Basis of a topology

Def: Basis, a basis for a topology on X is a collection $\mathcal{B}$ of subsets such that:

(1) for each $x \in X$, there is at least one basis element B s.t. $x \in B$.

(2) if $x \in B_1 \cap B_2$ then there exists a basis element B_3 s.t. $x \in B_3$ and $B_3 \subset B_1 \cap B_2$.

long story short: a basus is a collection of subsets of X which always contains every element of X.

and if any intersection of basis elements $B_i \cap B_j$ contains an element of x then there is a smaller B_K which contains this element.

How to generate a topology from a basis?

the topology $\mathcal{T}$ on X is formed as a collection of all unions of elements of the basis $\mathcal{B}$.

another look into the def. of a topo. basis

Given a topo space X,

we can have a collection of open sets $\mathcal{C}$ on X $\forall$ U openset in X there is an element $C \in \mathcal{C}$ such that $x \in C \subset \mathcal{C}$. then $\mathcal{C}$ is a basis for the topology of X.

in other words

a basis for a topology of X is a collection $\mathcal{C}$ of opensets of X such that at least one element of $\mathcal{C}$ contains any element of X.

How can we compare topologies based on their basis?

if for each $x \in X$ and basis $B \in \mathcal{B}$ which contains x there is a basis $B' \in \mathcal{B}'$ s.t. $x \in B' \subset B$

We say $\mathcal{T}'$ Generated by $\mathcal{B}'$ is finer than $\mathcal{T}$.

when talking about the real numbers which is its standard topology?

the basis for the standard topology on the real numbers is the set of open intervals over the real line.

What happens if you start with a given collection of sets and take finite intersections of them as well arbitrary unions?

↳ well, a subbasis happens...

Def: Subbasis

↳ a Subbasis $\underline{S}$ for a topological space $\underline{X}$, is a collection of subsets of X whose union equals X.

We can generate a topology $\mathcal{T}$ with the subbasis S by making a collection of all unions of finite intersections of elements of S.