~~Anova Manova 1-way~~ (Anova -1 way)

P_1 : $x_{11}, \dots, x_{1n_1}$ $\quad n = n_1 + n_2 + \dots + n_g$

P_2 : $x_{21}, \dots, x_{2n_2}$ $\quad x_{\ell j}$ es $(p \times 1) \sim N(\mu_\ell, \Sigma_\ell)$

$\vdots$

P_g : $x_{g1}, \dots, x_{gn_g}$ $\quad \ell = 1, \dots, g \quad j = 1, \dots, n_\ell$

- $H_0 : \mu_1 = \mu_2 = \dots = \mu_g$, $\mu_\ell = \mu + \underbrace{(\mu_\ell - \mu)}_{} \rightarrow \tau_\ell$
  $H_0 : \tau_1 = \dots = \tau_g = 0$.

$$x_{\ell j} = \bar{x} + \overbrace{(\bar{x}_\ell - \bar{x})}^{\tau_\ell} + \overbrace{(x_{\ell j} - \bar{x}_\ell)}^{e_{\ell j}}$$

mean $\quad$ treatment effect $\quad$ residual error

$SS_{mean} = \bar{x}$

$SS_{treat} = \tau_\ell$

$SS_{res} = e_{\ell j}$

$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$

Summary

	Sums	df
Treatment	$SS_{tr} = \sum_{\ell=1}^{g} n_\ell \bar{x}_\ell^2$	$g-1$
Residual	$SS_{res} = \sum_{\ell=1}^{g} \sum_{j=1}^{n_\ell} (x_{\ell j} - \bar{x}_\ell)^2$	$n-g$
Total	$SS_{total} = SS_{tr} + SS_{res}$	$n-1$

Test

$H_0 : \mu_1 = \dots = \mu_g$ ($H_0 : \tau_1 = \dots = \tau_g = 0$) $\quad F = \dfrac{\frac{SS_{tr}}{g-1}}{\frac{SS_{res}}{n-g}} \sim \underbrace{F_{g-1, n-g}^{(\alpha)}}_{V\text{-}C}$

Si $F \leq V\text{-}C$, Acepto H_0

Si $F > V\text{-}C$, Rechazo H_0

(Manova 1-way)

$$\underset{(Obs)}{x_{ij}} = \underset{mean}{\bar{x}} + \underset{\text{treatment effect}}{(\bar{x}_\ell - \bar{x})} + \underset{\text{residual error}}{(x_{ij} - \bar{x}_\ell)} \longrightarrow$$

Una ecuación por cada $x_1, x_2, \dots, x_p$

$B = H$, $W = E$

Summary

	SSCP	df
Treatment (H)	$B = \sum n_\ell (\bar{x}_\ell - \bar{x})(\bar{x}_\ell - \bar{x})'$	$g-1$
Residual (E)	$W = \sum^{g} \sum^{n_\ell} (x_{\ell j} - \bar{x}_\ell)(x_{\ell j} - \bar{x}_\ell)'$	$n-g$
Total (T)	$T = W + B$	$n-1$

Test

$H_0 : \tau_1 = \tau_2 = \dots = \tau_g = 0 \qquad \Lambda^* = \dfrac{|SSCP_e|}{|SSCP_e + SSCP_h|}$

Si $E.p \leq V\text{-}C$, Acepto H_0

Si $E.p > V.C$, Rechazo H_0 .

$p=1 \quad v_h \geq 1 \quad \left(\frac{v_e}{v_h}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{v_h, v_e}$

$p=2 \quad v_h \geq 1 \quad \left(\frac{v_e-1}{v_h}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2v_h, 2(v_e-1)}$

$v_h = g-1$

$v_e = n-g$

$p \geq 1 \quad v_h = 1 \quad \left(\frac{v_e+v_h-p}{p}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{p, (v_e+v_h-p)}$

$p \geq 2 \quad v_h = 2 \quad \left(\frac{v_e+v_h-p-1}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2p, 2(v_e+v_h-p-1)}$

En otro caso,

$$-\ln(\Lambda^*) \cdot \left(n-1-\frac{1}{2}(p+g)\right) \sim \chi^2_{p,g-1}$$

Bonferroni $m = pg(g-1)/2$, confianza de $(1-\alpha)100\%$. $(\tau_{li} - \tau_{l'i})$

pertenece $(\bar{x}_{li} - \bar{x}_{l'i}) \pm t_{v_e}(\alpha/2m)\sqrt{\frac{w_{ii}}{n-g}\left(\frac{1}{n_l}+\frac{1}{n_{l'}}\right)}$

