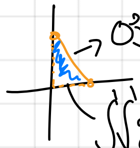
Solución Parcial Final:

1. $$f_{XY}(x,y) = \begin{cases} 2 & \text{Si } y+x \leq 1,\ x>0,\ y>0 \\ 0 & \text{d.l.c} \end{cases}$$

$$\operatorname{Cov}(X,Y) = E(XY) - E(X)\cdot E(Y)$$



Ojo Sin los bordes $x=0$ ni $y=0$

$$\int_0^1\int_0^1 2\,dx\,dy = 1$$

2

Marginales:

- $$f_X(x) = \int_0^{1-x} f_{XY}(x,y)\,dy$$
$$= 2\left[y\right]_0^{1-x} = 2(1-x) \quad \text{Para } x>0$$

- $$f_Y(y) = \int_0^{1-y} f_{XY}(x,y)\,dx$$
$$= 2\cdot\left[x\right]_0^{1-y} = 2(1-y) \quad \text{Para } y>0$$

Valores esperados:

- $$E(X) = \int_0^{1-y} \overbrace{x\cdot f_X(x)}^{2(x-x^2)}\,dx = 2\left[\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\right]_0^{1-y} = (1-y)^2 - \frac{2}{3}(1-y)^3$$

- $$E(Y) = \int_0^{1-x} y\cdot f_Y(y)\,dy = 2\left[\left(\frac{y^2}{2}-\frac{y^3}{3}\right)\right]_0^{1-x} = (1-x)^2 - \frac{2}{3}(1-x)^3$$

- $$E(XY) = \int_0^{1-x}\int_0^{1-y} xy\, f_{XY}(x,y)\,dx\,dy$$
$$= \int_0^{1-x}\int_0^{1-y} xy\cdot 2\,dx\,dy$$
$$= 2\int_0^{1-x} \frac{(1-y)^2}{2}\cdot y\,dy = \int_0^{1-x} (1^2-2y+y^2)\,y\,dy$$
$$= \int_0^{1-x} y-2y^2+y^3\,dy$$
$$= \frac{(1-x)^2}{2} - 2\frac{(1-x)^3}{3} + \frac{(1-x)^4}{4}$$

Luego:

$$\operatorname{Cov}(X,Y) = \frac{(1-x)^2}{2} - 2\frac{(1-x)^3}{3} + \frac{(1-x)^4}{4} - \left((1-y)^2-\frac{2}{3}(1-y)^3\right)\cdot\left((1-x)^2-\frac{2}{3}(1-x)^3\right)$$

Kolmogorov ← 2. # defectos tiene Poisson con $E(x) = 0.7$, $\alpha = 0.05$
o χ^2 $n = 60$

$$H_0: P(x) = \underbrace{\frac{\lambda^x \cdot e^{-\lambda}}{x!}}_{\text{Distr. Poisson.}}, \quad x = 0, 1, 2, \dots \quad E(x) = \lambda = 0.7$$

$$H_a: P(x) \neq \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

Num defectos	Frecuencia observada
0	32
1	15
Agrupar. ← 2,3	13

$P_0 = P(X=0) = e^{-0.7} = 0.4965$

$P_1 = P(X=1) = 0.7 \cdot e^{-0.7} = 0.34755$

$P_{2,3} = P(X \geq 2) = 1 - P(X < 2) = 1 - 0.4965 - 0.34755 = 0.15595$

$E(N_0) = 60 \cdot P_0 = 29.79$

$E(N_1) = 60 \cdot P_1 = 20.853$

$E(N_{2,3}) = 60 \cdot P_{2,3} = 9.357$

- Est. de Prueba:

$$\chi^2_{3-1\,gl} \; \sum_{i=1}^{3} \frac{(N_i - E(N_i))^2}{E(N_i)} = \frac{(32 - 29.79)^2}{29.79} + \frac{(15 - 20.853)^2}{20.853} + \frac{(13 - 9.357)^2}{9.357}$$

