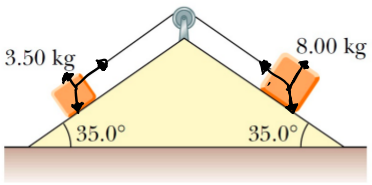
1.0.2 David Alsina y Juan José Caballero

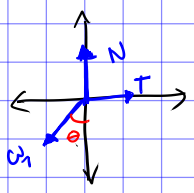
1.0.2. Problema 2

Dos objetos están conectados por una cuerda ligera que pasa por una polea sin fricción. **a)** Dibuje el diagrama de cuerpo libre para cada uno de los cuerpos. **b)** encuentre la aceleración y la tensión del sistema.

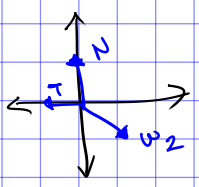


a) Diagramas

M_1 :



M_2 :



b) encuentre aceleración y tensión del sistema:

$$\sum F = m \cdot a$$

① M_1: $\sum F_x = T - w_1 \operatorname{sen}(35°) = m_1 a$

② M_2: $\sum F_x = -T + w_2 \operatorname{sen}(35°) = m_2 a$

Si ① + ② :

$$\begin{array}{rl} + & T - w_1 \operatorname{sen}(35°) = m_1 \cdot a \\ & -T + w_2 \operatorname{sen}(35°) = m_2 \cdot a \\ \hline & \operatorname{sen}(35°)(w_2 - w_1) = a(m_1 + m_2) \end{array}$$

③ $$\frac{\operatorname{sen}(35°)(w_2 - w_1)}{(m_1 + m_2)} = a$$

$$= \frac{\operatorname{sen}(35°)(g(8\,kg - 3.5\,kg))}{(8\,kg + 3.5\,kg)} = a$$

$$= \frac{\operatorname{sen}(35°)(g(4.5\,kg))}{11.5\,kg} = a$$

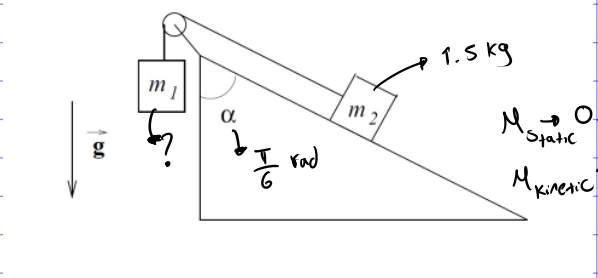
$$2.18\ m/s^2 = a$$

ahora que hay aceleración despejamos T:

$$T = \left((3.5\,kg)\ 2.18\ \frac{m}{s^2}\right) + (3.5\,kg) \cdot g \cdot \operatorname{sen}(35°)$$

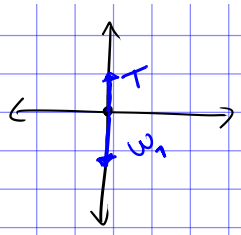
$$T = 27.181\ N$$

1.1 Fricción



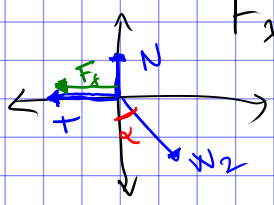
b

m_1:



$$F_y = T - w_1 = 0$$

m_2:



$$F_x = F_f + T - \operatorname{sen}(\alpha) w_2 = 0$$
$$F_y = N - \cos(\alpha) w_2 = 0$$

$$\alpha = \frac{\pi}{6} \text{ rad}$$

buscamos la Normal:

$$m_2: \quad F_y = N = \cos(\alpha) \cdot g \cdot m_2$$
$$N = 12.74 \text{ N}$$

Con la normal podemos obtener la fricción:

$$F_f = \mu_s \cdot N = \frac{1}{2} \cdot (12,74 \text{ N})$$
$$F_f = 6.37 \text{ N}$$

Con el Sistema m_1:

$$T = w_1$$
$$T = 9.81 \frac{m}{s^2} \cdot m_1$$

$$F_x = -F_f - \overbrace{g m_1}^{T} + \overbrace{\operatorname{sen}(\alpha) g m_2}^{w_2} = 0$$

