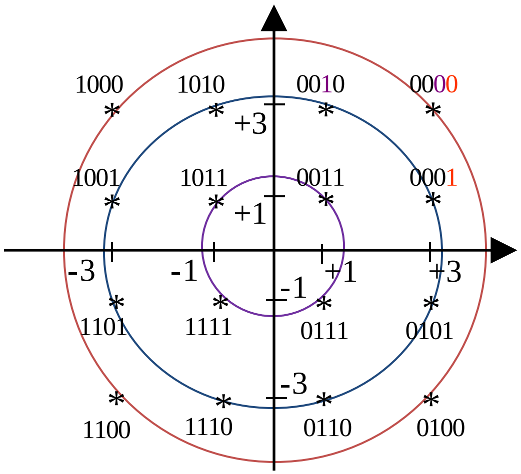




Linear Carrier Modulation

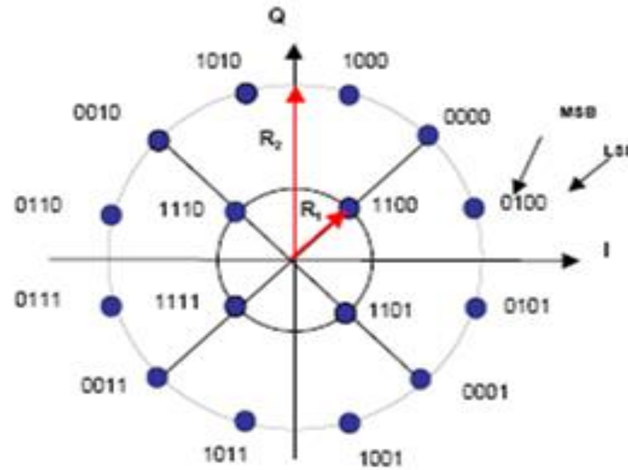
Examples

→ <u>Hybrid modulations :</u> M-APSK (DVB-S2)



16-QAM

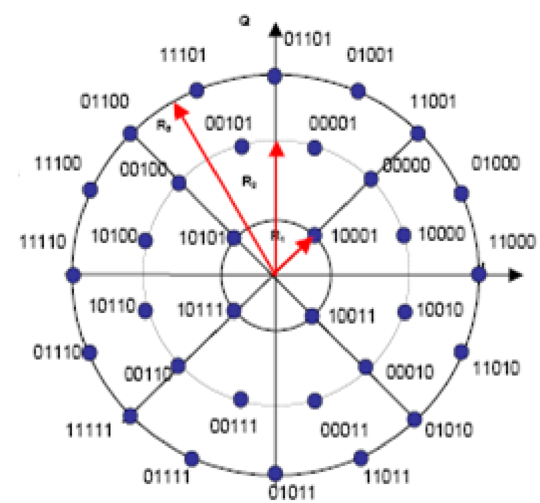
M-APSK



16-APSK (4-12 APSK)

$$d_k \in \begin{cases} R_1 e^{j\left(\frac{2\pi}{n_1} i + \theta_1\right)}, \; i = 0, \ldots, n_1 - 1 \\ R_2 e^{j\left(\frac{2\pi}{n_2} i + \theta_2\right)}, \; i = 0, \ldots, n_2 - 1 \\ \ldots \\ R_R e^{j\left(\frac{2\pi}{n_R} i + \theta_R\right)}, \; i = 0, \ldots, n_R - 1 \end{cases}$$

$$M = n_1 + n_2 + \ldots + n_R$$

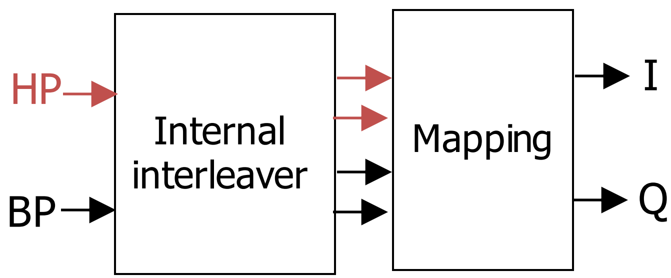


32-APSK (4-12-16 APSK)

Linear Carrier Modulation
Examples

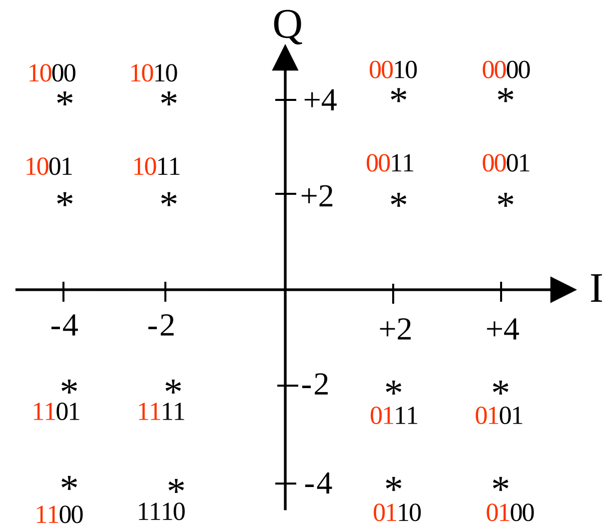
→ Hierarchical modulations : DVB-T and T2, DVB-H, DVB-S2

Example 1 : hierarchical 16-QAM (DVB-T or H)

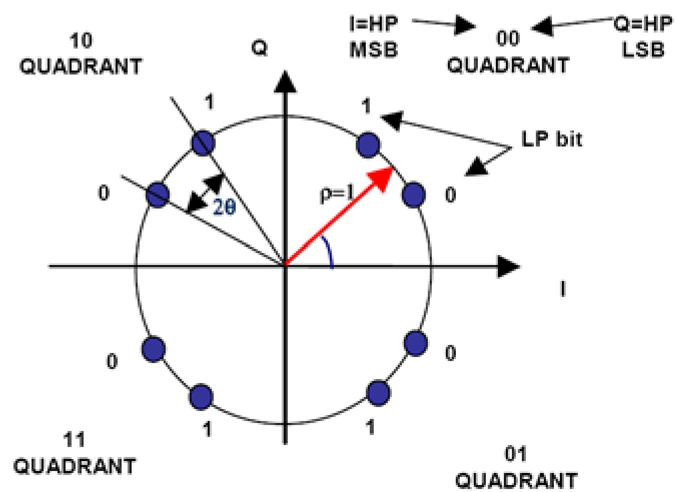


$$\alpha = \frac{\text{2 pts HP min distance}}{\text{2 pts min distance}}$$

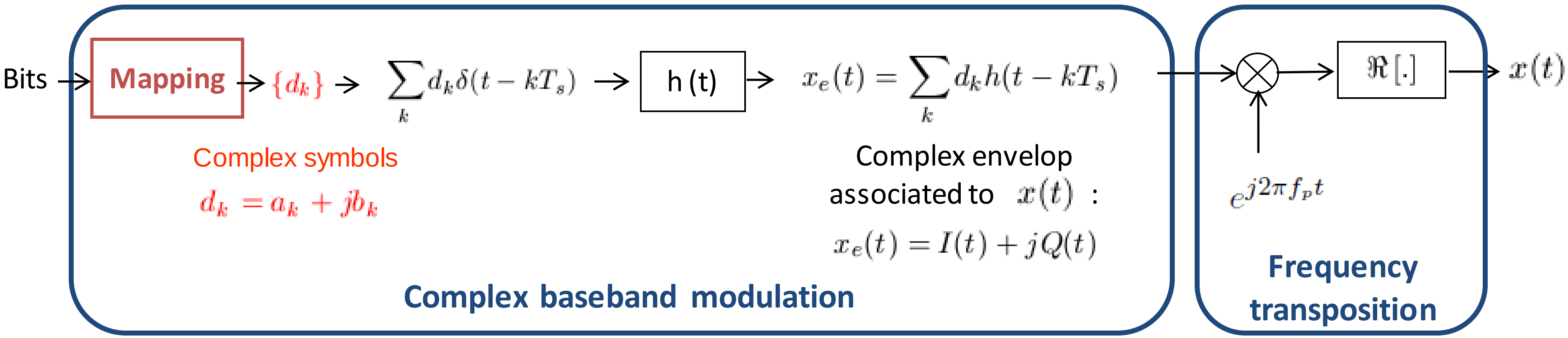
$\alpha = 2$



Example 2 : hierarchical 8-PSK (DVB-S2)



