Digital Communications
Short Introduction
(more details in moodle)

1) Definitions
2) Propagation channel
3) Elements of the communication channel
4) Performance criteria

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Basic digital transmission channel

Transmit a given bit rate Rb = Number of bits to be transmitted per second.

DVB example:

BER<10^{-10}, (QEF transmission)

Rb $\sim$ 30 à 40 Mbps

Obtain a given Bit Error Rate:

$$BER = \frac{\text{Number of erroneous bits}}{\text{Number of transmitted bits}}$$

Analog signal : sound, image …

↓

Digitization

↓

Binary information to transmit: 0 1 1 0 0 1 0 1 1 0

↓

B A S I C C H A N N E L

Transmitter

↓

Analog signal

↓

Propagation channel

↓

Corrupted analog signal

↓

Receiver

↓

Received Binary information: 0 1 0 1 0 1 1 1 1 1

Example:

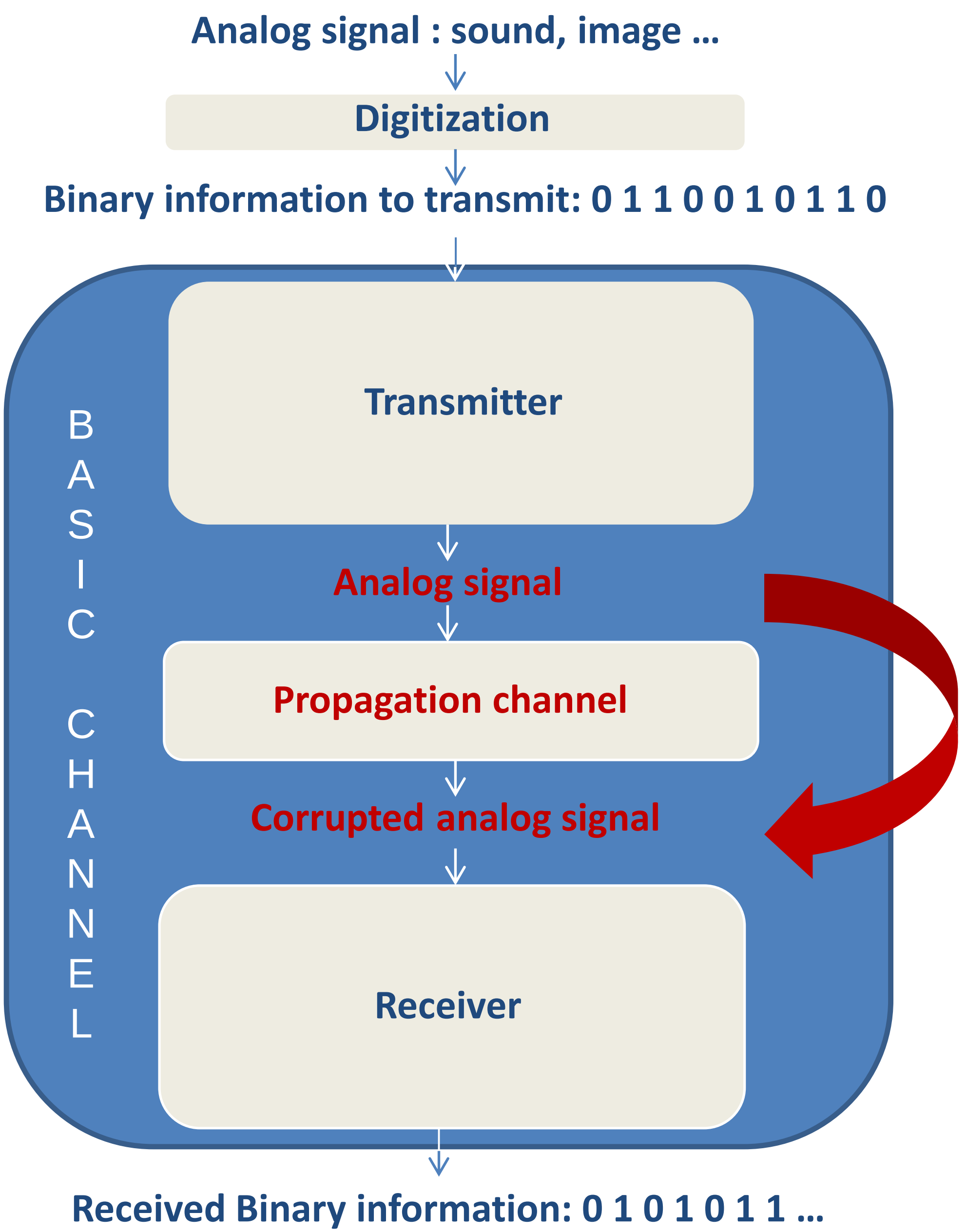
0 1 1 0 0 1 0 1 1 0

☺ ☺ ☹ ☹ ☺ ☺ ☹ ☺ ☺ ☹

0 1 0 1 0 1 1 1 1 1

BER = 4/10

Basic digital transmission channel



Propagation channel: **Additive noise**

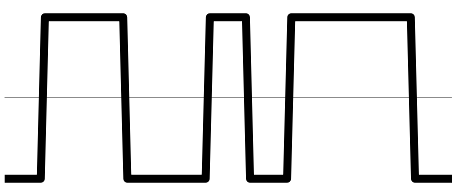
→ Additive noise: example of free space propagation

- → Other signals received in addition to the useful communication signal.
  - – Coming from natural sources: atmosphere (storm, lightning, thunder), earth, sky (sun, milky way)
  - – Coming from artificial sources: human activity.
- → Electronic devices in the receiver: amplifiers, antenna, etc.

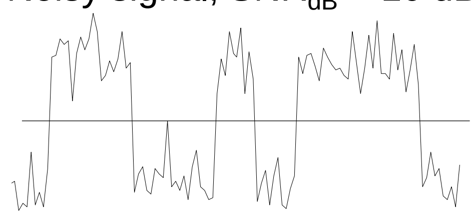
→ Examples of introduced distortions

<u>Examples :</u>

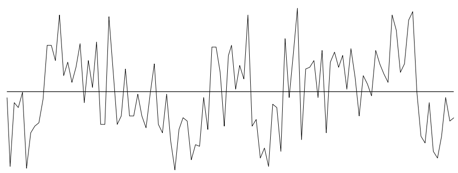
NRZ-type transmitted signal



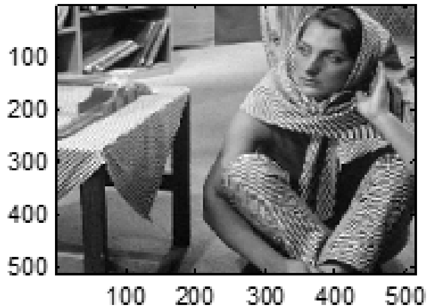
Noisy signal, SNR_{dB} = 10 dB



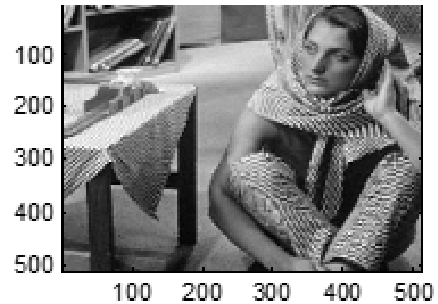
Noisy signal, SNR_{dB} = 0 dB



Transmitted image

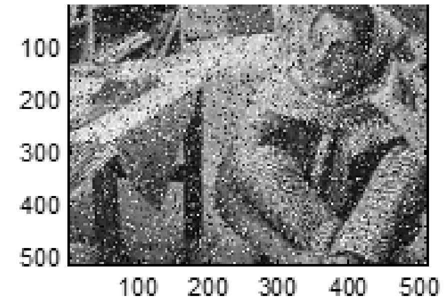


Received image, SNR_{dB} = 10 dB



BER = 2.38 10^{-6}

Received image, SNR_{dB} = 0 dB

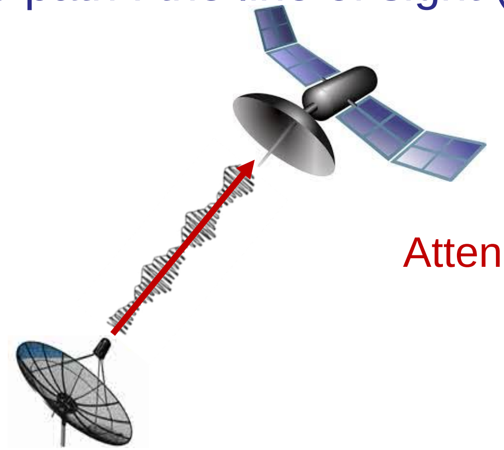


BER = 0.0784

Propagation channel

One or several paths between the transmitter and the receiver

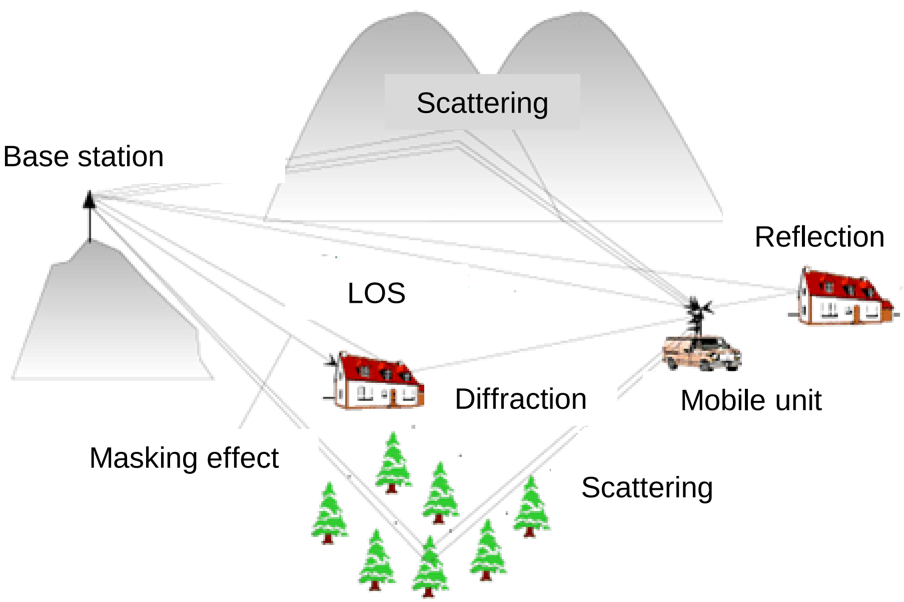
- Only one path : the line of sight (LOS) between the transmitter and the receiver



Attenuation and delay introduced by the channel

$$y(t) = \alpha \; x(t-\tau)+n(t)$$

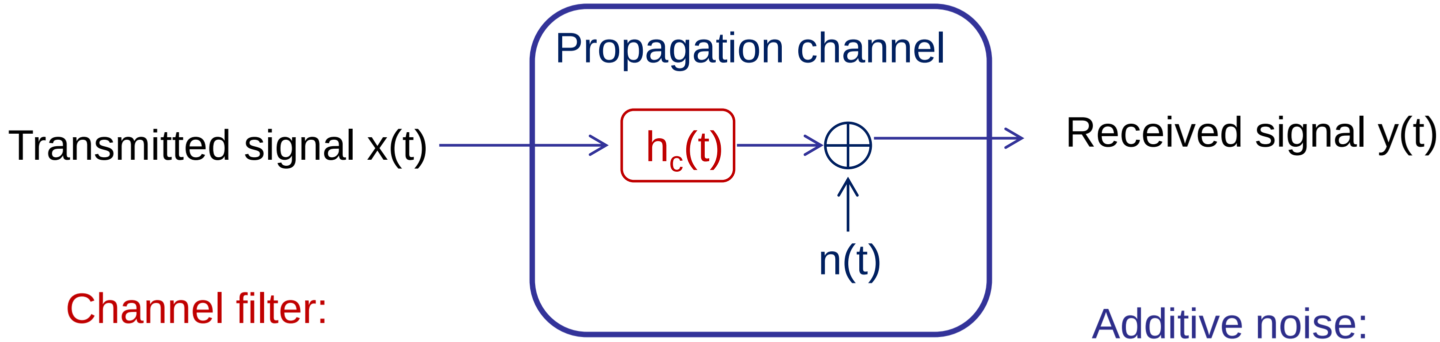
- Several paths between the transmitter and the receiver (« multi-paths » channel)



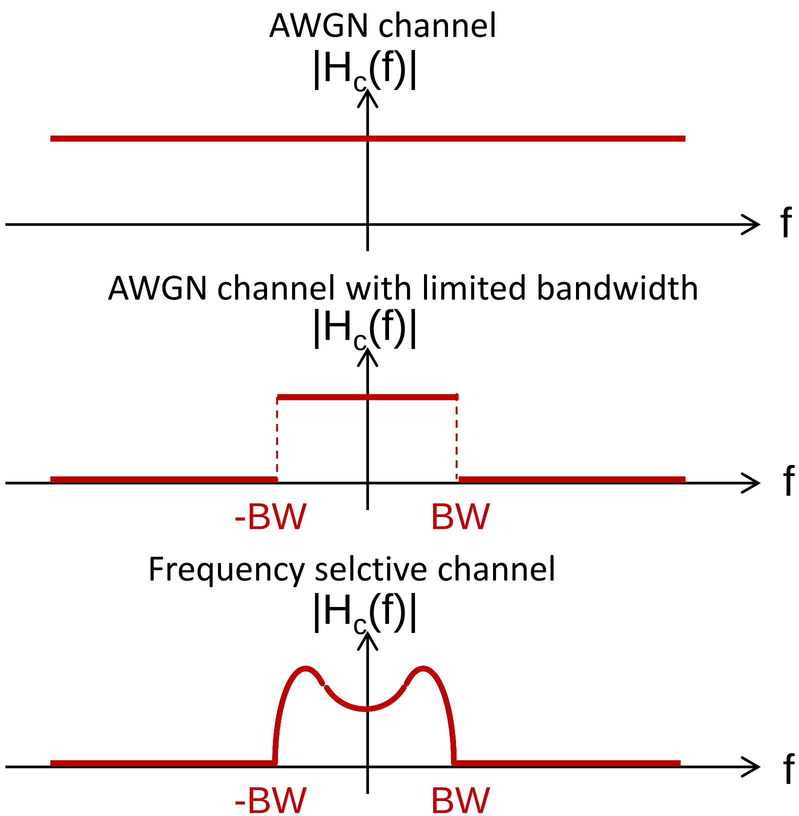
Several attenuations and delays introduced by the channel

$$y(t) = \sum_{k=0}^{N-1} \alpha_k \; x(t-\tau_k)+n(t)$$

Propagation channel : **model**

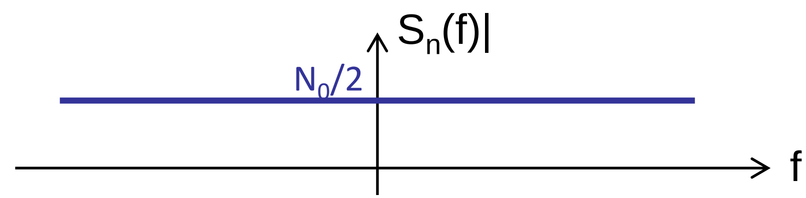


Channel filter:



Additive noise:

→ **White** noise: PSD = $N_0/2$ whatever ithe frequency



→ **Gaussien** noise, power σ^2

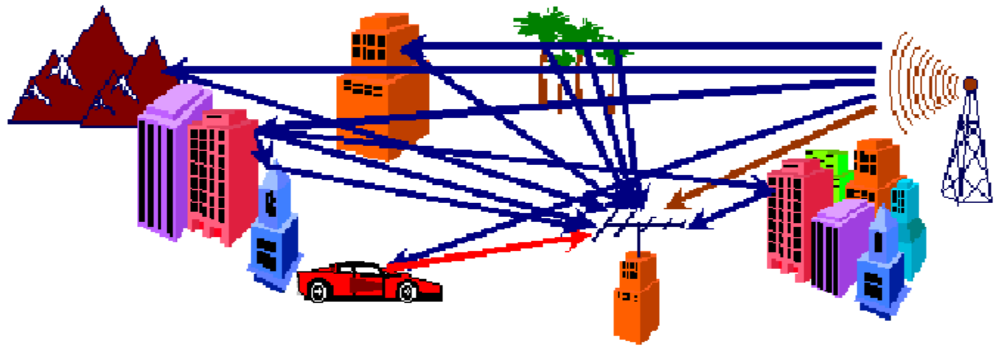
→ A degradation mesurment: the **signal to noise ratio** (SNR)

$$SNR_{dB} = 10 \log \frac{P_{\text{useful signal}}}{P_{\text{noise}}}$$

Mais aussi canal non stationnaire, sélectif en temps … (voir en 2A)

