Funciones de masa de probabilidad conjunta

$$p_{X,Y}(x,y) = P(X=x, Y=y)$$

$$\underbrace{p_X(x) = \sum_y p_{X,Y}(x,y) \;\; y \;\; p_Y(y) = \sum_x p_{X,Y}(x,y)}_{\text{Marginales a partir de la conjunta}}$$

- $p_Z(z) = \sum_{\{(x,y) \mid g(x,y)=z\}} p_{X,Y}(x,y)$ dado $Z = g(X,Y)$

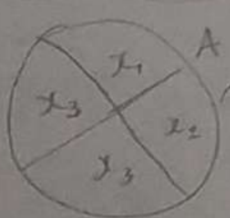
- $E[g(X,Y)] = \sum_x \sum_y g(x,y) \cdot p_{X,Y}(x,y)$

- $E(aX + bY + c)$
  $= a \cdot E(X) + b \cdot E(Y) + c$

Condicionamiento de vars Aleatorias

Condicional a eventos normalita:

- $P(X=x \mid A) = \dfrac{P(\{X=x \cap A\})}{P(A)}$



Teorema:

$$P(A) = \sum_x P\frac{(\{X=x\} \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

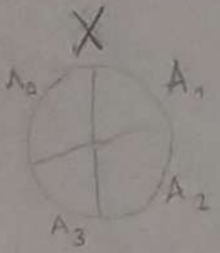
- $p_{X|Y}(x|y) = \dfrac{P(\{X=x\} \mid \{Y=y\})}{P(\{Y=y\})} = \dfrac{p_{X,Y}(x,y)}{p_Y(y)}$

$\Rightarrow p_{X,Y}(x,y) = p_{X|Y}(x,y) \cdot p_Y(y)$

$$P_x(x) = \sum_y P_y(y) \cdot P_{x|y}(x|y)$$

$$P_{(x,y)|z} = \frac{P_{x,y,z}(x,y,z)}{P(z)} \qquad P_{x|(y,z)} = \frac{P_{x,y,z}(x,y,z)}{P_{y,z}(y,z)}$$

$\boxed{\text{Teorema}}$



Si X tiene A_n particiones con $P(A_i) > 0$, $i = \dots, n$

entonces $P_X(x) = \sum_{i=1}^{n} P(A_i) \cdot P_{X|A_i}(x)$

$\boxed{\text{Teorema}}$ Sea B un evento t.q. $P(A_i \cap B) > 0$, $\forall i$

$$P_{X|B}(x) = \sum_{i=1}^{n} P(A_i|B) \cdot P_{X|(A_i \cap B)}(x)$$

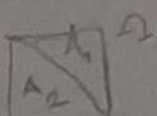
$$= \frac{\cancel{P(A_i \cap B)}}{P(B)} \cdot \frac{P(\{X = x\} \cap A_i \cap B)}{\cancel{P(A_i \cap B)}}$$

Valor esperado condicional

- $E(X|A) = \sum_x x \cdot P_{X|A}(x)$ es un evento
- $E(X|Y=y) = \sum_x x \cdot P_{X|Y}(x|Y=y)$

$\boxed{\text{Teorema}}$ $\cdot\ E(g(x)|A) = \sum_x g(x) \cdot P_{X|A}(x)$

$\boxed{\text{Teorema}}$ Sean A_n eventos que forman una partición de Ω, $P(A_i) > 0$, $\forall i \in \mathbb{N}$.



① $E(X) = \sum_{i=1}^{n} P(A_i) \cdot E(X|A_i)$

③ $P(A_i \cap B) > 0$

$\Rightarrow E(X|B) = \sum_i^n P(A_i|B) \cdot E(X|A_i \cap B)$

③ $E(X) = \sum_y P_Y(y) \cdot E(X|Y=y)$

Independencia ⊙ si $P(\{X=x\} \cap A) = P(\{X=x\}) \cdot P(A) = P_X(x) \cdot P(A)$ $\forall x \in X$.

Teorema → sea A un evento $P(A)>0$, y X indep de A entonces $P_{X|A}(x) = P_X(x)$

⊙ $P_{XY}(x,y) = P_X(x) \cdot P_Y(y) \quad \forall x \in X \ y \ \forall y \in Y$

Teorema → dado x e y independientes:

$$P_{X|Y}(x,y) = P_X(x)$$

Independencia condicional → $P_{(X,Y)|A}(x,y) = P_{X|A}(x) \cdot P_{Y|A}(y)$

- indep. condicional $\not\Rightarrow$ indep. X,Y
- indep. de X,Y $\not\Rightarrow$ indep. condicional

⊙ dado X y Y cond. indep y $P_{Y|A}(y)>0$ entonces $P_{X|(Y,A)}(x) = P_{X|A}(x)$

Teorema → X, Y v.a's indep y discretas

$$E(XY) = E(X)E(Y)$$

Teorema → X, Y v.a's indep. $f,g: \mathbb{R} \to \mathbb{R} \Rightarrow f(x)$ y $g(y)$ son indeps.

Corolario → si X,Y son indep. $f,g: \mathbb{R} \to \mathbb{R}$

$$E(f(x) \cdot g(y)) = E(f(x)) \cdot E(g(y))$$

Distribuciones de probabilidad

- Normal:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-(x-\mu)^2/2\sigma^2}$$

$$E[X] = \mu \qquad var(X) = \sigma^2$$

- Bernoulli:

$$P_X(k) \begin{cases} p & , \text{ si } k=1 \\ 1-p & , \text{ si } k=0 \end{cases}$$

$$E(X) = p \qquad Var(X) = p(1-p)$$

- Binomial

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k} , \quad k = 0, 1, \ldots, n$$

$$E(X) = np$$

- Geometrica $P_X(k) = (1-p)^{k-1}(p) \quad , \; k = 1, 2, 3, \ldots$