Workshop 7

Topics: State transition matrix of linear discrete systems.

Problem 1

(Ex. 4 Ch. 4 from Luenberger, Introduction to Dynamic Systems)

Consider the nonlinear difference equation of the form:

$$y(k+n) = F(y(k+n-1), \ldots, y(k), u(k+n-1), \ldots, u(k), k).$$

Find a state space representation of the difference equation. (Hint: The representation will be more than n-dimensional.)

Problem 2

Find the state-transition matrix $\Phi(k, 0)$ associated to the following system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \qquad k \in \mathbb{N}$$

Problem 3

Consider the following homogeneous linear system:

$$\mathbf{x}(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \qquad k \in \mathbb{N}$$

1. Find the state-transition matrix $\Phi(k, l)$
2. Find two linearly independent solutions
3. Find $\mathbf{x}(5)$ when

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Problem 4

Suppose you have a linear homogeneous discrete system with constant coefficients of the form:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k), \quad k \in \mathbb{N}^*$$

If the state transition matrix is given by:

$$\Phi(k, 0) = \begin{bmatrix} 1 & k & \frac{(k^2+k)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$$

Find $\mathbf{A}$.