

Basis of a topology

Def:

Basis, a basis for a topology on X is a collection \mathcal{B} of subsets such that:

- (1) for each $x \in X$, there is at least one basis element B s.t. $x \in B$.
- (2) if $x \in B_1 \cap B_2$ then there exists a basis element B_3 s.t. $x \in B_3$ and $B_3 \subset B_1 \cap B_2$.

long story short: a basis is a collection of subsets of X which always contains every element of X .

and if any intersection of basis elements $B_i \cap B_j$ contains an element of x then

there is a smaller B_k which contains this element.

How to generate a topology from a basis?

the topology \mathcal{T} on X is formed as a collection of all unions of elements of the basis \mathcal{B} .

another look into the def. of a topo. basis

Given a topo space X ,

we can have a collection of open sets \mathcal{G} on X
 ~~\forall~~ \bigcup openset in X there is an element $C \in \mathcal{G}$

in other words
Such that $x \in C \subset G$, then \mathcal{G} is a basis for the topology of X .

\mathcal{G} a basis for a topology of X is a collection of open sets of X such that at least one element of \mathcal{G} contains any element of X .

How can we compare topologies based on their basis?