(Anova 2 way) replications : $r = 1, \dots, n$

2 way → 2 Factors, Factor A : $l : 1, \dots, g$ Factor B : $k = 1, \dots, b$

$$x_{lkr} = \mu + \tau_l + \beta_k + \gamma_{lk} + e_{lkr}$$
$$= \bar{x} + (\bar{x}_{l..} - \bar{x}) + (\bar{x}_{.k.} - \bar{x}) + (\bar{x}_{lk.} - \bar{x}_{l..} - \bar{x}_{.k.} + \bar{x}) + (x_{lkr} - \bar{x}_{lk.})$$

Summary

	df	Sums
Factor A	$g-1$	$SS_A = \sum^{g} bn(\bar{x}_{l..} - \bar{x})^2$
Factor B	$b-1$	$SS_B = \sum^{b} gn(\bar{x}_{.k.} - \bar{x})^2$
Interacción	$(g-1)(b-1)$	$SS_{AB} = \sum^{g}\sum^{b} n(\bar{x}_{lk.} - \bar{x}_{l..} - \bar{x}_{.k.} + \bar{x})^2$
Residual	$gb(n-1)$	$SS_{res} = \sum^{g}\sum^{b}\sum^{n}(x_{lkr} - \bar{x}_{lk.})^2$
Total	$gbn-1$	$SS_{total} = SS_A + SS_B + SS_{AB} + SS_{res}$

Test $H_0 : \gamma_{11} = \dots = \gamma_{gb} = 0 \qquad F = \frac{SS_{AB}/((g-1)(b-1))}{SS_{res}/gb(n-1)} \sim F_{(g-1)(b-1), gb(n-1)}$

$H_0 : \tau_1 = \dots = \tau_g \qquad F = \frac{SS_A/(g-1)}{SS_{res}/gb(n-1)} \sim F_{g-1, gb(n-1)}$

$H_0 : \beta_1 = \dots = \beta_b \qquad F = \frac{SS_B/(b-1)}{SS_{res}/gb(n-1)} \sim F_{b-1, gb(n-1)}$

Si $F \leq$ V-C, Acepto Ho

Si $F >$ V-C, Rechazo Ho.

Manova 2-way

$x_{lkr} = \mu + \tau_l + \beta_k + \gamma_{lk} + e_{lkr} \rightarrow$ Una ecuación por cada $x_1, \dots, x_p$

$l: 1, \dots, g \quad k: 1, \dots, b \quad r: 1, \dots, n_{lk}$

$$(x_{lkr} - \bar{x}) = (\bar{x}_{l.} - \bar{x}) + (\bar{x}_{.k} - \bar{x}) + \underbrace{(\bar{x}_{lk.} - \bar{x}_{l.} - \bar{x}_{.k} + \bar{x})}_{(*)} + \underbrace{(x_{lkr} - \bar{x}_{lk.})}_{(o)}$$

Summary

Factor A	$SSCP_A = \sum_l bn(\bar{x}_{l.} - \bar{x})(\bar{x}_{l.} - \bar{x})'$	$g-1$
Factor B	$SSCP_B = \sum_k gn(\bar{x}_{.k} - \bar{x})(\bar{x}_{.k} - \bar{x})'$	$b-1$
Interaction	$SSCP_{AB} = \sum_l \sum_k n\, (*)(*)'$	$(g-1)(b-1)$
Residual	$SSCP_{res} = \sum_l \sum_k \sum_r (o)(o)'$	$gb(n-1)$
Total	$SSCP_{total} = SSCP_A + SSCP_B + SSCP_{AB} + SSCP_{res}$	$gbn-1$

Test

$H_0: \gamma_{11} = \dots = \gamma_{gb} \quad \Lambda^* = \dfrac{\det(SSCP_{res})}{\det(SSCP_{res} + SSCP_{AB})} \quad \nu_h = (g-1)(b-1)$

$H_0: \tau_1 = \dots = \tau_g = 0 \quad \Lambda^* = \dfrac{\det(SSCP_{res})}{\det(SSCP_{res} + SSCP_A)} \quad \nu_h = g-1$