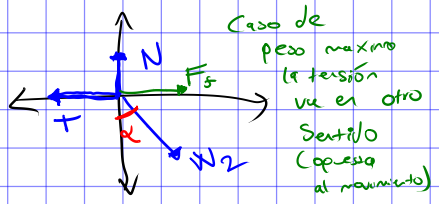
$$m_1 = \frac{\operatorname{sen}(\alpha) \cancel{g} m_2}{\cancel{g}} - \frac{F_f}{g}$$

$$m_1 = \operatorname{sen}(\alpha) m_2 - (F_f / g)$$

$$m_1 = \operatorname{sen}\left(\frac{\pi}{6}\right) 1.5 \text{ kg} - (6.37 \text{ N}) / \left(9.81 \frac{m}{s^2}\right)$$

$$m_1 = 0,1 \text{ kg}$$

(a)



$$F_x = F_f - \overbrace{g m_1}^{T} + \overbrace{\operatorname{sen}(\alpha) g m_2}^{W_2} = 0$$

$$m_1 = \frac{+\operatorname{sen}(\alpha)\cancel{g} m_2}{+\cancel{g}} + \frac{F_f}{g}$$

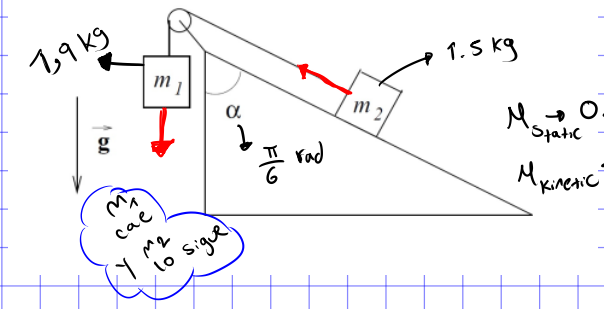
$$m_1 = \operatorname{sen}(\alpha) m_2 + F_f / g$$

$$m_1 = \operatorname{sen}\left(\tfrac{\pi}{6}\right) 1.5\,kg + \left(6.37\,N / 9.81\,\tfrac{m}{s^2}\right)$$

$$m_1 = 1.39\,kg \approx 1.4\,kg$$

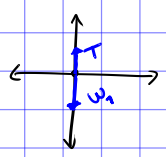
Situación Cinética

(c)



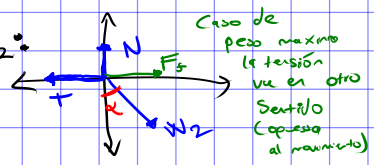
(d)

m_1:



$$F_y = T - W_1 = -m_1 \cdot a$$

m_2:



$$F_x = F_f + T - \operatorname{sen}(\alpha) W_2 = -m_2 a$$

$$F_y = N - \cos(\alpha) W_2 = 0$$

(0,45) (12.74N)

① $F_y = T - W_1 = -m_1 \cdot a$

② $F_x = F_f + T - \operatorname{sen}\left(\tfrac{\pi}{6}\right) W_2 = -m_2 a$

$$① - ② = -W_1 - F_f + \operatorname{sen}\left(\tfrac{\pi}{6}\right) W_2 = m_2 a - m_1 a$$

$$-W_1 - F_f + \operatorname{sen}\left(\tfrac{\pi}{6}\right) W_2 = a(m_2 - m_1)$$

$$\frac{-W_1 - F_f + \operatorname{sen}\left(\tfrac{\pi}{6}\right) W_2}{(m_2 - m_1)} = a$$

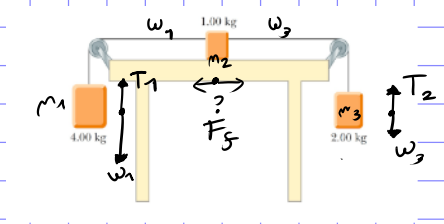
$$-27{,}51\,\tfrac{m}{s^2} = a$$

(e) dado $t = 2{,}5\ s$

$$dist = \frac{a}{2} t^2 = \frac{\left(-27{,}51 \frac{mt}{s^2}\right)}{2} t^2$$

$$= -85{,}96\ mt$$

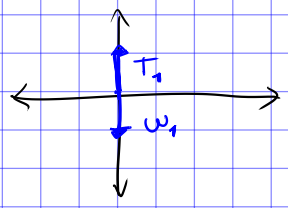
1.1.2 Problema 2



$$\mu_k = 0{,}35$$

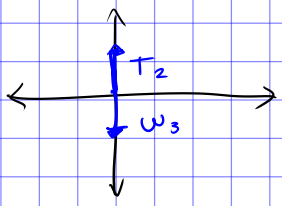
(a) Diagramas de cuerpo libre

m_1 :



