

# Introducción a la Visión por Computadora

**Santiago Alférez**

Matemáticas Aplicadas y Ciencias de la Computación

Escuela de Ingeniería, Ciencia y Tecnología

Universidad del Rosario

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**Sesión 5. Morfología Matemática**

- ✓ Mathematical morphology provides tools for the representation and description of image regions (e.g. boundary extraction, skeleton, convex hull).
- ✓ It provides techniques for pre- and post-processing of an image (morphological thinning, pruning, filtering).
- ✓ Its principles are based on set theory.
- ✓ Applications to both binary and gray-level images.

- The four horizontal and vertical neighbours of a pixel  $p$  are called 4-*neighbours* of  $p$  and are denoted by  $N_4(p)$ .
- The four diagonal neighbours of a pixel  $p$  are denoted by  $N_D(p)$ .
- Together  $N_4(p)$  and  $N_D(p)$  are called the 8-*neighbours* of pixel  $p$  and are denoted by  $N_8(p)$ .

## Adjacency of pixels

Let  $V$  be the set of intensity values used to define the adjacency (e.g.  $V=\{1\}$  for binary images).

- *4-adjacency*. Two pixels  $p$  and  $q$  with values in  $V$  are 4-adjacent if  $q$  is in  $N_4(p)$ .
- *8-adjacency*. Two pixels  $p$  and  $q$  with values in  $V$  are 8-adjacent if  $q$  is in  $N_8(p)$ .

## Adjacency of pixels

*m-adjacency* (mixed adjacency). Two pixels  $p$  and  $q$  with values in  $V$  are *m-adjacent* if

- $q$  is in  $N_4(p)$ , or
- $q$  is in  $N_D(p)$  and the set  $N_4(p) \cap N_4(q)$  has no pixels whose values are in  $V$ .

Mixed adjacency is a modification of the 8- *adjacency*. It is introduced to eliminate ambiguities of 8- *adjacency*.

## Adjacency of pixels

0   **1**   **1**  
 0   **1**   0  
 0   0   **1**

Pixels in a  
binary image

0   **1** -- **1**  
 0   **1**   0  
 0   0   **1**

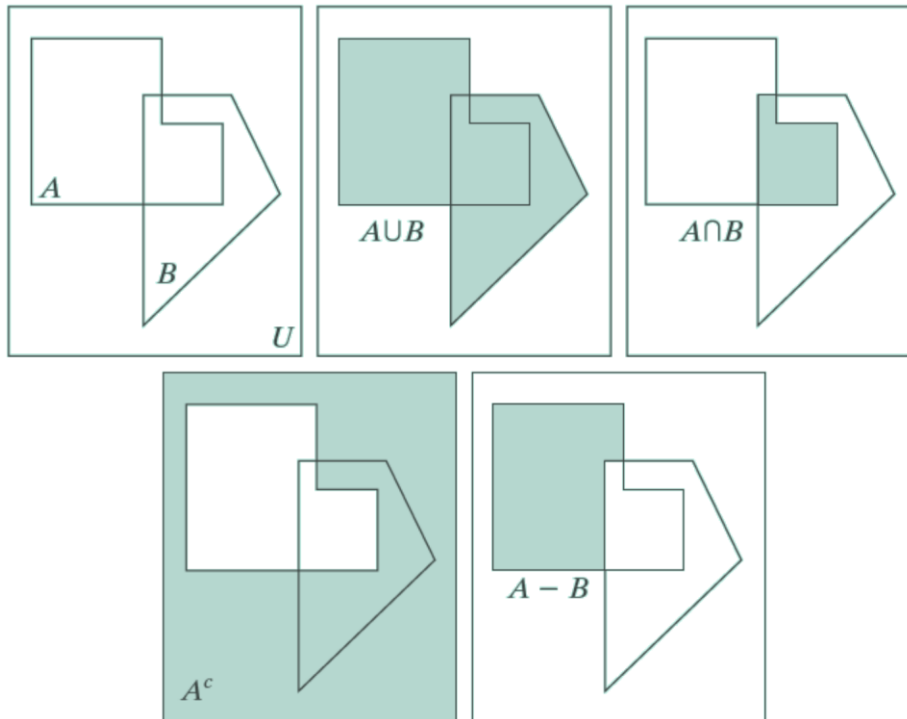
8-adjacency

0   **1** -- **1**  
 0   **1**   0  
 0   0   **1**

*m*-adjacency

The role of *m*-adjacency is to define a single path between pixels. It is used in many image analysis and processing algorithms.

## Basic set operations.



$$A \cup B = \{w \mid w \in A \text{ OR } w \in B\}$$

$$A \cap B = \{w \mid w \in A \text{ AND } w \in B\}$$

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

$$A^c = \{w \mid w \notin A\}$$

- ❑ The above operations assume that the images containing the sets are binary and involve only the pixel location.
- ❑ Union and intersection are different when we define set operations involving intensity values:

$$A \cup B = \left\{ \max_z(a, b) \mid a \in A, b \in B \right\}$$

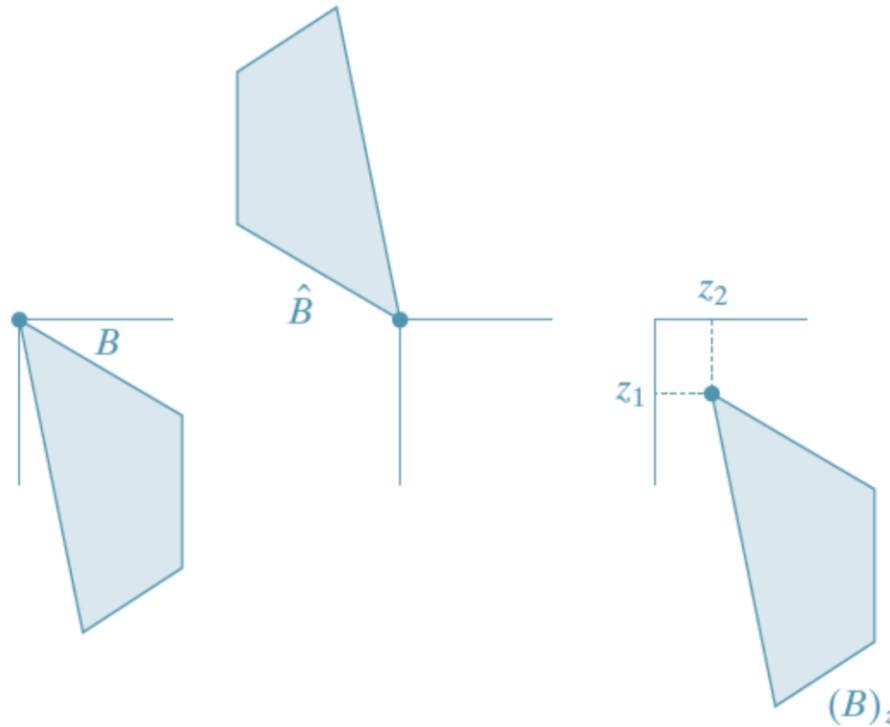
$$A \cap B = \left\{ \min_z(a, b) \mid a \in A, b \in B \right\}$$

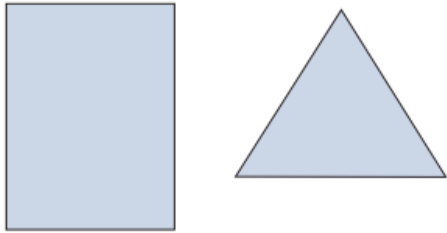
The elements of the sets are gray values on the same location  $z$ .



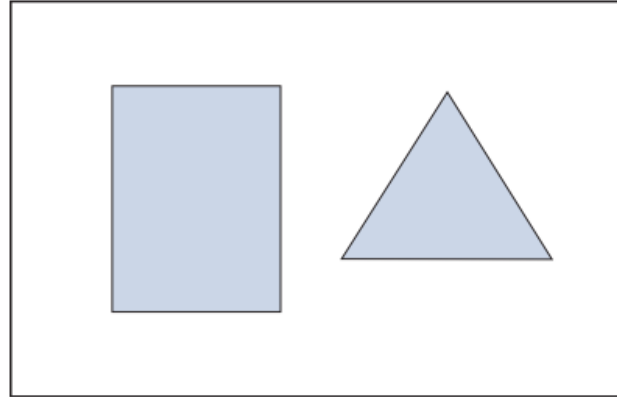
Set reflection:  $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

Set translation by  $z$ :  $(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$

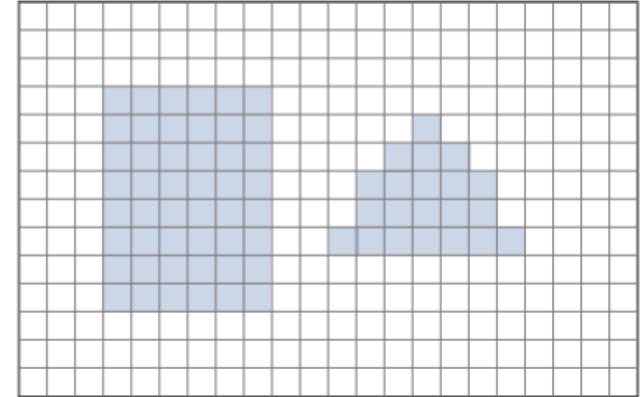




Objects represented  
as sets



Objects represented as  
a graphical image



Digital image



Structuring element  
represented as a set

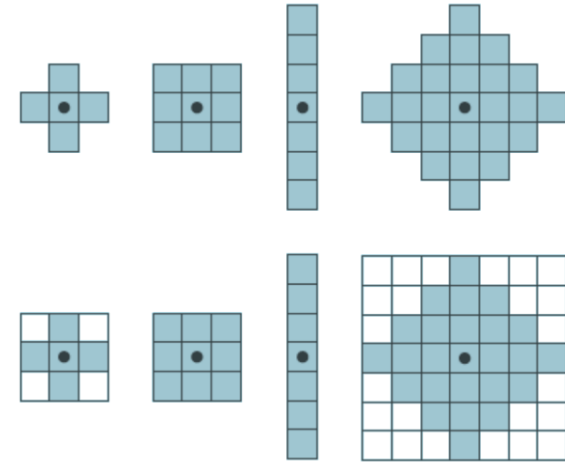


Structuring element  
represented as a graphical image



Digital  
structuring element

- Set reflection and translation are employed to structuring elements (SE).
- SE are small sets or subimages used to examine the image under study for properties of interest.
- The origin must be specified.
- Zeros are appended to SE to give them a rectangular form.



Note: blue-gray colour represents a value of one and white colour a zero value.

- The origin of the SE  $B$  visits every pixel in an image  $A$ .
- It performs an operation (generally non linear) between its elements and the pixels under it.
- It is then decided if the pixel will belong to the resulting set or not based on the results of the operation.
- Zero padding is necessary (like in *convolution*) to ensure that all of the elements of  $A$  are processed.

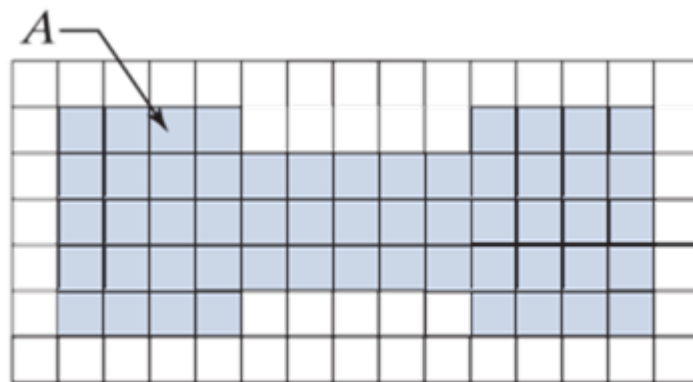
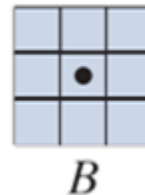
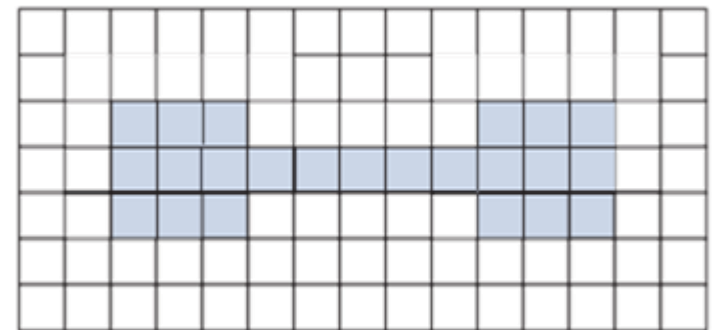
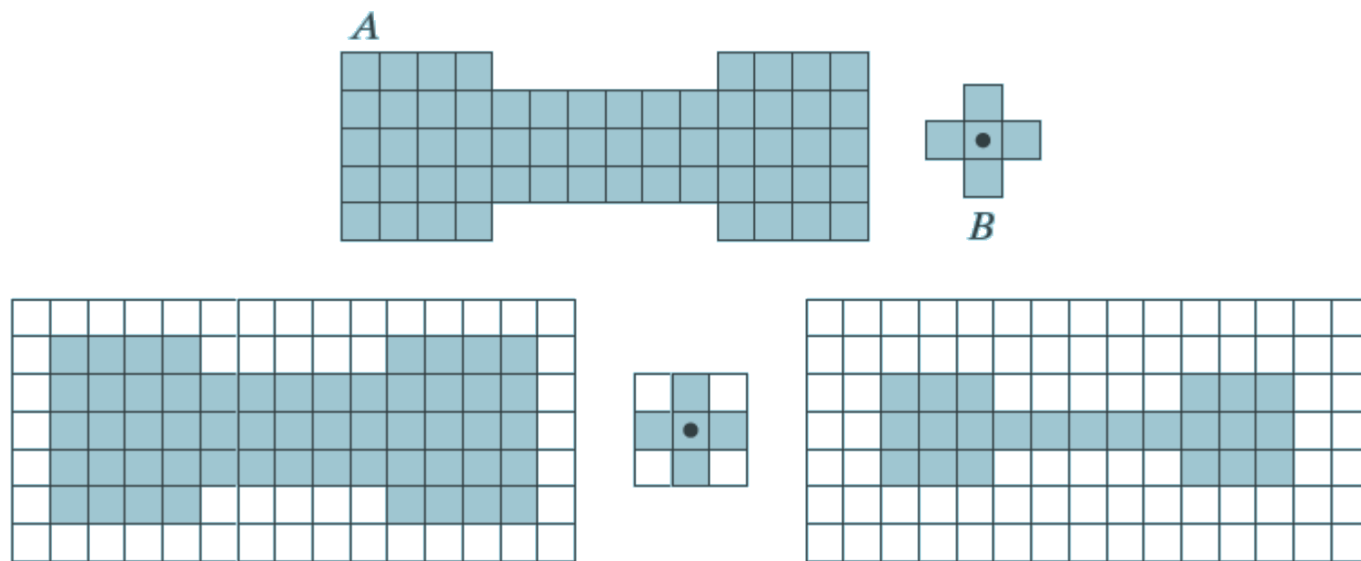
Image  $I$  $B$ 

Image after morphological operation

For example, it marks the pixel under its centre as belonging to the result if  $B$  is completely contained in  $A$  ( $A \in \mathbb{Z}^2, B \in \mathbb{Z}^2$ ).