$H_0: \beta_1 = \beta_2 = \dots = \beta_b = 0 \quad \Lambda^* = \dfrac{\det(SSCP_{res})}{\det(SSCP_{res} + SSCP_B)} \quad \nu_h = b-1$

$p = 1 \quad \nu_h \geq 1 \quad \left(\dfrac{\nu_e}{\nu_h}\right)\left(\dfrac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{\nu_h, \nu_e} \qquad \nu_e = gb(n-1)$

$p = 2 \quad \nu_h \geq 1 \quad \left(\dfrac{\nu_e - 1}{\nu_h}\right)\left(\dfrac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2\nu_h, 2(\nu_e - 1)}$

$p \geq 1 \quad \nu_h = 1 \quad \left(\dfrac{\nu_e + \nu_h - p}{p}\right)\left(\dfrac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{p, \nu_e + \nu_h - p}$

$p \geq 2 \quad \nu_h = 2 \quad \left(\dfrac{\nu_e + \nu_h - p - 1}{p}\right)\left(\dfrac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2p, 2(\nu_e + \nu_h - p - 1)}$

En otro caso, $\quad -\left(\nu_e - \dfrac{p+1-\nu_h}{2}\right) \ln(\Lambda^*) \sim \chi^2_{p\nu_h}$

Bonferroni

$\nu = gb(n-1)$

Factor 1: $\tau_{li} - \tau_{mi} \in (\bar{x}_{li} - \bar{x}_{mi}) \pm t_\nu\left(\dfrac{\alpha}{pg(g-1)}\right)\sqrt{\dfrac{E_{ii}}{\nu} \cdot \dfrac{2}{bn}}$

Factor 2: $\beta_{ki} - \beta_{qi} \in (\bar{x}_{ki} - \bar{x}_{qi}) \pm t_\nu\left(\dfrac{\alpha}{pb(b-1)}\right)\sqrt{\dfrac{E_{ii}}{\nu} \cdot \dfrac{2}{gn}}$

(Regresion lineal Multivariante I)

Modelo Clasico

$E(\varepsilon_j) = 0$
$Var(\varepsilon_j) = \sigma^2$
$cov(\varepsilon_i, \varepsilon_j) = 0, \ i \neq j$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & z_{11} & \dots & z_{1r} \\ 1 & z_{21} & \dots & z_{2r} \\ \vdots & \vdots & & \\ 1 & z_{n1} & \dots & z_{nr} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_r \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$E(\varepsilon) = 0$
$cov(\varepsilon) = \sigma^2 I$

$$\underset{(n\times 1)}{y} = \underset{(n\times(r+1))}{Z} \cdot \underset{((r+1)\times 1)}{\beta} + \underset{(n\times 1)}{\varepsilon}, \quad y = Z\beta + \varepsilon$$

Estimación de minimos cuadrados

Asumimos Z tiene rango completo $(r+1)$

$E(y) = \hat{y}$ Sea $\hat{y} = Z\hat{\beta}$ y $\hat{\beta} = (Z'Z)^{-1}Z'y$

$E(\beta) = \hat{\beta}$ $\hat{\varepsilon} = y - \hat{y}$, $Z'\hat{\varepsilon} = 0$ y $\hat{y}'\hat{\varepsilon} = 0$.

$cov(\hat{\beta}) = \sigma^2 (Z'Z)^{-1}$

Suma de residuos cuadrados $\rightarrow$ $SSE = \hat{\varepsilon}'\hat{\varepsilon} = y'y - y'Z\hat{\beta}$

Suma de descomposición de cuadrados

$CM = y'\left(\frac{1}{n} 1_n\right) y$

$SS_{regression} = SS_{model} - CM$

$SS_{uncorrected} = SS_{model} + SS_{error}$

Suma total corregida $\rightarrow$ $SS_{TOT\ corrected} = SS_{regression} + SS_{error}$

- Coeficiente de Determinación $\rightarrow$ $R^2 = \dfrac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \dfrac{SS_{regression}}{SS_{TOT.\ correted}}$.

