Workshop 5

Topics: Simulation of dynamical systems using Simulink.

Introduction

The aim of this workshop is to provide a first encounter with the simulation of dynamical systems using Simulink (complementary product of Matlab). If you have never used Simulink before, take a few moments (around 1 hour) to watch the following videos (last one is optional):

1. `https://www.youtube.com/watch?v=tv14EB2LqeA&ab_channel=MATLAB`

2. `https://www.youtube.com/watch?v=WLPvCefp6Qo&ab_channel=ChristopherLum`

3. `https://www.youtube.com/watch?v=Cvu2zWk3gYw&ab_channel=ChristopherLum`

4. `https://www.youtube.com/watch?v=sF_sjFqNFUk&ab_channel=ChristopherLum`

Problem:

Purpose: Simulation of dynamical systems (Romeo and Juliet).

For this problem, prepare a report containing your solutions, graphs, justifications and comments for each of the questions below. Upload your report to e-aulas along with the code you used to do the simulations.

The problem is based on the famous article from S. Strogatz, "Love Affairs and Differential Equations", Mathematics Magazine, v.61, no 1, 1988. Juliet and Romeo are in love, but in our version of this story, Juliet is an "indecisive lover". The more Romeo loves her, the more she begins to dislike him. But when he loses interest in her, her feelings of love grow. He, on the other hand, tends to echo what Juliet feels: love grows when she loves him and turns to hate when she hates him. A model simple for this particular romance is given by:

$$\dot{J}(t) = -aR(t),$$

$$\dot{R}(t) = bJ(t), \quad t \in [0, \infty),$$

where $J(t)$ denotes the amount of love (or hate, when negative) from Juliet to Romeo, $R(t)$ denotes the amount of love (or hate, when negative) from Romeo to Juliet, and a and b are positive parameters.

Problem 1

Using Simulink, simulate the system and comment the results. Try different values of a and b. Add your most interesting plots to the report.

Problem 2

Choose interesting values for your parameters and answer under which conditions (if they exist) will love never flourish between Juliet and Romeo.

Problem 3

Suppose that due to their stormy love, Romeo and Juliet decide to go to couples therapy. The therapist suggests that their love cannot depend solely on what the other feels. Moreover, he/she recommends that they be contained when it comes to love; according to the therapist, a love without measure is harmful. After this consultation, the equations that model the love of our characters are the following:

$$\dot{J}(t) = -aR(t) - cJ(t),$$
$$\dot{R}(t) = bJ(t) - dR(t), \quad t \in [0, \infty),$$

where c and d are positive parameters.

Analyse and describe the new model. You should mention explicitly what the new terms add to the dynamics.

Problem 4

Using Simulink, simulate the system and comment the results. Is the therapist correct in what he/she suggested? Try different values of the parameters. Add your most interesting plots to the report.

Problem 5

Now you play therapist: give Juliet and Romeo advice, then model the impact of your advice in the relationship between them by modifying the differential equations. Simulate and comment. Are you a good therapist?