$$y' = \gamma t y + 4e^{-t^2}$$

$$y' - \underbrace{\gamma t}_{p(t)} y = \underbrace{4e^{-t^2}}_{g(t)}$$

el factor integrante $\mu(t) = e^{\int p(t)\,dt}$

$$= e^{-\gamma \int t\,dt}$$

$$= e^{-\gamma \frac{t^2}{2}}$$

$$\int \mu(t)\, y(t)\, dt = \int \boxed{e^{-\gamma\frac{t^2}{2}} \cdot 4e^{-t^2}}$$

$$e^{-t^2\left(\frac{\gamma}{2}+1\right)}$$

Para evitar integrar podemos hacer $e^0 = 1$:

$$\frac{\gamma}{2} + 1 = 0$$

$$\gamma + 2 = 0$$

$$\gamma = -2$$

luego $\displaystyle\int e^{-t^2\left(\frac{-2}{2}+1\right)} = \int 1\,dt = t = \mu(t)y(t)$

$\therefore$ $y(t) = \dfrac{t}{e^{-\gamma\frac{t^2}{2}}} = \dfrac{t}{e^{t^2}}$ (dado que $\gamma = -2$)

$$X' = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}, \quad X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

buscamos los valores propios :

$$|X' - I\lambda| = 0, \quad \begin{vmatrix} 2-\lambda & 2 \\ -4 & 6-\lambda \end{vmatrix} = (2-\lambda)(6-\lambda) - (2\cdot -4)$$

$$= 12 - 2\lambda - 6\lambda + \lambda^2 + 8$$

$$= \lambda^2 - 8\lambda + 20$$

buscamos las raíces

$$\lambda^2 - 8\lambda + 20 = 0$$
$$\lambda^2 - 8\lambda = -20$$
$$\lambda^2 - 8\lambda + 16 = -4$$
$$(\lambda - 4)^2 = -4$$
$$\sqrt{(\lambda - 4)^2} = \sqrt{-4}$$
$$\lambda - 4 = \pm 2i$$
$$\lambda = 4 \pm 2i$$

Buscamos los auto vectores con $\lambda = 4 + 2i$

$$\begin{pmatrix} 2-4-2i & 2 \\ -4 & 6-4-2i \end{pmatrix} = \begin{pmatrix} -2-2i & 2 \\ -4 & 2-2i \end{pmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0$$

$$-\cancel{2}A(1+i) + \cancel{2}B = 0$$
$$\cancel{-}A(1+i) = \cancel{-}B$$
$$A(1+i) = B$$

$$\mathcal{E}_1 = \begin{pmatrix} 1 \\ 1+1i \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

el conjugado $\mathcal{E}_2$: $\mathcal{E}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$$e^{Mt}\left(\cos(\lambda t) + i\,sen(\lambda t)\right) = e^{4t}\left(\cos(2t) + i\,sen(2t)\right)$$

entonces para la solución general:

$$\mathcal{E}_1 \cdot e^{Mt}(\cos(\lambda t) + i\,sen(\lambda t)) = \begin{pmatrix} 1 \\ 1+1i \end{pmatrix} e^{4t}\left(\cos(2t) + i\,sen(2t)\right)$$

$$= e^{4t}\begin{pmatrix} \cos(2t) + i\,sen(2t) \\ \cos(2t) + i\,sen(2t) \\ i\cos(2t) - sen(2t) \end{pmatrix}$$

$$= e^{4t}\left(\begin{bmatrix} \cos(2t) \\ \cos(2t) - sen(2t) \end{bmatrix} + i\begin{bmatrix} sen(2t) \\ \cos(2t) + sen(2t) \end{bmatrix}\right)$$

$$y(t) = e^{4t}\left(c_1\begin{bmatrix} \cos(2t) \\ \cos(2t) - sen(2t) \end{bmatrix} + c_2\begin{bmatrix} sen(2t) \\ \cos(2t) + sen(2t) \end{bmatrix}\right)$$

$$\psi = \begin{bmatrix} \cos(2t) & \sin(2t) \\ \cos(2t) - \sin(2t) & \cos(2t) + \sin(2t) \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Bigg|_0$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - cb} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

$$\Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$