1. Suponga x proviene de una de las 2 poblaciones $\pi_1 \to \mu_1, \Sigma_1$
$\pi_2 \to \mu_2, \Sigma_2$.

Tenemos $f_1(x)$ y $f_2(x)$

Demostración: Sabemos que $f_i(x) = \frac{1}{2\pi^{p/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu_i)' \Sigma^{-1} (x-\mu_i)\right]$

- Si tenemos $\ln\left(\frac{f_1(x)}{f_2(x)}\right)$ entonces al hacer operaciones y aplicar propiedades obtenemos.

$$\ln\left(\frac{f_1(x)}{f_2(x)}\right) = -\frac{1}{2} \ln|\Sigma_1| - \frac{1}{2}(x-\mu_1)'\Sigma_1^{-1}(x-\mu_1) + \frac{1}{2}\ln|\Sigma_2|$$
$$+ \frac{1}{2}(x-\mu_2)'\Sigma_2^{-1}(x-\mu_2)$$

luego

$$= -\frac{1}{2}x'(\Sigma_1^{-1} - \Sigma_2^{-1})x + (\mu_1'\Sigma_1^{-1} - \mu_2'\Sigma_2^{-1})x - k$$

Donde k es $k = \frac{1}{2}\ln\left(\frac{|\Sigma_1|}{|\Sigma_2|}\right) + \frac{1}{2}(\mu_1'\Sigma_1^{-1}\mu_1 - \mu_2'\Sigma_2^{-1}\mu_2)$

Ahora bien, si $\Sigma_1 = \Sigma_2$

entonces $\ln\left(\frac{f_1(x)}{f_2(x)}\right) = (\mu_1-\mu_2)'\Sigma^{-1}x - \frac{1}{2}(\mu_1-\mu_2)'\Sigma^{-1}(\mu_1+\mu_2)$.

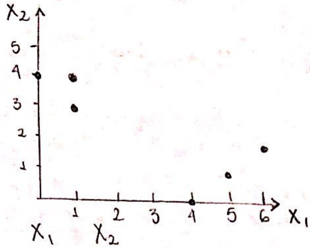
Debido a esto obtenemos una expresion para Q

Parcial 3 - AGD .

2. $n=6$, $p=2$.

obs	X_1	X_2
1	1	4
2	1	3
3	0	4
4	5	1
5	6	2
6	4	0.

a.



b. Del codigo en R obtenemos

label	obs	X_1	X_2
1	1	1	4
2	2	1	3
1	3	0	4
1	4	5	1
2	5	6	2
1	6	4	0

c. Vamos a calcular el centroide de cada grupo.

$$G_1 = \begin{array}{c} \begin{array}{cc} X_1 & X_2 \end{array} \\ \begin{pmatrix} 1 & 4 \\ 0 & 4 \\ 5 & 1 \\ 4 & 0 \end{pmatrix} \end{array} \qquad G_2 = \begin{array}{c} \begin{array}{cc} X_1 & X_2 \end{array} \\ \begin{pmatrix} 1 & 3 \\ 6 & 2 \end{pmatrix} \end{array}$$

$$\bar{X}_{11} = \frac{1+0+5+4}{4} = \frac{5}{2} \qquad C_1 = \left(\frac{5}{2}, \frac{9}{4}\right)$$

$$\bar{X}_{12} = \frac{4+4+1+0}{4} = \frac{9}{4}$$

$$\bar{X}_{21} = \frac{1+6}{2} = \frac{7}{2}$$

$$\bar{X}_{22} = \frac{3+2}{2} = \frac{5}{2} \qquad C_2 = \left(\frac{7}{2}, \frac{5}{2}\right)$$

d. Vamos a encontrar la distancia manhattan. $O_i \rightarrow$ observación i

$$d(C_1,O_1) = \left|\frac{5}{2}-1\right| + \left|\frac{9}{4}-4\right| = \frac{3}{2} + \frac{7}{4} = \frac{13}{4} = 3.25$$

$$d(C_1,O_2) = \left|\frac{5}{2}-1\right| + \left|\frac{9}{4}-3\right| = \frac{3}{2} + \frac{3}{4} = \frac{9}{4} = 2.25$$