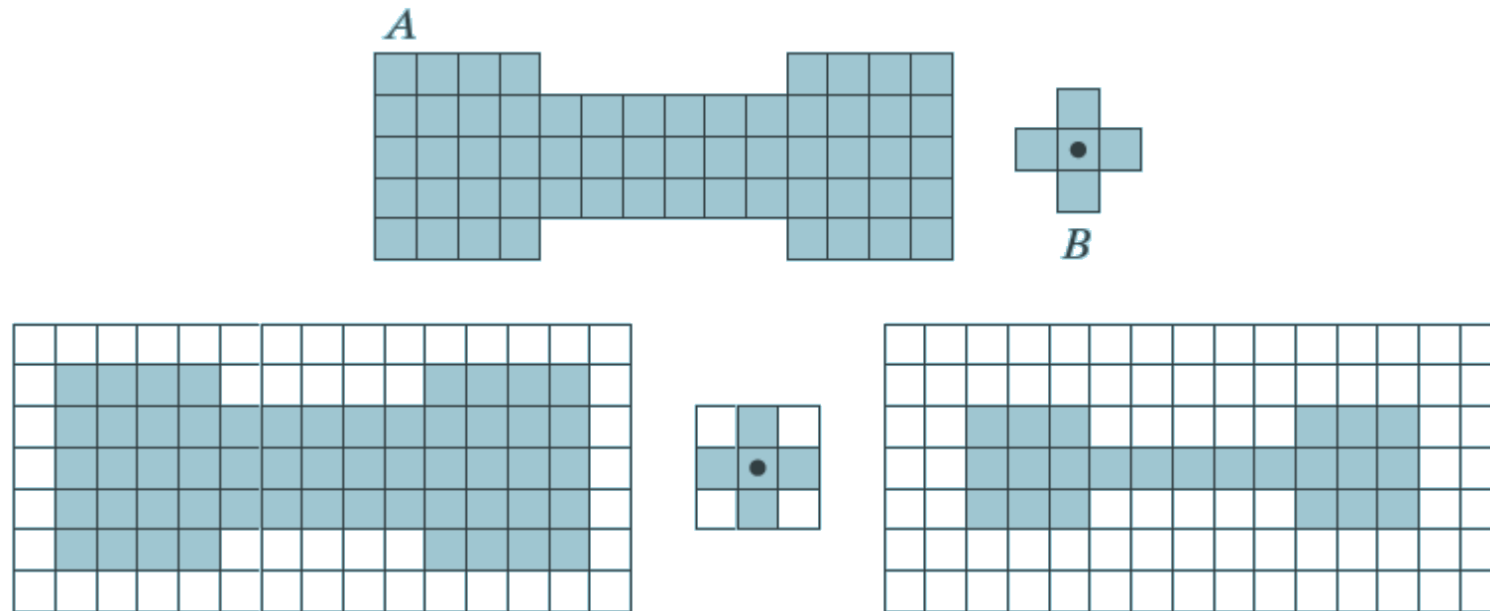
The erosion of a set  $A$  by a SE  $B$  is defined as

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

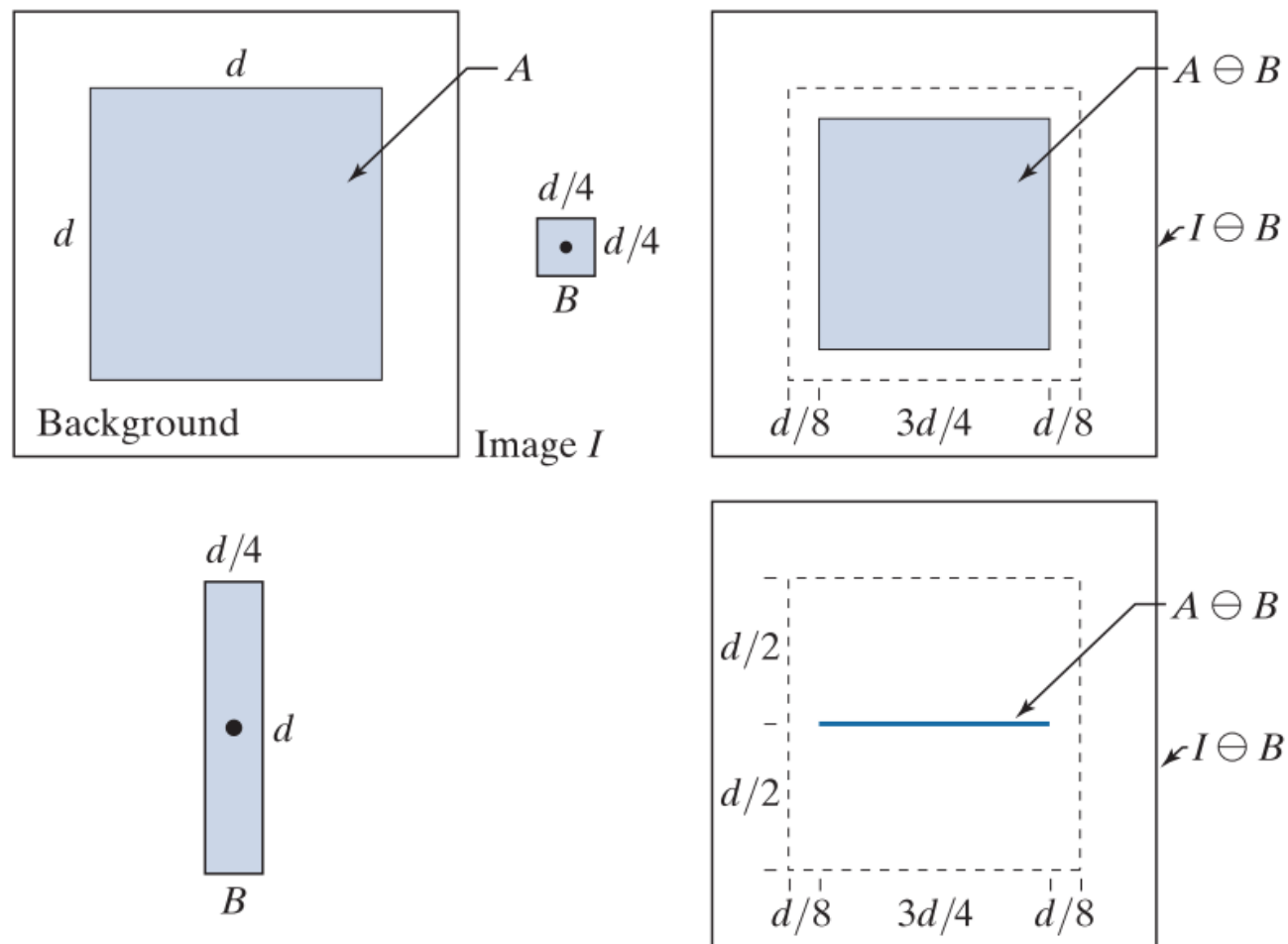
The result is the set of all points  $z$  such that  $B$  translated by  $z$  is contained in  $A$ .

Equivalently:

$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

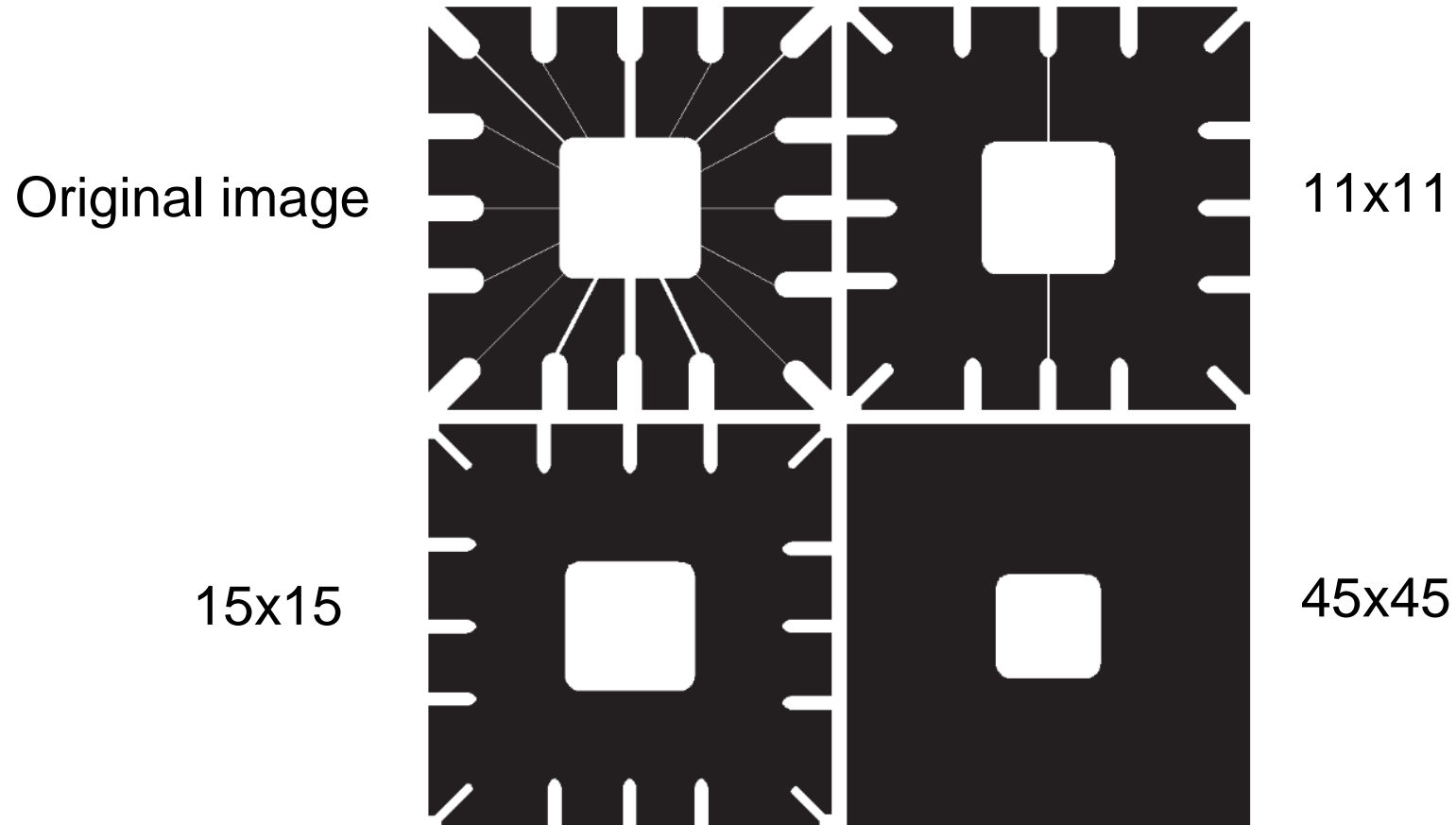


Erosion is a shrinking operation

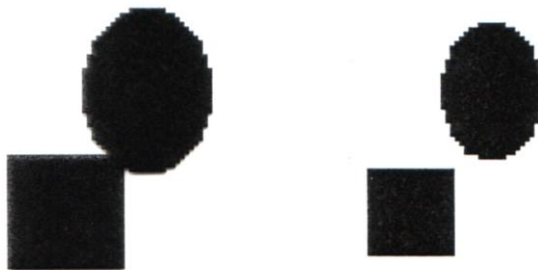




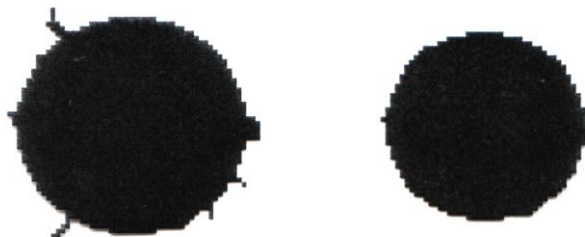
## Erosion by a square SE of varying size



