Movimiento circular

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T = \frac{\text{tiempo}}{\#\text{oscilaciones}}$$

$$\vec{r}(t) = r(t)\overbrace{\left[\cos(\theta(t))\,\hat{\imath} + \sin(\theta(t))\,\hat{\jmath}\right]}^{\hat{r}}$$

el movimiento circular uniforme asume que:

$$\frac{d\theta}{dt} = \dot{\theta} = \omega = cte$$

$$\frac{dr(t)}{dt} = \dot{r}(t) = 0 \quad , \text{ es decir } |\vec{r}| = cte$$

Con lo anterior para el mov. circular uniforme:

$$\frac{d\vec{r}(t)}{dt} = \dot{r}(t)\,\hat{r} + r(t)\,\dot{\theta}\left[-\sin(\theta(t))\,\hat{\imath} + \cos(\theta(t))\,\hat{\jmath}\right]$$
$$= \dot{r}(t)\,\hat{r} + r(t)\,\dot{\theta}\,\hat{\dot{r}}$$

dado $\dot{\theta} = cte$ y $\dot{r}(t) = 0$ en el circular uniforme:

$$\frac{d\vec{r}(t)}{dt} = \cancel{\dot{r}(t)\,\hat{r}}^{\,0} + r(t)\,\dot{\theta}\,\hat{\dot{r}} = r(t)\,\omega\,\hat{\dot{r}}$$
$$= r(t)\,\omega\left[-\sin(\theta(t))\,\hat{\imath} + \cos(\theta(t))\,\hat{\jmath}\right]$$

la Aceleración en el movimiento circular es:

$$\frac{d}{dt}\left(\frac{d\vec{r}(t)}{dt}\right) = \frac{d}{dt}\left(\dot{r}(t)\,\hat{r} + r(t)\,\dot{\theta}\,\hat{\dot{r}}\right) = \ddot{r}(t)\,\hat{r} + \dot{r}(t)\,\dot{\theta}\,\hat{\dot{r}} +$$
$$\dot{r}(t)\,\dot{\theta}\,\hat{\dot{r}} + r(t)\,\ddot{\theta}\,\hat{\dot{r}} +$$
$$r(t)\,\dot{\theta}\,(-\dot{\theta})\left[\cos(\theta(t))\,\hat{\imath} + \sin(\theta(t))\,\hat{\jmath}\right]$$

$$= \ddot{r}(t)\,\hat{r} + \dot{r}(t)\,\dot{\theta}\,\hat{\dot{r}} +$$
$$\dot{r}(t)\,\dot{\theta}\,\hat{\dot{r}} + r(t)\,\ddot{\theta}\,\hat{\dot{r}} - r(t)\,\dot{\theta}^2\,\hat{r}$$
$$= \hat{r}\left(\ddot{r}(t) - r(t)\,\dot{\theta}^2\right) +$$
$$\hat{\dot{r}}\left(2\dot{r}(t)\,\dot{\theta} + r(t)\,\ddot{\theta}\right) \quad \rightsquigarrow \hat{\dot{r}} = \vec{\theta}$$

- $\ddot{r}(t)$ → Aceleración lineal en dirección radial
- $-r(t)\dot{\theta}^2$ → Aceleración centripeta
- $r(t)\,\ddot{\theta}$ → Aceleración lineal en dirección tangencial
- $2\,\dot{r}\,\dot{\theta}$ → Aceleración coriolis

dado $\dot{\theta} = cte$ y $\dot{r}(t) = 0$ el circular uniforme tiene aceleración:

$$\frac{d}{dt}\left(\frac{d\vec{r}(t)}{dt}\right) = \hat{r}\left(\cancel{\ddot{r}(t)}^{\,0} - r(t)\,\dot{\theta}^2\right) + \hat{\dot{r}}\left(\cancel{2\dot{r}(t)\dot{\theta}}_{\,0} + \cancel{r(t)\ddot{\theta}}_{\,0}\right) = r(t)\,\dot{\theta}^2\,\hat{r} = r(t)\,\omega^2\,\hat{r}$$

Aceleración lineal en el uniforme:

$$|\vec{V}| = \omega\,\overbrace{r}^{cte}$$

Aceleración centripeta en el uniforme:

$$\ddot{\vec{r}}(t) = -r(t)\,\omega^2\,\hat{r} = -\frac{|\vec{V}|^2}{r}\,\hat{\vec{r}}$$

negativa porque va hacia adentro

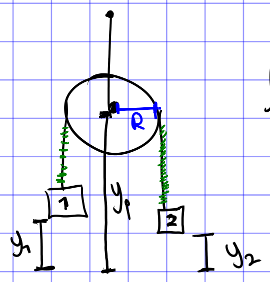
Angulo en función del tiempo:

$$\theta_{final} = \theta_i + \underbrace{\omega_0}_{\text{velocidad angular}} t + \frac{1}{2}\underbrace{\alpha}_{\text{aceleración angular}}{}^2 t$$

radio en función del tiempo:

$$r_{final} = r_0 + \dot{r}_0\,t + \frac{1}{2}\,\ddot{r}\,t^2$$

Ecuación de liguadura



$$l = (y_p - y_1) + \pi R + (y_p - y_2)$$

$$\ddot{l} = 0 = (a_p - a_1) + (a_p - a_2)$$

$$= 2a_p - a_1 - a_2 = 0$$

$$a_p = \frac{(a_1 + a_2)}{2}$$

Aceleración de la polea es cero

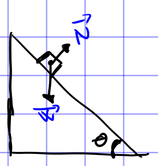
$$0 = \frac{(a_1 + a_2)}{2}$$

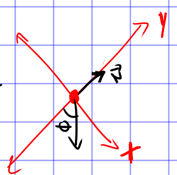
$$a_1 = -a_2$$

Lanzamiento parabólico

- $y(t) = Y_0 + V_{oy} \cdot t - \frac{1}{2} g t^2$
- $x(t) = x_0 + V_{ox} \cdot t$
- $V_{final} = \sqrt{V_{f_y}^2 + V_{f_x}^2}$
- $V_x(t) = V_{ox}$
- $V_y(t) = V_{oy} - gt$
- $V_{ox} = V \cdot \cos(\theta)$
- $V_{oy} = V \cdot \sin(\theta)$