Estimación de minimos cuadrados

$y = Z\beta + \varepsilon$ $\quad \hat{\beta} = (Z'Z)^{-1}Z'y$

$E(\hat{\varepsilon}) = 0$ $\quad cov(\hat{\varepsilon}) = \sigma^2(I - Z(Z'Z)^{-1}Z')$

$E(\hat{\varepsilon}'\hat{\varepsilon}) = (n-r-1)\sigma^2$, $\quad s^2 = \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n-r-1} = \dfrac{y'(I - Z(Z'Z)^{-1}Z')}{n-r-1}$, $\quad E(s^2) = \sigma^2$

Gauss $y = Z\beta + \varepsilon$, $E(\varepsilon) = 0$, $cov(\varepsilon) = \sigma^2 I$, $rank(Z) = r+1$, c vector

$c'\hat{\beta}$ y $a'y$, entonces $a'y = Zc'\hat{\beta} + \varepsilon$.

Inferencia

MLE de β es $\hat{\beta}$, $\hat{\beta} = (Z'Z)^{-1}Z'y \sim N_{r+1}(\beta, \sigma^2(Z'Z)^{-1})$

$\hat{\varepsilon} = y - Z\hat{\beta}$, $\quad \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{\sigma^2} = \dfrac{n\hat{\sigma}^2}{\sigma^2} = \dfrac{(n-r-1)s^2}{\sigma^2} \sim \chi^2_{n-r-1}$, $\hat{\sigma}^2$ MLE de σ^2

$\hat{y} = Z\hat{\beta}$, $\hat{\varepsilon} = y - \hat{y}$, $s^2 = \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n-r-1}$

- Intervalos de confianza

$(\beta - \hat{\beta})' Z'Z (\beta - \hat{\beta}) \leq (r+1)s^2 F_{r+1, n-r-1}(\alpha)$ $\rightarrow$ Region

$\hat{\beta}_i \pm \sqrt{\widehat{var}(\hat{\beta}_i)} \sqrt{(r+1) F_{r+1, n-r-1}(\alpha)}$

$\hat{\beta}_i \pm t_{n-r-1}\left(\frac{\alpha}{2}\right) \sqrt{\widehat{var}(\hat{\beta}_i)}$

$\widehat{var}(\hat{\beta}_i)$ entrada ii de $s^2(Z'Z)^{-1}$

Likelihood Ratio Test

Partimos $\rightarrow$ $Z = \left(\underset{n\times(q+1)}{Z_1} \middle| \underset{n\times(r-q)}{Z_2}\right)$ $\quad \beta = \begin{pmatrix} \underset{(q+1)\times 1}{\beta_{(1)}} \\ \hline \underset{(r-q)\times 1}{\beta_{(2)}} \end{pmatrix}$

$y = Z\beta + \varepsilon = Z_1\beta_{(1)} + Z_2\beta_{(2)} + \varepsilon$

- $H_0: \beta_{(2)} = 0 \Leftrightarrow y = Z_1\beta_{(1)} + \varepsilon$
- H_a: $y = Z\beta + \varepsilon$

$SS_{res}(Z) = (y - Z\hat{\beta})'(y - Z\hat{\beta})$ $\quad df(Z) = n-r-1$

$SS_{res}(Z_1) = (y - Z_1\hat{\beta}_{(1)})'(y - Z_1\hat{\beta}_{(1)})$ $\quad df(Z_1) = n-q-1$

Test $H_0: \beta_{(2)} = 0$ $\quad \hat{\sigma}^2 = \dfrac{(y-z\hat{\beta})'(y-z\hat{\beta})}{n-r-1}$ $\quad F = \dfrac{(SS_{res}(Z_1) - SS_{res}(Z))/r-q}{\hat{\sigma}^2}$

$V.C = F_{r-q,\,n-r-1}(\alpha)$

Si $F \leq V.C$, Acepto H_0
Si $F > V.C$, Rechazo H_0

Generalización $H_0: C\beta = A_0$. Sea C $(r-q)\times(r+1)$ con rango completo y $A_0 = 0$

$H_0: C\hat{\beta} = 0$, $C\beta \sim N_{r-q}(C\beta, \sigma^2 C(Z'Z)^{-1}C')$ $\quad s^2 = \dfrac{(y-z\hat{\beta})'(y-z\hat{\beta})}{n-r-1}$

Si $\dfrac{(C\hat{\beta})'(C(Z'Z)^{-1}C')^{-1}(C\hat{\beta})}{s^2} \leq (r-q)\,F_{r-q,\,n-r-1}(\alpha)$