Erosion can split apart joined objects



Erosion can strip away extrusions



**Watch out:** Erosion shrinks objects

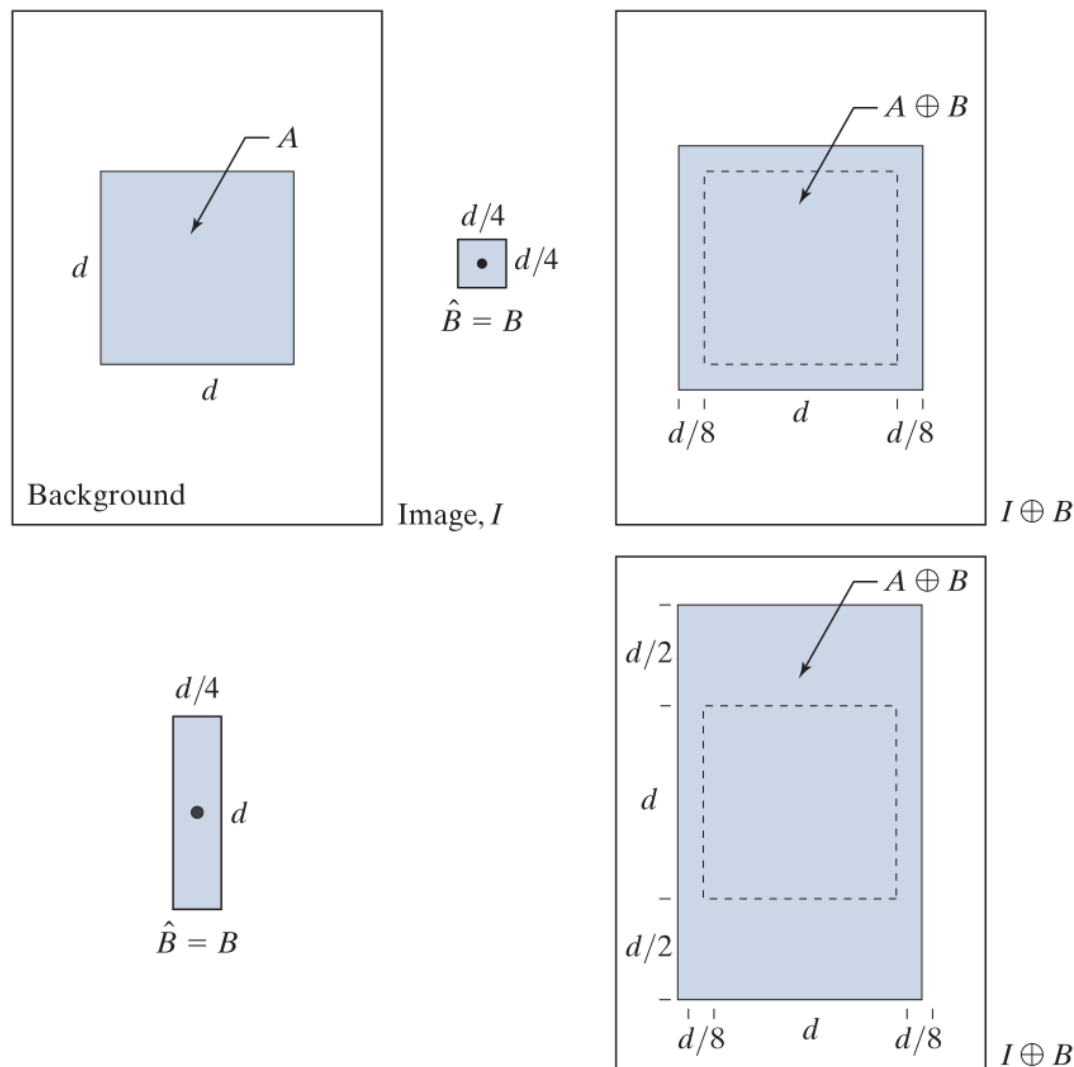
The dilation of a set  $A$  by a SE  $B$  is defined as

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

The result is the set of all points  $z$  such that the reflected  $B$  translated overlap with  $A$  at least one element.

Equivalently:  $A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$

Dilation is a thickening operation



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



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1	1	1
1	1	1
1	1	1

Dilation bridges gaps.

Contrary to low pass filtering it produces a binary image.

Dilation can repair breaks



Dilation can repair intrusions



**Watch out:** Dilation enlarges objects

Erosion and dilation are dual operations with respect to set complementation and reflection:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Also,

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

The duality is useful when the SE is symmetric:  
The erosion of an image is the dilation of its background.

More interesting morphological operations can be performed by combining erosions and dilations in order to reduce shrinking or thickening.

The most widely used of these *compound operations* are:

- Opening
- Closing

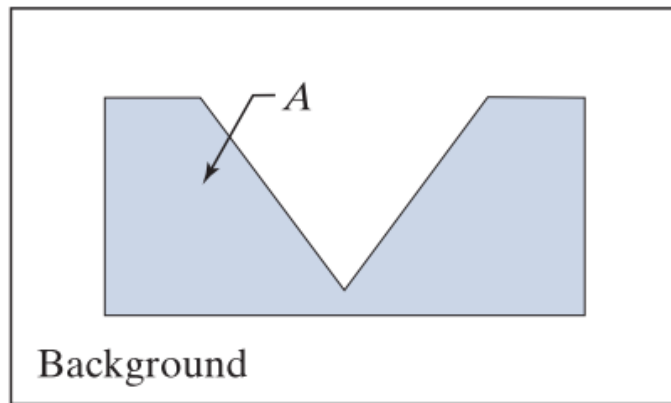


The opening of set  $A$  by structuring element  $B$  is defined as

$$A \circ B = (A \ominus B) \oplus B$$

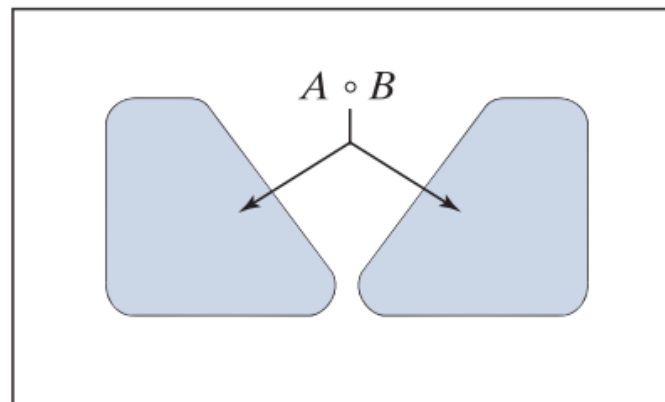
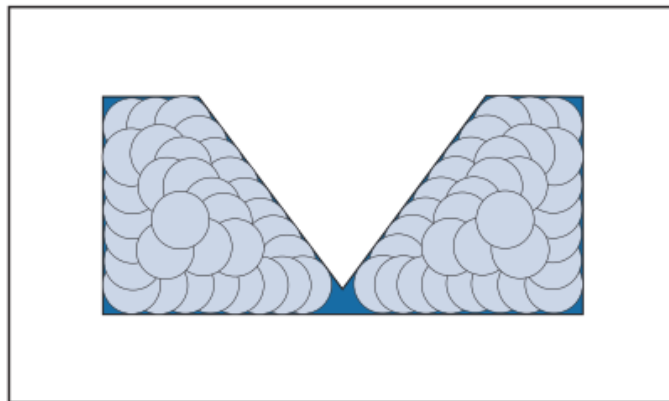
which is an erosion of  $A$  by  $B$  followed by a dilation of the result by  $B$ .

Geometric interpretation: The boundary of the opening is defined by points of the SE that reach the farthest into the boundary of  $A$  as  $B$  is “rolled” inside of this boundary.



$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

Image,  $I$

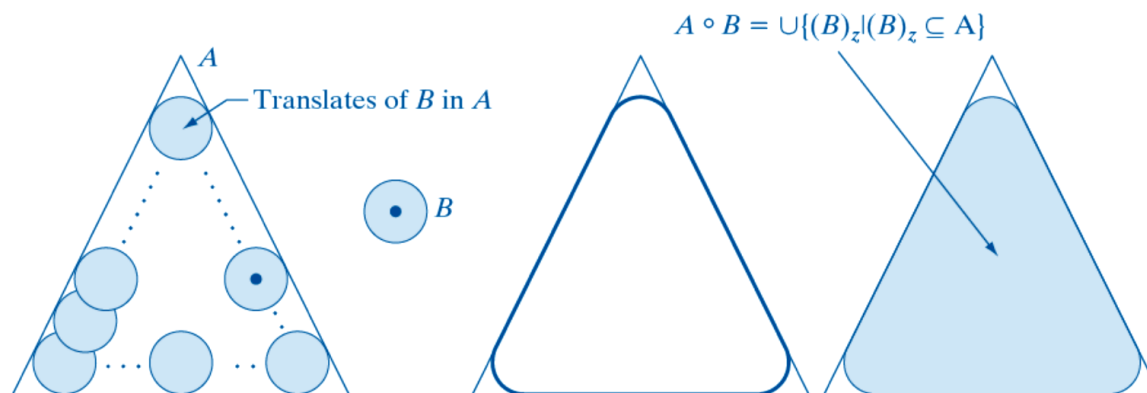


Notice the difference with the simple erosion:

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

If  $B$  translated by  $z$  lies inside  $A$ , then the result contains the whole set of points covered by the SE and not only its center as it is done in the erosion.

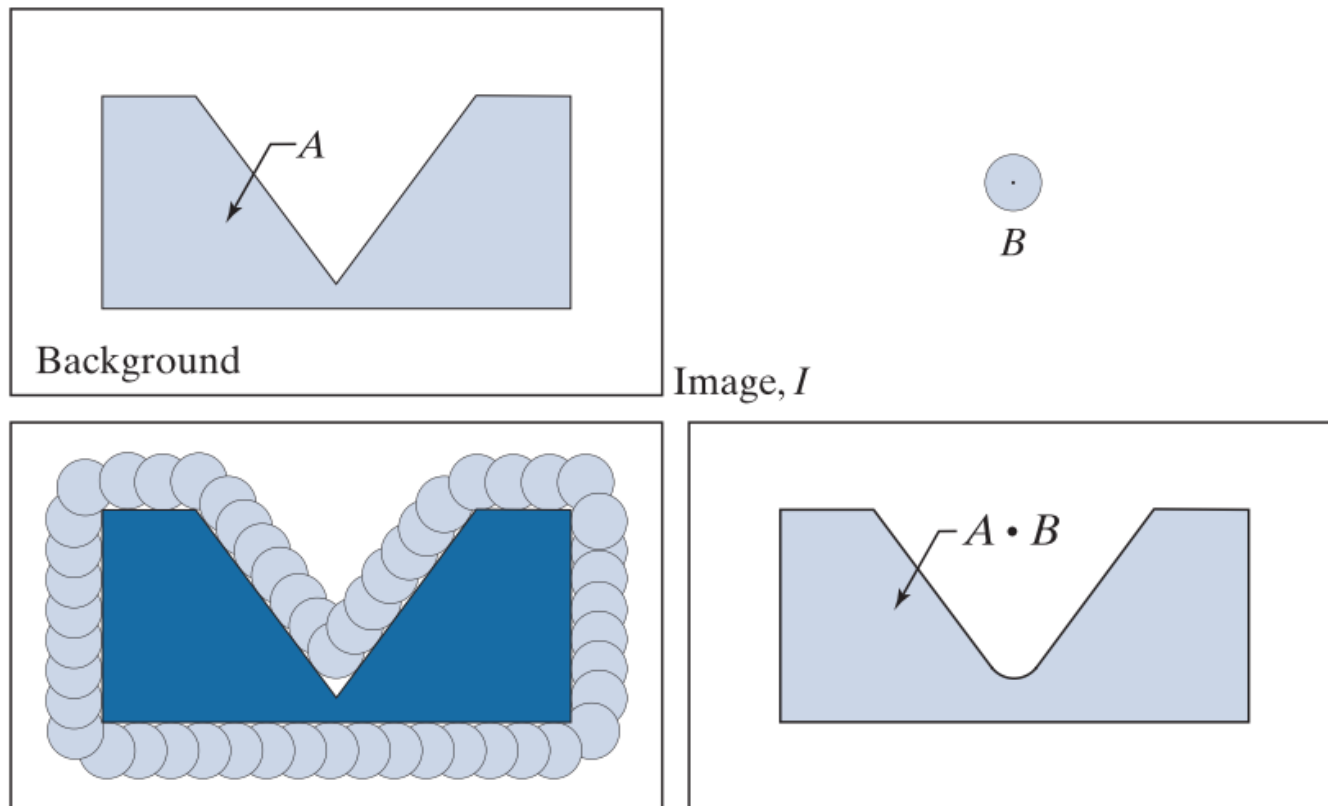


The closing of set  $A$  by structuring element  $B$  is defined as

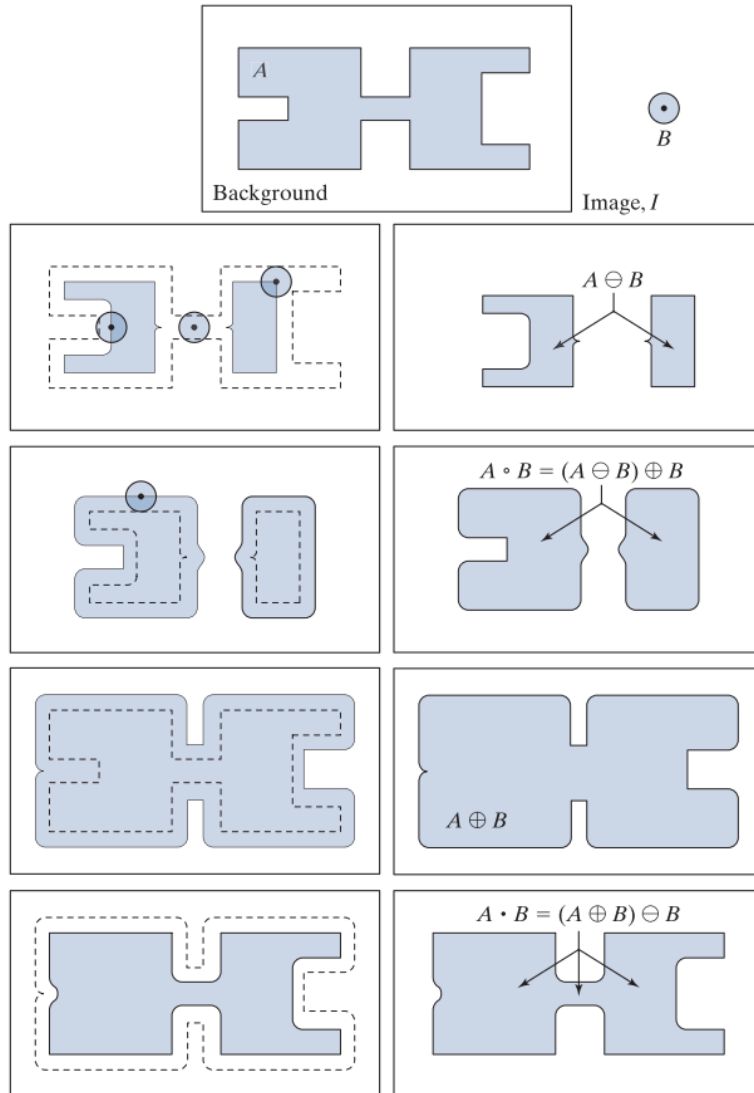
$$A \bullet B = (A \oplus B) \ominus B$$

which is a dilation of  $A$  by  $B$  followed by an erosion of the result by  $B$ .

