

Real Time Scheduling

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Main goals of the module

- A general overview of how real-time applications can be executed
 - ▶ Real-time scheduling
 - ★ Why is it different from non real-time scheduling?
 - ★ What are the features that have to be considered?
 - ★ What are the real-time scheduling algorithms?
 - ★ How can we be sure that it works?
 - ★ ...
 - ▶ Real-time operating system
 - ★ Illustration on OSEK

A definition of real-time

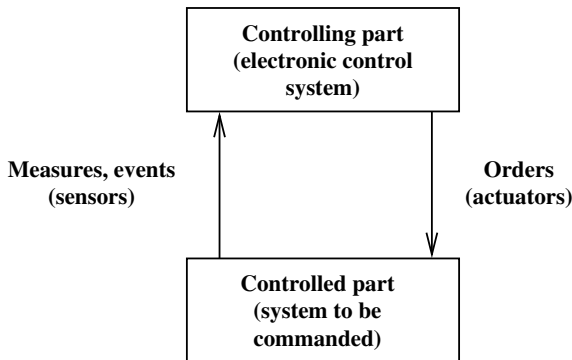
A real-time system is able, first to read all incoming data before they become useless, second to give an appropriate timely reaction

- Temporal correctness : correct result after the deadline = bad (useless) result
- The delay highly depends on the application
- \Rightarrow real-time is not real fast
- Two main classes of real-time systems :
 - ▶ Hard real-time : a missed deadline can have catastrophic consequences
 \Rightarrow strict timing constraints
 - ▶ Soft real-time : some (not frequent) missed deadlines can be tolerated
(\Rightarrow often a predefined rate of allowed missed deadlines)
- Mandatory to guarantee the functional behavior as well as the temporal behavior of the system

Application domains

- Factory automation
 - ▶ Control/command of a production line
 - ▶ Control process
 - ▶ ...
- Embedded systems
 - ▶ Avionics (flight control, ...)
 - ▶ Automotive
 - ▶ Space
 - ▶ Robotics
 - ▶ Home appliance
 - ▶ Phones
 - ▶ ...

Architecture overview



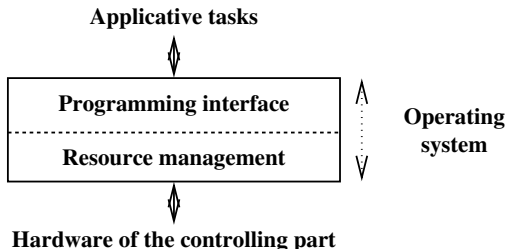
- Inputs and outputs are very important
- Hardware architecture of the controlling part : programmable logic controller (Schneider Electric, Siemens), ad hoc circuit, FPGA, micro-controller, micro-processor
- The controlling part can be centralised or distributed \Rightarrow industrial networks, fieldbuses (CAN, FIP, ...)

Processing overview

```
For each event of the controlled part do  
    Read the data from the controlled part  
    Compute the new system state  
    Send the orders associated to this state  
EndFor
```

- Synchronous behavior (time driven) : data are read at predefined instants \Rightarrow periodic tasks
- Asynchronous behavior (event driven) : immediate answer to events \Rightarrow non periodic tasks
- Hybrid behavior : mix of periodic and aperiodic tasks

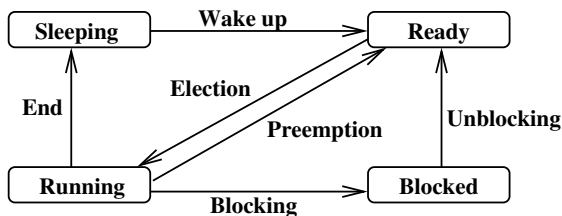
Software architecture



- More or less elaborated operating system, from nothing to a multitask operating system with graphical facilities
- Multitask operating system:
 - ▶ Scheduler to manage processor sharing
 - ▶ Control access mechanisms in order to share the other resources
 - ▶ Synchronisation and communication mechanisms in order to manage global consistency

Scheduler

- State diagram of a task



- Scheduler execution when an interrupt occurs:
 - ▶ The context of running task is saved and this task moves to ready state
 - ▶ One ready task is elected
 - ▶ The context of this elected task is restored
- Scheduling algorithm: activation time of the scheduler + election policy

Operating system

- General purpose operating system:
 - ▶ Goals:
 - ★ optimisation of resource utilisation
 - ★ minimisation of average response time
 - ▶ Features:
 - ★ round robin scheduler
 - ★ complex memory hierarchy
 - ▶ Examples : Linux, Solaris, Windows xx
- Real-time operating system:
 - ▶ Goals:
 - ★ predictable delays
 - ★ reliability
 - ▶ Features:
 - ★ priority based preemptive scheduling
 - ★ limited and controlled memory hierarchy
 - ★ execution of system primitives in bounded time
 - ▶ Examples: Tornado, LynxOS, QNX, RTAI, OSEK, TinyOS

Implementation of a real time system

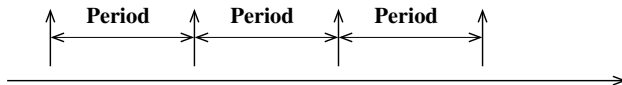
- Cross-development \Rightarrow two machines:
 - ▶ host machine:
 - ★ application development and debug
 - ★ code generation for the target machine \Rightarrow cross-compilation
 - ★ this code is loaded on the target machine
 - ★ no host machine at run-time
 - ▶ target machine:
 - ★ application execution
- Native development \Rightarrow one single machine:
 - ▶ application development and debug
 - ▶ application execution
 - ▶ Parts of the operating system can be removed at run-time

A real-time application in this module

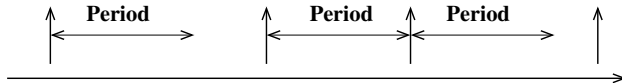
- A set of tasks (pieces of codes)
 - ▶ Can be fully independant
 - ▶ Can share resources, e.g. variables
 - ▶ Can communicate, synchronise, have precedence constraints
- We are only interested in the temporal correctness, not the fonctionnal one
- Each task is modelled by temporal values
- The set of tasks can be executed on a single core or on a multi-core or a multi-processor

Real-time tasks

- A task: a recurrent execution of a job.
- Request can be
 - ▶ Periodic: constant duration between consecutive releases



- ▶ Sporadic: variable duration between consecutive releases, with a known minimum

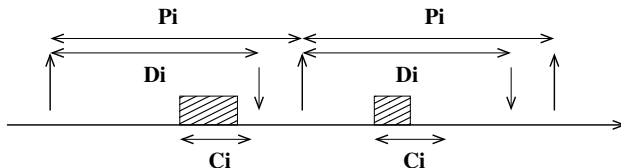


- ▶ Aperiodic: variable duration between consecutive releases.



Non concrete real-time task

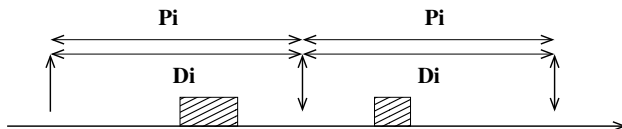
- Defined by the following features
 - ▶ A Worst-Case Execution Time (WCET): C_i
 - ★ Safe upper bound on the execution time of one job of the task
 - ▶ A release period: P_i
 - ★ (Minimum) duration between consecutive job releases of the task
 - ▶ A relative deadline D_i
 - ★ Maximum allowed response time for each job of the task



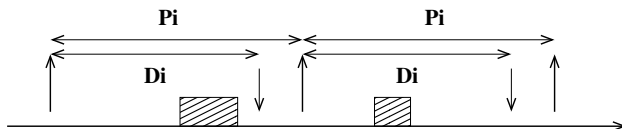
- The first release time of the task is unknown \Rightarrow non concrete task

Non concrete real-time task

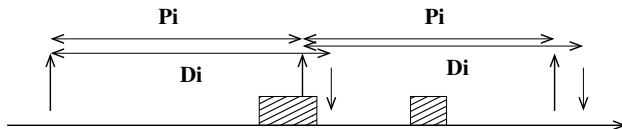
- Relationship between deadline and period
 - Implicit deadline ($D_i = P_i$): the simplest case



- Constrained deadline ($D_i \leq P_i$): the intermediate case

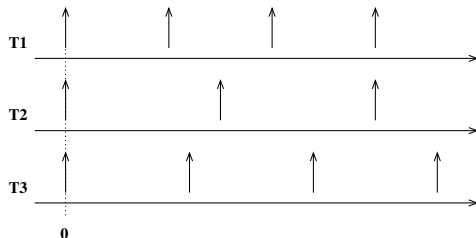


- Arbitrary deadline (D_i can be greater than P_i): the most complex case

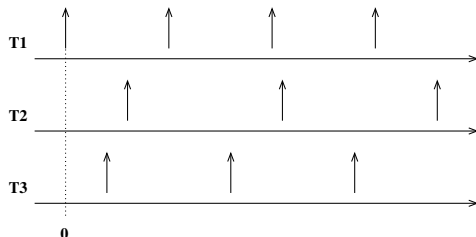


Concrete real-time task

- The first release time r_i of the task is known
- Synchronous system: all the tasks have the same first release time (0)



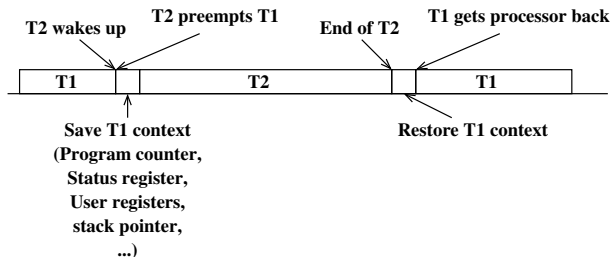
- Asynchronous system: tasks have different release times



Preemptive vs non preemptive tasks

- What is a preemption?