Experimento galileo





$$g \cancel{m} \operatorname{sen}(\theta) = \cancel{m} a$$

$$g \operatorname{sen}(\theta) = a$$

$$\sum F_x = w \operatorname{sen}(\theta) = ma$$

$$\sum F_y = N - w\cos(\theta) = 0$$

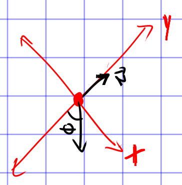
Fricción → Caso estático; Caso dinámico

$$\vec{F}_{fricción} = \underbrace{\mu}_{\text{coef. de fricción}} \overbrace{\vec{N}}^{\text{Normal}}$$

Caso estático

① $\sum F_y = N - mg\cos(\theta) = 0$

$= N = mg\cos(\theta)$



② $\sum F_x = mg \cdot \sin(\theta) - \underbrace{\vec{F}_{fricción}}_{\text{está estático}} = 0$

$= \cancel{mg} \cdot \sin(\theta) - \cancel{mg}\cos(\theta)\,\mu = 0$

$= \sin(\theta) - \mu\cos(\theta) = 0$

$= \mu = \frac{-\sin(\theta)}{-\cos(\theta)} = \tan(\theta)$

$\Rightarrow \mu = \tan(\theta)$ [si está quieto]

Caso dinámico

① $\sum F_y = N - mg\cos(\theta) = 0$

② $\sum F_x = mg\operatorname{sen}(\theta) - \vec{F}_f = m \cdot a$

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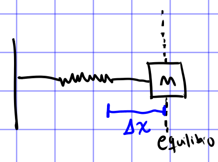
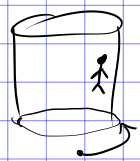
ley de Hooke

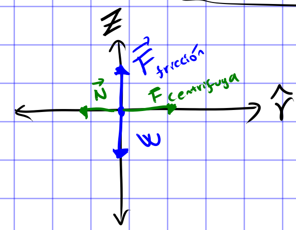


$$\underbrace{F}_{\text{fuerza restauradora}} = -k\cdot\underbrace{(\Delta x)}_{\text{desplazamiento respecto al punto de equilibrio}}$$

$$\sum F_x = -k(\Delta x) = m a$$
$$= -k(\Delta x) = m a$$
$$= \frac{-k(\Delta x)}{m} = a$$

Rotación 2



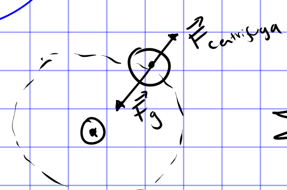


$$\sum F_{\hat{r}} = F_{centrífuga} - N = \omega^2 R m - N = \overset{\text{No sale volando}}{0}$$
$$= \omega^2 R m = N$$

$$\sum F_z = \vec{F}_{fricción} - W = \overset{\text{No se cae dentro del cilindro}}{0}$$
$$= N\mu = mg$$
$$= \omega^2 R \cancel{m} \mu = \cancel{m} g$$
$$= \omega^2 R \mu = g$$
$$= \omega = \sqrt{\frac{g}{R\mu}}$$

Fuerza gravitacional

$$|\vec{F}_g| = \left|\frac{\overset{\text{cte grav universal}}{G}\cdot m_1 \cdot m_2}{R^2}\right|$$

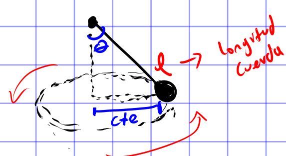


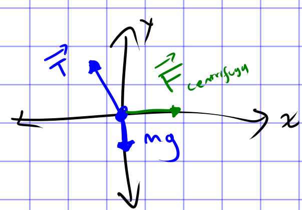
$$\sum F_R = \vec{F}_{centrífuga} - \vec{F}_{gravitacional} = 0$$
$$= \omega^2 R \cancel{m_2} - \frac{G\cdot m_1 \cdot \cancel{m_2}}{R^2} = 0$$
$$= \omega^2 = \frac{G m_1}{R^3}$$
$$= \frac{4\pi^2}{T^2} = \frac{G m_1}{R^3}$$

Aprox. cte. para cualquier planeta

$$= \frac{R^3}{T^2} = \frac{G m_1}{4\pi^2}$$

Péndulo cónico





$$\sum F_x = F_{centrífuga} - \vec{T}\,\mathrm{sen}(\theta) = 0$$
$$= m\omega^2 R = \vec{T}\,\mathrm{sen}(\theta)$$
$$= m\omega^2 (l\,\cancel{\mathrm{sen}(\theta)}) = \vec{T}\,\cancel{\mathrm{sen}(\theta)}$$
$$= m\omega^2 l = \vec{T}$$

$$\sum F_y = \vec{T}\cos(\theta) - mg = 0$$
$$= \vec{T}\cos(\theta) = mg$$
$$= \vec{T} = \frac{mg}{\cos(\theta)}$$