It has a similar geometric interpretation except that  $B$  is rolled on the outside of the boundary:



$$A \bullet B = \{w | (B)_z \cap A \neq \emptyset, \text{ for all translates of } (B)_z \text{ containing } w\}$$



Erosion: elements where the disk can not fit are eliminated

Opening: outward corners are rounded

Dilation: inward intrusions are reduced in depth

Closing: inward corners are rounded

Opening and closing are dual operations.

Erosion-Dilation duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Opening-Closing duality

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = A^c \bullet \hat{B}$$

Opening:

$$A \circ B \subseteq A$$
$$C \subseteq D \Rightarrow C \circ B \subseteq D \circ B$$
$$(A \circ B) \circ B = A \circ B$$

Closing:

$$A \subseteq A \bullet B$$
$$C \subseteq D \Rightarrow C \bullet B \subseteq D \bullet B$$
$$(A \bullet B) \bullet B = A \bullet B$$

The last properties, in each case, indicate that multiple openings or closings have no effect after the first application of the operator





1	1	1
1	1	1
1	1	1

 $B$ 

The image contains noise:

- Light elements on dark background.
- Dark elements on the light components of the fingerprint.

Objective: Eliminate noise while distorting the image as little as possible.

We will apply an opening followed by closing.

 $A$  $A \ominus B$ 

- ✓ Background noise completely removed (noise components smaller than the SE).
- ✓ The size of the dark noise elements in the fingerprint structure increased (inner dark structures).



$$A \ominus B$$



$$(A \ominus B) \oplus B = A \circ B$$

The dilation reduced the size of the inner noise or eliminated it completely.

However, new gaps were created by the opening between the fingerprint ridges.



$$A \circ B$$



$$A \circ B \oplus B$$

The dilation reduces the new gaps between the ridges but it also thickens the ridges.



$$A \circ B \oplus B$$



$$[A \circ B \oplus B] \ominus B = (A \circ B) \bullet B$$

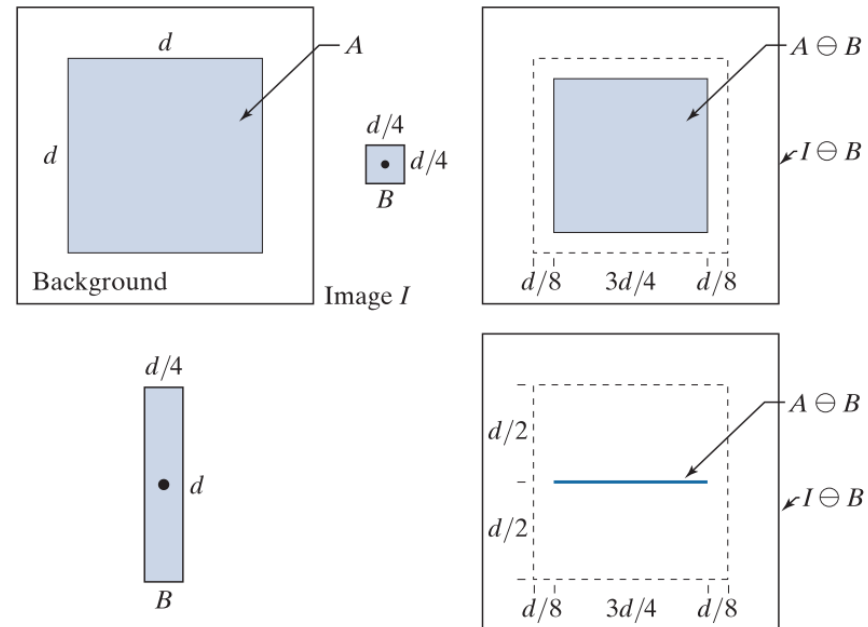
The final erosion (resulting to a closing of the opened image) eliminates makes the ridges thinner.

 $A$  $(A \circ B) \bullet B$ 

The final result is clean of noise but some ridges were not fully repaired.

We should impose conditions for maintaining the connectivity.

- Basic tool for shape detection.
- Erosion of  $A$  by  $B$ : the set of all locations of the *origin* of  $B$  that  $B$  is completely contained in  $A$ .
- Alternatively, it is the set of all locations that  $B$  found a match (hit) in  $A$ .



- There are many possible locations for the shape we search (the SE!). If we are looking for disjoint (disconnected) shapes it is natural to assume a background for it.
- Therefore, we seek to match  $B_1$  in  $A$  and simultaneously we seek to match the background of  $B_1$  ( $B_2$ ) in  $A^c$ .
- Mathematically, the hit-or-miss transformation is:

$$\begin{aligned} I \circledast B_{1,2} &= \left\{ z \mid (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c \right\} \\ &= (A \ominus B_1) \cap (A^c \ominus B_2) \end{aligned}$$

Where,  $I$  is a binary image composed of foreground ( $A$ ) and background pixels, respectively. And  $B_1$ , is a SE for detecting shapes in the foreground, and  $B_2$ , for detecting shapes in the background

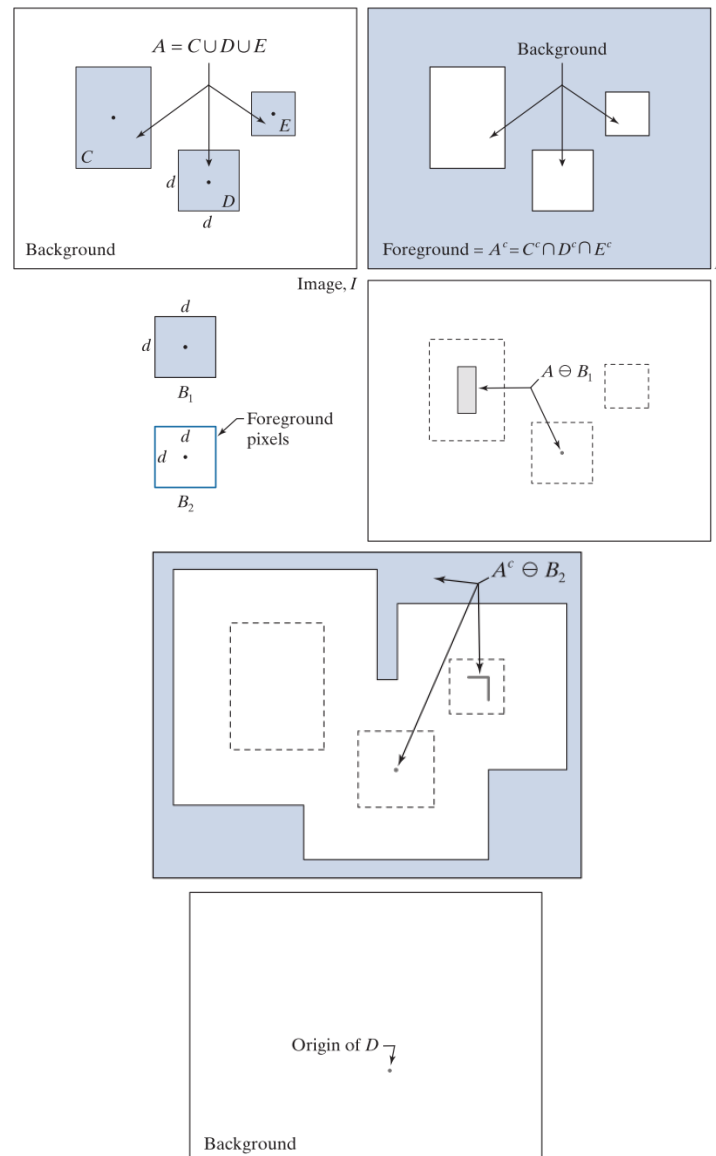


We seek to locate the shape  $D$  in the image  $A$ .

We define a thin background  $W$  for the shape.

We take the intersection of the two results

$$\text{Image: } I \circledast B_{1,2} = A \ominus B_1 \cap A^c \ominus B_2$$



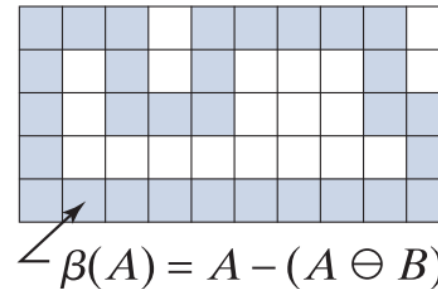
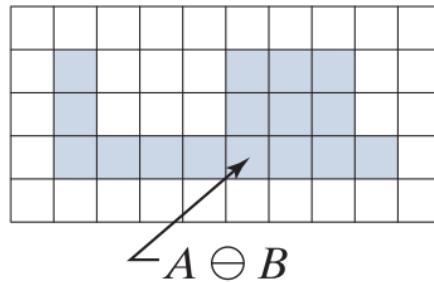
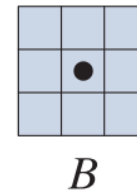
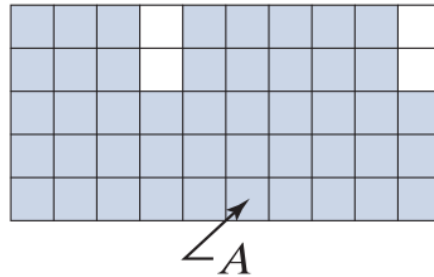
Using these morphological operations, we may extract image components for shape representation:

- Shape boundaries
- Region filling
- Connected components
- Convex hull
- Shape thinning and thickening
- Skeletons

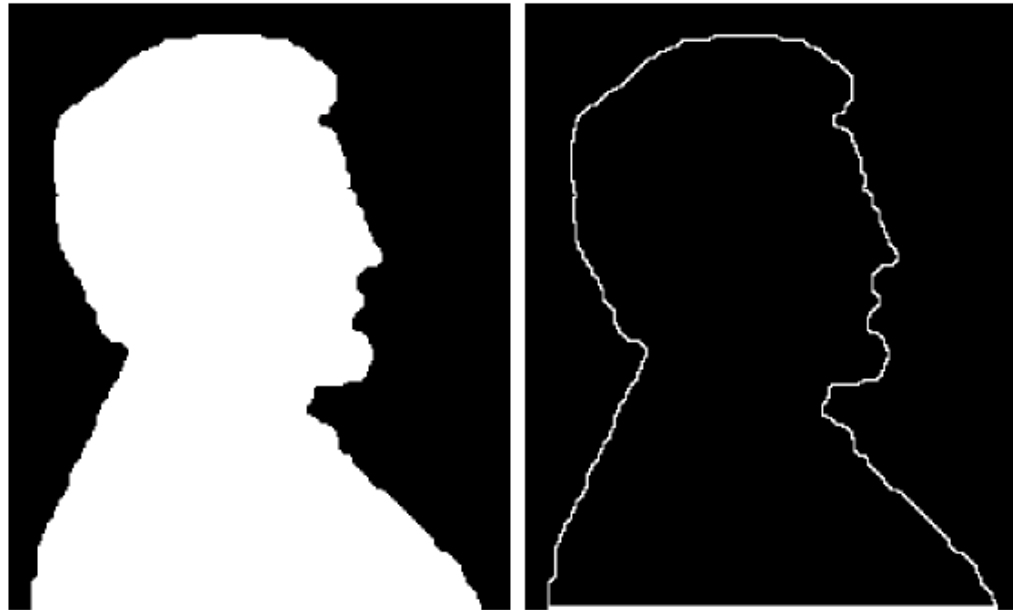
We may also accomplish a morphological image reconstruction.

The boundary of a set  $A$ , denoted by  $\beta(A)$ , may be obtained by:

$$\beta(A) = A - (A \ominus B)$$



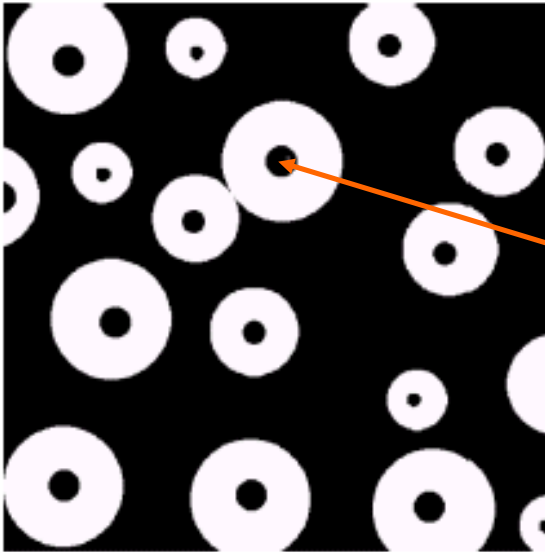
The boundary is one pixel thick due to the 3x3 SE.  
Other SE would result in thicker boundaries.



Original Image

Extracted Boundary

Given a pixel inside a boundary, region filling attempts to fill the area surrounded by that boundary with 1s.



Given a point inside here, can we fill the whole circle?

