Resumen — AED.

$X = (X)_{np}$ $n \times p$, $n \rightarrow$ Observaciones y $p \rightarrow$ variables.

$\bar{x} \rightarrow$ Vector de medias $\quad S_n = \frac{1}{n}(D'.D) \rightarrow$ Covarianzas Muestrales.

Matriz de correlación $\rightarrow R = V^{-1/2}.S.V^{-1/2}$. R tiene 1's en la diagonal.

$\Sigma \rightarrow$ Matriz de covarianzas $\quad D = X - (1).\bar{x}'$

$\Sigma = S = \frac{1}{n-1}(D'.D)$ $\quad V^{1/2} = \sqrt{Diag(S)} \rightarrow$ Desviación Estandar

Distancia Euclidiana $\rightarrow d(P,Q) = \sqrt{(x_1-y_1)^2+(x_2-y_2)^2+\cdots+(x_p-y_p)^2}$

Distancia Estadística $\rightarrow d(O,P) = \sqrt{\left(\frac{x_1}{\sqrt{S_{11}}}\right)^2+\left(\frac{x_2}{\sqrt{S_{22}}}\right)^2} = \sqrt{\frac{x_1^2}{S_{11}}+\frac{x_2^2}{S_{22}}}$

$c^2 = \frac{x_1^2}{S_{11}} + \frac{x_2^2}{S_{22}} \rightarrow$ Elipse $(0,0)$ $\quad$ P1 — Diap 31.

$$d(P,Q) = \sqrt{\frac{(x_1-y_1)^2}{S_{11}} + \frac{(x_2-y_2)^2}{S_{22}} + \cdots + \frac{(x_p-y_p)^2}{S_{pp}}}$$

$\tilde{x}_1 = x_1 \cos\theta + x_2 \operatorname{sen}\theta$
$\tilde{x}_2 = -x_1 \operatorname{sen}\theta + x_2 \cos\theta$
$\quad d(O,P) = \sqrt{\frac{\tilde{x}_1^2}{\tilde{S}_{11}} + \frac{\tilde{x}_2^2}{\tilde{S}_{22}}}$

$$d = \sqrt{a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2}$$

$$d(P,Q) = \sqrt{a_{11}(x_1-y_1)^2 + 2a_{12}(x_1-y_1)(x_2-y_2) + a_{22}(x_2-y_2)^2}$$

Propiedades $\rightarrow$
- $d(P,Q) = d(Q,P)$
- $d(P,Q) > 0$ si $P \neq Q$
- $d(P,Q) \leq d(P,R) + d(R,Q)$

Fórmulas $a_{11}, a_{12}, a_{22} \rightarrow$ P1 — Diapositiva 46

$\cos(\theta) = \frac{x'y}{\|x\|\|y\|}$ $\quad Proy\, x_y = \frac{(x'y)}{y'y}y = \frac{(x'y)}{\|y\|^2}y$

Longitud $\rightarrow$ Norma $\quad$ Matriz Ortogonal $\rightarrow A^{-1} = A'$

Descomposición espectral $\rightarrow A = \lambda_1 e_1 e_1' + \cdots + \lambda_k e_k e_k'$

Forma Cuadrática $\rightarrow x'Ax \geq 0$ es no negativa
$\quad > 0$ es definida positiva.

Función de densidad multivariable $\longrightarrow f_{x_1 \dots x_p}(x_1, \dots, x_p) = f_x(x)$
$\rightarrow$ función de densidad conjunta.

Independencia $\longrightarrow$ $\left.\begin{array}{l} f_{xy}(x,y) = f_x(x) \cdot f_y(y) \\ P_{xy}(x,y) = P_x(x)\, P_y(y) \end{array}\right\}$ p variables

- Si x_i y x_k independientes, entonces $\text{cov}(x_i, x_k) = 0$

$\frac{p(p-1)}{2} \rightarrow$ N° cov diferentes $\quad \Sigma = E((X-\mu)(X-\mu)')$

Partición:

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_q \\ \hline x_{q+1} \\ \vdots \\ x_p \end{pmatrix} = \begin{pmatrix} X^{(1)} \\ \hline X^{(2)} \end{pmatrix} \qquad \mu = E(X) = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_q \\ \hline \mu_{q+1} \\ \vdots \\ \mu_p \end{pmatrix} = \begin{pmatrix} \mu^{(1)} \\ \mu^{(2)} \end{pmatrix}$$

$E((X^{(1)} - \mu^{(1)})(X^{(2)} - \mu^{(2)})') = \Sigma_{12} \longrightarrow$ Cov de 1 y 2

$$\Sigma = E((X-\mu)(X-\mu)') = \begin{array}{c} \\ q \\ p-q \end{array} \begin{array}{c} \begin{array}{cc} q & p-q \end{array} \\ \left(\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right) \end{array}$$

$E((X^{(1)} - \mu^{(1)})^2) = \Sigma_{11} \quad E((X^{(2)} - \mu^{(2)})^2) = \Sigma_{22} \quad \Sigma_{12} = \Sigma_{21}'$

$c'X = c_1 x_1 + \dots + c_p x_p \qquad E(c'X) = c'\mu \qquad \mu = E(X)$
$\qquad \text{Var}(c'X) = c'\Sigma c \qquad \Sigma = \text{cov}(X)$

También para C matriz.

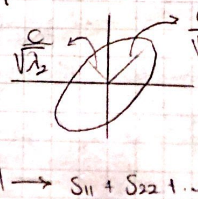
Geometría $\longrightarrow$ $X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix} = \begin{pmatrix} x_1' \\ \vdots \\ x_n' \end{pmatrix} \longrightarrow$ vector de Observaciones

$= (y_1 | y_2 | \dots | y_p) \longrightarrow$ vector de variables

Estimador para $\mu \longrightarrow \bar{x}$ insesgado, $\text{cov}(\bar{x}) = \frac{1}{n}\Sigma$

Estimador para $\Sigma \rightarrow \frac{n}{n-1} S_n \qquad$ Varianza Generalizada $\rightarrow |S|$

Int Geo $\longrightarrow |S| = (n-1)^{-p} (\text{volumen})^2$,



$(x-\bar{x})' S^{-1} (x-\bar{x}) = c^2$.

$|S| = (s_{11} s_{22} \cdots s_{pp}) |R|$

Var total $\longrightarrow s_{11} + s_{22} + \dots + s_{pp}$.

$\bar{x} = \frac{1}{n} X' 1 \qquad S = \frac{X'}{n-1} (I - \frac{1}{n}(1 1')) X$.