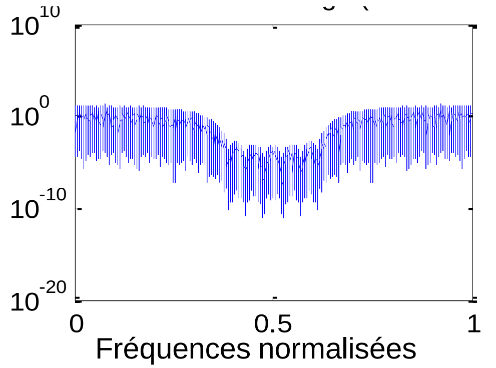
Propagation channel

Example of a frequency selective channel: DTV (DVB-T) transmission

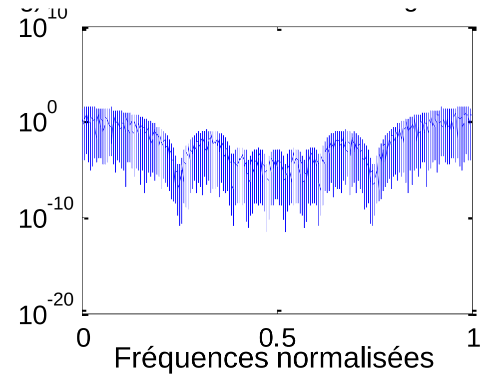


- Example of distorsions introduced by a frequency selective channel

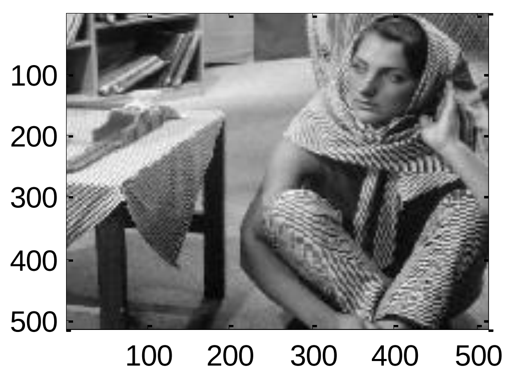
PSD of the transmitted signal:



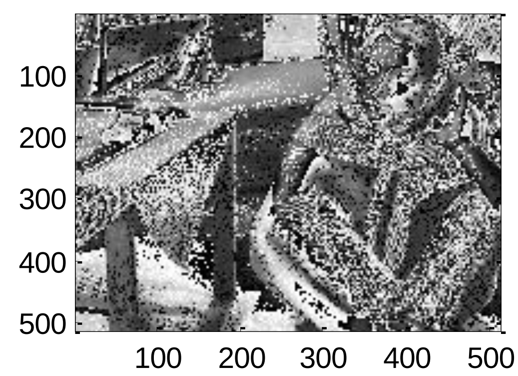
PSD of the received signal:



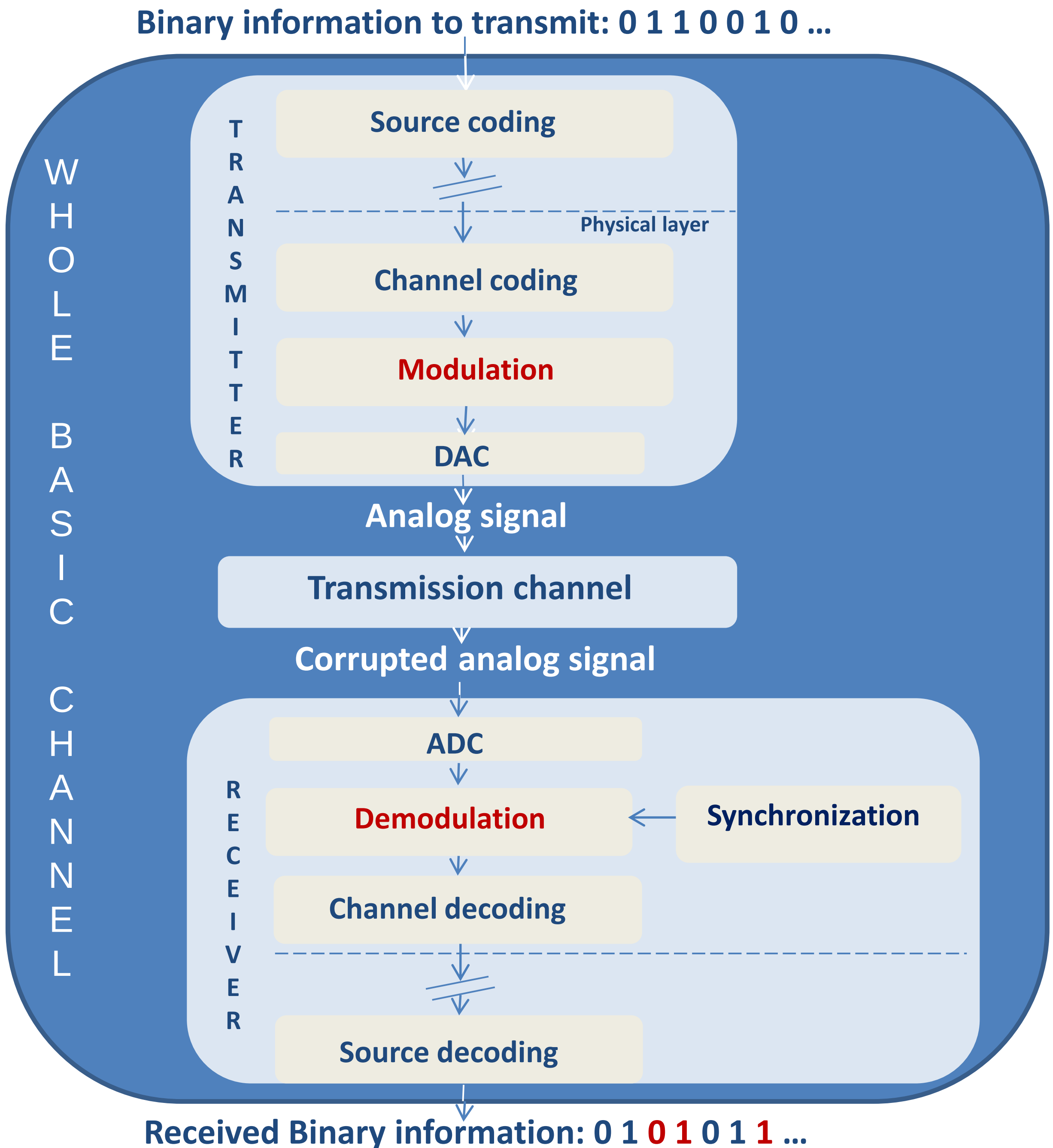
Transmitted image:



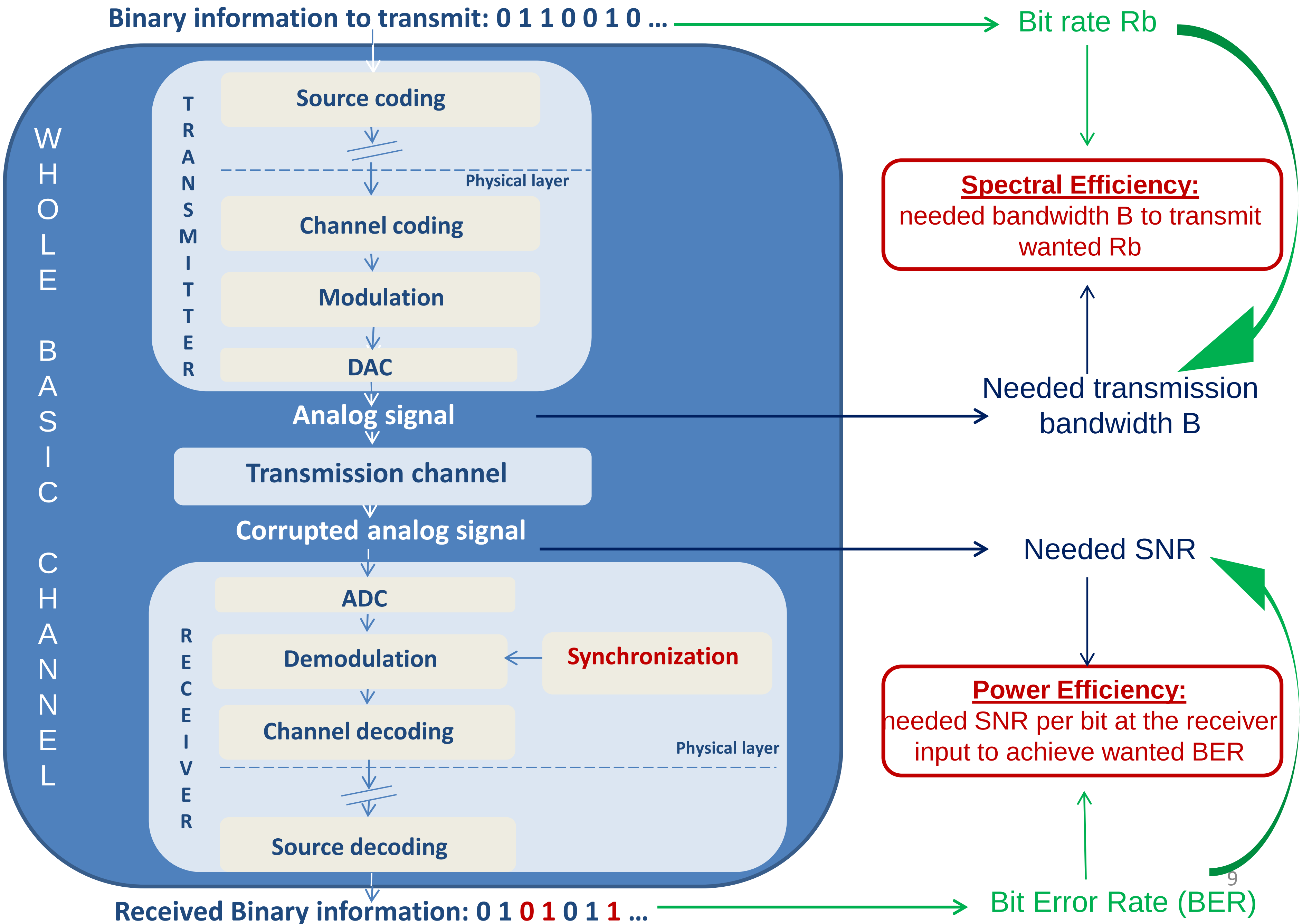
Received image:



Basic digital transmission channel: synchronization



Basic digital transmission channel: performance criteria



Digital Communications

Baseband Modulation/Demodulation: joint optimization

1) Signal generation => Spectral efficiency
2) Inter Symbol Interference(ISI) => Nyquist criterion
3) Noise influence => Matched filtering
4) BER computation => Power efficiency

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Baseband Digital Modulation

Some signal examples

→ Elementary coding with independent symbols

→ Coding using a level :

→ Unipolar NRZ :

→ Polar NRZ :

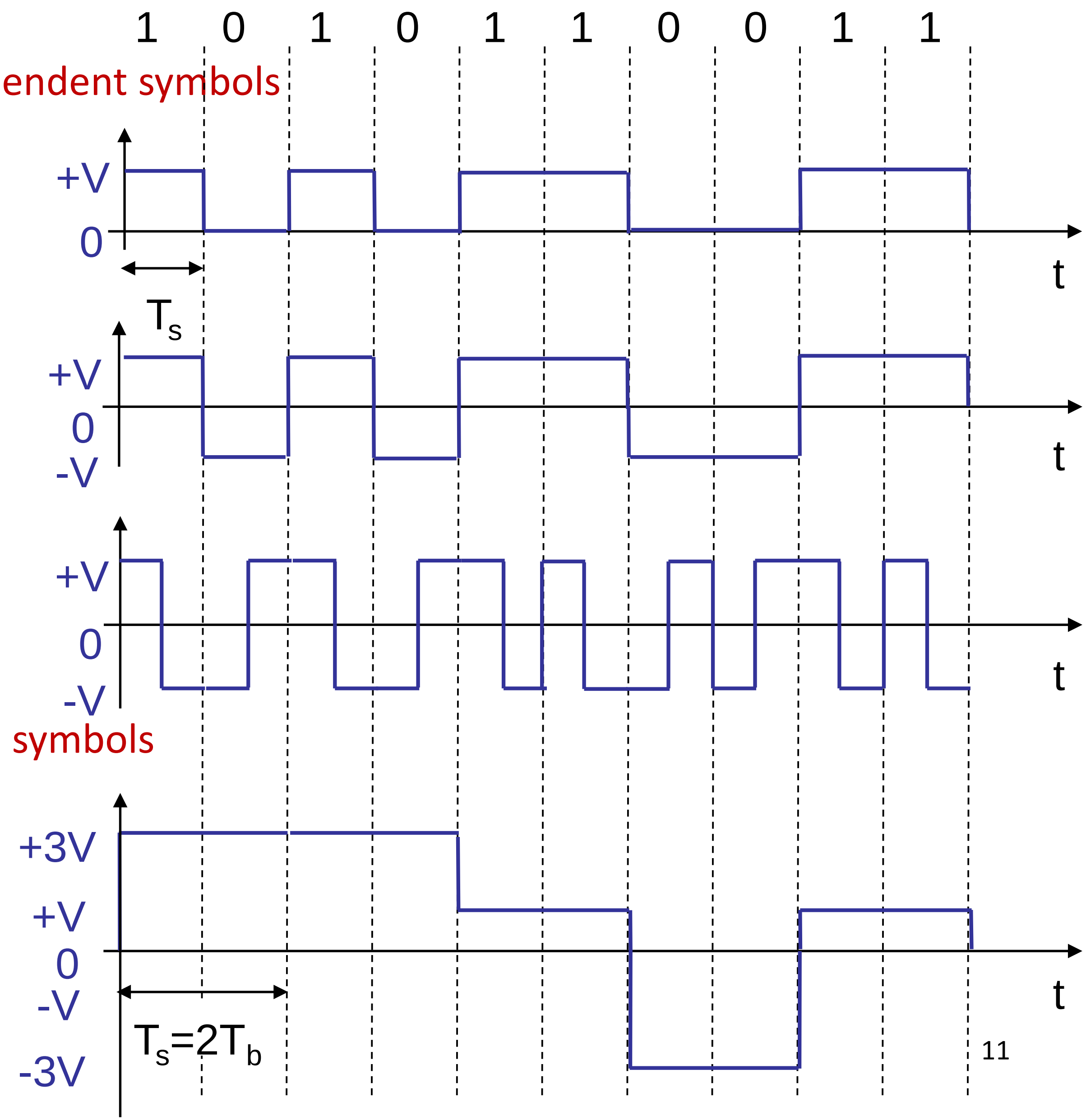
→ Coding with an edge

→ Biphase :

→ Bloc coding with independent symbols

→ Coding using a level :

→ Several levels NRZ :



Baseband Digital Modulation

General model

Binary information:
0 1 1 0 0 1 0 1 1 0 → **Baseband Modulation** → $x(t) = \sum_k a_k h(t - kT_s)$

Bit rate $R_b = 1/T_b$

Occupied bandwidth B ?