- Form a set  $X_0$  with zeros everywhere except at the seed point of the region.
- Then,

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1, 2, 3, \dots$$

- Where  $B$  is a 3x3 cross-shaped SE.
- The algorithm terminates when  $X_k = X_{k-1}$ .
- The set union of  $X_k$  and  $A$  contains all the filled holes and their boundaries.

This is a first example where the morphological operation (dilation) is conditioned.

The intersection of the result with the  $A^c$  limits the result inside the region of interest.

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1, 2, 3, \dots$$

a	b	c
d	e	f
g	h	i

(a) Set A (shown shaded) contained in image I.

(b) Complement of I.

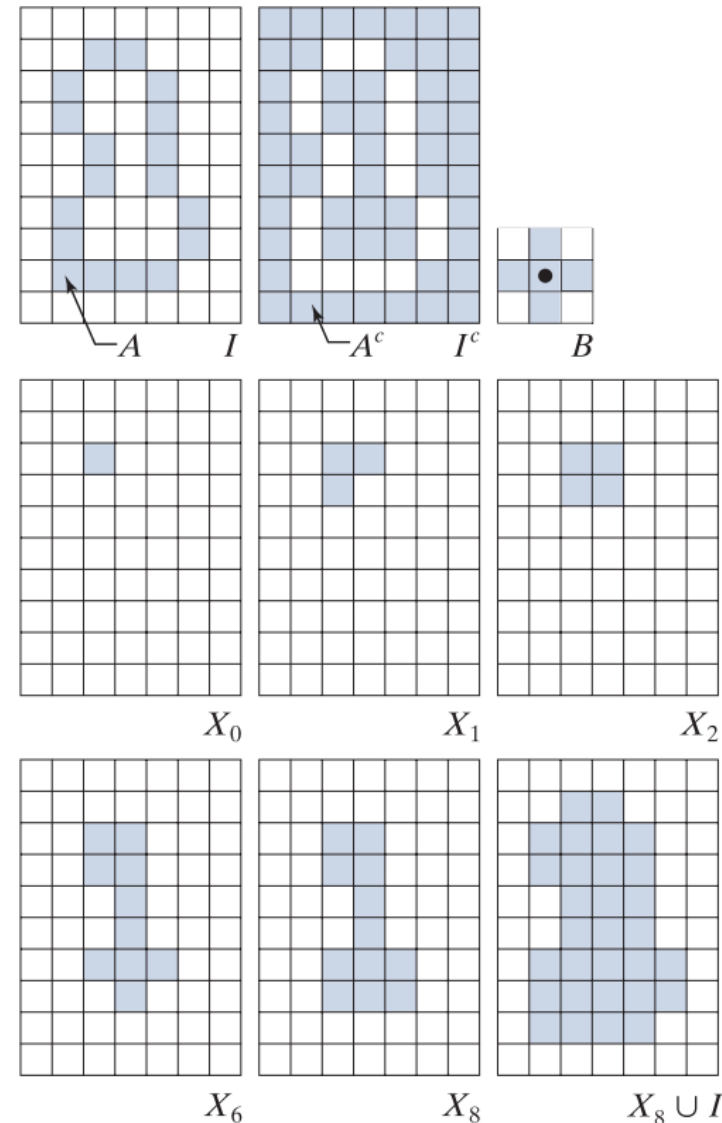
(c) Structuring element B. Only the foreground elements are used in computations

(d) Initial point inside hole, set to 1.

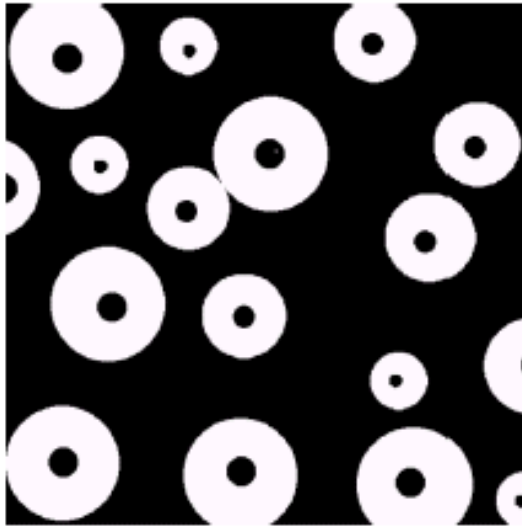
(e)–(h) Various steps of

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

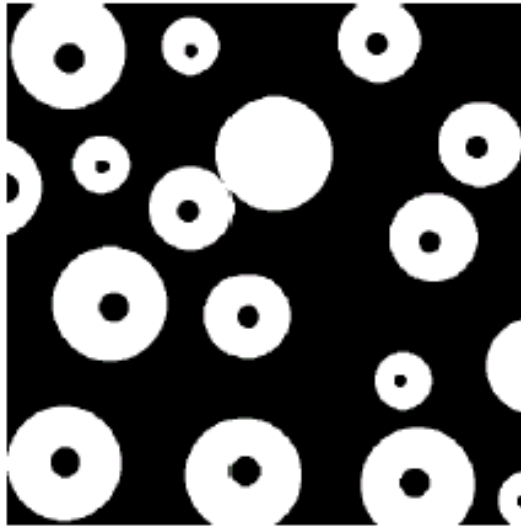
(i) Final result [union of (a) and (h)].



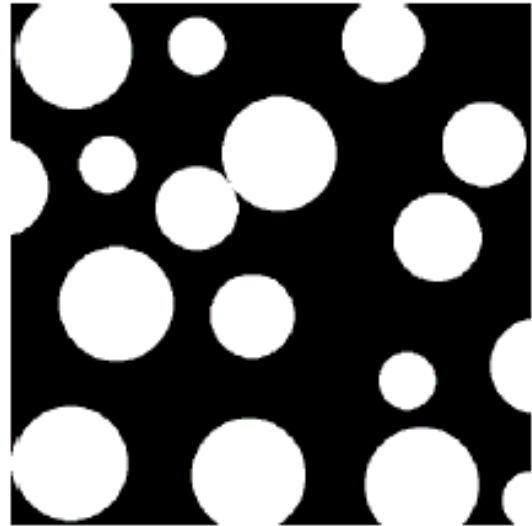




Original Image



One Region  
Filled



All Regions  
Filled

- Given a pixel on a connected component, find the rest of the pixels of that component.
- The algorithm may be applied to many connected components provided we know a pixel on each one of them.
- Disadvantage: We have to provide a pixel on the connected component.
- There are more sophisticated algorithms that detect the number of components without manual interaction. The purpose here is to demonstrate the flexibility of mathematical morphology.

- ✓ Form a set  $X_0$  with zeros everywhere except at the seed point of the connected components.
- ✓ Then,

$$X_k = (X_{k-1} \oplus B) \cap A, k = 1, 2, 3, \dots$$

- ✓ Where  $B$  is a 3x3 square-shaped SE.
- ✓ The algorithm terminates when  $X_k = X_{k-1}$ .
- ✓  $X_k$  contains all the connected components.

Note the similarity with region filling. The only difference is the use of  $A$  instead of  $A^c$ .

This is not surprising as we search for foreground objects.

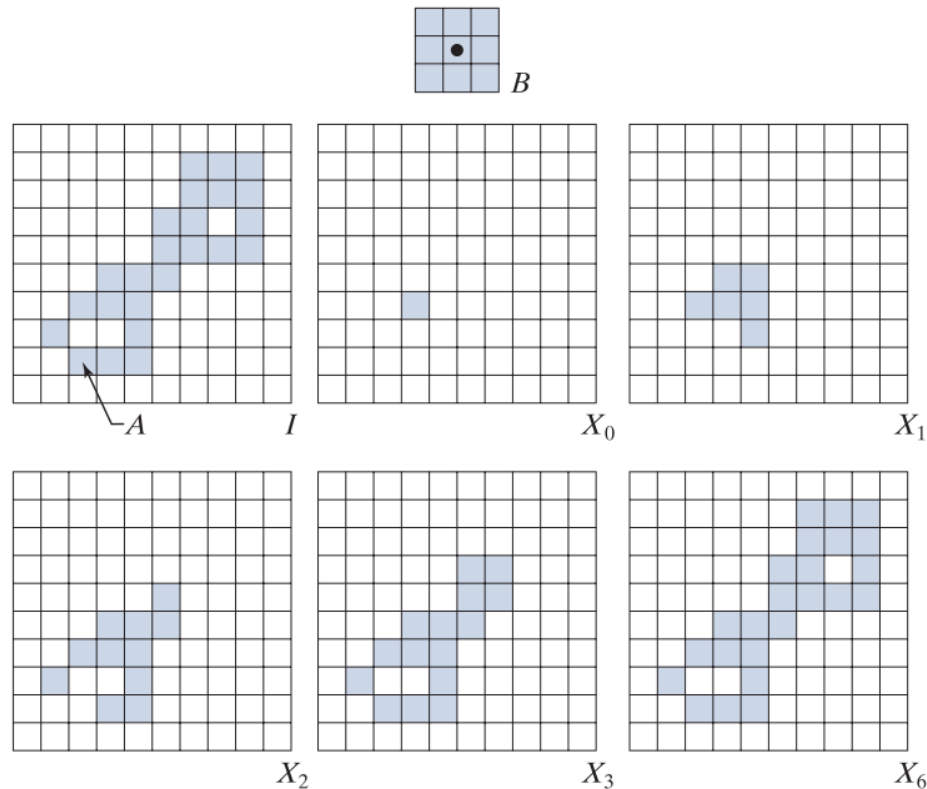
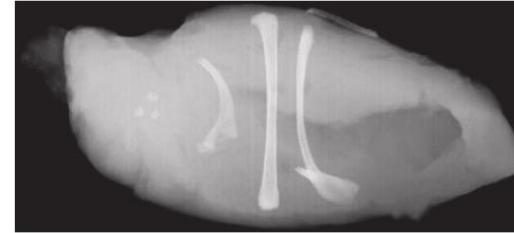


Image of chicken filet  
containing bone fragments



Result of simple thresholding  
(shown the negative)

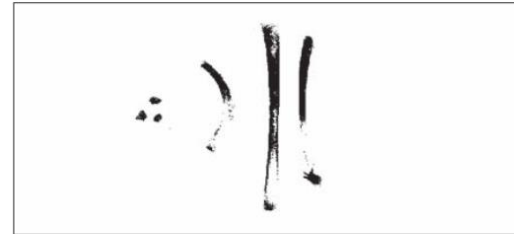


Image erosion to retain only  
objects of significant size.



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

15 connected components detected with four of them being significant in size. This is an indication to remove the chicken filet from packaging.

- A set  $A$  is convex if the straight line segment joining any two points in  $A$  lies entirely within  $A$ .
- The convex hull  $H$  of an arbitrary set is  $S$  the smallest convex set containing  $S$ .
- The difference  $H-S$  is called convex deficiency.
- The convex hull and the convex deficiency are useful quantities to characterize shapes.
- We present here a morphological algorithm to obtain the convex hull  $C(A)$  of a shape  $A$ .

The procedure requires four SE  $B^i$ ,  $i = 1, 2, 3, 4$ , and implements the following equation:

$$X_k^i = (X_{k-1} * B^i) \cup A, \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \dots$$

with  $X_0^i = A$

with  $i$  referring to the SE and  $k$  to the iteration.

Then, letting

$$D^i = X_k^i$$

The convex hull of  $A$  is

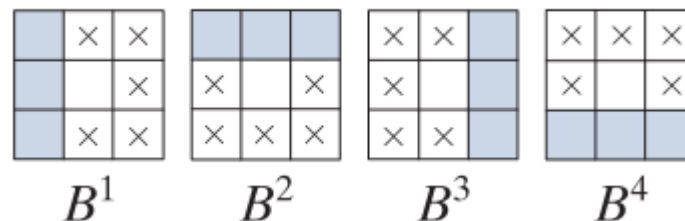
$$C(A) = \bigcup_{i=1}^4 D^i$$

$$X_k^i = (X_{k-1} * B^i) \cup A \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \dots \text{ with } X_0^i = A$$

$$D^i = X_k^i, C(A) = \bigcup_{i=1}^4 D^i$$

- The method consists of iteratively applying the hit-or miss transform to  $A$  with  $B^1$ .
- When no changes occur we perform the union with  $A$  and save the result to  $D^1$ .
- The procedure is then continued with  $B^2$  (applied to  $A$ ) and so on.
- The union of the results is the convex hull of  $A$ .
- Note that a simple implementation of the hit or miss is applied (no background match is required).



