Class Notes

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Nota: Remember this is in english. In exams you can turn to spanish. Workshops in english.

- 2 credits theory (wendsday - theory)
- 1 parctice (friday - practice)

work with matlab. 1h and a half, for each part, theory and practice.

This course will be theory driven, but in the road we will use some applications.

For the practical part of the exam we can use internet. For the theory we only can bring one sheet of paper.

1. What's a Dynamical sys?

- System: It's a collection of parts, which act as a whole.
- Dynamical: Phenomena that are constatly evolving.

We will deal with:

- Difference equations.
- Ordinay Differential equations.
- Stochastic processes. $\rightarrow$ Stochastic differential equations.

1.0.1. Difference equation (maps/iterative maps, they are functions)

Evolution is discrete (change occurs in discrete time). You can turn some continuos variable into discrete by modifiying the sampling period (i look my bank account once a month).

The most conventional way to model discrete behaviour is some sort of step wise funcion.

$n \geq 1$, variables, and we have eqns that relate values at a certain time to the adjacent values of the variables.

Ex:

$X_{k+1} = X_k + b, b \in \mathbb{R}$

The notation we will use, to emphasise that it's func of time:

$x(k+1) = x(k) + b, b \in \mathbb{R}, k \in \mathbb{Z}^*$

Notice that K is a discrete set. If we talk about solutions of $x(k+1) = x(k) + b, b \in \mathbb{R}$, we mean functions, but particularly: **Sequences** of numbers which satisfy the difference eqn.

A solution to $x(k+1) = x(k) + b, b \in \mathbb{R}$ is $0, b, 2b, 3b, ...$ Another solution is for instance $1, b+1, 2b+1, ...$. To get a solution we need a particular point.

2. Differential eqs:

Evolution is continuos. (In general) they relate derivatives of $n \geq 1$ variables to their present values.

$$\dot{x} = ax(t), t \in \mathbb{R}, \mathbb{R}^+, \mathbb{R}^*$$

Nota: We can take derivaties relative to some complex variable, but we won't do that here.

A solution to the above equation, is a continuos function, a function of a continous set. A particular solution would be: $x(t) = 0$, $x(t) = c \cdot e^{at}$.

Nota: The choice of which type of equation to use for model some problem, Difference or Differential, depends on your will. Difference eqn. are easier to simulate, Differential eqn. are harder to simulate and usually have error, so need some sort of aproximation. On maths Differential eqns are way easier than Difference ones.

3. Multivariable systems

More than one variable is generally necesary to model behaviour of systems with multiple parts.

To relate variables Linear Algebra is used many times (vector representations of the variables), other times schematic diagrams are also used (only for intuition).

Nota: All models are wrong, but some are better than others.

Ex:

1. **Geometric growth:**

$$x(k+1) = ax(k), k \in \mathbb{N}$$

A solution is $0, 0, \ldots$
Another solution is: $1, a, a^2, a^3, \ldots$

- if $a \geq 1$, $x(k)$ grows or decays to $-\infty, \infty$
- if $0 < a < 1$, decays to zero.
- if $0 > a > -1$, it osilates around zero, while decaying.

Another example:
k: month. $x(k)$: number of pairs of bunnies. $x(k+2) = x(k+1) + x(k)$.

Cohort population model example: Population divide into age groups od equal span (5 years). $x_0, x_1, x_2, \ldots, x_n$. x_0 represents the newborn individuals.

if timestep = age span:
certain death after 5(n+1) years, $x_{i+1}(k+1) = x_i(k)$.

if there is a survival rate β_i:
$x_{i+1}(k+1) = \beta_i x_i(k)$

Another example: Equations of force. $F = ma = m\dfrac{d^2x}{dt^2}$

Another example: Lotka-Volterra model predator-prey.
$N_1(t)$ population density of a prey at time t.
$N_2(t)$ population density of a predator at time t.

$$\textit{Some derivarive system I wasn't able to copy.} \tag{1}$$

4. Stages of modeling

1. Representation of the phenomena.
2. Generation analysis/approximation of solutions.
3. Exploration of structural relations. (describe the behaviour).
4. Control, Predictions.