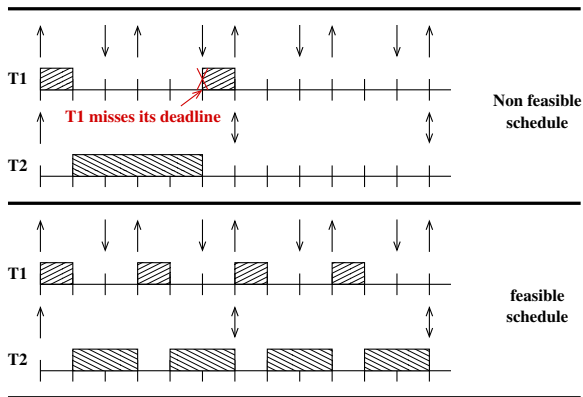
- ▶ the execution of a task is suspended at a point in its execution and will be resumed later at the same point
- ▶ Pros
 - ★ An urgent task can preempt a non-urgent time consuming one
- ▶ Cons
 - ★ context switch overhead



- ▶ Sometimes limited preemption: only possible at given point in the task

Real-time Scheduling

- Multitask system \Rightarrow tasks have to be scheduled
- Real time constraints (especially deadlines) have to be respected.
- Feasible schedule: the worst-case response time of every job of every task is not greater than the task's relative deadline
- Ex: T_1 (r_1 : 0, C_1 : 1, D_1 : 2, P_1 : 3), T_2 (r_2 : 0, C_2 : 4, D_2 : 6, P_2 : 6)

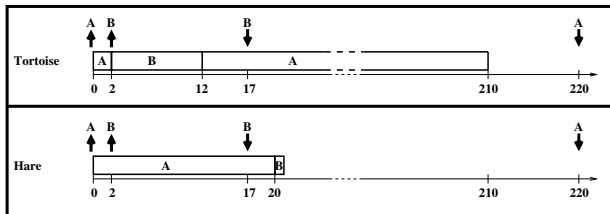


The tortoise and the hare (G. Le Lann)

- Two processors $Pr1$ (hare) et $Pr2$ (tortoise)
 - $Pr1$ 10 times faster than $Pr2$
 - $Pr1$: non preemptive scheduling *First Come First Served*
 - $P2$: preemptive scheduling *Earliest Deadline First*
- Two tasks

| | C^{hare} | $C^{tortoise}$ | Deadline | Release |
|---|------------|----------------|----------|---------|
| A | 20 | 200 | 220 | 0 |
| B | 1 | 10 | 15 | 2 |

- B respects its deadline on $P2$ but not on $P1$



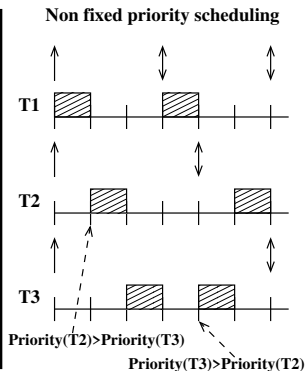
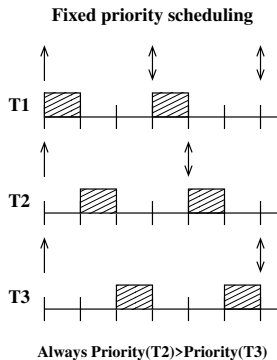
- Choosing the right scheduling policy is essential

Real-Time Scheduling

- Many work has been devoted to real-time scheduling problems
 - ▶ Scheduling policies
 - ▶ Feasibility tests
 - ▶ Complex task models
 - ▶ ...
- Feasibility problem is NP-hard, except for basic cases \Rightarrow
 - ▶ Exact test at exponential cost
 - ▶ Pessimistic test at polynomial or pseudo-polynomial cost
- Scheduling algorithms
 - ▶ On-line scheduling policies (priority driven)
 - ★ Knowledge of the active jobs only
 - ★ Simple scheduling policy in order to choose the highest priority ready job at specific instants (ends of job execution and/or job releases and/or ...)
 - ▶ Off-line scheduling policies (time driven)
 - ★ Knowledge of the past and the future
 - ★ off-line creation of a periodic feasible schedule, using meta-heuristics, branch and bound, ...

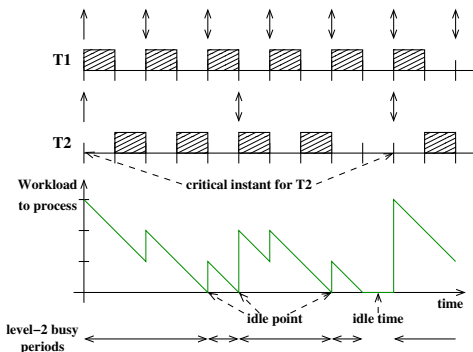
On-line fixed priority scheduling

- We mainly address on-line scheduling in this module
 - ▶ At any time, each ready job is assigned the priority of its task
- Task priorities are fixed off-line and don't change dynamically
- Ex: T_1 (r_1 : 0, C_1 : 1, D_1 : 3, P_1 : 3), T_2 (r_2 : 0, C_2 : 1, D_2 : 4, P_2 : 4), T_3 (r_3 : 0, C_3 : 2, D_3 : 6, P_3 : 6)



Critical instant and busy period

- We assume that tasks are ordered by priority level
 - ▶ Priority (T_1) > Priority (T_2) < Priority (T_3) > ...
- Critical instant for a task T_i : T_i is released simultaneously with all the higher priority tasks
- Level-i busy period: time period where the CPU is kept busy by tasks whose priority is higher or equal to priority of T_i
- Ex: T_1 (r_1 : 0, C_1 : 1, D_1 : 2, P_1 : 2), T_2 (r_2 : 0, C_2 : 2, D_2 : 5, P_2 : 5)



Results based on critical instant and busy period (1)

- A first theorem: the worst-case response time for a task T_i occurs during the longest level- i busy period
- A second theorem: the longest level- i busy period is initiated by the critical instant for task T_i
- Benefit of these two theorems: the worst-case is known for non concrete and synchronous task systems
 - ▶ build the first level- i busy period starting from the critical instant for T_i
 - ▶ claim the worst response time of T_i jobs in this busy period as the worst-case response time for T_i
- A busy period always ends iff the processor is not overloaded (its utilization ratio U doesn't exceed 1):

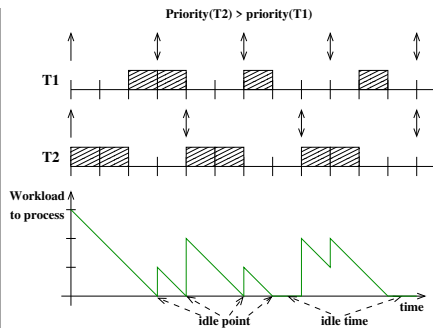
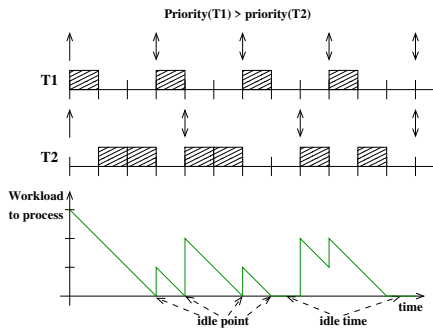
$$U = \sum_{i=1}^n \frac{C_i}{P_i} \leq 1$$

- ▶ Synchronous task system \Rightarrow idle time during the least common multiple of the periods ($LCM_{1 \leq i \leq n} T_i$)

$$LCM_{1 \leq i \leq n} T_i \times (1 - U)$$

Results based on critical instant and busy period (2)

- Idle times and instants are the same for any conservative scheduling algorithm



- Problem: building the level- i busy period time unit per time unit is exponential in time
 - When periods are co-prime numbers, we have

$$LCM_{1 \leq i \leq n} T_i = T_1 \times \dots \times T_n$$

Results based on critical instant and busy period (3)

- Specific case when $D_i \leq P_i$ and preemptive scheduling
 - ▶ If there are two jobs of T_i in a level- i busy period, then T_i misses its deadline \Rightarrow pseudo-polynomial test
- Find the smallest fixed point $R_i^{(*)}$ of the equation:

$$\begin{aligned}R_i^{(0)} &= C_i \\R_i^{(n+1)} &= C_i + \sum_{j \in hp(i)} \lceil \frac{R_i^{(n)}}{P_j} \rceil \times C_j\end{aligned}$$

- Ex: T_1 (r_1 : 0, C_1 : 1, D_1 : 2, P_1 : 2), T_2 (r_2 : 0, C_2 : 2, D_2 : 5, P_2 : 5)

$$\begin{aligned}R_2^{(0)} &= C_2 &= 2 \\R_2^{(1)} &= C_2 + \lceil \frac{R_2^{(0)}}{P_1} \rceil \times C_1 &= 2 + \lceil \frac{2}{2} \rceil \times 1 &= 3 \\R_2^{(2)} &= C_2 + \lceil \frac{R_2^{(1)}}{P_1} \rceil \times C_1 &= 2 + \lceil \frac{3}{2} \rceil \times 1 &= 4 \\R_2^{(3)} &= C_2 + \lceil \frac{R_2^{(2)}}{P_1} \rceil \times C_1 &= 2 + \lceil \frac{4}{2} \rceil \times 1 &= 4\end{aligned}$$

Results based on critical instant and busy period (4)

- What if $D_i > P_i$?

