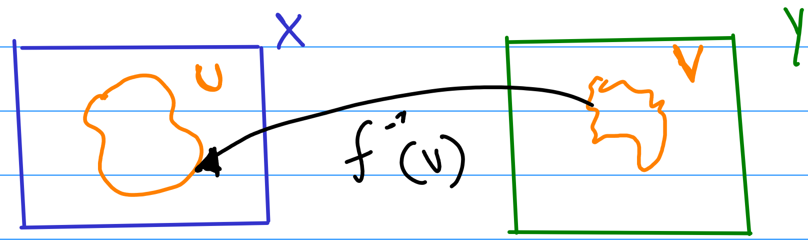
Continuous functions

Continuity of a function

Def 1

let X and Y be topological spaces. $f: X \to Y$ is said to be continuous if:

for each open set V of Y, the set $f^{-1}(V)$ is a open subset of X.



Def 2

if the topology of the range space Y is given by a basis β then:

f is continuous if $f^{-1}(B) \in X \ \forall B \in \beta$.

Def 3

if the topo. of the range space Y is given by a subbasis S then:

f is continuous if $f^{-1}(s) \in X \ \forall s \in S$.

Theorem 18.1

let X and Y be topological spaces and $f: X \to Y$.

then these are equivalent:

1. f is continuous
2. for every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.
3. for every closed set B of Y, the set $f^{-1}(B)$ is closed in X.
4. for each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

Homeomorphisms

Def:

- given a bijection $f: X \to Y$, if f and f^{-1} are continuous then f is called a homeomorphism.

- bijective correspondence $f: X \to Y$ such that $f(U)$ is open iif U is open.

Constructing Continuous functions

Theorem 18.2
Rules for constructing Continuous functions

Let X, Y, Z be topological spaces there are some general functions which are continuous.

(a) Constant function: if $f: X \to Y$ maps all X into a single point $y_0 \in Y$.

(b) Inclusion : if A is a subspace of X. inclusion function $j: A \mapsto X$ is continuous.

(c) composites : if $f: X \to Y$ and $g: Y \to Z$ are continuous, then the map $g \circ f: X \to Z$ is Cont.

(d) restricting the domain.

(e) restricting or expandin the range.

(f) local formulation of continuity.

Theorem 18.3 (pasting lemma)

also applies for open sets.

given $X = A \cup B$, where A and B are closed in X.

Let $f: A \to Y$ and $g: B \to Y$

Si $f(x) = g(x)$ para $x \in A \cap B$ entonces podemos definir :

$$h(x) := \begin{cases} f(x) & \text{si } x \in A \\ g(x) & \text{si } x \in B \end{cases}$$

Theorem 18.4 (maps into products)

Let $f: A \to X \times Y$ be given by the eqn :

$$f(a) = (f_1(a), f_2(a))$$

then f is continuous iff : f_1 and f_2 are continuous.

called coordinate functions of f.