$$= \underbrace{\frac{4.8841}{29.79}}_{0.16395} + \underbrace{\frac{34.2576}{20.853}}_{1.6428} + \underbrace{\frac{13.271}{9.357}}_{1.41834}$$

$$= 0.16395 + 1.6428 + 1.41834$$

$$= 3.225$$

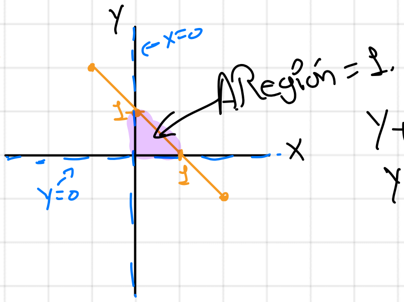
- RR

$\chi^2_{2, 0.05} = 5.99$ $\quad RR\{\chi^2 > 5.99\}$

$IP = P(\chi^2 > 3.225) = 0.1994$ Como Est. ∉ RR y $\overbrace{\text{IP pequeño}}^{IP > \alpha}$ no rechazo Ho. Entonces el jefe de control tiene razón con $\alpha = 0.05 < IP$.

3. $f_{XY}(x,y) = \begin{cases} cx+1 & \text{si } y+x \leq 1,\ x \geq 0,\ y \geq 0 \\ 0 & \text{dlc} \end{cases}$

a.



$Y+X=1$

$Y=1-X$ → Convención

b. Find c.

Usando $\iint f_{X}(x,y) = 1$ Por def tenemos:

$$\int_0^1\int_0^1 (cx+1\ dx\,dy = \int_0^1 c\left[\frac{x^2}{2}\right]_0^1 + [x]_0^1\ dy$$

$$= c\int_0^1 \frac{1}{2}+1\ dy \quad \left(\tfrac{3}{2}\right)$$

$$= c\left(\frac{3}{2}\left([y]_0^1\right)\right) = \frac{3}{2}c \cdot 1 = 1$$

$$\boxed{c = \frac{2}{3}}$$

c. $f_X(x) = \int_0^1 f_{XY}(x,y)\,dy$

$$= \int_0^1 \frac{2}{3}x+1\ dy = \frac{2}{3}x\cdot(1-0) + 1\cdot(1-0)$$

$$= \frac{2}{3}x+1$$

$$f_Y(y) = \int_0^1 \frac{2}{3}x+1\ dx$$

$$= \frac{2}{3}\left[\frac{x^2}{2}\right]_0^1 + [x]_0^1 = \frac{1}{3}+1 = \frac{4}{3}$$

4.

a. X Bernoulli con $P=\frac{1}{3}$, Y exp con $\lambda=3$, Z Poisson $\lambda=2$

$U = XY + (1-X)Z$

$$FGM(U) = E\left(e^{S(XY+(1-X)Z)}\right)$$

$X \cdot X = X^2$

$X^{1+1} = X \cdot X$ $\quad a^{bc} = (a^b)^c$

$$= E(e^{SXY}) \cdot E(e^{SZ-XZ})$$

$$e^{2(e^S-1)}$$

$$= E(e^{SXY}) \cdot E(e^{SZ}) \cdot E(e^{S(-XZ)})$$

$$= E(e^{SXY}) \cdot e^{2(e^S-1)} \cdot E(e^{S(XZ)})$$

b. $FGM(2Z+3) = E(e^{S(2Z+3)})$

Constante

$$= E(e^{S2Z}) \cdot E(e^{3S})$$

$$= e^{3S} \cdot e^{(2(e^{2S}-1))}$$

$$= e^{3S+2(e^{2S}-1)}$$

c. $FGM(Y+Z) = E(e^{S(Y+Z)})$

$$= E(e^{SY}) \cdot E(e^{SZ})$$

$$= \frac{3}{3-S} \cdot e^{2(e^S-1)}$$