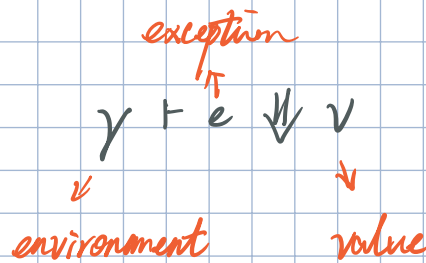


TD 1. Semantique Operationnelle et Interpretation

Environment



$$\frac{\gamma \vdash e \Downarrow v \neq \perp \quad v \in \text{dom}(op) \quad v' = op \ v}{\gamma \vdash op \ e \Downarrow v'}$$

$op = "!", " \pm", \text{etc.}$

Exercice 2.1 :

- environnement $\gamma : \{ n \mapsto 3 \}$
- expression $e = (-1 + n * 2)$
- value $v = 5 (-1 + 3 * 2)$

$$\frac{\{ n \mapsto 3 \} (n) = 3 \quad n \in \{ n \mapsto 3 \}}{\{ n \mapsto 3 \} \vdash n \Downarrow 3}$$

$$\frac{\{ n \mapsto 3 \} \vdash n \Downarrow 3 \quad \{ n \mapsto 3 \} \vdash 2 \Downarrow 2 \quad 3 * 2 \in \text{dom}(\times) \quad b = 3 * 2}{\{ n \mapsto 3 \} \vdash (n * 2) \Downarrow b}$$

$$\frac{\{ n \mapsto 3 \} \vdash 1 \Downarrow 1 \quad 1 \in \text{dom}(-) \quad -1 = -1}{\{ n \mapsto 3 \} \vdash (-1) \Downarrow (-1)} \quad \frac{\{ n \mapsto 3 \} \vdash (-1) \Downarrow (-1) \quad \{ n \mapsto 3 \} \vdash (n * 2) \Downarrow b \quad (-1) \times b \in \text{dom}(+)}{\{ n \mapsto 3 \} \vdash (-1 + n * 2) \Downarrow 5}$$

$$\{ n \mapsto 3 \} \vdash (-1 + n * 2) \Downarrow 5$$

Exercise 2.2

$\text{Expr} \rightarrow \text{let Ident} = \text{Expr in Expr}$

$$(\gamma \vdash \text{let } x = e_1 \text{ in } e_2)$$

分析过程:

$$(1) \gamma \vdash e_1 \Downarrow v_1$$

$$(2) \gamma \cup \{x \mapsto v_1\} : \cup \text{ union 并集}$$

$$(3) \gamma \cup \{x \mapsto v_1\} \vdash e_2 \Downarrow v_2$$

$$(4) \gamma \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v_2$$

Variante 1

$$\gamma \vdash e_1 \Downarrow v_1 \quad \gamma \cup \{x \mapsto v_1\} \vdash e_2 \Downarrow v_2$$

$$\gamma \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v_2$$

Variante 2

$$\gamma \vdash e_1 \Downarrow v_1 \quad v_1 = \perp_c$$

$$\gamma \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow \perp_c$$

Variante 3

$$\gamma \vdash e_1 \Downarrow v_1 \quad \gamma \cup \{x \mapsto v_1\} \vdash e_2 \Downarrow v_2 \quad v_2 = \perp_c$$

$$\gamma \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow \perp_c$$

For example:

$$\{x \mapsto 1\} \vdash x \Downarrow 1 \quad \{x \mapsto 1\} \vdash 1 \Downarrow 1 \quad \text{dom}(+)$$

$$\gamma \vdash 1 \Downarrow 1 \quad \{x \mapsto 1\} \vdash x + 1 \Downarrow 2$$

$$\gamma \vdash \text{let } x = 1 \text{ in } x + 1 \Downarrow 2$$

Exercise 2.3

$\text{Expr} \rightarrow \text{if Expr then Expr else Expr}$

$\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3$

分析过程:

1° $\gamma \vdash e_1 \Downarrow v_1$

2° $v_1 = \text{true}, \gamma \vdash e_2 \Downarrow v_2$

3° $\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3$

4° $v_1 = \text{false}, \gamma \vdash e_3 \Downarrow v_2$

5° $\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3$

Variante 1

$\gamma \vdash e_1 \Downarrow \text{true} \quad \gamma \vdash e_2 \Downarrow v_2$

$\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v_2$

$\gamma \vdash e_1 \Downarrow \text{false} \quad \gamma \vdash e_3 \Downarrow v_2$

Variante 2

$\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v_2$

$\gamma \vdash e_1 \Downarrow v \quad v \notin \{\text{true}, \text{false}\}$

Variante 3

$\gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow \perp_{\text{type}}$

Exercise 2.4

$\text{Expr} \rightarrow \text{fun Ident} \rightarrow \text{Expr}$
 $| (\text{Expr}) \text{ Expr}$

$$\gamma \vdash \text{fun } x \rightarrow e_B$$

$$\gamma \vdash (e_F) e_P$$

↓
function

↓
parameter

分析过程 =

$$(1) \quad \gamma \vdash e_P \Downarrow V_P$$

$$(2) \quad \gamma \vdash e_F \Downarrow \langle \text{fun } x \rightarrow e_B, \gamma_F \rangle$$

$$(3) \quad \gamma \cup \{x \mapsto V_P\}$$

$$(4) \quad \gamma_F \cup \{x \mapsto V_P\} \vdash e_B \Downarrow V_A$$

$$(5) \quad \gamma \vdash (e_F) e_P \Downarrow V_A$$

Variante 1

$$\frac{\gamma \vdash e_P \Downarrow V_P \quad \gamma \vdash e_F \Downarrow \langle \text{fun } x \rightarrow e_B, \gamma_F \rangle}{\gamma \vdash (e_F) e_P \Downarrow V_A}$$

Variante 2

$$\frac{\gamma \vdash \text{fun } x \mapsto e_B \Downarrow \langle \text{fun } x \rightarrow e_B, \gamma_F \rangle}{\gamma_F \cup \{x \mapsto V_P\} \vdash e_B \Downarrow V_A}$$