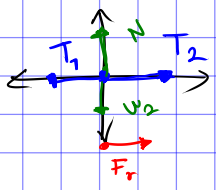
$$F_y = T_1 - w_1 = -m_1 a_1$$

m_3 :



$$F_y = T_2 - w_3 = m_3 a_3$$

m_2 :



$$F_x = F_r + w_3 - w_1 = -m_2 a_2$$

$$F_y = N - w_2 = 0$$

(b) Determine la aceleración de cada uno de los cuerpos y sus direcciones.

$$F_{1y} = T_1 = -m_1 a_1 + w_1$$

$$F_{3y} = T_2 = m_3 a_3 + w_3$$

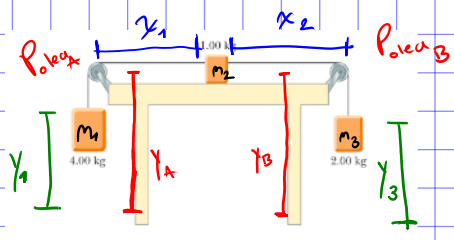
$$\rightarrow F_{2y} = N - w_2 = 0$$

$$\boxed{N = w_2 = 1\,Kg \cdot g = 9.81\ N}$$

$$\rightarrow F_{2x} = F_r + T_2 - T_1 = -m_2 \cdot a_2$$

$$= N(\mu_k) + g(m_1 + m_3) - m_1 a_1 + m_3 a_3 + m_2 a_2 = 0$$

$$= -m_1 a_1 + m_3 a_3 + m_2 a_2 = -62.3\ N \quad (1)$$



$\mu_k = 0{,}35$

$$l_1 = (y_A - y_1) + \frac{\pi}{2}R + (l_{mesa} - x_2)$$

$$\Rightarrow 0 = a_A - a_1 - a_{x_2}$$

está fija $\cancel{a_A} = a_1 + a_{x_2}$

$$-a_{x_2} = a_1$$

$$a_{x_2} = -a_1$$

$$l_2 = (y_B - y_3) + \frac{\pi}{2}R + x_2$$

$$\Rightarrow 0 = a_B - a_3 + a_{x_2}$$

está fija $\cancel{a_B} = -a_3 + a_{x_2}$

$$a_{x_2} = a_3$$

$$\Rightarrow -a_1 = a_3 = a_{x_2} = a_2$$

Retomando (1):

$$-m_1 a + m_3 a + m_2 a_2 = -62.3\,N$$
$$a(m_3 - m_1 + m_2) = -62.3\,N$$
$$a(2kg - 4kg + 1kg) = -62.3\,N$$
$$a(-1kg) = -62.3\,N$$
$$a = 62{,}3\,\frac{m}{s^2}$$

Tenemos que las aceleraciones son las mismas la aceleración de m_1 va hacia abajo y la de m_3 en sentido contrario. la de a_2 va hacia la izquierda.

(c) Determine la tensión en las 2 cuerdas

$$F_{1y} = T_1 = m_1 a + w_1 = 4\,kg\left(62{,}3\,\frac{m}{s^2} + 9{,}81\,\frac{m}{s^2}\right)$$
$$= 288{,}4\,N$$

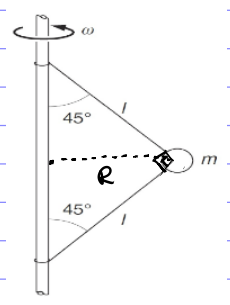
$$F_{3y} = T_2 = m_3 a + w_3 = 2\,kg\left(62{,}3\,\frac{m}{s^2} + 9{,}81\,\frac{m}{s^2}\right)$$
$$= 144{,}2\,N$$