Linear Carrier Modulation
Transmitter



$\rightarrow$ M-ASK: $d_k = a_k \in \{\pm 1, ..., \pm(M-1)\}$

$$x_e(t) = I(t)$$

$$x(t) = I(t)\cos(2\pi f_p t)$$

$\rightarrow$ M-QAM: $d_k = a_k + jb_k \ avec \ a_k \ et \ b_k \in \{\pm 1, ..., \pm(\sqrt{M}-1)\}$

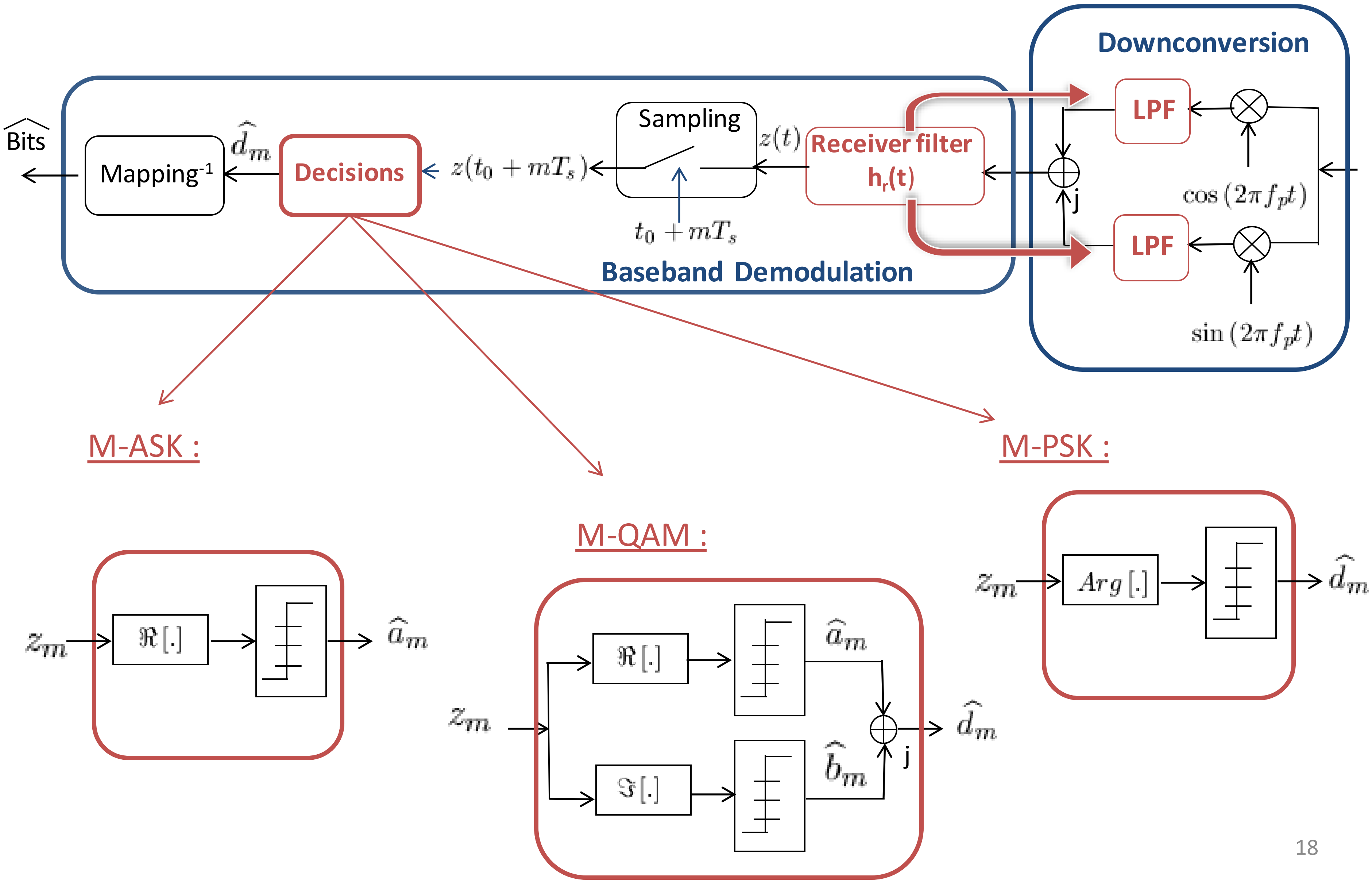
$\rightarrow$ M-PSK: $d_k \in \{e^{j\left(\frac{2\pi}{M}l + \frac{\pi}{M}\right)}\},\ l = 0, ..., M-1$

$$x_e(t) = I(t) + jQ(t)$$

$$x(t) = I(t)\cos(2\pi f_p t) - Q(t)\sin(2\pi f_p t)$$

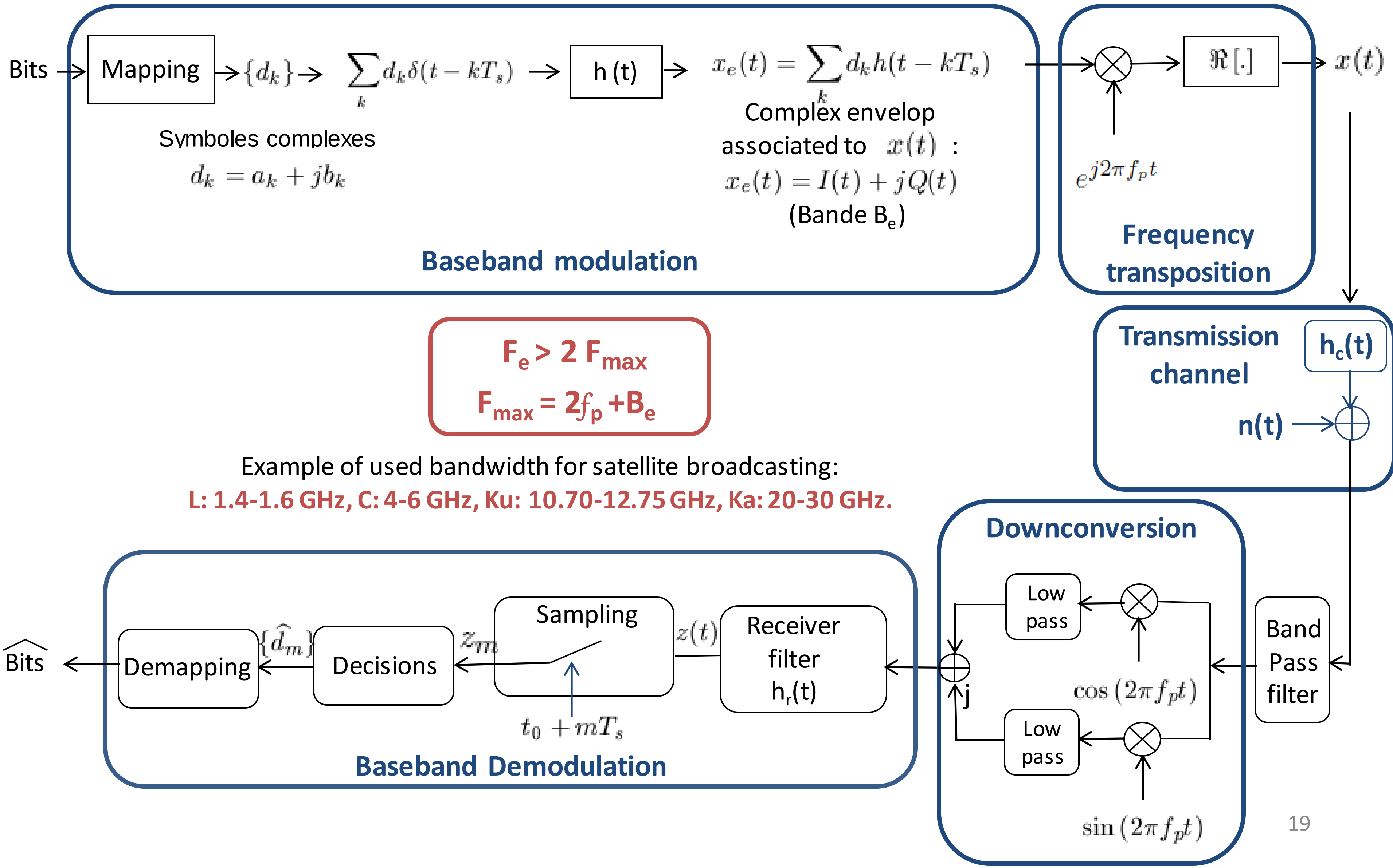
Linear Carrier Modulation

Receiver



Linear Carrier Modulation

Equivalent lowpass channel to reduce the processing time for digital implementations



