Workshop 2

Topics: Difference equations and discrete dynamical systems.

Problem 1

Express the following difference equations in terms of the differences $\Lambda^0(k),\ \Lambda^1(k), \ldots$. Keep in mind that an equation of order m

$$F(k, x(k), \ldots, x(k+m)) = 0$$

must be expressed as an equation involving the mth difference

$$G(k, \Lambda^0(k), \Lambda^1(k), \ldots, \Lambda^m(k)).$$

1. $x(k+1) = ax(k)$, where a is a constant and $k \in \mathbb{N}^*$.
2. $x(k+2)x(k) = 1$, $k \in \mathbb{N}^*$.

Problem 2

Consider the following difference equation:

$$x(k+1) = (k+1)x(k), \quad k \in \mathbb{N}^*.$$

1. Rewrite the equation in terms of the differences.
2. solution of the equation for $x(0) = 1$.
3. Find the general solution in terms of and arbitrary $x(0)$.

Problem 3

Consider the following difference equation:

$$x(k+1) = (k+1)x(k) + (k+1)!, \quad k \in \mathbb{N}^*.$$

1. Rewrite the equation in terms of the differences.
2. Find the solution of the equation for $x(0) = 1$.

Problem 4

An amount D_0 of a drug is administered every hour to a patient. It is known that each hour the body eliminates a fraction p of the drug present in the circulatory system. Denote by $D(k)$ the amount of drug present in the blood at the beginning of the kth hour.

1. Model the behavior of $D(k)$ in terms of a difference equation.

2. Solve the difference equation.

3. Find the limit value of $D(k)$ when $k \to \infty$.

4. Using Matlab, simulate the system for $D_0 = 2$ cc and $p = 0{,}25$. Verify your results of the previous item with those of your simulation. If there is a risk of overdosing for an amount of the drug present in the blood above $3cc$, will the patient risk overdosing with the current prescrition?

   Note: Use `stem()` instead of `plot()` to plot your results.

Problem 5

Suppose you ask for a loan of $\$500,000$ to be paid with a monthly interest rate of $1{,}2\,\%$.

1. Model the behavior of the system in terms of a difference equation.

2. Find the value of the monthly payments that will pay off the debt in 10 years.

3. Verify with a simulation using Matlab