1.2 Movimiento Circular

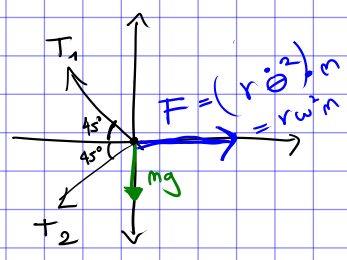
1.2.1 Problema 3

$\operatorname{sen}(45°) = \frac{R}{L}$

$L\operatorname{sen}(45°) = R$



a)



$$\sum F_x = -\cos(45°)\left(T_1 + T_2\right) + F = 0$$

$$\sum F_y = \operatorname{sen}(45°)\left(T_1 - T_2\right) - mg = 0$$

$$= \operatorname{sen}(45°)\left(T_1 - T_2\right) = mg$$

$$\sum F_x = -\cos(45°)\left(T_1 + T_2\right) + \left(R\dot{\theta}^2\right)m = 0$$

$$= -\cos(45°)\left(T_1 + T_2\right) + L\operatorname{sen}(45°)\,\omega^2 \cdot m = 0$$

$$= \left(L\operatorname{sen}(45°)\,\omega^2\right) \cdot m = \frac{\sqrt{2}}{2}\left(T_1 + T_2\right) \qquad (1)$$

$$\sum F_y = mg = \frac{\sqrt{2}}{2}\left(T_1 - T_2\right) \qquad (2)$$

$\boxed{(1) + (2):}$

$$m\left(L\operatorname{sen}(45°)\,\omega^2 + g\right) = -\sqrt{2}\,T_1$$

$$\frac{-m\left(L\operatorname{sen}(45°)\,\omega^2 + g\right)}{\sqrt{2}} = T_1$$

Con el valor de T_1:

$$\sum F_y = mg = \operatorname{sen}(45°)\left(T_1 - T_2\right)$$

$$\sum F_y = mg = \frac{\sqrt{2}}{2}\left(\frac{-m\left(L\operatorname{sen}(45°)\,\omega^2 + g\right)}{\sqrt{2}} - T_2\right)$$

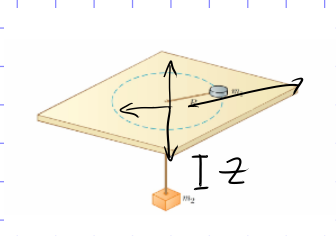
$$= mg - \cancel{\frac{\sqrt{2}}{2}} \cdot \left(\frac{-m\left(L\operatorname{sen}(45°)\,\omega^2 + g\right)}{\sqrt{2}}\right) = -\cancel{\frac{\sqrt{2}}{2}}\,T_2$$

$$= -\frac{2}{\sqrt{2}}\,mg + \left(\frac{-m\left(L\operatorname{sen}(45°)\,\omega^2 + g\right)}{\sqrt{2}}\right) = T_2$$

$$= -m\left(\frac{2}{\sqrt{2}}\,g + \frac{\left(L\operatorname{sen}(45°)\,\omega^2 + g\right)}{\sqrt{2}}\right) = T_2$$

1.2.2 Problema 4

- No hay fricción
- la masa m_1 está en equilibrio



a) ligadura

$$l = r + z$$
$$0 = \ddot{r} + \ddot{z}$$
$$\ddot{z} = -\ddot{r}$$

b) encuentre la Tensión

$$\Sigma F_x = \omega^2 R m_1 - T = \cancel{m_1 a}^{\,0} \quad \text{(está en equilibrio)}$$
$$(1) \quad = T = \omega^2 R m_1$$

$$\Sigma F_y = T - m_2 g = \cancel{m_2 a}^{\,0}$$
$$(2) \quad = T = m_2 g$$

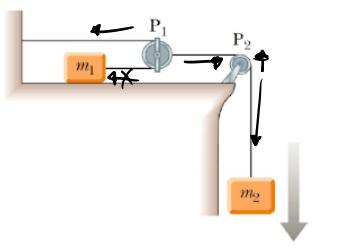
c) vel. necesaria para m_2 en equilibrio

$\boxed{\text{Ahora:}}$ $(1) = (2)$

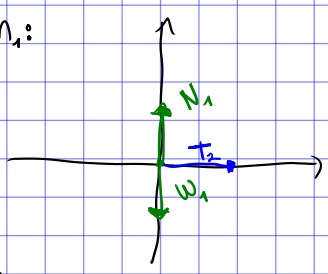
$$\omega^2 R m_1 = m_2 g$$
$$\omega = \sqrt{\frac{m_2 g}{m_1 R}}$$