Binary information
0 1 1 0 0 1 0 1 1 0 → **Mapping** → M-ary Symbols $\{a_k\}$ → $\sum_k a_k \delta(t - kT_s)$ → **Shaping filter h(t)** → **x(t)**

$a_k \in \{\pm V, \pm 3V, ..., \pm(M-1)V\}$

Baseband Modulation

Symbol rate = number of symbols transmitted per seconds:

$$\boxed{R_s = \frac{R_b}{\log_2(M)}}$$

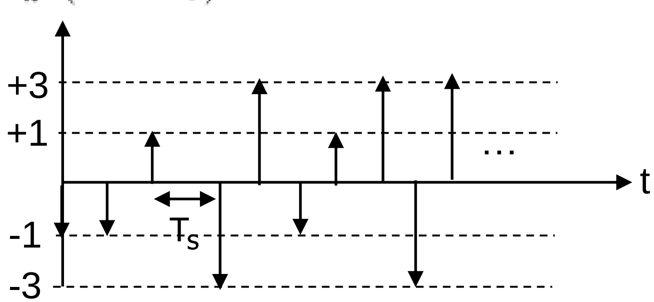
Symbol rate (symbols/s or bauds)

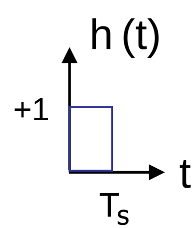
Bit Rate (bits/s)

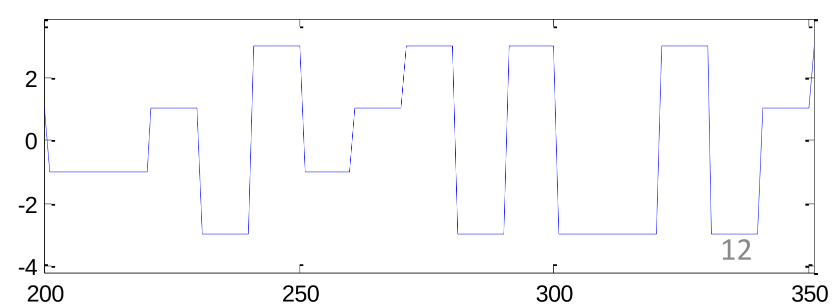
M=number of possible symbols

Example (NRZ, M=4):

$\sum_k a_k \delta(t - kT_s)$





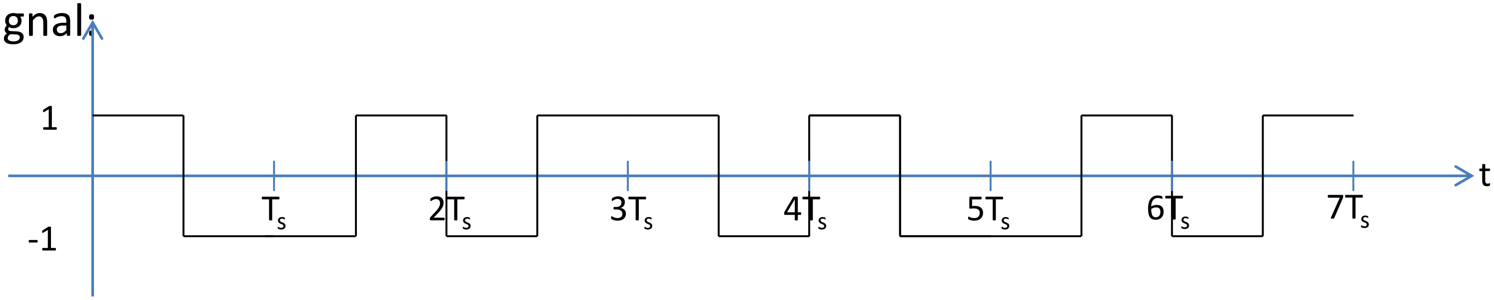


Generation of a polar NRZ

```
%Symbol duration in number of samples
Ns=4;
%Number of generated bits
nb_bits=100;
%Bit generation
bits=randint(1,nb_bits);
%Symbol generation : 0->-1, 1->1
Symboles=2*bits-1;
%Weighted Dirac delta function series
Diracs=kron(Symboles, [1 zeros(1,Ns-1)]);
%Shaping filter impulse response (for NRZ)
h=ones(1,Ns)
%Shaping filtering
y=filter(h,1,Diracs);
%Signal display
plot(y);
axis([0 nb_bits-1 -1.5 1.5]);
```

Bits to tansmit: 1001100

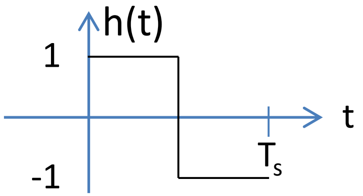
Generated signal:



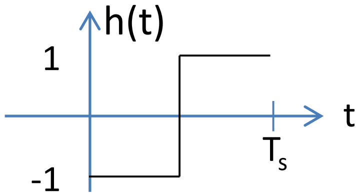
QUESTION

The mapping and the shaping filter impulse response h(t) are :

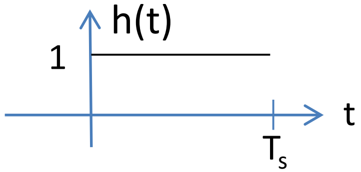
A Mapping : 0 -> -1, 1 -> +1



B Mapping : 0 -> +1, 1 -> -1



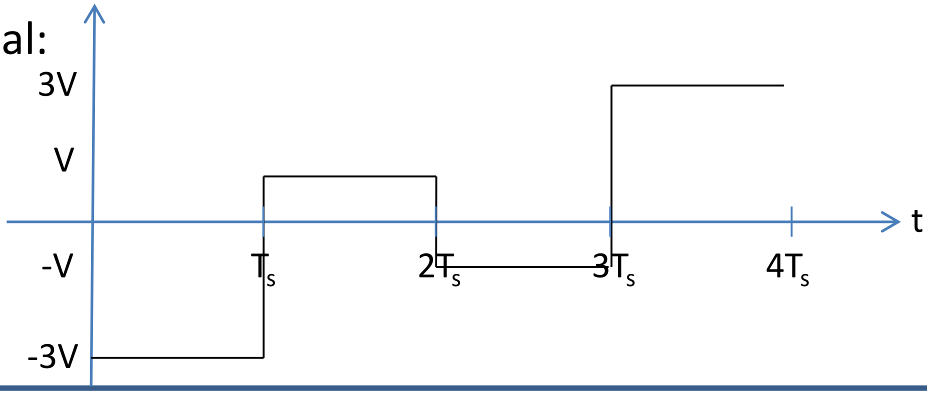
C Mapping : 0 -> -1, 1 -> +1



D Not enough elements to answer

Bits to tansmit : 00100111

Generated signal:



QUESTION

The symbol rate will be:

A The same as the bit rate

B Higher than the bit rate

C Lower than the bit rate

D Not enough elements to answer

Baseband Digital Modulation

General model

Binary information:
0 1 1 0 0 1 0 1 1 0

Bit rate $R_b = 1/T_b$

Baseband Modulation

$$x(t) = \sum_k a_k h(t - kT_s)$$

Occupied bandwidth B ?

Binary information
0 1 1 0 0 1 0 1 1 0

Mapping

M-ary Symbols

$\{a_k\}$

$a_k \in \{\pm V, \pm 3V, \ldots, \pm(M-1)V\}$

$$\sum_k a_k \delta(t - kT_s)$$

Shaping filter h(t)

x(t)

Cyclostationnary signal

Baseband Modulation

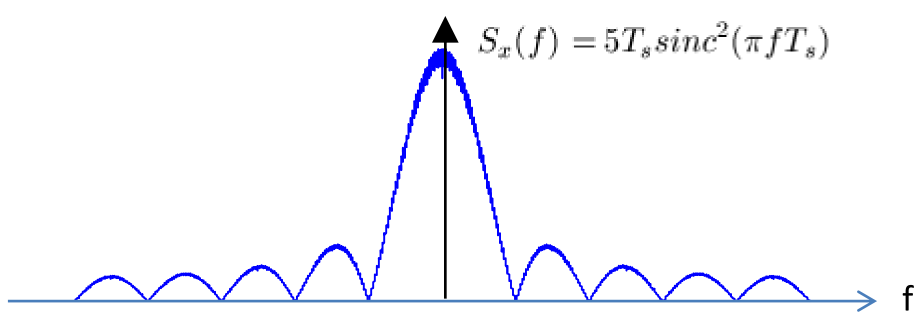
Baseband Pulse Amplitude Modulation : M-PAM = Linear Modulation, spectrum around frequency 0

$$S_x(f) = \frac{\sigma_a^2}{T_s}|H(f)|^2 + 2\frac{\sigma_a^2}{T_s}|H(f)|^2 \sum_{k=1}^{\infty} \mathfrak{Re}\left[R_a(k)e^{j2\pi f k T_s}\right] + \frac{|m_a|^2}{T_s^2}\sum_k \left|H\left(\frac{k}{T_s}\right)\right|^2 \delta\left(f - \frac{k}{T_s}\right)$$

where : $\sigma_a^2 = E\left[|a_k - m_a|^2\right]$; $m_a = E[a_k]$; $R_a(k) = \dfrac{E[a_m^* a_{m-k}] - |m_a|^2}{\sigma_a^2}$

Example (NRZ, M=4):

$a_k \in \{\pm 1, \pm 3\}$



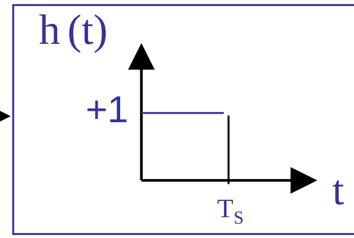
Baseband Digital Modulation

Some spectrum examples

→ Two level NRZ (GPS waveform)

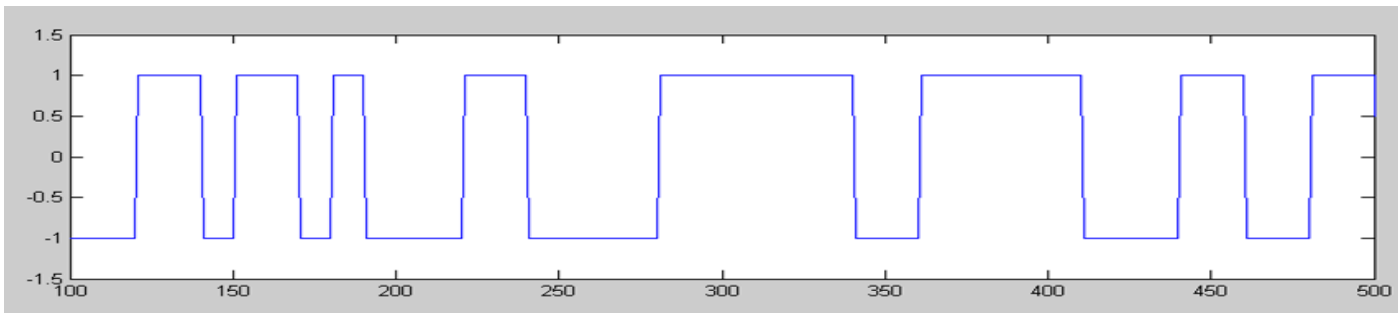
$a_k \in \{\pm 1\}$ equally likely

$$\sum_k a_k \delta(t - kT_s) \rightarrow$$

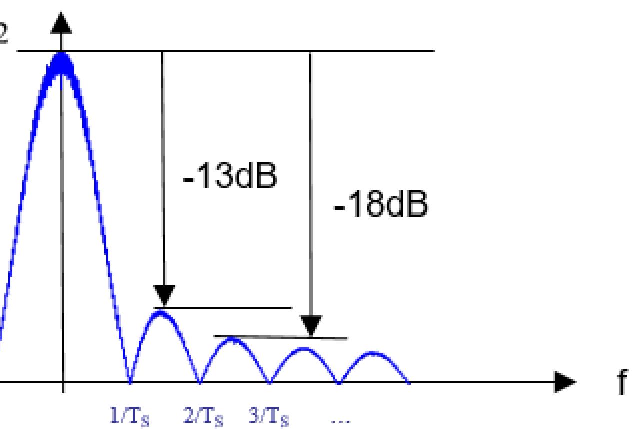


$$\rightarrow \quad x(t) = \sum_k a_k h(t - kT_s)$$

x(t)



$$S_X(f) = T_S \left\{ \frac{\sin(\pi f T_S)}{\pi f T_S} \right\}^2$$



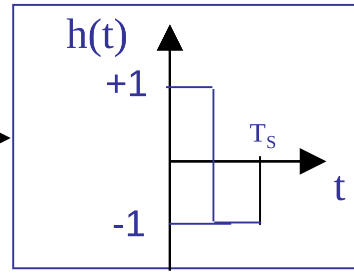
Baseband Digital Modulation

Some spectrum examples

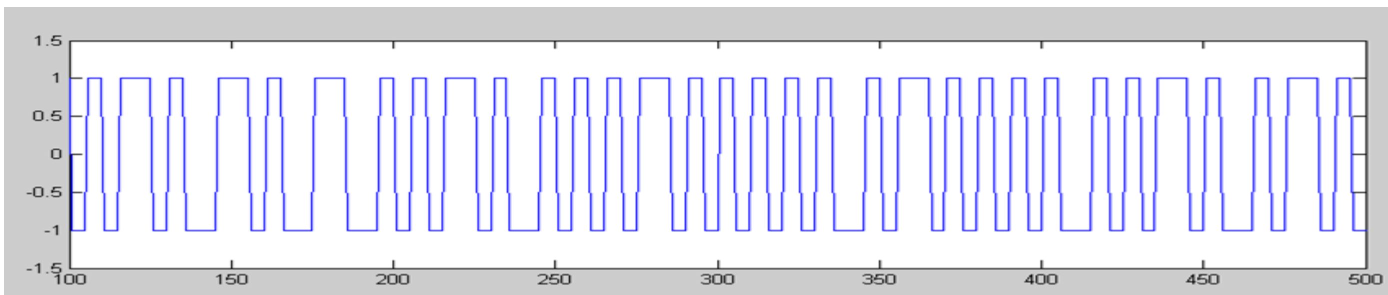
→ Biphase or Manchester (Ethernet waveform : IEEE802.3)

$a_k \in \{\pm 1\}$ equally likely

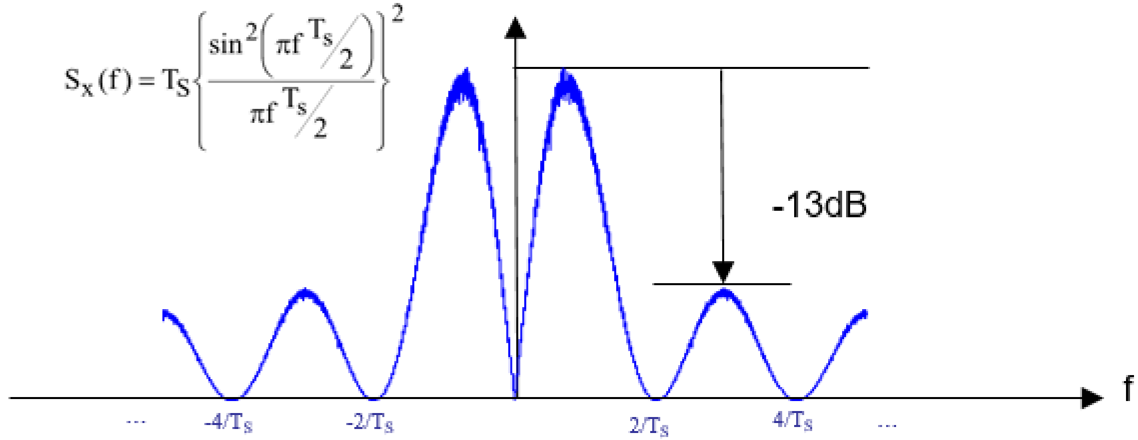
$$\sum_k a_k \delta(t-kT_s) \longrightarrow \boxed{h(t)} \longrightarrow x(t) = \sum_k a_k h(t-kT_s)$$



x(t)



$$S_X(f) = T_S\left\{\frac{\sin^2\left(\pi f \frac{T_S}{2}\right)}{\pi f \frac{T_S}{2}}\right\}^2$$

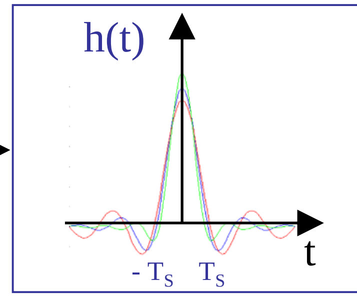


Baseband Digital Modulation

Some spectrum examples

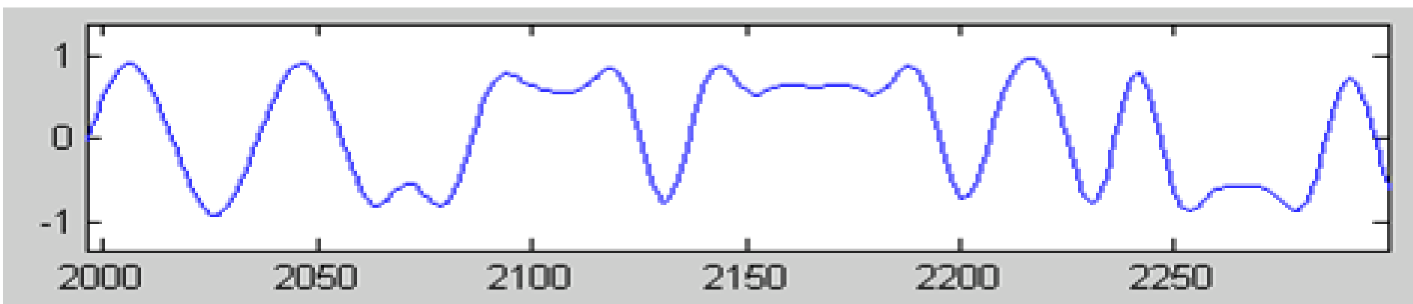
→ Square root raised cosine shaping (DVB-C, DVB-S waveform)