- ▶ $R_i^{(*)} \leq P_i \Rightarrow$ no difference with $D_i \leq P_i$
- ▶ $R_i^{(*)} > P_i \Rightarrow$ a second job of T_i has been released and the busy period is not over
- ▶ The response time of this job has to be computed
- ▶ General case: k jobs have to be tested, until a job finishes execution before the release of the following job
- ▶ Process: starting with $k = 1$, compute the latest possible ending time $R_i^{(*)}(k)$ of k^{th} job of T_i until $R_i^{(*)}(k) \leq k \times P_i$

$$R_i^{(0)}(k) = k \times C_i$$

$$R_i^{(n+1)}(k) = k \times C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i^{(n)}(k)}{T_j} \right\rceil \times C_j$$

Ex: $T_1 (C_1:26, D_1 = P_1:70), T_2 (C_2:62, D_2:118, P_2:100)$

$$T_1: R_1^{(0)}(1) = C_1 = 26$$

$$T_1: R_1^{(1)}(1) = C_1 = 26 = R_1^{(*)}(1)$$

$$T_2: R_2^{(0)}(1) = C_2 = 62$$

$$T_2: R_2^{(1)}(1) = C_2 + \lceil \frac{R_2^{(0)}(1)}{T_1} \rceil \times C_1 = 62 + \lceil \frac{62}{70} \rceil \times 26 = 88$$

$$T_2: R_2^{(2)}(1) = C_2 + \lceil \frac{R_2^{(1)}(1)}{T_1} \rceil \times C_1 = 62 + \lceil \frac{88}{70} \rceil \times 26 = 114$$

$$T_2: R_2^{(3)}(1) = C_2 + \lceil \frac{R_2^{(2)}(1)}{T_1} \rceil \times C_1 = 62 + \lceil \frac{114}{70} \rceil \times 26 = 114 = R_2^{(*)}(1)$$

$$T_2: R_2^{(0)}(2) = 2 \times C_2 = 124$$

...

$$T_2: R_2^{(2)}(2) = C_2 + \lceil \frac{R_2^{(1)}(2)}{T_1} \rceil \times C_1 = 124 + \lceil \frac{176}{70} \rceil \times 26 = 202$$

$$T_2: R_2^{(3)}(2) = C_2 + \lceil \frac{R_2^{(2)}(2)}{T_1} \rceil \times C_1 = 124 + \lceil \frac{202}{70} \rceil \times 26 = 202 = R_2^{(*)}(2)$$

\Rightarrow Response time of T_2 second job: 102

Ex: $T_1 (C_1:26, D_1 = P_1:70), T_2 (C_2:62, D_2:118, P_2:100)$

- $R_2^{(*)}(3) = 316$
 \Rightarrow Response time of T_2 second job: 116
- $R_2^{(*)}(4) = 404$
 \Rightarrow Response time of T_2 second job: 104
- $R_2^{(*)}(5) = 518$
 \Rightarrow Response time of T_2 second job: 118
- $R_2^{(*)}(6) = 606$
 \Rightarrow Response time of T_2 second job: 106
- $R_2^{(*)}(7) = 694$
 \Rightarrow Response time of T_2 second job: 94

Rate Monotonic

- Basic algorithm for periodic tasks with implicit deadlines
- The smaller the period of a task, the higher its priority
- Optimal algorithm within the class of fixed priority scheduling algorithms for independent periodic tasks with implicit deadlines
- Sufficient test for the schedulability of a configuration including n tasks:

$$U = \sum_{i=1}^n \frac{C_i}{P_i} \leq n \times (2^{\frac{1}{n}} - 1)$$

| | | | | | | |
|-------|---|------|------|------|------|-------|
| n | 1 | 2 | 3 | 5 | 10 | grand |
| bound | 1 | 0.83 | 0.78 | 0.74 | 0.72 | 0.69 |

- Necessary test for the schedulability of a configuration including n tasks:

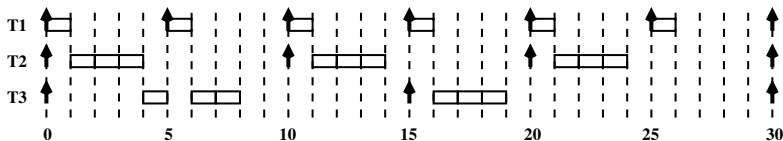
$$U \leq 1$$

Example with Rate Monotonic

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 1 | 5 | 5 |
| T_2 | 3 | 10 | 10 |
| T_3 | 3 | 15 | 15 |

- Scheduling sequence in the case of simultaneous releases



Rate Monotonic: worst-case response time computation

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 1 | 5 | 5 |
| T_2 | 3 | 10 | 10 |
| T_3 | 3 | 15 | 15 |

- Response time analysis

| | T_1 | T_2 | T_3 |
|-------|---|--|---|
| t_0 | C_1 = 1 | $C_1 + C_2$ = 4 | $C_1 + C_2 + C_3$ = 7 |
| t_1 | $C_1 \left\lceil \frac{t_0}{P_1} \right\rceil$ = 1 | $C_1 \left\lceil \frac{t_0}{P_1} \right\rceil + C_2 \left\lceil \frac{t_0}{P_2} \right\rceil$ = 4 | $C_1 \left\lceil \frac{t_0}{P_1} \right\rceil + C_2 \left\lceil \frac{t_0}{P_2} \right\rceil + C_3 \left\lceil \frac{t_0}{P_3} \right\rceil$ = 8 |
| t_2 | | | $C_1 \left\lceil \frac{t_1}{P_1} \right\rceil + C_2 \left\lceil \frac{t_1}{P_2} \right\rceil + C_3 \left\lceil \frac{t_1}{P_3} \right\rceil$ = 8 |
| | OK | OK | OK |

Rate Monotonic: Exercice

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 2 | 5 | 5 |
| T_2 | 4 | 10 | 10 |
| T_3 | 3 | 18 | 18 |

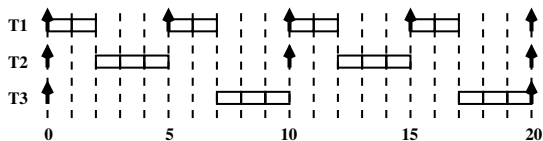
- Response time analysis for T_3 ?

A specific case: harmonic periods

- The sufficient and necessary condition becomes $U \leq 1$
- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 2 | 5 | 5 |
| T_2 | 3 | 10 | 10 |
| T_3 | 6 | 20 | 20 |

- Scheduling sequence in the synchronous case

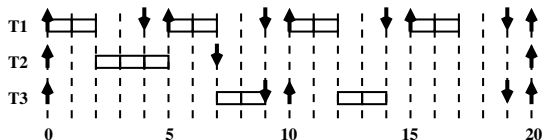


Deadline Monotonic

- In the case of constrained or arbitrary deadlines
- The smaller the deadline of a task, the higher its priority
- Example with the following task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 2 | 4 | 5 |
| T_2 | 3 | 7 | 20 |
| T_3 | 2 | 9 | 10 |

- Scheduling case in the synchronous case

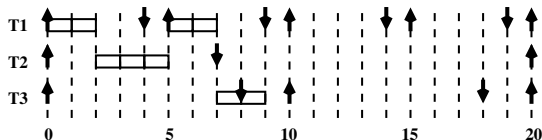


Deadline Monotonic: another example

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 2 | 4 | 5 |
| T_2 | 3 | 7 | 20 |
| T_3 | 2 | 8 | 10 |

- Scheduling sequence in the synchronous case

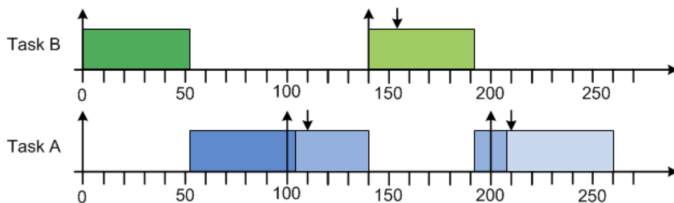


Non optimality of Deadline monotonic

- Deadline Monotonic is not an optimal fixed priority assignment
- Missed deadline if A has the highest priority