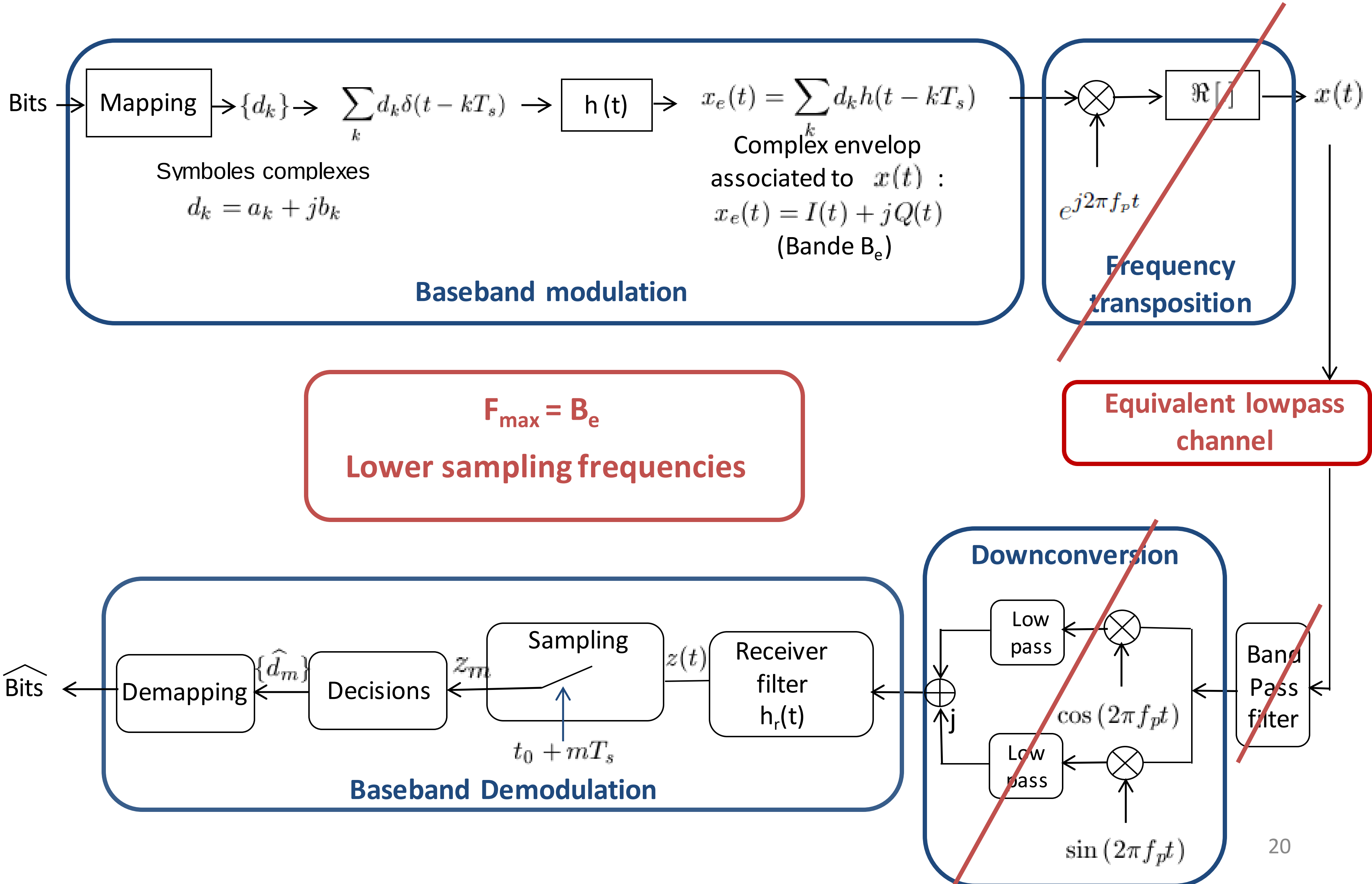
$$F_e > 2\,F_{max}$$
$$F_{max} = 2f_p + B_e$$

Example of used bandwidth for satellite broadcasting:

L: 1.4-1.6 GHz, C: 4-6 GHz, Ku: 10.70-12.75 GHz, Ka: 20-30 GHz.

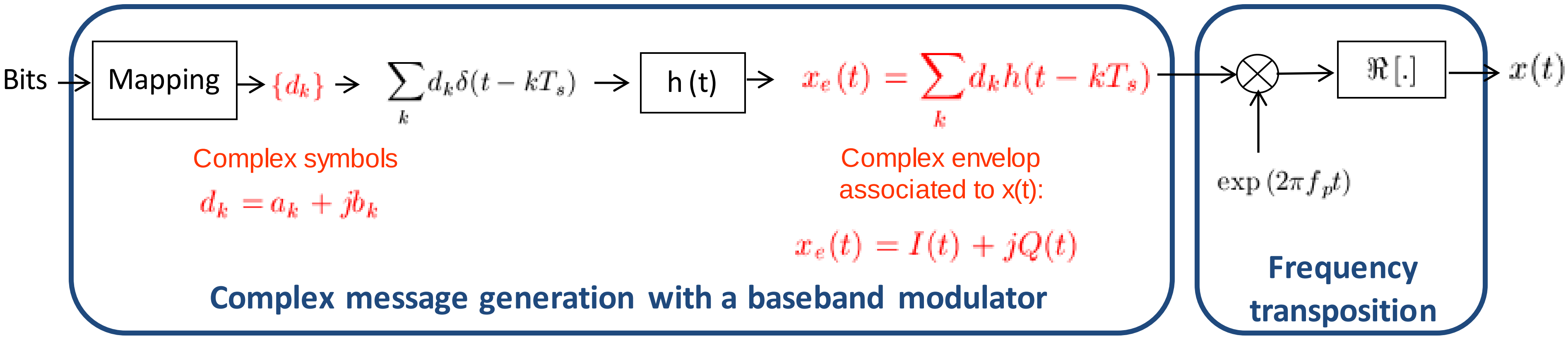
Linear Carrier Modulation

Equivalent lowpass channel to reduce the processing time for digital implementations



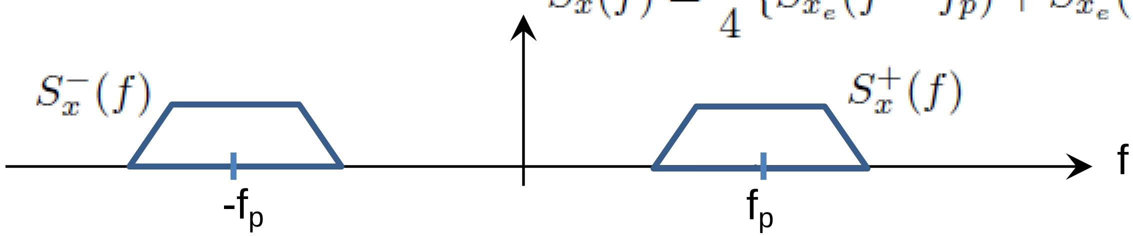
Linear Carrier Modulation

Complex envelop



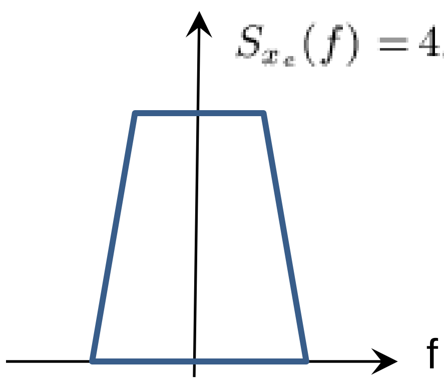
$\rightarrow$ PSD of the carrier-modulated signal:

$$S_x(f) = \frac{1}{4}\{S_{x_e}(f-f_p) + S_{x_e}(-f-f_p)\}$$



$\rightarrow$ PSD of the corresponding complex envelop:

$$S_{x_e}(f) = 4S_x^+(f+f_p)$$

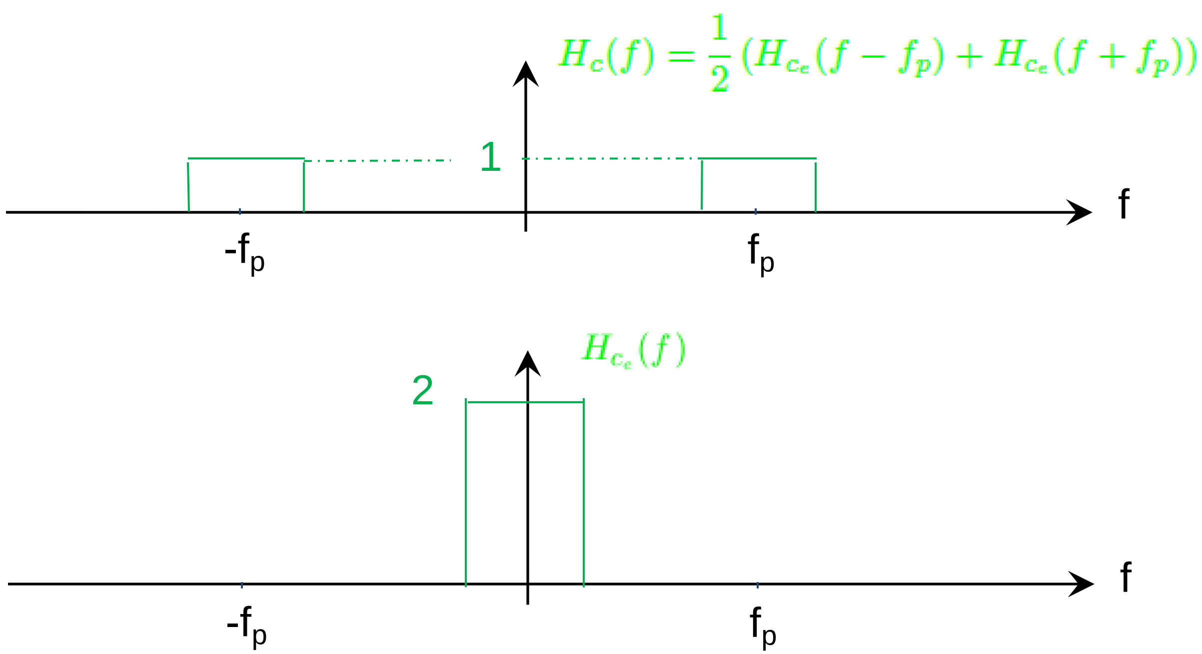


(or also $X_e(f) = 2X_e^+(f+f_p)$ for deterministic signals)

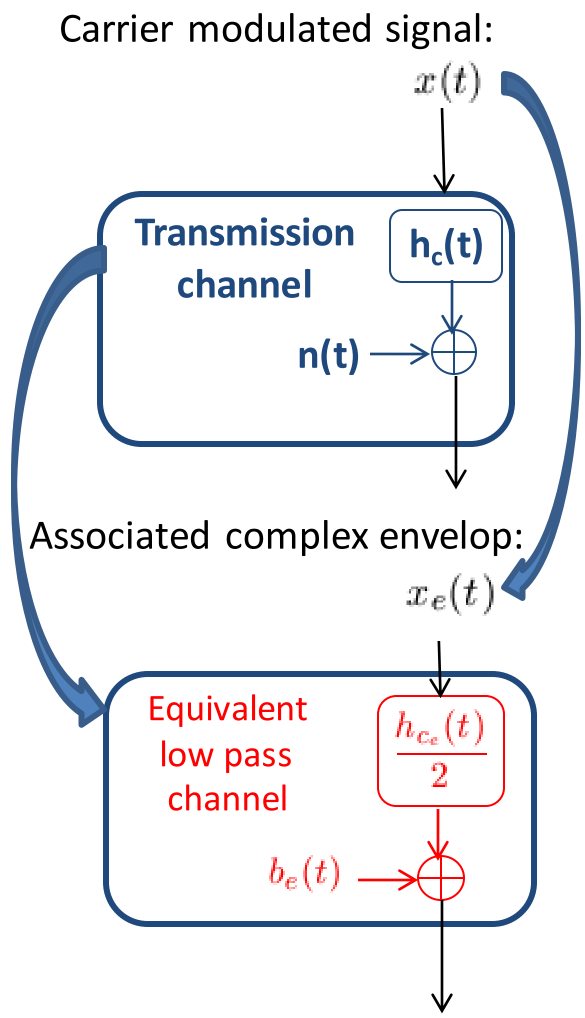
Equivalent lowpass channel: construction

→ Complex envelop associated to the bandpass channel:

$$h_{c_e}(t) = I_{c_e}(t) + jQ_{c_e}(t)$$



(remark: the channel is assumed to be ideal in the figure)

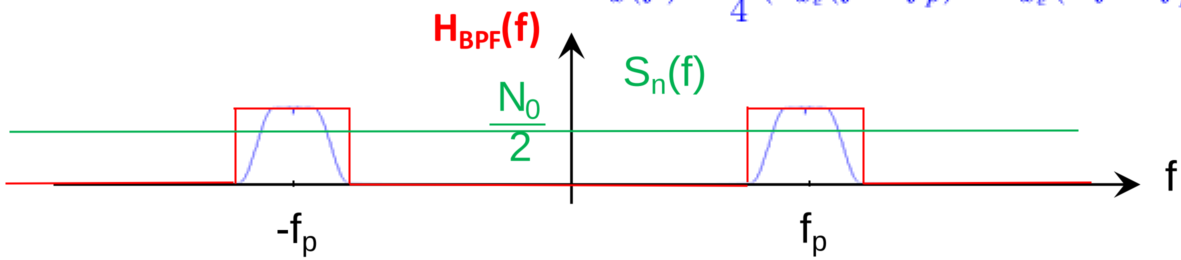


Linear Carrier Modulation

Equivalent lowpass channel: construction

$\longrightarrow$ Bandpass filtering:

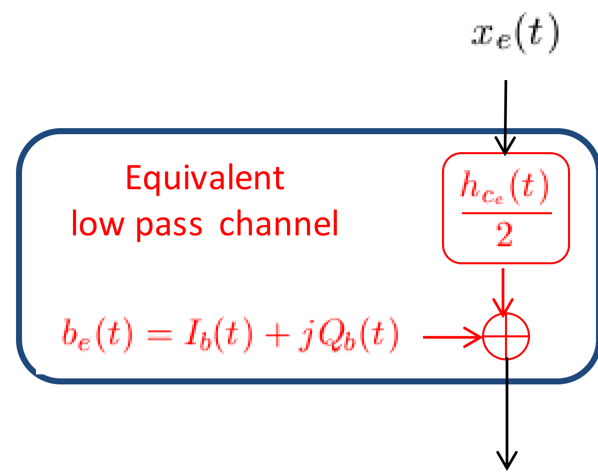
$$S_x(f) = \frac{1}{4}\left(S_{x_e}(f - f_p) + S_{x_e}(-f - f_p)\right)$$



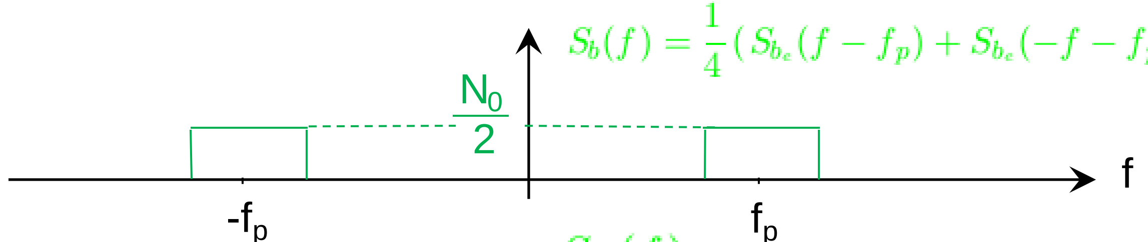
$\longrightarrow$ Complex envelop associated to the filtered noise:

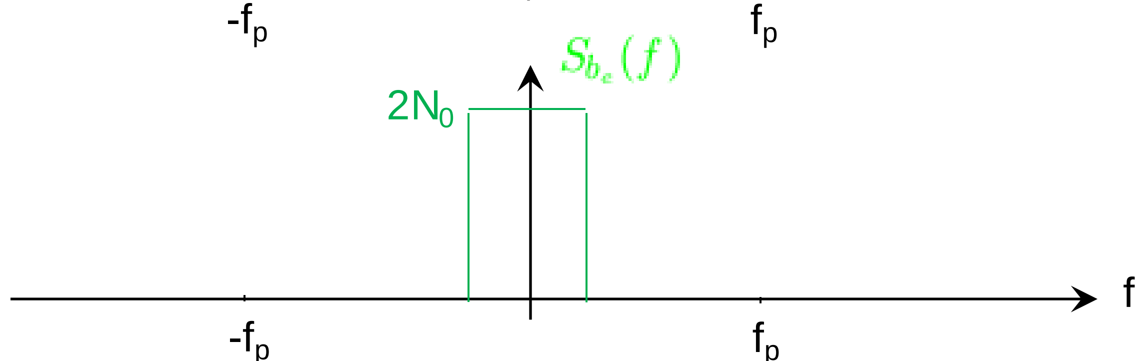
$$b_e(t) = I_b(t) + jQ_b(t)$$

$$S_{I_b}(f) = S_{Q_b}(f) = S^{+}_{b_e}(f - f_p) + S^{-}_{b_e}(f + f_p) = N_0$$



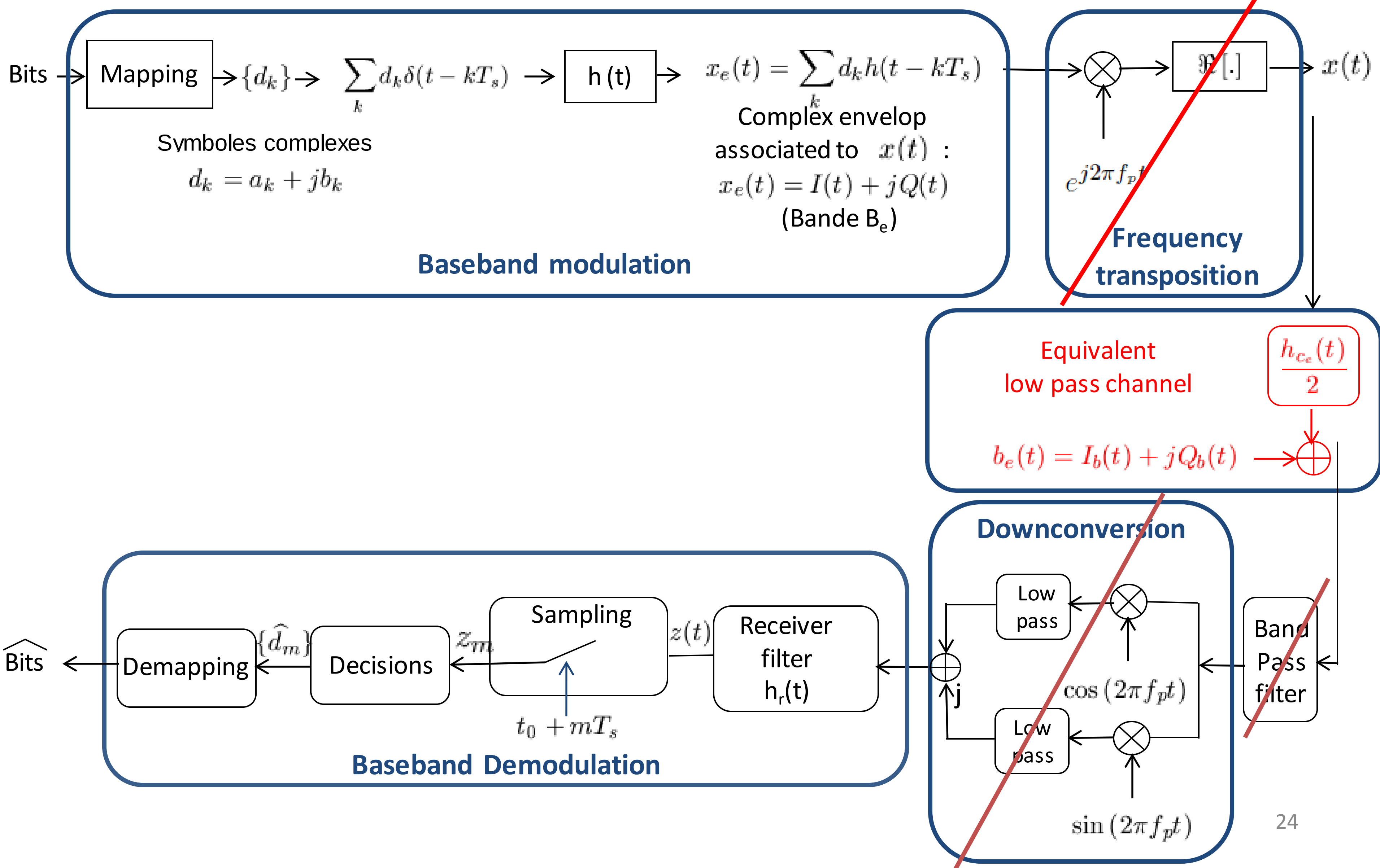
$$S_b(f) = \frac{1}{4}\left(S_{b_e}(f - f_p) + S_{b_e}(-f - f_p)\right)$$





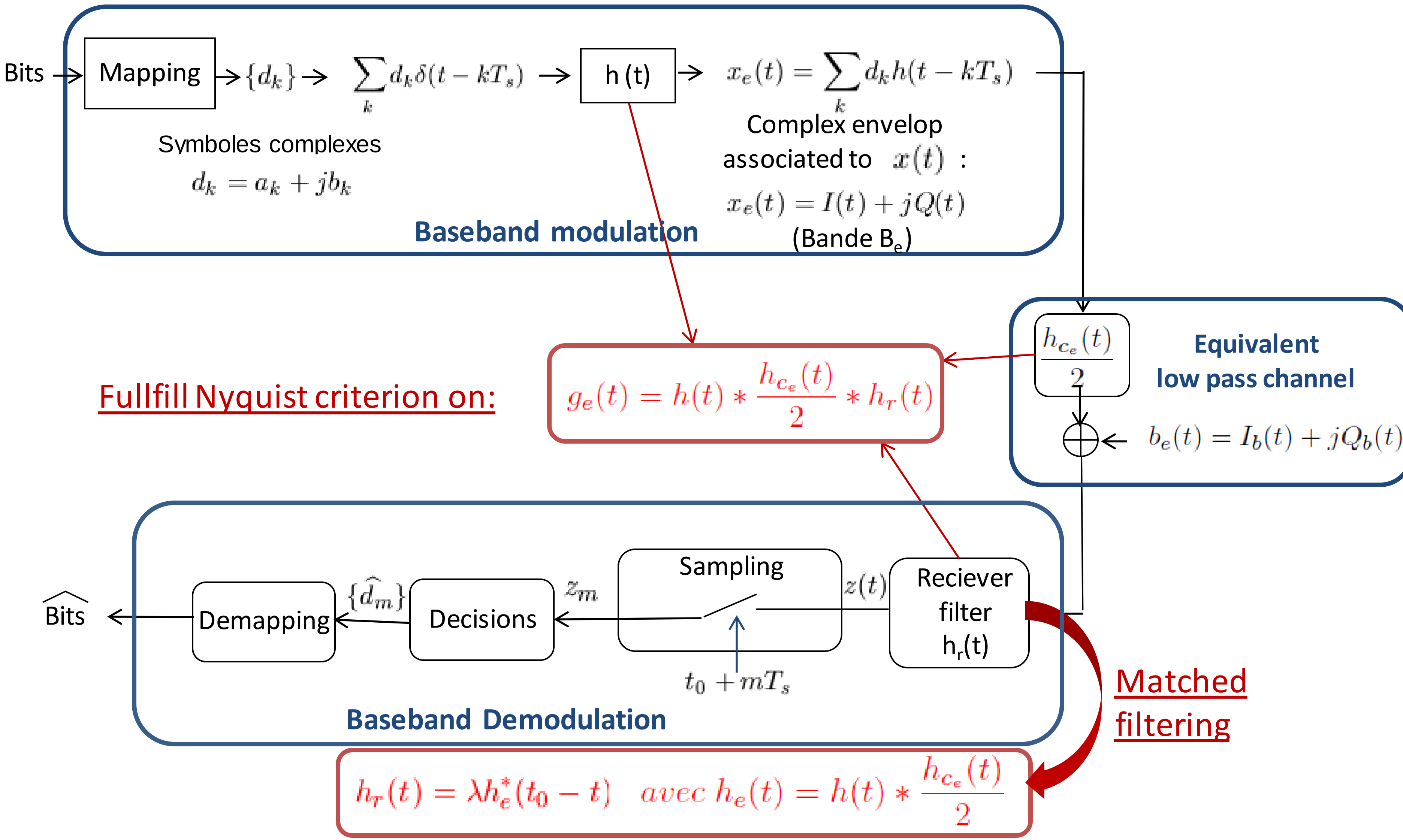
Linear Carrier Modulation

Equivalent lowpass channel: construction



Linear Carrier Modulation

Equivalent lowpass channel

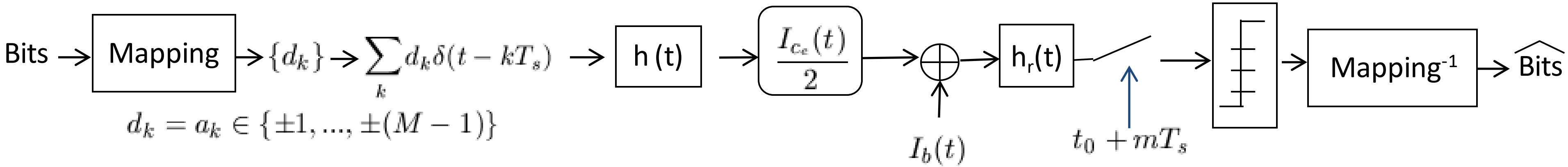


Baseband SER computations can be re-used

Linear Carrier Modulation

Performance (Hypothesis : Nyquist + Matched filtering)