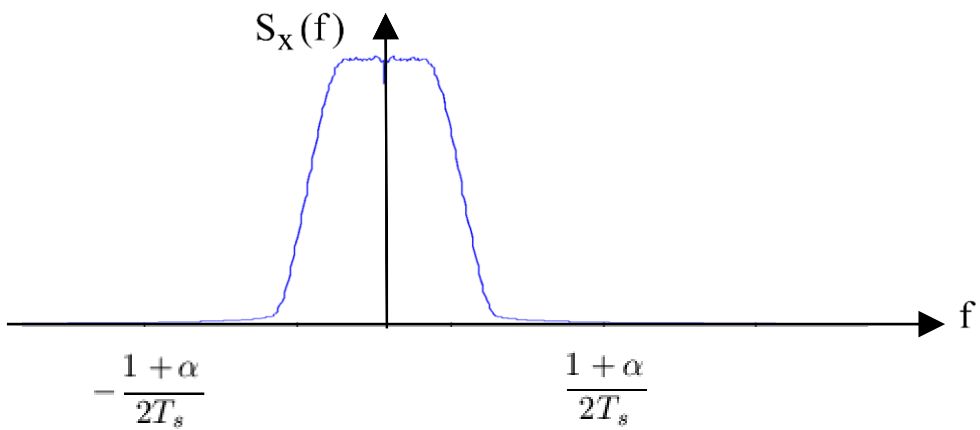
$$\sum_k a_k \delta(t - kT_s) \longrightarrow$$



$$\longrightarrow x(t) = \sum_k a_k h(t - kT_s)$$

x(t)





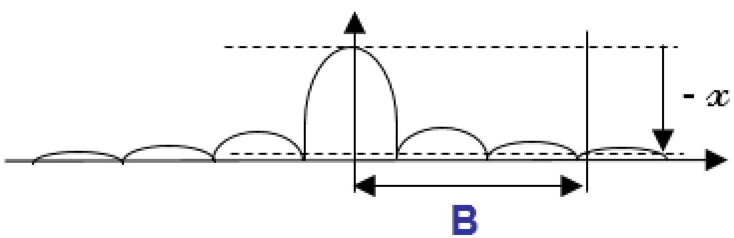
Spectral efficiency

→ <u>Bandwidth definition:</u>

- *Definition 1 :* frequency bandwidth B concentrating x % of the signal energy (typical values : 95 à 99 %)

$$\frac{\int_0^B S_x(f)df}{\int_0^\infty S_x(f)df} = \frac{x}{100}$$

- *Definition 2 :* frequency bandwidth beyond which the minimum rejection is of x dB (typical values: 20 à 30 dB)



→ <u>Spectral Efficiency (bits/s/Hz):</u>

$$\boxed{\eta = \frac{R_b}{B} = \frac{\log_2(M)}{k}}$$

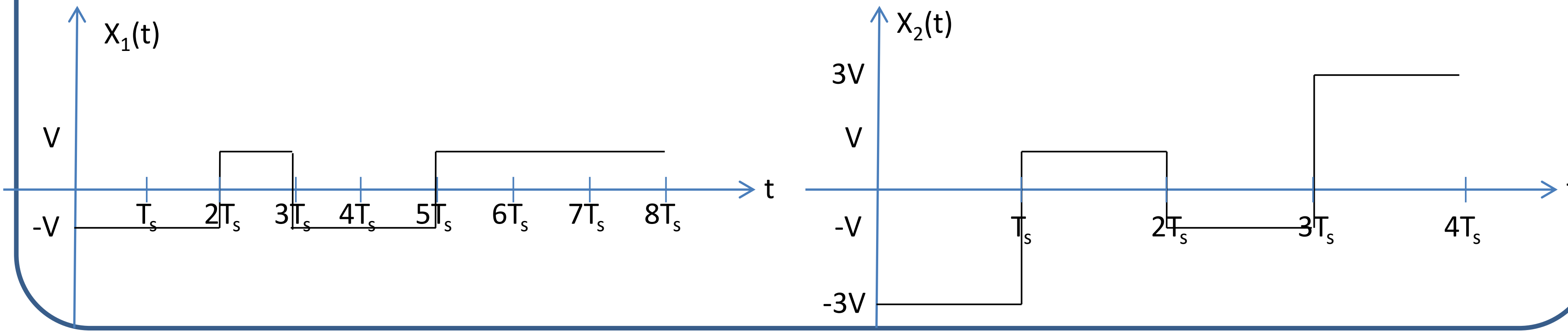
$a_k \in \{\pm V, \pm 3V, \dots, \pm(M-1)V\}$

M-ary symbols

$B = kR_s$

Bits to transmit: 00100111

Generated signals:



QUESTION

The transmission spectral efficiency will be:

A Higher if signal $x_1(t)$ is tranmitted

B Higher if signal $x_2(t)$ is transmitted

C The same for both signals transmission

D Not enough elements to answer

Bits to transmit: 00100111

Mapping: –V –V +V –V –V +V +V +V

QUESTION

By using a square root raised cosine filter, the obtained spectral efficiency will be:

A Higher than the one obtained using a rectangular shaping filter

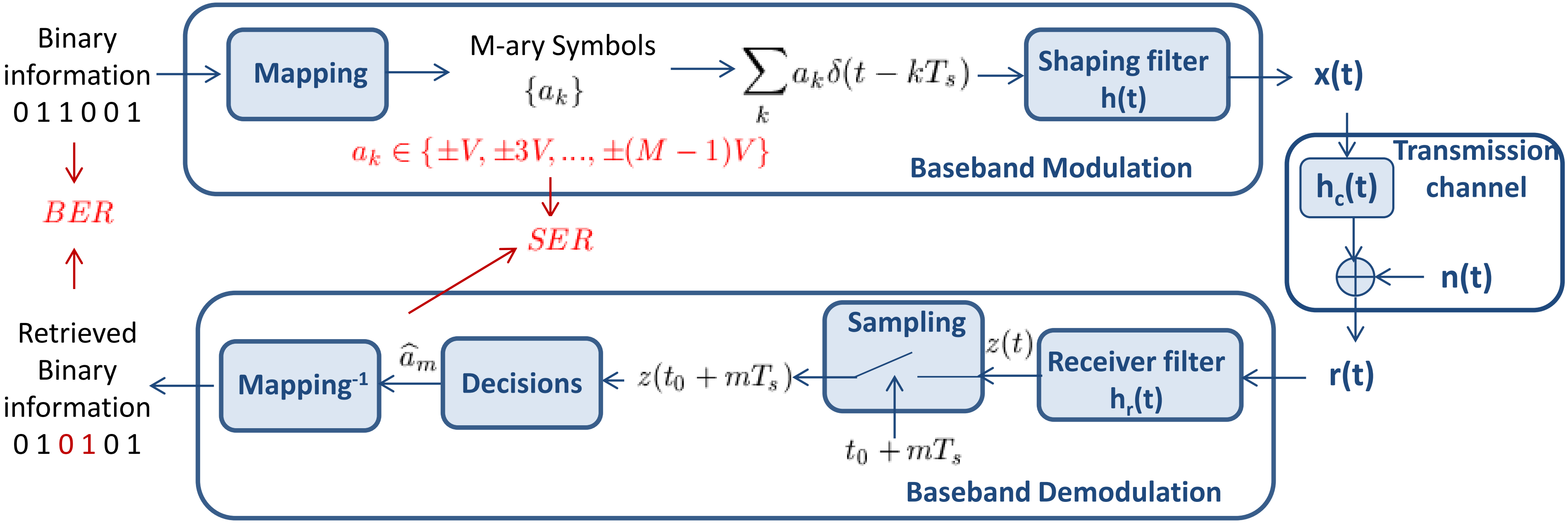
B Lower than the one obtained using a rectangular shaping filter

C The same as the one obtained using a rectangular shaping filter

D Not enough elements to answer

Baseband Digital Demodulation

Joint optimization with the modulation



$$z(t) = \sum_k a_k \delta(t-kT_s) * \underbrace{h(t) * h_c(t) * h_r(t)}_{g(t)} + \underbrace{n(t) * h_r(t)}_{w(t)}$$

$$z(t_0+mT_s) = a_m g(t_0) + \underbrace{\sum_{k \neq m} a_k g(t_0+(m-k)T_s)}_{\text{ISI at sampling instants}} + \underbrace{w(t_0+mT_s)}_{\text{Noise}} \longrightarrow \boxed{\text{Decisions}} \longrightarrow \{\hat{a}_k\}$$

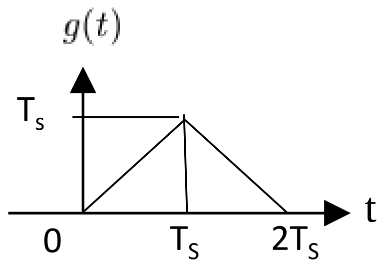
Term of Interest

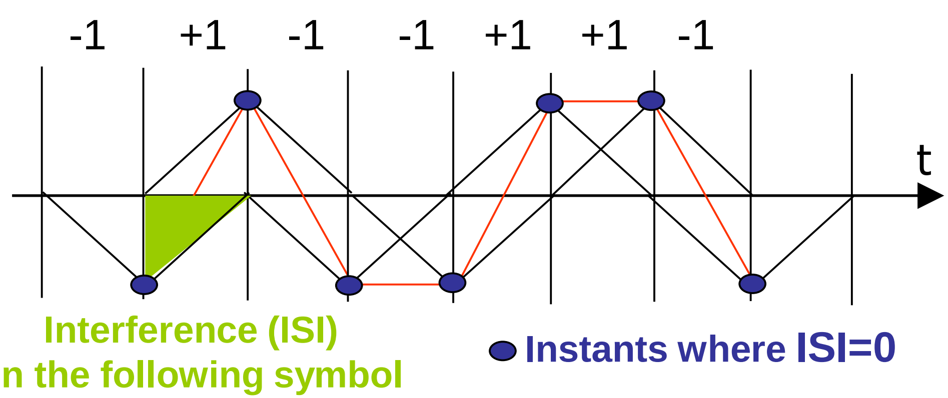
ISI at sampling instants
(Inter Symbol Interference)

Noise
(filtered and sampled)

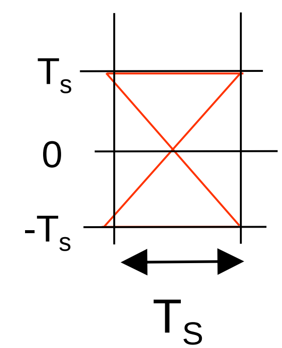
$\longrightarrow$ Interference visualization at the sampler input : example

On the signal :





Eye diagram



$\longrightarrow$ Interference suppression at t_0+mT_s : Nyquist criterion

- Time domain expression:

$$\boxed{\begin{cases} g(t_0) \neq 0 \\ g(t_0+pT_s) = 0 \ for \ p \in \mathbb{Z}^* \end{cases}} \Longrightarrow z(t_0+mT_s) = a_m g(t_0) + w(t_0+mT_s)$$

- Frequency domain expression:

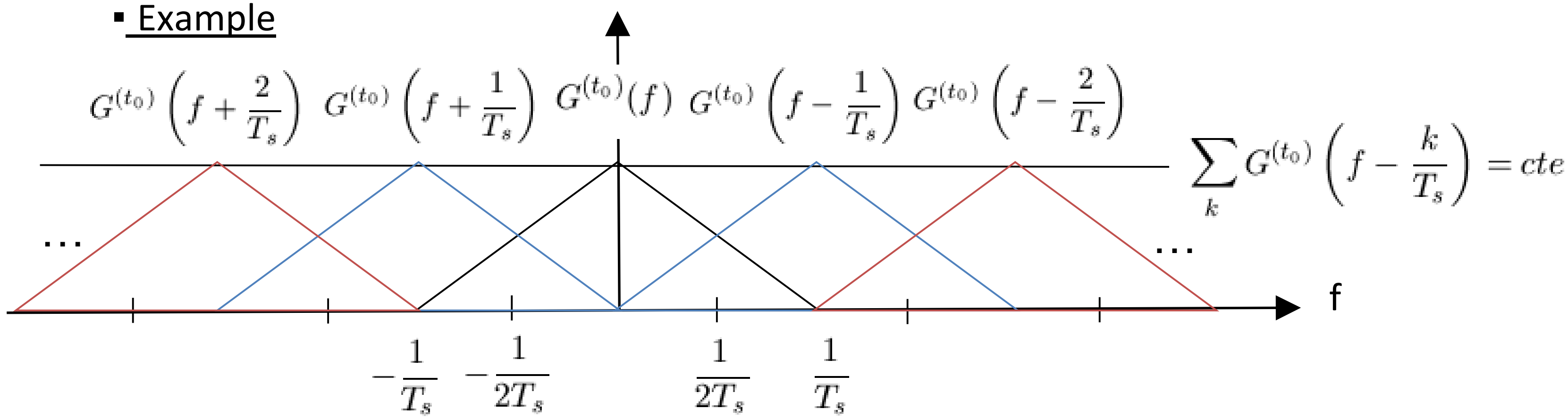
$$\begin{cases} g(t_0) \neq 0 \\ g(t_0+pT_s) = 0 \ for \ p \in \mathbb{Z}^* \end{cases} \xrightarrow{FT} \boxed{\sum_k G^{(t_0)}\left(f - \frac{k}{T_s}\right) = cte}$$

with $G^{t_0}(f) = FT\left[\frac{g(t+t_0)}{g(t_0)}\right]$

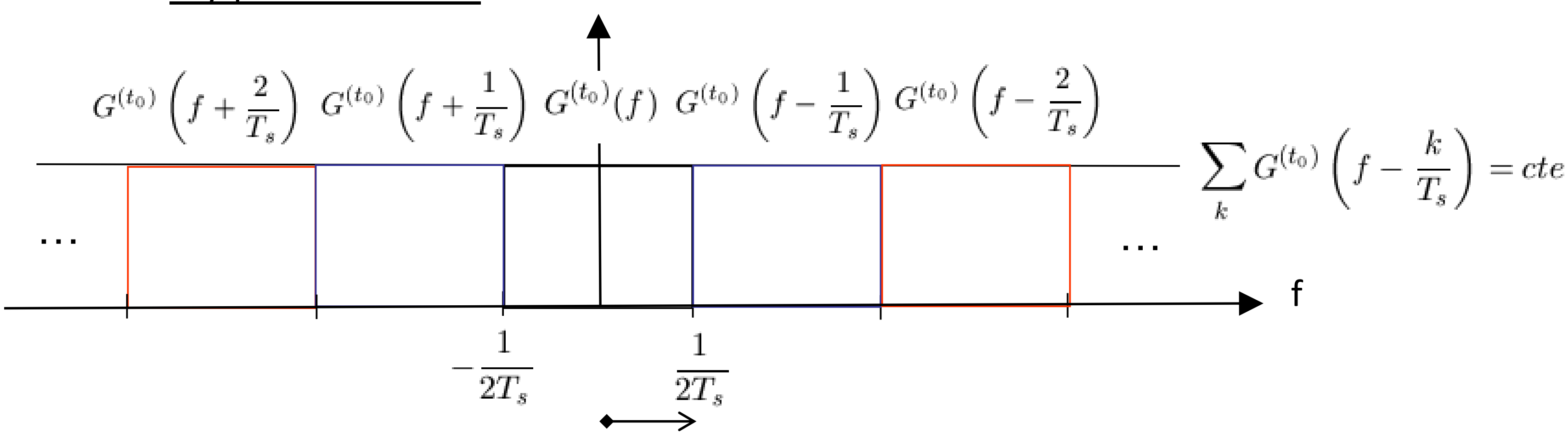
Joint optimization with the modulation

→ Interference suppression at t_0+mT_s : frequency domain Nyquist criterion

- Example



- Nyquist bandwidth



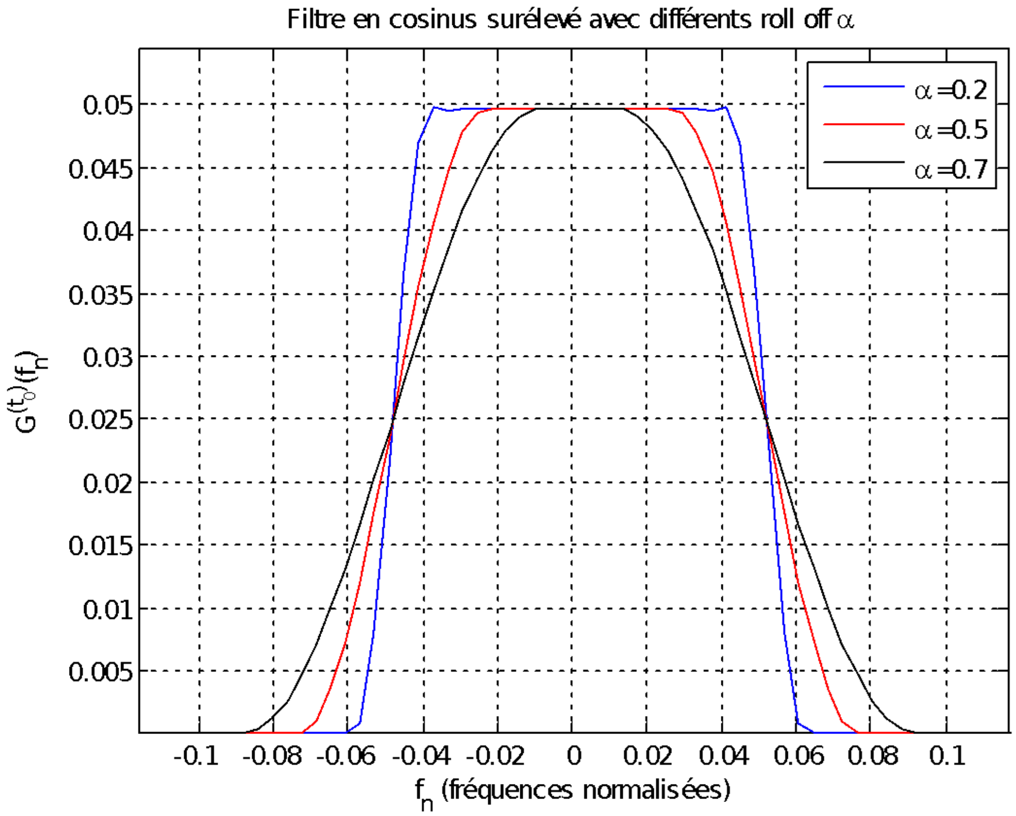
$$B_N=\frac{1}{2T_s}=\frac{R_s}{2}$$

Nyquist Bandwidth

⇒ Maximum symbol rate without interferences at time sampling instants : $R_s \leq 2B_N$