■ Tasks with arbitrary deadlines

| Task | Execution Time | Deadline | Period |
|------|----------------|----------|--------|
| A | 52 | 110 | 100 |
| B | 52 | 154 | 140 |



Dynamic priority algorithms

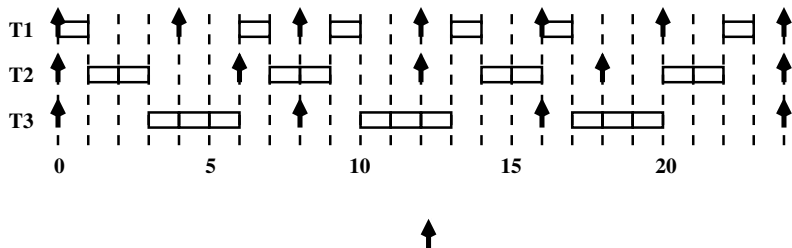
- The priority of a given task is not always the same
- Two classes of solutions
 - ▶ The priority of a job is fixed \Rightarrow job-level fixed priority scheduling
 - ★ Earliest Deadline first
 - ▶ The priority of a job may vary with time \Rightarrow job-level dynamic priority scheduling
 - ★ Least-Laxity First
 - ★ p-fair
 - ▶ Focus on Earliest Deadline First
 - ★ The most important (and analysed) dynamic priority algorithm
 - ★ The priority of a job is inversely proportional to its absolute deadline
 - ★ Two jobs with the same absolute deadline \Rightarrow random choice (doesn't matter)

Earliest Deadline First: example

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 1 | 4 | 4 |
| T_2 | 2 | 6 | 6 |
| T_3 | 3 | 8 | 8 |

- Scheduling sequence in the synchronous case



Earliest Deadline First vs Fixed Priority

- Some advantages of EDF
 - ▶ EDF can schedule all task sets that can be scheduled by Fixed Priority, but not vice versa
 - ▶ There is no need to define priorities in EDF
 - ★ Assuming Fixed Priority, basic priority assignment (e.g. Deadline Monotonic) are not optimal
 - ▶ EDF often has less preemptions
- Some disadvantages of EDF
 - ▶ EDF is not provided in many commercial RTOS
 - ▶ More implementation overhead with EDF
 - ▶ EDF is less controllable
 - ★ Reduce the response time of a task with Fixed Priority \Rightarrow increase its priority, nothing can be done with EDF

Schedulability analysis with EDF

- Task set with implicit deadlines for all the tasks
 - ▶ Necessary and sufficient condition

$$U = \sum_{i=1}^N \frac{C_i}{T_i} \leq 1$$

- ▶ \Rightarrow In this situation, EDF is obviously optimal
- Task sets with constrained or arbitrary deadlines
 - ▶ EDF is still optimal
 - ▶ The schedulability analysis becomes more complex
 - ▶ Based on the demand function (df)
 - ★ The demand function for a task T_i is a function of an interval $[t_1, t_2]$ that gives the amount of T_i computation that must be completed in $[t_1, t_2]$ for T_i to be schedulable

$$df_i(t_1, t_2) = \sum_{a_{ij} \geq t_1, d_{ij} \leq t_2} C_{ij}$$

- ★ For the entire task set

$$df(t_1, t_2) = \sum_{i=1}^N df_i(t_1, t_2)$$

Schedulability analysis with EDF

- For a set of synchronous periodic tasks, the worst case demand is found for intervals starting at 0

$$\forall t_1, t_2 > t_1 \quad df(t_1, t_2) \leq df(0, t_2 - t_1)$$

- Demand bound function

$$dbf(L) = \max_t (df(t, t + L) = df(0, L))$$

- Impossible to compute the dbf for all possible values of L
- For a set of synchronous periodic tasks, only consider one hyperperiod H and only consider task deadlines within H
 - ▶ A synchronous periodic task set TS is schedulable by EDF iff

$$\forall L \in \text{dead}(TS) \quad dbf(L) \leq L$$

Where $\text{dead}(TS)$ is the set of deadlines in $[0, H]$

- The synchronous case is a worst-case of any asynchronous one

Schedulability analysis with EDF

- Example with the following task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 1 | 4 | 4 |
| T_2 | 2 | 6 | 6 |
| T_3 | 3 | 8 | 8 |

- $H = 24$
- deadline set: 4, 6, 8, 12, 16, 18, 20, 24
- $dbf(4) = 1$
- $dbf(6) = 3$
- $dbf(8) = 7$
- $dbf(12) = 10$
- $dbf(16) = 14$
- $dbf(18) = 16$
- $dbf(20) = 17$
- $dbf(24) = 23$

Earliest Deadline First: exercice

- Task features

| | WCET | Deadline | Period |
|-------|------|----------|--------|
| T_1 | 2 | 4 | 5 |
| T_2 | 3 | 7 | 20 |
| T_3 | 2 | 8 | 10 |

- Is it schedulable with EDF?

Rate Monotonic / Earliest Deadline First: Mixed solution

- Trade-off between Fixed Priority simplicity and EDF better schedulability
- Rate Monotonic for tasks with shortest periods
- Other tasks with dynamic lower priorities
- Task features

| | WCET | Deasline | Period |
|-------|------|----------|--------|
| T_1 | 1 | 4 | 4 |
| T_2 | 1 | 5 | 5 |
| T_3 | 2 | 6 | 6 |
| T_4 | 2 | 10 | 10 |

Adding aperiodic tasks

- Up to now, only tasks with a known minimum duration between any two consecutive occurrences (the period)
- What if this minimum duration is unknown (i.e. aperiodic tasks)?
- Two cases have to be considered
 - ▶ Aperiodic jobs don't have deadlines
 - ★ Try to minimize their response time while ensuring that periodic tasks never miss deadlines
 - ▶ Aperiodic jobs do have deadlines
 - ★ Acceptance test: can it be executed before its deadline without jeopardising periodic jobs and already accepted aperiodic ones?
 - ★ If not, one solution is to reject the aperiodic job
 - ★ Another solution is to kill the least important job
 - ★ Approaches are classically based on Earliest Deadline First
- Presentation of some simple scheduling algorithms for the case without deadline

Background execution of aperiodic jobs without deadlines

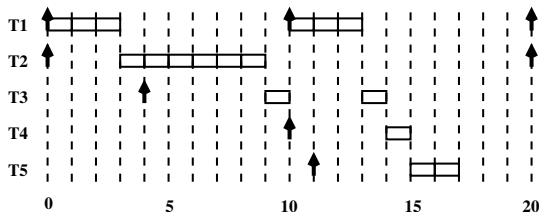
- The simplest solution
- Periodic tasks are scheduled using one of the previously presented algorithms (rate monotonic, deadline monotonic, earliest deadline first, ...)
- Aperiodic jobs are scheduled when there are no pending periodic ones
- Aperiodic jobs are scheduled following a "First Come First Served" policy
- Straightforward implementation
 - ▶ Periodic tasks are assigned priorities, based on the scheduling policy
 - ▶ All aperiodic jobs are assigned the same lowest priority
- This solution makes nothing to minimise the response time of aperiodic jobs

An example of the background execution approach

- Task features

| | (first) occurrence | WCET | Deadline | Period |
|-------|--------------------|------|----------|--------|
| T_1 | 0 | 3 | 10 | 10 |
| T_2 | 0 | 6 | 20 | 20 |
| T_3 | 4 | 2 | | |
| T_4 | 10 | 1 | | |
| T_5 | 11 | 2 | | |

- Task scheduling, assuming rate monotonic



Polling server for aperiodic jobs without deadlines

- Server: periodic task with a budget for the execution of pending aperiodic jobs
- Short period for the server \Rightarrow high priority task (consider for instance rate monotonic)
- Every time the server gets the processor

Budget $\leftarrow C_s$;

Next_Activation $\leftarrow t_{\text{current}} + P_s$;

While Pending aperiodic jobs **and** Budget > 0 **do**

 Execute one time unit of oldest pending aperiodic job;

 Budget \leftarrow Budget - 1;

EndWhile;

Budget $\leftarrow 0$;

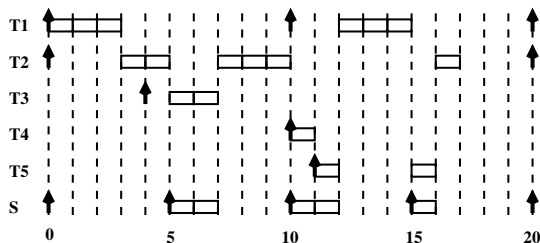
- Aperiodic request after the server dropped its budget \Rightarrow it has to wait for the next server activation

An example of the polling server approach

- Task features

| | (first) occurrence | WCET | Deadline | Period |
|-------|--------------------|------|----------|--------|
| T_1 | 0 | 3 | 10 | 10 |
| T_2 | 0 | 6 | 20 | 20 |
| S | 0 | 2 | 5 | 5 |
| T_3 | 4 | 2 | | |
| T_4 | 10 | 1 | | |
| T_5 | 11 | 2 | | |

- Task scheduling assuming rate monotonic



Deferable server for aperiodic jobs without deadlines

- The budget is not reset to 0 when there are no (more) pending aperiodic jobs
- Every time the server is activated

Budget $\leftarrow C_s$;

Next_Activation $\leftarrow t_{\text{current}} + P_s$;

While pending aperiodic jobs **and** Budget > 0 **do**

 Execute one time unit of oldest pending aperiodic job;

 Budget \leftarrow Budget - 1;

EndWhile;

- Aperiodic request while server is not executing, but has some remaining budget

While pending aperiodic jobs **and** Budget > 0 **do**

 Execute one time unit of new pending aperiodic job;

 Budget \leftarrow Budget - 1;

EndWhile;

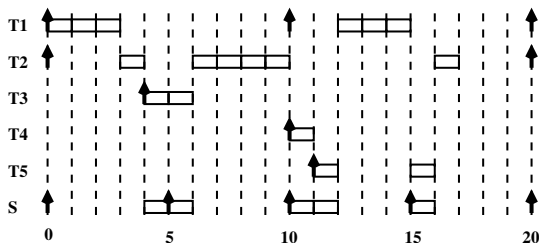
- Can lead to missed deadlines for periodic jobs!!!