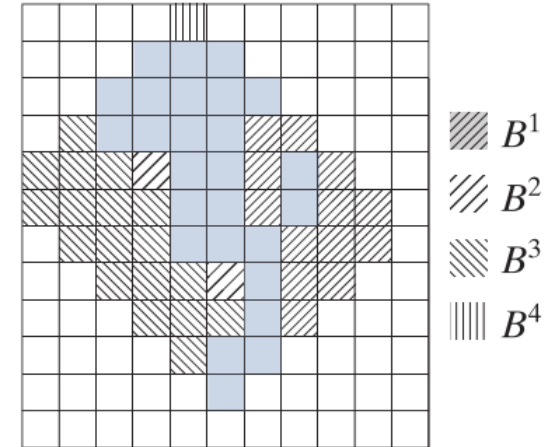
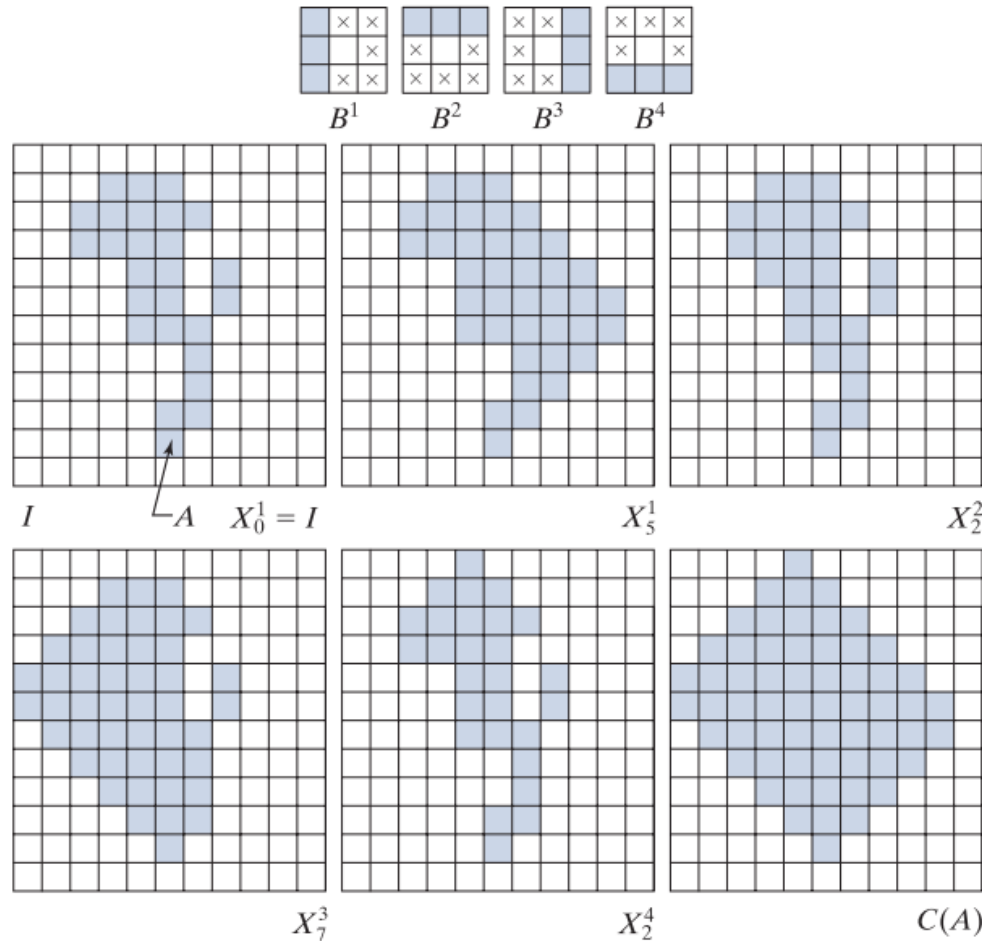


The hit-or-miss transform tries to find (“hit”) these structures in the image.

The SE has points with “don’t care” condition. For all the SE, a match is found in the image when these conditions hold:

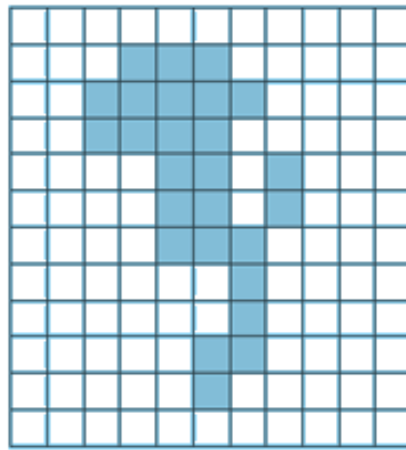
- the pixel in the 3x3 region in the image is 0
- the three shaded pixels under the mask are 1s

The remaining pixels do not matter.

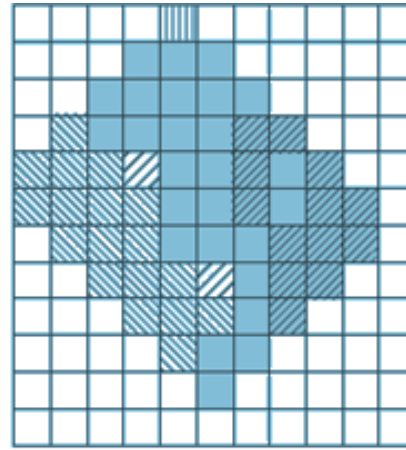


- Problem.  
The result is convex  
but greater than the  
true convex hull.

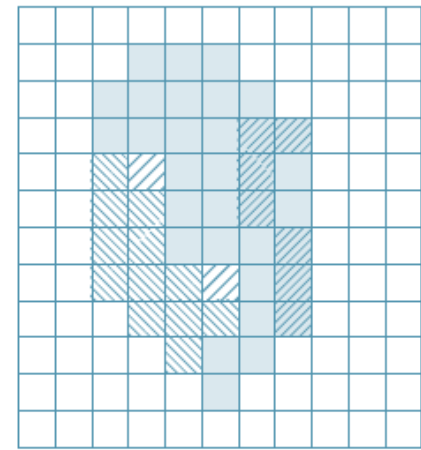
Solution: limit the growth so that it does not extend past the horizontal and vertical limits of the original set of points.



Original image



Initial convex hull



Refined convex hull

More complex boundaries have been imposed to images with finer details in their structure (e.g. The maximum of the horizontal vertical and diagonal dimensions could be used).

The thinning of a set  $A$ , by a SE  $B$  may be defined in terms of the hit-or-miss transform:

$$\begin{aligned} A \otimes B &= A - (A * B) \\ &= A \cap (A * B)^c \end{aligned}$$

No background match is required and the hit-or-miss part is reduced to simple erosion.

A more advanced expression is based on a sequence of SE  $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$ ,

where each  $B^i$  is a rotated version of  $B^{i-1}$ .

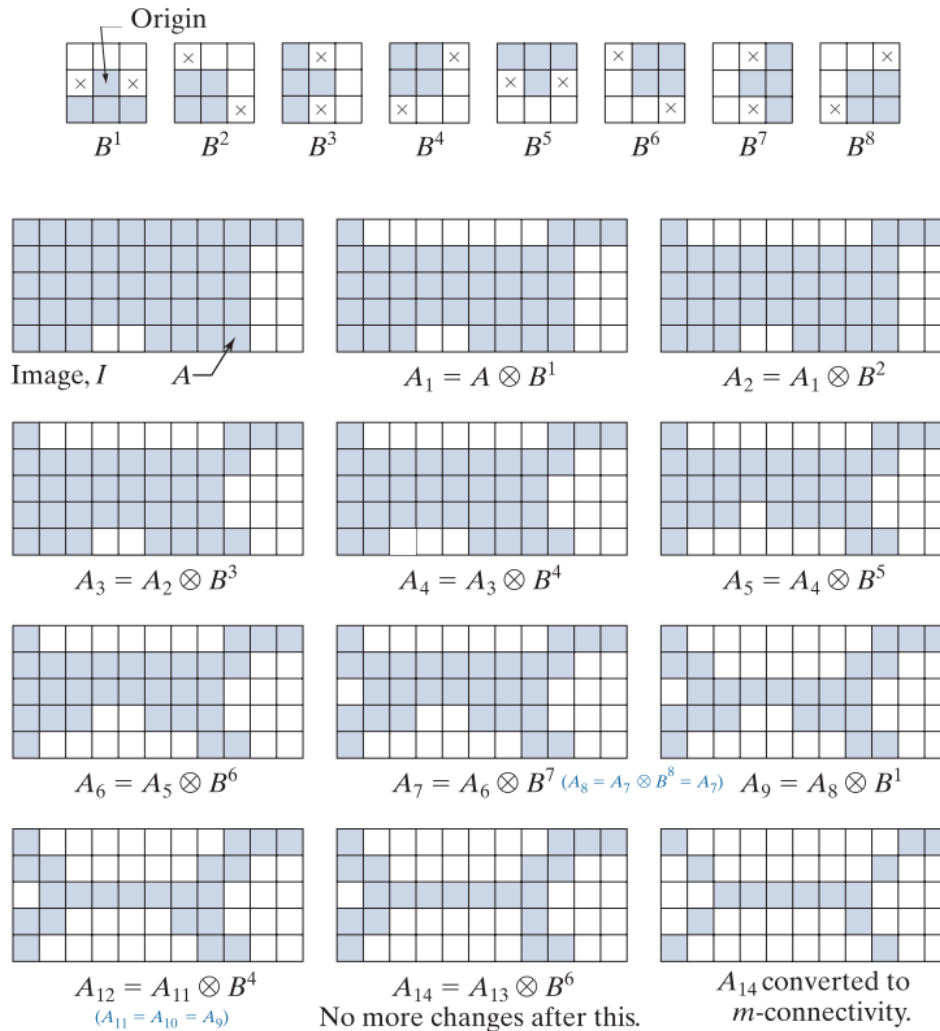
The thinning by a sequence of SE is defined by:

$$A \otimes \{B\} = ((\dots ((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$$

The process is to thin  $A$  by one pass by  $B^1$ , then thin the result with one pass of  $B^2$ , and so on, until we employ  $B^n$ .

The entire process is repeated until no further changes occur. Each individual thinning is performed by:

$$A - (A \circledast B)$$



- No change between the result of  $B^7$  and  $B^8$  at the first pass.
- No change between the results of  $B^1$ ,  $B^2$ ,  $B^3$ ,  $B^4$  at the second pass.
- No change occurs after the second pass by  $B^6$ .
- The final result is converted to  $m$ -connectivity to have a one pixel thick structure.

Thickening is a morphological dual of thinning:

$$A \odot B = A \cup (A \circledast B)$$

The SE have the same form as the ones used for thinning with the 1s and 0s interchanged.

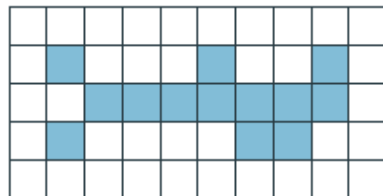
It may also be defined by a sequence of operations:

$$A \odot \{B\} = ((\dots ((A \odot B^1) \odot B^2) \dots) \odot B^n)$$

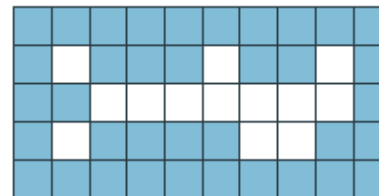
- ✓ In practice, a separate algorithm is seldom used for thickening.
- ✓ The usual process is to thin the background of the set in question and then take the complement of the result.
- ✓ The advantage is that the thinned background forms a boundary for the thickening process. Direct implementation of thickening has no stopping criterion.
- ✓ A disadvantage is that there may be isolated points needing post-processing.



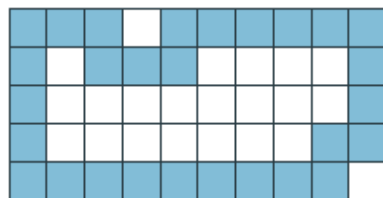
Original set  $A$



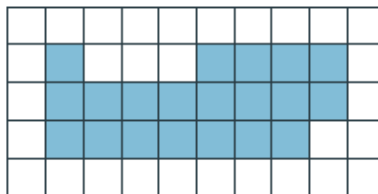
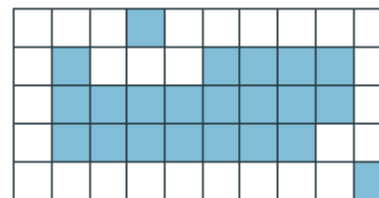
$A^c$



Thinning of  $A^c$



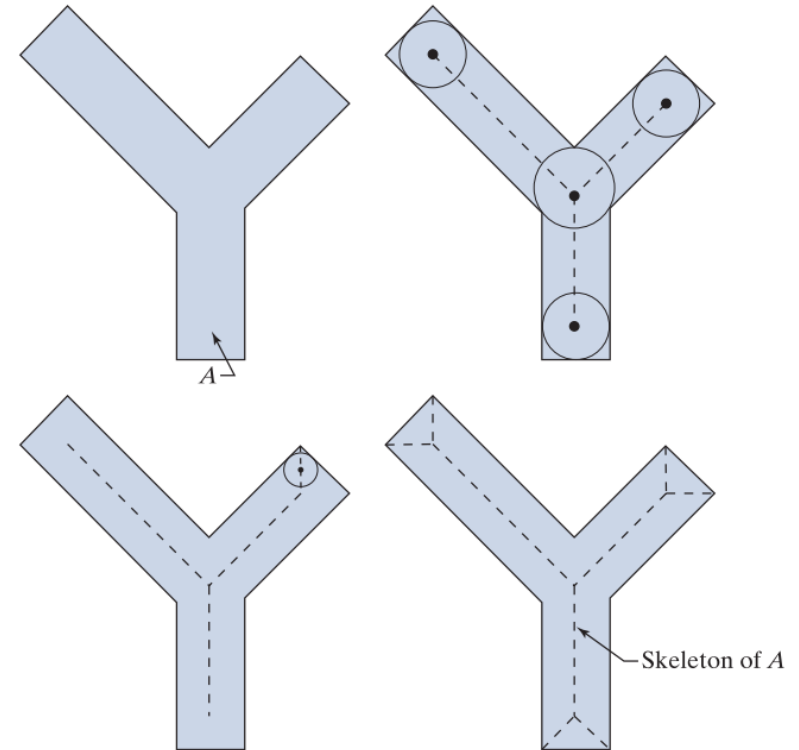
Thickened set  
obtained by  
complementing the  
result of thinning.



Elimination of disconnected points.

The notion of a skeleton  $S(A)$  of a set  $A$ , intuitively, has the following properties:

- A point  $z$  belongs to  $S(A)$  if one cannot find a *maximum disk* containing  $z$  and included in  $A$ .
- The maximum disk touches the boundary of  $A$  at two or more different points.



It may be shown that a definition of the skeleton may be given in terms of erosions and openings:

$$S(A) = \bigcup_{k=0}^K S_k(A), \text{ with } S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$\text{with } (A \ominus kB) = \underbrace{((\dots (A \ominus B) \ominus B) \ominus \dots) \ominus B)}_{k \text{ successive erosions}}$$

$K$  is the last iterative step before  $A$  erodes to an empty set:

$$K = \max\{k | A \ominus kb \neq \emptyset\}$$

The previous formulation allows the iterative reconstruction of  $A$  from the sets forming its skeleton by:

$$A = \bigcup_{k=0}^K (S_k(A) \oplus kB),$$

with  $S_k(A) \oplus B = \underbrace{((\dots(S_k(A) \oplus B) \oplus B) \oplus \dots) \oplus B)}_{k \text{ successive dilations of the set } S_k(A)}$

$k$	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{k=0}^K S_k(A)$	$S_k(A) \oplus kB$	$\bigcup_{k=0}^K S_k(A) \oplus kB$
0						
1						
2						

The skeleton is

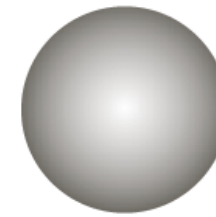
- thicker than essential.
- disconnected.