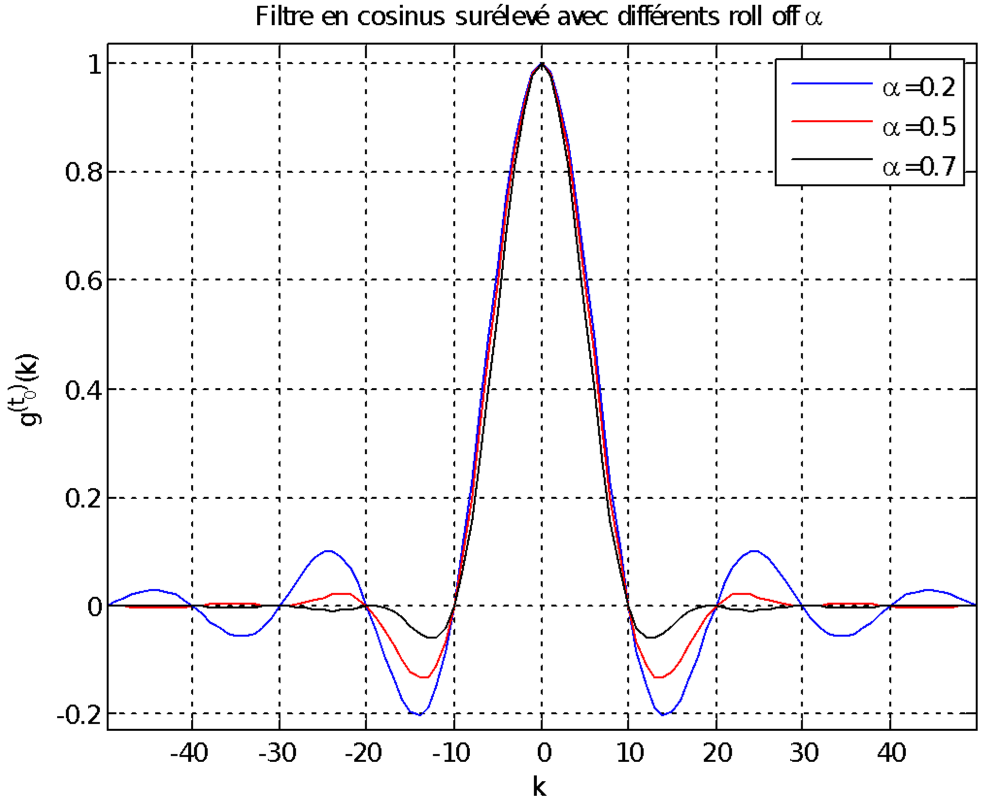
→ Example of Nyquist filter : raised cosine filter

$$G^{(t_0)}(f) = \begin{cases} T_S \text{ for } |f| \leq \dfrac{1-\alpha}{2T_S} \\ \dfrac{T_S}{2}\left[1+\cos\left(\dfrac{\pi T_S}{\alpha}\left(|f| - \dfrac{1-\alpha}{2T_S}\right)\right)\right] \text{pour } \dfrac{1-\alpha}{2T_s} \leq |f| \leq \dfrac{1+\alpha}{2T_S} \\ 0 \text{ elsewhere} \end{cases} \qquad 0 \leq \alpha \leq 1$$



→ <u>Example of Nyquist filter : raised cosine filter</u>

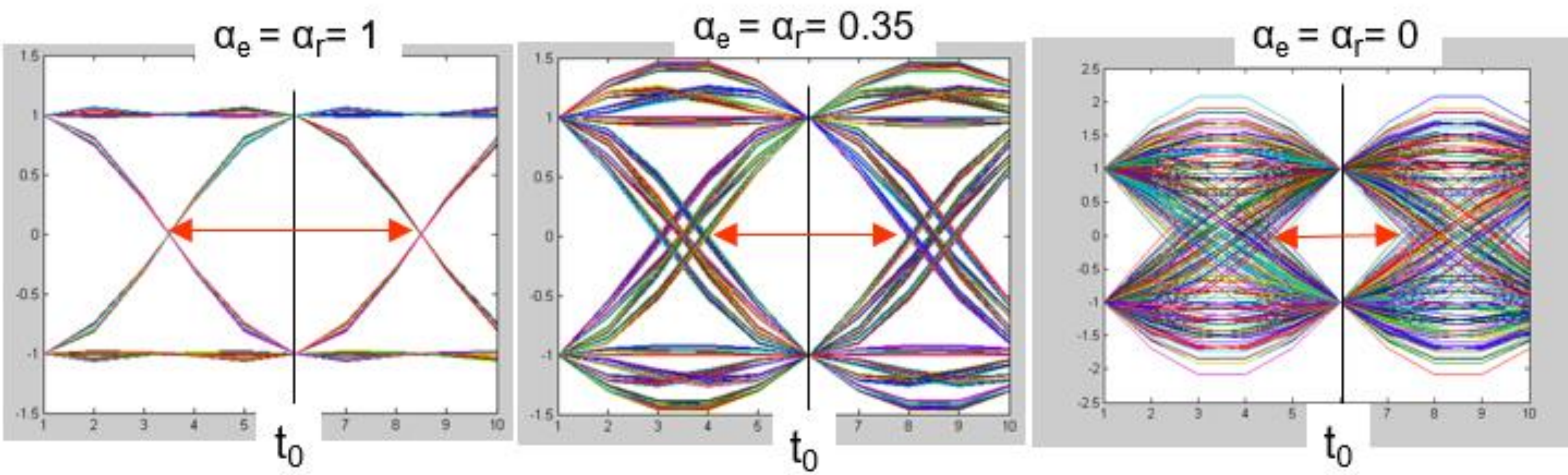
$$g^{(t_0)}(t) = \frac{1}{T_S}\,\mathrm{sinc}\left(\frac{\pi t}{T_S}\right)\frac{\cos\left(\frac{\alpha\pi t}{T_S}\right)}{1-\left(2\alpha\frac{t}{T_S}\right)^2} \qquad (0 \le \alpha \le 1)$$



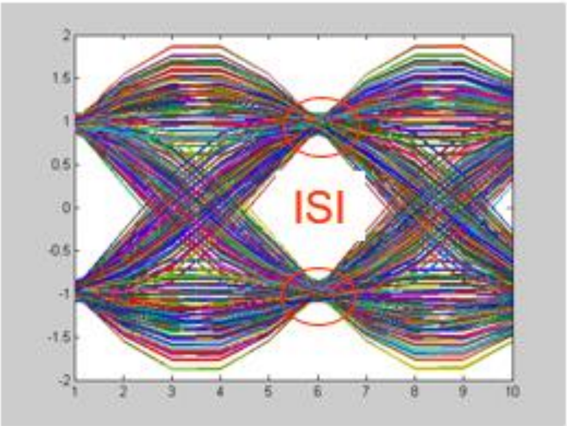
Some typical values: α=0.22 (UMTS), α=0.35 (DVB-S), α=0.15 (DVB-C)

→ Example of Nyquist filter : raised cosine filter

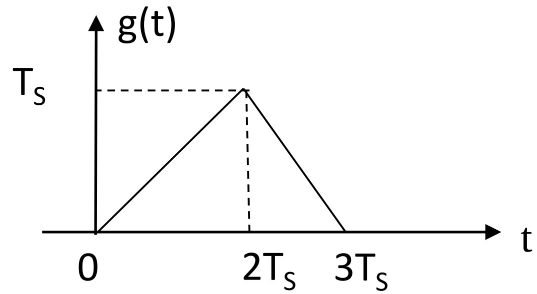
Some eyediagrams without noise (on $2T_s$)



Without noise, different roll off at the transmitter and receiver:



Let's consider $g(t)=h(t)*h_c(t)*h_r(t)$, the whole transmission channel impulse response:



(T_s = symbol duration)

QUESTION

The transmission channel:

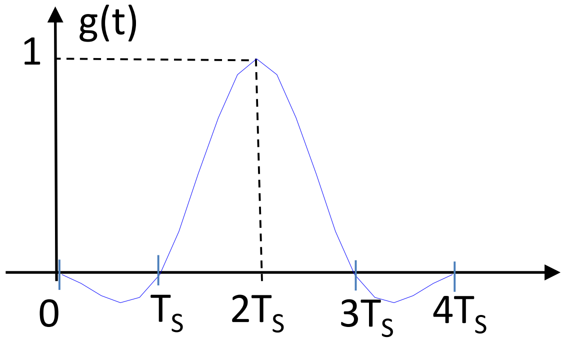
A Respect the Nyquist criterion

B Can respect the Nyquist criterion

C Cannot respect the Nyquist criterion

D Not enough elements to answer

Let's consider $g(t)=h(t)*h_c(t)*h_r(t)$, the whole transmission channel impulse response:



(T_s = symbol duration)

QUESTION

The transmission channel:

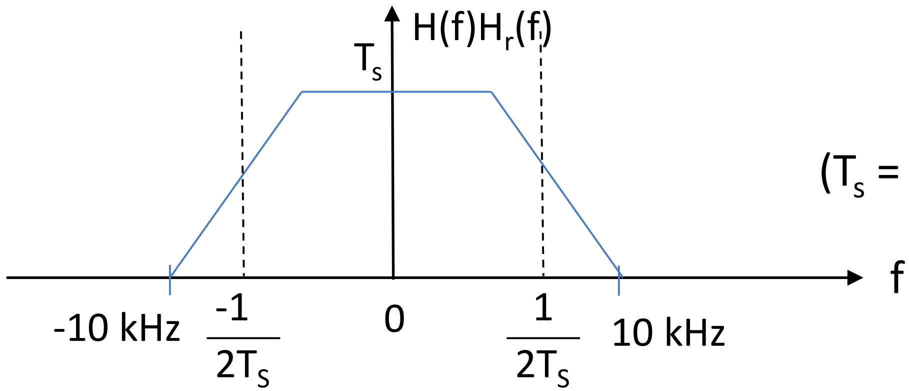
A. Respect the Nyquist criterion

B. Can respect the Nyquist criterion

C. Cannot respect the Nyquist criterion

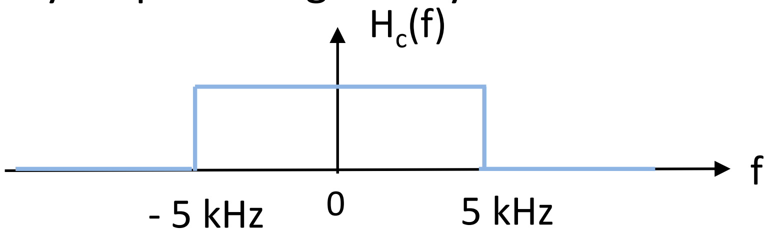
D. Not enough elements to answer

$H(f)H_r(f)$ is given below, where $H(f)$is the shaping filter frequency response and $H_r(f)$ is the receiver filter impulse response:



(T_s = symbol duration)

The channel frequency response is given by:



QUESTION

The transmission channel:

A Can respect the Nyquist criterion

B Cannot respect the Nyquist criterion

C Not enough elements to answer

Baseband Digital Demodulation

Joint optimization with the modulation

$$\underbrace{z(t_0+mT_s) = a_m g(t_0)}_{\text{Term of Interest}} + \underbrace{\sum_{k\neq m} a_k g(t_0+(m-k)T_s)}_{\text{ISI at sampling instants}} + \underbrace{w(t_0+mT_s)}_{\text{Noise}} \longrightarrow \boxed{\text{Decisions}} \longrightarrow \{\hat{a}_k\}$$

Term of Interest

ISI at sampling instants
(Inter Symbol Interference)

Noise
(filtered and sampled)

→ Interference suppression at t_0+mT_s: Nyquist criterion

Term of Interest

$$z(t_0+mT_s) = a_m g(t_0) + \underbrace{w(t_0+mT_s)}$$

Filtered and sampled noise:
w_m, variance σ^2

→ SNR maximisation at t_0+mT_s: matched filter (to the received waveform)

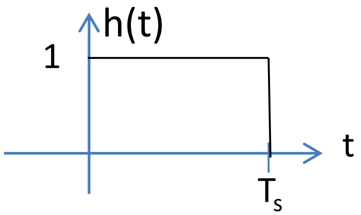
Maximize SNR_{t_0} ⇔ Maximize $\dfrac{|g(t_0)|^2}{\sigma^2}$

$$h_e(t) = h(t) * h_c(t)$$

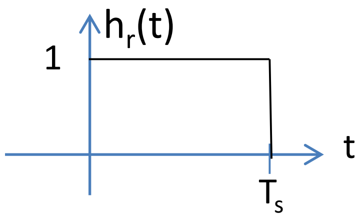
$$\left[\frac{|g(t_0)|^2}{\sigma^2}\right]_{max} \quad \text{for} \quad H_r(f) = \lambda H_e^*(f)e^{-j2\pi f t_0} \xrightarrow{FT^{-1}} \boxed{h_r(t) = \lambda h_e^*(t_0 - t)} \quad \text{Matched Filter}$$

(Cauchy-Schwarz inequality: $\left|\int_{-\infty}^{\infty} a(f)b^*(f)df\right|^2 \leq \int_{-\infty}^{\infty} a(f)a^*(f)df \int_{-\infty}^{\infty} b(f)b^*(f)df$, equality for $a(f) = kb(f)$)

The shaping filter impulse response is given below:



The receiver filter impulse response is given below:



QUESTION

The signal to noise ratio, computed after sampling is maximized:

A TRUE

B FALSE

C Not enough elements to answer

Baseband Digital Demodulation

Decision block

$\longrightarrow$ Decision rule: Maximum A Posteriori

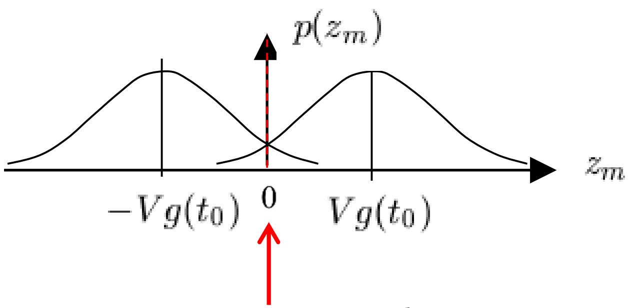
$$\boxed{\widehat{a}_m = \arg\max_{\widetilde{a}_m} P\left(\widetilde{a}_m | z_m\right)} \longrightarrow \widehat{a}_m = \arg\max_{\widetilde{a}_m} p\left(z_m | \widetilde{a}_m\right)$$

for equally likely symbols

Nyquist criterion is fulfilled: $z(t_0 + mT_s) \equiv z_m = a_m g(t_0) + w_m$

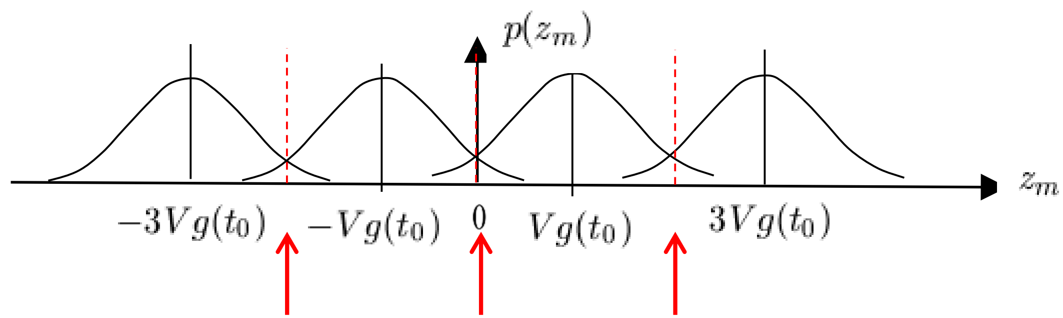
$$p(z_m|\widetilde{a}_m) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(z_m - \widetilde{a}_m g(t_0)\right)^2}{2\sigma^2}\right)$$