→ M-ASK



$d_k = a_k \in \{\pm 1, ..., \pm(M-1)\}$

$$SER = SER_I = 2\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)$$

→ Squared M-QAM

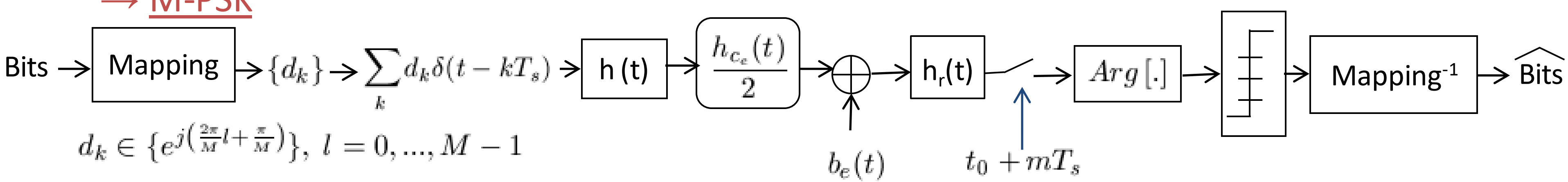
$$d_k = a_k + jb_k \; with \; a_k \; and \; b_k \in \{\pm 1, ..., \pm(\sqrt{M}-1)\}$$

⇔ two independen $\sqrt{M}$ -PAM transmissions

But !! Es = physical parameter = average symbol energy at the receiver input (M symbols d_k) !!

$$SER \simeq 2SER_I = 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right) = 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\log_2(M)}{M-1}\frac{E_b}{N_0}}\right)$$

→ M-PSK

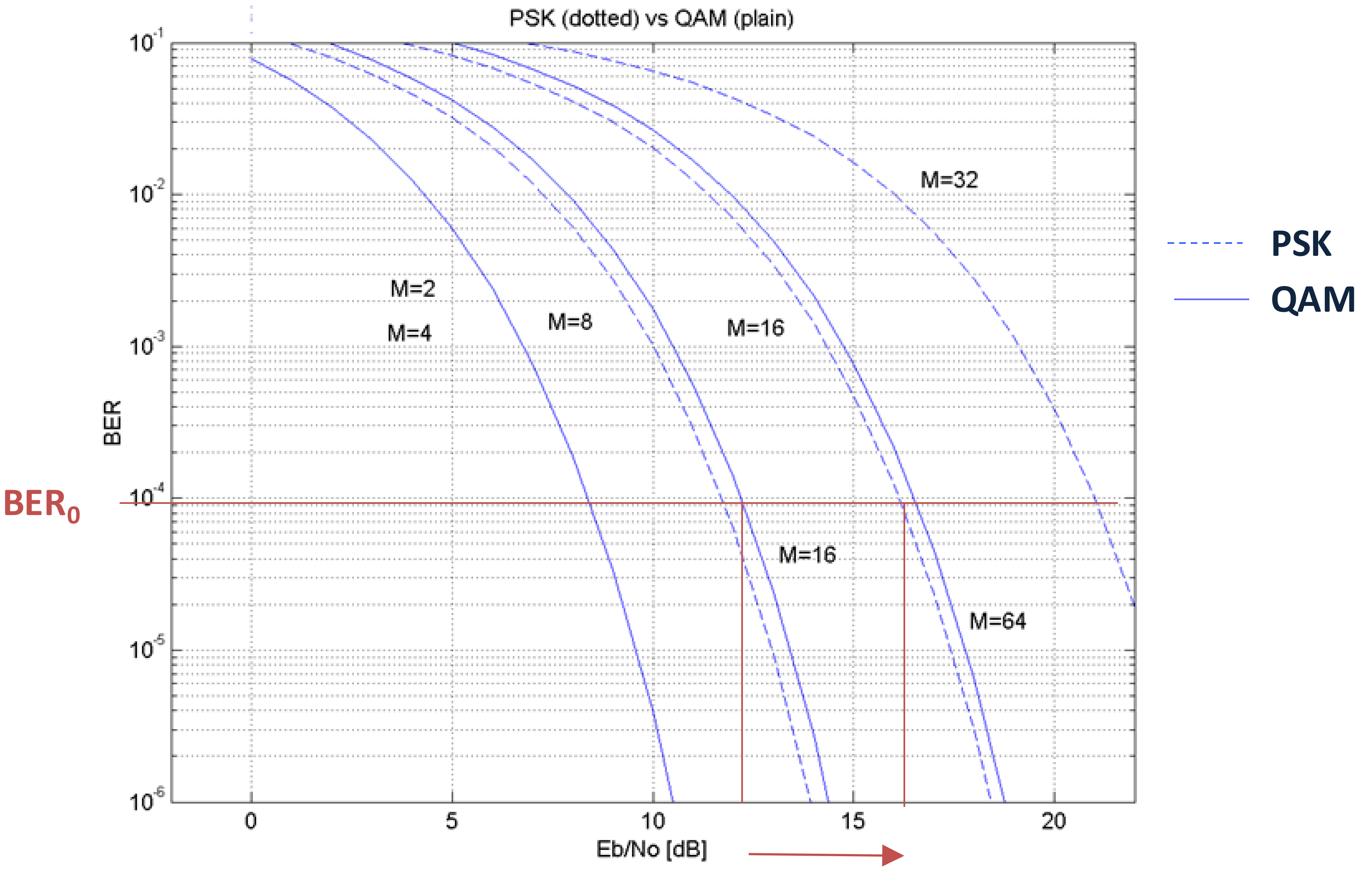


$d_k \in \{e^{j\left(\frac{2\pi}{M}l + \frac{\pi}{M}\right)}\}, \; l = 0, ..., M-1$

$$SER = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

Linear Carrier Modulation

BER comparison for M-QAM and M-PSK



Power efficiency for PSK
Same spectral efficiency

QUESTION

Assuming, for ech case, that the shaping filter is the same and that the transmission channel is optimzed (Gray Mapping, Nyquist, Matched filtering, optimal sampling and thresholds), a modulation 16-QAM will be:

More power efficient than a 16-PSK modulation:

A TRUE

B FALSE

More spectrally efficient than a 16-PSK modulation:

A TRUE

B FALSE

More power efficient than a QPSK modulation:

A TRUE

B FALSE

More spectrally efficient than a QPSK modulation:

A TRUE

B FALSE

QUESTION

Assuming, for ech case, that transmission channel is optimzed (Gray Mapping, Nyquist, Matched filtering, optimal sampling and thresholds), using a 16-QAM modulation with a retangular shaping filter is:

More power efficient than using a 16-QAM modulation with a square root raised cosine filter:

A TRUE

B FALSE

More spectrally efficient than using a 16-QAM modulation with a square root raised cosine filter :

A TRUE

B FALSE

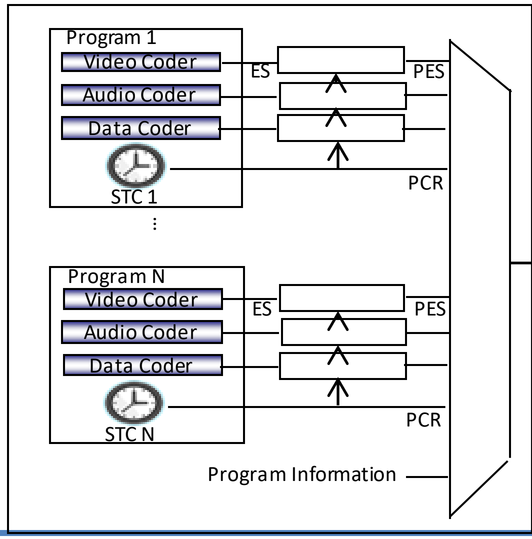
Example of physical layer on an AWGN channel:

Satellite Digital Video Broadcasting : DVB-S (1994)

Satellite Digital Video Broadcasting : DVB-S

Physical layer

Source coding and multiplexing

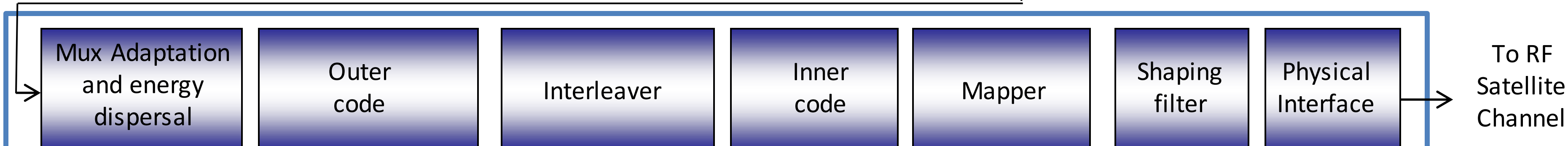


MPEG-2 System Transport stream generation

Transport Stream

Digital TV transmission must be « Quasi Error Free » (QEF) TEB < 10^{-10}

Physical layer



Reed Solomon RS(204,188, t=8)

Forney Convolutionnal Interleaving

Convolutional Code (7,1/2)

QPSK

SRRCF $\alpha = 0.35$

Table D.1: Example of System performance over 33 MHz transponder

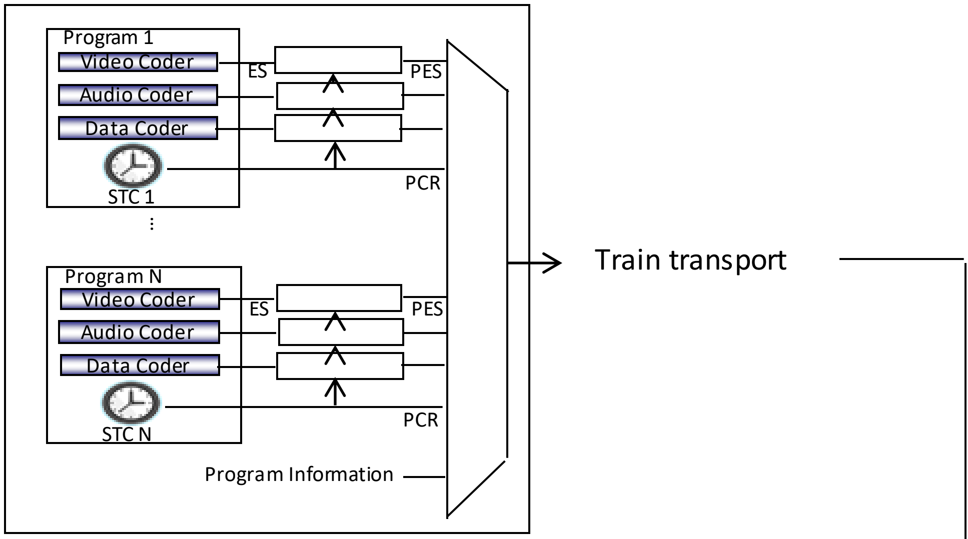
Bit rates

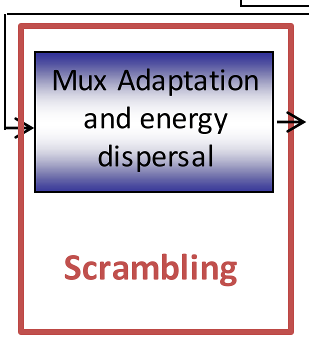
Bit Rate R_u (after MUX) [Mbit/s]	Bit Rate R'_u (after RS) [Mbit/s]	Symbol Rate [Mbaud]	Convolut. Inner Code Rate	RS Outer Code Rate	C/N (33 MHz) [dB]
23,754	25,776	25,776	1/2	188/204	4,1
31,672	34,368	25,776	2/3	188/204	5,8
35,631	38,664	25,776	3/4	188/204	6,8
39,590	42,960	25,776	5/6	188/204	7,8
41,570	45,108	25,776	7/8	188/204	8,4

Satellite Digital Video Broadcasting : DVB-S

Physical layer

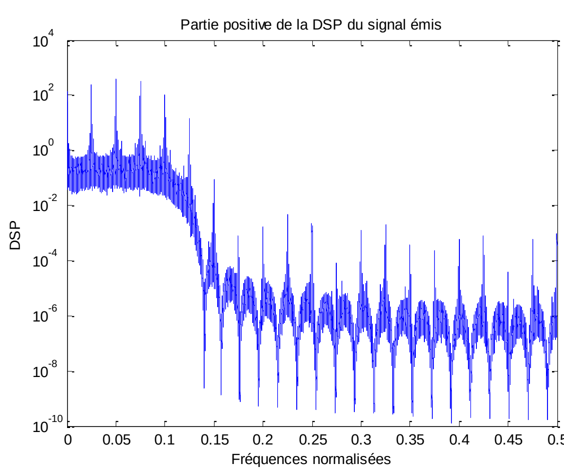
Codage source et multiplexage



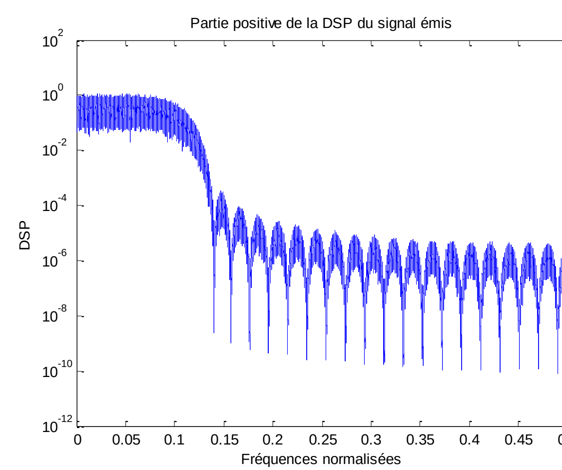


Example on an image

PSD of the unscramble signal :