$$d(C_1,O_3) = \left|\frac{5}{2}-0\right| + \left|\frac{9}{4}-4\right| = \frac{5}{2} + \frac{7}{4} = \frac{17}{4} = 4.25$$

$$d(C_1,O_4) = \left|\frac{5}{2}-5\right| + \left|\frac{9}{4}-1\right| = \frac{5}{2} + \frac{5}{4} = \frac{15}{4} = 3.75$$

$$d(C_1,O_5) = \left|\frac{5}{2}-6\right| + \left|\frac{9}{4}-2\right| = \frac{7}{2} + \frac{1}{4} = \frac{15}{4} = 3.75.$$

$$d(C_1,O_6) = \left|\frac{5}{2}-4\right| + \left|\frac{9}{4}-0\right| = \frac{3}{2} + \frac{9}{4} = \frac{15}{4} = 3.75$$

$$d(C_2,O_1) = \left|\frac{7}{2}-1\right| + \left|\frac{5}{2}-4\right| = 5/2 + 3/2 = 4$$

$$d(C_2,O_2) = \left|\frac{7}{2}-1\right| + \left|\frac{5}{2}-3\right| = 5/2 + 1/2 = 3$$

$$d(C_2,O_3) = \left|\frac{7}{2}-0\right| + \left|\frac{5}{2}-4\right| = 7/2 + 3/2 = 5$$

$$d(C_2,O_4) = \left|\frac{7}{2}-5\right| + \left|\frac{5}{2}-1\right| = 3/2 + 3/2 = 3$$

$$d(C_2,O_5) = \left|\frac{7}{2}-6\right| + |5/2-2| = \frac{5}{2} + 1/2 = 3$$

$$d(C_2,O_6) = \left|\frac{7}{2}-4\right| + |5/2-0| = \frac{1}{2} + \frac{5}{2} = 3$$

Nuevas etiquetas

label	obs
1	1
1	2
1	3
2	4
2	5
2	6.

e. Nuevos centroides

$$G_1 = \begin{array}{cc} x_1 & x_2 \\ \left(\begin{array}{c} 1 \\ 1 \\ 0 \end{array}\right. & \left.\begin{array}{c} 4 \\ 3 \\ 4 \end{array}\right) \end{array} \qquad G_2 = \begin{array}{cc} x_1 & x_2 \\ \left(\begin{array}{c} 5 \\ 6 \\ 4 \end{array}\right. & \left.\begin{array}{c} 1 \\ 2 \\ 0 \end{array}\right) \end{array}$$

$$C_1 = \left(\frac{2}{3}, \frac{11}{3}\right) \qquad C_2 = (5, 1)$$

Nuevas distancias.

$$d(C_1, O_1) = \left|\frac{2}{3} - 1\right| + \left|\frac{11}{3} - 4\right| = \frac{1}{3} + \frac{1}{3} = 2/3$$
$$d(C_1, O_2) = \left|\frac{2}{3} - 1\right| + \left|\frac{11}{3} - 3\right| = \frac{1}{3} + \frac{2}{3} = 1$$
$$d(C_1, O_3) = \left|\frac{2}{3} - 0\right| + \left|\frac{11}{3} - 4\right| = \frac{2}{3} + \frac{1}{3} = 1$$
$$d(C_1, O_4) = |2/3 - 5| + |11/3 - 1| = 13/3 + 8/3 = 7$$
$$d(C_1, O_5) = |2/3 - 6| + |11/3 - 2| = 16/3 + 5/3 = 7$$
$$d(C_1, O_6) = |2/3 - 4| + |11/3 - 0| = 10/3 + 11/3 = 7$$

$$d(C_2, O_1) = |5-1| + |1-4| = 4 + 3 = 7$$
$$d(C_2, O_2) = |5-1| + |1-3| = 4 + 2 = 6$$
$$d(C_2, O_3) = |5-0| + |1-4| = 5 + 3 = 8$$
$$d(C_2, O_4) = |5-5| + |1-1| = 0 + 0 = 0$$
$$d(C_2, O_5) = |5-6| + |1-2| = 1 + 1 = 2$$
$$d(C_2, O_6) = |5-4| + |1-0| = 1 + 1 = 2$$

las etiquetas no cambiaron.

f.