An example of the deferrable server approach

- Task features

| | (first) occurrence | WCET | Deadline | Period |
|-------|--------------------|------|----------|--------|
| T_1 | 0 | 3 | 10 | 10 |
| T_2 | 0 | 6 | 20 | 20 |
| S | 0 | 2 | 5 | 5 |
| T_3 | 4 | 2 | | |
| T_4 | 10 | 1 | | |
| T_5 | 11 | 2 | | |

- Task scheduling assuming rate monotonic

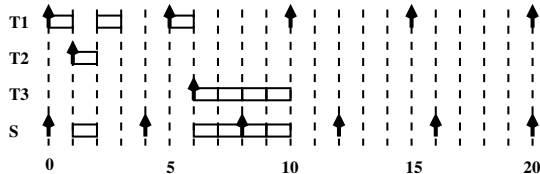


Another example of the deferrable server approach

- Task features

| | (first) occurrence | WCET | Deadline | Period |
|-------|--------------------|------|----------|--------|
| T_1 | 0 | 2 | 5 | 5 |
| S | 0 | 2 | 4 | 4 |
| T_2 | 1 | 1 | | |
| T_3 | 6 | 4 | | |

- Task scheduling assuming rate monotonic



Sporadic server for aperiodic jobs without deadlines

- No budget reset, the period starts when the server consumes budget
- Every time the server is activated

Budget \leftarrow Cs;

If pending aperiodic jobs **then**

Next_Activation \leftarrow t_current + Ps;

While pending aperiodic jobs **and** Budget > 0 **do**

Execute one time unit of oldest pending aperiodic job;

Budget \leftarrow Budget - 1;

EndWhile;

EndIf;

- Aperiodic request while server is not executing, but has some remaining budget

If Budget = Cs **Then** Next_Activation \leftarrow t_current + Ps;

While pending aperiodic jobs **and** Budget > 0 **do**

Execute one time unit of oldest pending aperiodic job;

Budget \leftarrow Budget - 1;

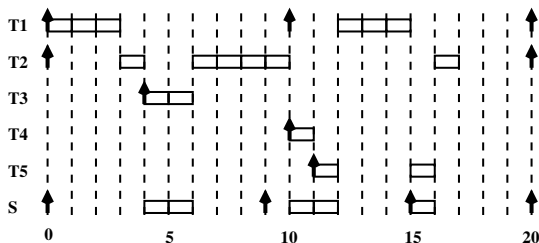
EndWhile;

An example of the sporadic server approach

- Task features

| | (first) occurrence | WCET | Deadline | Period |
|-------|--------------------|------|----------|--------|
| T_1 | 0 | 3 | 10 | 10 |
| T_2 | 0 | 6 | 20 | 20 |
| S | 0 | 2 | 5 | 5 |
| T_3 | 4 | 2 | | |
| T_4 | 10 | 1 | | |
| T_5 | 11 | 2 | | |

- Task scheduling assuming rate monotonic

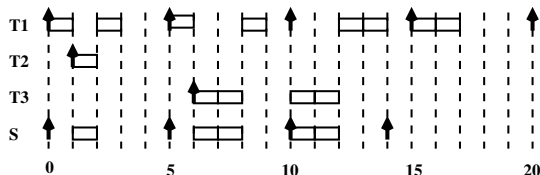


Another example of the sporadic server approach

- Task features

| | (first) occurrence) | WCET | Délai critique | Période |
|-------|---------------------|------|----------------|---------|
| T_1 | 0 | 2 | 5 | 5 |
| S | 0 | 2 | 4 | 4 |
| T_2 | 1 | 1 | | |
| T_3 | 6 | 4 | | |

- Task scheduling assuming rate monotonic



Exercise

- Task features, periodic tasks scheduled with rate monotonic

| | (first) occurrence | WCET | D | P |
|-------|--------------------|------|----|----|
| T_1 | 0 | 1 | 6 | 6 |
| T_2 | 0 | 4 | 12 | 12 |
| T_3 | 0 | 6 | 24 | 24 |
| T_4 | 1 | 2 | | |
| T_5 | 11 | 1 | | |
| T_6 | 13 | 2 | | |

- Response time of T_4 , T_5 and T_6 with the background approach
- Same question with a polling server with budget 1 and period 4
- Same question with a deferrable server with budget 1 and period 4
- Same question with a sporadic server with budget 1 and period 4

Precedence constraints with rate monotonic

- Precedence constraints between periodic tasks \Rightarrow defines an execution order
- one precedence constraint: n^{th} job of task T_i has to execute before n^{th} job of task T_j

$$T_i \rightarrow T_j$$

- Precedence constraints of a configuration are modelled by a precedence graph
- Tasks sharing precedence constraints typically have the same period
- Rate monotonic solution \Rightarrow precedence constraints are integrated in task features offline

- ▶ Activation dates and deadlines are modified offline for each task T_j

$$\begin{aligned} r_j^* &= \max(r_j, r_i^*) && \text{for every } T_i \rightarrow T_j \\ D_j^* &= D_j - (r_j^* - r_j) \end{aligned}$$

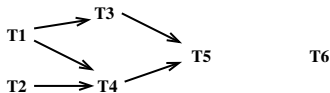
- ▶ Different priorities for tasks with the same period: T_i has a higher priority than T_j if $T_i \rightarrow T_j$

Example with precedence constraints and rate monotonic

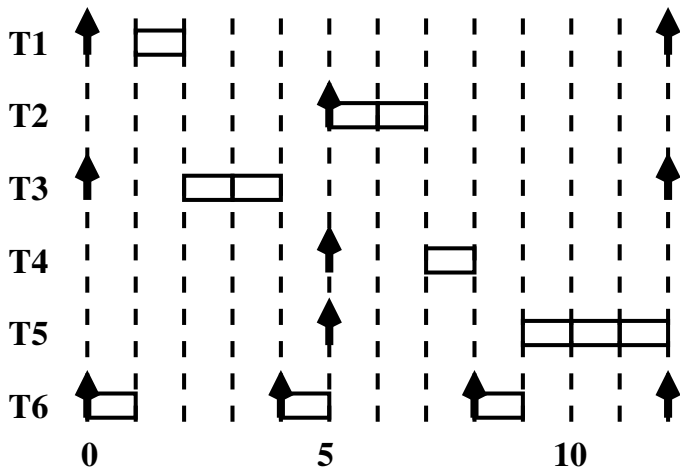
- Task features

| | first occurrence | | WCET | Deadline | | Period |
|-------|------------------|-------|------|----------|-------|--------|
| | r | r^* | | D | D^* | |
| T_1 | 0 | 0 | 1 | 12 | 12 | 12 |
| T_2 | 5 | 5 | 2 | 7 | 7 | 12 |
| T_3 | 0 | 0 | 2 | 12 | 12 | 12 |
| T_4 | 0 | 5 | 1 | 12 | 7 | 12 |
| T_5 | 0 | 5 | 3 | 12 | 7 | 12 |
| T_6 | 0 | 0 | 1 | 4 | 4 | 4 |

- Precedence constraints



Example with precedence constraints and rate monotonic



Precedence constraints with Earliest Deadline First

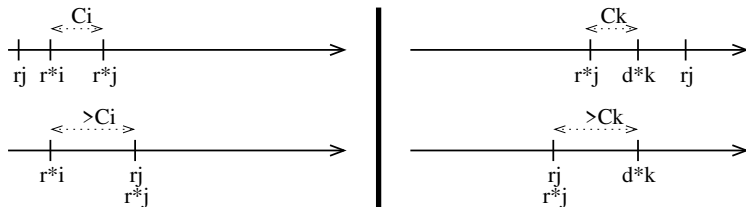
- Similar principle as with rate monotonic \Rightarrow off-line integration of constraints in task features

- Modification of the activation date of T_j

$$r_j^* = \max(r_j, \max(r_i^* + C_i)) \quad \text{for all } T_i \rightarrow T_j$$

- Modification of the deadline of T_j

$$d_j^* = \min(d_j, \min(d_k^* - C_k)) \quad \text{for all } T_j \rightarrow T_k$$