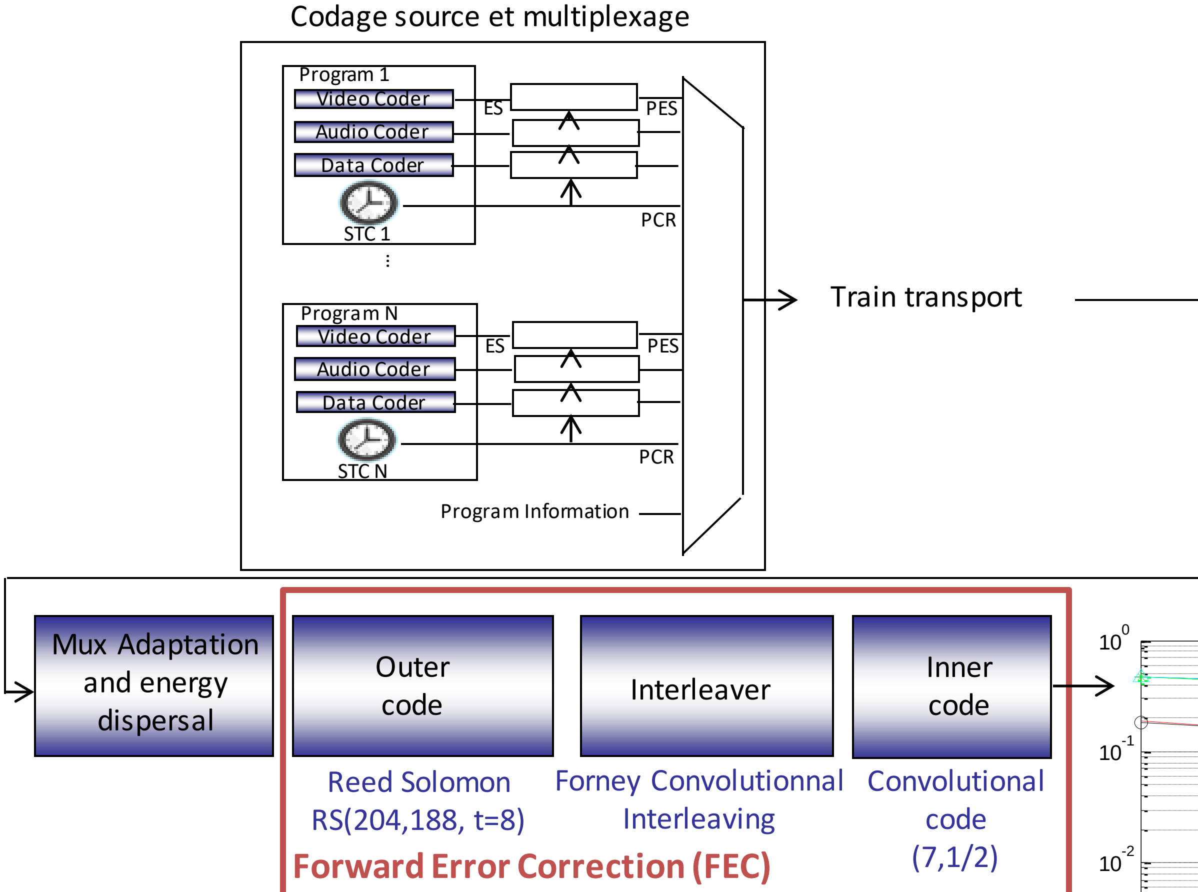


PSD of the scramble signal :

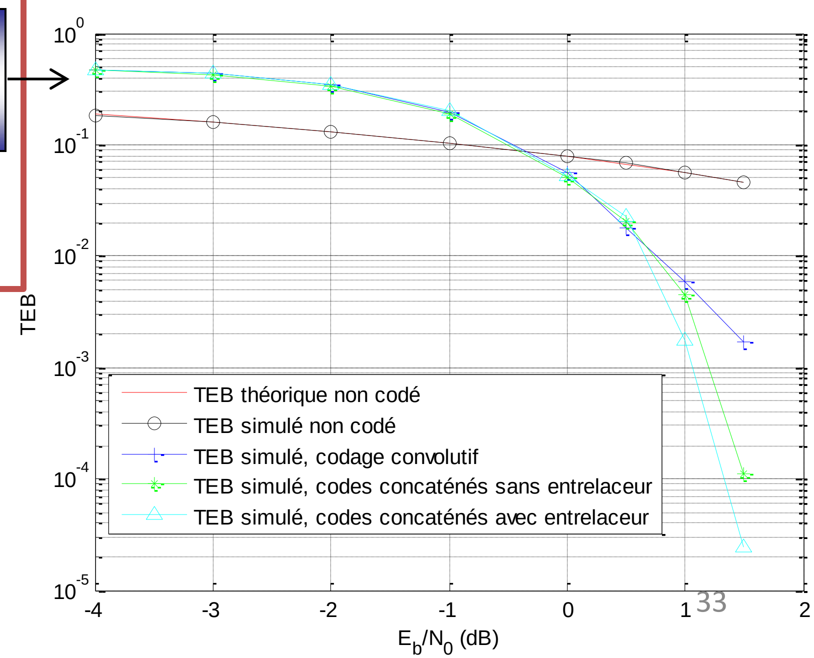


Satellite Digital Video Broadcasting : DVB-S

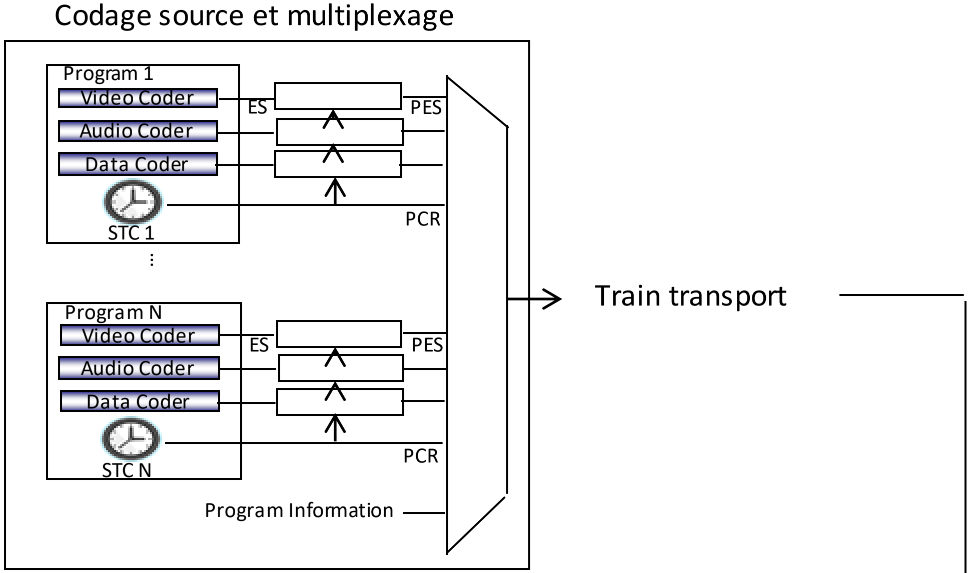
Physical layer



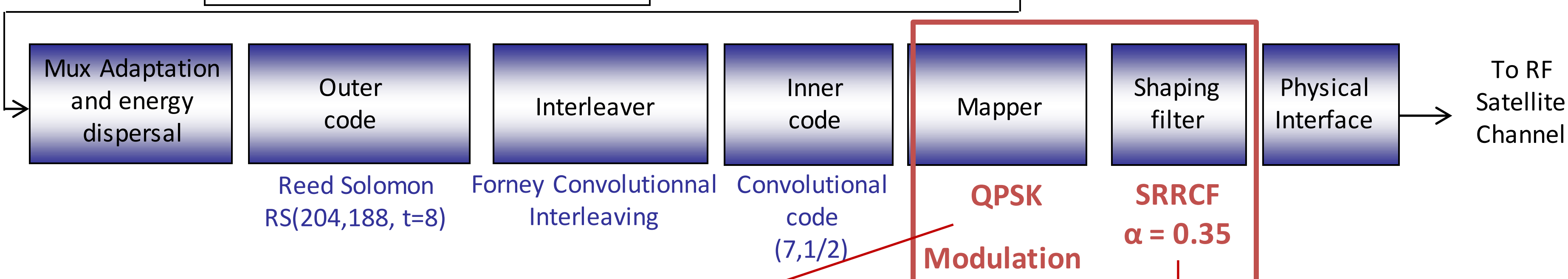
A digital TV transmission must be « Quasi Error Free » (QEF) : TEB < 10^{-10}

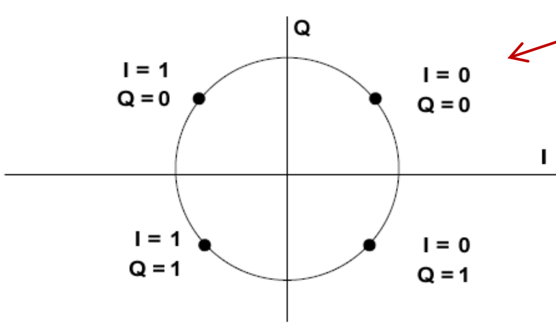


Satellite Digital Video Broadcasting : DVB-S
Physical layer



AWGN channel with non linearities





$$H(f)=\begin{cases} 1 \quad \text{pour } |f| < f_N(1-\alpha) \\ \left\{\dfrac{1}{2}+\dfrac{1}{2}\sin\left(\dfrac{\pi}{2f_N}\left[\dfrac{f_N-|F|}{\alpha}\right]\right)\right\}^{1/2} \quad \text{pour } f_N(1-\alpha)\le |f| \le f_N(1+\alpha) \\ 0 \quad \text{pour } |f| > f_N(1+\alpha) \end{cases}$$

References

→ Digital Communications, J. G. Proakis, Mac Graw Hill Book Cie

→ Telecommunications system engineering, Lindsay and Simon, Prentice Hall

→ Digital communication by satellite, J.J. Spilker, Prentice Hall

→ Digital Video Broadcasting (DVB): Framing structure, channel coding and modulation for 11/12 GHz satellite services, norme ETSI EN 300 421.

→ Digital Video Broadcasting (DVB): User guidelines for the second generation system for broadcasting, interactive services, news gathering and other broadband satellite applications (DVB-S2), norme ETSI EN 102 376.