1.0.1 Problema 1

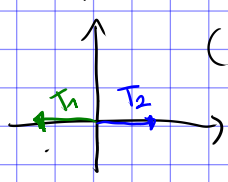
- No hay fricción
- Poleas de masa despreciable



m_1:

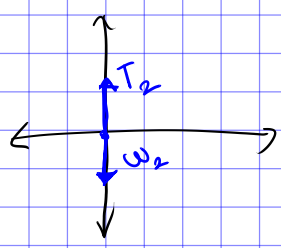


P_1:



(No hay peso)

m_2:

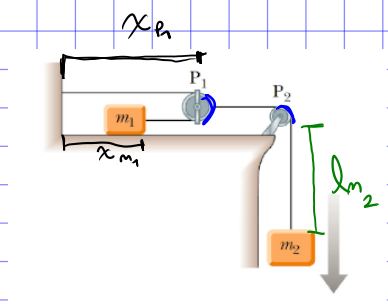


$$m_1 \begin{cases} \Sigma F_{m_1 x} = T_2 = m_1 \cdot a_1 \\ \Sigma F_{m_1 y} = N - \omega_1 = 0 \end{cases}$$

$$P_1 \begin{cases} \Sigma F_{P_1 x} = T_2 - T_1 = \cancel{m_{P_1} \cdot a}^{\,0} \\ T_2 = T_1 \end{cases}$$

$$m_2 \begin{cases} \Sigma F_{m_2 x} = 0 \\ \Sigma F_{m_2 y} = T_2 - \omega_2 = m_2 a_2 \\ T_2 = m_2 a_2 + \omega_2 \end{cases}$$

a) Si a_1 y a_2 son aceleraciones para los cuerpos ¿Qué relación hay entre ellos?



$$l_1 = (x_{P_1} - x_{m_1}) + (R_{P_1}\pi) + x_{P_1}$$

$$0 = a_{P_1} - a_{m_1} + a_{P_1} = a_{P_1} = \frac{a_{m_1}}{2}$$

$$l_2 = (x_{P_2} - x_{P_1}) + \left(\frac{R_{P_2}}{2}\pi\right) + l_{m_2}$$

$$0 = a_{P_2} - a_{P_1} + a_{m_2} = \underbrace{\cancel{a_{P_2}}}_{0} = \frac{a_{m_1}}{2} - a_{m_2}$$

$$0 = \frac{a_{m_1}}{2} - a_{m_2}$$

$$2\cdot(a_{m_2}) = a_{m_1}$$

b) Encuentre la tensión entre las cuerdas

$$m_1 \begin{cases} \sum F_{m_1 x} = T_2 = m_1 \cdot a_1 \\ \sum F_{m_1 y} = N - \omega_1 = 0 \end{cases}$$

$$P_1 \left[\sum F_{P_1 x} = T_2 - T_1 = \cancel{M_{P_1} \cdot a}^{\,0} \quad T_2 = T_1 \right.$$

$$m_2 \begin{cases} \sum F_{m_2 x} = 0 \\ \sum F_{m_2 y} = T_2 - \omega_2 = m_2 a_2 \\ T_2 = m_2 a_2 + \omega_2 \end{cases}$$

$$T_1 = T_2 = m_2 a_2 + \omega_2$$
$$= m_2(a_2 + g)$$

c) aceleraciones en términos de m_1, m_2 y g:

$$\boxed{\begin{aligned} 2\cdot(a_{m_2}) &= a_{m_1} \\ a_{P_1} &= 2a_{m_1} = 4a_{m_2} \end{aligned}}$$

Usaremos esto para despejar

$$\sum F_{m_1 x} = \quad T_2 = m_1 a_1$$
$$\sum F_{m_2 y} = \quad -T_2 = m_2 a_2 + \omega_2$$
$$0 = m_1 a_1 - m_2 a_2 - \omega_2$$

$$2a_{m_2} = a_{m_1}$$

$$0 = 2m_1 a_2 - m_2 a_2 - \omega_2$$
$$\omega_2 = a_2(2m_1 - m_2)$$
$$\boxed{\frac{\omega_2}{(2m_1 - m_2)} = a_2}$$

$$\therefore \quad a_1 = \frac{a_2}{2} = \frac{\omega_2}{2(2m_1 - m_2)} \quad y \quad a_{P_1} = \cancel{2}\frac{\omega_2}{\cancel{2}(2m_1 - m_2)} = a_2$$