The morphological formulation does not guarantee connectivity.

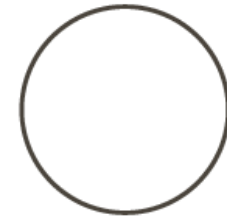
More assumptions are needed to obtain a maximally thin and connected skeleton.



- The image  $f(x,y)$  and the SE  $b(x,y)$  are take real or integer values.
- SE may be flat or nonflat.
- Due to a number of difficulties (result interpretation, erosion is not bounded by the image, etc.) we symmetrical flat SE with origin at the center are employed.
- Set reflection:  $\hat{b}(x, y) = -b(x, y)$



Nonflat SE



Flat SE



Intensity profile



Intensity profile

The erosion of image  $f$  by a SE  $b$  at any location  $(x,y)$  is defined as the minimum value of the image in the region coincident with  $b$  when the origin of  $b$  is at  $(x,y)$ :

$$[f \ominus b](x, y) = \min_{(s,t) \in b} \{f(x + s, y + t)\}$$

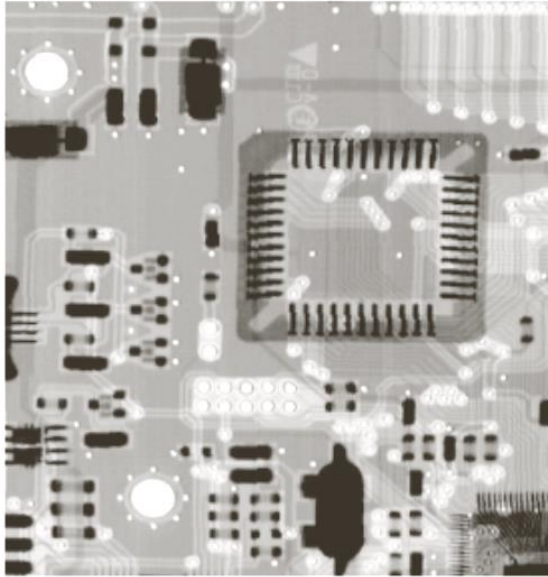
In practice, we place the center of the SE at every pixel and select the minimum value of the image under the window of the SE.

The dilation of image  $f$  by a SE  $b$  at any location  $(x,y)$  is defined as the maximum value of the image in the window outlined by  $b$ :

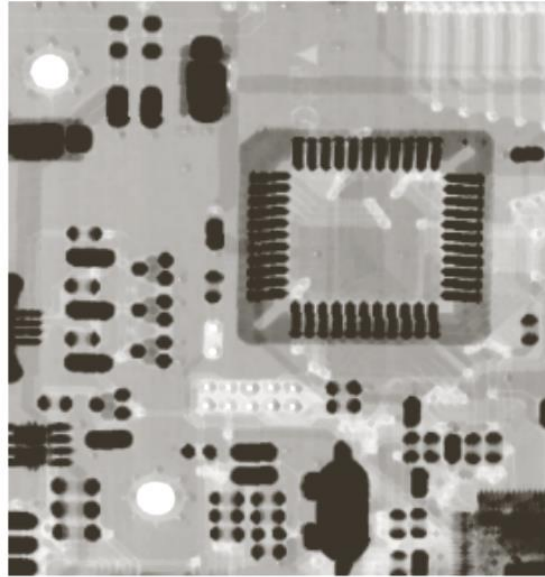
$$[f \oplus b](x, y) = \max_{(s,t) \in b} \{f(x - s, y - t)\}$$

The SE is reflected as in the binary case.





Original image



Erosion by a flat disk SE of radius 2:  
Darker background, small bright dots reduced, dark features grew.



Dilation by a flat disk SE of radius 2:  
Lighter background, small dark dots reduced, light features grew.

The erosion of image  $f$  by a nonflat SE  $b_N$  is defined as:

$$[f \ominus b_N](x, y) = \min_{(s, t) \in b_N} \{f(x + s, y + t) - b_N(s, t)\}$$

The dilation of image  $f$  by a nonflat SE  $b_N$  is defined as:

$$[f \oplus b_N](x, y) = \max_{(s, t) \in b_N} \{f(x - s, y - t) + b_N(s, t)\}$$

When the SE is flat the equations reduce to the previous formulas up to a constant.

As in the binary case, erosion and dilation are dual operations with respect to function complementation and reflection:

$$(f \ominus b)^c(x, y) = (f^c \oplus \hat{b})(x, y)$$

Similarly,

$$(f \oplus b)^c(x, y) = (f^c \check{\ominus} \hat{b})(x, y)$$

In what follows, we omit the coordinates for simplicity.

The opening of image  $f$  by SE  $b$  is:

$$f \circ b = (f \ominus b) \oplus b$$

The closing of image  $f$  by SE  $b$  is:

$$f \bullet b = (f \oplus b) \ominus b$$

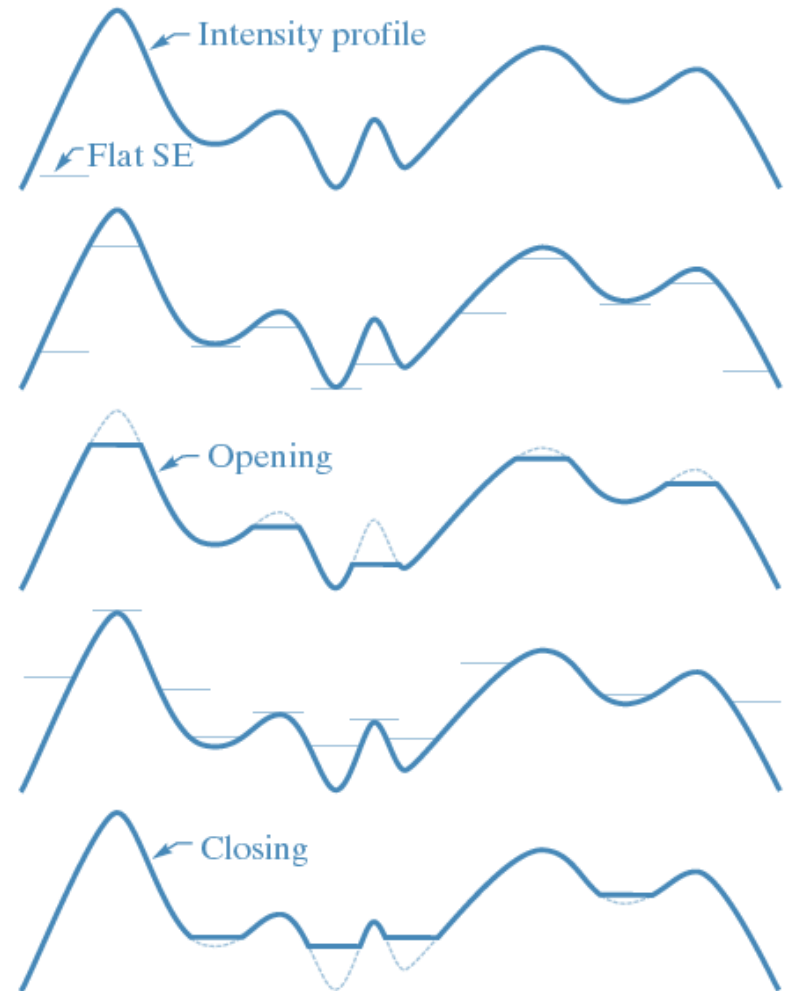
They are also duals with respect to function complementation and reflection:

$$(f \bullet b)^c = f^c \circ \hat{b} \qquad (f \circ b)^c = f^c \bullet \hat{b}$$

## Geometric interpretation of **opening**:

It is the highest value reached by any part of the SE as it pushes up against the under-surface of the image (up to the point it fits completely).

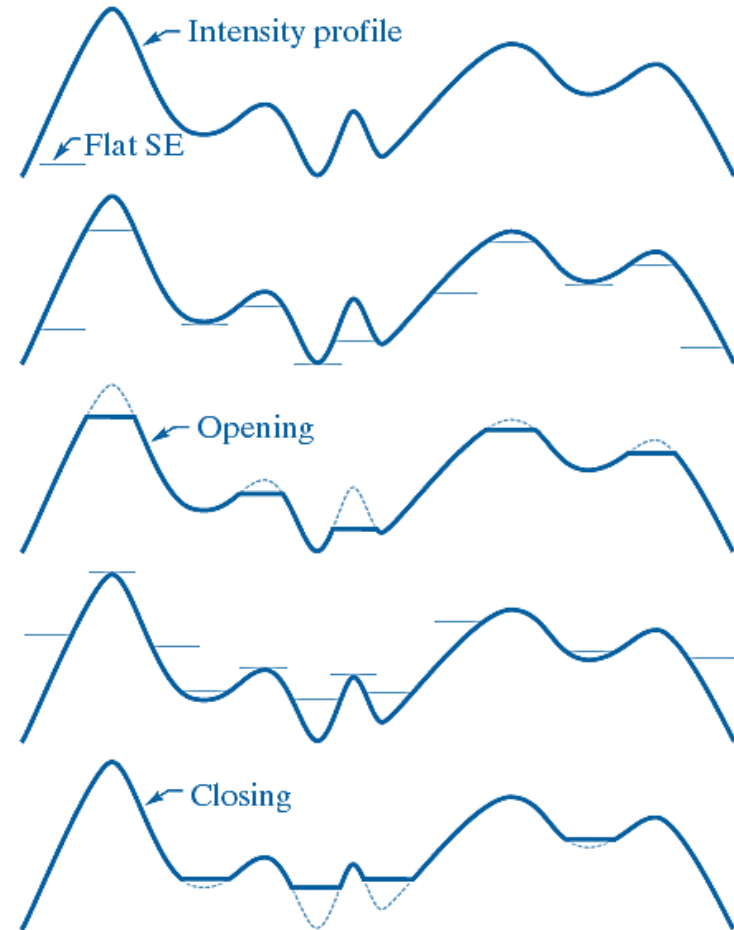
It removes small bright details.



## Geometric interpretation of **closing**:

It is the lowest value reached by any part of the SE as it pushes down against the upper side of the image intensity curve.

It highlights small dark regions of the image.



Properties of opening:

$$(1) \quad f \circ b \lhd f$$

$$(2) \quad \text{If } f_1 \lhd f_2, \text{ then } f_1 \circ b \lhd f_2 \circ b$$

$$(3) \quad (f \circ b) \circ b = f \circ b$$

The first property indicates that:

- the domain of the opening is a subset of the domain of  $f$  and

$$[f \circ b](x, y) \leq f(x, y)$$

Properties of closing:

$$(1) \quad f \leftarrow f \bullet b$$

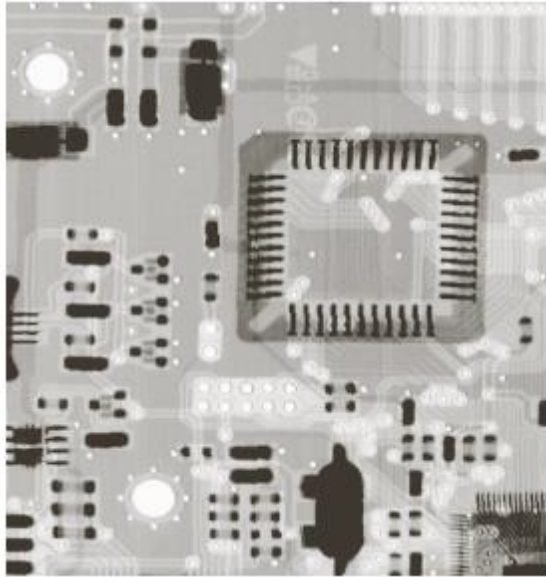
$$(2) \quad \text{If } f_1 \leftarrow f_2, \text{ then } f_1 \bullet b \leftarrow f_2 \bullet b$$

$$(3) \quad (f \bullet b) \bullet b = f \bullet b$$

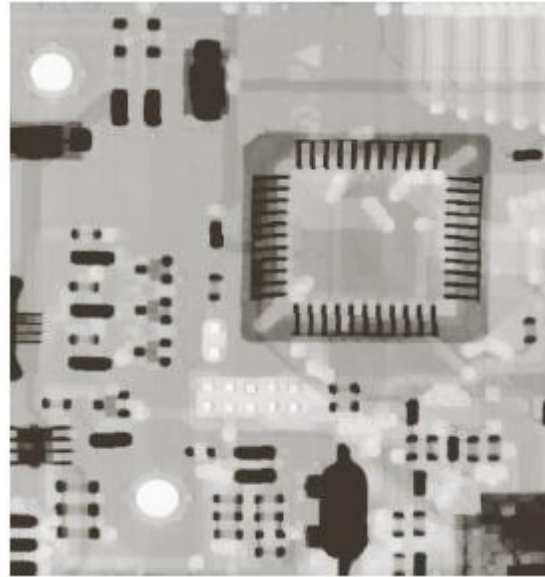
The first property indicates that:

- the domain of  $f$  is a subset of the domain of the closing and
- $f(x, y) \leq [f \bullet b](x, y)$

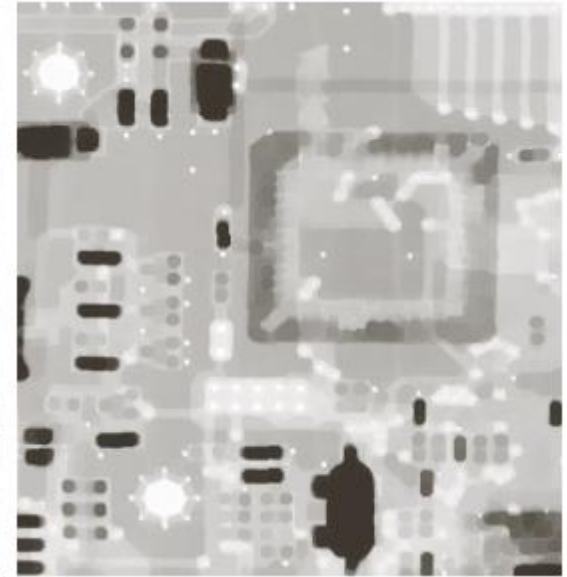




Original image



Opening by a flat disk  
SE of radius 3:  
Intensities of bright  
features decreased,  
Effects on background  
are negligible (as  
opposed to erosion).