Acepto H_0

Inferencias $z_0' = (1, z_{01}, \dots, z_{0r})$, $E(y_0|z_0) = \beta_0 + \beta_1 z_{01} + \dots + \beta_r z_{0r} = z_0'\beta$
$var(z_0'\hat{\beta}) = z_0'(z'z)^{-1}z_0\sigma^2$

- Intervalo de confianza para $E(y_0|z_0) = z_0'\beta$ $\longrightarrow$ $z_0'\hat{\beta} \pm t_{n-r-1}\left(\frac{\alpha}{2}\right)\sqrt{(z_0'(z'z)^{-1}z_0)s^2}$

$y_0 = z_0'\hat{\beta}$, $\quad var(y_0 - z_0'\hat{\beta}) = \sigma^2(1 + z_0'(z'z)^{-1}z_0)$

- Intervalo de predicción para $y_0 \longrightarrow z_0'\hat{\beta} \pm t_{n-r-1}\left(\frac{\alpha}{2}\right)\sqrt{s^2(1+z_0'(z'z)^{-1}z_0)}$

(Regresion lineal Multivariada Multiple)

Regresión

$E(\varepsilon_{(i)}) = 0$

$cov(\varepsilon_{(i)}, \varepsilon_{(j)}) = \sigma_{ik} I$

$$\underset{n\times m}{\begin{pmatrix} y_{11} & \dots & y_{1m} \\ \vdots & \ddots & \vdots \\ y_{n1} & \dots & y_{nm} \end{pmatrix}} = \underset{n\times(r+1)}{\begin{pmatrix} 1 & z_{10} & \dots & z_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{n1} & \dots & z_{nr} \end{pmatrix}} \underset{(r+1)\times m}{\begin{pmatrix} \beta_{01} & \beta_{02} & \dots & \beta_{0m} \\ \beta_{11} & & & \\ \vdots & & \ddots & \vdots \\ \beta_{r1} & & \dots & \beta_{rm} \end{pmatrix}} + \underset{n\times m}{\begin{pmatrix} \varepsilon_{11} & \dots & \varepsilon_{1m} \\ \vdots & \ddots & \vdots \\ \varepsilon_{n1} & \dots & \varepsilon_{nm} \end{pmatrix}}$$

$n\times m\ y \qquad n\times(r+1)\ Z \qquad (r+1)\times m\ \beta \qquad n\times m\ \varepsilon$

$\hat{\beta} = (z'z)^{-1}z'y$ $\quad SSCP_{res} = (y-z\hat{\beta})'(y-z\hat{\beta})$ $\quad \hat{y} = z\hat{\beta}$ $\quad \hat{\varepsilon} = y - \hat{y}$

Se cumple $z'\hat{\varepsilon} = 0$ y $\hat{y}'\hat{\varepsilon} = 0$, $y = \hat{y} + \varepsilon$, $y'y = \hat{y}'\hat{y} + \hat{\varepsilon}'\hat{\varepsilon}$

Estimador $E(\hat{\varepsilon}) = 0$, $E\left(\dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n-r-1}\right) = \Sigma$, $E(\hat{\beta}) = \beta$, $cov(\hat{\beta}_i, \hat{\beta}_k) = \sigma_{ik}(z'z)^{-1}$

Con $z_0'\beta_{(i)}$, $cov(z_0'(\hat{\beta}_{(i)} - \beta_{(i)}), z_0'(\hat{\beta}_{(k)} - \beta_{(k)})) = \sigma_{ik} z_0'(z'z)^{-1}z_0$.

$y_{0i} = z_0' + \varepsilon_{0i}$, $E(y_{0i} - z_0'\hat{\beta}_{(i)}) = 0$, $cov(y_{0i} - z_0'\hat{\beta}_{(i)}, y_{0k} - z_0'\hat{\beta}_{(k)}) = \sigma_{ik}(1 + z_0'(z'z)^{-1}z_0)$

$\hat{\Sigma} = \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n} = \dfrac{1}{n}(y-z\hat{\beta})'(y-z\hat{\beta})$, $n\hat{\Sigma} \sim W_{p,\,n-r-1}(\Sigma)$

Test $z = \left(\underset{n\times(q+1)}{z_1} \mid \underset{n\times(r-q)}{z_2}\right)$ $\quad H_0: \beta_{(2)} = 0$ $\quad \hat{\Sigma} = \frac{1}{n}(y-z\hat{\beta})'(y-z\hat{\beta})$