Binary case: $\widetilde{a}_m \in \{\pm V\}$

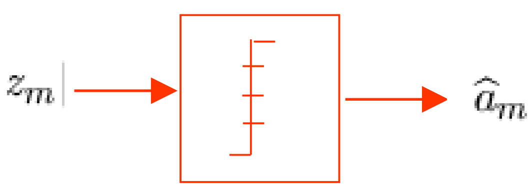


$$MAP\ decision\ rule \Longrightarrow \begin{cases} z_m \geq 0 : \widehat{a}_m = +V \\ z_m < 0 : \widehat{a}_m = -V \end{cases}$$

4-ary case: $\widetilde{a}_m \in \{\pm V, \pm 3V\}$



$$MAP\ decision\ rule \Longrightarrow \begin{cases} z_m \leq -2Vg(t_0) : \widehat{a}_m = -3V \\ -2Vg(t_0) < z_m \leq 0 : \widehat{a}_m = -V \\ 0 < z_m \leq 2Vg(t_0) : \widehat{a}_m = +V \\ z_m \geq 2Vg(t_0) : \widehat{a}_m = +3V \end{cases}$$



(Threshold detector or slicer)

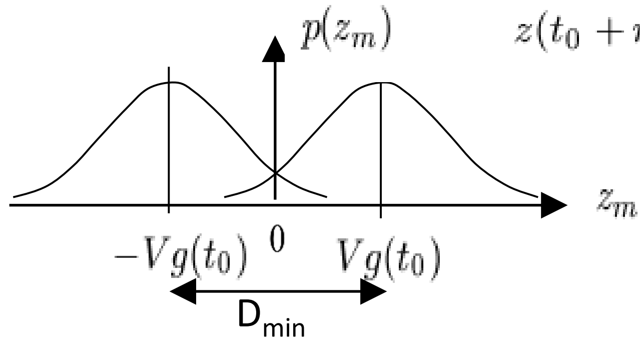
Baseband Digital Transmission (M-PAM)

Performances

→ Symbol Error Rate (SER)

- Binary case:

$$a_m \in \{\pm V\}$$



$$z(t_0 + mT_s) \equiv z_m = a_m g(t_0) + w_m$$

$$w_m \sim \mathcal{N}\left(0, \sigma^2\right)$$

Nyquist & decision threshold in 0

Matched filtering

$\frac{E_b}{N_0}$: SNR per bit at the receiver input

$$\boxed{SER = Q\left(\frac{D_{min}}{2\sigma}\right) = Q\left(\frac{Vg(t_0)}{\sigma}\right) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)}$$

- M-ary case

$$a_m \in \{\pm V, \pm 3V, ..., \pm (M-1)V\}$$

Nyquist & optimal decision thresholds

Matched filtering

$$\boxed{SER = 2\left(\frac{M-1}{M}\right) Q\left(\frac{Vg(t_0)}{\sigma}\right) = 2\left(\frac{M-1}{M}\right) Q\left(\sqrt{\frac{6log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)}$$

Obtained for an **M-PAM** (Baseband) modulation, in a **Nyquist channel**, with **matched filtering.**

$\longrightarrow$ Bit Error Rate (BER): Mapping optimization

« Natural » binary mapping

bits	symboles
00	-3
01	-1
10	+1
11	+3

On erroneous symbol = 2 erroneous bits

P_{e1}>>P_{e2}

Example (see exercises, for a 4-PAM with V=1, $N_0=10^{-3}\ V^2/Hz$, R_b=1kbps) :

$$P\left(\widehat{a}_k = -V / a_k = -3V\right) = Q(2) - Q(6) = 0.0228$$
$$P\left(\widehat{a}_k = +V / a_k = -3V\right) = Q(6) - Q(10) = 9.87\ 10^{-10}$$
$$P\left(\widehat{a}_k = +3V / a_k = -3V\right) = Q(10) = 7.62\ 10^{-24}$$

« Gray » mapping

bits	symboles
00	-3
01	-1
11	+1
10	+3

One erroneous symbol = 1 erroneous bit

GRAY Mapping $\Longrightarrow$ $BER \simeq \dfrac{SER}{\log_2(M)}$

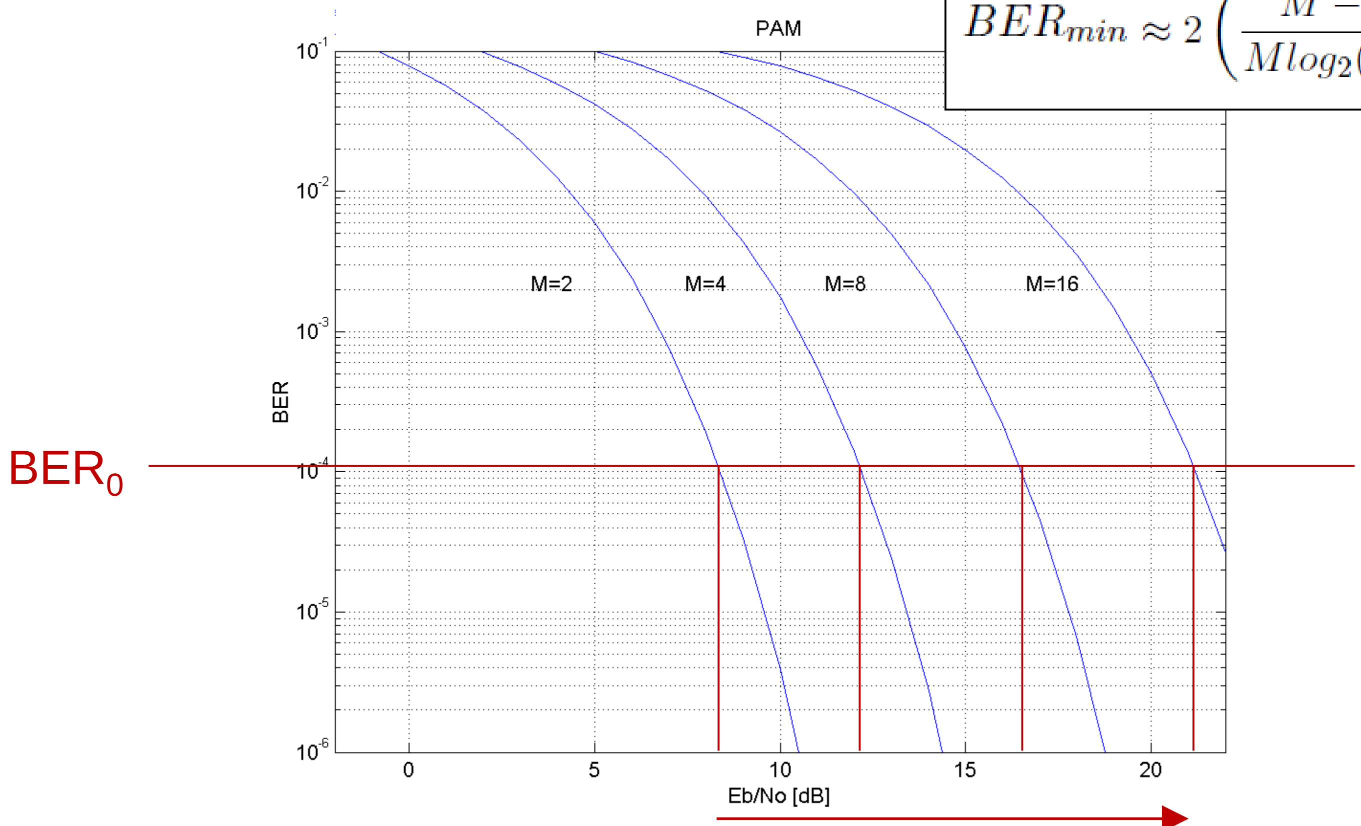
(BER = $\dfrac{\text{Number of erroneous bits}}{\text{Number of transmitted bits}} \approx \dfrac{\text{Number of erroneous symbols}}{\text{Number of transmitted symbols x Number of bits per symbol}}$)

Baseband Digital M-PAM Transmission

Power efficiency

→ BER = f(E_b/N_0) for M-PAM transmissions

$$BER_{min} \approx 2\left(\frac{M-1}{M\log_2(M)}\right) Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)$$

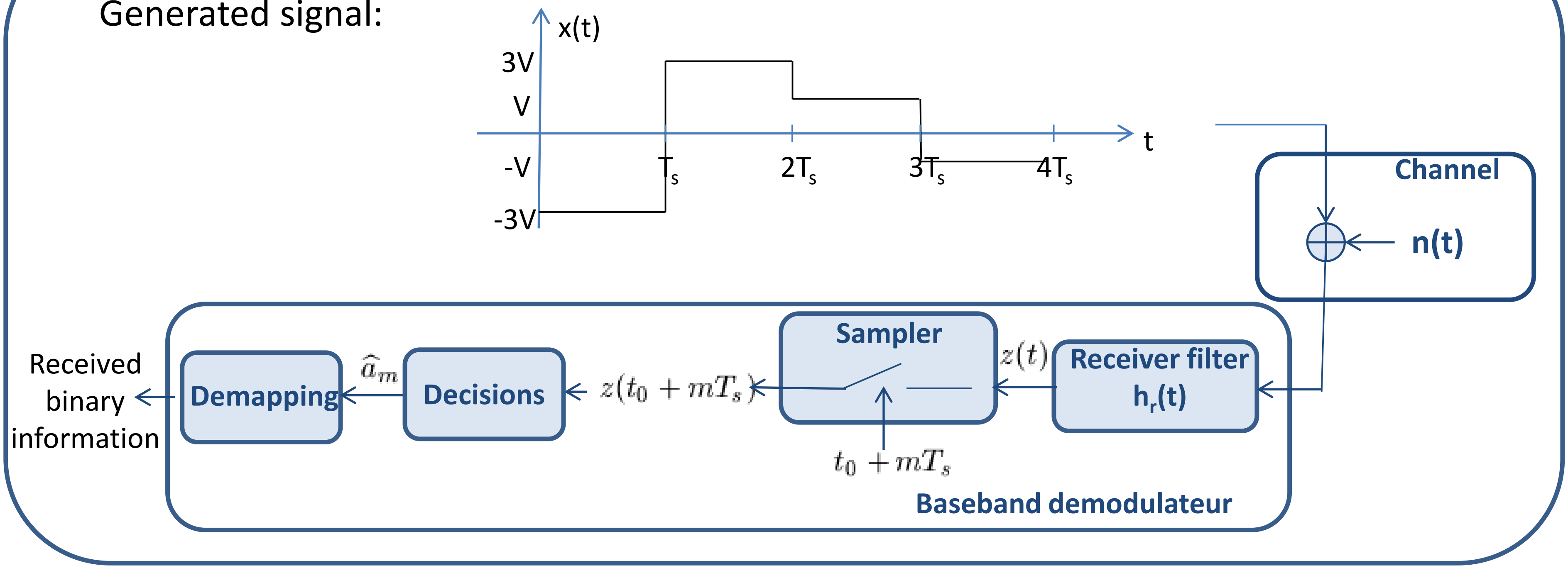


↘ Power efficiency
↗ Spectral efficiency

Obtained results for a M-ary baseband modulations (**M-PAM**), in a **Nyquist** channel, with **matched filtering** and **Gray mapping**

Bits to be transmitted: 00100111

Generated signal:



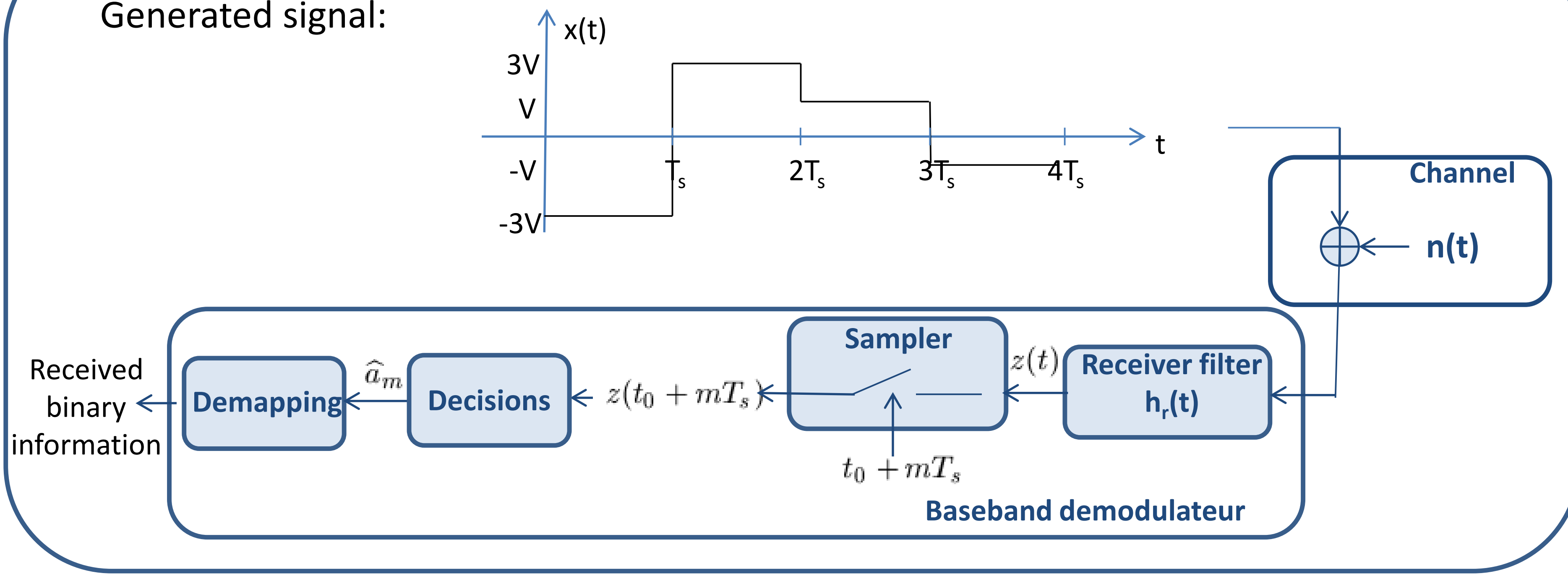
QUESTION

If we consider a receiver filter with a rectangular impulse response of T_s duration, $t_0=T_s$ and a threshold dectector with the following thresholds: –2V, 0, 2V, the symbol error rate will be the minimum one:

- A TRUE
- B FALSE

Bits to be transmitted: 00100111

Generated signal:



QUESTION

If we consider a receiver filter with a rectangular impulse response of T_s duration, t_0=T_s and a threshold dectector with the following thresholds: –2VT_s, 0, 2VT_s, the symbol error rate will be the minimum one:

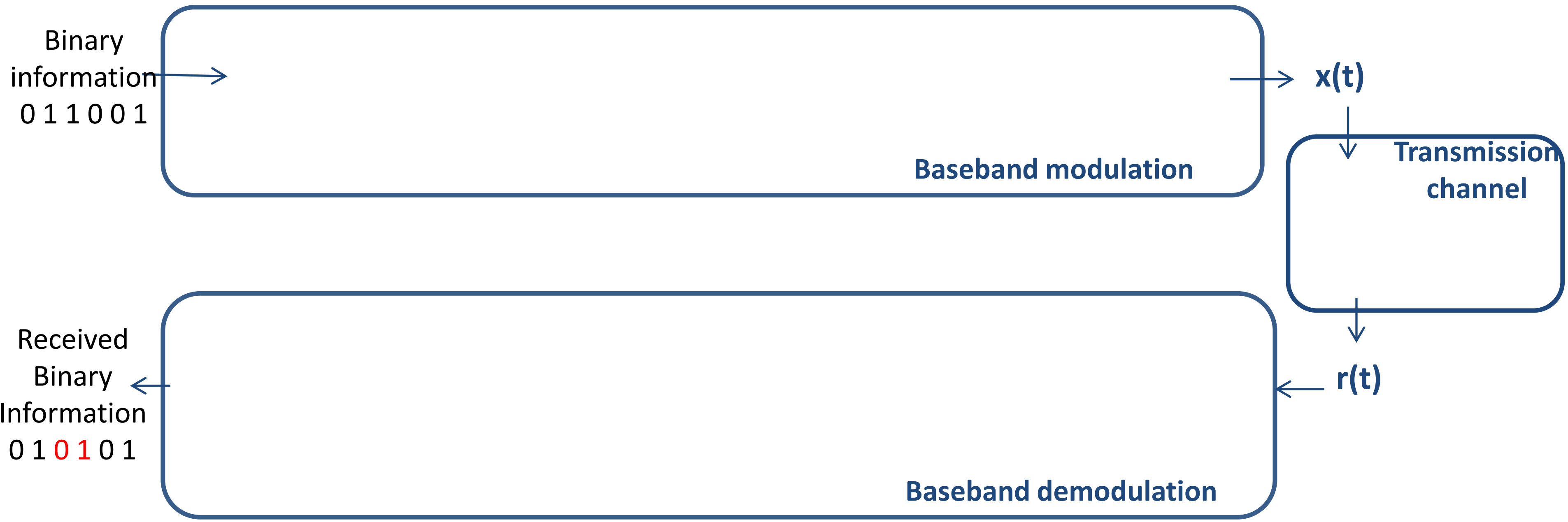
A TRUE

B FALSE

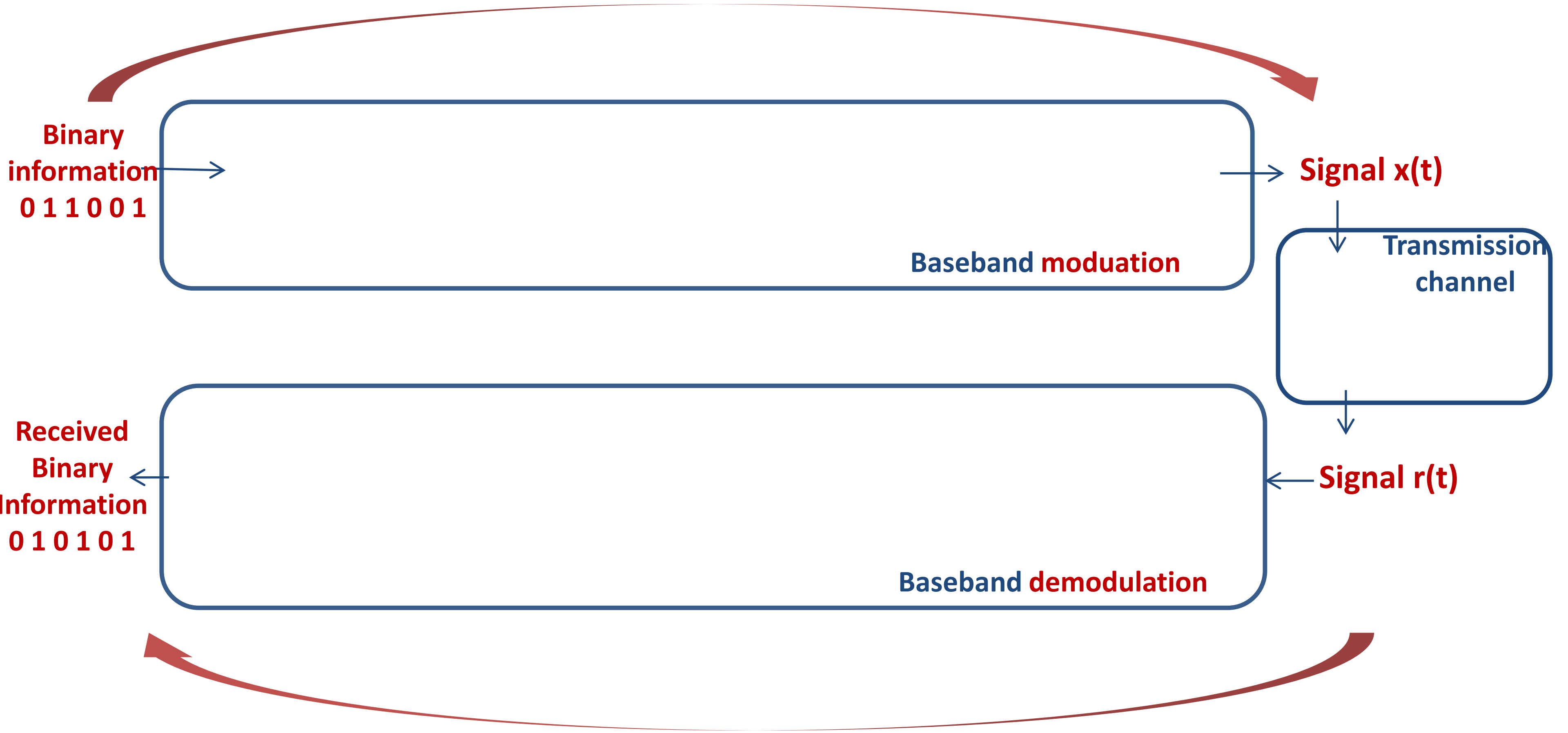
Baseband Modulation/Démodulation

Summary

Baseband Digital Modulation/Demodulation (M-PAM)



Baseband Digital Modulation/Demodulation (M-PAM)

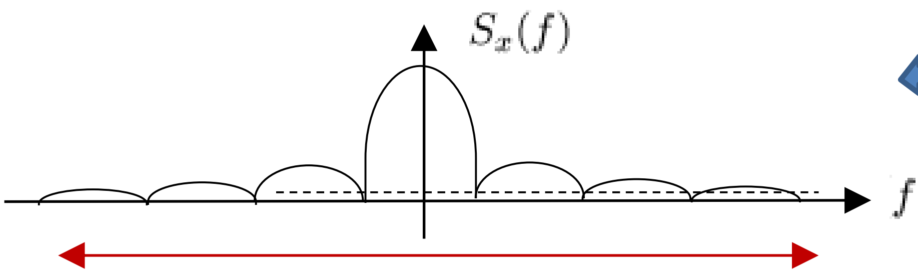


Baseband Digital Modulation/Demodulation (M-PAM)

Binary information
0 1 1 0 0 1

Baseband modulation

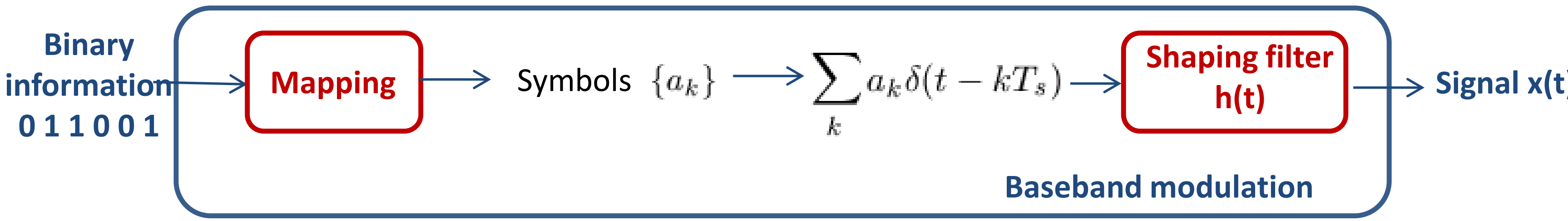
Signal x(t)



Power spectral density of the transmitted signal around frequency 0

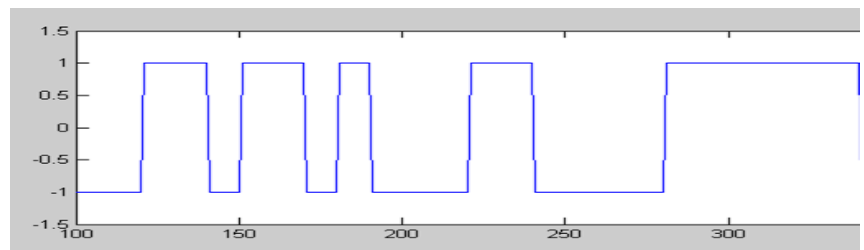
Baseband Digital Modulation/Demodulation (M-PAM)

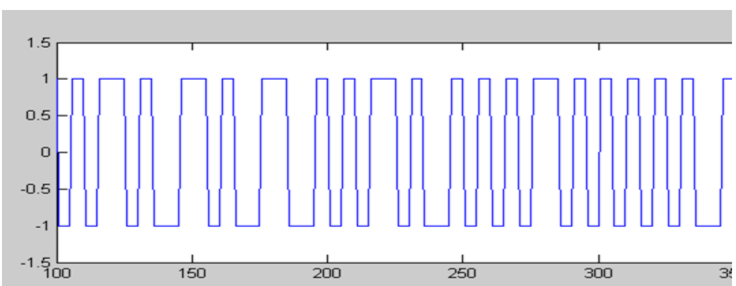
To be able to design the modulator in order to obtained the desired signal

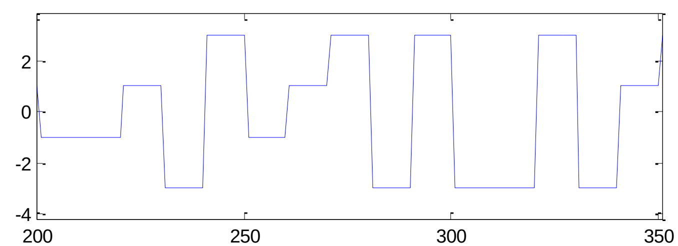


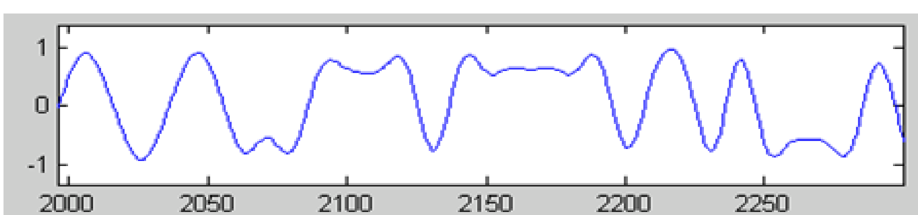
$$\boxed{x(t) = \sum_k a_k h(t - kT_s)}$$

Example of signals:

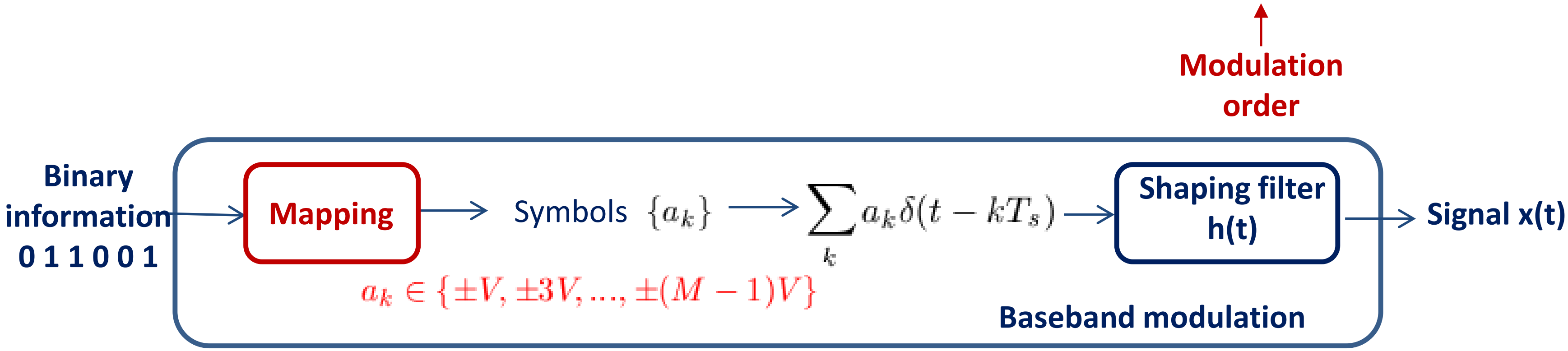








Digital Baseband Modulation/Demodulation (M-PAM)



Symbole Rate (symbols/s or bauds)

$$\boxed{R_s = \frac{R_b}{\log_2(M)}}$$

Binary Rate (bits/s)

M=number of possible symbols

Baseband Digital Modulation/Demodulation (M-PAM)

To be able to compare several modulation schemes in terms of spectral efficiency

Binary information
0 1 1 0 0 1 → **Mapping** → Symbols $\{a_k\}$ → $\sum_k a_k \delta(t - kT_s)$ → **Shaping filter h(t)** → **Signal x(t)**

$a_k \in \{\pm V, \pm 3V, \ldots, \pm(M-1)V\}$

Baseband modulation

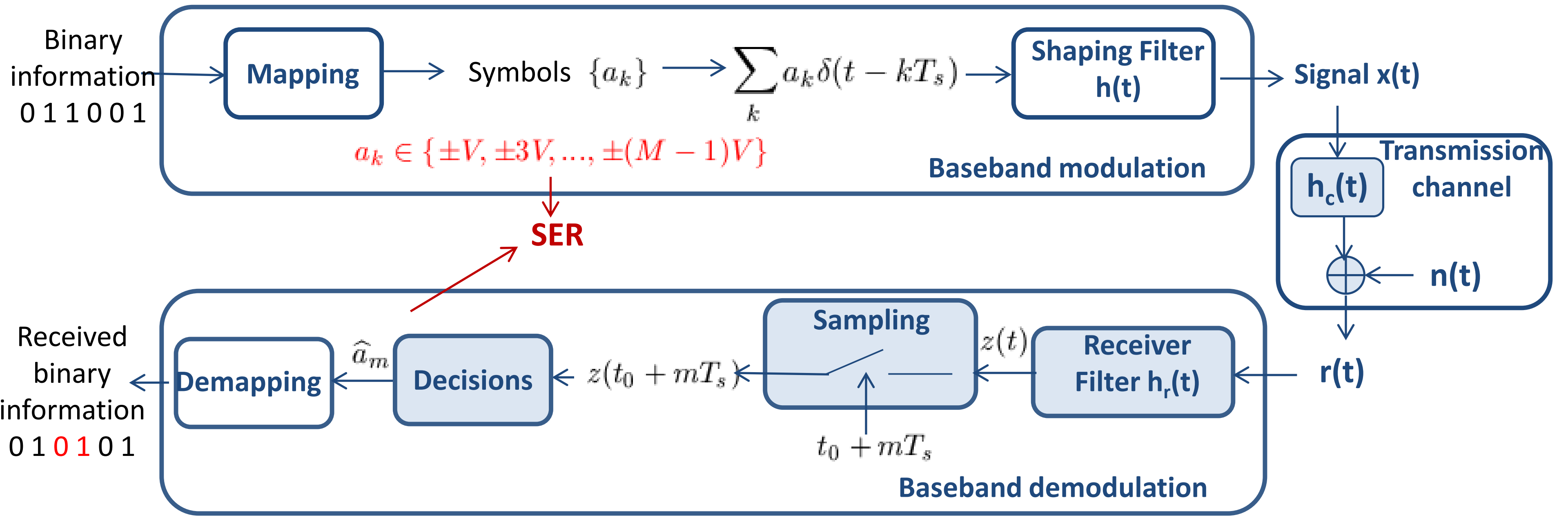
$$\eta = \frac{R_b}{B} = \frac{\log_2(M)}{k}$$

$$S_x(f) = \frac{\sigma_a^2}{T_s}|H(f)|^2 + 2\frac{\sigma_a^2}{T_s}|H(f)|^2 \sum_{k=1}^{\infty} Re\left[R_a(k)e^{j2\pi f k T_s}\right] + \frac{|m_a^2|}{T_s^2}\sum_k \left|H\left(\frac{k}{T_s}\right)\right|^2 \delta\left(f - \frac{k}{T_s}\right)$$

Power spectral density of x(t)

