Considere el sistema

$$\vec{X}' = \begin{pmatrix} 0 & 2 \\ -2 & -1 \end{pmatrix} \vec{X}$$

con condición inicial

$$\vec{X}_0 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

(a) Encuentre la solución general del sistema

(b) Determine la solución particular con la condición inicial dada

(c) Esboce las gráficas, (x(t),y(t)), de la solución particular mostrando la dirección de la trayectoria.

a) Primero buscamos los Autovalores

$$|A - I\lambda| = \begin{vmatrix} -\lambda & 2 \\ -2 & -1-\lambda \end{vmatrix} = -(-\lambda)(1+\lambda) - (2 \times -2)$$

$$= \lambda(1+\lambda) + 4$$

$$= \lambda^2 + \lambda + 4 = 0$$

buscamos las raíces:

$$\lambda^2 + \lambda = -4$$

$$\lambda^2 + \lambda + \frac{1}{4} = -\frac{15}{4}$$

$$\sqrt{\left(\lambda + \frac{1}{2}\right)^2} = \sqrt{-\frac{15}{4}}$$

$$\lambda + \frac{1}{2} = \pm \frac{\sqrt{15}\, i}{2}$$

$$\lambda = \underbrace{-\frac{1}{2}}_{\mu} \pm \underbrace{\frac{\sqrt{15}\, i}{2}}_{\lambda}$$

buscamos los autovectores:

$$\begin{vmatrix} \frac{1}{2} + \frac{\sqrt{15}}{2} i & 2 \\ -2 & -\frac{1}{2} + \frac{\sqrt{15}}{2} i \end{vmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$$

$$\frac{1}{2} V_1 \left(1 + \sqrt{15}\, i\right) + 2V_2 = 0$$

$$V_1 (1 + \sqrt{15}\, i) = -4V_2$$

$$V_1 = \frac{-4}{(1+\sqrt{15}\, i)} V_2$$

$$\mathcal{E}_1 = \begin{pmatrix} \frac{-4}{1 + i\sqrt{15}} \\ 1 \end{pmatrix}$$

$$e^{Mt}(\cos(\lambda t) + i\,\text{sen}(\lambda t)) = e^{-\frac{1}{2}t}\left(\cos\left(\frac{\sqrt{15}}{2}t\right) + i\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right)$$

luces para la solución general:

$$\xi_1 \cdot e^{Mt}(\cos(\lambda t) + i\,\text{sen}(\lambda t)) = \begin{pmatrix} \frac{-4}{1+i\sqrt{15}} \\ 1 \end{pmatrix} \cdot e^{-\frac{1}{2}t}\left(\cos\left(\frac{\sqrt{15}}{2}t\right) + i\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right)$$

$$= \begin{pmatrix} \frac{-1}{4} - \frac{i\sqrt{15}}{4} \\ 1 \end{pmatrix} \cdot e^{-\frac{1}{2}t}\left(\cos\left(\frac{\sqrt{15}}{2}t\right) + i\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right)$$

$$= e^{-\frac{1}{2}t}\begin{pmatrix} -\frac{1}{4}\cos\left(\frac{\sqrt{15}}{2}t\right) - \frac{1}{4} i\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right) - \frac{i\sqrt{15}}{4}\cos\left(\frac{\sqrt{15}}{2}t\right) + \frac{\sqrt{15}}{4}\text{sen}\left(\frac{\sqrt{15}}{2}t\right) \\ \cos\left(\frac{\sqrt{15}}{2}t\right) + i\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right) \end{pmatrix}$$

$$= e^{-\frac{1}{2}t}\left[\begin{bmatrix} \frac{1}{4}\left(-\cos\left(\frac{\sqrt{15}}{2}t\right) + \sqrt{15}\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right) \\ \cos\left(\frac{\sqrt{15}}{2}t\right) \end{bmatrix} + i\begin{bmatrix} -\frac{1}{4}\text{sen}\left(\frac{\sqrt{15}}{2}t\right) - \frac{\sqrt{15}}{4}\cos\left(\frac{\sqrt{15}}{2}t\right) \\ \text{sen}\left(\frac{\sqrt{15}}{2}t\right) \end{bmatrix}\right]$$

$$\psi = \begin{bmatrix} \frac{1}{4}\left(-\cos\left(\frac{\sqrt{15}}{2}t\right) + \sqrt{15}\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right) & -\frac{1}{4}\text{sen}\left(\frac{\sqrt{15}}{2}t\right) - \frac{\sqrt{15}}{4}\cos\left(\frac{\sqrt{15}}{2}t\right) \\ \cos\left(\frac{\sqrt{15}}{2}t\right) & \text{sen}\left(\frac{\sqrt{15}}{2}t\right) \end{bmatrix}$$

$$\psi = \left.\begin{bmatrix} \frac{1}{4}(-1) & -\frac{\sqrt{15}}{4} \\ 1 & 0 \end{bmatrix}\right|_0$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\psi = \left.\begin{bmatrix} -\frac{1}{4} & -\frac{\sqrt{15}}{4} \\ 1 & 0 \end{bmatrix}\right|_0 , \quad \left[\psi\big|_0\right]^{-1} = \frac{1}{15}\begin{bmatrix} 0 & 15 \\ -4\sqrt{15} & -\sqrt{15} \end{bmatrix}$$

$$\psi^{-1}\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{bmatrix} -1 \\ 3\sqrt{15} \end{bmatrix}$$

había mucho trabajo aritmético :(