Dimensional Analysis

Example:

free falling body on a vacumm chamber, we know that the speed of the body depends on height (h) and gravity (g) so:

$$V = v(g, h)$$

if we preted to play with units and forget about any previously known formula and we try to get dimensionless factors we can get something like this:

① $$\frac{V}{\sqrt{g}} = \frac{\frac{L}{T}}{\sqrt{\frac{L}{T^2}}} = \sqrt{L}$$

② $$\frac{\sqrt{L}}{\sqrt{h}} = \frac{\sqrt{L}}{\sqrt{L}} = \text{constant}$$

then we have:

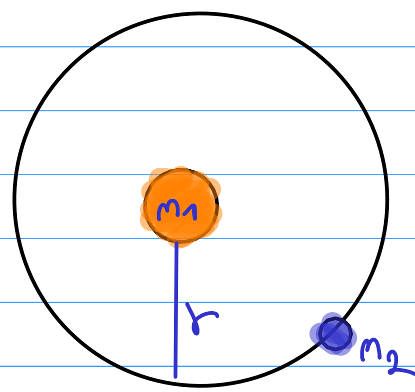
$$\text{constant (Cte)} = \frac{V}{\sqrt{g \cdot h}} \Rightarrow V = \text{Cte} \cdot \sqrt{g \cdot h}$$

This result from dimensional analisys also explains why free fall does not depend on the mass of the object.

↳ the reason is that if we were to add mass we couldn't cancel it because it only appears one time, and our equation wouldn't be dimensionless.

Example:

Another example to ilustrate the method.



We would like to get a dimensionless expression to explain the Period of oscillation T_R.

$$T_R = T_R(m_1, m_2, r)$$

Notice that there we have no chance of getting the desired dimension for T_R because there is no way to cancel the dimension of Lenght brought by the radious (r)

Unless... we use some extra constant.

$$T_R = T_R(m_1, m_2, r, G)$$

units of G are L^3/MT^2.

① $$\left(T_R\sqrt{G}\right) = T\cdot\sqrt{\frac{L^3}{MT^2}} = \sqrt{\frac{L^3}{M}}$$

② $$\frac{T_R\sqrt{G}}{\sqrt{r^3}} = \sqrt{\frac{1}{M}}$$

$$\frac{T_R\sqrt{G}}{\sqrt{r^3}} = T_{R_2}(m_1, m_2)$$

③ $$\frac{T_R\sqrt{G m_2}}{\sqrt{r^3}} = \sqrt{m_2}\cdot \underbrace{T_{R_2}(m_1, m_2)}$$

④ $$\frac{T_R\sqrt{G m_2}}{\sqrt{r^3}} = \sqrt{m_2}\cdot\left(\frac{m_1}{m_2}\right)$$

$$= T_{R_3}\cdot\left(\frac{m_1}{m_2}\right)$$

⑤ $$T_R = \sqrt{\frac{r^3}{G m_2}}\cdot T_{R_3}\cdot\left(\frac{m_1}{m_2}\right)$$

Although this method does not have a fix algorithm it can be described on this steps:

a. List all of the variables and parameters of the problem and their dimensions.
b. Anticipate how each variable qualitatively affects quantities of interest, that is, does an increase in a variable cause an increase or a decrease?
c. Identify one variable as depending on the remaining variables and parameters.
d. Express that dependence in a functional equation (i.e., analogs of eqs. (2.8) and (2.14)).
e. Choose and then eliminate one of the primary dimensions to obtain a revised functional equation.
f. Repeat steps (e) until a revised, *dimensionless* functional equation is found.
g. Review the final *dimensionless* functional equation to see whether the apparent behavior accords with the behavior anticipated in step "b".