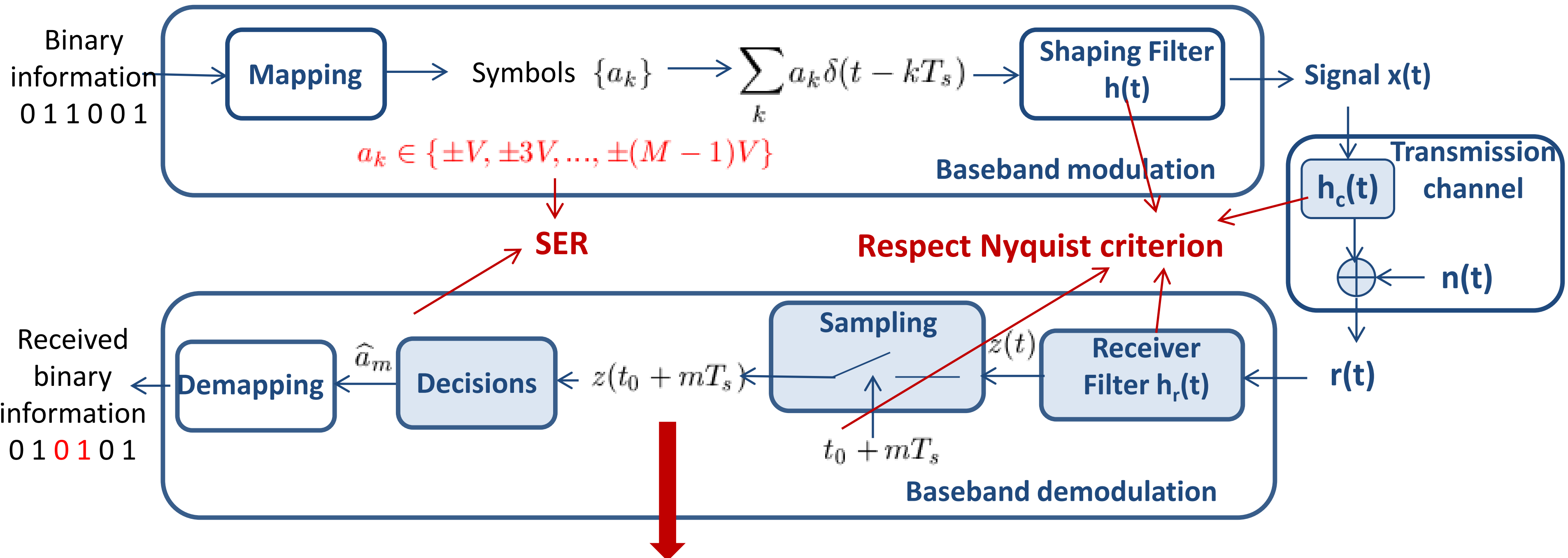
Baseband Digital Modulation/Demodulation (M-PAM)

To be able to design the demodulator allowing to minimize the symbol error rate (SER) for a given modulator



Baseband Digital Modulation/Demodulation (M-PAM)

To be able to design the demodulator allowing to minimize the symbol error rate (SER) for a given modulator

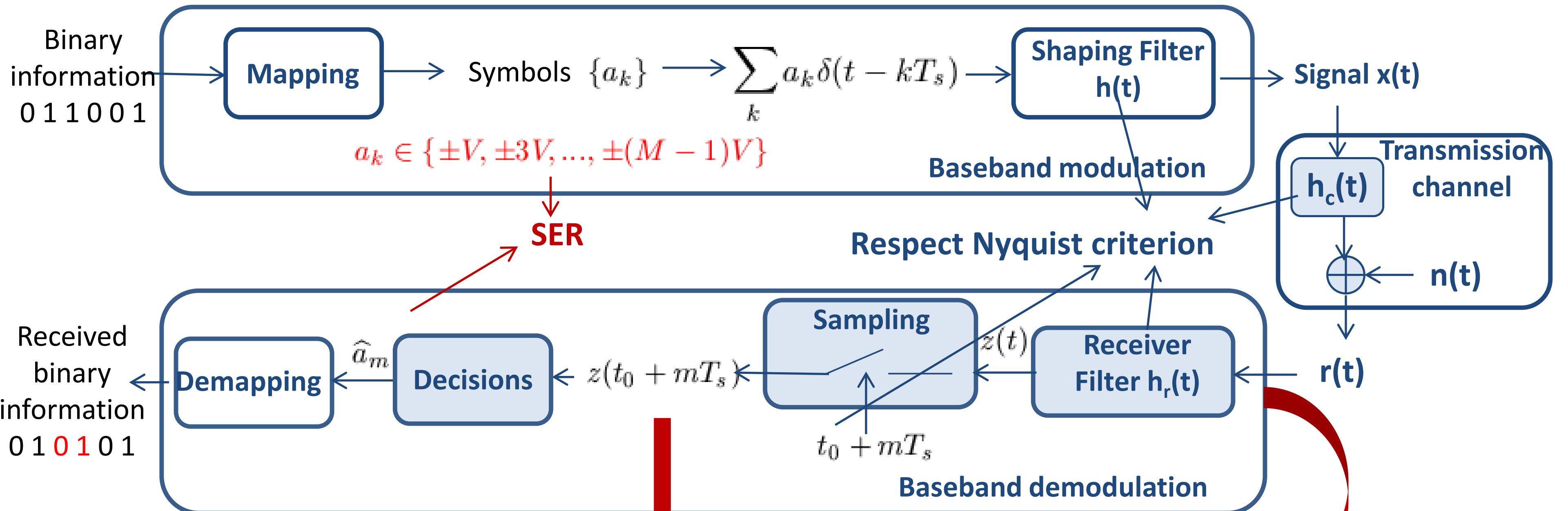


$$z(t_0 + mT_s) \equiv z_m = a_m g(t_0) + w_m$$

Useful term + noise (ISI term =0)

Baseband Digital Modulation/Demodulation (M-PAM)

To be able to design the demodulator allowing to minimize the symbol error rate (SER) for a given modulator



$$z(t_0+mT_s) \equiv z_m = a_m g(t_0) + w_m$$

Useful term + noise (ISI term =0)

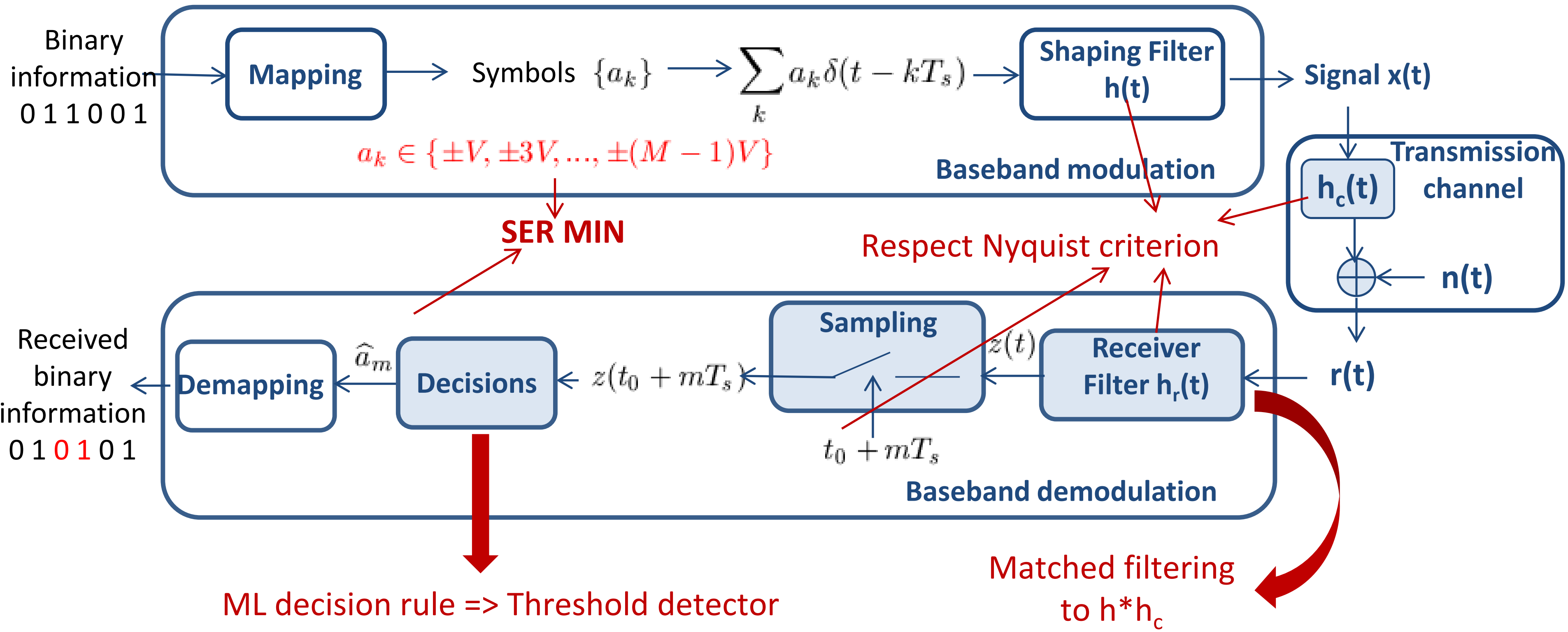
SNR is maximized

Matched filtering

($h_r(t)$ matched to $h*h_c$)

Baseband Digital Modulation/Demodulation (M-PAM)

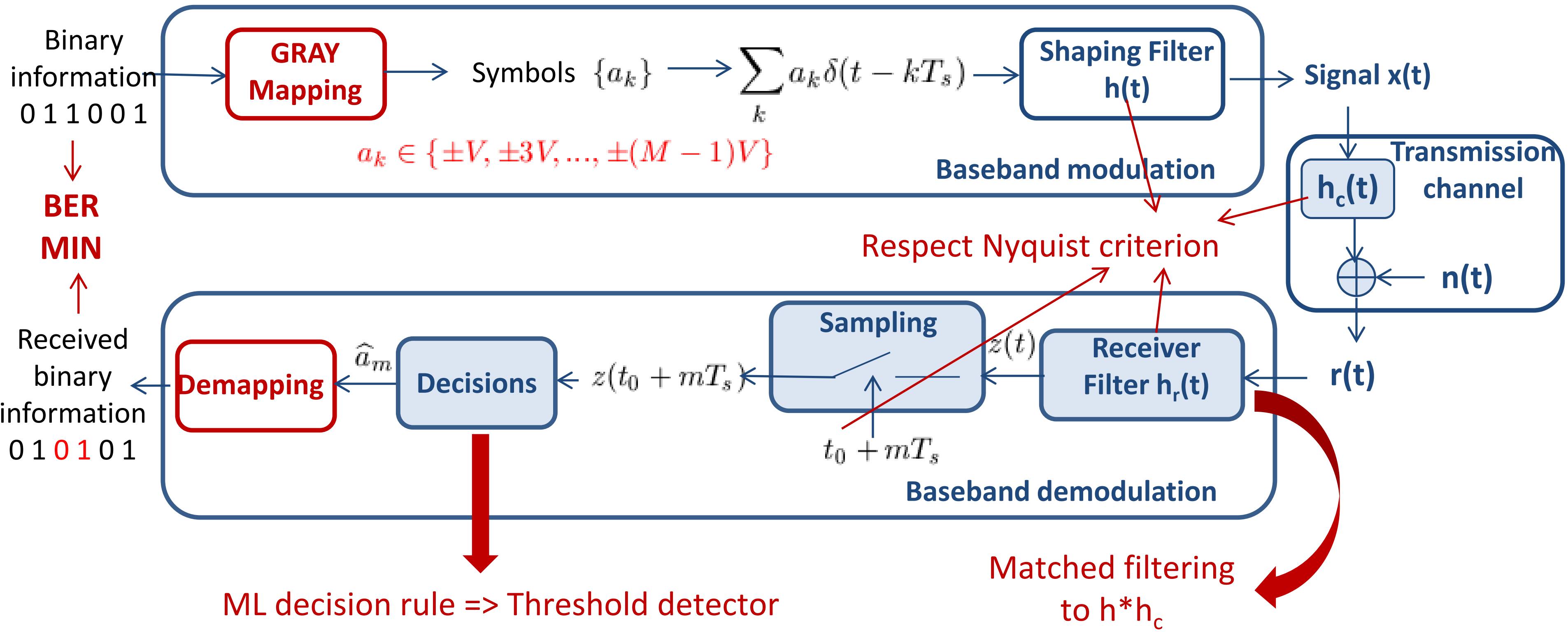
To be able to design the demodulator allowing to minimize the symbol error rate (SER) for a given modulator



$$\boxed{SER_{min} = 2\left(\frac{M-1}{M}\right) Q\left(\sqrt{\frac{6log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)}$$

Baseband Digital Modulation/Demodulation (M-PAM)

To be able to design a modulator/demodulator
allowing to minimize the bit error rate (BER)

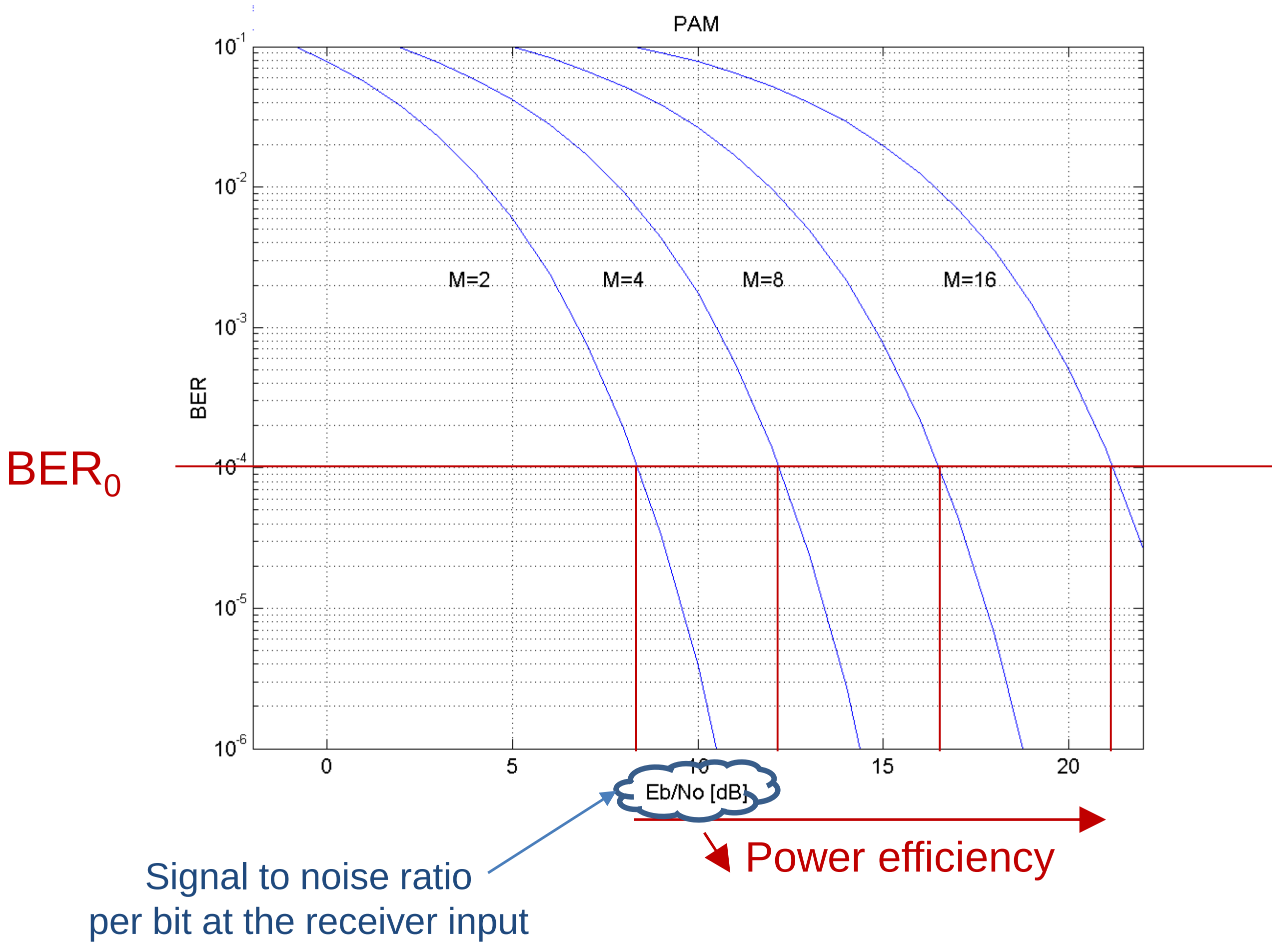


$$TEB_{min} \approx 2\left(\frac{M-1}{Mlog_2(M)}\right) Q\left(\sqrt{\frac{6log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)$$

Baseband Digital Modulation/Demodulation (M-PAM)

To be able to compare several modulation schemes in terms of power efficiency

$$BER_{min} \approx 2\left(\frac{M-1}{Mlog_2(M)}\right)Q\left(\sqrt{\frac{6log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)$$



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→ Digital Video Broadcasting (DVB): User guidelines for the second generation system for broadcasting, interactive services, news gathering and other broadband satellite applications (DVB-S2), norme ETSI EN 102 376.