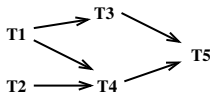


Example with precedence constraints and EDF

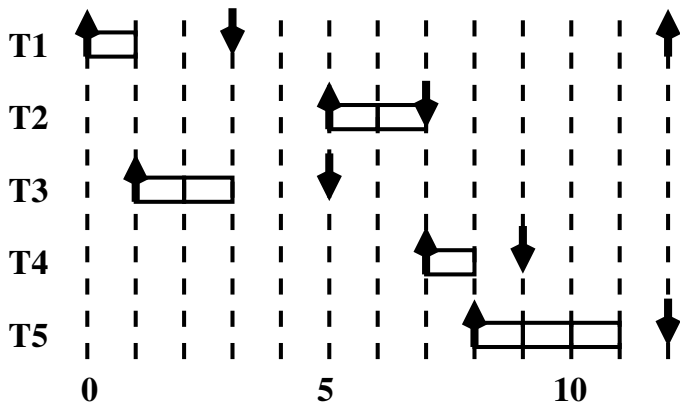
- Task features

| | first occurrence | | WCET | Deadline | | Period |
|-------|------------------|-------|------|----------|-------|--------|
| | r | r^* | | d | d^* | |
| T_1 | 0 | 0 | 1 | 5 | 3 | 12 |
| T_2 | 5 | 5 | 2 | 7 | 7 | 12 |
| T_3 | 0 | 1 | 2 | 5 | 5 | 12 |
| T_4 | 0 | 7 | 1 | 10 | 9 | 12 |
| T_5 | 0 | 8 | 3 | 12 | 12 | 12 |

- Precedence constraints



Example with precedence constraints and EDF



Dealing with shared resources

- Tasks often have to access to shared resources (i.e. shared variables) in mutual exclusion
- Critical sections are used to implement the access to these shared resources.
- A given task T_i cannot enter a critical section if the corresponding resource is not free
- Two potential problems
 - ▶ The deadlock: non cycle-free resource demands at a given time
 - ▶ Priority inversion: a task T_i waits for a resource earned by a low priority task T_j
- A deadlock should never occur in a critical real-time system
 - ▶ One solution: implement tasks such that they a deadlock can never occur (e.g. always take resources in the same order)
 - ▶ Another solution: provide a resource allocation mechanism that prevents deadlock (i.e. gives a resource only if it cannot lead to a deadlock)
- Priority inversion cannot be avoided, but should be upper bounded
 - ▶ Thanks to the resource allocation mechanism

Example of a deadlock

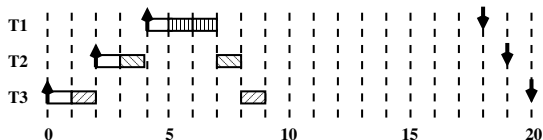
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 14 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-----------|--|
| | R_3 | R_3 | $R_1 R_3$ | |
| | R_2 | R_2 | $R_2 R_3$ | |
| | R_1 | R_1 | $R_1 R_2$ | |

- Scheduling result



- Deadlock at $t = 9$

Example of a priority inversion

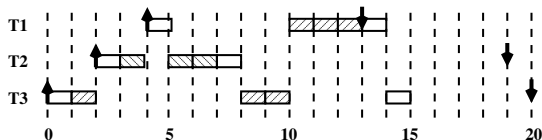
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-------|--|
| | R_1 | R_1 | R_1 | |
| | R_2 | R_2 | R_2 | |
| | R_1 | R_1 | R_1 | |

- Scheduling result



- Priority inversion between $t = 5$ and $t = 10$

The "super priority" approach

- The simplest solution to prevent deadlock and upper bound priority inversion
- A task asks for a resource \Rightarrow it gets it immediately
- Every task which owns a resource gets the highest priority in the system \Rightarrow no preemption within a critical section \Rightarrow at most one task in critical section at any time
- A deadlock cannot occur
- the duration of a priority inversion is upper bounded by the longest critical section
- No need to know resource utilisation by tasks beforehand
- Doesn't take into account which resource is used \Rightarrow can generate additional priority inversions

A first example of the super priority approach

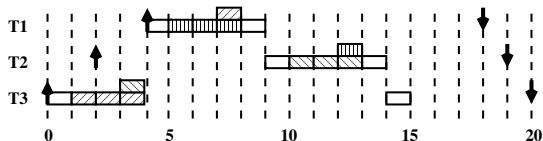
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 14 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-----------|--|
| | R_3 | R_3 | $R_1 R_3$ | |
| | R_2 | R_2 | $R_2 R_3$ | |
| | R_1 | R_1 | $R_1 R_2$ | |

- Scheduling result



- No more deadlock

A second example of the super priority approach

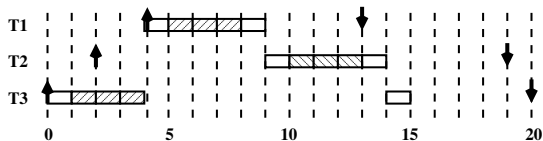
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-------|--|
| | R_1 | R_1 | R_1 | |
| | R_2 | R_2 | R_2 | |
| | R_1 | R_1 | R_1 | |

- Scheduling result



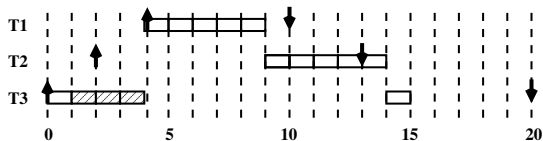
A third example of the super priority approach

- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

| Resource utilisation | | | | |
|----------------------|-------|-------|-------|--|
| | | | | |
| | | | | |
| | R_1 | R_1 | R_1 | |

- Scheduling result



The priority inheritance protocol

- Doesn't prevent deadlock, but upper bound priority inversion duration
- When a task T_i asks for an unavailable resource, the task T_j which owns the resource inherits the priority of blocked task T_i
- This rule is transitively applied
- No need to know resource utilisation by tasks beforehand

An example of the priority inheritance protocol

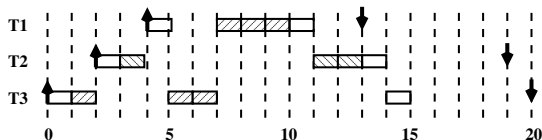
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-------|--|
| | R_1 | R_1 | R_1 | |
| | R_2 | R_2 | R_2 | |
| | R_1 | R_1 | R_1 | |

- Scheduling result



The “stack-based protocol”

- Based on the ceiling priority notion
 - ▶ The ceiling priority of a resource is the highest priority among the tasks which use this resource
 - ▶ Task requests in terms of resources have to be known beforehand
- A task owning no resource executes at its initial priority
- A task owning resources executes at the highest ceiling priority of these resources
- Every task asking for a resource gets it immediately
- It prevents deadlocks
- It upper bounds priority inversion duration

A first example of the “stack-based protocol”

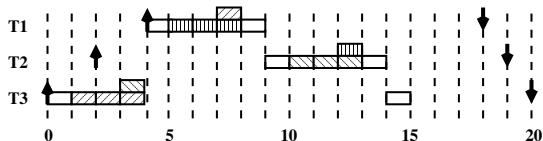
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 14 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

| | | | | |
|--|-------|-------|-----------|--|
| | R_3 | R_3 | $R_1 R_3$ | |
| | R_2 | R_2 | $R_2 R_3$ | |
| | R_1 | R_1 | $R_1 R_2$ | |

- Scheduling result



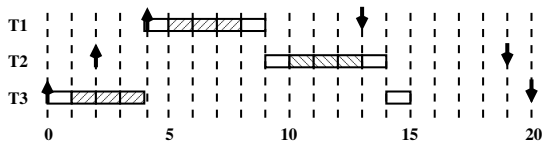
A second example of the “stack-based protocol”

- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

| | | | | |
|----------------------|-------|-------|-------|--|
| Resource utilisation | | | | |
| | R_1 | R_1 | R_1 | |
| | R_2 | R_2 | R_2 | |
| | R_1 | R_1 | R_1 | |

- Scheduling result



A third example of the “stack-based protocol”

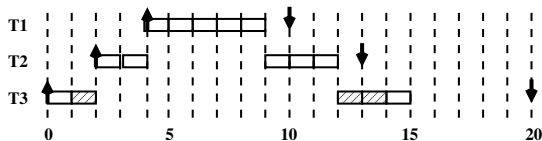
- Task features

| | First release | WCET | Deadline | Period |
|-------|---------------|------|----------|--------|
| T_1 | 4 | 5 | 9 | 20 |
| T_2 | 2 | 5 | 17 | 20 |
| T_3 | 0 | 5 | 20 | 20 |

Resource utilisation

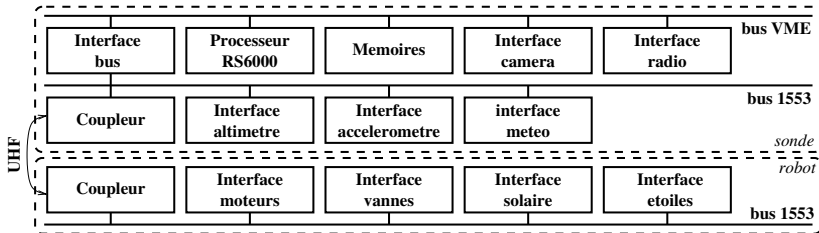
| | | | | |
|--|-------|-------|-------|--|
| | | | | |
| | | | | |
| | R_1 | R_1 | R_1 | |

