Clase_4_punto_4_v2

Thursday, February 3, 2022 10:02 AM

4. A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

$V_{max} = 500 \text{ gal}$ $\quad V_0 = 200 \text{ gal}$ $\quad r_0 = 100 \text{ Lb}$

$r = 3 \text{ gal/min}$ $\quad s = 2 \text{ gal/min}$ $\quad t \leq 300 \text{ min}$

$b = 1 \text{ Lb/gal}$

$$\frac{dQ(t)}{dt} + Q(t)\left(\frac{s}{V_0 + rt - st}\right) = br$$

$$M(t)\frac{dy}{dt} + M(t)\, y\, p(t) = M(t)\, g(t)$$

$$e^{\int p(t)} = M(t) \equiv e^{\int \frac{s}{V_0 + (r-s)t} dt}$$

La anterior integral es:

$$M(t) = e^{\frac{s \cdot \log(V_0 + t(r-s))}{r-s}} = \left(e^{\log(V_0 + t(r-s))}\right)^{\frac{s}{r-s}}$$

$$= (V_0 + t(r-s))^{\frac{s}{r-s}}$$

$$M\left(t \,\middle|\, s = 2\frac{\text{gal}}{\text{min}}, r = 3\frac{\text{gal}}{\text{min}}, V_0 = 200 \text{ gal}\right) = \left(200 \text{ gal} + t\frac{\text{gal}}{\text{min}}\right)^{\frac{2 \text{ gal/min}}{1 \text{ gal/min}}}$$

$$= \left(200 \text{ gal} + t\frac{\text{gal}}{\text{min}}\right)^2$$

Recordemos nuestra expresión:

$$\frac{d(M(t) \cdot Q(t))}{dt} = M(t)\frac{dQ(t)}{dt} + M(t)Q(t)\left(\frac{s}{V_0 + (r-s)t}\right) = br \cdot M(t)$$

$$\left(200 \text{ gal} + t\frac{\text{gal}}{\text{min}}\right)^2 \frac{dQ(t)}{dt} + \left(200 \text{ gal} + t\frac{\text{gal}}{\text{min}}\right)^2 Q(t)\left(\frac{s}{V_0 + (r-s)t}\right) = br \cdot \left(200 \text{ gal} + t\frac{\text{gal}}{\text{min}}\right)^2$$

$$\int \frac{d(\mu(t)\,Q(t))}{dt}\,dt = \mu(t)\,Q(t) = \int b\,r \cdot \left(200\ \text{gal} + t\ \tfrac{\text{gal}}{\text{min}}\right)^2 dt$$

$$= b\,r \int \left(200\ \text{gal} + t\ \tfrac{\text{gal}}{\text{min}}\right)^2 dt$$

$$= \frac{1\,\text{Lb}}{\text{min}} \cdot 40000t + 200t^2 + \frac{t^3}{3} + C$$

Ahora dividimos en $\mu(t)$:

$$\xrightarrow{\quad} (0+C)1 = 100$$

$$Q(t=0) = \frac{\frac{1\,\text{Lb}}{\text{min}} \cdot \left(\cancel{40000t}^{\,0} + \cancel{200t^2}^{\,0} + \cancel{\frac{t^3}{3}}^{\,0} + C\right)}{\left(200\ \text{gal} + \cancel{t}^{\,0}\ \tfrac{\text{gal}}{\text{min}}\right)^2} = \frac{1\,(C)}{(200)^2} = 100$$

$$\Rightarrow C = 4000000$$

$$Q(t) = \frac{\frac{1\,\text{Lb}}{\text{min}} \cdot \left(40000t + 200t^2 + \frac{t^3}{3} + 4000000\right)}{\left(200\ \text{gal} + t\ \tfrac{\text{gal}}{\text{min}}\right)^2}, \quad t \leq 300\ \text{min}$$

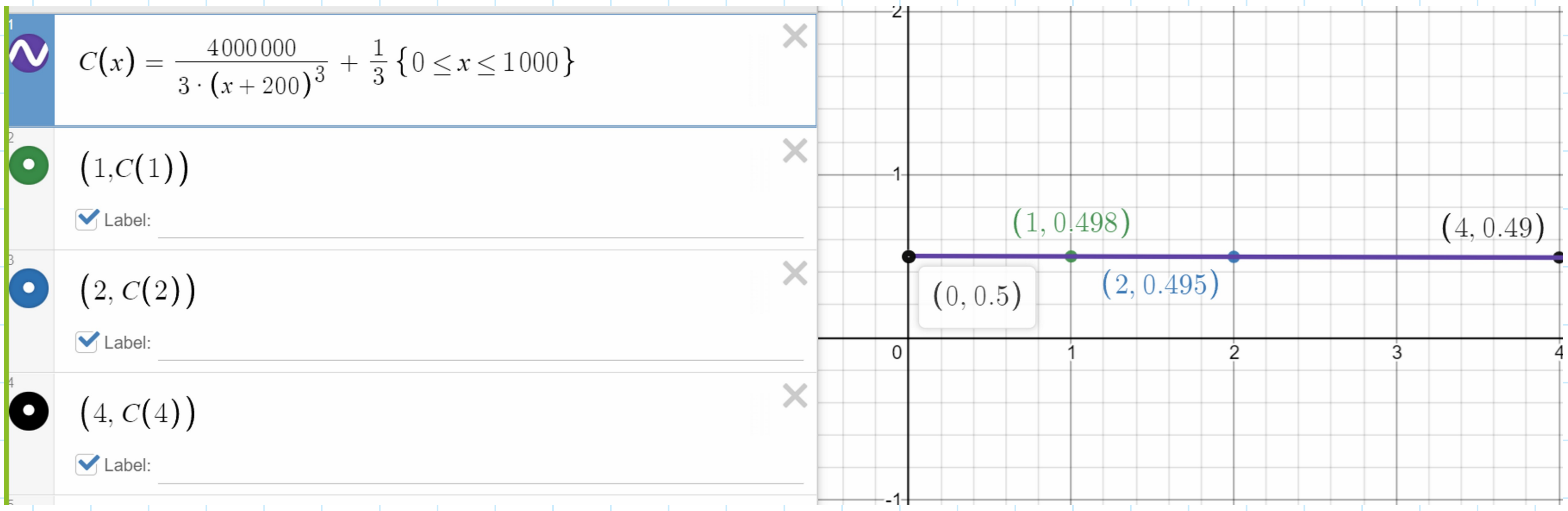
$\downarrow$

Lbs

$$Q(t) = \frac{x}{3} + \frac{4\times 10^6}{3(x+200)^2} + \frac{200}{3}$$

$$C(t) = \frac{Q(t)}{200\ \text{gal} + (r-s)\ \tfrac{\text{gal}}{\text{min}}\ t}, \quad t \leq 300\ \text{min}$$

$$= \frac{4\times 10^6}{3(x+200)^3} + \frac{1}{3}$$



1

$$C(x) = \frac{4000000}{3 \cdot (x+200)^3} + \frac{1}{3}\{0 \le x \le 1000\}$$

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$$Q(x) = \frac{x}{3} + \frac{4000000}{3(x+200)^2} + \frac{200}{3}$$

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