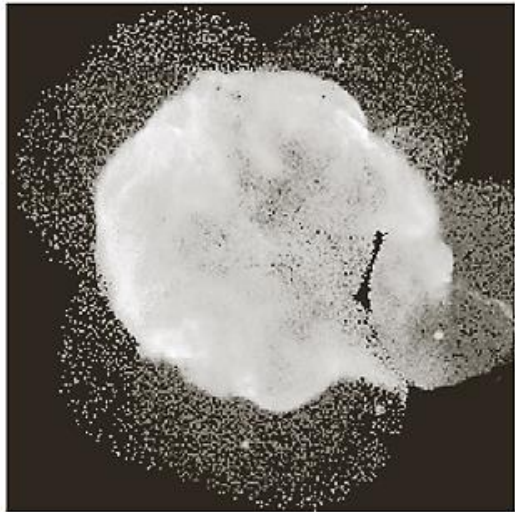


Closing by a flat disk SE  
of radius 5:  
Intensities of dark  
features increased,  
Effects on background  
are negligible (as  
opposed to dilation).

- Morphological smoothing
- Morphological gradient
- Top-hat transformation
- Bottom-hat transformation
- Granulometry
- Textural segmentation

Opening suppresses light details smaller than the SE and closing suppresses (makes lighter) dark details smaller than the SE.

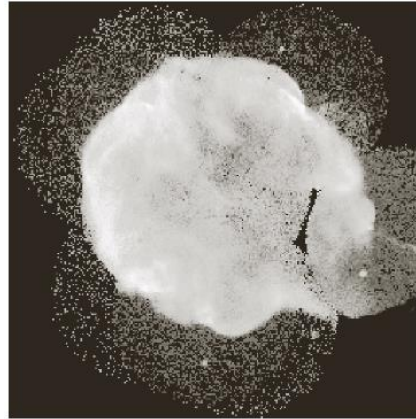
They are used in combination as *morphological filters* to eliminate undesired structures.



Cygnus Loop supernova.  
We wish to extract the  
central light region.

Opening followed by closing with disk SE of varying size

Original image



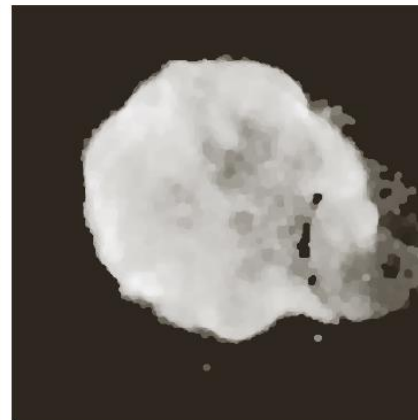
Radius 1



Radius 3



Radius 5



The difference of the dilation and the erosion of an image emphasizes the boundaries between regions:

$$g = (f \oplus b) - (f \ominus b)$$

- The difference of the dilation and the erosion of an image emphasizes the boundaries between regions.
- Homogeneous areas are not affected and the subtraction provides a derivative-like effect.
- The net result is an image with flat regions suppressed and edges enhanced.

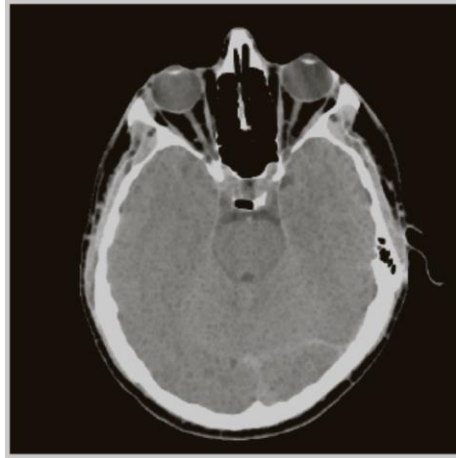
Original  
image



Dilation



Erosion



Difference



- Opening suppresses light details smaller than the SE.
- Closing suppresses dark details smaller than the SE.
- Choosing an appropriate SE eliminates image details where the SE does not fit.
- Subtracting the outputs of opening or closing from the original image provides the removed components.

Because the results look like the top or bottom of a hat these algorithms are called **top-hat** and **bottom-hat** transformations:

$$T_{\text{hat}}(f) = f - (f \circ b) \quad \text{Light details remain}$$

$$B_{\text{hat}}(f) = (f \bullet b) - f \quad \text{Dark details remain}$$

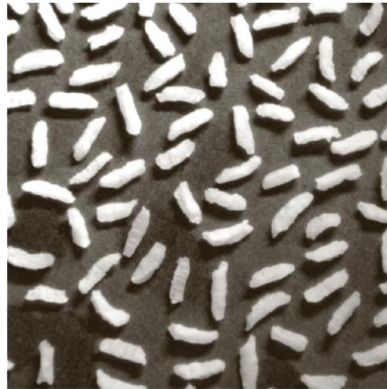
An important application is the correction of nonuniform illumination which is a pre-segmentation step.



Original image

Thresholded image  
(Otsu's method)

Opened image  
(disk SE  $r=40$ )  
Does not fit to grains  
and eliminates them



Top-hat  
(image - opening)  
Reduced  
nonuniformity



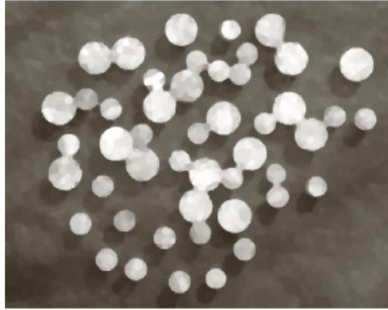
Thresholded top-hat

- Determination of the size distribution of particles in an image. Particles are seldom separated.
- The method described here measures their distribution indirectly.
- It applies openings with SE of increasing size.
- Each opening suppresses bright features where the SE does not fit.
- For each opening the sum of pixel values is computed and a histogram of the size of the SE vs the remaining pixel intensities is drawn.

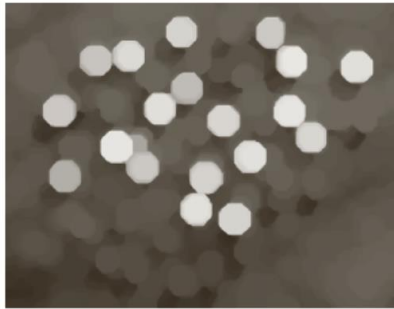
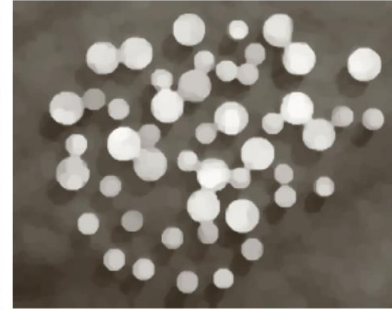
Image of  
wooden plugs



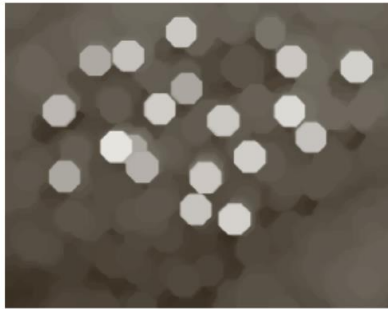
Smoothed  
image



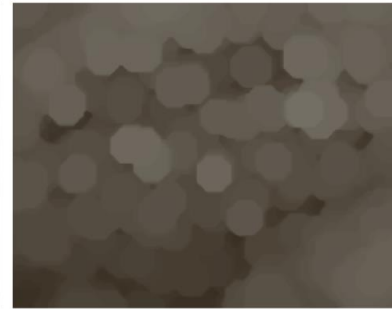
Opening by SE  
of radius 10



Opening by  
SE of radius  
20.  
Small dowels  
disappeared.



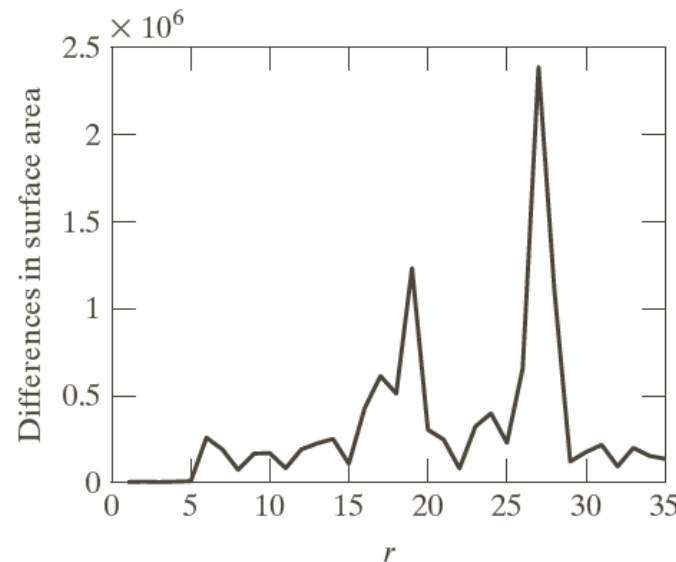
Opening by  
SE of radius  
25



Opening by  
SE of radius  
30  
Large dowels  
disappeared.

Histogram of the differences of the total image intensities between successive openings as a function of the radius of the SE.

There are two peaks indicating two dominant particle sizes (of radii 19 and 27 ).



- The objective is to find a boundary between the large and the small blobs (texture segmentation).
- The objects of interest are darker than the background.
- A closing with a SE larger than the blobs would eliminate them.



Closing with a SE of radius 30.

The small blobs disappeared as they have a radius of approximately 25 pixels.



The background is lighter than the large blobs. If we open the image with a SE larger than the distance between the large blobs then the blobs would disappear and the background would be dominant.



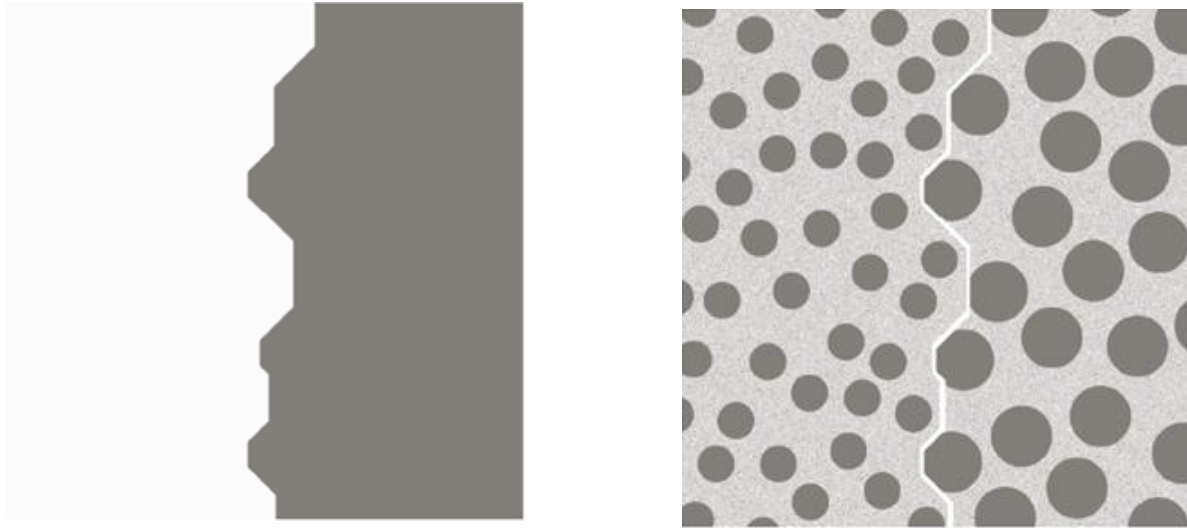
Opening with a SE of radius 60.

The lighter background was suppressed to the level of the blobs.

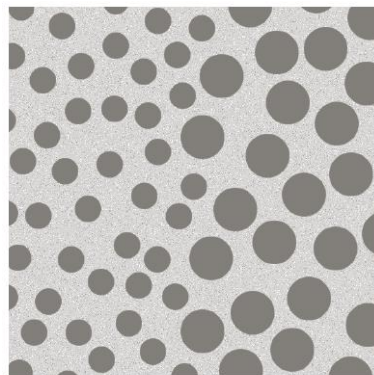




A morphological gradient with a 3x3 SE gives the boundary between the two regions which is superimposed on the initial image.



Original image



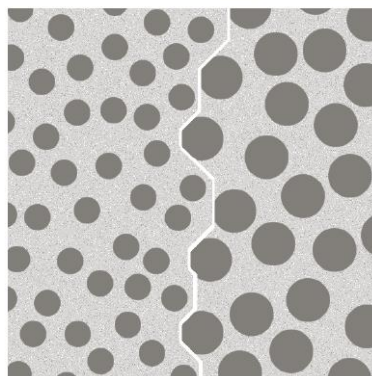
Opening with a SE  
of radius 60 (large  
blobs flooded the  
background)



Closing with a SE  
of radius 30 (small  
blobs are removed)



Morphological  
gradient  
superimposed  
onto the original  
image





## Ejercicio

Obtenga el rostro y las orejas de Shaun, usando una binarización simple y morfología matemática binaria.