- Scheduling sequence



A real example: the Pathfinder mission

- NASA mission (Jet Propulsion Laboratory) in order to characterize Mars environment
- Launched: 4 december, 1996
 - ▶ Lander: 264 Kg
 - ▶ Rover 11.5 Kg with six wheels, powered by solar panels and lithium batteries



- Landing on Mars: 4 juillet, 1997
- Some days without any problem, then the embedded computer resets several times \Rightarrow lots of data lost
- Remote detection and correction of the problem

A real example: the Pathfinder mission

- Multitask software using VxWorks RTOS (Wind River System)
- More than 25 periodic and aperiodic tasks
 - ▶ Mode management (fly, approach, landing, exploration)
 - ▶ Surface pointing
 - ▶ Fault analysis
 - ▶ Weather forecast management
 - ▶ 1553 bus control
 - ▶ Star analysis
 - ▶ ...
- Different phases in the mission (fly, approach, landing, exploration) \Rightarrow a subset of the tasks are executing in each phase
- The mode management task puts each task in the right state (asleep, awake)
- An internal real-time clock for the activation of periodic tasks
- Problem occurs during the execution phase

A real example: the Pathfinder mission

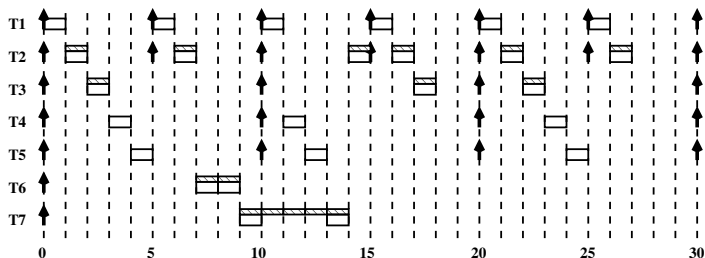
- Task features for the exploration phase

| Task name | WCET | Period | Priority |
|------------------------|----------------|--------------|----------|
| Bus_scheduling (T1) | 25 μs | 125 μs | 1 |
| Data_distribution (T2) | 25 μs | 125 μs | 2 |
| Guiding (T3) | 25 μs | 250 μs | 3 |
| Radio (T4) | 25 μs | 250 μs | 4 |
| Camera (T5) | 25 μs | 250 μs | 5 |
| Measures (T6) | 50 μs | 5000 μs | 6 |
| Weather (T7) | 50; 75 μs | 5000 μs | 7 |

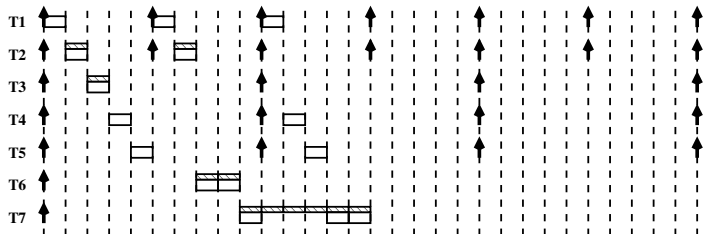
- One time unit: 25 μs
- T2, T3, T6 and T7 share a resource *data_buffer*
- Each task asks for the resource at the beginning of its execution and gives it back at the end
- The inheritance priority protocol is not used

A real example: the Pathfinder mission

- Worst-case when the execution time for Weather task is $50\ \mu s \Rightarrow$ ok

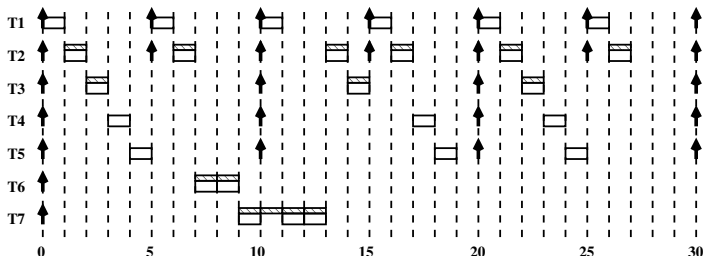


- Worst-case when the execution time for Weather task is $75\ \mu s \Rightarrow$ T2 misses a deadline



A real example: the Pathfinder mission

- Problem: too long priority inversion (between $t = 11$ et $t = 15$)
- Solution: Utilisation of the priority inheritance protocol
- Worst-case when the execution time for Weather task is $75 \mu s \Rightarrow$ ok

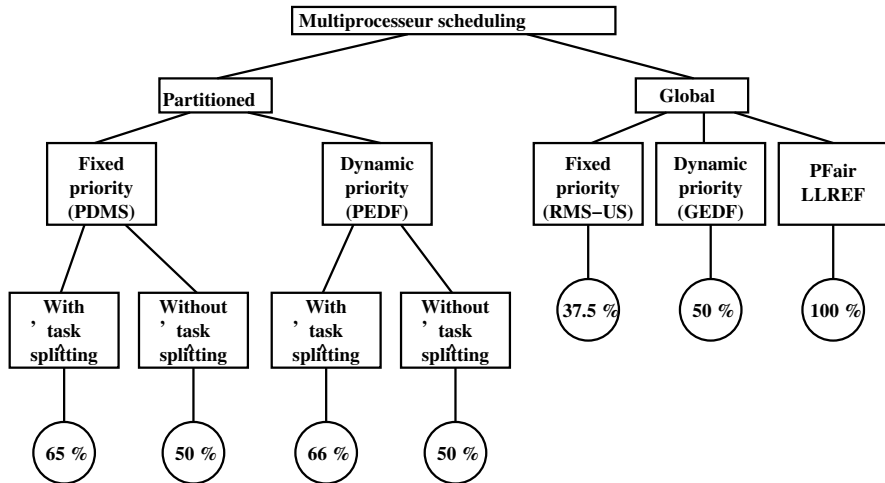


- Correction of the problem by uploading the modified code

Multiprocessor scheduling

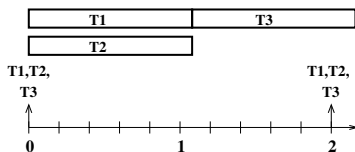
- Problem definition
 - ▶ Scheduling of a set of n periodic tasks on m processors
 - ▶ Each processor executes at most one task at any instant
 - ▶ Each task is executed by at most one processor at any instant
- Different criteria to classify the solutions
 - ▶ To which extent task migrations are allowed
 - ★ Partitioned scheduling No migrations: tasks statically distributed on processors
 - ★ Global scheduling
 - Task level migration: no migration of a job during its execution
 - Job level migration
 - ▶ Priorities assigned to tasks
 - ★ Task level fixed priorities (Rate Monotonic, ...)
 - ★ Job level fixed priorities (Earliest Deadline First, ...)
 - ★ Fully dynamic priorities (least laxity, PFAIR, DP-FAIR, ...)

Overview of the different solutions



Partitioned scheduling

- Tasks are statically distributed on the available processors
 - ▶ Optimal allocation: non polynomial problem (*bin packing*)
- Each processor executes independently its allocated task set
 - ▶ Benefit from the monoprocessor scheduling theory
- The bound on the utilisation factor for each processor can be quite low
 - ▶ $m + 1$ periodic tasks with implicit deadlines
 - ★ Worst-case execution time of one job: $1 + \epsilon$
 - ★ Period of a task: 2
 - ▶ not schedulable on m processors with a partitioned approach \Rightarrow on such a configuration we cannot exceed $U = \frac{m+1}{2}$ on m processors



Distribution of the tasks on the processors

- Divide the task set into m subsets such that each subset is schedulable on one processor
- One possible algorithm: Rate Monotonic First Fit
 - ▶ Tasks are sorted by increasing periods
 - ▶ Each task is allocated to the first processor which remains schedulable with this additional task
- Example of task distribution with RM first fit

| (P_i, C_i) | U_i | (P_i, C_i) | U_i | (P_i, C_i) | U_i |
|--------------|-------|--------------|-------|--------------|-------|
| (2, 1) | 0.500 | (4.5, 0.1) | 0.022 | (8, 1) | 0.125 |
| (2.5, 0.1) | 0.040 | (5, 1) | 0.200 | (8.5, 0.1) | 0.012 |
| (3, 1) | 0.333 | (6, 1) | 0.167 | (9, 1) | 0.111 |
| (4, 1) | 0.250 | (7, 1) | 0.143 | | |

P_1 : (2, 1), (2.5, 0.1), (4.5, 0.1), (6, 1), (8.5, 0.1)

P_2 : (3, 1), (4, 1), (7, 1)

P_3 : (5, 1), (8, 1), (9, 1)

Task splitting to improve the partitioning

- Comes to allow some task migrations
- Example: PDMS-HPTS-DS algorithm
 - ▶ Deadline Monotonic scheduling on each processor
 - ▶ Tasks are allocated by decreasing load
 - ▶ As soon as the task set allocated on the current processor is no more schedulable, a portion of the highest priority task is removed such that the task set is just schedulable
 - ▶ This portion is allocated to the next processor
- At most $m - 1$ splitted tasks for m processors
- The two portions of the splitted task must not execute simultaneously
 - ▶ The release time of the second portion must not be before the latest possible end of the first portion
 - ▶ Splitting the highest priority task minimises this latest possible end