$\hat{\Sigma}_1 = \frac{1}{n}(y - z_1\hat{\beta}_{(1)})'(y - z_1\hat{\beta}_{(1)})$

$\beta = \begin{pmatrix} \underset{(q+1)\times m}{\beta_{(1)}} \\ \hline \underset{(r-q)\times m}{\beta_{(2)}} \end{pmatrix}$ $\quad \hat{y} = z\hat{\beta} = z_1\hat{\beta}_{(1)} + z_2\hat{\beta}_{(2)}$ $\quad \Lambda^{2/n} = \dfrac{|\hat{\Sigma}|}{|\hat{\Sigma}_1|}$

Para n grande, $-\left(n-r-1-\frac{1}{2}(m-r+q+1)\right)\ln\left(\frac{|\hat{\Sigma}|}{|\hat{\Sigma}_1|}\right) \sim \chi^2_{m(r-q)}$ (6)

Generalización: $H_0: C\beta = \beta_{(2)} = 0$, $n(\hat{\Sigma}_1 - \hat{\Sigma}) \sim W_{r-q}(\Sigma)$

Con d predictores $\hat{\Sigma}_d = \frac{1}{n} SSCP_{error}$ $\quad AIC = n\ln(|\hat{\Sigma}_d|) - 2pd$

Alternative Test $\quad H_0: \beta_{(2)} = 0$, $E = (Y - Z\hat{\beta})'(Y - Z\hat{\beta}) = n\Sigma$

$H = n(\hat{\Sigma}_1 - \hat{\Sigma})$. Autovalores $\longrightarrow HE^{-1}$ $\quad \Lambda^* = \frac{|E|}{|E+H|}$

Pillai's Trace $\longrightarrow \text{trace}(H(H+E)^{-1})$ $\quad$ Hotelling-Lawley $\longrightarrow \text{trace}((HE)^{-1})$

Roy's largest root $\longrightarrow \frac{\lambda_1}{1+\lambda_1}$

Predicciones $\quad Y = Z\beta + \varepsilon$, $\hat{\beta} = (Z'Z)^{-1}$ $\quad \hat{\Sigma} = \frac{1}{n}\hat{\varepsilon}'\hat{\varepsilon} = \frac{1}{n}(Y - Z\hat{\beta})'(Y - Z\hat{\beta})$

$n\hat{\Sigma} \sim W_{p,n-r-1}(\Sigma)$ $\quad \hat{\beta}'z_0 = m\times 1$ vector

- T^2-estadistic, $T^2 = \left(\frac{\hat{\beta}'z_0 - \beta'z_0}{\sqrt{z_0'(Z'Z)^{-1}z_0}}\right)'\left(\frac{n}{n-(r+1)}\hat{\Sigma}\right)^{-1}\left(\frac{\hat{\beta}'z_0 - \beta'z_0}{\sqrt{z_0'(Z'Z)^{-1}z_0}}\right)$
- Region de confianza $\rightarrow T^2 \leq \frac{m(n-(r+1))}{n-r-m} F_{m,n-r-m}(\alpha) = V\_C$
- Intervalo de confianza para $E(Y_i) = z_0'\beta_{(i)}$ $\rightarrow z_0'\hat{\beta}_{(i)} \pm \sqrt{\frac{m(n-(r+1))}{n-r-m}F_{m,n-r-m}(\alpha)}\sqrt{z_0'(Z'Z)^{-1}z_0\left(\frac{n}{n-r-1}\right)\hat{\sigma}_{ii}}$ $\quad \hat{\sigma}_{ii} \to \hat{\Sigma}[i,i]$
- Region de Predicción para Y_0 $\rightarrow (Y_0 - \beta'z_0)'\left(\frac{n}{n-r-1}\hat{\Sigma}\right)^{-1}(Y_0 - \beta'z_0) \leq (1 + z_0'(Z'Z)^{-1}z_0)\, V\_C$
- Intervalo de Confianza (pred) para Y_{0i} $\rightarrow z_0'\beta_{(i)} \pm \sqrt{\frac{m(n-r-1)}{n-r-m}F_{m,n-r-m}(\alpha)}\sqrt{(1 + z_0'(Z'Z)^{-1}z_0)\left(\frac{n}{n-r-1}\right)\hat{\sigma}_{ii}}$ $\quad \hat{\sigma}_{ii} \to \hat{\Sigma}[i,i]$