Task splitting to improve the partitioning

- Example with five tasks on two processors

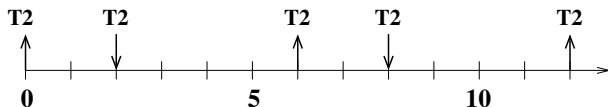
| | (P_i, D_i, C_i) | U_i | CH_i | | (P_i, D_i, C_i) | U_i | CH_i |
|-------|-------------------|-------|--------|-------|-------------------|-------|--------|
| T_1 | (4, 3, 1) | 0.25 | 0.33 | T_2 | (6, 2, 2) | 0.33 | 1 |
| T_3 | (4, 4, 1) | 0.25 | 0.25 | T_4 | (6, 4, 2) | 0.33 | 0.5 |
| T_5 | (6, 5, 1) | 0.16 | 0.2 | | | | |

- Allocation of the task with the highest load, i.e. T_2 on processor $P1$:
OK

P1 **T2**

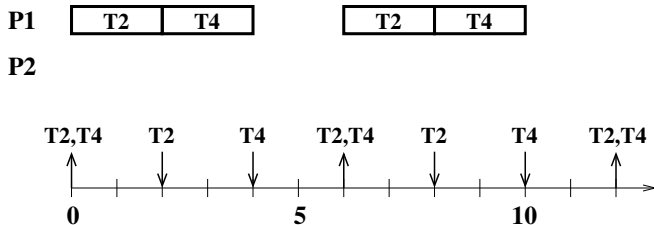
T2

P2

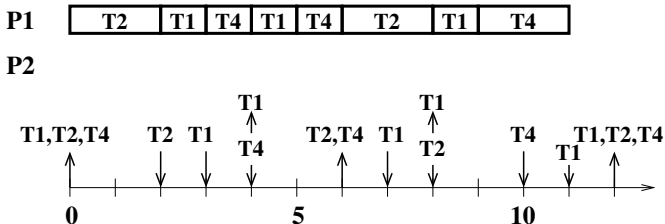


Task splitting to improve the partitioning

- Allocation of the remaining task with the highest load, i.e. T_4 on processor $P1$: OK

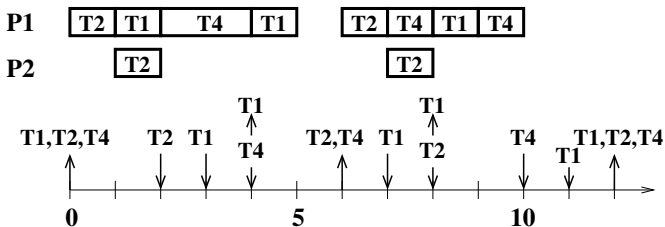


- Allocation of the remaining task with the highest load, i.e. T_1 on processor $P1$: KO (T_4 at 4 and 10)

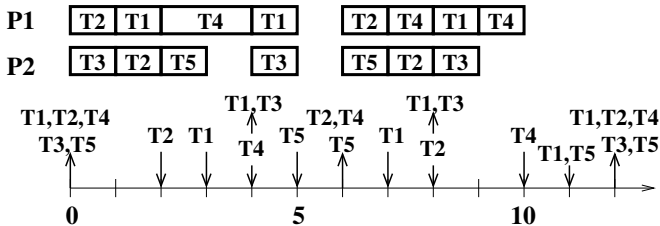


Task splitting to improve the partitioning

- Splitting of the highest priority task (T_2) such that the subset allocated to $P1$ becomes schedulable

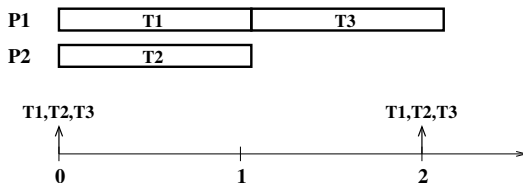


- Allocation of the two remaining tasks (T_3 and T_5) on processor $P2$

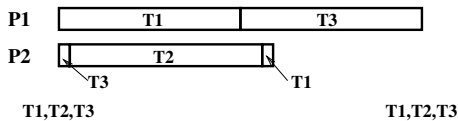


Global approaches without job migrations

- At any time, the m pending tasks with the highest priority are executed
- A job starts execution on a processor and never migrates
- Leads to a load balancing between processors
- Cannot schedule on m processors $m + 1$ periodic tasks avec a WCET of $1 + \epsilon$ and a period of 2
 - ▶ Example: three tasks on two processors



- ▶ Schedulable configuration if job migration is allowed migration des occurrences



Global approaches with job migrations

- A job can migrate on a different processor during its execution
- An ideal algorithm for tasks with implicit deadlines
 - ▶ At any time, every task gets the percentage of the processor equal to its utilisation factor (fluid model)
 - ▶ Example: a task T_i with $C_i = 8$ and $P_i = 11$ always gets $\frac{8}{11} = 72.73\%$ of the processor
 - ▶ Cannot be implemented like that
- An approximation: Pfair (Proportionate-fair)
 - ▶ The processor is allocated by discrete time units
 - ▶ The difference with the fluid model must never exceed one time unit

$$-1 < \text{difference}(T_i, t) < 1 \text{ for } t \in N^*$$

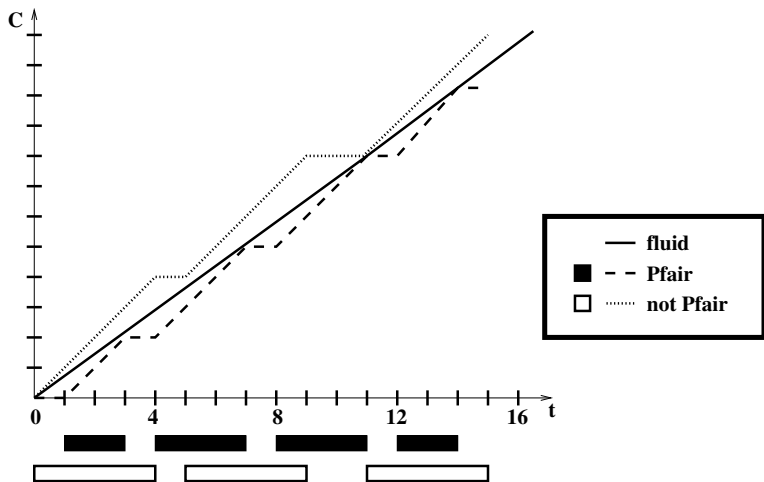
with

$$\text{difference}(T_i, t) = U_i \times t - \sum_{x=0}^{t-1} S(T_i, x)$$

et

$$S(T_i, x) = 1 \text{ if } T_i \text{ is executing during time unit } x \\ 0 \text{ otherwise}$$

Illustration of Pfair for $C_i = 8$ and $P_i = 11$



- Optimal algorithm
- Typically leads to a very high number of preemptions and migrations
⇒ very large overhead

Can a greedy algorithm be optimal?

- Modeling of the urgency of a task by a single value
 - ▶ *Earliest deadline first*
 - ▶ *Least laxity first*
- Fails for schedulable task configurations
- Example: *least laxity first* on two processors processeurs

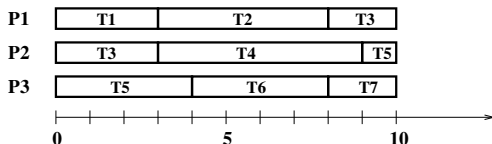
| | C | P |
|-------|---|----|
| T_1 | 9 | 10 |
| T_2 | 9 | 10 |
| T_3 | 8 | 40 |

- Utilisation factor of 2 for the configuration \Rightarrow no idle time on any processor for any valid scheduling sequence
- One idle time at $t = 9$ with *least laxity first* \Rightarrow fails
- No known greedy algorithm successfully schedules this configuration

Specific case: all the tasks have the same deadline

- Each task gets a slot equal to its WCET
- For m processors and a duration d_{dead} until the deadline
 - ▶ Building of a sequence with length $m \times d_{dead}$ with a slot of length its WCET for each task
 - ▶ Splitting of the sequence in m parts with the same length
- An example with three processors and seven tasks

| | C | P | | C | P |
|-------|---|----|-------|---|----|
| T_1 | 3 | 10 | T_5 | 5 | 10 |
| T_2 | 5 | 10 | T_6 | 4 | 10 |
| T_3 | 5 | 10 | T_7 | 2 | 10 |
| T_4 | 6 | 10 | | | |



DP-Fair : Deadline Partitionning

- Segmentation of the time, based on task deadlines
- In each portion, each task gets a slot corresponding to its utilisation factor
- The algorithm of the previous slide is applied in each portion
- Much less preemptions and migrations than with Pfair
- Example with three tasks on two processors

| | C | P |
|-------|---|----|
| T_1 | 9 | 10 |
| T_2 | 9 | 10 |
| T_3 | 8 | 40 |

